< A1 Solution >

#1번

Exercise 1

Suppose $f(x) = xe^x$, *find* $f'(x)$.

$f(x) = x \cdot e^x \qquad f'(x) = (x)'e^x + x \cdot (e^x)'$

$= e^x + xe^x$

#2번

Exercise 2

Suppose $f(x) = e^{2x}$, *find* $f'(x)$.

$f(x) = e^{2x}$, $A(x) = e^x$, $B(x) = 2x$

$f(x) = A(B(x))$

$f'(x) = A'(B(x)) \cdot B'(x)$

$= e^{2x} \cdot 2$

$= 2 \cdot e^{2x}$

cf) $f(x) = h(g(x))$

$\Rightarrow f'(x) = h'(g(x)) \cdot g'(x)$

$f(x) = e^{2x} \Rightarrow h(x) = e^x \cdot g(x) = 2x$

$f'(x) = e^{2x} \cdot 2 = 2e^{2x}$

#3번

Exercise 3

Derive $\int f'(x)g(x)\ dx = f(x)g(x) - \int f(x)g(x)'\ dx$. *(Hint: Use Theorem 2 above.)*

$$(f(x)\cdot g(x))' = f'(x)g(x) + f(x)\cdot g'(x)$$

antiderivative

$$\Rightarrow \quad f(x)g(x) = \int f'(x)g(x) + \int f(x)\cdot g'(x)$$

$$\int f'(x)g(x) = f(x)g(x) - \int f(x)\cdot g'(x)$$

#4번

Exercise 4

Find $\int xe^x\ dx$, *and evaluate* $\int_0^1 xe^x\ dx$. *(Hint: Use Exercise 3 above.)*

Exercise 3 $\int f'(x)g(x)\,dx = f(x)g(x) - \int (f(x)g'(x))\,dx$

$$\int xe^x\,dx = e^x x - \int (e^x \cdot 1)\,dx$$

$$= e^x x - e^x + C$$

$$\int_0^1 xe^x\,dx = \left[e^x x - e^x - C\right]_0^1$$

$$= (e^1\cdot 1 - e^1 - C) - (e^0\cdot 0 - e^0 - C)$$

$$= (0 - C) - (0 - 1 - C)$$

$$= 1$$

#5번

Exercise 5

Solve the followings.

$$(.6 \quad .4)\begin{pmatrix} .7 & .3 \\ .5 & .5 \end{pmatrix} =$$

$(x \quad y)\begin{pmatrix} a & b \\ c & d \end{pmatrix} = (ax+cy \quad bx+dy)$

$(.6 \quad .4)\begin{pmatrix} .7 & .3 \\ .5 & .5 \end{pmatrix} = (0.6\times0.7+0.4\times0.5 \quad 0.6\times0.3+0.4\times0.5) = (0.62 \quad 0.38)$

#6번

Exercise 6

What is P^2*?*

$$P = \begin{pmatrix} .7 & .3 \\ .5 & .5 \end{pmatrix}$$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x & y \\ z & w \end{pmatrix} = \begin{pmatrix} ax+bz & ay+bw \\ cx+dz & cy+dw \end{pmatrix}$

$\begin{pmatrix} 0.7 & 0.3 \\ 0.5 & 0.5 \end{pmatrix}\begin{pmatrix} 0.7 & 0.3 \\ 0.5 & 0.5 \end{pmatrix} = \begin{pmatrix} 0.64 & 0.36 \\ 0.6 & 0.4 \end{pmatrix}$

#7번

Exercise 7

Solve the followings.

$$(\pi_1 \quad \pi_2)\begin{pmatrix} .7 & .3 \\ .5 & .5 \end{pmatrix} = (\pi_1 \quad \pi_2)$$

$$\pi_1 + \pi_2 = 1$$

$[\pi_1 \ \pi_2]\begin{bmatrix} 0.7 & 0.3 \\ 0.5 & 0.5 \end{bmatrix} = [\overset{①}{0.7\pi_1 + 0.5\pi_2} \quad \overset{②}{0.3\pi_1 + 0.5\pi_2}] = [\pi_1 \ \pi_2]$

