

**A Deep Learning-Based Perception-Driven Vectorization
Approach For Semi-Structured Imagery**

A Special Problem by

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This special problem, entitled “**A DEEP LEARNING-BASED PERCEPTION-DRIVEN VECTORIZATION APPROACH FOR SEMI-STRUCTURED IMAGERY**”, prepared and submitted by **SEAN FRANCIS N. BALLAIS**, in partial fulfillment of the requirements for the degree of **BACHELOR OF SCIENCE IN COMPUTER SCIENCE** is hereby accepted.

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Abstract

Although the abstract is the first thing that appears in the thesis, it is best written last after you have written your conclusion. It should contain spell out your thesis problem and describe your solution clearly.

Make sure your abstract fits in one page !

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Chapter 1

Introduction

In computer graphics, most images are typically stored as a sequence of dots in a rectangular grid (see Fig. 1.1). Each dot is called a pixel, a small part of an image that holds one specific colour. Photographs, also called natural images [?], are one of, if not the most, common images that are stored in this manner. Many digital forms of art or any graphics work, such as paintings, posters, and icons, are also stored the same. Digital images stored in this manner are called *raster images*. These images are stored in various image formats. The most commonly used formats are JPEG, GIF, BMP, TIFF, and PNG. Each have their pros and cons, from quality of the resulting image to the file size. Nevertheless, they all still accomplish the task of holding raster image data. Everything you see in the displays of devices such as laptops and mobile devices is a raster image. Computer displays are collections of pixels, in the common definition of a dot on the screen, which computers map images to to be able display them. This is the reason why **all displayed** images are raster images.

A positive aspect of raster images is their simplicity. As mentioned earlier, raster images consists of a grid of pixels (also called a pixel matrix in other literature [?]). This pixel grid can simply be assigned a combination of colour values to create an



Figure 1.1: When zoomed in enough, each individual pixel of a raster image is visible. Meme image obtained from <http://thesismemes.tumblr.com/post/73483120281>.

image. As such, working with raster images can be analogous to painting in the real world [?]. Given the right combinations of colours, we can produce natural images, i.e. photographs [?]. Intuitively, this means that we can store fine details in a raster image [?]. This is in contrast to *vector images*, which use a series of points and mathematical calculations to form lines and shapes. Vector images are unable to display lush colour depth and keep granularity, as found in raster images, as they use solid colours or gradients [?][?]. There are studies that have been conducted in improving and utilizing *gradient meshes*, a vector graphics primitive that allows for intricate colour gradients in regular quadrilateral meshes first introduced by Adobe Illustrator, to produce photorealistic vector images. However, as noted in the paper by Jian, S, Liang, L., Wen, F., and Shum, H., simple gradient meshes are insufficient to keep the fine details of images [?][?]. It is also important to mention that vector graphics, despite represented as mathematical calculations, are still converted to raster format

in a process called *rasterization* for it to be displayed on-screen, since many modern screens are raster displays [?].

1.1 The Problems of Raster Graphics

With all the pros raster graphics have, it does not mean raster graphics are not without their caveats. Raster graphics have their own disadvantages which could affect the image quality and their use.

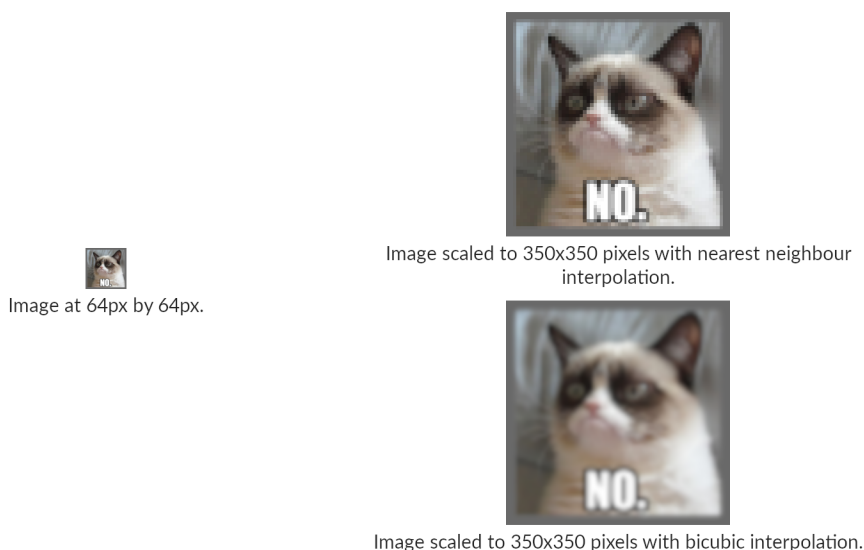


Figure 1.2: Different interpolation algorithms will produce different results. As seen in the image, the quality will also differ, from an image looking blocky to an image looking blurry. Cat meme image obtained from <https://www.hercampus.com/school/uwindsor/school-thoughts-told-grumpy-cat-memes>.

