B1_Exercise

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Contents

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Exercise

Exercise 1

Assume that D follows the following discrete distribution.

d	20	25	30	35
$P[D=d]$	0.1	0.2	0.4	0.3
$30 \wedge d$	20	25	30	30
$(30-d)^+$	10	5	0	0
$24 \wedge d$	20	24	24	24
$(24-d)^+$	4	0	0	0

$\sum_{d \in \{20, 25, 30, 35\}}$

d d

Answer the followings.

- $E[30 \wedge D] = \sum((30 \wedge D) \cdot P(D)) = 20 \times 0.1 + 25 \times 0.2 + 30 \times 0.4 + 30 \times 0.3 = 28$
- $E[(30-D)^+] = \sum((30-D)^+ \cdot P(D)) = 10 \times 0.1 + 5 \times 0.2 + 0 \times 0.4 + 0 \times 0.3 = 2$
- $E[24 \wedge D] = \sum((24 \wedge D) \cdot P(D)) = 20 \times 0.1 + 24 \times 0.2 + 24 \times 0.4 + 24 \times 0.3 = 23.6$
- $E[(24-D)^+] = \sum((24-D)^+ \cdot P(D)) = 4 \times 0.1 + 0 \times 0.2 + 0 \times 0.4 + 0 \times 0.3 = 0.4$

마찬가지

Exercise 2

Find your brother's optimal stock level by the above Theorem 2. Then, find his expected profit using the Remark 1.

good

Theorem2

- If D is a continuous r.v, with cdf $F(\cdot)$, then find y s.t. $F(y) = \frac{c_u}{c_o+c_u}$.
- If D is a discrete r.v, with cdf $F(\cdot)$, then find smallest y such that $F(y) \geq \frac{c_u}{c_o+c_u}$.

Remark 1

- $E[Profit]$=E(Sale Rev.)+E(salvage Rev.)-E(material Cost)

Answer

1) Finding optimal stock level

C_o=(Material Cost - Salvage Price)=$(1-\frac{1}{2}) = \frac{1}{2}$
C_u=(Retail Price - Material Cost)=(2-1)=1

$F(y) \geq \frac{1}{1+1/2}$
$F(y) \geq \frac{2}{3}$

d	11	12	13	14	15
$P(D=d)$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

$F_D(11) = \frac{1}{5} < \frac{2}{3}$

$F_D(12) = \frac{1}{5} + \frac{1}{5} = \frac{2}{5} < \frac{2}{3}$

$F_D(13) = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5} < \frac{2}{3}$

$F_D(14) = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{4}{5} \geq \frac{2}{3}$

Thus, y^*=14

2) Finding expected profit

Sale Revenue=$2 \cdot (D \wedge Y)$

Salvage Revenue=$\frac{1}{2} \cdot (Y - D)^+$

Material Cost=$\cancel{Y}$ $1 \cdot Y$

$E[Profit]$=E(Sale Rev.)+E(salvage Rev.)-E(material Cost)

$$=\sum_{\underset{d}{\cancel{D}}=11}^{15}(2 \cdot (\underset{d}{\cancel{D}} \wedge \underset{y}{\cancel{Y}}) \cdot P(\underset{d}{\cancel{D}})) + \sum_{\underset{d}{\cancel{D}}=11}^{15}(\frac{1}{2} \cdot (\underset{d}{\cancel{D}} - \underset{y}{\cancel{Y}})^+ \cdot P(\underset{d}{\cancel{D}})) - \sum_{\underset{d}{\cancel{D}}=11}^{15}(\underset{y}{\cancel{Y}} \cdot P(\underset{d}{\cancel{D}}))$$

D, Y는 r.v 자체이므로 여기에서는 등장하면 안되고 d, y로 변환해야 함

Using

~~Then~~, $y^* = 14$,

$$
\begin{aligned}
E[Profit] = &\textstyle\sum_{D=11}^{15}(2 \cdot (D \wedge 14) \cdot P(D)) + \sum_{D=11}^{15}(\frac{1}{2} \cdot (D-14)^+ \cdot P(D)) - \sum_{D=11}^{15}(14 \cdot P(D)) \\
=&2 \cdot (\textstyle\sum_{D=11}^{14}(D \cdot P(D)) + 14 \cdot P(15)) + \frac{1}{2} \cdot \sum_{D=11}^{14}((14-D) \cdot P(D)) - 14 \\
=&2 \cdot (\tfrac{11+12+13+14}{5} + \tfrac{14}{5}) + \tfrac{1}{2} \cdot (\tfrac{3+2+1+0}{5}) - 14 \\
=&\tfrac{61}{5} \\
=&12.2
\end{aligned}
$$

Thus, $E[Profit] = 12.2$

Exercise 3

D~$U(20, 40)$

$$f(x) = \begin{cases} \frac{1}{20} & 20 \leq x \leq 40 \\ 0 & otherwise \end{cases}$$

$$F(x) = \begin{cases} 0 & 20 < x \\ \frac{x-20}{20} & 20 \leq x \leq 40 \\ 1 & x > 40 \end{cases}$$

Thm2

If D is a continuous $r.v$, with cdf $F(\cdot)$,then find y $s.t.F(y) = \frac{c_u}{c_o+c_u}$

If D is a discrete $r.v$, with cdf $F(\cdot)$, then find smallest y such that $F(y) \geq \frac{c_u}{c_o+c_u}$

Uniform distribution is continuous, so we shall use $F(x^*) = \frac{c_u}{c_o+c_u}$

c_o=(Material Cost - Salvage Price)=$(1 - \frac{1}{2}) = \frac{1}{2}$
c_u=(Retail Price - Material Cost)=(2-1)=1

