Electric Vehicle Routing: Subpath-Based Decomposition

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Background and motivation

Biden administration plan seeks elimination of transportation emissions

calls for a transition to electric vehicles and more walkable neighborhoods by 2050

A 40-ton Mercedes-Benz e-truck just drove 1,000 km with only one stop to charge

Michelle Lewis | Oct 5 2023 - 10:48 am PT | 66 Comments

LOGISTICS REPORT

California's Electric-Truck Drive Draws Startups Building Charging Networks

aggressive emissions-slashing mandate means thousands of arging sites are needed in the coming years

Paul Berger Follow

y 29, 2023 7:00 am ET

Biden administration plan calls for $5 billion network of electric-vehicle chargers along interstates

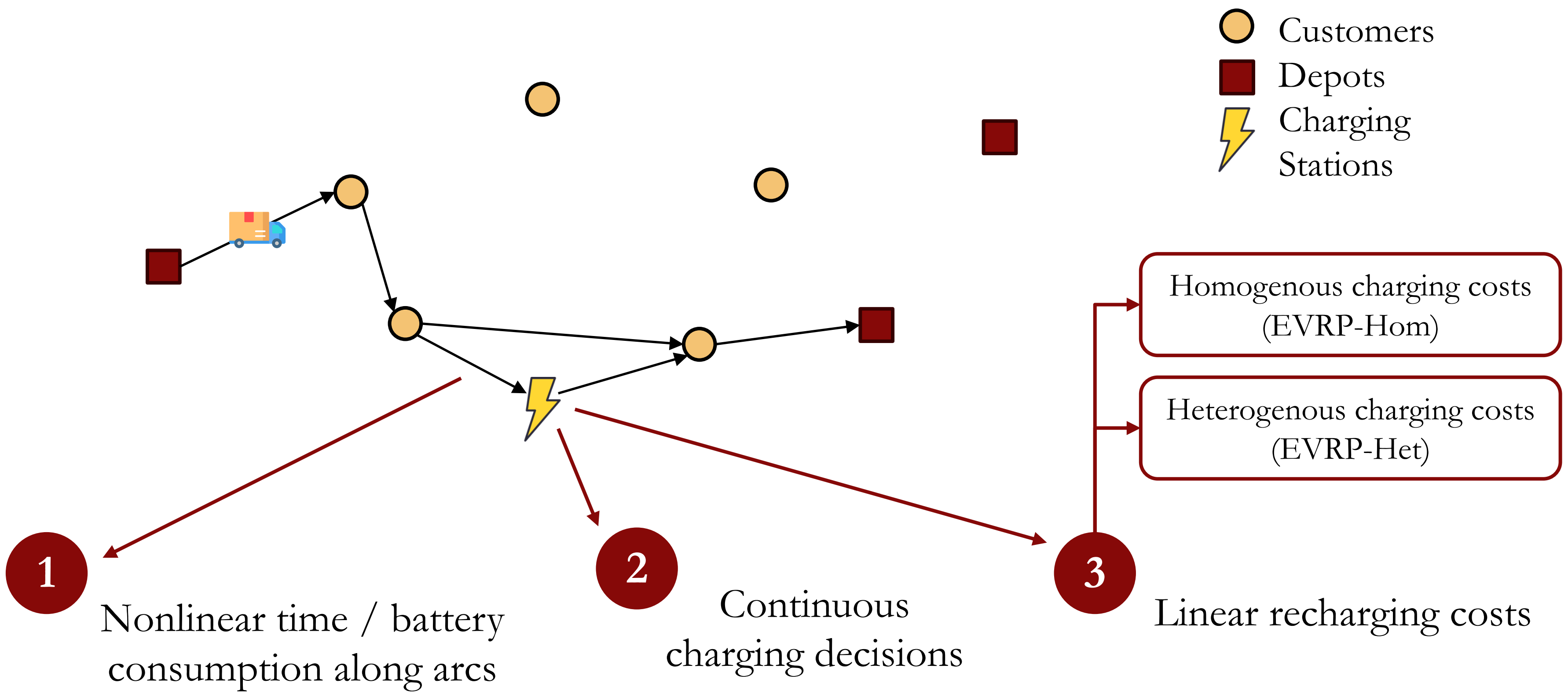
Grants included in the infrastructure law will help states build a charging network designed to reach highways in almost every corner of the country

By Ian Duncan

Updated February 10, 2022 at 1:46 p.m. EST | Published February 10, 2022 at 5:00 a.m. EST

New routing algorithms for electrified logistics

Electric Vehicle Routing Problem



Overview of literature

VRP: models and variants	VRP: approaches

VRP: models and variants

- VRP with Time Windows, Capacitated VRP

  routing (discrete) $\Rightarrow$ time, load (continuous)

- Electric VRP

  routing (discrete) ¦ charge (continuous)

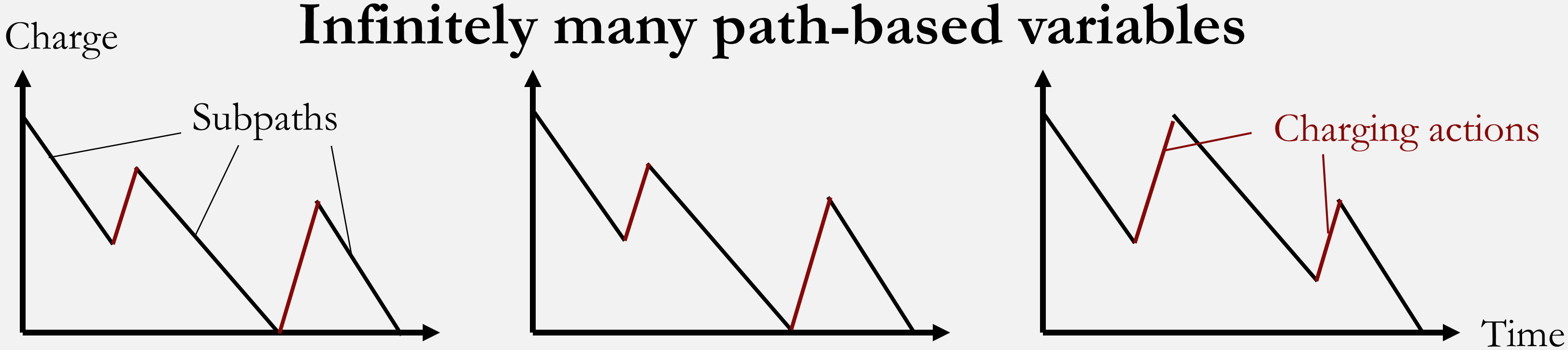
- EVRP-Het: **new**

VRP: approaches

- Metaheuristics
  - Large-scale neighborhood search
  - Genetic algorithms
- Exact methods
  - Column generation
  - Path-based label-setting
  - Branch-and-price-and-cut

