

Electric Vehicle Routing: Subpath-Based Decomposition

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INFORMS Optimization Society Conference 2024
22 March 2024

Motivation, problem setting

Biden administration plan seeks elimination of transportation emissions

calls for a transition to electric vehicles and more walkable neighborhoods by 2050

A 40-ton Mercedes-Benz e-truck just drove 1,000 km with only one stop to charge



Michelle Lewis | Oct 5 2023 - 10:48 am PT | 66 Comments

LOGISTICS REPORT

California's Electric-Truck Drive Draws Startups Building Charging Networks

An aggressive emissions-slashing mandate means thousands of charging sites are needed in the coming years

Paul Berger [Follow](#)

July 29, 2023 7:00 am ET

Biden administration plan calls for \$5 billion network of electric-vehicle chargers along interstates

Grants included in the infrastructure law will help states build a charging network designed to reach highways in almost every corner of the country



By Ian Duncan

Updated February 10, 2022 at 1:46 p.m. EST | Published February 10, 2022 at 5:00 a.m. EST

New routing algorithms for electrified logistics

Contributions

Electric vehicle routing: subpath-based decomposition algorithm

Modeling

Electric vehicle routing: semi-infinite set-partitioning formulation with continuous time and continuous charge

Optimization

- Subpath-based decomposition algorithm for column generation subproblem
- Iteratively growing ng -route neighborhoods to yield linear relaxation solution with elementary paths
- Cutting planes to strengthen linear relaxation

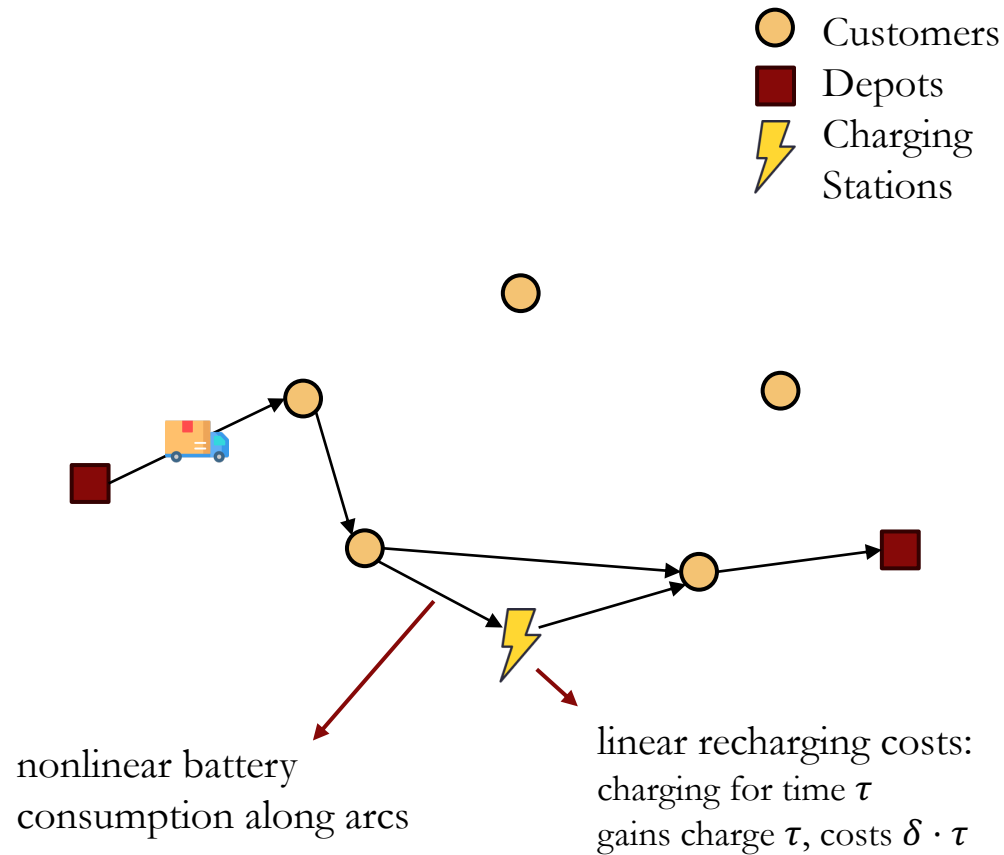
Computational results

Significantly outperforms path-based benchmark, and scales to realistic problem instances

Practical impact

Benefits over “business-as-usual” routing operations

The Electric VRP



Semi-infinite set-partitioning model

$$\begin{aligned} \min \quad & \sum_{p \in \mathcal{P}} c^p z^p && \text{(minimize total cost of paths)} \\ \text{s.t.} \quad & \sum_{p \in \mathcal{P}} \mathbb{1}(n_{\text{start}}^p = j) z^p = v_j^{\text{start}} && \forall \text{ depots } j \quad \text{(each depot } j \text{ starts with } v_j^{\text{start}} \text{ vehicles)} \\ & \sum_{p \in \mathcal{P}} \mathbb{1}(n_{\text{end}}^p = j) z^p \geq v_j^{\text{end}} && \forall \text{ depots } j \quad \text{(each depot } j \text{ ends with at least } v_j^{\text{end}} \text{ vehicles)} \\ & \sum_{p \in \mathcal{P}} \gamma_i^p z^p = 1 && \forall \text{ customers } j \quad \text{(each customer served once)} \\ & z^p \in \mathbb{Z}_+ && \forall p \in \mathcal{P} \end{aligned}$$