Raster graphics are **resolution-dependent**. This simply means that raster images are in their highest quality in the resolution they are initially created in and attempting to scale it up will gradually degrade the quality of the image as the image size or resolution grows larger. Rasters only have a finite number of pixels. Increasing the size of an image (also called upsampling [?] or single-image super resolution [?])

would entail moving the individual pixels into different locations depending on the scaling factor, the distance the individual pixels will be moved to horizontally and vertically. Upsampling will create empty pixels in between the shifted pixels when there is nothing done to substitute the empty pixels. This will create an unusable image [?]. We can utilize interpolation to fill these empty pixels with colour. *Classical* image upsampling approaches approximate colour and intensity values of these empty pixels are calculated based on the values of surrounding pixels, typically the shifted pixels. However, the specifics are dependent on the interpolation algorithm used in upscaling the images. Commonly used classical interpolation methods for resizing images include Nearest-Neighbour, Linear Interpolation, and Cubic Interpolation, which most, if not all, are readily available in popular raster image editing applications, such as Adobe Photoshop [?] and GIMP [?]. The results produced by these interpolation algorithms typically suffer from blurring of sharp edges and ringing artifacts due to the fact that the algorithms do not assume anything about the data [?][?]. See Fig. 1.2 for an example of blurring caused by scaling. There are adaptive image scaling techniques that consider image features such as edge information and texture to scale images with better quality than the classical methods. Examples of adaptive techniques are content-aware image resizing, seam curving, and warping-based methods. These techniques have their downsides as they take more computational time than their non-adaptive counterparts and may produce unexpected or even unsatisfactory results [?]. One of the latest advancements in upscaling images involves the use of artificial intelligence, specifically *neural networks*, as found in the works of AI Gigapixel and by Yang, C. Ma, C. and Yang, M.. They produce high quality upscaled images and is a significant improvement over previous non-AI based upscaling techniques.

However, they require high-end expensive hardware to produce results in the shortest amount of time possible. In the case of AI Gigapixel, a laptop with an integrated graphics card takes 20 minutes to produce a final high resolution image. For the work by Yang. C, Ma.C., and Yang. M., they utilized an Nvidia Titan Xp, a high-end GPU that costs \$1,200 as of November 18, 2018 [?], to upscale a 520x520px image 2x, 4x, and 8x its size, and took 0.8s, 2.1s, and 4.4s, respectively, to complete [?][?].

1.2 Vectorization To The Rescue

Vectorization is the process of converting raster images into vector images [?]. Vector graphics uses collections of geometric primitives, such as points, curves, and points, and mathematical calculations to form an image [?]. Unlike raster graphics which uses a large pixel matrix (which will require large spaces without using proper image compression), vector graphics are able to smoothly scale to different resolutions, large or small, without any degradation in image quality [?][?]. This makes them **resolution-independent**. Vector graphics innately have this property due to their reliance on mathematics, instead of context-free pixel grids. Each primitive have their own mathematic formulas which, obviously, stay the same no matter what the size of an image is. As such, the primitives can simply be re-rendered whenever the image is scaled [?][?]. Vector graphics also allow for easier editing [?][?], as you only need to modify individual polygons, lines, and curves, instead of dealing with individual pixels like you normally would when using raster images editors.

Vectorization would often be done manually. In a study by Hoshyari, et. al. [?], each of their raster images, which includes icons and small graphic illustrations, take 30-45 minutes to be vectorized by an artist. More than 7 million man hours are being

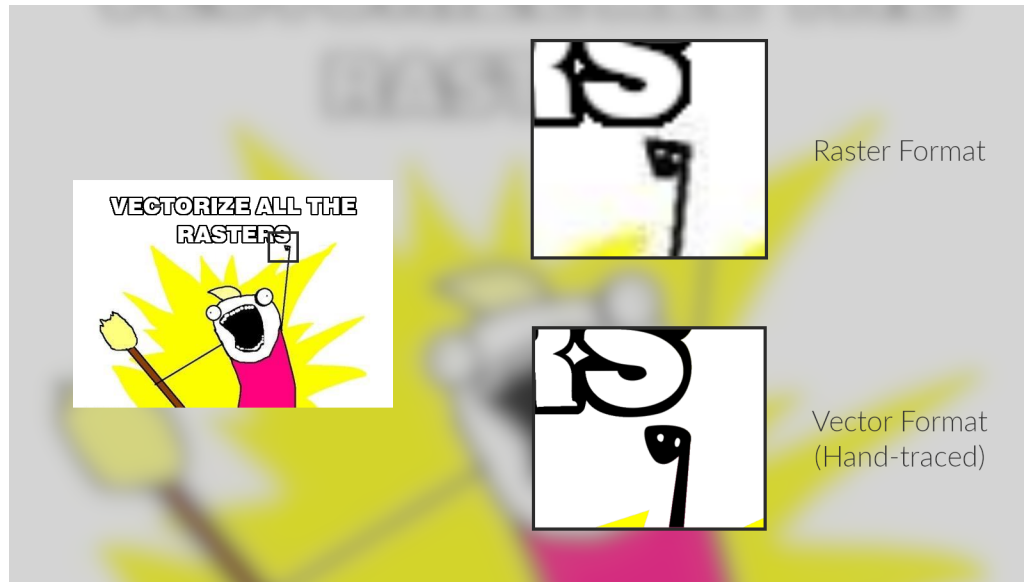


Figure 1.3: Zooming in closely at the same image, but one being in raster format (top right) and the other in vector (though hand-drawn; bottom left), quickly reveals the quality differences of both image formats. Raster images will show you individual pixels when close enough, but vectors will remain smooth.

spent on vectorizing raster graphics in the United States every year, according to a survey in the PhD dissertation of J.R. Diebel entitled, "Bayesian image vectorization: The probabilistic inversion of vector image rasterization" [?]. Demand is, therefore, there for a robust vectorization algorithm.