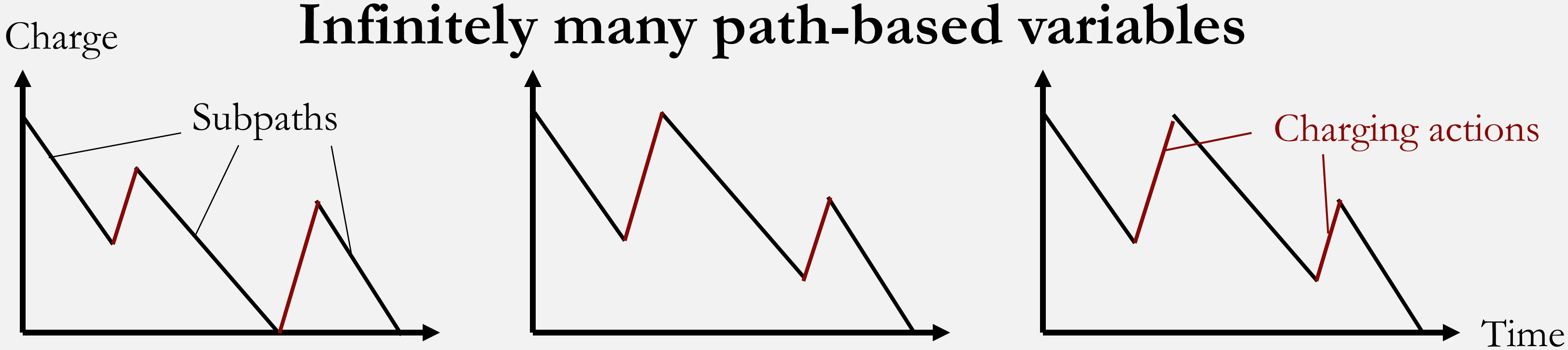
Challenges

Infinitely many path-based variables

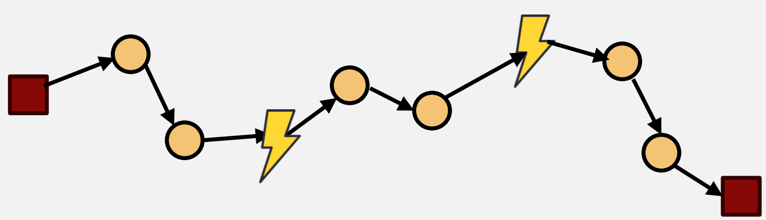


Challenges

Infinitely many path-based variables

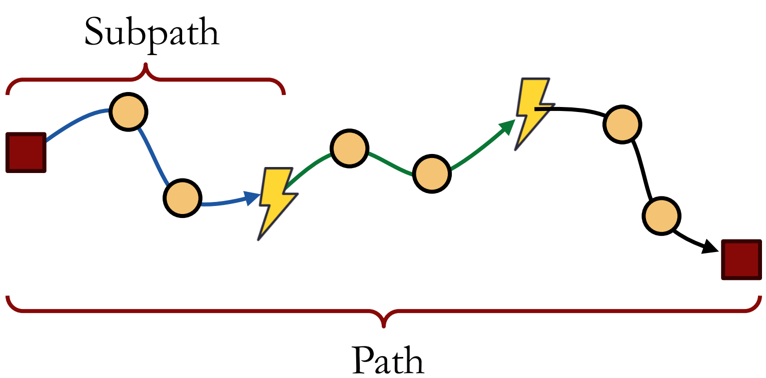


Long paths ⟶ Large DP in pricing problem

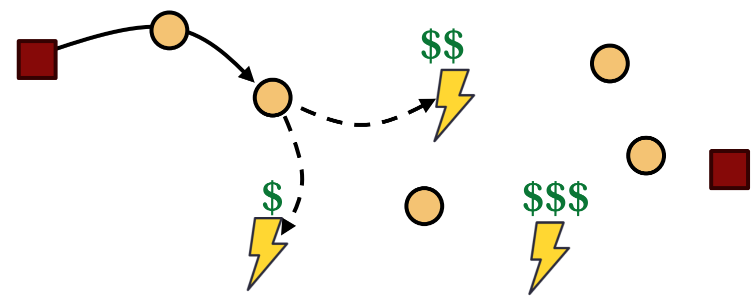


Contributions

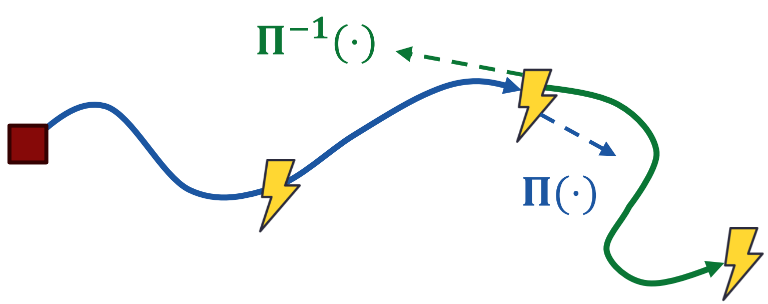
Two-level label-setting (2LLS)



Heterogenous charging costs



Acceleration



Forward and backward resources

Results + Impact

- 2-10x speedup
- 8% cost improvement over heuristics
- 5-20% charging cost saved (heterogenous charging costs)

Semi-infinite set-partitioning model

$$
\begin{aligned}
\min \quad & \sum_{p \in \mathcal{P}} c^p z^p \\
\text{s.t.} \quad & \sum_{p \in \mathcal{P}} \mathbb{1}\left(n^p_{\text{start}} = j\right) z^p = v_j^{\text{start}} && \forall \text{ depots } j \\
& \sum_{p \in \mathcal{P}} \mathbb{1}\left(n^p_{\text{start}} = j\right) z^p \geq v_j^{\text{end}} && \forall \text{ depots } j \\
& \sum_{p \in \mathcal{P}} \gamma_i^p z^p = 1 && \forall \text{ customers } i \\
& z^p \in \mathbb{Z}_{\geq 0} && \forall\, p \in \mathcal{P}
\end{aligned}
$$

- Set-partitioning formulation with path-based variables z^p
- Infinitely many variables
  - **Discrete** routing and timing decisions
  - **Continuous** charging decisions

