B1_Exercise

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2020-12-28

Contents

Exercise

Exercise 1

d	20	25	30	35
$P[D = d]$	0.1	0.2	0.4	0.3

- $E[30 \wedge D]$

d	20	25	30	35
$30 \wedge D$	20	25	30	30

$E[30 \wedge D] = 20 * 0.1 + 25 * 0.2 + 30 * 0.4 + 35 * 0.3 = 29.5$

- $E[(30 - D)^+]$

d	20	25	30	35
$(30 - D)^+$	10	5	0	0

$E[(30 - D)^+] = 10 * 0.1 + 5 * 0.2 + 0 * 0.4 + 0 * 0.3 = 2$

- $E[24 \wedge D]$

d	20	25	30	35
$24 \wedge D$	20	24	24	24

$E[24 \wedge D] = 20 * 0.1 + 24 * 0.2 + 24 * 0.4 + 24 * 0.3 = 23.6$

- $E[(24 - D)^+]$

d	20	25	30	35
$(24 - D)^+$	4	0	0	0

$E[(24 - D)^+] = 4 * 0.1 + 0 * 0.2 + 0 * 0.4 + 0 * 0.3 = 0.4$

Exercise 2

If D is a continuous $r.v$, with cdf $F(\cdot)$,then find $y\, s.t. F(y) = \frac{c_u}{c_o+c_u}$

If D is a discrete $r.v$, with cdf $F(\cdot)$, then find smallest y such that $F(y) \geq \frac{c_u}{c_o+c_u}$

$c_o = 0.5$ $c_u = 1$ $x^* =$ smallest $y\, s.t. F(y) \geq \frac{c_u}{c_o+c_u} = \frac{1}{0.5+1} = \frac{2}{3}$

d	11	12	13	14	15
$P(D = d)$	0.2	0.2	0.2	0.2	0.2
$P(D \geq d)$	0.2	0.4	0.6	0.8	1

14 is the minimum value for satisfying the condition.

$E[Profit] = E(SaleRev.) + E(salvageRev.) - E(materialCost)$

E[Sale Revenue]$=2 \cdot (D \wedge 14)$

E[Salvage Revenue]$=\frac{1}{2} \cdot (14 - D)^+$

Material Cost$=1 \cdot 14$

$$E[Profit] = \sum_{D=11}^{15}(2 \cdot (D \wedge 14) \cdot P(D)) + \sum_{D=11}^{15}(\tfrac{1}{2} \cdot (14 - D)^+ \cdot P(D)) - 1 \cdot 14$$

$$=2 \cdot (\tfrac{11+12+13+14+14}{5}) + \tfrac{1}{2} \cdot (\tfrac{3+2+1+0+0}{5}) - 14$$

$$=\tfrac{61}{5}$$

$$=12.2$$

Exercise 3

D~$U(20, 40)$

$$f(x) = \begin{cases} \frac{1}{20} & 20 \leq x \leq 40 \\ 0 & otherwise \end{cases}$$

$$F(x) = \begin{cases} 0 & 20 < x \\ \frac{x-20}{20} & 20 \leq x \leq 40 \\ 1 & x > 40 \end{cases}$$

If D is a continuous $r.v$, with $cdf\, F(\cdot)$,then find $y\, s.t. F(y) = \frac{c_u}{c_o+c_u}$

If D is a discrete $r.v$, with $cdf\, F(\cdot)$, then find smallest y such that $F(y) \geq \frac{c_u}{c_o+c_u}$

Uniform distribution is continuous $F(x^*) = \frac{c_u}{c_o+c_u}$

c_o=(Material Cost - Salvage Price)=$(1 - \frac{1}{2}) = \frac{1}{2}$
c_u=(Retail Price - Material Cost)=(2-1)=1

$F(x^*) = \frac{1}{1+1/2} = \frac{2}{3}$

$\frac{x^*-20}{20} = \frac{2}{3},\ x^* = \frac{100}{3}$

$E[Profit]$=E(Sale Rev.)+E(salvage Rev.)-E(material Cost)

Sale Revenue=$2 \cdot (D \wedge \frac{100}{3})$

Salvage Revenue=$\frac{1}{2} \cdot (\frac{100}{3} - D)^+$

Material Cost=$1 \cdot \frac{100}{3}$

$E[Profit] = E[2 \cdot (D \wedge \frac{100}{3})] + E[\frac{1}{2} \cdot (\frac{100}{3} - D)^+] - 1 \cdot \frac{100}{3}$

$= \int_{20}^{40} (2 \cdot (D \wedge \frac{100}{3}) \cdot \frac{1}{20}) dD + \int_{20}^{40} (\frac{1}{2} \cdot (\frac{100}{3} - D)^+ \cdot \frac{1}{20}) dD - \int_{20}^{40} (\frac{100}{30} \cdot \frac{1}{20}) dD$

$= \frac{1}{10} \cdot (\int_{20}^{\frac{100}{30}} (D) dD + \int_{\frac{100}{3}}^{40} (\frac{100}{3}) dD) + \frac{1}{40} \cdot (\int_{20}^{\frac{100}{3}} (\frac{100}{3} - D) dD + \int_{\frac{100}{3}}^{40} (0) dD) - \frac{100}{3}$

$= \frac{1}{10} \cdot ([\frac{1}{2} D^2]_{20}^{\frac{100}{3}} + \frac{100}{3} [D]_{\frac{100}{3}}^{40}) + \frac{1}{40} \cdot [\frac{100}{3} D - \frac{1}{2} D^2]_{20}^{\frac{100}{3}} - \frac{100}{3}$

