Operations Research, 2024 (112-2)
Final Project
Group N

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1 Introduction

There are many fires occurred every day. Each occurrence of fire requires fire truck to put out the fire. But if there are too many fires nearby, the station's fire trucks may be overwhelmed. Thus, if there is a way to allocate reasonably fire trucks in the city, the fires can be put out more efficiently.

In this project, we aim to appropriately reallocate the number of fire trucks in Taipei City. Since fires are unpredictable in nature and can occur in various forms and types, our goal is to strategically plan the deployment of different types of fire trucks to effectively address different types of fires. This ensures that fire trucks can be promptly dispatched to the locations where fires break out.

2 Data Process

2.1 Coordinate data

2.1.1 Fire locations

For the fire locations in each district, we choose a dot that is in the downtown of the district to represent the fire location.

2.1.2 Fire stations

For the fire station data, we gather it from open access from provided by government (https://data.gov.tw/dataset/128008). The dataset consists of the latitude and longitude of all 46 fire stations in Taipei City. We then convert the data into x and y coordinates respectively, which is shown as Figure 1 below.

區名	火災x座標(公里)	火災y座標(公里)		消防站	消防站x座標(公里)	消防站y座標(公里)
松山區	305.758	2771.65		1	302.552	2771.114
信義區	307.3562	2770.409		2	303.184	2770.719
大安區	304.7952	2769.499		3	301.717	2768.601
中山區	303.8061	2772.811		4	304.261	2767.18
中正區	302.3152	2769.577		5	301.807	2771.075
大同區	301.8184	2772.835		6	300.004	2769.06
萬華區	300.5294	2769.541		7	300.387	2770.144
文山區	306.697	2764.176		8	307.177	2766.126
南港區	310.8548	2771.873		9	304.631	2765.034
內湖區	308.455	2774.636		10	306.876	2764.656
士林區	303.0886	2776.264		11	306.021	2763.666
北投區	300.3012	2780.491		12	303.258	2769.481

Figure 1: Coordinates of fire locations and stations

2.2 Types of fire trucks and types of fire

2.2.1 Types of fire trucks

We researched information about fire truck types from the internet and found that there are many types available, most with similar functions. We simplified the categories into three main types: foam fire trucks, dry powder fire trucks, and water tank fire trucks. Each type is suited for different kinds of fires and has its own cost. Data was sourced from the Taipei City Fire Department website (https://www.119.gov.taipei/Content_List.aspx?n=B26CBB4DE8863422).

2.2.2 Types of fire

In general, there are four types of fires, we choose three of them as types of fires that may occur in Taipei City, namely type A fire (Regular), type B fire (Oil), type C fire (electric). The reason we didn't choose type D fire (metal) is that this kind of fire requires a special fire extinguisher to extinguisher. And we can't find a fire truck equipped with this kind of fire extinguisher. The correlation between fires and fire trucks is shown as Figure 2 below, "1" means that the fire truck can put out this fire, "0" means the opposite.

消防車種類	普通火災	油類火災	電氣火災	價格
泡沫消防車	1	1	0	900
乾粉消防車	0	1	1	1200
水箱消防車	1	0	0	700

Figure 2: Correlation between fire truck and fires

2.3 Population

The population data of each district is gathered from the Department of Civil Affairs, Taipei City Government (https://ca.gov.taipei/Default.aspx). The data is shown as Figure 3 below.

2.4 Number of fire cases

We collect the data of fire cases from the website of Taipei City Fire Department (https://pxweb.tfd.gov.tw/pxweb/Dialog/statfile9_n.asp) and average the data from the past 6 years. The data lists the number of fires that occurred due to different causes. We classify all the causes into three types of fires. The result is shown as Figure 4 below.

區名	平均人口數
松山區	193653
信義區	206356
大安區	291949
中山區	216125
中正區	150208
大同區	119869
萬華區	172944
文山區	260023
南港區	113682
內湖區	275843
士林區	266410
北投區	242050

Figure 3: Population of each district

平均每年	普通火災	油類火災	電氣火災
松山區	29	64	41
信義區	38	69	39
大安區	40	74	61
中山區	60	79	69
中正區	26	42	45
大同區	26	33	29
萬華區	47	80	51
文山區	47	87	38
南港區	20	32	20
內湖區	44	79	44
士林區	62	62	57
北投區	59	56	38

Figure 4: Number of fires of 3 types in each district

2.5 Distance

2.5.1 Actual Distance

Given the x and y coordinates of fire locations and fire stations, we can calculate the distance between each locations and stations with the built-in functions in excel.

