Team Note of Freshman Dognodap

sean9892, jjaewon9, pentagon03

Compiled on July 14, 2022

Contents

ALL BELOW HERE ARE USELESS IF YOU READ THE STATEMENT WRONG

1 Data Structure

1.1 Erasable Priority Queue

```
template<typename T, T inf>
struct pq_set{
  priority_queue<T, vector<T>, greater<T>> in, out; // min heap, inf = 1e18
  // priority_queue<T> in, out; // max heap, inf = -1e18
  pq_set(){ in.push(inf); }
  void insert(T v){ in.push(v); }
  void erase(T v){ out.push(v); }
  T top(){
  while(out.size() && in.top() == out.top()) in.pop(), out.pop();
    return in.top();
  }
  bool empty(){
  while(out.size() && in.top() == out.top()) in.pop(), out.pop();
    return in.top() == inf;
  }
};
```

1.2 Persistent Segment Tree

```
struct PSTNode{
  PSTNode *l, *r; int v;
  PSTNode(){ l = r = nullptr; v = 0; }
};
PSTNode *root[101010];
PST(){ memset(root, 0, sizeof root); } // constructor
void init(PSTNode *node, int s, int e){
  if(s == e) return;
  int m = s + e >> 1;
  node->l = new PSTNode; node->r = new PSTNode;
  init(node->l, s, m); init(node->r, m+1, e);
}
void update(PSTNode *prv, PSTNode *now, int s, int e, int x){
  if(s == e){ now->v = prv ? prv->v + 1 : 1; return; }
  int m = s + e >> 1;
  if(x <= m){
    now->l = new PSTNode; now->r = prv->r;
    update(prv->l, now->l, s, m, x);
  }
  else{
    now->r = new PSTNode; now->l = prv->l;
    update(prv->r, now->r, m+1, e, x);
  }
  int t1 = now->l ? now->l->v : 0;
  int t2 = now->r ? now->r->v : 0;
  now->v = t1 + t2;
}
int kth(PSTNode *prv, PSTNode *now, int s, int e, int k){
  if(s == e) return s;
  int m = s + e >> 1, diff = now->l->v - prv->l->v;
  if(k <= diff) return kth(prv->l, now->l, s, m, k);
  else return kth(prv->r, now->r, m+1, e, k-diff);
}
```

2 Flows, Matching

2.1 Hopcroft-Karp Bipartite Matching

```
const int MAXN = 50005, MAXM = 50005;
vector<int> gph[MAXN];
int dis[MAXN], l[MAXN], r[MAXM], vis[MAXN];
void clear(){ for(int i=0; i<MAXN; i++) gph[i].clear();  }
void add_edge(int l, int r){ gph[l].push_back(r); }
bool bfs(int n){
  queue<int> que;
  bool ok = 0;
  memset(dis, 0, sizeof(dis));
  for(int i=0; i<n; i++){
    if(l[i] == -1 && !dis[i]){
      que.push(i);
      dis[i] = 1;
    }
  }
  while(!que.empty()){
    int x = que.front();
    que.pop();
    for(auto &i : gph[x]){
      if(r[i] == -1) ok = 1;
      else if(!dis[r[i]]){
        dis[r[i]] = dis[x] + 1;
        que.push(r[i]);
      }
    }
  }
  return ok;
}
bool dfs(int x){
  if(vis[x]) return 0;
  vis[x] = 1;
  for(auto &i : gph[x]){
    if(r[i] == -1 || (!vis[r[i]] && dis[r[i]] == dis[x] + 1 && dfs(r[i]))){
      l[x] = i; r[i] = x;
      return 1;
    }
  }
  return 0;
}
int match(int n){
  memset(l, -1, sizeof(l));
  memset(r, -1, sizeof(r));
  int ret = 0;
  while(bfs(n)){
    memset(vis, 0, sizeof(vis));
    for(int i=0; i<n; i++) if(l[i] == -1 && dfs(i)) ret++;
  }
  return ret;
}
bool chk[MAXN + MAXM];
void rdfs(int x, int n){
  if(chk[x]) return;
  chk[x] = 1;
  for(auto &i : gph[x]){
```

```
    chk[i + n] = 1;
    rdfs(r[i], n);
  }
}
vector<int> getcover(int n, int m){ // solve min. vertex cover
  match(n);
  memset(chk, 0, sizeof(chk));
  for(int i=0; i<n; i++) if(l[i] == -1) rdfs(i, n);
  vector<int> v;
  for(int i=0; i<n; i++) if(!chk[i]) v.push_back(i);
  for(int i=n; i<n+m; i++) if(chk[i]) v.push_back(i);
  return v;
}
```

2.2 Dinic's Algorithm

```
const int MAXN = 505;
struct edg{ int pos, cap, rev; };
vector<edg> gph[MAXN];
void clear(){ for(int i=0; i<MAXN; i++) gph[i].clear(); }
void add_edge(int s, int e, int x){
  gph[s].push_back({e, x, (int)gph[e].size()});
  gph[e].push_back({s, 0, (int)gph[s].size()-1});
}
int dis[MAXN], pnt[MAXN];
bool bfs(int src, int sink){
  memset(dis, 0, sizeof(dis));
  memset(pnt, 0, sizeof(pnt));
  queue<int> que;
  que.push(src);
  dis[src] = 1;
  while(!que.empty()){
    int x = que.front();
    que.pop();
    for(auto &e : gph[x]){
      if(e.cap > 0 && !dis[e.pos]){
        dis[e.pos] = dis[x] + 1;
        que.push(e.pos);
      }
    }
  }
  return dis[sink] > 0;
}
int dfs(int x, int sink, int f){
  if(x == sink) return f;
  for(; pnt[x] < gph[x].size(); pnt[x]++){
    edg e = gph[x][pnt[x]];
    if(e.cap > 0 && dis[e.pos] == dis[x] + 1){
      int w = dfs(e.pos, sink, min(f, e.cap));
      if(w){
        gph[x][pnt[x]].cap -= w;
        gph[e.pos][e.rev].cap += w;
        return w;
      }
    }
  }
  return 0;
}
lint match(int src, int sink){
  lint ret = 0;
  while(bfs(src, sink)){
    int r;
    while((r = dfs(src, sink, 2e9))) ret += r;
  }
  return ret;
}
```

2.3 Min Cost Max Flow

```
const int MAXN = 100;
struct edg{ int pos, cap, rev, cost; };
vector<edg> gph[MAXN];
void clear(){
  for(int i=0; i<MAXN; i++) gph[i].clear();
}
void add_edge(int s, int e, int x, int c){
  gph[s].push_back({e, x, (int)gph[e].size(), c});
  gph[e].push_back({s, 0, (int)gph[s].size()-1, -c});
}
int dist[MAXN], pa[MAXN], pe[MAXN];
bool inque[MAXN];
bool spfa(int src, int sink){
  memset(dist, 0x3f, sizeof(dist));
  memset(inque, 0, sizeof(inque));
  queue<int> que;
  dist[src] = 0;
  inque[src] = 1;
  que.push(src);
  bool ok = 0;
  while(!que.empty()){
    int x = que.front();
    que.pop();
    if(x == sink) ok = 1;
    inque[x] = 0;
    for(int i=0; i<gph[x].size(); i++){
      edg e = gph[x][i];
      if(e.cap > 0 && dist[e.pos] > dist[x] + e.cost){
        dist[e.pos] = dist[x] + e.cost;
        pa[e.pos] = x;
        pe[e.pos] = i;
        if(!inque[e.pos]){
          inque[e.pos] = 1;
          que.push(e.pos);
        }
      }
    }
  }
  return ok;
}
int match(int src, int sink){
  int ret = 0;
  while(spfa(src, sink)){
    int cap = 1e9;
    for(int pos = sink; pos != src; pos = pa[pos]){
      cap = min(cap, gph[pa[pos]][pe[pos]].cap);
    }
```

```
    ret += dist[sink] * cap;
    for(int pos = sink; pos != src; pos = pa[pos]){
      int rev = gph[pa[pos]][pe[pos]].rev;
      gph[pa[pos]][pe[pos]].cap -= cap;
      gph[pos][rev].cap += cap;
    }
  }
  return ret;
}
```

2.4 Hell-Joseon style MCMF

```
const int MAXN = 100;
struct edg{ int pos, cap, rev, cost; };
vector<edg> gph[MAXN];
void clear(){ for(int i=0; i<MAXN; i++) gph[i].clear(); }
void add_edge(int s, int e, int x, int c){
  gph[s].push_back({e, x, (int)gph[e].size(), c});
  gph[e].push_back({s, 0, (int)gph[s].size()-1, -c});
}
int phi[MAXN], inque[MAXN], dist[MAXN];
void prep(int src, int sink){
  memset(phi, 0x3f, sizeof(phi));
  memset(dist, 0x3f, sizeof(dist));
  queue<int> que;
  que.push(src);
  inque[src] = 1;
  while(!que.empty()){
    int x = que.front();
    que.pop();
    inque[x] = 0;
    for(auto &i : gph[x]){
      if(i.cap > 0 && phi[i.pos] > phi[x] + i.cost){
        phi[i.pos] = phi[x] + i.cost;
        if(!inque[i.pos]){
          inque[i.pos] = 1;
          que.push(i.pos);
        }
      }
    }
  }
  for(int i=0; i<MAXN; i++){
    for(auto &j : gph[i]){
      if(j.cap > 0) j.cost += phi[i] - phi[j.pos];
    }
  }
  priority_queue<pi, vector<pi>, greater<pi> > pq;
  pq.push(pi(0, src));
  dist[src] = 0;
  while(!pq.empty()){
    auto l = pq.top();
    pq.pop();
    if(dist[l.second] != l.first) continue;
    for(auto &i : gph[l.second]){
      if(i.cap > 0 && dist[i.pos] > l.first + i.cost){
        dist[i.pos] = l.first + i.cost;
        pq.push(pi(dist[i.pos], i.pos));
      }
    }
  }
}
bool vis[MAXN];
int ptr[MAXN];
int dfs(int pos, int sink, int flow){
  vis[pos] = 1;
  if(pos == sink) return flow;
  for(; ptr[pos] < gph[pos].size(); ptr[pos]++){
    auto &i = gph[pos][ptr[pos]];
    if(!vis[i.pos] && dist[i.pos] == i.cost + dist[pos] && i.cap > 0){
      int ret = dfs(i.pos, sink, min(i.cap, flow));
      if(ret != 0){
        i.cap -= ret;
        gph[i.pos][i.rev].cap += ret;
        return ret;
      }
    }
  }
  return 0;
}
int match(int src, int sink, int sz){
  prep(src, sink);
  for(int i=0; i<sz; i++) dist[i] += phi[sink] - phi[src];
  int ret = 0;
  while(true){
    memset(ptr, 0, sizeof(ptr));
    memset(vis, 0, sizeof(vis));
    int tmp = 0;
    while((tmp = dfs(src, sink, 1e9))){
      ret += dist[sink] * tmp;
      memset(vis, 0, sizeof(vis));
    }
    tmp = 1e9;
    for(int i=0; i<sz; i++){
      if(!vis[i]) continue;
      for(auto &j : gph[i]){
        if(j.cap > 0 && !vis[j.pos]){
          tmp = min(tmp, (dist[i] + j.cost) - dist[j.pos]);
        }
      }
    }
    if(tmp > 1e9 - 200) break;
    for(int i=0; i<sz; i++){
      if(!vis[i]) dist[i] += tmp;
    }
  }
  return ret;
}
```

