Chapter 1

Determining the inclination of a ringed disk image

First we determine the center of the image. This is usually the location of highest intensity. But sometimes one has to hand-fit this.

Then we extract the intensity along radial rays as a function of distance r along the ray and angle of the ray ϕ. The image one then sees looks, for GW Lup, like Fig. 1.1. This is the intensity as a function of radius and angle $I_\nu(r, \phi)$.

To find out the inclination and position angle, we fit the curve corresponding to an ellipse to the ring-shaped local maximum seen in the figure. The radius of an ellipse as a function of angle ϕ in the image plane is

$$r_e(\phi) = \frac{r_{e0}}{\sqrt{\sin^2(\phi - \phi_{e0})/\cos^2 i + \cos^2(\phi - \phi_{e0})}} \tag{1.1}$$

The shape of this curve depends on three parameters: the deprojected radius r_{e0}, the inclination i and the offset of the angle ϕ_{e0} (the position angle) [**Check definition of sign of pa**].

We wish to find the best fit of this curve to the ring. Given the noise on the data, it will be not easy to use steepest descent methods to find the optimal fit. Instead we use an MCMC method, based on Bayesian inference.

Our model (denoted by M) is a perfect circle with radius r_M, inclination i_M and position angle ϕ_M, thus becoming an ellipse in the image plane. The intensity is assumed constant along the circle at a value of I_M. We compare this model to the data only along this ellipse. This comparison is done by computing the average intensity of the image along the path given by Eq. (1.1), shown as the red curve in Fig. (1.1):

$$\langle I_\nu \rangle_{\mathrm{path}} = \frac{1}{2\pi} \int_0^{2\pi} I_\nu(r_e(\phi), \phi) d\phi \tag{1.2}$$

This is to be compared to the value I_M. Due to the noise in the data the actual $\langle I_\nu \rangle_{\mathrm{path}}$ will, even along the correct path, not be identical to I_M. We can

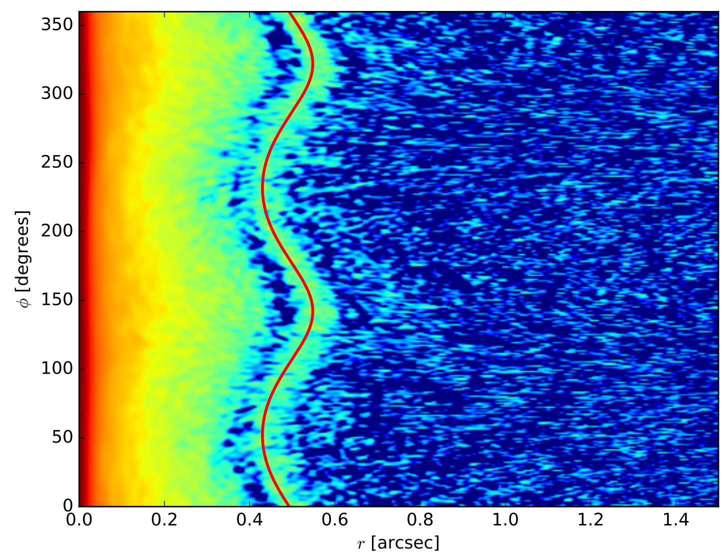


Figure 1.1: The intensity along the rays from the center for GW Lup, still uncorrected for the inclination. The red curve is the best-fitting fit of an inclined ring through the ring.

estimate the degree of this noise by simply computing the standard deviation of the intensity along the first guess of the correct path. This requires a bit of hand-work. So we fit by eye the correct path, and compute the average and the standard deviation of the intensity along that path:

$$\sigma^2_{\rm path} = \frac{1}{2\pi}\int_0^{2\pi}(I_\nu(r_e(\phi) - \langle I_\nu\rangle_{\rm path})^2 d\phi \tag{1.3}$$

