Average Price Options AOF20 on WTI Financial Futures

17 Nov 2019

Implied volatility of the monthly averaging price option on WTI

The CME Group Exchange lists prices of Asian options with monthly averaging, with underlying the WTI Financial Futures contracts (CS). These options are called WTI Average Price Options and are called AOX19 (Nov-2019), AOZ19 (Dec-2019), AOF20(Jan-2019), etc.

Figure 1 lists the settlement prices of the AOF20 (Jan-2020) option with expiry 31-Jan-2010 as of 15-Nov-2019. The data was obtained from the link below.

`https://www.cmegroup.com/trading/energy/crude-oil/west-texas-intermediate-wti-crude-oil-calendar-swap-futures_quotes_globex_options.html?optionExpiration=5361-F0&optionProductId=7613#optionExpiration=5361F0&optionProductId=2767&strikeRange=ATM`

Regressing $C - P$ vs $-K$ we determine the discount factor for the expiration date from the put-call parity

$$C(K) - P(K) = D(T)(F_0 - K) \tag{1}$$

Regression in Excel gives ($R^2 = 1$), see Fig. 2

$$D(T)F_0 = 56.417\,, \qquad D(T) = 0.9957 \tag{2}$$

This agrees perfectly with the futures value $F_0(Jan - 20) = 56.66$ seen at the top of Fig. 1.

Underlying Future	Charts	Last	Change	Prior Settle	High	Low	Volume	Hi / Low Limit	Updated
Jan 2020		-	-	56.66	-	-	0	No Limit / No Limit	16:37:15 CT 15 Nov 2019

Type: Average Price Options

Expiration: Jan 2020

Strike Range: At The Money

Calls									Puts							
Updated	Hi / Low Limit	Volume	High	Low	Prior Settle	Change	Last	Strike Price	Last	Change	Prior Settle	Low	High	Volume	Hi / Low Limit	Updated
16:36:37 CT 15 Nov 2019	No Limit / 0.01	0	-	-	3.08	-	-	5600.0	-	-	2.42	-	-	0	No Limit / 0.01	16:36:46 CT 15 Nov 2019
16:36:35 CT 15 Nov 2019	No Limit / 0.01	0	-	-	2.80	-	-	5650.0	-	-	2.64	-	-	0	No Limit / 0.01	16:37:15 CT 15 Nov 2019
16:37:17 CT 15 Nov 2019	No Limit / 0.01	0	-	-	2.54	-	-	5700.0	-	-	2.88	-	-	0	No Limit / 0.01	16:37:36 CT 15 Nov 2019
16:37:30 CT 15 Nov 2019	No Limit / 0.01	0	-	-	2.41	-	-	5725.0	-	-	3.00	-	-	0	No Limit / 0.01	16:37:30 CT 15 Nov 2019
16:36:56 CT 15 Nov 2019	No Limit / 0.01	0	-	-	2.29	-	-	5750.0	-	-	3.13	-	-	0	No Limit / 0.01	16:36:59 CT 15 Nov 2019
16:36:36 CT 15 Nov 2019	No Limit / 0.01	0	-	-	2.06	-	-	5800.0	-	-	3.39	-	-	0	No Limit / 0.01	16:37:39 CT 15 Nov 2019
16:37:10 CT 15 Nov 2019	No Limit / 0.01	0	-	-	1.95	-	-	5825.0	-	-	3.53	-	-	0	No Limit / 0.01	16:37:10 CT 15 Nov 2019
16:37:40 CT 15 Nov 2019	No Limit / 0.01	0	-	-	1.84	-	-	5850.0	-	-	3.67	-	-	0	No Limit / 0.01	16:37:40 CT 15 Nov 2019
16:37:36 CT 15 Nov 2019	No Limit / 0.01	0	-	-	1.64	-	-	5900.0	-	-	3.97	-	-	0	No Limit / 0.01	16:36:51 CT 15 Nov 2019
16:37:24 CT 15 Nov 2019	No Limit / 0.01	0	-	-	1.54	-	-	5925.0	-	-	4.12	-	-	0	No Limit / 0.01	16:37:24 CT 15 Nov 2019

Figure 1: Average Price Options AOF20 on the Jan-20 WTI Futures contract as of Fri 15-Nov-2019. *Prior Settle* means that these are settle prices for Thu 14-Nov-2019. Settle prices for Fri 15-Nov-2019 are given in Table 1.

