The Binomial Theorem and Binomial Series

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1 Recherches Sur la Serie $1 + \frac{m}{1}x + \frac{m(m-1)}{1 \cdot 2}x^2 + \frac{m(m-1)(m-2)}{1 \cdot 2 \cdot 3}x^3 + \ldots$

If we subject the reasoning which we generally use when it comes to infinite series to a more exact examination, we will find that it is, all things considered, unsatisfactory, and that consequently the number of theorems, concerning infinite series, which can be considered as rigorously founded, is very limited. We ordinarily apply the operations of analysis to infinite series in the same way as if the series were finite, which does not seem to me to be permitted without particular demonstration. If for example we have to multiply two infinite series one by the other, we put

$$
\begin{aligned}
&(u_0 + u_1 + u_2 + \ldots)(v_0 + v_1 + v_2 + \ldots) \\
&= u_0v_0 + (u_0v_1 + u_1v_0) + (u_0v_2 + u_1v_1 + u_2v_2) + \ldots + \\
&\quad (u_0v_n + u_1v_{n-1} + u_2v_{n-2} + \ldots + u_nv_0) + \ldots
\end{aligned}
$$