A zone plate is a diffractive optic that consists of several radially symmetric rings called *zones*. Zones alternate between opaque and transparent, and are spaced so that light transmitted by the transparent zones constructively interferes at the desired focus.

Contents

* Focusing a plane wave
  + Positioning of zone boundaries
  + Choosing the number of zones based on resolution
  + Choosing the number of zones based on lens size
  + Source bandwidth restrictions
* Point-to-point imaging
* Field of view
* References

**Focusing a plane wave**

One common use of a zone plate is to bring light from a distant source to focus. In this scenario, the incident wave can be approximated as a plane wave.

**Positioning of zone boundaries**

In this section, zone radii will be derived for a zone plate optic with focal length, (f).

To begin, consider a source point in the plane of the zone plate located a distance (r) from the center of the zone plate.

The distance l between the source point and the focus is called the optical path length, and is given by:

l=r2+f2−−−−−−√(1)(1)l=r2+f2

The optical pathlength from a source point at the center of the zone plate (r=0r=0) is simply equal to the focal length.

l0=f(2)(2)l0=f

The goal is to locate source points that constructively interfere at the focus. This is accomplished by requiring that optical path lengths ll differ by no more than λ/2λ/2 from the optical pathlength of the on-axis source l0l0.

l−l0<λ2(3)(3)l−l0<λ2

Source points satisfying this criterion define the *1st zone*, which is shaded in red below:

As source points move further away from the center of the zone plate, the quantity (l−l0)(l−l0) will increase beyond λ/2λ/2. These sources with optical pathlengths satisfying

λ2<l−l0<λ(4)(4)λ2<l−l0<λ

define the *2nd zone*, and will *destructively* interfere with the sources in the 1st zone. The 2nd zone is shaded in blue below:

By continuing in this manner, the definition of the *nth zone* can be generalized to be the collection of source points with optical pathlengths satisfying

(n−1)λ2<l−l0<nλ2(5)(5)(n−1)λ2<l−l0<nλ2

Here, nn is a positive integer from 11 to NN, where NN is the total number of zones in the zone plate.

Source points from odd zones (n=1,3,5,...n=1,3,5,...) will *constructively* interfere with the 1st zone, whereas source points from even zones (n=2,4,6,...n=2,4,6,...) will *destructively* interfere with the 1st zone. To maximize constructive interference, the zone plate is constructed by blocking the even numbered zones with an appropriate absorber and letting the odd zones pass through.

Note that the choice of the 1st zone as the reference is arbitrary. If instead a zone plate is constructed with the reversed tone — with transparent even zones and opaque odd zones — the effective lens will be the same.

In either case, pathlength differences satisfying

l−l0=nλ2(6)(6)l−l0=nλ2

represent the boundaries between opaque and transparent zones. Rewriting [(6)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqzone_boundaries)(6) in terms of the source point radii, gives

f2+r2n−−−−−−√−f=nλ2f2+rn2−f=nλ2

where rnrn represents the transition to the nnth zone. Solving for rnrn gives

r2n=nλ(f+nλ4)(7)(7)rn2=nλ(f+nλ4)

The zone plate geometry is constructed by alternating transparent and opaque zones, where the radius of the nnth zone is given by Eq. [(7)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqzone_radii)(7). All zone plates with radii given by Eq. [(7)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqzone_radii)(7) will bring plane wave illumination with wavelength λλ to focus at a distance ff past the lens.

The only remaining variable is the total number of zones NN in the zone plate. While NN does not affect the focal length of the lens, it has other important implications on properties of the zone plate including resolution, collection efficiency, and source bandwidth requirements. These relationships are explored in the following sections.

**Choosing the number of zones based on resolution**

When using a zone plate to focus light, resolution will usually be a primary design consideration. Here, resolution refers to the width ww of the focused beam. In a diffraction-limited optical system, the resolution is approximately given by

w=λ2NA(8)(8)w=λ2NA

where NANA is the numerical aperture of the system, defined as the sine of the half angle of the cone of light that leaves the zone plate. For a given wavelength λλ and focal length ff, increasing resolution translates to increasing the NANA of the system, and thus, increasing the number of zones. As the zone number nn gets larger, the width of the nnth zone gets smaller. The thinnest zone width is the NNth zone (the outer-most zone on the zone plate) and tends to be the most difficult zone to fabricate lithographically. It is therefore is useful to relate the resolution of the zone plate ww to the number of zones NN and width of the outer-most zone ΔrΔr.

