# Normative Calculus

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## Introduction

This document is an interpreted axiomatic system with the aim of applying existing logic to ethics. This system is based on Stoic virtue ethics.

- The system employs three normative operators:
  - *Required* or mandatory.
  - *Ought* which is advisable but not mandatory.
  - *Indifferent* which is completely optional.
- Normative propositions are always constructed from facts about the endeavours of individual agents.
- Conflicts between normative propositions are preserved as accurate representations of conflicts between and within agents. Inconsistencies are resolved using the strategies of rank-ordering and force choice.

All quotations taken from the *Appendix: A Calculus of Normative Constructs* in Becker (1999) and should be consulted for full details of the system. [1]

### Notation

- *Propositional* variables: $p, q, r, ...$
- *Predicate* constants: $A, B, ...U$ and variables: $X, Y, Z$
- *Individual* constants: $a, b, ...u$ and variables: $x, y, z$
- Existential and universal *Quantifiers:* $(\exists x)$ and: $(x)$
- Truth functional operators:
  - For negation: $\sim$

[1] Becker, Lawrence C. 'A New Stoicism'. Princeton University Press, 1999.

- For the inclusive disjunction (‘either p or q or both’): $\vee$
- For the exclusive disjunction (‘either p or q but not both’): $\veebar$
- For the conjunction ‘and’: $\&$
- For the material conditional ‘if p then q: $\supset$
- For the material biconditional ‘p if and only if q’: $\equiv$

- *Punctuation*: $()\,[]\,\{\}$
- *Modal* operators, read as:
  - 'it is necessary that': $\Box$
  - 'it is possible that': $\diamond$
- *Modal* operator subscripts:
  - 'it is logically possible that': $\diamond_l$
  - 'it is theoretically possible that': $\diamond_t$
  - 'it is practically possible that': $\diamond_p$
  - Note, when a subscript is not given, the operator is shorthand for the disjunction of all subscripts: $\sim\diamond p$ stands for $(\sim\diamond_l p \vee\ \sim\diamond_t p \vee\ \sim\diamond_p p)$
- *Normative* operators, read as:
  - 'it is required that': $R$
  - 'it is ought to be that': $O$
  - 'it is indifferent that': $I$
  - Notes:
    * Numerical subscripts (e.g. $R_{1,}O_2, I_3$) indicate that the norm is drawn from restricted, submoral considerations (etiquette, role obligations, etc.) The subscripts indicate the ordinal status of that *type* of norm relative to other submoral considerations. Priorities among norms of the same type or rank will be indicated with ordinal operators (see below) e.g. $R_2\ >\ R_2$
    * The subscript letter $_t$ will be used to identify norms that arise from transactions between individuals.
    * Normative operators without numerical subscripts are understood to refer to moral (ethical) norms.
- *Ordinal* operators, read as:
  - '... is subordinate to ..': $<$
  - '... is superordinate to ..': $>$
  - '... is coordinate to ..': $<>$
  - '... is subordinate or coordinate to ..': $\leq$
  - '... is superordinate or coordinate to ..': $\geq$

### Well-Formed Formulas

1. Any individual propositional variable or constant is a wff.
2. Any predicate constant or variable that is bound with individual constants or quantified variables is a wff.
3. If anything *w* is a wff, the $\sim w$,$\Box w$ , or $\diamond w$ is a wff, provided no quantifiers range over $\Box$, $\diamond$ , $R$, O or $I$.
4. If anything *w* is a wff, and if $Rw$, $Ow$ or $Iw$ is a construct as defined below, then $Rw$, $Ow$ or $Iw$ is a wff, provided no quantifiers range over $\Box$, $\diamond$ , $R$, O or $I$.
5. If anything *w* is a wff, and if anything *y* is a wff, then $w \vee y$, $w \veebar y$, $w \& y$, $w \supset y$, $w \equiv y$, $w < y$, $w <> y$, $w > y$, $w \leq y$, or $w \geq y$ is a wff, provided no quantifiers range over $\Box$, $\diamond$ , $R$, O or $I$ or within *w* or *y*.
6. Nothing is a wff unless its being so follows rules 1-5.

# a First Order Constructs

Rules governing the construction of well-formed normative propositions from descriptive bases.

