

Lecture B1. Newsvendor

Sim, Min Kyu, Ph.D., mksim@seoultech.ac.kr



서울과학기술대학교 데이터사이언스학과

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I. Problem and Solution

Motivation

Your brother will start part time job of selling newspapers at subway station in the morning. You are asked how many he should prepare for selling.

- What are the kind of information that you need in order to give him a good advice?

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The information

- ① (*demand*) How many customer?
 - ② (*retail price*) How much do you sell a copy at?
 - ③ (*material cost*) How much do you pay to the wholesaler?
 - ④ (*salvage value*) How much do you sell an unsold copy back to the wholesaler?
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- Suppose your brother gave you the following information.
 - ① 11, 12, 13, 14, or 15, equally likely.
 - ② \$2 per copy
 - ③ \$1 per copy
 - ④ \$0.5 per copy

Exercise 1

Assume that D follows the following discrete distribution.

d	20	25	30	35
$\mathbb{P}[D = d]$	0.1	0.2	0.4	0.3
$30 \wedge d$				
$(30 - d)^+$				
$24 \wedge d$				
$(24 - d)^+$				

Answer the followings.

- $\mathbb{E}[30 \wedge D] =$
- $\mathbb{E}[(30 - D)^+] =$
- $\mathbb{E}[24 \wedge D] =$
- $\mathbb{E}[(24 - D)^+] =$

Optimal economic decision

- Your brother's goal is to earn as much money as possible.
- In other words, the newsvendor wants to make *an optimal decision* that *maximizes his expected profit*.
- In some settings, the optimal decision is related to *minimize expected cost*.
- In this case, profit, *the objective function*, is composed in the following way.

$$\begin{aligned}\text{Profit} &= \text{Revenue} - \text{Cost} \\ &= \text{Sales Rev.} + \text{Salvage Rev.} - \text{Material Cost} \\ &= (\text{from Reg. Sale}) + (\text{from unsold item}) - (\text{for preparation})\end{aligned}$$

Solution - tabular method

Economic cost around decision making

- Your brother wants to the prepared **stock** to exactly match the customer **demand**.
- If **Stock > Demand**,
 - **Overstock cost occurs.**
 - In other words, prepared too much, excessive items, or over-prepared.
 - Overstock by how many?
 - Ex) holding cost, items lose its economic potential value
- If **Stock < Demand**,
 - **Understock cost occurs.**
 - In other words, prepared too less, lost opportunity, or lost sales.
 - Understock by how many?
 - Ex) stock-out, reputation loss, cancellation reward
- Newsvendor wants to find a optimal balance between overstock cost and understock cost.
- Just as all of the retail store you can think of.

Mathematical representation

- (number of units) If demand D is random and you prepare X unit, then
 - # of unit for sales:
 - # of unit for overstock:
 - # of unit for understock:
- (unit cost) If retail price is p , material cost is c , and salvage price is s , then
 - unit cost for overstock:
 - unit cost for understock:
- Cost = (# of units) \times (cost per unit)
 - overstock cost:
 - understock cost:

Formal treatment

Remark 1

Newsvendor problem aims to find the optimal number of preparation that maximizes the expected profit formulated as:

$$\mathbb{E}[\text{profit}] = \mathbb{E}(\text{sale rev.}) + \mathbb{E}(\text{salvage rev.}) - \mathbb{E}(\text{material cost})$$

Theorem 1

In newsvendor problem, maximizing the expected profit is equivalent to minimizing the expected economic cost (sum of the expected overstock cost and the expected understock cost).

- Two problems being mathematically equivalent to each other implies that a solution that solves the one problem solves the other problem, and vice versa.

Problem and Solution

Remark 2

By the above remark and theorem, newsvendor model is to find optimal x^* that minimize total expected economic cost. That is,

$$x^* = \operatorname{argmin}_x c_o \mathbb{E}[(X - D)^+] + c_u \mathbb{E}[(D - X)^+]$$

Theorem 2

The solution to the above problem can be found as:

- If D is a continuous r.v, with cdf $F(\cdot)$, then find y s.t. $F(y) = \frac{c_u}{c_o + c_u}$
- If D is a discrete r.v, with cdf $F(\cdot)$, then find smallest y such that $F(y) \geq \frac{c_u}{c_o + c_u}$

II. Exercises

Exercise 2

Find your brother's optimal stock level by the above Theorem 2. Then, find his expected profit using the Remark 1.

Exercise 3

Your brother is now selling milk. The customer demands follow $U(20, 40)$ gallons. Retail price is \$2 per gallon, material cost is \$1 per gallon, and salvage cost is \$0.5 per gallon. Find optimal stock level and expected profit.

Exercise 4

Lemonade sells for \$18 per gallon but only costs \$3 per gallon to make. If we run out of lemonade, it will be impossible to get more. On the other hand, leftover lemonade has a value of \$1. Express the following quantity using sale as X and demand as D .

- C_u
- C_o
- Expected economic cost
- Expected profit

Exercise 5

Prove Theorem 1. (Hint: you may use formulation from Exercise 4)

Discussion

On the nature of newsvendor

- What does $F(y) = \frac{c_u}{c_o + c_u}$ imply?
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- Buying a suit - department store vs outlet

News vendor in a big picture

- Newsvendor is characterized as an *optimal decision making* problem.
- Newsvendor is characterized as a “baby model” for a *decision making under uncertainty*.
- Since it is decision making *under uncertainty*, a random variable is involved.
- It can be viewed as *decision making at time 0*, then *results come out at time 1*. It is thus called as *one-period problem*.
- *Stochasticity* is the combined notion of *time* and *randomness*.
- In sum, newsvendor is *one-period stochastic decision making* problem, where
 - ① *one-period* specifies its time domain.
 - ② *stochastic* tells that we make decision considering future randomness.
 - ③ *decision making* obviously includes optimization aspect of the problem.

"Man can learn nothing unless he proceeds from the known to the unknown. - Claude Bernard"