Daipark_ch3_손민상

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Exercise(3-5)

Suppose each morning a factory posts the number of days worked in a row without any injuries. Assume that each day is injury free with probability 98/100. Furthermore, assume that whether tomorrow is injury free or not is independent of which of the preceding days were injury free. Let $X_0 = 0$ be the morning the factory first opened. Let X_n be the number posted on the morning after n full days of work.

(a) Is $X_n, n \geq 0$ a Markov chain? If so, give its state space, initial distribution, and transition matrix P. If not, show that it is not a Markov chain.

Whether tomorrow is injury free or not is independent of which of the preceding days were injury free.(markov property)

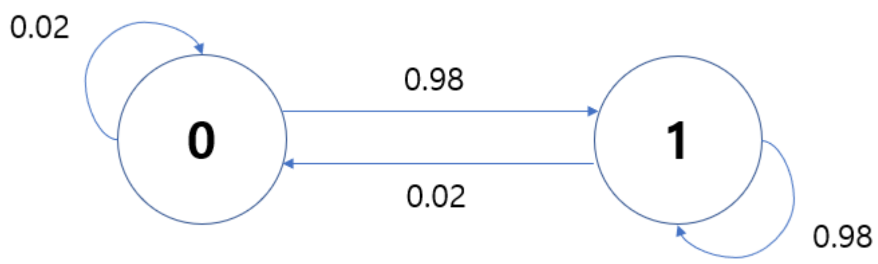
State space $S = \{0, 1\}$ 1 means injury free, 0 means not injury free.

Initial distribution $\pi_0 = \{1, 0\}$

Trainsition matrix

$$P = \begin{pmatrix} 0.98 & 0.02 \\ 0.98 & 0.02 \end{pmatrix}$$

(b) Is the Markov chain irreducible? Explain.



All states are connected in one transition. So it is irreducible.

(c) Is the Markov chain periodic or aperiodic? Explain and if it is periodic, also give the period.

```
import numpy as np
from numpy.linalg import matrix_power
P=np.array([[0.98,0.02],[0.98,0.02]])
matrix_power(P,2)
```

```
## array([[0.98, 0.02],
##        [0.98, 0.02]])
```

$$P^2 = \begin{pmatrix} 0.98 & 0.02 \\ 0.98 & 0.02 \end{pmatrix}$$

$P^2 = P^3 = P^\infty$

This Markov chain is periodic, the period is 1.

(d) Find the stationary distribution.

$P^2 = P^3 = P^\infty$

$s = \{0.98, 0.02\}$

(e) Is the Markov chain positive recurrent? If so, why? If not, why not?

No, it is not recurrent.

Exercise(3-6)

Consider the following transition matrix:

$$P = \begin{pmatrix} 0 & 0.5 & 0 & 0.5 \\ 0.6 & 0 & 0.4 & 0 \\ 0 & 0.7 & 0 & 0.3 \\ 0.8 & 0 & 0.2 & 0 \end{pmatrix}$$

(a) Is the Markov chain periodic? Give the period of each state.

```
import numpy as np
from numpy.linalg import matrix_power
P=np.array([[0,0.5,0,0.5],
           [0.6,0,0.4,0],
           [0,0.7,0,0.3],
           [0.8,0,0.2,0]])

print(matrix_power(P,200))
```

```
## [[0.6875 0.     0.3125 0.    ]
##  [0.     0.5625 0.     0.4375]
##  [0.6875 0.     0.3125 0.    ]
##  [0.     0.5625 0.     0.4375]]
```

```
print(matrix_power(P,201))
```

```
## [[0.     0.5625 0.     0.4375]
##  [0.6875 0.     0.3125 0.    ]
##  [0.     0.5625 0.     0.4375]
##  [0.6875 0.     0.3125 0.    ]]
```

```
print(matrix_power(P,202))
```

```
## [[0.6875 0.     0.3125 0.    ]
##  [0.     0.5625 0.     0.4375]
##  [0.6875 0.     0.3125 0.    ]
##  [0.     0.5625 0.     0.4375]]
```

$P^{200} = P^{202}$

∴This Markov chain is periodic, and the period is 2

(b) Is A$(\pi_1, \pi_2, \pi_3, \pi_4)$ = (33/96,27/96,15/96,21/96) the stationary distribution of the Markov Chain?

$(\pi_1, \pi_2, \pi_3, \pi_4) \cdot P$

$= (0.6\pi_2 + 0.8\pi_4, 0.5\pi_1 + 0.7\pi_3, 0.4\pi_2 + 0.2\pi_4, 0.5\pi_1 + 0.3\pi_3)$

$= (\frac{33}{96}, \frac{27}{96}, \frac{15}{96}, \frac{21}{96})$

$\therefore A$(◎_1,◎_2,◎_3,◎_4)$ is stationary distribution of the Markov Chain

(c) Is $P_{11}^{100} = \pi_1$? $P_{11}^{101} = \pi_1$? Is Give an expression for π_1 in terms of P_{11}^{100} and P_{11}^{101}.

```
import numpy as np
from numpy.linalg import matrix_power
print(matrix_power(P,100))
```

```
## [[0.6875 0.     0.3125 0.    ]
##  [0.     0.5625 0.     0.4375]
##  [0.6875 0.     0.3125 0.    ]
##  [0.     0.5625 0.     0.4375]]
```

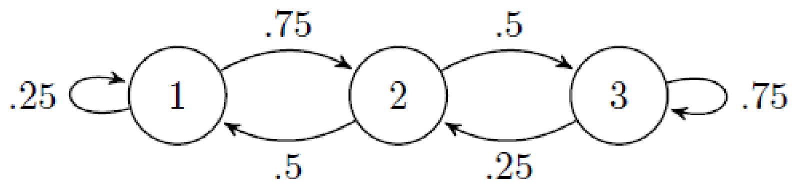
```
matrix_power(P,101)
```

```
## array([[0.    , 0.5625, 0.    , 0.4375],
##        [0.6875, 0.    , 0.3125, 0.    ],
##        [0.    , 0.5625, 0.    , 0.4375],
##        [0.6875, 0.    , 0.3125, 0.    ]])
```

$P_{11}^{100} = 0.6875 = 2 \cdot \pi_1, P_{11}^{101} = 0 = 0 \cdot \pi_1$

Exercise(3-14)

Suppose you are working as an independent consultant for a company. Your performance is evaluated at the end of each month and the evaluation falls into one of three categories: 1, 2, and 3. Your evaluation can be modeled as a discrete-time Markov chain and its transition diagram is as follows.



Denote your evaluation at the end of nth month by X_n and assume that $X_0 = 2$.

(a) What are state space, transition probability matrix and initial distribution of X_n?

State space $S = \{1, 2, 3\}$

Initial distribution $\pi_0 = \{0, 1, 0\}$

trainsition matrix

$$P = \begin{pmatrix} 0.25 & 0.75 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 0.25 & 0.75 \end{pmatrix}$$

(b) What is the stationary distribution?

$(\pi_1, \pi_2, \pi_3) \cdot P$

$= (0.25\pi_1 + 0.5\pi_2, 0.75\pi_1 + 0.25\pi_3, 0.5\pi_2 + 0.75\pi_3)$

$= (\pi_1, \pi_2, \pi_3)$

$\pi_1 + \pi_2 + \pi_3 = 1$

$\therefore \pi_1 = \frac{2}{11}, \pi_2 = \frac{3}{11}, \pi_3 = \frac{6}{11}$

Stationary distribution $= (\frac{2}{11}, \frac{3}{11}, \frac{6}{11})$

(c) What is the long-run fraction of time when your evaluation is either 2 or 3?

```
import numpy as np
from numpy.linalg import matrix_power


P=np.array([[0.25, 0.75, 0],
          [0.5, 0, 0.5],
          [0, 0.25, 0.75]])


matrix_power(P,100)
```

```
## array([[0.18181818, 0.27272727, 0.54545455],
##        [0.18181818, 0.27272727, 0.54545455],
##        [0.18181818, 0.27272727, 0.54545455]])
```

```
matrix_power(P,200)
```

```
## array([[0.18181818, 0.27272727, 0.54545455],
##        [0.18181818, 0.27272727, 0.54545455],
##        [0.18181818, 0.27272727, 0.54545455]])
```

$$P^{\infty} = \begin{pmatrix} 0.1818 & 0.2727 & 0.5454 \\ 0.1818 & 0.2727 & 0.5454 \\ 0.1818 & 0.2727 & 0.5454 \end{pmatrix}$$

Therefore, the long-run fraction of time when your evaluation is either 2 or 3

$= 0.2727 + 0.5454 = 0.8181$

Your monthly salary is determined by the evaluation of each month in the following way.

Salary when your evaluation is $n = \$5000 + n^2 * \$5000, \quad n = 1, 2, 3$

(d) What is the long-run average monthly salary?

Salary when your evaluation is 1 $= \$5000 + 1^2 * \$5000 = \$10000$

Salary when your evaluation is 2 $= \$5000 + 2^2 * \$5000 = \$25000$

Salary when your evaluation is 1 $= \$5000 + 3^2 * \$5000 = \$50000$

Average monthly salary $\quad 10000 * 0.1818 + 25000 * 0.2727 + 50000 * 0.5454 = \$35,905.5$