Homework 1

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1

(a)
$T = 1, 2, ..., n$
$I = 1, 2, ..., m$
Denote x_{it} as the amount of air conditioners that was produced in site i during month t, $\forall i \in I$, $\forall t \in T$
Denote y_t as the amount of air conditioners inventory that was left in month t, $\forall t \in T$

$min \sum_{t=1}^{6} \sum_{i \in I} \mathrm{C}_i^P x_{it} + \sum_{t=1}^{6} \mathrm{C}^H y_t$

$s.t.$ $y_{t-1} + \sum_{i \in I} x_{it}$ - $D_t = y_t$, $\forall t \in [1, 2, ..., 6]$
$\mathrm{y}_0 = I$
$x_{it} L_i \leq K$, $\forall i \in I$
$x_{it}, y_t \geq 0$, $\forall t \in [1, 2, ..., 6]$, $\forall i \in I$

(b)
Denote x_{it} as the amount of air conditioners that was produced in site i during month t, $\forall i \in I$, $\forall t \in T$
Denote y_t as the amount of air conditioners inventory that was left in month t, $\forall t \in T$
Denote z_t as the amount of air conditoners that was sold during month t, $\forall t \in T$

$max \sum_{t=1}^{6} 600 z_t$ - $\sum_{t=1}^{6} \sum_{i \in I} C^P x_{it} - \sum_{t=1}^{6} C^H y_t$

$s.t.$ $y_{t-1} + \sum_{i \in I} x_{it}$ - $D_t = y_t$, $\forall t \in [1, 2, ..., 6]$
$\mathrm{y}_0 = I$
$x_{it} L_i \leq K$, $\forall i \in I$
$z_t \leq D_t$ $\forall t \in [1, 2, ..., 6]$
$x_{it}, y_t, z_t \geq 0$, $\forall t \in [1, 2, ..., 6]$, $\forall i \in I$

(c)
$OptimalSolution$

sites	month 1	month 2	month 3	month 4	month 5	month 6
Hsinchu	1800	2000	2000	2000	2000	2000
Taoyuan	1600	1600	1600	1600	1600	1600

$ObjectiveValue$: 8784000

2

(a)
$I = [1, 2, ..., n]$
Denote x_i as whether district i has an ambulance, $\forall i \in I$
Denote w_i as $\min_{j \in I}(D_{ij}x_j + M(1 - x_j)) \leq B$, $\forall i \in I$
Denote y_i as the binary variable whether for district i, there exists a district that has at least one ambulance and the travel time is less than or equal to B

$$y_i = \begin{cases} 1 & \text{if w}_i \leq B \\ 0 & otherwise \end{cases}$$

$max \sum_{i \in I} H_i y_i$

$s.t.$ M is a very large number

$$w_i \leq D_{ij}x_j + M(1 - x_j),\ \forall j \in I, \forall i \in I$$

$$w_i - B \leq M(1 - y_i)$$

$$\sum_{i \in I} x_i = p$$

$$x_i, y_i \in [0, 1], \forall i \in I$$

(b)
$I = [1, 2, ..., n]$
Denote x_i as how many ambulances there are in district i, $\forall i \in I$
Denote y_{ij} as how many ambulances there are in district j and the travel time between district i and district j is less than or equal to B, $\forall i \in I$, $\forall j \in I$

$$y_{ij} = \begin{cases} x_j & \text{if } D_{ij} \leq B \\ 0 & otherwise \end{cases}$$

Denote w_i as the binary vairable whether for district i, the amount of ambulances that the travel time to district i is greater than or equal to 2, $\forall i \in I$

$$w_i = \begin{cases} 1 & \text{if } \sum_{j \in I} y_{ij} \geq 2 \\ 0 & otherwise \end{cases}$$

$max \sum_{i \in I} H_i w_i$

$s.t.$ M is a very large number

$\sum_{j \in I} -2 \leq M w_i, \forall i \in I$

$y_{ij} \leq x_j, \forall i \in I, \forall j \in I$

$D_{ij} - B \leq M(x_j - y_{ij}), \forall i \in I, \forall j \in I$

$\sum_{i \in I} x_i = p$

$x_i, w_i \in [0,1], \forall i \in I$

$y_{ij} \in [0,1], \forall i \in I, \forall j \in I$

(c)
$I = [1, 2, ..., n]$
Denote x_i as how many ambulances there are in district i, $\forall i \in I$
Denote y_{ijk} as how many ambulances there are in district j and the travel time between district i and district j is less than or equal to B_k,$\forall i \in I$, $\forall j \in I, \forall k \in [1,2,3]$

$$y_{ijk} = \begin{cases} x_j & \text{if } D_{ij} \leq B_k \\ 0 & otherwise \end{cases}$$

Denote A_k as the minimum value of ambulances needed, $A_1 = 2, A_2 = 1, A_3 = 3$
Denote w_{ik} as the binary vairable whether for district i, the amount of ambulances that the travel time to district i is greater than or equal to A_k, $\forall i \in I, \forall k \in [1,2,3]$

$$w_{ik} = \begin{cases} 1 & \text{if } \sum_{j \in I} y_{ijk} \geq A_k \\ 0 & otherwise \end{cases}$$

$z_i = \text{max}(w_{i2}, w_{i3}), \forall i \in I$
$v_i = \text{min}(w_{i1}, z_i), \forall i \in I$

$max \sum_{i \in I} H_i v_i$

$s.t.$ M is a very large number

$y_{ijk} \leq x_j, \forall i \in I, \forall k \in [1,2,3]$

$\sum_{i \in I} x_i = p$

$\sum_{j \in I} y_{ijk} - A_k \leq M w_{ik}, \forall i \in I, \forall k \in [1, 2, 3]$

