Electric Vehicle Routing: Subpath-Based Decomposition

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Motivation, problem setting

Biden administration plan seeks elimination of transportation emissions

calls for a transition to electric vehicles and more walkable neighborhoods by 2050

A 40-ton Mercedes-Benz e-truck just drove 1,000 km with only one stop to charge

Michelle Lewis | Oct 5 2023 - 10:48 am PT | 💬 66 Comments

LOGISTICS REPORT

California's Electric-Truck Drive Draws Startups Building Charging Networks

n aggressive emissions-slashing mandate means thousands of arging sites are needed in the coming years

Paul Berger [Follow]

y 29, 2023 7:00 am ET

Biden administration plan calls for $5 billion network of electric-vehicle chargers along interstates

Grants included in the infrastructure law will help states build a charging network designed to reach highways in almost every corner of the country

By Ian Duncan

Updated February 10, 2022 at 1:46 p.m. EST | Published February 10, 2022 at 5:00 a.m. EST

New routing algorithms for electrified logistics

Contributions

Electric vehicle routing: subpath-based decomposition algorithm

Modeling

Electric vehicle routing: semi-infinite set-partitioning formulation with continuous time and continuous charge

Optimization

- Subpath-based decomposition algorithm for column generation subproblem
- Iteratively growing *ng*-route neighborhoods to yield linear relaxation solution with elementary paths
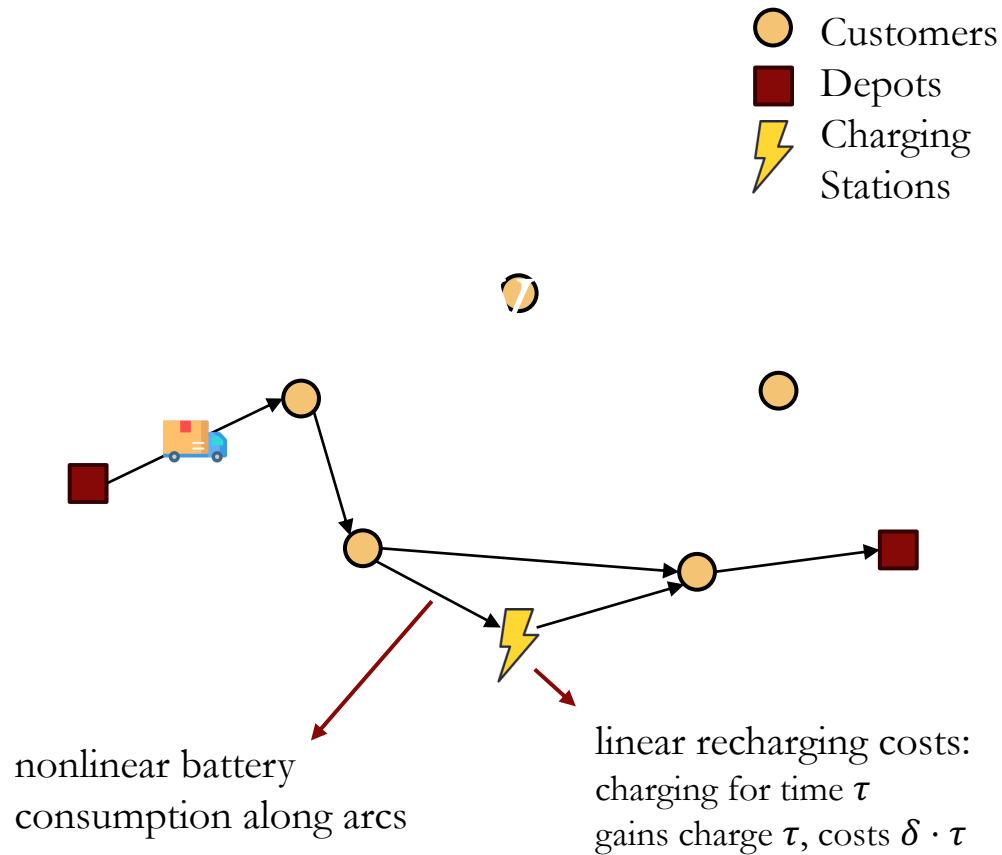- Cutting planes to strengthen linear relaxation

Computational results

Significantly outperforms path-based benchmark, and scales to realistic problem instances

Practical impact

Benefits over "business-as-usual" routing operations

The Electric VRP

Customers
Depots
Charging Stations

nonlinear battery consumption along arcs

linear recharging costs:
charging for time τ
gains charge τ, costs $\delta \cdot \tau$

Semi-infinite set-partitioning model

$$\min \quad \sum_{p \in \mathcal{P}} c^p z^p \qquad \qquad \text{(minimize total cost of paths)}$$

$$\text{s.t.} \quad \sum_{p \in \mathcal{P}} \mathbb{1}\left(n_{\text{start}}^p = j\right) z^p = v_j^{\textbf{start}} \quad \forall \text{ depots } j \qquad \text{(each depot } j \text{ starts with } v_j^{\textbf{start}} \text{ vehicles)}$$

$$\sum_{p \in \mathcal{P}} \mathbb{1}\left(n_{\text{end}}^p = j\right) z^p \geq v_j^{\text{end}} \quad \forall \text{ depots } j \qquad \text{(each depot } j \text{ ends with at least } v_j^{\textbf{end}} \text{ vehicles)}$$

$$\sum_{p \in \mathcal{P}} \gamma_i^p z^p = 1 \qquad \qquad \forall \text{ customers } j \quad \text{(each customer served once)}$$

$$z^p \in \mathbb{Z}_+ \qquad \qquad \forall \, p \in \mathcal{P}$$

- Set-partitioning formulation with path-based variables z^p

- Infinitely many variables
  - **Discrete** routing and timing decisions (as in traditional VRP)
  - **Continuous** charging decisions (new to E-VRP)

