B1. NEWSVENDOR

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Exercise 1

Assume that D follows the following discrete distribution

d	**20**	**25**	**30**	**35**
$P[D=d]$	0.1	0.2	0.4	0.3
$30 \wedge d$	20	25	30	30
$(30-d)^+$	10	5	0	0
$24 \wedge d$	20	24	24	24
$(24-d)^+$	4	0	0	0

Answer the followings.

- $E[30 \wedge D] = \sum(30 \wedge D) \times p(d) = 20 \times 0.1 + 25 \times 0.2 + 30 \times 0.4 + 30 \times 0.3 = 28$
- $E[(30-D)^+] = 10 \times 0.1 + 5 \times 0.2 + 0 \times 0.4 + 0 \times 0.3 = 2$
- $E[24 \wedge D] = 20 \times 0.1 + 24 \times 0.2 + 24 \times 0.4 + 24 \times 0.3 = 23.6$
- $E[(24-D)^+] = 4 \times 0.1 + 0 + 0 + 0 = 0.4$

Exercise 2

Find your brother's optimal stock level by the above Theorem 2. Then, find his expected profit using the Remark 1.

$C_{oversotck} = C_o = \$0.5,\ C_{understock} = C_u = \$1$

$x^* =$ Smallest $y\ s.t\ F(y) \geq \frac{C_u}{C_o+C_u} = \frac{1}{0.5+1} = \frac{2}{3}$

Prob.	**11**	**12**	**13**	**14**	**15**
$P(D=d)$	0.2	0.2	0.2	0.2	0.2
$P(D \leq d)$	0.2	0.4	0.6	0.8	1.0

$F(11) = \frac{1}{5} \quad < \frac{2}{3}$

$F(12) = \frac{2}{5} \quad < \frac{2}{3}$

$F(13) = \frac{3}{5} \quad < \frac{2}{3}$

$F(14) = \frac{4}{5} \quad \geq \frac{2}{3}$

$\therefore x^* = 14$

$E(profit) = E(sales) + E(salvage) - E(material)$

$E(profit) = E[(X \wedge D) \times 2] + E[(X - D)^+ \times 0.5] - E[X \times 1]$ (Here, $X = x^* = 14$)

$E(profit) = E[(14 \wedge D) \times 2] + E[(14 - D)^+ \times 0.5] - E[14]$

$= 2 \sum (14 \wedge D) \times p(d) + 0.5 \sum (14 - D)^+ \times p(d) - 14$

$= 2 \times (\frac{11}{5} + \frac{12}{5} + \frac{13}{5} + \frac{14}{5} + \frac{14}{5}) + 0.5 \times (\frac{3}{5} + \frac{2}{5} + \frac{1}{5} + \frac{0}{5} + \frac{0}{5}) - 14 = \frac{61}{5}$

Exercise 3

Your brother is now selling milk. The customer demands follow U (20, 40) gallons. Retail price is \$2 per gallon, material cost is \$1 per gallon, and salvage cost is \$0.5 per gallon. Find optimal stock level and expected profit.

$C_{oversotck} = C_o = \$1 - \$0.5 = \$0.5$

$C_{understock} = C_u = \$2 - \$1 = \$1$

$x^* =$ Smallest $y \; s.t \; F(y) \geq \frac{C_u}{C_o + C_u} = \frac{1}{0.5+1} = \frac{2}{3}$

$pdf \; f(x) = \begin{cases} \frac{1}{20} & if \; 20 \leq x \leq 40 \\ 0 & otherwise \end{cases}$

$cdf \; F(x) = \begin{cases} 0 & if \; x \leq 20 \\ \frac{x-20}{20} & if \; 20 \leq x \leq 40 \\ 1 & if \; x > 40 \end{cases}$

$F(x^*) = \frac{2}{3} \rightarrow \frac{1}{20}x - 1 = \frac{2}{3} \rightarrow x^* = \frac{100}{3}$

$E(profit) = E(sales) + E(salvage) - E(material)$

$E(profit) = E[(X \wedge D) \times 2] + E[(X - D)^+ \times 0.5] - E[X \times 1]$ (Here, $X = x^* = \frac{100}{3}$)

$E(profit) = \int_{40}^{20} (\frac{100}{3} \wedge x) \times 2f(x)dx + \int_{40}^{20} (\frac{100}{3} - x)^+ \times 0.5f(x)dx - \int_{40}^{20} \frac{100}{3}dx$

$= \int_{40}^{20} (\frac{100}{3} \wedge x) \times 2 \times \frac{1}{20}dx + \int_{40}^{20} (\frac{100}{3} - x)^+ \times 0.5 \times \frac{1}{20}dx - \int_{40}^{20} \frac{100}{3}dx$

$= \frac{1}{10}[\int_{20}^{\frac{100}{3}} xdx + \int_{\frac{100}{3}}^{40} \frac{100}{3}dx] + \frac{1}{40}[\int_{20}^{\frac{100}{3}} (\frac{100}{3} - x)dx + \int_{\frac{100}{3}}^{40} 0dx] - \int_{20}^{40} \frac{5}{3}dx$

$= \frac{80}{3}$

$\therefore E(profit) = \frac{80}{3}$

Exercise 4

Lemonade sells for \$18 per gallon but only costs \$3 per gallon to make. If we run out of lemonade, it will be impossible to get more. On the other hand, leftover lemonade has a value of \$1. Express the following quantity using sale as X and demand as D.

$SalesRevenue = \$18, MaterialCost = \$3, SalvageValue = \$1$

$C_{understock} = C_u = \$18 - \$3 = \$15$

$C_{oversotck} = C_o = \$3 - \$1 = \$2$

$E(EconomicCost) = 15 \times E[(D - X)^+] + 2 \times E[(X - D)^+]$

$E(Profit) = E(SalesRevenue) + E(SalvageValue) - E(MaterialCost)$

$= (X \wedge D) \times 15 + (X - D)^+ \times 2 - X \times 3)$

$= 15(X \wedge D) + 2(X - D)^+ - 3X$

Exercise 5

Prove Theorem 1. (Hint: you may use formulation from Exercise 4)

> **Theorem 1**
> *In newsvendor problem, maximizing the expected profit is equivalent to minimizing the expected economic cost (sum of the expected overstock cost and the expected understock cost).*

X = Stock (Supply), D = Demand

P = Retail price, C = Material cost, S = salvage value

$E(Profit) = E(SalesRevenue) + E(SalvageValue) - E(MaterialCost)$

$= (X \wedge D) \times (P - C) + (X - D)^+ \times (C - S) - X \times C$

$E(EconomicCost) = C_u \times (D - X)^+ + C_o \times (X - D)^+$

$= (P - C) \times (D - X)^+ + (C - S) \times (X - D)^+$

- $if\ X > D$

$E(profit) = D(P - C) + (X - D)(C - S) - CX$

$E(eco) = (X - D)(C - S)$

Mimize economic cost $\rightarrow$ minimize$X$ and maximize$D \Rightarrow$ maximizeprofit

- $if\ X = D$

  $E(profit) = X(P - 2C) = D(P - 2C)$

  $E(eco) = 0$

- $if\ X < D$

  $E(profit) = X(P - 2C)$

  $E(eco) = (D - X)(P - C)$

  Mimize economic cost $\rightarrow$ maximize$X$ and minimize$D \Rightarrow$ maximize profit

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