$D_{ij} - B_k \leq M(1 - y_{ijk}), \forall i \in I, \forall j \in I, \forall k \in [1, 2, 3]$

$x_i \in [0, 1], \forall i \in I$

$y_{ijk} \in [0, 1], \forall i \in I, \forall j \in I, \forall k \in [1, 2, 3]$

$w_{ik} \in [0, 1], \forall i \in I, \forall k \in [1, 2, 3]$

$z_i \geq w_{ik}, \forall k \in [2, 3], \forall i \in I$

$v_i \leq w_{i1}, \forall i \in I$

$v_i \leq z_i, \forall i \in I$

3

(a)
Denote x_{ij} as whether item i is in bag j, $\forall i \in I$, $\forall j \in [1, 2]$

$max \sum_{i \in I} \sum_{j=1}^{2} x_{ij} V_i$

$s.t. \sum_{i \in I} x_{ij} W_i \leq K \ \forall j \in [1, 2]$

$x_{ij} \in [0, 1], \forall i \in I, \forall j \in [1, 2]$

(b)
Denote x_{ij} as whether item i is in bag j, $\forall i \in I$, $\forall j \in [1, 2]$

$max \sum_{i \in I} \sum_{j=1}^{2} x_{ij} V_i$

$s.t. \ x_{2j} + x_{3j} \leq 1, \forall j \in [1, 2]$

$x_{4j} + x_{5j} + x_{6j} \leq 2, \forall j \in [1, 2]$

$\sum_{i=8}^{12} \sum_{j=1}^{2} x_{ij} \geq 2$

$\sum_{i=1}^{2} \sum_{j=1}^{2} x_{ij} \geq 1 - (x_{31} + x_{32})$

$\sum_{i \in I} x_{ij} W_i \leq K, \forall j \in [1, 2]$

$x_{ij} \in [0, 1], \forall i \in I, \forall j \in [1, 2]$

(c)
Denote x_{ij} as whether item i is in bag j, $\forall i \in I$, $\forall j \in [1, 2]$
Denote z_i as whether item i and item $i{+}1$ were both carried, $\forall i \in [1, 2, ..., n{-}1]$
Denote y_{ij} as whether item i and item $i + 1$ were both carried in bag j $\forall i \in [1, 2, ..., n - 1]$, $\forall j \in [1, 2]$

$$max \sum_{i\in I} \sum_{j=1}^{2} x_{ij} V_i + \sum_{i=1}^{n-1} A_i z_i + \sum_{i=1}^{n-1} \sum_{j=1}^{2} y_{ij} Bi$$

$$s.t. \sum_{i\in I} x_{ij} W_i \leq K,\ \forall j \in [1, 2]$$

$$z_i \leq \sum_{j=1}^{2} (x_{ij} + x_{(i+1)j}),\ \forall i \in [1, 2, ..., n - 1]$$

$$y_{ij} \leq x_{ij} + x_{(i+1)j},\ \forall j \in [1, 2],\ \forall i \in [1, 2, ..., n - 1]$$

$$x_{ij} \in [0, 1],\ \forall i \in I, \forall j \in [1, 2]$$

$$z_i \in [0, 1],\ \forall i \in [1, 2, ..., n - 1]$$

$$y_{ij} \in [0, 1],\ \forall i \in [1, 2, ..., n - 1], \forall j \in [1, 2]$$

4

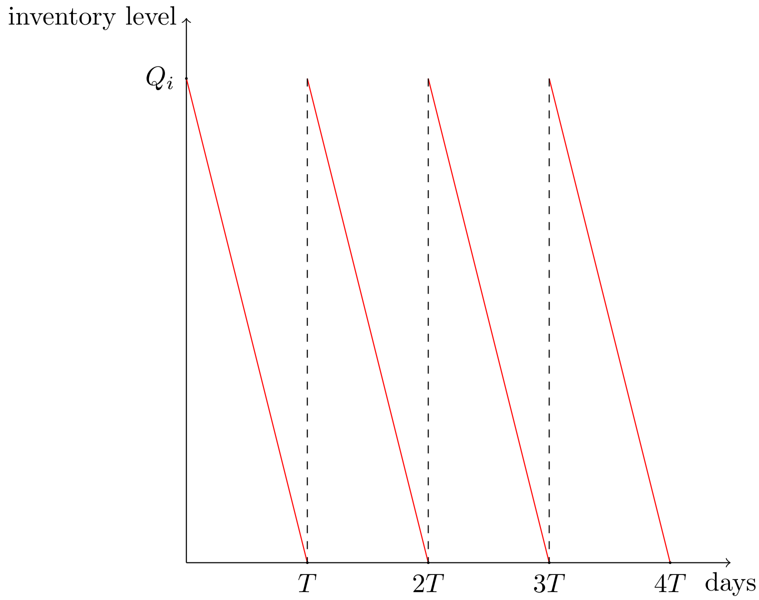
$I = [1, 2, ..., n]$
Denote T as how many days in a cycle
Denote Q_i as how many units of item i were purchased every cycle, $\forall i \in I$

$$min\ \frac{K}{T} + \sum_{i\in I} \frac{Q_i h_i}{2}$$

$$s.t.\ Q_i = D_i T,\ \forall i \in I$$

$$Q_i \geq 0, \forall i \in I$$

$$T \geq 0$$


inventory level
Q_i
T
$2T$
$3T$
$4T$
days