$F(x^*) = \frac{1}{1+1/2} = \frac{2}{3}$

$\frac{x^*-20}{20} = \frac{2}{3},\ x^* = \frac{100}{3}$

$E[Profit]$=E(Sale Rev.)+E(salvage Rev.)-E(material Cost)

Sale Revenue=$2 \cdot (D \wedge \frac{100}{3})$

Salvage Revenue=$\frac{1}{2} \cdot (\frac{100}{3} - D)^+$

Material Cost=$1 \cdot \frac{100}{3}$

$E[Profit] = E[2 \cdot (D \wedge \frac{100}{3})] + E[\frac{1}{2} \cdot (\frac{100}{3} - D)^+] - 1 \cdot \frac{100}{3}$

$= \int_{20}^{40}(2 \cdot (D \wedge \frac{100}{3}) \cdot \frac{1}{20})dD + \int_{20}^{40}(\frac{1}{2} \cdot (\frac{100}{3} - D)^+ \cdot \frac{1}{20})dD - \int_{20}^{40}(\frac{100}{30} \cdot \frac{1}{20})dD$

$= \frac{1}{10} \cdot (\int_{20}^{\frac{100}{30}}(D)dD + \int_{\frac{100}{3}}^{40}(\frac{100}{3})dD) + \frac{1}{40} \cdot (\int_{20}^{\frac{100}{3}}(\frac{100}{3} - D)dD + \int_{\frac{100}{3}}^{40}(0)dD) - \frac{100}{3}$

$= \frac{1}{10} \cdot ([\frac{1}{2}D^2]_{20}^{\frac{100}{3}} + \frac{100}{3}[D]_{\frac{100}{3}}^{40}) + \frac{1}{40} \cdot [\frac{100}{3}D - \frac{1}{2}D^2]_{20}^{\frac{100}{3}} - \frac{100}{3}$

$= \frac{80}{3}$

Exercise 4 답이 3가지로 나뉘었습니다 표현방식의 차이인 것 같습니다.

Lemonade sells for \$18 per gallon but only costs \$3 per gallon to make. If we run out of lemonade, it will be impossible to get more. On the other hand, leftover lemonade has a value of \$1. Express the following quantity using sale as X and demand as D.

c_u=(Retail Price - Material Cost)=(18-3)=15

c_o=(Material Cost - Salvage Price)=(3-1)=2

c_v=Material Cost = 3

1)

~~Expected economic cost $= 15 \cdot (D-X)^+ + 2 \cdot (X-D)^+$~~

~~Expected profit $= 15(D \wedge X) + 2(X-D)^+ - 3 \cdot X$~~

expected 이므로 E가 있어야 함

2)

Expected economic cost $= 15 \times \mathbb{E}[(D-E)^+] + 2 \times \mathbb{E}[(X-D)^+]$
Expected profit $= 18 \times \mathbb{E}[min(X,D)] + 1 \times \mathbb{E}[(X-D)^+] - 3 \times \mathbb{E}[X]$

정답

3)

economic cost 구할 때 material cost가 understock cost에 포함됨

Expected Economic Cost $= E[Cost]$

= ~~Manufacturing Cost~~+E[Cost associated with Understock Risk]+E[Cost assoiciated with Overstock Risk]

$$= c_v \cdot X + c_u \cdot E[(D-X)^+] + c_o \cdot E[(X-D)^+]$$
$$= 3X + 15\int_0^{\infty}((D-X)^+ \cdot f(D))dD + 2\int_0^{\infty}((X-D)^+ \cdot f(D))dD$$
$$= \cancel{3X} + 15\int_X^{\infty}((D-X) \cdot f(D))dD + 2\int_0^{X}((X-D) \cdot f(D))dD$$

Expected Profit $= E[Revenue] - E[Cost]$

$$= 18 \cdot E(X \wedge D) - (3X + 15\int_X^{\infty}(D-X) \cdot f(D)dD + 2\int_0^{X}(X-D) \cdot f(D)dD)$$

$$= 18 \cdot (\int_0^{X} D \cdot f(D)dD + \int_X^{\infty} X \cdot f(D)dD) - (3X + 15\int_X^{\infty}(D-X) \cdot f(D)dD + 2\int_0^{X}(X-D) \cdot f(D)dD)$$

Exercise 5 답이 3가지로 나뉘었습니다. 문제를 풀지 못한 학생들이 꽤 많았습니다.

Prove Theorem 1.

Theorem 1.

- Maximizing the expected profit is equivalent to minimizing the expected economic cost

1)

Set

- Selling Price : p
- Buying Price : c
- Salvage Value : s
- Holding Cost : h
- Market Demand : D
- Order Quantity : X

Expected Economic Cost = E[Manufacturing Cost] + E[Cost associated with understock Risk]
+ E[Cost associated with overstock Risk]
$= c \cdot E[X] + (p-c) \cdot E[(D-X)^{+}] + (c-s) \cdot E[(X-D)^{+}]$

Expected Profit = E[Revenue] - E[Cost]
= E[Revenue] - E[Ordering Cost] - E[Holding Cost] - E[Backorder Cost]
$= p \cdot E[(X \wedge D)] - c \cdot E[X] - h \cdot E[(X-D)^{+}] - (p-c) \cdot E[(D-X)^{+}]$
$= p \cdot E[(X \wedge D)] - (c \cdot E[X] + h \cdot E[(X-D)^{+}] + (p-c) \cdot E[(D-X)^{+}])$
$= p \cdot E[(X \wedge D)] - (c \cdot E[X] + (c-s) \cdot E[(X-D)^{+}] + (p-c) \cdot E[(D-X)^{+}])$
$= p \cdot E[(X \wedge D)]$-E[Cost]

DaiPark 이랑 mixing 됨.

