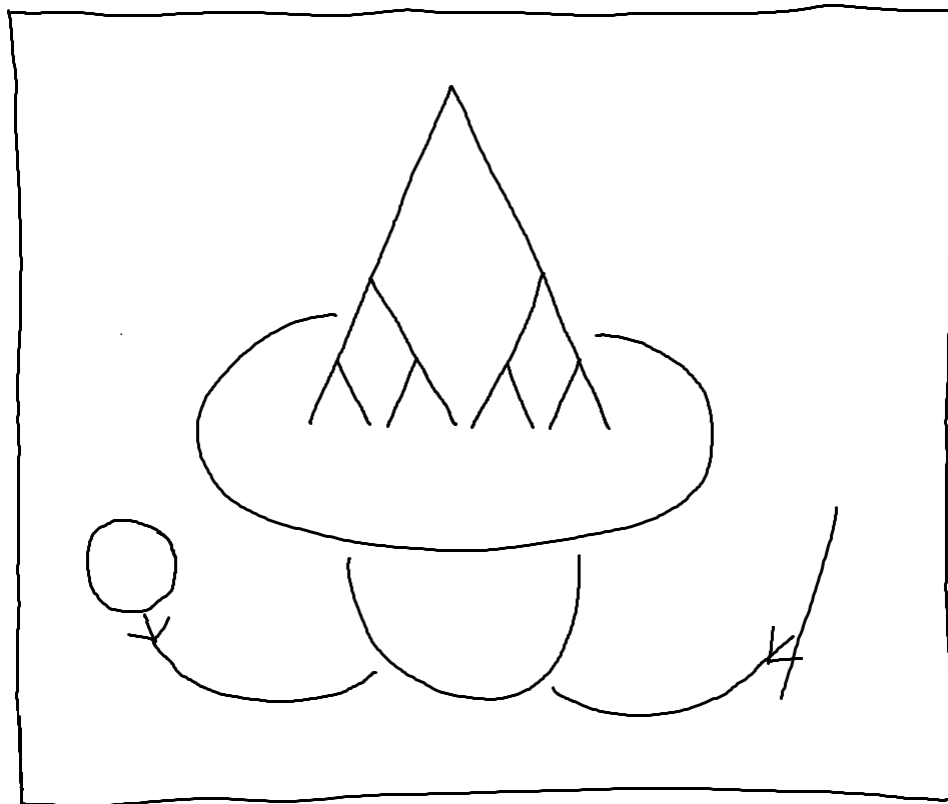# Treehehe
An interactive visualization of proof trees

by Chelsea Battell

Application Domain

- Proof: argument obeying logic rules (assumptions ↔ goal)
- Objective: gain insight on proofs and proof systems by exploring proof trees

Super-fast logic introduction:

Formula – an expression we build that is true or false

$$(p \wedge q) \rightarrow r$$

Inference rule – how to build a formula or take it apart, giving meaning to connectives
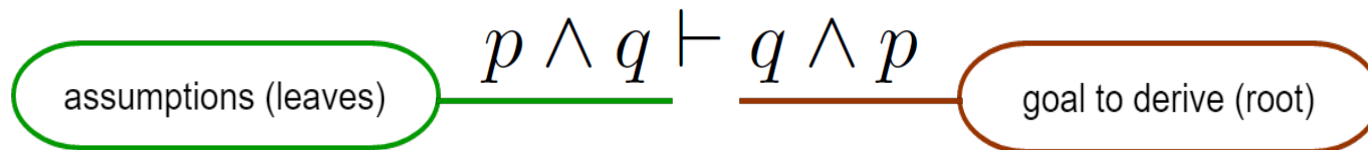
$$\frac{\text{If I have these things...}}{\text{... then I can make this thing}} \text{(rule name)} \qquad \frac{P_1 \quad P_2}{P_1 \wedge P_2} \wedge_I \qquad \frac{P_1 \wedge P_2}{P_i} \wedge_{E_i}$$

Proof tree – stick rules together, filling in variables consistently, with assumptions as leaves and goal as root

User Tasks

- Choose an example *sequent*

$$p \wedge q \vdash q \wedge p$$

assumptions (leaves) — goal to derive (root)

- Exploring a proof, optionally guided

$$\frac{\dfrac{p \wedge q}{q} \wedge_{E_2} \quad \dfrac{p \wedge q}{p} \wedge_{E_1}}{q \wedge p} \wedge_I$$

