$$\ln x = \log_e x$$

$$e \approx 2.718$$

$$\log_a(a) = 1$$

$$\Rightarrow a^1 = a$$

$$\ln x = a$$

$$e^a = x$$

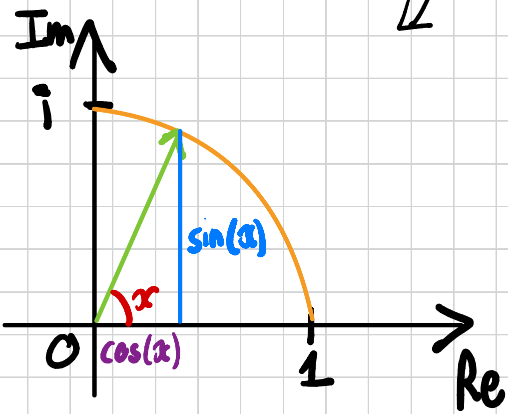
$$\underbrace{e^{i\omega t}}_{\cos(\omega t) + i\sin(\omega t)}$$

<u>in radians</u>

$$e^{ix}$$

inside a complex plane

$$e^{ix} = \cos(x) + i\sin(x)$$



Proof using power series: / maclaurin series

$$e^x = \sum_{n=0}^{\infty} \frac{(n)^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$i\sin(x) + \cos(x)$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} \dots$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \dots$$

$$\sin(x) + \cos(x) = 1 + x - \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} \dots$$

$$e^{ix} = \sum_{n=0}^{\infty} \frac{(ix)^n}{n!} = \frac{(ix)^0}{0!} + \frac{(ix)^1}{1!} + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \dots$$

$i^1 = i \quad i^2 = -1 \quad i^3 = -i \quad i^4 = 1 \quad i^5 = i \quad i^6 = -1$

let n be an integer

$i^{4n-3} = i \quad i^{4n-2} = -1 \quad i^{4n-1} = -i \quad i^{4n} = 1$

$$\text{Simplify} = e^{ix} = \sum_{n=0}^{\infty} \frac{(ix)^n}{n!} = 1 + ix + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4i} \dots$$

$$= 1 + ix - \frac{x^2}{2!} - \frac{ix^3}{3!} + \frac{x^4}{4!} \dots$$

$$= \underbrace{\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} \dots\right)}_{Re} + i\underbrace{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} \dots\right)}_{Im}$$

$= \cos(x) + i\sin(x)$

$$e^{i\pi} = \cos(\pi) + i\sin(\pi)$$

$$= -1 + i(0)$$

$$= -1$$

$$\therefore e^{i\pi} = -1$$

$$e^{i\pi} + 1 = 0$$

which is the Euler's identity

$$e^{\frac{\pi}{2}i} = \cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right) = i$$