Column generation

Restricted Master Problem

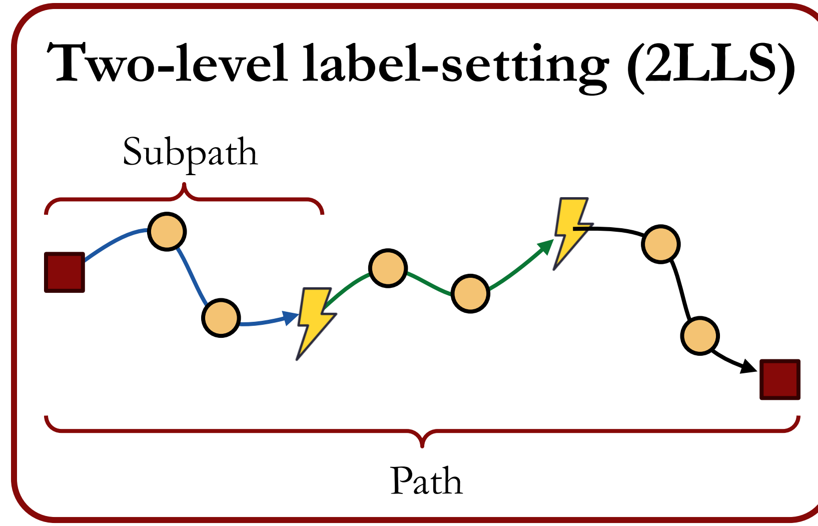
$$
\begin{aligned}
\min \quad & \sum_{p \in \mathcal{P}'} c^p z^p \\
\text{s.t.} \quad & \sum_{p \in \mathcal{P}'} \mathbb{1}\left(n^p_{\text{start}} = j\right) z^p = v^{\text{start}}_j && \forall \text{ depots } j && [\boldsymbol{\kappa}] \\
& \sum_{p \in \mathcal{P}'} \mathbb{1}\left(n^p_{\text{start}} = j\right) z^p \geq v^{\text{end}}_j && \forall \text{ depots } j && [\boldsymbol{\mu}] \\
& \sum_{p \in \mathcal{P}'} \gamma^p_i z^p = 1 && \forall \text{ customers } i && [\boldsymbol{\nu}] \\
& z^p \in \mathbb{R}_{\geq 0} \ \forall\, p \in \mathcal{P}'
\end{aligned}
$$

dual values $\boldsymbol{\kappa}, \boldsymbol{\mu}, \boldsymbol{\nu}$

paths not in $\mathcal{P}'$

Pricing Problem

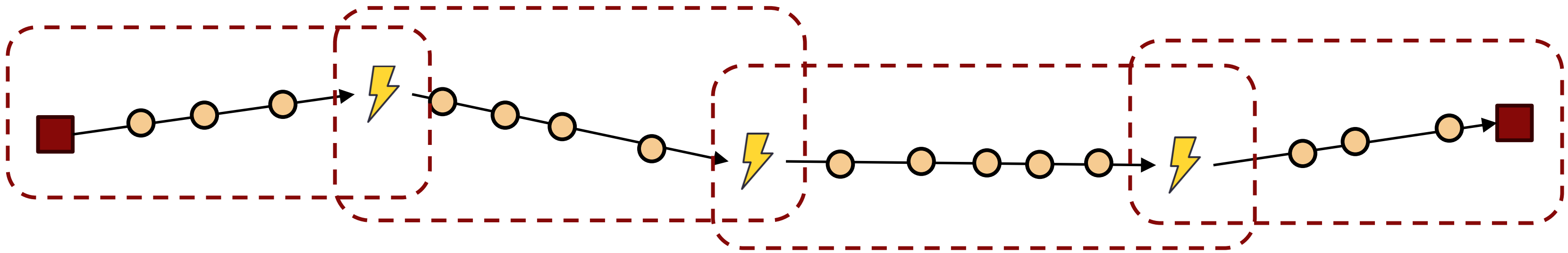
$$
\min_{p \in \mathcal{P}} \left\{ \bar{c}^p := c^p - \kappa_{\text{start}(p)} - \mu_{\text{end}(p)} - \sum_{i \in \mathcal{V}_C} \gamma^p_i \nu_i \right\}
$$



Two-level label-setting

in the pricing problem

Pricing problem in CG



- Finding paths of negative reduced cost via DP
  - Resource-Constrained Shortest Path Problem (RCSPP)[1]

Extend partial paths along edges

Prune "dominated" paths using domination criteria

p_1 dominates p_2 if $D(p_1) \leq D(p_2)$[2];

$$D(p) = \Big(\underbrace{\bar{c}(p)}_{\text{reduced cost}},\ \underbrace{t(p)}_{\text{time}},\ \underbrace{-b(p)}_{\text{(negative of) charge}}\Big)$$

[1] Irnich, S., & Desaulniers, G. (2005). Shortest Path Problems with Resource Constraints. In G. Desaulniers, J. Desrosiers, & M. M. Solomon (Eds.), *Column Generation* (pp. 33–65). Springer US. https://doi.org/10.1007/0-387-25486-2_2
[2] Desaulniers, G., Errico, F., Irnich, S., & Schneider, M. (2016). Exact Algorithms for Electric Vehicle-Routing Problems with Time Windows. Operations Research, 64(6), 1388–1405. https://doi.org/10.1287/opre.2016.1535

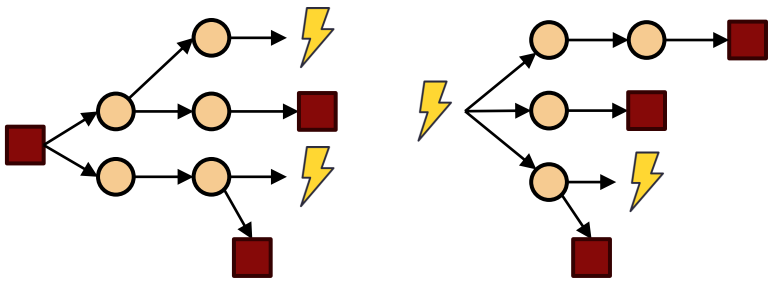
Key idea: two-level label-setting

Level 1: Generate **subpaths** s

- Label-setting, with domination criteria:

charge taken

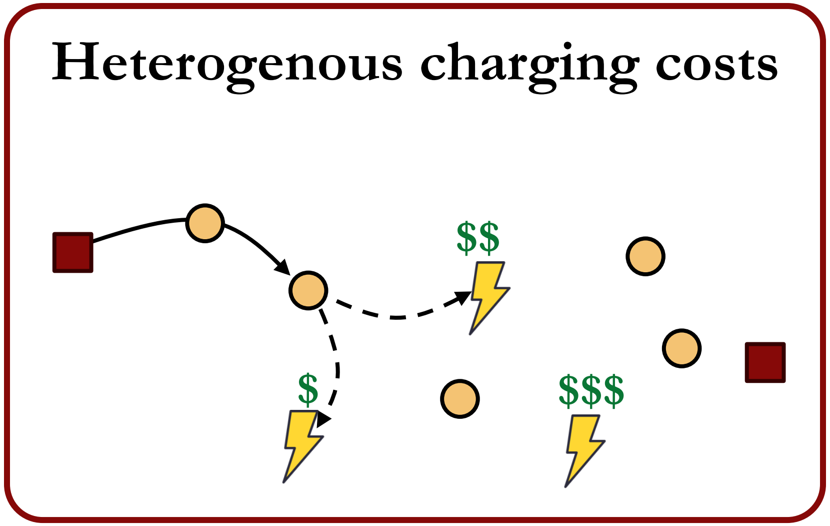
$$D(s) = \Big(\bar{c}(s),\ t(s),\ b(s)\Big)$$



