Long/Short Global Macro Strategies with Target Beta Using the 3-Factor Model

Theo Dimitrasopoulos*, Yuki Homma*

* Department of Financial Engineering; Stevens Institute of Technology Babbio School of Business

ABSTRACT

In this project we construct a Long/Short Global Macro Strategy based on French Fama 3-Factor Model with a Target Beta and evaluate its sensitivity to variation of Beta and its sensitivity to the length of the estimation for covariance matrix and the expected returns under different market scenario.
Several comparisons are drawn between different target betas as well as different term structures.

THEORIES

FRENCH FAMA 3-FACTOR MODEL

Historically, French Fama 3-Factor Model is regarded as a development of CAPM which explains a relationship between expected returns and risk factors. Sharpe, Lintner, and Black developed an asset pricing model referred as CAPM which illustrates expected returns on the securities are a positive linear function of market beta. The CAPM, developed theoretically, had an empirical success, and became the standard model for describing the cross-sectional structure of expected returns on equity. However, some researchers revealed there were some anomalies that cannot be explained by the CAPM. Banz found the market equity had a relationship between expected returns. Stocks with smaller market equity had higher rates of return, usually referred to as the small cap effect. Bhandari found leverage helps explain the cross-section of average stock returns. Stattman, Rosenberg, Reid and Lanstein found that average returns on U.S stocks were positively related to the ratio of a firm's book value of common equity to its market value.
In response to this criticism, Eugene Farmer, and Kenneth French, in a paper published in 1992, empirically showed that the four representative anomaly factors discovered at the time in the U.S. stock market: market equity, book-to-market ratio, leverage, and E/P (the inverse of P/E ratio), were aggregated into market equity and book-to-market ratio.
They advanced their study and proposed French Fama 3-factor model which describes a cross section of average stock returns with three factors, market risk premium, market equity, and book-to-market ratio.

Under the Farmer-French 3-factor model, the random return of security is given by the following formula

$$r_i - r_f = \alpha_i + \beta_i^{MKT}(r_m - r_f) + \beta_i^{SMB} r_i^{SMB} + \beta_i^{HML} r_i^{HML} + \epsilon_i \quad (1)$$

With $E[\epsilon_i] = 0$, $E[r_i] = R_i$, $E[r_m] = R_m$, $E[r_i^{SMB}] = R_{SMB}$, $E[r_i^{HML}] = R_{HML}$, expected return on a security can be written as

$$R_i - r_f = \alpha_i + \beta_i^{MKT}(R_M - r_f) + \beta_i^{SMB} R_{SMB} + \beta_i^{HML} R_{HML} \quad (2)$$

Where r_i is a random return on a security, r_f is a risk-free rate, r_M is a market return, and three βs, β_i^{MKT}, β_i^{SMB}, β_i^{HML} is a sensitivity measure for risk premium of market portfolio, risk factor of market equity, and risk factor of book-to-market ratio, respectively.

MARKOWITZ PORTFOLIO THEORY

Markowitz portfolio theory also known as modern portfolio theory is a theory on how risk-averse investors can construct portfolio to maximize expected return based on a given level of market risk. The theory can also be used to construct a portfolio that minimize risk for a given level of expected return. In mathematical format:

$$\min_{\omega} \omega^T \Sigma \omega \quad (3)$$

Subject to

$$e^T \omega = 1 \quad (4)$$
$$\rho^T \omega = \rho_T$$

Where ω is a vector of weights of securities, Σ is a covariance matrix , ρ is an expected return, ρ_T is a target return.

LINEAR REGRESSION

An approach for predicting a quantitative response Y n the basis of multiple predictor variable X_j that assume an approximately linear relationship between X_j and Y. For a model with p predictors, the linear regression takes the form

$$Y = \beta_0 + \sum_{j=1}^{p} \beta_j X_j + \varepsilon \quad (5)$$

with X_j is the j th predictor and β_j qualifies the relationship between that predictor and the response. Given estimates for $\beta's$, it can make predictors using the model

$$\hat{f}(X) = \hat{\beta}_0 + \sum_{j=1}^{p} \hat{\beta}_j X_j \quad (6)$$

where we estimate these parameters by minimizing the residual sum of squares

$$RSS = \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \sum_{j=1}^{p} \hat{\beta}_j X_j)^2 \quad (7)$$

INVESTMENT UNIVERSE AND BACKTESTING

DATA

We used following 12 ETFs from March 1st 2007 to June 30th 2020

1. CurrencyShares Euro Trust (FXE)
2. iShares MSCI Japan Index (EWJ)
3. SPDR GOLD Trust (GLD)
4. Powershares NASDAQ-100 Trust (QQQ)
5. SPDR S&P 500 (SPY)
6. iShares Lehman Short Treasury Bond (SHV)
7. PowerShares DB Agriculture Fund (DBA)
8. United States Oil Fund LP (USO)
9. SPDR S&P Biontech (XBI)
10. iShares S&P Latin America 40 Index (ILF)
11. iShares MSCI Pacific ex-Japan Index Fund (EPP)
12. SPDR DJ Euro Stoxx 50 (FEZ)

We chose S&P 500 (SPY) as the benchmark we considered.

The data for constructing French Fama 3-Factor Model is quoted from Ken French's website for the factors' historical values.

Analysis Period

Divided entire analysis period into 5 sub-periods:

1. Pre-Subprime Crisis : March 22, 2007 – March 3, 2008
2. During Subprime Crisis : March 3, 2008 – September 10, 2010
3. Post-Subprime Crisis : September 10, 2010 – January1, 2015
4. Pre-Covid19 Pandemic : January 1, 2015 – March 9, 2020
5. During Covid19 Pandemic : March 9, 2020 – October 30, 2020

Backtesting

Ran separate backtests for each sub-period to compare strategies. We compared with different perspectives.

(1) Impact on Beta Target

Compared our strategy in terms of target beta in the same sub-period. Changed target beta β_T^m ,and compare performance.

(2) Impact of various term structure given Beta

Compare portfolios' performance with different term structures and fixed beta. S_j^i represents a term structure with i days lookback period to estimate the expected return and j days lookback period to estimate the covariance matrix.

We also ran a backtest on whole period from March 1st, 2007 to November 30th, 2020.