Next we ask ourselves, what is the standard deviation of the value of $\langle I_\nu\rangle_{\rm path}$ if we repeat the experiment with a series of news datasets with new random noise of the same amplitude as. This is substantially less than $\sigma_{\rm path}$, because the integral over the path reduces the noise. We have roughly a number N_b of beams along the path, given by

$$N_b \simeq \frac{2\pi r_{e0}}{b} \tag{1.4}$$

where b is the FWHM beam size. The standard deviation of the average along the path is then

$$\sigma_{\rm tot} = \frac{\sigma_{\rm path}}{\sqrt{N_b}} \tag{1.5}$$

This leads to the probability of finding an path-average intensity $\langle I_\nu\rangle_{\rm path}$, given the model M, of

$$P(\langle I_\nu\rangle_{\rm path}|M) = \frac{1}{\sqrt{2\pi}\,\sigma_{\rm tot}}\exp\left(-\frac{(\langle I_\nu\rangle_{\rm path}-I_M)^2}{2\sigma_{\rm tot}^2}\right) \tag{1.6}$$

Now we can employ MCMC to find the most likely values of r_M, i_M and ϕ_M. But as a constraint we want to find the ring, not just any intensity value. Since we do not know I_M, but we want to find a ring, we can set I_M to the roughly a standard deviation above the value of $\langle I_\nu\rangle_{\rm path}$ along the by-eye fit. The precise value does not really matter, because the MCMC process will find the optimum nonetheless. We can also try to fit a gap instead of a ring, by setting the value of I_M to one standard deviation *below* the $I_\nu\rangle_{\rm path}$.

In practice the optimization using this likelihood is not always stable. A bit of experimentation shows that by multiplying the likelihood function by a boosting parameter $\psi \geq 1$ leads to better convergence. This essentially is equivalent to assuming that the measured standard deviation along the path $\sigma_{\rm path}$ is larger than the actual uncertainty. Or in other words, that due to the CLEAN algorithm a certain degree of blobbiness is introduced that is stronger than the actual data. We typically take $\psi = 2$.

[Perhaps it is possible to include the standard deviation as a free parameter, in the same way as the standard example in the emcee package?]

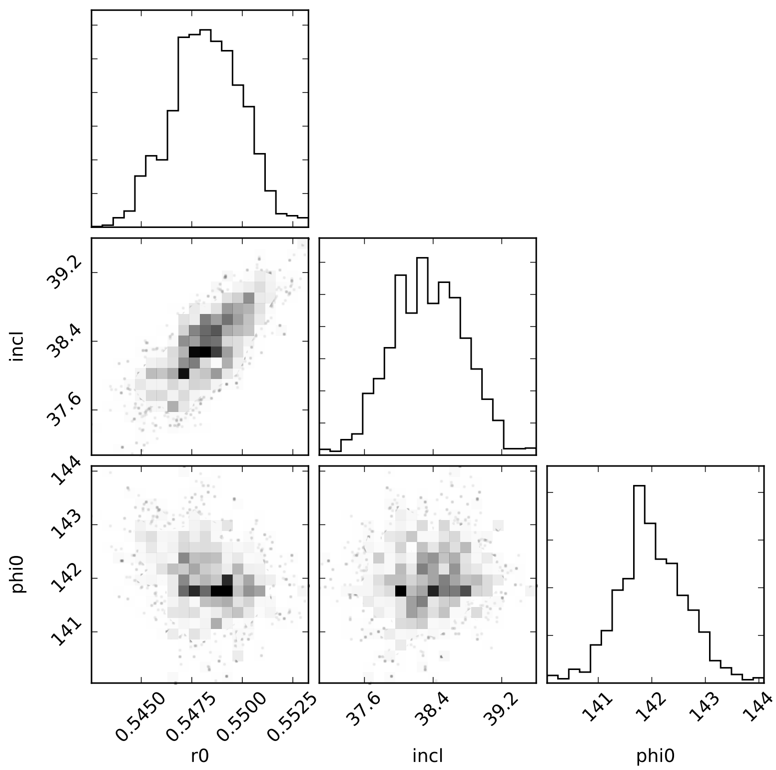


Figure 1.2: The result of the MCMC fitting of the ring radius, inclination and position angle for GW Lup.