Thus,

$E[profit] = \alpha - E[cost]$, $(\alpha \geq 0)$.

2)

- c_u = understock cost = $(p-c)(D-X)^+$
- c_o = overstock cost = $(c-s)(X-D)^+$

Expected Economic Cost = $(p-c)(D-X)^+$ $+(c-s)(X-D)^+$

Expected Profit = $p(D \wedge X) + s(X-D)^+ - cX$

we will minimize expected cost then the long-run average cost will be also guaranteed to be minimized

E[E_cost]=$c_o \int_y^\infty (X-D) f_D(x)dx + c_u \int_0^y (D-X) f(x)dx$

$\frac{d}{dy} E[cost] = c_u(F_D(y)-1) + C_o(F_D(y) = 0$

$F_D(y)(c_u + c_o) = c_u$

$F_D(y) = \frac{c_u}{c_u+c_o}$ (1)

$\frac{d^2}{d^2y} E(cost) = (c_u + c_o) f_D(y)$ (2)

(2) is always nonnegative because $0 \leq c_u, c_o$

Therefore, y* obtained from (1) minimizes the cost instead of maximizing it.

Since the profit maximization problem solved previously and the cost minimization problem solved now share the same logic, Maximizing the expected profit is equvalent to Minimizing the expected economic cost

3)

- p = Material Price
- c_u = Understock cost per unit
- c_o = Overstock cost per unit
- X = Sales
- D = Market Demand

Expected Economic Cost = $E[Cost]$

= Manufacturing Cost+E[Cost associated with Understock Risk]+E[Cost assoiciated with Overstock Risk]

$$= c_v \cdot X + c_u \cdot E[(D-X)^+] + c_o \cdot E[(X-D)^+]$$
$$= c_v \cdot X + c_u \cdot \int_0^\infty ((D-X)^+ \cdot f(D))dD + c_o \cdot \int_0^\infty ((X-D)^+ \cdot f(D))dD$$
$$= c_v \cdot X + c_u \cdot \int_X^\infty ((D-X) \cdot f(D))dD + c_o \cdot \int_0^X ((X-D) \cdot f(D))dD$$

Expected Profit = $E[Revenue] - E[Cost]$

$$= p \cdot E(X \wedge D) - (c_v \cdot X + c_u \cdot \int_X^\infty ((D-X) \cdot f(D))dD + c_o \cdot \int_0^X ((X-D) \cdot f(D))dD)$$

$$= p \cdot (\int_0^X D \cdot f(D)dD + \int_X^\infty X \cdot f(D)dD) - (c_v \cdot X + c_u \cdot \int_X^\infty ((D-X) \cdot f(D))dD + c_o \cdot \int_0^X ((X-D) \cdot f(D))dD)$$

E[Revenue] is decided by the sales.

So, we should consider only the cost

Claim $\pi^* = \underset{x}{\operatorname{argmin}} \; (p-c)(D-x)^+ + (c-s)(x-D)^+$ (LHS)

$= \underset{x}{\operatorname{argmax}} \; p\min(D, x) + s(x-D)^+ - cx$ (RHS)

Suppose $\pi^* = \underset{x}{\operatorname{argmin}} \; (p-c)(D-x)^+ + (c-s)(x-D)^+$.

$= \underset{x}{\operatorname{argmax}} \; -(p-c)(D-x)^+ - (c-s)(x-D)^+$ $\left(\because \underset{x}{\operatorname{argmin}} f(x) = \underset{x}{\operatorname{argmax}} -f(x)\right)$

(the goal is to work with above to match RHS up to constant)

$= \underset{x}{\operatorname{argmax}} \; -(p-c)\max(D-x, 0) - c(x-D)^+ + s(x-D)^+$

$= \underset{x}{\operatorname{argmax}} \; (p-c)\min(x-D, 0) - c\max(x-D, 0) + s(x-D)^+$ $\left(\because \max(x, 0) = -\min(-x, 0)\right)$

$= \underset{x}{\operatorname{argmax}} \; p\min(x-D, 0) - c\left[\min(x-D, 0) + \max(x-D, 0)\right] + s(x-D)^+$

$= \underset{x}{\operatorname{argmax}} \; p\min(x-D, 0) - c(x-D) + s(x-D)^+$ $\left(\because \min(x, y) + \max(x, y) = x + y\right)$

$= \underset{x}{\operatorname{argmax}} \; p\left[\min(x-D, 0) + D - D\right] - cx + cD + s(x-D)^+$

$= \underset{x}{\operatorname{argmax}} \; p\left[\min(x, D) - D\right] + s(x-D)^+ - cx + cD$

$= \underset{x}{\operatorname{argmax}} \; p\min(x, D) + s(x-D)^+ - cx - pD + cD$

$= \underset{x}{\operatorname{argmax}} \; \underbrace{p\min(x, D) + s(x-D)^+ - cx}_{\text{(RHS)}} \underbrace{-(p-c)D}_{\text{not affected by choice of } x.}$

① DaiPandas는 누구 참여인지 같이

ex) Exercise 1 (12번 이상)

② annotated 된 것을 각자에게 보내서

(다시 작성해서 확인)

sinw 꼭 기억하자.

