A Relation between Mass and Radius for 71 Exoplanets with $R_{\rm P} < 7 R_\oplus$

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1. Introduction

The Kepler Mission has found an arbundance of planets with $R < 4R_{\oplus}$. However, in many systems, it is difficult to measure the masses of such small planets because the gravitational acceleration these planets induce on their host stars or neighboring planets is too small to detect with current telescopes and instruments. How can we determine the composition of these planets? To understand the nature of small planets, we reverse the question: how do the mass of a planet, its composition, and the incident stellar flux from its star affect the radius of a planet?

Our work (Weiss et al. 2013) has shown that for planets between $\sim$20 and thousands of Earth masses, we can predict the radius of a planet from its mass and incident stellar flux, suggesting that planets of a given mass have a particular composition. However, below $\sim$20 $M_{\oplus}$, the large apparent scatter makes predicting planet radius harder. At 10 Earth masses, planets can have a radius of 2-8 $R_{\oplus}$, which spans a range in density from less dense than water to solid iron.

There are two explanations for the large apparent scatter in planet radius: either errors in radius and/or mass are underestimated for planets below 20 $M_{\oplus}$, or the (real) scatter in radius indicates a diversity of planet compositions. Identifying whether this apparent scatter is real or not is one of the most pressing issues in understanding planetary compositions today. Does the mass of a planet uniquely determine a planets composition, or is there diversity of the rock to volatile ratio at a given mass? If the scatter is only apparent, then it is possible that there is a one-to-one correspondence between planet mass and composition below 20 $M_{\oplus}$. However, if the scatter is real, then there must be a diversity of planet compositions at a given mass below 20 $M_{\oplus}$. Further refinements to the measurements of planet masses, especially below 20 $M_{\oplus}$, is necessary to test to what extent the apparent scatter of planetary masses at a given radius is real.

This updated examination of the mass-radius relation benefits from (1) the masses of 42 planets with $R_{\rm P} < 7R_{\oplus}$ characterized in **?**, (2) additional planets of all masses added to exoplanets.org between 27 September 2012 and [use most recent update] 2013, and (3) refined planet and stellar parameters for several systems, including 55 Cnc e (cite), Kepler-11 (**?**), GJ 3470 (**?**), HD 97658 b (**?**), [... other systems].

A plot of radius versus mass for 225 exoplanets is shown in Figure 1. The planets are colored by incident flux from the host star; planets that receive more than the median flux (448 Earth fluxes) are red, while those that receive less are blue. Most of the planets that have been discovered since the publication of Weiss et al. (2013) are in the low-mass branch of this diagram. For this reason, and also because the relation between mass and radius for small planets is both poorly understood and vital for determining planet composition, we focus on the low-mass portion of the population. In the rest of this paper, we examine planets for which $R_{\rm P} < 7R_{\oplus}$. We try to determine how their size correlates with their masses and densities (and therefore compositions), and we also investigate how other parameters might correlate with the physical properties of these small exoplanets.

2. Justification for a Mass-Radius Relation for Small Exoplanets

Is there a correlation between planet mass and radius for small exoplanets? By eye, we might judge that there is a correlation, but what is the probability that we have fooled ourselves into believing there is a correlation when there is none?

To answer this question, we calculated the probability that the data were uncorrelated. First, we calculated the correlation coefficient (Pearson R test) r = 0.61. For r = 0.61, the Student t value is 6.38. There are 71 planets with $R_{\rm P} < 7R_\oplus$, so there are 69 degrees of freedom in this sample. What is the probability that among data with 69 degrees of freedom, we find a Student t value of 6.38 if the data are uncorrelated? According to a Wolfram Alpha calculation, the probability of obtaining $t \geq 6.38$ for 69 degrees of freedom, given that the data are really uncorrelated, is 3.56×10^{-8}. We can safely say that the masses and radii of small planets are correlated.

Does this correlation also exist for planets with $R_{\rm P} < 4R_\oplus$? There are 58 planets smaller than 4 $R_\oplus$, leaving 56 degrees of freedom. The Pearson R coefficient is 0.51, and the Student t value is 4.4. The probability that these data are uncorrelated is 8.4×10^{-5}. Thus, the masses and radii of planets between the sizes of Earth and Neptune are correlated.

Another way to convince yourself that mass and radius are correlated for small planets is to examine Figure 2, which shows the weighted mean exoplanet mass in bins of width 1 $R_\oplus$. On average, exoplanet mass increases with increasing radius, indicating an underlying correlation in the individual exoplanet masses and radii.

3. The Updated Mass-Radius Relation for Small Exoplanets

Now that we have determined that a correlation between planet mass and radius exists, what is the best way to model the relation between exoplanet mass and radius? Figure 3 suggests that the data can be fit with a line; a more complicated fit is not warranted. To check this, we try a traditional power-law fit and obtain $M_{\rm P} \propto R_{\rm P}$ as the best result.

The best linear fit to the data for $R_{\rm P} < 7\ R_{\oplus}$ is:

$$M_{\rm P}/M_{\oplus} = -1.2 + 3.3\ R_{\rm P}/R_{\oplus}$$

There are 71 exoplanets in this sample. The reduced χ^2=6.1, and the RMS is 9.2 $M_{\oplus}$.