Vectorization have been applied in many cases including, but not limited to, 2D maps and natural images. There are multiple methods that can be utilized in the vectorization of raster images. Their results differ from one method to another and even from one input to another, as many vectorization methods are fine tuned to specific inputs, such as those by Hoshyari, S., et. al. (semi-structured images) [?], Kopf, J. and Lischinski, D. (pixel art) [?], and Bessmeltsev, M. and Solomon, J. (line drawings) [?]. Additionally, the ability to utilize GPUs for computational tasks has

allowed parallelization of vectorization such as in the paper where a GPU was used to vectorize a video stream in real time [?].

Many vectorization methods target natural images. As noted by Hoshyari, S., et. al. many of these natural image vectorization methods utilize *image segmentation* to identify the portions of the image that will be converted into geometric primitives. These primitives are then filled with solid colors (e.g. such as what was done by Birdal, T. and Bala, E. [?]), gradient meshes (which are used in Adobe Illustrator and Corel CorelDraw [?][?]), and/or diffusion curves (such was the case in the paper by Xie, G. Sun, X., Tong, X., and Nowrouzezahrat, D. [?]). These vectorization methods produce differing results whose image qualities vary. Some results are as close as possible to the original raster image. Typically, gradient meshes and diffusion curves were utilized to achieve these results [?][?][?]. Others produce results with obvious colour segmentations, as seen in results that purely utilize solid colours (see [?] for an example).

Natural images are not the only raster images that are being vectorized. Images called semi-structured images are also candidates for image vectorization. *Semi-structured images* (SSIs) are images that consists of distinctly coloured regions and have well-defined boundaries [?]. Logos, cartoons, clip art, computer icons, and even simple graphical illustrations (such as flat 2D art) can considered to be semi-structured images. 60% of the 10 million images to be vectorized are semi-structured images [?]. This makes the demand for vectorizing these types of images evident. Various methods for vectorizing semi-structured images have been proposed by numerous papers. The common methodology of these proposals is that they attempt to fit curves or Bezier splines on the boundaries of each region in SSIs and fill in

the appropriate colour. However, each of these methods naturally have their own unique ways of fitting curves. In the work of Kopf, J., and Lischinski, D. [?], they use similarity graphs and additional intermediate steps in identifying the regions of an image, though their work is targeted at pixel art. Another paper by Yang, M., et. al. would directly optimize the shapes of individual Bezier segments connecting each boundary transition vertices to produce high fidelity vectorized images [?][?]. The work by Hoshyari, S., et. al. can be seen as a complement to the aforementioned paper. Their work utilizes human perceptual cues, primarily guided by Gestalt psychology, to produce vectorizations of semi-structured images that align much more closely to what viewers expect from a raster image [?].

An alternative method we can perform for image vectorization is to use machine learning via convolutional neural networks, a type of neural network that is well suited for image classification for their ability to learn various image features [?], to create vectorizations of semi-structured images by having the computer learn to produce Bezier curves of image region boundaries that align well with the expected vectorization. This is the method that this paper is proposing. A similar work, in that convolutional neural networks were for vectorization, to this is that of the paper of Simo-Serra, E., Iizuka, S., Sasaki, K., and Ishikawa, H. where they convert and simplify paper-and-pencil sketch drawings to vectorized images [?].

1.3 B-Splines

Many image vectorization techniques, such as those of Hoshyari, S., et. al., and Yang, M., et. al., involves the use of fitting curves to pixels that form a border for a region in an image. The geometric primitive used in such approaches is the *bézier*

curve [?][?][?]. A collection of bézier curves, called a b-spline, is also used for image vectorization. B-Spline is short for *basis spline*. B-splines allow for curves with higher complexity, such as those with curves resembling squiggly lines.

A k -degree B-spline curve is defined to be

$$C(u) = \sum_{j=0}^n c_j N_j^k(u)$$

where c_j are the control points, u is the non-decreasing knot vector $u = (u_0, \dots, u_n)$ with u_0 and u_n having a multiplicity of $k + 1$, and a B-spline functions $N_j^k(u)$. The B-spline functions are referred to as the basis functions, from which the name of b-splines was obtained from. The basis functions are defined using the Cox-de Boor recursion formula [?] which is defined to be:

$$N_j^k = \begin{cases} 1 & , \text{ if } u_j \leq u \leq u_{j+1} \\ 0 & \text{ otherwise} \end{cases}$$

$$N_j^k = \frac{u - u_j}{u_{j+k} - u_j} N_{j-1}^k(u) + \frac{u_{j+k+1} - u}{u_{j+k+1} - u_{j+1}} N_{j+1}^{k-1}(u)$$

Knots in B-splines determine the basis functions, which affects the shape of the B-spline curve [?]. The number of knots in u , disregarding the multiplicity of u_0 and u_n , is defined to be

$$|u| = k + n + 1$$

B-Spline Approximation

Suppose that we are given a point sequence $p = (p_0, \dots, p_m)$ with each point being $p_i = (x_i, y_i)$. According to the work by Laube, P., et. al. [?], we can compute the

control points c_j of the B-spline curve C that will approximate p_i by using the least square problem defined as

$$\sum_{i=0}^m |p_i - C(t_i)|^2 \rightarrow \min$$

with precomputed parameters $t_i, i = 0, \dots, m$ combined in the parameter vector $t = (t_0, \dots, t_m)$ and end points $C(t_0) = c_0 = p_0$ and $C(t_m) = c_n = p_m$. This will give us a normal equation

$$(N^T N)c = q \quad (1.3.1)$$

where N is an $(m-1) \times (n-1)$ matrix

$$N = \begin{pmatrix} N_1^k(t_1) & \dots & N_{n-1}^k(t_1) \\ \vdots & \ddots & \vdots \\ N_1^k(t_{m-1}) & \dots & N_{n-1}^k(t_{m-1}) \end{pmatrix}$$

