CS61B Lecture #22

Today: Backtracking searches, game trees (*DSIJ*, Section 6.5

Searching by "Generate and Test"

- We've been considering the problem of searching a set of dat in some kind of data structure: "Is $x \in S$?"
- But suppose we *don't* have a set S, but know how to recogni we're after if we find it: "Is there an x such that $P(x)$?"
- If we know how to enumerate all possible candidates, can proach of *Generate and Test:* test all possibilities in turn.
- Can sometimes be more clever: avoid trying things that wor for example.
- What happens if the set of possible candidates is infinite?

Backtracking Search

- Backtracking search is one way to enumerate all possibilities
- Example: *Knight's Tour.* Find all paths a knight can travel on a board such that it touches every square exactly once and one knight move from where it started.
- In the example below, the numbers indicate position numbers starts at 0).
- Here, knight (N) is stuck; how to handle this?

6							
		5					
4	7						
	10		2				
8	3	0					
N		9		1			

General Recursive Algorithm

```
/** Append to PATH a sequence of knight moves starting at ROW,
 *  that avoids all squares that have been hit already and
 *  that ends up one square away from ENDROW, ENDCOL. B[i][j] i
 *  true iff row i and column j have been hit on PATH so far.
 *  Returns true if it succeeds, else false (with no change to
 *  Call initially with PATH containing the starting square, an
 *  the starting square (only) marked in B. */

boolean findPath(boolean[][] b, int row, int col,
                 int endRow, int endCol, List path) {
  if (path.size() == 64)   return isKnightMove(row, col, endRo
  for (r, c = all possible moves from (row, col)) {
    if (!b[r][c]) {
      b[r][c] = true; // Mark the square
      path.add(new Move(r, c));
      if (findPath(b, r, c, endRow, endCol, path))  return true
      b[r][c] = false; // Backtrack out of the move.
      path.remove(path.size()-1);
    }
  }
  return false;
}
```