However, we notice in Figure 2 that the weighted mean masses might follow a different trend above and below 4 $R_{\oplus}$. Also, there are very few planets with $4 < R_{\rm P} < 7$, and the scatter in their masses is large. This motivates us to find a linear fit between mass and radius on the subsample of exoplanets with $R_{\rm P} < 4$, obtaining:

$$M_{\rm P}/M_{\oplus} = -0.7 + 2.9\ R_{\rm P}/R_{\oplus}$$

There are 58 exoplanets in this sample. The reduced χ^2=4.3, and the RMS is 5.0 $M_{\oplus}$. This subsample of exoplanets and the linear fit are shown in Figure 5.

To illustrate how this population of exoplanets compares to our Solar System, we plot the Solar System planets in Figure 5 as blue triangles. A quadratic fit to the exoplanet population (pictured) happens to line up with the Solar System planets, but has a reduced χ^2 that is twice as large as the linear fit to the exoplanets. Since most of the exoplanets in this sample have $P < 50$ days, we do not expect them to behave the same way as Uranus and Neptune, which have orbital periods of tens of thousands of days. Therefore, the hefty masses of Uranus and Neptune compared to planets of similar size that are closer to their stars is not unreasonable. In fact, the difference in mass between Uranus and Neptune compared to closer-in planets of 4 $R_\oplus$ distinguished the quadratic fit as good for the solar system, as compared to the linear fit that is good for exoplanets.

4. Discussion

4.1. Using Negative Planet Masses for Statistical Soundness

? allow "negative" planet masses as Keplerian orbital solutions to avoid the Lutz-Kelker bias. A "negative" planet mass arises when, given the orbital period and phase determined by the transit, the RV data have the opposite sign from what is expected. This yields a negative semi-amplitude K in the orbital solution, which results in a negative planet mass. In a traditional analysis, the uncertainties in the RVs and the number of measurements combine to determine an upper limit to the planet mass. However, upper limits to planet masses are difficult to use statistically. The artifice of a negative planet mass allows the population of small planets to be treated statistically. Since there is no bias toward large or small planet masses in our sample, we can take the weighted mean mass of planets of a given radius.

4.2. Interpretation of the Mass-Radius Relation

The correlation between exoplanet mass and radius for $R_{\rm P} < 7R_\oplus$ and even for $R_{\rm P} < 4R_\oplus$ indicates that Earth-size planets are less massive than Neptune-size planets.

The data are better described by a linear fit than a quadratic or cubic fit. Since a cubic fit of $M_{\rm P} \propto R_{\rm P}^3$ describes how mass and radius relate for constant density, a fit of $M_{\rm P} \propto R_{\rm P}$ indicates that planet density decreases strongly as mass increases. This can be attributed to an increasing fraction of volatiles with increasing planet mass.

Previous work, including **?** and Weiss et al. (2013), suggest that the mass-radius relation is more like $M_{\rm P} \propto R_{\rm P}^2$ for small exoplanets. However, these studies include Saturn or Saturn-like planets. Such planets might better be described as part of the giant planet population. At any rate, using the properties of Saturn-like planets in a relation that attempts to describe the properties of planets that are more like Earth and Neptune can be, and is, misleading. On the other hand, Wu & Lithwick (2013) found $M_{\rm P}/M_\oplus \approx 3R_{\rm P}/R_\oplus$ in a sample of 22 pairs of planets that exhibited strong anti-correlated transit timing variations (TTVs).

Wu & Lithwick (2013) note that a linear relation between planet mass and radius is dimensionally consistent with a constant escape velocity from the planet (i.e. $v_{\rm esc}^2 \sim M_{\rm P}/R_{\rm P}$). While this is one interpretation of the linear mass-radius relation for small exoplanets, there are perhaps other valid physical interpretations. For instance, a constant $M_{\rm P}/R_{\rm P}$ implies that the gravitational potential energy of exoplanets is constant. This could be due to atmospheric particles escaping, but perhaps some other physical mechanism sets this mass-radius scaling.

4.3. The Possible Role of Photoevaporation in Sculpting Small Planets

Strong stellar irradiation can either inflate a planet, as with hot Jupiters (Seager et al. 2007), or it can photoionize and strip the planet's atmosphere, leaving a dense core (Lopez et al. 2012). Possible evidence for these processes can be seen in Figure 7, which shows that for massive planets ($M_{\mathrm{P}} > 60 M_{\oplus}$ or $R_{\mathrm{P}} > 7 R_{\oplus}$), high incident flux correlates with larger radius, whereas for low-mass planets ($M_{\mathrm{P}} < 60 M_{\oplus}$ or $R_{\mathrm{P}} < 7 R_{\oplus}$), high incident flux correlates with smaller radius.

