daipark_ch3

Kwon do yun

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ch3 - 5

Suppose each morning a factory posts the number of days worked in a row without any injuries. Assume that each day is injury free with probability 98/100. Furthermore, assume that whether tomorrow is injury free or not is independent of which of the preceding days were injury free. Let X0 = 0 be the morning the factory first opened. Let Xn be the number posted onthe morning after n full days of work.

(a) Is $X_n, n \geq 0$ a Markov chain? If so, give its state space, initial distribution, and transition matrix P. If not, show that it is not a Markov chain.

State space:

$S =$ {injury free,injury}

Initial distribution:

$a_0 = (1, 0)$

Transition matrix P:

$$P = \begin{pmatrix} 0.98 & 0.02 \\ 0.98 & 0.02 \end{pmatrix} \tag{1}$$

(b) Is the Markov chain irreducible? Explain.

This markov chain is irreducible. because it has only one class.

(c) Is the Markov chain periodic or aperiodic? Explain and if it is periodic, also give the period.

```
import numpy as np
a=np.matrix([[0.98,0.02],[0.98,0.02]])
a**2
```

```
## matrix([[0.98, 0.02],
##         [0.98, 0.02]])
```

```
a**3
```

```
## matrix([[0.98, 0.02],
##         [0.98, 0.02]])
```

```
a**100
```

```
## matrix([[0.98, 0.02],
##         [0.98, 0.02]])
```

```
a**101
```

```
## matrix([[0.98, 0.02],
##         [0.98, 0.02]])
```

$P^2 = P^3 = P^{100} = P^{101} = P^{\infty}$

This markov chain is aperiodic.

(d) Find the stationary distribution.

$s =$ {0.98,0.02}

(e) Is the Markov chain positive recurrent? If so, why? If not, why not?

No, it isn't recurrent.

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Consider the following transition matrix:

$$P = \begin{pmatrix} 0 & 0.5 & 0 & 0.5 \\ 0.6 & 0 & 0.4 & 0 \\ 0 & 0.7 & 0 & 0.3 \\ 0.8 & 0 & 0.2 & 0 \end{pmatrix} \tag{2}$$

(a) Is the Markov chain periodic? Give the period of each state.

```
p=np.matrix([[0,0.5,0,0.5],[0.6,0,0.4,0],[0,0.7,0,0.3],[0.8,0,0.2,0]])
p**1000
```

```
## matrix([[0.6875, 0.    , 0.3125, 0.    ],
##         [0.    , 0.5625, 0.    , 0.4375],
##         [0.6875, 0.    , 0.3125, 0.    ],
##         [0.    , 0.5625, 0.    , 0.4375]])
```

```
p**1001
```

```
## matrix([[0.    , 0.5625, 0.    , 0.4375],
##         [0.6875, 0.    , 0.3125, 0.    ],
##         [0.    , 0.5625, 0.    , 0.4375],
##         [0.6875, 0.    , 0.3125, 0.    ]])
```

```
p**1002
```

```
## matrix([[0.6875, 0.    , 0.3125, 0.    ],
##         [0.    , 0.5625, 0.    , 0.4375],
##         [0.6875, 0.    , 0.3125, 0.    ],
##         [0.    , 0.5625, 0.    , 0.4375]])
```

```
p**1003
```

```
## matrix([[0.    , 0.5625, 0.    , 0.4375],
##         [0.6875, 0.    , 0.3125, 0.    ],
##         [0.    , 0.5625, 0.    , 0.4375],
##         [0.6875, 0.    , 0.3125, 0.    ]])
```

$P^{1000} = P^{1002}\, P^{1001} = P^{1003}$

This markov chain is periodic, it has 2 period.

(b) Is $(\pi 1, \pi 2, \pi 3, \pi 4) = (33/96, 27/96, 15/96, 21/96)$ the stationary distribution of the Markov Chain?

```
k=np.matrix([[33/96,27/96,15/96,21/96]]).T
p*k
```

```
## matrix([[0.25   ],
##         [0.26875],
##         [0.2625 ],
##         [0.30625]])
```

```
p*k==k
```

```
## matrix([[False],
##         [False],
##         [False],
##         [False]])
```

It is not the stationary distribution of the Markov chain

(c) Is $P_{11}^{100} = \pi_1$? Is $P_{11}^{101} = \pi_1$? Give an expression for $\pi 1$ in terms of P_{11}^{100} and P_{11}^{101}.

```
p**100
```

```
## matrix([[0.6875, 0.    , 0.3125, 0.    ],
##         [0.    , 0.5625, 0.    , 0.4375],
##         [0.6875, 0.    , 0.3125, 0.    ],
##         [0.    , 0.5625, 0.    , 0.4375]])
```

```
p**101
```

```
## matrix([[0.    , 0.5625, 0.    , 0.4375],
##         [0.6875, 0.    , 0.3125, 0.    ],
##         [0.    , 0.5625, 0.    , 0.4375],
##         [0.6875, 0.    , 0.3125, 0.    ]])
```

$P_{11}^{100} = 0.6875$ $P_{11}^{101} = 0$

They are different from each other.

