Lecture A1 Solution

Reinforcement Learning Study

2021-01-13

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Exercise 1

Problem : Suppose $f(x) = xe^x$, find $f'(x)$.

$f(x) = xe^x$

$f'(x) = (x)'e^x + x(e^x)'$

$\therefore,\ f'(x) = e^x + xe^x$

Exercise 2

Problem : Suppose $f(x) = e^{2x}$, find $f'(x)$.

def $f(x) = h(g(x))$

$f(x) = e^{2x} => h(x) = e^x,\ g(x) = 2x$

$\therefore,\ f'(x) = e^{2x} \times 2 = 2e^{2x}$

Exercise 3

Derive $\int f'(x)g(x)dx = f(x)g(x) - \int f(x)g'(x)dx$.(Hint: Use Theorem 2 above.)

Solution

$(f(x) \cdot g(x))' = f'(x)g(x) + f(x)g'(x)$

antiderivative

$f(x) \cdot g(x) = \int f'(x)g(x)dx + \int f(x)g'(x)dx$

$\int f'(x)g(x)dx = f(x)g(x) - \int f(x)g'(x)dx$

Exercise 4

Find $\int xe^x dx$,and evaluate $\int_0^1 xe^x dx$.(Hint: Use Exercise 3 above.)

Solution

Exercise 3

$\int f'(x)g(x)dx = f(x) \cdot g(x) - \int f(x)g'(x)dx$

$$\begin{aligned}\int xe^x dx &= e^x x - \int (e^x \cdot 1)dx \\ &= e^x x - e^x + Cdx\end{aligned}$$

$$\begin{aligned}\int_0^1 xe^x dx &= [e^x x - e^x + C]_0^1 \\ &= (e^1 \cdot 1 - e^1 + C) - (e^0 \cdot 0 - e^0 + C)\end{aligned}$$

$$= (0 + C) - (0 - 1 + C)$$
$$= 1$$

Exercise 5

Solve the followings

$$\begin{pmatrix} .6 & .4 \end{pmatrix} \begin{pmatrix} .7 & .3 \\ .5 & .5 \end{pmatrix} =$$

$$\begin{pmatrix} 0.6 * 0.7 + 0.4 * 0.5 & 0.6 * 0.4 + 0.4 * 0.5 \end{pmatrix} = \begin{pmatrix} 0.62 & 0.38. \end{pmatrix}$$

Exercise 6

What is P^2?

$$P = \begin{pmatrix} .7 & .3 \\ .5 & .5 \end{pmatrix}$$

$$P = \begin{pmatrix} 0.7 & 0.3 \\ 0.5 & 0.5 \end{pmatrix} \begin{pmatrix} 0.7 & 0.3 \\ 0.5 & 0.5 \end{pmatrix} = \begin{pmatrix} 0.74 & 0.36 \\ 0.6 & 0.4 \end{pmatrix}$$

Exercise 7

Solve the followings.

$$\begin{pmatrix} \pi_1 & \pi_2 \end{pmatrix} \begin{pmatrix} .7 & .3 \\ .5 & .5 \end{pmatrix} \begin{pmatrix} \pi_1 & \pi_2 \end{pmatrix}$$
$$\pi_1 + \pi_2 = 1$$

(1)

$$\begin{aligned} 0.7\pi_1 + 0.5\pi_2 &= \pi_1 \\ 0.5\pi_2 &= 0.3\pi_1 \\ \pi_2 &= 0.6\pi_1 \end{aligned}$$

(2)

$$\pi_1 + \pi_2 = 1$$

From(1) & (2),

$\pi_1 + 0.6\pi_1 = 1$

Thus, $\pi_1 = \frac{5}{8}$**,** $\pi_2 = \frac{3}{8}$**.**

Exercise 8

Solve the following system of equations.

$$\begin{aligned}(1)\ \ x &= y \\ (2)\ \ y &= 0.5z \\ (3)\ \ z &= 0.6-0.4x \\ (4)\ \ x+y+z &= 1\end{aligned}$$

From (1) & (2) & (4),

$$\begin{aligned}y+y+z &= 1 \\ 2y+z &= 1 \\ z+z &= 1 \\ z &= \frac{1}{2}\end{aligned}$$