2.5 Circulation Problem

```
maxflow mf;
lint lsum;
void clear(){
  lsum = 0;
  mf.clear();
}
```

```
void add_edge(int s, int e, int l, int r){
  lsum += l;
  mf.add_edge(s + 2, e + 2, r - l);
  mf.add_edge(0, e + 2, l);
  mf.add_edge(s + 2, 1, l);
}
bool solve(int s, int e){
  mf.add_edge(e+2, s+2, 1e9); // to reduce as maxflow with lower bounds, in circulation
problem skip this line
  return lsum == mf.match(0, 1);
  // to get maximum LR flow, run maxflow from s+2 to e+2 again
}
```

2.6 Min Cost Circulation

```
// Cycle canceling (Dual of successive shortest path)
// Time complexity is ridiculously high (F * maxC * nm^2). But runs reasonably in practice
(V = 70 in 1s)
struct edg{ int pos, cap, rev, cost; };
vector<edg> gph[MAXN];
void clear(){ for(int i=0; i<MAXN; i++) gph[i].clear(); }
void add_edge(int s, int e, int x, int c){
  gph[s].push_back({e, x, (int)gph[e].size(), c});
  gph[e].push_back({s, 0, (int)gph[s].size()-1, -c});
}
int dist[MAXN], par[MAXN], pae[MAXN];
int negative_cycle(int n){
  bool mark[MAXN] = {};
  memset(dist, 0, sizeof(dist));
  int upd = -1;
  for(int i=0; i<=n; i++){
    for(int j=0; j<n; j++){
      int idx = 0;
      for(auto &k : gph[j]){
        if(k.cap > 0 && dist[k.pos] > dist[j] + k.cost){
          dist[k.pos] = dist[j] + k.cost;
          par[k.pos] = j;
          pae[k.pos] = idx;
          if(i == n){
            upd = j;
            while(!mark[upd]){
              mark[upd] = 1;
              upd = par[upd];
            }
            return upd;
          }
        }
        idx++;
      }
    }
  }
  return -1;
}
int match(int n){
  int rt = -1;
  int ans = 0;
  while(~(rt = negative_cycle(n))){
    bool mark[MAXN] = {};
    vector<pi> cyc;
    while(!mark[rt]){
      cyc.push_back(pi(par[rt], pae[rt]));
      mark[rt] = 1;
      rt = par[rt];
    }
    reverse(cyc.begin(), cyc.end());
    int capv = 1e9;
    for(auto &i : cyc){
      auto e = &gph[i.first][i.second];
      capv = min(capv, e->cap);
    }
    for(auto &i : cyc){
      auto e = &gph[i.first][i.second];
      e->cap -= capv;
      gph[e->pos][e->rev].cap += capv;
      ans += e->cost * capv;
    }
  }
  return ans;
}
```

3 Graph

3.1 2-SAT

```
strongly_connected scc;
int n; // = number of clauses
void init(int _n){ scc.clear(); n = _n; }
int NOT(int x){ return x >= n ? (x - n) : (x + n); }
void add_edge(int x, int y){ // input ~x to denote NOT
  if((x >> 31) & 1) x = (~x) + n;
  if((y >> 31) & 1) y = (~y) + n;
  scc.add_edge(x, y), scc.add_edge(NOT(y), NOT(x));
}
bool satisfy(vector<bool> &res){
  res.resize(n);
  scc.get_scc(2*n);
  for(int i=0; i<n; i++){
    if(scc.comp[i] == scc.comp[NOT(i)]) return 0;
    if(scc.comp[i] < scc.comp[NOT(i)]) res[i] = 0;
    else res[i] = 1;
  }
  return 1;
}
```

3.2 BCC

```
void color(int x, int p){
  if(p){
    bcc[p].push_back(x);
    cmp[x].push_back(p);
  }
  for(auto &i : gph[x]){
    if(cmp[i].size()) continue;
    if(low[i] >= dfn[x]){
      bcc[++c].push_back(x);
      cmp[x].push_back(c);
      color(i, c);
    }
```

```
    else color(i, p);
  }
}
```

3.3 Splay Tree + Link-Cut Tree

```
// Checklist 1. Is it link cut, or splay?
// Checklist 2. In link cut, is son always root?
void rotate(node *x){
  if(!x->p) return;
  push(x->p);  // if there's lazy stuff
  push(x);
  node *p = x->p;
  bool is_left = (p->l == x);
  node *b = (is_left ? x->r : x->l);
  x->p = p->p;
  if(x->p && x->p->l == p) x->p->l = x;
  if(x->p && x->p->r == p) x->p->r = x;
  if(is_left){
    if(b) b->p = p;
    p->l = b;
    p->p = x;
    x->r = p;
  }
  else{
    if(b) b->p = p;
    p->r = b;
    p->p = x;
    x->l = p;
  }
  pull(p); // if there's something to pull up
  pull(x);
  if(!x->p) root = x; // IF YOU ARE SPLAY TREE
  if(p->pp){ // IF YOU ARE LINK CUT TREE
    x->pp = p->pp;
    p->pp = NULL;
  }
}
void splay(node *x){
  while(x->p){
    node *p = x->p;
    node *g = p->p;
    if(g){
      if((p->l == x) ^ (g->l == p)) rotate(x);
      else rotate(p);
    }
    rotate(x);
  }
}
void access(node *x){
  splay(x);
  push(x);
  if(x->r){
    x->r->pp = x;
    x->r->p = NULL;
    x->r = NULL;
  }
  pull(x);
  while(x->pp){
    node *nxt = x->pp;
    splay(nxt);
    push(nxt);
    if(nxt->r){
      nxt->r->pp = nxt;
      nxt->r->p = NULL;
      nxt->r = NULL;
    }
    nxt->r = x;
    x->p = nxt;
    x->pp = NULL;
    pull(nxt);
    splay(x);
  }
}
node *root(node *x){
  access(x);
  while(x->l){
    push(x);
    x = x->l;
  }
  access(x);
  return x;
}
node *par(node *x){
  access(x);
  if(!x->l) return NULL;
  push(x);
  x = x->l;
  while(x->r){
    push(x);
    x = x->r;
  }
  access(x);
  return x;
}
node *lca(node *s, node *t){
  access(s);
  access(t);
  splay(s);
  if(s->pp == NULL) return s;
  return s->pp;
}
void link(node *par, node *son){
  access(par);
  access(son);
  son->rev ^= 1;  // remove if needed
  push(son);
  son->l = par;
  par->p = son;
  pull(son);
}
void cut(node *p){
  access(p);
  push(p);
```

```
  if(p->l){
    p->l->p = NULL;
    p->l = NULL;
  }
  pull(p);
}
```

3.4 Offline Dynamic MST

```
int n, m, q;
int st[MAXN], ed[MAXN], cost[MAXN], chk[MAXN];
pi qr[MAXN];

bool cmp(int &a, int &b){ return pi(cost[a], a) < pi(cost[b], b); }

void contract(int s, int e, vector<int> v, vector<int> &must_mst, vector<int> &maybe_mst){
  sort(v.begin(), v.end(), cmp);
  vector<pi> snapshot;
  for(int i=s; i<=e; i++) disj.uni(st[qr[i].first], ed[qr[i].first], snapshot);
  for(auto &i : v) if(disj.uni(st[i], ed[i], snapshot)) must_mst.push_back(i);
  disj.revert(snapshot);
  for(auto &i : must_mst) disj.uni(st[i], ed[i], snapshot);
  for(auto &i : v) if(disj.uni(st[i], ed[i], snapshot)) maybe_mst.push_back(i);
  disj.revert(snapshot);
}

void solve(int s, int e, vector<int> v, lint cv){
  if(s == e){
    cost[qr[s].first] = qr[s].second;
    if(st[qr[s].first] == ed[qr[s].first]){
      printf("%lld\n", cv);
      return;
    }
    int minv = qr[s].second;
    for(auto &i : v) minv = min(minv, cost[i]);
    printf("%lld\n",minv + cv);
    return;
  }
  int m = (s+e)/2;
  vector<int> lv = v, rv = v;
  vector<int> must_mst, maybe_mst;
  for(int i=m+1; i<=e; i++){
    chk[qr[i].first]--;
    if(chk[qr[i].first] == 0) lv.push_back(qr[i].first);
  }
  vector<pi> snapshot;
  contract(s, m, lv, must_mst, maybe_mst);
  lint lcv = cv;
  for(auto &i : must_mst) lcv += cost[i], disj.uni(st[i], ed[i], snapshot);
  solve(s, m, maybe_mst, lcv);
  disj.revert(snapshot);
  must_mst.clear(); maybe_mst.clear();
  for(int i=m+1; i<=e; i++) chk[qr[i].first]++;
  for(int i=s; i<=m; i++){
    chk[qr[i].first]--;
    if(chk[qr[i].first] == 0) rv.push_back(qr[i].first);
  }
  lint rcv = cv;
  contract(m+1, e, rv, must_mst, maybe_mst);
  for(auto &i : must_mst) rcv += cost[i], disj.uni(st[i], ed[i], snapshot);
  solve(m+1, e, maybe_mst, rcv);
  disj.revert(snapshot);
  for(int i=s; i<=m; i++) chk[qr[i].first]++;
}

int main(){
  scanf("%d %d",&n,&m);
  vector<int> ve;
  for(int i=0; i<m; i++){
    scanf("%d %d %d",&st[i],&ed[i],&cost[i]);
  }
  scanf("%d",&q);
  for(int i=0; i<q; i++){
    scanf("%d %d",&qr[i].first,&qr[i].second);
    qr[i].first--;
    chk[qr[i].first]++;
  }
  disj.init(n);
  for(int i=0; i<m; i++) if(!chk[i]) ve.push_back(i);
  solve(0, q-1, ve, 0);
}
```

4 Strings

4.1 Aho-Corasick Algorithm

```
const int MAXN = 100005, MAXC = 26;
int trie[MAXN][MAXC], fail[MAXN], term[MAXN], piv;
void init(vector<string> &v){
  memset(trie, 0, sizeof(trie));
  memset(fail, 0, sizeof(fail));
  memset(term, 0, sizeof(term));
  piv = 0;
  for(auto &i : v){
    int p = 0;
    for(auto &j : i){
      if(!trie[p][j]) trie[p][j] = ++piv;
      p = trie[p][j];
    }
    term[p] = 1;
  }
  queue<int> que;
  for(int i=0; i<MAXC; i++){
    if(trie[0][i]) que.push(trie[0][i]);
  }
  while(!que.empty()){
    int x = que.front();
    que.pop();
    for(int i=0; i<MAXC; i++){
      if(trie[x][i]){
        int p = fail[x];
        while(p && !trie[p][i]) p = fail[p];
        p = trie[p][i];
        fail[trie[x][i]] = p;
        if(term[p]) term[trie[x][i]] = 1;
        que.push(trie[x][i]);
```

```
      }
    }
  }
}
bool query(string &s){
  int p = 0;
  for(auto &i : s){
    while(p && !trie[p][i]) p = fail[p];
    p = trie[p][i];
    if(term[p]) return 1;
  }
  return 0;
}
```

