CS61B Lecture #17

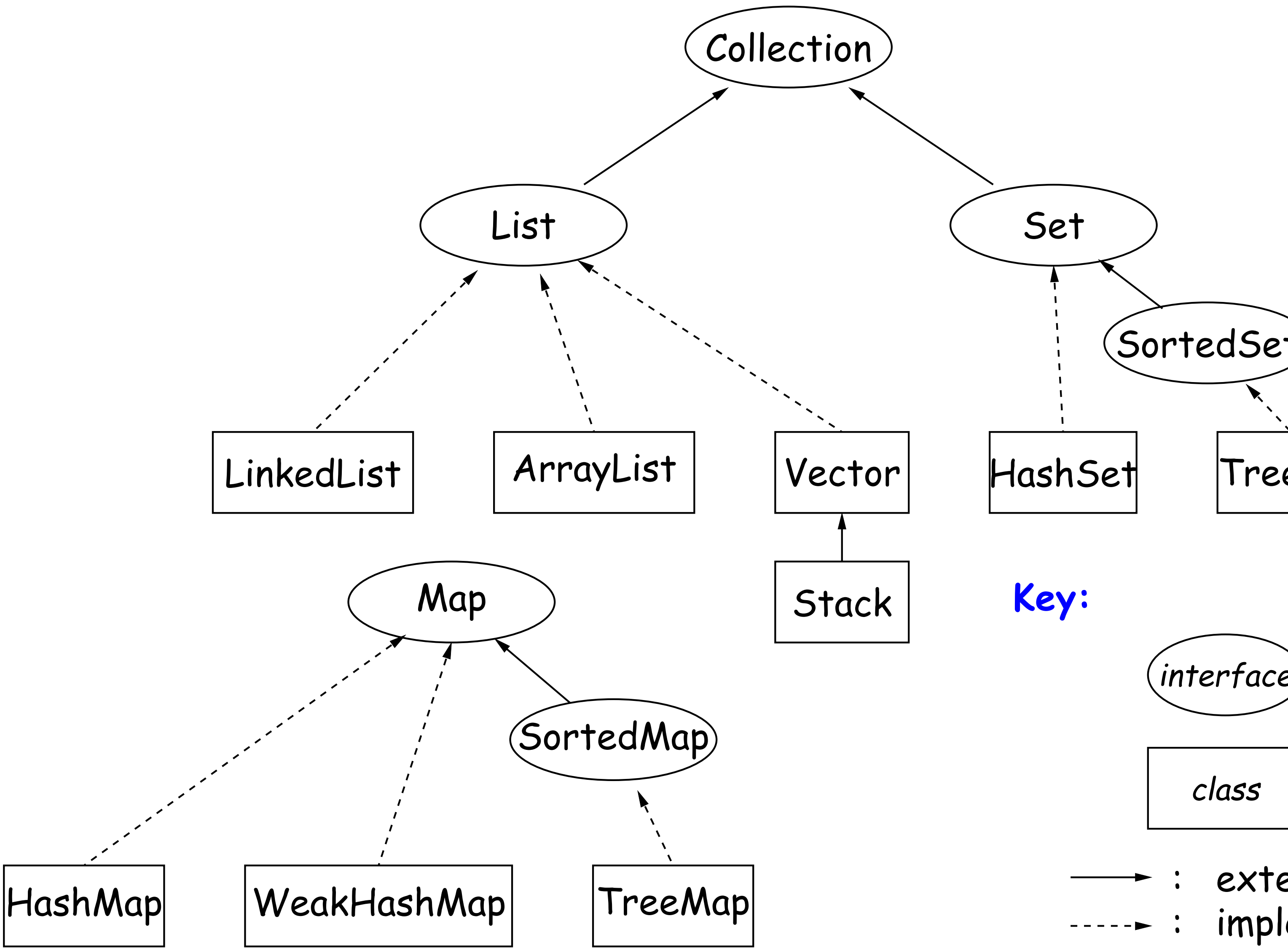
Topics

- Overview of standard Java Collections classes.
  - Iterators, ListIterators
  - Containers and maps in the abstract
- Amortized analysis of implementing lists with arrays.

