DaiPark Exercise

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#Exercise 5

Suppose each morning a factory posts the number of days worked in a row without any injuries. Assume that each day is injury free with probability 98/100. Furthermore, assume that whether tomorrow is injury free or not is independent of which of the preceding days were injury free. Let $X_o = 0$ be the morning the factory first opened. Let X_n be the number posted on the morning after n full days of work.

(a) Is $\{X_n, n \geq 0\}$ a Markov chain? If so, give its state space, initial distribution, and transition matrix P. If not, show that it is not a Markov chain.

yes, this is markov chain.

state space=$\{X_0, X_1, X_2, ...X_n\}$

initial distribution = (0.98, 0.02)

$$P = \begin{pmatrix} .02 & .98 & 0 & 0 & . & . & . \\ .02 & 0 & .98 & 0 & . & . & . \\ .02 & 0 & 0 & .98 & . & . & . \\ . & . & . & . & . & . & . \end{pmatrix}$$

(b) Is the Markov chain irreducible? Explain.

=This markov chain irreducible. Because, the class only one.

(c) Is the Markov chain periodic or aperiodic? Explain and if it is peri- odic, also give the period.

=aperiodic

(d) Find the stationary distribution.

v = (a,d,c,d,e,.....)

vP = v

$\therefore 0.02a + 0.98b = a, 0.02a + 0.98c = b....$

$a = b = c = ...$

So,v=$\{\frac{1}{n}\,\frac{1}{n}\,\frac{1}{n}\,\frac{1}{n}\,\frac{1}{n}.....\}$ the number of element is n

(e) Is the Markov chain positive recurrent? If so, why? If not, why not?

No, because the probability of getting back to X_0 is not 1.

#Exercise 6

Consider the following transition matrix:

$$P = \begin{pmatrix} 0 & 0.5 & 0 & 0.5 \\ 0.6 & 0 & 0.4 & 0 \\ 0 & 0.7 & 0 & 0.3 \\ 0.8 & 0 & 0.2 & 0 \end{pmatrix}$$

(a) Is the Markov chain periodic? Give the period of each state.

=periodic

(b) Is $(\pi_1, \pi_2, \pi_3, \pi_4) =$ (33/96, 27/96, 15/96, 21/96) the stationary dis-tribution of the Markov Chain?

=yes that's a stationary distribution

(c)Is $P_{11}^{100} = \pi_1$?, Is $P_{11}^{101} = \pi_1$? Give an expression for π_1 in terms of P_{11}^{100} and P_{11}^{101}.

$=$ In $\pi_1 = 0.34375$, but $P_{11}^{100} = 0.6875$ and $P_{11}^{101} = 0$

$=$ So,π_1 isn't same with $P_{11}^{100}, P_{11}^{101}$

#Exercise 14

Suppose you are working as an independent consultant for a company. Your performance is evaluated at the end of each month and the evaluation falls into one of three categories: 1, 2, and 3. Your evaluation can be modeled as a discrete-time Markov chain and its transition diagram is as follows.

Denote your evaluation at the end of nth month by X_n and assume that $X_0 = 2$. (a) What are state space, transition probability matrix and initial dis-tribution of X_n?

state space = [1,2,3]

transition probability matrix

$$P = \begin{pmatrix} 0.25 & 0.75 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 0.25 & 0.75 \end{pmatrix}$$

initial distribution (0,1,0)

(b) What is the stationary distribution?

stationary distribution(v)= $(\frac{2}{11}\ \frac{3}{11}\ \frac{6}{11})$

(c) What is the long-run fraction of time when your evaluation is either 2 or 3?

transition probability matrix (infinite)

$$P = \begin{pmatrix} 0.1818 & 0.2727 & 0.5454 \\ 0.1818 & 0.2727 & 0.5454 \\ 0.1818 & 0.2727 & 0.5454 \end{pmatrix}$$

evaluation 2 : 0.2727

evaluation 3 : 0.5454

Your monthly salary is determined by the evaluation of each month in the following way. Salary when your evaluation is n = $5000 + n^2 \times 5000$; n = 1, 2, 3

(d) What is the long-run average monthly salary?

salary = $(5000 + 1^2 \times 5000, 5000 + 2^2 \times 5000, 5000 + 3^2 \times 5000) \qquad = (10000, 25000, 50000)$

salary $\times$ long Rung Prob = (10000 25000 50000) $\times$ (0.1818 0.2727 0.5454)^T = 35905

answer is \$35905