2.5.2 Binary Variables

We set the service range of a fire station to be within a radius of 3 kilometers. It will be convenient to set up the binary variables of whether the distance is less than 3 kilometers in advance. The distance and binary variables are shown below as Figure 5 and 6, respectively, "1" means the distance is less than 3 kilometers, and "0" otherwise.

距離(公里)	1	2	3	4	5	6	7	8	9	10	11	12
1	3.250497	4.855653	2.764086	2.110113	1.555134	1.870831	2.562272	8.081885	8.33742	6.873856	5.17788	9.643352
2	2.737195	4.183701	2.020981	2.182538	1.434914	2.518396	2.904236	7.42644	7.757119	6.567064	5.545821	10.18835
3	5.062221	5.921946	3.206512	4.699834	1.144735	4.235214	1.514594	6.661909	9.705945	9.045544	7.784784	11.974
4	4.714012	4.472885	2.379733	5.649345	3.087353	6.159977	4.415785	3.867572	8.093358	8.554623	9.159344	13.8875
5	3.992621	5.589023	3.37833	2.647659	1.581857	1.760037	1.996351	8.456258	9.082923	7.541659	5.344925	9.535643
6	6.310041	7.474935	4.81127	5.34097	2.368319	4.188397	0.712324	8.285512	11.2095	10.12478	7.836605	11.43486
7	5.578143	6.974236	4.455138	4.336258	2.009837	3.048014	0.619586	8.685224	10.60963	9.234213	6.689771	10.34736
8	5.703344	4.286747	4.129177	7.486801	5.962089	8.586342	7.473474	2.008208	6.823065	8.605428	10.93133	15.92576
9	6.711303	6.026387	4.468018	7.820626	5.099194	8.292546	6.093945	2.237078	9.247032	10.33544	11.33543	16.05198
10	7.082793	5.773006	5.271089	8.713685	6.709481	9.616411	8.008905	0.51229	8.241113	10.10414	12.21024	17.14571
11	7.988331	6.873922	5.960409	9.4094	6.976595	10.08625	8.041971	0.846803	9.524729	11.23678	12.93478	17.77067
12	3.309768	4.201955	1.537305	3.374806	0.947675	3.649899	2.72926	6.322163	7.964486	7.32003	6.785115	11.40012
13	2.114172	1.844832	0.936531	3.798798	3.416046	5.117796	5.201602	5.446353	5.631598	5.781883	7.228296	12.2268
14	2.605043	2.796018	0.283787	3.736184	2.537467	4.705486	4.309796	5.37667	6.587011	6.521345	7.258253	12.1485
15	5.522279	4.214544	6.931221	7.512124	9.269995	9.492095	11.01281	9.03703	0.427354	3.87781	9.240677	13.89043

Figure 5: Distance between each fire locations and stations in kilometers

1/0	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	1	1	1	1	1	0	0	0	0	0
2	1	0	1	1	1	1	1	0	0	0	0	0
3	0	0	0	0	1	0	1	0	0	0	0	0
4	0	0	1	0	0	0	0	0	0	0	0	0
5	0	0	0	1	1	1	1	0	0	0	0	0
6	0	0	0	0	1	0	1	0	0	0	0	0
7	0	0	0	0	1	0	1	0	0	0	0	0
8	0	0	0	0	0	0	0	1	0	0	0	0
9	0	0	0	0	0	0	0	1	0	0	0	0
10	0	0	0	0	0	0	0	1	0	0	0	0
11	0	0	0	0	0	0	0	1	0	0	0	0
12	0	0	1	0	1	0	1	0	0	0	0	0
13	1	1	1	0	0	0	0	0	0	0	0	0
14	1	1	1	0	1	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	1	0	0	0
16	0	0	0	0	0	0	0	0	1	0	0	0
17	0	1	0	0	0	0	0	0	1	0	0	0

Figure 6: Binary variables of whether the distance is less than 3 kilometers

3 Parameters And Decision Variables Definition

We use two models for this project. The first model, called "region," uses integer programming to check if the distance is less than 3 kilometers. The second model, called "distance," calculates the exact distance between the fire and the fire station. We run both models to achieve our objectives: minimizing cost and minimizing the gap. Additionally, we aim to improve the gap by designing new constraints.

Firstly, we need to establish the parameters and decision variables, which are outlined as follows:

Parameters:

- Let I be the set of district, $I = \{1, 2, ..., 12\}$.
- Let J be the set of fire station, $J = \{1, 2, ..., 46\}$.

