Operations Research, Spring 2024 (112-2)

Final Project Proposal

Team N

B11705028 Siang-Ruei Hu

B11705044 Yen-Hung Chiang

B11705046 Yun-Shi Lee

B11705053 Ting-Yu Chen

B11705058 Chen-Hsiang Liao

Department of Information Management, Nation Taiwan University

March 30, 2024

1 Introduction

In this project, we are attempting to rearrange the locations of Taipei City fire department stations and allocate the number of fire trucks appropriately. Since the occurrence of fires is unpredictable in terms of both timing and location, we are striving to strategically plan the placement of fire stations to ensure that fire trucks can be dispatched promptly to any location where a fire breaks out.

The location of fire department facilities is equally important for emergency response effectiveness and the allocation of ambulance services because it directly impacts the efficiency and outcomes of emergency rescue operations. Failure to promptly and properly handle casualties can lead to a higher mortality rate. Therefore, we have decided to optimize the siting of fire department facilities through this project to enhance rescue efficiency and expand the scope of rescue operations.

We have organized the number of fires that occurred in each district of Taipei City over the past 6 years, assessing which areas require more firefighting resources. We have then strategically increased the number of fire stations in proximity to these areas to minimize the distance between fire locations and fire stations.

To address this issue, we will divide Taipei City into coordinate axes and determine the locations for setting up fire stations to efficiently allocate fire trucks, and we will establish a mixed linear integer program to optimize this problem.

Figure 1: Fire stations in Taipei

Figure 2: Fire stations in Taipei

2 Problem description

We first converted the entire map of Taipei City into a two-dimensional vector graphic, making it easier to mark the locations of each fire and fire station. Next, given the amounts of fire occur in each district, we assume an even distribution of fires within each district.

For each fire station, we set that its service range must be within a radius of 3 kilometers. Therefore, fire locations located more than 3 kilometers away from a fire station will not be taken into consideration.

We assume that the maximum amount of fire trucks in Taipei City is fixed. Each fire station must be allocated at least one fire truck, and a maximum of five. And if a fire station has more fire trucks, its total distance between it and the fire locations that the station serve will decrease.

The objective value is to minimize the distance between each fire location and each fire station.

3 Mathematical model

Let $I = \{1, 2, ..., n\}$ be the set of fire stations, and let $J = \{1, 2, ..., m\}$ be the set of fire locations.

Denote x_i as the x-coordinate of the station i, $\quad \forall i \in I$
Denote y_i as the y-coordinate of the station i, $\quad \forall i \in I$
Denote v_j as the x-coordinate of the fire location j, $\quad \forall j \in J$
Denote w_j as the y-coordinate of the fire location j, $\quad \forall j \in J$

Denote d_{ij} as the distance between the fire station i and the fire location $j, \forall i \in I, \forall j \in J$

$$d_{ij} = \sqrt{(x_i - v_j)^2 + (y_i - w_j)^2}, \quad \forall i \in I, \forall j \in J$$

To determine whether the distance between station i and fire location j is less than or equal to 3 km. We denote z_{ij} as the binary variable whether the abovementioned discription is true.

$$z_{ij} = \begin{cases} 1 & \text{if } d_{ij} \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Denote q_i as the number of fire trucks that is allocated to fire station $i, q_i \in \{1, 2, 3, 4, 5\}$, $\forall i \in I$. And if a station has q fire trucks, then the total sum of distance for the station will be divided by q.

Denote X as the maximum value of the x-coordinate on the map and denote Y as the maximum value of the y-coordinate on the map.

$$\min \quad \sum_{i=1}^{n} \sum_{j=1}^{m} d_{ij} z_{ij} \frac{1}{q_i}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} q_i \leq F, \text{ where } F \text{ is the total amount of fire trucks}$$

$$d_{ij} = \sqrt{(x_i - v_j)^2 + (y_i - w_j)^2}, \quad \forall i \in I, \forall j \in J$$

$$d_{ij} - 3 \leq M(1 - z_{ij}), \quad \forall i \in I, \forall j \in J$$

$$q_i \in \{1, 2, 3, 4, 5\}, \quad \forall i \in I$$

$$x_i, v_i \geq 0, x_i, v_i \leq X, \quad \forall i \in I$$

$$y_i, w_i \geq 0, y_i, w_i \leq Y, \quad \forall i \in I$$

$$d_{ij} \geq 0, \quad \forall i \in I, \forall j \in J$$

$$z_{ij} \in \{0, 1\}$$

4 Expected results

We will first collect the data from National Police Agency. The data includes the frequency of fires and the location of fires in the past 6 years. We will use those data in our program to solve the question. We will also use GIS to generate coordinates and apply it into our program. After collect the data, we will write code to solve the program.

We plan to solve the problem by using Gurobi Optimizer with Python to get the optimal value. And all the x-coordinate and y-coordinate of the best solution demonstrate the best locations of fire stations.