Level 2: Extend **paths** p along subpaths s

$$D(p) = \Big(\bar{c}(p),\ t(p),\ -b(p)\Big)$$

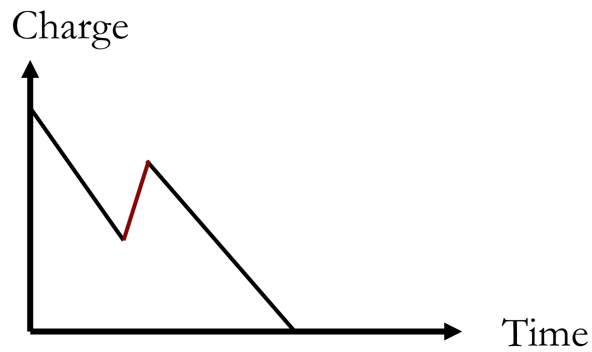
- WLOG, the charging decision between subpaths is the minimum possible
- Reduced cost of path = r.c. of subpaths + cost of charging



Heterogenous charging costs

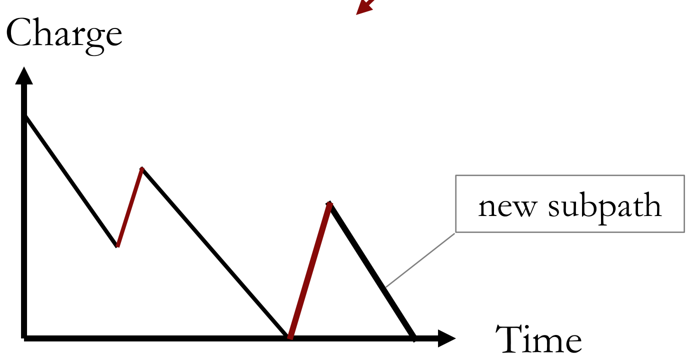
Two-level label-setting:
determining charging times

Earlier:
WLOG, charge to minimum needed

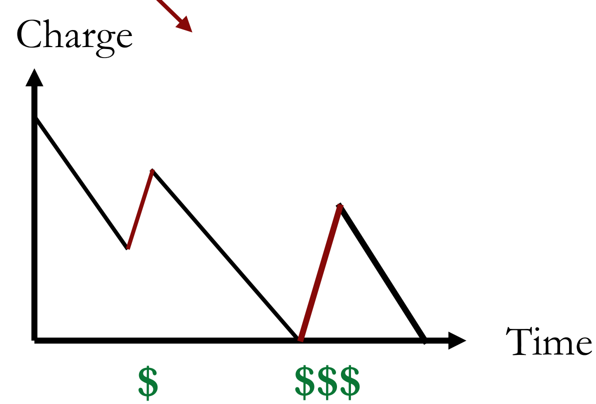


Now:
Need to rebalance charging actions dynamically!

homogenous charging costs



heterogenous charging costs



Two-level label-setting:
determining charging times

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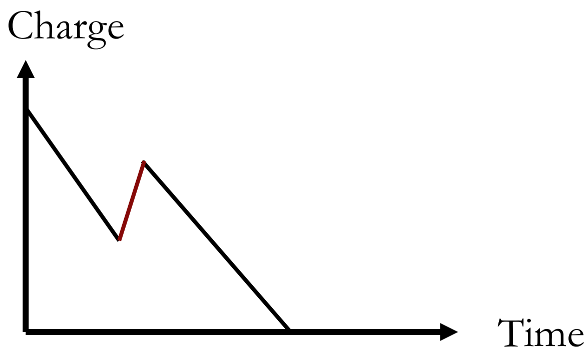

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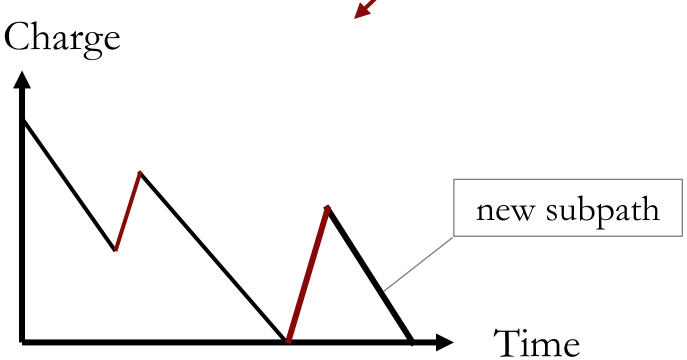
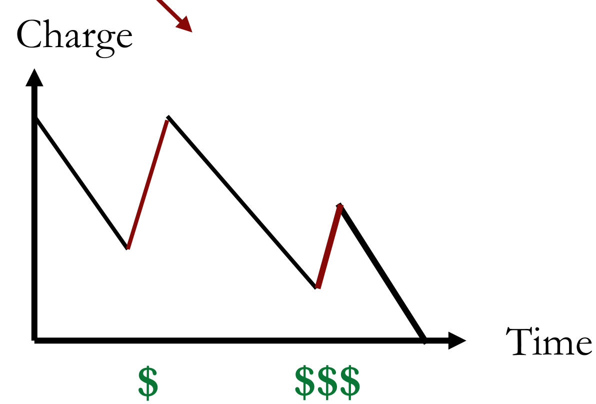
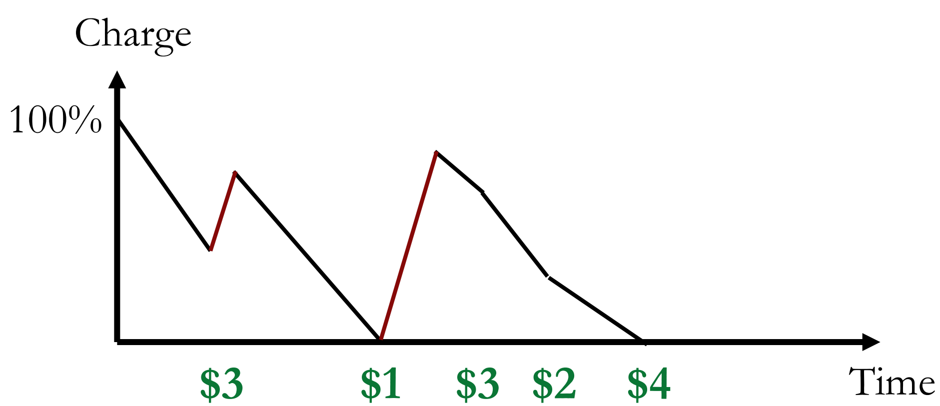
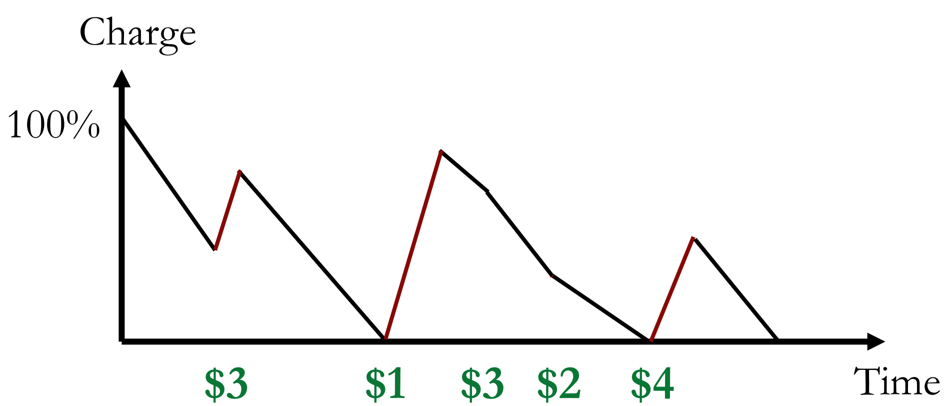


