

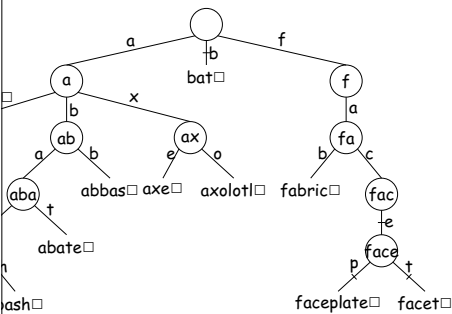
Efficient Use of Keys: the Trie

much about cost of comparisons.  
worst case is length of string.  
ould throw extra factor of key length,  $L$ , into costs:  
comparisons really means  $\Theta(ML)$  operations.  
for key  $X$ , keep looking at same chars of  $X$   $M$  times.  
better? Can we get search cost to be  $O(L)$ ?

multi-way decision tree, with one decision per character

Adding Item to a Trie

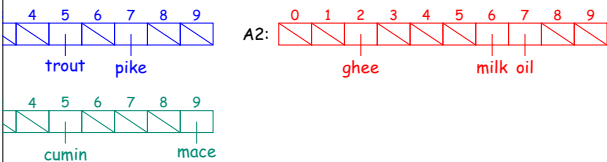
adding bat and faceplate.  
added.



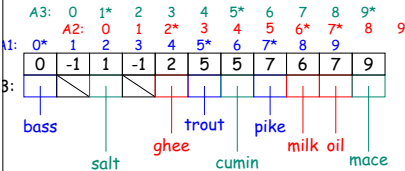
Scrunching Example

(unrelated to Tries on preceding slides)

arrays, each indexed 0..9



them, but keep track of original index of each item:



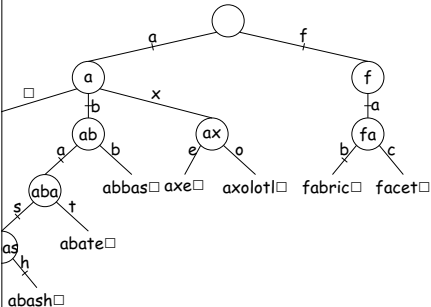
CS61B Lecture #31

ed search structures (DS(IJ), Chapter 9

om Numbers (DS(IJ), Chapter 11)

The Trie: Example

e, abash, abate, abbas, axotl, axe, fabric, facet}  
show paths followed for "abash" and "fabric"  
l node corresponds to a possible prefix.  
n path to node = that prefix.



A Side-Trip: Scrunching

obvious implementation for internal nodes is array in-  
character.

performance,  $L$  length of search key.

independent of  $N$ , number of keys. Is there a depen-

arrays are *sparsely populated* by non-null values—waste of

arrays on top of each other!

empty) entries of one array to hold non-null elements of

markers to tell which entries belong to which array.

Probabilistic Balancing: Skip Lists

can be thought of as a kind of n-ary search tree in which we put the keys at "random" heights.  
thought of as an ordered list in which one can skip large

Example:

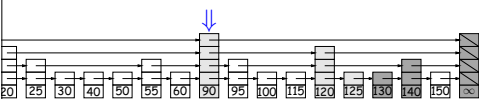


start at top layer on left, search until next step would then go down one layer and repeat.  
we search for 125 and 127. Gray nodes are looked at; nodes are overshoots.  
the nodes were chosen randomly so that there are about nodes that are  $> k$  high as there are that are  $k$  high.  
is fast with high probability.

Probabilistic Balancing: Skip Lists

can be thought of as a kind of n-ary search tree in which we put the keys at "random" heights.  
thought of as an ordered list in which one can skip large

Example:

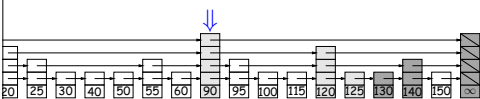


start at top layer on left, search until next step would then go down one layer and repeat.  
we search for 125 and 127. Gray nodes are looked at; nodes are overshoots.  
the nodes were chosen randomly so that there are about nodes that are  $> k$  high as there are that are  $k$  high.  
is fast with high probability.

Probabilistic Balancing: Skip Lists

can be thought of as a kind of n-ary search tree in which we put the keys at "random" heights.  
thought of as an ordered list in which one can skip large

Example:



start at top layer on left, search until next step would then go down one layer and repeat.  
we search for 125 and 127. Gray nodes are looked at; nodes are overshoots.  
the nodes were chosen randomly so that there are about nodes that are  $> k$  high as there are that are  $k$  high.  
is fast with high probability.

