DaiPark Exercise

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CH3

Q5

Suppose each morning a factory posts the number of days worked in a row without any injuries. Assume that each day is injury free with probability 98/100. Furthermore, assume that whether tomorrow is injury free or not is independent of which of the preceding days were injury free. Let $X_0 = 0$ be the morning the factory first opened. Let X_n be the number posted on the morning after n full days of work.

(a) Is $\{X_n,\ n \geq 0\}$ a Markov chain? If so, give its state space, initial distribution, and transition matrix P. If not, show that it is not a Markov chain.

State space - {Injury free, Injury not free}

Initial distribution - [1,0]

transition matrix P

$$P = \begin{pmatrix} 0.98 & 0.02 \\ 0.98 & 0.02 \end{pmatrix}$$

(b) Is the Markov chain irreducible? Explain.

Yes this Markov chain is irrdeucible. We can reach any states in this Markov chain.

(c) Is the Markov chain periodic or aperiodic? Explain and if it is periodic, also give the period.

```r
#library(expm)
p<-matrix(c(0.98,0.98,0.02,0.02),nrow=2)
for (i in 1:5){
  print(p%^%i)
}
```

```
##      [,1] [,2]
## [1,] 0.98 0.02
## [2,] 0.98 0.02
##      [,1] [,2]
## [1,] 0.98 0.02
## [2,] 0.98 0.02
##      [,1] [,2]
## [1,] 0.98 0.02
## [2,] 0.98 0.02
##      [,1] [,2]
## [1,] 0.98 0.02
## [2,] 0.98 0.02
```

```
##      [,1] [,2]
## [1,] 0.98 0.02
## [2,] 0.98 0.02
```

this Markov chain is aperiodic

(d) Find the stationary distribution

```
p<-matrix(c(0.98,0.98,0.02,0.02),nrow=2)
x_1 <- eigen(t(p))$vectors[,1]
v<- x_1/sum(x_1)
v
```

```
## [1] 0.98 0.02
```

```
v%*%p
```

```
##      [,1] [,2]
## [1,] 0.98 0.02
```

stationary distribution ($\pi 1,\ \pi 2$) = (0.98,0.02)

(e) Is the Markov chain positive recurrent? If so, why? If not, why not?

Q6

Consider the following transition matrix:

$$P = \begin{pmatrix} 0 & .5 & 0 & .5 \\ .6 & 0 & .4 & 0 \\ 0 & .7 & 0 & .3 \\ .8 & 0 & .2 & 0 \end{pmatrix}$$

(a) Is the Markov chain periodic? Give the period of each state.

```r
#library(expm)
p<-matrix(c(0,0.6,0,0.8,0.5,0,0.7,0,0,0.4,0,0.2,0.5,0,0.3,0),nrow=4)
for (i in 1:5){
  print(p%^%i)
}
```

```
##      [,1] [,2] [,3] [,4]
## [1,]  0.0  0.5  0.0  0.5
## [2,]  0.6  0.0  0.4  0.0
## [3,]  0.0  0.7  0.0  0.3
## [4,]  0.8  0.0  0.2  0.0
##      [,1] [,2] [,3] [,4]
## [1,] 0.70 0.00 0.30 0.00
## [2,] 0.00 0.58 0.00 0.42
## [3,] 0.66 0.00 0.34 0.00
## [4,] 0.00 0.54 0.00 0.46
##       [,1]  [,2]  [,3]  [,4]
## [1,] 0.000 0.560 0.000 0.440
## [2,] 0.684 0.000 0.316 0.000
## [3,] 0.000 0.568 0.000 0.432
## [4,] 0.692 0.000 0.308 0.000
##        [,1]   [,2]   [,3]   [,4]
## [1,] 0.6880 0.0000 0.3120 0.0000
## [2,] 0.0000 0.5632 0.0000 0.4368
## [3,] 0.6864 0.0000 0.3136 0.0000
## [4,] 0.0000 0.5616 0.0000 0.4384
##         [,1]    [,2]    [,3]    [,4]
## [1,] 0.00000 0.56240 0.00000 0.43760
## [2,] 0.68736 0.00000 0.31264 0.00000
## [3,] 0.00000 0.56272 0.00000 0.43728
```

```
## [4,] 0.68768 0.00000 0.31232 0.00000
```

```r
print(p%^%100)
```

```
##        [,1]   [,2]   [,3]   [,4]
## [1,] 0.6875 0.0000 0.3125 0.0000
## [2,] 0.0000 0.5625 0.0000 0.4375
## [3,] 0.6875 0.0000 0.3125 0.0000
## [4,] 0.0000 0.5625 0.0000 0.4375
```

Markov chain is periodic.