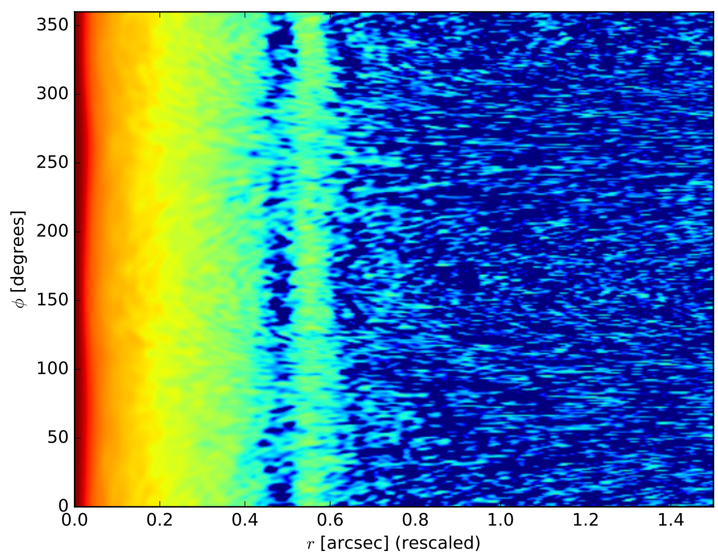
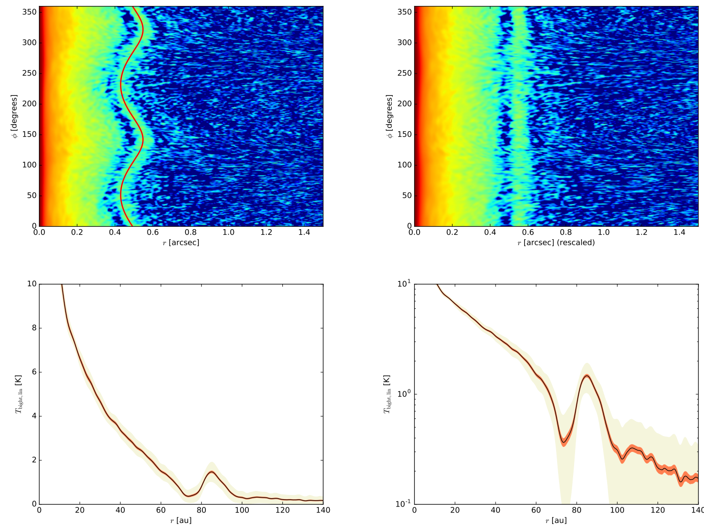


Figure 1.3: The deprojected version of the $(r, \phi)-$image of GW Lup.

