B1_Exercise

Tae Hyeon Kwon, undergrad(ITM)

2020-12-29

Contents

Exercise

Exercise 1

Problem : Assume that D follows the following discrete distribution.

d	20	25	30	35
P[D=d]	0.1	0.2	0.4	0.3
$30 \wedge d$	20	25	30	30
$(30-d)^+$	10	5	0	0
$24 \wedge d$	20	24	24	24
$(24-D)^+$	4	0	0	0

$\mathbb{E}[30 \wedge D] = 20 \times 0.1 + 25 \times 0.5 + 30 \times 0.4 + 30 \times 0.3 = 28$

$\mathbb{E}[(30-D)^+] = 10 \times 0.1 + 5 \times 0.2 + 0 \times 0.4 + 0 \times 0.3 = 2$

$\mathbb{E}[24 \wedge D] = 20 \times 0.1 + 24 \times 0.2 + 24 \times 0.4 + 24 \times 0.3 = 23.6$

$\mathbb{E}[(24-D)^+] = 4 \times 0.1 + 0 \times 0.2 + 0 \times 0.4 + 0 \times 0.3 = 0.4$

Exercise 2

Problem : Find your brother's optimal stock level by the above Theorem 2. Then,find his expected profit using the Remark 1.

$Price(p) = 2$

$Cost(c) = 1$

$Salvage\ Cost(s) = 0.5$

$$\begin{aligned} C_u &= understock\ cost \\ &= p - c \\ &= 2 - 1 \\ &= 1 \end{aligned}$$

$$\begin{aligned} C_o &= overstock\ cost \\ &= c - s \\ &= 1 - 0.5 \\ &= 0.5 \end{aligned}$$

by Theorem 2, $F(y) \geq (\frac{c_u}{c_o+c_u}) = (\frac{1}{1+0.5}) = \frac{2}{3} = 0.6666$

find smallest 'y' qualified that formula

	11	12	13	14	15
$P(D = d)$	0.2	0.2	0.2	0.2	0.2
$P(D \leq d)$	0.2	0.4	0.6	0.8	1.0

$\therefore\ y = 14\ \ (smallest\ y)$

Expected profit = $\mathbb{E}[profit] = \mathbb{E}(sale\ rev.) + \mathbb{E}(salvage\ rev.) - \mathbb{E}(material\ cost)$

stock=14/demand	11	12	13	14	15
sales quantity	11	12	13	14	14
left quantity	3	2	1	0	0
material quantity	14	14	14	14	14

1. $\mathbb{E}(sale\ rev.) = 0.2 \times (11 \times 2 + 12 \times 2 + 13 \times 2 + 14 \times 2 + 14 \times 2) = 25.6$
2. $\mathbb{E}(salvage\ rev.) = 0.2 \times (3 \times 0.5 + 2 \times 0.5 + 1 \times 0.5 + 0 \times 0.5 + 0 \times 0.5) = 0.6$
3. $\mathbb{E}(material\ cost) = 0.2 \times (14 \times 1 + 14 \times 1 + 14 \times 1 + 14 \times 1 + 14 \times 1) = 14$

1+2-3 $= 25.6 + 0.6 - 14 = 12.2$

$Answer : Expected\ profit = 12.2$

Exercise 3

Problem : Your brother is now selling milk. The customer demands follow (20, 40) gallons. Retail price is \$2 per gallon, material cost is \$1 per gallon, and salvage cost is \$0.5 per gallon. Find optimal stock level and expected profit.

$$f(x) = \begin{Bmatrix} \frac{1}{20} & (20 \leq x \leq 40) \\ 0 & otherwise \end{Bmatrix}$$

$$F(x) = \begin{Bmatrix} 0 & (20 \leq x) \\ \frac{x-20}{20} & (20 \leq x \leq 40) \\ 1 & (x > 40) \end{Bmatrix}$$

