# Notes on Complex Analysis

October 7, 2023

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# 1  Basic Algebraic Properties

## 1.1  Exercises

**2.2**

Some interesting properties

$$z + \bar{z} = (a + ib) + (a - ib) = 2a = 2\Re(z)$$

Similarly,

$$z - \bar{z} = (a + ib) - (a - ib) = 2ib = 2\Im(z)$$

Following the same mechanics,

$$\Re(iz) = \Re(i(a + ib)) = \Re(ai - b) = -\Im(z)$$

And

$$\Im(iz) = \Im(ai - b) = \Re(z)$$

## 1.2  Further Algebraic Properties

$$|z|^2 = |z\bar{z}| = (a + ib)(a - ib) = a^2 + b^2$$

Powers,

$$(a + ib)^n = [r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

$$r = |z| = \sqrt{a^2 + b^2}$$

$$\theta = \tan^- 1\frac{y}{x}$$

Nth roots: for any positive integer n, the nt distinct roots of $(a + ib)^n = r^n(\cos nx + i\sin nx)$ are

$$r^{\frac{1}{n}}\left[\cos\frac{x + 2\pi k}{n} + i\sin\frac{x + 2\pi k}{n}\right]$$

for $k = 0, 1, \ldots, n - 1$.