

## CS61B Lecture #14: Integers

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## Integer Types and Literals

Type	Bits	Signed?	Literals
byte	8	Yes	Cast from int: (byte) 3
short	16	Yes	None. Cast from int: (short) 4096
char	16	No	'a' // (char) 97 '\n' // newline ((char) 10) '\t' // tab ((char) 8) '\' // backslash 'A', '\101', '\u0041' // == (char) 65
int	32	Yes	123 0100 // Octal for 64 0x3f, 0xffffffff // Hexadecimal 63, -1 (!)
long	64	Yes	123L, 01000L, 0x3fL 1234567891011L

- Negative numerals are just negated (positive) literals.
- " $N$  bits" means that there are  $2^N$  integers in the domain of the type:
  - If signed, range of values is  $-2^{N-1} \dots 2^{N-1} - 1$ .
  - If unsigned, only non-negative numbers, and range is  $0 \dots 2^N - 1$ .

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## Overflow

- **Problem:** How do we handle overflow, such as occurs in  $10000 * 10000 * 10000$ ?
- Some languages throw an exception (Ada), some give undefined results (C, C++)
- Java *defines* the result of any arithmetic operation or conversion on integer types to "wrap around"—*modular arithmetic*.
- That is, the "next number" after the largest in an integer type is the smallest (like "clock arithmetic").
- E.g., (byte) 128 == (byte) (127+1) == (byte) -128
- In general,
  - If the result of some arithmetic subexpression is supposed to have type  $T$ , an  $n$ -bit integer type,
  - then we compute the real (mathematical) value,  $x$ ,
  - and yield a number,  $x'$ , that is in the range of  $T$ , and that is equivalent to  $x$  modulo  $2^n$ .
  - (That means that  $x - x'$  is a multiple of  $2^n$ .)

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## Modular Arithmetic

- Define  $a \equiv b \pmod{n}$  to mean that  $a - b = kn$  for some integer  $k$ .
- Define the binary operation  $a \bmod n$  as the value  $b$  such that  $a \equiv b \pmod{n}$  and  $0 \leq b < n$  for  $n > 0$ . (Can be extended to  $n \leq 0$  as well, but we won't bother with that here.) This is *not* the same as Java's % operation.
- Various facts: (Here, let  $a'$  denote  $a \bmod n$ ).

$$\begin{aligned}
 a'' &= a' \\
 a' + b'' &= (a' + b')' = a + b' \\
 (a' - b')' &= (a' + (-b'))' = (a - b)' \\
 (a' \cdot b')' &= a' \cdot b' = a \cdot b' \\
 (a^k)' &= ((a')^k)' = (a \cdot (a^{k-1}))', \text{ for } k > 0.
 \end{aligned}$$

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## Modular Arithmetic: Examples

- (byte) (64\*8) yields 0, since  $512 - 0 = 2 \times 2^8$ .
- (byte) (64\*2) and (byte) (127+1) yield -128, since  $128 - (-128) = 1 \times 2^8$ .
- (byte) (101\*99) yields 15, since  $9999 - 15 = 39 \times 2^8$ .
- (byte) (-30\*13) yields 122, since  $-390 - 122 = -2 \times 2^8$ .
- (char) (-1) yields  $2^{16} - 1$ , since  $-1 - (2^{16} - 1) = -1 \times 2^{16}$ .

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## Modular Arithmetic and Bits

- Why wrap around?
- Java's definition is the natural one for a machine that uses binary arithmetic.
- For example, consider bytes (8 bits):

Decimal	Binary
101	1100101
$\times 99$	1100011
9999	100111 00001111
- 9984	100111 00000000
15	00001111

- In general, bit  $n$ , counting from 0 at the right, corresponds to  $2^n$ .
- The bits to the left of the vertical bars therefore represent multiples of  $2^8 = 256$ .
- So throwing them away is the same as arithmetic modulo 256.

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## Negative numbers

- Why this representation for -1?

$$\begin{array}{r|l} 1 & 00000001_2 \\ + -1 & 11111111_2 \\ = 0 & 10000000_2 \end{array}$$

Only 8 bits in a byte, so bit 8 falls off, leaving 0.

- The truncated bit is in the  $2^8$  place, so throwing it away gives an equal number modulo  $2^8$ . All bits to the left of it are also divisible by  $2^8$ .
- On unsigned types (`char`), arithmetic is the same, but we choose to represent only non-negative numbers modulo  $2^{16}$ :

$$\begin{array}{r|l} 1 & 0000000000000001_2 \\ + 2^{16} - 1 & 1111111111111111_2 \\ = 2^{16} + 0 & 1000000000000000_2 \end{array}$$

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## Conversion

- In general Java will silently convert from one type to another if this makes sense and no information is lost from value.
- Otherwise, cast explicitly, as in (byte) x.
- Hence, given

```
byte aByte; char aChar; short aShort; int anInt; long aLong;
```

```
// OK:
```

```
aShort = aByte; anInt = aByte; anInt = aShort;
anInt = aChar; aLong = anInt;
```

```
// Not OK, might lose information:
```

```
anInt = aLong; aByte = anInt; aChar = anInt; aShort = anInt;
aShort = aChar; aChar = aShort; aChar = aByte;
```

```
// OK by special dispensation:
```

```
aByte = 13; // 13 is compile-time constant
aByte = 12+100 // 112 is compile-time constant
```

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## Promotion

- Arithmetic operations (+, \*, ...) *promote* operands as needed.
- Promotion is just implicit conversion.
- For integer operations,
  - if any operand is `long`, promote both to `long`.
  - otherwise promote both to `int`.
- So,

```
aByte + 3 == (int) aByte + 3 // Type int
aLong + 3 == aLong + (long) 3 // Type long
'A' + 2 == (int) 'A' + 2 // Type int
aByte = aByte + 1 // ILLEGAL (why?)
```

- But fortunately,

```
aByte += 1; // Defined as aByte = (byte) (aByte+1)
```

- Common example:

```
// Assume aChar is an upper-case letter
char lowerCaseChar = (char) ('a' + aChar - 'A'); // why cast?
```

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## Bit twiddling

- Java (and C, C++) allow for handling integer types as sequences of bits. No "conversion to bits" needed: they already are.
- Operations and their uses:

Mask	Set	Flip	Flip all
00101100	00101100	00101100	
& 10100111	10100111	~ 10100111	~ 10100111
00100100	10101111	10001011	01011000

- Shifting:

Left	Arithmetic Right	Logical Right
10101101 << 3	10101101 >> 3	10101100 >>> 3
01101000	11110101	00010101

- What is:  
 $(-1) >>> 29?$   
 $x << n?$   
 $x >> n?$   
 $(x >>> 3) \& ((1 << 5) - 1)?$

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 $x << n? = x \cdot 2^n.$   
 $x >> n?$   
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