By examining the edge of the zone plate, it can be seen that the alternating zones locally resemble a simple diffraction grating with pitch d=2Δrd=2Δr. The angle θθ of the first order diffracted light is given by the grating equation:

sinθ=λd=λ2Δr(9)(9)sin⁡θ=λd=λ2Δr

Since sinθsin⁡θ is simply the numerical aperture NANA of the zone plate, Eqs. [(8)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqres_limit)(8) and [(9)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqgrating_eq)(9) can be combined in the following way

w=λ2sinθ=λ2⋅2Δrλ=Δr(10)w=λ2sin⁡θ=λ2⋅2Δrλ(10)=Δr

Eq. [(10)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqw_eq_dr)(10) reveals the simple yet useful result that the resolution of a zone plate is set by the width of the outermost zone.

Because this result was derived using a simple model based on a diffraction grating, one might wonder how it compares with the zone radii equation in Eq. [(7)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqzone_radii)(7).

For zone plates with low numerical aperture, ff is large compared with nλ/4nλ/4, and Eq. [(7)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqzone_radii)(7) can be simplified to

r2n≃nλf(11)(11)rn2≃nλf

Writing Eq. [(11)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqzone_radii_approx)(11) twice, once for rNrN and once for rN−1rN−1 gives

r2N−r2N−1=Nλf−(N−1)λf=λf(12)rN2−rN−12=Nλf−(N−1)λf(12)=λf

Setting rN−1 =rN−ΔrrN−1 =rN−Δr in Eq. [(12)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqouter_zone_width_intermediate)(12) and expanding gives

2rNΔr−(Δr)2=λf(13)(13)2rNΔr−(Δr)2=λf

Since Δr≪rNΔr≪rN, Eq. [(13)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqouter_zone_width_intermediate_2)(13) simplifies to

Δr≃λf2rN(14)(14)Δr≃λf2rN

This manipulation of Eq. [(7)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqzone_radii)(7), provides an expression relating the outer zone width of a zone plate, ΔrΔr, to its radius, rNrN.

Recognizing that rN/frN/f is simply the zone plate NANA, Eq. [(14)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqouter_zone_width_final)(14) can be rewritten as

Δr≃λ2NA(15)(15)Δr≃λ2NA

Solving Eq. [(8)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqres_limit)(8) for NANA and inserting the result into Eq. [(15)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqouter_zone_width_using_NA)(15) gives Δr≃wΔr≃w, which agrees with the result in Eq. [(10)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqw_eq_dr)(10).

Eq. [(14)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqouter_zone_width_final)(14) also provides a convenient representation for the zone plate radius:

rN≃λf2Δr=λf2w(16)(16)rN≃λf2Δr=λf2w

Finally, it remains to compute the NN, the number of zones in the zone plate. Writing Eq. [(11)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqzone_radii_approx)(11) in terms of the outermost zone gives

N=r2Nλf=λf4w2(17)(17)N=rN2λf=λf4w2

Eq. [(17)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqN_from_w)(17) shows that the number of zones NN has an inverse square relation to the resolution. Therefore, doubling the resolution of a zone plate while preserving its focal length requires a *quadrupling* of the number of zones.

The table below is a summary of the important properties of a zone plate written in terms of the resolution ww.

Zone plate parameters as a function of resolution ww

* Choose wavelength, λλ
* Choose focal length, ff
* Choose resolution, ww

|  |  |  |
| --- | --- | --- |
| **Parameter** | **Symbol** | **Value** |
| Number of zones | NN | λf4w2λf4w2 |
| Zone plate radius | rNrN | λf2wλf2w |
| Outer zone width | ΔrΔr | ww |
| Radius of nthnth zone transition | rnrn | nλf−−−−√nλf |

**Choosing the number of zones based on lens size**

In some applications, it may be desirable to directly specify the physical size of the lens. For example, in scenarios where photon flux is limited, constructing the zone plate aperture to match the intrinsic width of the incident beam ensures that the maximum amount of light will enter the system.

