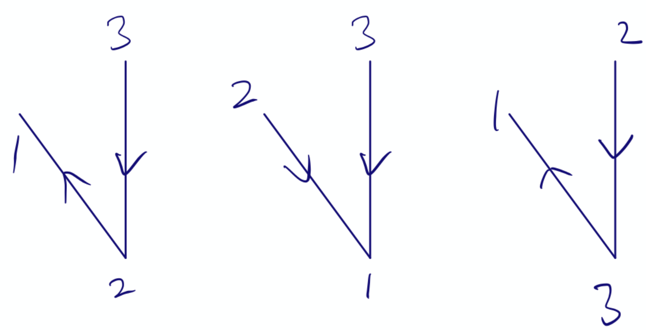
Notes on $\det' A$

Consider the $n \times n$ symmetric matrix A_n. We rewrite every diagonal element 0 as the sum of the remaining elements in the row (column). We delete the n-th row and column at first. For the remaining $(n-1) \times (n-1)$ symmetric matrix, we can still let every diagonal element as the sum of the remaining elements in the row (column) by using SL(2) gauge to send σ_n to infinity. Now we can use the matrix tree theorem [1]. We choose to continue to delete the 1-th row and column. Then the determinant of the remaining $(n-2) \times (n-2)$ symmetric matrix can be written as a sum of all labelled trees made up by $(n-1)$ nodes and $(n-2)$ directed edges. All directions are determined by requiring all flows end at node 1. See figure 1 as an example of 4pt. Each directed edge $i \to j$ represents a factor $\frac{s_{ij}}{\sigma_{ij}}$ and each



verse

Figure 1: 4pt example

labelled tree denoted as $\mathcal{T}$ represents a product of $(n-2)$ such factors denoted as $P(\mathcal{T})$.

Consider the soft limit of particle i, i.e. $s_{ij} \to \tau \hat{s}_{ij}$ with $\tau \to 0$ for any $j \neq i$. Obviously, if the node i is linked by r edges in a labelled tree $\mathcal{T}$, the corresponding product $P(\mathcal{T})$ will vanish at order $\mathcal{O}(\tau^r)$. Thus we only need to consider such labelled trees where the node i is only linked by one edge, i.e. the node i is on the ends of the tree. If we remove the node i and the only edge connecting to it, the remaining part is still a labelled tree of lower points. For any such $(n-2)$-pt labelled tree, we can see the node i can connect to any node the $(n-2)$-pt labelled tree through an edge, resulting in a sum factor $\sum_{j \neq i} \frac{s_{ij}}{\sigma_{ij}} = 0$. This explains the Alder zero.

[1] Note that the matrix tree theorem itself doesn't require the matrix is symmetric.