Notes on General Relativity

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Contents

1 Special Relativity

1.1 Invariance of Interval

1.1.1 Exercises

1.14.8

1. Derive equation 1.3 from equation 1.2 for general $\{M_{\alpha\beta}, \alpha, \beta = 0, 1, 2, 3\}$.
2. Since $\Delta\bar{s}^2 = 0$ in equation 1.3 for any $\{\Delta x^i\}$, replace Δx^i by $-\Delta x^i$ in equation 1.3 and subtract the resulting equation from equation 1.3 to establish that $M_{0i} = 0$ for $i = 1, 2, 3$.
3. Use equation 1.3 with $\Delta\bar{s}^2 = 0$ to establish equation 1.4b. (Hint: Δx, Δy, and Δz are arbitrary.)

$$\Delta\bar{s}^2 = \sum_{\alpha=0}^{3}\sum_{\beta=0}^{3} M_{\alpha\beta}\Delta x^\alpha \Delta x^\beta = M_{\alpha\beta}\Delta x^\alpha \Delta x^\beta$$

Writing it all out we have

$$\begin{bmatrix} M_{00}\Delta x^0\Delta x^0 & M_{01}\Delta x^0\Delta x^1 & M_{02}\Delta x^0\Delta x^2 & M_{03}\Delta x^0\Delta x^3 \\ M_{10}\Delta x^1\Delta x^0 & M_{11}\Delta x^1\Delta x^1 & M_{12}\Delta x^1\Delta x^2 & M_{13}\Delta x^1\Delta x^3 \\ M_{20}\Delta x^2\Delta x^0 & M_{21}\Delta x^2\Delta x^1 & M_{22}\Delta x^2\Delta x^2 & M_{23}\Delta x^2\Delta x^3 \\ M_{30}\Delta x^3\Delta x^0 & M_{31}\Delta x^3\Delta x^1 & M_{32}\Delta x^3\Delta x^2 & M_{33}\Delta x^3\Delta x^3 \end{bmatrix}$$

The above can be broken down into diagonal and off-diagonal components. The diagonal elements are

$$M_{00}\Delta r^2 + \sum_{i=1}^{3}\sum_{j=1}^{3} M_{ij}\Delta x^i \Delta x^j$$

And the off-diagonal elements are of the form

$$\sum_{i=1}^{3} M_{0i}\Delta t \Delta x^i$$

With the above explanation we can see how equation 1.3 comes from 1.2.