Video Compression
視訊壓縮

Yu-Lun (Alex) Liu

劉育綸

with slides by Wen-Hsiao Peng,

Shao-Yi Chien,

Hsueh-Ming Hang,

and Aggelos K. Katsaggelos

Week	Date	Topic	Assignments
1	2025-09-01		
2	2025-09-08	Introduction to Image and Video Processing	
3	2025-09-16	Signals and Systems	#1 – Color Transform, due: 2025-09-29 1:19pm
4	2025-09-22	Fourier Transform and Sampling	
5	2025-09-29	教師節補假	
6	2025-10-06	中秋節	
7	2025-10-13	Fourier Transform and Sampling	#2 – 2D-DCT, due: 2025-10-27 1:59pm
8	2025-10-20	Motion Estimation	Final project assigned (group together in fours)
9	2025-10-27	Lossless Compression	#3 – MEMC, due: 2025-11-10 1:59pm
10	2025-11-03	Image Compression	
11	2025-11-10	Video Compression	#4 – Entropy coding, due: 2025-11-24 1:59pm
12	2025-11-17	Learning-based Image/Video Compression	
13	2025-11-24	Paper Presentation	
14	2025-12-01	Guest Lecturer – MEDIATEK Meta NETFLIX	
15	2025-12-08	Guest Lecturer – MEDIATEK Meta NETFLIX	
16	2025-12-15	Final Project Presentation	

Fourier Transform and Sampling

with slides by Wen-Hsiao Peng, Shao-Yi Chien, Hsueh-Ming Hang, and Aggelos K. Katsaggelos

Fourier Transform and Sampling

- 2D Fourier Transform
- Sampling
- Discrete Fourier Transform
- Filtering in the Frequency Domain
- Change of Sampling Rate
- The Discrete Cosine Transform
- The Two-Dimensional Discrete Cosine Transform

Fourier Transform and Sampling

- **2D Fourier Transform**
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Eigenfunctions of LSI Systems

$$x(n_1,n_2) = e^{j(\omega_1' n_1+\omega_2' n_2)} \longrightarrow \boxed{h(n_1,n_2)} \longrightarrow y(n_1,n_2) = ?$$

$$y(n_1,n_2) = x(n_1,n_2) ** h(n_1,n_2)$$

$$= \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} e^{j\omega_1'(n_1-k_1)} e^{j\omega_2'(n_2-k_2)} h(k_1,k_2)$$

$$= e^{j\omega_1' n_1} e^{j\omega_2' n_2} \underbrace{\sum_{k_1} \sum_{k_2} h(k_1,k_2) e^{-j\omega_1' k_1} e^{-j\omega_2' k_2}}_{H(\omega_1',\omega_2') \,\triangleq\, \text{frequency response}}$$

2D Fourier Transform

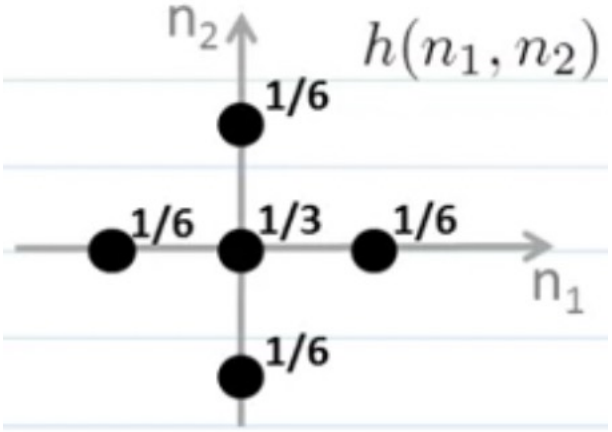
$$X(\omega_1,\omega_2) = \sum_{n_1=-\infty}^{\infty}\sum_{n_2=-\infty}^{\infty} x(n_1,n_2)e^{-j\omega_1 n_1}e^{-j\omega_2 n_2}$$

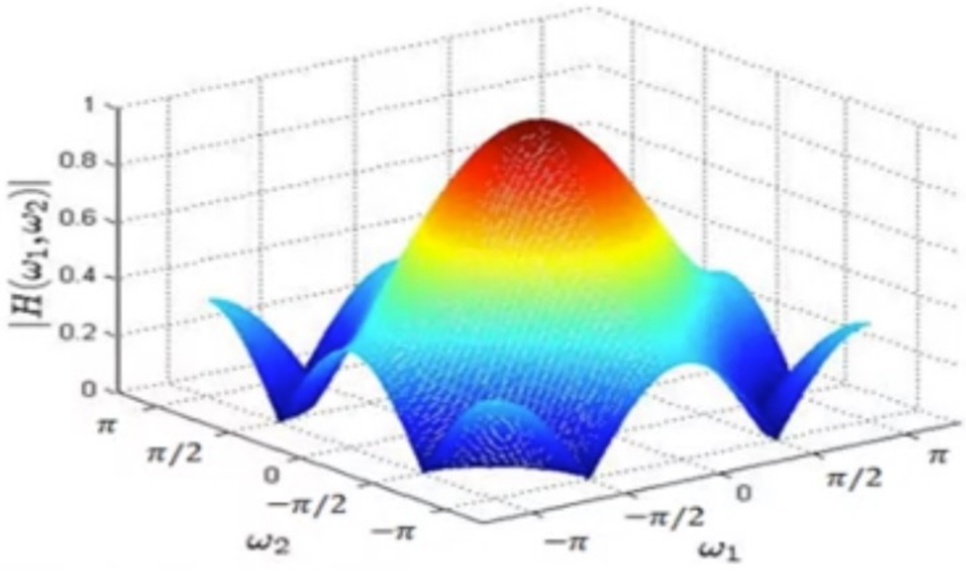
$$x(n_1,n_2) = \frac{1}{4\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} X(\omega_1,\omega_2)\, e^{j\omega_1 n_1}e^{j\omega_2 n_2}d\omega_1 d\omega_2$$

Properties

- $X(\omega_1,\omega_2) = X(\omega_1+2\pi,\omega_2+2\pi)$
- $x(n_1-m_1,n_2-m_2) \leftrightarrow e^{-j\omega_1 m_1 - j\omega_2 m_2}X(\omega_1,\omega_2)$
- $x(n_1,n_2)e^{j\theta_1 n_1 + j\theta_2 n_2} \leftrightarrow X(\omega_1-\theta_1,\omega_2-\theta_2)$
- for $x(n_1,n_2)$ real: $|X(\omega_1,\omega_2)| = |X(-\omega_1,-\omega_2)|$ and $\arg X(\omega_1,\omega_2) = -\arg X(-\omega_1,-\omega_2)$
- $\sum_{n_1}\sum_{n_2}|x(n_1,n_2)|^2 = \frac{1}{4\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}|X(\omega_1,\omega_2)|^2 d\omega_1 d\omega_2$

Example





$$H(\omega_1, \omega_2) = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} h(n_1, n_2) e^{-j\omega_1 n_1} e^{-j\omega_2 n_2}$$

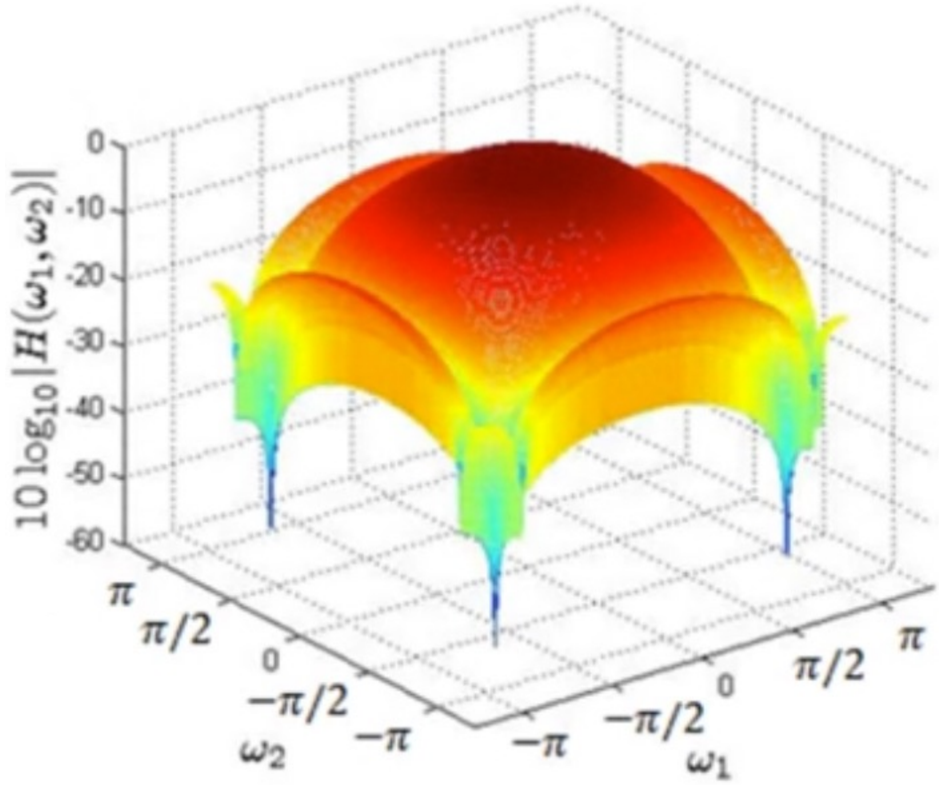
$$= h(0,0) + h(-1,0)e^{j\omega_1} + h(1,0)e^{-j\omega_1} + h(0,-1)e^{j\omega_2} + h(0,1)e^{-j\omega_2}$$

$$= \frac{1}{3} + \frac{1}{6}e^{j\omega_1} + \frac{1}{6}e^{-j\omega_1} + \frac{1}{6}e^{j\omega_2} + \frac{1}{6}e^{-j\omega_2}$$

$$= \frac{1}{3} + \frac{1}{6} \cdot 2\cos\omega_1 + \frac{1}{6} \cdot 2\cos\omega_2$$

$$= \frac{1}{3}(1 + \cos\omega_1 + \cos\omega_2)$$

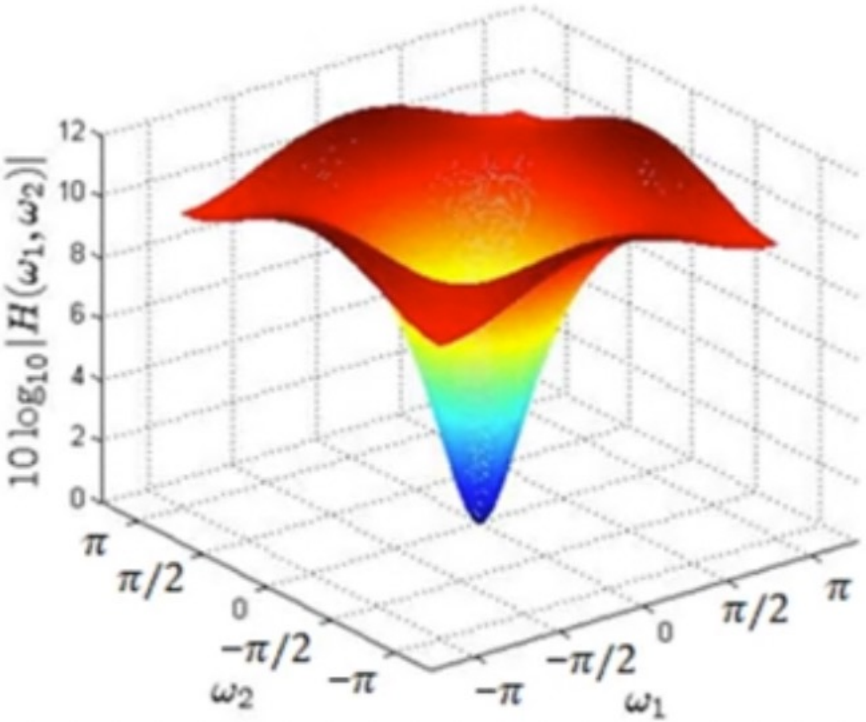
Example



$$h(n_1, n_2) = \begin{bmatrix} 0.075 & 0.124 & 0.075 \\ 0.124 & 0.204 & 0.124 \\ 0.075 & 0.124 & 0.075 \end{bmatrix}$$

$H(\omega_1, \omega_2) = 0.204 + 0.124 \cdot 2 \cdot \cos\omega_1 + 0.124 \cdot 2 \cdot \cos\omega_2 + 0.075 \cdot 2 \cdot \cos(\omega_1 + \omega_2) + 0.075 \cdot 2 \cdot \cos(\omega_1 - \omega_2)$

Example



$$h(n_1,n_2)=\begin{bmatrix}-1 & -1 & -1\\ -1 & 9 & -1\\ -1 & -1 & -1\end{bmatrix}$$

$$H(\omega_1,\omega_2)$$
$$=9-2\cdot\cos\omega_1-2\cdot\cos\omega_2-2\cdot\cos(\omega_1+\omega_2)\ \ -2\cdot\cos(\omega_1-\omega_2)$$

The Convolution Theorem

$$x(n_1,n_2) \longrightarrow \boxed{h(n_1,n_2)} \longrightarrow y(n_1,n_2) =$$

$$y(n_1,n_2) = T[x(n_1,n_2)]$$

$$= T\left[\frac{1}{4\pi^2}\iint_{-\pi}^{\pi} X(\omega_1,\omega_2)e^{j\omega_1 n_1}e^{j\omega_2 n_2}d\omega_1 d\omega_2\right]$$

$$= \frac{1}{4\pi^2}\iint_{-\pi}^{\pi} X(\omega_1,\omega_2)T[e^{j\omega_1 n_1}e^{j\omega_2 n_2}]\,d\omega_1 d\omega_2$$

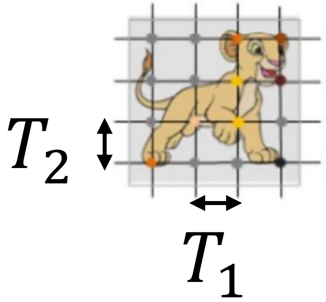
$$= \frac{1}{4\pi^2}\iint_{-\pi}^{\pi} X(\omega_1,\omega_2)H(\omega_1,\omega_2)\,e^{j\omega_1 n_1}e^{j\omega_2 n_2}d\omega_1 d\omega_2$$

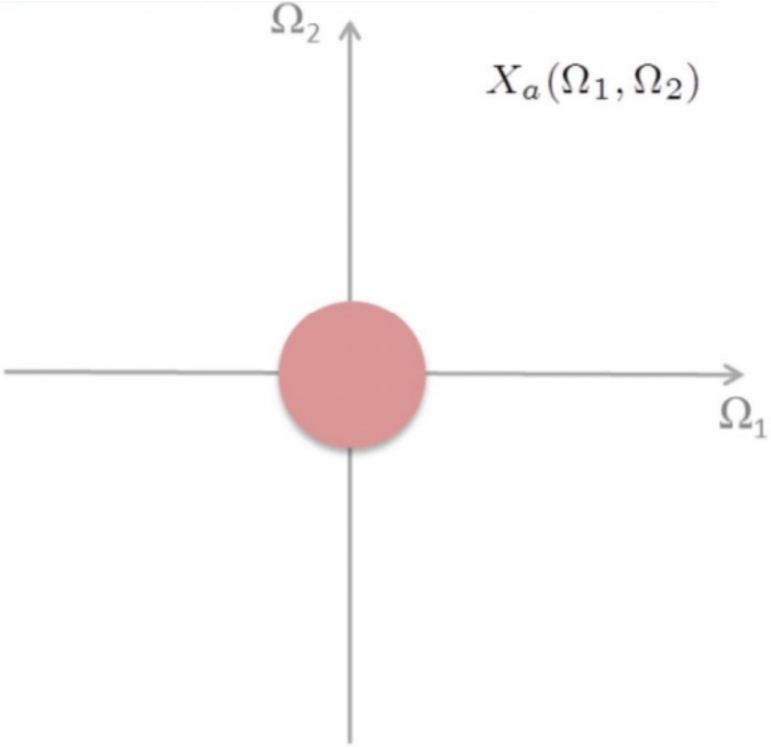
$$\Rightarrow Y(\omega_1,\omega_2) = X(\omega_1,\omega_2)\cdot H(\omega_1,\omega_2)$$

Fourier Transform and Sampling

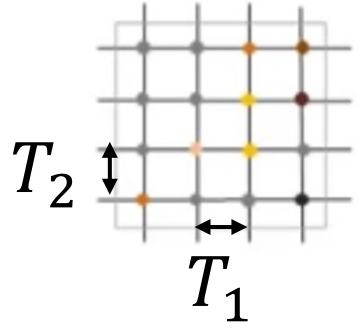
- 2D Fourier Transform
- **Sampling**
- Discrete Fourier Transform
- Filtering in the Frequency Domain
- Change of Sampling Rate
- The Discrete Cosine Transform
- The Two-Dimensional Discrete Cosine Transform

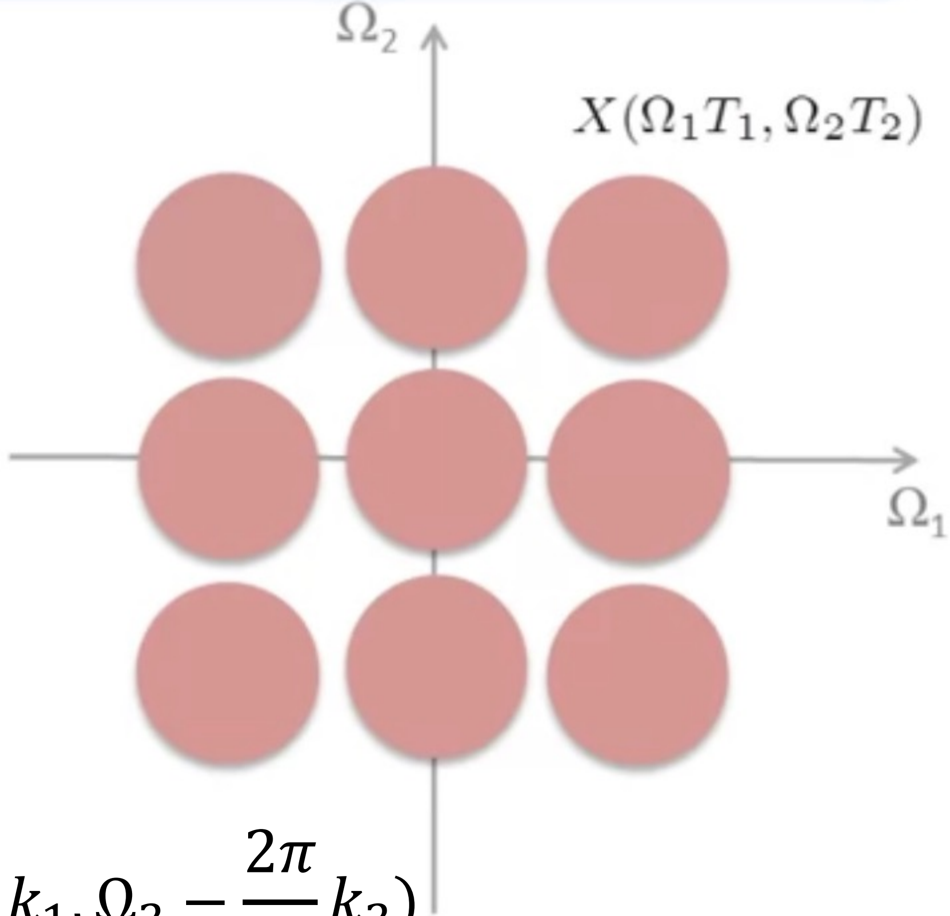
2D Sampling





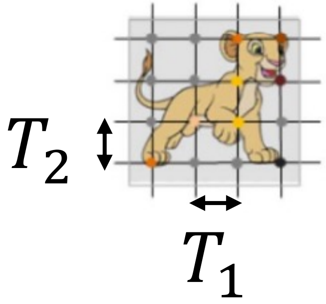
2D Sampling

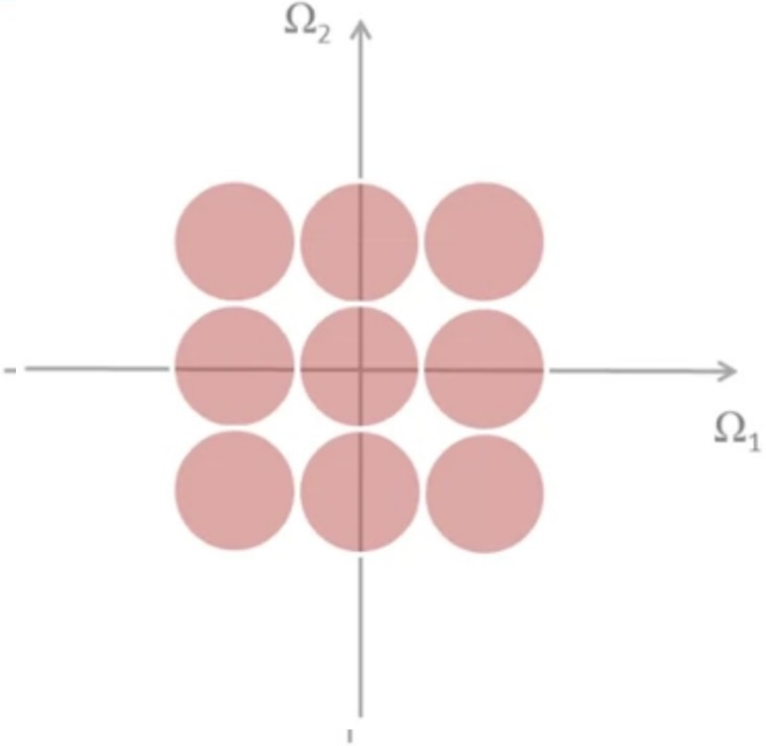




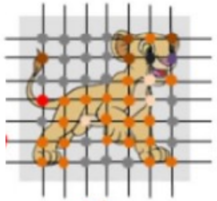
$$X(\Omega_1 T_1, \Omega_2 T_2) = \frac{1}{T_1 T_2} \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} X_a\left(\Omega_1 - \frac{2\pi}{T_1} k_1, \Omega_2 - \frac{2\pi}{T_2} k_2\right)$$