- Set-partitioning formulation with path-based variables z^p
- Infinitely many variables
 - **Discrete** routing and timing decisions (as in traditional VRP)
 - **Continuous** charging decisions (new to E-VRP)

Column generation

Restricted Master Problem

$$\begin{aligned}
 \min \quad & \sum_{p \in \mathcal{P}'} c^p z^p \\
 \text{s.t.} \quad & \sum_{p \in \mathcal{P}'} \mathbb{1}(n_{\text{start}}^p = j) z^p = v_j^{\text{start}} \quad \forall \text{ depots } j \quad [\kappa] \\
 & \sum_{p \in \mathcal{P}'} \mathbb{1}(n_{\text{end}}^p = j) z^p \geq v_j^{\text{end}} \quad \forall \text{ depots } j \quad [\mu] \\
 & \sum_{p \in \mathcal{P}'} \gamma_i^p z^p = 1 \quad \forall \text{ customers } j \quad [\nu] \\
 & z^p \in \mathbb{Z}_+ \quad \forall p \in \mathcal{P}'
 \end{aligned}$$

dual values κ, μ, ν

Pricing Problem

$$\min_{p \in \mathcal{P}} \left\{ \bar{c}^p := c^p - \kappa_{\text{start}(p)} - \mu_{\text{end}(p)} - \sum_{i \in \mathcal{V}_C} \gamma_i^p \nu_i \right\}$$

paths not in \mathcal{P}'

Challenges:

1. How to solve the pricing problem?
2. How to ensure finite convergence?

Challenges

1. **The pricing problem in CG**

2. Finite termination of CG

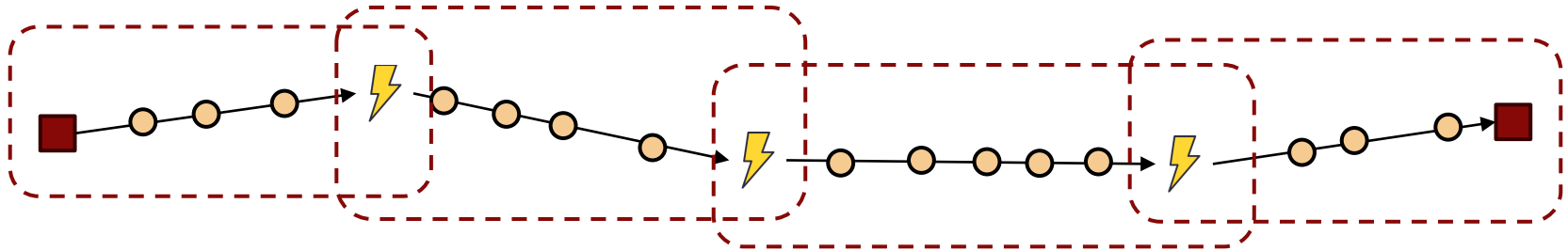
3. Path elementarity

4. Eliminating fractional solutions

} Required for correctness of CG
in solving LP relaxation of EVRP

} Acceleration strategies and
relaxation tightening

Pricing problem in CG



- Finding paths of negative reduced cost via DP
 - Resource-Constrained Shortest Path Problem (RCSPP)^[1]

Extend partial paths
along edges

Prune “dominated” paths
using domination criteria

$$D(p) = (\bar{c}(p), t(p), -b(p))$$

reduced cost time (negative of)
charge

Challenges:

1. Grows exponentially with no. of customers
2. How to determine charging time?

→ Extra labels^[2]

[1] Irnich, S., & Desaulniers, G. (2005). Shortest Path Problems with Resource Constraints. In G. Desaulniers, J. Desrosiers, & M. M. Solomon (Eds.), *Column Generation* (pp. 33–65). Springer US. https://doi.org/10.1007/0-387-25486-2_2

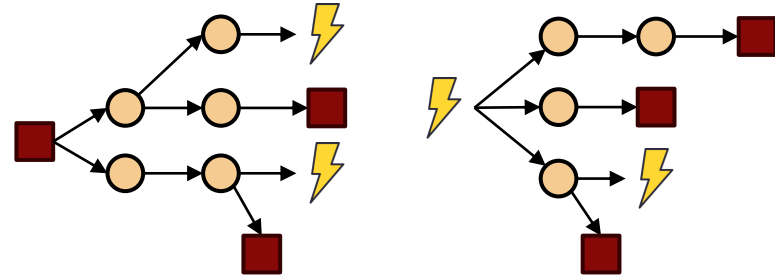
[2] Desaulniers, G., Errico, F., Irnich, S., & Schneider, M. (2016). Exact Algorithms for Electric Vehicle-Routing Problems with Time Windows. *Operations Research*, 64(6), 1388–1405. <https://doi.org/10.1287/opre.2016.1535>

