Video Compression
視訊壓縮

Yu-Lun (Alex) Liu
劉育綸

with slides by Wen-Hsiao Peng,
Shao-Yi Chien,
Hsueh-Ming Hang,
and Aggelos K. Katsaggelos

Week	Date	Topic	Assignments
1	2025-09-01		
2	2025-09-08	Introduction to Image and Video Processing	
3	2025-09-16	Signals and Systems	#1 – Color Transform, due: 2025-09-29 1:19pm
4	2025-09-22	Fourier Transform and Sampling	
5	2025-09-29	教師節補假	
6	2025-10-06	中秋節	
7	2025-10-13	Fourier Transform and Sampling	#2 – 2D-DCT, due: 2025-10-27 1:59pm
8	2025-10-20	Motion Estimation	Final project assigned (group together in fours)
9	2025-10-27	Lossless Compression	#3 – MEMC, due: 2025-11-10 1:59pm
10	2025-11-03	Image Compression	
11	2025-11-10	Video Compression	#4 – Entropy coding, due: 2025-11-24 1:59pm
12	2025-11-17	Learning-based Image/Video Compression	
13	2025-11-24	Paper Presentation	
14	2025-12-01	Guest Lecturer – MEDIATEK Meta NETFLIX	
15	2025-12-08	Guest Lecturer – MEDIATEK Meta NETFLIX	
16	2025-12-15	Final Project Presentation	

Signals and Systems

with slides by Wen-Hsiao Peng, Shao-Yi Chien, Hsueh-Ming Hang, and Aggelos K. Katsaggelos

Signals and Systems

- 2D and 3D Discrete Signals
- Complex Exponential Signals
- Linear Shift-Invariant Systems
- 2D Convolution
- Filtering in the Spatial Domain
- Fundamentals of Color Image Processing

Signals and Systems

- **2D and 3D Discrete Signals**
- Complex Exponential Signals
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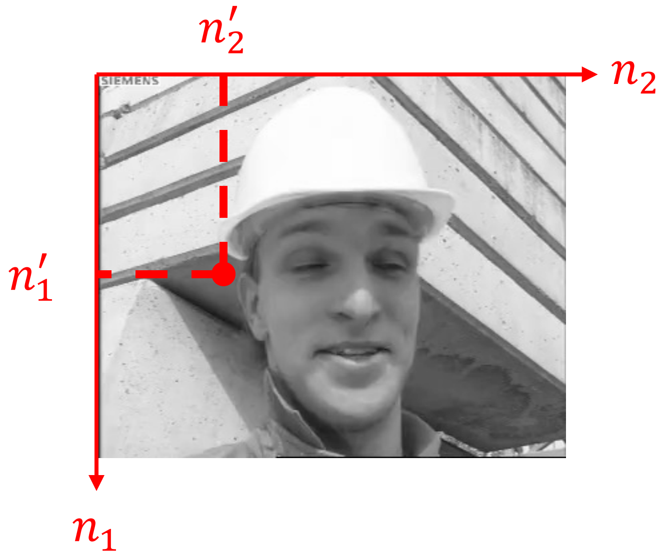
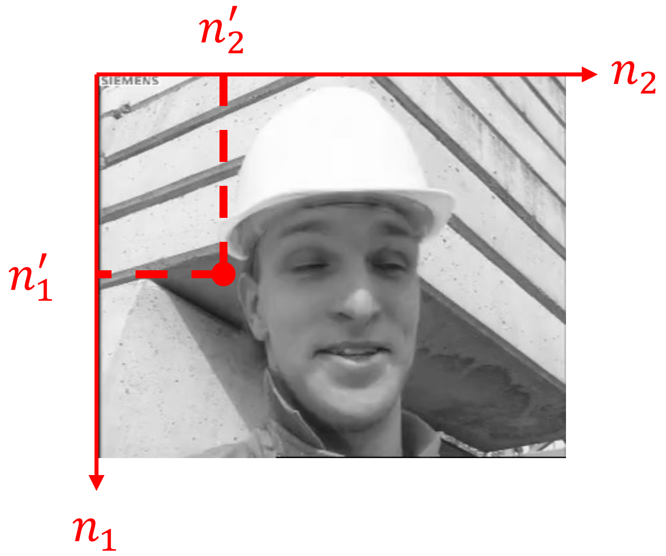
2D and 3D Discrete Signals

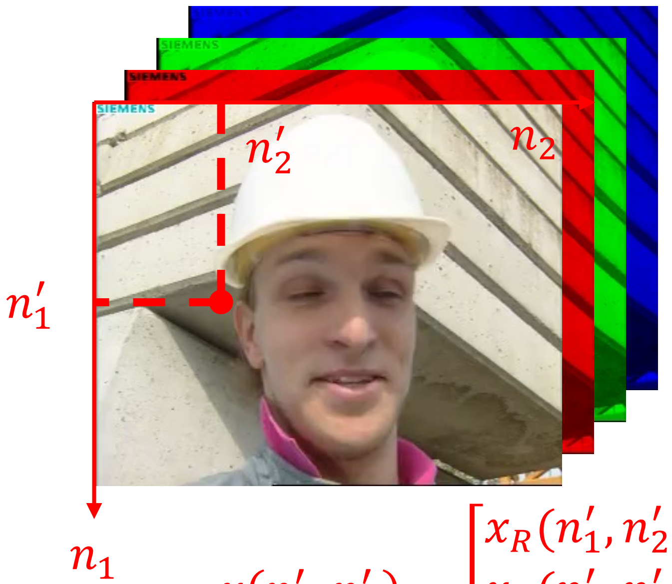
$$x(n_1, n_2), -\infty < n_1, n_2 < \infty, n_1, n_2 = 0, \pm 1, \pm 2, \ldots$$



2D and 3D Discrete Signals

$$x(n_1, n_2), -\infty < n_1, n_2 < \infty, n_1, n_2 = 0, \pm 1, \pm 2, \ldots$$
$$x(n_1, n_2, n_3), -\infty < n_1, n_2, n_3 < \infty, n_1, n_2, n_3 = 0, \pm 1, \pm 2, \ldots$$





$$x(n_1', n_2') = \begin{bmatrix} x_R(n_1', n_2') \\ x_G(n_1', n_2') \\ x_B(n_1', n_2') \end{bmatrix}$$

2D and 3D Discrete Signals

$$x(n_1, n_2), -\infty < n_1, n_2 < \infty, n_1, n_2 = 0, \pm 1, \pm 2, \ldots$$
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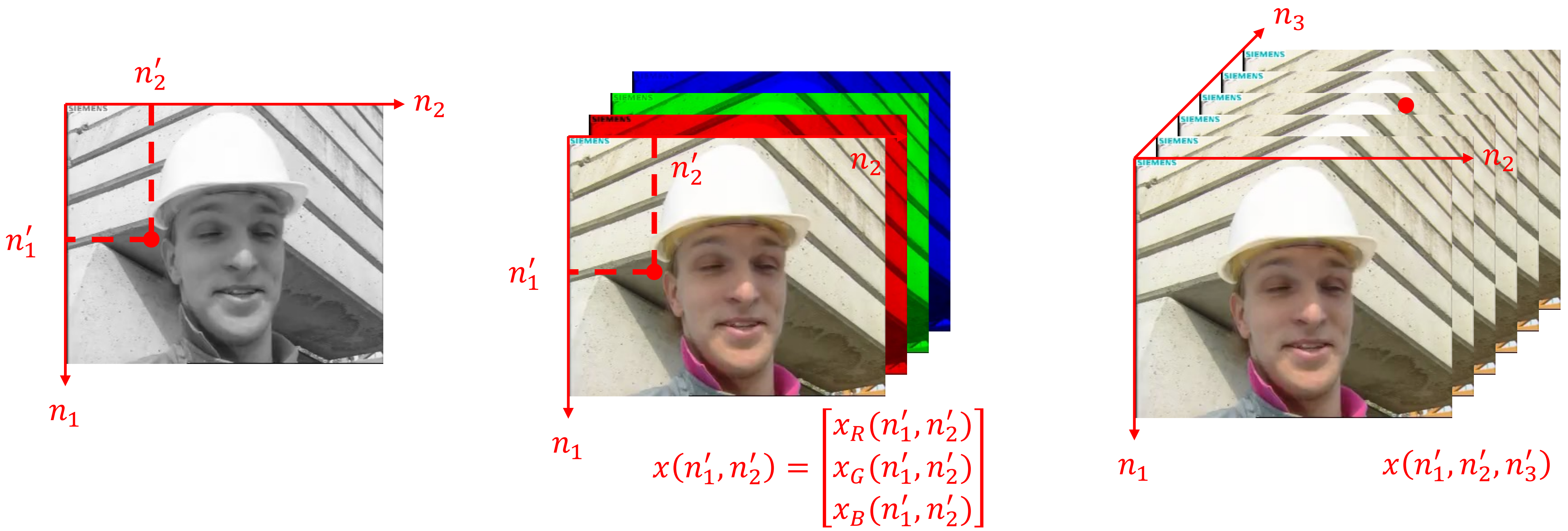
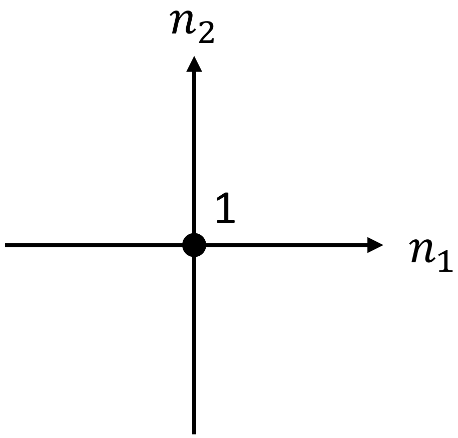


Discrete Unit Impulse



- $\delta(n_1, n_2) = \begin{cases} 1, & \text{for } n_1 = n_2 = 0 \\ 0, & \text{otherwise} \end{cases}$
- $\delta(n_1 - n_1', n_2 - n_2') = \begin{cases} 1, & \text{for } \begin{matrix} n_1 - n_1' = 0 & \rightarrow n_1 = n_1' \\ n_2 - n_2' = 0 & \rightarrow n_2 = n_2' \end{matrix} \\ 0, & \text{otherwise} \end{cases}$

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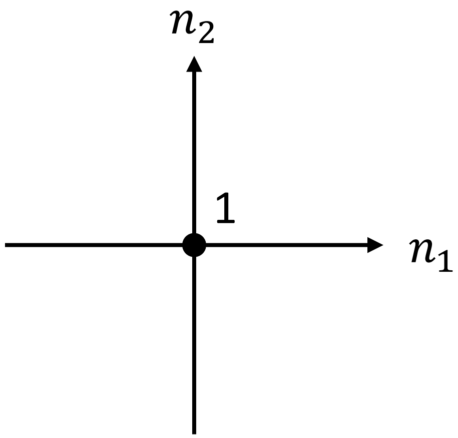

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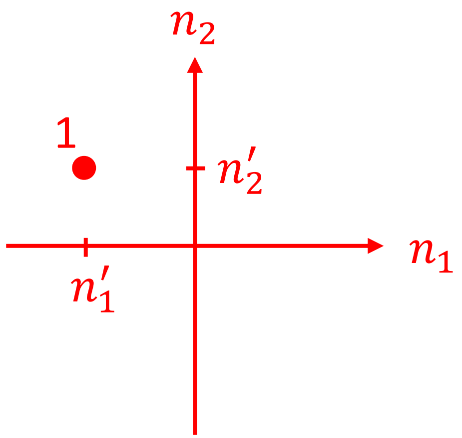
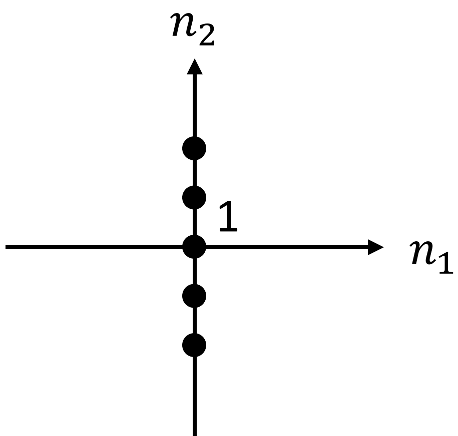
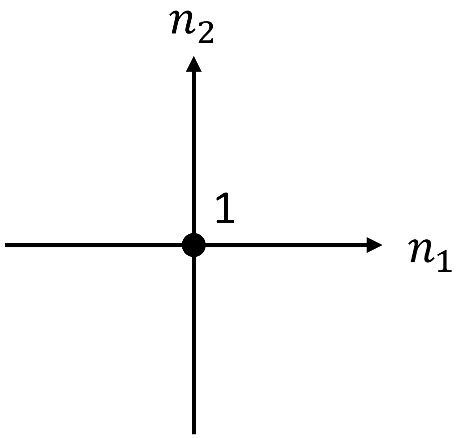


- Separable signals: $g(n_1, n_2) = f_1(n_1) \cdot f_2(n_2)$



Discrete Unit Impulse



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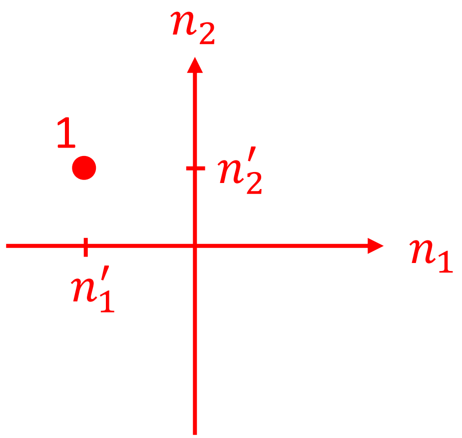

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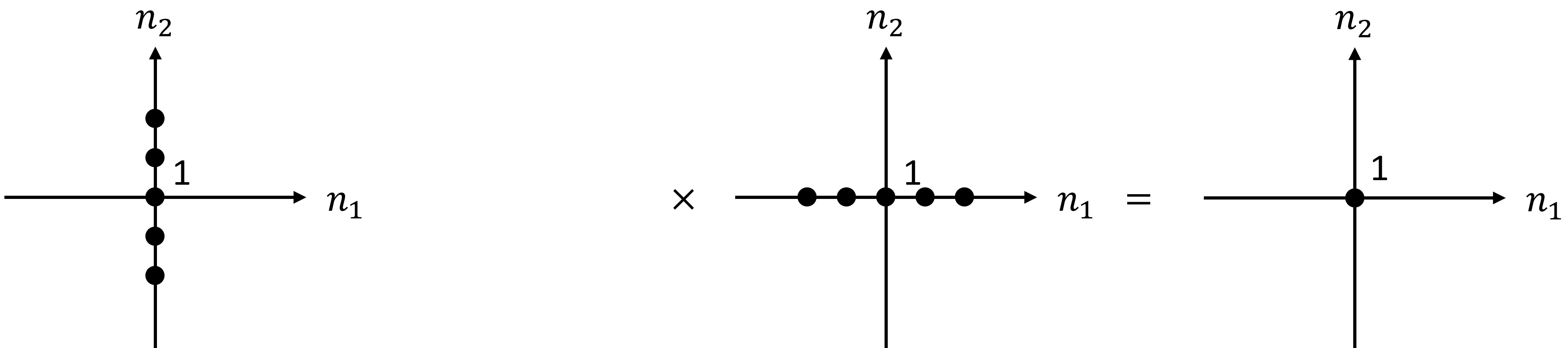
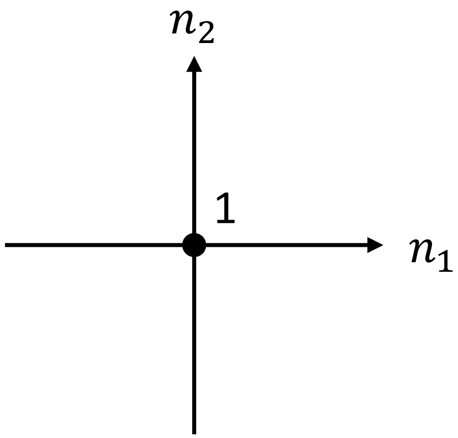