Table 1: The settlements of the Average Price Options AOF20 for Thu 14-Nov-2019 and Fri 15-Nov-2019. The forward of the underlying for each day is $F_0 = 56.66$ and $F_0 = 57.57$.

	Settle Thu 14-Nov-2019		Settle Fri 15-Nov-2019	
Strike	Call	Put	Call	Put
5600	3.08	2.42	3.58	2.02
5650	2.80	2.64	3.28	2.21
5700	2.54	2.88	2.99	2.42
5725	2.41	3.00	2.85	2.53
5750	2.29	3.13	2.72	2.65
5800	2.06	3.39	2.46	2.88
5825	1.95	3.53	2.33	3.01
5850	1.84	3.67	2.21	3.14
5900	1.64	3.97	1.98	3.41
5925	1.54	4.12	1.87	3.55

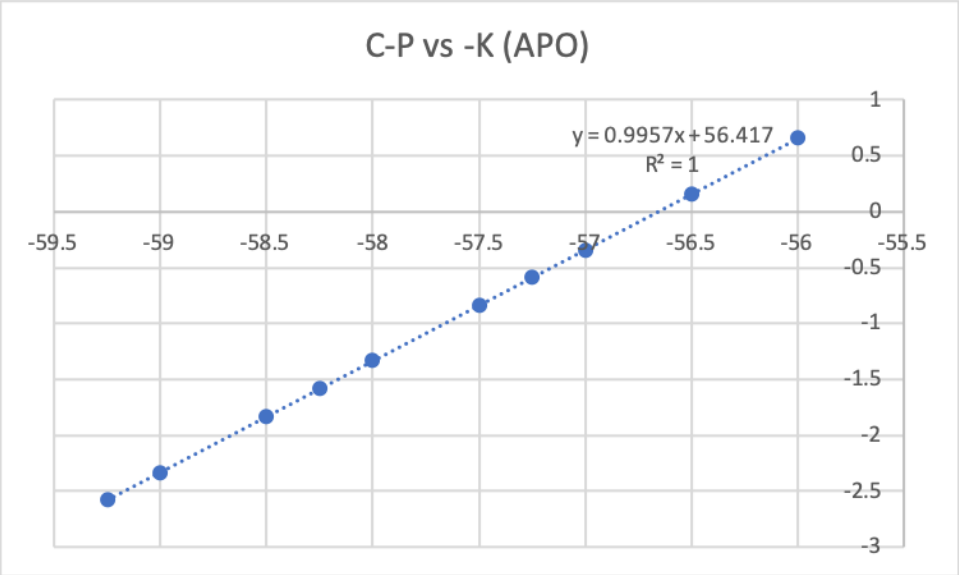

Figure 2: Regressing $C - P$ vs $-K$ for the Average Price Options AOF20 (Jan-2020). This regression is used to obtain the discount factor for maturity 31-Jan-2020 in Eq. (2).

Next we imply the Black volatility for the Asian options, separately for calls and puts. This is the volatility which reproduces the market price of the Asian option when substituted into the Black-Scholes equation with spot price $F_0 = 56.66$.

The time to expiry (31-Jan-2020) is 77 calendar days or 51 business days. Using the business days this gives $T = \frac{51}{252} = 0.202$.

The results for the Black vols vs K are shown in Figure 3. The regression of these Black vols against the log-strike $x = \log(K/F_0)$ with $F_0 = 56.66$ is shown in the lower plot of Figure 3

$$\Sigma(x) = 0.2684 - 0.313x\,. \tag{3}$$

The intercept is the ATM Asian vol Σ_{ATM} and the slope is the Asian implied vol skew s_A.

Underlyings

Some comments about the different types of futures. The underlying of the Asian option AOF20 is the WTI Financial Futures contract CSF20 (Jan-2020 contract). This is strictly speaking a futures contract on a monthly calendar swap, paying on 31-Jan-2020 the delivery price equal to the arithmetic average of the nearby CL futures on each trading day of Jan-2020. The last day of trading of this contract is 31-Jan-2020.

