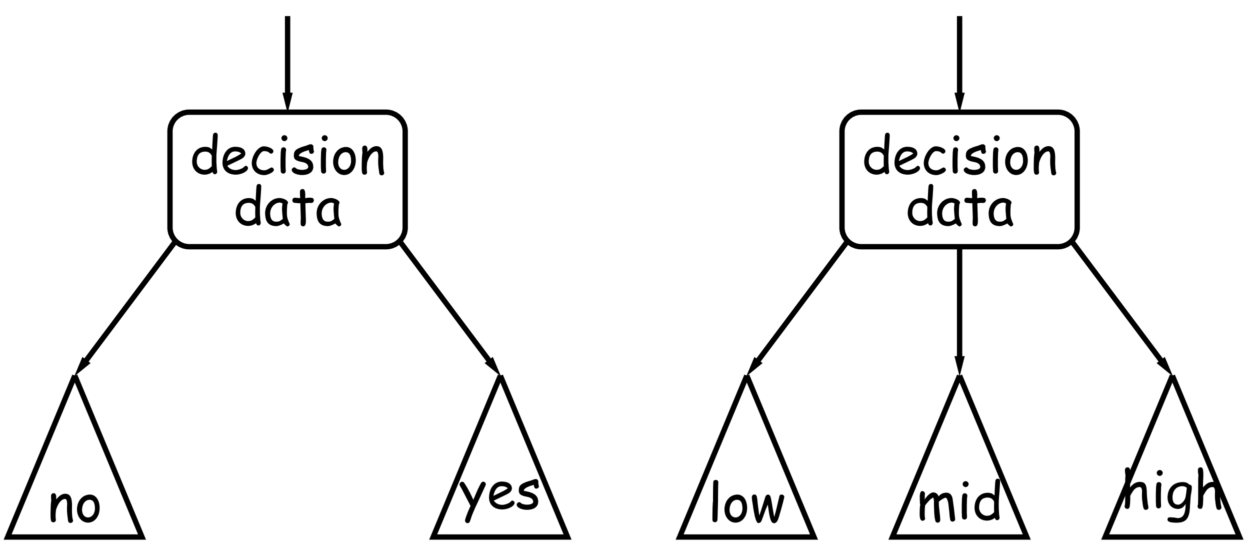
CS61B Lecture #21: Tree Searching

Divide and Conquer

- Much (most?) computation is devoted to finding things in r
  to various forms of query.
- Linear search for response can be expensive, especially wh
  set is too large for primary memory.
- Preferable to have criteria for *dividing* data to be search
  pieces recursively
- We saw the figure for $\lg N$ algorithms: at 1 μsec per com
  could process 10^{300000} items in 1 sec.
- Tree is a natural framework for the representation:



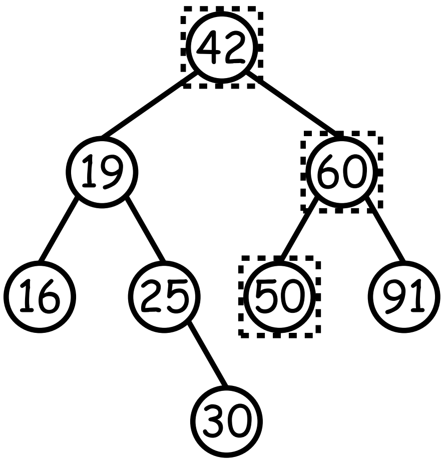
Binary Search Trees

Binary Search Property:

- Tree nodes contain *keys,* and possibly other data.
- All nodes in left subtree of node have *smaller* keys.
- All nodes in right subtree of node have *larger* keys.
- "Smaller" means any complete transitive, anti-symmetric ord keys:
  - exactly one of $x \prec y$ and $y \prec x$ true.
  - $x \prec y$ and $y \prec z$ imply $x \prec z$.
  - (To simplify, won't allow duplicate keys this semester).
- E.g., in dictionary database, node label would be (*word, defi* *word* is the key.
- For concreteness here, we'll just use the standard Java cor of calling `.compareTo`.