$price(p) = 2$

$material\ cost(c) = 1$

$salvage\ cost = 0.5$

$$\begin{Bmatrix} C_u = 1 \\ C_o = 0.5 \end{Bmatrix}$$

we must find 'y' qualified that $F(y) = \frac{2}{3}$

$\therefore\ \frac{x-20}{20} = \frac{2}{3}$

$=> x - 20 = \frac{40}{3}$

$=> x = \frac{40}{3} + 20 = \frac{100}{3}$

$\therefore\ y = \frac{100}{3}$

1. $\mathbb{E}(sale\ rev.) = \mathbb{E}[min(x, \frac{100}{3})] \times 2 = [\int_{20}^{\frac{100}{3}} \frac{1}{20} x\ dx + \int_{\frac{100}{3}}^{40} \frac{100}{3} \times \frac{1}{20}\ dx] \times 2 = \frac{520}{9}$
2. $\mathbb{E}(salvage\ rev.) = \mathbb{E}[(\frac{100}{3} - X)^{+}] \times 0.5 = [\int_{20}^{\frac{100}{3}} (\frac{100}{3} - x) \times \frac{1}{20}\ dx + \int_{\frac{100}{3}}^{40} 0 \times \frac{1}{20}\ dx] \times 0.5 = \frac{20}{9}$
3. $\mathbb{E}(material\ cost) = \frac{100}{3} \times 1 = \frac{100}{3}$

1+2-3 $= \frac{520}{9} + \frac{20}{9} - \frac{100}{3} =\$\frac{80}{3}$

$Answer : optimal\ stock\ level = \frac{100}{3},\ \ expected\ profit =\$\frac{80}{3}$

Exercise 4

Problem : Lemonade sells for $18 per gallon but only costs $3 per gallon to make. If we run out of lemonade, it will be impossible to get more. On the other hand, leftover lemonade has a value of $1. Express the following quantity using sals as X and demand as D.

$*C_u$

$*C_o$

∗Expected economic cost

∗Expected profit

$price(p) = 18$

$cost(c) = 3$

$Salvage\ value(s) = 1$

1. $C_u = 18 - 3 = 15$
2. $C_o = 3 - 1 = 2$
3. Expected economic cost $= 2\mathbb{E}[(X - D)^+] + 15\mathbb{E}[(D - X)^+]$
4. Expected profit $= \mathbb{E}[profit] = \mathbb{E}[revenue] - \mathbb{E}[cost] - \mathbb{E}[economic\ cost]$

$$= 18\mathbb{E}[X \wedge D] - 3 \times X - (2\mathbb{E}[(X - D)^+] + 15\mathbb{E}[(D - X)^+])$$

Exercise 5

Problem : Prove Theorem 1.

holding cost = -(salvage value) but, holding cost is already include that economic cost. So, we assume that salvage value is 0.

$c_u = p - c$

$c_o = c$

$F(y) = \frac{c_u}{c_u + c_o} = \frac{p-c}{c}$

$\mathbb{E}[profit] = \mathbb{E}[revenue] - \mathbb{E}[cost] - \mathbb{E}[economic\ cost]$

$\mathbb{E}[revenue] = p \times \mathbb{E}[D \wedge y] = p \times (\int_0^y xf(x)\ dx + \int_y^\infty yf(x)\ dx)$

$\mathbb{E}[cost] = c \times y = cy$

$\mathbb{E}[economic\ cost] = c_o\mathbb{E}[(y-D)^+] + c_u\mathbb{E}[(D-y)^+]$

$= c_o \int_0^y (y-D)f(x)\ dx + c_u \int_y^\infty (D-y)f(x)\ dx$

$\frac{d}{dy}\mathbb{E}[profit] = \frac{d}{dy}(\mathbb{E}[revenue] - \mathbb{E}[cost] - \mathbb{E}[economic\ cost])$

$= \frac{d}{dy}[p \times (\int_0^y xf(x)\ dx + \int_y^\infty yf(x)\ dx) + (cy) - (c_o \int_0^y (y-D)f(x)\ dx + c_u \int_y^\infty (D-y)f(x)\ dx)]$

$= p \times (yf(y) + 1 - F(y) - yf(y)) + c - [c_o(F(y) + yf(y) - yf(y)) + c_u(-yf(y) - 1 + F(y) + yf(y)]$

$= p(1 - F(y)) + c - (c_o(F(y) - c_u(F(y) - 1))$

Then, $F(y) = \frac{p-c}{c},\ \ p(1 - F(y)) + c = 0$

Assume that 'y' is minimum point

$\frac{d}{dy}\mathbb{E}[economic\ cost] = 0$

$\therefore \frac{d}{dy}\mathbb{E}[profit] = 0$

To, maximum/minimum discrimination when find second derivative, the result always negative to be maximum point