Column generation

Restricted Master Problem

$$\min \quad \sum_{p \in \mathcal{P}'} c^p z^p$$

$$\text{s.t.} \quad \sum_{p \in \mathcal{P}'} \mathbb{1}\left(n^p_{\text{start}} = j\right) z^p = v^{\text{start}}_j \quad \forall \text{ depots } j \quad [\boldsymbol{\kappa}]$$

$$\sum_{p \in \mathcal{P}'} \mathbb{1}\left(n^p_{\text{end}} = j\right) z^p \geq v^{\text{end}}_j \quad \forall \text{ depots } j \quad [\boldsymbol{\mu}]$$

$$\sum_{p \in \mathcal{P}'} \gamma^p_i z^p = 1 \quad \forall \text{ customers } j \quad [\boldsymbol{\nu}]$$

$$z^p \in \mathbb{Z}_+ \quad \forall\, p \in \mathcal{P}'$$

dual values $\boldsymbol{\kappa}, \boldsymbol{\mu}, \boldsymbol{\nu}$

Pricing Problem

$$\min_{p \in \mathcal{P}} \left\{ \bar{c}^p := c^p - \kappa_{\text{start}(p)} - \mu_{\text{end}(p)} - \sum_{i \in \mathcal{V}_C} \gamma^p_i \nu_i \right\}$$

paths not in $\mathcal{P}'$

Challenges:
1. How to solve the pricing problem?
2. How to ensure finite convergence?

Challenges

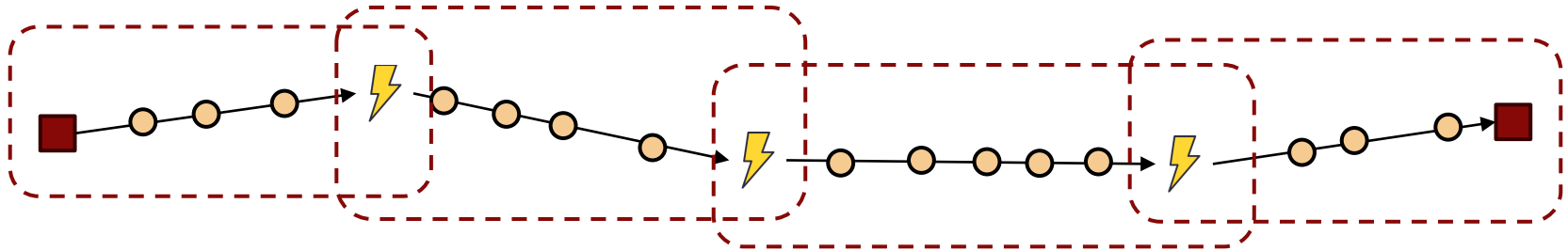1. **The pricing problem in CG**

2. **Finite termination of CG**

3. **Path elementarity**

4. **Eliminating fractional solutions**

Required for correctness of CG in solving LP relaxation of EVRP

Acceleration strategies and relaxation tightening

Pricing problem in CG

- Finding paths of negative reduced cost via DP

  - Resource-Constrained Shortest Path Problem (RCSPP)[1]

Extend partial paths along edges

Prune "dominated" paths using domination criteria

$$D(p) = \Big(\bar{c}(p), \ t(p), \ -b(p) \Big)$$

reduced cost   time   (negative of) charge

Challenges:

1. Grows exponentially with no. of customers

2. How to determine charging time?   ⟶   Extra labels[2]

[1] Irnich, S., & Desaulniers, G. (2005). Shortest Path Problems with Resource Constraints. In G. Desaulniers, J. Desrosiers, & M. M. Solomon (Eds.), *Column Generation* (pp. 33–65). Springer US. https://doi.org/10.1007/0-387-25486-2_2
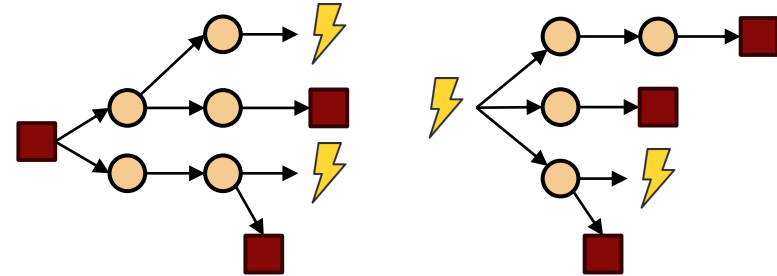[2] Desaulniers, G., Errico, F., Irnich, S., & Schneider, M. (2016). Exact Algorithms for Electric Vehicle-Routing Problems with Time Windows. Operations Research, 64(6), 1388–1405. https://doi.org/10.1287/opre.2016.1535

