FE 620: Asian options project

Asian options

1. Build a Monte Carlo pricer for an Asian option under the Black Scholes model. The pricer should be able to handle both options with arithmetic average

$$A_n = \frac{1}{n}\sum_{i=1}^{n} S_i \tag{1}$$

and with geometric average

$$G_n = (S_1 S_2 \cdots S_n)^n \tag{2}$$

2. Test the pricer for arithmetic average Asian options on the test cases in Table B of the Linetsky paper[1] reproduced in Figure 1.

B. Asian call option prices (q = 0 and K = 2.0)

Case	r	σ	T	S_0	[redacted]	[redacted]	EE	MC
1	0.02	0.10	1	2.0	[redacted]	[redacted]	0.0559860415 (400)	0.05602 (0.00017)
2	0.18	0.30	1	2.0	[redacted]	[redacted]	0.2183875466 (57)	0.2185 (0.00059)
3	0.0125	0.25	2	2.0	[redacted]	[redacted]	0.1722687410 (41)	0.1725 (0.00063)
4	0.05	0.50	1	1.9	[redacted]	[redacted]	0.1931737903 (24)	0.1933 (0.00084)
5	0.05	0.50	1	2.0	[redacted]	[redacted]	0.2464156905 (23)	0.2465 (0.00095)
6	0.05	0.50	1	2.1	[redacted]	[redacted]	0.3062203648 (23)	0.3064 (0.00106)
7	0.05	0.50	2	2.0	[redacted]	[redacted]	0.3500952190 (13)	0.3503 (0.00146)

Figure 1: Seven scenarios for Asian options with arithmetic averaging. The column EE shows very precise price results for these instruments. These results can be used to test a MC pricer.

3. Test the pricer for geometric average Asian options against the closed form result in the Black-Scholes model.

The call and put Asian option prices with geometric averaging are

$$G_C(K,T) = e^{-rT}[G_0 N(d_1) - K N(d_2)] \tag{3}$$

$$G_P(K,T) = e^{-rT}[K N(-d_2) - G_0 N(-d_1)] \tag{4}$$

with

$$d_{1,2} = \frac{1}{\Sigma_G \sqrt{T}}\left(\log\frac{G_0}{K} \pm \frac{1}{2}\Sigma_G^2 T\right)\ , \quad \Sigma_G = \frac{1}{\sqrt{3}}\sigma \tag{5}$$

and

$$G_0 = S_0 e^{\frac{1}{2}(r-q)T - \frac{1}{12}\sigma^2 T}\ . \tag{6}$$

4. Hedging exercise. Generate a sample of e.g. 10 daily stock prices S_i following from the Black-Scholes model. For each day, price: Asian call option price $C(t_i)$, Delta of the option $\Delta(t_i)$.

Plot the option prices $C(t_i)$ vs t_i, and compare with the plot of the prices of the hedged portfolio $C(t_i) - \Delta(t_i)S_i$. Show which has the least variability.

[1] V. Linetsky, Exotic spectra, Risk magazine, April 2002.