Key idea: generate-and-stitch

Step 1: Generate subpaths s

- Label-setting, with domination criteria:

$$D(s) = (\bar{c}(s), t(s), b(s))$$



Step 2: Extend paths p along subpaths s

- A subpath valid at time 0 is still valid at time t with the same reduced cost
- Charging action between subpaths is the minimum possible
- Reduced cost of path =
r.c. of subpaths + r.c. of charging actions

A closer look at domination

Our work:

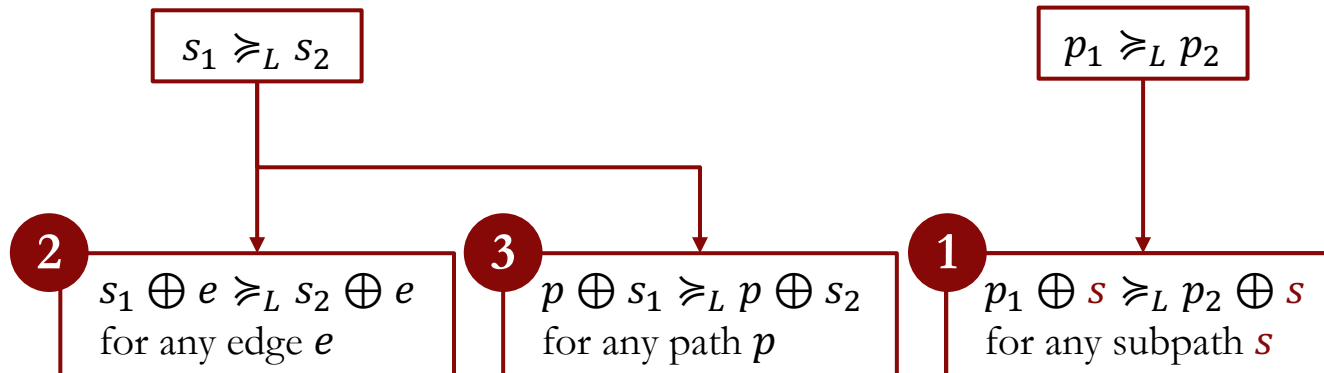
Def: $s_1 \succcurlyeq_L s_2$ if: $L(s_1) \leq L(s_2)$
 $D(s) = (\bar{c}(s), t(s), b(s))$

reduced time charge
cost taken taken

Traditionally^[1]:

Def: $p_1 \succcurlyeq_L p_2$ if: $L(p_1) \leq L(p_2)$
 $D(p) = (\bar{c}(p), t(p), -b(p))$

reduced time (negative of)
cost charge



Key results

Theorem 1: Generate-and-stitch finds negative reduced-cost paths, or certifies that none exists

Proof (sketch):

- Use properties **1** **2** **3** arising from domination criteria

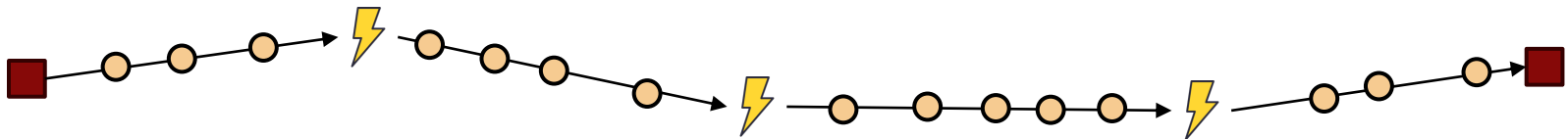
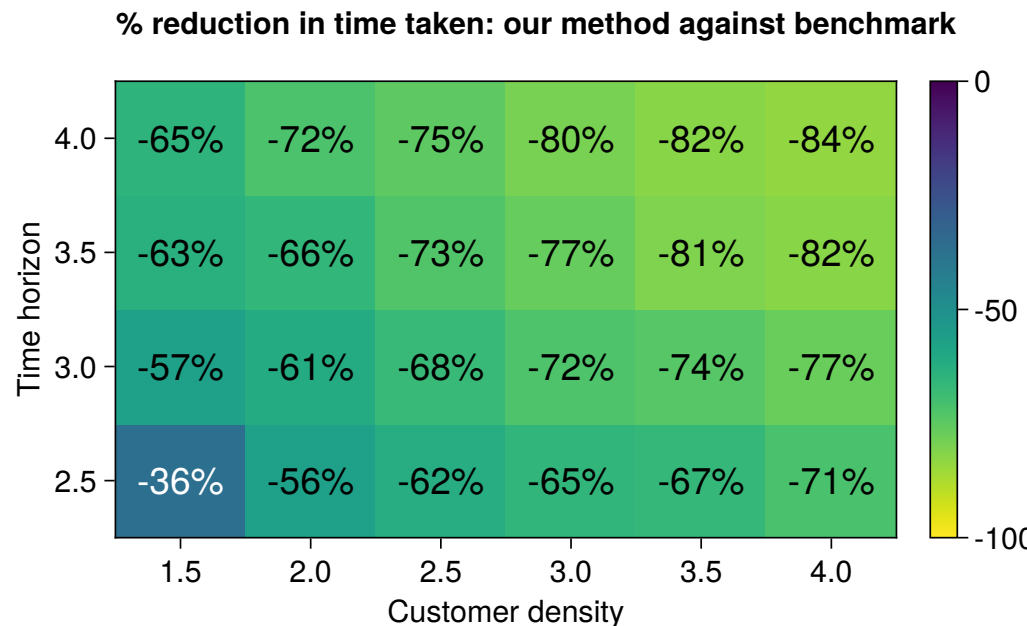
Theorem 2: With generate-and-stitch, CG converges **finitely** to LP optimum of EVRP

Proof (sketch):

- infinitely many paths, but finitely many subpath sequences
- once a path is added to RMP, no other path w/ same subpath sequence will be added in future iterations

Comparison to benchmark

- Significant speedups against path-based benchmark
- Stronger improvement with:
 - Higher customer density
≈ **longer subpaths**
 - Longer time horizon
≈ **more subpaths per path**



Challenges

1. The pricing problem in CG
2. Finite termination of CG
3. **Path elementarity**
4. Eliminating fractional solutions

The elementarity constraint

- Ideally, each path serves each customer at most once! (elementarity)
- Affects the structure of the label-setting in the pricing problem:

Option 1:

Ignore elementarity

Option 2:

Enforce elementarity

Q: can we get the
best of both worlds?

One binary resource per customer^[1]:

$$D(p) = \left(\bar{c}(p), t(p), -b(p), \overbrace{\gamma_1^p, \dots, \gamma_n^p} \right)$$

+ Computationally cheap!

— Good LP solutions, but
bad IP solutions

— Expensive: NP-hard^[2]

+ Better IP solutions

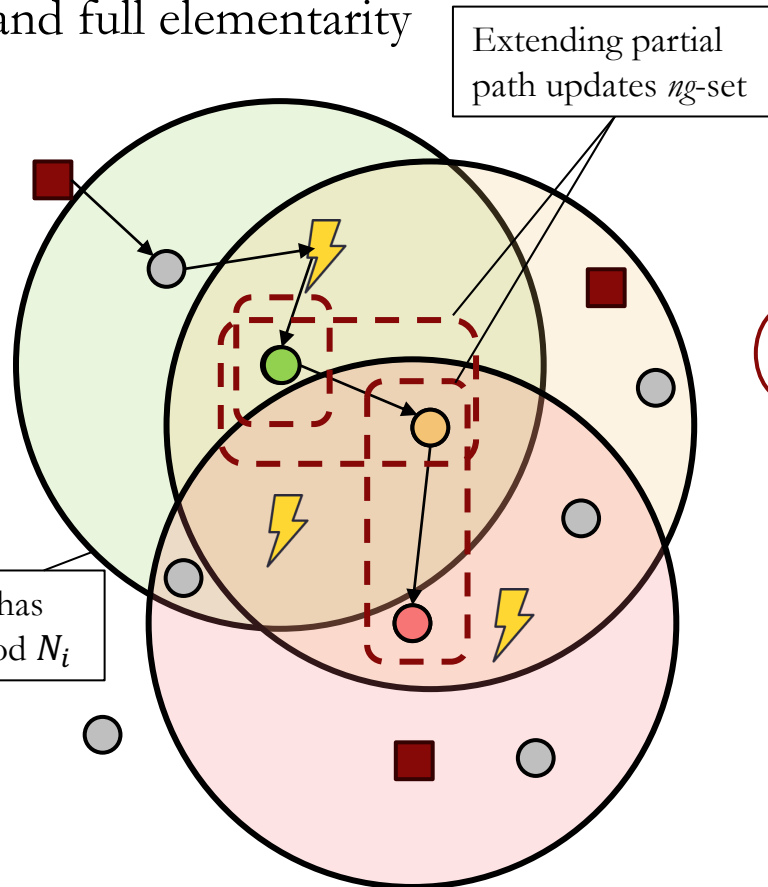
[1] Beasley, J. E., & Christofides, N. (1989). An algorithm for the resource constrained shortest path problem. *Networks*, 19(4), 379–394. <https://doi.org/10.1002/net.3230190402>

[2] Dror, M. (1994). Note on the Complexity of the Shortest Path Models for Column Generation in VRPTW. *Operations Research*, 42(5), 977–978. <https://doi.org/10.1287/opre.42.5.977>

Adaptive^[2] *ng*-relaxations^[1]