Key idea: generate-and-stitch

Step 1: Generate **subpaths** s

- Label-setting, with domination criteria:

$$D(s) = \Big(\bar{c}(s), \ t(s), \ b(s) \Big)$$

Step 2: Extend **paths** p along subpaths s

- A subpath valid at time 0 is still valid at time t with the same reduced cost
- Charging action between subpaths is the minimum possible
- Reduced cost of path =
  r.c. of subpaths + r.c. of charging actions
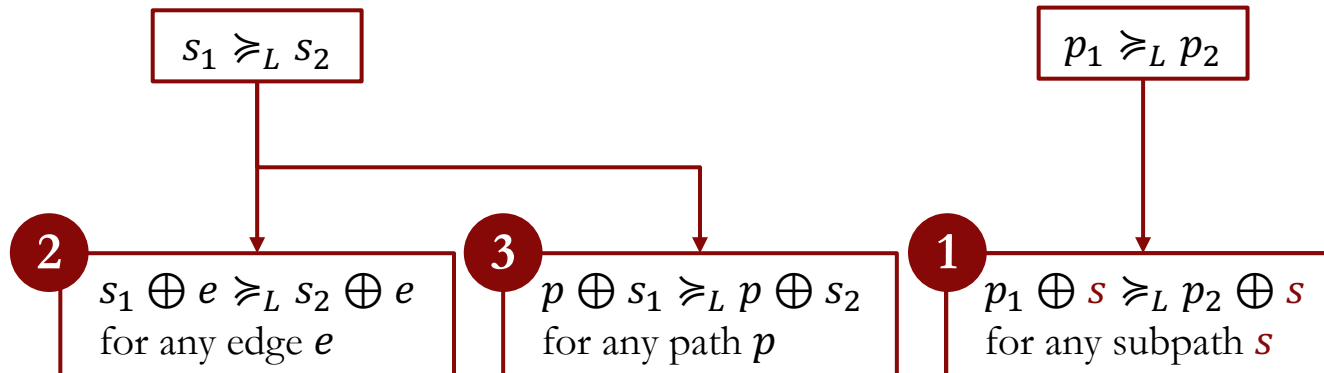
A closer look at domination

Our work:

Def: $s_1 \succcurlyeq_L s_2$ if: $L(s_1) \leq L(s_2)$

$$D(s) = \Big(\bar{c}(s), \ t(s), \ b(s) \Big)$$

reduced   time   charge
cost   taken   taken

Traditionally[1]:

Def: $p_1 \succcurlyeq_L p_2$ if: $L(p_1) \leq L(p_2)$

$$D(p) = \Big(\bar{c}(p), \ t(p), \ -b(p) \Big)$$

reduced   time   (negative of)
cost   charge

$s_1 \succcurlyeq_L s_2$

$p_1 \succcurlyeq_L p_2$

2
$s_1 \oplus e \succcurlyeq_L s_2 \oplus e$
for any edge e

3
$p \oplus s_1 \succcurlyeq_L p \oplus s_2$
for any path p

1
$p_1 \oplus s \succcurlyeq_L p_2 \oplus s$
for any subpath s

[1] Irnich, S., & Desaulniers, G. (2005). Shortest Path Problems with Resource Constraints. In G. Desaulniers, J. Desrosiers, & M. M. Solomon (Eds.), *Column Generation* (pp. 33–65). Springer US. https://doi.org/10.1007/0-387-25486-2_2

Key results

Theorem 1: Generate-and-stitch finds negative reduced-cost paths,
or certifies that none exists

Proof (sketch):
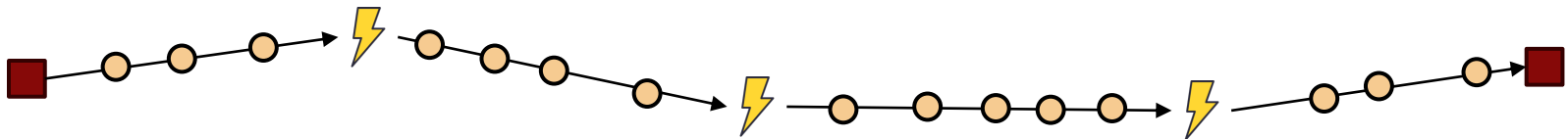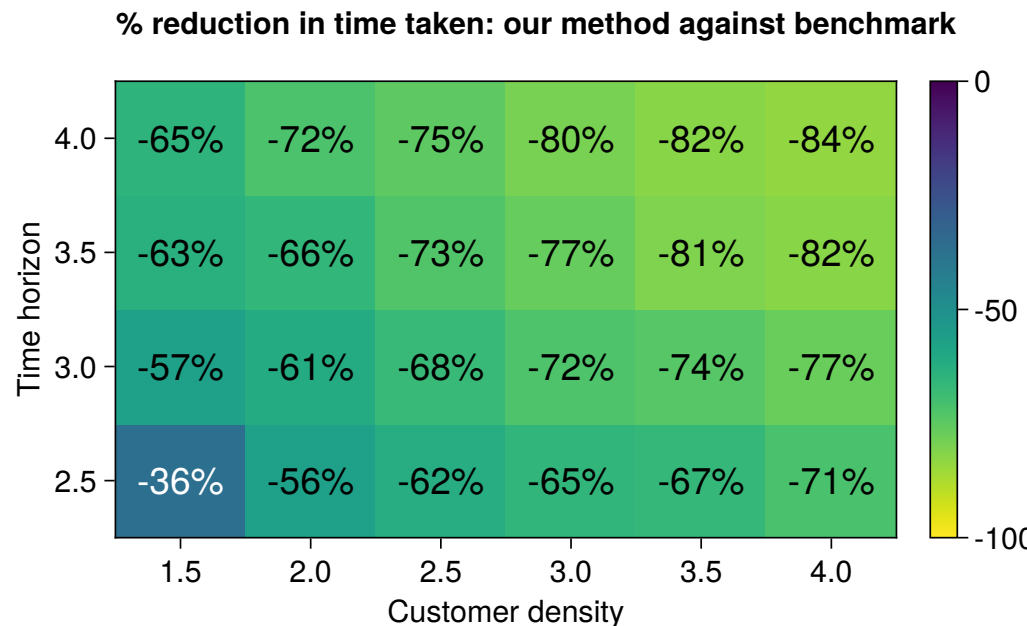- Use properties **1** **2** **3** arising from domination criteria

