OPERATIONS RESEARCH CENTER

Electric Vehicle Routing: Subpath-Based Decomposition

Sean Lo seanlo@mit.edu
MIT Operations Research Center,
Cambridge, MA

Alexandre Jacquillat alexjacq@mit.edu
MIT Sloan School of Management,
Cambridge, MA

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Motivation, problem setting

Biden administration plan seeks elimination of transportation emissions

calls for a transition to electric vehicles and more walkable neighborhoods by 2050

A 40-ton Mercedes-Benz e-truck just drove 1,000 km with only one stop to charge

Michelle Lewis | Oct 5 2023 - 10:48 am PT | 66 Comments

LOGISTICS REPORT

California's Electric-Truck Drive Draws Startups Building Charging Networks

n aggressive emissions-slashing mandate means thousands of
arging sites are needed in the coming years

Paul Berger Follow

y 29, 2023 7:00 am ET

Biden administration plan calls for $5 billion network of electric-vehicle chargers along interstates

Grants included in the infrastructure law will help states build a charging network designed to reach highways in almost every corner of the country

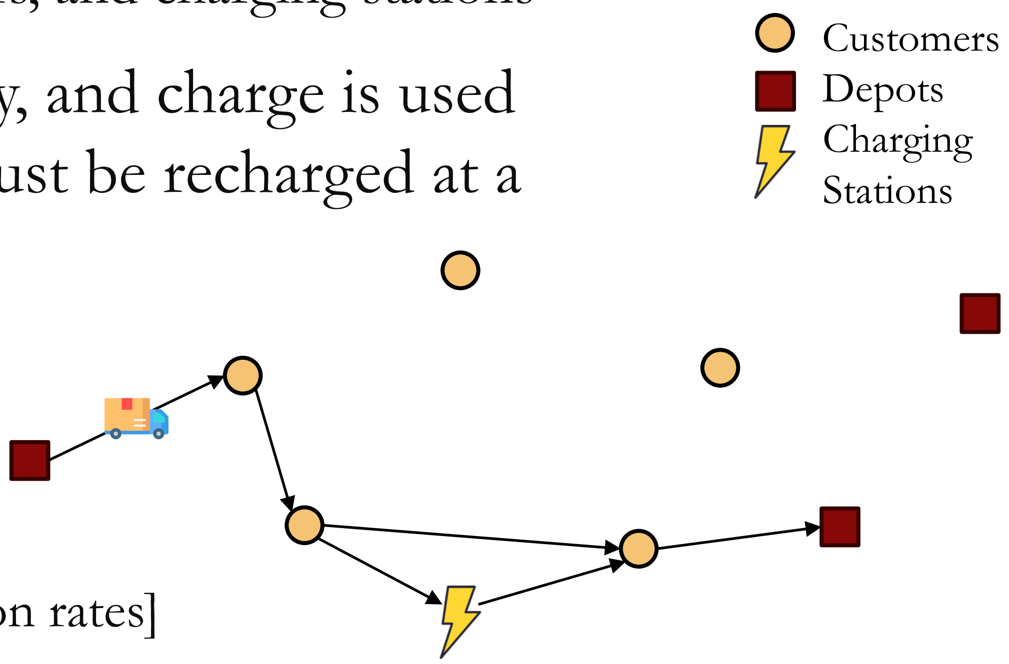
By Ian Duncan

Updated February 10, 2022 at 1:46 p.m. EST | Published February 10, 2022 at 5:00 a.m. EST

New routing algorithms for electrified logistics

Problem description

- Vehicle routing problem with electric vehicles, in continuous time and charge
  - multiple depots, customers, and charging stations
- Each vehicle has a battery, and charge is used between locations and must be recharged at a charging station
- <u>Assumptions:</u>
  - No time windows
  - Linear charging dynamics
    [Possibly non-linear depletion rates]



