

C2_DTMC Python

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Exercises

p.12 Method 1 - eigen-decomposition

R code

```
P <- array(c(0.7, 0.5, 0.3, 0.5), dim = c(2,2))
eigen(t(P)) # eigen-decomposition for P^t

x_1 <- eigen(t(P))$vectors[,1]
x_1

v <- x_1/sum(x_1)
v
```

Python code

```
P=np.matrix([[0.7,0.5], [0.3,0.5]])
eg_val, eg_vec = np.linalg.eig(P)
print(eg_val)
```

```
## [1.  0.2]
```

```
print(eg_vec)
```

```
## [[ 0.85749293 -0.70710678]
##   [ 0.51449576  0.70710678]]
```

```
x_1 = eg_vec[:,0]
print(x_1)
```

```
## [[0.85749293]
##   [0.51449576]]
```

```
v = x_1/sum(x_1)
print(v)
```

```
## [[0.625]
##  [0.375]]
```

p.15 Method 2 - system of linear equation

R code

```
P <- array(c(0.7, 0.5, 0.3, 0.5), dim = c(2,2))
n <- nrow(P) # n=|S|
I <- diag(n) # identity matrix
A <- cbind(P-I, rep(1,n))
b <- array(c(rep(0,n),1), dim = c(1, n+1))

v <- solve(A %*% t(A), A %*% t(b))
v
```

Python code

```
P = np.array([[0.7,0.3],[0.5,0.5]])
n = P.shape[0] #nrow
I = np.identity(n)
rep=np.array([[1],[1]])
A = np.hstack([P-I, rep])
b=np.array([0,0,1])
sol = np.linalg.solve(np.dot(A,A.T),np.dot(A,b.T))
print(sol)
```

```
## [0.625 0.375]
```

p.17 Limiting probabilities Motivation

R code

```
library(expm)
P <- array(c(0.7,0.5,0.3,0.5), dim = c(2,2))
P %*% P
P %^% 3
P %^% 4
P %^% 20
```

Python code

```
P = np.array([[0.7,0.3],[0.5,0.5]])
print(np.dot(P,P))
```

```
## [[0.64 0.36]
##  [0.6  0.4  ]]
```

```
print(np.linalg.matrix_power(P,3))
```

```
## [[0.628 0.372]
##  [0.62  0.38  ]]
```

```
print(np.linalg.matrix_power(P,4))
```

```
## [[0.6256 0.3744]
##  [0.624  0.376  ]]
```

```
print(np.linalg.matrix_power(P,20))
```

```
## [[0.625 0.375]
##  [0.625 0.375]]
```

p.19 The limiting distribution may not exist.

R code

```
library(expm)
P <- array(c(0, 1, 1, 0), dim = c(2,2))
P
P %>% 2
P %>% 3
```

Python code

```
P1 = np.array([[0,1],[1,0]])
print(P1)
```

```
## [[0 1]
##  [1 0]]
```

```
print(np.dot(P1,P1))
```

```
## [[1 0]
##  [0 1]]
```

```
print(np.linalg.matrix_power(P1,3))
```

```
## [[0 1]
##  [1 0]]
```