① $0.7\pi_1 + 0.5\pi_2 = \pi_1$
$3\pi_1 = 5\pi_2 \quad \pi_2 = \frac{3}{5}\pi_1$

② $0.3\pi_1 + 0.5\pi_2 = \pi_2$
$3\pi_1 = 5\pi_2 \quad \pi_2 = \frac{3}{5}\pi_1$

$\pi_1 + \frac{3}{5}\pi_1 = \frac{8}{5}\pi_1 = 1$

$\therefore \pi_1 = \frac{5}{8}, \ \pi_2 = \frac{3}{5} \times \frac{5}{8} = \frac{3}{8}$

#8번

Exercise 8

Solve the following system of equations.

$$x = y \quad \cdots ①$$
$$y = 0.5z \quad \cdots ②$$
$$z = 0.6 + 0.4x \quad \cdots ③$$
$$x + y + z = 1 \quad \cdots ④$$

②→① $x = 0.5z \quad \cdots ⑤$

⑤→③ $z = 0.6 + 0.4 \cdot 0.5z$, $z = 0.6 + 0.2z$ $\quad z = \frac{0.6}{0.8} = \frac{3}{4}$

$y = \frac{3}{8}$, $x = \frac{3}{8}$.

but. $\frac{3}{8} + \frac{3}{8} + \frac{6}{8} = \frac{12}{8}$. this pair does not satisfy the equation ④

⇒ impossible

Z 항 "-" 로 고쳤을 때

Exercise 8

Solve the following system of equations.

$$x = y$$
$$y = 0.5z$$
$$z = 0.6 + 0.4x$$
$$x + y + z = 1$$

x=y, y=0.5z,

z=0.6-0.4x

x + y + z =1

-> 0.5z + 0.5z + z =1

z = 0.5

x = y =0.25

#9번

Exercise 9

Solve the following system of equations.

$$(\pi_0 \quad \pi_1 \quad \pi_2)\begin{pmatrix} -2 & 2 & \\ 3 & -5 & 2 \\ & 3 & -3 \end{pmatrix} = (0 \quad 0 \quad 0)$$

$$\pi_0 + \pi_1 + \pi_2 = 1$$

$(\pi_0 \quad \pi_1 \quad \pi_2)\begin{pmatrix} -2 & 2 & 0 \\ 3 & -5 & 2 \\ 0 & 3 & -3 \end{pmatrix} = (-2\pi_0+3\pi_1 \quad 2\pi_0-5\pi_1+3\pi_2 \quad 2\pi_1-3\pi_2)$

$-2\pi_0+3\pi_1=0$ ---①

$2\pi_0-5\pi_1+3\pi_2=0$ ---②

$2\pi_1-3\pi_2=0$ ---③

$\pi_0+\pi_1+\pi_2=1$ ---④

①$\Rightarrow \pi_0=\frac{3}{2}\pi_1$ ③$\Rightarrow \pi_2=\frac{2}{3}\pi_1$

④$\Rightarrow \frac{3}{2}\pi_1+\pi_1+\frac{2}{3}\pi_1=1 \quad \therefore \pi_1=\frac{6}{19}$

$\pi_0=\frac{9}{19} \quad \pi_2=\frac{4}{19}$

②$\Rightarrow \frac{18}{19}-\frac{30}{19}+\frac{12}{19}=0$

#10번 (1. 황재훈)

Exercise 10

Solve the following system of equations.

$$(\pi_1 \quad \pi_2 \quad \pi_3 \quad \pi_4)\begin{pmatrix} .7 & .3 & & \\ .5 & .5 & & \\ & & .6 & .4 \\ & & .3 & .7 \end{pmatrix} = (\pi_1 \quad \pi_2 \quad \pi_3 \quad \pi_4)$$

$$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1 \quad , \quad \pi_1 + \pi_2 = \alpha$$

$0.7\pi_1 + 0.5\pi_2 = \pi_1 \rightarrow 5\pi_1 = 3\pi_2$
$0.3\pi_1 + 0.5\pi_2 = \pi_2 \rightarrow 3\pi_1 = 5\pi_2$ } $\pi_2 = \frac{3}{5}\pi_1$

$0.6\pi_3 + 0.3\pi_4 = \pi_3 \rightarrow 3\pi_4 = 4\pi_3$
$0.4\pi_3 + 0.7\pi_4 = \pi_4 \rightarrow 4\pi_3 = 3\pi_4$ } $\pi_4 = \frac{4}{3}\pi_3$

$\pi_1 + \pi_2 + \pi_3 + \pi_4 = \pi_1 + \frac{3}{5}\pi_1 + \pi_3 + \frac{4}{3}\pi_3$

