B1_Exercises

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Exercise 1

Assume that D foloows the follwing discrete distribution.

d	20	25	30	35
$P[D=d]$	0.1	0.2	0.4	0.3
$30 \wedge d$	20	25	30	30
$(30-d)^+$	10	5	0	0
$24 \wedge d$	20	24	24	24
$(24-d)^+$	4	0	0	0

Answer the followings.

- $E[30 \wedge D] = 0.1 * 20 + 0.2 * 25 + 0.4 * 30 + 0.3 * 30 = 28$
- $E[(30-D)^+] = 0.1 * 10 + 0.2 * 5 + 0.4 * 0 + 0.3 * 0 = 2$
- $E[24 \wedge D] = 0.1 * 20 + 0.2 * 45 + 0.4 * 24 + 0.3 * 24 = 23.6$
- $E[(24-D)^+] = 0.1 * 4 + 0.2 * 0 + 0.4 * 0 + 0.3 * 0 = 0.4$

Exercise 2

Find your brother's optimal stock level by the above Theorem 2. Then, find his expected profit using the Remark 1.

$c_o = 0.5$

$c_u = 1$

$x^* = smallest(y)$

$f(y) \geq \frac{c_u}{c_o+c_u} = \frac{1}{0.5+1} = \frac{2}{3}$

d	11	12	13	14	15
$P(D=d)$	0.2	0.2	0.2	0.2	0.2
$P(D \leq d)$	0.2	0.4	0.6	0.8	1.0

$F_D(11) = \frac{1}{5} < \frac{2}{3}$

$F_D(12) = \frac{1}{5} + \frac{1}{5} < \frac{2}{3}$

$F_D(13) = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} < \frac{2}{3}$

$F_D(14) = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} \geq \frac{2}{3}$

thus, Optimal stock : $x^* = 14$

$E[Profit]$=E(Sale Rev.)+E(salvage Rev.)-E(material Cost)

E(Sale Rev.)$=$ $2 * P(11) * (11 \wedge 14) + 2 * P(12) * (12 \wedge 14) + 2 * P(13) * (13 \wedge 14) + 2 * P(14) * (14 \wedge 14) + 2 * P(15) * (15 \wedge 14) = 25.6$

E(salvage Rev.)$=$ $0.5 * P(11) * (14 - 11)^+ + 0.5 * P(12) * (14 - 12)^+ + 0.5 * P(13) * (14 - 13)^+ + 0.5 * P(14) * (14 - 14)^+ + 0.5 * P(15) * (14 - 15)^+ = 0.6$

E(matarial Cost.)$=$ $1 * 14 = 14$

so, $E[Profit] = 25.6 + 0.6 - 14 = 12.2$

Exercise 3

Your brother is now selling milk. The customer demands follow $U(20, 40)$ gallons. Retail price is \$2 gallon, meterial cost is \$1 per gallon, and salvage cost is \$0.5 per gallon. Find optimal stock level and expected profit

$c_o = 1 - 0.5 = 0.5$

$c_u = 2 - 1 = 1$

$x^* = smallest(y)$

$U(20, 40)$

$$f(x) = \begin{cases} \frac{1}{20} & 20 \leq x \leq 40 \\ 0 & otherwise \end{cases}$$

$$F(x) = \begin{cases} 0 & x \leq 20 \\ \frac{x-20}{20} & 20 \leq x \leq 40 \\ 1 & x > 40 \end{cases}$$

$F(y) = \frac{c_u}{c_o + c_u} = \frac{2}{3}$

$F(y) = \frac{y-20}{20}$

$\frac{y-20}{20} = \frac{2}{3}$

$y = \frac{100}{3}$

$x^* = \frac{100}{3}$

$E[Profit] = E$(sale Rev.)+E(salvage Rev.)-E(matarial Cost.)

E(sale Rev.) $= \int_{20}^{40} 2 \cdot (D \wedge \frac{100}{3}) \cdot \frac{1}{20} dD$
$= \frac{1}{10} \cdot (\int_{20}^{\frac{100}{3}} (D) dD + \int_{\frac{100}{3}}^{40} (\frac{100}{3}) dD)$
$= \frac{520}{9}$

E(salvage Rev.) $= \int_{20}^{40} 0.5 \cdot (\frac{100}{3} - D)^+ \cdot \frac{1}{20} dD$
$= \frac{1}{40} \cdot (\int_{20}^{\frac{100}{3}} (\frac{100}{3} - D) dD)$
$= \frac{100}{9}$

E(matarial Cost.) $= 1 \cdot \frac{100}{3}$

$E[Profit] = \frac{520}{9} + \frac{100}{9} - \frac{100}{3} = \frac{320}{9}$

Exercise 4

Lemonade sells for \$18 per gallon but only costs \$3 per gallon to make. If we run out of lemonade, it will be impossible to get more. On the other hand, leftover lemonade has a value of \$1. Express the following quantity using sale as X and demand as D.