INVESTMENT STRATEGY

Objective function

We considered the following investment strategy:

$$\max_{\omega} \rho^T \omega - \lambda(\omega - \omega_p)^{\mathrm{T}} \Sigma (\omega - \omega_p) \tag{8}$$

With constraints

$$\sum_{i=1}^{n} \beta_i^m \omega_i = \beta_T^m \tag{9}$$

$$\sum_{i=1}^{n} \omega_i = 1$$

where

- ω_i : weight allocated to each security S_i, and ω is a vector of weights.
- ρ : a vector of expected returns of security S_i
- Σ : the covariance matrix between securities returns derived from the Factor model
- ω_p : composition of a reference Portfolio, the previous portfolio when rebalancing the portfolio.
- λ : small regularization parameter to limit turnover.
- $\beta_i^m = \frac{cov(r_i, r_M)}{\sigma^2(r_M)}$: the Beta of security S_i defined in the CAPM model.
- β_T^m : the Portfolio's Target Beta

Term structures

We analyzed the following combinations of term structures and Target Beta

- S_{60}^{60}
- S_{120}^{60}
- S_{60}^{90}
- S_{120}^{90}
- S_{60}^{120}
- S_{120}^{120}

Target Beta

- $\beta_T^m = \{-1.0, -0.5, 0.5, 1.0, 1.5, 2.0\}$

PERFORMANCE AND RISK METRICS

We introduced following metrics to compare portfolio's performance and a degree of risks. We assumed that each year has 250 trading days to annualize these metrics.

Performance Metrics

- Cumulated Return
- Annual Arithmetic Mean / Geometric Mean Return
- Annual Min Return
- Max 10-days Drawdown
- Sharpe Ratio

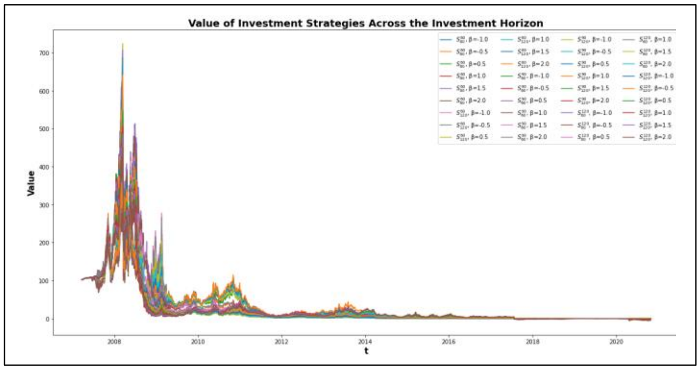
Risk Metrics

- Volatility
- Daily VaR
- Annual VaR
- Modified VaR
- Annual CVaR
- Skewness
- Kurtosis

RESULTS AND DISCUSSIONS

In this section, we illustrate the 1) the evolution of cumulated daily profit and loss, 2) the distribution of daily returns, and 3) Performance and Risk comparison during the whole investment horizon. The results of 5-sub period analysis are shown in Appendix section and also please refer to the files we submit simultaneously.

1. The evolution of cumulated daily Profit and Loss as summing that investing $100 at the first allocation date



2. The distribution of daily returns

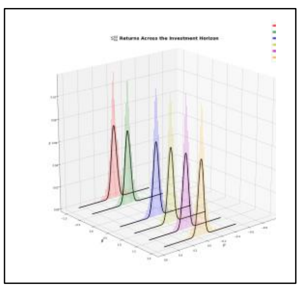


Figure 2: Distribution of daily returns of S_{60}^{60}

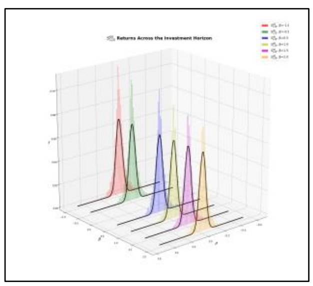


Figure 3: Distribution of daily returns of S_{120}^{60}

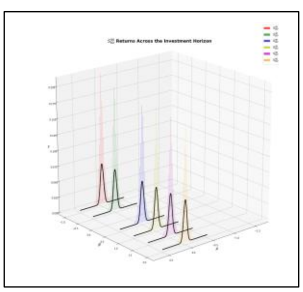


Figure 2: Distribution of daily returns of S_{60}^{90}

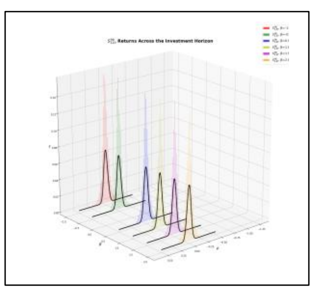


Figure 3: Distribution of daily returns of S_{120}^{90}

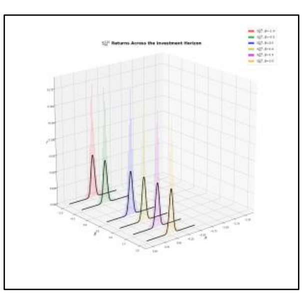


Figure 2: Distribution of daily returns of S_{60}^{120}

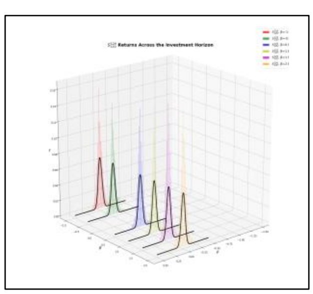


Figure 3: Distribution of daily returns of S_{120}^{120}

3. Comparison of Strategies' Performances and Risks

	S^{60}_{60}	S^{60}_{60}	S^{60}_{60}	S^{60}_{60}	S^{60}_{60}	S^{60}_{60}
	b=-1.0	b=-0.5	b=0.5	b=1.0	b=1.5	b=2.0
Cumulative Retur	0.638525625	0.683192482	0.754013823	0.779089143	0.797733507	0.801894903
Arithmetic Mean	0.032390709	0.081951489	0.173905644	0.218357395	0.264544256	0.300696997
Geometric Mean	-0.448191239	-0.380688499	-0.282185201	-0.249505219	-0.225878585	-0.220680267
Minimum Return	-78.23072119	-69.955303	-76.06813949	-79.44488715	-82.7668602	-85.8584402
Max 10-day Draw	-0.611033922	-0.59189803	-0.616536435	-0.634282574	-0.653581012	-0.670360319
Volatility	0.980654214	0.962201492	0.955090037	0.966694829	0.988828364	1.018086406
Sharpe Ratio (An	-0.028153952	0.022813817	0.119261682	0.163813222	0.206855167	0.236420991
Kurtosis (Annual)	5.456298994	4.73151552	3.822763641	3.6430633	3.728458051	4.019736774
Skew (Annual)	0.344832908	0.311519182	0.256406631	0.218898408	0.185811527	0.145716872
mVaR (Annual)	-140.4686899	-139.8662919	-141.5511402	-144.416909	-148.2474743	-153.0420619
VaR (Daily)	-10.18875789	-9.976970501	-9.866208598	-9.969152182	-10.18093189	-10.47084126
VaR (Annual)	-161.0984073	-157.7497547	-155.9984552	-157.6261362	-160.9746673	-165.5585369
CVaR (Annual)	-224.3636746	-219.0479595	-218.3124599	-221.8859406	-226.6336049	-232.4192154