## a.1 Best Means

If e is an Endeavour for agent s, and g is a Goal of e, and c is a course of conduct or state of being that is a practically possible Means by which s can achieve g, and there is no other Means by which s can achieve g that is equal to or superordinate to c, then s ought to Undertake to do or be c in e.

$\{eEs \; \& \; gGe \; \& \; \diamond_p \, (cMsg) \; \& \; [\sim (\exists x)(xMsg) \; \& \; (x \geq c)]\} \; / \therefore \; O\,(sUce)$

## a.2 Multiple Means

If e is an Endeavour for agent s, and g is a Goal of e, and c is a course of conduct or state of being that is a practically possible Means by which s can achieve g, and b is another (coordinate) Means by which s can achieve g, then (again, nothing-else-considered), s ought to make a Arbitrary choice between c and b.

$\{eEs \; \& \; gGe \; \& \; \diamond_p \, (cMsg) \; \& \, [(bMsg) \; \& v \, (b <> c)]\} \; / \therefore \; O_1 \, (sAcb)$

## a.3 Desires

From desires to norms:

If e is an Endeavour for agent s, and d is a Desire of s to do or be c in e, and d is a Sufficient reason for e for s to Undertake c, then (nothing-else-considered) s ought to Undertake to do or be c in e.

$(eEs \; \& \; dDsc \; \& \; dSesc) \; / \therefore \; O_1 \, (sUce)$

## a.4 Commitments

From commitments to norms:

If e an Endeavour for agent s, and c is defined as a Commitment for s in e, then (nothing-else-considered) s is required to Undertake to do or be c in e.

$(eEs \ \& \ cCse) \ / \therefore \ R_1 \, (sUce)$

## a.5 Appropriateness

From standards to norms:

If e is an Endeavour for agent s, and c is defined as a Standard or fitting conduct or character for s in e, then (nothing-else-considered) s ought not to Undertake to do or be anything other than c in e.

$(eEs \ \& \ cSse) \ / \therefore \sim O_1 \, (sU \sim ce)$

## a.6 Ideals

From ideals to norms:

If e is an Endeavour for agent s, and c is defined as Ideal conduct or character for s in e, then (nothing-else-considered) s ought to Undertake to do or be c in e.

$(eEs \ \& \ cIse) \ / \therefore \ O_1 \, (sUce)$

## a.7 Transactional Commitments

From transactions to norms:

If e an Endeavour for agent s, and t is a Transaction in e that Generates commitment c for s, then s is *transactionally* required to Undertake to do or be c in e[2].

$(eEs \ \& \ cCse \ \& \ tTe \ \& \ tGcs) \ / \therefore \ R_{t1} \, (sUce)$

## a.8 Transactional Standards

For standards, or fittingness:

If e is an Endeavour for agent s, and t is a Transaction in e that Generates a Standard of fittingness c for s, then *transactionally* s ought not to Undertake to do or be anything other than c in e.

$(eEs \ \& \ cSse \ \& \ tTe \ \& \ tGcs) \ / \therefore \sim O_{t1} \, (sU \sim ce)$

## a.9 Transactional Ideals

And for ideals:

If e is an Endeavour for agent s, and t is a Transaction in e that Generates an Ideal c for s, then *transactionally* s ought to Undertake to do or be c in e.

$(eEs \ \& \ cIse \ \& \ tTe \ \& \ tGcs) \ / \therefore \ O_{t1} \, (sUce)$

[2]Normative operators without $_t$ will be understood as nontransactional or 'structural'.

### a.10 Amendments

From amendments to norms, for when changing the nature of a given endeavour:

If e is an Endeavour for agent s, f is a given Factor or element in e, g is a licit amenDment to f in e, then g replaces f in e.

$(eEs \ \& \ fFe \ \& \ gDfe) \ / \therefore \ R_{t1}(gRfe)$

## b Ranking Same-Level, Same-Endeavour Norms

Rules governing the ranking of normative propositions within a given ordinal level.

### b.1 Requirements over Oughts

Requirements and oughts:

If an agent s is required to do c in e but (from other considerations within the endeavour) ought to do b, and undertaking both is logically or theoretically or practically impossible, then the requirement dominates the ought.