heterogenous charging costs



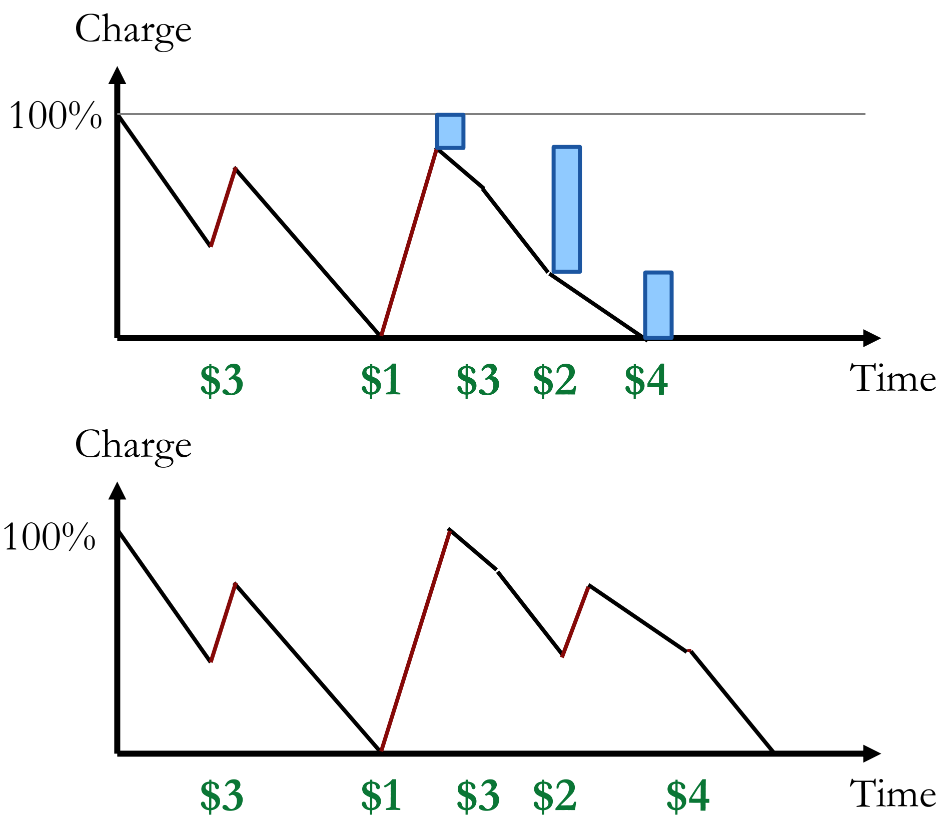
2L-LS: extra resources





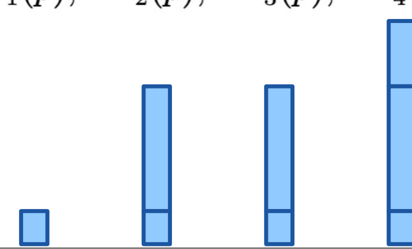
- 2L-LS decouples charging and routing decisions
- In stitching subpaths to paths, rebalance charging actions dynamically (DP)

2L-LS: extra resources



- 2L-LS decouples charging and routing decisions
- In stitching subpaths to paths, rebalance charging actions dynamically (DP)
- Extra resources in 2nd level representing "rebalancing opportunities":

$$D(p) = \Big(\bar{c}(p),\ t(p),\ -b(p),\ -Z_1(p), -Z_2(p), -Z_3(p), -Z_4(p), \ldots\Big)$$



Theory:

domination for paths and subpaths

A closer look at domination[1]

1st level (subpaths)

Def: $s_1 \succcurlyeq s_2$ if: $D(s_1) \leq D(s_2)$

$$D(s) = \Big(\bar{c}(s),\ t(s),\ b(s)\Big)$$

reduced cost | time taken | charge taken

2nd level (paths)

Def: $p_1 \succcurlyeq p_2$ if: $D(p_1) \leq D(p_2)$

$$D(p) = \Big(\bar{c}(p),\ t(p),\ -b(p)\Big)$$

reduced cost | time | (negative of) charge

$s_1 \succcurlyeq s_2$

2 $s_1 \oplus e \succcurlyeq s_2 \oplus e$ for any edge e

3 $p \oplus s_1 \succcurlyeq p \oplus s_2$ for any path p

$p_1 \succcurlyeq p_2$

1 $p_1 \oplus s \succcurlyeq p_2 \oplus s$ for any subpath s

Rigorous and generalizable framework for domination criteria

[1] Irnich, S., & Desaulniers, G. (2005). Shortest Path Problems with Resource Constraints. In G. Desaulniers, J. Desrosiers, & M. M. Solomon (Eds.), *Column Generation* (pp. 33–65). Springer US. https://doi.org/10.1007/0-387-25486-2_2

Key results

> **Theorem 1:** Two-level label-setting finds negative reduced-cost paths, or certifies that none exists

