

# CS61B Lecture #31

## Today:

- More balanced search structures (*DS(IJ)*, Chapter 9)

## Coming Up:

- Pseudo-random Numbers (*DS(IJ)*, Chapter 11)

# Really Efficient Use of Keys: the Trie

- Haven't said much about cost of comparisons.
- For strings, worst case is length of string.
- Therefore should throw extra factor of key length,  $L$ , into costs:
  - $\Theta(M)$  comparisons really means  $\Theta(ML)$  operations.
  - So to look for key  $X$ , keep looking at same chars of  $X$   $M$  times.
- Can we do better? Can we get search cost to be  $O(L)$ ?

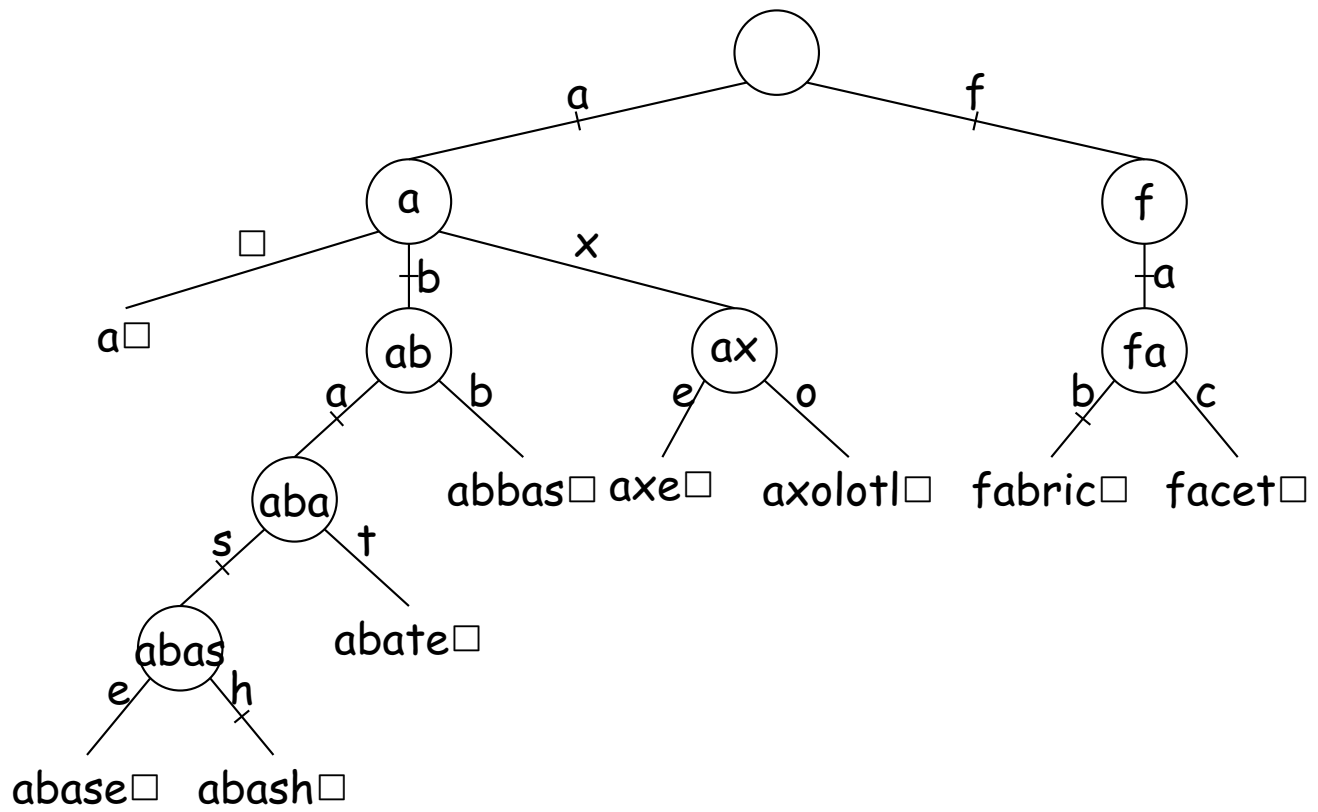
**Idea:** Make a *multi-way decision tree*, with one decision per character of key.