- Let $i \in I$ denotes the ith district, i is the index.
- Let $j \in J$ denotes the jth fire station, j is the index.
- Let p_i be the population of district i, $i \in I$.
- Let c_k be the cost of fire truck k, $k \in \{1, 2, 3\}$.
- Let f_{ir} be the number of fire r cases in district i, $r \in \{1, 2, 3\}$.
- Let x_{ij} be 1 if station j covers district i, $i \in I$, $j \in J$.
- Let y_{kr} be 1 if truck k can handle fire r, $k \in \{1, 2, 3\}$, $r \in \{1, 2, 3\}$.
- (A truck can be set for P people and H fires).

Decision variables:

- Let t_{jk} be the number of fire truck k in station j, $k \in \{1, 2, 3\}, j \in J$.

4 Integer Program

We want to minimize the total cost while deploying the fire truck.

The objective function we want to minimize is:

$$
\begin{aligned}
\min \quad & \sum_{j \in J} \sum_{k \in K} c_k t_{jk} & \\
\text{s.t.} \quad & P \sum_{j \in J} \sum_{k \in K} x_{ij} t_{jk} \geq p_i & \forall i \in I \\
& H \sum_{j \in J} \sum_{k \in K} x_{ij} y_{kr} t_{jk} \geq f_{ir} & \forall i \in I \quad \forall r \in \{1, 2, 3\} \\
& t_{jk} \geq 0 & \forall j \in J \quad \forall k \in \{1, 2, 3\}
\end{aligned}
$$

We define the objective value of the above model as c_{min}.

After optimize the model, we found out that the result is far different from what we expected. It's really unbalanced in the distribution of fire trucks between different fire stations.

Therefore, we want to improve the performance of the model by minimizing the gap between the result from out first model and the parameters. In this way, we can equally distribute the fire trucks to the fire stations.

The objective function we want to minimize is:

$$
\begin{aligned}
\min \quad & g & \\
\text{s.t.} \quad & g \geq w_j & \forall j \in J \\
& w_j \geq \sum_{k \in K} c_k t_{jk} - \frac{c_{min}}{|J|} & \forall j \in J \\
& P \sum_{j \in J} \sum_{k \in K} x_{ij} t_{jk} \geq p_i & \forall i \in I \\
& H \sum_{j \in J} \sum_{k \in K} x_{ij} y_{kr} t_{jk} \geq f_{ir} & \forall i \in I \quad \forall r \in K \\
& t_{jk} \geq 0 & \forall j \in J \quad \forall k \in K \\
& w_j \geq 0 & \forall j \in J
\end{aligned}
$$

Moreover, in another model that we previously mentioned to as distance, it is different from region. We add some pre-processes and rewrite some model constraints.

The pre-processes we add in distance model:

$$
H_{ij} = ttlH(1 - \frac{D_{ij}}{20})
$$
$$
P_{ij} = ttlP(1 - \frac{D_{ij}}{20})
$$

H_{ij} represent a parameter to determine the weight of how many people a fire truck can serve. If the distance between station j and district i is too long, the weight would be low, which means that station j can only serve few people in district i.

P_{ij} represent a parameter to determine the weight of how many fires a fire truck can cover. If the distance between station j and district i is too long, the weight would be low, which means that station j can only cover few fires in district i.

The revised version in distance model of minimizing cost:

$$
\begin{aligned}
\min \quad & \sum_{j\in J}\sum_{k\in K} c_k t_{jk} & \\
\text{s.t.} \quad & \sum_{j\in J}\sum_{k\in K} P_{ij} t_{jk} \geq p_i & \forall i \in I \\
& \sum_{j\in J}\sum_{k\in K} H_{ij} Y_{kr} t_{jk} \geq f_{ir} & \forall i \in I \quad \forall r \in R \\
& t_{jk} \geq 0 & \forall j \in J \quad \forall k \in \{1,2,3\}
\end{aligned}
$$

The revised version in distance model of minimizing gap:

$$
\begin{aligned}
\min \quad & g & \\
\text{s.t.} \quad & g \geq w_j & \forall j \in J \\
& w_j \geq \sum_{k\in K} c_k t_{jk} - \frac{c_{min}}{|J|} & \forall j \in J \\
& \sum_{j\in J}\sum_{k\in K} P_{ij} t_{jk} \geq p_i & \forall i \in I \\
& \sum_{j\in J}\sum_{k\in K} H_{ij} Y_{kr} t_{jk} \geq f_{ir} & \forall i \in I \quad \forall r \in R \\
& t_{jk} \geq 0 & \forall j \in J \quad \forall k \in K \\
& w_j \geq 0 & \forall j \in J
\end{aligned}
$$

5 Performance Analysis

We will talk about how the two model perform respectively.

