# Daipark_chapter3

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## 차 례

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## Exercise 5

a) Is $\{X_n, n \geq 0\}$ a Markov chain? If so, give its **state space**, **initial distribution** and **transition matrix P**. If not,m show that it is not a Markov chain

This is Markov chain.
States = [injury free,injury]
initial distribution = $X_0 = (1, 0)$
transition matrix P = $\begin{pmatrix} 0.98 & 0.02 \\ 0.98 & 0.02 \end{pmatrix}$

b) Is the Markov chain irreducible? Explain.

This Markov chain is irreducible.
The state does not stay in one state.

c) Is the Markov chain periodic or aperiodic? Explain and if it is periodic, also give the period. this is aperiod

```
from numpy.linalg import matrix_power
p = np.array([[0.98,0.02],[0.98,0.02]])
matrix_power(p,100)
```

```
## array([[0.98, 0.02],
##        [0.98, 0.02]])
```

```
matrix_power(p,101)
```

```
## array([[0.98, 0.02],
##        [0.98, 0.02]])
```

d) Find the stationary distribution.

```
p = array(c(0.7,0.5,0.3,0.5),dim= c(2,2))
p
```

```
##      [,1] [,2]
## [1,]  0.7  0.3
## [2,]  0.5  0.5
```

```
t(p)
```

```
##      [,1] [,2]
## [1,]  0.7  0.5
## [2,]  0.3  0.5
```

```
import numpy as np
p = np.array([[0.98,0.02],[0.98,0.02]])
p.T
```

```
## array([[0.98, 0.98],
##        [0.02, 0.02]])
```

```
eig_value, eig_vector = np.linalg.eig(p.T)
print('Eigenvalue : {}'.format(eig_value))
```

```
## Eigenvalue : [1. 0.]
```

```
print('Eigenvector : {}'.format(eig_vector))
#
```

```
## Eigenvector : [[ 0.99979182 -0.70710678]
##  [ 0.02040391  0.70710678]]
```

```
x_1 = eig_vector[:,0]
v = x_1/np.sum(x_1)
print('stationary distribution : {}'.format(v))
```

```
## stationary distribution : [0.98 0.02]
```

e) Is the Markov chain positive recurrent? If so, why? If not, why not?

No absorbing states exists. so, the markov chain is recurrent

### Exercise 6

$$P = \begin{pmatrix} 0 & 0.5 & 0 & 0.5 \\ 0.6 & 0 & 0.4 & 0 \\ 0 & 0.7 & 0 & 0.3 \\ 0.8 & 0 & 0.2 & 0 \end{pmatrix}$$

a) Is the Markov chain periodic? Give the period of each state. this is periodic

```
import numpy as np
from numpy.linalg import matrix_power
p = np.array([0,0.5,0,0.5,0.6,0,0.4,0,0,0.7,0,0.3,0.8,0,0.2,0]).reshape(4,4)
matrix_power(p,2)
```

```
## array([[0.7 , 0.  , 0.3 , 0.  ],
##        [0.  , 0.58, 0.  , 0.42],
##        [0.66, 0.  , 0.34, 0.  ],
##        [0.  , 0.54, 0.  , 0.46]])
```

```
matrix_power(p,151)
```

```
## array([[0.    , 0.5625, 0.    , 0.4375],
##        [0.6875, 0.    , 0.3125, 0.    ],
##        [0.    , 0.5625, 0.    , 0.4375],
##        [0.6875, 0.    , 0.3125, 0.    ]])
```

b) Is $(\pi_1, \pi_2, \pi_3, \pi_4) = (33/96, 27/96, 15/96, 21/96)$ the stationary distribution of the Markov chain?
yes, this is stationary distribution

```
import numpy as np
p = np.array([0,0.5,0,0.5,0.6,0,0.4,0,0,0.7,0,0.3,0.8,0,0.2,0]).reshape(4,4)
v = np.array([33/96, 27/96,15/96,21/96]).reshape(1,4)
print(np.dot(v,p))
```

```
## [[0.34375 0.28125 0.15625 0.21875]]
```

```
print(v)
```

```
## [[0.34375 0.28125 0.15625 0.21875]]
```

c) Is $P_{11}^{100} = \pi_1$? Is $p_{11}^{101} = \pi_1$? Give an expression for $\pi_1$ in terms of $P_{11}^{100} = \pi_1$ and $p_{11}^{101} = \pi_1$

### Exercise 14

a) What are state space, transition probability matrix and initial distribution of $X_n$

state space = [1,2,3]

$$\begin{pmatrix} 0.25 & 0.75 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 0.25 & 0.75 \end{pmatrix}$$

initial distribution = [0,1,0]

b) What is the stationary distribution?

```
import numpy as np
p = np.array([[0.25,0.75,0],[0.5,0,0.5],[0,0.25,0.75]])
eig_value, eig_vector = np.linalg.eig(p.T)
print('Eigenvalue : {}'.format(eig_value))
```

```
## Eigenvalue : [-0.55901699  0.55901699  1.        ]
```

```
print('Eigenvector : {}'.format(eig_vector))
```

```
## Eigenvector : [[ 0.5         0.5         0.28571429]
##  [-0.80901699  0.30901699  0.42857143]
##  [ 0.30901699 -0.80901699  0.85714286]]
```

```
x_1 = eig_vector[:,2]
x_1
```

```
## array([0.28571429, 0.42857143, 0.85714286])
```

```
v = x_1/np.sum(x_1)

print('stationary distribution : {}'.format(v))
```

```
## stationary distribution : [0.18181818 0.27272727 0.54545455]
```

c) What is the long-run fraction of time when your evaluation is either 2 or 3?

long-run??

salary when your evaluation is n = $5000 + n^2 \times \$5000, n = 1, 2, 3,$

```
evalutaion_2 = v[1]
evalutaion_3 = v[2]
print('the long-run fraction for time (evalutaion 2) : {:f}'.format(evalutaion_2))
```

```
## the long-run fraction for time (evalutaion 2) : 0.272727
```

```
print('the long-run fraction for time (evalutaion 2) : {:f}'.format(evalutaion_3))
```

```
## the long-run fraction for time (evalutaion 2) : 0.545455
```

d) What is the long-run average monthly salary?

average$\cdots$?

Daipark_ch3.Rmd

```
"Hello"
```

```
## [1] "Hello"
```