From Eq. [(16)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqrn_to_w)(16), the zone plate radius rNrN can be directly related to the zone plate resolution ww

w=λf2rN(18)(18)w=λf2rN

All other design parameters can be found by using the resolution relations above. The following table is a concise summary of the main zone plate parameters written in terms of the zone plate radius rNrN

Zone plate parameters as a function of zone plate radius rNrN

* Choose wavelength, λλ
* Choose focal length, ff
* Choose radius, rNrN

|  |  |  |
| --- | --- | --- |
| **Parameter** | **Symbol** | **Value** |
| Zone number | NN | r2NλfrN2λf |
| Resolution | ww | λf2rNλf2rN |
| Outer zone width | ΔrΔr | λf2rNλf2rN |
| Radius of nthnth zone transition | rnrn | nλf−−−−√nλf |

**Source bandwidth restrictions**

So far, this analysis has assumed that the zone plate is illuminated by a monochromatic source. Real sources are never perfectly monochromatic; rather, they have an intrinsic bandwidth ΔλΔλ. Since zone plates are based on diffraction, system performance is sensitive to source bandwidth.

The effect of finite bandwidth on zone plate performance can be explored by revisiting the grating equation (Eq. [(9)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqgrating_eq)(9)) at the edge of the zone plate. Under the small-angle approximation, sinθ=tanθ=rN/fsin⁡θ=tan⁡θ=rN/f, giving

λf=d⋅rN(19)(19)λf=d⋅rN

Since the outer grating period dd and the radius rNrN are fixed, Eq. [(19)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqfwav_const)(19) shows that the product λfλf is a constant. This result shows that each source wavelength has a different focal length. Furthermore, λλ and ff are inversely related; for a given zone plate, long wavelengths will focus more quickly than short wavelengths.

Writing Eq. [(19)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqfwav_const)(19) twice, once at the wavelength λλ, and once at the wavelength λ+Δλλ+Δλ gives

λf=(λ+Δλ)(f−Δf)(20)(20)λf=(λ+Δλ)(f−Δf)

Expanding Eq. [(20)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqdeltaf_intermediate)(20) and dropping the second order term ΔλΔfΔλΔf, reveals that the relative change in focal length is simply equal to the relative bandwidth of the source.

Δλλ=Δff(21)(21)Δλλ=Δff

In order to maintain near diffraction-limited performance, the magnitude of the focus shift ΔfΔf due to the finite bandwidth ΔλΔλ should be less than the depth of focus (DOF) of the system, λ/NA2λ/NA2.

Δλλ≤λ/NA2f(22)(22)Δλλ≤λ/NA2f

Since NA=rN/fNA=rN/f,

Δλλ≤λfr2N(23)(23)Δλλ≤λfrN2

Finally, using that r2N=NfλrN2=Nfλ from Eq. [(11)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqzone_radii_approx)(11) gives:

Δλλ≤1N(24)(24)Δλλ≤1N

**Point-to-point imaging**

Eq. [(7)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqzone_radii)(7) describes the zone radii, rnrn, of a zone plate designed to focus a plane wave dd past the zone plate. For generalized point-to-point imaging, the equation for the zone radii needs to be modified. The new geometry is shown below.

In the generalized case, the on-axis optical pathlength from the object point to the image point is p+qp+q. For rays off of the optical axis, the optical pathlength grows by Δp+ΔqΔp+Δq where

Δp=p2+r2−−−−−−√−pΔp=p2+r2−p

Δq=q2+r2−−−−−−√−qΔq=q2+r2−q

The beginning of the first opaque zone is placed at the radius where Δp+Δq=λ/2Δp+Δq=λ/2, such that the change in pathlength from the axial ray just reaches the critical amount of λ/2λ/2, as discussed above. The radius of the nthnth zone transition is given by

nλ2=p2+r2n−−−−−−√+q2+r2n−−−−−−√−q−p(25)(25)nλ2=p2+rn2+q2+rn2−q−p

In the limit of p→∞p→∞, Eq. [(25)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqzone_radii_pq)(25) simplifies to Eq. [(7)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqzone_radii)(7)

**Field of view**

For zone plate-based optical systems, the field of view (FOV) is often expressed as the maximum angular size of the object over which the system exhibits good performance.

Zone plates are always designed with specific conjugates in mind; i.e., the location of the object and image points are known a-priori and the zone locations are calculated based on their values. See, for example, Eq. [(25)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqzone_radii_pq)(25).

For small viewing angles from the designed conjugate points, the imaging condition is approximately satisfied on a pair of spherical conjugate *surfaces*, known as the object-side spherical conjugate surface and the image-side spherical conjugate surface. As shown below,points on the object-side spherical conjugate surface of radius pp are imagedto an image-side spherical conjugate surface of radius qq. *The field angle*θθ*is exaggerated for illustrative purposes*.

Since samples and detectors are often planar, their surfaces can't coexist with the ideal spherical conjugate surfaces;they can only intercept the spherical conjugate surfaces at one point, which is typically along the optical axis.

