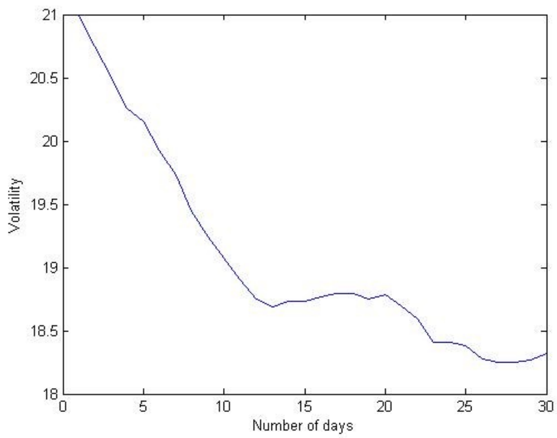
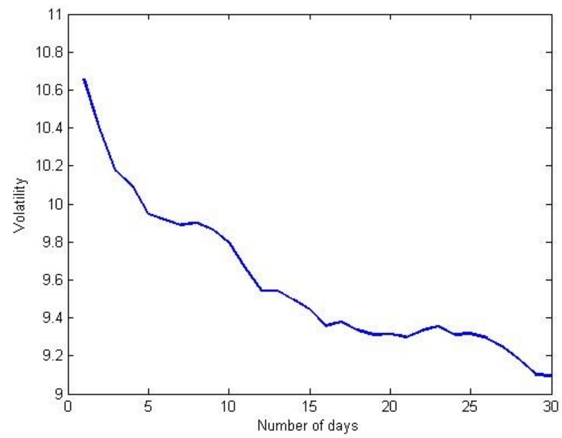
Frequency Arbitrage

Frequencygram:

Historical volatility tends to depend on the sampling frequency: SPX historical vols over last 5 (left) and 2 (right) years, averaged over the starting dates.





Can we take advantage of this pattern?

If weekly historical vol < daily historical vol :

- buy strip of T options, Δ-hedge daily
- sell strip of T options, Δ-hedge weekly

Adding up :

- do not buy nor sell any option;
- play intra-week mean reversion until T;

Final P&L : Net total position

Daily Vol / weekly Vol Arbitrage:

- On each leg: always keep $a invested in the index and update every Δt
- Resulting spot strategy: follow each week a mean reverting strategy
- **Prove that we should keep each day the following exposure**:

$$\alpha.\left(\frac{1}{S_{t_{i,j}}}-\frac{1}{S_{t_{i,1}}}\right)$$

where $t_{i,j}$ is the j-th day of the i-th week

- It amounts to follow an intra-week mean reversion strategy