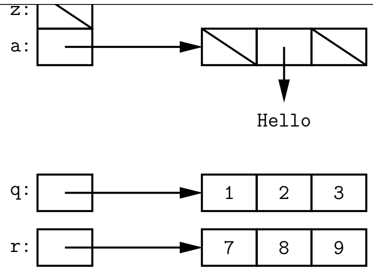
after expanding and collecting terms?

components are

- **length**, a fixed integer.
- a sequence of **length** simple containers of the same type, numbered from 0.
- (.length field usually implicit in diagrams.)

- Arrays are anonymous, like other structured containers.
- Always referred to with pointers.
- For array pointed to by `A`,
  - Length is `A.length`
  - Numbered component i is `A[`i`]` (i is the *index*)
  - Important feature: index can be *any integer expression*.



```
int[] x, y, z;
String[] a;
x = new int[3];
y = x;
a = new String[3];
x[1] = 2;
y[1] = 3;
a[1] = "Hello";
```

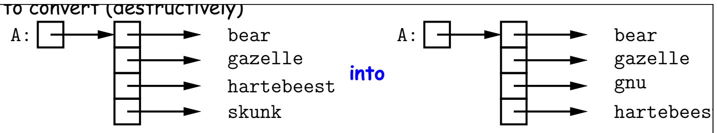
```
int[] q;
q = new int[] { 1, 2,
3 };
// Short form for
declarations:
```

```
static int sum(int[] A) {
  int N;
  N = 0;                                          //
New (1.5) syntax
  for (int i = 0; i < A.length; i += 1)           for
(int x : A)
    N += A[i];                                       N
+= x;
  return N;
}


// For the hard-core: could have written

int N, i;
for (i=0, N=0; i<A.length; N += A[i], i +=
1)
  { }   // or just ;
```

To convert (destructively)


A: bear gazelle hartebeest skunk

into

A: bear gazelle gnu hartebees

```
/** Insert X at location K in ARR, moving items K,
K+1, ... to locations
 *  K+1, K+2, ....  The last item in ARR is lost.
*/
static void insert (String[] arr, int k, String x)
{
  for (int i = arr.length-1; i > k; i -= 1) // Why
backwards?
    arr[i] = arr[i-1];
  /* Alternative to this loop:
        System.arraycopy(arr, k,  arr, k+1,  arr.length-k-1);*/
                          from      to         # to copy
  arr[k] = x;
}
```

including at the top of the source file:

```
import static java.lang.System.arraycopy;
```

- This means "define the simple name `arraycopy` to be the equivalent of `java.lang.System.arraycopy` in the current source file."
- Can do the same for `out` so that you can write

  `out.println(...);`

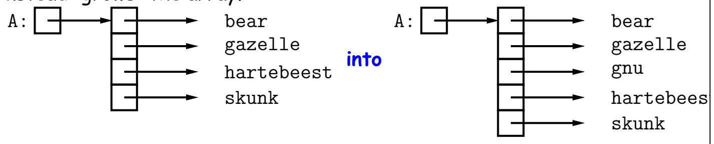
  in place of

  `System.out.println(...);`
- Finally, a declaration like

  `import static java.lang.Math.*;`

  means "take all the (public) static definitions in `java.lang.Math` and make them available in this source file by their simple names (the name after the last dot)."

description above, so that `A = insert2 (A, 2, "gnu")` does *not* shove "skunk" off the end, but instead "grows" the array.

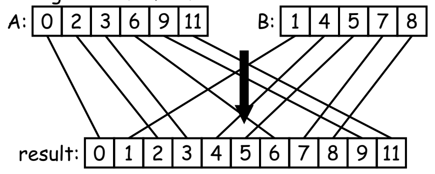

A: bear gazelle hartebeest skunk

into

A: bear gazelle gnu hartebees skunk

```
/** Return array, r, where r.length = ARR.length+1;
r[0..K-1]
 *  the same as ARR[0..K-1], r[k] = x, r[K+1..] same
as ARR[K..]. */
static String[] insert2(String[] arr, int k, String
x) {
  String[] result = new String[arr.length + 1];
  arraycopy(arr, 0, result, 0, k);
  arraycopy(arr, k, result, k+1, arr.length-k);
  result[k] = x;
  return result;
}
```

and B, produce their *merge*: a sorted array containing all items from A and B.


A: 0 2 3 6 9 11

B: 1 4 5 7 8

result: 0 1 2 3 4 5 6 7 8 9 11

and B, produce their *merge*: a sorted array containing all from A and B.

