Dynamics of Self-Driving Cars

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Consider a car of length moving at speed in the +y direction from the origin of the xy-plane. Suppose the steering wheel is turned an angle CW from the neutral position. The path of the car lies on a circle with turn radius . The car lies on the sector of a circle, which for small turn angles has arc length and sector angle . Using the relationship for circle arc length, we obtain the car turn radius as . The car therefore lies on the path , which represents a circle through the origin. The angular frequency on the circle is , with period where is the circumference of the circle . Therefore, we have and the dynamics of the car become the following.

However, cars often drive straight with which produces a singularity. However, as , we perform limits of the functions and , which are known power series. Rewriting the original trajectory in this form gives us:

A plot of these trajectories for different turn angles are shown below, including a perfect straight-line path! The remaining dynamics of the system involve forward acceleration and turning:

Diagram

Description automatically generated with medium confidence

Full Nonlinear State Space Dynamics of the Car

State consists of with constant parameters , and controls which affect respectively. The position of the car is then an output of the nonlinear dynamics as shown below. With or initial zero frame rotation, the dynamics are: