Lecture B1. Excercise

Baek, Jong min

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차 례

Exercise 1

Assume that D flows the following discrete distribution.

d	20	25	30	35
$\mathbb{P}[D=d]$	0.1	0.2	0.4	0.3
$3 \wedge d$	20	25	30	30
$(30-d)^+,$	10	5	0	0
$24 \wedge d$	20	24	24	24
$(24-d)^+$	4	0	0	0

Answer the followings.

$\mathbb{E}[30 \wedge D] = 0.1 \times 20 + 0.25 \times 25 + 0.4 \times 30 + 0.3 \times 80 = 28$

$\mathbb{E}[(30-D)^+] = 0.1 \times 10 + 0.2 \times 5 + 0.4 \times 0 + 0.3 \times 0 = 2$

$\mathbb{E}[24 \wedge D] = 0.1\, times 20 + 0.2 \times 24 + 0.4 \times 24 + 0.3 \times 24 = 23.6$

$\mathbb{E}[(24-D)^+] = 0.1 \times 4 + 0.2 \times 0 + 0.4 \times 0 + 0.3 \times 0 = 0.4$

Exercise 2

Find your brother's optimal stock level by the above Theorem 2. Then, find his expected profit using the Remark 1.

Answer

1) Find Optimal stock level

$$p = 2,\ s = 0.5,\ c = 1$$
$$c_o = 0.5$$
$$c_u = 1$$
$$x^* = smallest\, y\ s.t\ F(y) \geq \frac{C_u}{C_o + c_u} = \frac{2}{3}$$

If D is a discrete random variable

d	11	12	13	14	15
$\mathbb{P}[D = d]$	0.2	0.2	0.2	0.2	0.2
$\mathbb{P}[D \leq d]$	0.2	0.4	0.6	0.8	1.0

$$F_D(11) = \frac{1}{5} \quad < \frac{1}{3}(0.66)$$
$$F_D(12) = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}(0.4) \quad < \frac{2}{3}(0.66)$$
$$F_D(13) = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}(0.6) \quad < \frac{2}{3}(0.66)$$
$$F_D(14) = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{4}{5}(0.8) \quad > \frac{2}{3}(0.66)$$
$$F_D(15) = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{5}{5}(1.0) \quad > \frac{2}{3}(0.66)$$

The optimal order quantity $y^* = 14$

2) Find Expected profit

d	11	12	13	14	15
$\mathbb{P}[D = d]$	0.2	0.2	0.2	0.2	0.2
$(14 \wedge D)$	11	12	13	14	14
$(14 - D)^+$	3	2	1	0	0

$$\mathbb{E}[profit] = \mathbb{E}[sale\, rev.] + \mathbb{E}[salvage\, rev.] - \mathbb{E}[material\, cost]$$

$$\mathbb{E}[14 \wedge D] = 2 \times \sum_{d=11}^{15} (14 \wedge d) \times \mathbb{P}[D = d] = 25.6$$
$$\mathbb{E}[(14 - D)^+] = 0.5 \times \sum_{d=11}^{15} (14 - d)^+ \times \mathbb{P}[D = d] = 0.6$$
$$\mathbb{E}[14] = 1 \times 14 = 14$$

$$\mathbb{E}[profit] = 25.6 + 0.6 - 14 = 12.2$$

Exercise 3

Your brother is now selling milk. The customer demands follow $U(20, 40)$ gallons. *Retail price* is $\$2$ per gallon, *material cost* is $\$1$per gallon, and *salvage cost* is $\$0.5$ per gallon.Find optimal stock level and expected profit.

Answer

1) Find Optimal stock level

$c_u = p - c = 1$
$c_o = c - s = 0.5$
$F(y) = \frac{c_u}{c_o + c_u} = \frac{1}{1+0.5} = \frac{2}{3}$

Optimal stock level $= \frac{x-20}{40-20} = \frac{2}{3} = \frac{100}{3}$

2) Find Expected profit

$$\mathbb{E}[profit] = \mathbb{E}[sale\,rev.] + \mathbb{E}[salvage\,rev.] - \mathbb{E}[material\,cost]$$

$\mathbb{E}[sale\,rev.] = p \times \mathbb{E}[min(D, \frac{100}{3})] = \int_{20}^{\frac{100}{3}} x\frac{1}{20}dx + \int_{\frac{100}{3}}^{40} \frac{100}{3}\frac{1}{20}dx = \frac{310}{9}$
$\mathbb{E}[salvage\,rev.] = s \times \mathbb{E}[(\frac{100}{3} - D)^+] = \int_{20}^{\frac{100}{3}} 0\frac{1}{20}dx + \int_{\frac{100}{3}}^{40} (\frac{100}{3} - D)\frac{1}{20}dx = \frac{40}{3} * \frac{1}{2} = \frac{20}{3}$
$\mathbb{E}[material\,cost] = c \times \mathbb{E}[\frac{100}{3}] = 1 \times \frac{100}{3} = \frac{100}{3}$

$\mathbb{E}[profit] = \frac{46}{9}$

Exercise 4

Lemonade sells for $\$18$ per gallon but only costs $\$3$ per gallon to make. If we run out of lemonade, it will be impossible to get more. On the other hand, leftover lemonade has a value of $\$1$\$. Express the following quantity using sale as X and demand as D.

Answer

$c_u =$ lemonade($\$18$) - material cost($\$3$) = 15
$c_o =$ material cost($\$3$) - salvage cost($\$1$) = 2
Expected economic cost = $15 \times \mathbb{E}[(D - E)^+] + 2 \times \mathbb{E}[(X - D)^+]$
Expected profit $= 18 \times \mathbb{E}[min(X, D)] + 1 \times \mathbb{E}[(X - D)^+] - 3 \times \mathbb{E}[X]$

Exercise 5

Prove Theorem 1.(Hint: you may use formulation from Exercise 4)

DaiPark

Writing

B1.Rmd

```r
"Hello"
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## [1] "Hello"
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