S^{60}_{120}	S^{60}_{120}	S^{60}_{120}	S^{60}_{120}	S^{60}_{120}	S^{60}_{120}
b=-1.0	b=-0.5	b=0.5	b=1.0	b=1.5	b=2.0
0.645533238	0.698297572	0.788941528	0.818043917	0.841627098	0.859639743
0.070231322	0.125906583	0.230732524	0.272318564	0.315002962	0.361915572
-0.437295677	-0.35885215	-0.236950707	-0.200758604	-0.172358797	-0.151196142
-78.88453344	-76.26550236	-79.507063	-80.43721654	-82.30847341	-85.00489159
-0.618196885	-0.616814872	-0.629936969	-0.640813063	-0.652429732	-0.663134253
1.004887405	0.982165802	0.96534079	0.971241133	0.985751554	1.010895546
0.010181561	0.067103317	0.176862436	0.218605408	0.258688877	0.298661492
5.298589122	4.691889466	3.834240719	3.727404231	3.800445383	4.124816312
0.221447512	0.188285337	0.171228861	0.18310061	0.186276575	0.183046484
-147.6817482	-146.1337808	-145.1037824	-145.6105617	-147.3052995	-150.2501417
-10.42571874	-10.16707654	-9.950116104	-9.99486277	-10.12874044	-10.37154755
-164.8450873	-160.755595	-157.3251494	-158.0326563	-160.1494482	-163.9885655
-234.9542928	-228.6657893	-222.7937149	-223.9580735	-226.0882325	-229.725476

S^{90}_{60}	S^{90}_{60}	S^{90}_{60}	S^{90}_{60}	S^{90}_{60}	S^{90}_{60}
b=-1.0	b=-0.5	b=0.5	b=1.0	b=1.5	b=2.0
0.653210906	0.693511028	0.765358482	0.793130216	0.802448208	0.79861286
0.060284244	0.105187594	0.203043341	0.250887528	0.282318771	0.308801646
-0.425492723	-0.365720374	-0.267267984	-0.231660464	-0.219991115	-0.22477787
-95.99020037	-89.47072692	-76.25503707	-75.48515463	-79.07622538	-88.35203129
-0.63025057	-0.595222751	-0.602506606	-0.614125355	-0.629086702	-0.646109734
0.987756472	0.972781477	0.971219829	0.982741593	1.00123557	1.029971808
0.000287767	0.046451947	0.147282146	0.194239797	0.222044419	0.241561608
8.349615251	7.203754894	5.275006489	4.656963782	4.32390753	4.264588341
0.528191831	0.493860754	0.375844445	0.304990927	0.232490004	0.158611872
-130.0983638	-131.0991443	-137.4944452	-142.1322857	-147.4477941	-153.9055914
-10.25148503	-10.07773931	-10.02235123	-10.12307409	-10.30289384	-10.59124286
-162.0902105	-159.3430494	-158.467287	-160.0598553	-162.9030552	-167.4622534
-226.0608165	-222.4015392	-222.2083129	-225.4860783	-229.6080521	-236.2049898

S^{90}_{120}	S^{90}_{120}	S^{90}_{120}	S^{90}_{120}	S^{90}_{120}	S^{90}_{120}
b=-1.0	b=-0.5	b=0.5	b=1.0	b=1.5	b=2.0
0.67275128	0.709906247	0.794600265	0.826333189	0.842692612	0.842509338
0.117252266	0.155360218	0.261867376	0.311627083	0.35070351	0.378357671
-0.396065519	-0.342387692	-0.229810411	-0.190684452	-0.17109445	-0.171311811
-92.58917459	-86.87036989	-74.37131173	-77.0333908	-80.77968546	-86.6815629
-0.604105022	-0.607610864	-0.623441254	-0.634912663	-0.648289777	-0.661557638
1.01288172	0.997553215	0.990954959	1.000922393	1.018977762	1.044143657
0.056524138	0.095594116	0.203709941	0.251395197	0.285289356	0.304898343
6.378948266	5.734692843	4.75082429	4.466469692	4.283149139	4.305905363
0.355512942	0.330892242	0.262785395	0.221285315	0.168791143	0.111724463
-142.3472824	-141.9676108	-144.3097596	-147.2561409	-151.6375356	-156.9396507
-10.49007497	-10.31536984	-10.20412554	-10.28791255	-10.4601114	-10.71084973
-165.8626486	-163.100318	-161.3413911	-162.6661802	-165.3888831	-169.3534041
-235.52639	-231.630405	-229.7208078	-231.8985046	-235.5486391	-240.6163646

S^{120}_{60}	S^{120}_{60}	S^{120}_{60}	S^{120}_{60}	S^{120}_{60}	S^{120}_{60}
b=-1.0	b=-0.5	b=0.5	b=1.0	b=1.5	b=2.0
0.750548061	0.801623003	0.866178128	0.880857711	0.891374041	0.893518382
0.180175385	0.233686561	0.312926806	0.340350916	0.371113105	0.400468457
-0.286786971	-0.221019096	-0.14362343	-0.126826993	-0.114964699	-0.112563023
-73.55044995	-67.58849597	-72.71769485	-75.9868056	-78.32494048	-77.72370449
-0.587243027	-0.597511243	-0.626790499	-0.642665441	-0.653763323	-0.651391076
0.965329824	0.952988223	0.954620652	0.964889482	0.98326315	1.008921034
0.124491529	0.182254677	0.264950068	0.290552359	0.316408791	0.337457983
5.05030193	4.654695751	4.176914339	4.067627531	4.030644979	3.946694233
0.251194393	0.247020655	0.207719006	0.167220202	0.126189257	0.077321767
-140.7973336	-139.5218735	-141.2812111	-143.9999267	-147.8308738	-153.1553154
-9.97022488	-9.820431132	-9.805717137	-9.901573808	-10.0804096	-10.3355856
-157.643097	-155.2746499	-155.0420012	-156.5576283	-159.3852704	-163.4199573
-222.2672138	-218.8447293	-219.9685524	-223.0906939	-227.8579687	-234.3136161

S^{120}_{120}	S^{120}_{120}	S^{120}_{120}	S^{120}_{120}	S^{120}_{120}	S^{120}_{120}	SPY
b=-1.0	b=-0.5	b=0.5	b=1.0	b=1.5	b=2.0	
0.78523435	0.843400363	0.941106936	0.964597474	0.971645955	0.958208976	1.085139148
0.253905768	0.310890175	0.414003418	0.447941536	0.475549245	0.493435472	0.10318669
-0.241656201	-0.170255507	-0.060691137	-0.036041792	-0.02876213	-0.042685742	0.08172158
-79.61695643	-73.60237313	-76.41287659	-79.57630774	-82.74035344	-85.90155808	-27.35591157
-0.625734059	-0.633176106	-0.653390486	-0.666781403	-0.68013162	-0.694412225	-0.249483005
0.993893866	0.979872183	0.97322458	0.981941474	1.001118256	1.030235247	0.207044877
0.195097057	0.256043777	0.363742784	0.395076027	0.415085074	0.420715048	0.208586132
5.45863345	5.014715637	4.246273609	3.9939182	3.91295303	3.996545622	14.86576961
0.245764727	0.243041104	0.199064289	0.153867164	0.101400901	0.0359592	-0.046914821
-143.8703487	-142.4137352	-143.5434866	-146.4290288	-150.8512486	-156.9730998	-27.46710382
-10.23788353	-10.06922265	-9.958822557	-10.03592888	-10.22438124	-10.52012987	-2.112606505
-161.8751519	-159.2083892	-157.4628105	-158.6819684	-161.6616619	-166.3378583	-33.40324178
-229.7913456	-225.6573768	-222.4222077	-225.371844	-231.4884888	-239.3888292	-51.41788848

OVERALL CONCLUSION

Portfolios with higher beta tend to have a higher expected return along with a higher risk exposure. That is not always the case as we saw in the results above. A beta model with different re-estimation periods for the covariance and expected returns matrices does not suffice for a portfolio with consistent performance against the market benchmark. Assuming this strategy alone will be used during portfolio construction we observe that:

- Pre-Housing Crisis, S^{120}_{120}**($\boldsymbol{\beta^m_T}$=0.5)** is a top performer, with a low drawdown and large Sharpe Ratio.
- During the Subprime Crisis, S^{120}_{120}**($\boldsymbol{\beta^m_T}$=2)** yielding the highest return and low volatility.
- Post-Housing Crisis, S^{60}_{60}**($\boldsymbol{\beta^m_T}$=2)** has the largest Sharpe ratio and yields the highest cumulative return
- In the period before COVID-19, S^{120}_{60}**($\boldsymbol{\beta^m_T}$=1.5)** has the highest return with the lowest volatility.
- During COVID-19, S^{120}_{90}**($\boldsymbol{\beta^m_T}$ = -1.0)** was the best performer. The target beta is to be expected, as a fully de-correlated portfolio performs better than the market when the latter is under stress.

When looking at the portfolio sensitivity to the structure term and beta, we do not observe a clear relationship, but its return has a more significant relationship to the estimation term. The first day of each lookback period also plays an important role in the return of each strategy.

APPENDIX

I. The evolution of cumulated daily Profit and Loss as summing that investing $100 at the first allocation date. (5 subperiod analysis)

Pre-Subprime Crisis

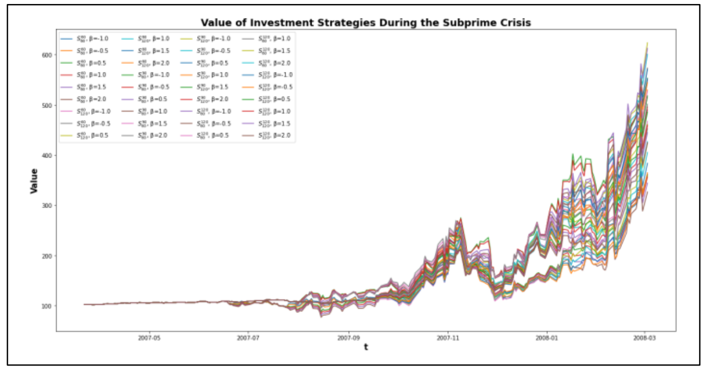


Figure 1: the evolution of cumulated daily profit and loss for several portfolio strategies (Pre-Subprime Crisis)

During Subprime Crisis

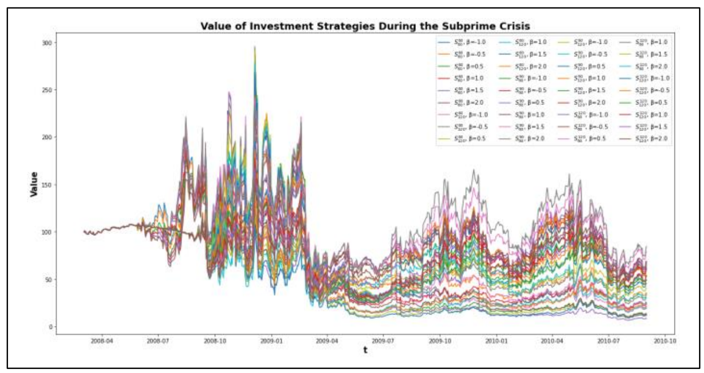


Figure 2: the evolution of cumulated daily profit and loss for several portfolio strategies (During Subprime Crisis)