Contributions

Electric vehicle routing: subpath-based decomposition algorithm

Modeling	Electric vehicle routing: Semi-infinite set-partitioning formulation with continuous time and continuous charge
Optimization	• Subpath-based decomposition algorithm for column generation subproblem • Acceleration strategy via adaptive route relaxations to obtain elementary paths • Cutting planes to strengthen linear relaxation
Computational results	Significantly outperforms path-based benchmark, and scales to realistic problem instances
Practical impact	Benefits over separately optimizing routing and charging

Semi-finite set-partitioning model

$$
\begin{aligned}
\min \quad & \sum_{p \in \mathcal{P}} c^p z^p & & \text{(minimize total cost of paths)} \\
\text{such that} \quad & \sum_{p \in \mathcal{P}} \alpha_j^p z^p = v_j^{\text{start}} & \forall\, j \in \mathcal{V}_D \quad & \text{(each depot } j \text{ starts with } v_j^{\text{start}} \text{ vehicles)} \\
& \sum_{p \in \mathcal{P}} \beta_j^p z^p \geq v_j^{\text{end}} & \forall\, j \in \mathcal{V}_D \quad & \text{(each depot } j \text{ ends with at least } v_j^{\text{start}} \text{ vehicles)} \\
& \sum_{p \in \mathcal{P}} \gamma_i^p z^p = 1 & \forall\, i \in \mathcal{V}_C \quad & \text{(each customer served once)} \\
& z^p \in \{0, 1\} & \forall\, p \in \mathcal{P} \quad &
\end{aligned}
$$

- Set-partitioning formulation with path-based variables z^p
- Infinitely many variables
  - **Discrete** routing and timing decisions (as in traditional VRP)
  - **Continuous** charging decisions (new to E-VRP)

Column generation

Restricted Master Problem

$$\begin{aligned}
\min \quad & \sum_{p \in \mathcal{P}'} c^p z^p \\
\text{such that} \quad & \sum_{p \in \mathcal{P}'} \alpha_j^p z^p = v_j^{\text{start}} && \forall\, j \in \mathcal{V}_D \quad [\boldsymbol{\kappa}] \\
& \sum_{p \in \mathcal{P}'} \beta_j^p z^p \geq v_j^{\text{end}} && \forall\, j \in \mathcal{V}_D \quad [\boldsymbol{\mu}] \\
& \sum_{p \in \mathcal{P}'} \gamma_i^p z^p = 1 && \forall\, i \in \mathcal{V}_C \quad [\boldsymbol{\nu}] \\
& z^p \in \{0, 1\} && \forall\, p \in \mathcal{P}'
\end{aligned}$$

dual values $\boldsymbol{\kappa}, \boldsymbol{\mu}, \boldsymbol{\nu}$

Subproblem

$$\min_{p \in \mathcal{P}} \left\{ \bar{c}^p := c^p - \kappa_{\text{start}(p)} - \mu_{\text{end}(p)} - \sum_{i \in \mathcal{V}_C} \gamma_i^p \nu_i \right\}$$

paths not in $\mathcal{P}'$

Traditionally: solves an E-RCSPP by dynamic programming

- **Q:** How to guarantee finite termination?

Subpath-based decomposition in the pricing problem

Master problem variables

Pricing problem variables		Path	Subpath
	Path	Column generation e.g. [1] for EVRPTW	CG for extended formulations [4]
	Subpath	**This work** **Q:** How can you build paths from subpaths?	CG *on* extended formulations, e.g. [2] for PDP, [3] for ride-sharing

[1] Desaulniers, G., Errico, F., Irnich, S., & Schneider, M. (2016). Exact Algorithms for Electric Vehicle-Routing Problems with Time Windows. *Operations Research*, *64*(6), 1388–1405. https://doi.org/10.1287/opre.2016.1535

[2] Alyasiry, A. M., Forbes, M., & Bulmer, M. (2019). An Exact Algorithm for the Pickup and Delivery Problem with Time Windows and Last-in-First-out Loading. *Transportation Science*, *53*(6), 1695–1705. https://doi.org/10.1287/trsc.2019.0905

