Asian Options Pricing Project Proposal

Theo Dimitrasopoulos

William Kraemer

Snehal Rajguru

Vaikunth Seshadri

Spring 2020

Advisor: Dr. Dan Pirjol

Submitted in partial fulfillment of the requirements for FE 620: Pricing and Hedging

Stevens Institute of Technology

1. Abstract

This project proposal details the plan of four graduate students to create an exotic option valuation software. Starting in early March 2020, the team will be researching the characteristics of Asian Options. Using numerical techniques such as the Monte Carlo method on modified Black-Scholes models, the team will reveal the sensitivities of Asian Options through a Python implementation and properly assess their value. The team will download financial data and option prices from the CME Group Database and Yahoo Finance to evaluate the accuracy and limitations of their model. Mathematical discourse about the Asian Option is included to show the direction of the project, and a set of definition in pseudocode is provided in the project's current stages. Moreover, this document provides a detailed timeline of tasks to be completed by each team member. Additionally, a discussion of our motivation to study Asian Options is provided along with topics for further study.

2. Introduction

1.1 Choice of Option

Asian options characteristics, mechanics and usage is the main focus of this study. An Asian option is an exotic option type where the payoff depends on the average price of the underlying asset over a certain period of time, as opposed to standard options (American and European) where the payoff depends on the price of the underlying asset at a specific point in time (maturity). Since Asian options depend on the average price of the underlying asset, pricing them requires more steps compared to pricing vanilla options.

There are several types of Asian option: 1) by averaging: arit

1.2 Project Definition

In this project, we price Asian options using the Monte Carlo method. Asian options are appropriate to meet the hedging needs of users of commodities,energies, or foreign currencies who will be exposed to the risk of average prices during a future period. Since the volatility for the average of the underlying asset prices is lower than the volatility for the underlying asset prices, Asian options are less expensive than corresponding vanilla options and are therefore more attractive for some investors. Asian options are also useful in thinly-traded markets to prevent the manipulation of the underlying asset price.
We price the derivative based on fictitious data from Yahoo finance. We compute valuations and sensitivities with respect to market parameters.

1.3 Pricing the Option

Similar to vanilla options, Asian options have the following payoffs:

- Asian put option: max(K - S_{avg}, 0)
- Asian call option: max(S_{avg} - K, 0)

These are the "fixed-strike Asian options". The

Where K is the strike price and S_{avg} is the average price of the underlying asset. We will assume that the average price S_{avg} is the geometric average,

$$\mathbf{S_{avg}} = \{\prod_{1}^{N} S_{t_i}\}^{1/N}$$

Why do you need this assumptio

Where S_{t_i} = the value of the asset at time t_i with i = 1,...., N.

This assumption is crucial in the calculation of the value of the option. If S_{avg} is defined as the geometric average of stock prices, S_{avg} is lognormally distributed since the product of lognormally distributed random variables also follows the lognormal distribution. Furthermore, the continuously compounded growth rate is $\frac{1}{2}(r - q - \frac{\sigma^2}{6})T$, making the expected growth of S_{avg},

The Levy approximaton assumes that S_avg is log-normal.

$$E[S_{avg}] = S_0 e^{\frac{1}{2}(r-q-\frac{\sigma^2}{6})T} \textbf{ and the volatility = } \sigma\sqrt{\frac{T}{3}}$$

For geometric average options, because the role of S_{avg} is the same as S_T in the payoff function, the lognormal distribution of S_{avg} and the Black-Scholes formula can be used to derive the price formula for geometric average options in a straightforward manner.

Using the Black-Scholes formula and adapting it to Asian options, the value of the option can be defined as,

$$c = S_0 A_j N\left(d_{n-j} + \sigma\sqrt{T_{2,n-j}}\right) - Ke^{-rT} N(d_{n-j})$$

$$p = Ke^{-rT} N(-d_{n-j}) - S_0 A_j N\left(-d_{n-j} - \sigma\sqrt{T_{2,n-j}}\right)$$

where,

$$d_{n-j} = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \frac{\sigma^2}{2}\right) T_{1,n-j} + \ln(B_j)}{\sigma\sqrt{T_{2,n-j}}}$$

$$A_j = e^{-r\left(T - T_{1,n-j}\right) - \sigma^2 (T_{1,n-j} - T_{2,n-j})/2} B_j$$

$$T_{1,n-j} = \frac{n-j}{n}(T - \frac{(n-j-1)h}{2})$$

$$T_{2,n-j} = \left(\frac{n-j}{n}\right)^2 T - \frac{(n-j)(n-j-1)(4n-4j+1)}{6n^2} h$$

$$B_j = \left(\prod_{j=1}^{n} \frac{S(T-(n-j)h}{S}\right)^{1/n}, B_0 = 1$$

For arithmetic averages however, the value of the asian options are a little different. Since the arithmetic averages are not lognormal, the best way to price the options is to estimate them.