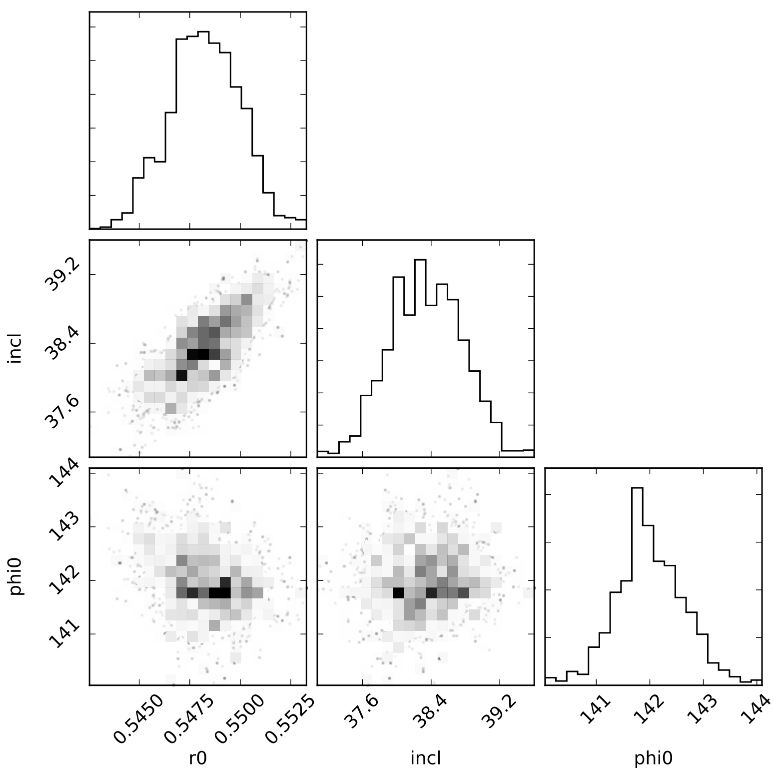
Appendix A

Results for various sources

A.1 GW Lup

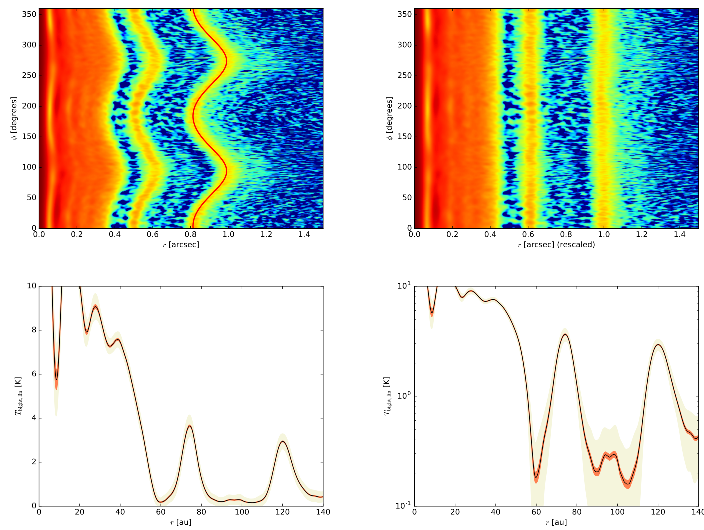


ϕ [degrees]
r [arcsec]
r [arcsec] (rescaled)
r [au]

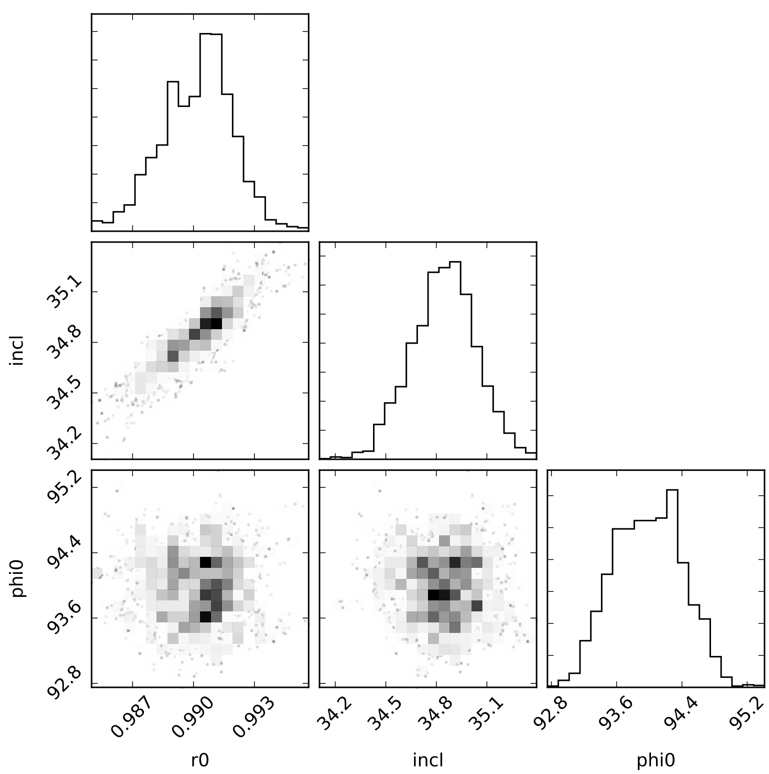


incl
phi0
r0
0.5450
0.5475
0.5500
0.5525
37.6
38.4
39.2
141
142
143
144

A.2 AS 209

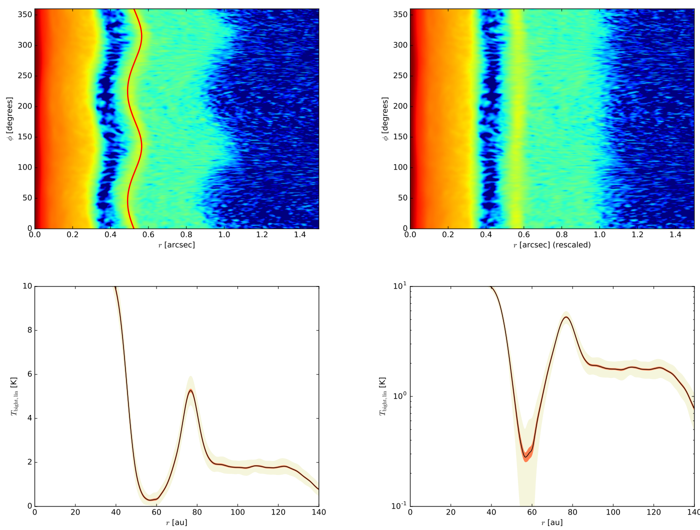


ϕ [degrees]
r [arcsec]
r [arcsec] (rescaled)
$T_{\rm right,Ss}$ [K]
r [au]