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Suppose you are working as an independent consultant for a company. Your performance is evaluated at the end of each month and the evaluation falls into one of three categories: 1, 2, and 3. Your evaluation can be modeled as a discrete-time Markov chain and its transition diagram is as follows. Denote your evaluation at the end of nth month by X_n and assume that $X_0 = 2$.

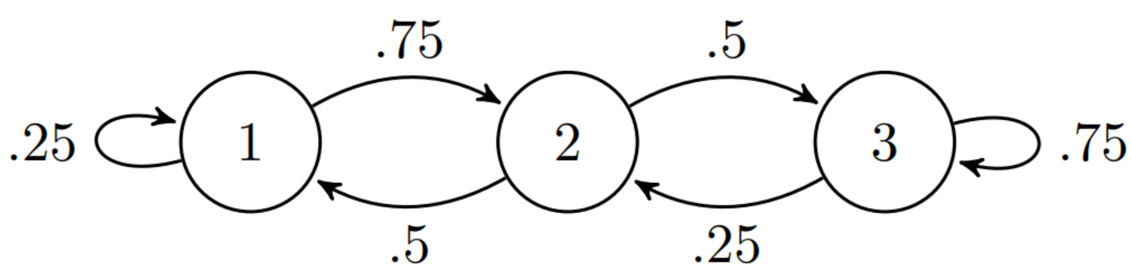

그림 1: Diagram

(a) What are state space, transition probability matrix and initial distribution of X_n?

State space:

$S =${1,2,3}

Transition matrix:

$$P = \begin{pmatrix} 0.25 & 0.75 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 0.25 & 0.75 \end{pmatrix} \tag{3}$$

(b) What is the stationary distribution?

```
p=np.matrix([[0.25,0.75,0],[0.5,0,0.5],[0,0.25,0.75]])
p**2
```

```
## matrix([[0.4375, 0.1875, 0.375 ],
##         [0.125 , 0.5   , 0.375 ],
##         [0.125 , 0.1875, 0.6875]])
```

```
p**3
```

```
## matrix([[0.203125, 0.421875, 0.375   ],
##         [0.28125 , 0.1875  , 0.53125 ],
##         [0.125   , 0.265625, 0.609375]])
```

```
p**100
```

```
## matrix([[0.18181818, 0.27272727, 0.54545455],
##         [0.18181818, 0.27272727, 0.54545455],
##         [0.18181818, 0.27272727, 0.54545455]])
```

```
p**101
```

```
## matrix([[0.18181818, 0.27272727, 0.54545455],
##         [0.18181818, 0.27272727, 0.54545455],
##         [0.18181818, 0.27272727, 0.54545455]])
```

$P^2 = P^3 = P^{100} = P^{101} = P^{\infty}$

stationary distribution $= (0.18181818, 0.27272727, 0.54545455)$

This markov chain is aperiodic.

(c) What is the long-run fraction of time when your evaluation is either 2 or 3?

```
p=np.matrix([[0.25,0.75,0],[0.5,0,0.5],[0,0.25,0.75]])
p**2
```

```
## matrix([[0.4375, 0.1875, 0.375 ],
##         [0.125 , 0.5   , 0.375 ],
##         [0.125 , 0.1875, 0.6875]])
```

```
p**3
```

```
## matrix([[0.203125, 0.421875, 0.375   ],
##         [0.28125 , 0.1875  , 0.53125 ],
##         [0.125   , 0.265625, 0.609375]])
```

```
p**100
```

```
## matrix([[0.18181818, 0.27272727, 0.54545455],
##         [0.18181818, 0.27272727, 0.54545455],
##         [0.18181818, 0.27272727, 0.54545455]])
```

```
p**101
```

```
## matrix([[0.18181818, 0.27272727, 0.54545455],
##         [0.18181818, 0.27272727, 0.54545455],
##         [0.18181818, 0.27272727, 0.54545455]])
```

$P^2 = P^3 = P^{100} = P^{101} = P^{\infty}$

$= 0.27272727 + 0.54545455 = 0.81818182$

Your monthly salary is determined by the evaluation of each month in the following way.

Salary when your evaluation is $n = \$5000 + n^2 \times \$5000, \ n = 1, 2, 3$

(d) What is the long-run average monthly salary?

$n = 1$:

$= \$5000 + 1^2 \times \$5000 = \$10000$

$n = 2$:

$= \$5000 + 2^2 \times \$5000 = \$25000$

$n = 3$:

$= \$5000 + 3^2 \times \$5000 = \$50000$

Average monthly salary $=\ 10000 \cdot 0.18181818 + 25000 \cdot 0.272727 + 50000 \cdot 0.54545455 = \$35909.0843$

```
"daipark_ch3"
```