Numerical tests for Asian options in the BS model

The table below compares the results of 2nd order and 3rd order moment matching for Asian options under the 7 scenarios of [1], with the very precise results of Linetsky [2]. All the decimal points shown here from Linetsky are exact. The Asian option prices in [2] correspond to continuous time averaging. The PZ column shows the asymptotic results from the method of [3].

Scenario	r	T	S_0	K	σ	Lévy	3^{rd} MM	Linetsky	PZ
1	0.02	1	2	2	0.1	0.056054	0.055990	0.055986	0.055998
2	0.18	1	2	2	0.3	0.219829	0.218407	0.218387	0.218480
3	0.0125	2	2	2	0.25	0.155968	0.172403	0.172269	0.172460
4	0.05	1	1.9	2	0.5	0.195379	0.193528	0.193174	0.193692
5	0.05	1	2	2	0.5	0.249791	0.246684	0.246416	0.246944
6	0.05	1	2.1	2	0.5	0.310646	0.306366	0.306220	0.306744
7	0.05	2	2	2	0.5	0.359204	0.350233	0.350095	0.351517

The relative errors of the 2nd order and 3rd order moment matching are shown in the table below. They are defined as

$$\text{err} = \frac{\text{Price} - \text{Linetsky}}{\text{Linetsky}} \tag{1}$$

We see dramatic improvement going to 3rd order moment matching from 2nd order MM. The PZ method has slightly larger errors than 3rd order MM, at least for these 7 standard scenarios.

Scenario	$2ndMM$	$3rdMM$	PZ
1	0.12%	0.01%	0.02%
2	0.66%	0.01%	0.04%
3	-9.46%	0.08%	0.11%
4	1.14%	0.18%	0.27%
5	1.37%	0.11%	0.21%
6	1.45%	0.05%	0.17%
7	2.60%	0.04%	0.41%

References

[1] M. Fu, D. Madan and T.Wang, Pricing continuous time Asian options: a comparison of MC and Laplace transform inversion methods, J. Comp. Finance 2, 49-74 (1998).

[2] V. Linetsky, Exotic spectra, Risk 15, 85-89 (2002).

[3] D. Pirjol, L. Zhu, Asymptotic expansion for the discrete time average of the geometric Brownian motion and Asian options, 2015.