Finding

- Searching for 50 and 49:

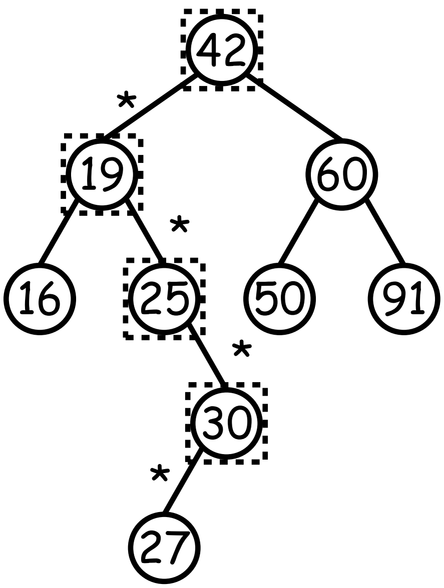



```
/** Node in T containing L, or null i
static BST find(BST T, Key L) {
  if (T == null)
    return T;
  if (L.compareTo(T.label()) == 0)
    return T;
  else if (L.compareTo(T.label()) < 0
    return find(T.left(), L);
  else
    return find(T.right(), L);
}
```

- Dashed boxes show which node labels we look at.
- Number looked at proportional to height of tree.

Inserting

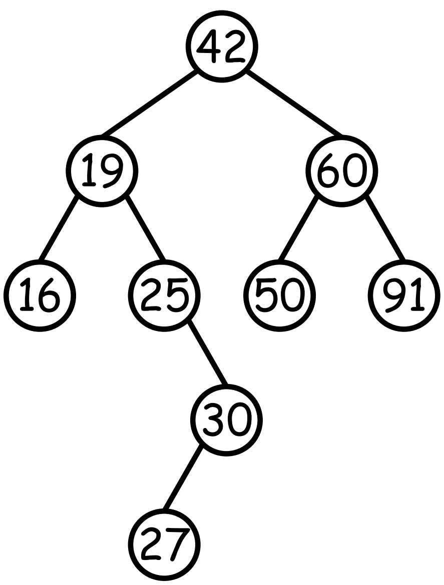
- Inserting 27



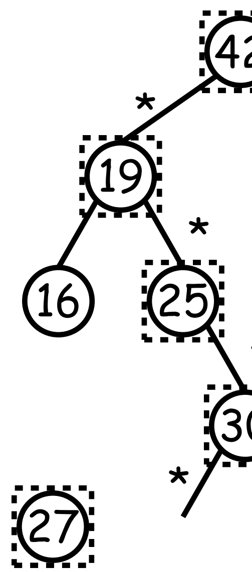

```
/** Insert L in T, replacing existing
 *  value if present, and returning
 *  new tree. */
static BST insert(BST T, Key L) {
  if (T == null)
    return new BST(L);
  if (L.compareTo(T.label()) == 0)
    T.setLabel(L);
  else if (L.compareTo(T.label()) < 0)
    T.setLeft(insert(T.left(), L));
  else
    T.setRight(insert(T.right(), L));
  return T;
}
```

- Starred edges are set (to themselves, unless initially null).
- Again, time proportional to height.

