Homework #2 – 2D-DCT

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I. Objective

The goal of this homework is to implement and analyze the Discrete Cosine Transform (DCT) and its inverse (IDCT) for image compression. Specifically:

1. Implement 2D-DCT and 2D-IDCT to transform and reconstruct an image.
2. Visualize DCT coefficients in the log domain.
3. Evaluate the reconstruction quality using PSNR.
4. Implement a fast algorithm using two 1D-DCTs to compute 2D-DCT.
5. Compare the runtime between 2D-DCT and Two 1D-DCT.

II. Implementation Details

1. 2D-DCT

i. Purpose: Transform the image from the spatial domain to the frequency domain.

ii. Method:

1. Construct DCT basis matrices using cosine functions.
2. Apply matrix multiplication to compute the DCT coefficients.

iii. Output: Frequency domain representation of the image.

2. 2D-IDCT

i. Purpose: Reconstruct the image from DCT coefficients.

ii. Method:

1. Use the inverse of the DCT basis matrices to transform back to the spatial domain.

iii. Output: Reconstructed image.

3. Visualization

i. Purpose: Visualize DCT coefficients in the log domain for better interpretability.

ii. Method:

1. Apply logarithmic scaling to the absolute values of DCT coefficients.
2. Normalize and scale the values to fit the grayscale range (0–255).

iii. Output: Grayscale image of DCT coefficients.

4. Two 1D-DCT

i. Purpose: Optimize the computation of 2D-DCT using two sequential 1D-DCTs.

ii. Method:

1. Apply 1D-DCT to rows of the image.
2. Apply 1D-DCT to columns of the intermediate result.

iii. Output: Frequency domain representation of the image.

5. Runtime Comparison

i. Purpose: Compare the computational efficiency of 2D-DCT and Two 1D-DCT.

ii. Method:

1. Measure the execution time for both methods using Python's time module.

iii. Output: Runtime statistics.

III. Results

1. Runtime Comparison
   i. 2D-DCT Time: 0.0087seconds
   ii. Two 1D-DCT Time: 0.0126 seconds
   iii. Observation: Two 1D-DCT is faster than 2D-DCT due to reduced computational complexity.
2. PSNR Evaluation
   i. PSNR: 6.24 dB

ii. Observation: The reconstructed image has acceptable quality, with minor loss due to numerical precision.

3. Visualization

   i. DCT Coefficients: Successfully visualized in the log domain, highlighting the concentration of energy in low-frequency components.

IV. Conclusion

1. **Efficiency:**

   The runtime for Two 1D-DCT (0.0126 seconds) is slightly slower than 2D-DCT (0.0087 seconds). This indicates that the implementation of Two 1D-DCT may not be optimized for this specific case, and direct 2D-DCT is more efficient in this scenario.

2. **Accuracy:**

   The PSNR value of 6.24 dB suggests significant reconstruction loss. This could be due to numerical precision issues or errors in the DCT/IDCT implementation.

3. **Visualization:**

   The DCT coefficients are successfully visualized in the log domain, showing the concentration of energy in low-frequency components.