Discrete Unit Impulse



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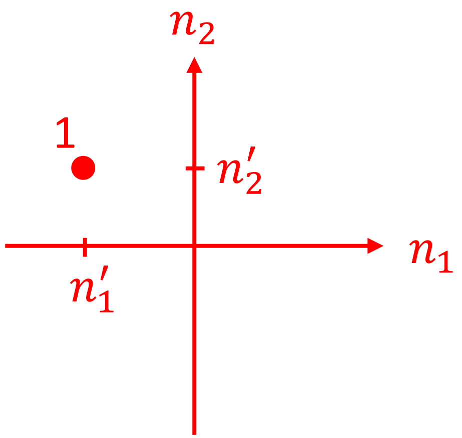

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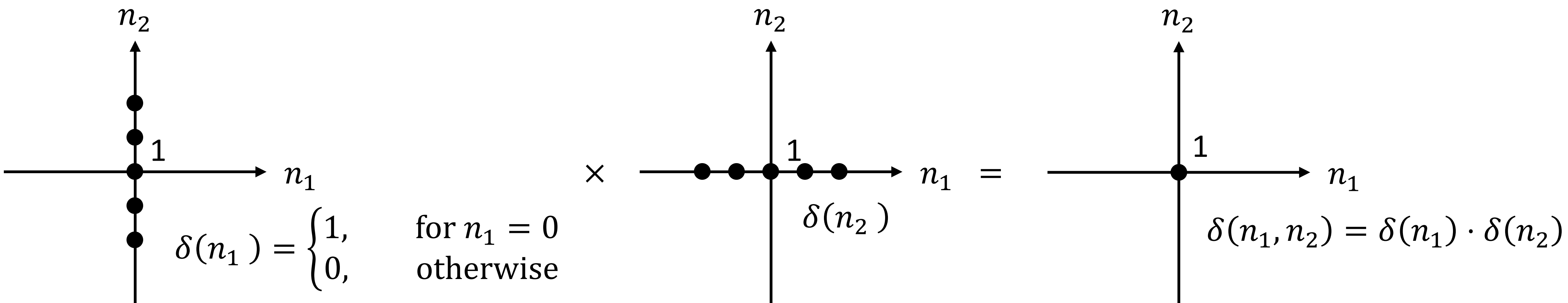
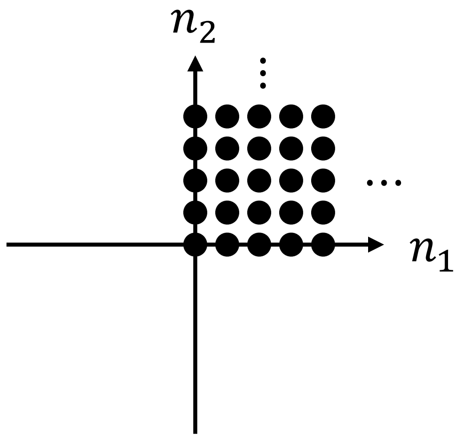


Discrete Unit Step

- $u(n_1, n_2) = \begin{cases} 1, & \text{for } n_1 \geq 0, n_2 \geq 0 \\ 0, & \text{otherwise} \end{cases}$



- $u(n_1 - n_1', n_2 - n_2') = \begin{cases} 1, \ \text{for} & \begin{matrix} n_1 - n_1' \geq 0 & \rightarrow n_1 \geq n_1' \\ n_2 - n_2' \geq 0 & \rightarrow n_2 \geq n_2' \end{matrix} \\ 0, & \text{otherwise} \end{cases}$

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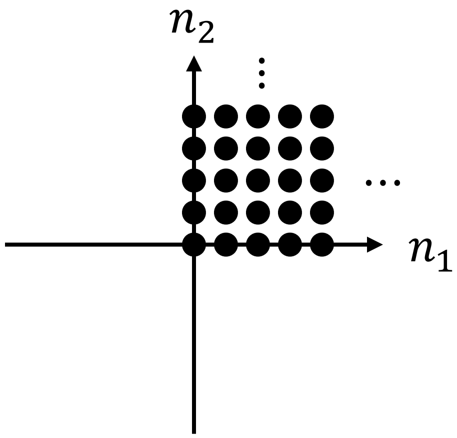

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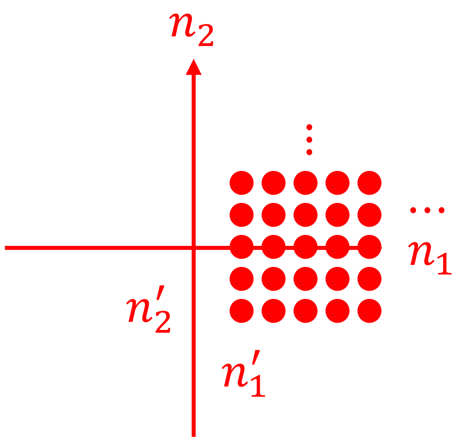
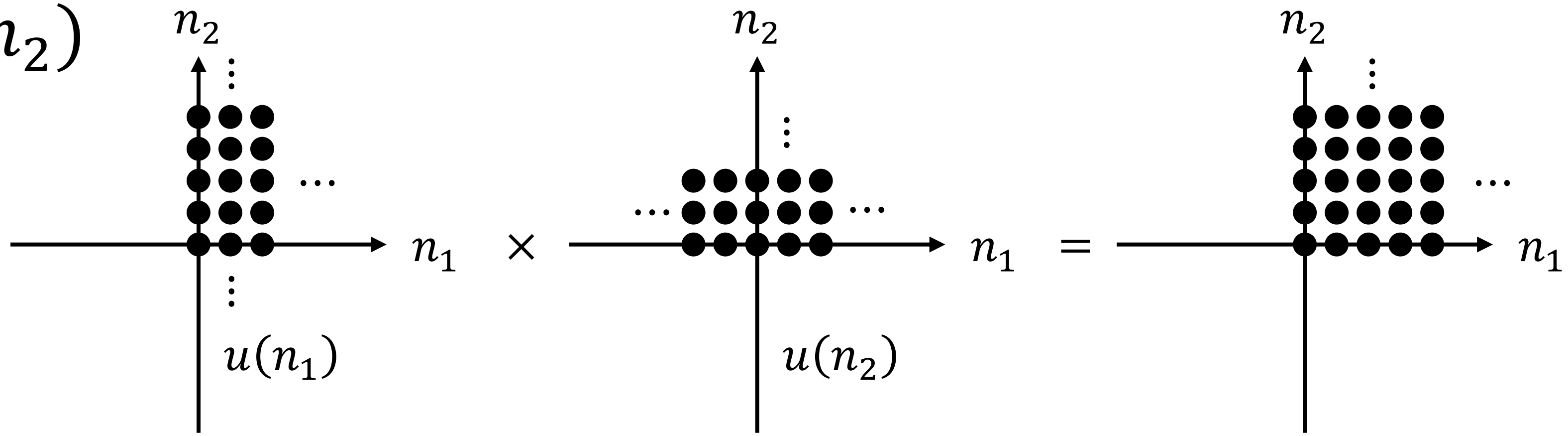


- $u(n_1, n_2) = u(n_1) \cdot u(n_2)$



Signals and Systems

- 2D and 3D Discrete Signals
- **Complex Exponential Signals**
- Linear Shift-Invariant Systems
- 2D Convolution
- Filtering in the Spatial Domain
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Exponential Sequences

- $x(n_1, n_2) = e^{j\omega_1 n_1} \cdot e^{j\omega_2 n_2}$
  - Eigen-function of LSI systems

$$e^{j\omega_1 n_1 + j\omega_2 n_2} \longrightarrow \boxed{\text{LSI}} \longrightarrow A \cdot e^{j\omega_1 n_1 + j\omega_2 n_2} e^{j\phi}$$

Exponential Sequences

- $x(n_1, n_2) = e^{j\omega_1 n_1} \cdot e^{j\omega_2 n_2}$
  - Eigen-function of LSI systems
  - "Building Blocks" of any signal

$$e^{j\omega_1 n_1 + j\omega_2 n_2} \longrightarrow \boxed{\text{LSI}} \longrightarrow A \cdot e^{j\omega_1 n_1 + j\omega_2 n_2} e^{j\phi}$$

- $x(n_1, n_2) = e^{j\omega_1 n_1} \cdot e^{j\omega_2 n_2}$

$$= \cos(\omega_1 n_1 + \omega_2 n_2) + j \sin(\omega_1 n_1 + \omega_2 n_2)$$

Euler's formula

$$\left|e^{j\omega_1 n_1}\right| = \left|e^{j\omega_2 n_2}\right| = 1$$

Exponential Sequences

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Euler's formula

$$\left|e^{j\omega_1 n_1}\right| = \left|e^{j\omega_2 n_2}\right| = 1$$

  - Periodically w.r.t. ω_1, ω_2

$$e^{j(\omega_1 + 2\pi) n_1} \cdot e^{j(\omega_2 + 2\pi) n_2} = e^{j\omega_1 n_1} \cdot \underline{e^{j2\pi n_1}} \cdot e^{j\omega_2 n_2} \cdot e^{j2\pi n_2} = e^{j\omega_1 n_1} e^{j\omega_2 n_2}$$

Exponential Sequences

Periodicity of $e^{j\omega_1 n_1} \cdot e^{j\omega_2 n_2}$ w.r.t. n_1, n_2

If periodic w/ periods N_1, N_2

$e^{j\omega_1(n_1+N_1)} \cdot e^{j\omega_2(n_2+N_2)} = e^{j\omega_1 n_1} \cdot e^{j\omega_2 n_2}$

$\Rightarrow e^{j\omega_1 n_1} \cdot e^{j\omega_1 N_1} \cdot e^{j\omega_2 n_2} \cdot e^{j\omega_2 N_2} = e^{j\omega_1 n_1} \cdot e^{j\omega_2 n_2}$

$\Rightarrow e^{j\omega_1 N_1} \cdot e^{j\omega_2 N_2} = 1$

Exponential Sequences

Periodicity of $e^{j\omega_1 n_1} \cdot e^{j\omega_2 n_2}$ w.r.t. n_1, n_2

If periodic w/ periods N_1, N_2

$$e^{j\omega_1(n_1+N_1)} \cdot e^{j\omega_2(n_2+N_2)} = e^{j\omega_1 n_1} \cdot e^{j\omega_2 n_2}$$

$$\Rightarrow e^{j\omega_1 n_1} \cdot e^{j\omega_1 N_1} \cdot e^{j\omega_2 n_2} \cdot e^{j\omega_2 N_2} = e^{j\omega_1 n_1} \cdot e^{j\omega_2 n_2}$$

$$\Rightarrow e^{j\omega_1 N_1} \cdot e^{j\omega_2 N_2} = 1$$

$$\Rightarrow \omega_1 N_1 = k_1 \cdot 2\pi \Rightarrow N_1 = k_1 \cdot \boxed{\frac{2\pi}{\omega_1}}$$

$$\Rightarrow \omega_2 N_2 = k_2 \cdot 2\pi \Rightarrow N_2 = k_2 \cdot \boxed{\frac{2\pi}{\omega_2}}$$

rational number $\frac{p}{q}$

integer

Exponential Sequences

Periodicity of $e^{j\omega_1 n_1} \cdot e^{j\omega_2 n_2}$ w.r.t. n_1, n_2

If periodic w/ periods N_1, N_2

$$e^{j\omega_1(n_1+N_1)} \cdot e^{j\omega_2(n_2+N_2)} = e^{j\omega_1 n_1} \cdot e^{j\omega_2 n_2}$$

$$\Rightarrow e^{j\omega_1 n_1} \cdot e^{j\omega_1 N_1} \cdot e^{j\omega_2 n_2} \cdot e^{j\omega_2 N_2} = e^{j\omega_1 n_1} \cdot e^{j\omega_2 n_2}$$

$$\Rightarrow e^{j\omega_1 N_1} \cdot e^{j\omega_2 N_2} = 1$$

$$\Rightarrow \omega_1 N_1 = k_1 \cdot 2\pi \Rightarrow N_1 = k_1 \cdot \boxed{\frac{2\pi}{\omega_1}}$$

$$\Rightarrow \omega_2 N_2 = k_2 \cdot 2\pi \Rightarrow N_2 = k_2 \cdot \boxed{\frac{2\pi}{\omega_2}}$$

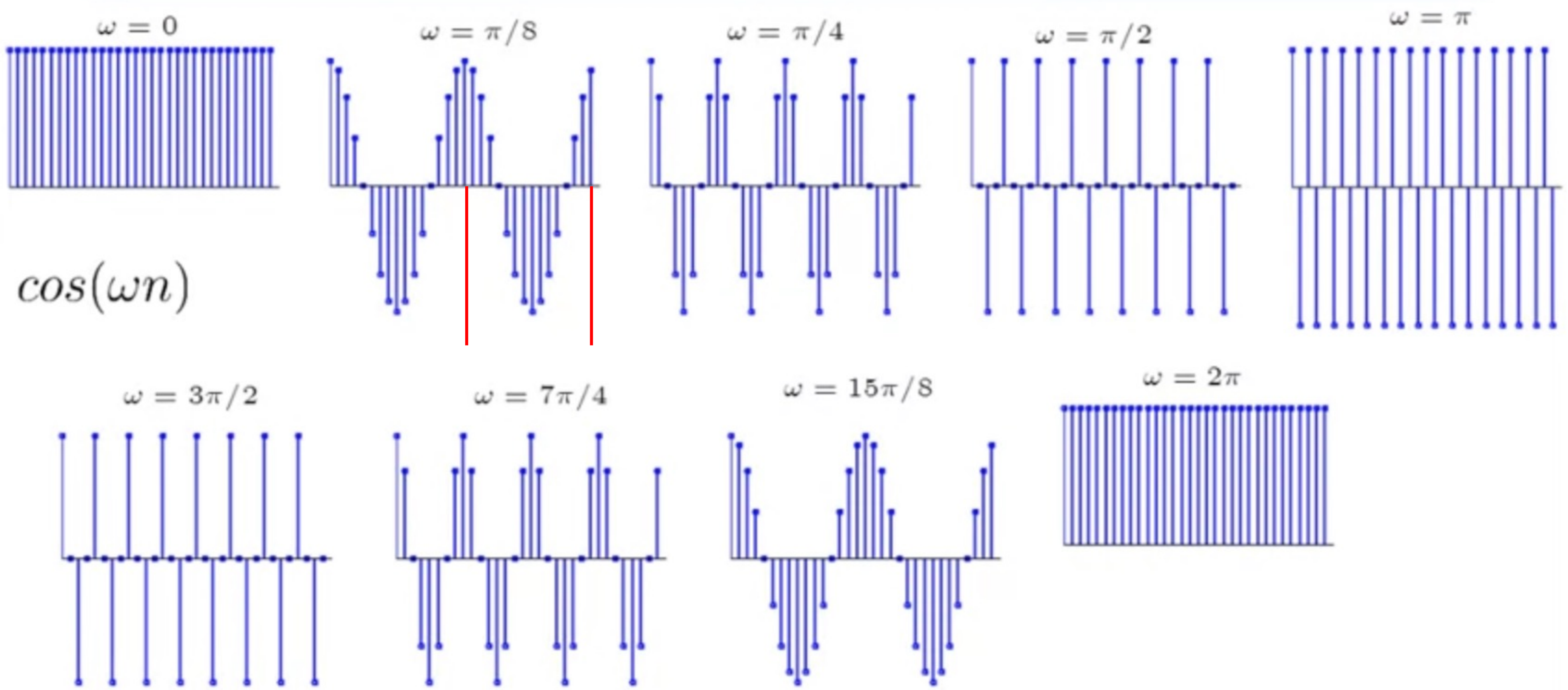
rational number $\frac{p}{q}$ integer

$e^{j3n_1} \cdot e^{j4n_2}$ $\quad \omega_1 = 3, \omega_2 = 4 \quad \frac{2\pi}{\omega_1} = \frac{2\pi}{3}$ Not a rational number