4.2 Suffix Array

```
const int MAXN = 500005;
int ord[MAXN], nord[MAXN], cnt[MAXN], aux[MAXN];
void solve(int n, char *str, int *sfx, int *rev, int *lcp){
  int p = 1;
  memset(ord, 0, sizeof(ord));
  for(int i=0; i<n; i++){
    sfx[i] = i;
    ord[i] = str[i];
  }
  int pnt = 1;
  while(1){
    memset(cnt, 0, sizeof(cnt));
    for(int i=0; i<n; i++) cnt[ord[min(i+p, n)]]++;
    for(int i=1; i<=n || i<=255; i++) cnt[i] += cnt[i-1];
    for(int i=n-1; i>=0; i--)
      aux[--cnt[ord[min(i+p, n)]]] = i;
    memset(cnt, 0, sizeof(cnt));
    for(int i=0; i<n; i++) cnt[ord[i]]++;
    for(int i=1; i<=n || i<=255; i++) cnt[i] += cnt[i-1];
    for(int i=n-1; i>=0; i--)
      sfx[--cnt[ord[aux[i]]]] = aux[i];
    if(pnt == n) break;
    pnt = 1;
    nord[sfx[0]] = 1;
    for(int i=1; i<n; i++){
      if(ord[sfx[i-1]] != ord[sfx[i]] || ord[sfx[i-1] + p] != ord[sfx[i] + p]){
        pnt++;
      }
      nord[sfx[i]] = pnt;
    }
    memcpy(ord, nord, sizeof(int) * n);
    p *= 2;
  }
  for(int i=0; i<n; i++) rev[sfx[i]] = i;
  int h = 0;
  for(int i=0; i<n; i++){
    if(rev[i]){
      int prv = sfx[rev[i] - 1];
      while(str[prv + h] == str[i + h]) h++;
      lcp[rev[i]] = h;
    }
    h = max(h-1, 0);
```

```
  }
}
```

4.3 Manacher's Algorithm

```
const int MAXN = 1000005;
int aux[2 * MAXN - 1];
void solve(int n, int *str, int *ret){
  // *ret : number of nonobvious palindromic character pair
  for(int i=0; i<n; i++){
    aux[2*i] = str[i];
    if(i != n-1) aux[2*i+1] = -1;
  }
  int p = 0, c = 0;
  for(int i=0; i<2*n-1; i++){
    int cur = 0;
    if(i <= p) cur = min(ret[2 * c - i], p - i);
    while(i - cur - 1 >= 0 && i + cur + 1 < 2*n-1 && aux[i-cur-1] == aux[i+cur+1]){
      cur++;
    }
    ret[i] = cur;
    if(i + ret[i] > p){
      p = i + ret[i];
      c = i;
    }
  }
}
```

4.4 Suffix Array (Linear time)

Should be **revised**.

```
class SuffixArray {
public:
  int A[7 * N / 10], B[7 * N / 10], cnt[N + 2], SAV[N];
  int mem[5 * N]; int* mem_pt = mem;
  void clear(int n){
    int *ptr = mem;
    while(ptr != mem_pt){
      *ptr = 0;
      ptr++;
    }
    mem_pt = mem;
    for(int i=0; i<n+10 && i < 7 * N / 10; i++) A[i] = B[i] = 0;
    for(int i=0; i<n+2; i++) cnt[i] = 0;
    for(int i=0; i<n; i++) SAV[i] = 0;
  }
  inline int* mloc(size_t sz) {
    int* ret = mem_pt; mem_pt = mem_pt + sz;
    return ret;
  }
  void rsort(int* a, int* b, int* dat, int n, int k) {
    for (int i = 0; i <= k; i++) cnt[i] = 0;
    for (int i = 0; i < n; i++) SAV[i] = dat[a[i]], cnt[SAV[i]]++;
    for (int i = 1; i <= k; i++) cnt[i] += cnt[i - 1];
    for (int i = n - 1; i >= 0; i--) b[--cnt[SAV[i]]] = a[i];
  }

#define I(x) ((x)%3==1)?((x)/3):((x)/3+num1)
#define I2(x) (x<num1)?(3*x+1):(3*(x-num1)+2)
```

```
  static int cmp(int x, int y, int str[], int A[], int num1) {
    if (x % 3 == 1) {
      if (y % 3 == 2) return A[I(x)] < A[I(y)];
      else return str[x] < str[y] || str[x] == str[y] && A[I(x + 1)] < A[I(y + 1)];
    }
    else {
      return str[x] < str[y] || str[x] == str[y] && cmp(x + 1, y + 1, str, A, num1);
    }
  }
  void make(int* str, int* sa, int n, int k) {
    if (n == 0) return;
    int num1 = (n + 2) / 3, num2 = n / 3;
    int num = num1 + num2;
    str[n] = str[n + 1] = str[n + 2] = 0;
    int *nsa = mloc(num), *nstr = mloc(num + 3);

    for (int i = 0, j = 0; i < n; i++) if (i % 3) A[j++] = i;
    if (n % 3 == 1) A[num - 1] = n;
    rsort(A, B, str + 2, num, k); rsort(B, A, str + 1, num, k); rsort(A, B, str, num, k);

    int cnt = 1;
    nstr[I(B[0])] = 1;
    for (int i = 1; i < num; i++) {
      int c = B[i], p = B[i - 1];
      if (str[p] != str[c] || str[p + 1] != str[c + 1] || str[p + 2] != str[c + 2]) cnt++;
      nstr[I(c)] = cnt;
    }
    if (cnt == num) for (int i = 0; i < num; i++) nsa[nstr[i] - 1] = i;
    else make(nstr, nsa, num, cnt);

    for (int i = 0, j = 0; i < num; i++) if (nsa[i] < num1) A[j++] = 3 * nsa[i];
    rsort(A, B, str, num1, k);
    for (int i = 0; i < num; i++) A[nsa[i]] = i, nsa[i] = I2(nsa[i]);
    A[num] = -1;
    merge(B, B + num1, nsa + (n % 3 == 1), nsa + num, sa, [&](int x, int y) {
      return cmp(x, y, str, A, num1);
    });
    return;
  }
}sa;
```

4.5 eertree

```
int nxt[MAXN][26];
int par[MAXN], len[MAXN], slink[MAXN], ptr[MAXN], diff[MAXN], series[MAXN], piv;
void clear(int n = MAXN){
  memset(par, 0, sizeof(int) * n);
  memset(len, 0, sizeof(int) * n);
  memset(slink, 0, sizeof(int) * n);
  memset(nxt, 0, sizeof(int) * 26 * n);
  piv = 0;
}
void init(int n, char *a){
  par[0] = 0;
  par[1] = 1;
  a[0] = -1;
  len[0] = -1;
  piv = 1;
  int cur = 1;
  for(int i=1; i<=n; i++){
    while(a[i] != a[i - len[cur] - 1]) cur = slink[cur];
    if(!nxt[cur][a[i]]){
      nxt[cur][a[i]] = ++piv;
      par[piv] = cur;
      len[piv] = len[cur] + 2;
      int lnk = slink[cur];
      while(a[i] != a[i - len[lnk] - 1]){
        lnk = slink[lnk];
      }
      if(nxt[lnk][a[i]]) lnk = nxt[lnk][a[i]];
      if(len[piv] == 1 || lnk == 0) lnk = 1;
      slink[piv] = lnk;
      diff[piv] = len[piv] - len[lnk];
      if(diff[piv] == diff[lnk]) series[piv] = series[lnk];
      else series[piv] = piv;
    }
    cur = nxt[cur][a[i]];
    ptr[i] = cur;
  }
}
int query(int s, int e){
  int pos = ptr[e];
  while(len[pos] >= e - s + 1){
        if(len[pos] % diff[pos] == (e - s + 1) % diff[pos] &&
           len[series[pos]] <= e - s + 1) return true;
        pos = series[pos];
        pos = slink[pos];
  }
  return false;
}
vector<pi> minimum_partition(int n){ // (odd min, even min)
  vector<pi> dp(n + 1);
  vector<pi> series_ans(n + 10);
  dp[0] = pi(1e9 + 1, 0);
  for(int i=1; i<=n; i++){
    dp[i] = pi(1e9 + 1, 1e9);
    for(int j=ptr[i]; len[j] > 0;){
      int slv = slink[series[j]];
      series_ans[j] = dp[i - (len[slv] + diff[j])];
      if(diff[j] == diff[slink[j]]){
        series_ans[j].first = min(series_ans[j].first, series_ans[slink[j]].first);
        series_ans[j].second = min(series_ans[j].second, series_ans[slink[j]].second);
      }
      auto val = series_ans[j];
      dp[i].first = min(dp[i].first, val.second + 1);
      dp[i].second = min(dp[i].second, val.first + 1);
      j = slv;
    }
  }
  return dp;
}
```

4.6 Circular LCS

```
string s1, s2;
int dp[4005][2005];
int nxt[4005][2005];
int n, m;
void reroot(int px){
  int py = 1;
  while(py <= m && nxt[px][py] != 2) py++;
  nxt[px][py] = 1;
  while(px < 2 * n && py < m){
    if(nxt[px+1][py] == 3){
      px++;
      nxt[px][py] = 1;
    }
    else if(nxt[px+1][py+1] == 2){
      px++;
      py++;
      nxt[px][py] = 1;
    }
    else py++;
  }
  while(px < 2 * n && nxt[px+1][py] == 3){
    px++;
    nxt[px][py] = 1;
  }
}

int track(int x, int y, int e){ // use this routine to find LCS as string
  int ret = 0;
  while(y != 0 && x != e){
    if(nxt[x][y] == 1) y--;
    else if(nxt[x][y] == 2) ret += (s1[x] == s2[y]), x--, y--;
    else if(nxt[x][y] == 3) x--;
  }
  return ret;
}

int solve(string a, string b){
  n = a.size(), m = b.size();
  s1 = "#" + a + a;
  s1 = '#' + b;
  for(int i=0; i<=2*n; i++){
    for(int j=0; j<=m; j++){
      if(j == 0){
        nxt[i][j] = 3;
        continue;
      }
      if(i == 0){
        nxt[i][j] = 1;
        continue;
      }
      dp[i][j] = -1;
      if(dp[i][j] < dp[i][j-1]){
        dp[i][j] = dp[i][j-1];
        nxt[i][j] = 1;
      }
      if(dp[i][j] < dp[i-1][j-1] + (s1[i] == s2[j])){
        dp[i][j] = dp[i-1][j-1] + (s1[i] == s2[j]);
        nxt[i][j] = 2;
      }
      if(dp[i][j] < dp[i-1][j]){
        dp[i][j] = dp[i-1][j];
        nxt[i][j] = 3;
      }
    }
  }
  int ret = dp[n][m];
  for(int i=1; i<n; i++){
    reroot(i), ret = max(ret, track(n+i, m, i));
  }
  return ret;
}
```

5 Geometry

5.1 Buldozer

```
Point v[2020];
struct Line{
  ll i, j, dx, dy;
  Line(int i, int j) : i(i), j(j) {
    dx = v[i].x - v[j].x; dy = v[i].y - v[j].y;
  }
  bool operator < (const Line &t) const {
    ll a = dy * t.dx, b = t.dy * dx;
    return tie(a, i, j) < tie(b, t.i, t.j);
  }
};
int ccw(Point a, Point b, Point c){}
int ccw(Line a, Line b){
  ll res = a.dx*b.dy - b.dx*a.dy;
  if(!res) return 0; return res > 0 ? 1 : -1;
}
int idx[2020]; vector<Line> line;
void bulldozer(int n){
  sort(v+1, v+n+1); for(int i=1; i<=n; i++) idx[i] = i;
  for(int i=1; i<=n; i++) for(int j=1; j<i; j++) line.emplace_back(i, j);
  for(int i=0, j=0; i<line.size(); ){
    int ed = i;
    while(ed < line.size() && !ccw(line[i], line[ed])) ed++;
    for(int j=i; j<ed; j++){
      int a = line[j].i, b = line[j].j;
      swap(idx[a], idx[b]); swap(v[idx[a]], v[idx[b]]);
      update(idx[a]); update(idx[b]);
    }
    ans = merge(ans, query()); i = ed;
  }
}
```