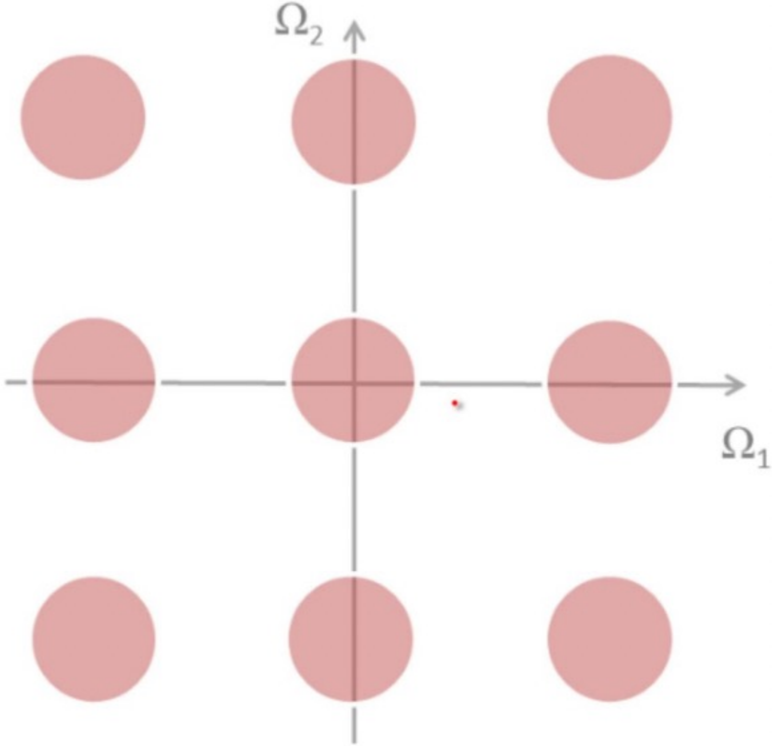
Critically Sampled



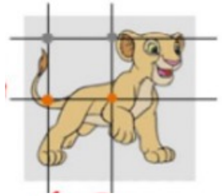


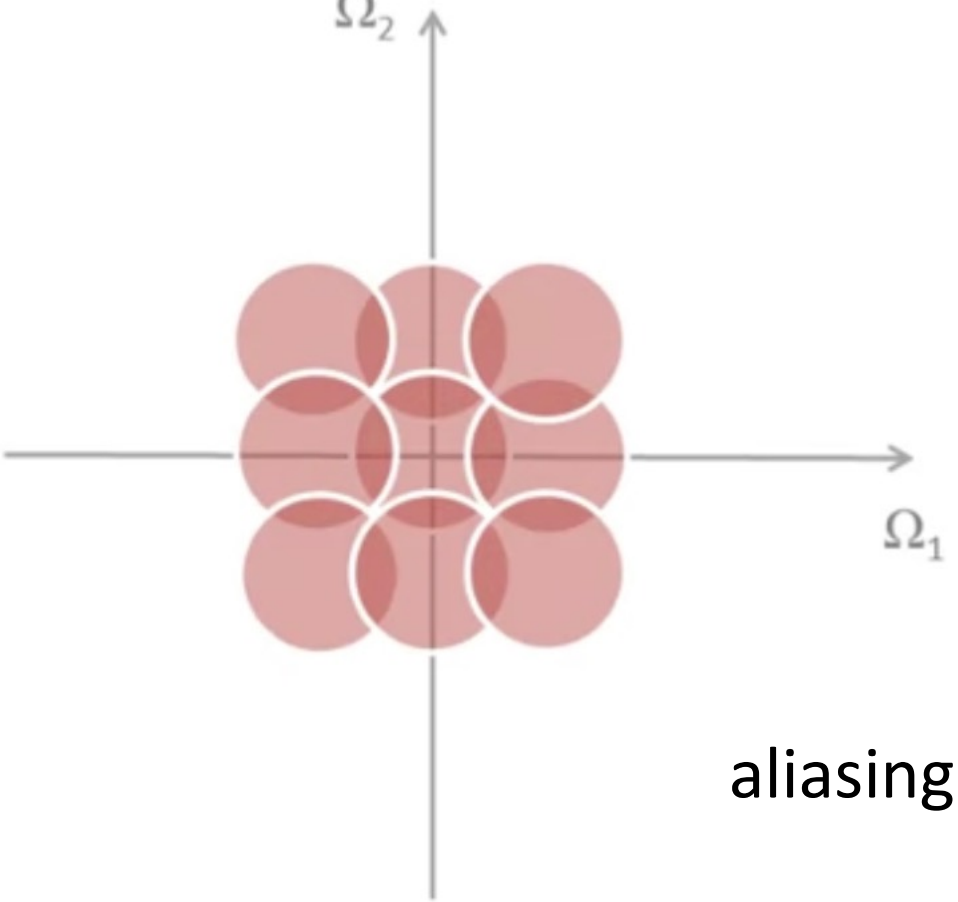
Over-Sampled





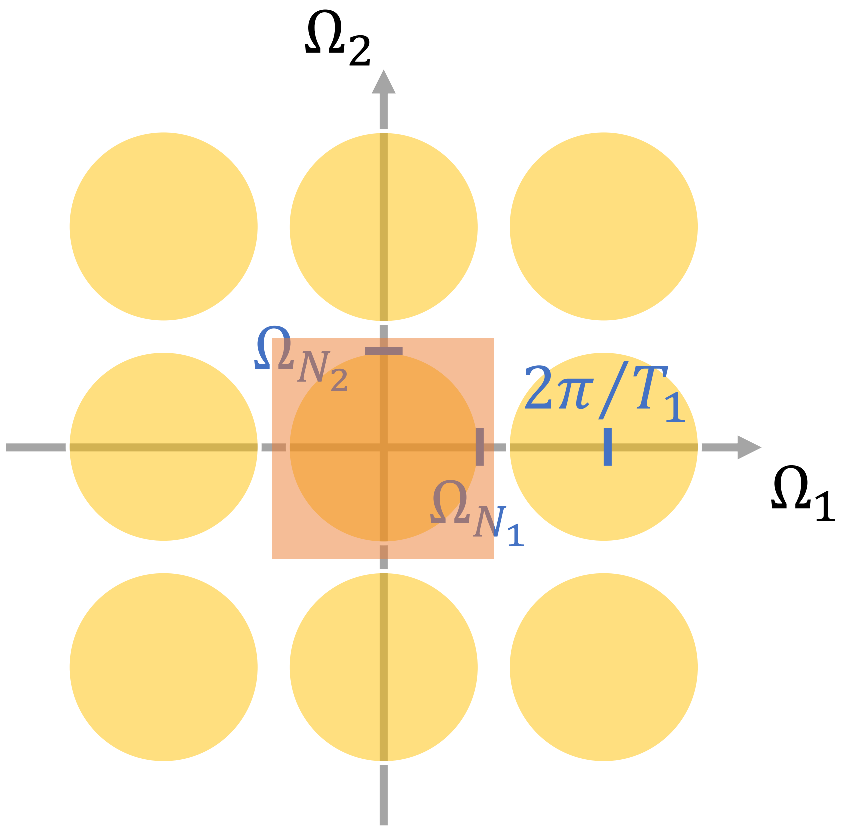
Under-Sampled





aliasing

2D Nyquist Theorem

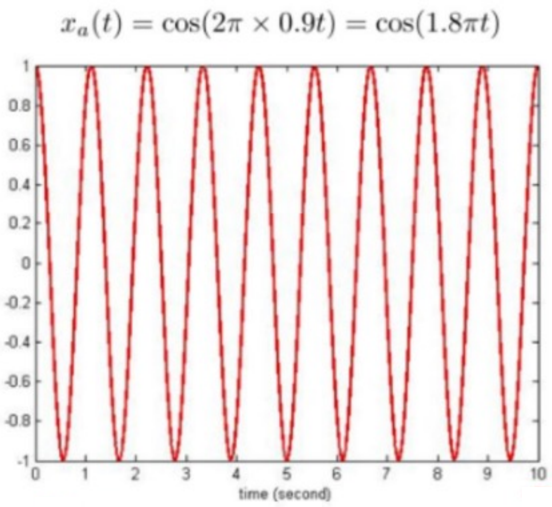


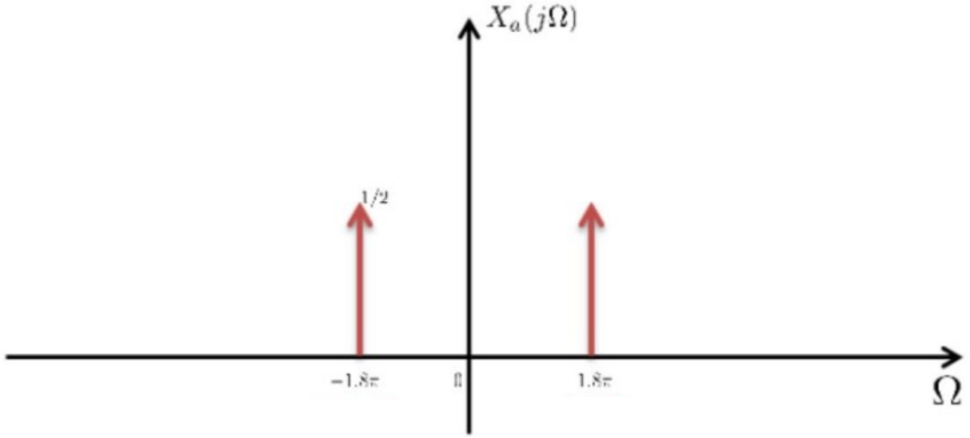
$$\frac{2\pi}{T_1} - \Omega_{N_1} \geq \Omega_{N_1} \Rightarrow \frac{2\pi}{T_1} \geq 2\Omega_{N_1}$$

$$\frac{2\pi}{T_2} - \Omega_{N_2} \geq \Omega_{N_2} \Rightarrow \frac{2\pi}{T_2} \geq 2\Omega_{N_2}$$

$$F(\Omega_1, \Omega_2) = \begin{cases} T_1 T_2, & \begin{array}{r} |\Omega_1| < \pi/T_1 \\ |\Omega_2| < \pi/T_2 \end{array} \\ 0, & \text{otherwise} \end{cases}$$

Analog Signal





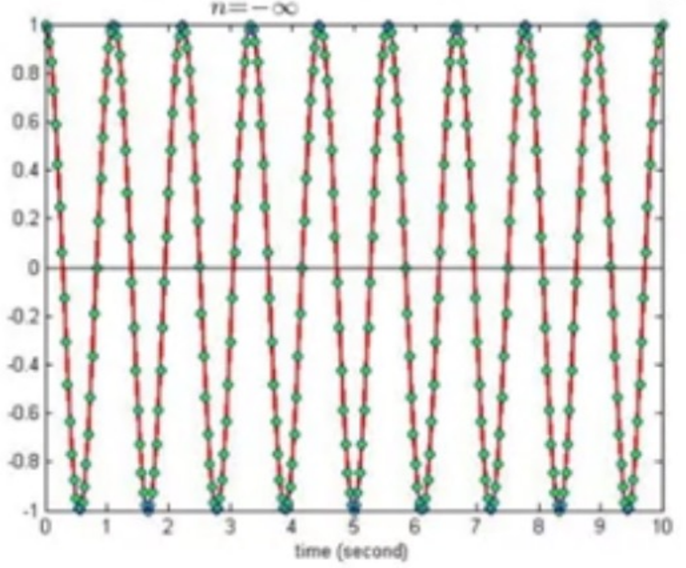
$$f = 0.9 \text{ Hz}$$
$$\Omega = 2\pi f = 1.8\pi \text{ rads/sec}$$

Nyquist freq: $\begin{cases} 1.8\text{Hz} \\ 2\pi \cdot 1.8 \dfrac{\text{rads}}{\text{sec}} \end{cases}$

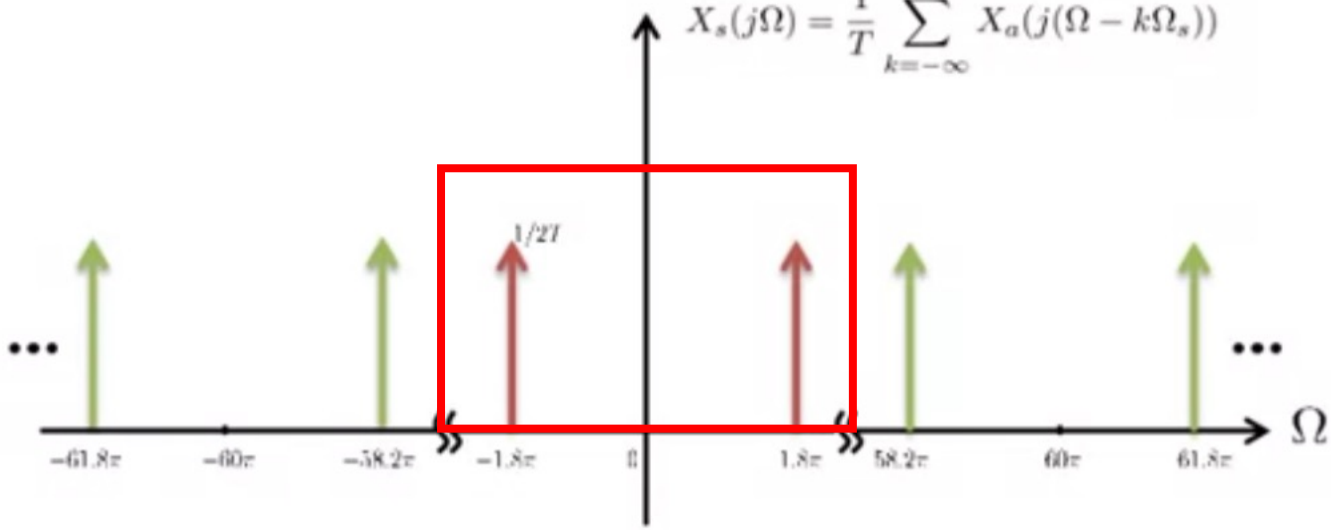
Over-Sampling

$$\Omega_N = 1.8 \times 2\pi = 3.6\pi \text{ rads/sec}$$

$$x_s = \sum_{n=-\infty}^{\infty} x_a(nT)\delta(t - nT) \quad T = \frac{1}{30}\text{s}, \Omega_s = \frac{2\pi}{T} = 60\pi \text{ rads/s}$$

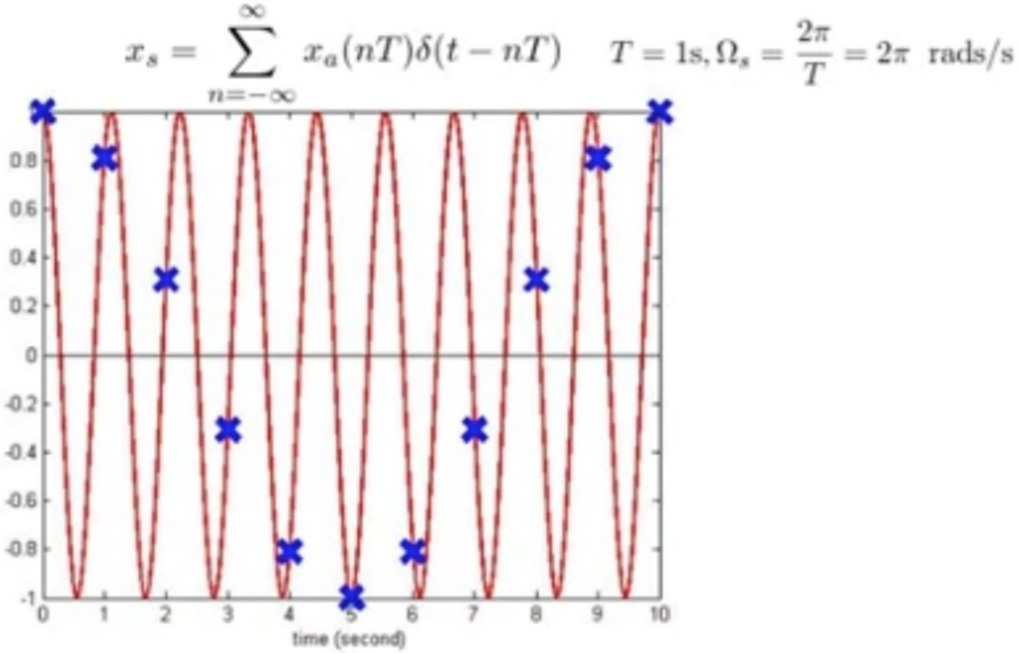


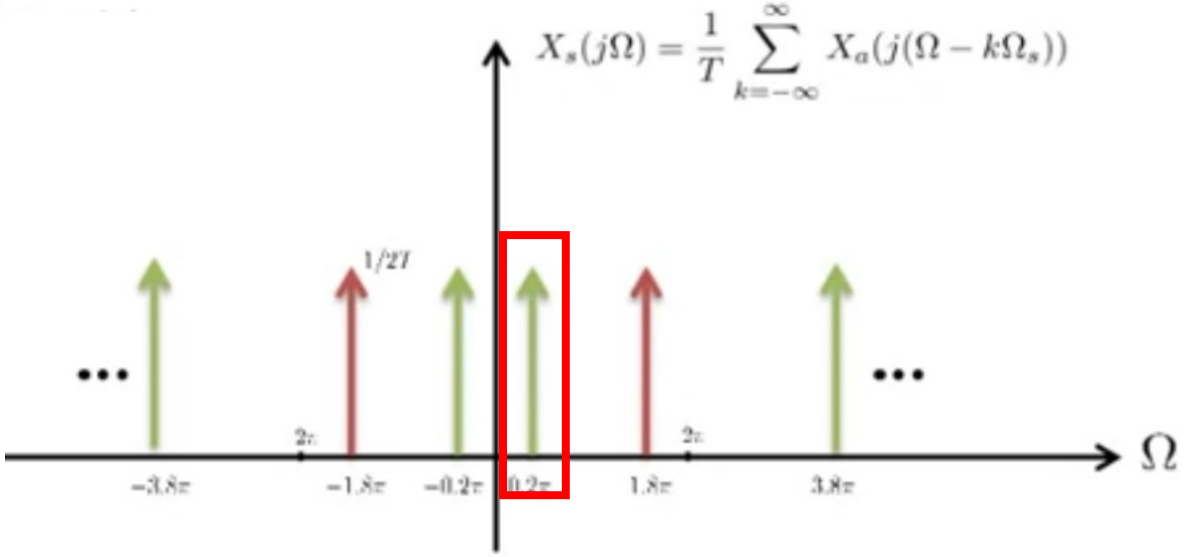
$$X_s(j\Omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_a(j(\Omega - k\Omega_s))$$