5.1 Region

5.1.1 Cost

After running the Gurobi, we get the result of the cost of the region model is 61,300. The objective value of the cost is shown as Figure 7 below.

```
Optimal solution found (tolerance 1.00e-04)
Best objective 6.130000000000e+04, best bound 6.130000000000e+04, gap 0.0000%
```

Figure 7: The cost of region model

5.1.2 Allocation of fire trucks

After running the Gurobi, we get the result of the allocation of the fire trucks at every fire stations of region model. The objective value of the fire trucks allocation is shown as Figure 8 below. Moreover, we plot the results into a graph by Tableau. The graph is shown as Figure 9 below.

	Station	Truck 1	Truck 2	Truck 3
0	Station 1	-0.0	-0.0	-0.0
1	Station 2	3.0	7.0	3.0
2	Station 3	-0.0	-0.0	-0.0
3	Station 4	-0.0	-0.0	-0.0
4	Station 5	-0.0	-0.0	-0.0
5	Station 6	-0.0	-0.0	-0.0
6	Station 7	-0.0	-0.0	-0.0
7	Station 8	6.0	-0.0	-0.0
8	Station 9	-0.0	-0.0	-0.0
9	Station 10	-0.0	-0.0	-0.0
10	Station 11	-0.0	5.0	-0.0
11	Station 12	-0.0	-0.0	-0.0
12	Station 13	-0.0	-0.0	-0.0
13	Station 14	-0.0	-0.0	-0.0
14	Station 15	-0.0	-0.0	-0.0
15	Station 16	-0.0	-0.0	-0.0
16	Station 17	-0.0	-0.0	-0.0
17	Station 18	-0.0	-0.0	-0.0
18	Station 19	-0.0	-0.0	-0.0
19	Station 20	-0.0	-0.0	-0.0
20	Station 21	-0.0	-0.0	-0.0
21	Station 22	-0.0	-0.0	-0.0
22	Station 23	-0.0	-0.0	-0.0
23	Station 24	-0.0	-0.0	-0.0

Figure 8: The allocation of region model

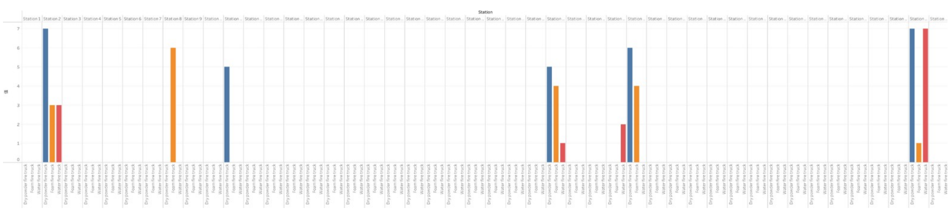

Figure 9: The graph of region model by Tableau

5.1.3 Gap

After running the Gurobi, we get the result of the gap of region model. The objective value of the gap shown as Figure 10 below. After we add the new constraint in order to reduce the gap, the optimal solution is more balanced than the previous one. Also, we plot the results into a graph by Tableau. The graph is shown as Figure 11 below.

```
Maximum gap: 2667.391304347826
Budget per station: 1332.608695652174
```

Figure 10: The allocation of region model

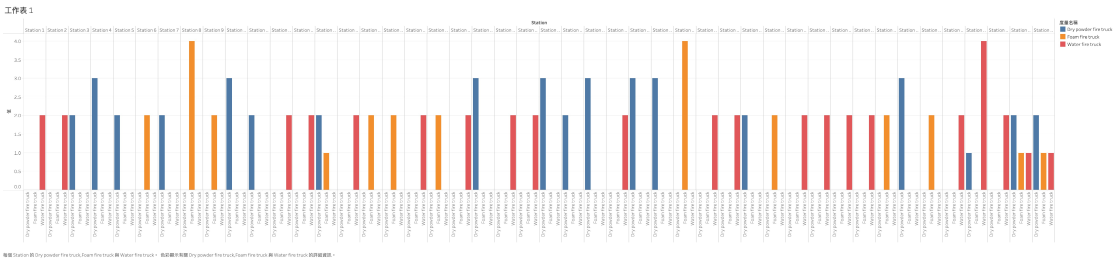

Figure 11: The graph of the adjusted gap of region model by Tableau

5.2 Distance

5.2.1 Cost

After running the Gurobi, we get the result of the cost of the distance model is 110,200. The objective value of the cost shown as Figure 12 below.

```
Optimal solution found (tolerance 1.00e-04)
Best objective 1.102000000000e+05, best bound 1.102000000000e+05, gap 0.0000%
```

Figure 12: The cost of distance model

5.2.2 Allocation of fire trucks

After running the Gurobi, we get the result of the allocation of the fire trucks at every fire stations of distance model. The objective value of the fire trucks allocation shown as

Figure 13 below. Moreover, we plot the results into a graph by Tableau. The graph is shown as Figure 14 below.

