**Recreation** Prove that for every acute angle $\alpha > 0$,

$$\tan\alpha + \cot\alpha \geq 2$$

**Announcements**

- **Today:** More pointer hacking.
- **Handing in labs and homework:** We'll be lenient about accepting late homework and labs for lab1, lab2, and hw0. Just get it done: part of the point is getting to understand the tools involved. We will *not* accept submissions by email.
- We will feel free to interpret the absence of a central repository for you or a lack of a lab1 submission from you as indicating that you intend to drop the course.
- Project 0 to be released tonight.





laration means that the variable's value may not be changed after the variable is initialized.

- Is the following class valid?

```
public class Issue {

    private final IntList aList = new
IntList(0, null);

    public void modify(int k) {
        this.aList.head = k;
    }
}
```

Why or why not?



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- Is the following class valid?

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public class Issue {

    private final IntList aList = new
IntList(0, null);

    public void modify(int k) {
        this.aList.head = k;
    }
}
```

Why or why not?

**Answer:** This is *valid*. Although `modify` changes the `head` variable of the object pointed to by `aList`, it does *not* modify the contents of `aList` itself (which is a pointer).

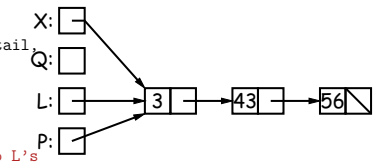


original list to save time or space:



```
dincrList(IntList P, int n)
{
  if (P == null)
    return null;
  else {
    P.head += n;
    P.tail = dincrList(P.tail,
n);
    return P;
  }
}
/** Destructively add N to L's
items. */
static IntList
dincrList(IntList L, int n)
{
  // 'for' can do more than
count!
  for (IntList p = L; p !=
null; p = p.tail)
    p.head += n;
  return L;
}
```

```
/* IntList.list from
HW #1 */
Q = dincrList(X, 2);
```



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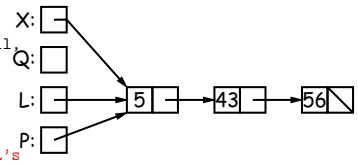




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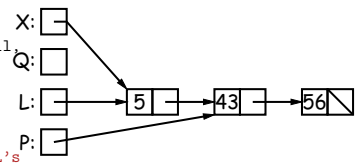




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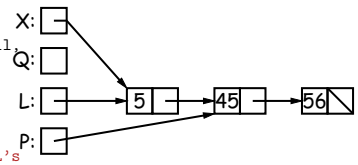

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    return P;
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static IntList
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{
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count!
  for (IntList p = L; p !=
null; p = p.tail)
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  return L;
}
```

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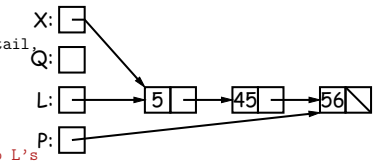




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    return P;
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/** Destructively add N to L's
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static IntList
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  for (IntList p = L; p !=
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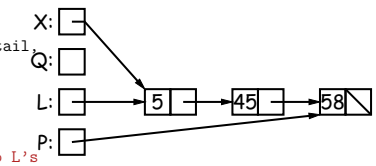




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n);
    return P;
  }
}
/** Destructively add N to L's
items. */
static IntList
dincrList(IntList L, int n)
{
  // 'for' can do more than
count!
  for (IntList p = L; p !=
null; p = p.tail)
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  return L;
}
```

```
/* IntList.list from
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Q = dincrList(X, 2);
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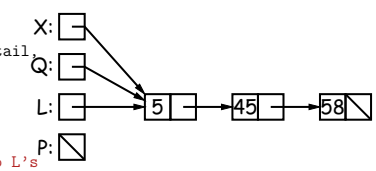

If L is the list [2, 1, 2, 9, 2], we want `removeAll(L,2)` to be the new list [1, 9].

```
/** The list resulting from removing all instances
of X from L
 *  non-destructively. */
static IntList removeAll(IntList L, int x)
{
  if (L == null)
     return /*( null with all x's removed
)*/;
  else if (L.head == x)
     return /*( L with all x's removed (L!=null,
L.head==x) )*/;
  else
     return /*( L with all x's removed (L!=null, L.head!=x) )*/;
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If L is the list [2, 1, 2, 9, 2], we want `removeAll(L,2)` to be the new list [1, 9].

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static IntList removeAll(IntList L, int x)
{
  if (L == null)
     return null;
  else if (L.head == x)
     return removeAll(L.tail, x);
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If L is the list [2, 1, 2, 9, 2], we want `removeAll(L,2)` to be the new list [1, 9].

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/** The list resulting from removing all instances
of X from L
 *  non-destructively. */
static IntList removeAll(IntList L, int x)
{
  if (L == null)
     return null;
  else if (L.head == x)
     return removeAll(L.tail, x);
  else
     return new IntList(L.head, removeAll(L.tail,
x));
}
```