Under-Sampling

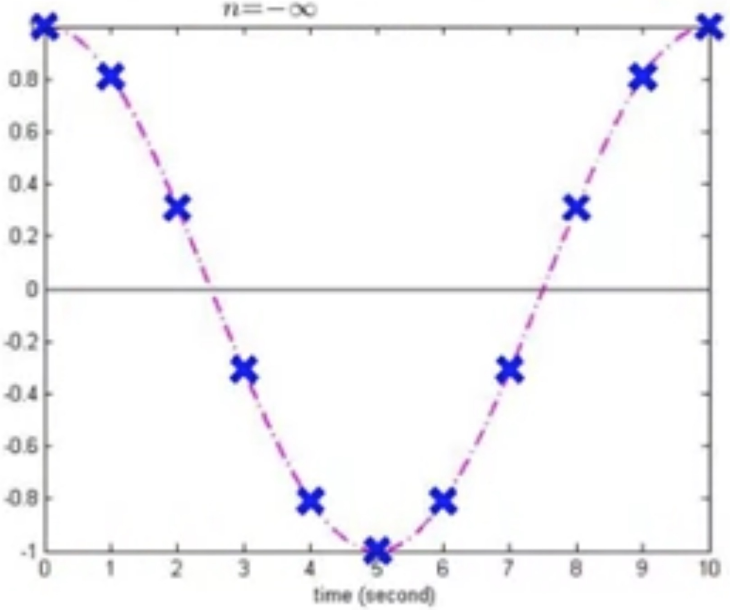
$$\Omega_N = 3.6\pi \text{ rads/sec}$$

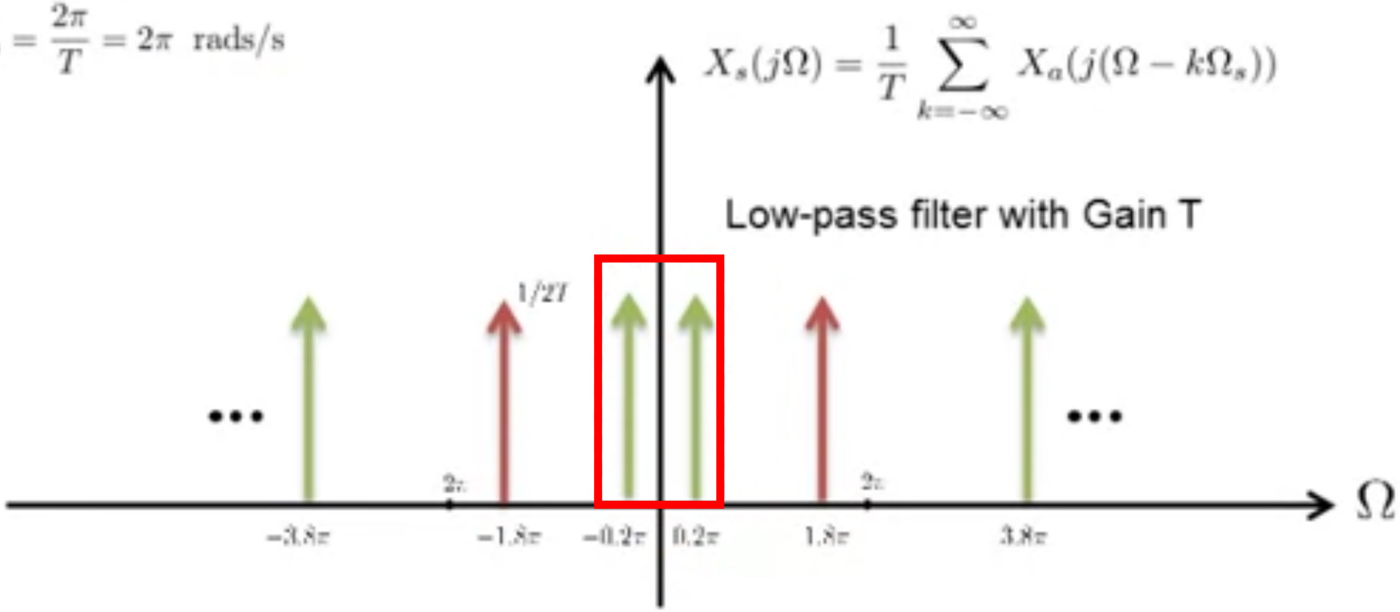




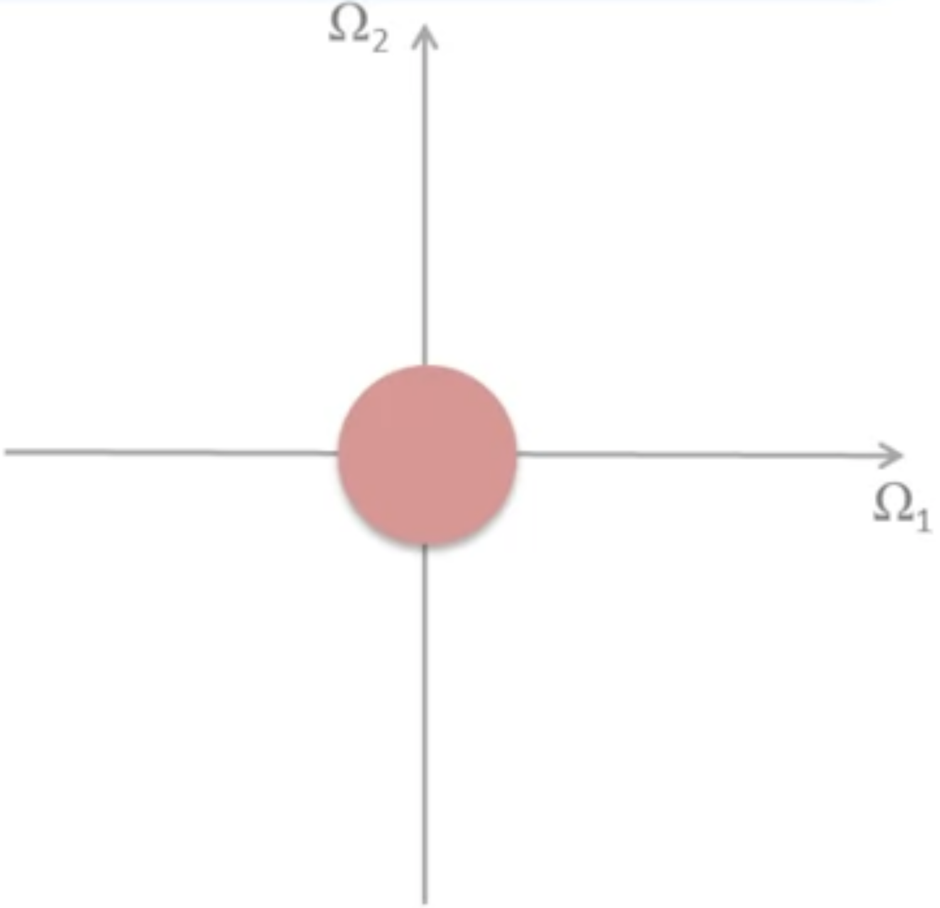
Reconstructed Aliased Signal

$$x_s = \sum_{n=-\infty}^{\infty} x_a(nT)\delta(t - nT) \qquad T = 1\text{s}, \Omega_s = \frac{2\pi}{T} = 2\pi \ \text{rads/s}$$





Sampling in the Frequency Domain



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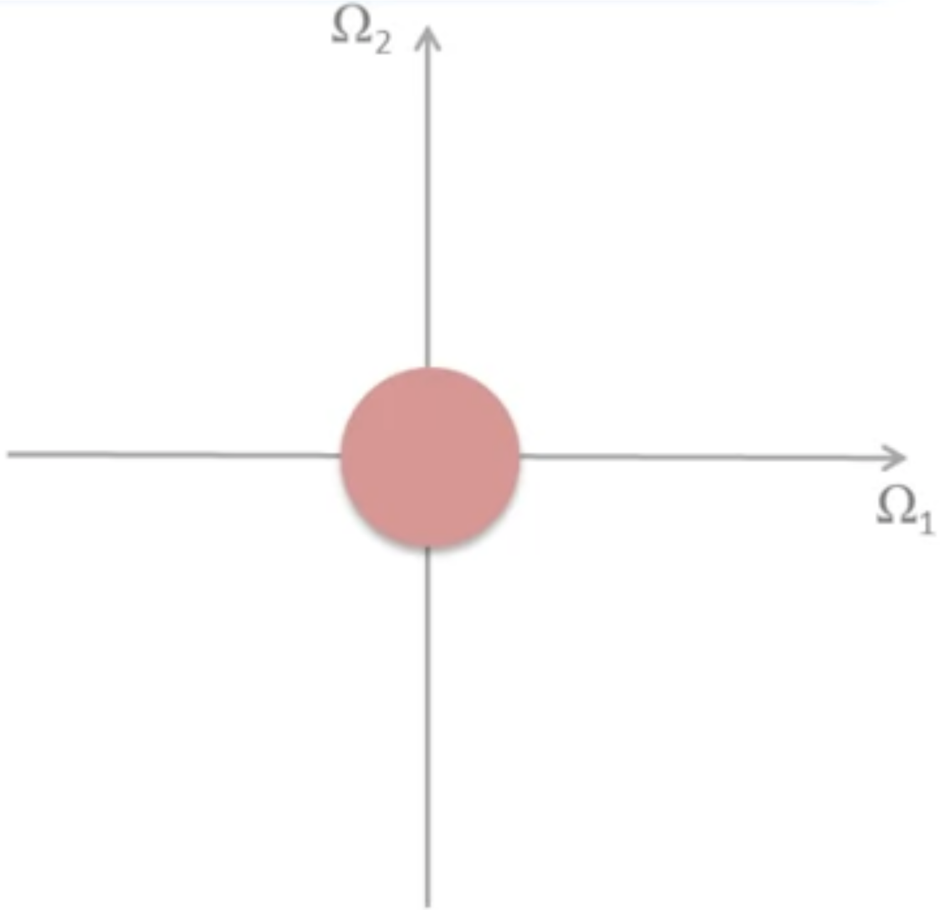
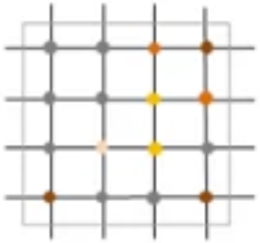
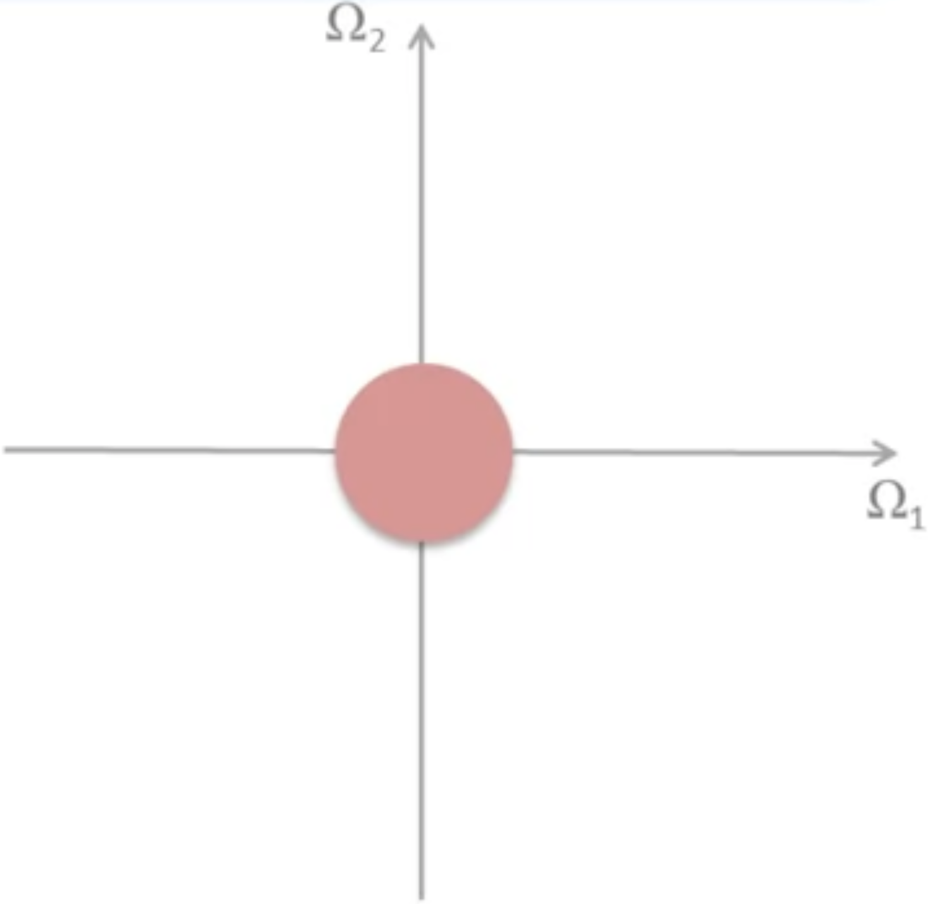
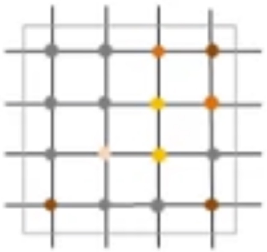


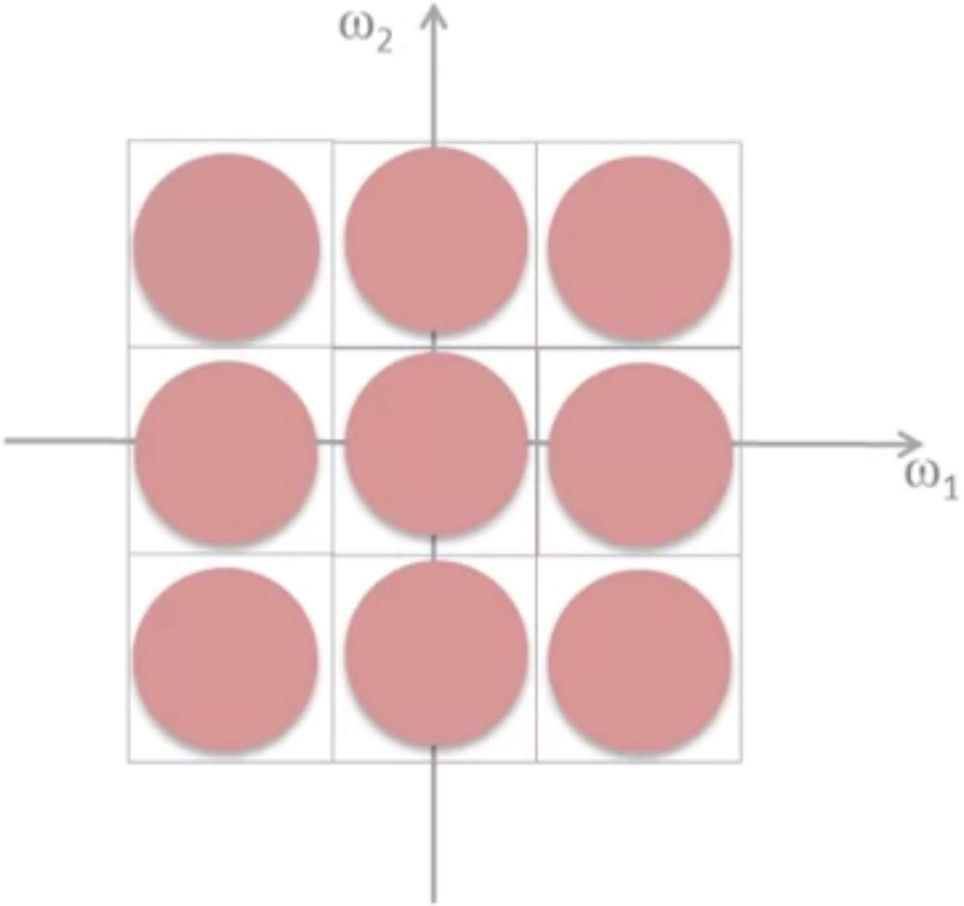
Sampling in the Frequency Domain





Sampling in the Frequency Domain





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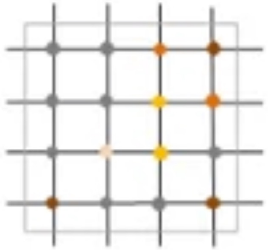
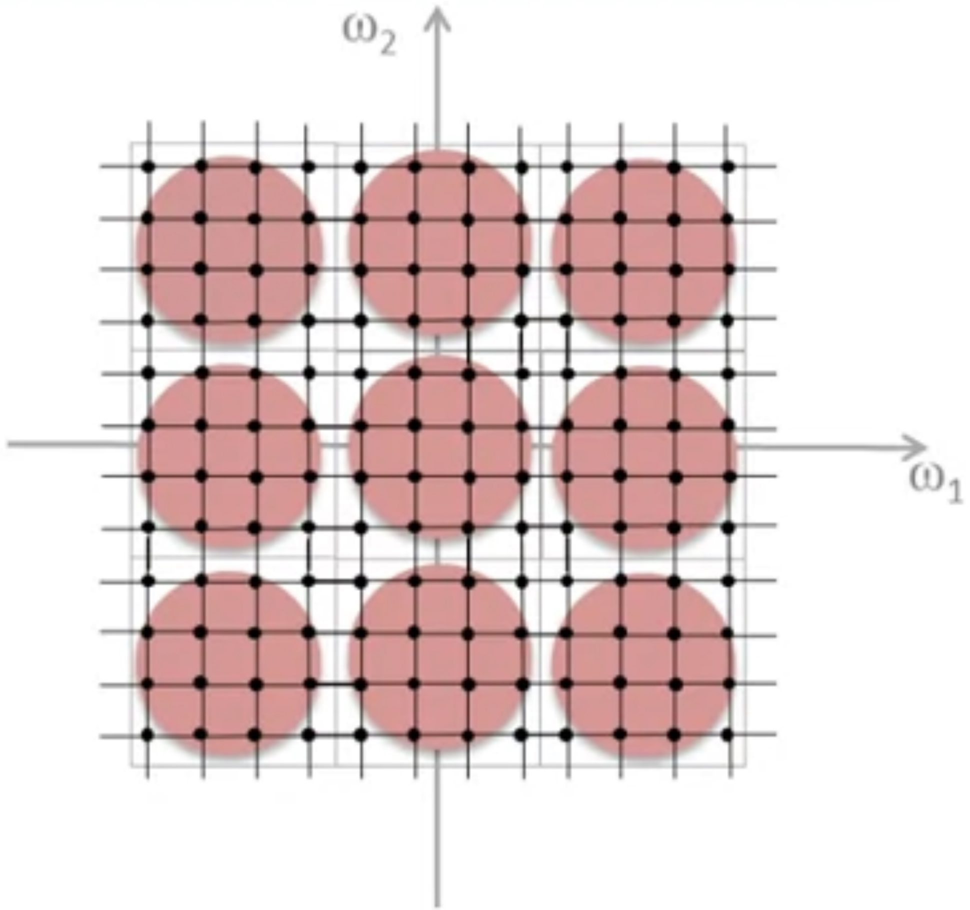
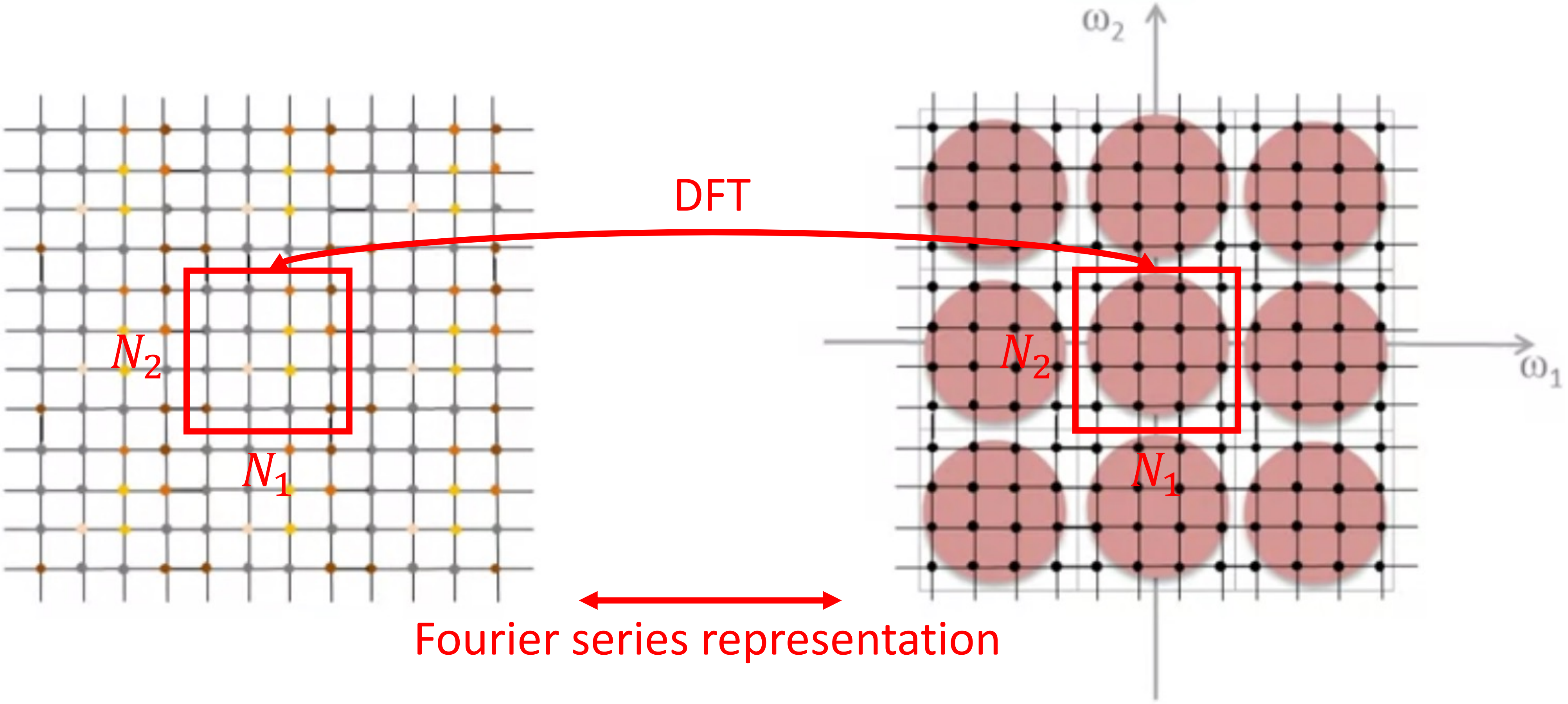




Sampling in the Frequency Domain



Fourier Transform and Sampling

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- **Discrete Fourier Transform**
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Discrete Fourier Transform (DFT)

$$X(\omega_1, \omega_2) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} x(n_1, n_2) e^{-j\omega_1 n_1} e^{-j\omega_2 n_2}$$

$$X(k_1, k_2) = X(\omega_1, \omega_2) \Big|_{\omega_1 = \frac{2\pi}{N_1}k_1, \omega_2 = \frac{2\pi}{N_2}k_2} \quad \begin{cases} k_1 = 0, \dots, N_1 - 1 \\ k_2 = 0, \dots, N_2 - 1 \end{cases}$$

$$X(k_1, k_2) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} x(n_1, n_2) e^{-j\frac{2\pi}{N_1}n_1 k_1} e^{-j\frac{2\pi}{N_2}n_2 k_2} \quad \begin{cases} k_1 = 0, \dots, N_1 - 1 \\ k_2 = 0, \dots, N_2 - 1 \end{cases}$$

$$x(n_1, n_2) = \frac{1}{N_1 N_2} \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} X(k_1, k_2)\, e^{j\frac{2\pi}{N_1}n_1 k_1} e^{j\frac{2\pi}{N_2}n_2 k_2} \quad \begin{cases} n_1 = 0, \dots, N_1 - 1 \\ n_2 = 0, \dots, N_2 - 1 \end{cases}$$