DaiPark Exercises

alternative

$\min(D, Y) + \max(Y-D, 0)$

$= \min(D-Y, 0) + Y + \max(Y-D, 0)$

$= D - Y + Y = Y$

$(\because \min(X,Y) + \max(X,Y) = X + Y)$

Exercise 1

Show that $(D \wedge Y) + (Y - D)^+ = Y$

i) $D > Y$

$$(D \wedge Y) + (Y - D)^+ = Y + 0 = Y$$

ii) $Y > D$

$$(D \wedge Y) + (Y - D)^+ = D + (Y - D) = Y$$

Exercise 2

Let D **be a discrete a random variable with the following pmf.**

d	5	6	7	8	9
$P(D = d)$	0.1	0.3	0.4	0.1	0.1

$= \sum P(D=d)\min(d, 7) = 0.1\min(5, 7) + 0.3\min(6, 7) + \cdots\cdots$

(a)

$$\begin{aligned}
E[min(D, 7)] &= E[min(5, 7)] + E[min(6, 7)] + E[min(7, 7)] + E[min(8, 7)] + E[min(9, 7)] \\
&= E[5] + E[6] + E[7] + E[7] + E[7] \\
&= 5 \cdot \frac{1}{10} + 6 \cdot \frac{3}{10} + 7 \cdot \frac{4}{10} + 7 \cdot \frac{1}{10} + 7 \cdot \frac{1}{10} \\
&= \frac{5 + 18 + 28 + 7 + 7}{10} \\
&= \frac{65}{10} \\
&= 6.5
\end{aligned}$$

이어서 연산

마찬가지.

(b)

$$\begin{aligned}
E[(7 - D)^+] &= E[(7 - 5)^+] + E[(7 - 6)^+] + E[(7 - 7)^+] + E[(7 - 8)^+] + E[(7 - 9)^+] \\
&= E[2] + E[1] + E[0] + E[0] + E[0] \\
&= 2 \cdot \frac{1}{10} + 1 \cdot \frac{3}{10} + 0 \cdot \frac{4}{10} + 0 \cdot \frac{1}{10} + 0 \cdot \frac{1}{10} \\
&= \frac{2 + 3}{10} \\
&= \frac{5}{10} \\
&= 0.5
\end{aligned}$$

Exercise 3

Let D be a Poisson random variable with parameter 3. $\cancel{\backslash\ D \sim Poi(3)}$

$P(D=k) = \frac{3^k e^{-3}}{k!}$ for $k = 0, 1, 2, ...$

$E(X) = Var(X) = 3$

(a)

$$
\begin{aligned}
E[min(D,2)] &= \sum_{x=-\infty}^{\infty} min(\overset{x}{\cancel{D}}, 2)p(x) \\
&= \sum_{x=-\infty}^{0} min(\overset{x}{\cancel{D}}, 2)p(x) + \sum_{x=0}^{\infty} min(D,2)p(x) \\
&= 0 + \sum_{x=0}^{\infty} min(D,2)p(x) \\
&= 0 + 3 \\
&= 3
\end{aligned}
$$

✓ why?

(b)

$$
\begin{aligned}
E[(3-D)^+] &= \sum_{D=-\infty}^{\infty} (3-D)^+ p(D) \\
&= \sum_{D=-\infty}^{0} (3-D)^+ (D) + \sum_{D=0}^{\infty} (3-D)^+ p(D) \\
&= 0 + \sum_{D=0}^{\infty} (3-D)^+ p(D) \\
&= \sum_{D=0}^{3} (3-D)^+ p(D) + \sum_{D=4}^{\infty} (3-D)^+ p(D) \\
&= \sum_{D=0}^{3} (3-D) p(D) + 0 \\
&= 3
\end{aligned}
$$

Exercise 4 (b) 번 문제의 답이 2가지로 나뉘었습니다 (D-S)⁻ 해석하는 방식에 따라 답이 나뉘었습니다.

Let D be a continuous random variable and uniformly distributed between 5 and 10. $\backslash D$

$\sim U(5, 10)$

$$f_d(x) = \begin{cases} \frac{1}{5} & 5 \leq x \leq 10 \\ 0 & otherwise \end{cases}$$

$$F_d(x) = \begin{cases} 0 & 5 < x \\ \frac{x-5}{5} & 5 \leq x \leq 10 \\ 1 & x > 10 \end{cases}$$

$\int_{-\infty}^{\infty} max(x, 8) f_D(x)\, dx$ 로부터 시작

$=$

(a)

이건 notation을 틀렸죠. 작성한 사람이 이거 대신 해서 넘어가고 말았

$$\begin{aligned}
E[max(D,8)] &= \int_{-\infty}^{\infty} max(D,8) f_D X d_x \\
&= \int_{-\infty}^{0} max(D,8) f_D X d_x + \int_{0}^{\infty} max(D,8) f_D X d_x \\
&= 0 + \int_{0}^{\infty} max(D,8) f_D X d_x \\
&= \int_{0}^{5} max(D,8) f_D X d_x + \int_{5}^{10} max(D,8) f_D X d_x + \int_{10}^{\infty} max(D,8) f_D X d_x \\
&= 0 + \int_{5}^{10} max(D,8) \frac{1}{5} d_x + 0 \\
&= \int_{5}^{8} max(D,8) \frac{1}{5} d_x + \int_{8}^{10} max(D,8) \frac{1}{5} d_x \\
&= \int_{5}^{8} 8 \frac{1}{5} d_x + \int_{8}^{10} D \frac{1}{5} d_x \\
&= \frac{8}{5} [x]_5^8 + \frac{1}{5} [\frac{1}{2} D^2]_8^{10} \\
&= \frac{24}{5} + \frac{18}{5} \\
&= \frac{42}{5}
\end{aligned}$$