# The Trie: Example

- Set of keys

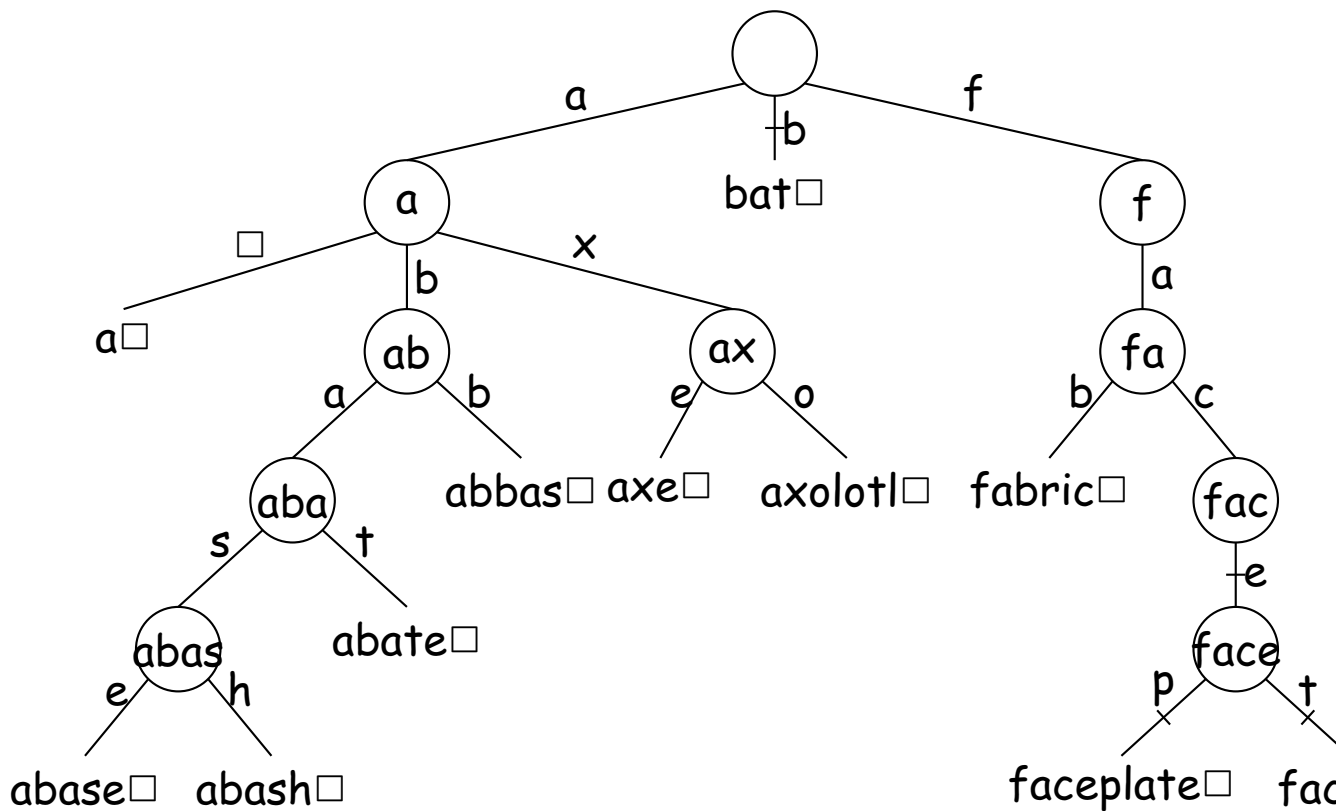
`{a, abase, abash, abate, abbas, axolotl,  
axe, fabric, facet}`

- Ticked lines show paths followed for "abash" and "fabric"
- Each internal node corresponds to a possible prefix.
- Characters in path to node = that prefix.