Post Subprime Crisis

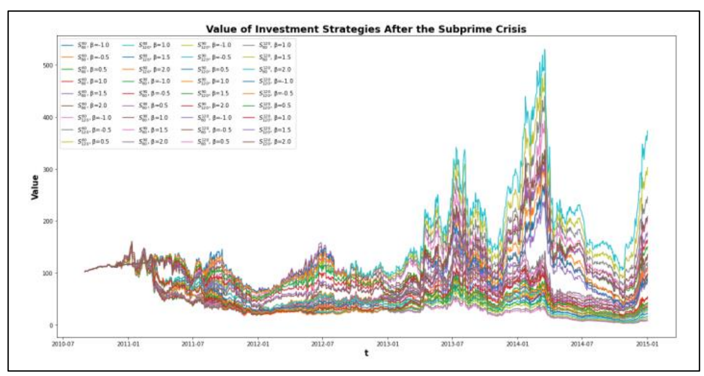


Figure 3: the evolution of cumulated daily profit and loss for several portfolio strategies (Post-Subprime Crisis)

Pre-Covi9 Crisis

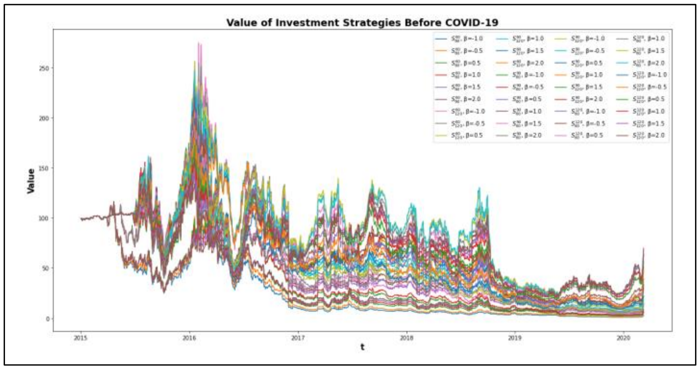


Figure 3: the evolution of cumulated daily profit and loss for several portfolio strategies (Pre-Covid19 Crisis)

DuringCovid19 Crisis

II.

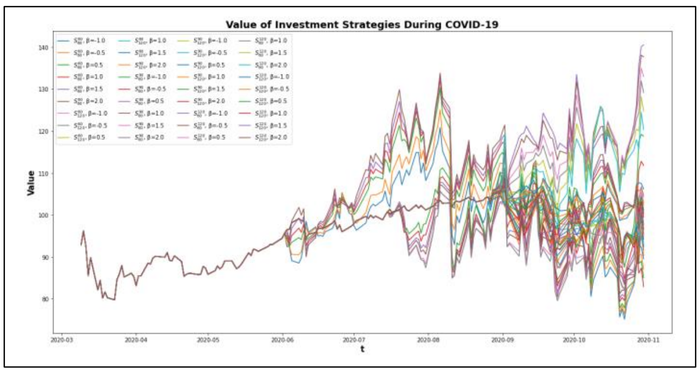


Figure 3: the evolution of cumulated daily profit and loss for several portfolio strategies (DuringCovid19 Crisis)

