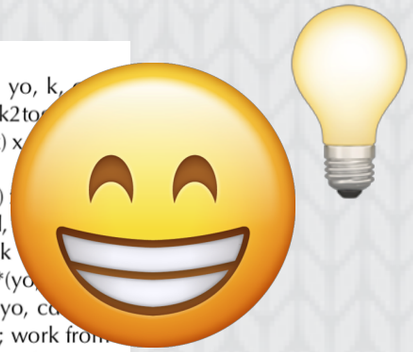
Knitting Math Formalized

Chelsea Battell

Presentation Goals

Basic knowledge of knitting and knitting patterns



Construction Note

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Garter Tab Cast On

Cast on 3 sts. K 6 rows, slipping the first st of each row. Without turning, pick up and knit 3 sts along the vertical edge of strip, then 3 sts from the cast on edge. 9 sts. Turn. Sl, k2, p3, k3, turn.

Continue with either written directions or charts.

Written Directions

— Chart A —

Row 1 (RS): Sl, k2, (yo, k) x 3, yo, k3. (13 sts)
Row 2 and all WS Rows: Sl, k2, p to final 3 sts, k3.
Row 3: Sl, k2, (yo, k) x 7, yo, k3. (21 sts)
Row 5: Sl, k2, yo, k2tog, (k, yo) x 2, k, ssk, yo, k, yo, k2tog, (k, yo) x 2, k, ssk, yo, k3. (25 sts)
Row 7: Sl, k2, yo, k2tog, k2, yo, k, yo, k2, ssk, yo, k, yo, k2tog, k2, yo, k, yo, k2, ssk, yo, k3. (29 sts)
Row 9: Sl, k2, *yo, k, yo, ssk, k5, k2tog; work from *, (yo, k) x 2, *yo, k, yo, ssk, k5, k2tog; work from *, yo, k, yo, k3. (33 sts)
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Row 13: Sl, k2, yo, k2tog, k, *yo, k, yo, k2, ssk, k, k2tog, k2; work from *, (yo, k) x 2, ssk, yo, k, yo, k2tog, k, *yo, k, yo, k2, ssk, k, k2tog, k2; work from *, (yo, k) x 2, ssk, yo, k3. (45 sts)
Row 15: Sl, k2, yo, k2tog, k2, *yo, k, yo, k3, cdd, k3; work from *, yo, k, yo, k2, ssk, yo, k, yo, k2tog, k2, *yo, k, yo, k3, cdd, k3; work from *, yo, k, yo, k2, ssk, yo, k3. (49 sts)

Work rows 9-16 10 more times, (249 sts). To adjust the overall size of the shawl you may continue to repeat rows 9-16, make sure your stitch count total is a multiple of 40 +9.