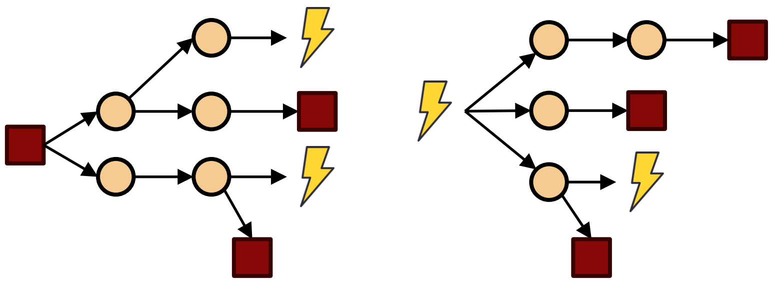
[3] Zhang, W., Jacquillat, A., Wang, K., & Wang, S. (2022). Routing Optimization with Vehicle-Customer Coordination. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.4208397

[4] Sadykov, R., & Vanderbeck, F. (2013). Column generation for extended formulations. *EURO Journal on Computational Optimization*, *1*(1), 81–115. https://doi.org/10.1007/s13675-013-0009-9

Key idea: generate-and-stitch

Step 1: Generate **subpaths**
- Label-setting, with charge and time taken as domination criteria

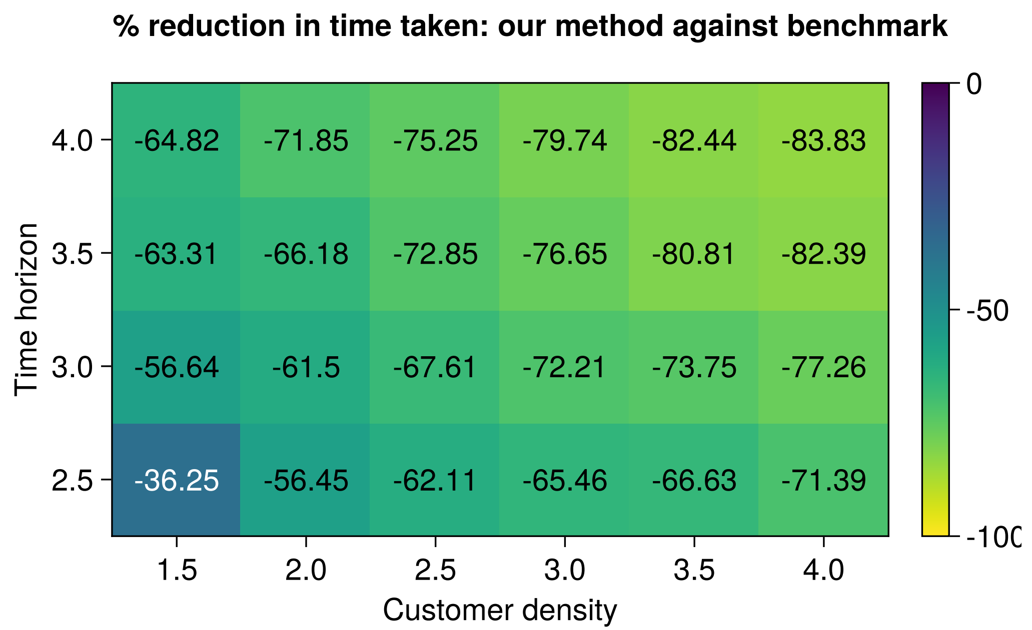
Step 2: Stitch subpaths into **paths**
- A subpath valid at time 0 is still valid at time t with the same reduced cost*
- Charging action between subpaths is the minimum possible
- Reduced cost of path = r.c. of subpaths + r.c. of charging actions