Pre-Subprime

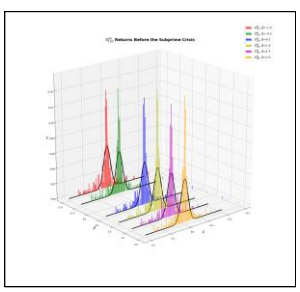


Figure 2: Distribution of daily returns of S_{120}^{60}

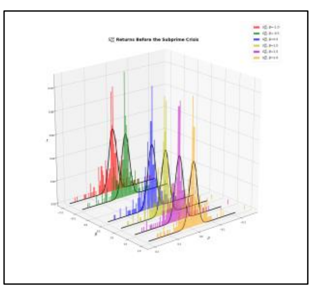


Figure 3: Distribution of daily returns of S_{60}^{60}

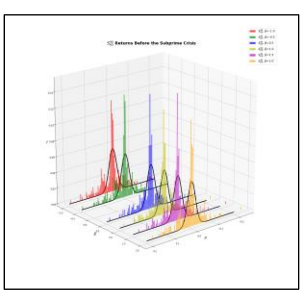


Figure 4: Distribution of daily returns of S_{60}^{90}

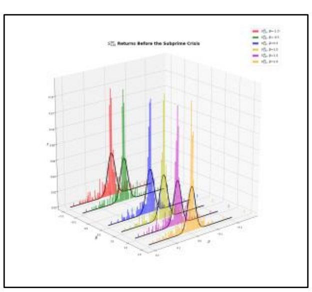


Figure 5: Distribution of daily returns of S_{120}^{90}

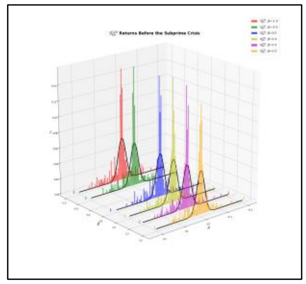


Figure 6: Distribution of daily returns of S_{60}^{120}

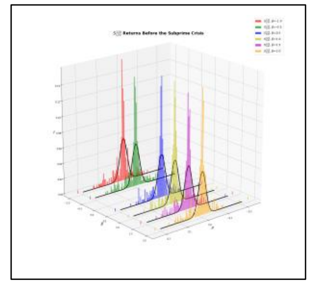


Figure 7: Distribution of daily returns of S_{120}^{120}

During Subprime Crisis

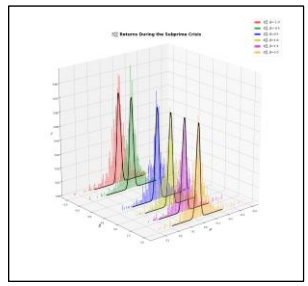


Figure 8: Distribution of daily returns of S_{60}^{60}

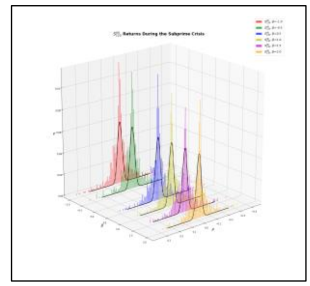


Figure 4: Distribution of daily returns of S_{60}^{90}

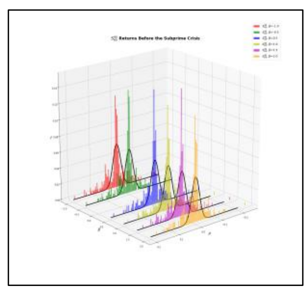


Figure 10: Distribution of daily returns of S_{60}^{90}

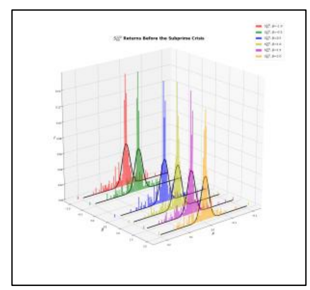


Figure 11: Distribution of daily returns of S_{60}^{120}

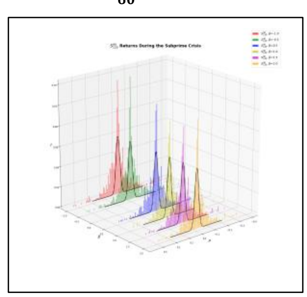


Figure 12: Distribution of daily returns of S_{120}^{90}

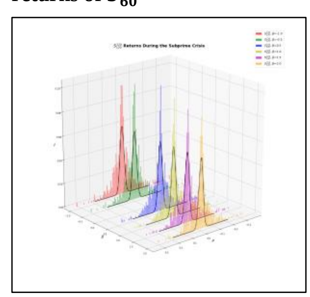
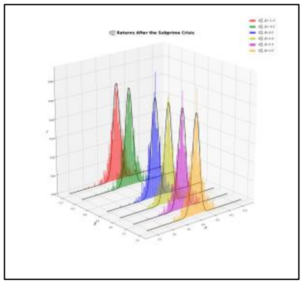


