Notes on Calculus

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1.1.1 Exercises

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Given $\vec{x}, \vec{y} \in \mathbb{R}^{\kappa}$,

$$\begin{aligned} |\vec{x}+\vec{y}|^2 &= (\vec{x}+\vec{y})(\vec{x}+\vec{y}) \\ &= \vec{x}\cdot\vec{x} + \vec{y}\cdot\vec{y} + 2\vec{x}\cdot\vec{y} \\ &= |\vec{x}|^2 + |\vec{y}|^2 + 2\vec{x}\cdot\vec{y} \end{aligned}$$

Similarly,

$$\begin{aligned} |\vec{x}-\vec{y}|^2 &= (\vec{x}-\vec{y})(\vec{x}-\vec{y}) \\ &= \vec{x}\cdot\vec{x} + \vec{y}\cdot\vec{y} - 2\vec{x}\cdot\vec{y} \\ &= |\vec{x}|^2 + |\vec{y}|^2 - 2\vec{x}\cdot\vec{y} \end{aligned}$$

Hence

$$|\vec{x}+\vec{y}|^2 + |\vec{x}-\vec{y}|^2 = 2\left(|\vec{x}|^2 + |\vec{y}|^2\right)$$