The underlying of this futures are the Crude Oil (CL) Futures, with physical delivery. Their last trading day is 3 days before the 25-th of the month previous to the contract month. During January 2020 two of these futures are trading:

- CLG20 (Feb-2020 contract) trades from 2-Jan-2020 (1-Jan is a holiday) until 21-Jan-2020, for a total of 13 trading days.
- CLH20 (Mar-2020 contract) trades all month long. However this is the front contract only from 22-Jan-2020 until 31-Jan-2020, for a total of 8 trading days.

The settlement prices of all these futures contracts as of Thu 14-Nov-2019 and Fri 15-Nov-2019 are listed in the Table 2.

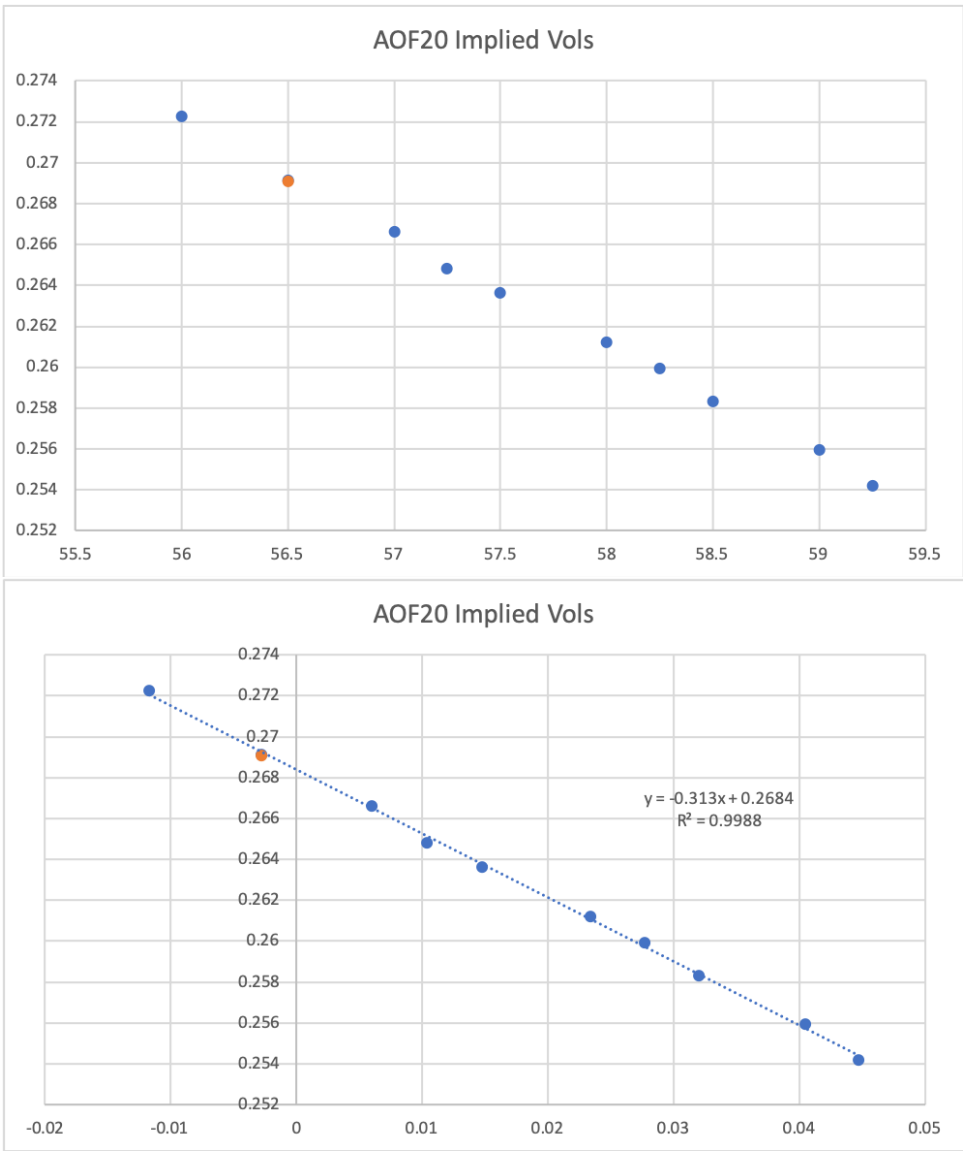

Figure 3: The Black vols of the WTI Asian options AOF20 with maturity 31-Jan-2020, obtained as explained in text. Blue dots: call options, red dots: put options. Above: vols vs strike K. Below: vols vs log-strike $x = \log(K/F_0)$ with $F_0 = 56.66$.