# Adding Item to a Trie

- Result of adding bat and faceplate.
- New edges ticked.



## A Side-Trip: Scrunching

- For speed, obvious implementation for internal nodes is array indexed by character.
- Gives  $O(L)$  performance,  $L$  length of search key.
- [Looks as if independent of  $N$ , number of keys. Is there a dependence?]
- **Problem:** arrays are *sparsely populated* by non-null values—waste of space.

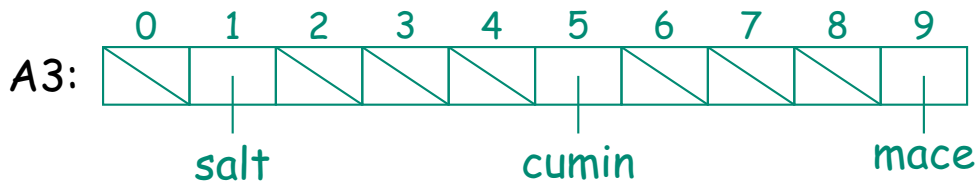
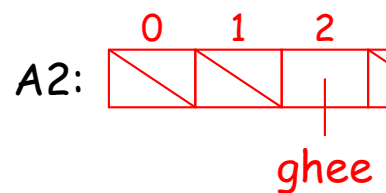
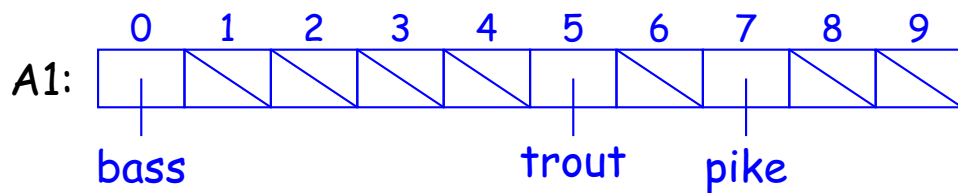
**Idea:** Put the arrays on top of each other!

- Use null (0, empty) entries of one array to hold non-null elements of another.
- Use extra markers to tell which entries belong to which array.

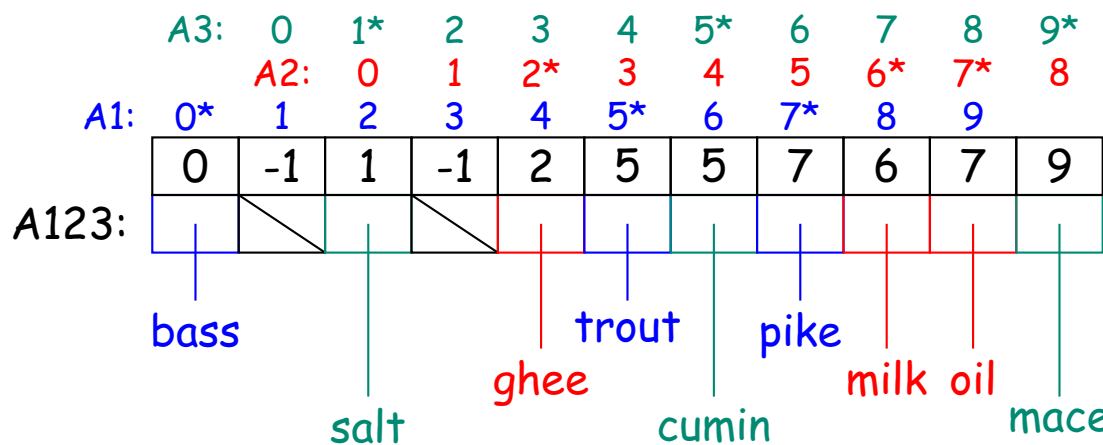
## Scrunching Example

**Small example:** (unrelated to Tries on preceding slides)

- Three leaf arrays, each indexed 0..9



- Now overlay them, but keep track of original index of each item:



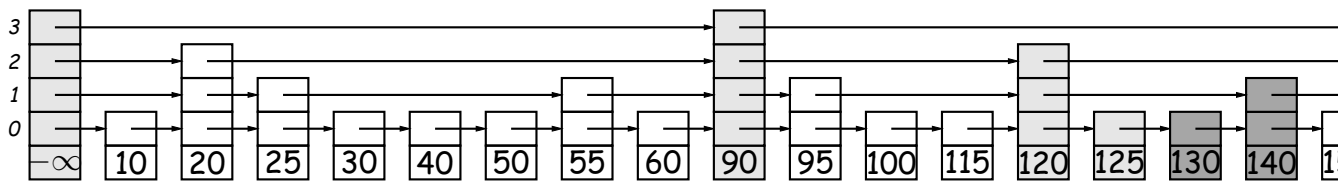
# Practicum

- The scrunching idea is cute, but
  - Not so good if we want to expand our trie.
  - A bit complicated.
  - Actually more useful for representing large, sparse, fixed tables with many rows and columns.
- Furthermore, number of children in trie tends to drop drastically when one gets a few levels down from the root.
- So in practice, might as well use linked lists to represent set of node's children...
- ...but use arrays for the first few levels, which are likely to have more children.



# Probabilistic Balancing: Skip Lists

- A *skip list* can be thought of as a kind of n-ary search tree in which we choose to put the keys at “random” heights.
- More often thought of as an ordered list in which one can skip large segments.
- Typical example:



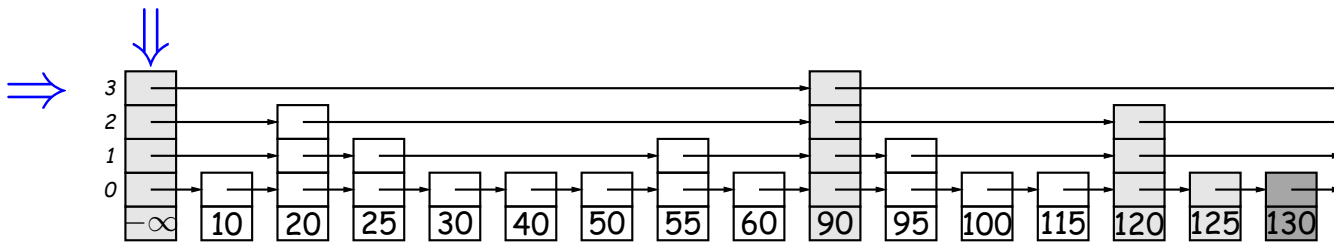
- To search, start at top layer on left, search until next step would overshoot, then go down one layer and repeat.
- In list above, we search for 125 and 127. Gray nodes are looked at; darker gray nodes are overshoots.
- Heights of the nodes were chosen randomly so that there are about 1/2 as many nodes

that are  $> k$  high as there are that are  $k$  high.

- Makes searches fast *with high probability*.