and c and q are vectors defined to be

$$c = \begin{pmatrix} c_1 \\ \vdots \\ c_{n-1} \end{pmatrix}, \quad q = \begin{pmatrix} \sum_{i=1}^{m-1} N_1^k(t_i) q_i \\ \vdots \\ \sum_{i=1}^{m-1} N_{n-1}^k(t_i) q_i \end{pmatrix}$$

and

$$q_i = p_i - N_0^k(t_i)p_0 - N_n^k(t_i)p_m$$

for $i = 1, \dots, m-1$. If there are no constraints for end point interpolation, then 1.3.1 reduces to

$$(N^T N)c = N^T p \quad (1.3.2)$$

The control points c_j can be computed using 1.3.1 if

$$\sum_{l=1}^{m-1} N_i^k(t_l) N_j^k(t_l) \neq 0 \quad (1.3.3)$$

Chapter 2

Review of Related Literature

2.1 Great Deluge-Based Works

Many previous works dealing with course timetabling utilize an optimizing heuristic (or derivatives of it) called **Great Deluge**. Great Deluge was introduced by Gunter Dueck, and is a heuristic that is similar to Hill Climbing and Simulated Annealing. To understand how it works, imagine that you are in a point in some area with mountainous terrain. This area constitutes your solution space, with higher points in the area having higher values and, thus, having a better solution. The initial point you are located in in the area represents the initial solution that is generated. Imagine as well that it is raining endlessly and the water level W is continuously rising at a constant rate R_w , where $R_w > 0$. W can start at any value that is greater than 0. Assuming that we are attempting to maximize some function Q , which evaluates a solution based on some criteria, your goal is to locate the relatively highest point in the location. This point can be thought of as the local optimum. Locating the highest point involves walking around the area that is not below the current water level. This will force you to walk to a higher and higher point since W is constantly rising. Once you are no longer able to proceed to a higher level, it means that you are now in a

local optimum. Going outside the analogy and back to a technical perspective, this local optimum would now be the *relatively* best solution for your problem. Every "walk" or move to a higher point is nuanced relative to the analogy. A single walk means construction of a new solution S_{new} with basis on the current solution $S_{current}$. If $Q(S_{new}) \geq W$ [5], then we accept S_{new} as the new current solution and "walk" towards it, and we increase W by R_w . Otherwise, we simply generate a new S_{new} . These walks are performed until $Q(S_{new})$ is not greater than $Q(S_{curr})$ for a long time or we have reached the maximum number of moves/iterations [8]. Great Deluge can also be adapted to minimize Q . Instead of W increasing, it will be decreasing by the same rate. A solution S will now be accepted if $Q(S) \leq W$. Conversely, the algorithm will stop when $Q(S_{new})$ is not lesser than $Q(S_{curr})$. The second stopping condition for the algorithm still applies in this case [5][12][13]. This minimization case is the one adapted by Great Deluge-based works that focus on course timetabling.

One of the earliest works that uses Great Deluge was that of Burke, E., Bykov, Y., Newall, J., and Petrovic, S. [5]. Their work explores the use of Great Deluge in university course timetabling. In their work, they defined Q as the sum of the number of soft constraints violated. They also, however, slightly modified Great Deluge. We will be referring to the modified version as the Extended Great Deluge (EGD). Instead of simply obtaining a single new solution S in an iteration and comparing $Q(S)$ to W . Burke, E., et. al. extended Great Deluge to take multiple neighbouring solutions N on every iteration. A random solution S_N from N will be taken and it will be the one to be compared to W and the current solution. S_N will be accepted as the current solution if $Q(S_N) \leq Q(S_{curr})$. S_N can still be accepted only under the condition that $Q(S_N) \leq W$. In addition to taking multiple solutions in an iteration, a few more

extensions have been added to Great Deluge by Burke, et. al.. The initial value for W is set to $Q(S_i)$, where S_i is the initial solution. Computing for R_w is made easier by using the following equation:

$$R_w = \frac{W - Q(S')}{N_{moves}}$$

N_{moves} denotes the number of moves the algorithm will perform before terminating. The desired number of violations the desired solution S' should have is denoted by $Q(S')$. Through their work, Burke, E., et. al. observed that there is a tradeoff between the quality of produced timetables and the amount of searching time given to the algorithm. The authors noted that even though in some situations, a user would require obtaining the results quickly, it is naturally more preferable that one gives the algorithm more in order for it provide high quality timetables. As the authors have mentioned, course timetabling normally only occurs once or twice in a year. Thus, a long amount of time for searching "seems to be quite acceptable". EGD was compared to Simulated Annealing (SA), Threshold Acceptance (TA), Hill-Climbing (HC), and 21 algorithms submitted for the International Timetabling Competition (ITC) of 2002. The data set used for comparing the algorithms was the one provided by the aforementioned competition. The experiments of Burke, E., et. al. show that EGD produce less scattered results than SA, TA, and HC. This proves the effectiveness of EGD as a heuristic for course timetabling. Further proof of this effectiveness is shown by comparing the heuristic to the 21 other algorithms participating in the ITC. EGD generated the best results for 8 out of 23 data sets among the participating algorithms [5].

