$$\text{posterior } P(\theta, \text{event}) = \frac{\text{likelihood} \times \text{prior}}{\text{marginal}}$$

$$= \frac{\binom{N}{m} p^m (1-p)^{N-m} \times p^{a-1} (1-p)^{1-b} \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}}{\int_0^1 \binom{N}{m} \theta^m (1-\theta)^{N-m} \theta^{a-1} (1-\theta)^{b-1} \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} d\theta}$$

$$= \frac{p^{m+a-1} (1-p)^{N-m+b-1}}{\int_0^1 \theta^{m+a-1} (1-\theta)^{N-m+b-1} d\theta}$$

$$\int_0^1 \beta(\theta, m+a-1, N-m+b-1)\, d\theta = \int_0^1 \theta^{m+a-1} (1-\theta)^{N-m+b-1} \frac{\Gamma(a+N+b)}{\Gamma(m+a)\Gamma(N-m+b)} d\theta$$

$$= \frac{\Gamma(a+N+b}{\Gamma(m+a)\Gamma(N-m+b)} \int_0^1 \theta^{m+a-1} (1-\theta)^{N-m+b-1} d\theta = 1$$

$$\Rightarrow \int_0^1 \theta^{m+a-1} (1-\theta)^{N-m+b-1} d\theta = \frac{\Gamma(m+a)\Gamma(N-m+b)}{\Gamma(a+N+b)}$$

$$\therefore P(\theta, \text{event}) = \frac{p^{m+a-1} (1-p)^{N-m+b-1}}{\int_0^1 \theta^{m+a-1} (1-\theta)^{N-m+b-1} d\theta} = \frac{p^{m+a-1} (1-p)^{N-m+b-1}}{\frac{\Gamma(m+a)\Gamma(N-m+b)}{\Gamma(a+N+b)}}$$

$$= \frac{\Gamma(a+N+b)}{\Gamma(m+a)\Gamma(N-m+b)} p^{m+a-1} (1-p)^{N-m+b-1}$$

$$= \beta(p, a+m, b+N-m)$$