1D Discrete Cosine

$$N = \frac{2\pi}{\omega} = 16$$

$$\cos(\pi n) = (-1)^n$$



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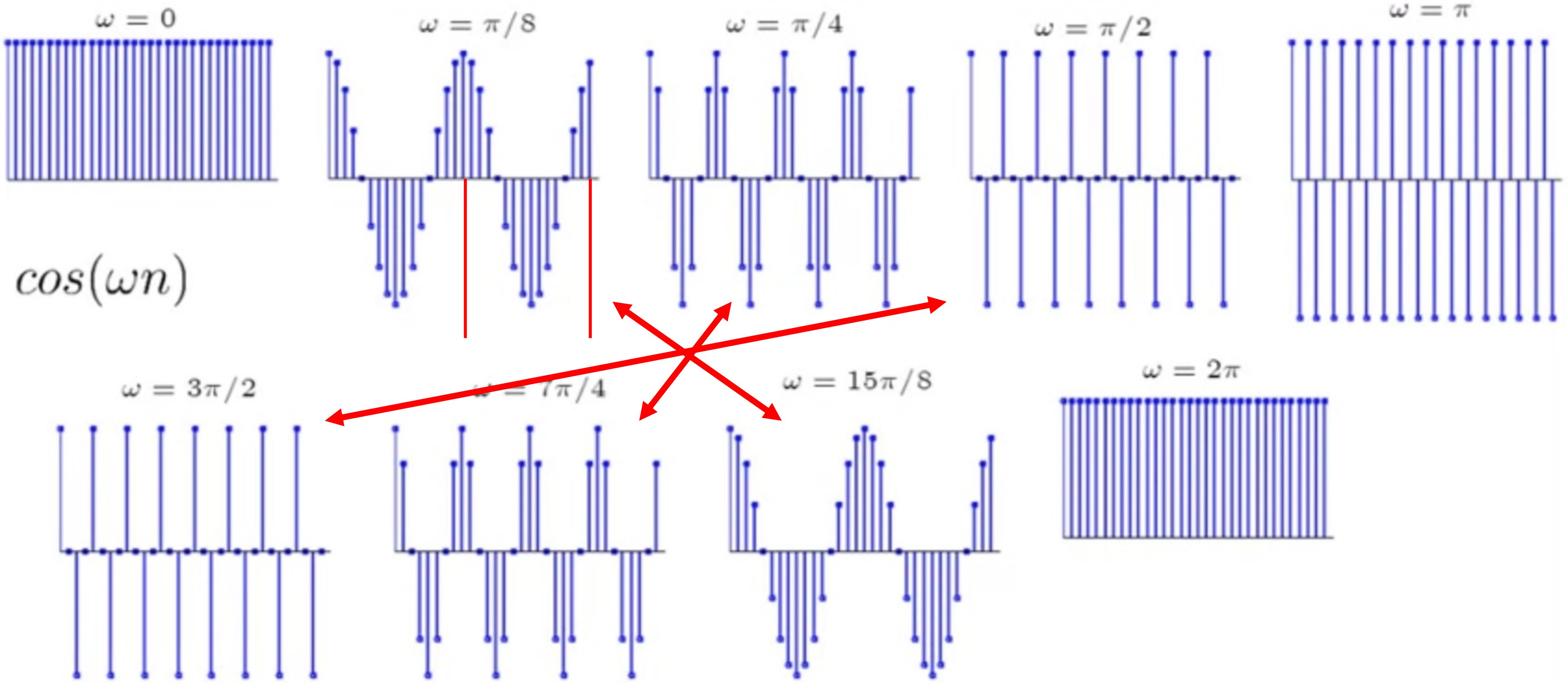
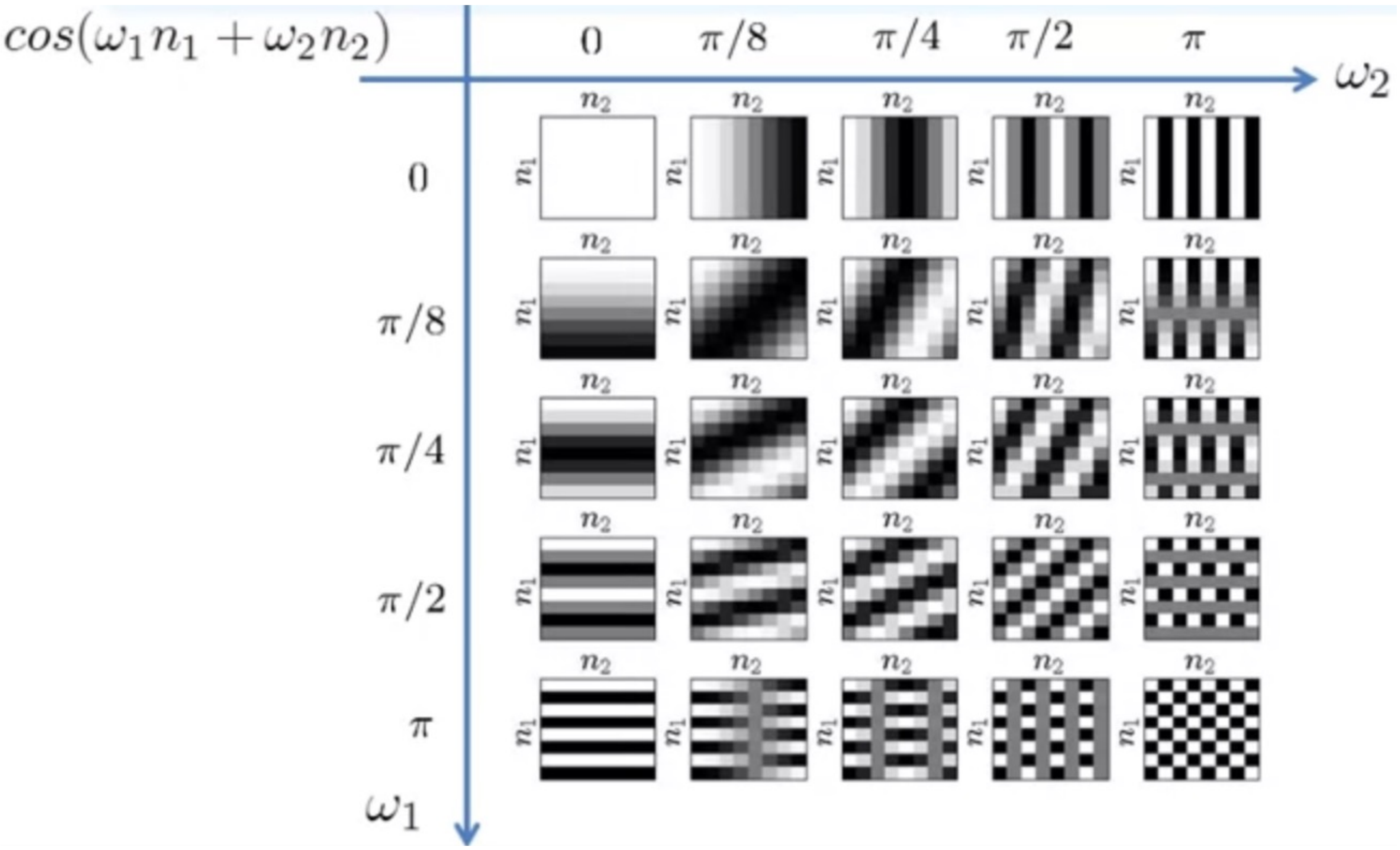


2D Discrete Cosine



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2D Systems

$$x(n_1, n_2) \longrightarrow \boxed{\mathbf{T}[\cdot]} \longrightarrow y(n_1, n_2) = \mathbf{T}[x(n_1, n_2)]$$

- System Properties
  - Stability
  - With/out Memory
  - Causality
  - **Linearity**
  - **Spatial Invariance**

Linear Systems

If
$\mathbf{T}[\alpha_1 x_1(n_1, n_2) + \alpha_2 x_2(n_1, n_2)] = \alpha_1 \mathbf{T}[x_1(n_1, n_2)] + \alpha_2 \mathbf{T}[x_2(n_1, n_2)]$,
then $\mathbf{T}[\cdot]$ is linear

Linear Systems

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then $\mathbf{T}[\cdot]$ is linear

- $\alpha_2 = 0 \quad \mathbf{T}[\alpha_1 x_1(n_1,n_2)] = \alpha_1 \mathbf{T}[x_1(n_1,n_2)]$
- $\alpha_1 = -\alpha_2, x_1(n_1,n_2) = x_2(n_1,n_2)$
  $\mathbf{T}[\alpha_1 x_1(n_1,n_2) - \alpha_1 x_1(n_1,n_2)] = \mathbf{T}[0] = \alpha_1 \mathbf{T}[x_1(n_1,n_2)] - \alpha_1 \mathbf{T}[x_1(n_1,n_2)] = 0$

Linear Systems

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$\mathbf{T}[\alpha_1 x_1(n_1,n_2) + \alpha_2 x_2(n_1,n_2)] = \alpha_1\mathbf{T}[x_1(n_1,n_2)] + \alpha_2\mathbf{T}[x_2(n_1,n_2)]$,
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- $\alpha_2 = 0 \quad \mathbf{T}[\alpha_1 x_1(n_1,n_2)] = \alpha_1\mathbf{T}[x_1(n_1,n_2)]$
- $\alpha_1 = -\alpha_2, x_1(n_1,n_2) = x_2(n_1,n_2)$

$$\mathbf{T}[\alpha_1 x_1(n_1,n_2) - \alpha_1 x_1(n_1,n_2)] = \mathbf{T}[0] = \alpha_1\mathbf{T}[x_1(n_1,n_2)] - \alpha_1\mathbf{T}[x_1(n_1,n_2)] = 0$$
$$\mathbf{T}[0] = 0$$

- $y(n_1,n_2) = \mathbf{T}[x(n_1,n_2)] = 255 - x(n_1,n_2)$

$\mathbf{T}[0] = 255 \neq 0 \Rightarrow \mathbf{T}[\cdot]$ is non-linear

Spatially Invariant Systems

$$\mathbf{T}[x(n_1, n_2)] = y(n_1, n_2)$$

if $\mathbf{T}[x(n_1 - k_1, n_2 - k_2)] = y(n_1 - k_1, n_2 - k_2)$,
then $\mathbf{T}[\cdot]$ is spatially invariant

- $y(n_1, n_2) = \mathbf{T}[x(n_1, n_2)] = 255 - x(n_1, n_2)$ Non-linear

Spatially Invariant Systems

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$\mathbf{T}[x(n_1 - k_1, n_2 - k_2)] = 255 - x(n_1 - k_1, n_2 - k_2) = y(n_1 - k_1, n_2 - k_2)$ SI

- $y(n_1, n_2) = \underbrace{c(n_1, n_2)}_{\text{gain}} \cdot x(n_1, n_2)$

Linear
Not spatially-invariant, or spatially-varying

Linear and Spatially Invariant (LSI) Systems

$$\delta(n_1, n_2) \longrightarrow \boxed{\text{LSI}} \longrightarrow h(n_1, n_2) \triangleq \text{impulse response of the LSI system}$$

Linear and Spatially Invariant (LSI) Systems

$$\delta(n_1,n_2) \longrightarrow \boxed{\text{LSI}} \longrightarrow h(n_1,n_2) \triangleq \text{impulse response of the LSI system}$$

$$x(n_1,n_2) \longrightarrow \boxed{h(n_1,n_2)} \longrightarrow y(n_1,n_2) = x(n_1,n_2) \underbrace{**}_{\text{2D discrete convolution}} h(n_1,n_2)$$

$$y(n_1,n_2) = x(n_1,n_2) ** h(n_1,n_2) = \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} x(k_1,k_2)h(n_1-k_1,n_2-k_2)$$

$$= h(n_1,n_2) ** x(n_1,n_2)$$

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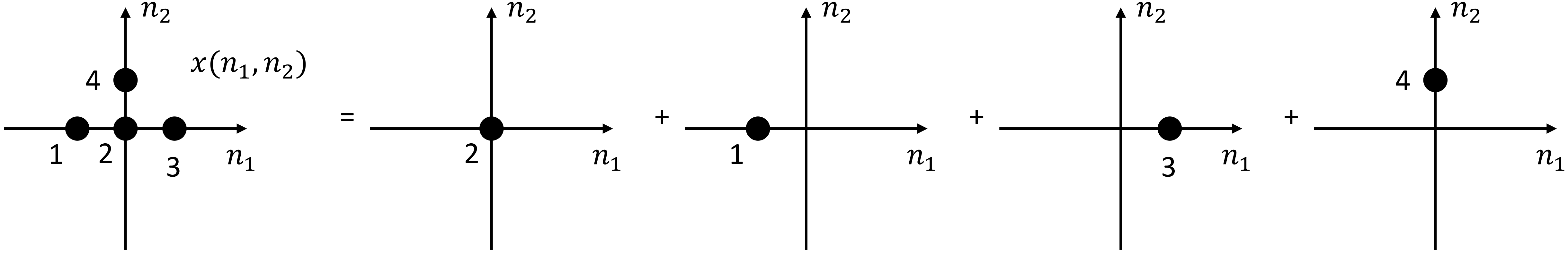
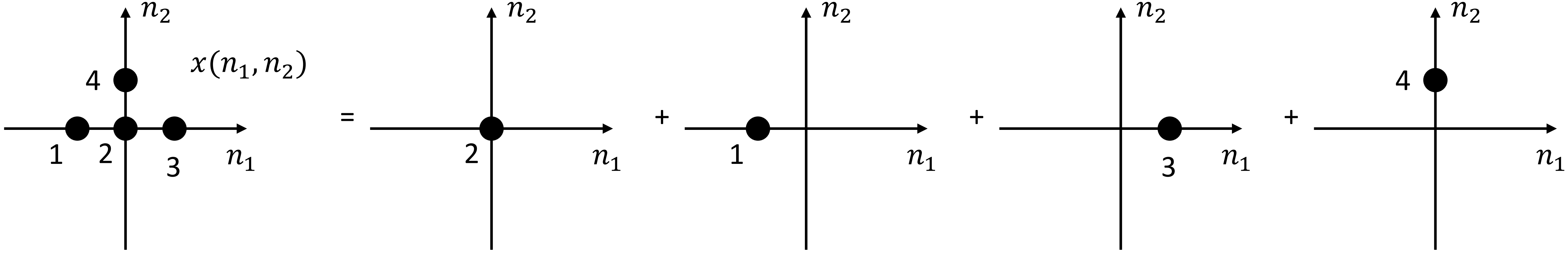
Derivation of Convolution

$$x(n_1, n_2) = \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} x(k_1, k_2)\delta(n_1 - k_1, n_2 - k_2)$$



Derivation of Convolution

$$x(n_1, n_2) = \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} x(k_1, k_2)\delta(n_1 - k_1, n_2 - k_2)$$

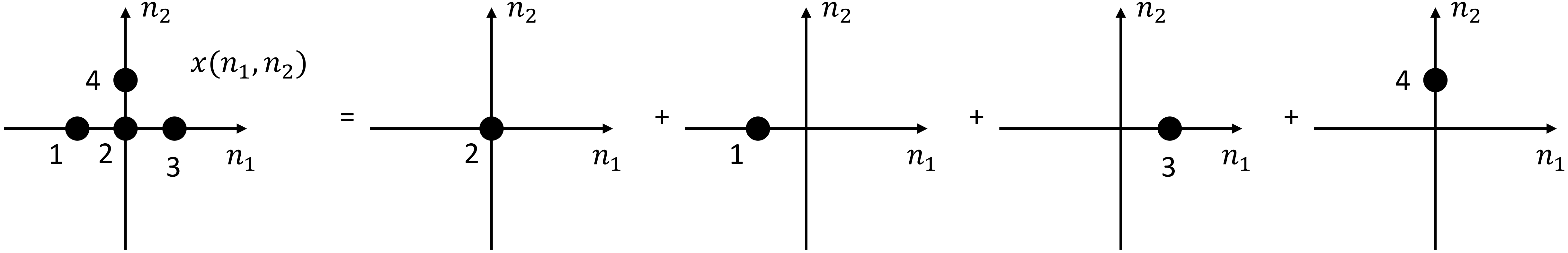


$$x(0,0)\delta(n_1, n_2) + x(-1,0)\delta(n_1+1, n_2) + x(1,0)\delta(n_1+1, n_2) + x(0,1)\delta(n_1, n_2-1)$$

$$y(n_1, n_2) = \mathbf{T}[x(n_1, n_2)] = \mathbf{T}\left[\sum_{k_1} \sum_{k_2} x(k_1, k_2)\delta(n_1 - k_1, n_2 - k_2)\right]$$