$R_n(sUce) \ \& \ O_n(sUbe)$
$\sim \diamond(sUce \ \& \ sUbe)$
$/ \therefore \ R_n(sUce) \ > \ O_n(sUbe)$

### b.2 Requirements over Indifference

Requirements and indifference:

If an agent s is required to do c in e but (from other considerations within the endeavour) b is a matter of indifference, and undertaking both is logically or theoretically or practically impossible, then the requirement dominates the indifference.

$R_n(sUce) \ \& \ I_n(sUbe)$
$\sim \diamond(sUce \ \& \ sUbe)$
$/ \therefore \ R_n(sUce) \ > \ I_n(sUbe)$

### b.3 Oughts over Indifference

Oughts and indifference:

If an agent s ought to do c in e but (from other considerations within the endeavour) b is a matter of indifference, and undertaking both is logically or theoretically or practically impossible, then the ought dominates the indifference.

$O_n(sUce) \ \& \ I_n(sUbe)$
$\sim \diamond(sUce \ \& \ sUbe)$
$/ \therefore \ O_n(sUce) \ > \ I_n(sUbe)$

## b.4 Coordinate Conflicting Norms of the Same Type

Within an endeavour, there may be conflicts between requirements, between oughts, or between judgements about indifference in cases where it is logically or theoretically or practically impossible to carry out all the norms. Coordinate conflicting requirements or oughts are replaced by a judgement of indifference together with a requirement to choose. Coordinate conflicting indifference generate a requirement to choose.

### b.4.1 Coordinate Conflicting Requirements

$R_n\,(sUce)\ \&\ R_n\,(sUbe)$
  $R_n\,(sUce)\ <>\ R_n\,(sUbe)$
  $\sim\diamond(sUce\ \&\ sUbe)$
  $/\therefore\ I_n\,(sUce)\ \&\ I_n\,(sUbe)\ \&\ R_n\,[(sUce)\ \veebar\ (sUbe)]$

### b.4.2 Coordinate Conflicting Oughts

$O_n\,(sUce)\ \&\ O_n\,(sUbe)$
  $O_n\,(sUce)\ <>\ O_n\,(sUbe)$
  $\sim\diamond(sUce\ \&\ sUbe)$
  $/\therefore\ I_n\,(sUce)\ \&\ I_n\,(sUbe)\ \&\ R_n\,[(sUce)\ \veebar\ (sUbe)]$

### b.4.3 Coordinate Conflicting Indifference

$I_n\,(sUce)\ \&\ I_n\,(sUbe)$
  $I_n\,(sUce)\ <>\ I_n\,(sUbe)$
  $\sim\diamond(sUce\ \&\ sUbe)$
  $/\therefore\ I_n\,(sUce)\ \&\ I_n\,(sUbe)\ \&\ R_n\,[(sUce)\ \veebar\ (sUbe)]$

## b.5 Superordinate Conflicting Norms of the Same Type

If the norms are not coordinate, the superordination relation simply falls through into the conclusion.

### b.5.1 Superordinate Conflicting Requirements

$R_n\,(sUce)\ \&\ R_n\,(sUbe)$
  $R_n\,(sUce)\ >\ R_n\,(sUbe)$
  $\sim\diamond(sUce\ \&\ sUbe)$
  $/\therefore\ R_n\,(sUce)\ >\ R_n\,(sUbe)$

### b.5.2 Superordinate Conflicting Oughts

$O_n\,(sUce)\ \&\ O_n\,(sUbe)$
  $O_n\,(sUce)\ >\ O_n\,(sUbe)$
  $\sim\diamond(sUce\ \&\ sUbe)$

$/ \therefore \ O_n\,(sUce) \ > \ O_n\,(sUbe)$

# c Rules of Escalation

Rules governing the construction of normative propositions of ordinal level n + 1 from normative propositions of level n.

When normative propositions from different endeavours conflict at level n, we will represent the resolution of the conflict in terms of rules for generating normative propositions at level n+1. Forced choices will be made at level n to choose between conflicting courses of conduct of states of being, either within or across endeavours.

## c.1 Escalation from Endogenous Rankings

### c.1.1 Superordination for Coincident Endogenous Rankings

If both e and f are Endeavours for agent s, and s is required at level n to Undertake both c in e and b in f, but although undertaking both is logically or theoretically or practically impossible, each endeavour Defines c as superordinate to b, then s ought, at level n + 1, to Undertake c in e.