— Chart B —

Row 1 (RS): Sl, k2, *yo, k, yo, ssk, k2, yo, ssk, yo, k2, cdd, k2, yo, k2tog, yo, k2, k2tog; work from * 6 times, (yo, k) x 2, *yo, k, yo, ssk, k2, yo, ssk, yo, k2, cdd, k2, yo, k2tog, yo, k2, k2tog; work from * 6 times, yo, k, yo, k3. (253 sts)
Row 2 and all WS Rows: Sl, k2, p to final 3 sts, k3.
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(285 sts)
Row 5: Sl, k2, *(yo, k)
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Row 9: Sl, k2, *yo, k2tog, k2, yo, k, yo, k2, ssk, yo, k, yo, k2tog, k2, yo, k, yo, k2, ssk, yo, ssk, k2, yo, cdd, yo, k2, k2tog; work from * 6 times, (yo, k2tog, k2, yo, k, yo, k2, ssk, yo, k) x 2, *yo, k2tog, k2, yo, k, yo, k2, ssk, yo, k, yo, k2tog, k2, yo, k, yo, k2, ssk, yo, ssk, k2, yo, cdd, yo, k2, k2tog; work from * 6 times, yo, k2tog, k2, yo, k, yo, k2, ssk, yo, k, yo, k2tog, k2, yo, k, yo, k2, ssk, yo, k3. (437 sts)
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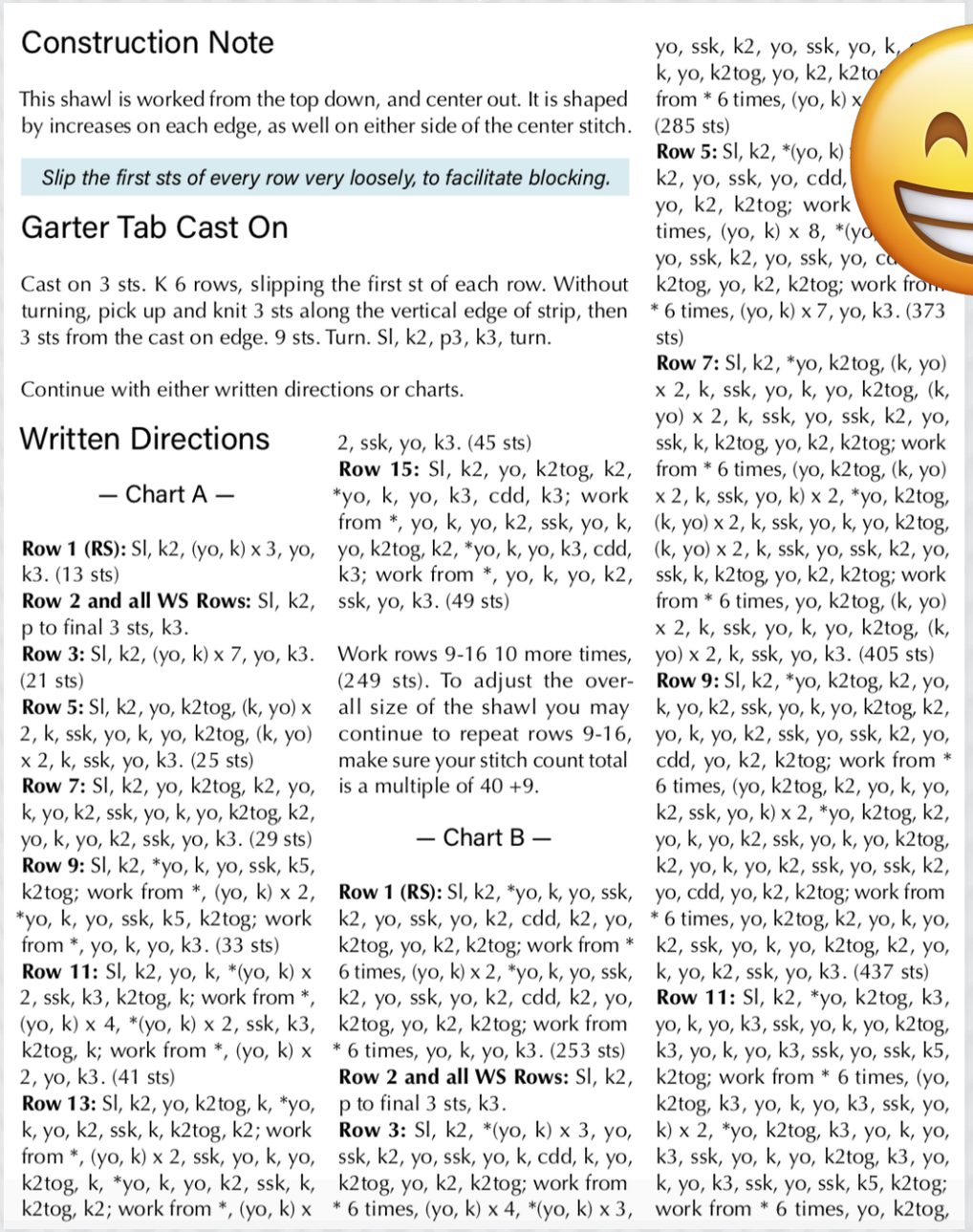

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Work rows 9-16 10 more times, (249 sts). To adjust the overall size of the shawl you may continue to repeat rows 9-16, make sure your stitch count total is a multiple of 40 +9.

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Row 5: Sl, k2, *(yo, k) k2, yo, ssk, yo, cdd, yo, k2, k2tog; work times, (yo, k) x 8, *(yo yo, ssk, k2, yo, ssk, yo, cd k2tog, yo, k2, k2tog; work fro * 6 times, (yo, k) x 7, yo, k3. (373 sts)
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Know how linear logic can be used…