- $c_u = (18 - 3) \cdot (D - X)^+$
- $c_o = (3 - 1) \cdot (X - D)^+$
- Expected economic cost $= 15 \cdot (D - X)^+ + 2 \cdot (X - D)^+$
- Expected profit $= 15(D \wedge X) + 2(X - D)^+ - 3 \cdot X$

Exercise 5

Prove Theorem 1. (Hint: you may use formulation from Exercise 4)

- retail price $= p$
- material cost $= c$
- salvage price $= s$
- demand $= D$
- stock $= X$
- understock cost $= (p - c)(D - X)^+ = c_u$
- overstock cost $= (c - s)(X - D)^+ = c_o$

Expected economic cost$ = c_u + c_o$

$= (p - c)(D - X)^+ + (c - s)(X - D)^+$

if D is X, Expected economic cost is minimum.

Expectd profit $= p(D \wedge X) + s(X - D)^{+} - c \cdot X$

Expected profit is maximum value when D is X.

Therefore, when Expected economic cost is minimal, Expectd profit is maximum.

DaiPark Exercise 8 (p.20)

Suppose that a bakery specializes in chocolate cakes. Assume the cakes retail at $20 per cake, but it takes $10 to prepare each cake. Cakes cannot be sold after one week, and they have a negligible salvage value. It is estimated that the weekly demand for cakes is: 15 cakes in 5% of the weeks, 16 cakes in 20% of the weeks, 17 cakes in 30% of the weeks, 18 cakes in 25% of the weeks, 19 cakes in 10% of the weeks, and 20 cakes in 10% of the weeks. How many cakes should the bakery prepare each week? What is the bakery's expected optimal weekly profit?

optimal weekly profit $= 20 \cdot (\sum_{D=15}^{20} (D \wedge X) \cdot P(D)) - 10 \cdot X$
$= 20 \cdot ((15 \wedge X) \cdot \frac{5}{100} + (16 \wedge X) \cdot \frac{20}{100} + (17 \wedge X) \cdot \frac{30}{100} + (18 \wedge X) \cdot \frac{25}{100} + (19 \wedge X) \cdot \frac{10}{100} + (20 \wedge X) \cdot \frac{10}{100}) - 10 \cdot X$
$= (15 \wedge X) + 4(16 \wedge X) + 6(17 \wedge X) + 5(18 \wedge X) + 2(19 \wedge X) + 2(20 \wedge X) - 10 \cdot X$

$X = 15, 15 + 4 \cdot 15 + 6 \cdot 15 + 5 \cdot 15 + 2 \cdot 15 + 2 \cdot 15 - 10 \cdot 15 = 150$

$X = 16, 15 + 4 \cdot 16 + 6 \cdot 16 + 5 \cdot 16 + 2 \cdot 16 + 2 \cdot 16 - 10 \cdot 16 = 159$

$X = 17, 15 + 4 \cdot 16 + 6 \cdot 17 + 5 \cdot 17 + 2 \cdot 17 + 2 \cdot 17 - 10 \cdot 17 = 164$

$X = 18, 15 + 4 \cdot 16 + 6 \cdot 17 + 5 \cdot 18 + 2 \cdot 18 + 2 \cdot 18 - 10 \cdot 18 = 163$

$X = 19, 15 + 4 \cdot 16 + 6 \cdot 17 + 5 \cdot 18 + 2 \cdot 19 + 2 \cdot 19 - 10 \cdot 19 = 157$

$X = 20, 15 + 4 \cdot 16 + 6 \cdot 17 + 5 \cdot 18 + 2 \cdot 19 + 2 \cdot 20 - 10 \cdot 20 = 149$

therefore, the bakery should prepare 17 cakes each week. and the bakery's expected optimal weekly profit is 163$

DaiPark Exercise 14 (p.23)

A store sells a particular brand of fresh juice. By the end of the day, any unsold juice is sold at a discounted price of \$2 per gallon. The store gets the juice daily from a local producer at the cost of \$5 per gallon, and it sells the juice at \$10 per gallon. Assume that the daily demand for the juice is uniformly distributed between 50 gallons to 150 gallons.

(a) What is the optimal number of gallons that the store should order from the distribution each day in order to maximize the expected profit each day?

(b) If 100 gallons are ordered, what is the expected profit per day?

(a)

$c_u = 5$

$c_o = 3$

$F(y) = \frac{c_u}{c_u + c_o} = \frac{5}{8}$

$U(50, 150)$

$$f(y) = \begin{cases} \frac{1}{100} & 50 \leq x \leq 150 \\ 0 & otherwise \end{cases}$$

$$F(y) = \begin{cases} 0 & y \leq 50 \\ \frac{y-50}{150-50} & 50 \leq y \leq 150 \\ 1 & y > 150 \end{cases}$$

$\frac{y-50}{100} = \frac{5}{8}$

$y = \frac{225}{2}$

therefore, the optimal number of gallons is 112.5 gallons.

(b)

Expectd profit X:100 $= 10(D \wedge 100) \cdot f(y) + 2(100 - D)^{+} \cdot f(y) - 5 \cdot 100$

$= \int_{50}^{100} 10 \cdot (D \wedge 100) \cdot \frac{1}{100} dD + \int_{100}^{150} 10 \cdot 100 \cdot \frac{1}{100} dD + \int_{50}^{100} 2 \cdot (100 - D) \cdot \frac{1}{100} dD - 5 \cdot 100$

$= 900 - 500 = 400$ If 100 gallons are ordered, the expected profit per day is 400\$

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"Man can learn nothing unless he proceeds from the known to the unknown. - Claude Bernard"
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