Derivation of Convolution

$$x(n_1, n_2) = \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} x(k_1, k_2)\delta(n_1 - k_1, n_2 - k_2)$$



$$x(0,0)\delta(n_1, n_2) + x(-1,0)\delta(n_1 + 1, n_2) + x(1,0)\delta(n_1 - 1, n_2) + x(0,1)\delta(n_1, n_2 - 1)$$

$$y(n_1, n_2) = \mathbf{T}[x(n_1, n_2)] = \mathbf{T}\left[\sum_{k_1}\sum_{k_2} \underbrace{x(k_1, k_2)}_{\text{weight}}\delta(n_1 - k_1, n_2 - k_2)\right]$$

linearity

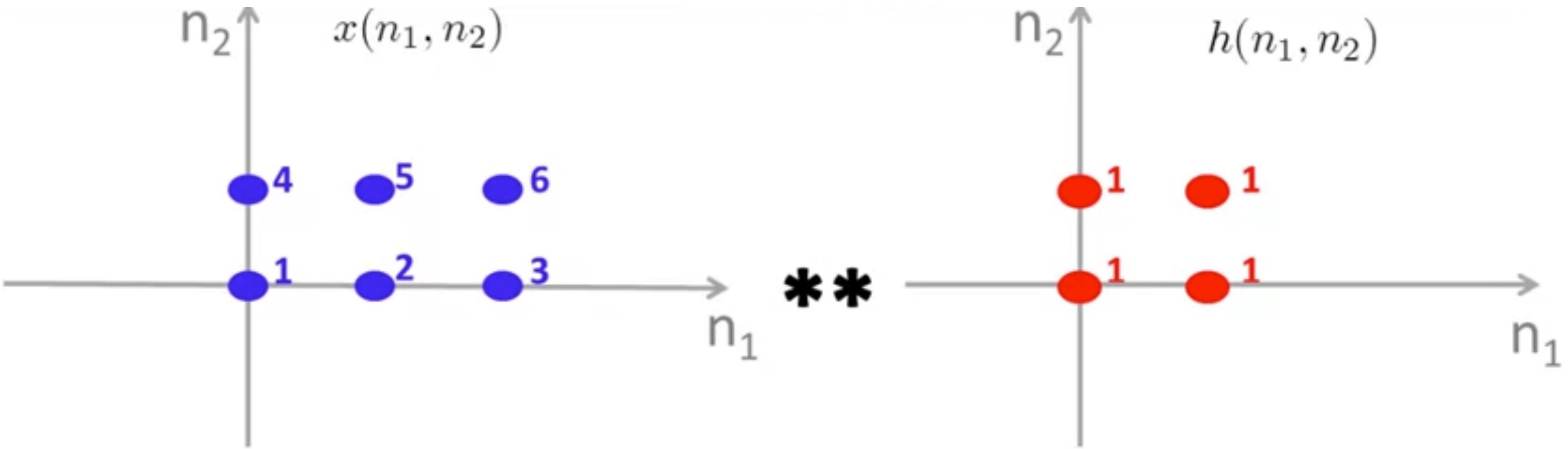
$$= \sum_{k_1}\sum_{k_2} x(k_1, k_2)\,\mathbf{T}[\delta(n_1 - k_1, n_2 - k_2)]$$

SI

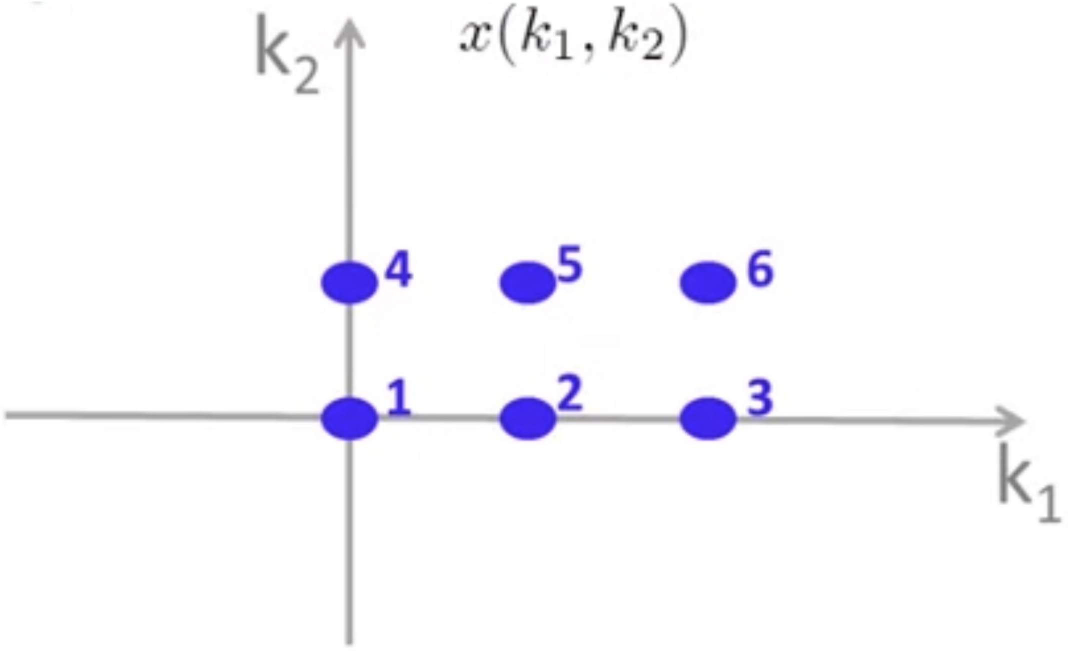
$$= \sum_{k_1}\sum_{k_2} x(k_1, k_2) \cdot h(n_1 - k_1, n_2 - k_2) = x(n_1, n_2) ** h(n_1, n_2) = h(n_1, n_2) ** x(n_1, n_2)$$

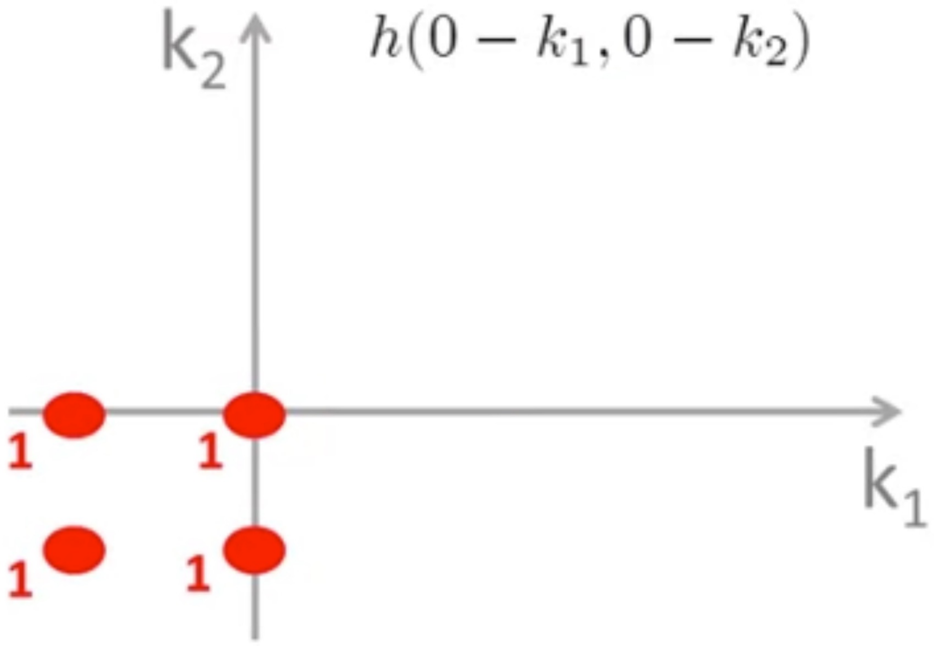
2D Convolution Example

$$y(n_1, n_2) = x(n_1, n_2) ** h(n_1, n_2) = \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} x(k_1, k_2)\delta(n_1 - k_1, n_2 - k_2)$$