5.2 Rotating Calipers

```
Point v[2020];
struct Line{
  ll i, j, dx, dy;
  Line(int i, int j) : i(i), j(j) {
    dx = v[i].x - v[j].x; dy = v[i].y - v[j].y;
```

```
  }
  bool operator < (const Line &t) const {
    ll a = dy * t.dx, b = t.dy * dx;
    return tie(a, i, j) < tie(b, t.i, t.j);
  }
};
int ccw(Point a, Point b, Point c){}
int ccw(Line a, Line b){
  ll res = a.dx*b.dy - b.dx*a.dy;
  if(!res) return 0; return res > 0 ? 1 : -1;
}
int idx[2020]; vector<Line> line;
void bulldozer(int n){
  sort(v+1, v+n+1); for(int i=1; i<=n; i++) idx[i] = i;
  for(int i=1; i<=n; i++) for(int j=1; j<i; j++) line.emplace_back(i, j);
  for(int i=0, j=0; i<line.size(); ){
    int ed = i;
    while(ed < line.size() && !ccw(line[i], line[ed])) ed++;
    for(int j=i; j<ed; j++){
      int a = line[j].i, b = line[j].j;
      swap(idx[a], idx[b]); swap(v[idx[a]], v[idx[b]]);
      update(idx[a]); update(idx[b]);
    }
    ans = merge(ans, query()); i = ed;
  }
}
```

5.3 Delaunay

```
using lll = __int128; // using T = ll; (if coords are < 2e4)
// return true if p strictly within circumcircle(a,b,c)
bool inCircle(P p, P a, P b, P c) {
  a -= p, b -= p, c -= p; // assert(cross(a,b,c)>0);
  lll x = (lll)norm(a)*cross(b,c)+(lll)norm(b)*cross(c,a)
      +(lll)norm(c)*cross(a,b);
  return x*(cross(a,b,c)>0?1:-1) > 0;
}
P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point
using Q = struct Quad*;
struct Quad {
  bool mark; Q o, rot; P p;
  P F() { return r()->p; }
  Q r() { return rot->rot; }
  Q prev() { return rot->o->rot; }
  Q next() { return r()->prev(); }
};
Q makeEdge(P orig, P dest) {
  Q q[]{new Quad{0,0,0,orig}, new Quad{0,0,0,arb},
      new Quad{0,0,0,dest}, new Quad{0,0,0,arb}};
  FOR(i,4) q[i]->o = q[-i & 3], q[i]->rot = q[(i+1) & 3];
  return *q;
}
void splice(Q a, Q b) { swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o); }
Q connect(Q a, Q b) {
  Q q = makeEdge(a->F(), b->p);
  splice(q, a->next()); splice(q->r(), b);
  return q;
}
pair<Q,Q> rec(const vP& s) {
  if (sz(s) <= 3) {
    Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.bk);
    if (sz(s) == 2) return { a, a->r() };
    splice(a->r(), b);
    auto side = cross(s[0], s[1], s[2]);
    Q c = side ? connect(b, a) : 0;
    return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
  }

#define H(e) e->F(), e->p
#define valid(e) (cross(e->F(),H(base)) > 0)
  Q A, B, ra, rb;
  int half = sz(s) / 2;
  tie(ra, A) = rec({all(s)-half});
  tie(B, rb) = rec({sz(s)-half+all(s)});
  while ((cross(B->p,H(A)) < 0 && (A = A->next())) ||
      (cross(A->p,H(B)) > 0 && (B = B->r()->o)));
  Q base = connect(B->r(), A);
  if (A->p == ra->p) ra = base->r();
  if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (inCircle(e->dir->F(), H(base), e->F())) { \
      Q t = e->dir; \
      splice(e, e->prev()); \
      splice(e->r(), e->r()->prev()); \
      e = t; \
    }
  while (1) {
    DEL(LC, base->r(), o);  DEL(RC, base, prev());
    if (!valid(LC) && !valid(RC)) break;
    if (!valid(LC) || (valid(RC) && inCircle(H(RC), H(LC))))
      base = connect(RC, base->r());
    else base = connect(base->r(), LC->r());
  }
  return {ra, rb};
}
V<AR<P,3>> triangulate(vP pts) {
  sor(pts); assert(unique(all(pts)) == end(pts)); // no duplicates
  if (sz(pts) < 2) return {};
  Q e = rec(pts).f; V<Q> q = {e};
  while (cross(e->o->F(), e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.pb(c->p); \
  q.pb(c->r()); c = c->next(); } while (c != e); }
  ADD; pts.clear();
  int qi = 0; while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
  V<AR<P,3>> ret(sz(pts)/3);
  FOR(i,sz(pts)) ret[i/3][i%3] = pts[i];
  return ret;
}
```

5.4 Dual Graph

```
const int MV = 101010, ME = 101010; // MAX_V, MAX_E
p pt[MV]; // vertex's coord
vector<p> g[MV]; // g[s].emplace_back(e, edge_id);
vector<int> dual_pt; // coord compress
```

```
int par[ME * 2]; // Union Find
void uf_init(){ iota(par, par+ME*2, 0); }
int find(int v){return v == par[v] ? v : par[v] = find(par[v]);}
void merge(int u, int v){ u = find(u); v = find(v); if(u != v)par[u] = v; }
p base; // sort by angle
bool cmp_angle(const p &_a, const p &_b){
  p a = pt[_a.x], b = pt[_b.x];
  if((a > base) != (b > base)) return a > b;
  return ccw(a, base, b) > 0;
}
void addEdge(int s, int e, int id){
  g[s].emplace_back(e, id); g[e].emplace_back(s, id);
}
int out; //outer face
void getDual(int n, int m){
  uf_init();
  for(int i=1; i<=n; i++){
    base = pt[i]; sort(all(g[i]), cmp_angle);
    // up, left : *2+1 / down, right : *2
    for(int j=0; j<g[i].size(); j++){
      int k = j ? j - 1 : g[i].size()-1;
      int u = g[i][k].y << 1 | 1, v = g[i][j].y << 1;
      p p1 = pt[g[i][k].x], p2 = pt[g[i][j].x];
      if(p1 > base) u ^= 1;
      if(p2 > base) v ^= 1;
      merge(u, v);
    }
  }
  int mn_idx = min_element(pt+1, pt+n+1) - pt;
  out = find(g[mn_idx][0].y << 1 | 1);
  for(int i=1; i<=m; i++){
    dual_pt.push_back(find(i << 1));
    dual_pt.push_back(find(i << 1 | 1));
  }
  compress(dual_pt);
  // @TODO coord compress
}
```

5.5 Smallest Enclosing Circle / Sphere

```
namespace cover_2d{
  double eps = 1e-9;
  using Point = complex<double>;
  struct Circle{ Point p; double r; };
  double dist(Point p, Point q){ return abs(p-q); }
  double area2(Point p, Point q){ return (conj(p)*q).imag(); }
  bool in(const Circle& c, Point p){ return dist(c.p, p) < c.r + eps; }
  Circle INVAL = Circle{Point(0, 0), -1};
  Circle mCC(Point a, Point b, Point c){
    b -= a; c -= a;
    double d = 2*(conj(b)*c).imag(); if(abs(d)<eps) return INVAL;
    Point ans = (c*norm(b) - b*norm(c)) * Point(0, -1) / d;
    return Circle{a + ans, abs(ans)};
  }
  Circle solve(vector<Point> p) {
    mt19937 gen(0x94949); shuffle(p.begin(), p.end(), gen);
    Circle c = INVAL;
    for(int i=0; i<p.size(); ++i) if(c.r<0 ||!in(c, p[i])){
      c = Circle{p[i], 0};
      for(int j=0; j<=i; ++j) if(!in(c, p[j])){
        Circle ans{(p[i]+p[j])*0.5, dist(p[i], p[j])*0.5};
        if(c.r == 0) { c = ans; continue; }
        Circle l, r; l = r = INVAL;
        Point pq = p[j]-p[i];
        for(int k=0; k<=j; ++k) if(!in(ans, p[k])) {
          double a2 = area2(pq, p[k]-p[i]);
          Circle c = mCC(p[i], p[j], p[k]);
          if(c.r<0) continue;
          else if(a2 > 0 && (l.r<0||area2(pq, c.p-p[i]) > area2(pq, l.p-p[i]))) l = c;
          else if(a2 < 0 && (r.r<0||area2(pq, c.p-p[i]) < area2(pq, r.p-p[i]))) r = c;
        }
        if(l.r<0&&r.r<0) c = ans;
        else if(l.r<0) c = r;
        else if(r.r<0) c = l;
        else c = l.r<=r.r?l:r;
      }
    }
    return c;
  }
};

namespace cover_3d{
  double enclosing_sphere(vector<double> x, vector<double> y, vector<double> z){
    int n = x.size();
    auto hyp = [](double x, double y, double z){
      return x * x + y * y + z * z;
    };
    double px = 0, py = 0, pz = 0;
    for(int i=0; i<n; i++){
      px += x[i];
      py += y[i];
      pz += z[i];
    }
    px *= 1.0 / n;
    py *= 1.0 / n;
    pz *= 1.0 / n;
    double rat = 0.1, maxv;
    for(int i=0; i<10000; i++){
      maxv = -1;
      int maxp = -1;
      for(int j=0; j<n; j++){
        double tmp = hyp(x[j] - px, y[j] - py, z[j] - pz);
        if(maxv < tmp){
          maxv = tmp;
          maxp = j;
        }
      }
      px += (x[maxp] - px) * rat;
      py += (y[maxp] - py) * rat;
      pz += (z[maxp] - pz) * rat;
      rat *= 0.998;
    }
    return sqrt(maxv);
  }
```

```
};
```

5.6 3D Convex Hull

```
struct vec3{
  ll x, y, z;
  vec3(): x(0), y(0), z(0) {}
  vec3(ll a, ll b, ll c): x(a), y(b), z(c) {}
  vec3 operator*(const vec3& v) const{ return vec3(y*v.z-z*v.y, z*v.x-x*v.z, x*v.y-y*v.x); }
  vec3 operator-(const vec3& v) const{ return vec3(x-v.x, y-v.y, z-v.z); }
  vec3 operator-() const{ return vec3(-x, -y, -z); }
  ll dot(const vec3 &v) const{ return x*v.x+y*v.y+z*v.z; }
};

struct twoset {
  int a, b;
  void insert(int x) { (a == -1 ? a : b) = x; }
  bool contains(int x) { return a == x || b == x; }
  void erase(int x) { (a == x ? a : b) = -1; }
  int size() { return (a != -1) + (b != -1); }
} E[MAXN][MAXN]; // i < j

struct face{
  vec3 norm;
  ll disc;
  int I[3];
};

face make_face(int i, int j, int k, int ii, vector<vec3> &A){ // p^T * norm < disc
  E[i][j].insert(k); E[i][k].insert(j); E[j][k].insert(i);
  face f; f.I[0]=i, f.I[1]=j, f.I[2]=k;
  f.norm = (A[j]-A[i])*(A[k]-A[i]);
  f.disc = f.norm.dot(A[i]);
  if(f.norm.dot(A[ii])>f.disc){
    f.norm = -f.norm;
    f.disc = -f.disc;
  }
  return f;
}

vector<face> get_hull(vector<vec3> &A){
  int N = A.size();
  vector<face> faces; memset(E, -1, sizeof(E));
  faces.push_back(make_face(0,1,2,3,A));
  faces.push_back(make_face(0,1,3,2,A));
  faces.push_back(make_face(0,2,3,1,A));
  faces.push_back(make_face(1,2,3,0,A));
  for(int i=4; i<N; ++i){
    for(int j=0; j<faces.size(); ++j){
      face f = faces[j];
      if(f.norm.dot(A[i])>f.disc){
        E[f.I[0]][f.I[1]].erase(f.I[2]);
        E[f.I[0]][f.I[2]].erase(f.I[1]);
        E[f.I[1]][f.I[2]].erase(f.I[0]);
        faces[j--] = faces.back();
        faces.pop_back();
      }
    }
    int nf = faces.size();
    for(int j=0; j<nf; ++j){
      face f=faces[j];
      for(int a=0; a<3; ++a) for(int b=a+1; b<3; ++b){
        int c=3-a-b;
        if(E[f.I[a]][f.I[b]].size()==2) continue;
        faces.push_back(make_face(f.I[a], f.I[b], i, f.I[c], A));
      }
    }
  }
  return faces;
}
```