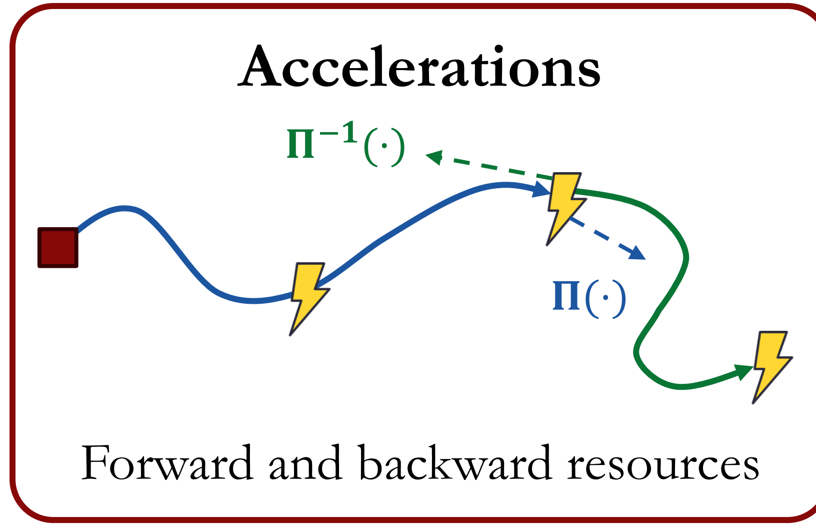
Proof idea:

- Use properties 1 2 3 arising from domination criteria

> **Theorem 2:** With two-level label-setting, CG converges **finitely** to LP optimum of EVRP

Proof idea:

- Infinitely many paths, but **finitely** many <u>subpath sequences</u> (sequences of subpaths, w/ routing but w/o charging decisions)



Accelerations:

Incorporating *ng*-relaxations and subset-row cuts

The *ng*-route relaxation[1]

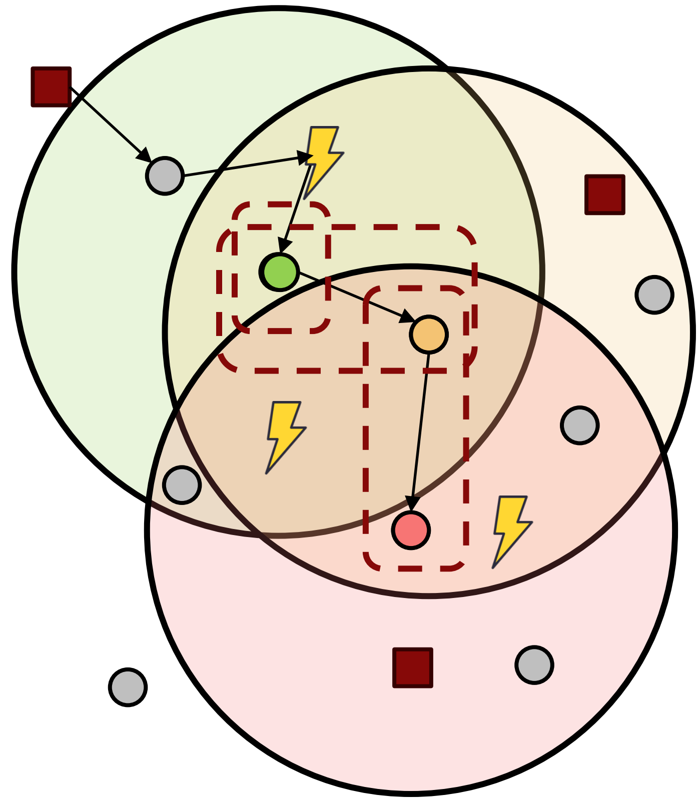
ng-route relaxation: intermediate between no and full elementarity

- Each customer i has a neighborhood N_i ($i \in N_i$)
- Each partial path has forward *ng*-set $\Pi(p)$: "nodes p cannot visit next"

$$\Pi(p) = \left\{ n_r \,\middle|\, n_r \in \bigcap_{\rho=r+1}^{m} N_{n_\rho},\ r \in \{0, \cdots, m-1\} \right\} \cup \{n_m\}$$

- Domination criteria uses *ng*-sets:

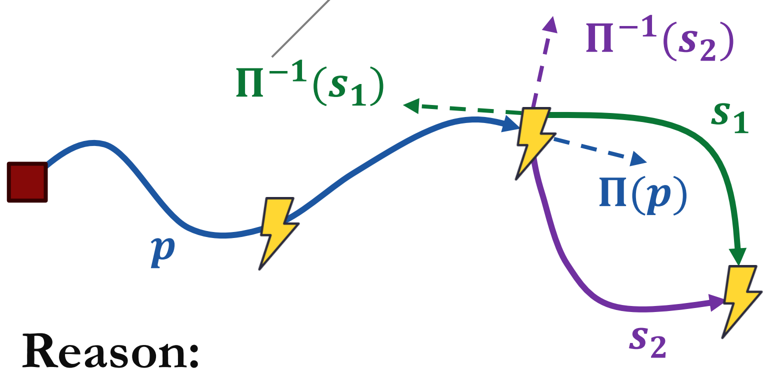
$$D(p) = \left(\bar{c}(p),\ t(p),\ -b(p),\ \{\mathbb{1}\,(i \in \Pi(p))\}_i \right)$$



[1] Baldacci, R., Mingozzi, A., & Roberti, R. (2011). New Route Relaxation and Pricing Strategies for the Vehicle Routing Problem. *Operations Research*, *59*(5), 1269–1283.

ng-routes in two-level label-setting

- **1st level (subpaths):**
  Also include <u>backward *ng*-set</u>[1]:
  "nodes which appear in all <u>previous</u> *ng*-neighborhoods"



Reason:

- Require this property:

$\boxed{s_1 \succcurlyeq s_2} \rightarrow$ 3 $\boxed{p \oplus s_1 \succcurlyeq p \oplus s_2 \text{ for any path } p}$

- $p \oplus s$ feasible $\Leftrightarrow \Pi(p) \cap \Pi^{-1}(s) = \{⚡\}$

- **2nd level (paths):**
  Only uses forward *ng*-set

Domination **properties** for subpaths and paths

↓

Design of domination **criteria** for variants

[1] Baldacci, R., Mingozzi, A., & Roberti, R. (2011). New Route Relaxation and Pricing Strategies for the Vehicle Routing Problem. *Operations Research*, *59*(5), 1269–1283.