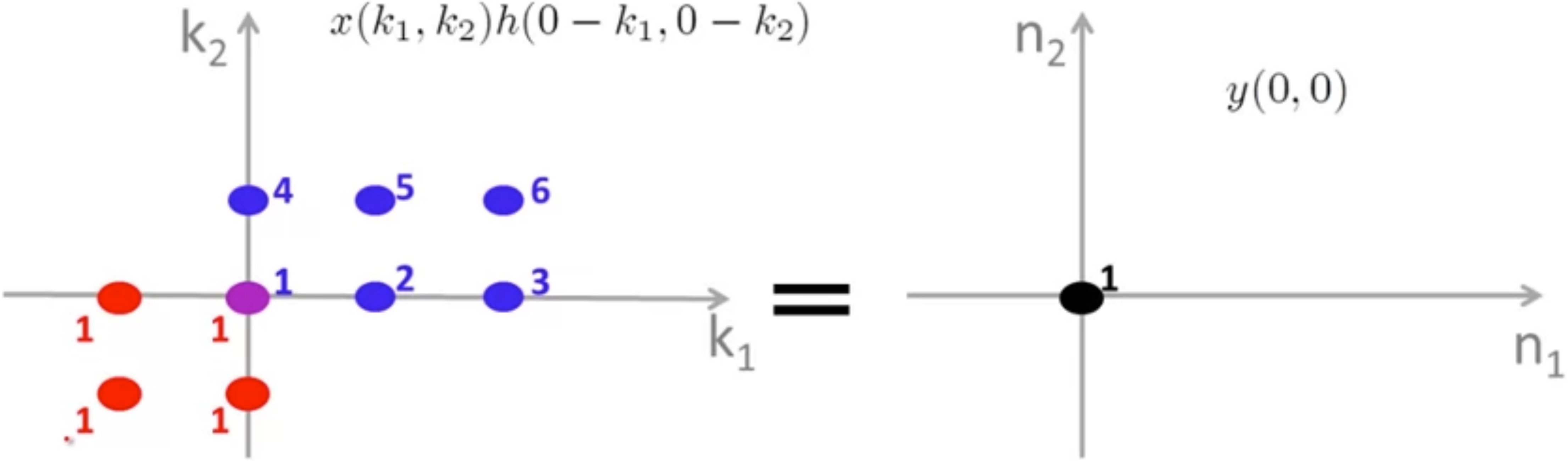


2D Convolution Example



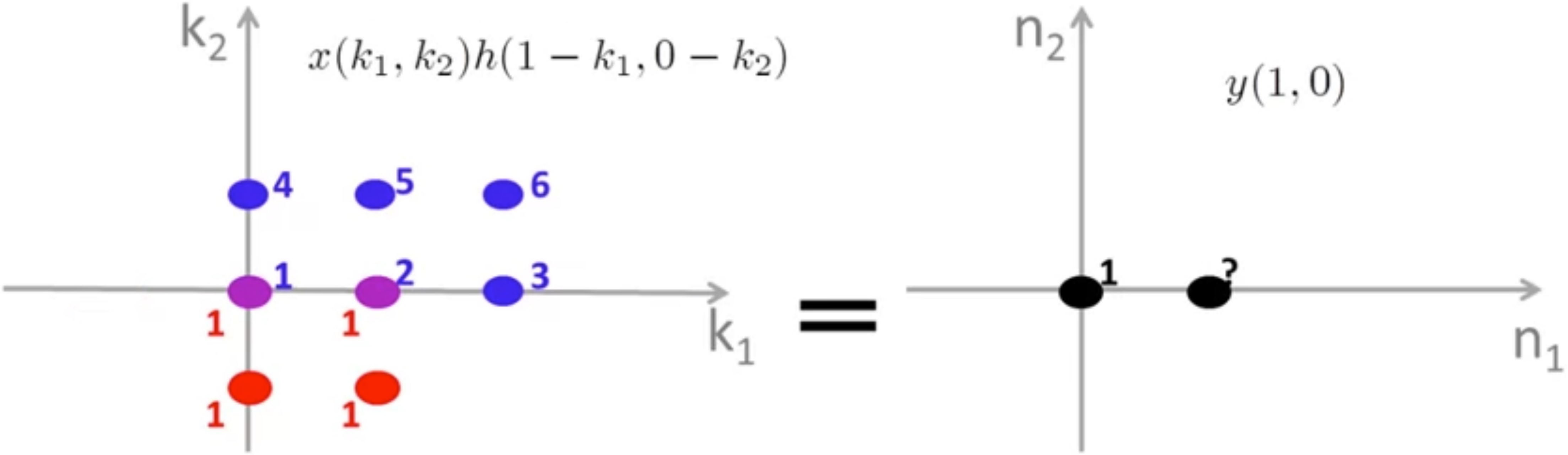


2D Convolution Example



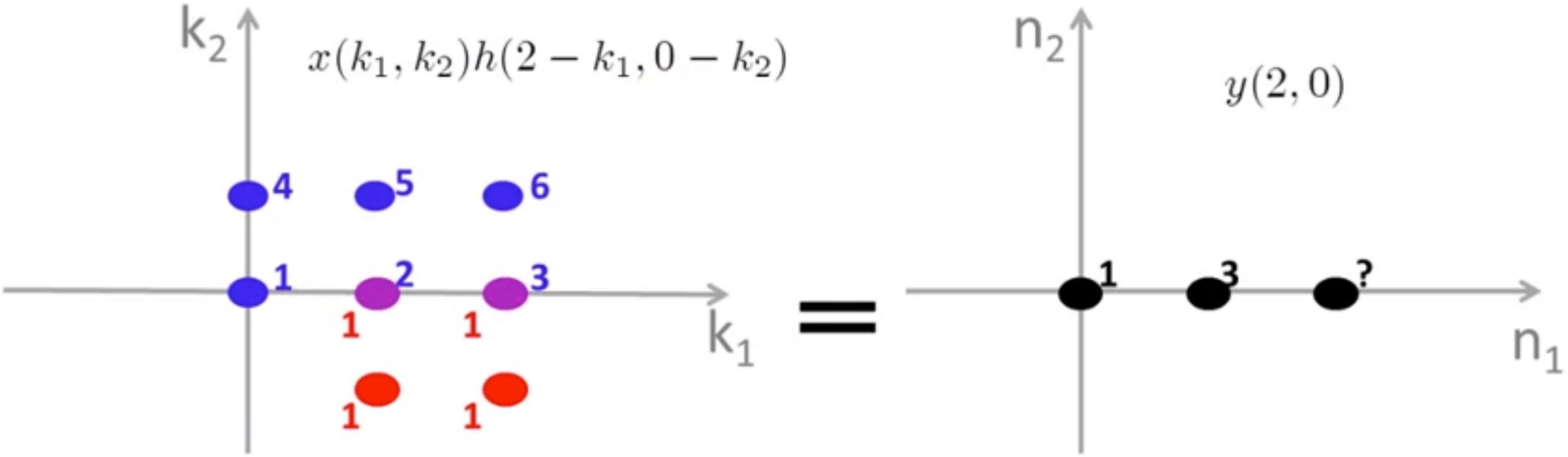
1×1 = 1

2D Convolution Example



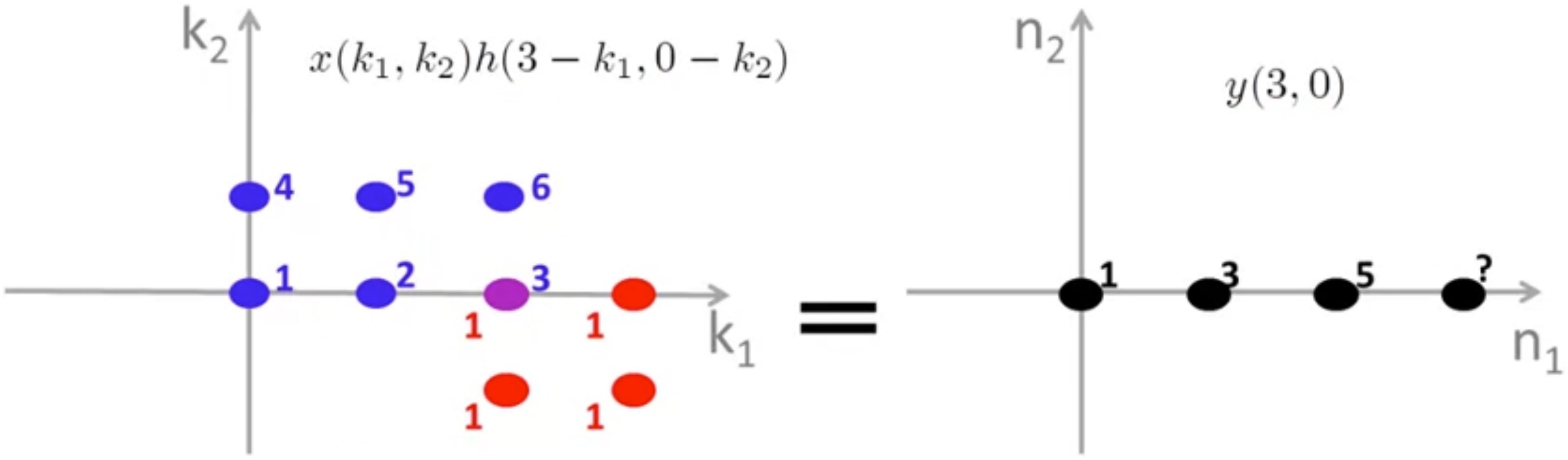
$(1 \times 1) + (1 \times 2) = 3$

2D Convolution Example



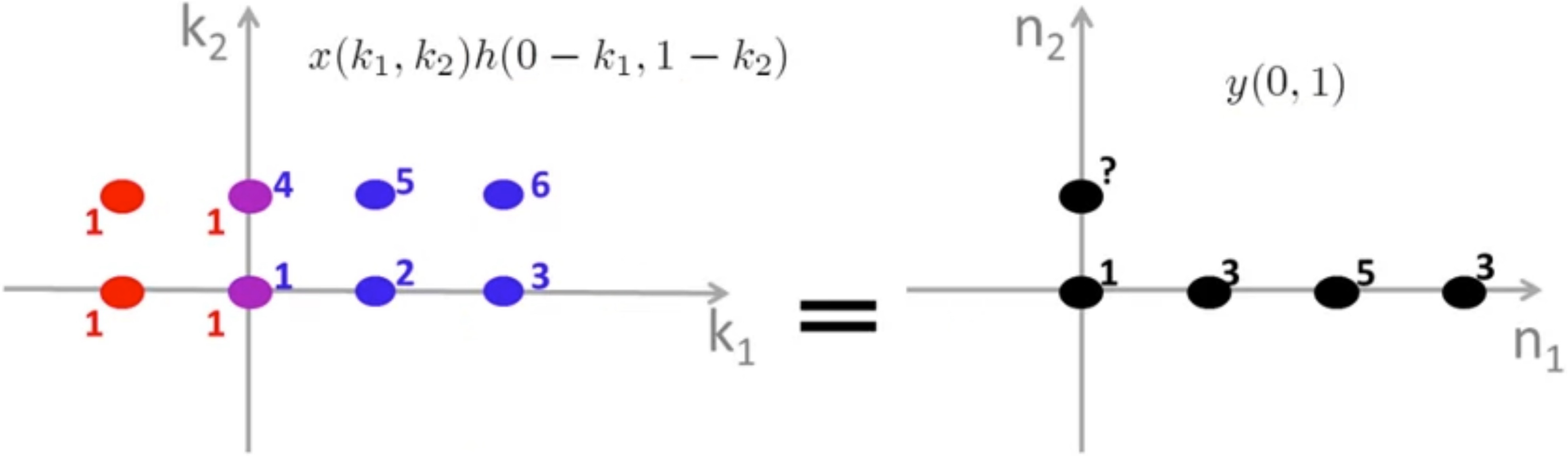
$(1 \times 2) + (1 \times 3) = 5$

2D Convolution Example



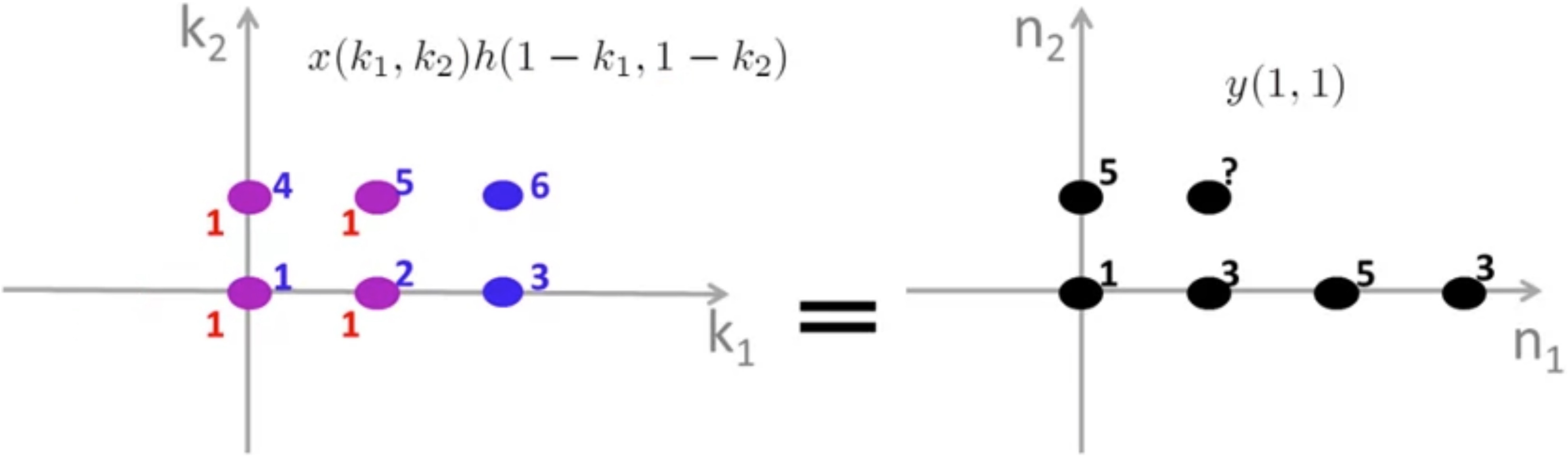
$1 \times 3 = 3$

2D Convolution Example



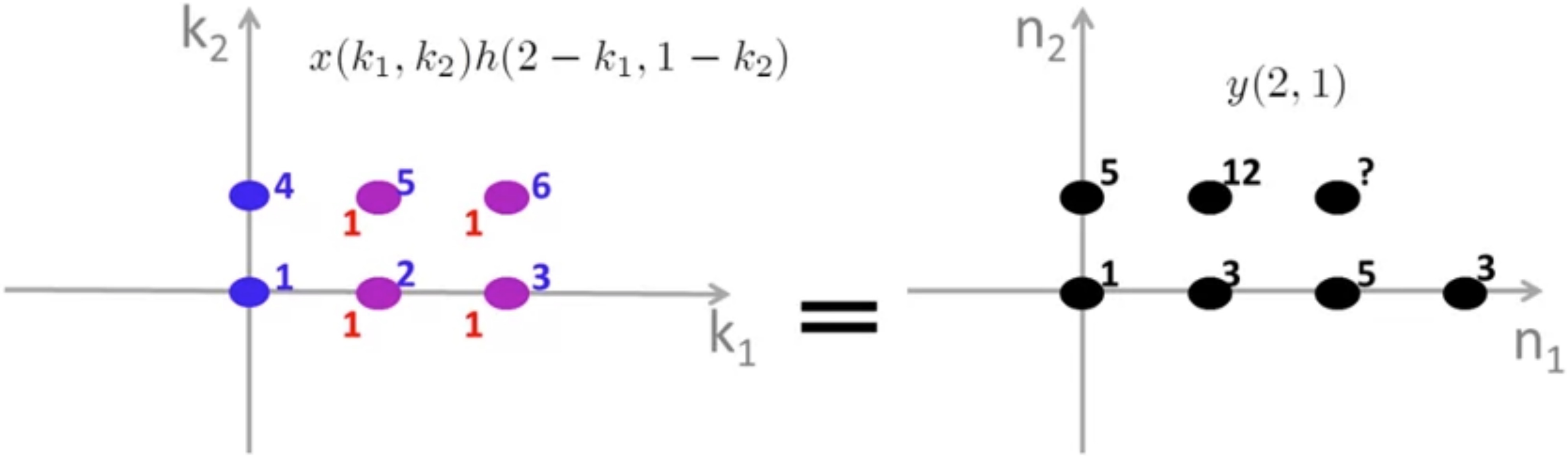
$(1 \times 1) + (1 \times 4) = 5$

2D Convolution Example



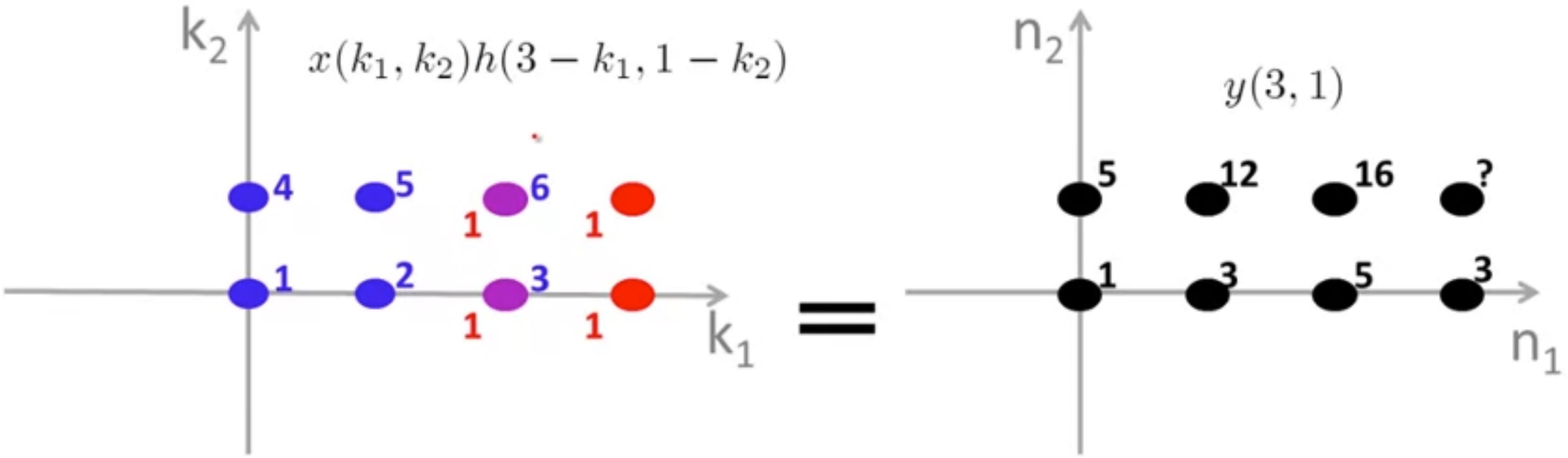
$(1\times1) + (1\times2) + (1\times4) + (1\times5) = 12$

2D Convolution Example



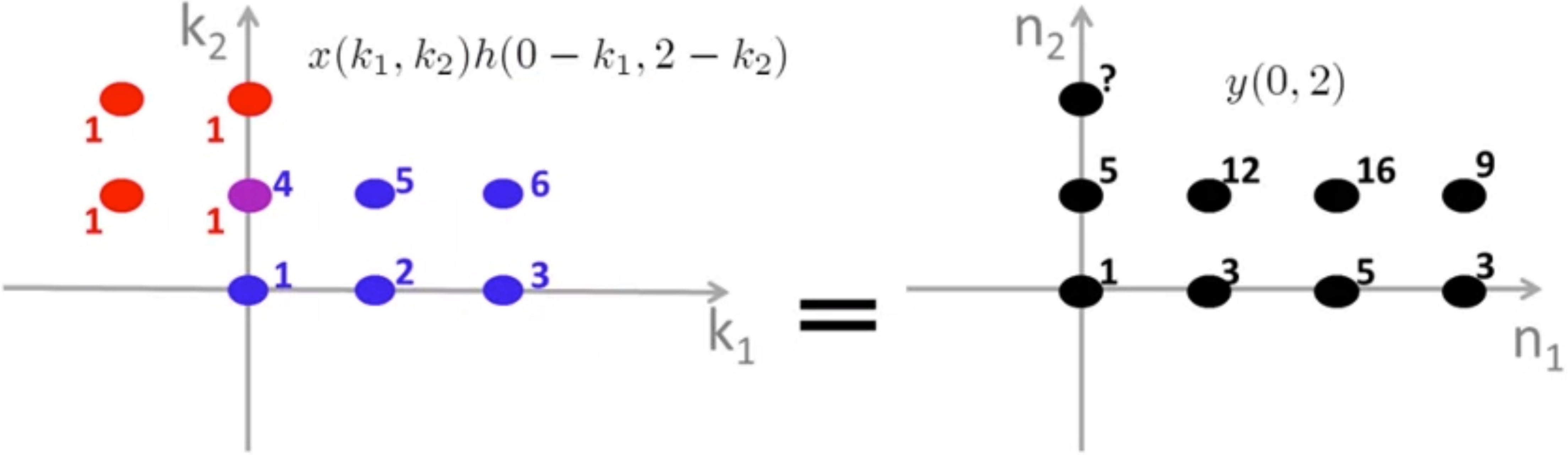
$$(1 \times 2) + (1 \times 3) + (1 \times 5) + (1 \times 6) = 16$$

2D Convolution Example



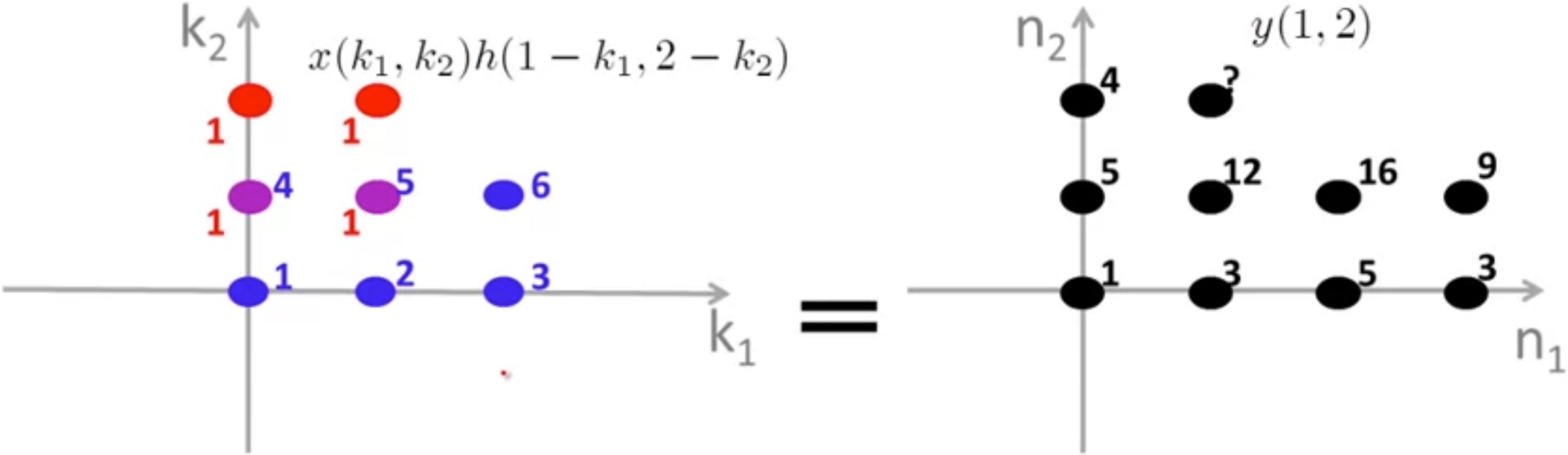
$$(\color{red}{1}\times\color{blue}{3}) + (\color{red}{1}\times\color{blue}{6}) = 9$$