(b_1) (a) (b) 문제의 답이 각각 2가지로 나뉘었습니다

$$
\begin{aligned}
E[(D-8)^-] &= \int_{-\infty}^{\infty} (D-8)^- f_D(x) d_x \quad dx \\
&= \int_{-\infty}^{0} (D-8)^- f_D(x) d_x + \int_{0}^{\infty} (D-8)^- f_D(x) d_x \\
&= 0 + \int_{0}^{\infty} (D-8)^- f_D(x) d_x \\
&= \int_{0}^{5} (D-8)^- f_D(x) d_x + \int_{5}^{10} (D-8)^- f_D(x) d_(x) + \int_{10}^{\infty} (D-8)^- f_D(x) d_x \\
&= 0 + \int_{5}^{10} (D-8)^- \frac{1}{5} d_x + 0 \\
&= \int_{5}^{8} (D-8)^- \frac{1}{5} d_x + \int_{8}^{10} (D-8)^- \frac{1}{5} d_x \\
&= \int_{5}^{8} (D-8) \frac{1}{5} d_x + 0 \\
&= \frac{1}{5}[\frac{1}{2}D^2 - 8D]_5^8 \\
&= -0.9
\end{aligned}
$$

how?

(b_2)

$\mathbb{E}[(D-8)^-] = \int_5^8 (8-D) f(x)\ dx + \int_8^{10} 0 \times f(x)\ dx$

$= \int_5^8 (8-X) f(x)\ dx + 0 = \int_5^8 (8-X)\frac{1}{5}\ dx = \int_5^8 (\frac{8}{5} - \frac{1}{5}x)\ dx$

$= \frac{32}{5} - \frac{55}{10} = \frac{9}{10}$

Exercise 5 (a) (b) 문제의 답이 각각 2가지로 나뉘었습니다

Let D be an exponential random variable with parameter 7. $\backslash\ D \sim exp(7)$

$$f_d(x) = \begin{cases} 7e^{-7x} & x \geq 0 \\ 0 & otherwise \end{cases}$$

$$F_d(x) = \begin{cases} 1 - e^{-7x} & x \geq 0 \\ 0 & otherwise \end{cases}$$

(a_1) 계산방식의 차이인 것 같습니다.

$$\begin{aligned}
E[max(D,3)] &= \int_{-\infty}^{\infty} max(\overset{x}{\cancel{D}},3)f_D(x)\cancel{d_x}\ dx \\
&= \int_{-\infty}^{0} max(\overset{x}{\cancel{D}},3)f_D(x)d_x + \int_{0}^{\infty} max(\overset{x}{\cancel{D}},3)f_D(x)\cancel{d_x}\ dx \\
&= 0 + \int_{0}^{\infty} max(D,3)f_D(x)d_x \\
&= \int_{0}^{3} max(D,3)f_D(x)d_x + \int_{3}^{\infty} max(D,3)f_D(x)d_x \\
&= \int_{0}^{3} 3 \cdot 7e^{-7x}d_x + \int_{3}^{\infty} x \cdot 7e^{-7x}d_x \\
&= 21\int_{0}^{3} e^{-7x}d_x + 7\int_{3}^{\infty} x \cdot e^{-7x}d_x \\
&= 3 - 3e^{-21} - 3e^{-21} + \frac{1}{7}e^{-21} \\
&= 3 - \frac{41}{7}e^{-21}
\end{aligned}$$

차이.

(a_2)

$\mathbb{E}[max(D,3)] = \int_{-\infty}^{3} 3 \times 7e^{-7x}\ dx + \int_{3}^{\infty} x \times 7e^{-7x}\ dx$

$= \int_{0}^{3} 21e^{-7x}\ dx + \int_{3}^{\infty} 7xe^{-7x}\ dx$

$= [-3e^{-7x}]_{0}^{3} + [-xe^{-7x}]_{3}^{\infty} - \int_{3}^{\infty} -e^{-7x}\ dx$

$= (-3e^{21} - (-3)) + (0 - (-3e^{21})) + \int_{3}^{\infty} e^{-7x}\ dx$

$= 3 + [-\frac{e^{-7x}}{7}]_{7}^{\infty}$

$= 3 + (0 - (-\frac{e^{-21}}{7}))$

$= 3 + \frac{e^{-21}}{7}$

(b_1) (D-4)- 해석하는 방식의 차이인 것 같습니다.

→ Dai Park의 definition을 보고 그에 맞게 하여 이는 답안을 보내드림

$$
\begin{aligned}
E[(D-4)^-] &= \int_{-\infty}^{\infty} (D-4)^- f_D(x) d_x \\
&= \int_{-\infty}^{0} (D-4)^- f_D(x) d_x + \int_{0}^{\infty} (D-4)^- f_D(x) d_x \\
&= 0 + \int_{0}^{\infty} (D-4)^- f_D(x) d_x \\
&= \int_{0}^{4} (D-4)^- f_D(x) d_x + \int_{4}^{\infty} (D-4)^- f_D(x) d_x \\
&= 0 + \int_{4}^{\infty} (D-4) f_D(x) d_x \\
&= \int_{4}^{\infty} (x-4) 7 \cdot e^{-7x} d_x \\
&= \frac{1}{7} e^{-28}
\end{aligned}
$$

(b_2)

$$
\begin{aligned}
\mathbb{E}[(D-4)^-] &= \textstyle\int_0^4 (4-x) 7 e^{-7x} \, dx + \int_4^\infty 0 \times f(x) \, dx \\
&= \textstyle\int_0^4 28 e^{-7x} \, dx - \int_0^4 7x e^{-7x} \, dx \\
&= (-4e^{-28} + 4) - (0 + 4e^{-28}) + (0 + \frac{e^{-28}}{7}) \\
&= 4 + \frac{e^{-28}}{7}
\end{aligned}
$$