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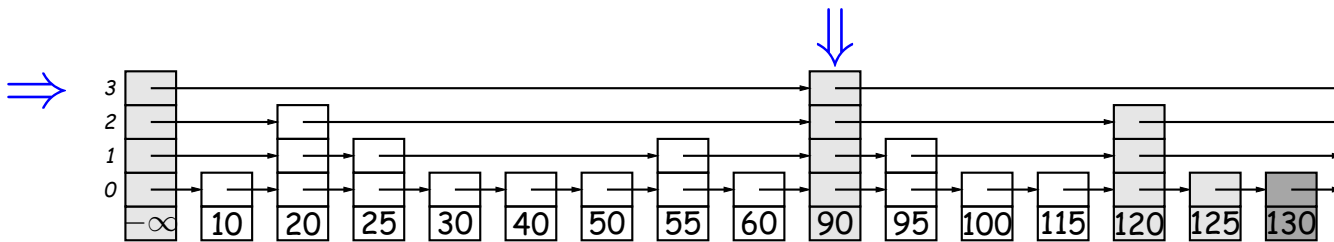
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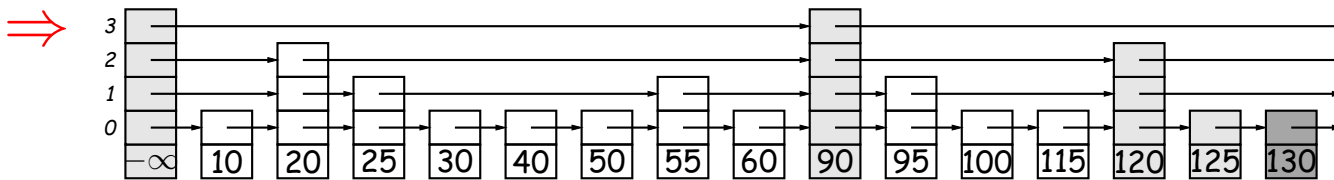
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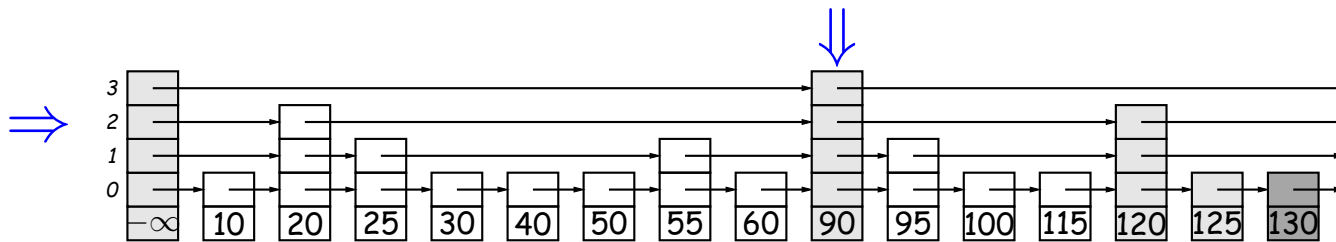
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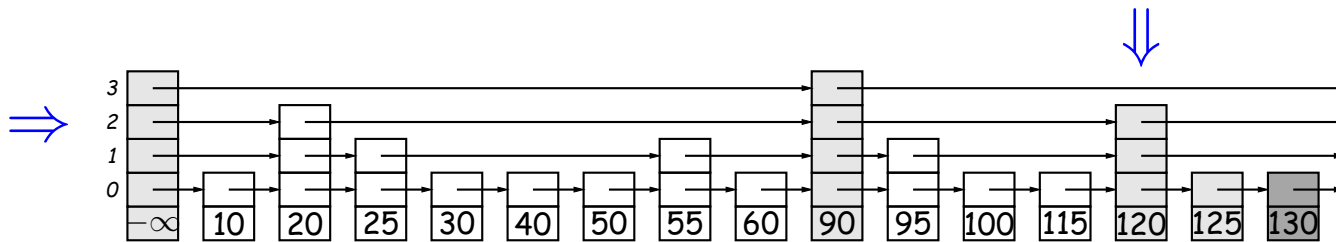