As shown below in blue, when planar samples and detectors are used, there is a separation of δpδp between the object and the object-side spherical conjugate surface at the angular field point θθ. Likewise, there is a separation of δqδq between the detector and the image-side spherical conjugate surface for the same angular field point θθ.

δp=p(secθ−1)(26)(26)δp=p(sec⁡θ−1)

δq=q(secθ−1)(27)(27)δq=q(sec⁡θ−1)

When zone plates are used to image planar samples, the separation, δpδp, between the object and the object-side spherical conjugate surface (for non-zero θθ) increases the effective object distance from pp to p+δpp+δp. This causes the image of the object point to form a distance δ′qδq′ inside of the image-side conjugate surface, as shown below:

For any optical system, the change of the image distance, δ′qδq′, due to a small change of the object distance, δpδp, is given by†

δ′q=−m2δp(28)(28)δq′=−m2δp

where mm is the magnification, qpqp.

The total distance between the detector (image-side planar surface) and the focused image of the object pointis δq+δ′qδq+δq′. Using Eqs. [(27)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqfov_deltaq)(27), [(26)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqfov_deltap)(26), and [(28)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqfov_deltaqprime)(28), this quantity can be written in terms of pp, qq, mm, and θθ:

δq+δ′q=(q+m2p)(secθ−1)(29)(29)δq+δq′=(q+m2p)(sec⁡θ−1)

In order for the imaging fidelity to remain high at the angular field point θθ, δq+δ′qδq+δq′ needs to be less than the image-side DOF of the system (λ/NA2iλ/NAi2). Satisfying this criterion leads to one quarter wave of defocus at the angular field point θθ; if better wavefront quality is desired, a stricter criterion must be met. For example, a wavefront quality of λ20λ20 is achieved when δq+δ′qδq+δq′ is less than DOF/5

Using the small-angle approximation secθ≃(1+θ/2)1/2sec⁡θ≃(1+θ/2)1/2 and the criterion δq+δ′q<δq+δq′< DOF/ηη where ηη is a scaling factor, Eq. [(29)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqfov_totaldist)(29) can be solved for θθ, the angular field of view.

θ=2(ληNA2i1(q+m2p)+1)2−2(30)(30)θ=2(ληNAi21(q+m2p)+1)2−2

The transverse field of view (half-width) can be computed by multiplying the angular field of view by the object distance pp. The [zone plate design app](http://zoneplate.lbl.gov/#design) on this website solves for the field of view of every zone plate it calculates using this equation.

**Example**

The [SHARP](http://sharp.lbl.gov/) EUV mask inspection microscope at the Advanced Light Source at Lawrence Berkeley National Laboratory has the following geometry:

SHARP imaging geometry.*All*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *λ* (nm) | *p* (m) | *q* (m) | Magnification | *NAo* |
| 13 | 500e-6 | 0.45 | 900 | 0.0825 |

Inserting these values into Eq. [(30)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqfov_final)(30) with ηη = 5 (for a λ/20λ/20 wavefront) and multiplying by the object distance pp gives a transverse FOV of 3 μm (full width). This value agrees well with what is observed in practice.

**References**

A. Fresnel: Calcul de l'intensite de la lumiere au centre de l'ombre d'un ecran, Oeuvres completes d' Augustin Fresnel, Vol.1, Note 1, 365 (1866)

J.L. Soret: Ueber die von Kreisgittern erzeugten Diffraktionsphaenomene, Ann.Phys.Chem 156, 99 (1875)

D. Attwood, [*Soft X-rays and Extreme Ultaviolet Radiation: Principles and Applications*](http://www.amazon.com/Soft-X-Rays-Extreme-Ultraviolet-Radiation/dp/052102997X). Cambridge University Press, 1999.

**Footnotes**

**†** The imaging equation is:

1p+1q=1f(31)(31)1p+1q=1f

Solving Eq. [(31)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqimg_eqn)(31) for qq and differentiating the result with respect to pp gives:

dqdp=−1(pf−1)2(32)(32)dqdp=−1(pf−1)2

Multiplying both sides of Eq. [(31)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqimg_eqn)(31) by pp and using m≡q/pm≡q/p shows that the denominator of the right side of Eq. [(32)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqdq_dp)(32) is the inverse of the magnification. This result is equivalent to Eq. [(28)](http://zoneplate.lbl.gov/theory#mjx-eqn-eqfov_deltaqprime)(28)