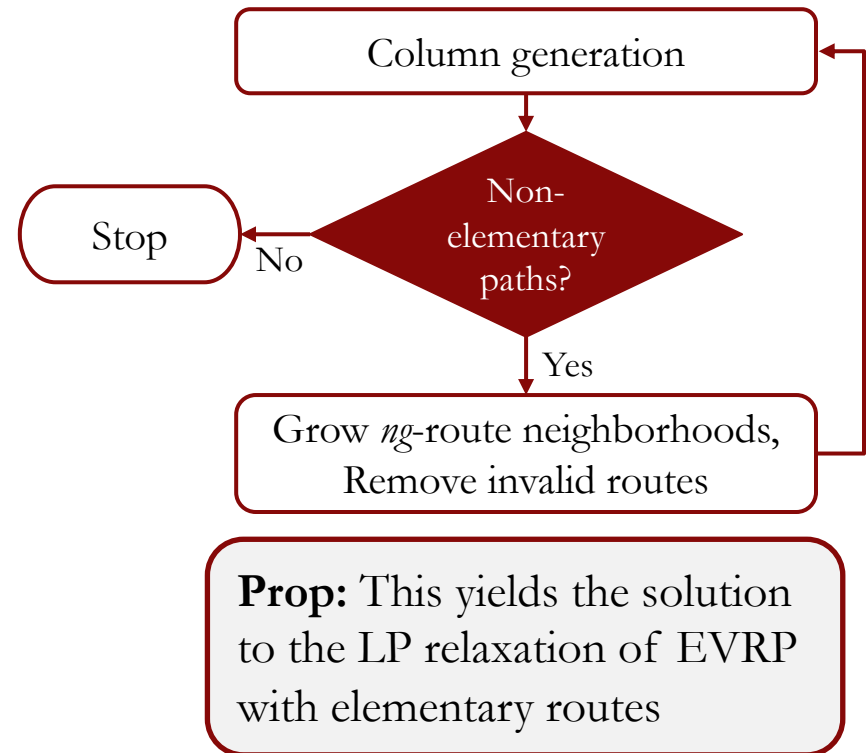
ng-route relaxation^[1]:

Interpolates between
no and full elementarity



Nested *ng*-route relaxations

Start with a loose *ng*-relaxation,
tighten when necessary^[2]!



[1] Baldacci, R., Mingozzi, A., & Roberti, R. (2011). New Route Relaxation and Pricing Strategies for the Vehicle Routing Problem. *Operations Research*, 59(5), 1269–1283.

[2] Martinelli, R., Pecin, D., & Poggi, M. (2014). Efficient elementary and restricted non-elementary route pricing. *European Journal of Operational Research*, 239(1), 102–111. <https://doi.org/10.1016/j.ejor.2014.05.005>

ng-routes in generate-and-stitch

Traditionally:

- Forward labelling keeps track of **forward** *ng*-sets [1]

$$\Pi(P) = \left\{ i_r : i_r \in \bigcap_{s=r+1}^k N_{i_s}, r = 1, \dots, k-1 \right\} \cup \{i_k\}.$$

- Backward labelling (bidirectional label setting) tracks **backward** *ng*-sets [1]

$$\Pi^{-1}(\bar{P}) = \{i_k\} \cup \left\{ i_r : i_r \in \bigcap_{s=k}^{r-1} N_{i_s}, r = k+1, \dots, h \right\}.$$

Our work:

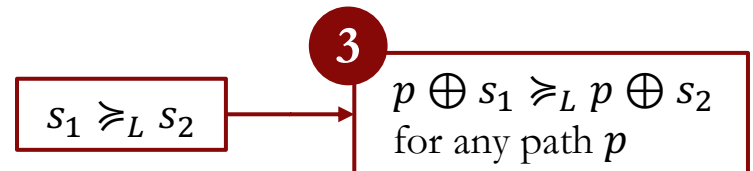
- Path domination criteria include forward *ng*-set inclusions:

$$L(p) = \left(\bar{c}(p), t(p), -b(p), \{ \mathbb{1}(i \in \Pi(p)) \}_i \right)$$

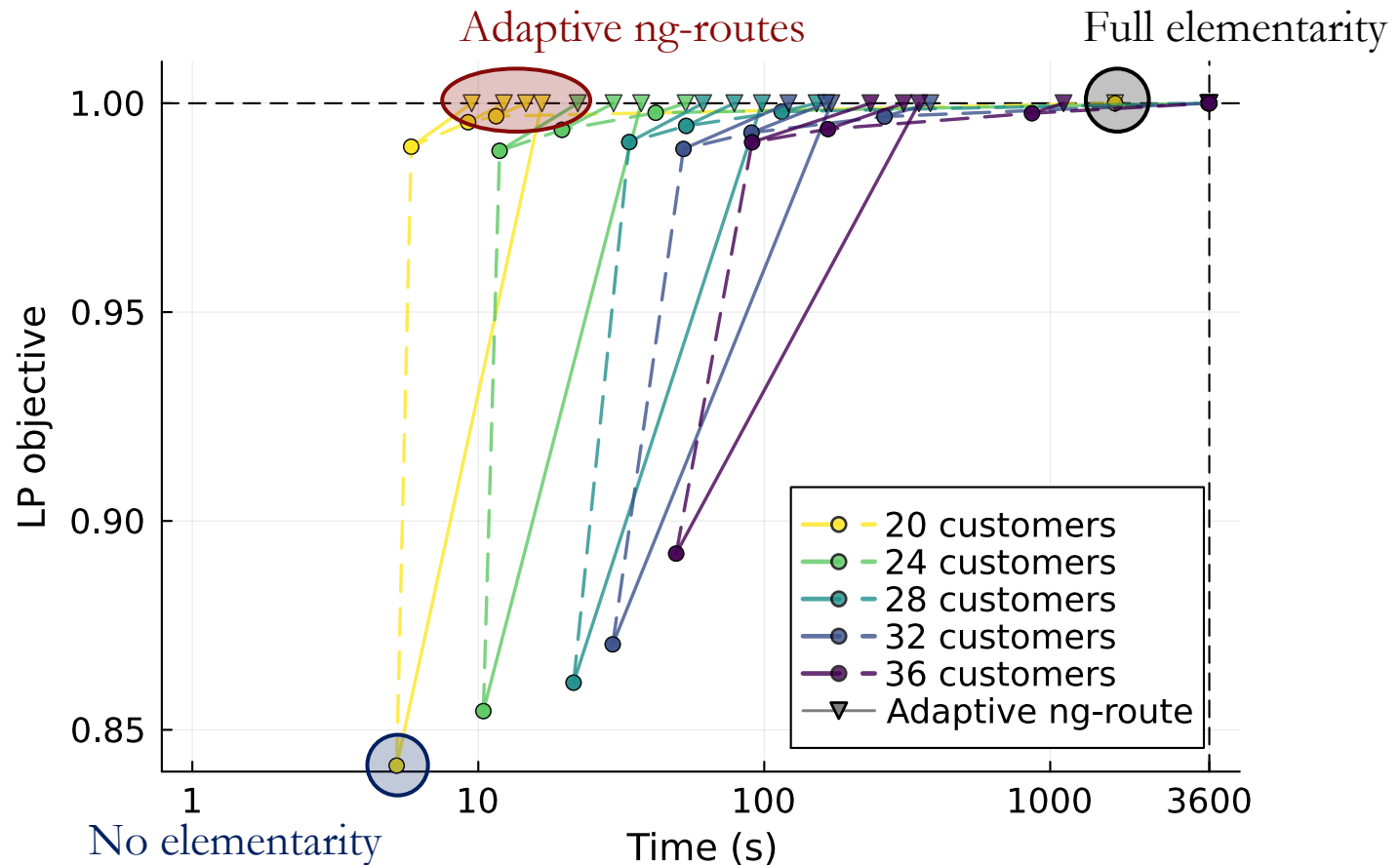
- Subpath domination criteria include **both** forward and backward *ng*-sets:

$$L(s) = \left(\bar{c}(s), t(s), b(s), \{ \mathbb{1}(i \in \Pi(p)) \}_i, \{ \mathbb{1}(i \in \Pi^{-1}(p)) \}_i \right)$$

Reason: need $L(s)$ to satisfy:



Benefits of adaptive *ng*-relaxations



Challenges



1. The pricing problem in CG
2. Finite termination of CG
3. Path elementarity
4. **Eliminating fractional solutions**

Subset-row cuts

- Consider a cut defined by a subset S of customers:
 - At most $\lfloor n/k \rfloor$ routes visiting at least k out of n customers^[1]
(Chvatal-Gomory cut of rank 1)

$$\sum_{p \in \mathcal{P}} \sum_{i \in S} \gamma_i^p z^p = |S| \implies \sum_{p \in \mathcal{P}} \left\lfloor \frac{1}{k} \sum_{i \in S} \gamma_i^p \right\rfloor z^p \leq \left\lfloor \frac{|S|}{k} \right\rfloor$$

- Non-robust* cuts which change subproblem structure (new duals)
 - Track resource $\sum_{i \in S} \gamma_i^p \pmod{k}$ for each subset S
 - When resource hits 0, subtract dual from reduced cost
 - Track $\sum_{i \in S} \gamma_i^s \pmod{k}$ and $\sum_{i \in S} \gamma_i^p \pmod{k}$ for subpaths and paths respectively:

$$D(s) = \left(\dots, \left\{ \sum_{i \in S} \gamma_i^s \right\}_S \right)$$

$$D(p) = \left(\dots, \left\{ \sum_{i \in S} \gamma_i^p \right\}_S \right)$$

Limited-memory subset-row cuts

- Limited-memory subset-row cuts^[1] include a *memory neighborhood* for each cut
 - Smaller state space
 - Weaker cuts
- Requires tracking forward and backward criteria