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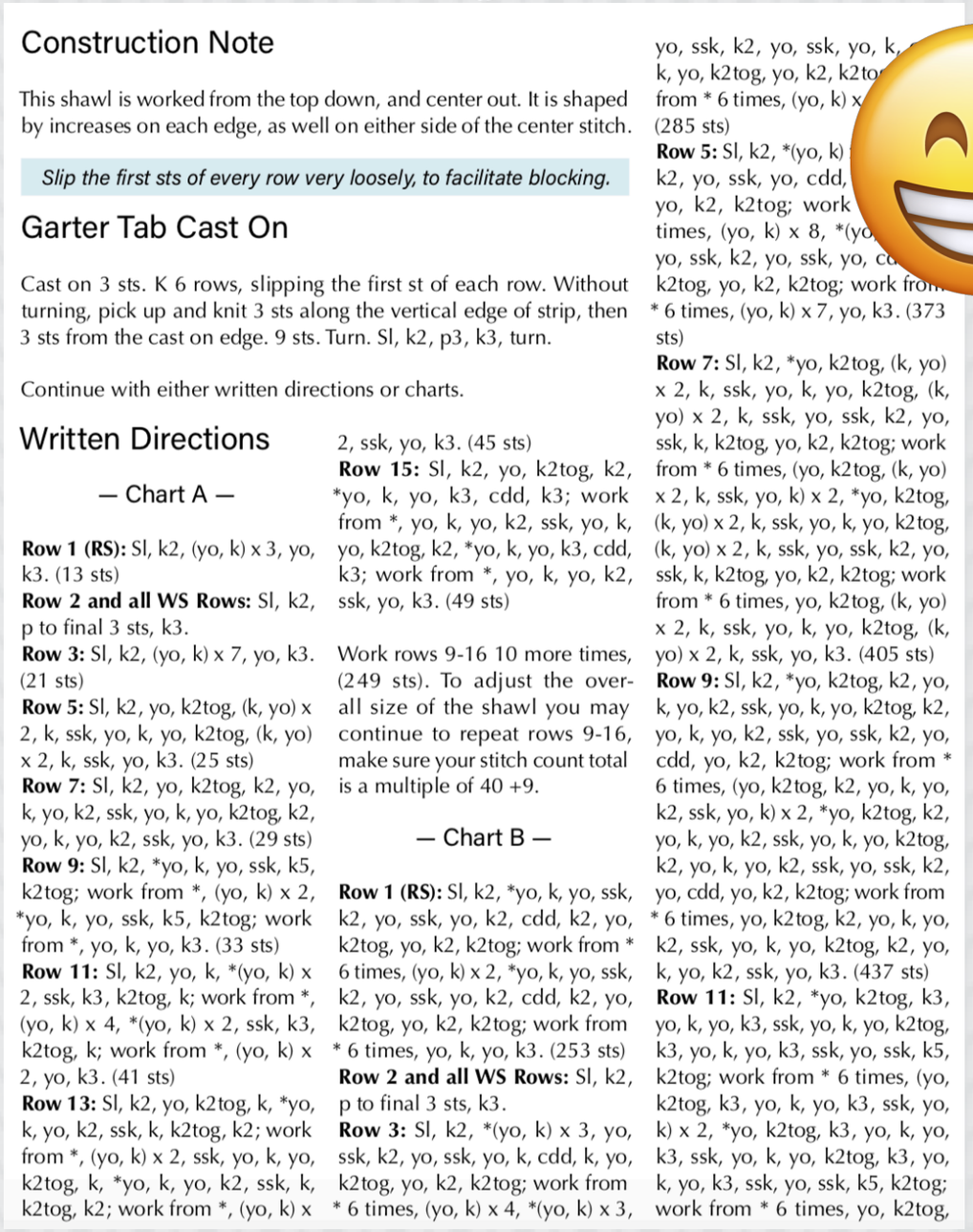

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Know how linear logic can be used…

…in defining correct knitting patterns

Benefits of Knitting

Why care?

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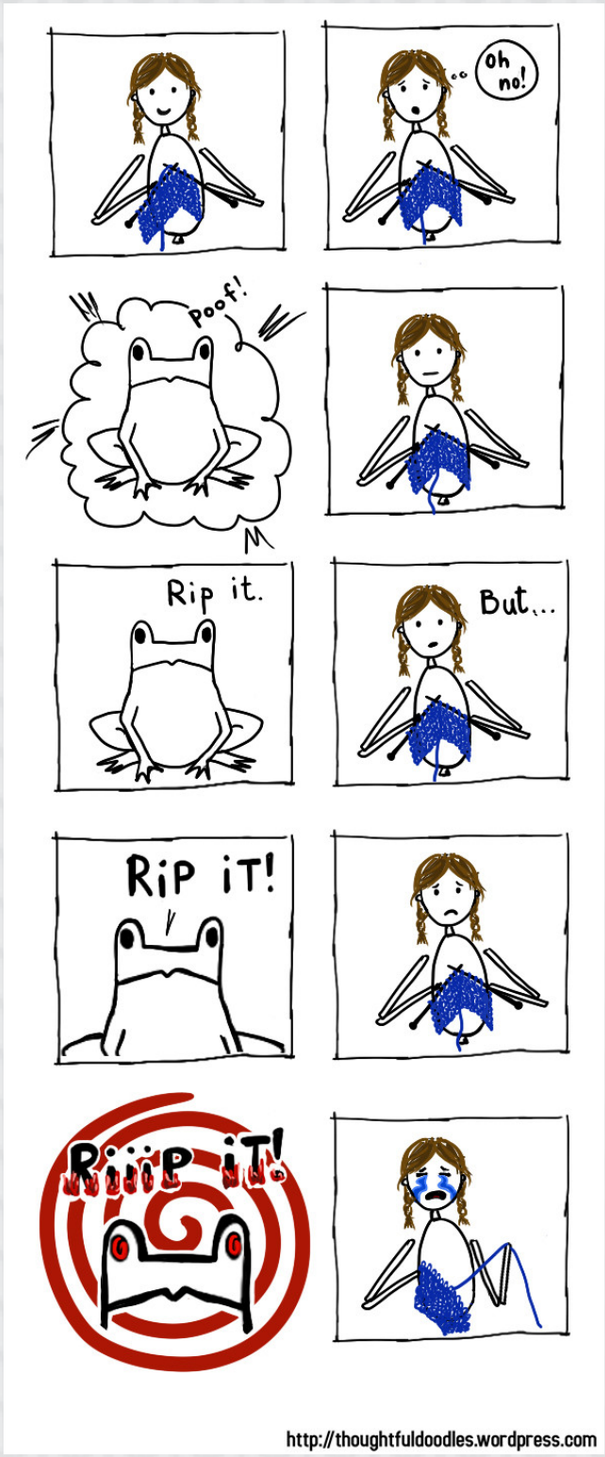
Why care?

