# Notes on Calculus

## September 16, 2023

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### 1 Setting the Stage

### 1.1 Euclidean Spaces and Vectors

### 1.1.1 Exercises

### 1.1.2

Given  $\vec{x}, \vec{y} \in \mathbb{R}^{\kappa}$ ,

$$\begin{aligned} |\vec{x} + \vec{y}|^2 &= (\vec{x} + \vec{y})(\vec{x} + \vec{y}) \\ &= \vec{x} \cdot \vec{x} + \vec{y} \cdot \vec{y} + 2\vec{x} \cdot \vec{y} \\ &= |\vec{x}|^2 + |\vec{y}|^2 + 2\vec{x} \cdot \vec{y} \end{aligned}$$

Similarly,

$$|\vec{x} - \vec{y}|^2 = (\vec{x} - \vec{y})(\vec{x} - \vec{y})$$
$$= \vec{x} \cdot \vec{x} + \vec{y} \cdot \vec{y} - 2\vec{x} \cdot \vec{y}$$
$$= |\vec{x}|^2 + |\vec{y}|^2 - 2\vec{x} \cdot \vec{y}$$

Hence

$$|\vec{x} + \vec{y}|^2 + |\vec{x} - \vec{y}|^2 = 2(|\vec{x}|^2 + |\vec{y}|^2)$$