Data Types in the Abstract

- Most of the time, should *not* worry about implementation structures, search, etc.
- What they do for us—their specification—is important.
- Java has several standard types (in `java.util`) to represen tions of objects
  - Six interfaces:
    - `Collection`: General collections of items.
    - `List`: Indexed sequences with duplication
    - `Set`, `SortedSet`: Collections without duplication
    - `Map`, `SortedMap`: Dictionaries (key $\mapsto$ value)
  - Concrete classes that provide actual instances: `LinkedLis` `HashSet`, `TreeSet`.
  - To make change easier, purists would use the concrete ty for **new**, interfaces for parameter types, local variables.

Collection Structures in java.util



The Collection Interface

- Collection interface. Main functions promised:
  - Membership tests: `contains` ($\in$), `containsAll` ($\subseteq$)
  - Other queries: `size`, `isEmpty`
  - Retrieval: `iterator`, `toArray`
  - *Optional* modifiers: `add`, `addAll`, `clear`, `remove`, `removeA`
    difference), `retainAll` (intersect)

Side Trip about Library Design: Optional Opera

- Not all `Collections` need to be modifiable; often makes se to get things from them.
- So some operations are optional (`add`, `addAll`, `clear`, `remove`, `retainAll`)
- The library developers decided to have *all* Collections im these, but allowed implementations to throw an exception:

  `UnsupportedOperationException`

- An alternative design would have created separate interfac

```
interface Collection { contains, containsAll, size, iterator,
interface Expandable extends Collection { add, addAll }
interface Shrinkable extends Collection { remove, removeAll, .
interface ModifiableCollection
   extends Collection, Expandable, Shrinkable { }
```

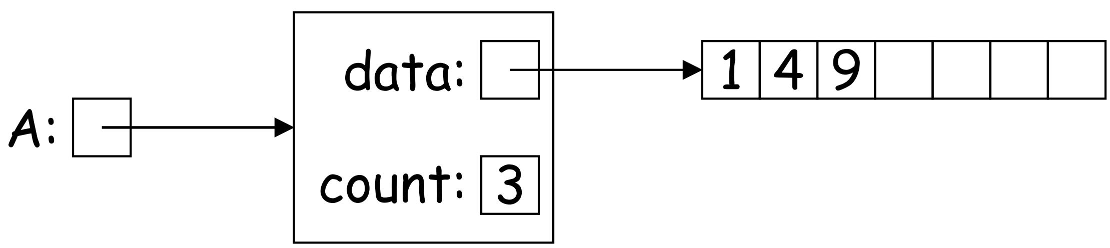
- You'd soon have lots of interfaces. Perhaps that's why the do it that way.

The List Interface

- Extends `Collection`
- Intended to represent *indexed sequences* (generalized arra
- Adds new methods to those of `Collection`:
  - Membership tests: `indexOf`, `lastIndexOf`.
  - Retrieval: `get(`i`)`, `listIterator()`, `sublist(`B, E`)`.
  - Modifiers: `add` and `addAll` with additional index to say *w* add. Likewise for removal operations. `set` operation to `get`.
- Type `ListIterator<Item>` extends `Iterator<Item>`:
  - Adds `previous` and `hasPrevious`.
  - `add`, `remove`, and `set` allow one to iterate through a list, in removing, or changing as you go.
  - **Important Question:** What advantage is there to saying rather than `LinkedList L` or `ArrayList L`?

Implementing Lists (I): ArrayLists

- The main concrete types in Java library for interface `L`
  `ArrayList` and `LinkedList`:
- As you might expect, an `ArrayList`, `A`, uses an array to ho
  For example, a list containing the three items 1, 4, and 9 m
  represented like this:



- After adding four more items to `A`, its `data` array will be f
  the value of `data` will have to be replaced with a pointer t
  bigger array that starts with a copy of its previous values.
- Question: For best performance, how big should this new ar
- If we increase the size by 1 each time it gets full (or by c
  stant value), the cost of N additions will scale as $\Theta(N^2)$
  makes `ArrayList` look much worse than `LinkedList` (which
  `IntList`-like implementation.)