Same as before, but use front-to-back iteration rather than recursion.



```
non-destructively. */
static IntList removeAll(IntList
L, int x) {
  IntList result, last;
  result = last = null;
  for ( ; L != null; L = L.tail)
{
    if (x == L.head)
      continue;
    else if (last == null)
      result = last = new
IntList(L.head, null);
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      last = last.tail = new
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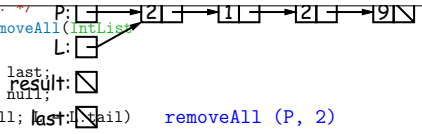


removeAll (P, 2)



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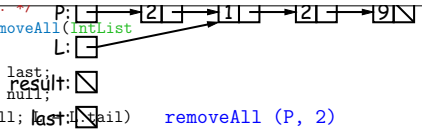


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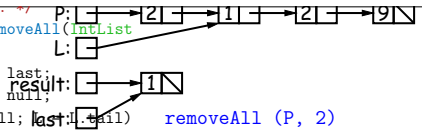


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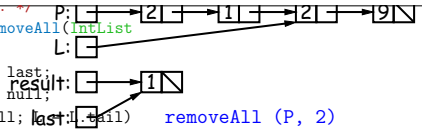


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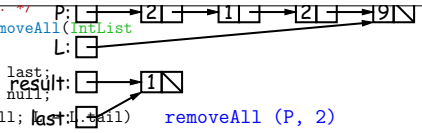


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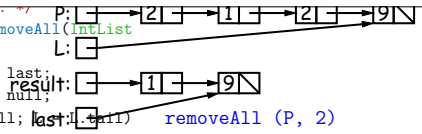


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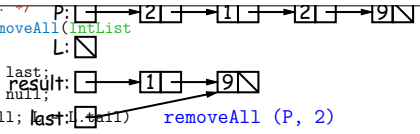


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  if (L == null)
     return /*( null with all x's removed
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  else if (L.head == x)
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  else {
     /*{ Remove all x's from L's tail. }*/;
     return L;
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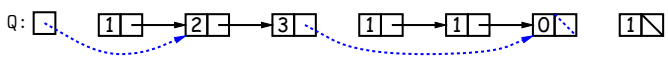


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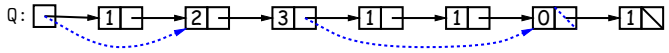


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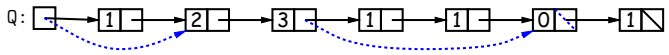


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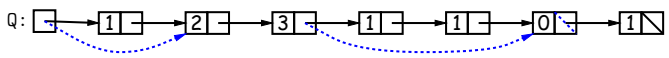


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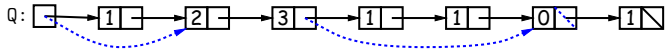


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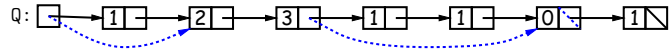


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  else if (L.head == x)
     return dremoveAll(L.tail, x);
  else {
     L.tail = dremoveAll(L.tail, x);
     return L;
  }
}
```





```
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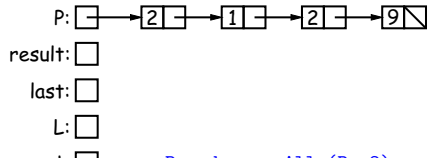
 *  destructively. */
static IntList dremoveAll(IntList L,
int x) {
  IntList result, last;
  result = last = null;
  while (L != null) {
    IntList next = L.tail;
    if (x != L.head) {
      if (last == null)
        result = last = L;
      else
        last = last.tail = L;
      L.tail = null;
    }
    L = next;
  }
  return result;
}
```






P = dremoveAll (P, 2)

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      if (last == null)
        result = last = L;
      else
        last = last.tail = L;
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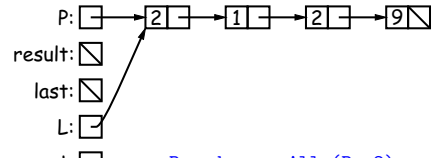


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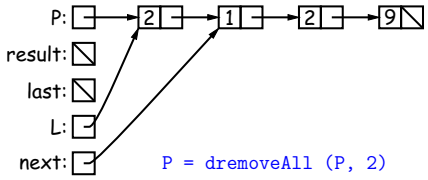


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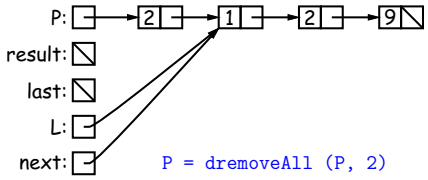