Building upon the work of Burke, et. al. is that of Landa-Silva, D., and Obit, J.'s

[12]. The primary contribution of Landa-Silva, D., and Obit, J. is the introduction of a modification of the Extended Great Deluge by Burke, E., et. al., the Non-Linear Great Deluge. Key to this heuristic is the use of an equation, instead of a constant rate, in determining the amount of reduction in the water level in an iteration. We refer to this amount as the decay rate. The next water level W is computed with the following equation:

$$W = W \times (\exp^{-\delta(\text{rand}[\text{min}, \text{max}])}) + P$$

P is "the minimum expected penalty corresponding to the best solution". Another key part of the equation is $\exp^{-\delta(\text{rand}[\text{min}, \text{max}])}$, which controls the speed the water level decreases. Besides from contributing an equation-based decay rate for the water level, another modification they added to the Extended Great Deluge is the conditional increase of the water level. If the current solution obtained S_{curr} is about to converge with the water level W , i.e. when $W - Q(S_{curr}) < 1$, then the Non-Linear Great Deluge algorithm allows the water level to rise for a certain amount. This amount is a random value from the interval $[W_{min}, W_{max}]$. The intended increase is to allow the algorithm to "accept slightly worse solutions to explore different areas of the search space in the hopes of finding better solutions". The work of Landa-Silva, D. and Obit, J. uses three moves in generating solutions:

- **M1:** Selects one random class and gives it a random but feasible timeslot-room pair.
- **M2:** Selects two random classes and swaps their timeslot-room pairs while maintaining feasibility.

- **M3:** Looks for a class which violates soft constraints based on their current timeslot-room pair. It then moves this class to a random timeslot-room pair but still maintaining feasibility.

It should be noted that these three neighbourhood moves always maintain compliance with the hard constraints. Another key modification the authors introduced is the three-step heuristic for generating an initial solution. This heuristic is based on the work by Chiarandini, M., Birattari, M., Socha, K., and Rossi-Doria, O. [6]. The three steps are as follows:

1. All classes will first be assigned a random timeslot. However, unassigned classes with the highest number of conflicts will be assigned first. A class has a conflict with another class if it has students also taking the other class. The maximum bipartite matching algorithm [1] is then used to assign each event to a room. This first step of creating an initial timetable S does not guarantee feasibility. But the solution will be further improved by the later steps.
2. Moves M1 and M2 are then used to improve the current S . At this step, feasibility and satisfaction of the hard constraints are sought. As such, "a move is only accepted if it improves the satisfaction of the hard constraints". This step performed continuously until there are no more improvements to S after 10 iterations.
3. Tabu search [4] is then used to further refine S . In this step, classes that were assigned t_{iter} iterations ago will be stored in the tabu list. t_{iter} is computed as $t_{iter} = \text{ran}(10) + \delta \times N_v$, where $\text{ran}(10)$ is a random number from the interval $(0, 10)$, $\delta = 0.6$, and N_v is the number of classes that partook in violating the

hard constraints. The termination condition for this step is when after 500 iterations, no solution has been produced that is better than the current best.

Only step 1 is run once. The other steps, steps 2 and 3, are performed repeatedly in order until a feasible solution/timetable is obtained. In all 11 experiment instances obtained from the work of Socha, K., Knowles, J., and Sampels, M. [16], which has 5 small instances, 5 medium instances, and 1 large instance, Non-Linear Great Deluge produced results better than Great Deluge. Interestingly enough, in 4 of these experiment cases, NLGD performed the best compared to the best known literature at the time NLGD was introduced [12].

2.2 Machine Learning-Based Works

All related works encountered that use machine learning that were proposed do not use a purely machine-based approach. Rather, they combine heuristics with machine learning.

An example of such works, is that of Obit, J., Landa-Silva, D., Sevaux, M., and Ouelhadj, D. [13]. Their work is built upon their previous work on Non-Linear Great Deluge for course timetabling [12]. An addition to their work compared to previous studies is the use of reinforcement learning as part of their solution generation process. Previous works, relative to their's, randomly select moves. But, Obit, J., et. al. uses reinforcement learning to determine which moves is best to further refine the current solution. Two types of reinforcement learning are employed and investigated in their work: **(a)** reinforcement learning with static memory length, and **(b)** reinforcement learning with dynamic memory length. Initially, all moves are given equal weights of 0.01, with the weight for each move i denoted by w_i and thus, equal probabilities

of being chosen for an iteration. The probability for each move is denoted by p_i is computed by

$$p_i = \frac{w_i}{\sum_{i=1}^n w_i}$$

n is the number of moves there are. In the type with static memory length, on every iteration, the moves are punished or rewarded depending on their performance. A move is rewarded by giving its weight 1 point. It is punished by giving it no points at all. The weights are updated every learning period lp computed using $lp = \max(N_{moves}/500, n)$, where the total number of feasible moves performed is denoted by N_{moves} . During this learning period, the water level is also increased by a certain amount. Performance of each move is computed based on the number of it was called, number of times it generates solutions that has different fitness values, and the number of times it produces solutions that have been accepted. On the other hand, reinforcement learning (RL) with dynamic memory length, the probability p_i of a move being chosen is computed through

$$p_i = \frac{w_i + w_{min}}{\sum_{i=1}^n w_i + w_{min}}$$

In this equation, $w_{min} = \min(0, w_i)$. Unlike its counterpart, this type of RL updates the weights of the moves every time a feasible move is performed. The performed move is rewarded when it produces an improve solution, and punished otherwise. Differing from its counterpart once again, this type of RL uses a piecewise function R to compute the reward/punishment for the currently selected move i at the current iteration j . In the function, Δ is the difference between the best solution so far and the current generated solution.