$$c \approx e^{-rT}\left[\left(\frac{1}{n}\sum_{i=1}^{n} e^{\mu_i+\sigma_i^2/2} N\left(\frac{\mu-\ln(\widehat{K})}{\sigma_x}+\frac{\sigma_{xi}}{\sigma_x}\right)\right)-KN\left(\frac{\mu-\ln(\widehat{K})}{\sigma_x}\right)\right]$$

$$p \approx e^{-rT}\left[KN\left(-\frac{\mu-\ln(\widehat{K})}{\sigma_x}\right)-\left(\frac{1}{n}\sum_{i=1}^{n} e^{\mu_i+\sigma_i^2/2} N\left(\frac{\mu-\ln(\widehat{K})}{\sigma_x}+\frac{\sigma_{xi}}{\sigma_x}\right)\right)\right]$$

where,

This looks again like an approximation. It is ok to use approximations but we have

$$\mu_i = \ln(S)+(r-\sigma^2/2)(t_1+(i-1)\Delta t)$$

$$\sigma_i = \sigma\sqrt{(t_1+(i-1)\Delta t)}$$

$$\sigma_{xi} = \sigma^2(t_1+\Delta t((i-1)-i(i-1)/2n)$$

$$\mu = \ln(\mathrm{S})+(r-\sigma^2/2)(t_1+(n-1)\Delta t/2)$$

$$\sigma_x = \sigma\sqrt{t_1+\Delta t(n-1)(2n-1)/6n}$$

$$\widehat{K} = 2K-\frac{1}{n}\sum_{i=1}^{n} e^{\mu_i+\frac{\sigma_{xi}(\ln(K)-\mu)}{\sigma_x^2}+\frac{\sigma_i^2-\sigma_{xi}^2/\sigma_x^2}{2}}$$

1.4 Pseudocode

We will be constructing the following definitions to simulate the price and confidence intervals of an Asian Option (the derivation of a closed form for the arithmetic mean is impossible since it does not follow a lognormal distribution, but we will be valuing the Arithmetic Asian option by conditioning on the geometric mean of the underlying price):

#Calculate the variance of the lognormal distribution

> **def** variance_lognormal(r,σ,T, S_0):
>
> > **calculate** μ_{log} σ_{log}
> >
> > **return** values under the above parameters

#Calculate the expected value of the lognormal distribution

> **def** expected_lognormal(r, σ, T, S_0)
>
> **return** expected value under the above parameters

#Monte Carlo simulation with random lognormal sampling

> **def** monte_carlo_lognormal(iterations, K, T, r):
>
> > *lognormal* = random lognormal sampling using the μ_{log} and σ_{log}
> >
> > *price* = e^{-rT} * maximum(*lognormal* - K, 0)
> >
> > - upper_confidence_interval = []
> > - mean_price = []
> > - lower_confidence_interval = []

for i in iterations:

- upper_confidence_interval = append mean + 95% i.e. 1.96 within mean
- mean_price = append the mean(price)
- lower_confidence_interval = append mean - 95% i.e. 1.96 within mean

return upper_confidence_interval, mean_price, lower_confidence_interval

We are also planning on working with the definitions for both the exact geometric and arithmetic pricing based on the Black-Scholes-Merton model so as to familiarize ourselves with the mathematical underpinnings of the pricing methodology. We will then benchmark the accuracy of the simulation against stock data from Yahoo Finance.

We are also interested in a Monte Carlo simulation of arithmetic average Calls, and potential tests of the numerical difference between the approximations under the two distributions. However, we will mainly focus on the geometric mean pricing model, and then proceed with the arithmetic mean pricing formula if time permits.

3. Project Schedule

Regarding the project schedule, there are several important deadlines to meet. Namely, the project proposal, the final report and the project presentation. These deadlines will allow us to make a schedule to complete the various aspects of this project and allow for enough time to receive feedback from Dr. Pirjol as required. As a group, we have currently identified the following steps as required for the fulfillment of the project requirements:

- Literature Review
- Price Modeling
- Writing a general description of the project
- Data Analysis
- Writing the Price Modelling segment of the final report
- Analyzing results from the real data
- Hedging Analysis
- Writing the associate sections of the report
- Programming and Debugging
- Completing the report
- Delivering the presentation

With a long list of tasks, each with its own unique challenges, proper delegation and a sound schedule is very important. We have constructed a Gantt chart that will show the schedule as well as the expected duration of these tasks. In the table below, the highlighted cells indicate the time that we will spend working on a certain task as well as the deadlines for the project deliverables. The best method, however, will be to consistently tackle the project to ensure timely completion.

Task / Week	03/03 - 03/10	03/11 - 03/18	03/18 - 03/25	03/25 - 04/01	04/01 - 04/08	04/08 - 04/15	04/15 - 04/22
Literature Review							
Project Proposal		Due: 03/13					
Price Modeling							
Data Analysis							
Analyzing Real Data							
Hedging Analysis							
Programming							
Debugging							
Final Report							Due: 04/17
Final Presentation							Due: 04/22

Table 1: Gantt Chart with key deadlines for each milestone.

4. Resources

1. Hull, John. Options, Futures, and Other Derivatives. Upper Saddle River, N.J: Pearson/Prentice Hall, 2017. Print.
2. Wiklund, Erik. "Asian option pricing and volatility." KTH Royal Institute of Technology in Stocholm. 2012. Online. Accessed March 2020.
3. Curran Michael. Valuing Asian and Portfolio Options by Conditioning on the Geometric Mean Price. The Institute of Management Sciences, Management Science/Vol. 40, No. 12, 1994. Online. Accessed March 2020.
4. Shreve, Steven E. Stochastic Calculus for Finance II : Continuous-Time Models. New York ; London :Springer, 2004. Print.
5. Bertsekas, Dimitri, and John Tsitsiklis. Introduction to Probability. 2nd ed. Athena Scientific, 2008. Print.