2D Convolution Example



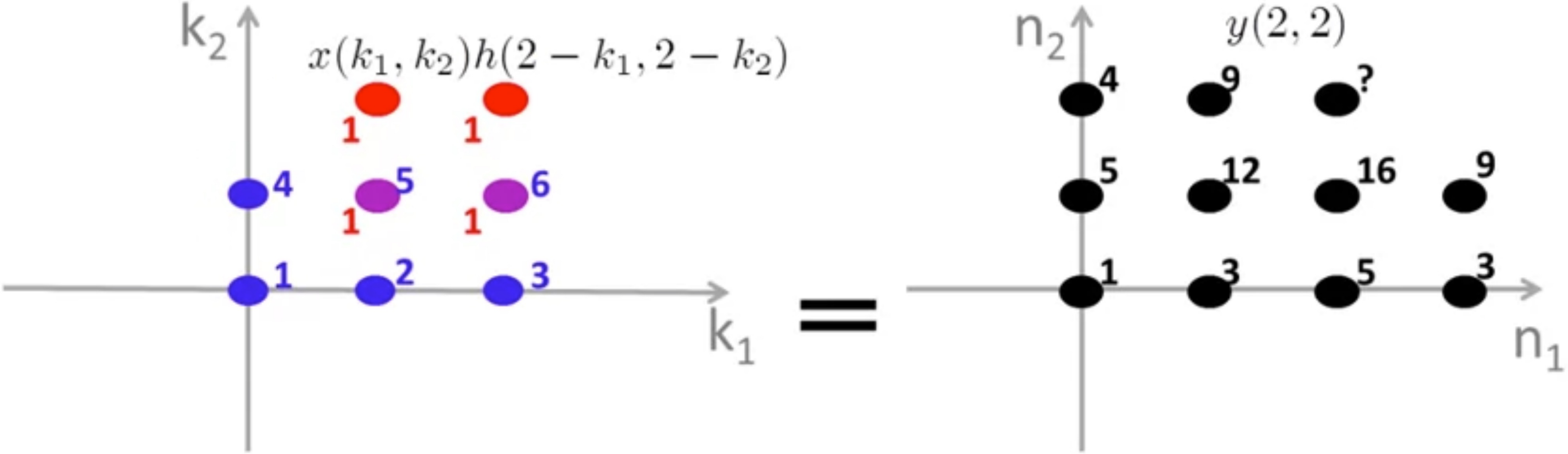
$1 \times 4 = 4$

2D Convolution Example



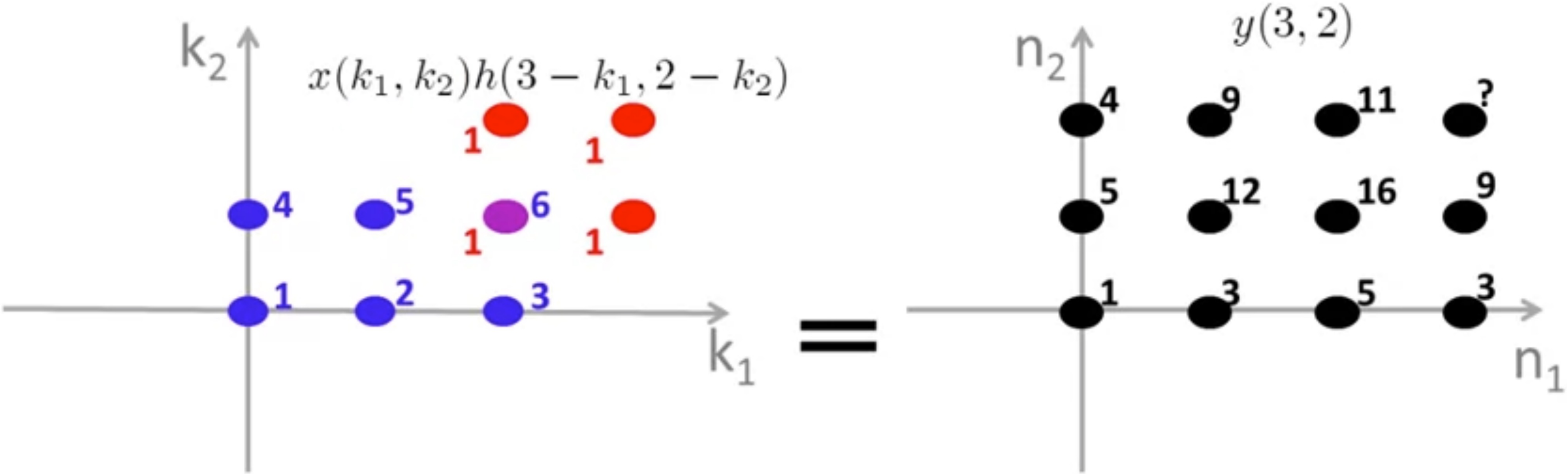
$(1 \times 4) + (1 \times 5) = 9$

2D Convolution Example



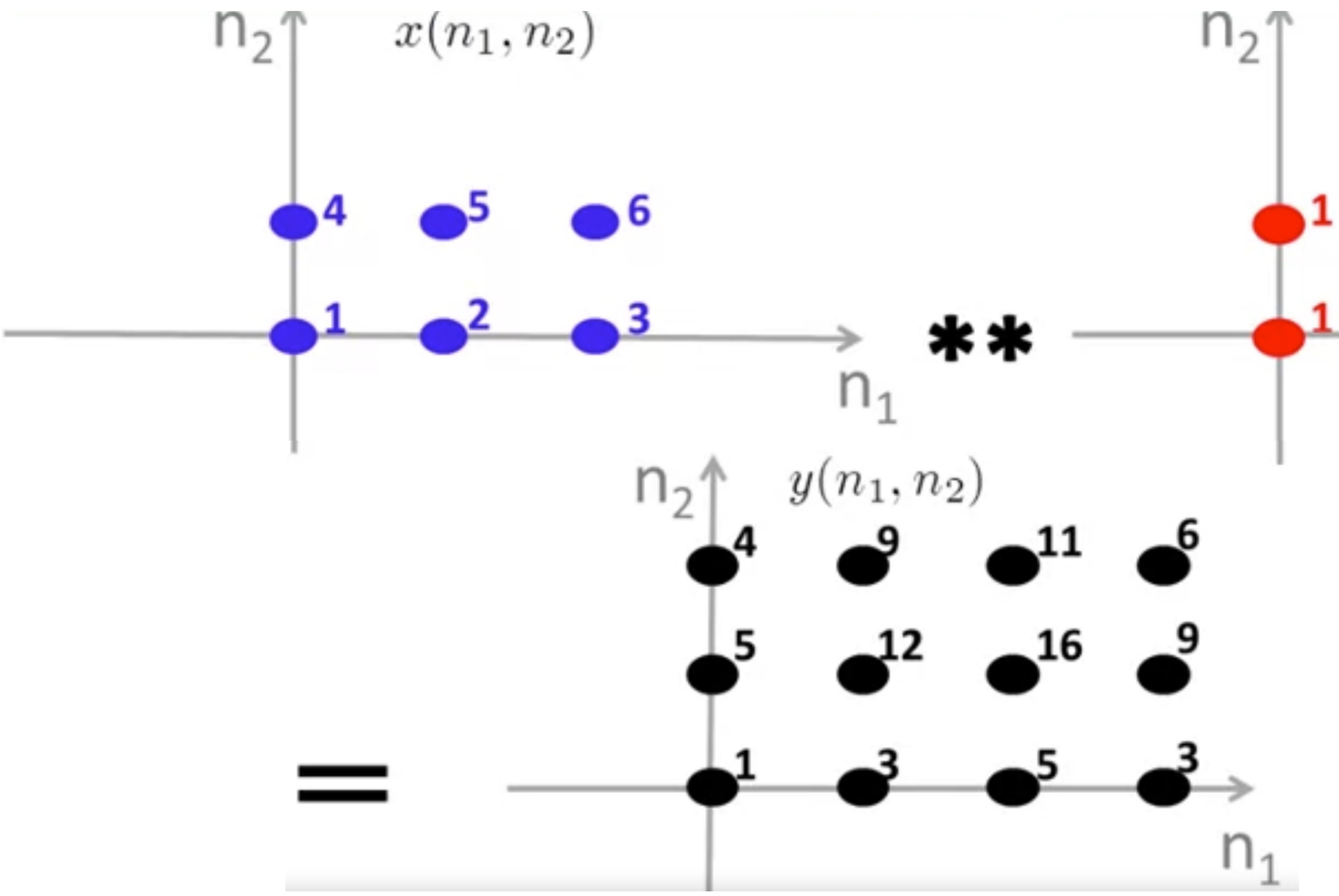
$(1 \times 5) + (1 \times 6) = 11$

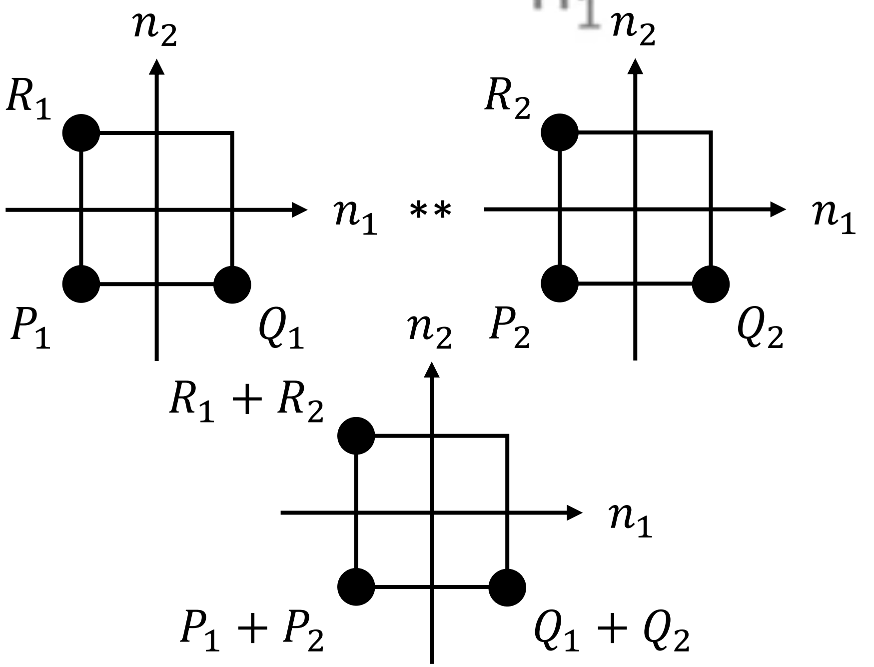
2D Convolution Example



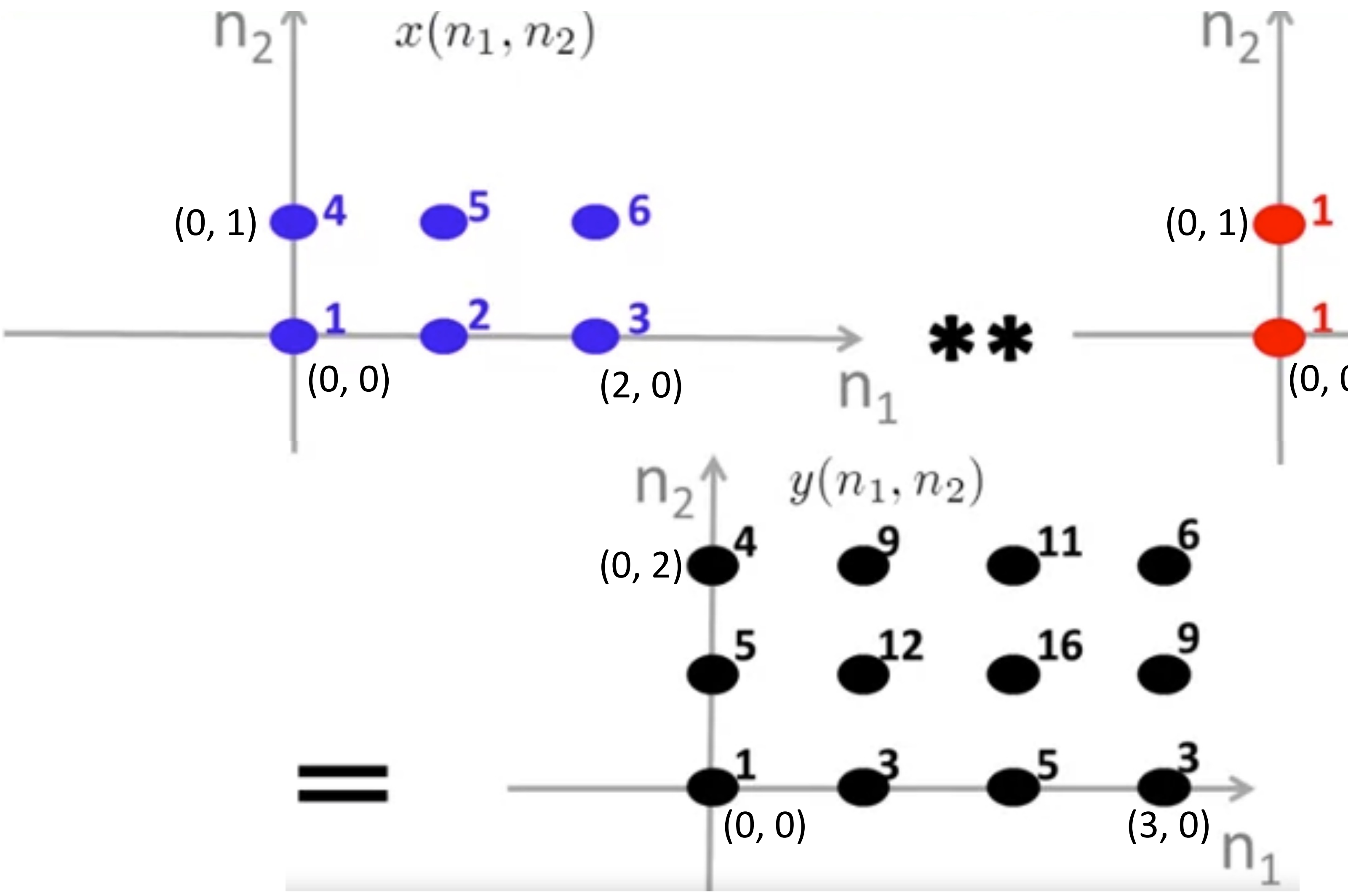
$1 \times 6 = 6$

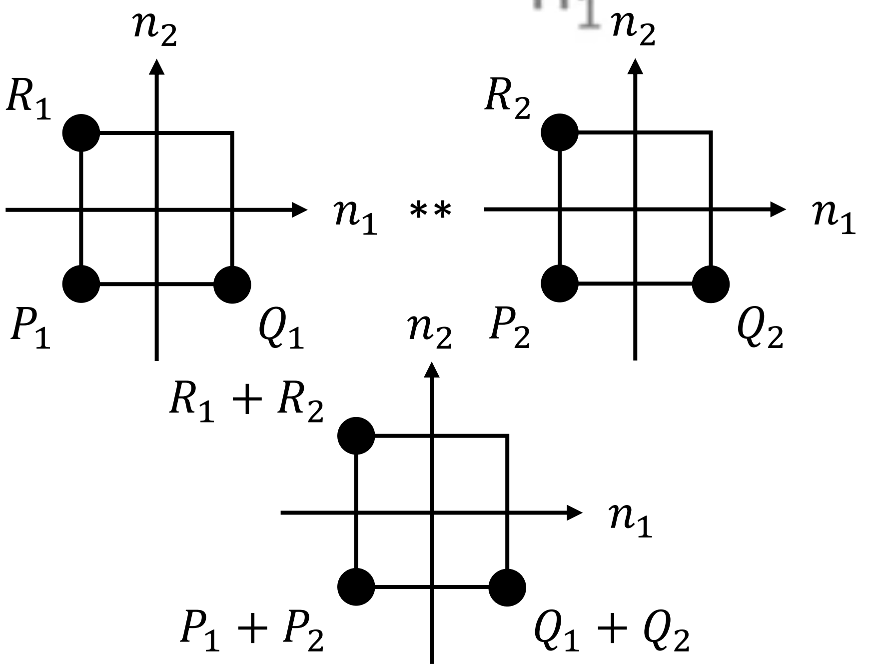
2D Convolution Example





2D Convolution Example

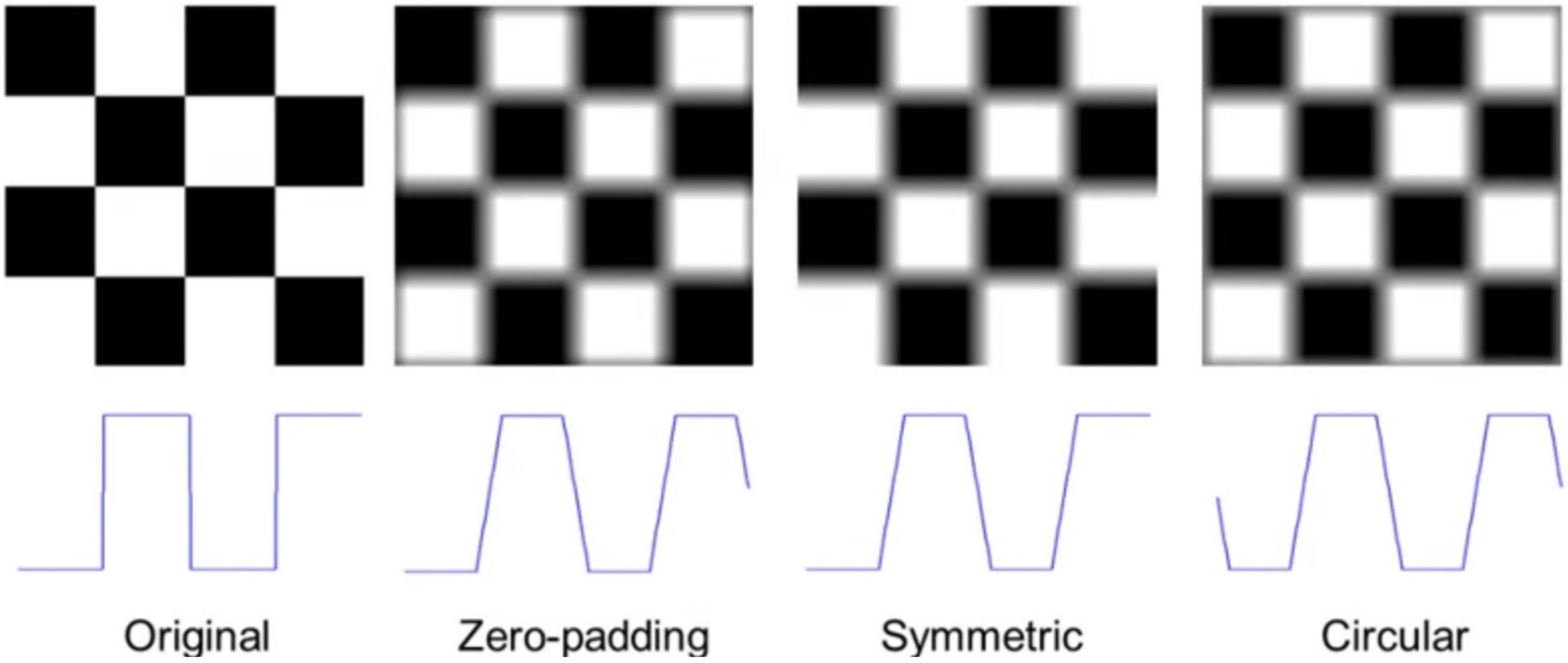




Signals and Systems

- 2D and 3D Discrete Signals
- Complex Exponential Signals
- Linear Shift-Invariant Systems
- 2D Convolution
- **Filtering in the Spatial Domain**
- Fundamentals of Color Image Processing

Boundary Effects



A 15x15 symmetric flat filter is used (amplitude of each sample = 1/225)

Spatial Filtering (LPF)

$$\begin{bmatrix} 0.075 & 0.124 & 0.075 \\ 0.124 & 0.204 & 0.124 \\ 0.075 & 0.124 & 0.075 \end{bmatrix}$$

Spatial Filtering (LPF)

Original

Flat 5x5 LPF

Spatial Filtering (HPF)

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Noise Reduction

$$\begin{bmatrix} 0.075 & 0.124 & 0.075 \\ 0.124 & 0.204 & 0.124 \\ 0.075 & 0.124 & 0.075 \end{bmatrix}$$

Noise Reduction

$$\begin{bmatrix} 0.075 & 0.124 & 0.075 \\ 0.124 & 0.204 & 0.124 \\ 0.075 & 0.124 & 0.075 \end{bmatrix}$$

Noise Reduction

Median Filtering

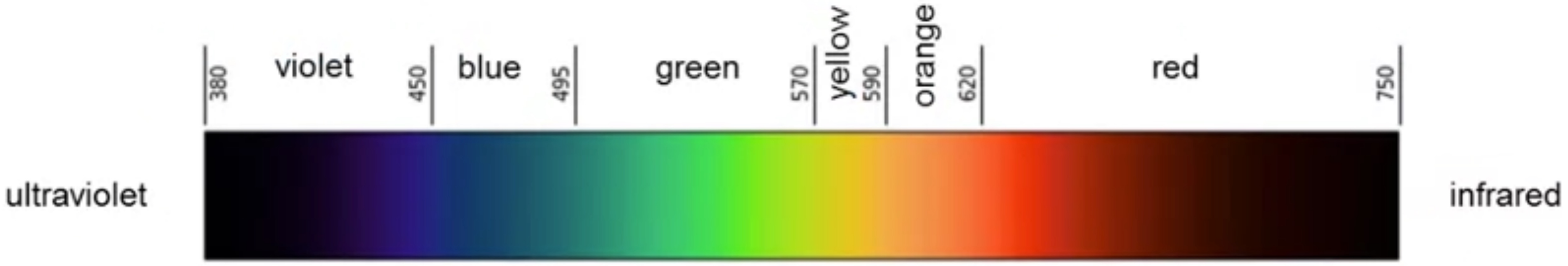