Theorem 2: With generate-and-stitch,
CG converges **finitely** to LP optimum of EVRP

Proof (sketch):
- infinitely many paths, but finitely many subpath sequences
- once a path is added to RMP, no other path w/ same subpath sequence
  will be added in future iterations

Comparison to benchmark

- Significant speedups against path-based benchmark
- Stronger improvement with:
  - Higher customer density
    ≈ **longer subpaths**
  - Longer time horizon
    ≈ **more subpaths per path**

% reduction in time taken: our method against benchmark

Time horizon	1.5	2.0	2.5	3.0	3.5	4.0
4.0	-65%	-72%	-75%	-80%	-82%	-84%
3.5	-63%	-66%	-73%	-77%	-81%	-82%
3.0	-57%	-61%	-68%	-72%	-74%	-77%
2.5	-36%	-56%	-62%	-65%	-67%	-71%

Customer density

Challenges

1. The pricing problem in CG

2. Finite termination of CG

3. **Path elementarity**

4. Eliminating fractional solutions

The elementarity constraint

- Ideally, each path serves each customer at most once! (elementarity)

- Affects the structure of the label-setting in the pricing problem:

Option 1:
Ignore elementarity

Option 2:
Enforce elementarity

Q: can we get the best of both worlds?

One binary resource per customer[1]:

$$D(p) = \left(\bar{c}(p), \; t(p), \; -b(p), \; \gamma_1^p, \ldots, \gamma_n^p \right)$$

+ Computationally cheap!

— Good LP solutions, but bad IP solutions
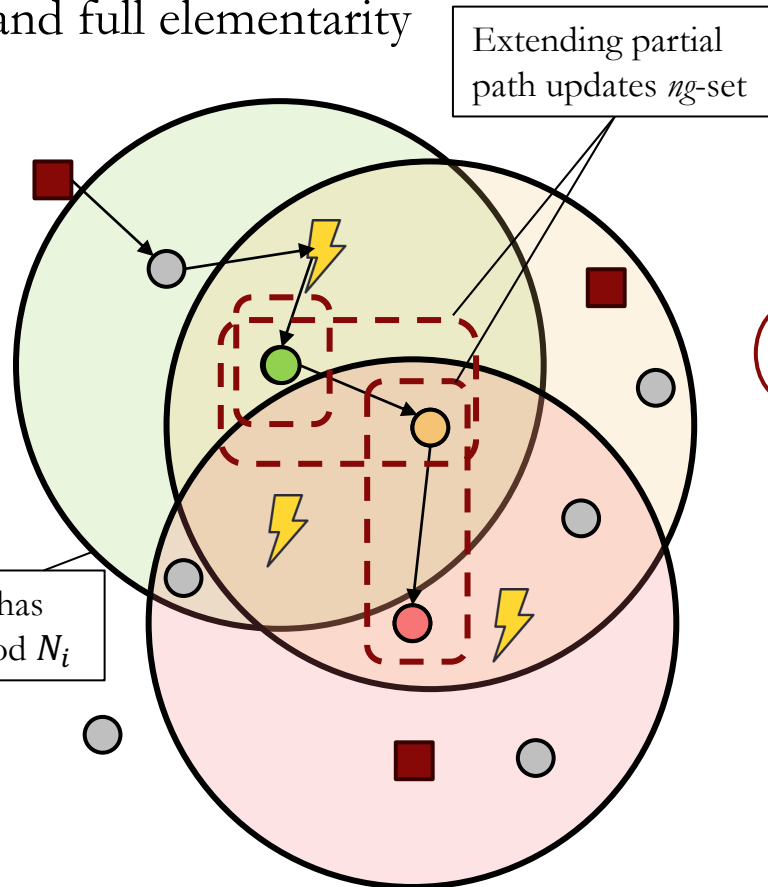
— Expensive: NP-hard[2]

+ Better IP solutions

[1] Beasley, J. E., & Christofides, N. (1989). An algorithm for the resource constrained shortest path problem. *Networks*, *19*(4), 379–394. https://doi.org/10.1002/net.3230190402

[2] Dror, M. (1994). Note on the Complexity of the Shortest Path Models for Column Generation in VRPTW. *Operations Research*, *42*(5), 977–978. https://doi.org/10.1287/opre.42.5.977
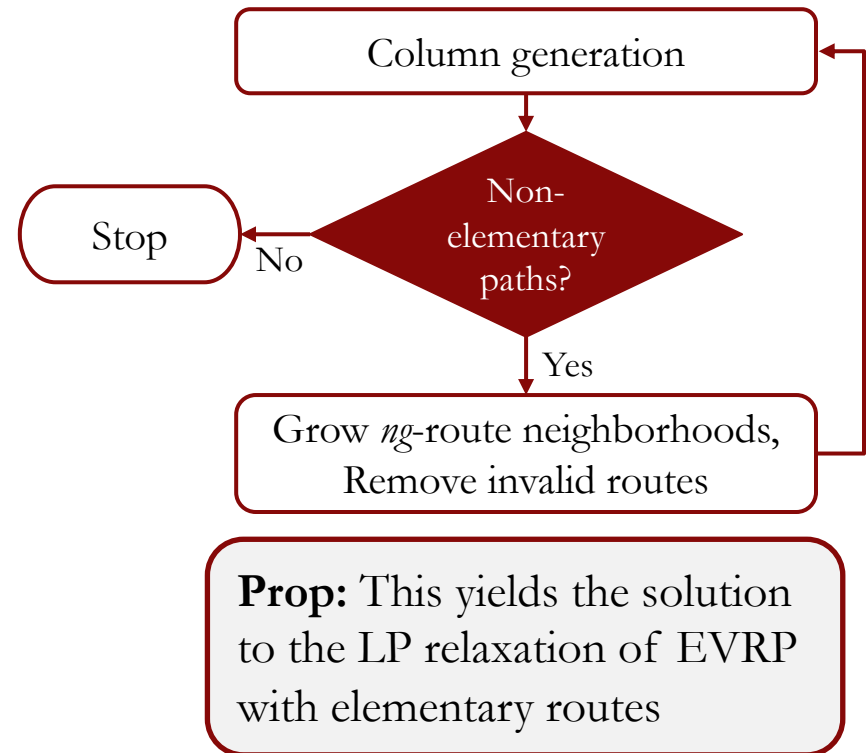
Adaptive[2] *ng*-relaxations[1]