Thus, $x=\frac{1}{4}, y=\frac{1}{4}, z=\frac{1}{2}$

Exercise 9

Solve the following system of equations.

$$\begin{pmatrix}\pi_0 & \pi_1 & \pi_2\end{pmatrix}\begin{pmatrix}-2 & 2 & \\ 3 & -5 & 2 \\ & 3 & -3\end{pmatrix} = \begin{pmatrix}0 & 0 & 0\end{pmatrix}$$

$$\pi_0+\pi_1+\pi_2=1$$

$$\begin{pmatrix}\pi_0 & \pi_1 & \pi_2\end{pmatrix}\begin{pmatrix}-2 & 2 & \\ 3 & -5 & 2 \\ & 3 & -3\end{pmatrix} = \begin{pmatrix}-2\pi_0+3\pi_1 & 2\pi_0-5\pi_1+3\pi_2 & 2\pi_1-3\pi_2\end{pmatrix}$$

$$-2\pi_0 + 3\pi_1 = 0 \tag{1}$$

$$2\pi_0 - 5\pi_1 + 3\pi_2 = 0 \tag{2}$$

$$2\pi_1 - 3\pi_2 = 0 \tag{3}$$

$$\pi_0 + \pi_1 + \pi_2 = 1 \tag{4}$$

(1) equals $\pi_0 = \frac{3}{2}\pi_1$.

(2) equals $\pi_2 = \frac{3}{2}\pi_1$. Using above two equations, (4) equals $\frac{3}{2}\pi_1 + \pi_1 + \frac{2}{3}\pi_1 = 1$. Thus, $\pi_1 = \frac{6}{19}$. Using (1), $\pi_0 = \frac{9}{19}$. Using (3), $\pi_2 = \frac{4}{19}$.

(2) equals $\frac{18}{19} - \frac{30}{19} + \frac{12}{19} = 0$.

$\therefore \pi_0 = \frac{9}{19}, \pi_1 = \frac{6}{19}, \pi_2 = \frac{4}{19}$.

Exercise 10

Solve the following system of equations.

$$\begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \end{pmatrix} \begin{pmatrix} .7 & .3 & & \\ .5 & .5 & & \\ & & .6 & .4 \\ & & .3 & .7 \end{pmatrix} = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \end{pmatrix}$$

**$$ ⚜_1 + ⚜_2 + ⚜_3 + ⚜_4 = 1 **

⚜_1 + ⚜_2 = ⚜$$

$$0.7\pi_1 + 0.5\pi_2 = \pi_1 \tag{5}$$

$$0.3\pi_1 + 0.5\pi_2 = \pi_2 \tag{6}$$

$$0.6\pi_3 + 0.3\pi_4 = \pi_3 \tag{7}$$

$$0.4\pi_3 + 0.7\pi_4 = \pi_4 \tag{8}$$

(5) equals $5\pi_2 = 3\pi_1$.

(6) equals $3\pi_1 = 5\pi_2$.

(7) equals $4\pi_3 = 3\pi_4$.

(8) equals $4\pi_3 = 3\pi_4$.

By using (5) and (6), $\pi_2 = \frac{3}{5}\pi_1$. By using (7) and (8), $\pi_4 = \frac{4}{3}\pi_3$.

$$\pi_1 + \pi_2 + \pi_3 + \pi_4 = \pi_1 + \frac{3}{5}\pi_1 + \pi_3 + \frac{4}{3}\pi_3 = \frac{3}{5}\pi_1 + \frac{7}{3}\pi_3 = 1 \tag{9}$$

(9) equals $\pi_3 = \frac{3}{7} - \frac{24}{35}\pi_1$.