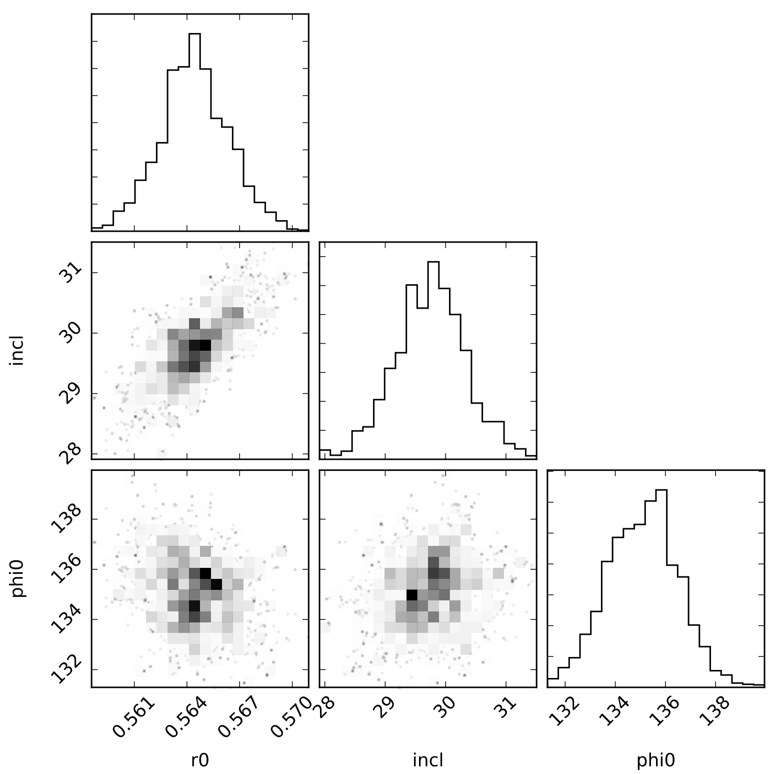


incl
35.1
34.8
34.5
34.2
phi0
95.2
94.4
93.6
92.8
0.987
0.990
0.993
r0
34.2
34.5
34.8
35.1
incl
92.8
93.6
94.4
95.2
phi0

A.3 Elias 24



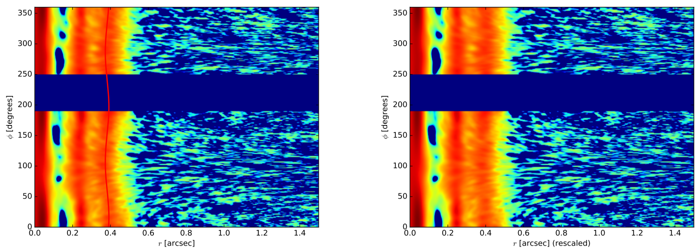
φ [degrees]
r [arcsec]
r [arcsec] (rescaled)
r [au]