Figure 13: Distribution of daily returns of S_{120}^{120}

Post-Subprime Crisis



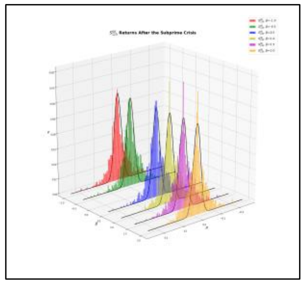


Figure 8: Distribution of daily returns of S_{60}^{60}

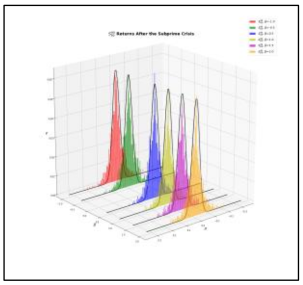


Figure 10: Distribution of daily returns of S_{60}^{90}

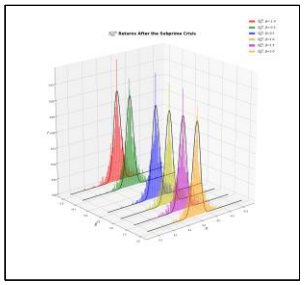


Figure 11: Distribution of daily returns of S_{60}^{120}

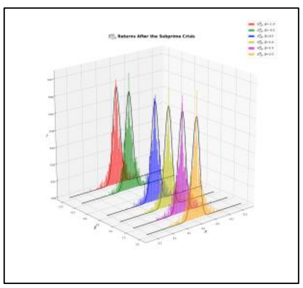


Figure 12: Distribution of daily returns of S_{120}^{90}

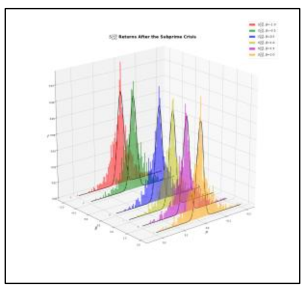


Figure 13: Distribution of daily returns of S_{120}^{120}

Pre-COVID Crisis

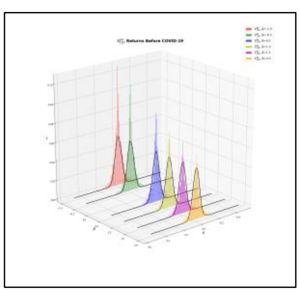


Figure 2: Distribution of daily returns of S_{120}^{60}

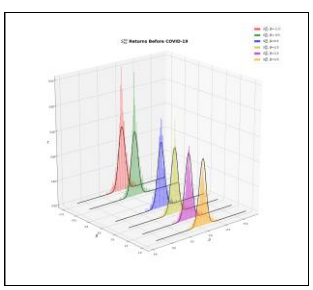


Figure 3: Distribution of daily returns of S_{60}^{60}

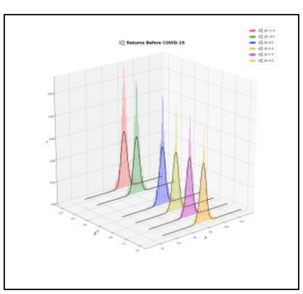


Figure 4: Distribution of daily returns of S_{60}^{90}

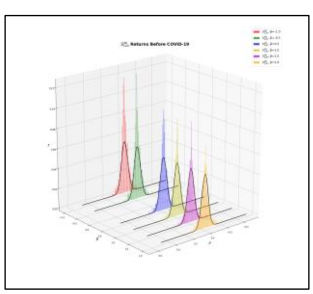


Figure 5: Distribution of daily returns of S_{120}^{90}

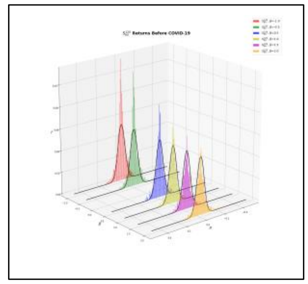


Figure 6: Distribution of daily returns of S_{60}^{120}

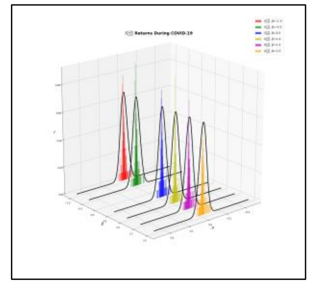


Figure 7: Distribution of daily ly returns of S_{120}^{120}

During Covid19 Crisis

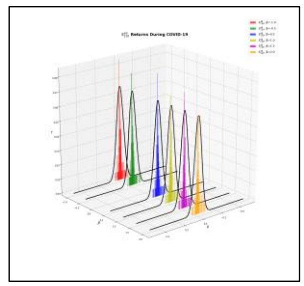


Figure 2: Distribution of daily returns of S_{120}^{60}

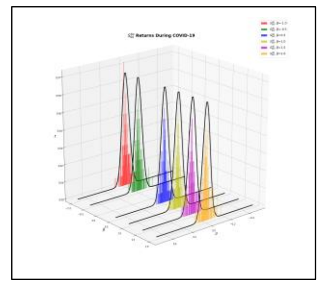


Figure 3: Distribution of daily returns of S_{60}^{60}

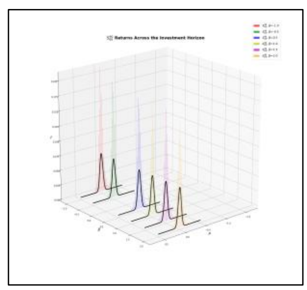


Figure 4: Distribution of daily

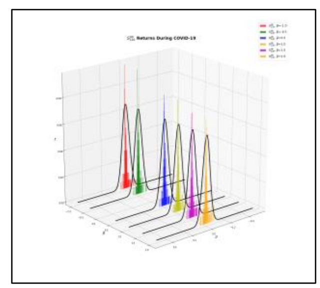


Figure 5: Distribution of daily r

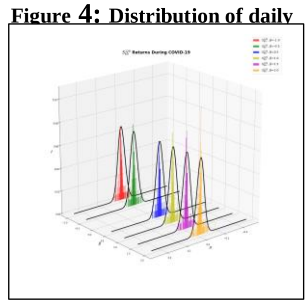


Figure 6: Distribution of daily returns of S_{60}^{120}

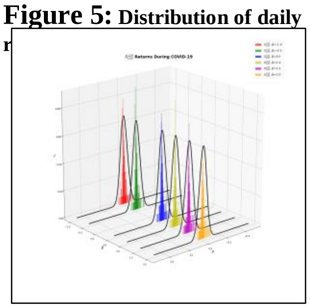


Figure 7: Distribution of daily returns of S_{120}^{120}

III. The distribution of daily returns

The following tables show the best performer of our strategies. Please refer to the data in deliberate/data/xxx.exec.csv file to see entire table.

I. Pre-Subprime Period

	S^{120}_{120}	S^{120}_{120}	S^{120}_{120}	S^{120}_{120}	S^{120}_{120}	S^{120}_{120}	SPY
	b=-1.0	b=-0.5	b=0.5	b=1.0	b=1.5	b=2.0	
Cumulative Retur	6.0938863	5.800365502	5.176351992	4.892516087	4.744144629	4.485051749	0.964412341
Arithmetic Mean	2.096576313	2.027689962	1.891994238	1.836191775	1.813920231	1.768169502	-0.021356888
Geometric Mean	1.813834358	1.764116017	1.649518563	1.592759023	1.561769172	1.505263562	-0.03623371
Minimum Return	-47.72926669	-49.20107634	-52.35153712	-54.72030433	-57.10126549	-59.46562173	-7.408435644
Max 10-day Draw	-0.326741916	-0.312174507	-0.284386724	-0.270850117	-0.26270685	-0.260931	-0.074810431
Volatility	0.754964147	0.728849546	0.698353059	0.698927286	0.71082329	0.724709088	0.172296237
Sharpe Ratio (An	2.697580173	2.699720366	2.623306671	2.541311251	2.467449021	2.357041648	-0.4721919
Kurtosis (Annual)	4.384091045	5.041310821	6.255009432	6.642518352	6.892797617	6.767124314	0.472457853
Skew (Annual)	0.234184878	0.251717726	0.23339572	0.179284696	0.152295173	0.066082901	-0.277512626
mVaR (Annual)	-99.13768138	-94.34386968	-89.38283429	-90.37688512	-92.45191576	-96.75683945	-29.64524056
VaR (Daily)	-7.01523715	-6.77112234	-6.50814666	-6.536441309	-6.669103676	-6.831857471	-1.800934954
VaR (Annual)	-110.9206386	-107.0608445	-102.902834	-103.3502116	-105.4477878	-108.0211513	-28.47528186
CVaR (Annual)	-166.9619167	-158.1509072	-147.2696124	-148.2404085	-152.2498555	-157.5936754	-40.17965124

II. During subprime Crisis

	S^{120}_{120}	S^{120}_{120}	S^{120}_{120}	S^{120}_{120}	S^{120}_{120}	S^{120}_{120}	SPY
	b=-1.0	b=-0.5	b=0.5	b=1.0	b=1.5	b=2.0	
Cumulative Retur	0.396176487	0.50776768	0.73646339	0.816788729	0.855882611	0.860330597	0.940681933
Arithmetic Mean	-0.152497426	0.045976488	0.357985586	0.456497601	0.515733013	0.556408325	-0.009393771
Geometric Mean	-0.924183043	-0.676813448	-0.305708684	-0.202292922	-0.155573623	-0.150393294	-0.061142728
Minimum Return	-101.3616279	-91.98998681	-84.19510194	-81.22947619	-77.24028534	-79.11109333	-24.6119145
Max 10-day Draw	-0.592406188	-0.591261136	-0.603286065	-0.616173167	-0.630207922	-0.645269179	-0.249483005
Volatility	1.214214221	1.178523466	1.133071613	1.129775095	1.141422733	1.172451833	0.322236709
Sharpe Ratio (An	-0.175008184	-0.011899222	0.26298919	0.350952683	0.39926751	0.423393363	-0.215350298
Kurtosis (Annual)	4.614699602	4.28405284	3.642023535	3.332574593	3.091148133	2.950269252	7.822812725
Skew (Annual)	-0.432278487	-0.397559803	-0.374567511	-0.371777149	-0.345930623	-0.288699698	0.39426041
mVaR (Annual)	-203.8710894	-196.3386488	-187.5472831	-186.9923502	-188.332592	-191.7896885	-44.27004428
VaR (Daily)	-12.69243037	-12.24175102	-11.64411322	-11.57041485	-11.66789068	-11.97441528	-3.355975603
VaR (Annual)	-200.684945	-193.5590789	-184.1095955	-182.944322	-184.4855502	-189.3321297	-53.06263339
CVaR (Annual)	-319.4600091	-306.2211848	-290.9369938	-288.0490885	-287.0504959	-290.1420382	-77.65164134