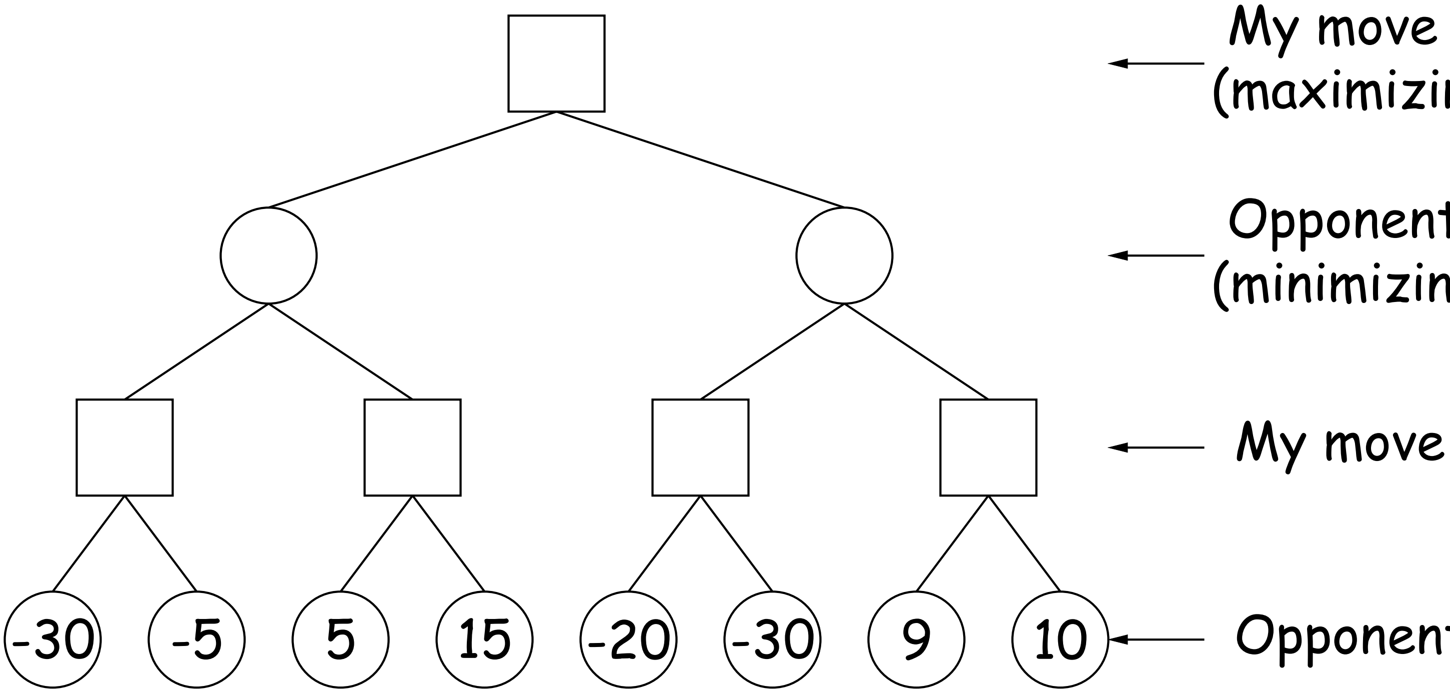
Another Kind of Search: Best Move

- Consider the problem of finding the *best* move in a two-perso
- One way: assign a *heuristic value* to each possible move c highest (aka *static evaluation*). Examples:
  - number of black pieces − number of white pieces in chec
  - weighted sum of white piece values − weighted sum c pieces in chess (Queen=9, Rook=5, etc.)
  - Nearness of pieces to strategic areas (center of board).
- But this is misleading. A move might give us more pieces, bu a devastating response from the opponent.
- So, for each move, look at *opponent's* possible moves, ass picks the best one for him, and use that as the value.
- But what if *you* have a great response to his response?
- How do we organize this sensibly?

Game Trees

- Think of the space of possible continuations of the game as

- Each node is a position, each edge a move.



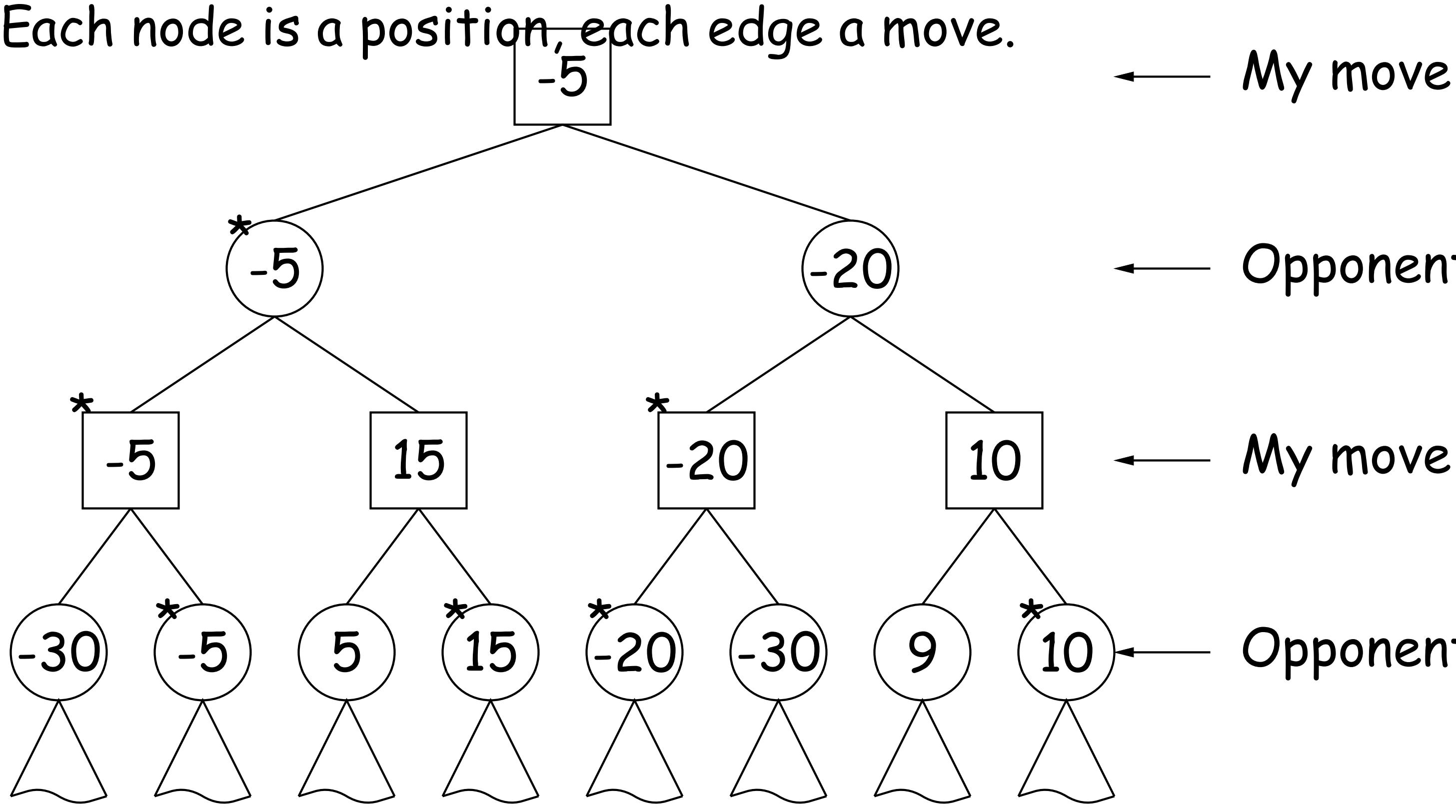
- Suppose numbers at the bottom are the values of those fi
tions *to me*. Smaller numbers are of more value to *my oppon*

- What should I move? What value can I get if my opponent
well as possible?

