CS229 Fall 2017

Problem Set #4 Solutions: EM, DL & RL

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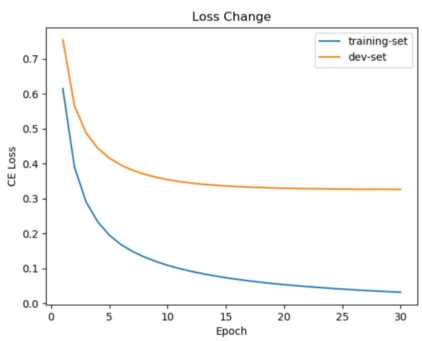
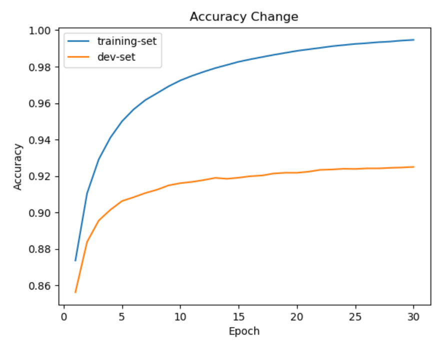
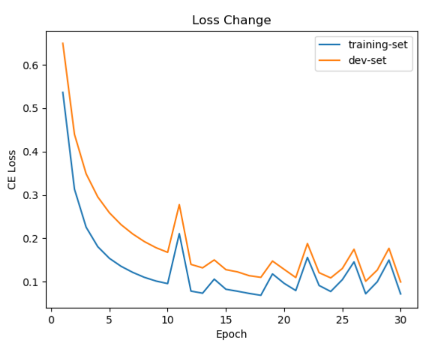
Neural Networks: MNIST image classification

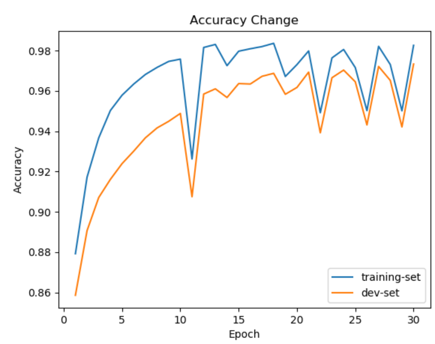
(a) Training the BP neural network version 1 without regularization.





(b) Training the BP neural network version 2 with regularization ($\lambda = 0.25$). From the





result we can see, with regularization of W, the overfitting is prevented but the training process has more fluctuation.

(c) The accuracy of test set is 0.928700 (without regularization) and 0.972300 (with regularization).

EM Convergence

Since EM algorithm has converged, the lower bound of $l(\theta)$ is maxmized. Let LB represent the lower bound and we have (drop down some scripts for convenience)

$$
\begin{aligned}
\bigtriangledown_\theta LB|_{\theta=\theta^*} &= \Sigma_i \Sigma_z Q_z \frac{Q_z}{p(x,z;\theta^*)} \frac{1}{Q_z} \bigtriangledown_\theta p(x,z;\theta)|_{\theta=\theta^*} \\
&= \Sigma_i \Sigma_z \frac{\bigtriangledown_\theta p(x,z;\theta)|_{\theta=\theta^*}}{p(x;\theta^*)} \\
&= \Sigma_i \frac{\bigtriangledown_\theta p(x;\theta)|_{\theta=\theta^*}}{p(x;\theta^*)} \\
&= \Sigma_i \bigtriangledown_\theta [logp(x;\theta)]_{\theta=\theta^*} \\
&= \bigtriangledown_\theta l(\theta) \\
&= 0
\end{aligned}
$$

So when EM algorithm has converges, the $l(\theta)$ acheives the maxima.

PCA

Since the line between projection and $x^{(i)}$ is perpendicular with the line between Origin point and projection, minimizing the distance can be written:

$$argmin_{\mu:\mu^T\mu=1} \Sigma_{i=1}^m ||{x^{(i)}}^2 - {x^{(i)}}^T \mu||_2$$

Because ${x^{(i)}}^2$ is a constant, so the formula is equivalent to maxmizing the variance.

Independent components analysis

The W matrix is as below shows:

$$
\begin{bmatrix}
72.15081922 & 28.62441682 & 25.91040458 & -17.2322227 & -21.191357 \\
13.45886116 & 31.94398247 & -4.03003982 & -24.0095722 & 11.89906179 \\
18.89688784 & -7.80435173 & 28.71469558 & 18.14356811 & -21.17474522 \\
-6.0119837 & -4.15743607 & -1.01692289 & 13.87321073 & -5.26252289 \\
-8.74061186 & 22.55821897 & 9.61289023 & 14.73637074 & 45.28841827
\end{bmatrix}
$$

The S can be derived by XW^T.

Markov decision processes

(a) let $s^a = ||V_1 - V_2||_\infty$ and $s^b = ||B(V_1) - B(V_2)||_\infty$, thus when $s = s^a, s^b$ the two formulas get their maximas. With Bellman equation, we have: (assume $B(V_1) < B(V_2)$)

$$\begin{aligned}||B(V_1) - B(V_2)||_\infty &= \gamma(\Sigma_{s'} P_{s^a a^1}(s')V_1(s') - \Sigma_{s'} P_{s^a a^2}(s')V_2(s')) \\ &\leq \gamma(\Sigma_{s'} P_{s^a a^2}(s')V_1(s') - \Sigma_{s'} P_{s^a a^1}(s')V_2(s')) \\ &= \gamma\Sigma_{s'} P_{s^a a^2}(V_1(s') - V_2(s')) \\ &\leq (V_1(s^b) - V_2(s^b))\end{aligned}$$

(b) Assume the two points are V_1 and V_2, so $||B(V_1) - B(V_2)||_\infty = ||V_1 - V_2||_\infty \leq \gamma||V_1 - V_2||_\infty$, which can't be true because $\gamma < 1$.

Reinforcement Learning: The inverted pendulum

Finish the code in control.py and make 5 training trails to get an average of 158 failure times to get converged.