Thus, $\pi_4 = \frac{4}{7} - \frac{32}{35}\pi_1$.

$$\therefore \pi_1 = \frac{5}{8}\alpha, \pi_2 = \frac{3}{8}\alpha, \pi_3 = \frac{3}{7} - \frac{3}{7}\alpha, \pi_4 = \frac{4}{7} - \frac{4}{7}\alpha$$

Exercise 11

Solve following and express π_i **for** $i = 0, 1, 2, ...$

$$\begin{aligned}
\pi_0 + \pi_1 + \pi_2 + ... &= 1 \\
0.02\pi_0 + 0.02\pi_1 + 0.02\pi_2 + ... &= \pi_0 \\
0.98\pi_0 &= \pi_1 \\
0.98\pi_1 &= \pi_2 \\
0.98\pi_2 &= \pi_3 \\
... &= ...
\end{aligned}$$

Solution:

$$\begin{aligned}
\pi_0 + \pi_1 + \pi_2 + ... &= 1 && (10) \\
0.02\pi_0 + 0.02\pi_1 + 0.02\pi_2 + ... &= \pi_0 && (11) \\
0.98\pi_0 &= \pi_1 && (12) \\
0.98\pi_1 &= \pi_2 && (13) \\
0.98\pi_2 &= \pi_3 && (14) \\
... &= ...
\end{aligned}$$

From (3)-(5),

$$\begin{aligned}
\pi_1 &= (0.98)\pi_0 \\
\pi_2 &= (0.98)\pi_1 = (0.98)^2\pi_0 \\
\pi_3 &= (0.98)\pi_2 = (0.98)^3\pi_0 \\
\pi_i &= (0.98)^i\pi_0
\end{aligned}$$

Thus (1), becomes

$$\begin{aligned}
\pi_0 + \pi_1 + \pi_2 + ... &= \pi_0(1 + 0.98 + 0.98^2 + ...) \\
\pi_0(\frac{1}{1 - 0.98})) &= 1 \\
\pi_0 &= 0.02 \\
\therefore \pi_i &= (0.02)(0.98)^i
\end{aligned}$$

Excercise 12 (Infinite geometric series)

Simplify the following. When $|r| < 1, S = a + ar + ar^2 + ar^3 + ...$

Solution :

$$\begin{aligned} S &= a + ar^2 + ar^3 + ... && (1) \\ rS &= ar^2 + ar^3 + ar^4 + ... && (2) \end{aligned}$$

Subtract (1) and (2)

$(1 - r)S = a$

$\therefore S = \frac{a}{1-r}$

Exercise 13 (Finite geometric series)

Simplify the following. When $r \neq 1, S = a + ar + ar^2 + ar^3 + ... + ar^{n-1}$

Solution:

$S = a + ar + ar^2 + ... + ar^{n-1}$

$rS = ar + ar^2 + ar^3 + ... + ar^n$

$S - rs = a - ar^n$

$S = \frac{a(1-r^n)}{(1-r)}$

Exercise 14 (Power series)

Simplify the following. When $|r| \neq 1, S = r + 2r^2 + 3r^3 + 4r^4 + ...$

Solution:

$S = r + 2r^2 + 3r^3 + 4r^4 + ...$

$rS = r^2 + 2r^3 + 3r^4 + ...$

$(1 - r)S = r + r^2 + r^3 + ...$

$(1 - r)S = \frac{r}{1-r}$

$S = \frac{r}{(1-r)^2}$

Exercise 15

During the first hour $(0 \leq t \geq 1)$, $\lambda(t)$ increases linearly from 0 to 60. After the first hour, $\lambda(t)$ is constant at 60. Draw plot for $\lambda(t)$ and express the function in math form.

Solution:

$$\lambda(t) = \begin{cases} 60t & t \leq 60 \\ 60 & t > 60 \end{cases}$$

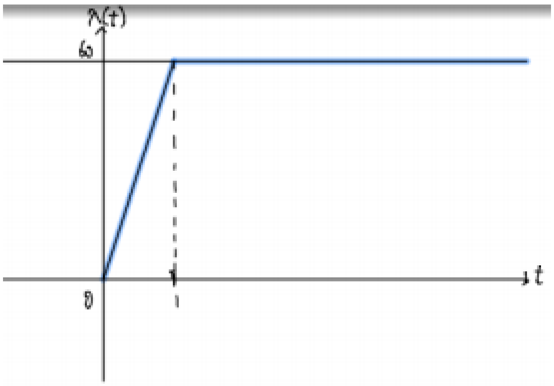

그림 1: graph

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