Another way to describe how incident flux relates to exoplanet size is that for low levels of incident stellar flux (less than 100 times what Earth receives), you can find planets ranging in size from Earth to Jupiter. However, at higher incident flux, there are only hot Earths (which might have been photo-evaporated) and hot Jupiters (which have been inflated). In other words, there are no hot Neptunes. Given the detection of hot and cold Earths, hot and cold Jupiters, and cold Neptunes, it seems unlikely that a detection bias causes the dearth of hot Neptunes; rather, their absence is astrophysical.

How significant is the decrease in planet size with increasing planet flux for small exoplanets? For $R_{\mathrm{P}} < 7 R_{\oplus}$, we find a correlation Pearson R coefficient between planet radius and incident stellar flux of -0.19. Over a sample of 71 planets, this corresponds to a Student-t value of -1.6, indicating a probability of () that these variables are uncorrelated.

4.4. Do Small Planets form around Small Stars?

Although several studies find that planet radius correlates with stellar mass, we find no correlation in this sample (Pearson R = 0.05). This is partly because most of our planets are smaller than $4R_{\oplus}$. However, some planets in our sample have $4R_{\oplus} < R_{\rm P} < 7R_{\oplus}$, in which range Wu & Lithwick (2013) find a correlation between planet size and stellar mass.

4.5. Goodness of Fit

The large reduced χ^2 values for the linear and quadratic mass-radius relations indicate that these relations are not sufficient models to explain the variation in planet mass at a given radius. Either a diversity of planet compositions, or correlation between the residuals and some other parameter, is required to explain the large scatter in planet mass.

We examine the possibility that the residuals to the mass-radius fit correlate with some other parameter. The correlation between the residual (exoplanet mass - predicted mass) and each of: the incident flux from the star, the semi-major axis, and the orbital period, were all about -0.2 (+0.2 for correlation with incident flux). The correlation between the residuals and stellar mass was 0.00. However, the correlation between residuals and stellar radius was -0.11.

Table 1. Exoplanets with Measured Mass and Radius

Name	Planet Mass ($M_\oplus$)	Planet Radius ($R_\oplus$)	Incident Flux ($F_\oplus$)	Period (d)	First Ref.	Orbital Ref.
55 Cnc e	8.380	2.210	2439.690	0.737	McArthur et al. (2004)	Endl et al. (2012)
CoRoT-7 b	5.021	1.679	1779.433	0.854	Queloz et al. (2009); Léger et al. (2009)	Queloz et al. (2009)
CoRoT-8 b	68.673	6.389	88.184	6.212	Bordé et al. (2010)	Bordé et al. (2010)
GJ 1214 b	6.260	2.800	16.631	1.580	Charbonneau et al. (2009)	Carter et al. (2011)
GJ 3470 b	13.900	4.830	38.335	3.337	Bonfils et al. (2012)	**?**
GJ 436 b	23.105	4.222	29.882	2.644	Butler et al. (2004)	Maness et al. (2007)
HAT-P-11 b	26.231	4.730	97.355	4.888	Bakos et al. (2010)	Bakos et al. (2010)
HAT-P-26 b	18.640	6.333	163.050	4.235	Hartman et al. (2011)	Hartman et al. (2011)
HD 97658 b	7.862	2.340	48.052	9.491	Howard et al. (2011)	**?**
Kepler-10 b	4.539	1.416	3572.048	0.837	Batalha et al. (2011)	Batalha et al. (2011)
Kepler-11 b	1.900	1.800	126.384	10.304	Lissauer et al. (2011)	**?**
Kepler-11 c	2.900	2.870	91.413	13.025	Lissauer et al. (2011)	Lissauer et al. (2011)
Kepler-11 d	7.300	3.120	43.562	22.687	Lissauer et al. (2011)	Lissauer et al. (2011)
Kepler-11 e	8.000	4.190	27.524	31.996	Lissauer et al. (2011)	Lissauer et al. (2011)
Kepler-11 f	2.000	2.490	16.745	46.689	Lissauer et al. (2011)	Lissauer et al. (2011)
Kepler-18 b	6.900	2.000	462.244	3.505	Borucki et al. (2011)	Cochran et al. (2011)
Kepler-18 c	17.299	5.490	163.493	7.642	Borucki et al. (2011)	Cochran et al. (2011)
Kepler-18 d	16.399	6.980	67.364	14.859	Borucki et al. (2011)	Cochran et al. (2011)
Kepler-20 b	8.474	1.908	343.928	3.696	Borucki et al. (2011)	Gautier et al. (2012)
Kepler-20 c	15.734	3.067	81.783	10.854	Borucki et al. (2011)	Gautier et al. (2012)
Kepler-20 d	7.528	2.748	5.937	77.612	Borucki et al. (2011)	Gautier et al. (2012)
Kepler-36 b	4.461	1.485	217.365	13.840	Borucki et al. (2011)	Carter et al. (2012)
Kepler-36 c	8.101	3.676	175.646	16.239	Carter et al. (2012)	Carter et al. (2012)
Kepler-4 b	24.544	4.002	1123.918	3.213	Borucki et al. (2010)	Borucki et al. (2010)
Kepler-68 b	8.300	2.310	409.092	5.399	Borucki et al. (2011)	Gilliland et al. (2013)
Kepler-68 c	4.377	0.952	189.764	9.605	Batalha et al. (2013)	Gilliland et al. (2013)
KOI-94 b	9.400	1.770	1155.374	3.743	Weiss et al. (2013)	Weiss et al. (2013)
KOI-94 c	8.300	4.280	295.035	10.424	Borucki et al. (2011)	Weiss et al. (2013)
KOI-94 d	105.000	11.400	106.760	22.343	Borucki et al. (2011)	Weiss et al. (2013)
KOI-94 e	38.000	6.640	32.631	54.320	Borucki et al. (2011)	Weiss et al. (2013)
KOI-41.01	0.85000000	2.2000000	1156.6217	12.815900		
KOI-41.02	7.3400000	1.3000000	65.166782	6.8870500		
KOI-41.03	-5.3100000	1.6000000	85.869041	35.333100		
KOI-69.01	2.5900000	1.5000000	172.33078	4.7267400		
KOI-82.01	8.9300000	2.2000000	806.96474	16.145700		
KOI-82.02	3.8000000	1.2000000	147.34753	10.311700		
KOI-82.03	8.1200000	0.90000000	447.82440	27.453600		
KOI-82.04	-2.4500000	0.60000000	447.27347	7.0714200		
KOI-82.05	0.41000000	0.50000000	2950.4414	5.2869600		
KOI-104.01	10.840000	3.5000000	1189.7862	2.5080600		
KOI-108.01	18.690000	3.4000000	348.15857	15.965400		