Exercise 6 (a) (c) 문제의 답이 각각 2가지로 나뉘었습니다

$E[Profit] = E(SaleRev.) + E(salvageRev.) - E(materialCost)$

(a_1) Optimal solution을 구해야 하는지 아닌지의 차이인 것 같습니다.

d	5	6	7	8	9
$P(D=d)$	0.1	0.3	0.4	0.1	0.1

E[Sale Revenue]=$30 \cdot (D \wedge 7)$

Salvage Revenue=$5 \cdot (7-D)^+$

Material Cost=$10 \cdot 7$

$E[Profit] = \sum_{D=5}^{9}(30 \cdot (D \wedge 7) \cdot P(D)) + \sum_{D=5}^{9}(5 \cdot (7-D)^+ \cdot P(D)) - 10 \cdot 7$

$=30 \cdot (5*0.1+6*0.3+7*0.4+7*0.1+7*0.1)+5 \cdot (2*0.1+1*0.3+0*0.4+0*0.1+0*0.1)-70$

$=127.5$

(a_2)

- Selling Price : 30
- Buying Price : 10
- Salvage Value : 5
- C_u=Selling Price - Buying Price = 30 - 10 = 20
- C_o=Buying Price - Salvage Value = 10 - 5 = 5

Thus,

$$\begin{aligned} F(y) &>= \frac{C_u}{C_o + C_u} \\ &>= \frac{20}{20+5} \\ &>= \frac{4}{5} \end{aligned}$$

$F_D(5) = \frac{1}{10} < \frac{1}{10}$

$F_D(6) = \frac{1}{10} + \frac{3}{10} = \frac{2}{5} < \frac{2}{5}$

$F_D(7) = \frac{1}{10} + \frac{3}{10} + \frac{4}{10} = \frac{4}{5} \geq \frac{4}{5}$

Thus, $Y^* = 7$

$$
\begin{aligned}
E[profit] &= E(salesrev.) + E(salvagerev.) - E(materialcost) \\
&= E[30(Y \wedge D)] + E[5(Y-D)^+] - E[10Y] \\
&= 30\sum_{D=5}^{9}(7 \wedge D)p(D) + 5\sum_{D=5}^{9}(7-D)^+p(D) - 10\sum_{D=5}^{9}7p(D) \\
&= 30\sum_{D=5}^{9}(7 \wedge 7)p(D) + 5\sum_{D=5}^{9}(7-7)^+p(D) - 10\sum_{D=5}^{9}7p(D) \\
&= 30\sum D = 5^9 7 \cdot p(D) - 10\sum D = 5^9 7 \cdot p(D) \\
&= 30 \times 7\frac{1+3+4+1+1}{10} - 10 \times 7\frac{1+3+4+1+1}{10} \\
&= 210 - 70 \\
&= 140
\end{aligned}
$$

(b)

Since $D \sim U(5, 10)$,

$$f_d(x) = \begin{cases} \frac{1}{5} & 5 \le x \le 10 \\ 0 & otherwise \end{cases}$$

$$F_d(x) = \begin{cases} 0 & x < 5 \\ \frac{x-5}{5} & 5 \le x \le 10 \\ 1 & x > 10 \end{cases}$$

In this problem, sale revenue $= 30 \cdot (D \wedge 7)$, salvage revenue $= 5 \cdot (7-D)^+$, material cost $= 7 \cdot 10 = 70$ cents for preparing 7 pounds of banana.

$$
\begin{aligned}
E[Profit] = E(saleRevenue) + E(salvageRevenue) - E(materialCost) \qquad (1) \\
= E[30 \cdot (D \wedge 7)] + E[5 \cdot (7-D)^+] - 7 \cdot 10 \qquad (2) \\
= \int_5^{10}(30 \cdot (D \wedge 7) \cdot \frac{1}{5})d_D + \int_5^{10}(5 \cdot (7-D)^+ \cdot \frac{1}{5})d_D - 70 \qquad (3) \\
= 6 \cdot \int_5^{10}(D \wedge 7)d_D + \int_5^{10}(7-D)^+ d_D - 70 \qquad (4) \\
= 6 \cdot (\int_5^{7}(D)d_D + \int_7^{10}(7)d_D) + \int_5^{7}(7-D)d_D - 70 \qquad (5) \\
= 6 \cdot ([\frac{1}{2}D^2]_5^7 + 7[D]_7^{10}) + [7D - \frac{1}{2}D^2]_5^7 - 70 \qquad (6) \\
= 130 \qquad (7)
\end{aligned}
$$

(c_1) D~$exp(\frac{1}{7})$ Optimal solution을 구해야 하는지 아닌지의 차이인 것 같습니다.

$E[Profit] = 30 \cdot \int_0^{\infty}(D \wedge 7) \cdot f(D)dD + 5 \cdot \int_0^{\infty}(7-D)^{+} \cdot f(D)dD - 10 \cdot 7$

$=30 \cdot \int_0^{7} D \cdot f(D)dD + 30 \cdot \int_7^{\infty} 7 \cdot f(D)dD + 5 \cdot \int_0^{7}(7-D) \cdot f(D)dD + 5 \cdot \int_7^{\infty} 0 \cdot f(D)dD - 10 \cdot 7$

$=\frac{1}{7} \cdot (25 \cdot \int_0^{7} D \cdot e^{-\frac{1}{7}D} + 30 \cdot \int_7^{\infty} 7 \cdot e^{-\frac{1}{7}D}dD + 5 \cdot \int_0^{7} 7 \cdot e^{-\frac{1}{7}D}dD)$