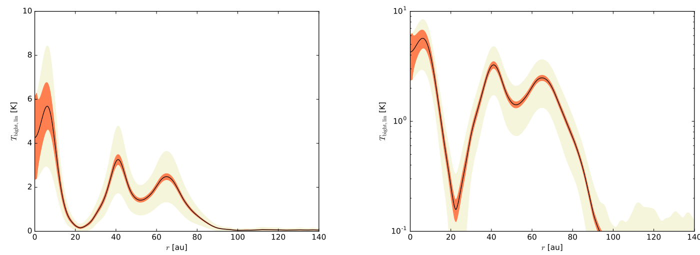


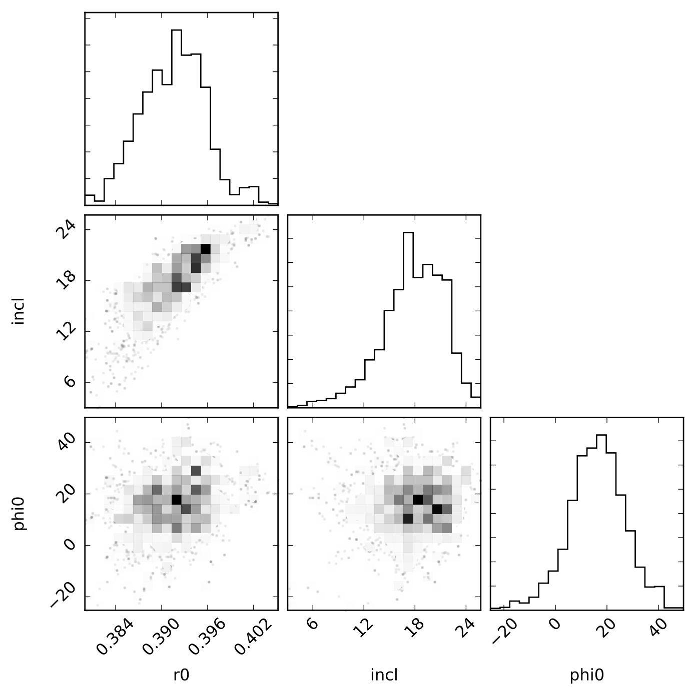
incl
phi0
r0
incl
phi0

A.4 HD 143006

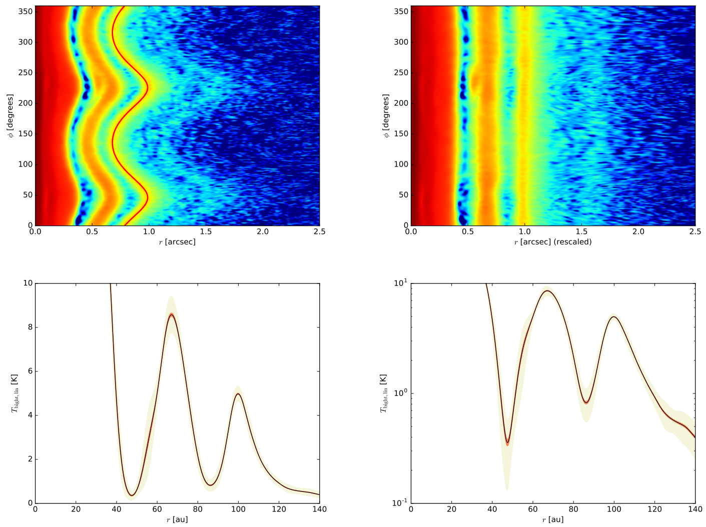
Note: Removed the blob.



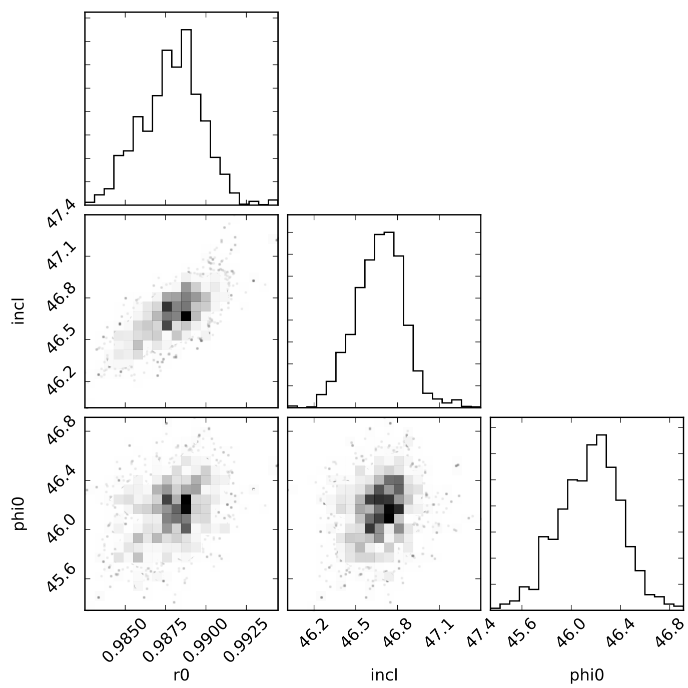




A.5 HD 163296



φ [degrees]
r [arcsec]
r [arcsec] (rescaled)
r [au]



incl
phi0
r0
incl
phi0