_ng_-route relaxation[1]:
Interpolates between
no and full elementarity

Nested _ng_-route relaxations
Start with a loose *ng*-relaxation,
tighten when necessary[2]!

Extending partial
path updates *ng*-set

Customer i has
neighborhood N_i

Column generation

Non-elementary
paths?

Stop

No

Yes

Grow *ng*-route neighborhoods,
Remove invalid routes

Prop: This yields the solution
to the LP relaxation of EVRP
with elementary routes

[1] Baldacci, R., Mingozzi, A., & Roberti, R. (2011). New Route Relaxation and Pricing Strategies for the Vehicle Routing Problem. *Operations Research*, *59*(5), 1269–1283.
[2] Martinelli, R., Pecin, D., & Poggi, M. (2014). Efficient elementary and restricted non-elementary route pricing. *European Journal of Operational Research*, *239*(1), 102–111. https://doi.org/10.1016/j.ejor.2014.05.005

ng-routes in generate-and-stitch

Traditionally:

- Forward labelling keeps track of **forward** *ng*-sets [1]

$$\Pi(P) = \left\{ i_r : i_r \in \bigcap_{s=r+1}^{k} N_{i_s}, \ r = 1, \ldots, k-1 \right\} \bigcup \{i_k\}.$$

- Backward labelling (bidirectional label setting) tracks **backward** *ng*-sets [1]

$$\Pi^{-1}(\bar{P}) = \{i_k\} \cup \left\{ i_r : i_r \in \bigcap_{s=k}^{r-1} N_{i_s}, \ r = k+1, \ldots, h \right\}.$$
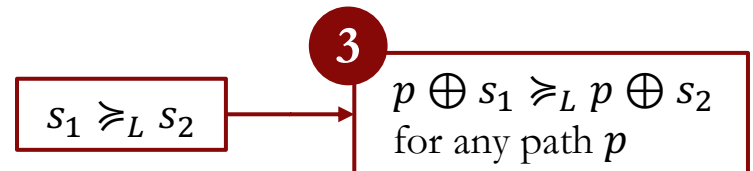
Our work:

- Path domination criteria include forward *ng*-set inclusions:

$$L(p) = \left(\bar{c}(p), \ t(p), \ -b(p), \ \{\mathbb{1}\,(i \in \Pi(p))\}_i \right)$$
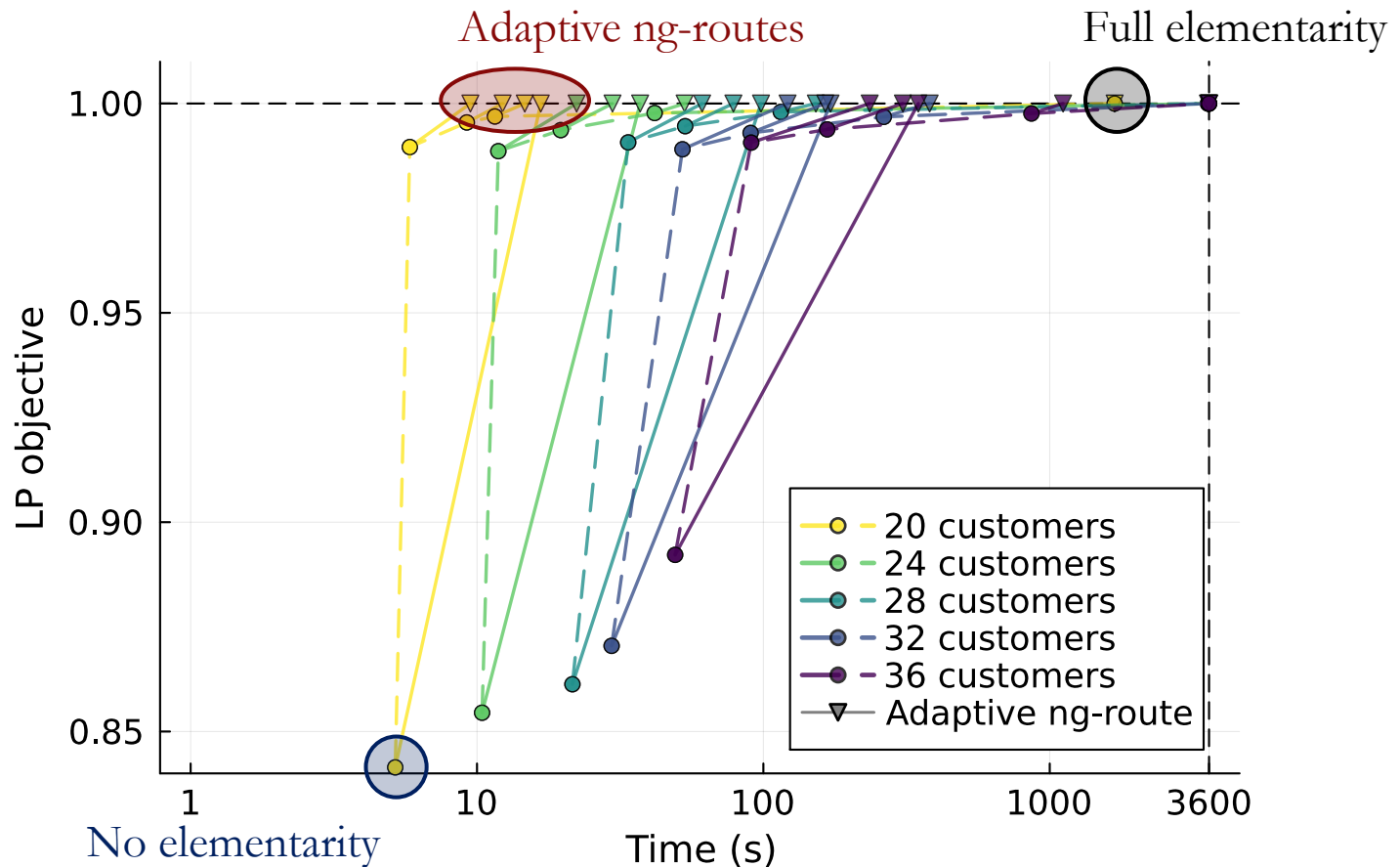
- Subpath domination criteria include **both** forward and backward *ng*-sets:

