B1_(Jeong,wonryeol)

Jeong, wonryeol

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Ex1

d $P[D=d]$	20 0.1	25 0.2	30 0.4	35 0.3
$30 \wedge d$	20	25	30	30
$(30-d)^+$	10	5	0	0
$24 \wedge d$	20	24	24	24
$(24-d)^+$	4	0	0	0

- $E[30 \wedge D] = \sum((30 \wedge D) * P(D)) = 28$
- $E[(30-D)^+] = \sum((30-d)^+ * P(D)) = 2$
- $E[24 \wedge D] = \sum((24 \wedge D) * P(D)) = 23.6$
- $E[(24-d)^+] = \sum(((24-d)^+) * P(D)) = 0.4$

Ex2

Find your brother's optimal stock level by the above Theorem 2. Then, find his expected profit using the Remark 1.

Theorem 2

- If D is a continuous r.v, with cdf $F(\cdot)$, then find y s.t. $F(y) = \frac{c_u}{c_o+c_u}$.
- If D is a discrete r.v, with cdf $F(\cdot)$, then find smallest y such that $F(y) \geq \frac{c_u}{c_o+c_u}$.

Remark1

- $E[Profit]$=E(Sale Rev.)+E(salvage Rev.)-E(material Cost)

Table

d	11	12	13	14	15
$P(D=d)$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

- c_o (Overcost) is \$ (2-1)*(X-D)^+ \$
- c_u (Undercost) is \$ (1-0.5)*(D-X)^+ \$

$F(y) = \frac{1}{0.5+1}$ and it also can be written as $\frac{2}{3}$

To find smallest x^* we need to calculate cummulative function.

$F_D(11) = \frac{1}{5} < \frac{2}{3}$

$F_D(12) = \frac{1}{5} + \frac{1}{5} = \frac{2}{5} < \frac{2}{3}$

$F_D(13) = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5} < \frac{2}{3}$

$F_D(14) = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{4}{5} \geq \frac{2}{3}$

Therefore, 14 is most optimal number

$E[Profit]{=}E(SaleRev.) + E(salvageRev.) - E(materialCost)$

$=E[2 \cdot (D \wedge Y)] + E[\frac{1}{2} \cdot (Y-D)^+] - E[Y]$

in this case, Y is 14 (optimal solution)

$=2 \cdot (\frac{11+12+13+14}{5} + \frac{14}{5}) + \frac{1}{2} \cdot (\frac{3+2+1+0}{5}) - 14$

`= 12.2`

Ex3

in this case, there are 3 cases to show cummulative density probability

$$f_d(x) = \begin{cases} \frac{1}{20} & 20 \leq x \leq 40 \\ 0 & otherwise \end{cases}$$

$$F_d(x) = \begin{cases} 0 & 20 < x \\ \frac{x-20}{20} & 20 \leq x \leq 40 \\ 1 & x > 40 \end{cases}$$

$F(Y) = \frac{C_u}{C_o+C_u}$

$C_u = (MaterialCost - SalvagePrice) = 0.5$

$C_o = (RetailPrice - MaterialCost) = 1$

$F(Y) = \frac{0.5}{1+0.5}$

$F(Y) = \frac{2}{3}$

Theorem 2

- If D is a continuous r.v, with cdf $F(\cdot)$, then find y s.t. $F(y) = \frac{c_u}{c_o+c_u}$.
- If D is a discrete r.v, with cdf $F(\cdot)$, then find smallest y such that $F(y) \geq \frac{c_u}{c_o+c_u}$.

As following continous condition, we should make a equation as $F(y) = \frac{2}{3}$

$\frac{x-20}{20} = \frac{2}{3}$

After calculate , we can get those optimal number

$Y^* = \frac{100}{3}$

Therefore, we can calculate as below

$E[Profit] = \int_{20}^{40}(2 * (D \wedge \frac{100}{3}) * \frac{1}{20})d_D + \int_{20}^{40}(\frac{1}{2} * (\frac{100}{3} - D)^+ * \frac{1}{20})d_D - \int_{20}^{40}(1 * \frac{100}{30} * \frac{1}{20})d_D$

$2 * \frac{1}{20} * \int_{20}^{40}(D)d_D + \frac{1}{2} * \frac{1}{20}\int_{20}^{\frac{100}{3}}(\frac{100}{3} - D)d_D + 1 * \frac{1}{20} * \int_{20}^{40}(\frac{100}{30})d_D$

After calculate

$E[Profit] = \frac{80}{3}$

#Ex4

$C_u = (RetailPrice - MaterialCost) = (18 - 3) = 15$

$C_o = (MaterialCost - SalvagePrice) = (3 - 1) = 2$

$ExpectedCost = E[ManufacturingCost] + E[understockcost] + E[overstockcost]$
$= C_m \cdot E[X] + C_u \cdot E[(D - X)^+] + C_o \cdot E[(X - D)^+]$
$= 3E[X] + 15E[(D - X)^+] + 2E[(X - D)^+]$ $ExpectedProfit = E[Sale] - E[Cost]$

$$= 18E[(X \wedge D)] - (3E[X] + 15E[(D - X)^+] + 2E[(X - D)^+])$$

#Ex5

Prove Theorem 1. $ExpectedCost = E[ManufacturingCost] + E[understockcost] + E[overstockcost]$

to show how above equation process, we have to define some items

- Selling Price : p
- manufacturing cost : c

- Salvage value : s
- Leftover Cost : h
- Demand : D
- How many make : X

$ExpectedProfit = E[Revenue] - E[Cost]$

To figure out cost details, decomposition is needed

$$
\begin{aligned}
ExpectedCost = E[ManufacturingCost] + E[understockcost] + E[overstockcost] \\
&= E[Revenue] - E[ManufacturingCost] - E[UnderstockCost] - E[OverstockCost] \\
&= p \cdot E[(X \wedge D)] - (c \cdot E[X] + (c - s) \cdot E[(X - D)^+] + (p - c) \cdot E[(D - X)^+]) \\
&= p \cdot E[(X \wedge D)] - E[Cost]
\end{aligned}
$$

Therefore ,

$E[profit] = \alpha - E[cost]$ $(\alpha \geq 0)$.