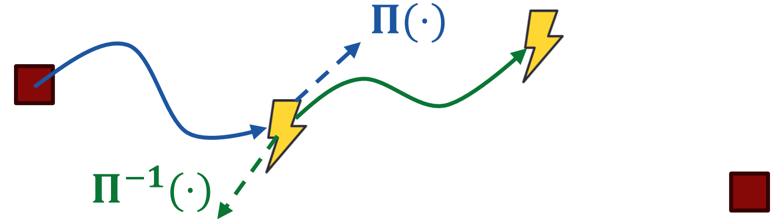
Subset-row cuts in 2L-LS

- Subset-row cuts[1]:
  - At most $\lfloor n/k \rfloor$ routes visiting at least k out of n customers[1] (Chvatal-Gomory cut of rank 1): $$\sum_{p\in\mathcal{P}}\sum_{i\in S}\gamma_i^p z^p = |S| \implies \sum_{p\in\mathcal{P}}\left\lfloor \frac{1}{k}\sum_{i\in S}\gamma_i^p \right\rfloor z^p \leq \left\lfloor \frac{|S|}{k} \right\rfloor$$
  - Track resource $\sum_{i\in S}\gamma_i^p \pmod k$ for each subset S in label-setting
- *Limited-memory* subset-row cuts (*lm*-SRCs)[2]
  - Only track $\sum_{i\in S}\gamma_i^p \pmod k$ inside a memory set M (zero it out when p steps outside M)
  - Not symmetric – requires forward/backward trackers $\vec{\alpha}(p), \overleftarrow{\alpha}(p)$

Domination **properties** for subpaths and paths → Design of domination **criteria** for variants

[1] Jepsen, M., Petersen, B., Spoorendonk, S., & Pisinger, D. (2008). Subset-Row Inequalities Applied to the Vehicle-Routing Problem with Time Windows. Operations Research, 56(2), 497–511. https://doi.org/10.1287/opre.1070.0449
[2] Pecin, D., Pessoa, A., Poggi, M., & Uchoa, E. (2017). Improved branch-cut-and-price for capacitated vehicle routing. *Mathematical Programming Computation*, *9*(1), 61–100. https://doi.org/10.1007/s12532-016-0108-8

Accelerations



Forward and backward resources
(from bidirectional label-setting)

Accelerations:

incorporating *ng*-relaxations…

- Additional resources from bidirectional LS
- Higher-dimensional domination

…and subset-row cuts

- Forward and backward cut quantities $\overrightarrow{\alpha_q}, \overleftarrow{\alpha_q}$ for cut q
- Weaker domination comparison for reduced costs

Results + Impact

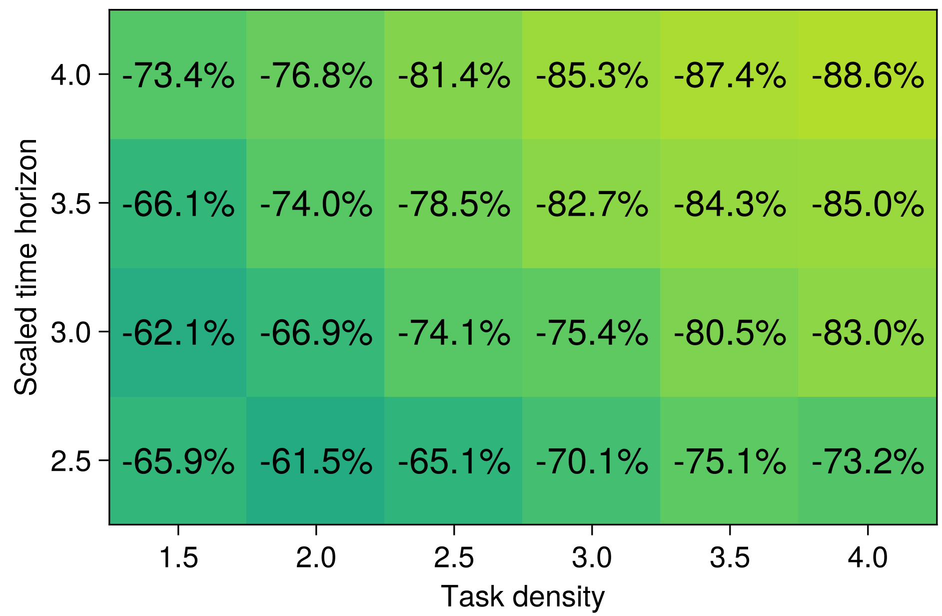
- 2-10x speedup
- 8% cost improvement over heuristics
- 5-20% charging cost saved (heterogenous charging costs)

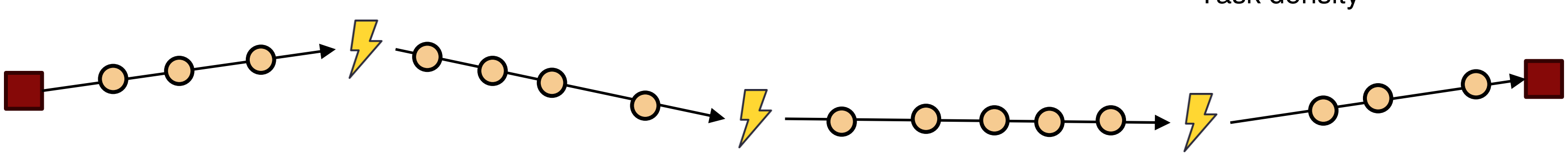
Results, practical impact

Result 1: 2L-LS is faster than LS

- Significant speedups against path-based benchmark
- Stronger improvement with:
  - Higher customer density ≈ **longer subpaths**
  - Longer time horizon ≈ **more subpaths per path**

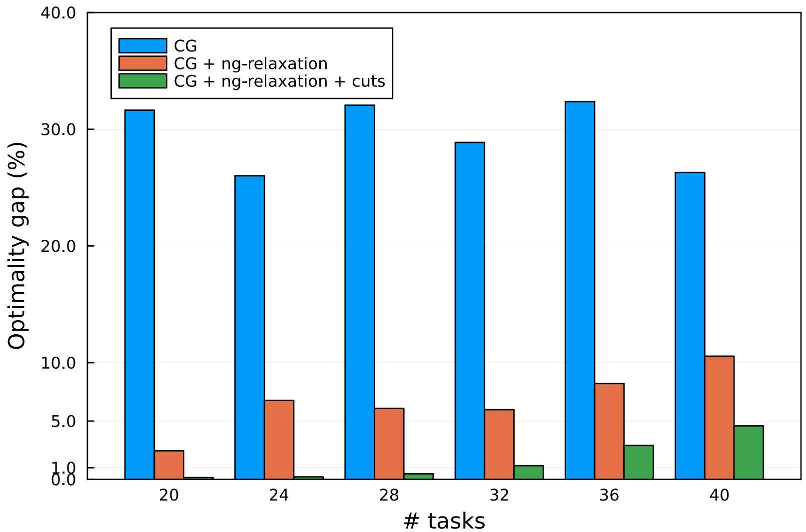
% time reduction vs. path-based benchmark