Table 1—Continued

Name	Planet Mass ($M_\oplus$)	Planet Radius ($R_\oplus$)	Incident Flux ($F_\oplus$)	Period (d)	First Ref.	Orbital Ref.
KOI-108.02	-22.540000	5.1000000	163.05049	179.61200		
KOI-116.01	10.440000	2.5000000	308.73816	13.570800		
KOI-116.02	11.170000	2.6000000	604.88523	43.844500		
KOI-116.03	0.15000000	0.80000000	416.85849	6.1648600		
KOI-116.04	-13.340000	0.90000000	298.50217	23.980200		
KOI-122.01	13.000000	3.4000000	1191.9507	11.523100		
KOI-123.01	1.3000000	2.4000000	618.50330	6.4816300		
KOI-123.02	2.2200000	2.5000000	1914.9410	21.222700		
KOI-148.01	3.9400000	1.9000000	1903.1669	4.7780000		
KOI-148.02	14.610000	2.7000000	537.46087	9.6739500		
KOI-148.03	7.9300000	2.0000000	1030.1836	42.896100		
KOI-153.01	-5.7000000	2.2000000	1818.3791	8.9250700		
KOI-153.02	7.1000000	1.8000000	439.71133	4.7540000		
KOI-244.01	24.600000	5.2000000	226.04191	12.720400		
KOI-244.02	9.6000000	2.7000000	1560.3683	6.2385000		
KOI-245.01	-5.9800000	1.9000000	1367.9423	39.792200		
KOI-245.02	3.3500000	0.80000000	1611.6302	21.302000		
KOI-245.03	-0.42000000	0.30000000	2337.2286	13.367500		
KOI-246.01	7.8900000	2.3000000	926.85884	5.3987500		
KOI-246.02	2.1800000	1.0000000	1299.0590	9.6050400		
KOI-261.01	8.4600000	2.7000000	4070.4875	16.238500		
KOI-283.01	16.130000	2.4000000	1637.1673	16.092000		
KOI-283.02	17.020000	0.80000000	909.50346	25.516900		
KOI-292.01	3.5100000	1.5000000	584.57028	2.5866400		
KOI-299.01	3.5500000	2.0000000	1299.4980	1.5416800		
KOI-305.01	6.1500000	1.5000000	142.83632	4.6035800		
KOI-321.01	6.3500000	1.4000000	344.40060	2.4262900		
KOI-321.02	2.7100000	0.80000000	725.85388	4.6233200		
KOI-1442.01	0.060000000	1.1000000	11.621547	0.66931000		
KOI-1612.01	0.48000000	0.80000000	1420.2428	2.4650200		
KOI-1925.01	2.6900000	1.2000000	418.26336	68.958400		
Kic843...	1.32	1.1				

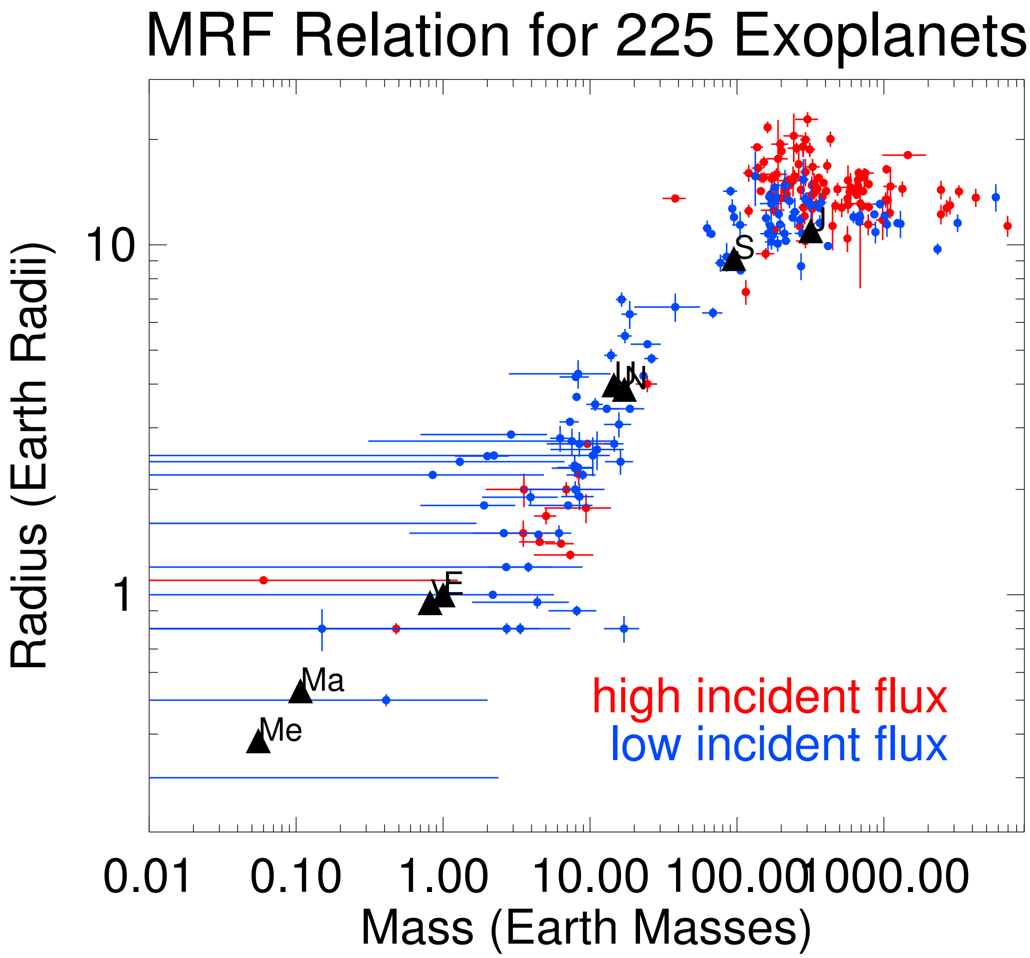

Fig. 1.— Radius vs. mass for exoplanets with measured masses and radii. Planets receiving lower than the median incident flux from this sample (448 times the incident flux at Earth) are blue; those receiving higher than the median incident flux are red. The Solar System planets are over plotted as black triangles for comparison. For giant planets ($M_{\mathrm{P}} >\sim M_{\mathrm{Sat}}$), planet radius correlates with incident flux, whereas for the smaller planets, radius and incident flux appear anti-correlated. See Figure 7 for detail.

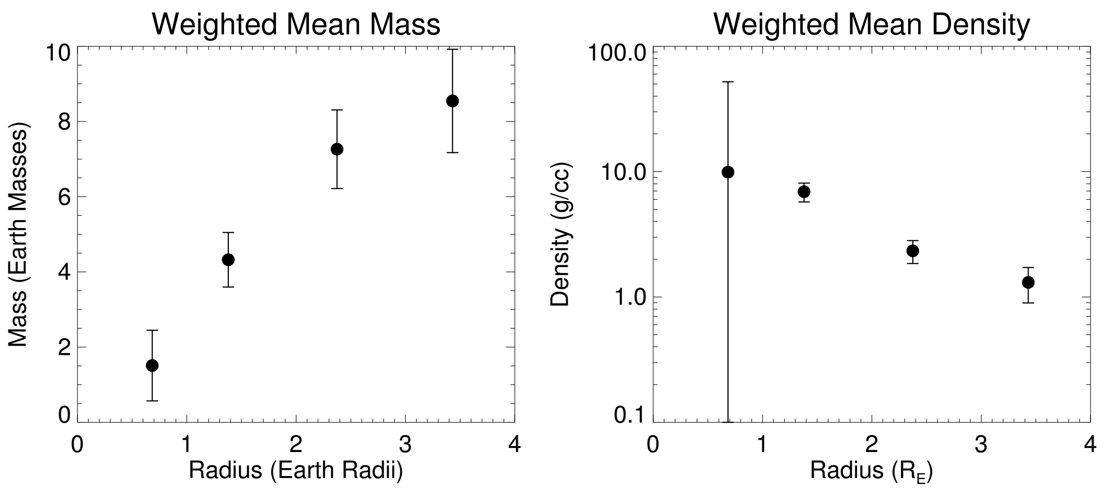

Fig. 2.— The weighted mean mass (left) and density (right) for exoplanets with $R_{\rm P} < 7R_\oplus$ in bins of 1 $R_\oplus$. The error bars are the weighted uncertainties in the means. Larger exoplanet radii correlate with larger masses but lower densities, indicating that large planets have a larger mass fraction of volatiles than small planets.