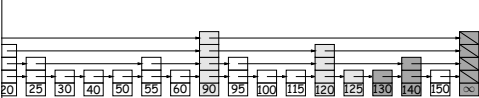
Practicum

ing idea is cute, but  
od if we want to expand our trie.  
plicated.  
more useful for representing large, sparse, fixed tables  
rows and columns.  
e, number of children in trie tends to drop drastically  
ts a few levels down from the root.  
ce, might as well use linked lists to represent set of  
en...  
rays for the first few levels, which are likely to have  
n.

Probabilistic Balancing: Skip Lists

can be thought of as a kind of n-ary search tree in which we put the keys at "random" heights.  
thought of as an ordered list in which one can skip large

Example:

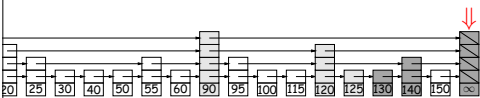


start at top layer on left, search until next step would then go down one layer and repeat.  
we search for 125 and 127. Gray nodes are looked at; nodes are overshoots.  
the nodes were chosen randomly so that there are about nodes that are  $> k$  high as there are that are  $k$  high.  
is fast with high probability.

Probabilistic Balancing: Skip Lists

can be thought of as a kind of n-ary search tree in which we put the keys at "random" heights.  
thought of as an ordered list in which one can skip large

Example:

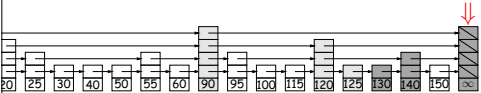


start at top layer on left, search until next step would then go down one layer and repeat.  
we search for 125 and 127. Gray nodes are looked at; nodes are overshoots.  
the nodes were chosen randomly so that there are about nodes that are  $> k$  high as there are that are  $k$  high.  
is fast with high probability.

robabilistic Balancing: Skip Lists

an be thought of as a kind of n-ary search tree in which  
put the keys at "random" heights.  
thought of as an ordered list in which one can skip large

ple:

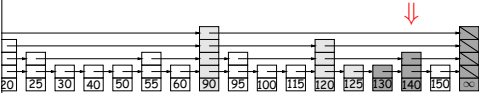


tart at top layer on left, search until next step would  
then go down one layer and repeat.  
, we search for 125 and 127. Gray nodes are looked at;  
nodes are overshoots.  
he nodes were chosen randomly so that there are about  
nodes that are  $> k$  high as there are that are  $k$  high.  
hes fast *with high probability*.

robabilistic Balancing: Skip Lists

an be thought of as a kind of n-ary search tree in which  
put the keys at "random" heights.  
thought of as an ordered list in which one can skip large

ple:

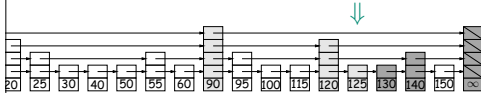


tart at top layer on left, search until next step would  
then go down one layer and repeat.  
, we search for 125 and 127. Gray nodes are looked at;  
nodes are overshoots.  
he nodes were chosen randomly so that there are about  
nodes that are  $> k$  high as there are that are  $k$  high.  
hes fast *with high probability*.

robabilistic Balancing: Skip Lists

an be thought of as a kind of n-ary search tree in which  
put the keys at "random" heights.  
thought of as an ordered list in which one can skip large

ple:

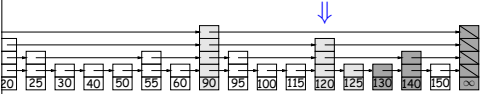


tart at top layer on left, search until next step would  
then go down one layer and repeat.  
, we search for 125 and 127. Gray nodes are looked at;  
nodes are overshoots.  
he nodes were chosen randomly so that there are about  
nodes that are  $> k$  high as there are that are  $k$  high.  
hes fast *with high probability*.

robabilistic Balancing: Skip Lists

an be thought of as a kind of n-ary search tree in which  
put the keys at "random" heights.  
thought of as an ordered list in which one can skip large

ple:

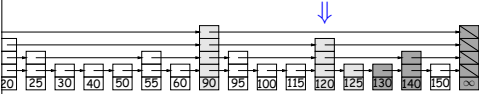


tart at top layer on left, search until next step would  
then go down one layer and repeat.  
, we search for 125 and 127. Gray nodes are looked at;  
nodes are overshoots.  
he nodes were chosen randomly so that there are about  
nodes that are  $> k$  high as there are that are  $k$  high.  
hes fast *with high probability*.

robabilistic Balancing: Skip Lists

an be thought of as a kind of n-ary search tree in which  
put the keys at "random" heights.  
thought of as an ordered list in which one can skip large

ple:

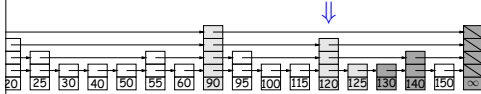


tart at top layer on left, search until next step would  
then go down one layer and repeat.  
, we search for 125 and 127. Gray nodes are looked at;  
nodes are overshoots.  
he nodes were chosen randomly so that there are about  
nodes that are  $> k$  high as there are that are  $k$  high.  
hes fast *with high probability*.

robabilistic Balancing: Skip Lists

an be thought of as a kind of n-ary search tree in which  
put the keys at "random" heights.  
thought of as an ordered list in which one can skip large

ple:



tart at top layer on left, search until next step would  
then go down one layer and repeat.  
, we search for 125 and 127. Gray nodes are looked at;  
nodes are overshoots.  
he nodes were chosen randomly so that there are about  
nodes that are  $> k$  high as there are that are  $k$  high.  
hes fast *with high probability*.

## Summary

Search trees allows us to realize  $\Theta(\lg N)$  performance.

Red-black trees:

$\Theta(\lg N)$  performance for searches, insertions, deletions.

Good for external storage. Large nodes minimize # of rotations

$\Theta(\lg N)$  performance for searches, insertions, and deletions, independent of length of key being processed.

Good to manage space efficiently.

*Idea:* scrunched arrays share space.

Red-black trees achieve  $\Theta(\lg N)$  performance for searches, insertions, deletions.

Implementation.

Look for *interesting ideas*: probabilistic balance, random structures.

9-39 2018

CS61B: Lecture #31 20

## Structures that Implement Abstractions

Linked lists, circular buffers

Set

Queue: heaps

Set: binary search trees, red-black trees, B-trees, arrays or linked lists

Ordered Set: hash table

Map: hash table

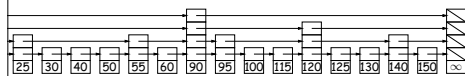
Ordered Map: red-black trees, B-trees, sorted arrays or linked lists

9-39 2018

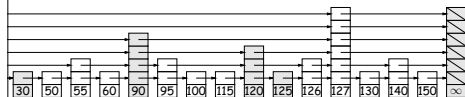
CS61B: Lecture #31 22

## Example: Adding and deleting

Initial list:



Now, we add 126 and 127 (choosing random heights for insertion) and remove 20 and 40:

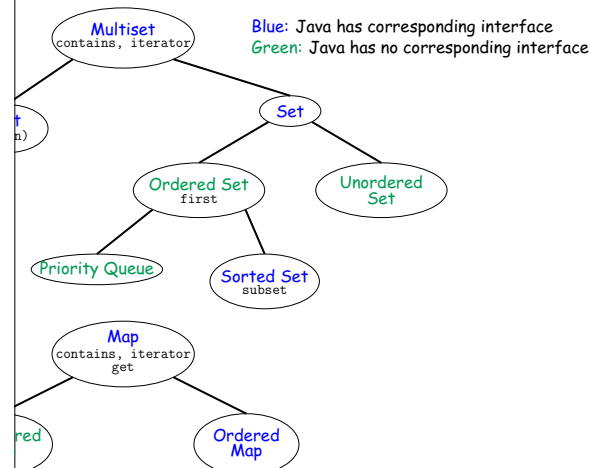


These nodes here have been modified.

9-39 2018

CS61B: Lecture #31 19

## Summary of Collection Abstractions



9-39 2018

CS61B: Lecture #31 21

## Corresponding Classes in Java

Collection

ArrayList, LinkedList, Stack, ArrayBlockingQueue,

Set

Queue: PriorityQueue

Set (SortedSet): TreeSet

Ordered Set: HashSet

Map: HashMap

Ordered Map (SortedMap): TreeMap

9-39 2018

CS61B: Lecture #31 23