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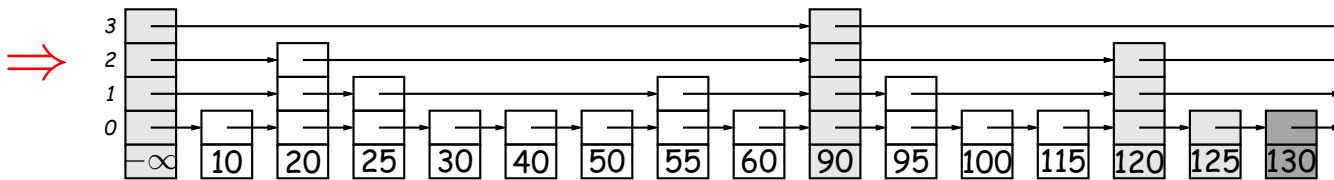
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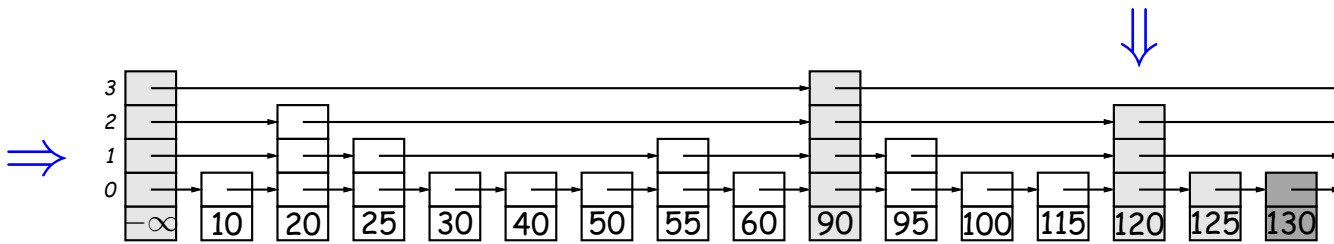
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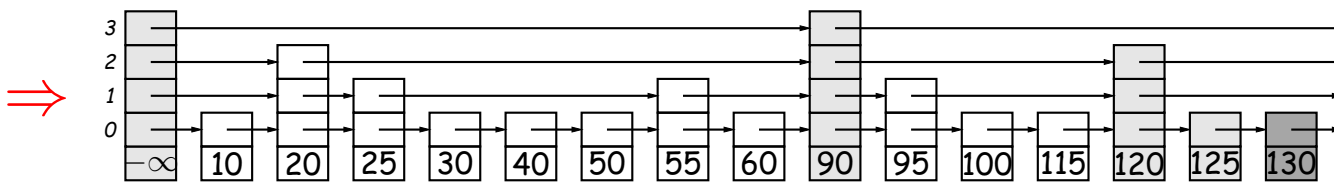
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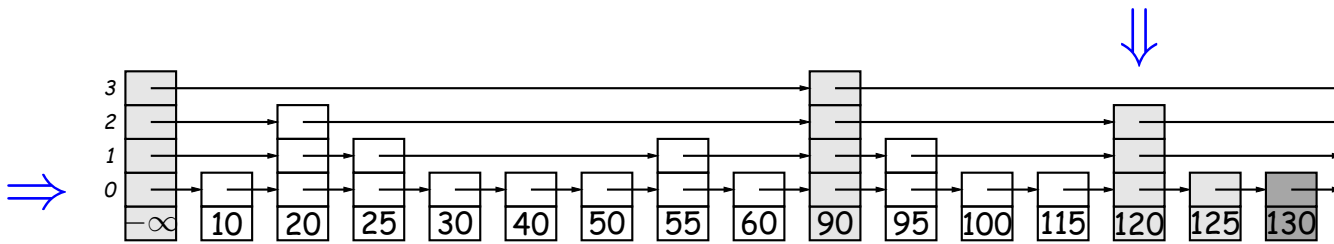