Stress relief

Knitting Rage

What's the problem?



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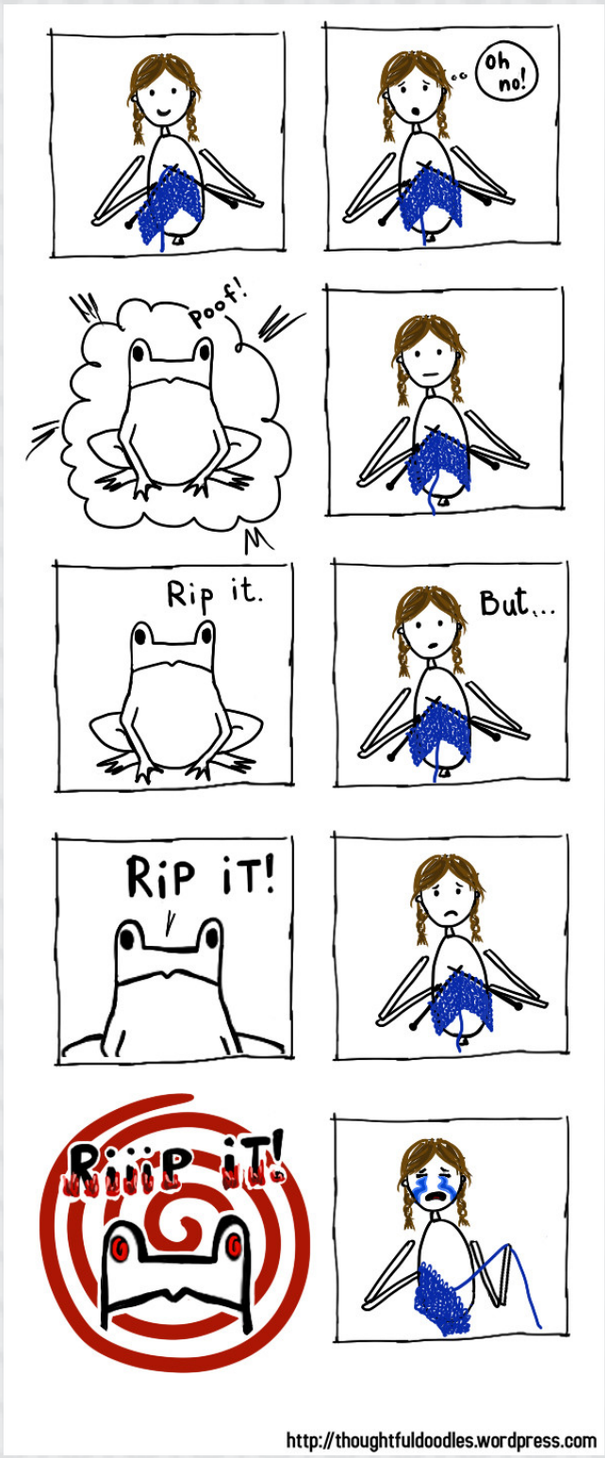
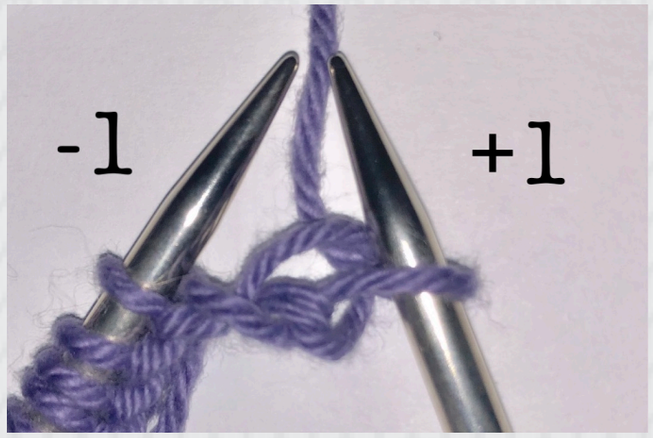


Let's avoid pattern errors

Knitting 101



Focused Linear Logic

$$\frac{\Gamma; \Delta \vdash [A] \quad \Gamma; \Delta', [B] \vdash C}{\Gamma; \Delta, \Delta', [A \multimap B] \vdash C} \multimap_L$$

Persistent context (application formulas in here)

Linear context

Linear implication

$$\frac{}{\Gamma; [P^-] \vdash P^-} \; id_{P^-}$$

+ Focusing and blurring as needed (it all works out)

Blocks World Variant

There are some blocks on a table.