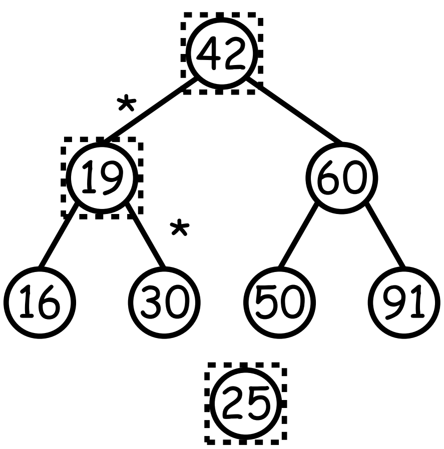
Deletion



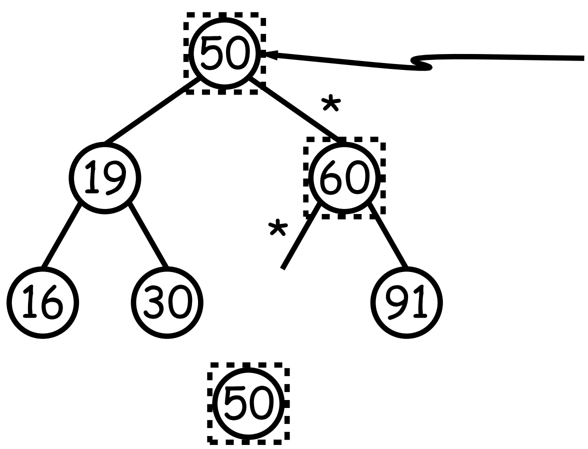
Initial



Remov



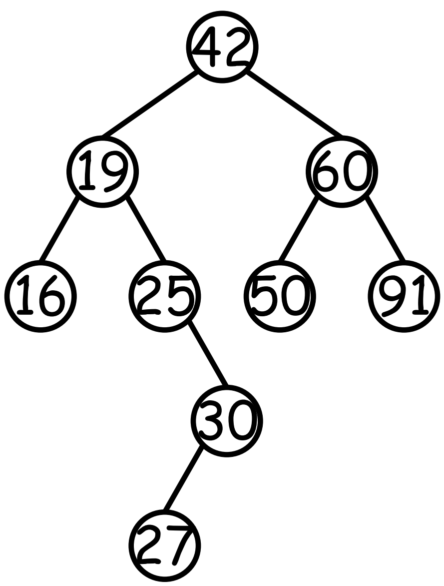
Remove 25



formerly contained

Remove 42

Deletion Algorithm

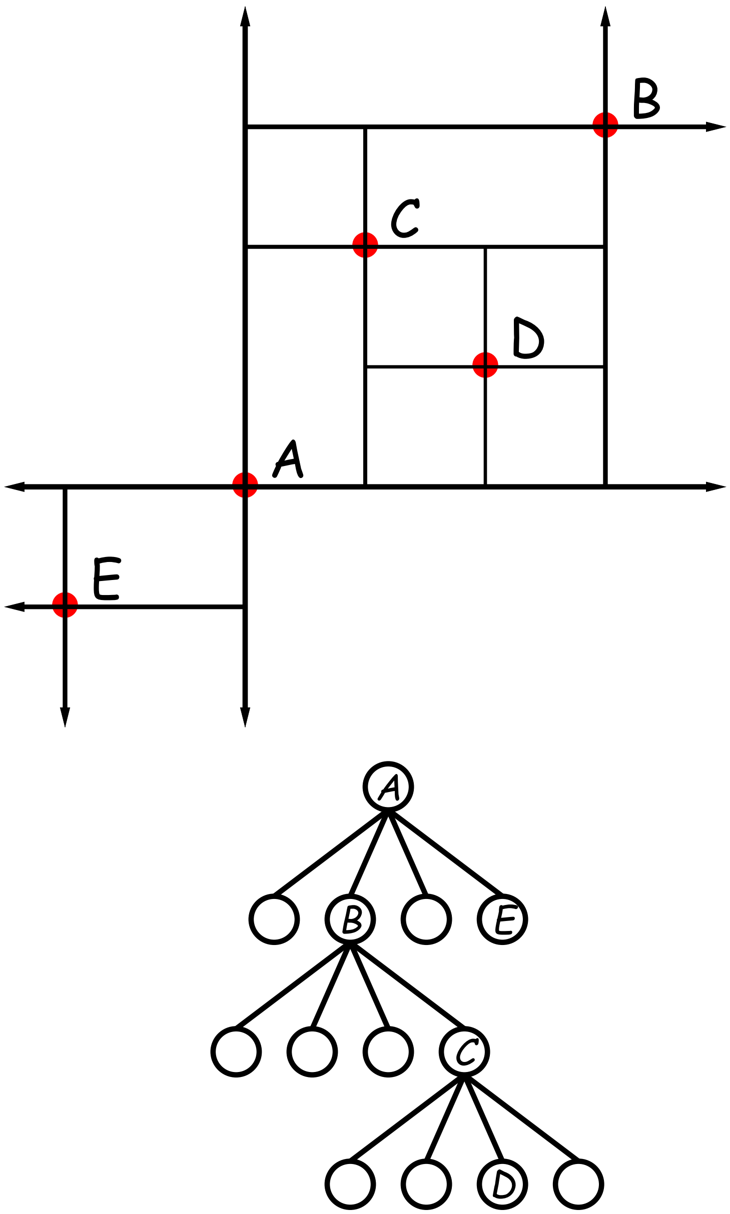


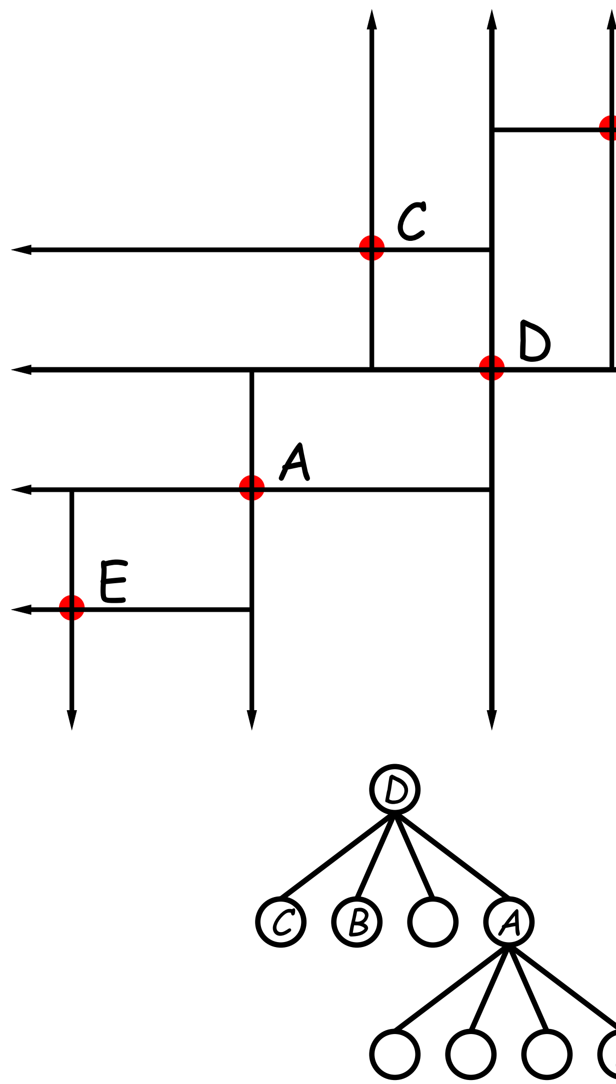

```
/** Remove L from T, returning new tree.
static BST remove(BST T, Key L) {
  if (T == null)
    return null;
  if (L.compareTo(T.label()) == 0) {
     if (T.left() == null)
         return T.right();
     else if (T.right() == null)
         return T.left();
     else {
         Key smallest = minVal(T.right()
         T.setRight(remove(T.right(), sma
         T.setLabel(smallest);
     }
  }
  else if (L.compareTo(T.label()) < 0)
    T.setLeft(remove(T.left(), L));
  else
    T.setRight(remove(T.right(), L));
  return T;
}
```

More Than Two Choices: Quadtrees

- Want to *index* information about 2D locations so that items
  retrieved by position.
- Quadtrees do so using standard data-structuring trick: *Di*
  Conquer.
- Idea: divide (2D) space into four *quadrants,* and store item
  appropriate quadrant. Repeat this recursively with each q
  that contains more than one item.
- Original definition: a quadtree is either
  - Empty, or
  - An item at some position (x, y), called the root, plus
  - four quadtrees, each containing only items that are nor
    northeast, southwest, and southeast of (x, y).
- Big idea is that if you are looking for point (x', y') and the roo
  the point you are looking for, you can narrow down which of t
  subtrees of the root to look in by comparing coordinates $(x$
  (x', y').