$$R = \begin{cases} 1 & \text{if } \Delta < 0 \\ -1 & \text{if } \Delta > 0 \\ 0.1 & \text{if } \Delta = 0 \text{ and new solution} \\ -0.1 & \text{if } \Delta = 0 \text{ and no new solution} \end{cases}$$

Each weight w_i is then computed at each iteration h with the formula below

$$w_i = \sum_{j=n_{timeslots}}^h \sigma^j R$$

$n_{timeslots}$ is the number of timeslots there are. The parameter σ is a random value between $(0.5, 1.0]$. The parameter's value is set every learning period lp . Similar to the previous type of RL, the lp is still determined using the same formula, and the water level increases on every learning period. The experiments of this work included an experiment comparing the two types of reinforcement learning employed. Both types of RL produced optimal results in the small problem instances and good results in the medium instances. In the large problem instance, the static memory RL produced a better result than its counterpart. When comparing to other Great Deluge-based works, namely EGD, NLGD, ENLGD, and GD, the static memory RL produced the best solution in all problem instances except the large instance, in which its solution's score has a difference of only 1 from the best solution, which was generated by the Extended Great Deluge. The dynamic memory RL produced the worst solution among the algorithms compared for the large problem, but came in second overall in all of the medium problem instances, but one in which it came in third. When comparing the algorithm to hyper-heuristic-based algorithms with the same problem instances, the algorithm, specifically the static memory RL-based algorithm, produced the best results, except for the large instance. The authors also noted that the value

of lp affects the quality of the best solution the algorithm produces, with the sweet spot being either $lp = 2500$ or $lp = 5000$. When the algorithm is finally compared to course timetabling algorithms that are known to produce the best results for a specific problem instance, the algorithm, specifically the static memory RL one, produced new best solutions in all medium problem instances. This proves that the algorithm is a viable solution for course timetabling.

2.3 Genetic Algorithm-Based Works

Great Deluge is a metaheuristic [8] that has been used for solving course timetabling problems [5][12][13]. Aside from Great Deluge, another metaheuristic that has seen use in the course timetabling problem and its variations is the Genetic Algorithm ((add other citations on GA)). The genetic algorithm is an optimization algorithm whose behaviour is based on how nature works, particularly on how reproduction works at a genetic level. In the algorithm, an initial population is first generated. This population does not necessarily contain the locally optimal solution but it is where the relatively best solution will be obtained from. This initial population is referred to as the first generation. From this population, a certain number of individuals will be randomly selected to be bred with one another and be the parents of the next generation. Selection of individuals is dependent on an individual's fitness, with the most fit individuals usually being selected. Calculating this fitness is dependent on the problem. In the context of course timetabling, fitness is based on the constraints that have been violated by the current solution/timetable generated [3][15][10][11][9][7][14]. The breeding process involves selection of parents and having genes from the parents crossover and/or mutate to produce new offspring. Crossover

is when genes from both parents are combined to produce an offspring. On the other hand, mutation is done by changing a random gene from either parent to create an offspring. Determining which genes from either parent to apply onto the offspring and which to mutate is dependent on implementation. Once a population of new generations is established, the cycle repeats. This continuous reproduction of generations eventually produces solutions "moves" towards the optimal solution [2]. Despite being able to produce feasible solutions for university timetabling, it should be noted that using a genetic algorithm approach may require more time executing compared to other approaches due to its population-based property. When compared to simulated annealing, another approach for university timetabling, the genetic algorithm takes more time executing [10]. However, approaches utilizing the genetic algorithm can see an improvement in execution time when they utilize graphics processing units (GPUs). The work of Yousef, A., Salama, C., Jad, M., El-Gafy, T., Matar, M., and Habashi, S. showed that it is possible to speed up genetic algorithms using GPUs. In their work, they accelerated the computation of the fitness function. Their experiments show that execution speed can be improved by up to 59 times in very large problem instances and by 280% overall when utilizing the GPU [17].

A work that utilizes genetic algorithms is the work of Alves, S., Oliveira, S., and Rocha Neto, A. [3]. Unlike the previous works which tackles the problems as organizing a single set of lectures onto a table of time slots (which can be thought of timetabling for a semester), Alves. S., et. al.'s work organizes a timetable for entire courses throughout multiple semesters. We refer to a course here as a degree a student takes up in college. It is important to note that the perspective in which they tackle the problem is that of the Brazilian university academia's. Unique to

their approach is that they only consider a single course in timetabling at a time. This means that the classes of other courses are not **directly** taken into account. Instead, what is considered is the unavailability of agents, which are students and professors. This unavailability determined from the assignments of the agents from the timetabling of the previous courses. Each course will contain timetables for each semester. In constructing the timetable for a course, a genetic algorithm is applied on each course, with each individual being a timetable for the course. Specific rules for selection of parents have not been specified in the paper but it can be presumed that the fittest individuals are chosen for breeding. Fitness F is computed using the following function:

$$F(C) = 1 - \frac{AM_C + AU_C + AL_C}{AM_{wc} + AU_{wc} + AL_{wc}}$$

C is a course timetable. AM represents the number of times an agent has been assigned to concurrent events. AU stands for the number of times agents have assigned to classes whose timeslots they are not available in. Lastly, AL represents the number of times a class has been given more than three consecutive timeslots. The numerator represents the values for C , while the denominator denotes the worst case values of those variables. Crossover is performed using the OX operator from the work of Chinnasri, W., Krootjohn, S., and Sureerattanan, N. [7]. Mutation is performed in a course by selecting a random semester timetable and swapping two random timeslots. The timetable of a course is then stored in a global timetable. This timetabling is performed until all courses have been timetabled. The final solution is the global timetable. According to the experimental results, the approach of Alves, S., et. al. produces timetables that have a fitness close to 1. However, producing

fit timetables required performing the approach multiple times with different values. In their work, the authors obtained the the parameter values by performing 15 tests with each test having different values for the parameters. It should be highlighted that the stop criteria has been set to when a fitness of 1 has been achieved or the execution time has reached 10 minutes. Alves, S., et. al. found that, in their problem context, the parameters that produced the best results are: (a) 25% mutation rate, (b) 50% OX crossover rate, and (c) a population size of 75. Using these parameters, the authors managed to have their approach produce 493 generations and having a run time of just 3.7 minutes, with all timetables for each course having a fitness of approximately 1.