$$L(s) = \left(\bar{c}(s), \ t(s), \ b(s), \right.$$
$$\left. \{\mathbb{1}\,(i \in \Pi(p))\}_i, \{\mathbb{1}\,(i \in \Pi^{-1}(p))\}_i \right)$$

Reason: need $L(s)$ to satisfy:

3

$$s_1 \succcurlyeq_L s_2 \longrightarrow p \oplus s_1 \succcurlyeq_L p \oplus s_2$$
for any path p

[1] Baldacci, R., Mingozzi, A., & Roberti, R. (2011). New Route Relaxation and Pricing Strategies for the Vehicle Routing Problem. *Operations Research*, *59*(5), 1269–1283.

Benefits of adaptive *ng*-relaxations

Challenges

1. **The pricing problem in CG**

2. **Finite termination of CG**

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4. **Eliminating fractional solutions**

Subset-row cuts

- Consider a cut defined by a subset S of customers:
  - At most $\lfloor n/k \rfloor$ routes visiting at least k out of n customers[1] (Chvatal-Gomory cut of rank 1)

$$\sum_{p \in \mathcal{P}} \sum_{i \in S} \gamma_i^p z^p = |S| \implies \sum_{p \in \mathcal{P}} \left\lfloor \frac{1}{k} \sum_{i \in S} \gamma_i^p \right\rfloor z^p \leq \left\lfloor \frac{|S|}{k} \right\rfloor$$

- *Non-robust* cuts which change subproblem structure (new duals)
  - Track resource $\sum_{i \in S} \gamma_i^p \ (mod \ k)$ for each subset S
  - When resource hits 0, subtract dual from reduced cost
  - Track $\sum_{i \in S} \gamma_i^s \ (mod \ k)$ and $\sum_{i \in S} \gamma_i^p \ (mod \ k)$ for subpaths and paths respectively:

$$D(s) = \left(\ldots, \ \{\textstyle\sum_{i \in S} \gamma_i^s\}_S \right)$$
$$D(p) = \left(\ldots, \ \{\textstyle\sum_{i \in S} \gamma_i^p\}_S \right)$$

[1] Jepsen, M., Petersen, B., Spoorendonk, S., & Pisinger, D. (2008). Subset-Row Inequalities Applied to the Vehicle-Routing Problem with Time Windows. Operations Research, 56(2), 497–511. https://doi.org/10.1287/opre.1070.0449

Limited-memory subset-row cuts

- Limited-memory subset-row cuts[1] include a *memory neighborhood* for each cut

  - Smaller state space

  - Weaker cuts

- Requires tracking forward and backward criteria
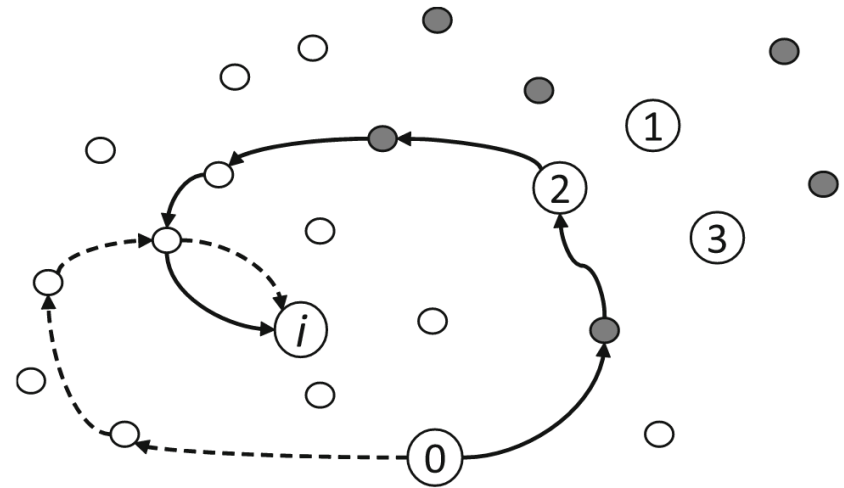
Fig. 2 Example illustrating the performance gain in the pricing when using lm-SRCs

