Data Structures
Homework #2

Due: Apr 12, 2021

1. Give a recursive algorithm to compute the product of two positive integers, n and m, using only addition and subtraction.

2. Suppose $a(n)$ is $O(f(n))$ and $b(n)$ is $O(g(n))$. Please verify the following conclusions. If the conclusion is true, then please prove the correctness; otherwise, please illustrate the incorrectness by a counterexample.

   (a) $a(n) + b(n)$ is $O(f(n) + g(n))$.

   (b) the product $a(n)b(n)$ is $O(f(n)g(n))$.

   (c) $a(n) - b(n)$ is $O(f(n) - g(n))$.

   (d) $f(n) = O((f(n))^2)$.

3. Please consider the following pseudo-code and answer the questions, where A is an $I \times J$ matrix.

   Input array $A[I][J]$
   create array $B[J][I]$
   for $i = 1$ to I
       for $j = 1$ to J
           $B[j][i] = A[i][j]$

   (a) Please state what this pseudo-code does.

   (b) Please give the number of operations (steps) for each statement. Then, sum up these numbers to derive the total number of operations.

   (c) What is the asymptotic time of this program if $I = J = n$.

4. Let S be a set of n lines in the plane such that no two are parallel and no three meet in the same point. Show, by induction, that the lines in S determine $\Theta(n^2)$ intersection points. (**hint**: Consider the contribution made by one line.)

5. (**Programming problem 1**)
   Consider the recursive approach in above problem 1.

   (a) Implement the approach as a function named as `product_rec` using Python.

   (b) Please have an iterative version for the approach and write a function for this version with function name `product_ite`.

   (c) Compare these two function with the same input in terms of running time and write what you have observed.

Note that we will use an in-built python library `timeit` and the module function `timeit.timeit()` for measuring the running time.

6. (**Programming problem 2**)
   Consider the **selection sort** we discussed in class.

   (a) Implement the **selection sort** using iterative approach in Python and name the function as `select_sort_ite`.

   (b) Please implement the recursive version of **selection sort** in Python with the function name of `select_sort_rec`.

   (c) Compare these two functions with the same input in terms of running time and write what you have observed.

About submitting this homework

1. For problem 1, 2, 3, and 4, Please

   (1) write all of your solutions on the papers of size **A4**,

   (2) leave you name and student ID on the first page, and

   (3) hand in your solutions for problem 1, 2, 3, and 4 to me **in class**.

2. For problem 5 and 6, please upload the completed `.ipynb` file with the filename as `HW2_studentID.ipynb` to **i-school(Plus)** (`https://istudy.ntut.edu.tw/learn/index.php`).

3. There will be some **penalty** on the things you miss to submit. **Late work** is not acceptable. Remember, the **deadline** is the **midnight of April 12**, 2021.

4. Honest Policy: We encourage students to discuss their work with the peer. However, each student should write the program or the problem solutions on her/his own. Those who copy others work will get 0 on the homework grade.