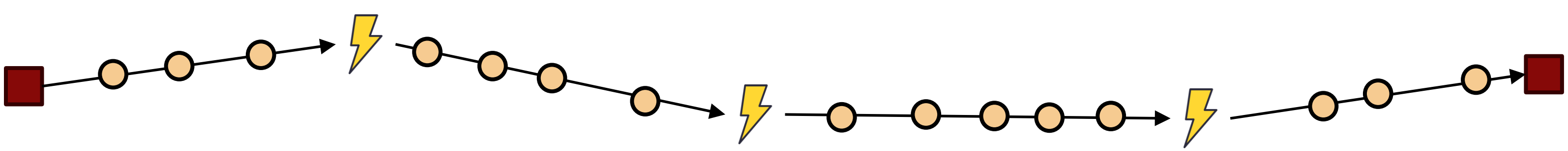


Theorem: with this, CG finitely converges to LP optimum of EVRP

Comparison to benchmark

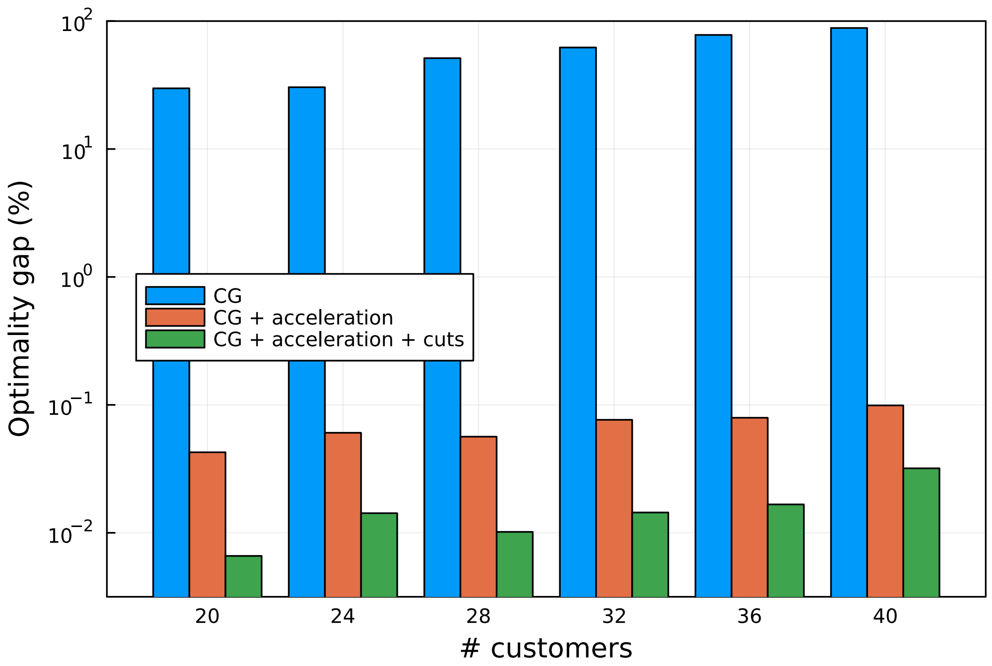
- Significant speedups against path-based benchmark
- Stronger improvement with:
  - Higher customer density ≈ **longer subpaths**
  - Longer time horizon ≈ **more subpaths per path**