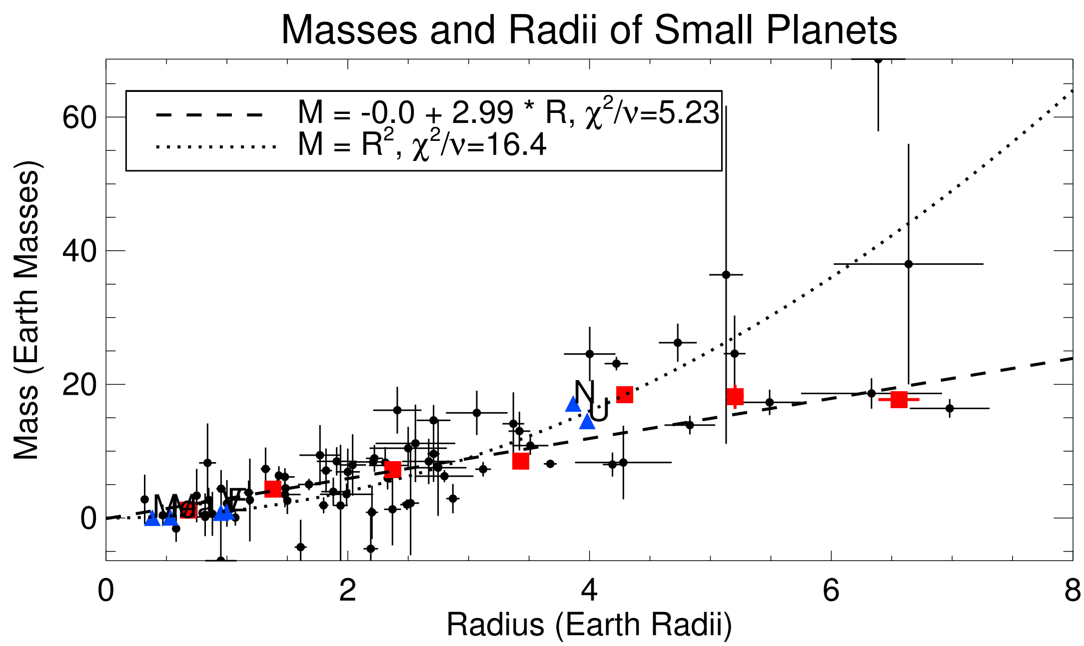

Fig. 3.— Mass vs. radius for planets with $R_{\rm P} < 7R_\oplus$ and 1σ uncertainties. The dashed line is the best linear fit to the data. The dotted line is a simple quadratic fit. Red squares represent the weighted mean mass in bins of 1 $R_\oplus$, and error bars are the weighted uncertainty in the mean mass (as in Figure 2), to guide the eye.