$= \frac{80}{3}$

Exercise 4

c_u=(Retail Price - Material Cost)=(18-3)=15

c_o=(Material Cost - Salvage Price)=(3-1)=2

c_v=Material Cost = 3

Expected Economic Cost = $E[Cost]$

= Manufacturing Cost+E[Cost associated with Understock Risk]+E[Cost assoiciated with Overstock Risk]

$$= c_v \cdot X + c_u \cdot E[(D-X)^+] + c_o \cdot E[(X-D)^+]$$
$$= 3X + 15\int_0^\infty ((D-X)^+ \cdot f(D))dD + 2\int_0^\infty ((X-D)^+ \cdot f(D))dD$$
$$= 3X + 15\int_X^\infty ((D-X) \cdot f(D))dD + 2\int_0^X ((X-D) \cdot f(D))dD$$

Expected Profit = $E[Revenue] - E[Cost]$

$$= 18 \cdot E(X \wedge D) - (3X + 15\int_X^\infty (D-X) \cdot f(D)dD + 2\int_0^X (X-D) \cdot f(D)dD)$$
$$= 18 \cdot (\int_0^X D \cdot f(D)dD + \int_X^\infty X \cdot f(D)dD) - (3X + 15\int_X^\infty (D-X) \cdot f(D)dD + 2\int_0^X (X-D) \cdot f(D)dD)$$

Exercise 5

- p = Material Price
- c_u = Understock cost per unit
- c_o = Overstock cost per unit
- X = Sales
- D = Market Demand

Expected Economic Cost = $E[Cost]$

= Manufacturing Cost+E[Cost associated with Understock Risk]+E[Cost assoiciated with Overstock Risk]

$$= c_v \cdot X + c_u \cdot E[(D - X)^+] + c_o \cdot E[(X - D)^+]$$
$$= c_v \cdot X + c_u \cdot \int_0^\infty ((D - X)^+ \cdot f(D))dD + c_o \cdot \int_0^\infty ((X - D)^+ \cdot f(D))dD$$
$$= c_v \cdot X + c_u \cdot \int_X^\infty ((D - X) \cdot f(D))dD + c_o \cdot \int_0^X ((X - D) \cdot f(D))dD$$

Expected Profit = $E[Revenue] - E[Cost]$

$$= p \cdot E(X \wedge D) - (c_v \cdot X + c_u \cdot \int_X^\infty ((D - X) \cdot f(D))dD + c_o \cdot \int_0^X ((X - D) \cdot f(D))dD)$$

$$= p \cdot (\int_0^X D \cdot f(D)dD + \int_X^\infty X \cdot f(D)dD) - (c_v \cdot X + c_u \cdot \int_X^\infty ((D - X) \cdot f(D))dD + c_o \cdot \int_0^X ((X - D) \cdot f(D))dD)$$

E[Revenue] is decided by the sales.

So, we should consider only the cost

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Exercise 6

$E[Profit] = E(SaleRev.) + E(salvageRev.) - E(materialCost)$

(a)

d	5	6	7	8	9
$P(D=d)$	0.1	0.3	0.4	0.1	0.1

E[Sale Revenue]=$30 \cdot (D \wedge 7)$

Salvage Revenue=$5 \cdot (7-D)^+$

Material Cost=$10 \cdot 7$

$$E[Profit] = \sum_{D=5}^{9}(30 \cdot (D \wedge 7) \cdot P(D)) + \sum_{D=5}^{9}(5 \cdot (7-D)^+ \cdot P(D)) - 10 \cdot 7$$
$$=30 \cdot (5*0.1+6*0.3+7*0.4+7*0.1+7*0.1)+5 \cdot (2*0.1+1*0.3+0*0.4+0*0.1+0*0.1)-70$$
$$=127.5$$

(b)

$D \sim U(5,10)$

$$E[Profit] = \sum_{D=5}^{9}(30 \cdot (D \wedge 7) \cdot P(D)) + \sum_{D=5}^{9}(5 \cdot (7-D)^+ \cdot P(D)) - 10 \cdot 7$$
$$=30 \cdot (5*0.2+6*0.2+7*0.2+7*0.2+7*0.2)+5 \cdot (2*0.2+1*0.2+0*0.2+0*0.2+0*0.2)-70$$
$$=125$$

(c)

$D \sim exp(\frac{1}{7})$

$$E[Profit] = 30 \cdot \int_0^{\infty}(D \wedge 7) \cdot f(D)dD + 5 \cdot \int_0^{\infty}(7-D)^+ \cdot f(D)dD - 10 \cdot 7$$
$$=30 \cdot \int_0^7 D \cdot f(D)dD + 30 \cdot \int_7^{\infty} 7 \cdot f(D)dD + 5 \cdot \int_0^7 (7-D) \cdot f(D)dD + 5 \cdot \int_7^{\infty} 0 \cdot f(D)dD - 10 \cdot 7$$
$$=\frac{1}{7} \cdot (25 \cdot \int_0^7 D \cdot e^{-\frac{1}{7}D} + 30 \cdot \int_7^{\infty} 7 \cdot e^{-\frac{1}{7}D}dD + 5 \cdot \int_0^7 7 \cdot e^{-\frac{1}{7}D}dD)$$
$$=\frac{1}{7} \cdot (25 \cdot (49 - 98e^{-1}) + 1470e^{-1} + 245 - 245 \cdot e^{-1})$$
$$=\frac{1}{7} \cdot (1470 - 1225e^{-1})$$

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