For conflicting requirements:
$eEs \ \& \ R_n\,(sUce)$
$fEs \ \& \ R_n\,(sUbf)$
$\sim\diamond(sUce \ \& \ sUbf)$
$R_n\,(sUce) \ > \ R_n\,(sUbf)$
$/ \therefore \ O_{n+1}\,(sUce)$
for conflicts between oughts, requirements and oughts, etc.

This rule applies, *mutatis mutandis*, for any ordering > or < of any norms (**R**, **O**, **I**) governing $\mathrm{sUce}$ and $\mathrm{sUbf}$.

### c.1.2 Equivalence for Coincident Endogenous Rankings

When the various endogenous rankings agree that neither of the conflicting norms at level n dominates each other, they in effect agree that the choice between the norms is arbitrary. We will represent this by deriving norms of indifference at n + 1, coupled with a requirement to choose.

For conflicting requirements:
$eEs \ \& \ R_n\,(sUce)$
$fEs \ \& \ R_n\,(sUbf)$
$\sim\diamond(sUce \ \& \ sUbf)$
$R_n\,(sUce) \ <> \ R_n\,(sUbf)$
$/ \therefore \ I_{n+1}\,(sUce) \ \& \ I_{n+1}\,(sUbf) \ \& \ R_{n+1}\,[(sUce) \ \veebar \ (sUbf)]$
for conflicts between oughts, requirements and oughts, etc.

This rule applies, *mutatis mutandis*, for any ordering > or < of any norms (**R**, **O**, **I**) governing $\mathrm{sUce}$ and $\mathrm{sUbf}$.

## c.2 Forced Choice Under Indifference

If it is a matter of indifference at level n whether agent s Undertakes c in e, or b in f, but s is required to choose exactly one, then s ought at n + 1 to make an Arbitrary choice between c and b.

### c.2.1 Forced Choice Under Indifference Within an Endeavour

$I_n\,(sUce)\ \&\ I_n\,(sUbe)$
$\sim \diamond (sUce\ \&\ sUbe)$
$\&\ R_n\,[(sUce)\ \veebar\ (sUbe)]$
$/ \therefore\ O_{n+1}\,(sAcb)$

### c.2.2 Forced Choice Under Indifference Across Endeavours

$I_n\,(sUce)\ \&\ I_n\,(sUbf)$
$\sim \diamond (sUce\ \&\ sUbf)$
$\&\ R_n\,[(sUce)\ \veebar\ (sUbf)]$
$/ \therefore\ O_{n+1}\,(sAcb)$

## c.3 Escalation from Exogenous Rankings

### c.3.1 Comprehensiveness

Some endeavours are embedded in more comprehensive endeavours, for example a board game may be embedded in a childcare operation. In such cases, where norms at level n conflict, those from the more comprehensive endeavour are elevated to n + 1. Thus, if both e and f are Endeavours for agent s, but although undertaking both is logically or theoretically or practically impossible, endeavour e is defined as more Comprehensive than f for s, then s is required, at level n + 1, to Undertake c in e.

This rule preserves the normative operator from e, to represent the fact that e is more comprehensive. The less comprehensive norm is left intact at level n to represent the fact that the norm from f has not been altered but merely overridden.

For requirements:
$eEs\ \&\ R_n\,(sUce)$
$fEs\ \&\ R_n\,(sUbf)$
$\sim \diamond (sUce\ \&\ sUbf)$
$eCfs$
$/ \therefore\ R(sUce)$

For other norms, this rule applies, *mutatis mutandis*, for any combination of norms. Whatever the norm from the more comprehensive and controlling of the endeavours at n, it escalates to n + 1 as a norm of the same type.

### c.3.2 Exogenous Assessment

Exogenous rankings also arise from the fact that some of our endeavours are designed to assess or evaluate others, peer review, judicial review and other types of activity where we monitor and assess our own activities in terms of their prudence, effects on others, law, morality and so forth. When we adopt an endeavour as an assessment mechanism for other target endeavours, we use it to construct norms that are about those of its targets, and superordinate to them. We will represent that situation here by saying that when the norms of an assessment (or critical endeavour) conflict an level n with those of its target endeavour, the target's norms are rejected, and the norms from the assessment (critique) are elevated to n + 1.