	Station	Truck 1	Truck 2	Truck 3
0	Station 1	0.000000	0.000000	0.000000
1	Station 2	0.000000	0.000000	0.000000
2	Station 3	0.000000	0.000000	0.000000
3	Station 4	0.000000	0.000000	0.000000
4	Station 5	0.000000	0.000000	0.000000
5	Station 6	0.000000	0.000000	0.000000
6	Station 7	0.000000	0.000000	0.000000
7	Station 8	0.000000	0.000000	0.000000
8	Station 9	0.000000	0.000000	0.000000
9	Station 10	3.172484	0.000000	0.000000
10	Station 11	0.000000	0.000000	0.000000
11	Station 12	0.000000	0.000000	0.000000
12	Station 13	0.000000	0.000000	0.000000
13	Station 14	3.285532	0.000000	0.000000
14	Station 15	0.000000	0.000000	0.000000
15	Station 16	0.000000	0.000000	0.000000
16	Station 17	0.000000	0.000000	0.000000
17	Station 18	0.000000	0.000000	0.000000
18	Station 19	0.000000	0.000000	0.000000
19	Station 20	0.000000	0.000000	0.000000
20	Station 21	0.000000	0.000000	0.000000
21	Station 22	0.000000	3.885246	103.185687
22	Station 23	0.000000	0.000000	0.000000
23	Station 24	0.000000	0.000000	0.000000

Figure 13: The allocation of distance model

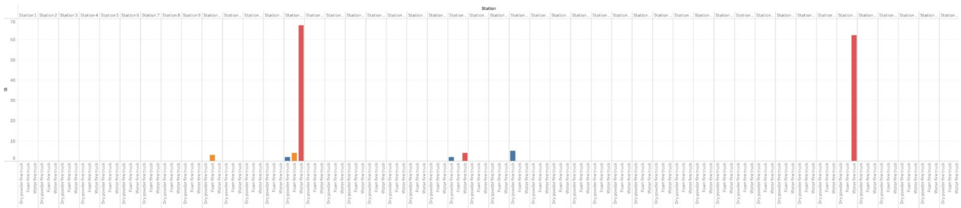

Figure 14: The graph of distance model by Tableau

5.2.3 Gap

After running the Gurobi, we get the result of the gap of distance model. The objective value of the gap shown as Figure 15 below. After we add the new constraint in order to reduce the gap, the optimal solution is more balanced than the previous one. Also, we plot the results into a graph by Tableau. The graph is shown as Figure 16 below.

```
Maximum gap: 409.64211780323194
Budget per station: 2390.357882196768
```

Figure 15: The allocation of distance model

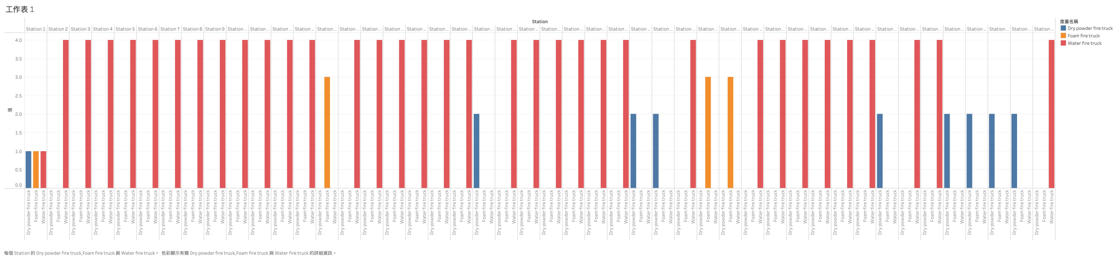

Figure 16: The graph of the adjusted gap of distance model by Tableau

6 Conclusion and Future Works

6.1 Conclusion

After we get the result of two model, we can compare the two model on the minimum cost. We can find out that the region model has the lower minimum cost and has the better balanced allocation. In the region model, the standard deviation before the gap adjustment is 3.58. And the standard deviation of the gap function is only 0.75, which means that this function actually help distribute fire trucks evenly across districts. Besides, We found that the two model at deploying the fire trucks is different. In region model, 3 types of fire trucks deploy evenly. However, in distance model, mainly dispatched by water tank fire truck.

6.2 Future works

- Get the data that the break out location of every fire.
- Precisely know the cost of allocating fire trucks to each fire station.
- Adjust the parameters of people each fire truck can rescue and the number of fires it can handle.