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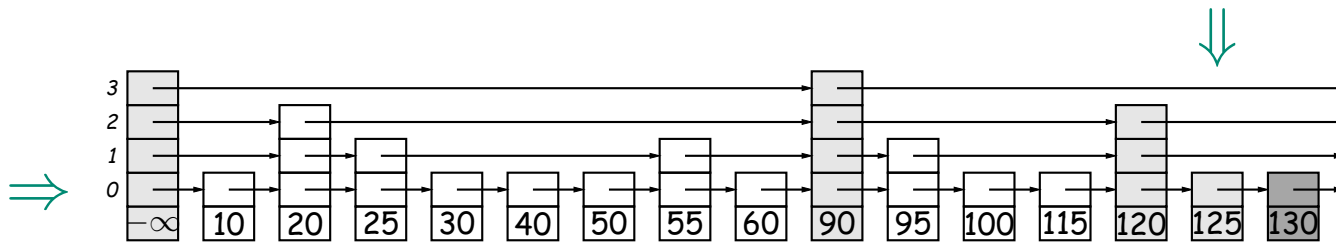
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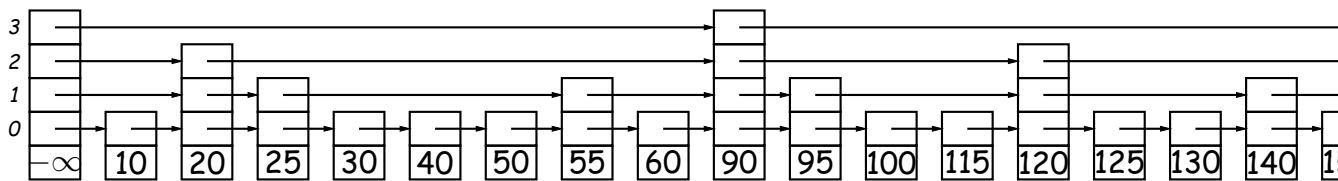
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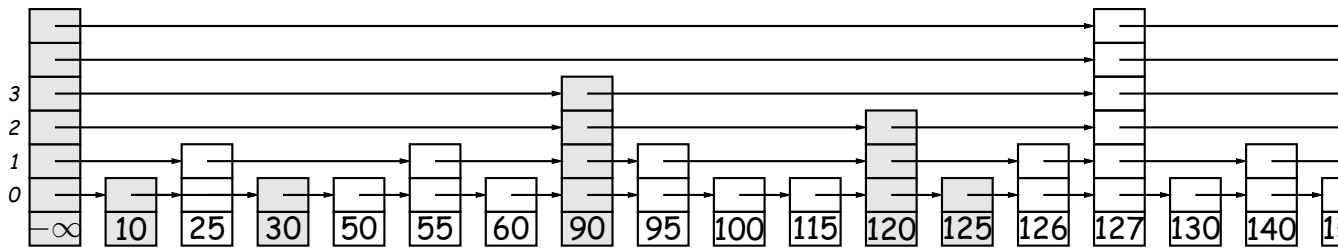
- Makes searches fast *with high probability*.