FT linear shifts <-> DFT circular shifts

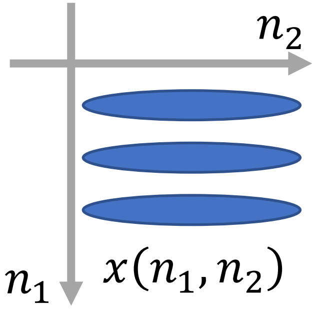
Fast Fourier Transforms (FFTs)

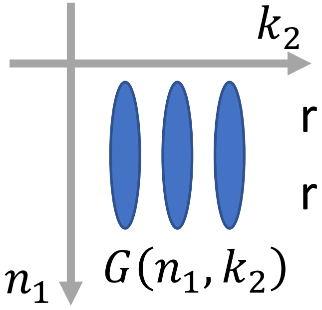
$$X(k_1, k_2) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} x(n_1, n_2) e^{-j\frac{2\pi}{N_1}n_1 k_1} e^{-j\frac{2\pi}{N_2}n_2 k_2} \quad \begin{cases} k_1 = 0, \dots, N_1 - 1 \\ k_2 = 0, \dots, N_2 - 1 \end{cases}$$

A. Direct Computations
   For each (k_1, k_2): $N_1 N_2$ mults; Total $N_1^2 N_2^2$ mults ($N_1 = N_2 = N$: N^4)

B. Row-Column Decomposition

$$X(k_1, k_2) = \sum_{n_1=0}^{N_1-1} \left[\sum_{n_2=0}^{N_2-1} x(n_1, n_2) e^{-j\frac{2\pi}{N_2}n_2 k_2} \right] e^{-j\frac{2\pi}{N_1}n_1 k_1}$$





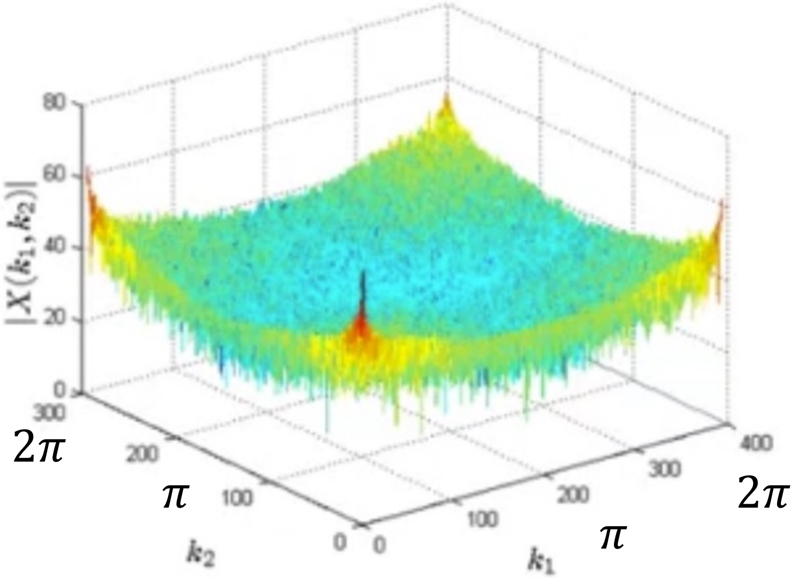
r|c directly: $N_1 \cdot N_2^2 + N_2 \cdot N_1^2 = N_1 N_2 (N_1 + N_2)$ ($N_1 = N_2 = N$: $2N^3$)

r|c FFT: $N_1 \cdot \frac{N_2}{2} \log_2 N_2 + N_2 \cdot \frac{N_1}{2} \log_2 N_1 = \frac{N_1 N_2}{2} \log(N_1 N_2)$ ($N_1 = N_2 = N$: $N^2 \log_2 N$)

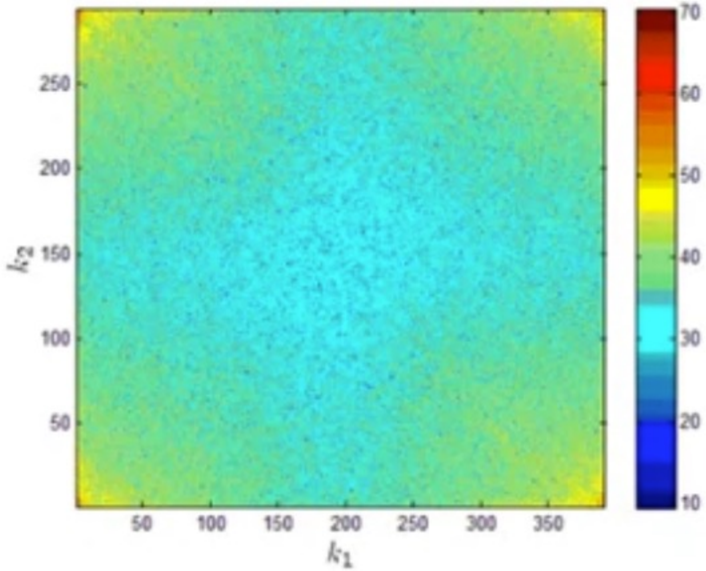
DFT

N_2

N_1

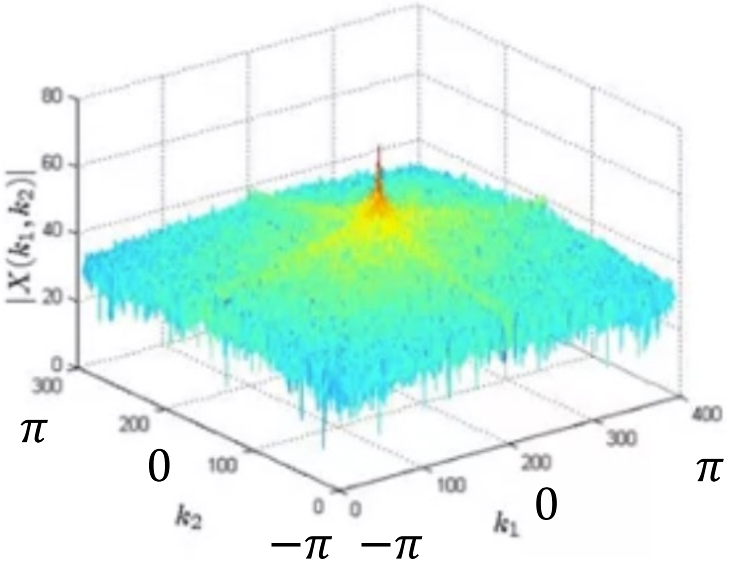


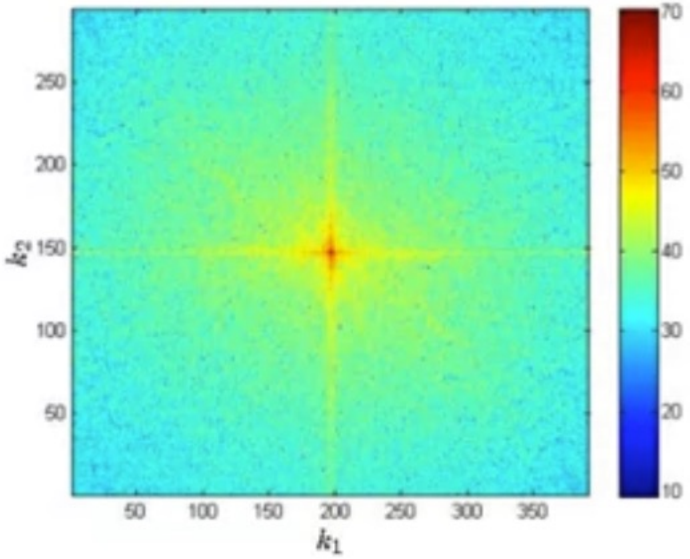
$|X(k_1,k_2)|$
80
60
40
20
0
300
200
100
0
2π
π
k_2
0
100
200
300
400
π
2π
k_1



k_2
250
200
150
100
50
50
100
150
200
250
300
350
k_1
70
60
50
40
30
20
10

DFT (centered)



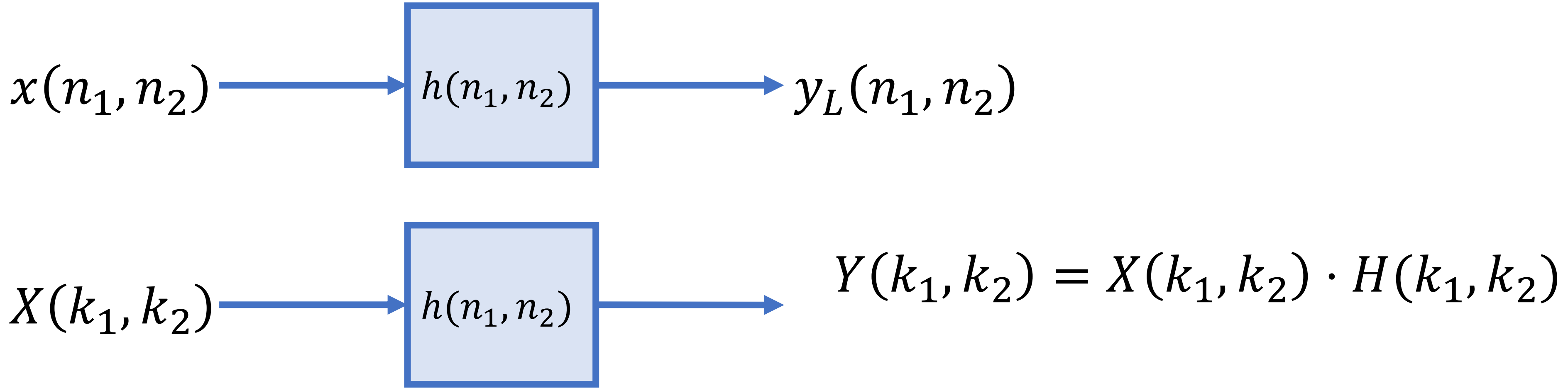


$$(-1)^{n_1+n_2}$$
$$= e^{-j\pi(n_1+n_2)}$$

Fourier Transform and Sampling

- 2D Fourier Transform
- Sampling
- Discrete Fourier Transform
- **Filtering in the Frequency Domain**
- Change of Sampling Rate
- The Discrete Cosine Transform
- The Two-Dimensional Discrete Cosine Transform

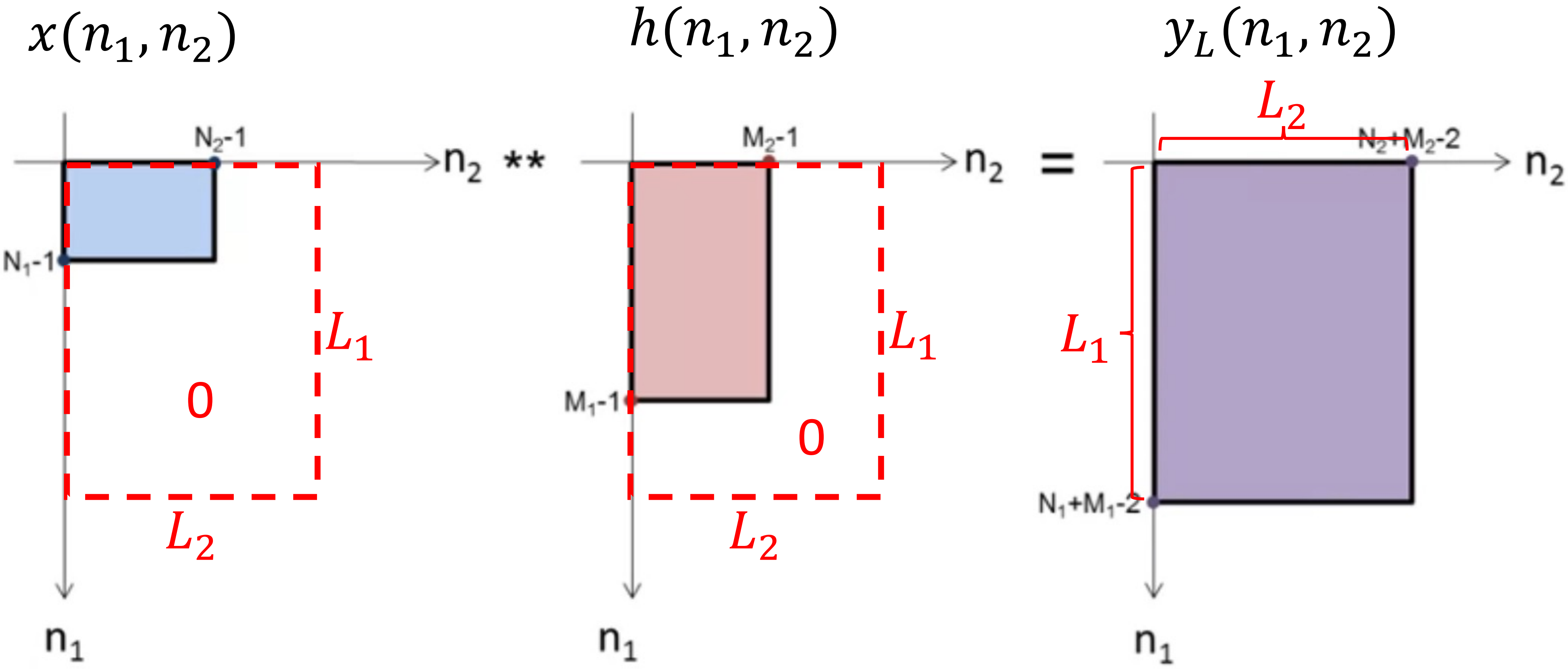
2D Circular Convolution



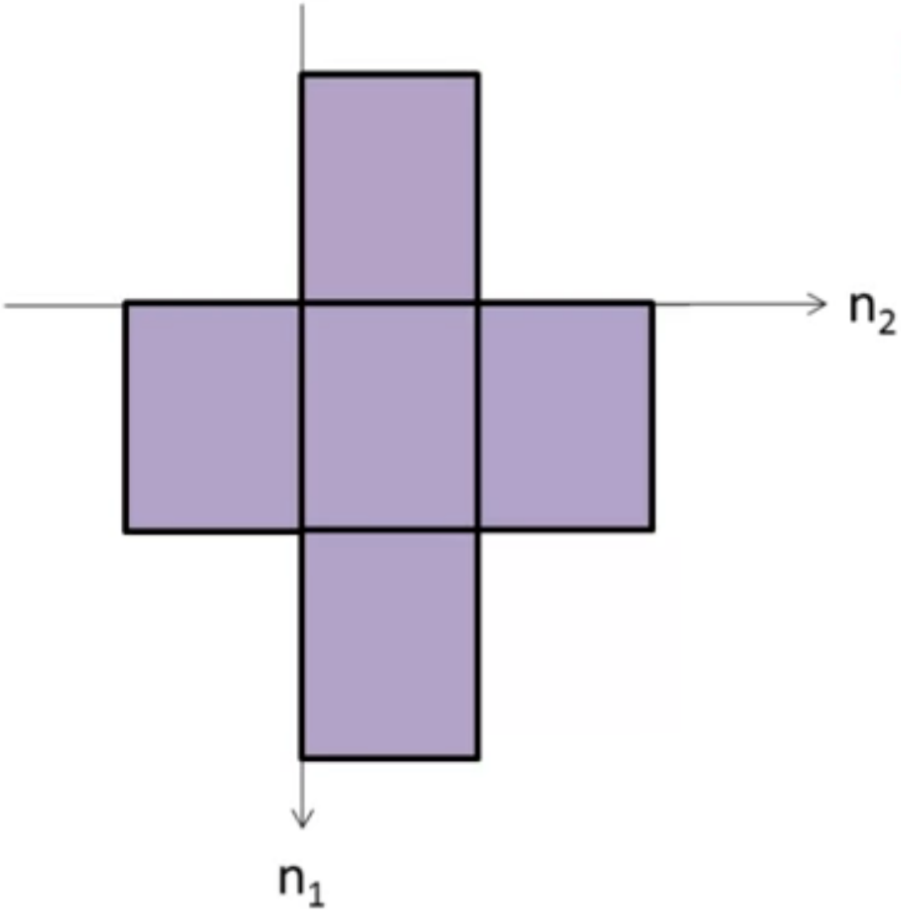
Circular Convolution

$$y(n_1, n_2) = \sum_{r_1} \sum_{r_2} y_L(n_1 - r_1 N_1, n_2 - r_2 N_2) \begin{cases} n_1 = 0, \dots, N_1 - 1 \\ n_2 = 0, \dots, N_2 - 1 \end{cases}$$