Classical Quadtree: Example

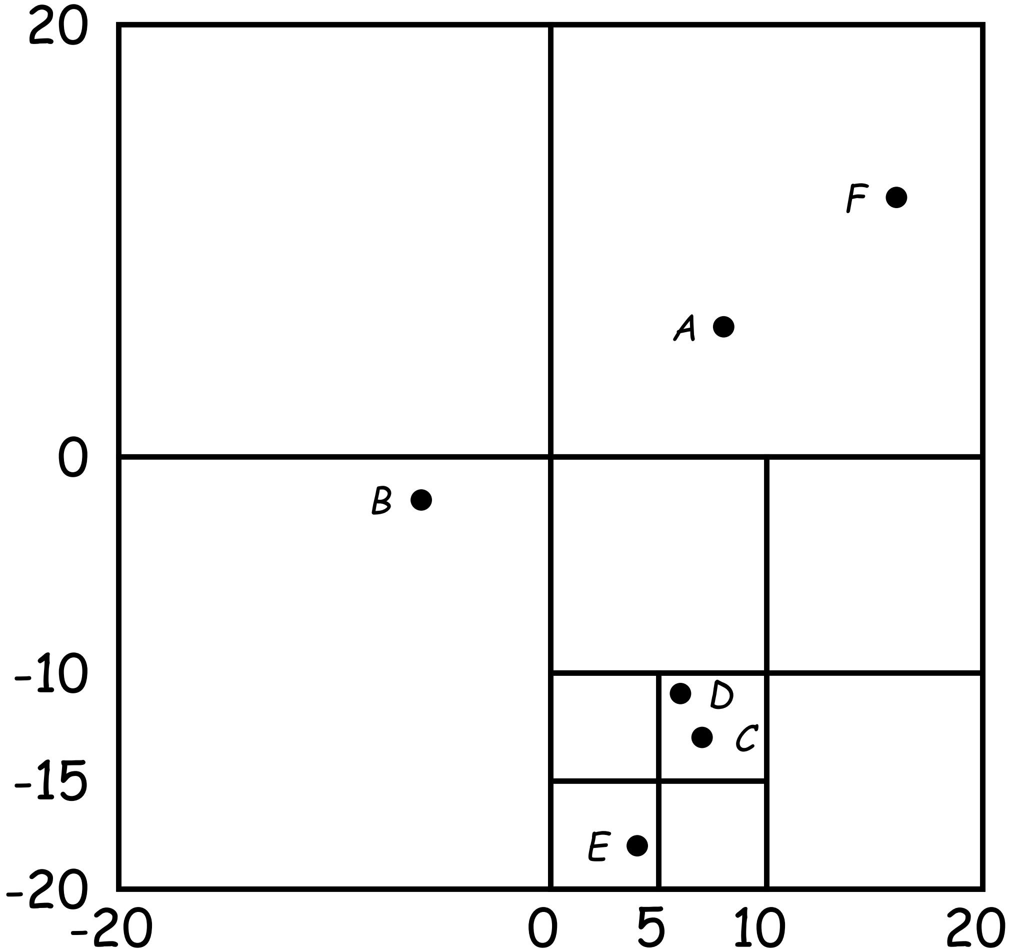




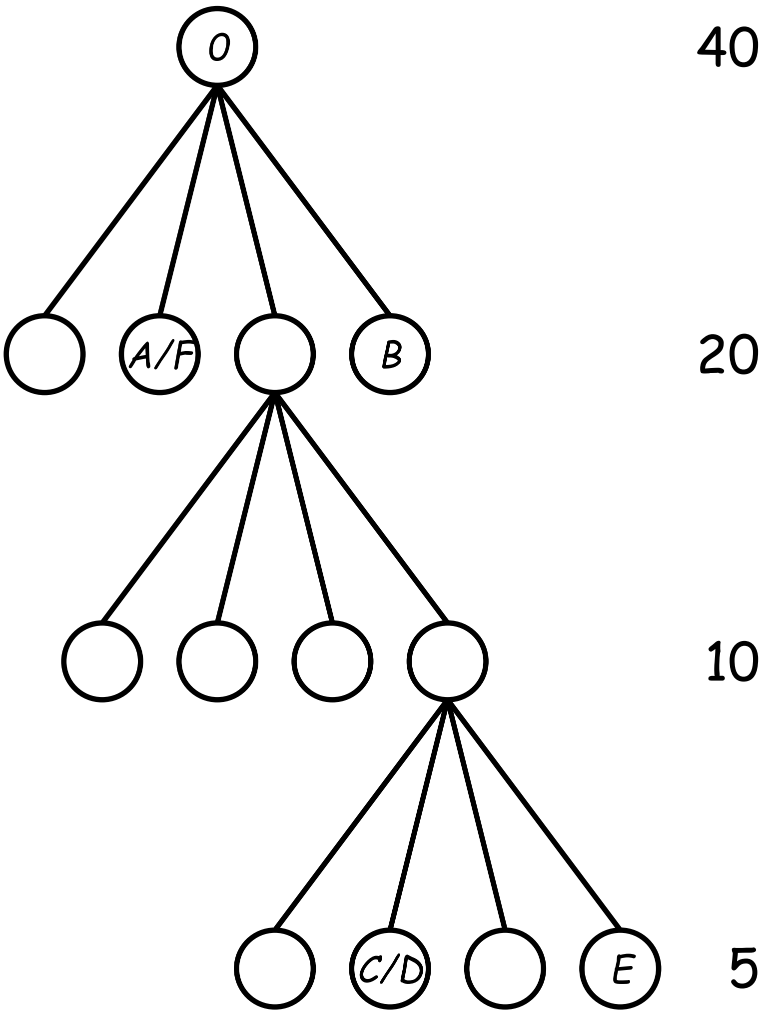
Point-region (PR) Quadtrees

- If we use a Quadtree to track moving objects, it may be u
be able to *delete* items from a tree: when an object mov
subtree that it goes in may change.
- Difficult to do with the classical data structure above, so w
fine instead:
- A quadtree consists of a bounding rectangle, B and either
  - Zero up to a small number of items that lie in that rectan
  - Four quadtrees whose bounding rectangles are the four qu
of B (all of equal size).
- A completely empty quadtree can have an arbitrary boundin
angle, or you can wait for the first point to be inserted.

Example of PR Quadtree



(≤ 2 points per leaf)


0
A/F
B
C/D
E
40
20
10
5

Navigating PR Quadtrees

- To find an item at (x, y) in quadtree T,
  1. If (x, y) is outside the bounding rectangle of T, or T is
     then (x, y) is not in T.
  2. Otherwise, if T contains a small set of items, then $(x, y$
     iff it is among these items.
  3. Otherwise, T consists of four quadtrees. Recursively
     (x, y) in each (however, step #1 above will cause all but
     these bounding boxes to reject the point immediately).
- Similar procedure works when looking for all items within sor
  angle, R:
  1. If R does not intersect the bounding rectangle of T,
     empty, then there are no items in R.
  2. Otherwise, if T contains a set of items, return those tha
     R, if any.
  3. Otherwise, T consists of four quadtrees. Recursively
     points in R in each one of them.

Insertion into PR Quadtrees

Various cases for inserting a new point N, assuming maximum oc
of a region is 2, showing initial state $\Longrightarrow$ final state.