Game Trees, Minimax

- Think of the space of possible continuations of the game as

- Each node is a position, each edge a move.

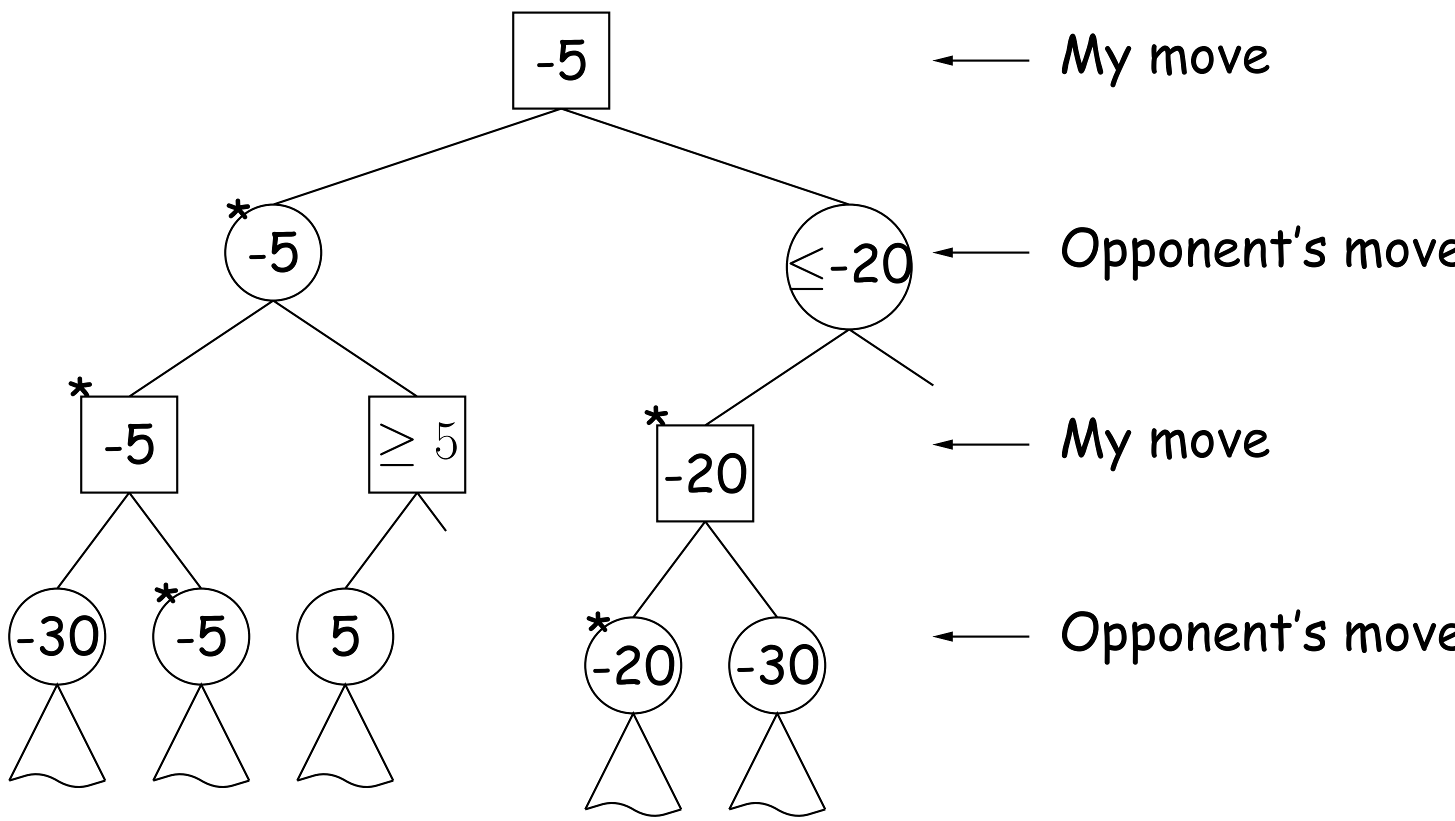


- Numbers are the values we guess for the positions (larger better for me). Starred nodes would be chosen.