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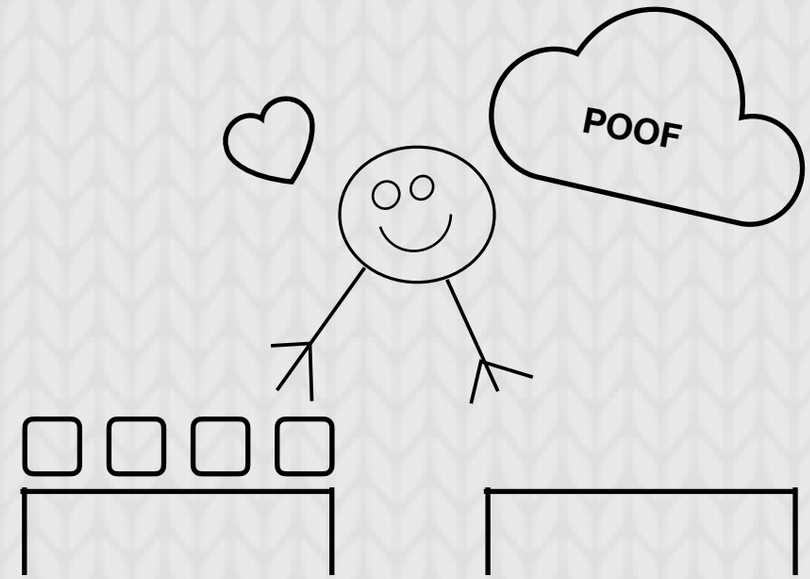
Robot moves blocks from the left table to the right table.



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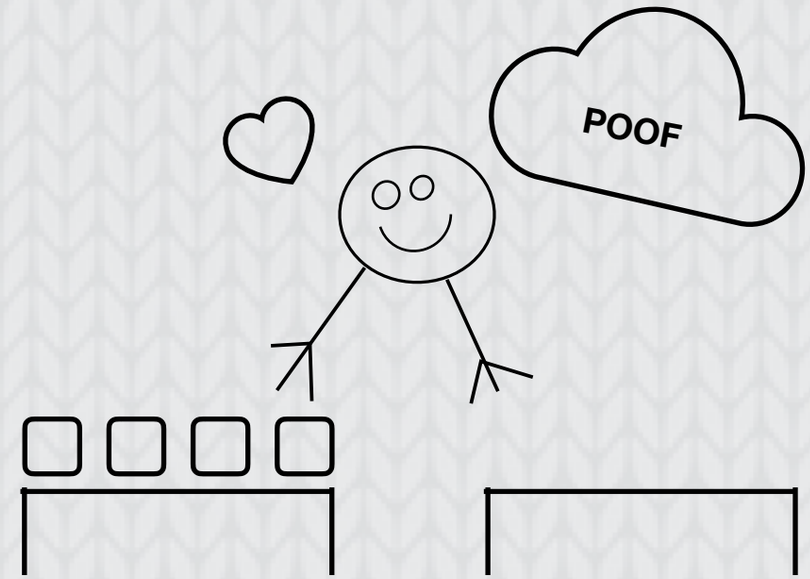

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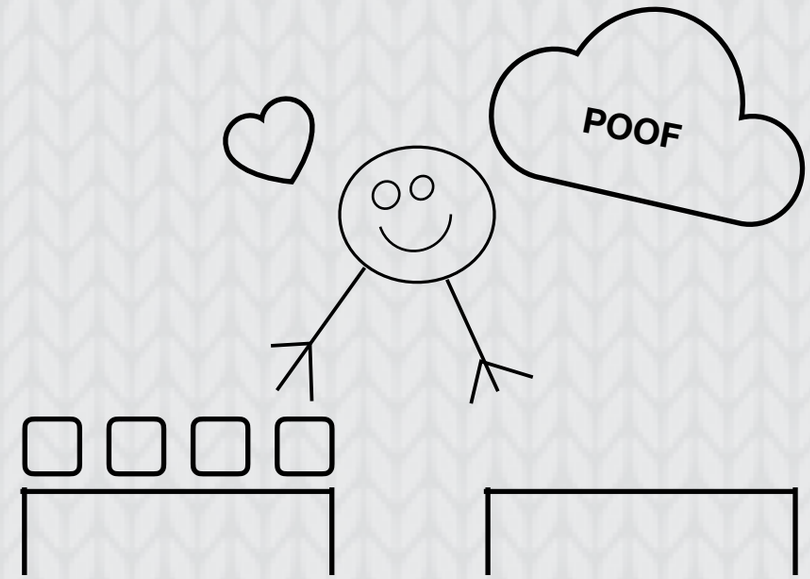
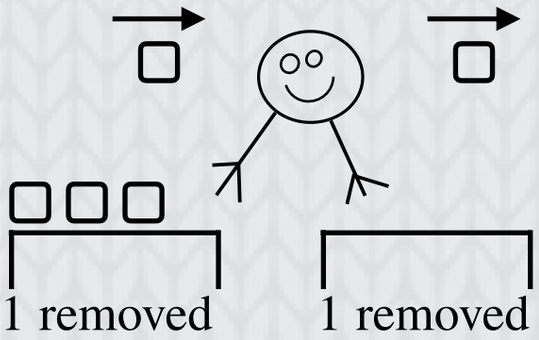