ROS of Linear Convolution

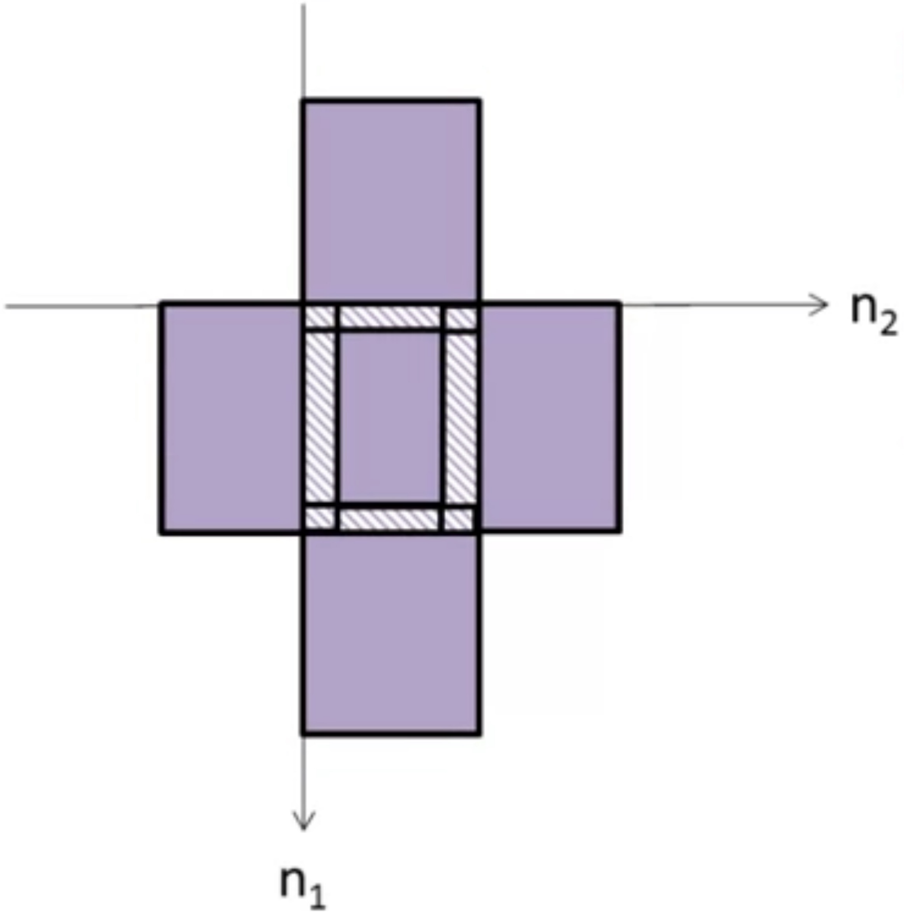


Spatial Aliasing



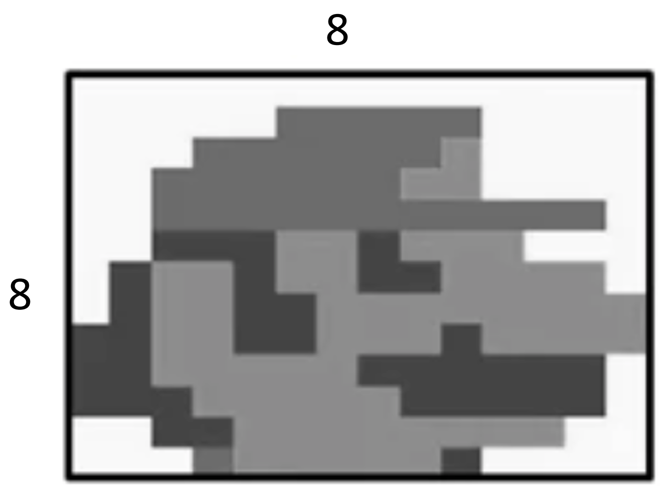
L_1, L_2
$L_1' < L_1$
$L_2' < L_2$

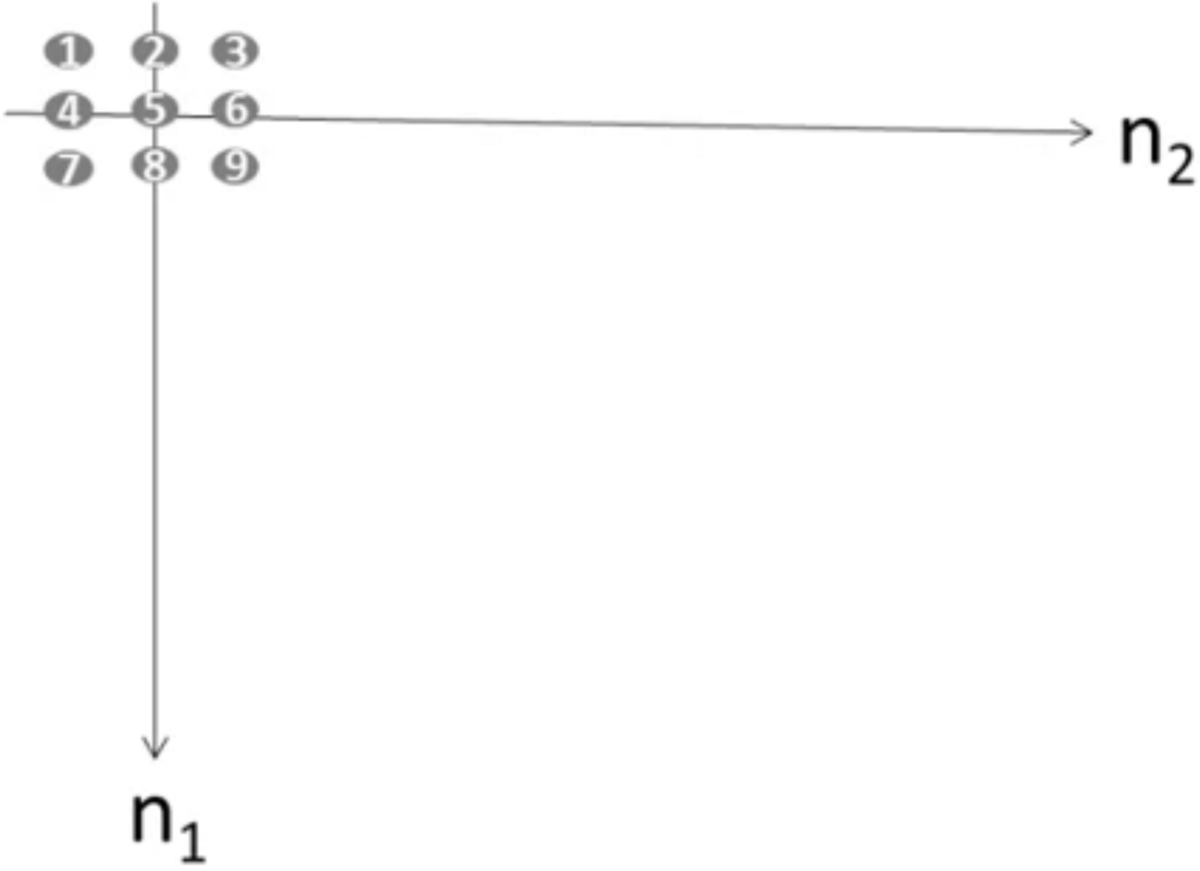
Spatial Aliasing



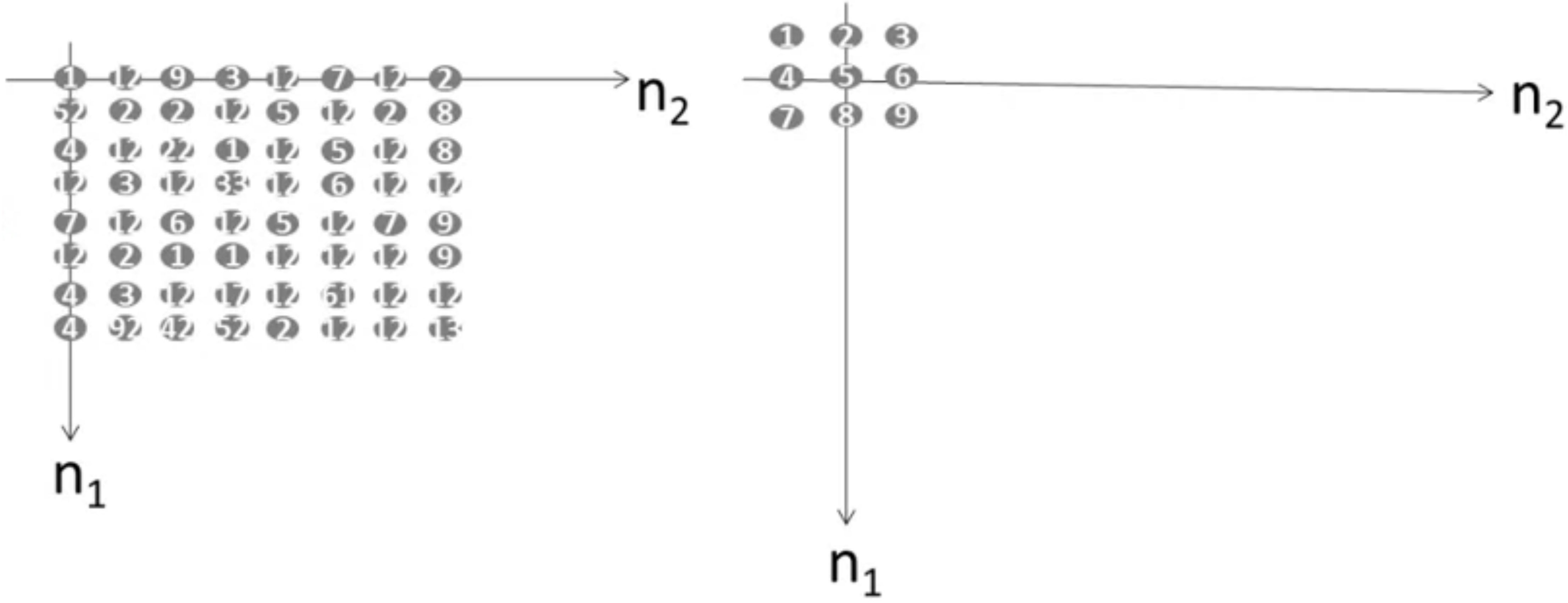
L_1, L_2
$L_1' < L_1$
$L_2' < L_2$

Use of DFT for Filtering

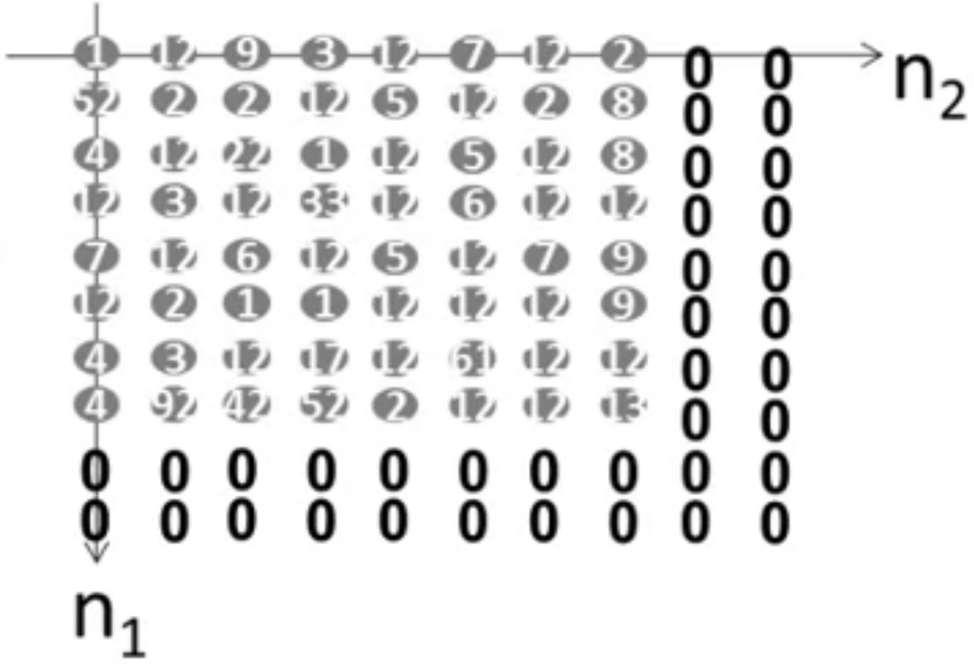


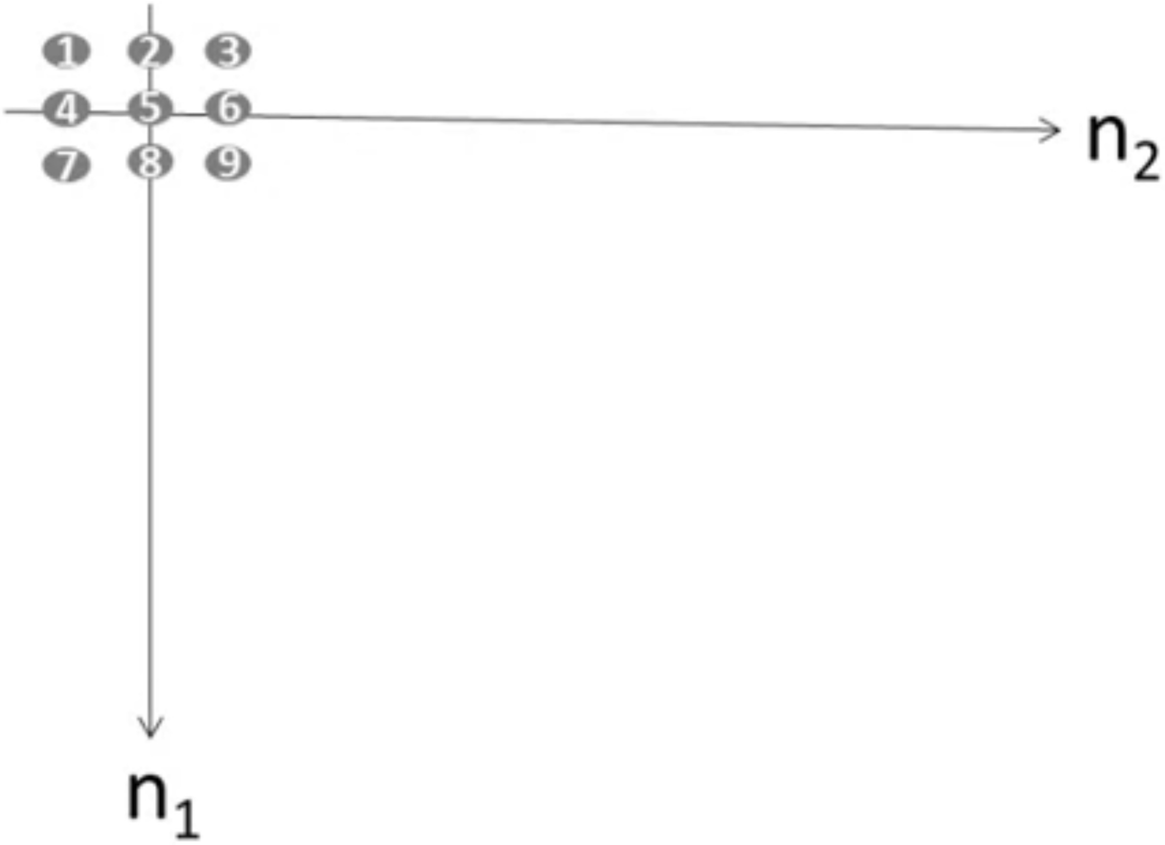


Use of DFT for Filtering

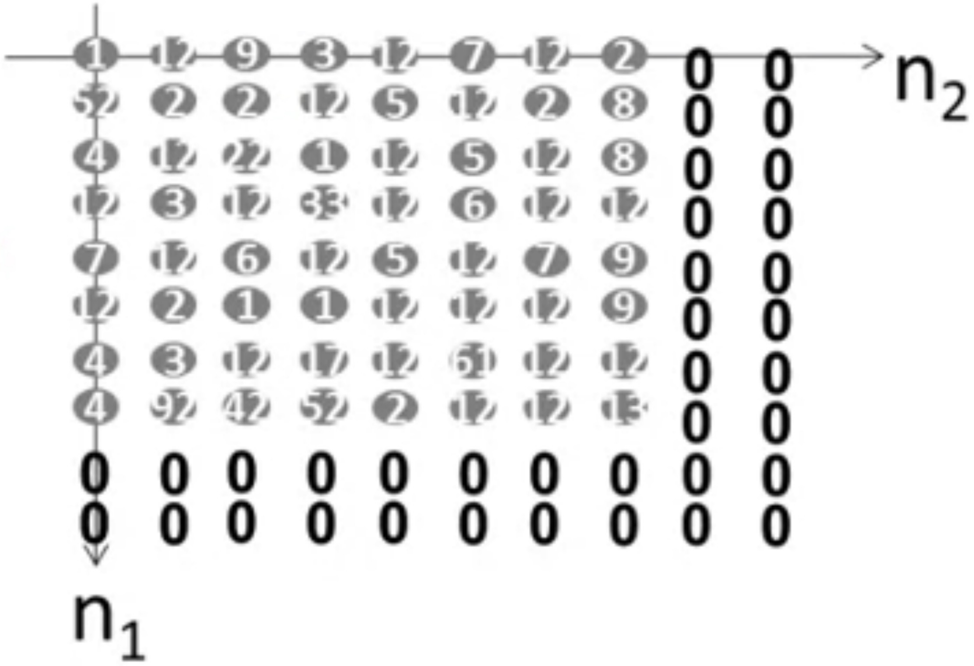


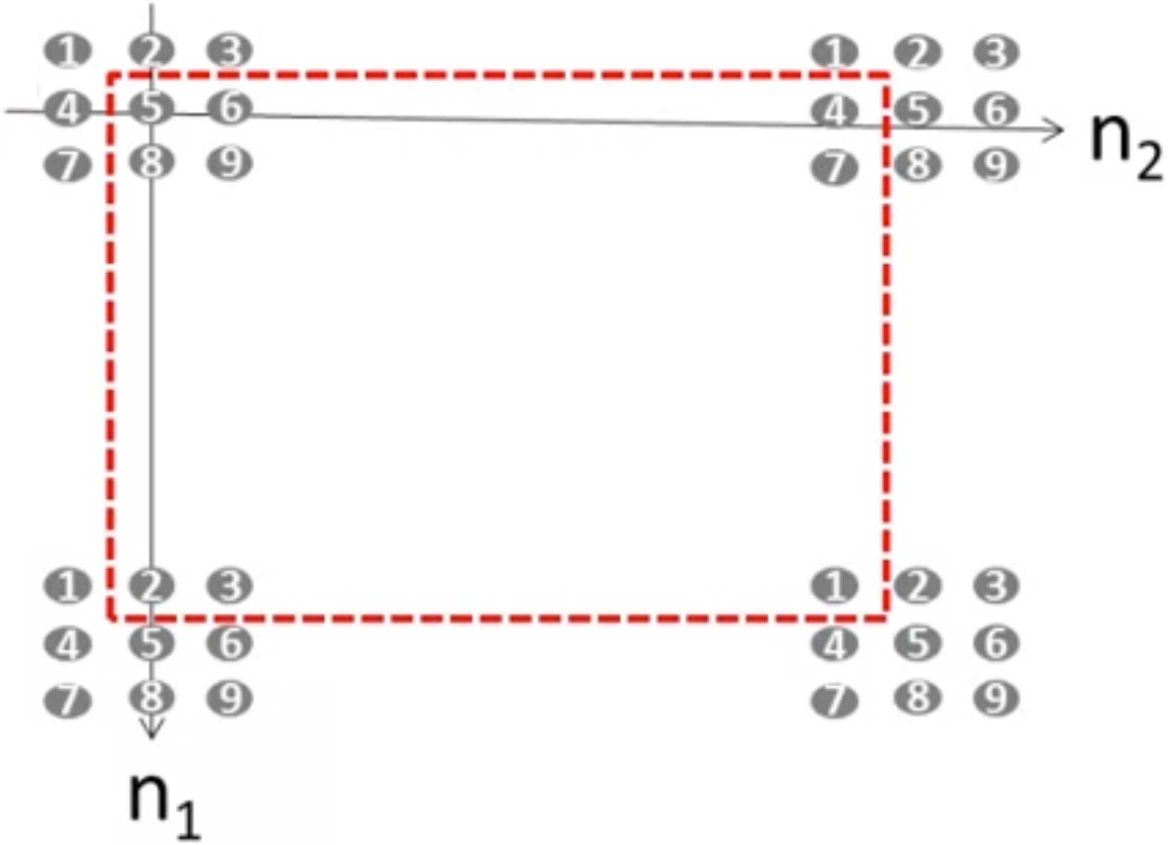
Use of DFT for Filtering



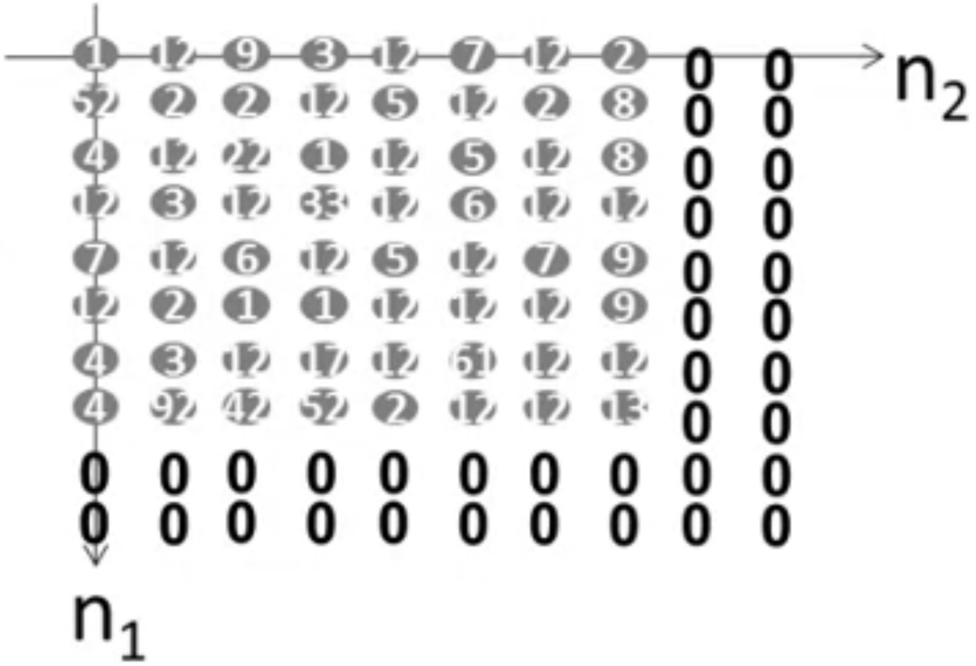


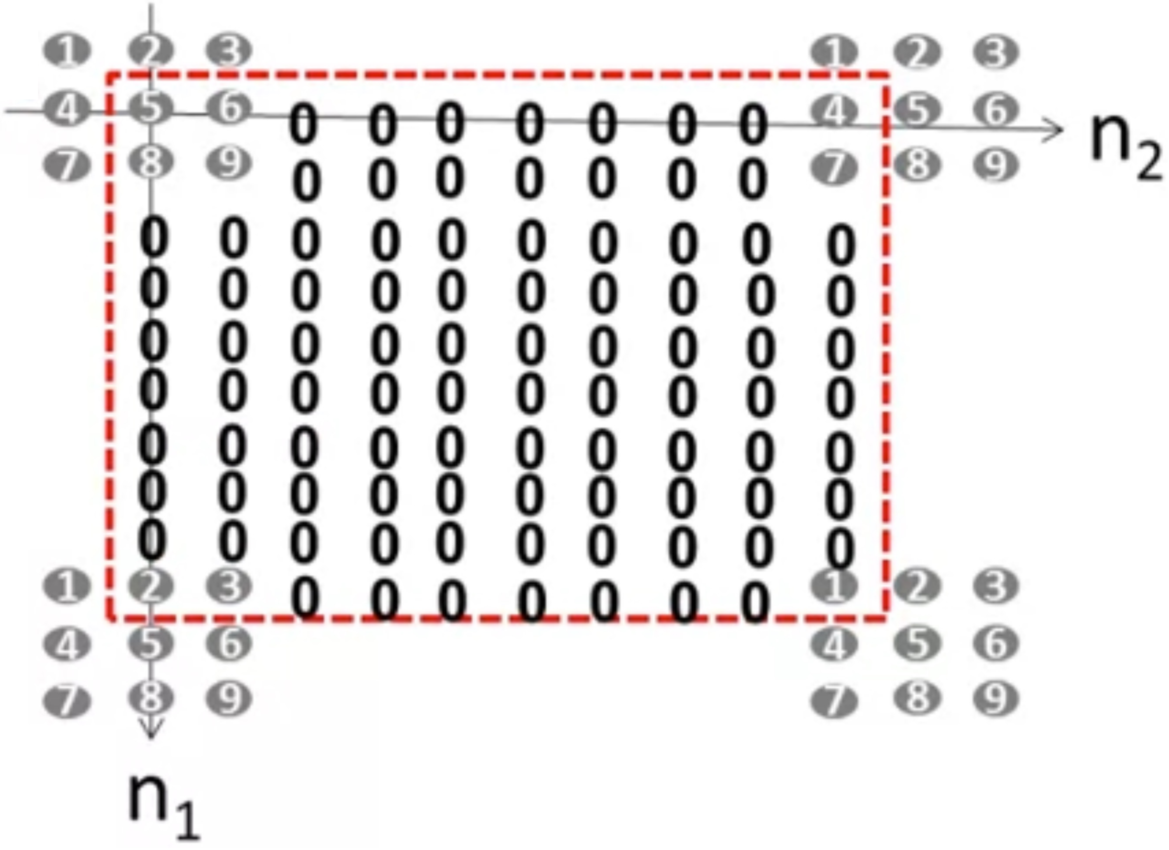
Use of DFT for Filtering





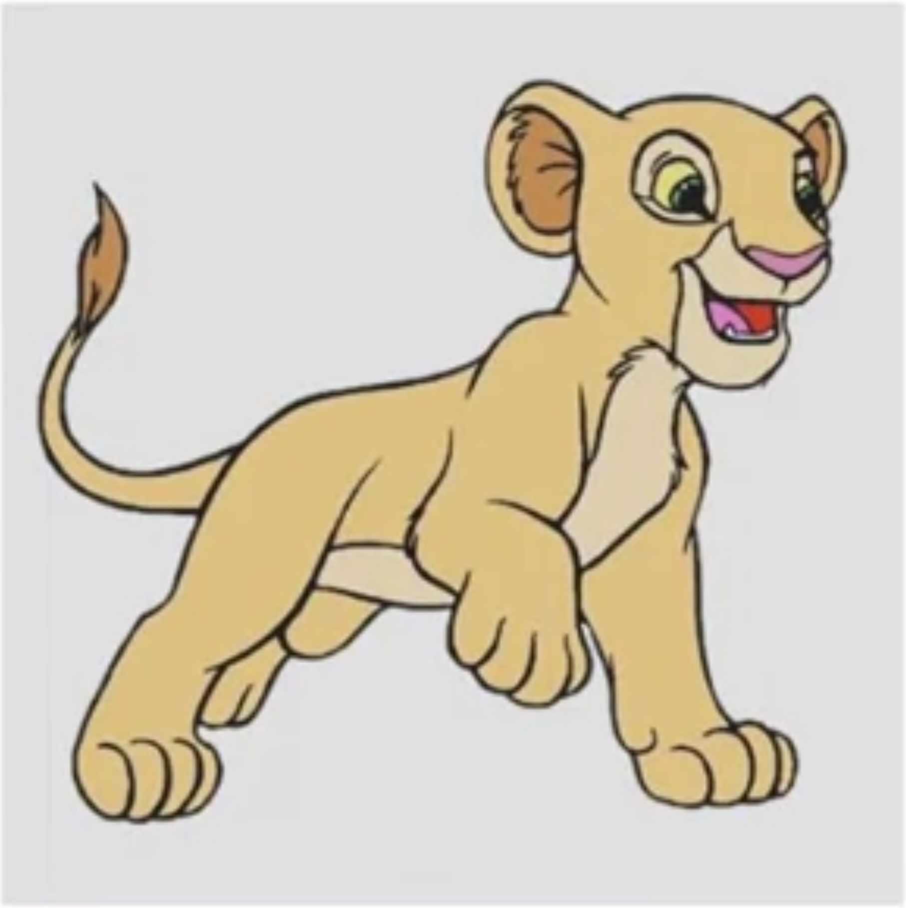