5.7 Dynamic Convex Hull Trick

```
using line_t = double;
const line_t is_query = -1e18;

struct Line {
  line_t m, b;
  mutable function<const Line*()> succ;
  bool operator<(const Line& rhs) const {
    if (rhs.b != is_query) return m < rhs.m;
    const Line* s = succ();
    if (!s) return 0;
    line_t x = rhs.m;
    return b - s->b < (s->m - m) * x;
  }
};

struct HullDynamic : public multiset<Line> { // will maintain upper hull for maximum
  bool bad(iterator y) {
    auto z = next(y);
    if (y == begin()) {
      if (z == end()) return 0;
      return y->m == z->m && y->b <= z->b;
    }
    auto x = prev(y);
    if (z == end()) return y->m == x->m && y->b <= x->b;
    return (x->b - y->b)*(z->m - y->m) >= (y->b - z->b)*(y->m - x->m);
  }
  void insert_line(line_t m, line_t b) {
    auto y = insert({ m, b });
    y->succ = [=] { return next(y) == end() ? 0 : &*next(y); };
    if (bad(y)) { erase(y); return; }
    while (next(y) != end() && bad(next(y))) erase(next(y));
    while (y != begin() && bad(prev(y))) erase(prev(y));
  }
  line_t query(line_t x) {
    auto l = *lower_bound((Line) { x, is_query });
    return l.m * x + l.b;
  }
}H;
```

5.8 Half-plane Intersection

```
const double eps = 1e-8;
typedef pair<long double, long double> pi;
bool z(long double x){ return fabs(x) < eps; }
```

```
struct line{
  long double a, b, c;
  bool operator<(const line &l)const{
    bool flag1 = pi(a, b) > pi(0, 0);
    bool flag2 = pi(l.a, l.b) > pi(0, 0);
    if(flag1 != flag2) return flag1 > flag2;
    long double t = ccw(pi(0, 0), pi(a, b), pi(l.a, l.b));
    return z(t) ? c * hypot(l.a, l.b) < l.c * hypot(a, b) : t > 0;
  }
  pi slope(){ return pi(a, b); }
};
pi cross(line a, line b){
  long double det = a.a * b.b - b.a * a.b;
  return pi((a.c * b.b - a.b * b.c) / det, (a.a * b.c - a.c * b.a) / det);
}
bool bad(line a, line b, line c){
  if(ccw(pi(0, 0), a.slope(), b.slope()) <= 0) return false;
  pi crs = cross(a, b);
  return crs.first * c.a + crs.second * c.b >= c.c;
}
bool solve(vector<line> v, vector<pi> &solution){ // ax + by <= c;
  sort(v.begin(), v.end());
  deque<line> dq;
  for(auto &i : v){
    if(!dq.empty() && z(ccw(pi(0, 0), dq.back().slope(), i.slope()))) continue;
    while(dq.size() >= 2 && bad(dq[dq.size()-2], dq.back(), i)) dq.pop_back();
    while(dq.size() >= 2 && bad(i, dq[0], dq[1])) dq.pop_front();
    dq.push_back(i);
  }
  while(dq.size() > 2 && bad(dq[dq.size()-2], dq.back(), dq[0])) dq.pop_back();
  while(dq.size() > 2 && bad(dq.back(), dq[0], dq[1])) dq.pop_front();
  vector<pi> tmp;
  for(int i=0; i<dq.size(); i++){
    line cur = dq[i], nxt = dq[(i+1)%dq.size()];
    if(ccw(pi(0, 0), cur.slope(), nxt.slope()) <= eps) return false;
    tmp.push_back(cross(cur, nxt));
  }
  solution = tmp;
  return true;
}
```

5.9 Point-in-polygon test / Point-to-polygon tangent

```
// C : counter_clockwise(C[0] == C[N]), N >= 3
// return highest point in C <- P(clockwise) or -1 if strictly in P
// polygon is strongly convex, C[i] != P
int convex_tangent(vector<pi> &C, pi P, int up = 1){
  auto sign = [&](lint c){ return c > 0 ? up : c == 0 ? 0 : -up; };
  auto local = [&](pi P, pi a, pi b, pi c) {
    return sign(ccw(P, a, b)) <= 0 && sign(ccw(P, b, c)) >= 0;
  };
  int N = C.size()-1, s = 0, e = N, m;
  if( local(P, C[1], C[0], C[N-1]) ) return 0;
  while(s+1 < e){
    m = (s+e) / 2;
    if( local(P, C[m-1], C[m], C[m+1]) ) return m;
    if( sign(ccw(P, C[s], C[s+1])) < 0){ // up
      if( sign(ccw(P, C[m], C[m+1])) > 0) e = m;
      else if( sign(ccw(P, C[m], C[s])) > 0) s = m;
      else e = m;
    }
    else{ // down
      if( sign(ccw(P, C[m], C[m+1])) < 0) s = m;
      else if( sign(ccw(P, C[m], C[s])) < 0) s = m;
      else e = m;
    }
  }
  if( s && local(P, C[s-1], C[s], C[s+1]) ) return s;
  if( e != N && local(P, C[e-1], C[e], C[e+1]) ) return e;
  return -1;
}
```

5.10 kd-tree

```
typedef pair<int, int> pi;
struct node{
  pi pnt;
  int spl, sx, ex, sy, ey;
}tree[270000];

pi a[100005];
int n, ok[270000];

lint sqr(int x){ return 1ll * x * x; }
bool cmp1(pi a, pi b){ return a < b; }
bool cmp2(pi a, pi b){ return pi(a.second, a.first) < pi(b.second, b.first); }

// init(0, n-1, 1) : Initialize kd-tree
// set dap = INF, and call solve(1, P). dap = (closest point from P)
void init(int s, int e, int p){ // Initialize kd-tree
  int minx = 1e9, maxx = -1e9, miny = 1e9, maxy = -1e9;
  int m = (s+e)/2;
  for(int i=s; i<=e; i++){
    minx = min(minx, a[i].first);
    miny = min(miny, a[i].second);
    maxx = max(maxx, a[i].first);
    maxy = max(maxy, a[i].second);
  }
  tree[p].spl = (maxx - minx < maxy - miny);
  sort(a+s, a+e+1, [&](const pi &a, const pi &b){
    return tree[p].spl ? cmp2(a, b) : cmp1(a, b);
  });
  ok[p] = 1;
  tree[p] = {a[m], tree[p].spl, minx, maxx, miny, maxy};
  if(s <= m-1) init(s, m-1, 2*p);
  if(m+1 <= e) init(m+1, e, 2*p+1);
}

lint dap = 3e18;

void solve(int p, pi x){ // find closest point from point x (L^2)
  if(x != tree[p].pnt) dap = min(dap, sqr(x.first - tree[p].pnt.first) + sqr(x.second -
tree[p].pnt.second));
  if(tree[p].spl){
    if(!cmp2(tree[p].pnt, x)){
      if(ok[2*p]) solve(2*p, x);
```

```
      if(ok[2*p+1] && sqr(tree[2*p+1].sy - x.second) < dap) solve(2*p+1, x);
    }
    else{
      if(ok[2*p+1]) solve(2*p+1, x);
      if(ok[2*p] && sqr(tree[2*p].ey - x.second) < dap) solve(2*p, x);
    }
  }
  else{
    if(!cmp1(tree[p].pnt, x)){
      if(ok[2*p]) solve(2*p, x);
      if(ok[2*p+1] && sqr(tree[2*p+1].sx - x.first) < dap) solve(2*p+1, x);
    }
    else{
      if(ok[2*p+1]) solve(2*p+1, x);
      if(ok[2*p] && sqr(tree[2*p].ex - x.first) < dap) solve(2*p, x);
    }
  }
}
```

6 Math

6.1 FFT / NTT

```
typedef complex<double> base;
void fft(vector<base> &a, bool inv){
  int n = a.size(), j = 0;
  vector<base> roots(n/2);
  for(int i=1; i<n; i++){
    int bit = (n >> 1);
    while(j >= bit){
      j -= bit;
      bit >>= 1;
    }
    j += bit;
    if(i < j) swap(a[i], a[j]);
  }
  double ang = 2 * acos(-1) / n * (inv ? -1 : 1);
  for(int i=0; i<n/2; i++){
    roots[i] = base(cos(ang * i), sin(ang * i));
  }
  /* In NTT, let prr = primitive root. Then,
  int ang = ipow(prr, (mod - 1) / n);
  if(inv) ang = ipow(ang, mod - 2);
  for(int i=0; i<n/2; i++){
    roots[i] = (i ? (1ll * roots[i-1] * ang % mod) : 1);
  }
  XOR Convolution : set roots[*] = 1.
  OR Convolution : set roots[*] = 1, and do following:
    if (!inv) {
        a[j + k] = u + v;
        a[j + k + i/2] = u;
    } else {
        a[j + k] = v;
        a[j + k + i/2] = u - v;
    }
  */
  for(int i=2; i<=n; i<<=1){
    int step = n / i;
    for(int j=0; j<n; j+=i){
      for(int k=0; k<i/2; k++){
        base u = a[j+k], v = a[j+k+i/2] * roots[step * k];
        a[j+k] = u+v;
        a[j+k+i/2] = u-v;
      }
    }
  }
  if(inv) for(int i=0; i<n; i++) a[i] /= n; // skip for OR convolution.
}

vector<lint> multiply(vector<lint> &v, vector<lint> &w){
  vector<base> fv(v.begin(), v.end()), fw(w.begin(), w.end());
  int n = 2; while(n < v.size() + w.size()) n <<= 1;
  fv.resize(n); fw.resize(n);
  fft(fv, 0); fft(fw, 0);
  for(int i=0; i<n; i++) fv[i] *= fw[i];
  fft(fv, 1);
  vector<lint> ret(n);
  for(int i=0; i<n; i++) ret[i] = (lint)round(fv[i].real());
  return ret;
}
vector<lint> multiply(vector<lint> &v, vector<lint> &w, lint mod){
  int n = 2; while(n < v.size() + w.size()) n <<= 1;
  vector<base> v1(n), v2(n), r1(n), r2(n);
  for(int i=0; i<v.size(); i++){
    v1[i] = base(v[i] >> 15, v[i] & 32767);
  }
  for(int i=0; i<w.size(); i++){
    v2[i] = base(w[i] >> 15, w[i] & 32767);
  }
  fft(v1, 0);
  fft(v2, 0);
  for(int i=0; i<n; i++){
    int j = (i ? (n - i) : i);
    base ans1 = (v1[i] + conj(v1[j])) * base(0.5, 0);
    base ans2 = (v1[i] - conj(v1[j])) * base(0, -0.5);
    base ans3 = (v2[i] + conj(v2[j])) * base(0.5, 0);
    base ans4 = (v2[i] - conj(v2[j])) * base(0, -0.5);
    r1[i] = (ans1 * ans3) + (ans1 * ans4) * base(0, 1);
    r2[i] = (ans2 * ans3) + (ans2 * ans4) * base(0, 1);
  }
  fft(r1, 1);
  fft(r2, 1);
  vector<lint> ret(n);
  for(int i=0; i<n; i++){
    lint av = (lint)round(r1[i].real());
    lint bv = (lint)round(r1[i].imag()) + (lint)round(r2[i].real());
    lint cv = (lint)round(r2[i].imag());
    av %= mod, bv %= mod, cv %= mod;
    ret[i] = (av << 30) + (bv << 15) + cv;
    ret[i] %= mod;
    ret[i] += mod;
    ret[i] %= mod;
  }
  return ret;
```

```
}
```