Remark: In order to solve this recursively, it is useful to *generalize* the original function to allow merging *portions* of the arrays.

```
/** Assuming A and B are sorted, returns their merge.
*/
public static int[] merge(int[] A, int[] B) {
   return mergeTo(A, 0, B, 0);
}
```

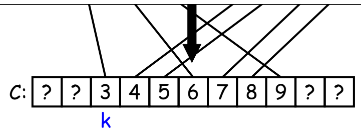
```
int L1) {
   int N = A.length - L0 + B.length - L1; int[]
C = new int[N];
   if (L0 >= A.length) arraycopy(B, L1, C, 0,
N);
   else if (L1 >= B.length) arraycopy(A, L0, C,
0, N);
   else if (A[L0] <= B[L1]) {
      C[0] = A[L0]; arraycopy(mergeTo(A, L0+1,
B, L1), 0, C, 1, N-1);
   } else {
      C[0] = B[L1]; arraycopy(mergeTo(A, L0, B,
L1+1), 0, C, 1, N-1);
   }
   return C;
}
```

What is wrong with this implementation?

```
   return mergeTo(A, 0, B, 0, new int[A.length+B.length],
0);
}

/** Merge A[L0..] and B[L1..] into C[K..], assuming
A and B sorted. */
static int[] mergeTo(int[] A, int L0, int[] B, int
L1, int[] C, int k){
   ...
}
```

This last method merges *part* of A with part of B into part of C. For example, consider a possible call mergeTo(A, 3, B, 1, C, 2)



```
   return mergeTo(A, 0, B, 0, new int[A.length+B.length],
0);
}

/** Merge A[L0..] and B[L1..] into C[K..], assuming
A and B sorted. */
static int[] mergeTo(int[] A, int L0, int[] B, int
L1, int[] C, int k){
   if (??) {
      return C;
   } else if (??) {
      C[k] = A[L0];
      return mergeTo(A, ??, B, ??, C, ??)
   } else {
      C[k] = B[L1];
      return mergeTo(A, ??, B, ??, C, ??)
   }
}
```

```
   return mergeTo(A, 0, B, 0, new int[A.length+B.length],
0);
}

/** Merge A[L0..] and B[L1..] into C[K..], assuming
A and B sorted. */
static int[] mergeTo(int[] A, int L0, int[] B, int
L1, int[] C, int k){
   if (L0 >= A.length && L1 >= B.length) {
      return C;
   } else if (??) {
      C[k] = A[L0];
      return mergeTo(A, ??, B, ??, C, ??)
   } else {
      C[k] = B[L1];
      return mergeTo(A, ??, B, ??, C, ??)
   }
}
```

```
   return mergeTo(A, 0, B, 0, new int[A.length+B.length],
0);
}

/** Merge A[L0..] and B[L1..] into C[K..], assuming
A and B sorted. */
static int[] mergeTo(int[] A, int L0, int[] B, int
L1, int[] C, int k){
   if (L0 >= A.length && L1 >= B.length) {
      return C;
   } else if (L1 >= B.length || (L0 < A.length &&
A[L0] <= B[L1])) {
      C[k] = A[L0];
      return mergeTo(A, ??, B, ??, C, ??)
   } else {
      C[k] = B[L1];
      return mergeTo(A, ??, B, ??, C, ??)
   }
}
```

```
   return mergeTo(A, 0, B, 0, new int[A.length+B.length],
0);
}

/** Merge A[L0..] and B[L1..] into C[K..], assuming
A and B sorted. */
static int[] mergeTo(int[] A, int L0, int[] B, int
L1, int[] C, int k){
   if (L0 >= A.length && L1 >= B.length) {
      return C;
   } else if (L1 >= B.length || (L0 < A.length &&
A[L0] <= B[L1])) {
      C[k] = A[L0];
      return mergeTo(A, L0 + 1, B, L1, C, k + 1);
   } else {
      C[k] = B[L1];
      return mergeTo(A, ??, B, ??, C, ??)
   }
}
```

```
   return mergeTo(A, 0, B, 0, new int[A.length+B.length],
0);
}

/** Merge A[L0..] and B[L1..] into C[K..], assuming
A and B sorted. */
static int[] mergeTo(int[] A, int L0, int[] B, int
L1, int[] C, int k){
   if (L0 >= A.length && L1 >= B.length) {
      return C;
   } else if (L1 >= B.length || (L0 < A.length &&
A[L0] <= B[L1])) {
      C[k] = A[L0];
      return mergeTo(A, L0 + 1, B, L1, C, k + 1);
   } else {
      C[k] = B[L1];
      return mergeTo(A, L0, B, L1 + 1, C, k + 1);
   }
}
```

approaches in languages like C and Java. Array manipulation is most often iterative:

```
public static int[] merge(int[] A, int[] B) {
   int[] C = new int[A.length + B.length];
   // mergeTo(A, 0, B, 0, C, 0)
   int L0, L1, k;
   L0 = L1 = k = 0;

   while (??) {
       if (L1 >= B.length || (L0 < A.length && A[L0]
<= B[L1])) {
           C[k] = A[L0];
           ??
       } else {
           C[k] = B[L1];
           ??
       }
   }
   return C;
}
```