Expanding Vectors Efficiently

- When using array for expanding sequence, best to *double* t
  of array to grow it. Here's why.
- If array is size s, doubling its size and moving s elements to
  array takes time proportional to $2s$.
- In all cases, there is an additional $\Theta(1)$ cost for each add
  account for actually assigning the new value into the array.
- When you add up these costs for inserting a sequence of N
  the *total* cost turns out to be proportional to N, as if each a
  took constant time, even though some of the additions actua
  time proportional to N all by themselves!

Amortized Time

- Suppose that the actual costs of a sequence of N operat $c_0, c_1, \ldots, c_{N-1}$, which may differ from each other by arbitrar and where $c_i \in O(f(i))$.
- Consider another sequence $a_0, a_1, \ldots, a_{N-1}$, where $a_i \in O(g(i$
- If

$$\sum_{0 \leq i < k} a_i \geq \sum_{0 \leq i < k} c_i \text{ for all } k,$$

  we say that the operations all run in $O(g(i))$ *amortized time*.
- That is, the actual cost of a given operation, c_i, may be arb larger than the amortized time, a_i, as long as the *total* am time is always greater than or equal to the total actual t matter where the sequence of operations stops—i.e., no matt k is.
- In cases of interest, the amortized time bounds are much le the actual individual time bounds: $g(i) \ll f(i)$.
- E.g., for the case of insertion with array doubling, $f(i) \in O$ $g(i) \in O(1)$.

Amortization: Expanding Vectors (II)

To Insert Item #	Resizing Cost	Cumulative Cost	Resizing Cost per Item	Ar After
0	0	0	0	
1	2	2	1	
2	4	6	2	
3	0	6	1.5	
4	8	14	2.8	
5	0	14	2.33	
⋮	⋮	⋮	⋮	
7	0	14	1.75	
8	16	30	3.33	
⋮	⋮	⋮	⋮	
15	0	30	1.88	
⋮	⋮	⋮	⋮	
2^m+1 to $2^{m+1}-1$	0	$2^{m+2}-2$	≈ 2	
2^{m+1}	2^{m+2}	$2^{m+3}-2$	≈ 4	

- If we spread out (*amortize*) the cost of resizing, we average about 4 time units for resizing on each item: "amortized time is 4 units." Time to add N elements is $\Theta(N)$, *not* $\Theta(N^2$

Demonstrating Amortized Time: Potential Meth

- To formalize the argument, associate a *potential,* $\Phi_i \geq 0$*,* to operation that keeps track of "saved up" time from cheap ope that we can "spend" on later expensive ones. Start with $\Phi_0 =$
- Now we pretend that the cost of the i^{th} operation is actuall *amortized cost*, defined

$$a_i = c_i + \Phi_{i+1} - \Phi_i,$$

where c_i is the real cost of the operation. Or, looking at pot

$$\Phi_{i+1} = \Phi_i + (a_i - c_i)$$

- On cheap operations, we artificially set $a_i > c_i$ so that we crease Φ ($\Phi_{i+1} > \Phi_i$).
- On expensive ones, we typically have $a_i \ll c_i$ and greatly dec (but don't let it go negative—may not be "overdrawn").
- We try to do all this so that a_i remains as we desired (e.g., expanding array), without allowing $\Phi_i < 0$.
- Requires that we choose a_i so that Φ_i always stays ahead of

Application to Expanding Arrays

- When adding to our array, the cost, c_i, of adding element # the array already has space for it is 1 unit.
- The array does not initially have space when adding items 1, 16,...—in other words at item 2^n for all $n \geq 0$. So,
  - $c_i = 1$ if $i \geq 0$ and is not a power of 2; and
  - $c_i = 2i + 1$ when i is a power of 2 (copy i items, clear ar items, and then add item #i).
- So on each operation #2^n we're going to need to have save least $2 \cdot 2^n = 2^{n+1}$ units of potential to cover the expense of ex the array, and we have this operation and the preceding operations in which to save up this much potential (everythi the preceding doubling operation).
- So choose a_0 = 1 and $a_i = 5$ for $i > 0$. Apply $\Phi_{i+1} = \Phi_i + (a_i -$ here is what happens:

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
c_i	1	3	5	1	9	1	1	1	17	1	1	1	1	1	1	1	33	1
a_i	1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
Φ_i	0	0	2	2	6	2	6	10	14	2	6	10	14	18	22	26	30	2

Pretend
5 never
true cur