Use of DFT for Filtering





Frequency Filtering

$$y(n_1, n_2) = x(n_1, n_2) + \cos(0.1\pi n_2)$$



Fourier Transform and Sampling

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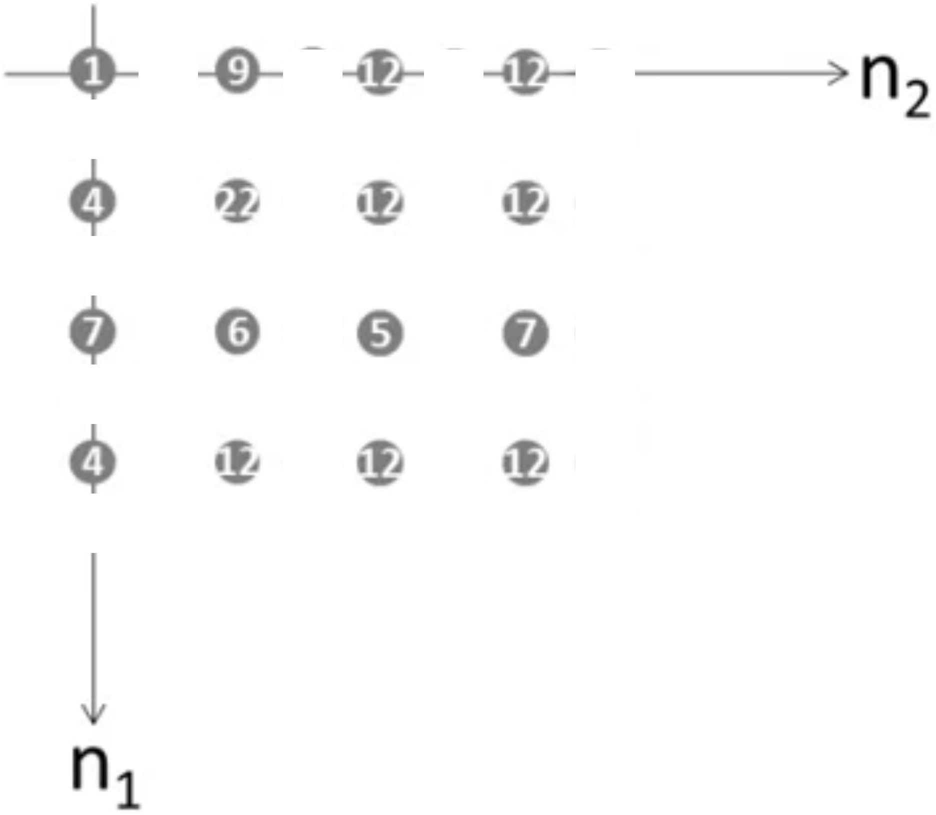
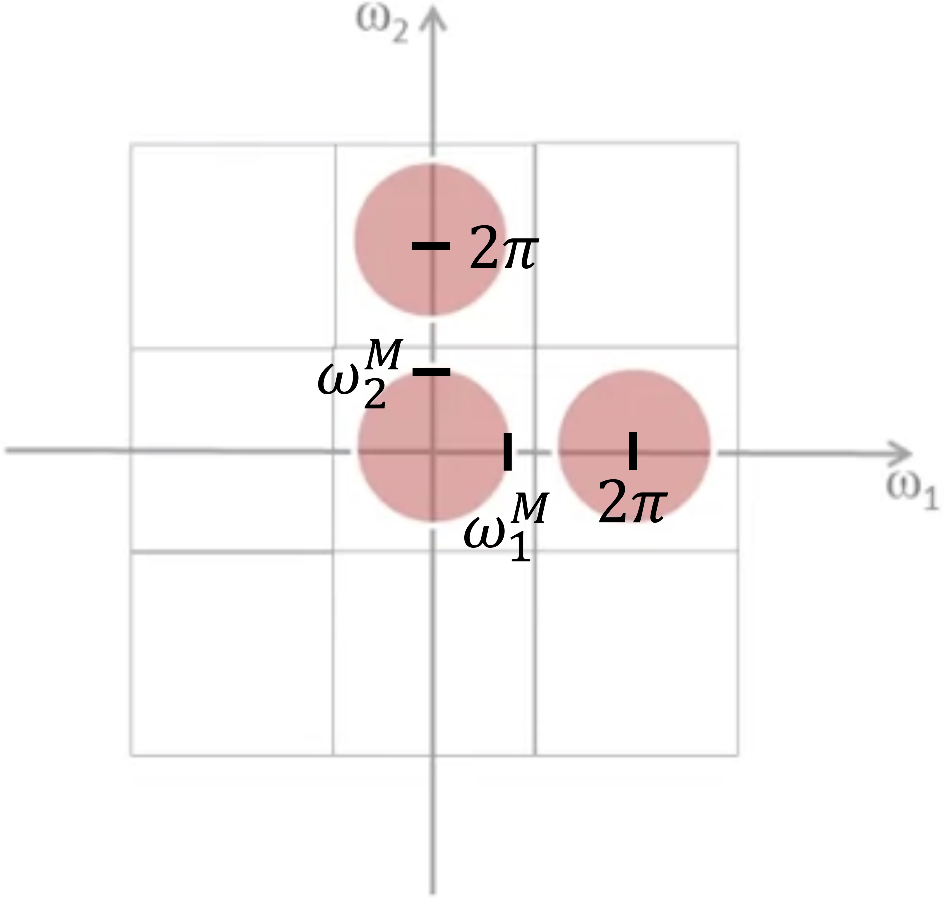
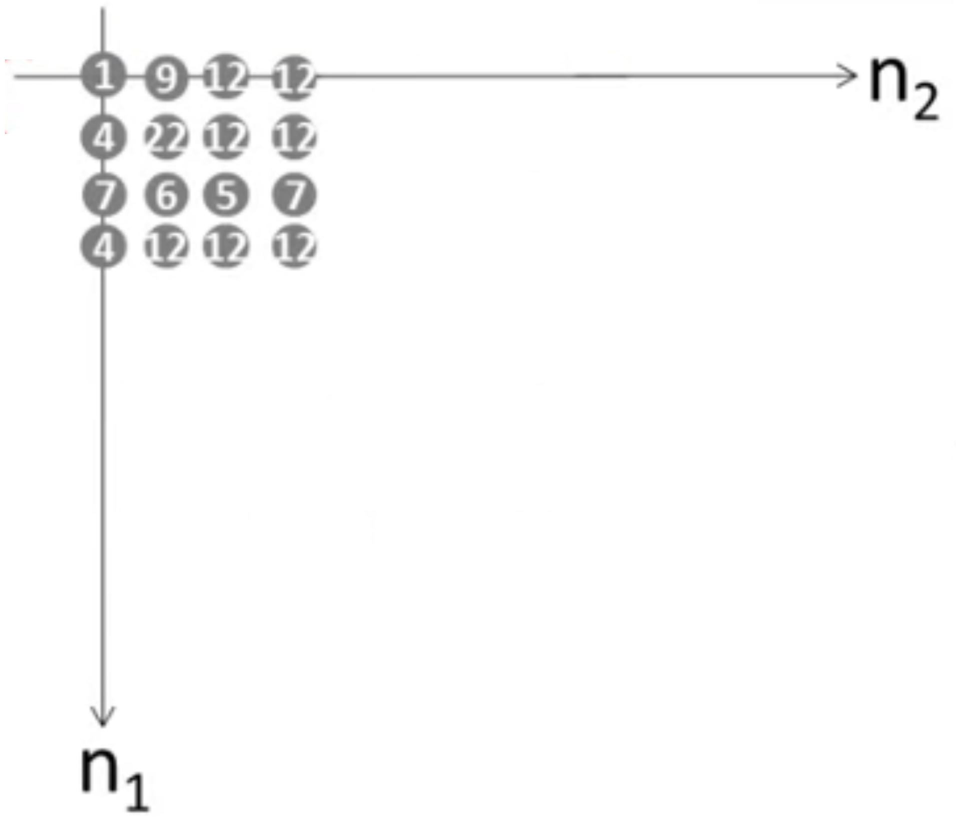
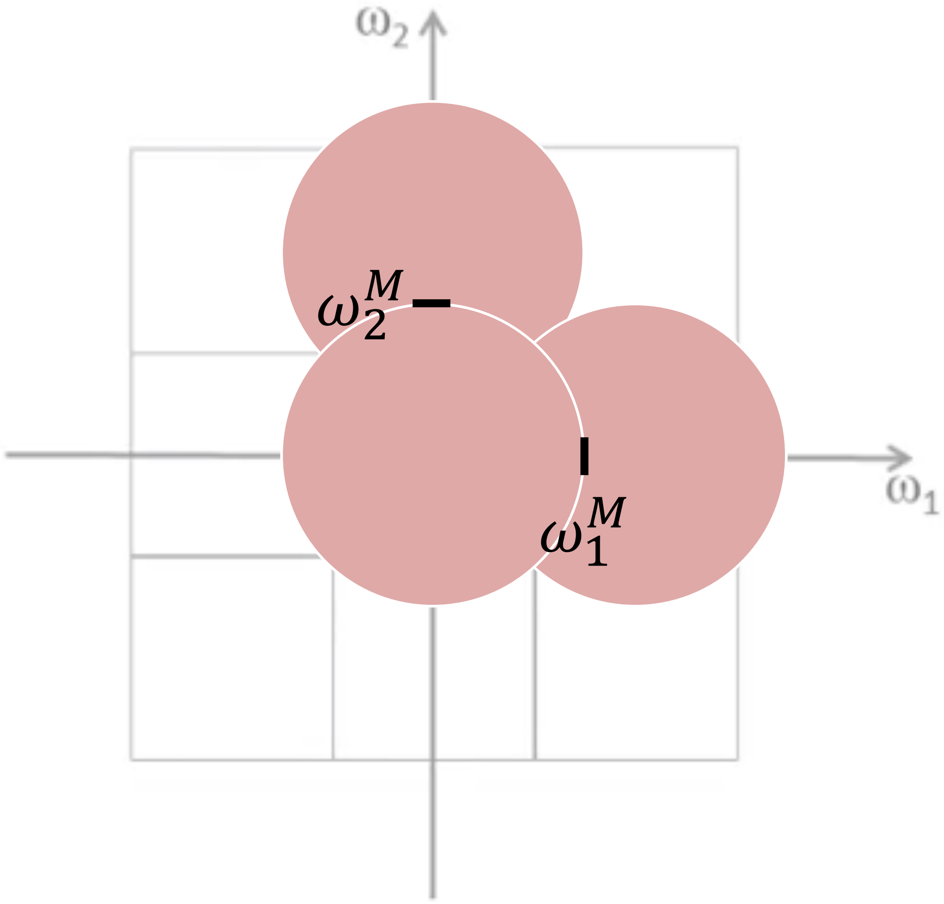
Down-Sampling





Down-Sampling

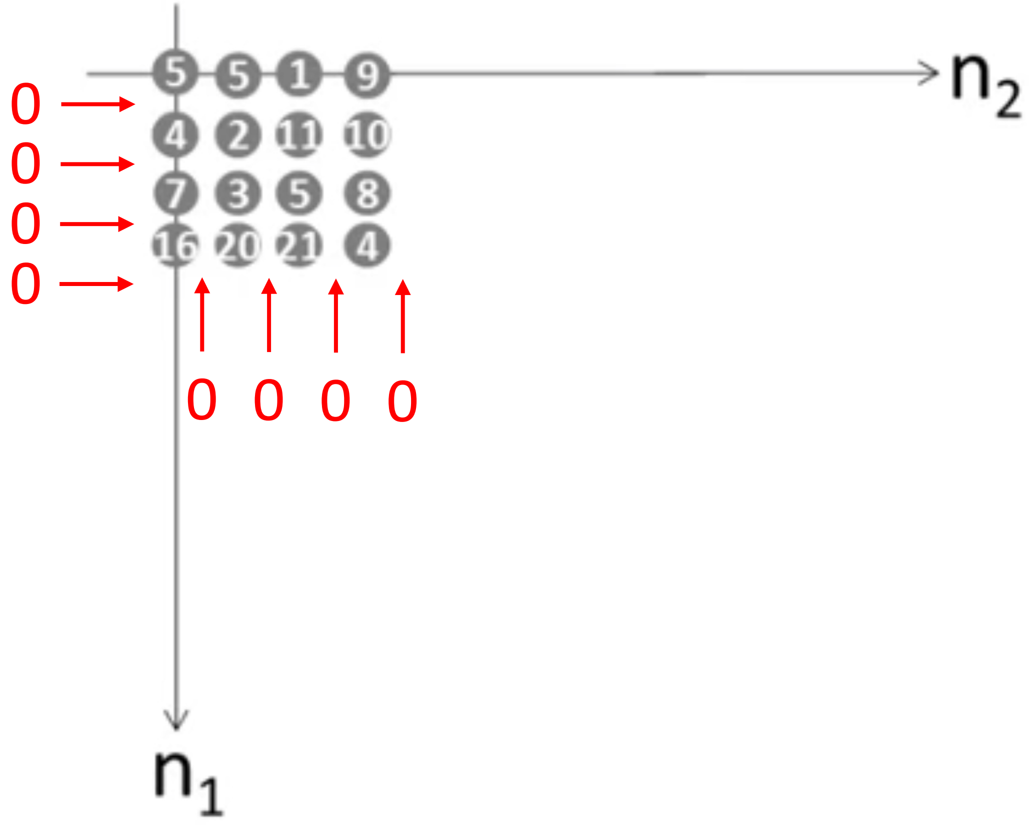




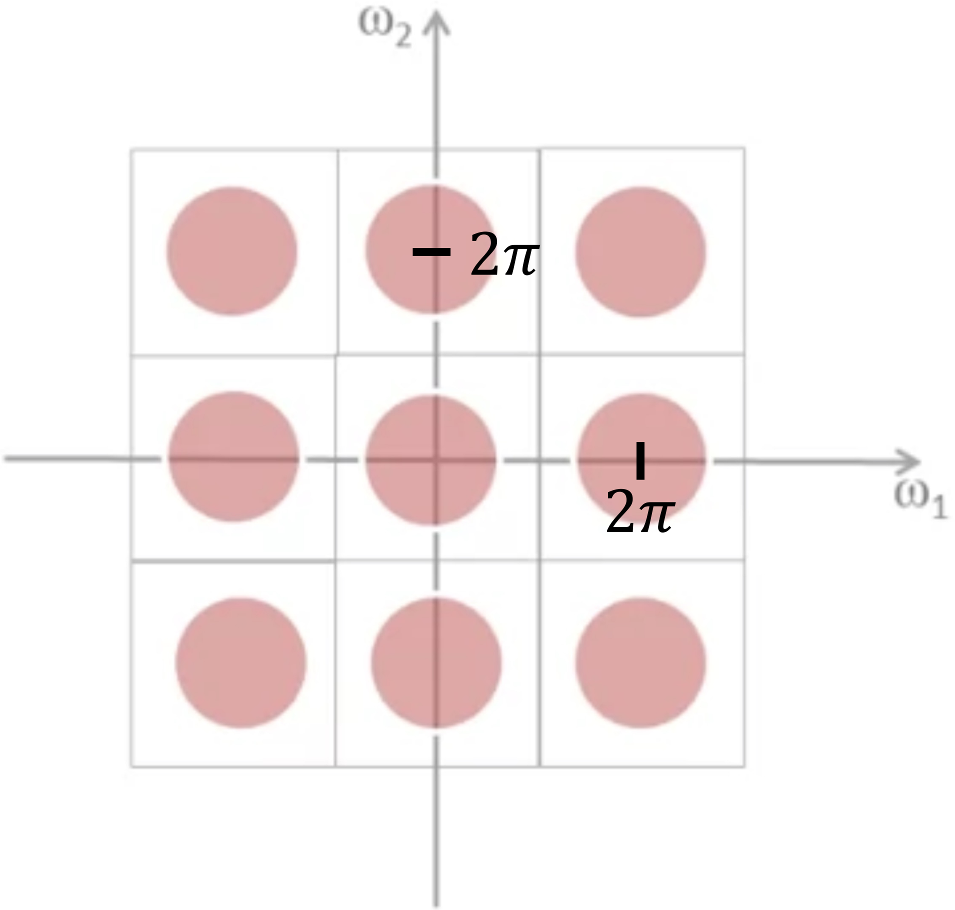
Down-Sampling Example

Resolution of original image: 4200 x 3000 pixels (not shown due to space limitations)
Left image: Down-sampled by a factor of 10, direct pixel removal.
Right image: LPF by a Gaussian 11x11 filter, then down-sampled by a factor of 10