III. Post Subprime crisis

	S^{60}_{60}	S^{60}_{60}	S^{60}_{60}	S^{60}_{60}	S^{60}_{60}	S^{60}_{60}
	b=-1.0	b=-0.5	b=0.5	b=1.0	b=1.5	b=2.0
Cumulative Retur	0.703824957	0.766728492	0.832081602	0.86130932	0.9033088	0.942696368
Arithmetic Mean	-0.006903516	0.071012972	0.152561714	0.192289686	0.251207115	0.310828957
Geometric Mean	-0.350978992	-0.265481467	-0.183757197	-0.149257009	-0.101670133	-0.05900407
Minimum Return	-58.3098973	-57.61298546	-56.75884551	-56.50015281	-58.41400425	-61.24821035
Max 10-day Draw	-0.511376933	-0.501704969	-0.48163136	-0.471003931	-0.461561245	-0.453685926
Volatility	0.826876005	0.817466896	0.816439342	0.822465805	0.835925964	0.855830553
Sharpe Ratio (An	-0.080911183	0.01347207	0.113372433	0.160845211	0.228736901	0.293082499
Kurtosis (Annual)	2.62037538	2.390598161	2.060063734	1.871975271	1.722667336	1.673768117
Skew (Annual)	0.014267021	-0.024854901	-0.103854231	-0.131665661	-0.140316144	-0.133546728
mVaR (Annual)	-131.3443914	-130.6450286	-132.3268987	-134.0117951	-136.3059907	-139.1351318
VaR (Daily)	-8.60472583	-8.475676577	-8.432367462	-8.479169375	-8.595628004	-8.778846067
VaR (Annual)	-136.0526613	-134.0122135	-133.3274362	-134.0674395	-135.9088121	-138.805744
CVaR (Annual)	-187.5621103	-186.7664879	-188.8864708	-189.8282298	-191.3275242	-194.2755738

IV. pre-Covid19 crisis

S^{60}_{120}	S^{60}_{120}	S^{60}_{120}	S^{60}_{120}	S^{60}_{120}	S^{60}_{120}
b=-1.0	b=-0.5	b=0.5	b=1.0	b=1.5	b=2.0
0.551384554	0.555366761	0.558071371	0.567907164	0.57347106	0.576906063
-0.167365729	-0.163758928	-0.145853266	-0.131796943	-0.118830691	-0.106956554
-0.594614537	-0.58743531	-0.582588544	-0.565157546	-0.555429885	-0.549471104
-132.4251466	-142.6342431	-161.3001361	-156.9348675	-154.4640715	-149.6649849
-0.612306161	-0.646227474	-0.708102804	-0.693149692	-0.684532362	-0.668444168
0.914198167	0.902110154	0.897771988	0.896815282	0.9014298	0.910843898
-0.248705081	-0.248039474	-0.229293483	-0.21386449	-0.198385599	-0.183298757
13.91108444	13.77176559	15.54847366	13.54356336	12.1542404	10.27208515
0.234998306	-0.136356123	-0.816137175	-0.833886553	-0.853379619	-0.798294015
-119.5642236	-127.8128603	-140.1270254	-143.9214955	-147.5469313	-151.1939907
-9.577320345	-9.450126422	-9.397834354	-9.382259238	-9.42507742	-9.518262323
-151.4307309	-149.4196183	-148.5928082	-148.346544	-149.0235588	-150.4969415
-203.6155468	-201.4706347	-203.1483457	-205.2004395	-208.2543362	-212.405187

V. During Covid crisis

	S^{90}_{120}	$S^{90}_{	$S^{90}_{	$S^{90}_{120}	$S^{90}_{	$S^{90}_{
	b=-1.0	b=-0.5	b=0.5	b=1.0	b=1.5	b=2.0
Cumulative Returns (Ann	0.932968642	0.898237	0.846352	0.811492355	0.783118	0.754031
Arithmetic Mean Returns	-0.023359842	-0.06159	-0.11485	-0.150432396	-0.17697	-0.20319
Geometric Mean Returns	-0.069374062	-0.1073	-0.16676	-0.208793075	-0.24435	-0.28216
Minimum Return (Annual	-21.36068073	-21.3607	-21.3607	-21.36068073	-21.3607	-21.3785
Max 10-day Drawdown	-0.170906187	-0.17091	-0.17091	-0.170906187	-0.17091	-0.17266
Volatility	0.300843074	0.2998	0.319584	0.338949183	0.364139	0.394045
Sharpe Ratio (Annual)	-0.277087457	-0.40556	-0.54711	-0.620837595	-0.65077	-0.66792
Kurtosis (Annual)	4.667160953	4.664931	3.441082	2.731752013	2.299728	2.149678
Skew (Annual)	-1.211072351	-1.23365	-1.10358	-0.986509029	-0.92879	-0.89501
mVaR (Annual)	-56.32640554	-56.536	-60.3683	-63.7203268	-68.3487	-73.8225
VaR (Daily)	-3.139004736	-3.14344	-3.37057	-3.586250386	-3.85892	-4.18052
VaR (Annual)	-49.63202275	-49.7022	-53.2933	-56.70359739	-61.0149	-66.0998
CVaR (Annual)	-85.33378554	-84.9434	-88.9303	-93.41026889	-100.728	-108.553

References

[1] Eugene F. Fama, Kenneth R. French, "The Cross-Section of Expected Stock Return" ,The Journal of Finance, vol. 47, No.2 , 1992, pp. 427–465.

[2] Eugene F. Fama, Kenneth R. French, "Common risk factors in the returns on stocks and bonds" ,Journal of Financial Economics, vol. 33, No.1 , 1993, pp. 3–56.

[3] Eugene F. Fama, Kenneth R. French, "Size and Book-to-Market Factors in Earnings and Return" ,The Journal of Finance, vol. 50, No.1 , 1995, pp. 131–155.

[4] Eugene F. Fama, Kenneth R. French, "Multifactor Explanation of Asset Pricing Anomalies" ,The Journal of Finance, vol. 51, No.1 , 1996, pp. 55–84.

[5] Francis, Kim, "Modern Portfolio Thoery", Wiley Finance

Authors

Theo Dimitrasopoulos – MSc. Financial Engineering, Stevens Institute of Technology 2021, Bachelor of Science in Engineering, Structural Engineering, Princeton University 2017; tdimitr1(at)stevens(dot)edu

Yuki Homma – MSc. Financial Engineering, Stevens Institute of Technology 2021, Bachelor of Science, Physics, Keio University 2019; yhomma(at)stevens(dot)e