(b) Is ($\pi 1$, $\pi 2$, $\pi 3$, $\pi 4$) = (33/96, 27/96, 15/96, 21/96) the stationary distribution of the Markov Chain?

```r
p<-matrix(c(0,0.6,0,0.8,0.5,0,0.7,0,0,0.4,0,0.2,0.5,0,0.3,0),nrow=4)
pi<-matrix(c(33/96,27/96,15/96,21/96),nrow=1)
pi
```

```
##         [,1]    [,2]    [,3]    [,4]
## [1,] 0.34375 0.28125 0.15625 0.21875
```

```r
pi%*%p
```

```
##         [,1]    [,2]    [,3]    [,4]
## [1,] 0.34375 0.28125 0.15625 0.21875
```

($\pi 1$, $\pi 2$, $\pi 3$, $\pi 4$) is stationary distribution.

(c) Is $P_{11}^{100} = \pi 1$? Is $P_{11}^{101} = \pi 1$? Give an expression for $\pi 1$ in terms of $P_{11}^{100} = \pi 1$ and $P_{11}^{101} = \pi 1$

```r
p<-matrix(c(0,0.6,0,0.8,0.5,0,0.7,0,0,0.4,0,0.2,0.5,0,0.3,0),nrow=4)
p%^%100
```

```
##        [,1]   [,2]   [,3]   [,4]
## [1,] 0.6875 0.0000 0.3125 0.0000
## [2,] 0.0000 0.5625 0.0000 0.4375
## [3,] 0.6875 0.0000 0.3125 0.0000
## [4,] 0.0000 0.5625 0.0000 0.4375
```

```
p%^%101
```

```
##        [,1]   [,2]   [,3]   [,4]
## [1,] 0.0000 0.5625 0.0000 0.4375
## [2,] 0.6875 0.0000 0.3125 0.0000
## [3,] 0.0000 0.5625 0.0000 0.4375
## [4,] 0.6875 0.0000 0.3125 0.0000
```

$P_{11}^{100} = 0.6875 \neq \pi 1$ - No. $P_{11}^{101} = 0 \neq \pi 1$ - No.

Q14

Suppose you are working as an independent consultant for a company. Your performance is evaluated at the end of each month and the evaluation falls into one of three categories: 1, 2, and 3. Your evaluation can be modeled as a discrete-time Markov chain and its transition diagram is as follows.
Denote your evaluation at the end of nth month by Xn and assume that $X_0 = 2$.

(a) What are state space, transition probability matrix and initial distribution of X_n?

State space - {1,2,3}

Transition probability matrix P

$$P = \begin{pmatrix} 0.25 & 0.75 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 0.25 & 0.75 \end{pmatrix}$$

Initial distribution = [0,1,0]

(b) What is the stationary distribution?

```
p<-matrix(c(0.25,0.5,0,0.75,0,0.25,0,0.5,0.75),nrow=3)
x_1 <- eigen(t(p))$vectors[,1]
v<- x_1/sum(x_1)
v
```

```
## [1] 0.1818182 0.2727273 0.5454545
```

```
v%*%p
```

```
##           [,1]      [,2]      [,3]
## [1,] 0.1818182 0.2727273 0.5454545
```

stationary distribution ($\pi 1$, $\pi 2$, $\pi 3$) = (0.182,0.273,0.545)

(c) What is the long-run fraction of time when your evaluation is either 2 or 3?

```r
p<-matrix(c(0.25,0.5,0,0.75,0,0.25,0,0.5,0.75),nrow=3)
p%^%100
```

```
##           [,1]      [,2]      [,3]
## [1,] 0.1818182 0.2727273 0.5454545
## [2,] 0.1818182 0.2727273 0.5454545
## [3,] 0.1818182 0.2727273 0.5454545
```

long run 0.273 of evaluation 2, 0.545 of evaluation 3.

Your monthly salary is determined by the evaluation of each month in the following way.Salary when your evaluation is n = $5000 + n^2 * $5000; n = 1,2,3

(d) What is the long-run average monthly salary?

n=1, salary = $10,000, n=2, salary = $25,000, n=3, salary = $50,000

long run average salary = ($10,000, $25,000, $50,000) * P^{∞}

=$10,000 * 0.182 + $25,000 * 0.273, $50,000 * 0.545 = $35,895