[1] Pecin, D., Pessoa, A., Poggi, M., & Uchoa, E. (2017). Improved branch-cut-and-price for capacitated vehicle routing. *Mathematical Programming Computation*, *9*(1), 61–100. https://doi.org/10.1007/s12532-016-0108-8
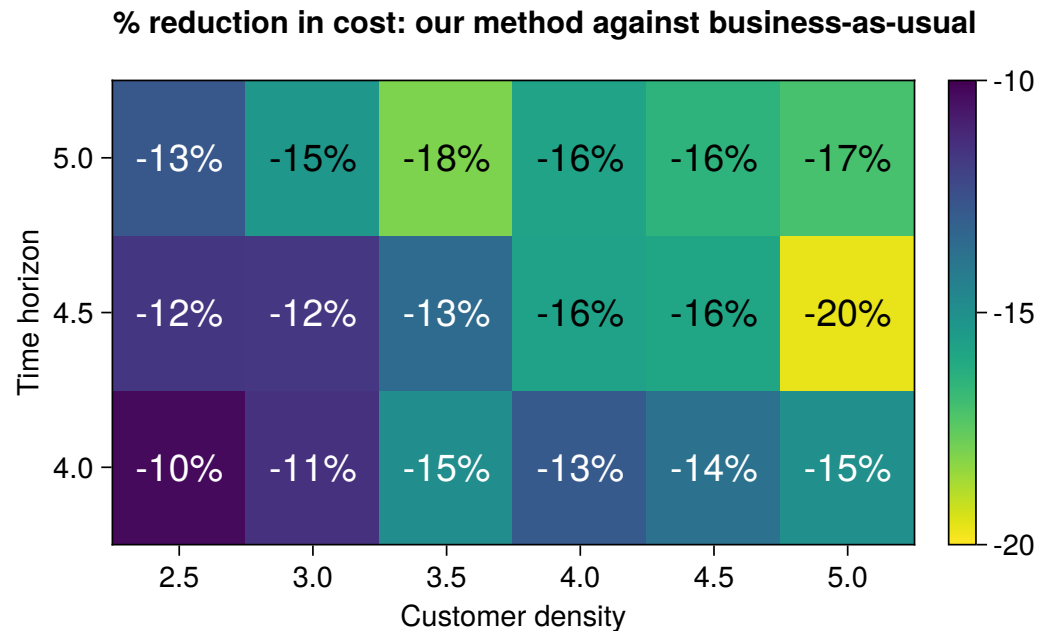
Computational results

- Cuts further close the optimality gap at the cost of more time

- IP solution obtained typically optimal

- *lm*-SR3 cuts provides an intermediate approach
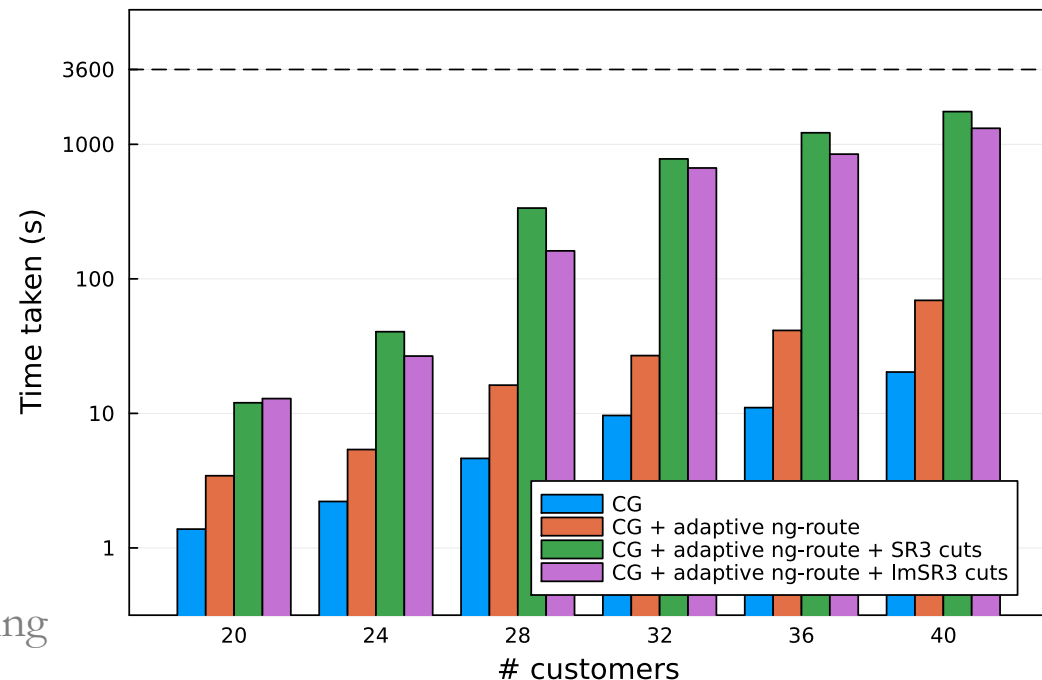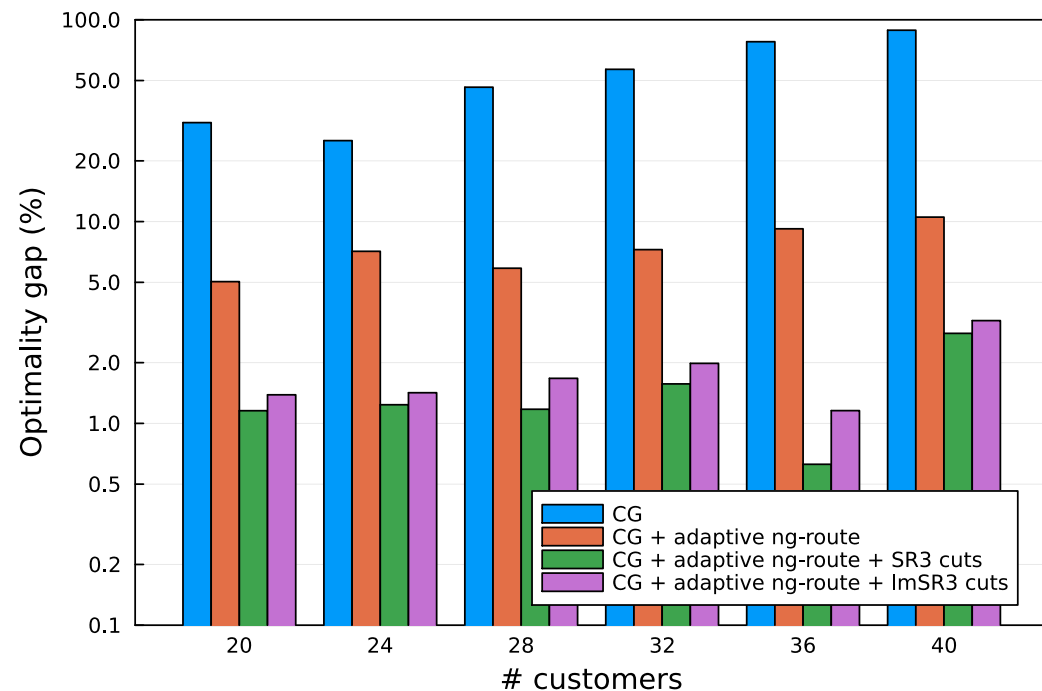
The benefits of optimization