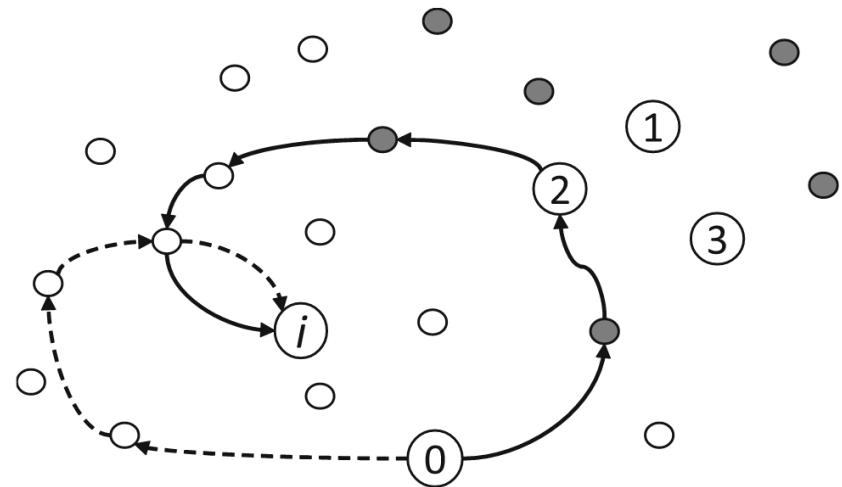
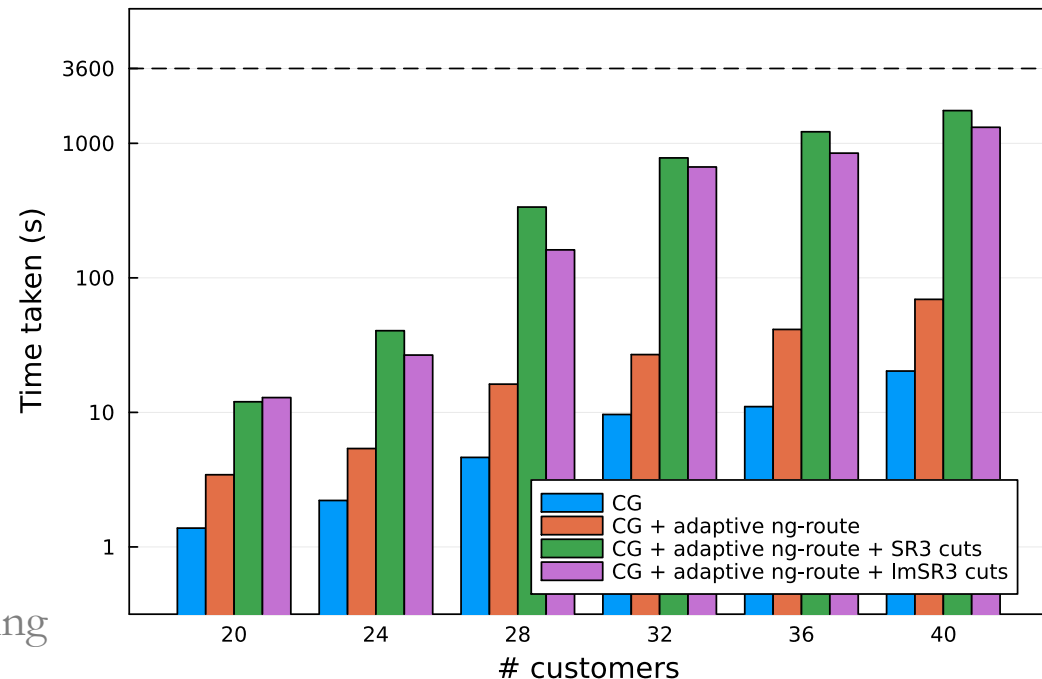
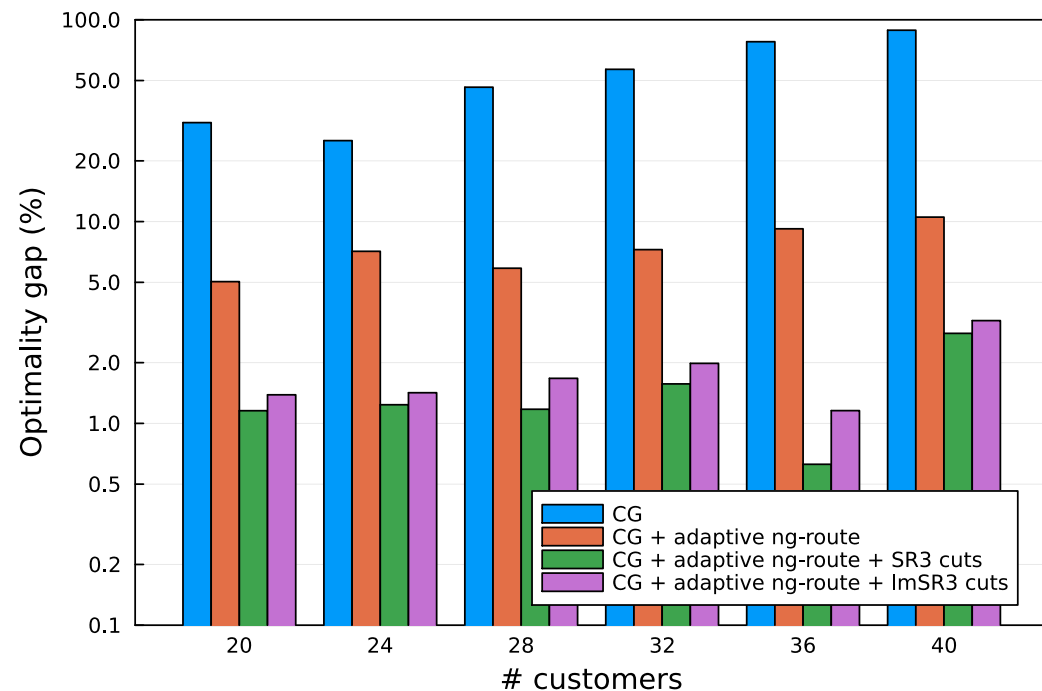


Fig. 2 Example illustrating the performance gain in the pricing when using lm-SRCs