6.2 Hell-Joseon style FFT

```
#include <smmintrin.h>
#include <immintrin.h>
#pragma GCC target("avx2")
#pragma GCC target("fma")
__m256d mult(__m256d a, __m256d b){
  __m256d c = _mm256_movedup_pd(a);
  __m256d d = _mm256_shuffle_pd(a, a, 15);
  __m256d cb = _mm256_mul_pd(c, b);
  __m256d db = _mm256_mul_pd(d, b);
  __m256d e = _mm256_shuffle_pd(db, db, 5);
  __m256d r = _mm256_addsub_pd(cb, e);
  return r;
}
void fft(int n, __m128d a[], bool invert){
  for(int i=1, j=0; i<n; ++i){
    int bit = n>>1;
    for(;j>=bit;bit>>=1) j -= bit;
    j += bit;
    if(i<j) swap(a[i], a[j]);
  }
  for(int len=2; len<=n; len<<=1){
    double ang = 2*3.14159265358979/len*(invert?-1:1);
    __m256d wlen; wlen[0] = cos(ang), wlen[1] = sin(ang);
    for(int i=0; i<n; i += len){
      __m256d w; w[0] = 1; w[1] = 0;
      for(int j=0; j<len/2; ++j){
        w = _mm256_permute2f128_pd(w, w, 0);
        wlen = _mm256_insertf128_pd(wlen, a[i+j+len/2], 1);
        w = mult(w, wlen);
        __m128d vw = _mm256_extractf128_pd(w, 1);
        __m128d u = a[i+j];
        a[i+j] = _mm_add_pd(u, vw);
        a[i+j+len/2] = _mm_sub_pd(u, vw);
      }
    }
  }
  if(invert){
    __m128d inv; inv[0] = inv[1] = 1.0/n;
    for(int i=0; i<n; ++i) a[i] = _mm_mul_pd(a[i], inv);
  }
}
vector<int64_t> multiply(vector<int64_t>& v, vector<int64_t>& w){
  int n = 2; while(n < v.size()+w.size()) n<<=1;
  __m128d* fv = new __m128d[n];
  for(int i=0; i<n; ++i) fv[i][0] = fv[i][1] = 0;
  for(int i=0; i<v.size(); ++i) fv[i][0] = v[i];
  for(int i=0; i<w.size(); ++i) fv[i][1] = w[i];
  fft(n, fv, 0); // (a+bi) is stored in FFT
  for(int i=0; i<n; i += 2){
    __m256d a;
    a = _mm256_insertf128_pd(a, fv[i], 0);
    a = _mm256_insertf128_pd(a, fv[i+1], 1);
    a = mult(a, a);
    fv[i] = _mm256_extractf128_pd(a, 0);
    fv[i+1] = _mm256_extractf128_pd(a, 1);
  }
  fft(n, fv, 1);
  vector<int64_t> ret(n);
  for(int i=0; i<n; ++i) ret[i] = (int64_t)round(fv[i][1]/2);
  delete[] fv;
  return ret;
}
```

6.3 NTT Polynomial Division

```
vector<lint> get_inv(int n, const vector<lint> &p){
  vector<lint> q = {ipow(p[0], mod - 2)};
  for(int i=2; i<=n; i<<=1){
    vector<lint> res;
    vector<lint> fq(q.begin(), q.end()); fq.resize(2*i);
    vector<lint> fp(p.begin(), p.begin() + i); fp.resize(2*i);
    fft(fq, 0); fft(fp, 0);
    for(int j=0; j<2*i; j++){
      fp[j] *= fq[j] * fq[j] % mod;
      fp[j] %= mod;
    }
    fft(fp, 1);
    res.resize(i);
    for(int j=0; j<i; j++){
      res[j] = mod - fp[j];
      if(j < i/2) res[j] += 2 * q[j];
      res[j] %= mod;
    }
    q = res;
  }
  return q;
}
vector<lint> poly_divide(const vector<lint> &a, const vector<lint> &b){
  assert(b.back() != 0); // please trim leading zero
  int n = a.size(), m = b.size();
  int k = 2; while(k < n-m+1) k <<= 1;
  vector<lint> rb(k), ra(k);
  for(int i=0; i<m && i<k; ++i) rb[i] = b[m-i-1];
  for(int i=0; i<n && i<k; ++i) ra[i] = a[n-i-1];
  vector<lint> rbi = get_inv(k, rb);
  vector<lint> res = multiply(rbi, ra);
  res.resize(n - m + 1);
  reverse(res.begin(), res.end());
  return res;
}
```

6.4 Black Box Linear Algebra + Kitamasa

```
vector<int> berlekamp_massey(vector<int> x){
  vector<int> ls, cur;
  int lf, ld;
  for(int i=0; i<x.size(); i++){
    lint t = 0;
    for(int j=0; j<cur.size(); j++){
      t = (t + 1ll * x[i-j-1] * cur[j]) % mod;
    }
    if((t - x[i]) % mod == 0) continue;
    if(cur.empty()){
```

```
      cur.resize(i+1);
      lf = i;
      ld = (t - x[i]) % mod;
      continue;
    }
    lint k = -(x[i] - t) * ipow(ld, mod - 2) % mod;
    vector<int> c(i-lf-1);
    c.push_back(k);
    for(auto &j : ls) c.push_back(-j * k % mod);
    if(c.size() < cur.size()) c.resize(cur.size());
    for(int j=0; j<cur.size(); j++){
      c[j] = (c[j] + cur[j]) % mod;
    }
    if(i-lf+(int)ls.size()>=(int)cur.size()){
      tie(ls, lf, ld) = make_tuple(cur, i, (t - x[i]) % mod);
    }
    cur = c;
  }
  for(auto &i : cur) i = (i % mod + mod) % mod;
  return cur;
}
int get_nth(vector<int> rec, vector<int> dp, lint n){
  int m = rec.size();
  vector<int> s(m), t(m);
  s[0] = 1;
  if(m != 1) t[1] = 1;
  else t[0] = rec[0];
  auto mul = [&rec](vector<int> v, vector<int> w){
    int m = v.size();
    vector<int> t(2 * m);
    for(int j=0; j<m; j++){
      for(int k=0; k<m; k++){
        t[j+k] += 1ll * v[j] * w[k] % mod;
        if(t[j+k] >= mod) t[j+k] -= mod;
      }
    }
    for(int j=2*m-1; j>=m; j--){
      for(int k=1; k<=m; k++){
        t[j-k] += 1ll * t[j] * rec[k-1] % mod;
        if(t[j-k] >= mod) t[j-k] -= mod;
      }
    }
    t.resize(m);
    return t;
  };
  while(n){
    if(n & 1) s = mul(s, t);
    t = mul(t, t);
    n >>= 1;
  }
  lint ret = 0;
  for(int i=0; i<m; i++) ret += 1ll * s[i] * dp[i] % mod;
  return ret % mod;
}
int guess_nth_term(vector<int> x, lint n){
  if(n < x.size()) return x[n];
  vector<int> v = berlekamp_massey(x);
  if(v.empty()) return 0;
  return get_nth(v, x, n);
}
struct elem{int x, y, v;}; // A_(x, y) <- v, 0-based. no duplicate please..
vector<int> get_min_poly(int n, vector<elem> M){
  // smallest poly P such that A^i = sum_{j < i} {A^j \times P_j}
  vector<int> rnd1, rnd2;
  mt19937 rng(0x14004);
  auto randint = [&rng](int lb, int ub){
    return uniform_int_distribution<int>(lb, ub)(rng);
  };
  for(int i=0; i<n; i++){
    rnd1.push_back(randint(1, mod - 1));
    rnd2.push_back(randint(1, mod - 1));
  }
  vector<int> gobs;
  for(int i=0; i<2*n+2; i++){
    int tmp = 0;
    for(int j=0; j<n; j++){
      tmp += 1ll * rnd2[j] * rnd1[j] % mod;
      if(tmp >= mod) tmp -= mod;
    }
    gobs.push_back(tmp);
    vector<int> nxt(n);
    for(auto &i : M){
      nxt[i.x] += 1ll * i.v * rnd1[i.y] % mod;
      if(nxt[i.x] >= mod) nxt[i.x] -= mod;
    }
    rnd1 = nxt;
  }
  auto sol = berlekamp_massey(gobs);
  reverse(sol.begin(), sol.end());
  return sol;
}
lint det(int n, vector<elem> M){
  vector<int> rnd;
  mt19937 rng(0x14004);
  auto randint = [&rng](int lb, int ub){
    return uniform_int_distribution<int>(lb, ub)(rng);
  };
  for(int i=0; i<n; i++) rnd.push_back(randint(1, mod - 1));
  for(auto &i : M){
    i.v = 1ll * i.v * rnd[i.y] % mod;
  }
  auto sol = get_min_poly(n, M)[0];
  if(n % 2 == 0) sol = mod - sol;
  for(auto &i : rnd) sol = 1ll * sol * ipow(i, mod - 2) % mod;
  return sol;
}
```

6.5 Gaussian Elimination

```
int n, inv;
vector<int> basis[505];
lint gyesu = 1;

void insert(vector<int> v){
```

```
  for(int i=0; i<n; i++){
    if(basis[i].size()) inv ^= 1; // inversion num increases
    if(v[i] && basis[i].empty()){
      basis[i] = v;
      return;
    }
    if(v[i]){
      lint minv = ipow(basis[i][i], mod - 2) * v[i] % mod;
      for(auto &j : basis[i]) j = (j * minv) % mod;
      gyesu *= minv;
      gyesu %= mod;
      for(int j=0; j<basis[i].size(); j++){
        v[j] += mod - basis[i][j];
        while(v[j] >= mod) v[j] -= mod;
      }
    }
  }
  puts("0");
  exit(0);
}

// Sample: Calculates Determinant in Z_p Field
int main(){
  scanf("%d",&n);
  for(int i=0; i<n; i++){
    vector<int> v(n);
    for(int j=0; j<n; j++) scanf("%d",&v[j]);
    if(i % 2 == 1) inv ^= 1;
    insert(v);
  }
  if(inv) gyesu = mod - gyesu;
  gyesu = ipow(gyesu, mod - 2);
  for(int i=0; i<n; i++) gyesu = gyesu * basis[i][i] % mod;
  cout << gyesu % mod << endl;
}
```