Thus, if both e and f are Endeavours for agent s, but although undertaking both is logically or theoretically or practically impossible, endeavour e is defined as exogenous aSsessment of f for s, then s is required, at level n + 1, to Undertake c in e.

For conflicting requirements:
$eEs \ \& \ R_n\,(sUce)$
$fEs \ \& \ R_n\,(sUbf)$
$\sim \diamond(sUce \ \& \ sUbf)$
$eSfs$
$/ \therefore \ R_{n+1}(sUce)$

For other norms, this rule applies, *mutatis mutandis*, for any combination of norms. Whatever conflicting norms emerges from the assessment endeavour at n, it escalates to n + 1 as a norm of the same type.

# d Transcendence

Rules governing the construction of normative propositions without ordinal subscripts (moral or ethical norms).

## d.1 Transcendent Assessment

Normative propositions representing practical reasoning all-things-considered are written (or may be rewritten) with unsubscripted normative operators, and are interpreted as representing moral norms. Thus, if endeavour e for s is an aSsessment, and e for s is Practical reasoning all-things-considered, and these things generate a normative proposition via the rule of exogenous assessment, then that normative proposition is unsubscripted.

For requirements:
$eEs \ \& \ ePs \ \& \ R_n\,(sUce)$
$fEs \ \& \ R_n\,(sUbf)$
$\sim \diamond(sUce \ \& \ sUbf)$
$eSfs$
$/ \therefore \ R(sUce)$

## d.2 Transcendent Comprehensiveness

The same logic as above holds for applications of the rule of comprehensiveness.

For requirements:

$eEs\ \&\ ePs\ \&\ R_n\,(sUce)$

$fEs\ \&\ R_n\,(sUbf)$

$\sim\diamond(sUce\ \&\ sUbf)$

$eCfs$

$/\therefore\ R(sUce)$

For other norms, this rule applies, *mutatis mutandis*, for any combination of norms. Whatever the proposition from practical reasoning all-things-considered, it escalates to an unsubscripted one of the same type.

## d.3 Conflicting Moral Requirements

Forced Moral Choice:

Conflicts amongst unsubscripted, coordinate normative propositions may force an arbitrary choice. When this happens the arbitrary choice has effectively become the more encompassing norm. Thus, if it is impossible for s to do both c and b, in the same or different endeavours, and if both c and b are unsubscripted requirements for s, and there is an unsubscripted requirement that s do one or the other, then the forced-choice requirements generates a moral requirement to make an Arbitrary choice between the two.

$\sim\diamond(sUce\ \&\ sUbf)$

$R\,(sUce)\ \ \&\ R\,(sUbf)$

$R\,(sUce)\ \ <>\ \ R\,(sUbf)$

$R(sUce\ \veebar\ sUbf)$

$/\therefore\ R(sAcb)$

## d.4 Conflicting Moral Oughts

If both c and b are unsubscripted oughts for s that are jointly impossible to undertake, and a choice between them is forced, we get a moral requirement to make an Arbitrary choice between the two.

$\sim\diamond(sUce\ \&\ sUbf)$

$O\,(sUce)\ \ \&\ O\,(sUbf)$

$O\,(sUce)\ \ <>\ \ O\,(sUbf)$

$R(sUce\ \veebar\ sUbf)$

$/\therefore\ R(sAcb)$

## d.5 Conflicting Moral Indifference

If both c and b are unsubscripted indifferences, and a choice is forced, we get a moral requirement to make an Arbitrary choice.

$\sim\diamond(sUce\ \&\ sUbf)$

$I\,(sUce)\ \ \&\ I\,(sUbf)$

$I\,(sUce)\ \ <>\ \ I\,(sUbf)$
$R(sUce\ \veebar\ sUbf)$
$/\therefore\ R(sAcb)$

### d.6 Closure for Moral Indifference

If it is a matter of indifference whether or not s Undertakes c in e, but a choice between undertaking c or ˜c is forced, we use a conjunction of them to represent the forced choice, which then generates a moral requirement for s to make an Arbitrary choice between c and ˜c.