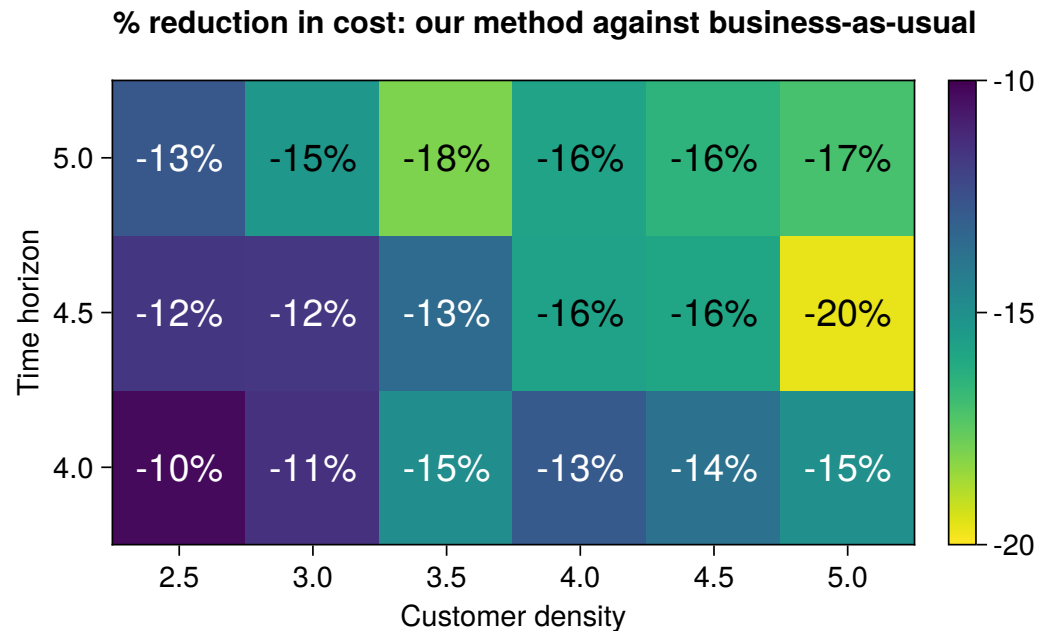
Computational results

- Cuts further close the optimality gap at the cost of more time
- IP solution obtained typically optimal
- *lm*-SR3 cuts provides an intermediate approach



The benefits of optimization

- Improvement compared to business-as-usual solution:
 - Solve a VRP w/o charge
 - Then optimize charging stations with fixed routes
- Benefit of **jointly** optimizing charging and routing decisions



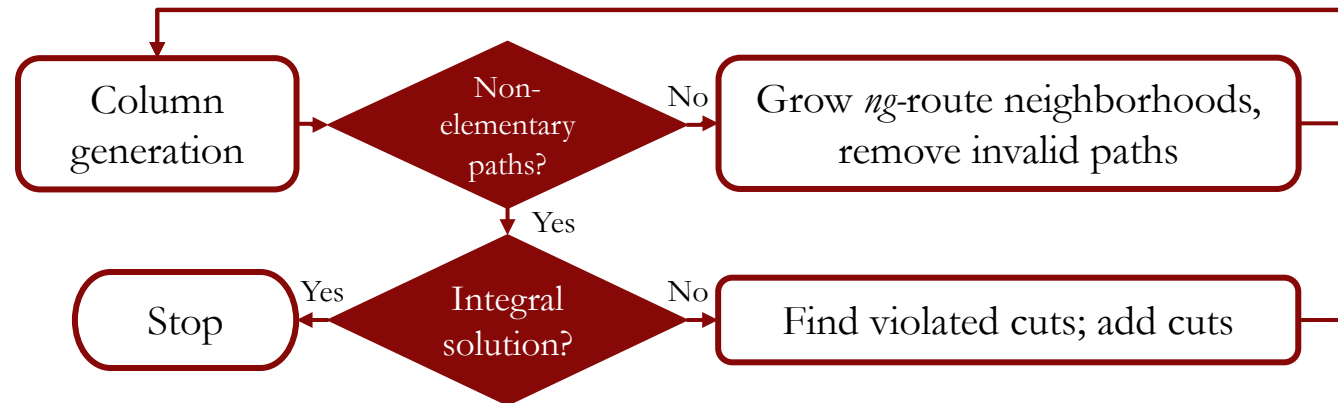
Summary

Electric vehicle routing: subpath-based decomposition algorithm

Modeling

Electric vehicle routing: semi-infinite set-partitioning formulation with continuous time and continuous charge

Optimization



Computational results

Significantly outperforms path-based benchmark, and scales to realistic problem instances

Practical impact

Benefits over “business-as-usual” routing operations

Additional slides

Comparison with literature

	[1]:	Our work:
Setting	<ul style="list-style-type: none">• Continuous time and charge• Single / multiple recharges, partial / full recharging• Time windows• No charging costs	<ul style="list-style-type: none">• Continuous time and charge• Multiple recharges, partial recharging• No time windows• Linear constant / heterogenous charging costs (charging τ at i costs $\delta_i \cdot \tau$)

Comparison with literature

	[1]:	Our work:
Methods	<ul style="list-style-type: none">• Bidirectional label-setting, with bidirectional criteria• <i>ng</i>-route relaxation• 2-path cuts and subset-row cuts• Branching	<ul style="list-style-type: none">• Unidirectional, two-level label-setting, with bidirectional criteria• Adaptive tightening of <i>ng</i>-route relaxations• SRC and lm-SRC• No branching