Do given instructions clear the left table?

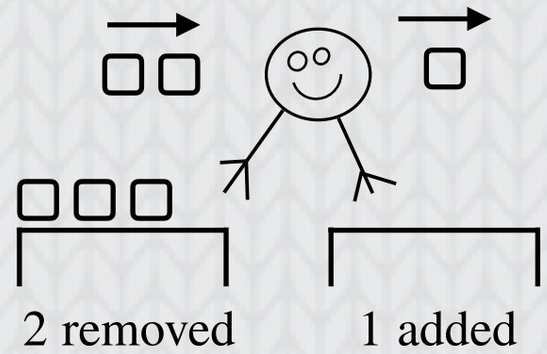
***move(m, x, y)* means move *m* removes *x* from left and adds *y* to right**

Blocks Moves: Basic



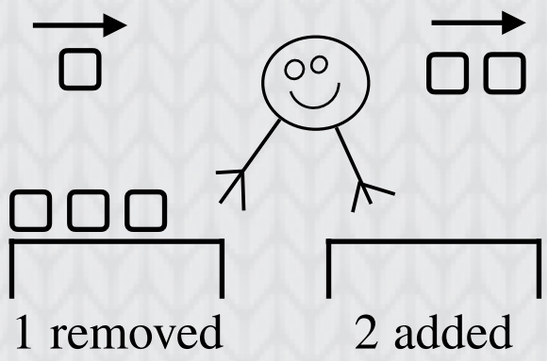
$move(\texttt{SAME}, 1, 1)$

$$\frac{}{\Gamma \vdash move(\texttt{SAME}, 1, 1)}\ move\_same$$



$move(\texttt{DEC}, 2, 1)$

$$\frac{}{\Gamma \vdash move(\texttt{DEC}, 2, 1)}\ move\_dec$$



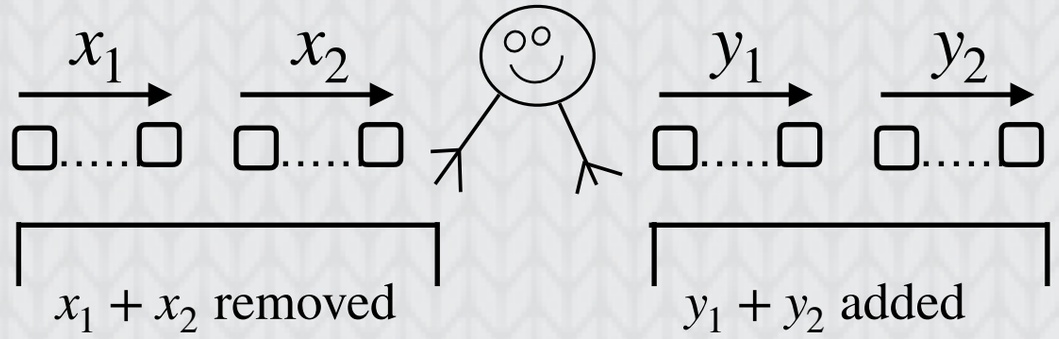
$move(\texttt{INC}, 1, 2)$

$$\frac{}{\Gamma \vdash move(\texttt{INC}, 1, 2)}\ move\_inc$$

Block Move: SEQ

Formula:

$$move(m_1, x_1, y_1) \multimap move(m_2, x_2, y_2) \multimap move(\mathtt{SEQ}(m_1, m_2), x_1 + x_2, y_1 + y_2)$$