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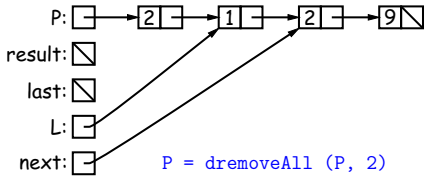


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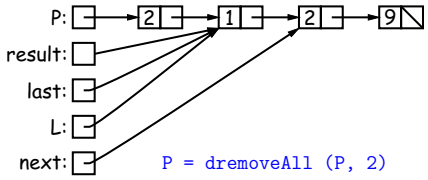


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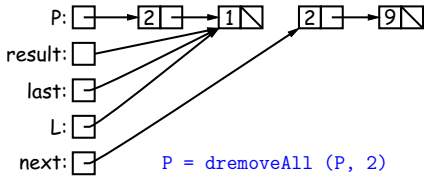


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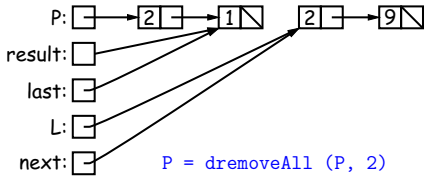


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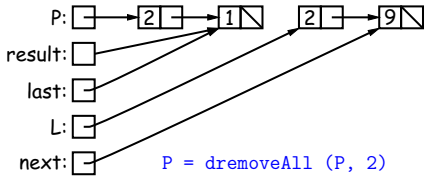


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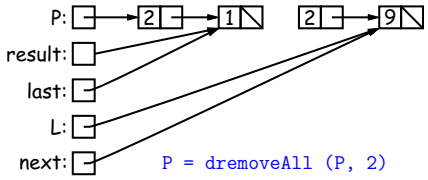


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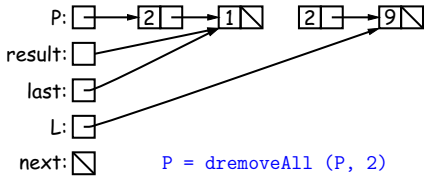


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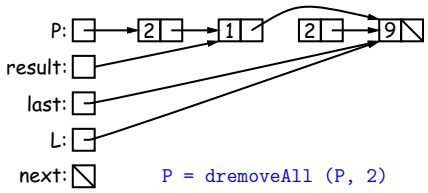


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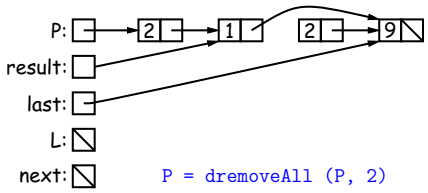


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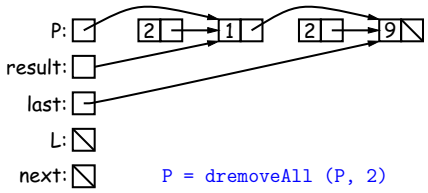


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```

- Try to give a description of how things look on *any arbitrary iteration* of the loop.
- This description is known as a *loop invariant,* because it is always true at the start of each iteration.
- The loop body then must
  - Start from any situation consistent with the invariant;
  - Make progress in such a way as to make the invariant true again.

```
// Invariant must be true here
while (condition) { // condition must
not have side-effects.
    // (Invariant will necessarily be
true here.)
    loop body
    // Invariant must again be true here
```



ever *Invariant* is true and *condition* false, our job is done!



equivalent recursive procedure:

```
/** Assuming Invariant, produce a situation
where Inveriant
 *  is true and condition is false. */
void loop() {
    // Invariant assumed true here.
    if (condition) {
        loop body
        // Invariant must be true here.
        loop()
        // Invariant true here and con-
dition false.
    }
}
```

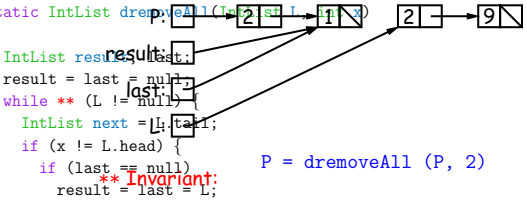
- Here, the invariant is the precondition of the function **loop**.
- The loop maintains the invariant while making the condition false.







```
/** The list resulting from removing all
X's from L
 *  destructively. */
static IntList dremoveAll(IntList L, int x)
{
  IntList result = last;
  result = last = null;
  while ** (L != null) {
    IntList next = L.tail;
    if (x != L.head) {
      if (last == null)
        result = last = L;
      else
        last = last.tail = L;
      L.tail = null;
    }
    L = next;
  }
  return result;
}
```




P = dremoveAll (P, 2)

** Invariant:

- result points to the list of items in the final result except for those from L onward.
- L points to an unchanged tail of the original list of items in L.
- last points to the last item in result or is null if result is null.