Table 2: The settlements of the relevant futures contracts as of COB 15-Nov-2019. The Nov-2019 Crude Oil Futures contract stopped trading before 25-Oct-2019.

	Settle Thu 14-Nov-2019		Settle Fri 15-Nov-2019	
Date	WTI Fin Fut's	Crude Oil Fut's	WTI Fin Fut's	Crude Oil Fut's
Nov-2019	(CSX19) 56.83	-	57.31	-
Dec-2019	(CSZ19) 56.84	(CLZ19) 56.77	57.78	57.72
Jan-2020	(CSF20) 56.66	(CLF20) 56.88	57.57	57.83
Feb-2020	(CSG20) 56.37	(CLG20) 56.77	57.25	57.69
Mar-2020	(CSH20) 56.03	(CLH20) 56.48	56.85	57.37

While both CL contracts are trading, the futures price of the CS contracts are related to the futures prices of the CL contracts, as

$$CSF20 = \frac{13}{21} \cdot CLG20 + \frac{8}{21} \cdot CLH20\,. \tag{4}$$

This expresses the averaging rule for the WTI Financial Futures, in terms of the nearby CL contract described above. This relation holds indeed with the values shown in Table 2.

Black volatility of the WTI futures

We would like to compare the Asian vols obtained above with the futures Black vols obtained from the American option LOF20 on the Jan-2020 Crude Oil futures contract, see the plot in Fig. 4. This plot is obtained with the *QuickVol* tool at the CME Group, accessible at this link.

`https://www.cmegroup.com/tools-information/quikstrike/pricing-volatility-strategy-tools/quikvol-tool.html`

Actually LOF20 (Jan-2020) option expires on 16-Dec-2019. Its underlying is CLF20 (Jan-2020 Crude Oil Futures contract). A more appropriate option would be LOG20 (Feb-2020) which expires on 15-Jan-2020 or LOH20 (Mar-2020) which expires on 14-Feb-2020.

The underlying of LOF20 (Jan-20) option shown in figure 4 is the CLF20 which settled at 57.83 on Fri 15-Nov-2019, as seen in Table 2. Same for LOG20 which has the underlying CLG20 which settled at 57.69 on the same day.

Visual reading off the plots of Fig. 4 gives the ATM volatility $\sigma_{ATM} = 0.30$.

The Asian option AOF20 considered is a forward start Asian option, with averaging period $[T_1, T_2]$ starting on 1-Jan-2020 and ending on 31-Jan-2020. The time $T_1 = 47$ days, and $T_2 = 77$ days (calendar days) or $T_1 = 21$ and $T_2 = 51$ business days. The ratio $\tau := T_1/T_2 = 0.41$ (using business days).

The short maturity asymptotics for Asian options prices gives the following prediction for the implied volatility of a forward start Asian option in the Black-Scholes model (no local vol prediction available yet), see Remark 3.15 in the AMF paper [2]

$$\Sigma_{LN}(K, S_0, \tau) = \sigma\sqrt{\frac{1+2\tau}{3}}\left(1 + \frac{(1-\tau)^2}{10(1+2\tau)^2}x + O(x^2)\right) \tag{5}$$

Using $\sigma = 0.30$ and $\tau = 0.41$ gives the ATM forward start Asian option vol $\Sigma(0) = 0.234$, which is somewhat smaller than the 0.268 we get directly from Asian option prices by regression.

The formula (5) implies an increasing Asian smile (positive skew), while the data in Fig. Fig:AsianVol shows a decreasing smile. The explanation is that the formula holds only in the Black-Scholes model, and the observed decreasing smile is an effect of the smile in the underlying futures contract.

In [1] we give also a prediction for the skew of the Asian implied vol, with averaging starting at time zero (corresponding to $\tau = 0$). (No such prediction is made in [2] for the forward start case.)

$$\Sigma_{ATM} = \frac{1}{\sqrt{3}}\sigma_{ATM} \tag{6}$$

$$s_A = \frac{1}{\sqrt{3}}\sigma_{ATM}\left(\frac{1}{10} + 12 s_E\right) \tag{7}$$

References

[1] D. Pirjol and L. Zhu, Short maturity Asian options in local volatility models, *SIAM Journal on Financial Mathematics*, **7**, 947-992 (2016).

