5. Injury free probability is 0.98 & independent $\Rightarrow$ It is Markov property

$X_0 = 0 \Rightarrow$ Initial distribution is $a_0 = (1, 0)$, $P = \begin{bmatrix} 0.98 & 0.02 \\ 0.02 & 0.98 \end{bmatrix}$

5-b) Yes, Because it consists of only one group (injury, not injury) that is no longer divided, with no state of absorption

5-c) Aperiodic. Because $P^{\infty} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$

5-d) $(a, b)\begin{pmatrix} 0.98 & 0.02 \\ 0.02 & 0.98 \end{pmatrix} = (a, b)$ $\quad 0.98a + 0.02b = a \Rightarrow 0.02a = 0.02b$ & $a+b=1$

$\therefore (0.5, 0.5)$

5-e) No it is null-recurrent. Because it multiply itself, the more value changes approximately 0.5

6-a) Yes $P^{11}, P^{13} = \begin{pmatrix} 0 & 0.5625 & 0 & 0.4375 \\ 0.6875 & 0 & 0.3125 & 0 \\ 0 & 0.5625 & 0 & 0.4375 \\ 0.6875 & 0 & 0.3125 & 0 \end{pmatrix}$, P^{10} & P^{12}: $\begin{pmatrix} 0.6875 & 0 & 0.3125 & 0 \\ 0 & 0.5625 & 0 & 0.4375 \\ 0.6875 & 0 & 0.3125 & 0 \\ 0 & 0.5625 & 0 & 0.4375 \end{pmatrix}$

$\Rightarrow d(i) = 2$

6-b) If stationary distribution: $aP = a$ $\therefore (\pi_1, \pi_2, \pi_3, \pi_4) \times P = (\pi_1, \pi_2, \pi_3, \pi_4)$

$\Rightarrow (33/96, 27/96, 15/96, 21/96) \times \begin{pmatrix} 0 & 0.5 & 0 & 0.5 \\ 0.6 & 0 & 0.4 & 0 \\ 0 & 0.7 & 0 & 0.1 \\ 0.8 & 0 & 0.2 & 0 \end{pmatrix} = (33/96, 27/96, 15/96, 21/96)$ (using R)

$\therefore$ It is stationary distribution

6-c) $P_{11}^{100} = \dfrac{0.6875(33+15)}{96} = 0.34375$, $P_{11}^{101} = \dfrac{0.6875}{96}(27+21) = 0.34375 = 33/96$

$\therefore P_{11}^{100} = P_{11}^{101} = 33/96$

14-a) $a_0 = (0,1,0)$ - initial distribution.

transition probability matrix: $P = \begin{pmatrix} 0.25 & 0.75 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 0.25 & 0.75 \end{pmatrix}$

14-b) $xP = x \Rightarrow P^t x^t = 1 \cdot x^t$ $\quad \therefore x = \left(\frac{2}{11}, \frac{3}{11}, \frac{6}{11}\right)$ = stationary dist

14-c) $P^{\infty} = \begin{bmatrix} 0.1818182 & 0.272727 & 0.5454545 \\ " & " & " \\ " & " & " \end{bmatrix}$ $\quad \therefore$ 2: $\frac{3}{11}$ 3: $\frac{6}{11}$

14-d) long-run avg $= \frac{2}{11}(5000 + 1^2 \times 5000) + \frac{3}{11}(5000 + 2^2 \times 5000) + \frac{6}{11}(5000 + 3^2 \times 5000)$

$= \$35909.09$