## Example: Adding and deleting

- Starting from initial list:



- In any order, we add 126 and 127 (choosing random heights for them), and remove 20 and 40:



- Shaded nodes here have been modified.

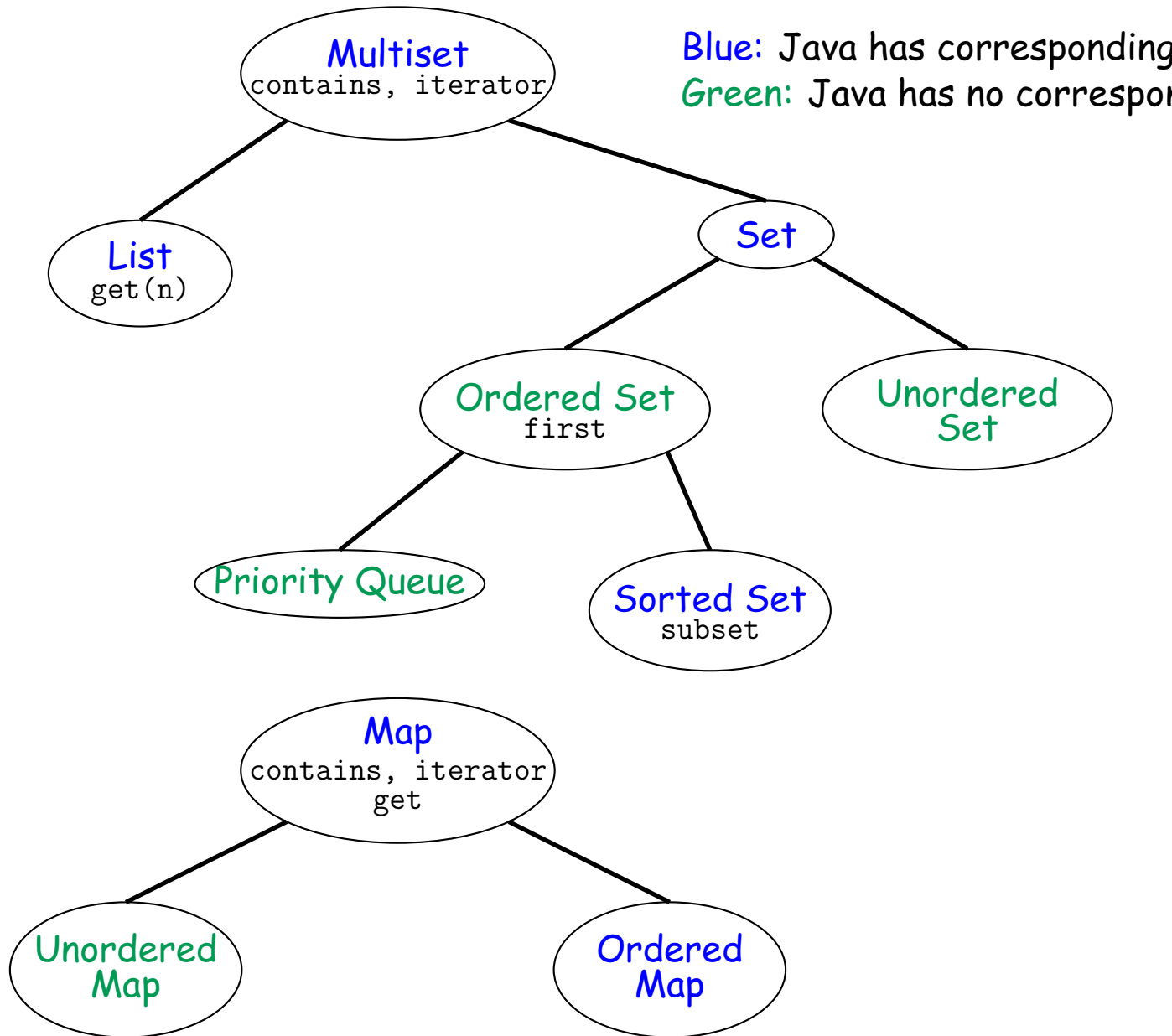
# Summary

- Balance in search trees allows us to realize  $\Theta(\lg N)$  performance.
- B-trees, red-black trees:
  - Give  $\Theta(\lg N)$  performance for searches, insertions, deletions.
  - B-trees good for external storage. Large nodes minimize # of I/O operations
- Tries:
  - Give  $\Theta(B)$  performance for searches, insertions, and deletions, where  $B$  is length of key being processed.
  - But hard to manage space efficiently.
- *Interesting idea:* scrunched arrays share space.
- Skip lists:
  - Give probable  $\Theta(\lg N)$  performance for searches, insertions, deletions



- Easy to implement.
- Presented for *interesting ideas*: probabilistic balance, randomized data structures.

# Summary of Collection Abstractions



# Data Structures that Implement Abstractions

## Multiset

- **List**: arrays, linked lists, circular buffers
- **Set**
  - **OrderedSet**
    - \* **Priority Queue**: heaps
    - \* **Sorted Set**: binary search trees, red-black trees, B-trees, sorted arrays or linked lists
  - **Unordered Set**: hash table

## Map

- **Unordered Map**: hash table
- **Ordered Map**: red-black trees, B-trees, sorted arrays or linked lists

# Corresponding Classes in Java

## Multiset (Collection)

- **List**: ArrayList, LinkedList, Stack, ArrayBlockingQueue, ArrayDeque
- **Set**
  - **OrderedSet**
    - \* **Priority Queue**: PriorityQueue
    - \* **Sorted Set** (SortedSet): TreeSet
  - **Unordered Set**: HashSet

## Map

- **Unordered Map**: HashMap
- **Ordered Map** (SortedMap): TreeMap