Lecture C1. Discrete Time Markov Chain 1

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Excersie 1

Let's revisit Coke & Pepsi DTMC. Describe the following

1.State Space

2.Trainsition Probability Matrix

3.Transition Diagram

4.Initial Distribution

Solution:

- State space: a set of all possible state that S can take

  S can take Coke or Pepsi, S= {c,p}

- Trainsition Probability Matrix

$$P_{2,2} = \begin{pmatrix} 0.7 & 0.3 \\ 0.5 & 0.5 \end{pmatrix}$$

- Initial Distribution : The information of where the chain starts at time 0.

  a_0= distribution of S_0 in row vector.

  EX) Suppose you always drink coke on day 0 then,

  $S_0 = c \Leftrightarrow \mathbb{P}(S_0 = c) = 1, \mathbb{P}(S_0 = p) = 0 \Leftrightarrow$a_0=(1,0)

EX) Suppose you drink coke with probability 0.6 and pepsi with probability 0.4 then,

$\mathbb{P}(S_0 = c) = 0.6, \mathbb{P}(S_0 = p) = 0.4$ ⇔a_0=(0.6,0.4)

- Transition Diagram

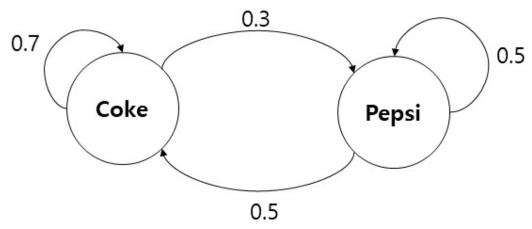

그림 1: Transition Diagram

Excersise 2

Suppose, $\mathbb{P}(S_0 = c) = 0.6, \mathbb{P}(S_0 = p) = 0.4$ then, what is $\mathbb{P}(S_1 = c) = ?$

Solution:

1)

a_0=(0.6,0.4)

$\mathbb{P}(S_1 = c) = \mathbb{P}(S_1 = c, S_0 = c) + \mathbb{P}(S_1 = c, S_0 = p)$

=$\mathbb{P}(S_1 = c|S_0 = c)\mathbb{P}(S_0 = c) + \mathbb{P}(S_1 = c|S_0 = p)\mathbb{P}(S_0 = p)$

= 0.7(0.6)+0.5(0.4) = 0.62

2)

$$a_o P = \begin{pmatrix}\mathbb{P}(S_0 = c) & \mathbb{P}(S_0 = p)\end{pmatrix} \begin{pmatrix} \mathbb{P}(S_1 = c|S_0 = c) & \mathbb{P}(S_1 = p|S_0 = c) \\ \mathbb{P}(S_1 = c|S_0 = p) & \mathbb{P}(S_1 = p|S_0 = p) \end{pmatrix}$$

$$= \begin{pmatrix}.6 & .4\end{pmatrix} \begin{pmatrix} .7 & .3 \\ .5 & .5 \end{pmatrix} = 0.62$$

Excercise 3

Suppose, $\mathbb{P}(S_0 = c) = 0.6, \mathbb{P}(S_0 = p) = 0.4$ then, what is $\mathbb{P}(S_2 = c) = ?$

Solution:

$\mathbb{P}(S_0 = c) = 0.6, \mathbb{P}(S_0 = p) = 0.4$ ⇔a_0=(0.6,0.4)

cf) $(AB)C = A(BC)$

$a_1 = a_o P$

$a_2 = a_1 P = (a_0 P)P = a_0 P^2$

$$a_0 P^2 = \begin{pmatrix}.6 & .4\end{pmatrix} \begin{pmatrix} .64 & .36 \\ .6 & .4 \end{pmatrix} = \begin{pmatrix}.624 & .376\end{pmatrix}$$

Thus, $\mathbb{P}(S_2 = c) = .376$

Excercise 4

Suppose, $\mathbb{P}(S_0 = c) = 0.6, \mathbb{P}(S_0 = p) = 0.4$ then, what is $\mathbb{P}(S_2 = p) = ?$

Solution:

$S_0 = p \Leftrightarrow \mathbb{P}(S_0 = c) = 0, \mathbb{P}(S_0 = p) = 1 \Leftrightarrow a_0 = (0, 1)$

cf) $(AB)C = A(BC)$

$a_1 = a_o P$

$a_2 = a_1 P = (a_0 P)P = a_0 P^2$

$$a_0 P^2 = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} .64 & .36 \\ .6 & .4 \end{pmatrix} = \begin{pmatrix} .6 & .4 \end{pmatrix}$$

Thus, Thus, $\mathbb{P}(S_2 = p) = .4$

DTMC Simulator p.25 in python

```
import numpy as np

def soda_simul(this_state):
    u= np.random.uniform()

    if(this_state=="c"):
        if(u<=0.7):
            next_state="c"
        else:
            next_state="p"

    else:
        if(u<=0.5):
            next_state="c"
        else:
            next_state="p"

    return(next_state)
```

```
for i in range(5):
    path="c" # coke today (day=0)
    for n in range(9):
        this_state=path[-1]
        next_state=soda_simul(this_state)
```

```
        path+=next_state
    print(path)
```

```
## ccpppccccc
## cccccccccc
## cccppppccc
## cpcccccccp
## cpcccccccp
```

```
def cost_eval(path):
    cost_one_path=path.count('c')*1.5+path.count('p')*1
    return cost_one_path
```

```
import numpy as np

MC_N=10000
spending_records=np.zeros(MC_N)

for i in range(len(spending_records)):
    path="c" # coke today
    for n in range(9):
        this_state=path[-1]
        next_state=soda_simul(this_state)
        path+=next_state
    spending_records[i]=cost_eval(path)

print(np.mean(spending_records))
```

```
## 13.35165
```

```
"Done, Lecture C1. Discrete Time Markov Chain 1 "
```

```
## [1] "Done, Lecture C1. Discrete Time Markov Chain 1 "
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