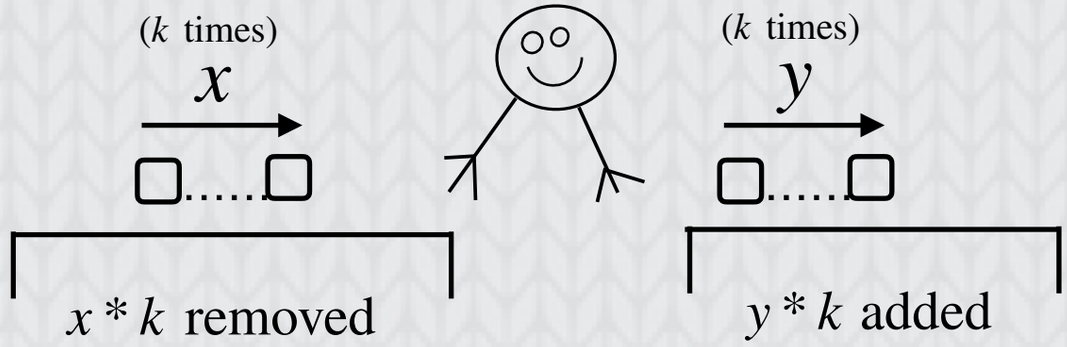
Derived rule:

$$\frac{\Gamma; \Delta_1 \vdash move(m_1, x_1, y_1) \qquad \Gamma; \Delta_2 \vdash move(m_2, x_2, y_2)}{\Gamma; \Delta_1, \Delta_2 \vdash move(\mathtt{SEQ}(m_1, m_2), x_1 + x_2, y_1 + y_2)}\ move\_seq$$

Block Move: REP

Formula:

$$move(m, x, y) \multimap move(\mathrm{REP}(m, k), x * k, y * k)$$



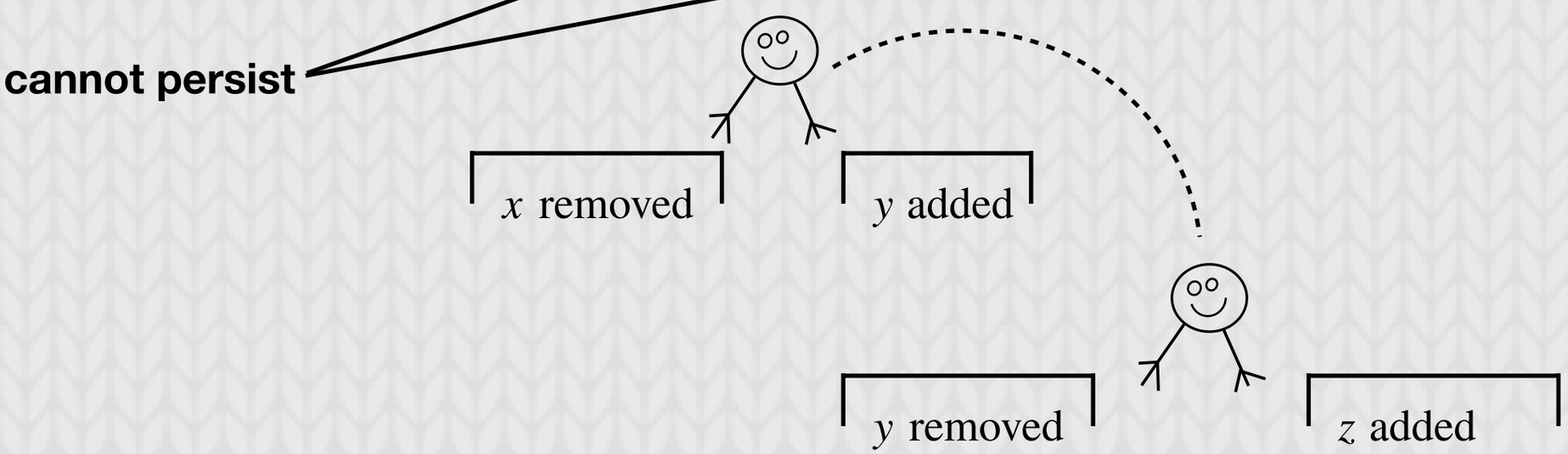
Derived rule:

$$\frac{\Gamma; \Delta \vdash move(m, x, y)}{\Gamma; \Delta \vdash move(\mathrm{REP}(m, k), x * k, y * k)}\ move\_rep$$

Blocks Program

Formulas:

$$move(head, x, y) \multimap program(tail, y, z) \multimap program(head :: tail, x, z)$$



Derived rule:

$$\frac{\Gamma; \Delta_1 \vdash move(head, x, y) \qquad \Gamma; \Delta_2 \vdash program(tail, y, z)}{\Gamma; \Delta_1, \Delta_2 \vdash program(head :: tail, x, y)}\ prog\_check$$