Chapter 3

Statement of the Problem

Raster images are composed of a pixel matrix. This makes their data representation simple. However, this reduces the amount of detail and quality they have. This limitation is clearer when scaling raster images. This motivates the use of an alternative form of representing images. One form is vector images, which uses mathematical equations to represent an image. The process of converting a raster image to a vector image is called *vectorization*. Semi-structured imagery, such as those used in graphic designs, is one of the classes of images that would benefit from vectorization. This process will allow such images to be easily scaled without sacrificing quality nor detail.

There have been numerous works that tackle image vectorization for semi-structured images. Many of these works primarily use a curve optimization algorithm such as variants of NEWUOA and conjugate gradient. A machine learning approach has been used in one of the works, but as a preprocessing step only [?]. Deep learning have been used to solve problems in multiple domains such as natural language processing (NLP), object detection, and playing board games. No other known work has applied deep learning as a core step in image vectorization. This study will deal with such application.

Chapter 4

Objectives

This study is primarily aims to apply deep learning via artificial neural networks to the problem of vectorizing semi-structured imagery. However, there are still some key objectives that this study seeks to accomplish:

1. To develop an approach that considers Gestalt psychology.
2. To evaluate the effectiveness of using deep learning for image vectorization.
3. To evaluate the accuracy of results obtained from the proposed approach to the target raster inputs.
4. To evaluate and compare the results of the proposed approach to that of previous semi-structured image vectorization methods.
5. To evaluate and compare the speed of the proposed approach compared to previous semi-structured image vectorization methods.

Chapter 5

Proposed Methodology

The proposed methodology will be based on the frameworks proposed by Hoshyari, S., et. al. [?], Yang, M., et. al. [?], Xiong, X., et. al. [?], and Laube, P., et. al. [?]. Additionally, human perception will also be taken account. Thus, the Gestalt psychology principles of accuracy, simplicity, continuity, and closure, as taken from Hoshyari, S., et. al., will be taken into account as well.

5.1 Image Preprocessing

Before a raster input image can be fitted with curves, it must preprocessed to simplify the vectorization procedure and align it with .

Corner Detection

The first step in the approach is detecting the corners in the input image. This is an important step as it will allow us to enforce the simplicity principle in Gestalt psychology and make sure the resulting vectorization be C^0 continuous should the raster input be as such.

This stage will be based from the corner detection classifier of the work by Hoshyari, S., et. al. [?]. A random forest classifier is used in the said work. Supervised learning is used as corners are manually annotated. Annotated corners will not be specific pixels. Rather, corners will be between at least two pixels. Training data is available publicly provided by the researchers. However, such data is limited only to quantized data (i.e. aliased data). The target raster input is expected to be anti-aliased data. As such, the training data will have to be built from scratch to support anti-aliased data.

Region Segmentation

In line again with the simplicity principle, the input image must be divided into regions. This will result in simpler curves being used in the final vectorization output. The additional benefit of segmenting the input into multiple distinct regions is the possibility for parallelism to be used in the vectorization approach. Since each region is distinct and independent from one another, multiple regions can be vectorized at the same time. Thus, speeding up the vectorization process.

Due to the nature of semi-structured imagery where each region will only contain a single colour, a scanline-based approach can be used in this stage. The scanline algorithm will be based off of the scanline algorithm used for boundary pixel detection in the work of Xiong, X., et. al..

For each line l_i , where $0 \leq i < h \mid h$ is the height of input image, in the raster input, each pixel p_i will be assigned to a pixel set r_{ij} , where j is the index to a set in l_i . Each pixel set will contain horizontally adjacent pixels that have the same or near-same colours. We include pixels whose colours are within a certain threshold k from the colour of the pixels that is most prominent in the set. This threshold is

necessary due to the fact that certain parts of a regions may contain an anti-aliased pixel. Each l_i will have a pixel set vector r_i containing all pixel sets of l_i :

$$r_i = (r_{i0}, r_{i1}, \dots, r_{i(j-1)}, r_{ij})$$

Consequently, a region vector r will contain all pixel set vectors.

$$r = (r_0, \dots, r_i)$$

Note that there is an opportunity to utilize parallelism, as shown in the work of Xiong, X., et. al. [?], during this step due to the independent nature of every line in the raster input.

Once we obtain all the pixel sets for each line, we iterate through r , $(h - 2)$ times. For each iteration, we process r_i and r_{i+1} . If there are any r_{ij_α} whose pixels are vertically adjacent and have the same colour (or within k) to another pixel set $r_{(i+1)j_\beta}$, then those two pixel sets are merged into one. By the end of the iterations, we have obtained a set of regions that we can individually vectorize.