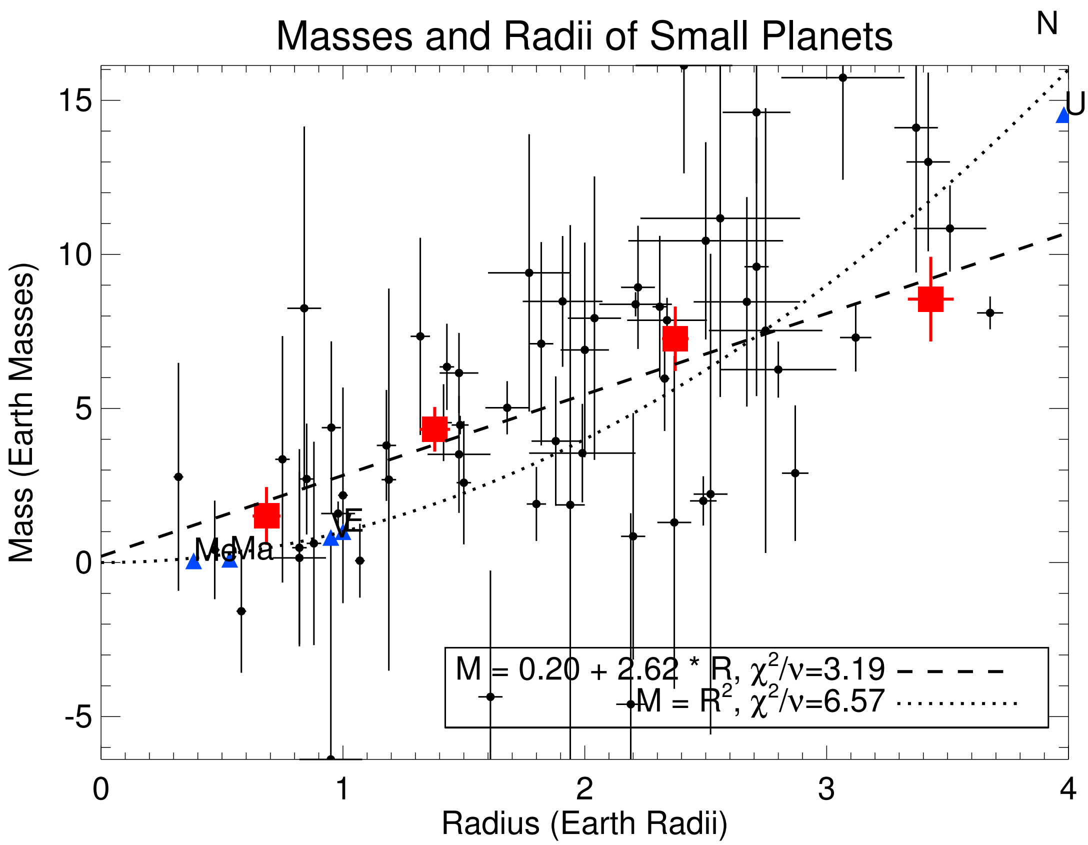

Fig. 4.— Mass vs. radius for planets with $R_{\rm P} < 4R_\oplus$ and 1σ uncertainties. The dashed line is the best linear fit to the data. The dotted line is a simple quadratic fit. Red squares represent the weighted mean mass in bins of 1 $R_\oplus$, and error bars are the weighted uncertainty in the mean mass (as in Figure 2), to guide the eye. The solar system planets are plotted as blue triangles for reference. Note that a linear fit goes through the exoplanet population (and the weighted mean of that population), whereas the quadratic fit goes through the Solar System planets. This is not surprising since Uranus and Neptune are much colder than the exoplanets in this sample.

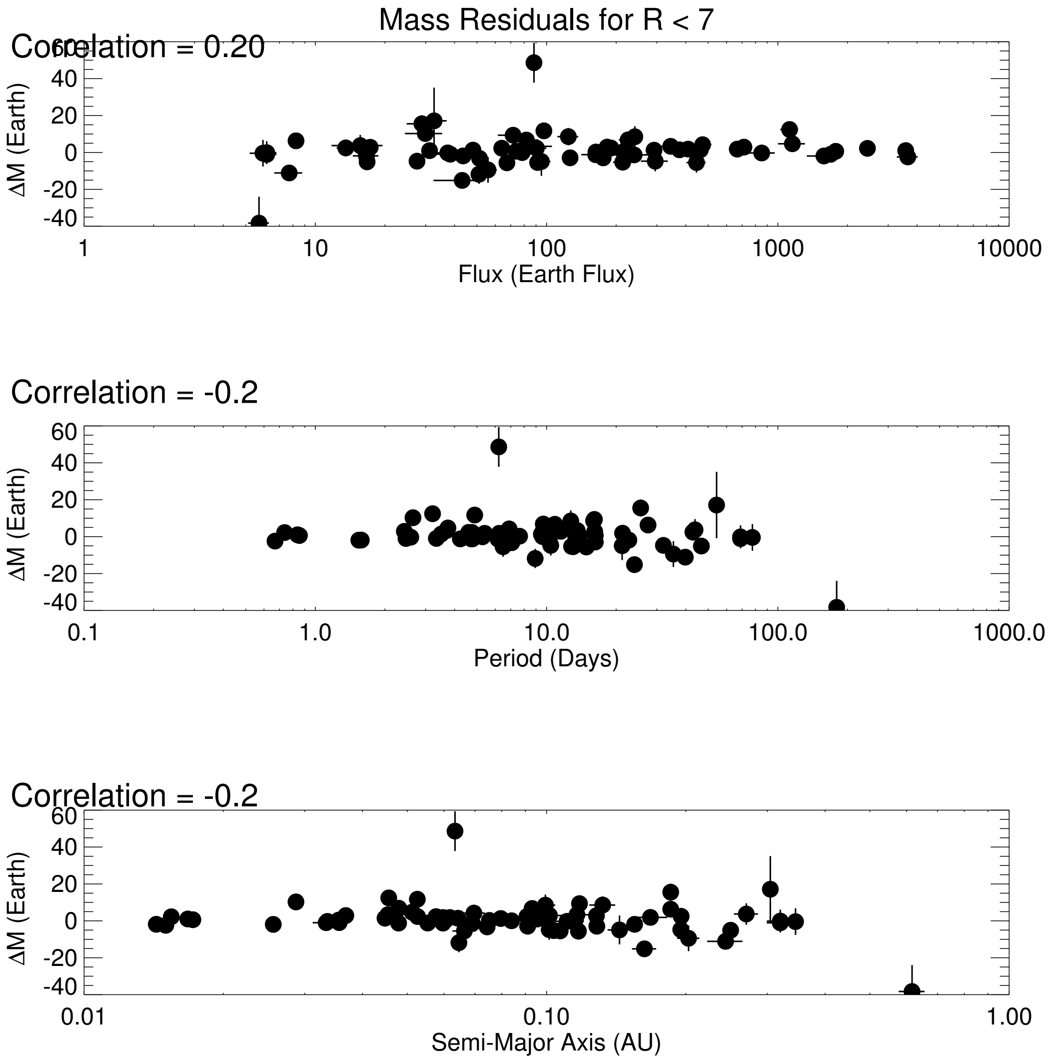

Fig. 5.— Mass residuals (measured minus predicted mass) vs. radius for planets with $R_{\rm P} < 7R_{\oplus}$ and 1σ uncertainties. The correlation between mass residuals and orbital parameters is too small to be significant. For $R_{\rm P} < 4R_{\oplus}$, the correlations are slightly larger (top to bottom: 0.24, -0.3, -0.3), but still not large enough to rule out and uncorrelated underlying distribution.