6.6 Simplex Algorithm

```
using T = long double;
const int N = 410, M = 30010;
const T eps = 1e-7;
int n, m;
int Left[M], Down[N];
// time complexity: exponential. fast $O(MN^2)$ in experiment. dependent on the modeling.
// Ax <= b, max c^T x. 최댓값: v, 답 추적: sol[i]. 1 based
T a[M][N], b[M], c[N], v, sol[N];
bool eq(T a, T b) { return fabs(a - b) < eps;  }
bool ls(T a, T b) { return a < b && !eq(a, b); }
void init(int p, int q) {
  n = p; m = q; v = 0;
  for(int i = 1; i <= m; i++){
    for(int j = 1; j <= n; j++) a[i][j]=0;
  }
  for(int i = 1; i <= m; i++) b[i]=0;
  for(int i = 1; i <= n; i++) c[i]=sol[i]=0;
}
void pivot(int x,int y) {
  swap(Left[x], Down[y]);
  T k = a[x][y]; a[x][y] = 1;
  vector<int> nz;
  for(int i = 1; i <= n; i++){
    a[x][i] /= k;
    if(!eq(a[x][i], 0)) nz.push_back(i);
  }
  b[x] /= k;

  for(int i = 1; i <= m; i++){
    if(i == x || eq(a[i][y], 0)) continue;
    k = a[i][y]; a[i][y] = 0;
    b[i] -= k*b[x];
    for(int j : nz) a[i][j] -= k*a[x][j];
  }
  if(eq(c[y], 0)) return;
  k = c[y]; c[y] = 0;
  v += k*b[x];
  for(int i : nz) c[i] -= k*a[x][i];
}
// 0: found solution, 1: no feasible solution, 2: unbounded
int solve() {
  for(int i = 1; i <= n; i++) Down[i] = i;
  for(int i = 1; i <= m; i++) Left[i] = n+i;
  while(1) { // Eliminating negative b[i]
    int x = 0, y = 0;
    for(int i = 1; i <= m; i++) if (ls(b[i], 0) && (x == 0 || b[i] < b[x])) x = i;
    if(x == 0) break;
    for(int i = 1; i <= n; i++) if (ls(a[x][i], 0) && (y == 0 || a[x][i] < a[x][y])) y = i;
    if(y == 0) return 1;
    pivot(x, y);
  }
  while(1) {
    int x = 0, y = 0;
    for(int i = 1; i <= n; i++)
      if (ls(0, c[i]) && (!y || c[i] > c[y])) y = i;
    if(y == 0) break;
    for(int i = 1; i <= m; i++)
      if (ls(0, a[i][y]) && (!x || b[i]/a[i][y] < b[x]/a[x][y])) x = i;
    if(x == 0) return 2;
    pivot(x, y);
  }
  for(int i = 1; i <= m; i++) if(Left[i] <= n) sol[Left[i]] = b[i];
  return 0;
}
```

6.7 De Bruijn Sequence

```
// Create cyclic string of length k^n that contains every length n string as substring.
alphabet = [0, k - 1]
int res[10000000]; // >= k^n
int aux[10000000]; // >= k*n
int de_bruijn(int k, int n) { // Returns size (k^n)
  if(k == 1) {
    res[0] = 0;
    return 1;
  }
  for(int i = 0; i < k * n; i++)
    aux[i] = 0;
```

```cpp
  int sz = 0;
  function<void(int, int)> db = [&](int t, int p) {
    if(t > n) {
      if(n % p == 0)
        for(int i = 1; i <= p; i++)
          res[sz++] = aux[i];
    }
    else {
      aux[t] = aux[t - p];
      db(t + 1, p);
      for(int i = aux[t - p] + 1; i < k; i++) {
        aux[t] = i;
        db(t + 1, t);
      }
    }
  };
  db(1, 1);
  return sz;
}
```

6.8 Discrete Kth root

```cpp
/*
 * Solve x for x^P = A mod Q
 * (P, Q-1) = 1 -> P^-1 mod (Q-1) exists
 * x has solution iff A^((Q-1) / P) = 1 mod Q
 * PP | (Q-1) -> P < sqrt(Q), solve lgQ rounds of discrete log
 * else -> find a s.t. s | (Pa - 1) -> ans = A^a */
using LL = long long;
LL mul(LL x, LL y, LL mod){ return (__int128) x * y % mod; }
LL add(LL x, LL y, LL mod){ return (x + y) % mod; }
LL pw(LL x, LL y, LL mod){
  LL ret = 1, piv = x;
  while(y){
    if(y & 1) ret = mul(ret, piv, mod);
    piv = mul(piv, piv, mod);
    y >>= 1;
  }
  return ret % mod;
}
void gcd(LL a, LL b, LL &x, LL &y, LL &g){
  if (b == 0) {
    x = 1, y = 0, g = a;
    return;
  }
  LL tx, ty;
  gcd(b, a%b, tx, ty, g);
  x = ty; y = tx - ty * (a / b);
}
LL P, A, Q, g; // x^P = A mod Q
const int X = 1e5;
LL base, ae[X], aXe[X], iaXe[X];
unordered_map<LL, LL> ht;
#define FOR(i, c) for (int i = 0; i < (c); ++i)
#define REP(i, l, r) for (int i = (l); i <= (r); ++i)
void build(LL a) { // ord(a) = P < sqrt(Q)
  base = a;
  ht.clear();
  ae[0] = 1; ae[1] = a; aXe[0] = 1; aXe[1] = pw(a, X, Q);
  iaXe[0] = 1; iaXe[1] = pw(aXe[1], Q-2, Q);
  REP(i, 2, X-1) {
    ae[i] = mul(ae[i-1], ae[1], Q);
    aXe[i] = mul(aXe[i-1], aXe[1], Q);
    iaXe[i] = mul(iaXe[i-1], iaXe[1], Q);
  }
  FOR(i, X) ht[ae[i]] = i;
}

LL dis_log(LL x) {
  FOR(i, X) {
    LL iaXi = iaXe[i];
    LL rst = mul(x, iaXi, Q);
    if (ht.count(rst)) return i*X + ht[rst];
  }
}
LL main2() {
    cin >> P >> A >> Q;
  LL t = 0, s = Q-1;
  while (s % P == 0) {
    ++t;
    s /= P;
  }
  if (A == 0) return 0;
  if (t == 0) {
    // a^{P^-1 mod phi(Q)}
    LL x, y, _;
    gcd(P, Q-1, x, y, _);
    if (x < 0) {
      x = (x % (Q-1) + Q-1) % (Q-1);
    }
    LL ans = pw(A, x, Q);
    if (pw(ans, P, Q) != A) while(1);
    return ans;
  }
  // A is not P-residue
  if (pw(A, (Q-1) / P, Q) != 1) return -1;
  for (g = 2; g < Q; ++g) {
    if (pw(g, (Q-1) / P, Q) != 1)
      break;
  }
  LL alpha = 0;
  {
    LL y, _;
    gcd(P, s, alpha, y, _);
    if (alpha < 0) alpha = (alpha % (Q-1) + Q-1) % (Q-1);
  }
  if (t == 1) {
    LL ans = pw(A, alpha, Q);
    return ans;
  }
  LL a = pw(g, (Q-1) / P, Q);
  build(a);
  LL b = pw(A, add(mul(P%(Q-1), alpha, Q-1), Q-2, Q-1), Q);
  LL c = pw(g, s, Q);
```

```
  LL h = 1;
  LL e = (Q-1) / s / P; // r^{t-1}
  REP(i, 1, t-1) {
    e /= P;
    LL d = pw(b, e, Q);
    LL j = 0;
    if (d != 1) {
      j = -dis_log(d);
      if (j < 0) j = (j % (Q-1) + Q-1) % (Q-1);
    }
    b = mul(b, pw(c, mul(P%(Q-1), j, Q-1), Q), Q);
    h = mul(h, pw(c, j, Q), Q);
    c = pw(c, P, Q);
  }
  return mul(pw(A, alpha, Q), h, Q);
}
```

6.9 Miller-Rabin Test + Pollard Rho Factorization

```
namespace miller˙rabin{
    lint mul(lint x, lint y, lint mod){ return (__int128) x * y % mod; }
  lint ipow(lint x, lint y, lint p){
    lint ret = 1, piv = x % p;
    while(y){
      if(y&1) ret = mul(ret, piv, p);
      piv = mul(piv, piv, p);
      y >>= 1;
    }
    return ret;
  }
  bool miller_rabin(lint x, lint a){
    if(x % a == 0) return 0;
    lint d = x - 1;
    while(1){
      lint tmp = ipow(a, d, x);
      if(d&1) return (tmp != 1 && tmp != x-1);
      else if(tmp == x-1) return 0;
      d >>= 1;
    }
  }
  bool isprime(lint x){
    for(auto &i : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}){
      if(x == i) return 1;
      if(x > 40 && miller_rabin(x, i)) return 0;
    }
    if(x <= 40) return 0;
    return 1;
  }
}

namespace pollard˙rho{
  lint f(lint x, lint n, lint c){
    return (c + miller_rabin::mul(x, x, n)) % n;
  }
  void rec(lint n, vector<lint> &v){
    if(n == 1) return;
    if(n % 2 == 0){
      v.push_back(2);
      rec(n/2, v);
      return;
    }
    if(miller_rabin::isprime(n)){
      v.push_back(n);
      return;
    }
    lint a, b, c;
    while(1){
      a = rand() % (n-2) + 2;
      b = a;
      c = rand() % 20 + 1;
      do{
        a = f(a, n, c);
        b = f(f(b, n, c), n, c);
      }while(gcd(abs(a-b), n) == 1);
      if(a != b) break;
    }
    lint x = gcd(abs(a-b), n);
    rec(x, v);
    rec(n/x, v);
  }
  vector<lint> factorize(lint n){
    vector<lint> ret;
    rec(n, ret);
    sort(ret.begin(), ret.end());
    return ret;
  }
};
```

6.10 Highly Composite Numbers, Large Prime

< 10^k	number	divisors	2 3 5 71113171923293137
1	6	4	1 1
2	60	12	2 1 1
3	840	32	3 1 1 1
4	7560	64	3 3 1 1
5	83160	128	3 3 1 1 1
6	720720	240	4 2 1 1 1 1
7	8648640	448	6 3 1 1 1 1
8	73513440	768	5 3 1 1 1 1 1
9	735134400	1344	6 3 2 1 1 1 1
10	6983776800	2304	5 3 2 1 1 1 1 1
11	97772875200	4032	6 3 2 2 1 1 1 1
12	963761198400	6720	6 4 2 1 1 1 1 1 1
13	9316358251200	10752	6 3 2 1 1 1 1 1 1 1
14	97821761637600	17280	5 4 2 2 1 1 1 1 1 1
15	866421317361600	26880	6 4 2 1 1 1 1 1 1 1 1
16	8086598962041600	41472	8 3 2 2 1 1 1 1 1 1 1
17	74801040398884800	64512	6 3 2 2 1 1 1 1 1 1 1 1
18	897612484786617600	103680	8 4 2 2 1 1 1 1 1 1 1 1

< 10^k	prime	# of prime	< 10^k	prime
1	7	4	10	9999999967
2	97	25	11	99999999977
3	997	168	12	999999999989

4	9973	1229	13	9999999999971
5	99991	9592	14	99999999999973
6	999983	78498	15	999999999999989
7	9999991	664579	16	9999999999999937
8	99999989	5761455	17	99999999999999997
9	999999937	50847534	18	999999999999999989

NTT Prime:

$998244353 = 119 \times 2^{23} + 1$. Primitive root: 3.
$985661441 = 235 \times 2^{22} + 1$. Primitive root: 3.
$1012924417 = 483 \times 2^{21} + 1$. Primitive root: 5.