often iterative:

```
public static int[] merge(int[] A, int[] B) {
   int[] C = new int[A.length + B.length];
   // mergeTo(A, 0, B, 0, C, 0)
   int L0, L1, k;
   L0 = L1 = k = 0;

   while (L0 < A.length || L1 < B.length) {
       if (L1 >= B.length || (L0 < A.length && A[L0]
<= B[L1])) {
           C[k] = A[L0];
           ??
       } else {
           C[k] = B[L1];
           ??
       }
   }
   return C;
}
```

often iterative:

```
public static int[] merge(int[] A, int[] B) {
   int[] C = new int[A.length + B.length];
   // mergeTo(A, 0, B, 0, C, 0)
   int L0, L1, k;
   L0 = L1 = k = 0;

   while (L0 < A.length || L1 < B.length) {
       if (L1 >= B.length || (L0 < A.length && A[L0]
<= B[L1])) {
           C[k] = A[L0];
           L0 += 1; k += 1;
       } else {
           C[k] = B[L1];
           L1 += 1; k += 1;
       }
   }
   return C;
}
```

```
public static int[] merge(int[] A, int[] B) {
   int[] C = new int[A.length + B.length];
   int L0, L1;
   L0 = L1 = 0;
   for (int k = 0; k < C.length; k += 1) {
       if (L1 >= B.length || (L0 < A.length && A[L0]
<= B[L1])) {
           C[k] = A[L0]; L0 += 1;
       } else {
           C[k] = B[L1]; L1 += 1;
       }
   }
   return C;
}
```

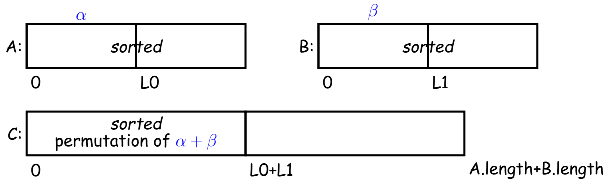
Invariant (true after `int k = 0`):

$0 \leq L0 < A.length \ \wedge \ 0 \leq L1 < B.length \wedge \ C.length = A.length + B.length \ \wedge \ k = L0 + L$

$\wedge \ C[0:k]$ is a permutation of A[0:L0] + B[0:L1]

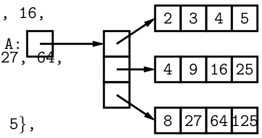
$\wedge \ C[0:k], A, B$ are sorted.

```
public static int[] merge(int[] A, int[] B) {
   int[] C = new int[A.length + B.length];
   int L0, L1;  L0 = L1 = 0;
   while (L0 + L1 < C.length) {
       if (L1 >= B.length || (L0 < A.length && A[L0]
< B[L1])) {
           C[L0 + L1] = A[L0]; L0 += 1;
       } else {
           C[L0 + L1] = B[L1]; L1 += 1;
       }
   }
   return C;
}
```

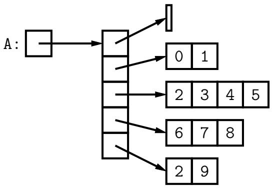


$$A = \begin{array}{|r|r|r|r|} \hline 2 & 3 & 4 & 5 \\ \hline 4 & 9 & 16 & 25 \\ \hline 8 & 27 & 64 & 125 \\ \hline \end{array} \quad ?$$

```
   A[2] = new int[] {8, 27, 64,
125};
 // or
   int[][] A;
   A = new int[][] { {2, 3, 4,
5},
                     {4, 9, 16,
25},
                     { 8, 27, 64,
125} };
 // or
   int[][] A = { {2, 3, 4, 5},
                 {4, 9, 16, 25},
                 {8, 27, 64, 125}
};
 // or
   int[][] A = new A[3][4];
   for (int i = 0; i < 3; i += 1)
       for (int j = 0; j < 4; j
+= 1)
           A[i][j] = (int)
Math.pow(j + 2, i + 1);
```



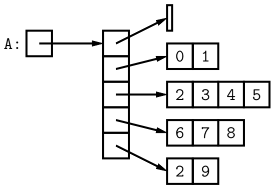
```
   int[][] A = new
int[5][];
   A[0] = new int[] {};
   A[1] = new int[] {0,
1};
   A[2] = new int[] {2,
3, 4, 5};
   A[3] = new int[] {6,
7, 8};
   A[4] = new int[] {9};
```



- What does this print?

```
   int[][] ZERO = new
int[3][];
   ZERO[0] = ZERO[1] =
ZERO[2] =
       new int[] {0, 0,
0};
   ZERO[0][1] = 1;
   System.out.println(ZERO[2][1]);
```

```
    int[][] A = new
int[5][];
    A[0] = new int[] {};
    A[1] = new int[] {0,
1};
    A[2] = new int[] {2,
3, 4, 5};
    A[3] = new int[] {6,
7, 8};
    A[4] = new int[] {9};
```



- What does this print?

```
    int[][] ZERO = new
int[3][];
    ZERO[0] = ZERO[1] =
ZERO[2] =
         new int[] {0, 0,
0};
    ZERO[0][1] = 1;
    System.out.println(ZERO[2][1]);
```