$=\frac{1}{7} \cdot (25 \cdot (49 - 98e^{-1}) + 1470e^{-1} + 245 - 245 \cdot e^{-1})$

$=\frac{1}{7} \cdot (1470 - 1225e^{-1})$

(c_2) D~$exp(\frac{1}{7})$

$f_D(x) = \begin{cases} \frac{1}{7}e^{-\frac{1}{7}x} & x \geq 0 \\ 0 & otherwise \end{cases}$

$F_D(x) = \begin{cases} 1-e^{-\frac{1}{7}x} & x \geq 0 \\ 0 & otherwise \end{cases}$

$F_D(Y) = 1 - e^{-\frac{1}{7}Y} = \frac{4}{5}$

Thus, $Y^{+} = 7ln5$

$$\begin{aligned} E[profit] &= E(salesrev.) + E(salvagerev.) - E(materialcost) \\ &= E[30(Y \wedge D)] + E[5(Y-D)^{+}] - E[10Y] \\ &= E[30(7ln5 \wedge 7)] + E[5(7ln5-7)^{+}] - E[10(7ln5)] \\ &= 30E[7] + 5E[7ln5-7] - 10E[7ln5] \\ &= 30\int_0^{\infty} 7 \cdot \frac{1}{7}e^{-\frac{1}{7}x}d_x + 5\int_0^{\infty}(7ln5-7) \cdot \frac{1}{7}e^{-\frac{1}{7}x}d_x - 10\int_0^{\infty} 7ln5 \cdot \frac{1}{7}e^{-\frac{1}{7}x}d_x \\ &= 210 + 35ln5 - 35 - 70ln5 \\ &= 175 - 35ln5 \end{aligned}$$

Exercise 7

Demand	10	11	12	13	14
P[D=d]	0.1	0.2	0.4	0.2	0.1

(a)

mean demand $= \sum x \overset{p(x)}{\cancel{P(x)}} = 10(0.1) + 11(0.2) + 12(0.4) + 13(0.2) + 14(0.1)$ =12

(b)

Demand	10	12	13	14	Expected Profit	
14	$10(18)+$ $1(1)-$ $11(3)=148$	$11(18)+$ $0(0)-$ $11(3)=147$	$11(18)+$ $0(0)-$ $11(3)=147$	$11(18)+$ $0(0)-$ $11(3)=147$	$11(18)+$ $0(0)-$ $11(3)=147$	$0.1(148)+$ $0.2(147)+$ $0.4(147)+$ $0.2(147)+$ $0.1(147)=$ 147.1

Thus, Expected profit is \$ 147.1 If 11 gallons are prepared

(c)

$c_o = 3 - 1 = 2$

$c_u = 18 - 3 = 15$

If D is a discrete r.v with cdf F(x), then find smallest y s.t $F(y) \geq \frac{c_u}{c_u+c_o} = \frac{15}{15+2} = \frac{15}{17} = 0.88235$

Demand	10	11	12	13	14
$P[D = d]$	0.1	0.2	0.4	0.2	0.1
$P[D \leq d]$	0.1	0.3	0.7	0.9	1.0

Thus, 13 is the best amount of lemonade before the game to order

(d)

Demand$\sim N(1000, 200^2)$

If D is a continuous r.v with cdf F(x), then find y s.t $F(y) = \frac{c_u}{c_u+c_o} = \frac{15}{15+2} = \frac{15}{17} = 0.88235$

then in standard Normal Distribution $P(Z \leq 1.175) = 0.88$

thus, $\frac{X-100}{200} = 1.175$

$X = 1.175(200) + 1000 = 1235$

we should prepare 1235 gallons.

Exercise 8 풀이방식이 2가지로 나뉘었습니다 첫번째 답변에서 Expected profit 구할 때 실수한 거 같습니다.

(1) optimal weekly profit $= 20 \cdot (\sum_{D=15}^{20} (D \wedge X) \cdot P(D)) - 10 \cdot X$

$= 20 \cdot ((15 \wedge X) \cdot \frac{5}{100} + (16 \wedge X) \cdot \frac{20}{100} + (17 \wedge X) \cdot \frac{30}{100} + (18 \wedge X) \cdot \frac{25}{100} + (19 \wedge X) \cdot \frac{10}{100} + (20 \wedge X) \cdot \frac{10}{100}) - 10 \cdot X$

$= (15 \wedge X) + 4(16 \wedge X) + 6(17 \wedge X) + 5(18 \wedge X) + 2(19 \wedge X) + 2(20 \wedge X) - 10 \cdot X$

$X = 15,\ 15 + 4 \cdot 15 + 6 \cdot 15 + 5 \cdot 15 + 2 \cdot 15 + 2 \cdot 15 - 10 \cdot 15 = 150$

$X = 16,\ 15 + 4 \cdot 16 + 6 \cdot 16 + 5 \cdot 16 + 2 \cdot 16 + 2 \cdot 16 - 10 \cdot 16 = 159$

$X = 17,\ 15 + 4 \cdot 16 + 6 \cdot 17 + 5 \cdot 17 + 2 \cdot 17 + 2 \cdot 17 - 10 \cdot 17 = 164$

$X = 18,\ 15 + 4 \cdot 16 + 6 \cdot 17 + 5 \cdot 18 + 2 \cdot 18 + 2 \cdot 18 - 10 \cdot 18 = 163$

$X = 19,\ 15 + 4 \cdot 16 + 6 \cdot 17 + 5 \cdot 18 + 2 \cdot 19 + 2 \cdot 19 - 10 \cdot 19 = 157$