$= \frac{8}{5}\pi_1 + \frac{7}{3}\pi_3 = 1$

$\rightarrow \frac{7}{3}\pi_3 = 1 - \frac{8}{5}\pi_1$

$\pi_3 = \frac{3}{7} - \frac{24}{35}\pi_1 \rightarrow \pi_4 = \frac{4}{3}\left(\frac{3}{7} - \frac{24}{35}\pi_1\right) = \frac{4}{7} - \frac{32}{35}\pi_1$

$\therefore$
$\pi_1 = \text{free}$
$\pi_2 = \frac{3}{5}\pi_1$
$\pi_3 = \frac{3}{7} - \frac{24}{35}\pi_1$
$\pi_4 = \frac{4}{7} - \frac{32}{35}\pi_1$

(2. 신희섭)

Exercise 10

Solve the following system of equations.

$$(\pi_1 \quad \pi_2 \quad \pi_3 \quad \pi_4)\begin{pmatrix} .7 & .3 & & \\ .5 & .5 & & \\ & & .6 & .4 \\ & & .3 & .7 \end{pmatrix} = (\pi_1 \quad \pi_2 \quad \pi_3 \quad \pi_4)$$

$$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$$

$.7\pi_1 + .5\pi_2 = \pi_1 \Rightarrow .3\pi_1 - .5\pi_2 = 0$
$.3\pi_1 + .5\pi_2 = \pi_2 \Rightarrow .3\pi_1 - .5\pi_2 = 0$ } same equation

$.6\pi_3 + .3\pi_4 = \pi_3 \Rightarrow .4\pi_3 - .3\pi_4 = 0$
$.4\pi_3 + .7\pi_4 = \pi_4 \Rightarrow .4\pi_3 - .3\pi_4 = 0$ } same equation

$\Rightarrow$ 4 variable, 3 equation : indeterminate.

(3. 권도윤)

Exercise 10

Solve the following system of equations.

$$(\pi_1 \quad \pi_2 \quad \pi_3 \quad \pi_4)\begin{pmatrix} .7 & .3 & & \\ .5 & .5 & & \\ & & .6 & .4 \\ & & .3 & .7 \end{pmatrix} = (\pi_1 \quad \pi_2 \quad \pi_3 \quad \pi_4)$$

$$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$$

$0.1\pi_1 + 0.5\pi_2 = \pi_1$
$0.3\pi_1 + 0.5\pi_2 = \pi_2$
$\Rightarrow 0.3\pi_1 = 0.5\pi_2$
$\pi_2 = \frac{3}{5}\pi_1$

$0.6\pi_3 + 0.3\pi_4 = \pi_3$
$0.4\pi_3 + 0.1\pi_4 = \pi_4$
$\Rightarrow 0.4\pi_3 = 0.3\pi_4$
$\pi_4 = \frac{4}{3}\pi_3$

$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$
$\Rightarrow \pi_1 + \frac{3}{5}\pi_1 + \pi_3 + \frac{4}{3}\pi_3 = 1$
$\frac{8}{5}\pi_1 + \frac{7}{3}\pi_3 = 1$

$\therefore \frac{8}{5}\pi_1 + \frac{7}{3}\pi_3 = 1,\ \pi_2 = \frac{3}{5}\pi_1,\ \pi_4 = \frac{4}{3}\pi_3$
$(\pi_1, \pi_3 \in R)$

#11번

Exercise 11

Solve following and express π_i for $i = 0, 1, 2, \ldots$

$$\begin{aligned} \pi_0 + \pi_1 + \pi_2 + \ldots &= 1 \\ 0.02\pi_0 + 0.02\pi_1 + 0.02\pi_2 + \ldots &= \pi_0 \\ 0.98\pi_0 &= \pi_1 \\ 0.98\pi_1 &= \pi_2 \\ 0.98\pi_2 &= \pi_3 \\ \ldots &= \ldots \end{aligned}$$

$r = 0.98$

$\pi_1 = (0.98)\pi_0$
$\pi_2 = (0.98)\pi_1 = (0.98)^2\pi_0$
$\pi_3 = (0.98)\pi_2 = (0.98)^3\pi_0$
$\therefore \pi_i = (0.98)^i\pi_0$

$\sum_{i=0}^{\infty} \pi_i = \frac{a}{1-r}$ ($r = 0.98$, $a = \pi_0$, Infinite series)

$\frac{\pi_0}{1-0.98} = 1$

$\frac{\pi_0}{0.02} = 1 \quad \therefore \pi_0 = 0.02$

$\therefore \pi_i = (0.02)(0.98)^i$

From ③ – ⑤, $\pi_i = (0.98)^i \pi_0$.