Signals and Systems

- 2D and 3D Discrete Signals
- Complex Exponential Signals
- Linear Shift-Invariant Systems
- 2D Convolution
- Filtering in the Spatial Domain
- **Fundamentals of Color Image Processing**

Importance of Color

- Color is a powerful descriptor that can be used for various tasks, e.g., segmentation, object detection, tracking, and identification
- Humans can distinguish thousands of color shades and intensities, as compared to about only two dozen shades of gray

Color Spectrum



Light Characterization

- Achromatic light
  - Intensity
- Chromatic light
  - Radiance: total amount of energy that flows from the light sources (Watts)
  - Luminance: perceived amount of energy (lumens)
  - Brightness: hard to measure; embodies the achromatic notion of intensity; used to describe color sensation

Light Absorption by the Human Eye

Cones

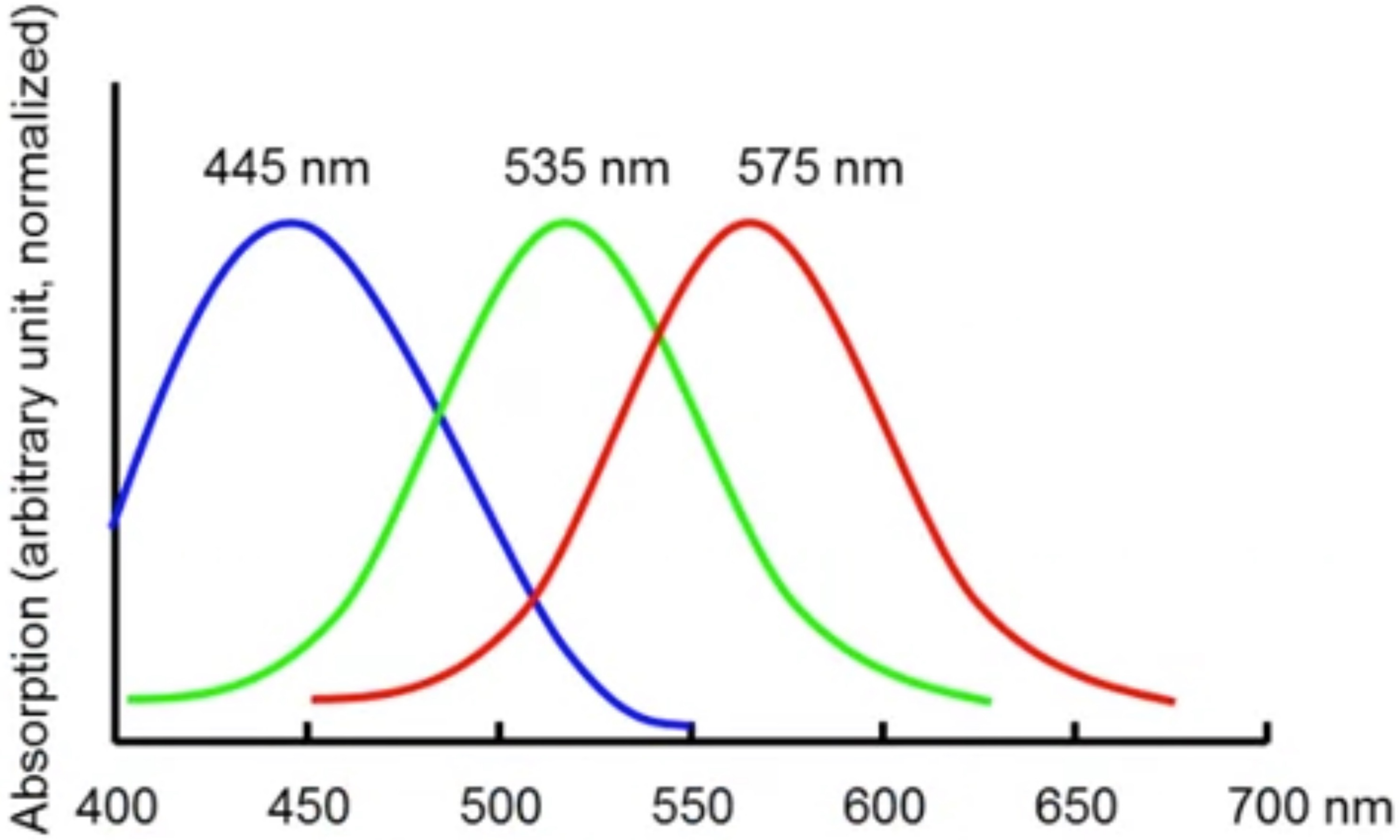
6-7 millions

65% -> R

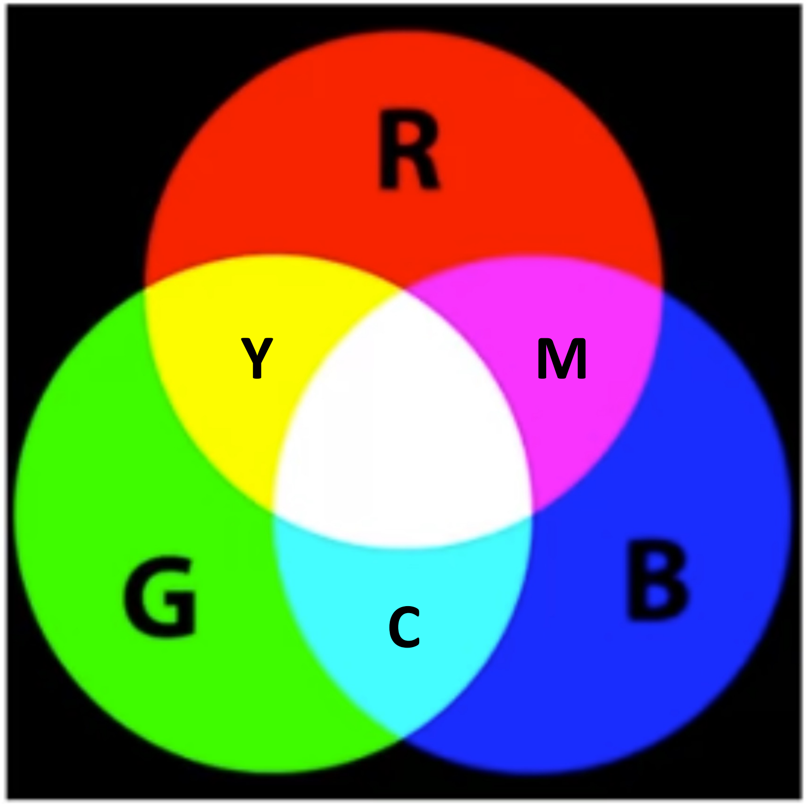
33% -> G

2% -> B

Primary colors



Primary Colors of Light

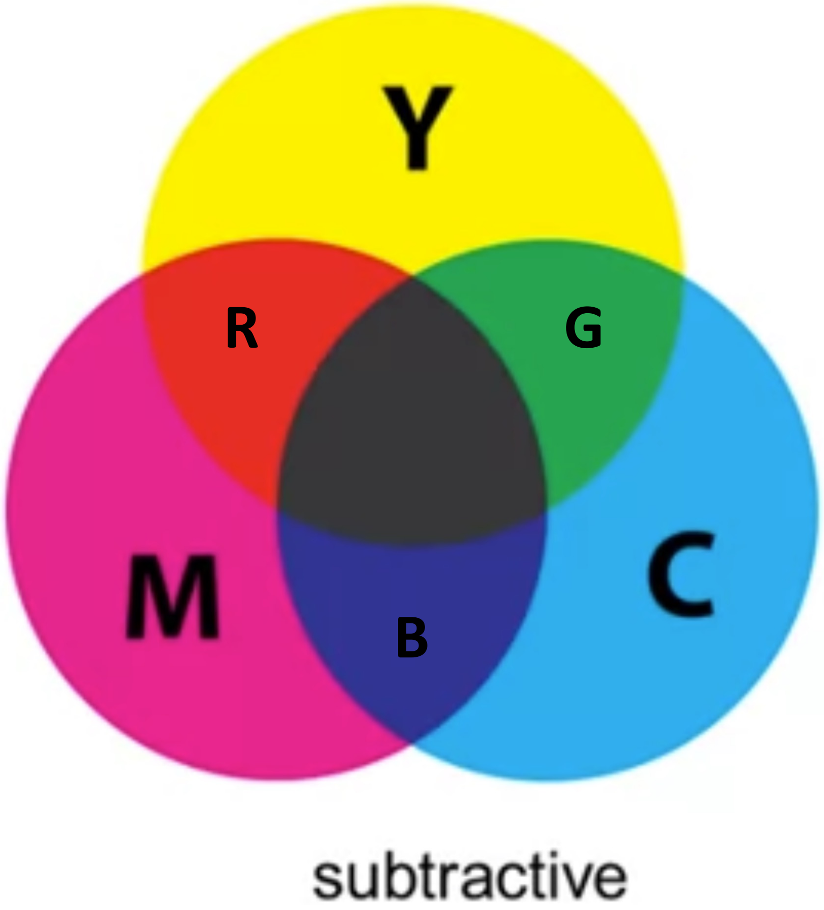


additive

Primary Colors of Pigments

$C = 1 - R$

$[0, 1]$



Color Distinguishing Characteristics

- Brightness
  - Hard to measure; embodies the achromatic notion of intensity; used to describe color sensation
- Hue
  - Indicates the dominant wavelength in a mixture of light waves
- Saturation
  - Refer to the relative purity or the amount of white light mixed with its hue

