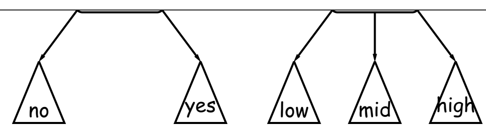
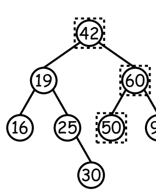
ing things in response to various forms of query.

- Linear search for response can be expensive, especially when data set is too large for primary memory.
- Preferable to have criteria for *dividing* data to be searched into pieces recursively
- We saw the figure for $\lg N$ algorithms: at 1 μsec per comparison, could process 10^{300000} items in 1 sec.
- Tree is a natural framework for the representation:

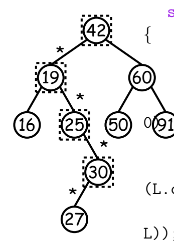


- Tree nodes contain *keys,* and possibly other data.
- All nodes in left subtree of node have *smaller* keys.
- All nodes in right subtree of node have *larger* keys.
- "Smaller" means any complete transitive, anti-symmetric ordering on keys:
  - exactly one of $x \prec y$ and $y \prec x$ true.
  - $x \prec y$ and $y \prec z$ imply $x \prec z$.
  - (To simplify, won't allow duplicate keys this semester).
- E.g., in dictionary database, node label would be (*word, definition*): *word* is the key.
- For concreteness here, we'll just use the standard Java convention of calling `.compareTo`.

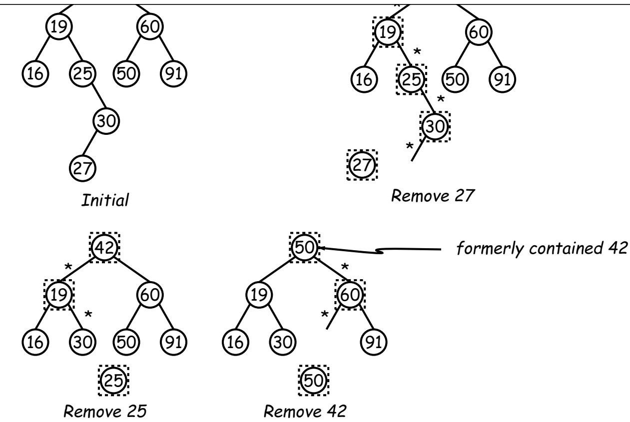


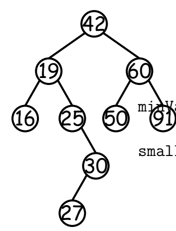

```
   /** Node in T containing L, or
null if none */
   static BST find(BST T, Key L) {
     if (T == null)
       return T;
     if (L.compareTo(T.label()) ==
0)
       return T;
     else if
(L.compareTo(T.label()) < 0)
       return find(T.left(), L);
     else
       return find(T.right(), L);
   }
```

- Dashed boxes show which node labels we look at.
- Number looked at proportional to height of tree.




```
   /** Insert L in T, replacing
existing
    *  value if present, and
returning
    *  new tree. */
   static BST insert(BST T, Key L)
   {
     if (T == null)
       return new BST(L);
     if (L.compareTo(T.label()) ==
0)
       T.setLabel(L);
     else if
(L.compareTo(T.label()) < 0)
       T.setLeft(insert(T.left(),
L));
     else
       T.setRight(insert(T.right(),
L));
     return T;
   }
```

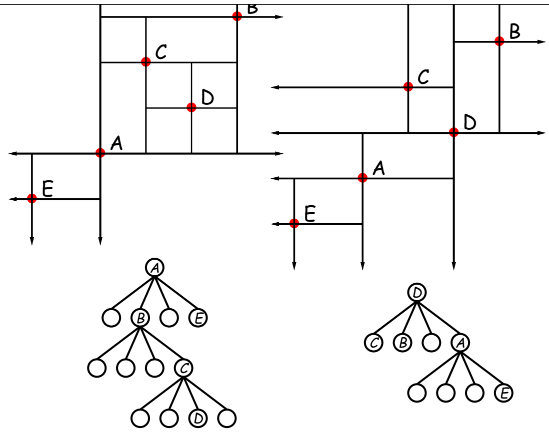




```
static BST remove(BST T, Key L) {
  if (T == null)
    return null;
  if (L.compareTo(T.label()) == 0) {
     if (T.left() == null)
         return T.right();
     else if (T.right() == null)
         return T.left();
     else {
         Key smallest =
minVal(T.right());  // ??
         T.setRight(remove(T.right(),
smallest));
         T.setLabel(smallest);
     }
  }
  else if (L.compareTo(T.label()) <
0)
    T.setLeft(remove(T.left(), L));
  else
    T.setRight(remove(T.right(),
L));
  return T;
}
```

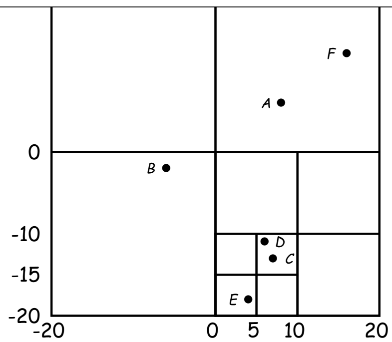

tions so that items can be retrieved by position.

- Quadtrees do so using standard data-structuring trick: *Divide and Conquer*.
- Idea: divide (2D) space into four *quadrants,* and store items in the appropriate quadrant. Repeat this recursively with each quadrant that contains more than one item.
- Original definition: a quadtree is either
  - Empty, or
  - An item at some position (x, y), called the root, plus
  - four quadtrees, each containing only items that are northwest, northeast, southwest, and southeast of (x, y).
- Big idea is that if you are looking for point (x', y') and the root is not the point you are

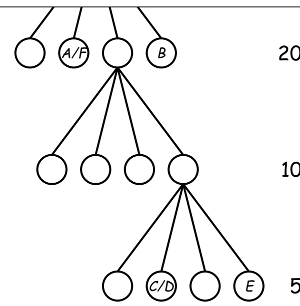


jects, it may be useful to be able to *delete* items from a tree: when an object moves, the subtree that it goes in may change.

- Difficult to do with the classical data structure above, so we'll define instead:
- A quadtree consists of a bounding rectangle, B and either
  - Zero up to a small number of items that lie in that rectangle, or
  - Four quadtrees whose bounding rectangles are the four quadrants of B (all of equal size).
- A completely empty quadtree can have an arbitrary bounding rectangle, or you can wait for the first point to be inserted.



(≤ 2 points per leaf)



1. If (x, y) is outside the bounding rectangle of T, or T is empty, then (x, y) is not in T.
2. Otherwise, if T contains a small set of items, then (x, y) is in T iff it is among these items.
3. Otherwise, T consists of four quadtrees. Recursively look for (x, y) in each (however, step #1 above will cause all but one of these bounding boxes to reject the point immediately).

- Similar procedure works when looking for all items within some rectangle, R:

1. If R does not intersect the bounding rectangle of T, or T is empty, then there are no items in R.
2. Otherwise, if T contains a set of items, return those that are in R, if any.

suming maximum occupancy of a region is 2, showing initial state $\Longrightarrow$ final state.