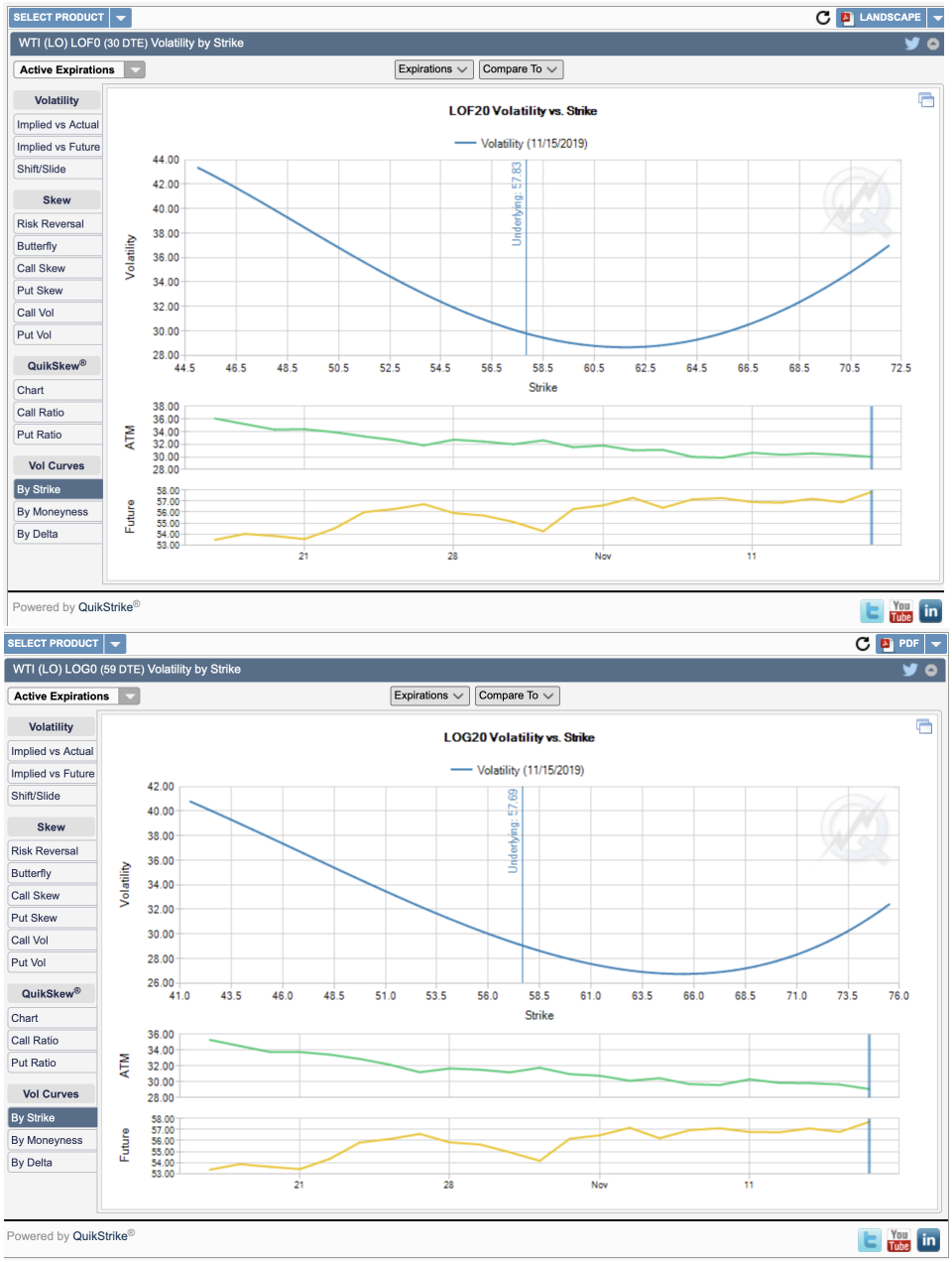

Figure 4: LOF0 (Jan-20) and LOG0 (Feb-20) Volatility vs Strike as of 15-Nov-2019, as implied from American options on the respective futures contracts.

[2] D. Pirjol, J. Wang and L. Zhu, Short maturity forward start Asian options in Local Volatility models, *Applied Mathematical Finance* **26**(3), 187-221 (2019).