Homework 0

B11705028 IM2 Siang-Ruei Hu

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1

(a) If $f(x)$ is maximized at $x = 2$, the critical point of $f(x)$ should lands on $x = 2$. We can set the derivative $f'(x)$ equal to zero to find the critical points.

$f'(x) = 2ax + 8$
$2ax + 8 = 0$
$2ax = -8$
$x = -\frac{4}{a}$

Now, we want to find the values of a such that $x = 2$ is a critical point. $2 = -\frac{4}{a}$
$a = -2$

So, $a = -2$ is the value for which $f(x)$ is maximized at $x = 2$.

(b) First, find the antiderivative of $f(x)$:

$$G(x) = \int f(x)\,dx = \frac{a}{3}x^3 + 4x^2 + 6x + C$$

where C is the constant of integration.

Then $F(x) = G(x) - G(a)$

$$F(t) = G(t) - G(a) = \left(\frac{a}{3}x^3 + 4x^2 + 6x + C\right)\Big|_0^t = \frac{a}{3}t^3 + 4t^2 + 6t$$

Therefore, $F(t) = \frac{a}{3}t^3 + 4t^2 + 6t$ for all $t > 0$.

(c) If the determinant of the matrix equals to 0, then the inverse of the matrix doesn't exist.

$det(A) = (1\times1\times4+a\times1\times1+3\times0\times2)-(1\times1\times3+2\times1\times1+4\times0\times a) = a-1$

If $a - 1 = 0$, then a equals to 1.

2

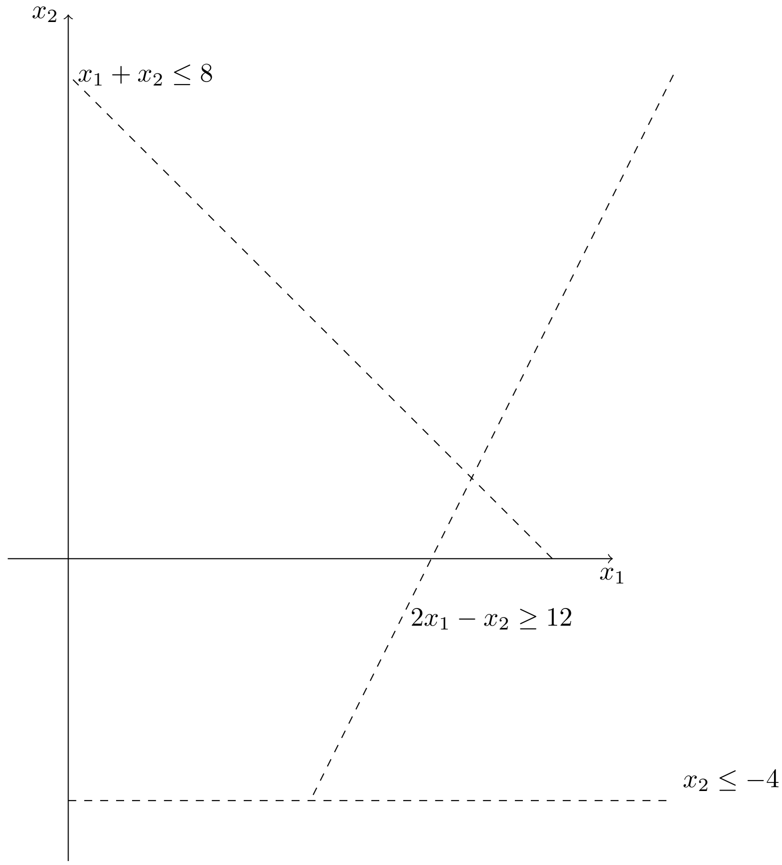
(a) Algorithm isPrime(n):

```
if n ≤ 1:
    Print "n is not a prime number."
if n ≤ 2:
    Print "n is a prime number."
if 2 or 3 can divide n:
    Print "n is not a prime number."
i = 5;w = 2;
while i × i ≤ n:
    if i can divide n:
        Print "n is not a prime number."
    i = i + w;
    w = 6 − w;
Print "n is a prime number."
```