Up-Sampling

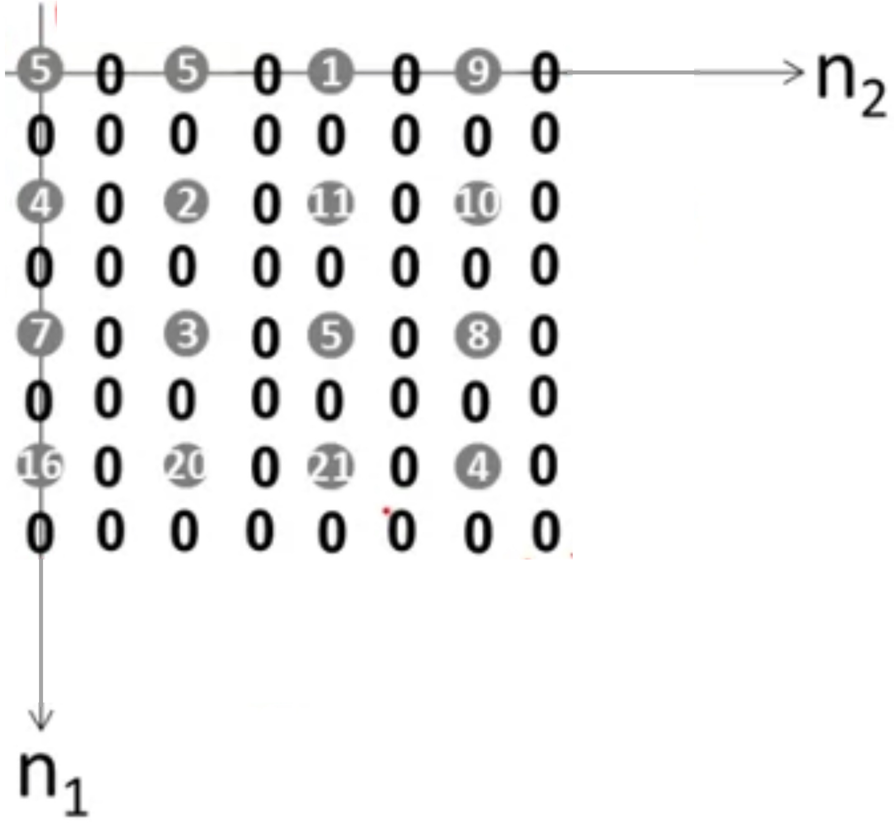


5 5 1 9
4 2 11 10
7 3 5 8
16 20 21 4
0
0
0
0
0 0 0 0
n_2
n_1

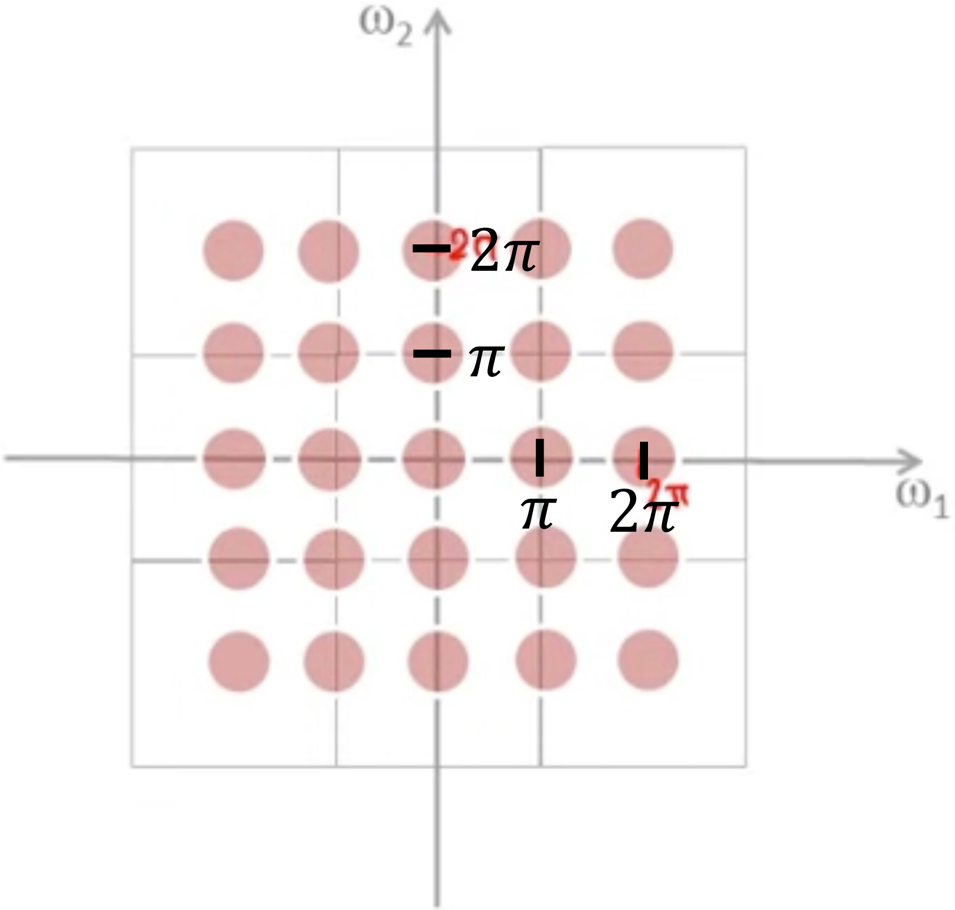


ω_2
ω_1
2π
2π

Up-Sampling

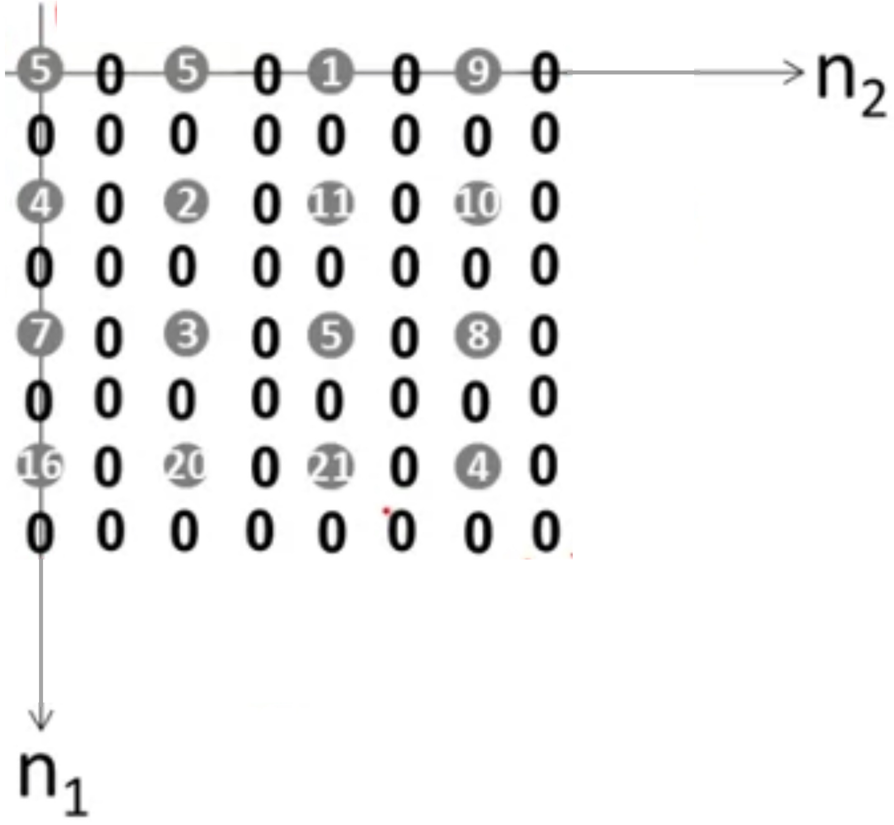


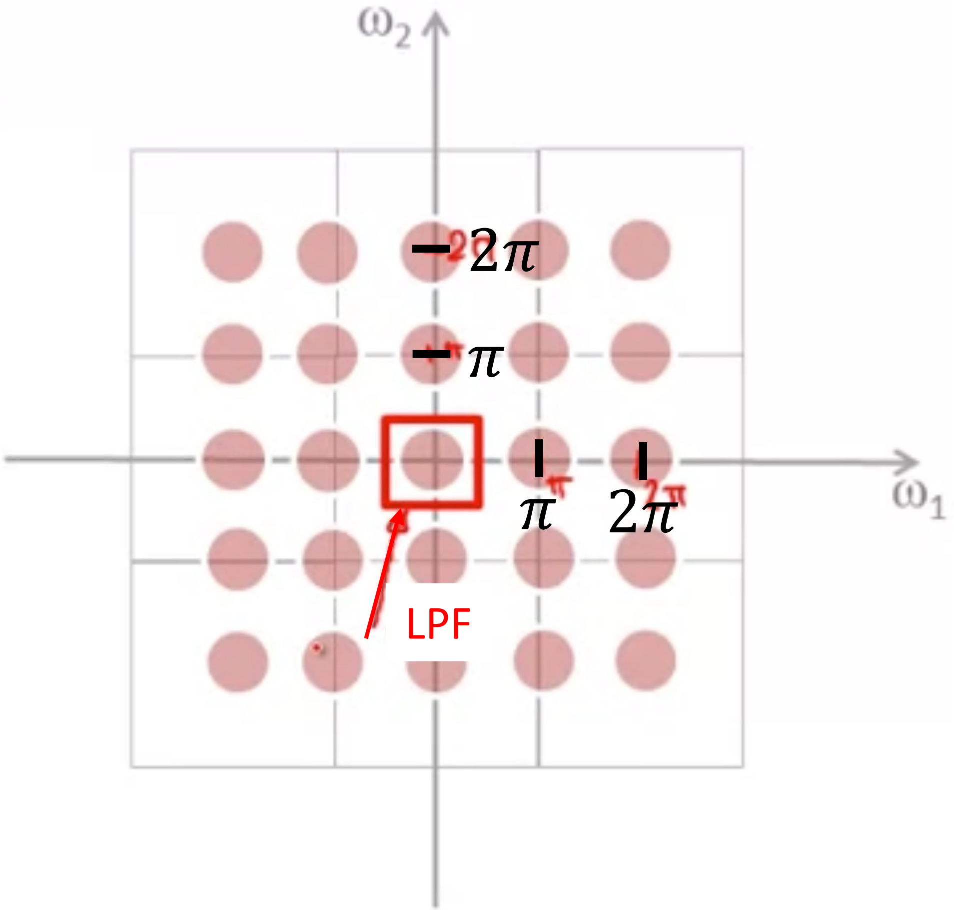
5 0 5 0 1 0 9 0
0 0 0 0 0 0 0 0
4 0 2 0 11 0 10 0
0 0 0 0 0 0 0 0
7 0 3 0 5 0 8 0
0 0 0 0 0 0 0 0
16 0 20 0 21 0 4 0
0 0 0 0 0 0 0 0
n_2
n_1



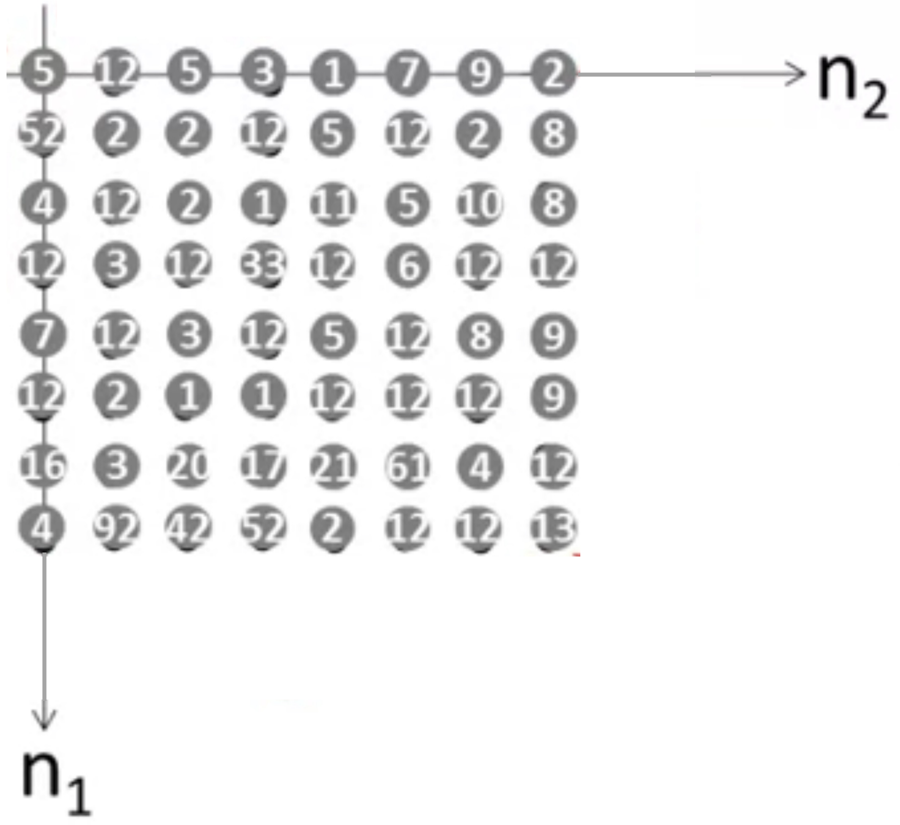
ω_2
2π
π
π
2π
ω_1

Up-Sampling

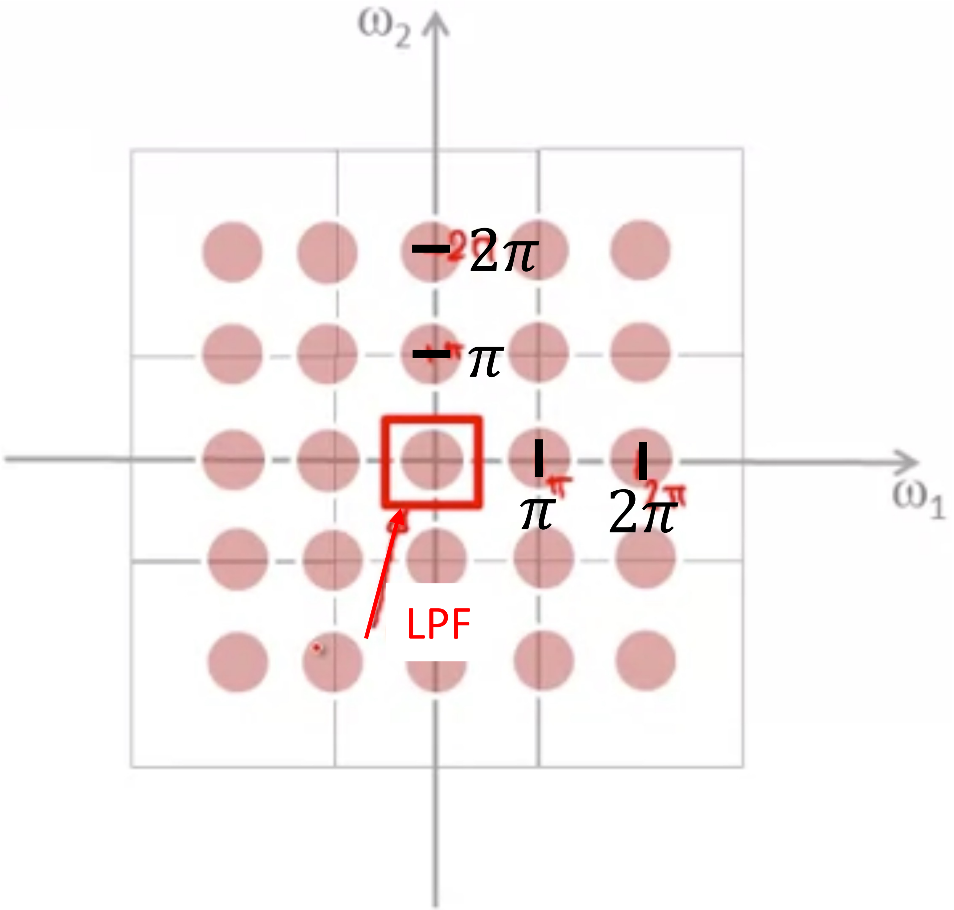

5 0 5 0 1 0 9 0
0 0 0 0 0 0 0 0
4 0 2 0 11 0 10 0
0 0 0 0 0 0 0 0
7 0 3 0 5 0 8 0
0 0 0 0 0 0 0 0
16 0 20 0 21 0 4 0
0 0 0 0 0 0 0 0
n_2
n_1


ω_2
2π
π
π
2π
ω_1
LPF

Up-Sampling



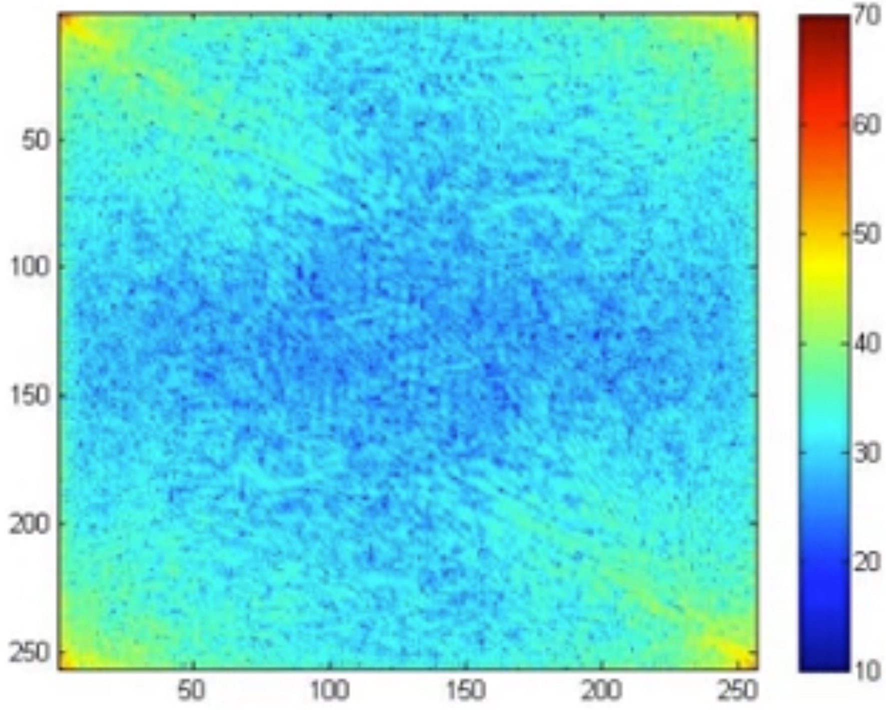
5 12 5 3 1 7 9 2
52 2 2 12 5 12 2 8
4 12 2 1 11 5 10 8
12 3 12 33 12 6 12 12
7 12 3 12 5 12 8 9
12 2 1 1 12 12 12 9
16 3 20 17 21 61 4 12
4 92 42 52 2 12 12 13
n_2
n_1



ω_2
2π
π
π
2π
ω_1
LPF

Up-Sampling Example

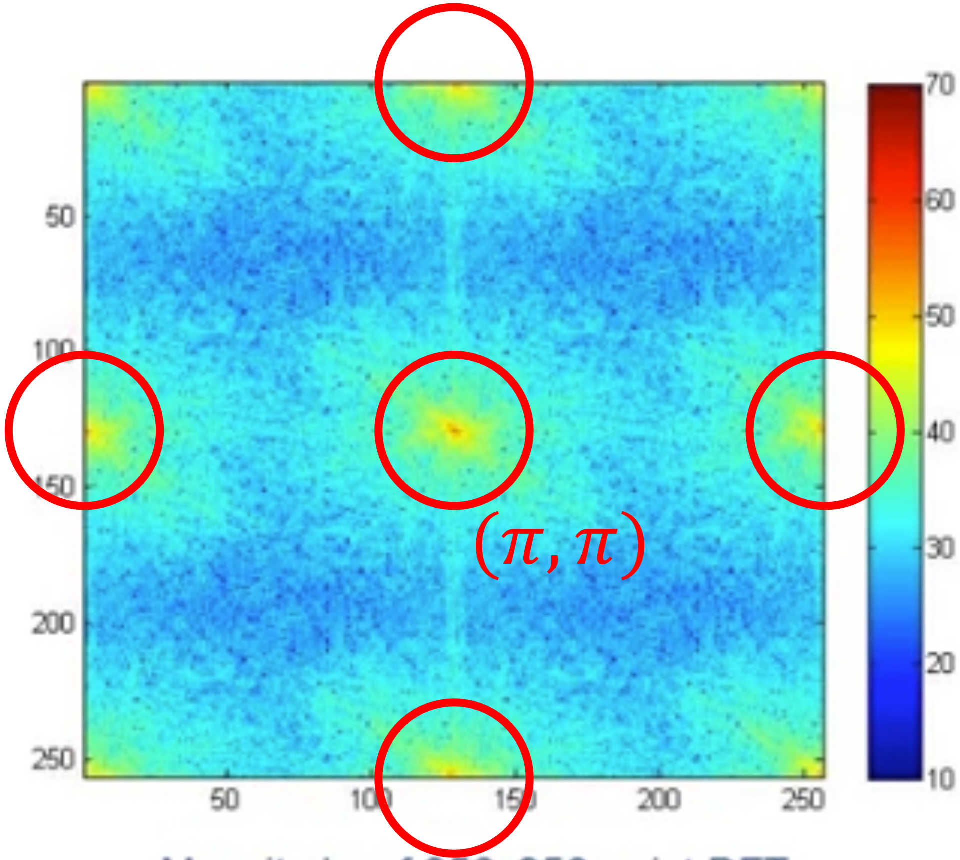
128x128 input image



Magnitude of the 256x 256-point DFT, obtained by zero padding the input image to size 256x256 and then taking the DFT

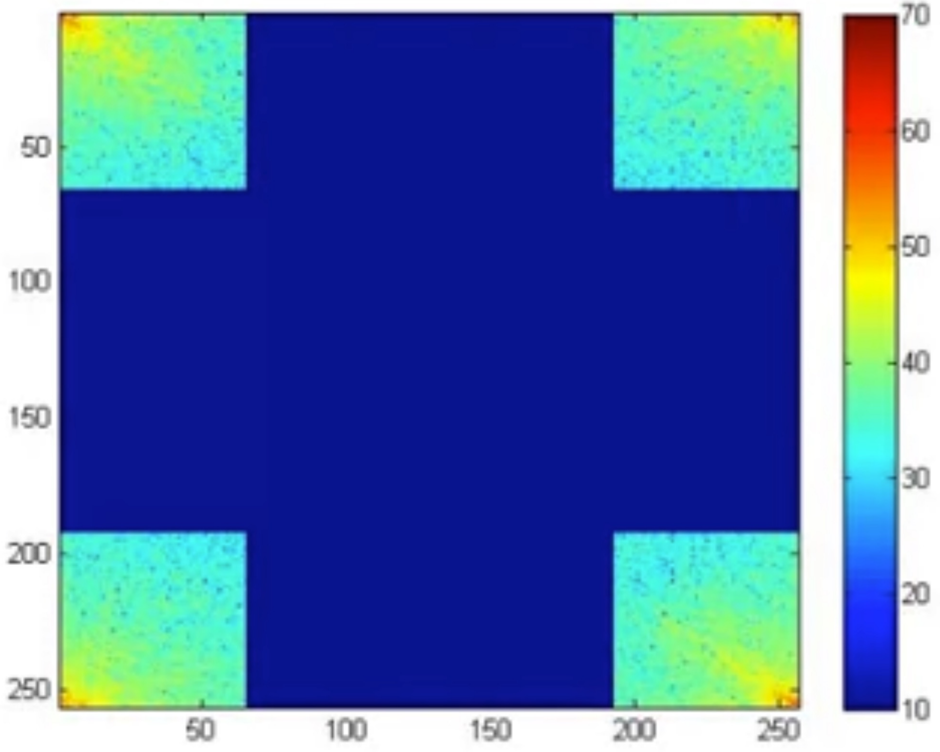
Up-Sampling Example

256x256 image obtained by inserting zero-columns and zero-rows to the original 128x128 image