- Improvement compared to business-as-usual solution:
  - Solve a VRP w/o charge
  - Then optimize charging stations with fixed routes

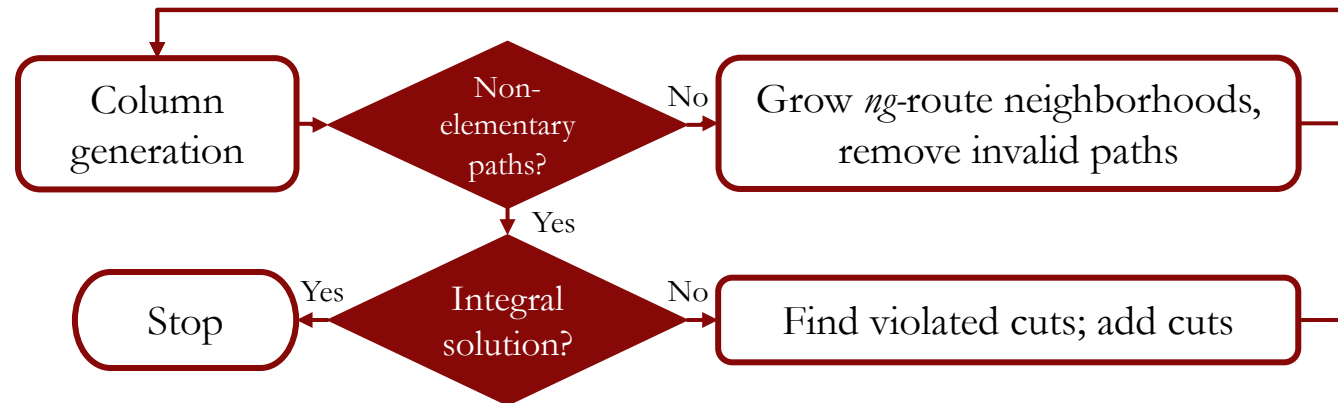- Benefit of **jointly** optimizing charging and routing decisions

% reduction in cost: our method against business-as-usual

Summary

Electric vehicle routing: subpath-based decomposition algorithm

Modeling

Electric vehicle routing: semi-infinite set-partitioning formulation with continuous time and continuous charge

Optimization

Computational results

Significantly outperforms path-based benchmark, and scales to realistic problem instances

Practical impact

Benefits over "business-as-usual" routing operations

Additional slides

Comparison with literature

Setting

[1]:

- Continuous time and charge

- Single / multiple recharges, partial / full recharging

- Time windows

- No charging costs

Our work:

- Continuous time and charge

- Multiple recharges, partial recharging

- **No** time windows

- Linear constant / heterogenous charging costs
  (charging τ at i costs $\delta_i \cdot \tau$)

[1] Desaulniers, G., Errico, F., Irnich, S., & Schneider, M. (2016). Exact Algorithms for Electric Vehicle-Routing Problems with Time Windows. Operations Research, 64(6), 1388–1405. https://doi.org/10.1287/opre.2016.1535

Comparison with literature

	[1]:	**Our work:**
Methods	• Bidirectional label-setting, with bidirectional criteria	• Unidirectional, two-level label-setting, with bidirectional criteria
	• *ng*-route relaxation	• Adaptive tightening of *ng*-route relaxations
	• 2-path cuts and subset-row cuts	• SRC and lm-SRC
	• Branching	• No branching

[1] Desaulniers, G., Errico, F., Irnich, S., & Schneider, M. (2016). Exact Algorithms for Electric Vehicle-Routing Problems with Time Windows. Operations Research, 64(6), 1388–1405. https://doi.org/10.1287/opre.2016.1535