$\therefore \frac{d^2}{dy^2}\mathbb{E}[profit] = -f(y)(c_o + c_u)$

But, $f(y), (c_o + c_u) is\ always\ positive$

So, the result of second derivative is negative

$\therefore cost\ minimum\ point = profit\ maximum\ point$

DaiPark Exercise

1. Show that $(D \wedge y) + (y - D)^{+} = y$

if $(y < D)$

$(D \wedge y) = y, \;\; (y - D)^{+} = 0$

$\therefore (D \wedge y) + (y - D)^{+} = y$

else $(y > D)$

$(D \wedge y) = D, \;\; (y - D)^{+} = y - D$

$\therefore (D \wedge y) + (y - D)^{+} = y$

$Answer : (D \wedge y) + (y - D)^{+} = y \; always \; qualified$

2. Let D be a discrete random variable with the following pmf.

d	5	6	7	8	9
$Pr(D = d)$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

$Find$

(a)$\mathbb{E}[min(D, 7)$

(b)$\mathbb{E}[(7 - D)^{+}]$

(a)$5 \times \frac{1}{10} + 6 \times \frac{3}{10} + 7 \times \frac{4}{10} + 8 \times \frac{1}{10} + 9 \times \frac{1}{10} = \frac{34}{5}$

(b)$2 \times \frac{1}{10} + 1 \times \frac{3}{10} + 0 + 0 + 0 = \frac{1}{2}$

4. Let D be a continuous random variable and uniformly distributed between 5 and 10. Find

(a)$\mathbb{E}[max(D, 8)]$

(b)$\mathbb{E}[(D - 8)^{-}]$

$$f(x) = \begin{Bmatrix} \frac{1}{5} & (5 \leq x \leq 10) \\ 0 & otherwise \end{Bmatrix}$$

(a)$\mathbb{E}[max(D, 8)] = \int_5^8 8f(x)\;dx + \int_8^{10} xf(x)\;dx$

$= \int_5^8 \frac{8}{5}\;dx + \int_8^{10} \frac{1}{10}x^2\;dx$

$= \frac{24}{5} + \frac{18}{5} = \frac{42}{5}$

(a)$\frac{42}{5}$

(b)$\mathbb{E}[(D - 8)^{-}] = \int_5^8 (8 - D)f(x)\;dx + \int_8^{10} 0 \times f(x)\;dx$

$= \int_5^8 (8 - X)f(x)\;dx + 0 = \int_5^8 (8 - X)\frac{1}{5}\;dx = \int_5^8 (\frac{8}{5} - \frac{1}{5}x)\;dx$

$= \frac{32}{5} - \frac{55}{10} = \frac{9}{10}$

(b)$\frac{9}{10}$

5. Let D be an exponential random variable with parameter 7. Find

(a)$\mathbb{E}[max(D, 3)]$

(b)$\mathbb{E}[(D - 4)^{-}]$

$f(x) = 7e^{-7x}$

(a)$\mathbb{E}[max(D,3)] = \int_{-\infty}^{3} 3 \times 7e^{-7x}\ dx + \int_{3}^{\infty} x \times 7e^{-7x}\ dx$

$= \int_{0}^{3} 21e^{-7x}\ dx + \int_{3}^{\infty} 7xe^{-7x}\ dx$

$= [-3e^{-7x}]_0^3 + [-xe^{-7x}]_3^\infty - \int_3^\infty -e^{-7x}\ dx$

$= (-3e^{21} - (-3)) + (0 - (-3e^{21})) + \int_3^\infty e^{-7x}\ dx$

$= 3 + [-\frac{e^{-7x}}{7}]_7^\infty$

$= 3 + (0 - (-\frac{e^{-21}}{7}))$

$= 3 + \frac{e^{-21}}{7}$

(a)$3 + \frac{e^{-21}}{7}$

(b)$\mathbb{E}[(D-4)^-] = \int_0^4 (4-x)7e^{-7x}\ dx + \int_4^\infty 0 \times f(x)\ dx$

$= \int_0^4 28e^{-7x}\ dx - \int_0^4 7xe^{-7x}\ dx$

$= (-4e^{-28} + 4) - (0 + 4e^{-28}) + (0 + \frac{e^{-28}}{7})$

$= 4 + \frac{e^{-28}}{7}$

(b)$4 + \frac{e^{-28}}{7}$