- I always choose child (next position) with maximum value; o chooses minimum value ("Minimax algorithm")

Alpha-Beta Pruning

- We can *prune* this tree as we search it.



- At the '≥ 5' position, I know that the opponent will not ch
move here (since he already has a -5 move).
- At the '≤ -20' position, my opponent knows that I will never
to move here (since I already have a -5 move).

Cutting off the Search

- If you could traverse game tree to the bottom, you'd be
  force a win (if it's possible).
- Sometimes possible near the end of a game.
- Unfortunately, game trees tend to be either infinite or im
  large.
- So, we choose a maximum *depth,* and use a heuristic value co
  on the position alone (called a *static valuation*) as the value
  depth.
- Or we might use *iterative deepening*, repeating the searc
  creasing depths until time is up.
- Much more sophisticated searches are possible, however (tak

Overall Search Algorithm

- Depending on whose move it is (maximizing player or minimizin
we'll search for a move estimated to be optimal in one dire
the other.
- Search will be exhaustive down to a particular depth in th
tree; below that, we guess values.
- Also pass α and β limits:
  - High player does not care about exploring a position furth
he knows its value is larger than what the minimizing playe
he can get (β), because the minimizing player will never al
position to come about.
  - Likewise, minimizing player won't explore a positions whos
is less than what the maximizing player knows he can get
- To start, a maximizing player will find a move with

  findMax(*current position*, *search depth* $-\infty$, $+\infty$)

- minimizing player:

  findMin(*current position*, *search depth* $-\infty$, $+\infty$)

Some Pseudocode for Searching (One Level)

- The most basic kind of game-tree search is to assign some h
  value to any given position, looking at just the next possible

```
Move simpleFindMax(Position posn, double alpha, double
    if (posn.maxPlayerWon())
        return artificial "Move" with value +∞;
    else if (posn.minPlayerWon())
        return artificial "Move" with value −∞;
    Move bestSoFar = artificial "Move" with value −∞;
    for (each M = a legal move for maximizing player from pos
         Position next = posn.makeMove(M);
         next.setValue(heuristicEstimate(next));
         if (next.value() >= bestSoFar.value()) {
              bestSoFar = next;
              alpha = max(alpha, next.value());
              if (beta <= alpha) break;
         }
    }
    return bestSoFar;
}
```

One-Level Search for Minimizing Player

```
Move simpleFindMin(Position posn, double alpha, double
    if (posn.maxPlayerWon())
        return artificial "Move" with value +∞;
    else if (posn.minPlayerWon())
        return artificial "Move" with value −∞;
    Move bestSoFar = artificial "Move" with value +∞;
    for (each M = a legal move for minimizing player from posn
         Position next = posn.makeMove(M);
         next.setValue(heuristicEstimate(next));
         if (next.value() <= bestSoFar.value()) {
              bestSoFar = next;
              beta = min(beta, next.value());
              if (beta <= alpha) break;
         }
    }
    return bestSoFar;
}
```

Some Pseudocode for Searching (Maximizing Pla

```
/** Return a best move for maximizing player from POSN
 *  to depth DEPTH.  Any move with value >= BETA is al
 *  "good enough". */
Move findMax(Position posn, int depth, double alpha, d
    if (depth == 0 || gameOver(posn))
        return simpleFindMax(posn, alpha, beta);
    Move bestSoFar = artificial "Move" with value −∞;
    for (each M = a legal move for maximizing player from pos
         Position next = posn.makeMove(M);
         Move response = findMin(next, depth-1, alpha,
         if (response.value() >= bestSoFar.value()) {
              bestSoFar = next;
              next.setValue(response.value());
              alpha = max(alpha, response.value());
              if (beta <= alpha) break;
         }
    }
    return bestSoFar;
}
```

Some Pseudocode for Searching (Minimizing Play

```
/** Return a best move for minimizing player from POSN
 *  to depth DEPTH.  Any move with value <= ALPHA is al
 *  "good enough". */
Move findMin(Position posn, int depth, double alpha, d
    if (depth == 0 || gameOver(posn))
        return simpleFindMin(posn, alpha, beta);
    Move bestSoFar = artificial "Move" with value +∞;
    for (each M = a legal move for minimizing player from posn
         Position next = posn.makeMove(M);
         Move response = findMax(next, depth-1, alpha,
         if (response.value() <= bestSoFar.value()) {
             bestSoFar = next;
             next.setValue(response.value());
             beta = min(beta, response.value());
             if (beta <= alpha) break;
         }
    }
    return bestSoFar;
}
```