Blocks Example

$$\dfrac{\dfrac{\dfrac{}{\Gamma;\cdot \vdash \mathit{move}(\texttt{INC}, 1, 2)}\,\mathit{move\_inc} \quad \dfrac{\dfrac{\dfrac{}{\Gamma;\cdot \vdash \mathit{move}(\texttt{SAME}, 1, 1)}\,\mathit{move\_same}}{\Gamma;\cdot \vdash \mathit{move}(\texttt{REP}(\texttt{SAME}, 2), 2, 2)}\,\mathit{move\_rep} \quad \dfrac{\dfrac{\dfrac{}{\Gamma;\cdot \vdash \mathit{move}(\texttt{DEC}, 2, 1)}\,\mathit{move\_dec} \quad \dfrac{}{\Gamma;\cdot \vdash \mathit{program}([\,], 1, 1)}\,\mathit{prog\_empty}}{\Gamma;\cdot \vdash \mathit{program}(\texttt{DEC} :: [\,], 2, 1)}\,\mathit{prog\_check}}{\Gamma;\cdot \vdash \mathit{program}(\texttt{REP}(\texttt{SAME}, 2) :: \texttt{DEC} :: [\,], 2, 1)}\,\mathit{prog\_check}}{\Gamma;\cdot \vdash \mathit{program}(\texttt{INC} :: \texttt{REP}(\texttt{SAME}, 2) :: \texttt{DEC} :: [\,], 1, 1)}\,\mathit{prog\_check}}{\Gamma;\cdot \vdash \exists X, Y : \mathit{nat}\,.\,\mathit{program}(\texttt{INC} :: \texttt{REP}(\texttt{SAME}, 2) :: \texttt{DEC} :: [\,], X, Y)}\,\exists_R \text{ (twice)}$$

This is a tiny example 😬

Map to Knitting

move becomes stitch
program becomes pattern

SAME → **knit: move(K, 1, 1)**
purl: move(P, 1, 1)
slip: move(S, 1, 1)

INC → **yarnover: move(YO, 0, 1)**
make-one: move(M1, 0, 1)
knit-front-back: move(KFB, 1, 2)

DEC → **knit-together: move(K2T, 2, 1)**
slip-knit-pass: move(SKP, 2, 1)

Example

```
REP(K, 4) ::
SEQ(REP(K, 2), SEQ(YO, REP(K, 2)) ::
SEQ(K, 5) ::
SEQ(REP(K, 2), SEQ(YO, REP(K, 3)) ::
SEQ(K, 6) ::
SEQ(REP(K, 2), SEQ(YO, REP(K, 4)) ::
SEQ(K, 7) ::
...
```

row: K4.
row: K2, YO, K2.
row: K5.
row: K2, YO, K3.
row: K6.
row: K2, YO, K4.
row: K7.
...

Proof?

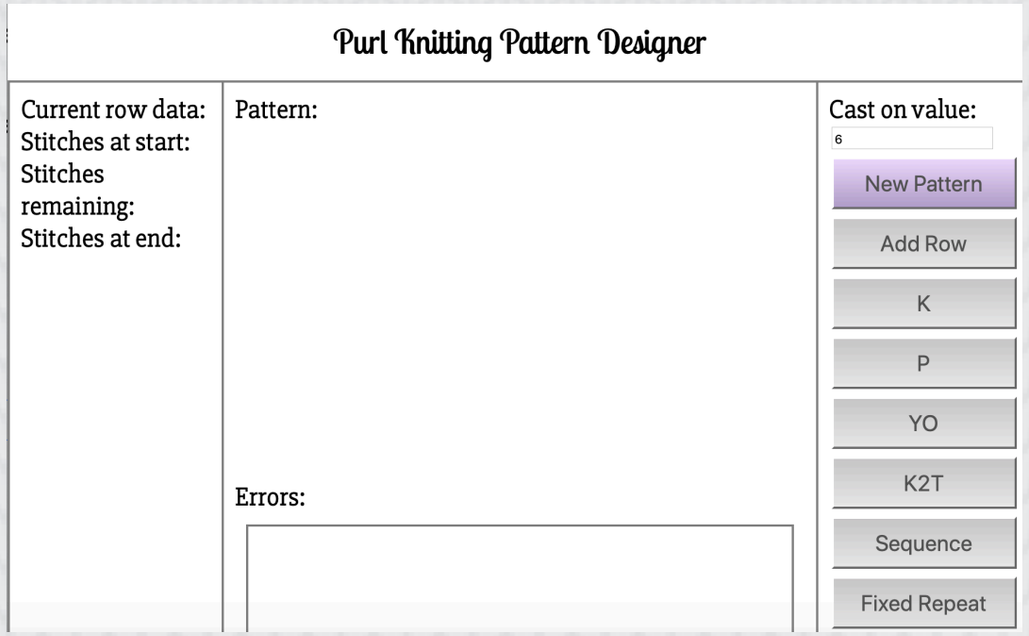
Future Work

Unidentified repeats very useful but complicated to reason about

row: K4.
row: K2, YO, *K; to end.
row: *K; to end.
(repeat last two rows until 44 stitches on needle)

Implement it!

GUI to build pattern



Thank you!

Questions?