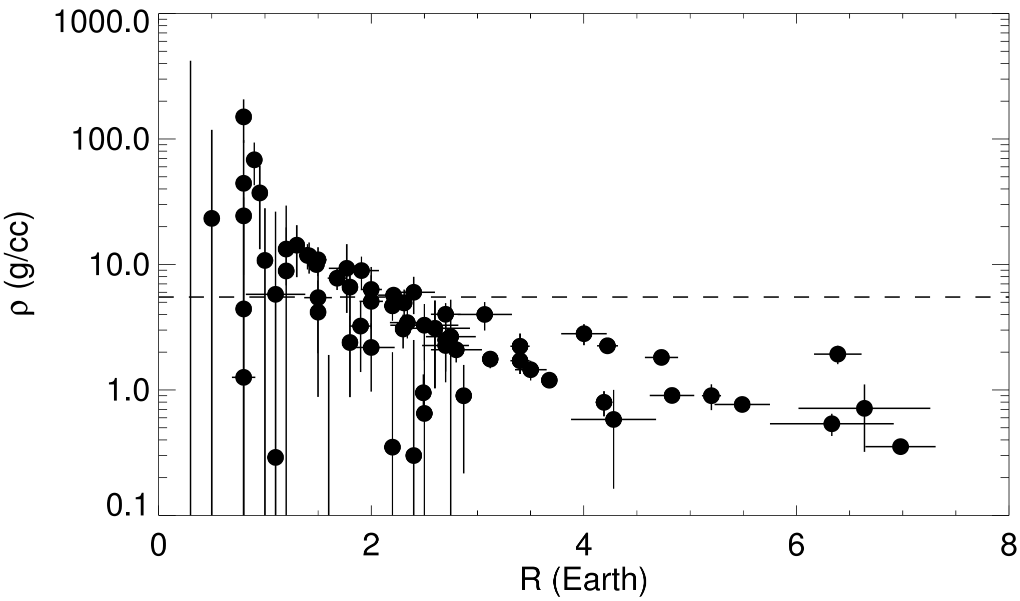

Fig. 6.— Density vs. radius for planets with $R_{\rm P} < 7R_\oplus$ and 1σ uncertainties. Smaller planets are denser than larger planets. For reference, the density of Earth ($5.5\ \mathrm{g\,cm^{-3}}$) is shown as a dashed line.

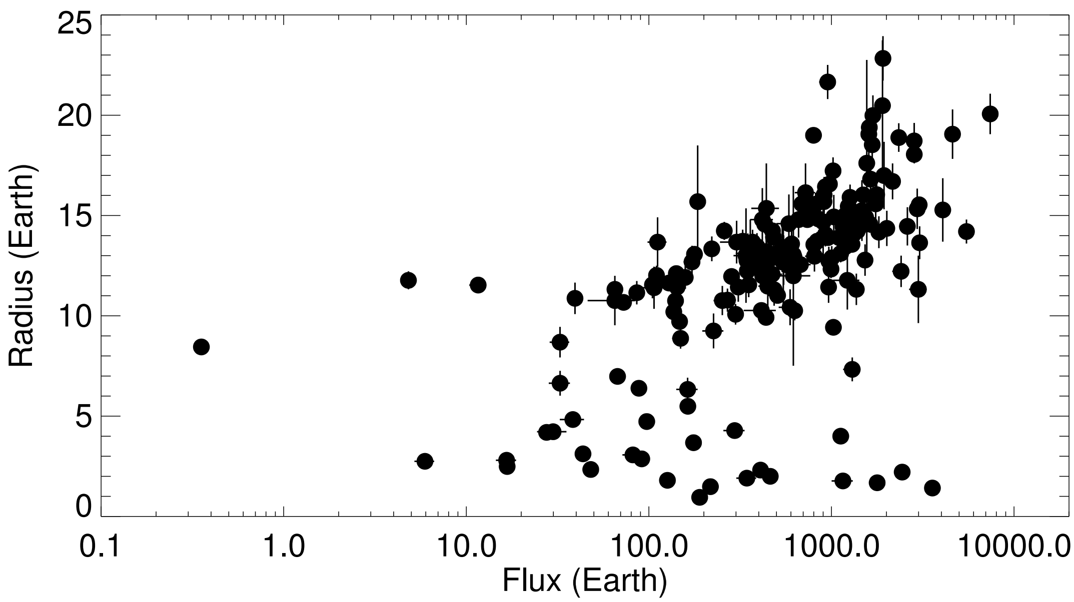

Fig. 7.— Radius vs. flux and 1σ uncertainties for exoplanets with measured masses and radii. Giant planets are orange; low-mass planets are green. For giant planets, radius increases with increasing incident flux from the star; for small planets, radius slightly decreases with increasing flux, especially above a few hundred Earth fluxes. Note that the division between the high and low mass groups is at 7 $R_{\oplus}$, and so the green points correspond to the sample of small planets analyzed above.

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