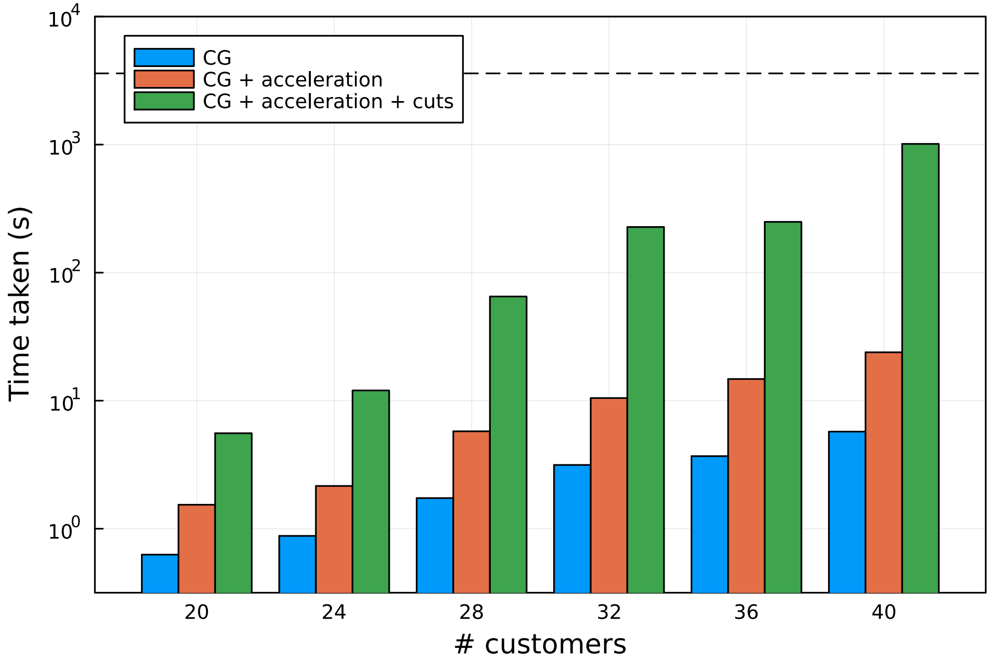




Computational results

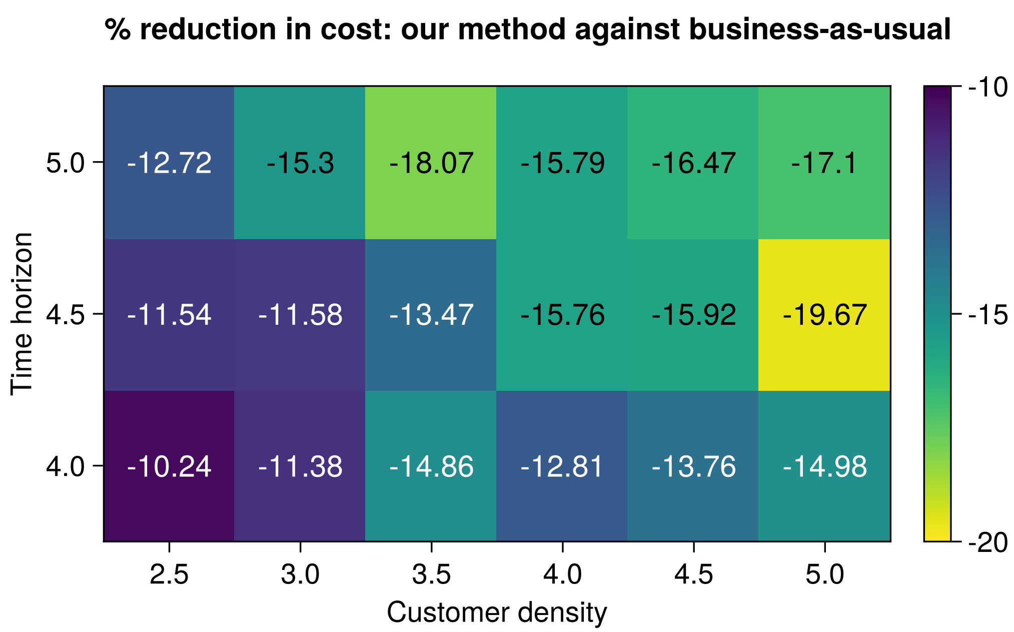
- Also implement an acceleration strategy for finding the LP relaxation of elementary paths (orange)
- Also introduce a cutting-plane strategy (green)
- Navigates tradeoff between optimality and time





The benefits of optimization

- Benefit of **jointly** optimizing charging and routing decisions
- Improvement compared to business-as-usual solution:
  - Solve a VRP w/o charge
  - Then optimize charging stations with fixed routes



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