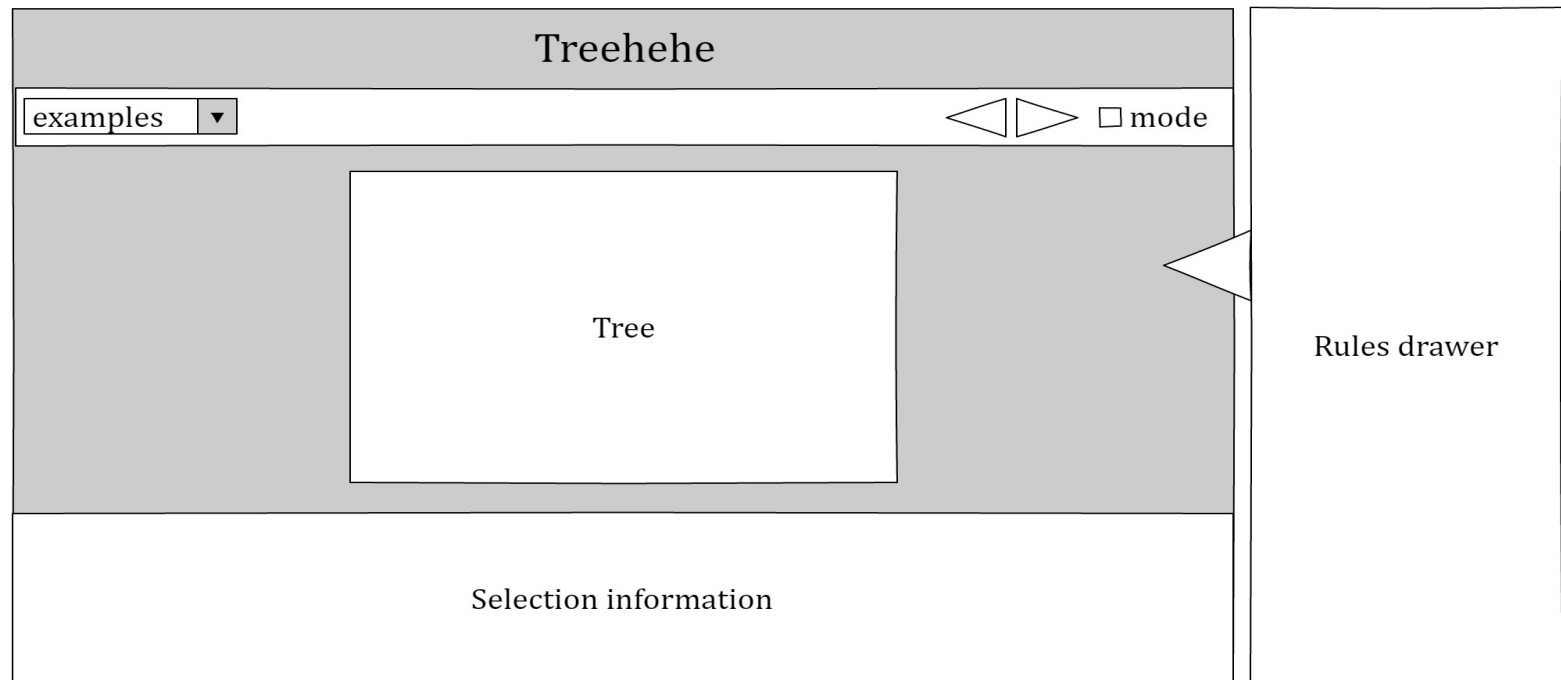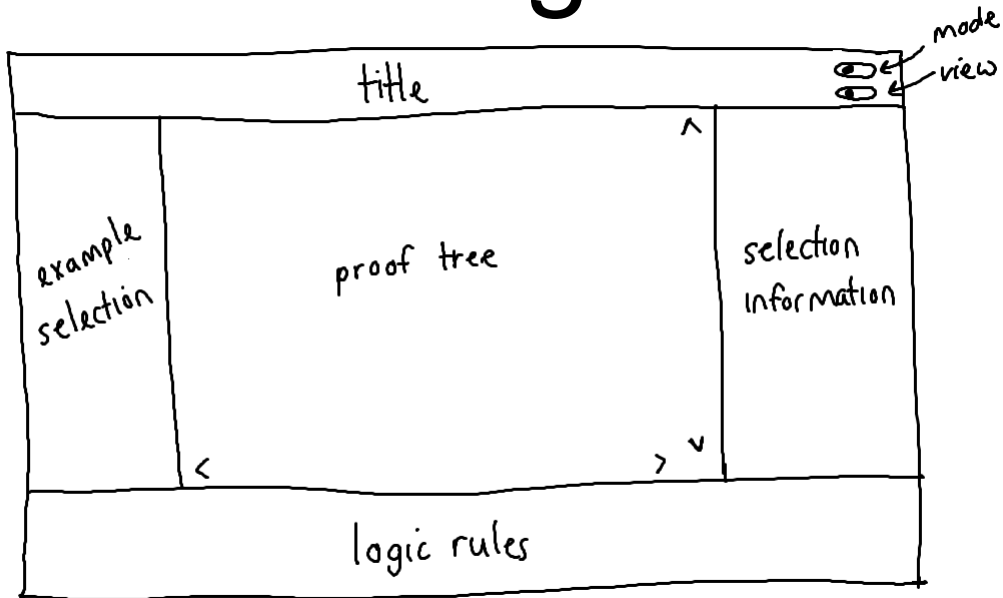- Viewing supplementary info on selected node

- Reviewing rule sets

InfoVis Elements

- Tilford-Reingold-Walker algorithm for layout

- Focus+context: children ← focus → parent

- Interaction:

  - Overview/explore: full tree view and navigation

  - Abstract/elaborate: detail view vs structure view

  - Select: display information on focused node

- Colour

  - To highlight a focused node and related nodes

  - Categorizing nodes (e.g. children, visited)

Design Sketches: Page

title   mode   view

example selection

proof tree

selection information

logic rules

Treehehe

examples ▼ ◁▷ ☐ mode

Tree

Rules drawer

Selection information

Design Sketches: Proof

$$\frac{\overline{}\ u}{\boxed{p \wedge r}}\wedge_{E_1}$$
$$\frac{\boxed{p}\qquad\boxed{p \to q}}{\boxed{q}}\to_E$$

$$\frac{\overline{}\ u}{\boxed{p \wedge r}}\wedge_{E_2}$$
$$\frac{\boxed{r}\qquad\boxed{r \to s}}{\boxed{s}}\wedge_E$$

$$\frac{\boxed{q}\qquad\boxed{s}}{\boxed{q \wedge s}}\wedge_I$$

$$\frac{}{\boxed{(p \wedge r) \to (q \wedge s)}}\to_{I^u}$$
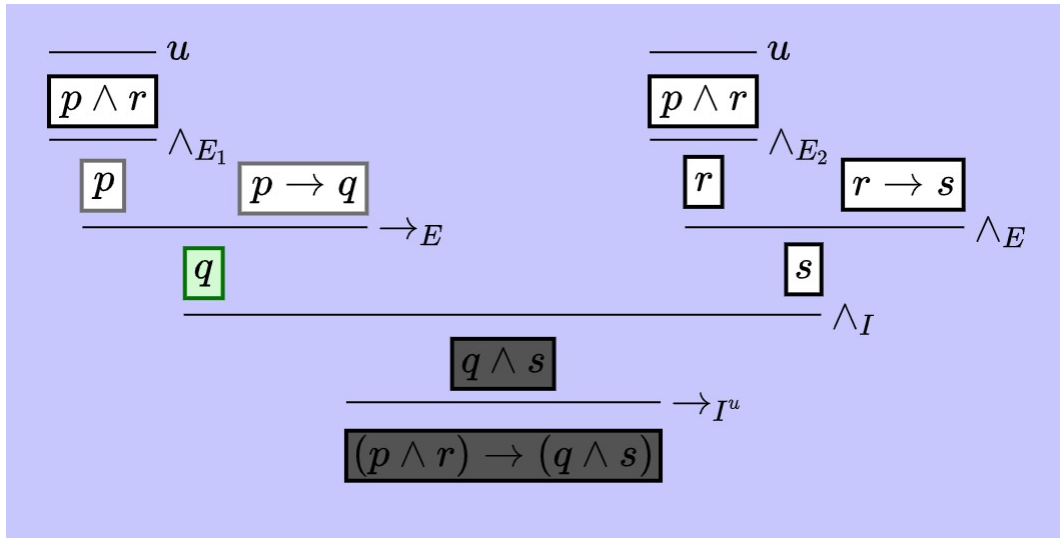
Selection

Selected proposition: q

Derived by $\to_E$ applied to $p, p \to q$

Rules

$$\frac{P_1 \qquad P_2}{P_1 \wedge P_2}\wedge_I$$

$$\frac{P_1 \wedge P_2}{P_1}\wedge_{E_1}$$

$$\frac{P_1 \wedge P_2}{P_2}\wedge_{E_2}$$

$$\frac{\begin{array}{c}\overline{P_1}\\ \vdots \\ P_2\end{array}}{P_1 \to P_2}\to_{I^u}$$

$$\frac{P_1 \qquad P_1 \to P_2}{P_2}\to_E$$

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Technology

- Main languages: HTML, CSS, JavaScript
  - Fast prototyping, easy sharing
- Laying out math: MathJax
- Tree structure: D3 trees
  - Handles most of layout computation
  - Modifications:
    - Bottom rooted, no links, add inference lines & side content
  - Challenges:
    - Unconventional tree components
    - Varied node widths
    - Sequencing layout using LaTeX

Demo

- Trees for different logics
  - Natural deduction
  - Logic programming
- Remaining tasks:
  - Structure view
  - Minimizing subtrees

Discussion

- Consequences of logic choice

    - Simpler logics see fewer benefits in documentation and insight, but are easier to understand

    - Other visualizations may be better for natural deduction

- Reasoning direction

    - Forward (assumptions to goal) more natural

    - Backward (goal to assumptions) more conducive to automatic reasoning

- Impact of different traversals on cognition

- Remaining tasks:

    - Hiding subtrees

    - Structure view