$\sim\diamond[(sUce\ \&\ \sim(sUce))]$
$IsUce\ \&\ I\sim(sUce)$
$R\,(sUce\ \veebar\ \sim sUse)$
$/\therefore\ R(sAc\sim c)$

## e Axioms of Stoic Normative Logic

Stoics add the following postulates specific to their ethical doctrines.

### e.1 Axiom of Encompassment

The exercise of agency through practical intelligence, including practical reasoning all-things-considered, is the most comprehensive of an agents endeavours. That is, in every case, if endeavour x for agent s is the exercise of Agency through practical intelligence and endeavour y for s is not, then x is more comprehensive than y for s.

$\vdash\ (x)\ (y)\ [(xEs\ \&\ yEs\ \&\ xAs\ \&\ y\sim As)\ \supset\ xCys]$

### e.2 Axiom of Finality

There is no assessment endeavour exogenous to the exercise of practical reasoning all-things-considered. That is, in every case, if endeavour x for agent s is pracTical reasoning all-things-considered and endeavour y for s is not, then y is not an aSsessment mechanism for x for s.

$\vdash\ (x)\ (y)\ [(xEs\ \&\ yEs\ \&\ xTs\ \&\ y\sim Ts)\ \ y\sim Sys]$

### e.3 Axiom of Moral Priority

Unsubscripted norms are superordinate to subscripted ones. If N stands for any normative operator and P stands for a wff of any form, then:

$\vdash\ NP\ >\ N_nP$

### e.4 Axiom of Moral Rank

The order R > O > I holds for unsubscripted (i.e. moral) normative propositions.

$\vdash\ RP\ >\ OP\ >\ IP$

### e.5 Axiom of Closure

If no normative proposition at any level can be constructed for a given course of conduct or state of being (that is, if a situation is normatively open), the relevant proposition is closed with an unsubscripted ought-not. That is, if it is not the case that either a subscripted or unsubscripted normative operator N governs c for s in e, then s ought not to Undertake c in e.

$\vdash\ [N\,(sUce)\ \vee\ N_n\,(sUce)]\ \supset\ O\sim(sUce)$

### e.6 Axiom of Futility

If any norm represented by an atomic normative proposition (not a conjunction of them), is logically, theoretically, or practically impossible to carry out, it yields an unsubscripted prohibition. That is, if any normative operator N governs c for s in e, and it is logically, theoretically, or practically impossible for s to Undertake c in e, then s is required not to Undertake c in e, and that requirement dominates the original norm.

$\vdash\ [N\,(sUce)\ \&\ \sim\diamond(sUce)]\ \supset\ R\sim(sUce)\ \&\ [R\sim(sUce)\ >\ N\,(sUce)]$

## f Immediate Inferences

### f.1 From Stronger to Weaker

If the stronger proposition is proved, we can conclude the weaker counterparts are proved:

$R_n w\ /\therefore\ O_n w$

$I_n w\ /\therefore \sim O_n w\ \&\ \sim R_n w$

### f.2 From Opposites

#### f.2.1 Contradictories

Contradictories can be neither jointly true nor jointly false:

$I_n w\ /\therefore \sim(R_n w)$ and vice versa.

$I_n w\ /\therefore \sim(R_n \sim w)$

$I_n w\ /\therefore \sim(O_n w)$

$I_n w\ /\therefore \sim(O_n \sim w)$

$R_n w\ /\therefore \sim(I_n w)$

$R_n \sim w\ /\therefore \sim(I_n w)$

$O_n w\ /\therefore \sim(I_n w)$

$O_n \sim w\ /\therefore \sim(I_n w)$

#### f.2.2 Contraries

Contraries cannot be jointly true but can be jointly false:

$R_n w\ /\therefore \sim(R_n \sim w)$

$R_n w\ /\therefore \sim(O_n \sim w)$

and:
$R_n \sim w \; / \therefore \sim (R_n w)$
$R_n \sim w \; / \therefore \sim (O_n w)$
and:
$O_n w \; / \therefore \sim (O_n \sim w)$
$O_n \sim w \; / \therefore \sim (O_n w)$
Note $I_n w$ and $I_n \sim w$ are definitional equivalents and thus not contraries.