$X = 20,\ 15 + 4 \cdot 16 + 6 \cdot 17 + 5 \cdot 18 + 2 \cdot 19 + 2 \cdot 20 - 10 \cdot 20 = 149$

— 확률이 안들어갔음.

therefore, the bakery should prepare 17 cakes each week. and the bakery's expected optimal weekly profit is 163$

(2)

1. Optimal Stock
   retail price = \$20, material cost = \$10, salvage value = \$0
   $c_o = \$10 \rightarrow$ (material cost - salvage value)
   $c_u = \$10 \rightarrow$ (retail price - material cost)
   $x^* =$ Smallest Y s.t $F(y) \geq \frac{c_u}{c_o + c_u}$

	15	16	17	18	19	20
$P(D = d)$	0.05	0.2	0.3	0.25	0.1	0.1
$P(D \leq d)$	0.05	0.25	0.55	0.8	0.9	1.0

$x^* =$ Smallest Y s.t $F(y) \geq \frac{1}{2}$

$x^* = 17$

$\therefore$ Optimal stock $= 17$

2. Expected Profit
   $\mathbb{E}[profit] = \mathbb{E}[sale\ rev.] + \mathbb{E}[salvage\ rev.] - \mathbb{E}[material\ cost]$
   $\mathbb{E}[sale\ rev.] = \sum_{i=15}^{20} (i \wedge 17) \times P(i) \times \$20$
   $\mathbb{E}[salvage\ rev.] = 0$
   $\mathbb{E}[material\ cost] = 17 \times \$10 = \$170$
   $\mathbb{E}[profit] = \$334 + \$0 - \$170 = \$164$

$\therefore$ Expected Profit $= \$164$

Exercise 14

(a)

- Salvage Value= s= \$2 per gallon
- Wholesale price= c =\$5 per gallon
- retail price = p = \$10 per gallon
- Demand distribution = U(50,150)

$c_o = c - s = 5 - 2 = 3$
$c_u = p - c = 10 - 5 = 5$
$F(y) = \frac{c_u}{c_u + c_o} = \frac{5}{8}$

$U(50, 150)$

$$f(y) = \begin{cases} \frac{1}{100} & 50 \leq x \leq 150 \\ 0 & otherwise \end{cases}$$

$$F(y) = \begin{cases} 0 & y \leq 50 \\ \frac{y-50}{150-50} & 50 \leq y \leq 150 \\ 1 & y > 150 \end{cases}$$

$\frac{y-50}{100} = \frac{5}{8}$
$y = \frac{225}{2}$

therefore, the optimal number of gallons is 112.5 gallons.

(b) when X=100 = E[profit] = ... 로 시작

Expectd profit ~~X.100~~ $= 10(D \wedge 100) \cdot f(y) + 2(100 - D)^+ \cdot f(y) - 5 \cdot 100$
$= \int_{50}^{100} 10 \cdot (D \wedge 100) \cdot \frac{1}{100} dD + \int_{100}^{150} 10 \cdot 100 \cdot \frac{1}{100} dD + \int_{50}^{100} 2 \cdot (100 - D) \cdot \frac{1}{100} dD - 5 \cdot 100$
$= 900 - 500 = 400$

If 100 gallons are ordered, the expected profit per day is 400\$

Exercise 16

d	100	200	300	400
$Pr(D=d)$	.2	.4	.3	.1

(a)

- C_o=(Material Cost + Salvage Cost, overstock Cost)=(100+50)=150
- C_u=(penalty Cost + Material Cost, understock Cost)=(500+100)=600

Unit have discrete properties that can count. $\therefore F(y) \geq \frac{c_u}{c_o+c_u}$

$F(y) \geq \frac{600}{150+600} = 0.8$

From problem's table,

$F_D(100) = 0.2 < 0.8$

$F_D(200) = 0.2 + 0.4 < 0.8$

$F_D(300) = 0.2 + 0.4 + 0.3 \geq 0.8$

Thus, x^*=300

Expected Economic Cost = E[Cost associated with understock Risk] + E[Cost associated with overstock Risk]

$= E[cost] = arg\cancel{min}(600 \times E[(D-X)^+] + 150 \times E[(X-D)^+])$

$= argmin(\sum_{D=100}^{400}(600 \times (D \wedge 300) \times P(D)) + \sum_{D=100}^{400}(150 \times (D-300)^+ \times P(D)))$

$=600 \times (0.2 \times 100 + 0.4 \times 200 + 0.3 \times 300 + 0.1 \times 300) + 150 \times (0.2 \times 200 + 0.4 \times 100 + 0.3 \times 0) = 126,000$

$\therefore$ Expected Economic Cost = 126,000$

(b)

- C_o=(Material Cost + Salvage Cost, Overstock Cost)=(100+50)=150
- C_u=(Penalty Cost + Material Cost, Understock Cost)=(500+100)=600

Unit have discrete properties that can count. $\therefore F(y) \geq \frac{c_u}{c_o+c_u}$

$F(y) \geq \frac{600}{150+600} = 0.8$

From problem's table,

$F_D(100) = 0.2 < 0.8$

$F_D(200) = 0.2 + 0.4 < 0.8$

$F_D(300) = 0.2 + 0.4 + 0.3 \geq 0.8$

Thus, x^*=300

Expected Economic Cost = E[Cost associated with understock Risk] + E[Cost associated with overstock Risk] + E[Fixed cost]

$= E[cost] = \cancel{argmin}(600 \times E[(D - X)^+] + 150 \times E[(X - D)^+] + 20000 \times (D - 100) \vee 1)$

Note that the `echo = FALSE` parameter was added to the code chunk to prevent printing of the R code that generated the plot.