(b) Time Complexity : $O(\sqrt{n})$

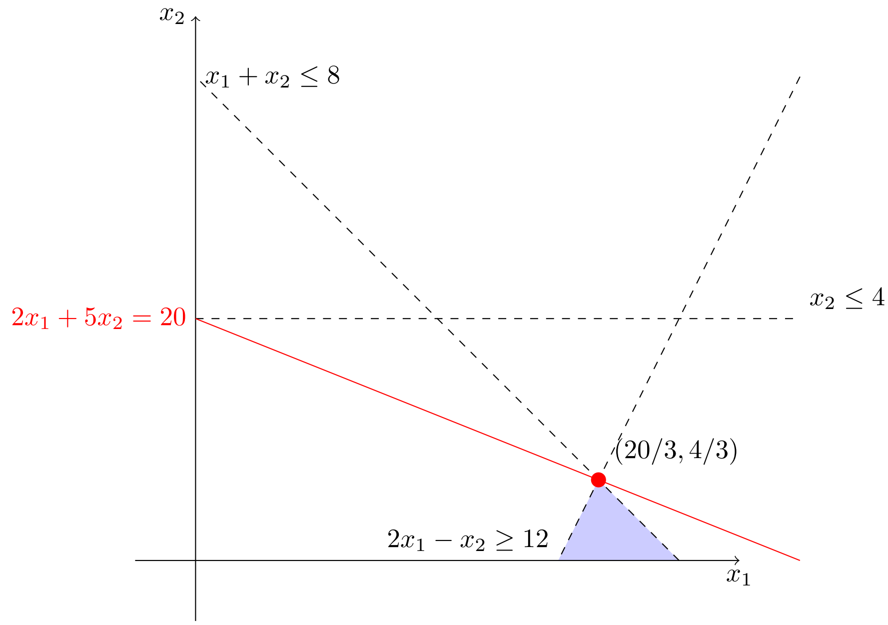
3

(a)



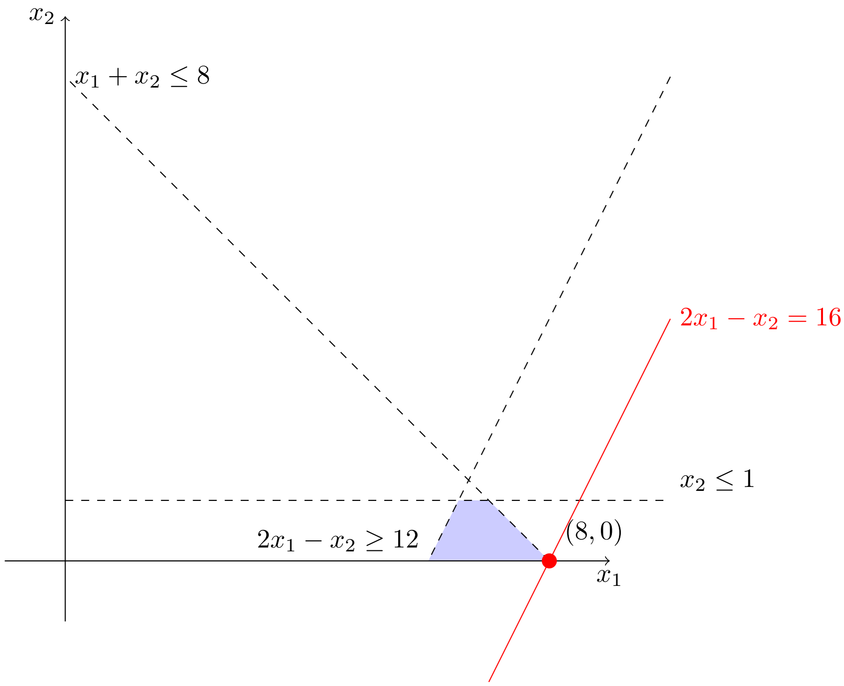
$x_2 \leq -4$ and $x_2 \geq 0$, contradictory

(b)



There is only one optimal solution, which is $(x_1, x_2) = (\frac{20}{3}, \frac{4}{3})$.
There are two constraints binding at that optimal solution, one is $x_1 + x_2 \leq 8$, the other one is $2x_1 - x_2 \geq 12$

(c)



There is only one optimal solution, which is $(x_1, x_2) = (8, 0)$.
There are two constraints binding at that optimal solution, one is $x_1 + x_2 \leq 8$, the other one is $x_2 > 0$.

4

(a) Denote x_{11} as the amount of oil that is sent from Kaohsiung to Hsinchu in million barrels, x_{12} as the amount from Kaohsiung to Taichung, x_{21} as the amount from Taipei to Hsinchu, x_{22} as the amount from Taipei to Taichung.

$max\ P_{11}x_{11} + P_{12}x_{12} + P_{21}x_{21} + P_{22}x_{22}$

$$
\begin{aligned}
s.t.\ & x_{11} + x_{12} \leq K_1 \\
& x_{21} + x_{22} \leq K_2 \\
& x_{11}\ ,\ x_{12}\ ,\ x_{21}\ ,\ x_{22} \leq D \\
& x_{11}\ ,\ x_{12}\ ,\ x_{21}\ ,\ x_{22} \geq 0
\end{aligned}
$$

(b) Denote x_{ij} as the amount of oil that is sent form city i to city j in million barrels.

$max \sum_{i=1}^{n} \sum_{j=1}^{m} \mathrm{P}_{ij} x_{ij}$

$s.t. \sum_{j=1}^{m} \mathrm{x}_{ij} \leq \mathrm{K}_i \ \forall i \in [1, 2, ..., n]$

$\sum_{i=1}^{n} \mathrm{x}_{ij} \leq \mathrm{D} \ \forall j \in [1, 2, ..., m]$

$x_{ij} \geq 0 \ \forall i \in [1, 2, ..., n] \ \forall j \in [1, 2, ..., m]$