Trichromatic Coefficients

X, Y, Z tristimulus values

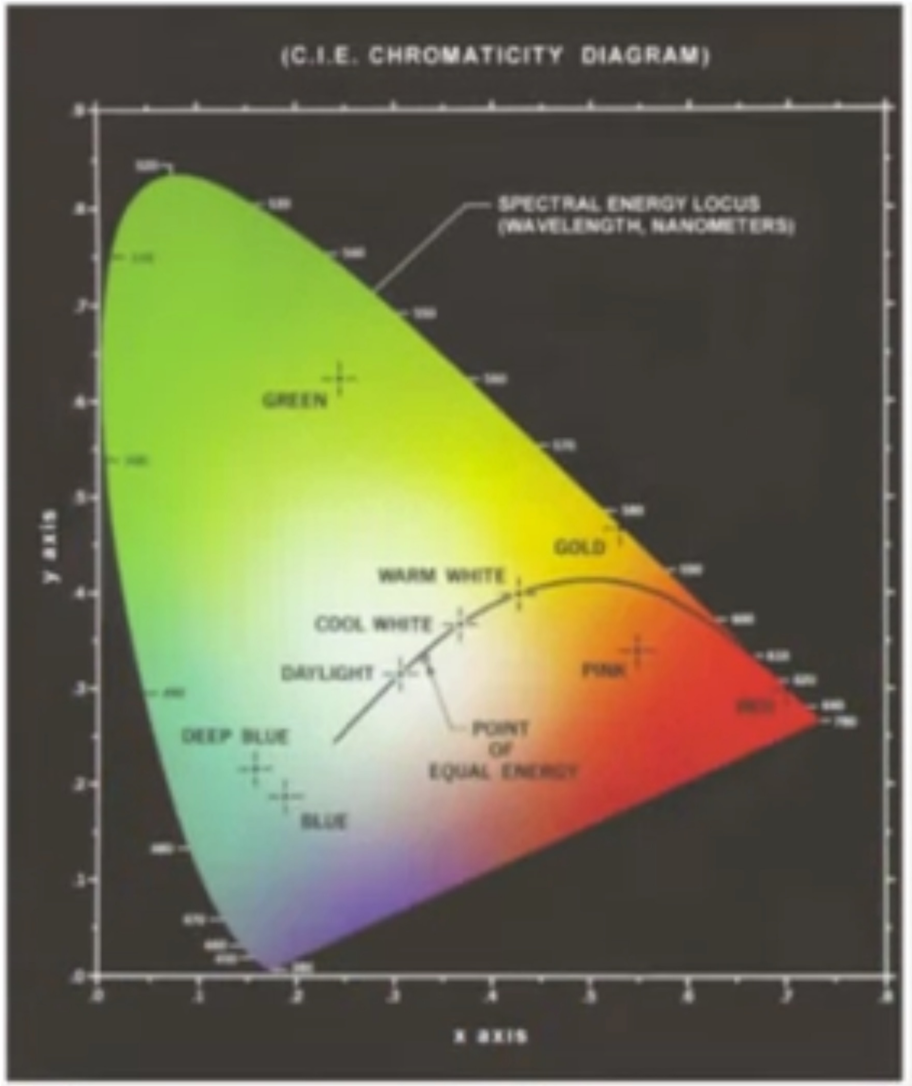
x, y, z trichromatic coefficients

$$x = \frac{X}{X + Y + Z}$$

$$y = \frac{Y}{X + Y + Z}$$

$$z = \frac{Z}{X + Y + Z}$$

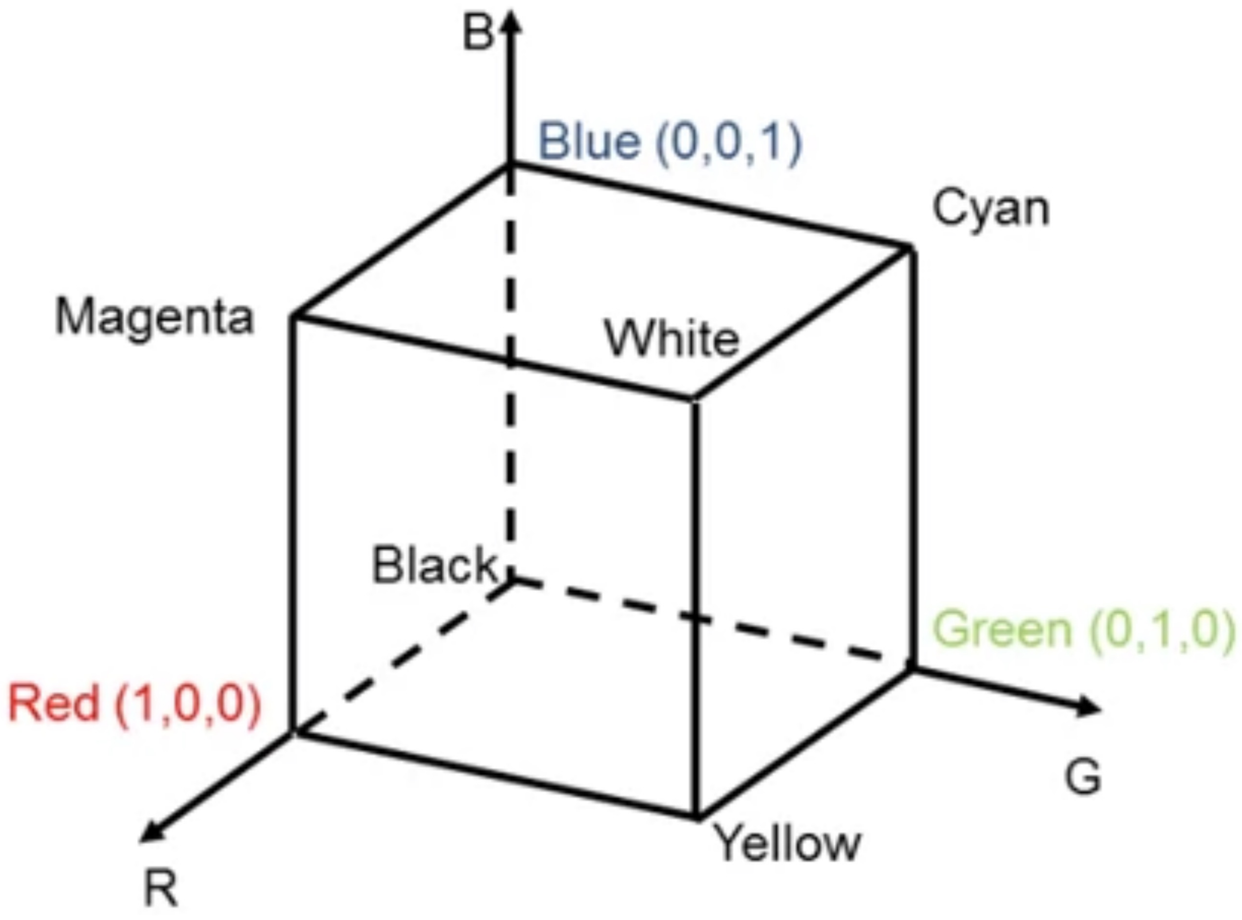
$$x + y + z = 1$$



Color Models

- They specify a coordinate system and a subspace within that system where each color is represented by a single point
  - RGB
  - CMY and CMYK
  - HSI
  - YUV/YCbCr

RGB Model



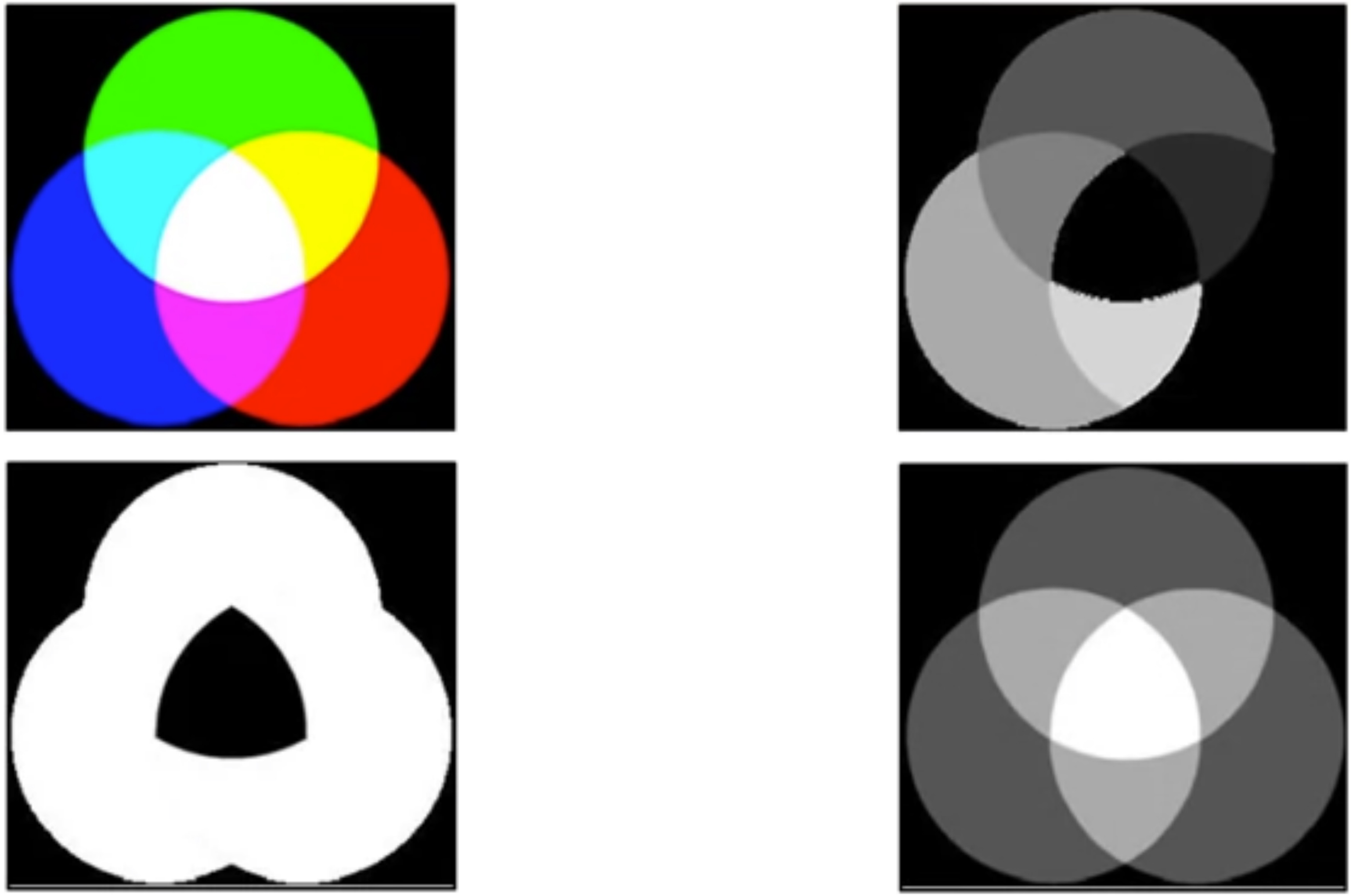
RGB 24-bit color cube

CMY Model

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

- Equal amounts of C, M, Y should produce black, but they produce instead in practice a muddy-looking black
- Therefore, a fourth color “black” K is added to produce the CMYK color model

HSI Model



H

S

I

YUV/YCbCr Model

- YUV is used by the PAL, NTSC, and SECAM composite color video standards
- YIQ is derived from the YUV color space
- YCbCr (ITU-R BT.601) is a scaled and offset version of YUV

$$\begin{cases} Y = 0.299R + 0.587G + 0.114B \\ Cb = 128 - 0.168736R - 0.331264G + 0.5B \\ Cr = 128 + 0.5R - 0.418688G - 0.081312B \end{cases}$$

- JPEG conversion formulae (there are other conversion formulae, for analog color, digital color, etc)
- This takes input RGB values from 0 to 255, and output Y, Cb, Cr in the range 0 to 255.

Example of Color Spaces

Original

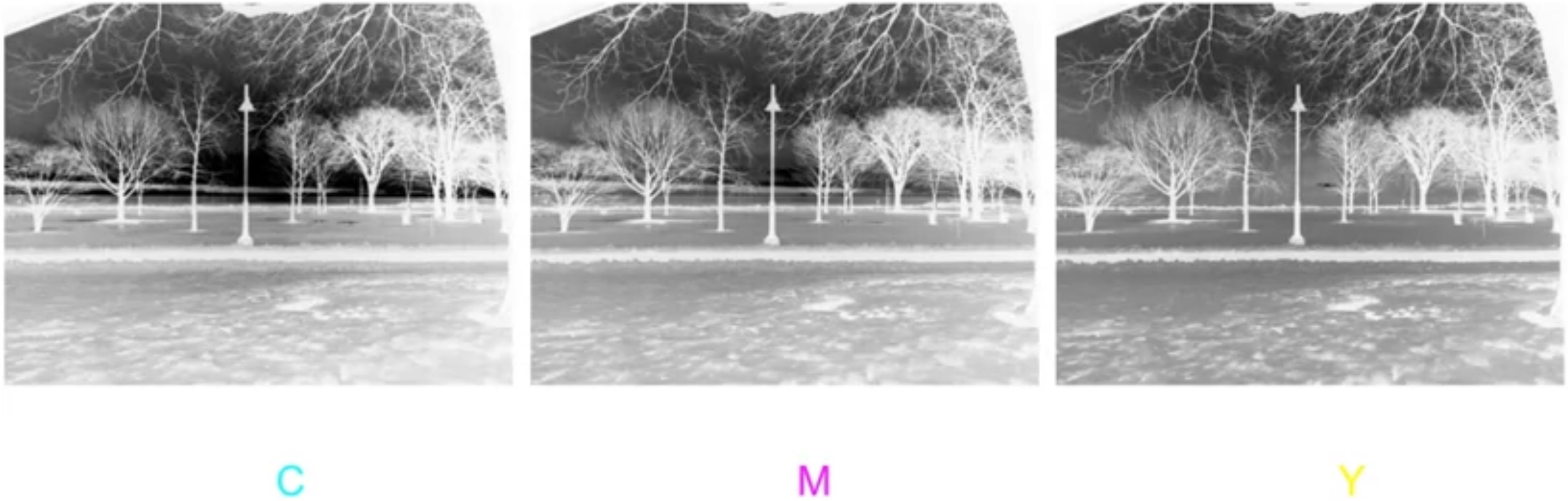
Example of Color Spaces

R

G

B

Example of Color Spaces



Example of Color Spaces

H S I

Example of Color Spaces

Y Cb Cr

Color Image and Video Processing

- Choice of appropriate color space
- Independent channel processing
- Multi-channel processing

RGB vs HIS Filtering

RGB filtered separately

Signals and Systems

- 2D and 3D Discrete Signals
- Complex Exponential Signals
- Linear Shift-Invariant Systems
- 2D Convolution
- Filtering in the Spatial Domain
- Fundamentals of Color Image Processing

Homework #1 – Color Transform

- Please represent “lena.png” in terms of RGB, YUV, and YCbCr.
- In any language you are comfortable with (C/C++/Python/MATLAB).
- Output 8 grayscale images representing R, G, B, Y, U, V, Cb, Cr, respectively.
- **Do not** use any ready-made functions to transform the color.
- You are allowed to use image reading/writing APIs.
- Deadline: 2025/09/29 1:19 PM
- Upload to E3 with required files :
  - **VC_HW1_[student_id].pdf**: Report PDF
  - **VC_HW1_[student_id].zip**: Zipped source code (C/C++/Python/MATLAB)

Homework #1 – Color Transform

lena.png