7 Miscellaneous

7.1 Mathematics

- **Tutte Matrix**. For a simple undirected graph G, Let M be a matrix with entries $A_{i,j} = 0$ if $(i,j) \notin E$ and $A_{i,j} = -A_{j,i} = X$ if $(i,j) \in E$. X could be any random value. If the determinants are non-zero, then a perfect matching exists, while other direction might not hold for very small probability.

- **Cayley's Formula**. Given a degree sequence $d_1, d_2 \cdots, d_n$ for each labeled vertices, there exists $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\cdots(d_n-1)!}$ spanning trees. Summing this for every possible degree sequence gives n^{n-2}.

- **Kirchhoff's Theorem**. For a multigraph G with no loops, define Laplacian matrix as $L = D - A$. D is a diagonal matrix with $D_{i,i} = deg(i)$, and A is an adjacency matrix. If you remove any row and column of L, the determinant gives a number of spanning trees.

- **Green's Theorem**. Let C is positive, smooth, simple curve. D is region bounded by C.
  $\oint_C (Ldx + Mdy) = \iint_D (\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y})$

  To calculate area, $\dfrac{\partial M}{\partial x} - \dfrac{\partial L}{\partial y} = 1$, common selection is $M = \dfrac{1}{2}x, \quad L = -\dfrac{1}{2}y$.

  Line integral of circle parametrized by $(x,y) = (x_C + r_C\cos\theta,\ y_C + r_C\sin\theta)$, when $\theta = t\theta_i + (1-t)\theta_f$, is given as follows.: $\dfrac{1}{2}(r_C(x_C(\sin\theta_f - \sin\theta_i) - y_C(\cos\theta_f - \cos\theta_i)) + (\theta_f - \theta_i)r_C^2)$.

  Line integral of line parametrized by $(x,y) = t(x_1,y_1) + (1-t)(x_2,y_2)$ is given as follows.: $\dfrac{1}{2}(x_1y_2 - x_2y_1)$.

- **Burnside's lemma / Pólya enumeration theorem**. let G and H be groups of permutations of finite sets X and Y. Let $c_m(g)$ denote the number of cycles of length m in $g \in G$ when permuting X. The number of colorings of X into $|Y| = n$ colors with exactly r_i occurrences of the i-th color is the coefficient of $w_1^{r_1} \dots w_n^{r_n}$ in the following polynomial:
  $P(w_1, \dots, w_n) = \frac{1}{|H|}\sum_{h\in H}\frac{1}{|G|}\sum_{g\in G}\prod_{m\geq 1}(\sum_{h^m(b)=b}(w_b^m))^{c_m(g)}$

  When $H = \{I\}$ (No color permutation):
  $P(w_1, \dots, w_n) = \frac{1}{|G|}\sum_{g\in G}\prod_{m\geq 1}(w_1^m + \dots + w_n^m)^{c_m(g)}$

  Without the occurrence restriction:
  $P(1, \dots, 1) = \frac{1}{|G|}\sum_{g\in G} n^{c(g)}$
  where $c(g)$ could also be interpreted as the number of elements in X that are fixed up to g.

- **Pick's Theorem**. $A = i + \frac{b}{2} - 1$, where: P is a simple polygon whose vertices are grid points, A is area of P, i is # of grid points in the interior of P, and b is # of grid points on the boundary of P. If h is # of holes of P ($h+1$ simple closed curves in total), $A = i + \frac{b}{2} + h - 1$.

```
// number of (x, y) : (0 <= x < n && 0 < y <= k/d x + b/d)
// argument should be positive
ll count_solve(ll n, ll k, ll b, ll d) {
  if (k == 0) {
    return (b / d) * n;
  }
  if (k >= d || b >= d) {
    return ((k / d) * (n - 1) + 2 * (b / d)) * n / 2 + count_solve(n, k % d, b % d, d);
  }
  return count_solve((k * n + b) / d, d, (k * n + b) % d, k);
}
```

- **Xudyh Sieve**. $F(n) = \sum_{d|n} f(d)$
  $S(n) = \sum_{i\leq n} f(i) = \sum_{i\leq n} F(i) - \sum_{d=2}^{n} S\left(\lfloor \frac{n}{d} \rfloor\right)$
  Preprocess $S(1)$ to $S(M)$ (Set $M = n^{\frac{2}{3}}$ for complexity)
  $S(n) = \sum f(i) = \sum_{i\leq n}\left[F(i) - \sum_{j|i, j\neq i} f(j)\right] = \sum F(i) - \sum_{i/j=d=2}^{n}\sum_{dj\leq n} f(j)$
  $S(n) = \sum if(i) = \sum_{i\leq n} i\left[F(i) - \sum_{j|i, j\neq i} f(j)\right] = \sum iF(i) - \sum_{i/j=d=2}^{n}\sum_{dj\leq n} djf(j)$
  $\sum_{d|n}\varphi(d) = n \qquad \sum_{d|n}\mu(d) = \text{if } (n > 1) \text{ then } 0 \text{ else } 1 \qquad \sum_{d|n}(\mu(\frac{n}{d})\sum_{e|d} f(e)) = f(n)$

7.2 Popular Optimization Technique

- CHT. DnC optimization. Mo's ~~algorithm~~ trick (on tree). IOI 2016 Aliens trick. IOI 2009 Regions trick.

- Knuth's $O(n^2)$ Optimal BST : minimize $D_{i,j} = Min_{i\leq k<j}(D_{i,k} + D_{k+1,j}) + C_{i,j}$. Quadrangle Inequality : $C_{a,c} + C_{b,d} \leq C_{a,d} + C_{b,c}$, $C_{b,c} \leq C_{a,d}$. Now monotonicity holds.

- Sqrt batch processing - Save queries in buffer, and update in every sqrt steps (cf : IOI 2011 Elephant. hyea calls it "ainta technique")

- Dynamic insertion in static set (Make $O(\log n)$ copy. Merge like binomial heap.)

- Offline insertion / deletion in insert-only set (Pair insertion-deletion operation, and regard it as range query)

- Atcoder Median Pyramid : Reduce the input to binary, and solve the easier problem.

- LP Duality. max $c^T x$ sjt to $Ax \leq b$. Dual problem is min $b^T x$ sjt to $A^T x \geq c$. By strong duality, min max value coincides.

7.3 Fast LL Division / Modulo

```
inline void fasterLLDivMod(unsigned long long x, unsigned y, unsigned &out_d, unsigned
&out_m) {
  unsigned xh = (unsigned)(x >> 32), xl = (unsigned)x, d, m;
#ifdef __GNUC__
  asm(
    "divl %4; \n\t"
    : "=a" (d), "=d" (m)
    : "d" (xh), "a" (xl), "r" (y)
  );
#else
  __asm {
    mov edx, dword ptr[xh];
    mov eax, dword ptr[xl];
    div dword ptr[y];
    mov dword ptr[d], eax;
    mov dword ptr[m], edx;
  };
```

```
#endif
  out_d = d; out_m = m;
}
//x < 2^32 * MOD !
inline unsigned Mod(unsigned long long x){
  unsigned y = mod;
  unsigned dummy, r;
  fasterLLDivMod(x, y, dummy, r);
  return r;
}
```

7.4 Bit Twiddling Hack

```
int __builtin_clz(int x);// number of leading zero
int __builtin_ctz(int x);// number of trailing zero
int __builtin_clzll(long long x);// number of leading zero
int __builtin_ctzll(long long x);// number of trailing zero
int __builtin_popcount(int x);// number of 1-bits in x
int __builtin_popcountll(long long x);// number of 1-bits in x

lsb(n): (n & -n); // last bit (smallest)
floor(log2(n)): 31 - __builtin_clz(n | 1);
floor(log2(n)): 63 - __builtin_clzll(n | 1);

// compute next perm. ex) 00111, 01011, 01101, 01110, 10011, 10101..
long long next_perm(long long v){
  long long t = v | (v-1);
  return (t + 1) | (((~t & -~t) - 1) >> (__builtin_ctz(v) + 1));
}
```

7.5 Fast Integer IO

```
static char buf[1 << 19]; // size : any number geq than 1024
static int idx = 0;
static int bytes = 0;
static inline int _read() {
  if (!bytes || idx == bytes) {
    bytes = (int)fread(buf, sizeof(buf[0]), sizeof(buf), stdin);
    idx = 0;
  }
  return buf[idx++];
}
static inline int _readInt() {
  int x = 0, s = 1;
  int c = _read();
  while (c <= 32) c = _read();
  if (c == '-') s = -1, c = _read();
  while (c > 32) x = 10 * x + (c - '0'), c = _read();
  if (s < 0) x = -x;
  return x;
}
```

7.6 OSRank in g++

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace ¨gnu˙pbds;

typedef
tree<int, null_type, less<int>, rb_tree_tag, tree_order_statistics_node_update> ordered_set;
ordered_set X;
X.insert(1); X.insert(2); X.insert(4); X.insert(8); X.insert(16);

cout<<*X.find_by_order(1)<<endl; // 2
cout<<*X.find_by_order(2)<<endl; // 4
cout<<*X.find_by_order(4)<<endl; // 16
cout<<(end(X)==X.find_by_order(6))<<endl; // true

cout<<X.order_of_key(-5)<<endl;  // 0
cout<<X.order_of_key(1)<<endl;   // 0
cout<<X.order_of_key(3)<<endl;   // 2
cout<<X.order_of_key(4)<<endl;   // 2
cout<<X.order_of_key(400)<<endl; // 5
```

7.7 Nasty Stack Hacks

```
// 64bit ver.
int main2(){ return 0; }
int main(){
  size_t  sz = 1<<29;  // 512MB
  void* newstack = malloc(sz);
  void* sp_dest = newstack + sz - sizeof(void*);
  asm  __volatile__("movq %0, %%rax\n\t"
  "movq %%rsp , (%%rax)\n\t"
  "movq %0, %%rsp\n\t": : "r"(sp_dest): );
  main2();
  asm  __volatile__("pop %rsp\n\t");
  return  0;
}
```

7.8 C++ / Environment Overview

```
// vimrc : set nu sc ci si ai sw=4 ts=4 bs=2  mouse=a syntax on

// compile : g++ -o PROB PROB.cpp -std=c++11 -Wall -O2
// options : -fsanitize=address -Wfatal-errors

struct StupidGCCCantEvenCompileThisSimpleCode{
  pair<int, int> array[1000000];
}; // https://gcc.gnu.org/bugzilla/show_bug.cgi?id=68203

// how to use rand (in 2018)
mt19937 rng(0x14004);
int randint(int lb, int ub){ return uniform_int_distribution<int>(lb, ub)(rng); }

// comparator overload
auto cmp = [](seg a, seg b){ return a.func() < b.func(); };
set<seg, decltype(cmp)> s(cmp);
map<seg, int, decltype(cmp)> mp(cmp);
priority_queue<seg, vector<seg>, decltype(cmp)> pq(cmp); // max heap

// hash func overload
struct point{
    int x, y;
    bool operator==(const point &p)const{ return x == p.x && y == p.y; }
};
struct hasher {
    size_t operator()(const point &p)const{ return p.x * 2 + p.y * 3; }
};
unordered_map<point, int, hasher> hsh;
```