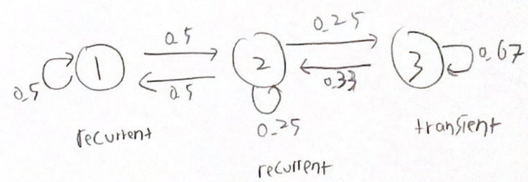
A few definitions (4) - Classifications of state

- A state i is said to be *recurrent* if, starting from i, the probability of getting back to i is 1.
  (There is always a way to get back to state i).
- A state i is said to be *absorbing state*, as a special case of recurrent state, if $\mathbf{P}_{ii} = 1$.
  (You can never leave the state i if you get there once).
- A state i is said to be *transient* if, starting from i, the probability of getting back to i is less than 1.
  (It is possible that the process cannot come back to state i)
- Remark: Recurrence and Transience are class property
  - If $i \leftrightarrow j$, then i is recurrent if and only if j is recurrent.
  - If $i \leftrightarrow j$, then i is transient if and only if j is transient.

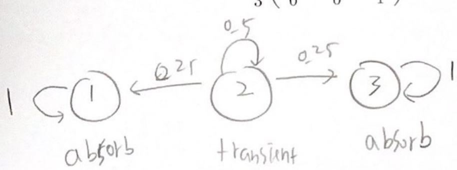
Example

$$\mathbf{P} = \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/4 & 1/4 \\ 0 & 1/3 & 2/3 \end{pmatrix}$$



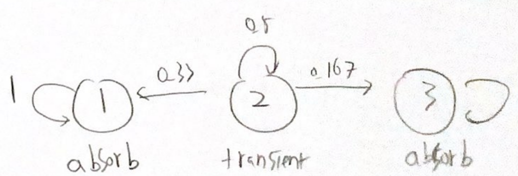
Example

$$\mathbf{P} = \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{pmatrix} 1 & 0 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 0 & 1 \end{pmatrix}$$



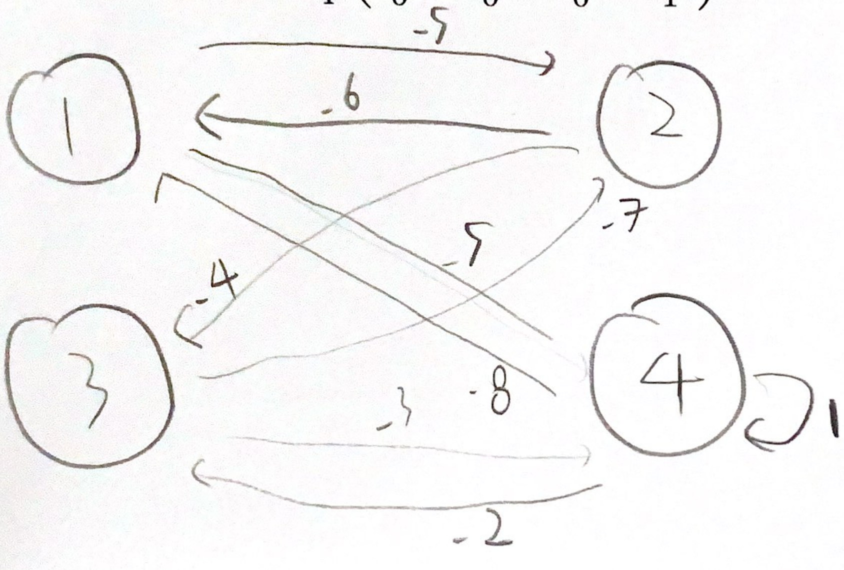
Example

$$\mathbf{P} = \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{pmatrix} 1 & 0 & 0 \\ 1/3 & 1/2 & 1/6 \\ 0 & 0 & 1 \end{pmatrix}$$



Example

$$\mathbf{P} = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



1: recurrent

2: recurrent

3: transient

4 absorb