Magnitude of 256x256-point DFT

Up-Sampling Example



LPF spectrum (blue area corresponds to zero values)

Inverse DFT

Up-Sampling Example

$\frac{a+b}{2}$

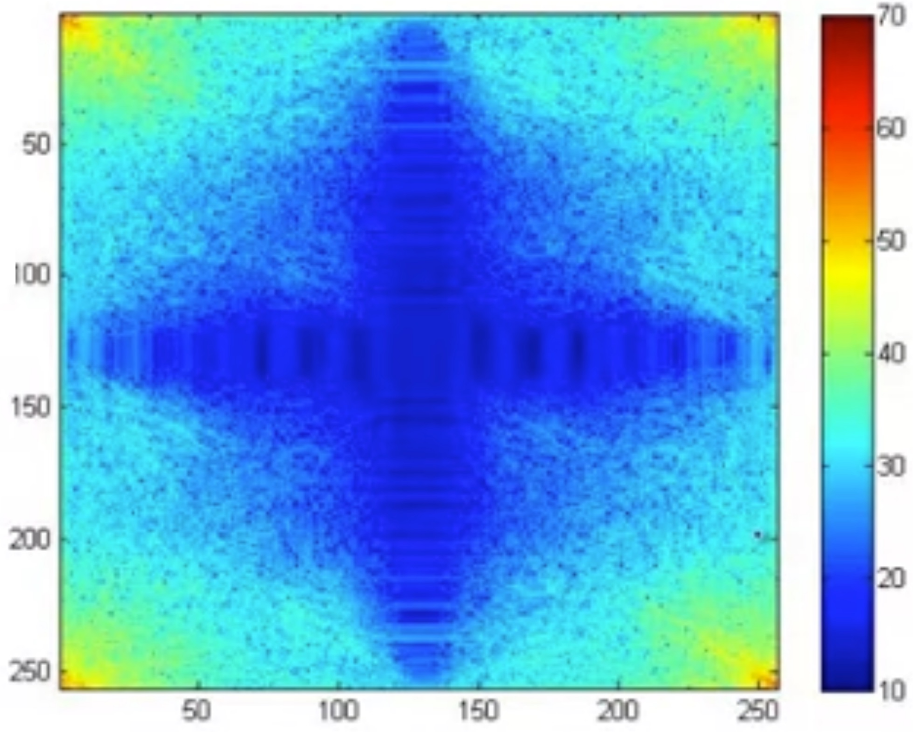
a b

c d

$\frac{a+b+c+d}{4}$

Up-sampled 256x256 image, with filter

0.25	0.5	0.25
0.5	1	0.5
0.25	0.5	0.25



Spectrum

$$H(\omega_1, \omega_2) = 1 + \cos\omega_1 + \cos\omega_2 + \frac{1}{2}\cos(\omega_1 + \omega_2) + \frac{1}{2}\cos(\omega_1 - \omega_2)$$

Fourier Transform and Sampling

- 2D Fourier Transform
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- **The Discrete Cosine Transform**
- The Two-Dimensional Discrete Cosine Transform

In general --

$x[n] = \sum_{k=0}^{N-1} b_k c_k[n]\,, \quad n = 0,1,2,\ldots,N$

b_k: coefficients $\quad c_k[n]$: "basis function"

DFT – complex sinusoids $c_k[n] = e^{j2\pi\left(\frac{k}{N}\right)n}$

Why?

LSI systems

$$e^{j\omega n} \longrightarrow \boxed{h(n)} \longrightarrow H(\omega)e^{j\omega n}$$

$$x[n] = \sum_{k=0}^{N-1} c_k e^{j\omega n} \longrightarrow \boxed{\text{LSI}} \longrightarrow y[n] = \sum_{k=0}^{N-1} H(\omega) c_k e^{j\omega n}$$

Output coefficients : product of input coefficients and frequency response

Filtering!

Compression

$$x[n] = \sum_{k=0} b_k c_k[n], \qquad n = 0, 1, \dots, N-1$$

If most $b_k \approx 0$, store / transmit large ones

$x[n]$: N values. If L, b_k are significant, save factor $\frac{N}{L}$

JPEG, MP3, MPEG, etc.

Discrete Cosine Transform

$$x[n] = \frac{1}{N} X^c[0] + \frac{1}{N} 2X^c[k] \cos(2\pi \left(\frac{k + 1/2}{2N}\right) n) , \qquad n = 0,1, \dots , N - 1$$

$$b_0 = \frac{1}{N} X^c[0], c_0[n] = 1 \quad b_k = \frac{1}{N} 2X^c[k], c_k[n] = \cos(2\pi \left(\frac{k + 1/2}{2N}\right) n)$$

$$X^c[k] = \sum_{n=0}^{N-1} x[n] \cos(2\pi (\frac{k}{2N})(n + 1/2))$$

$X^c[k]$ real!

$$\cos(2\pi \hat{f} n) \longrightarrow \boxed{\text{LSI}} \longrightarrow \left|H(\hat{f})\right| \cos(2\pi \hat{f} n + \phi(\hat{f})) \neq A(\hat{f}) \cos(2\pi \hat{f} n)$$

Fourier Transform and Sampling

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Objectives

- Interpret 2D-DCT: represent image as a sum of basis images
- Compression: set some DCT coefficients to zeros
- JPEG

Two-dimensional DCT

$$x[m,n] = \frac{1}{N}\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} a_k a_l X^c[k,l] \underbrace{\cos\left(2\pi(\frac{l}{2N})(n+\frac{1}{2})\right)\cos\left(2\pi(\frac{k}{2N})(m+\frac{1}{2})\right)}_{c_{k,l}[m,n]}$$

$$a_i = \begin{cases} 1 & i = 0 \\ \sqrt{2} & \text{otherwise} \end{cases}$$

$$x[m,n] = \sum_{k=0}^{N-1}\sum_{l=0}^{N-1} b_{k,l} c_{k,l}[m,n]$$

Weighted sum of “basis” images

$c_{k,l}[m,n]$: Product of a vertical (frequency $\frac{k}{2N}$) and horizontal (frequency $\frac{l}{2N}$) cosine

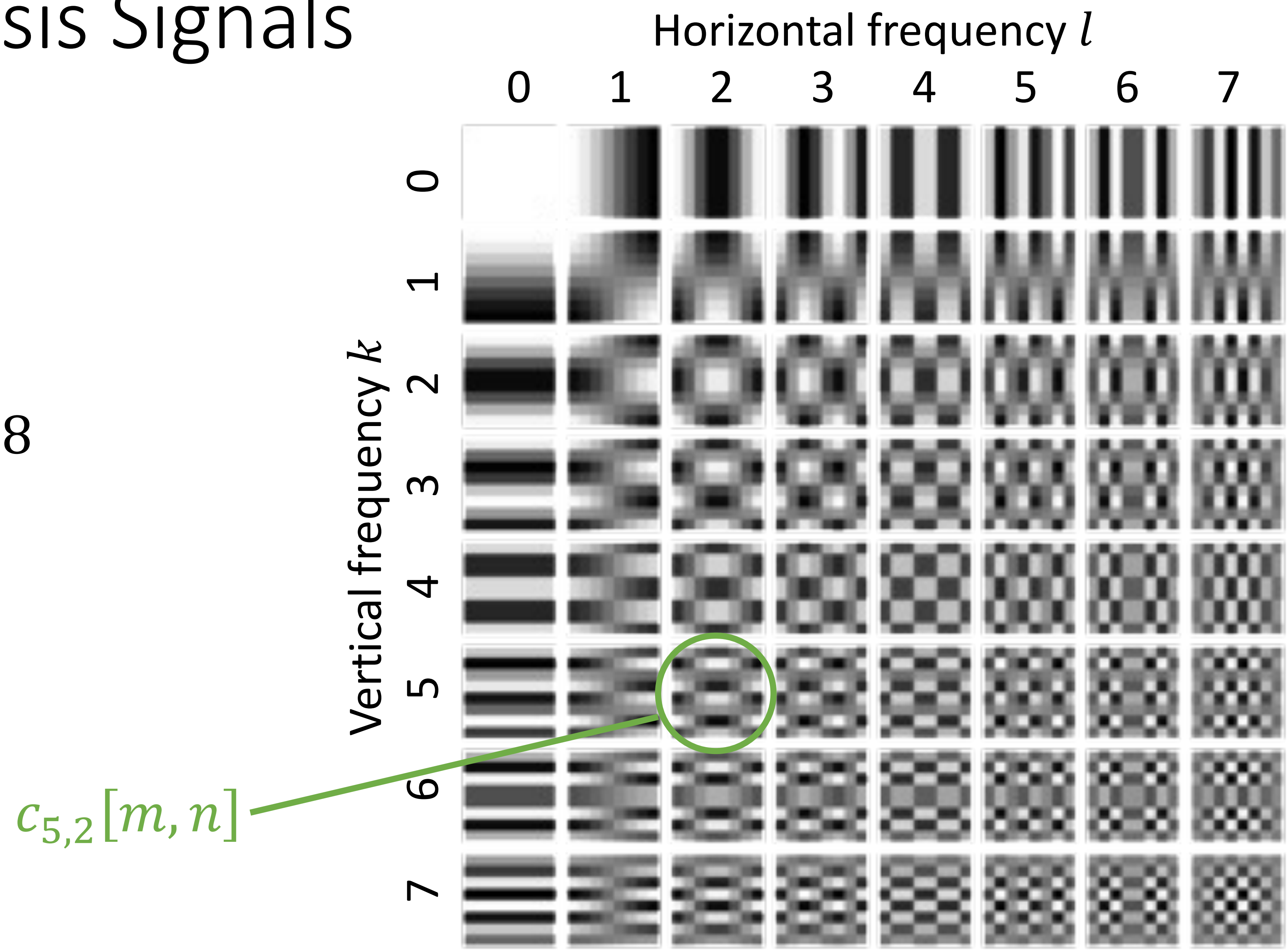
2D DCT Basis Signals

$c_{k,l}[m,n]$

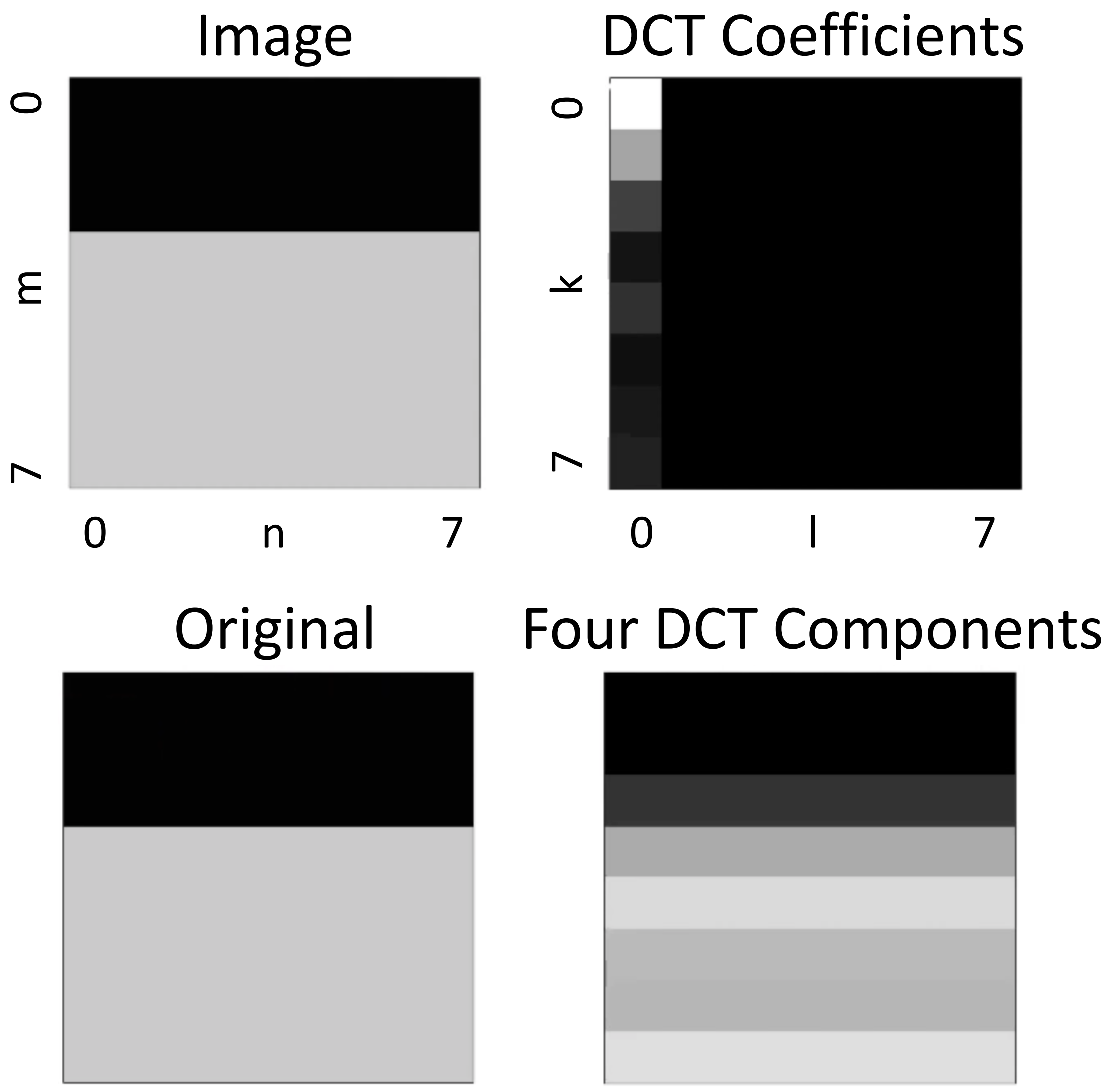
$N = 8$

(8×8 image)

Each entry is 8×8



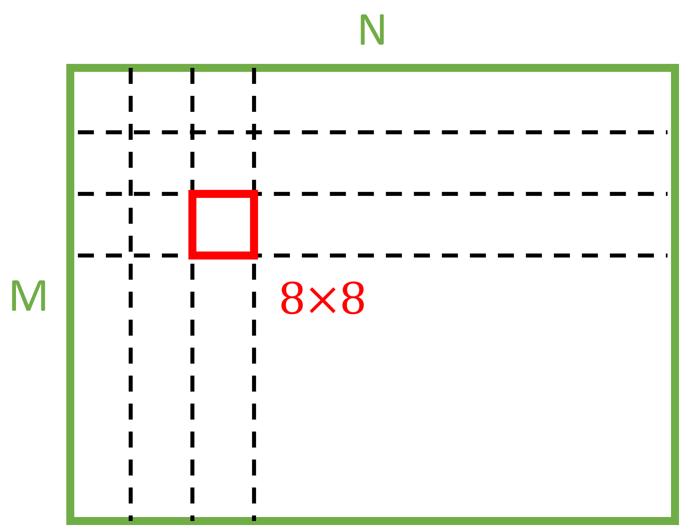
Example: Horizontal Edge



Compression
by factor
64/4=16

JPEG Image Compression

1. Break up $M \times N$ image into 8×8 blocks



2. Take 2D-DCT of each block to find DCT coefficients
3. Save important/significant DCT coefficients (eye is less sensitive to high spatial frequencies)
   - Allocate bits to each coefficients based on its significance/importance
   - More bits -> more accurate coefficient

JPEG Decoding

JPEG file has bits for each $8{\times}8$ block

1. Read bits for block
2. Convert bits to DCT coefficients
3. Take inverse DCT to find $8{\times}8$ image
4. Put blocks together

Two-dimensional DCT & IDCT

$$F(u,v) = \frac{2}{N} C(u) C(v) \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y) \cos\frac{(2x+1)u\pi}{2N} \cos\frac{(2y+1)v\pi}{2N}$$

$$f(x,y) = \frac{2}{N} \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} C(u) C(v) F(u,v) \cos\frac{(2x+1)u\pi}{2N} \cos\frac{(2y+1)v\pi}{2N}$$

$$\text{Where } C(u), C(v) = \begin{cases} \frac{1}{\sqrt{2}} & \text{for } u, v = 0 \\ 1 & \text{otherwise} \end{cases}$$

$$0 \le x, y, u, v \le N-1, N^2\text{: frame size}$$

One-dimensional DCT & IDCT

$$F(u,v) = \sqrt{\frac{2}{N}} C(u) \sum_{x=0}^{N-1} f(x) \cos \frac{(2x+1)u\pi}{2N}$$

$$f(x,y) = \sqrt{\frac{2}{N}} \sum_{u=0}^{N-1} C(u)F(u) \cos \frac{(2x+1)u\pi}{2N}$$

$$\text{Where } C(u) = \begin{cases} \frac{1}{\sqrt{2}} \text{ for } u = 0 \\ 1 \quad \text{otherwise} \end{cases}$$

$$0 \le x, u \le N-1, N^2\text{: frame size}$$

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Homework #2 – 2D-DCT

- 2D-DCT
  - Implement 2D-DCT to transform “lena.png” to DCT coefficients (visualize in log domain).
    - Convert the input image to gray-scale first.
    - Visualize the coefficients in log domain. Feel free to scale and clip the coefficients for visualization.
  - Implement 2D-IDCT to reconstruct the image.
  - Evaluate the PSNR.
- Two 1D-DCT
  - Implement a fast algorithm by two 1D-DCT to transform “lena.png ” to DCT coefficients.
- Compare the runtime between 2D-DCT and two 1D-DCT.
- Do **not** use any functions for DCT and IDCT, e.g., cv2.dct
  - Although, you can still use these functions to validate your output.
- Deadline: 2023/10/23 11:59 PM
- Upload to E3 with required files :
  - **VC_HW2_[student_id].pdf**: Report PDF
  - **VC_HW2_[student_id].zip**: Zipped source code (C/C++/Python/MATLAB)

Two-dimensional DCT & IDCT

$$F(u,v) = \frac{2}{N} C(u)C(v) \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y) \cos\frac{(2x+1)u\pi}{2N} \cos\frac{(2y+1)v\pi}{2N}$$

$$f(x,y) = \frac{2}{N} \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} C(u)C(v)F(u,v) \cos\frac{(2x+1)u\pi}{2N} \cos\frac{(2y+1)v\pi}{2N}$$

$$\text{Where } C(u), C(v) = \begin{cases} \frac{1}{\sqrt{2}} & \text{for } u, v = 0 \\ 1 & \text{otherwise} \end{cases}$$

$$0 \le x, y, u, v \le N-1, N^2\text{: frame size}$$

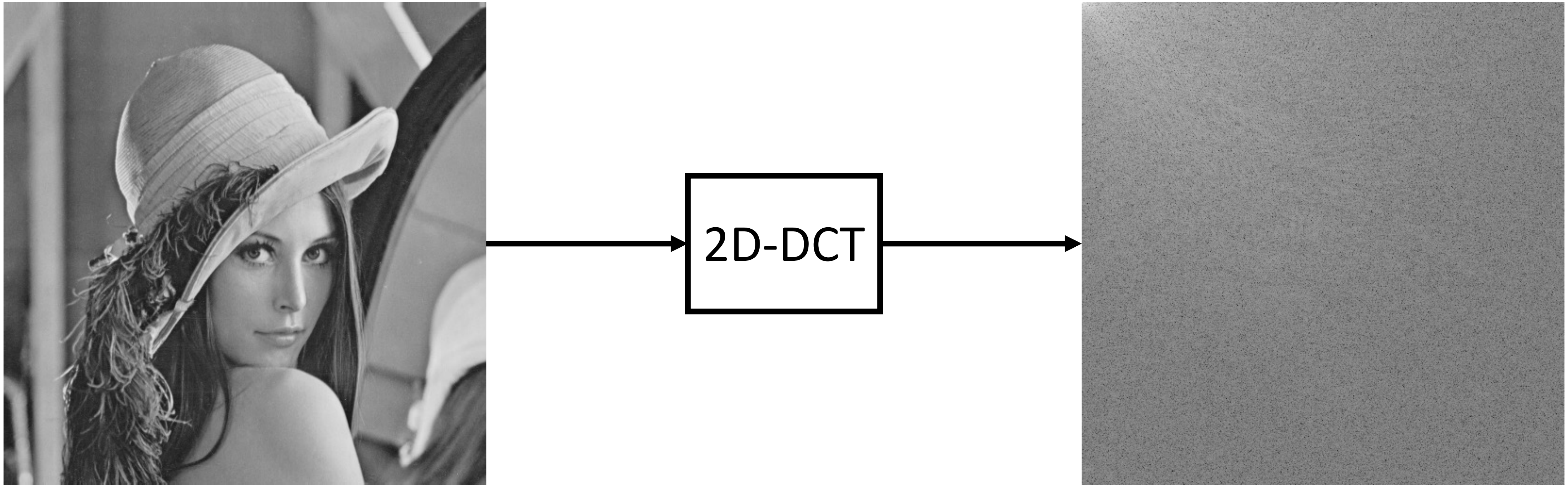
One-dimensional DCT & IDCT

$$F(u,v) = \sqrt{\frac{2}{N}}C(u)\sum_{x=0}^{N-1} f(x)\cos\frac{(2x+1)u\pi}{2N}$$

$$f(x,y) = \sqrt{\frac{2}{N}}\sum_{u=0}^{N-1} C(u)F(u)\cos\frac{(2x+1)u\pi}{2N}$$

$$\text{Where } C(u) = \begin{cases} \frac{1}{\sqrt{2}} & \text{for } u = 0 \\ 1 & \text{otherwise} \end{cases}$$

$$0 \leq x, u \leq N-1, N^2\text{: frame size}$$


2D-DCT