Thus, ① becomes

$$\pi_0(1 + 0.98 + 0.98^2 + \cdots +)$$

$$= \pi_0\left(\frac{1}{1-0.98}\right) = 1$$

$$\therefore \pi_0 = 0.02.$$

$$\therefore \pi_i = (0.02) \times (0.98)^i \quad \text{for all } i$$

$$S = a + ar + ar^2 + \cdots$$
$$-\ rS = \quad ar + ar^2 + \cdots$$
$$(1-r)S = a \Rightarrow S = \frac{a}{1-r}$$

간단한 풀이.

#12번

Exercise 12 (Infinite geometric series)

Simplify the following. When $|r| < 1$, $S = a + ar + ar^2 + ar^3 + \ldots$

$$S_n = a + ar + ar^2 + \cdots + ar^{n-1}$$
$$-\ rS_n = ar + ar^2 + \cdots + ar^{n-1} + ar^n$$
$$(1-r)S = a - ar^n$$
$$S_n = \frac{a(1-r^n)}{1-r}$$

$S = a \cdot r^{n-1}$

만약 n이 무한(infinite)이면

$$\therefore \sum_{n=1}^{\infty} S_n = \lim_{n\to\infty} S_n = \frac{a}{1-r}$$

문제에서 무한으로 주어짐

#13번

Exercise 13 (Finite geometric series)

Simplify the following. When $r \neq 1$, $S = a + ar + ar^2 + ar^3 + \ldots + ar^{n-1}$

good

$$S = a + ar + \cdots + ar^{n-1}$$
$$r \cdot S = ar + ar^2 + ar^3 + \cdots + ar^n$$
$$S - r \cdot S = a - ar^n$$
$$S = \frac{a(1-r^n)}{(1-r)}$$

$S = r + 2r^2 + 3r^3 + 4r^4 + \cdots$

$-\quad rS = \quad r^2 + 2r^2 + 3r^3 + \cdots$

$(1-r)S = r + r^2 + \cdots$

$\Rightarrow (1-r)S = \frac{r}{1-r} \Rightarrow S = \frac{r}{(1-r)^2}$

0 in 2 post

#14번

Exercise 14 (Power series)

Simplify the following. When $|r| < 1$, $S = r + 2r^2 + 3r^3 + 4r^4 + \ldots$

$r + r^2 + r^3 + r^4 + \cdots = \frac{r}{1-r}$

$r^2 + r^3 + r^4 + \cdots = \frac{r^2}{1-r}$

$r^3 + r^4 + \cdots = \frac{r^3}{1-r}$

$\vdots \quad = \quad \vdots$

$+$

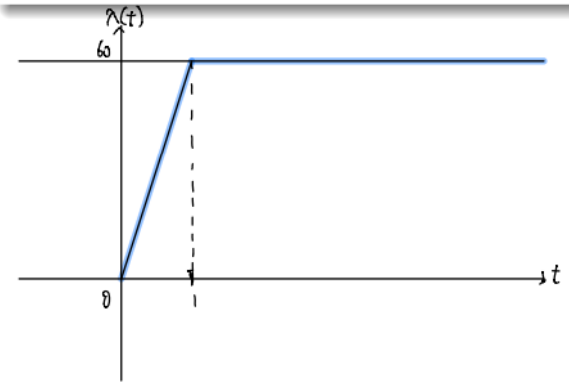
$r + 2r^2 + 3r^3 + 4r^4 + \cdots = \frac{1}{1-r}(r + r^2 + r^3 + \cdots)$

$S = \frac{1}{1-r} \cdot \frac{r}{1-r} = \boxed{\frac{r}{(1-r)^2}}$

#15번

Exercise 15

During the first hour $(0 \le t \le 1)$, $\lambda(t)$ *increases linearly from* 0 *to* 60. *After the first hour,* $\lambda(t)$ *is constant at* 60. *Draw plot for* $\lambda(t)$ *and express the function in math form.*



$$\lambda(t) = \begin{cases} 60t & \text{if } t \le 60 \\ 60 & \text{if } t > 60 \end{cases}$$

$(t \ge 0)$

time always positive (hour)