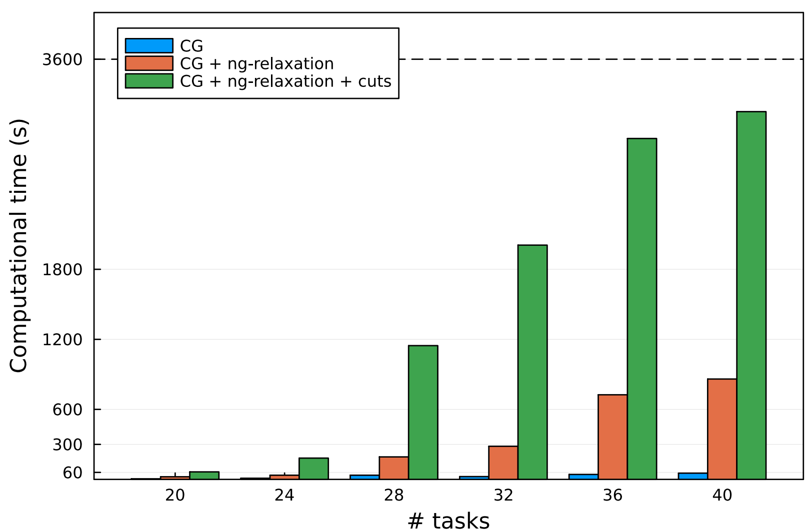


Result 2: Scalability

Optimality gap (%)



Computational time



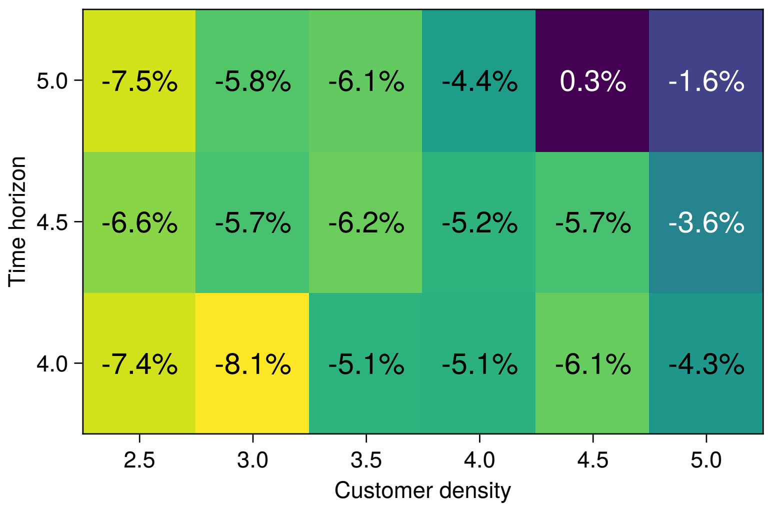
- Baseline column generation algorithm is fast and weak
- Iteratively tightening *ng*-relaxations gives the LP relaxation w/ elementary paths (and good solution quality)
- Cuts further narrow the gap, but take a long time

Result 3: Practical benefits

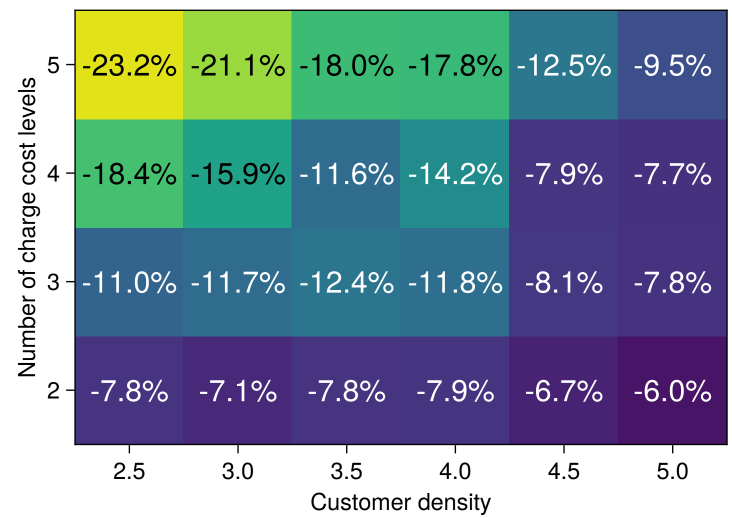
a Jointly optimizing charging and routing decisions, v.s. route-then-charge heuristic

b Using EVRP-Het algorithm instead of EVRP-Hom, with heterogenous charging costs

% cost reduction



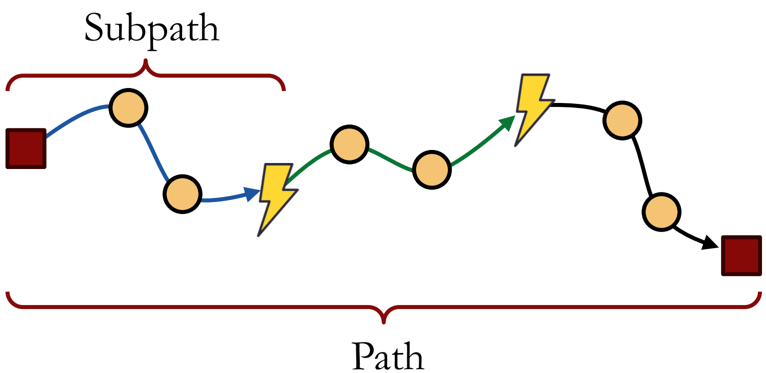
% reduction in charging costs



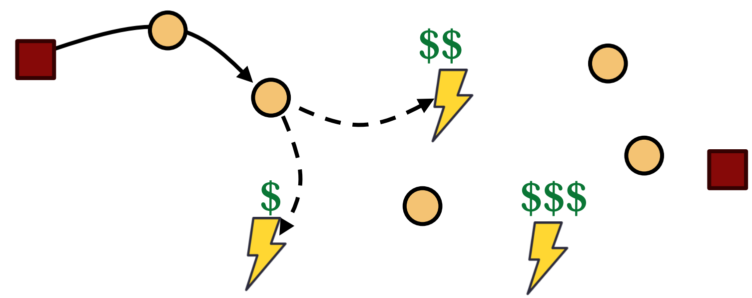
Benefits from large-scale optimization algorithms to support emerging vehicle technologies and operating models toward sustainable logistics

Contributions

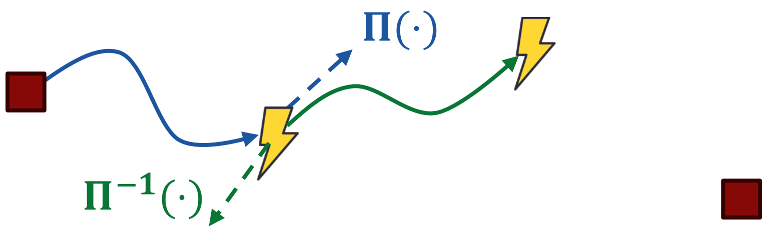
Two-level label-setting (2LLS)



Heterogenous charging costs



Accelerations



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Additional slides