Fractions!

C. Godsalve
email:seagods@hotmail.com

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1 The Embarrassment of it All

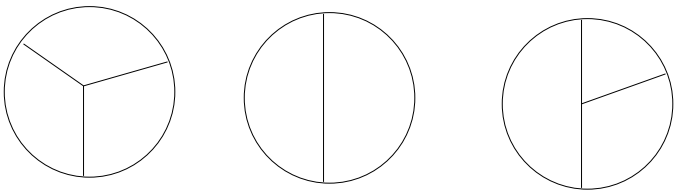
Oh, hell, there you are, you were sure you knew how to "do fractions", but now you have to do something with fractions, and you have plain forgot how to do them. You just can't figure it out. Fractions have a way of doing that to people, if you don't use them for a while, they can disappear out of your head altogether. Also, you might be able to "do fractions", but do you understand why the rules work. Understanding why the rules work and where they come from is the key to making fractions stick, even if you don't use them for years.

2 How To "Do" Fractions

Why is adding fractions so darn tricky? Suppose you need to get the answer to this question.

$$\frac{1}{3}+\frac{1}{2}=? \tag{1}$$

Well, you only have to take a look at the three circles below to see it's tricky. The first



circle is split into thirds, the second circle is split into halves. Take a third from the first

circle, take a half from the second one (do it on paper and cut them out if you want) and put them together. Clearly, the answer is not in thirds or halves.

So what is to be done? Well, to be able to think it out, you need to be able to multiply fractions! Oddly enough, that's easier than adding fractions. All you have to remember is that *a times sign means of.* That's right, five times three means five lots of three, a half times a quarter means a half of a quarter. Remembering that a times means of can get you out of a lot of messy thinking, get into the habit! What's a half of a quarter? An eighth, right? Draw a circle, split into quarters, split each quarter in two, and you get eight equal bits. So, for instance

$$\frac{1}{3} \times \frac{1}{2} = \frac{1 \times 1}{3 \times 2}. \tag{2}$$

This is another thing to remember. To multiply by a half is the same as dividing by 2, and dividing by a half is the same as multiplying by two. So for instance

$$\frac{1}{6} \times \frac{1}{4} = \frac{1}{6} \div 4. \tag{3}$$

Now, back to adding fractions, now we can multiply fractions, we know that

$$\frac{1}{3} \times \frac{1}{2} = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}. \tag{4}$$

But how does that help us? It tells us that a half can be split up into three sixths $\frac{1}{3}\ of\ a\ \frac{1}{2} = \frac{1}{6}$ and a third can be split up into two sixths $\frac{1}{2}\ of\ a\ \frac{1}{3} = \frac{1}{6}$. Now,

$$\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}. \tag{5}$$

Remember, the problem was that the answer was not in thirds or halves, but something else. Now, we are adding sixths and sixths, so the answer will certainly be in sixths. The number 6 on the bottom is the *common denominator* .

Lets try another fraction

$$\frac{4}{15} + \frac{2}{5} =? \tag{6}$$

Now you could say

$$\frac{1}{15} \times \frac{1}{5} = \frac{1}{5} \times \frac{1}{15} = \frac{1}{75}, \tag{7}$$

and split the each fifteenth into five seventy-fifths and each fifth into fifteen seventy-fifths, and you wouldn't be wrong, but you would be making life harder than necessary. The number 75 is a common denominator, but so is 75 million! Multiplying the two denominators always gives you a common denominator, but it's not necessarily the lowest one. In this case, each fifth can be split up into three fifteenths. So, the *lowest common denominator* is the lowest number that both the denominators go into. In this case both five and fifteen go into fifteen. So, we have

$$\frac{4}{15} + \frac{2}{5} = \frac{4}{15} + 2 \times \frac{3}{15} = \frac{4+6}{15} = \frac{2}{3}. \tag{8}$$

What was that last bit? If you divide or multiply both the top and bottom of a fraction by the same thing (as long as it's not zero!) the fraction is unchanged so a half is six twelfths is a million over two million. In this case I divided top and bottom by five.

Back to multiplication, and division. So

$$\frac{3}{4} \times \frac{2}{15} = 3 \times \frac{1}{4} \times 2 \times \frac{1}{15} = 3 \times 2 \times \frac{1}{4} \times \frac{1}{15} = 6 \times \frac{1}{60} = \frac{6}{60} = \frac{1}{10} \tag{9}$$

We can cope with numbers in the numerator (the top) quite easily. So, the general rule for addition, subtraction and multiplication are

$$\frac{a}{b} + \frac{c}{d} = \frac{a \times d + c \times b}{b \times d} \tag{10}$$

The number $b \times d$ is a common denominator, each bth is split into d *lots of* $d \times b$ths and each dth is split into b *lots of* $d\times$ bths. (By a bth I mean the fraction 1/b in the same way as a tenth is 1 over ten.)

$$\frac{a}{b} - \frac{c}{d} = \frac{a \times d - c \times b}{b \times d} \tag{11}$$

works just the same way, and for multiplication

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} \tag{12}$$

But what about division. That's easy too. Remember, dividing by ten and multiplying by a tenth are the same thing. So it is with division too, dividing by a tenths is the same as multiplying by 10. So, one divided by a tenth is the same as multiplying by ten. (How many tenths go into one? of course one divided by a tenth is ten). What if there's a number on top, well, that's really a multiplication, so for example

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \div (c \times \frac{1}{d}) = \frac{a}{b} \div c \times d = \frac{a}{b} \times \frac{d}{c} \tag{13}$$

So, to divide by, turn whatever is on the right of the $\div$ sign upside down and multiply. Oh, and one last thing. Whole numbers are fractions too! Yes

$$5 = \frac{5}{1}\,! \tag{14}$$