5.2 Curve Approximation

The core step of the proposed approach is curve approximation. This stage fits curves to the region boundaries of the raster input. This stage is based on the work by Laube, P., et. al. on curve approximation on point sequences using deep learning [?].

Point Sequence Generation

The work of Laube, P., et. al. takes a point sequence as input. As such, for this proposed approach, we must generate point sequences from the region we will be

vectorizing. For every region we ought to vectorize, we treat the center of boundary pixels and the detected corners of each region as a point sequence. However, we must also take into account the fact that certain portions of a region may be a corner where the curves in such segment would have C_0 continuity. As such, the point sequence we generate must take into account corners. This would implore us to take note of the following cases during point sequence generation:

1. For regions with no corners, a random pixel will be selected as both start and end point of the point sequence. The expectation is that there will be no difference in the resulting curve from choosing a different start and end point.
2. For regions with a single corner, the corner point will be selected as the start and end point of the point sequence. This is to ensure that the resulting vector output will have a corner at that point.
3. For regions with two or more corners and assuming n is the number of corners, the point sequence will be divided at those corners into separate point sequences. This will result in $(n + 1)$ new point sequences. Each new point sequence will have their start and end points be the corner points they are adjacent to.

Each point sequence will then be passed to the next stage to be parametrized and have a curve approximated for.

Point Sequence Curve Approximation

The point sequences obtained from the previous step are now to be fitted with curves, specifically B-splines. As provided by the framework by Laube, P., et. al., two neural networks will be used in this stage and some preprocessing will be performed on

the point sequences. The two neural networks are a point parametrization network (PPN), which approximates parametric values to point sequences, and a knot selection network (KSN), which predicts new knot values for knot vector refinement.

Sequence Segmentation

The input point sequence must first be split. This is to ensure that real data and training data match in terms of complexity. Let us define a function $\hat{k}(p)$ that measures the complexity of a point sequence p , where k_i is the curvature at point p_i , given its total curvature:

$$\hat{k}(p) = \sum_{i=0}^{m-1} \frac{(|k_i| + |k_{i+1}|) \|p_{i+1} - p_i\|_2}{2}$$

A point sequence p is split into point sequence segments $p^s, s = 1, \dots, r$ at the median, if $k(\hat{p}) > \hat{k}_t$ for a threshold \hat{k}_t . This \hat{k}_t will be set, as per the original authors have done, to the 98th percentile of $\hat{k}(\cdot)$ of the training set. This process is performed $r - 1$ times, until each p^s satisfies $k(\hat{p}) > \hat{k}_t$.

Sub/Supersampling and Normalization

To be able to approximate parametric and knot values using the PPN and KSN, the number of points per segment p^s must equal the input size l of the aforementioned networks. As such, all segments p^s are either subsampled or supersampled.

If a segment p^s has a number of points greater than l , then p^s is subsampled. This process involves drawing points in p^s such that the drawn indices i are equally distributed and include the first and last point. If, on the other hand, the number of points in p^s is less than l , then *temporary* points are linearly interpolated between consecutive points p_i^s and p_{i+1}^s . This interpolation is performed until the number of

points equal l .

The sampled segments are then normalized to \bar{p}^s , which consists of the points

$$\bar{p}_i^s = \frac{p_i^s - \min(p^s)}{\max(p^s) - \min(p^s)}$$

where $\min(p^s)$ and $\max(p^s)$ are the minimum and maximum coordinates of p^s respectively.

Parametrization of Point Segments

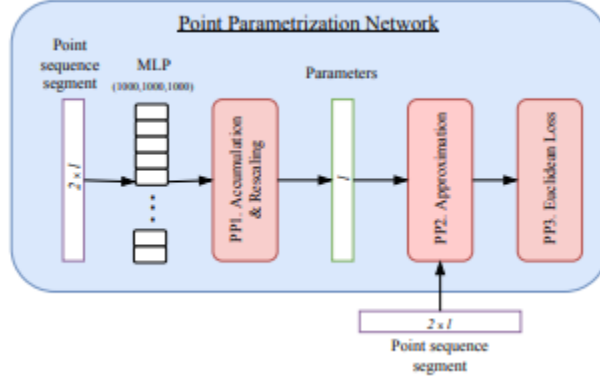


Figure 5.1: The network architecture of the Point Parametrization Network (PPN)

The PPN will be responsible for parametrization of point segments. For every \bar{p}^s , the PPN generates a parametrization $\bar{t}^s \subset [0, 1]$, which will be rescaled to $[u_{s-1}, u_s]$ and adapted to sampling of \bar{p}^s .

In supersampled segments, the parameters t_i^s of temporary points are simply removed from \bar{t}^s . In a subsampled p^s , for every point p_i that was removed from the segment, a parameter \bar{t}_i is inserted to \bar{t}^s .

$$t_i = t_\alpha^s + (t_\beta^s) \frac{\text{chordlen}(p_\alpha^s, p_i)}{\text{chordlen}(p_\alpha^s, p_\beta^s)}$$

$chordlen$ is the length of the polygon defined by a point sequence. In the subsampled segment, with parameters t_α^s and t_β^s , p_α^s and p_β^s are the closest neighbours of p_i .

The initialization of the parametric step requires an initial knot vector. We first define $u_0 = 0$ and $u_n = 1$. For each segment (except the last one), one knot u_i is added.

$$u_i = u_{i-1} + \frac{chordlen(p^s)}{chordlen(p)}, i = 1, \dots, r - 1$$

This yields a start and end knot for every point sequence segment.

The PPN Architecture The PPN, as stated earlier, takes in an input of segments p , which can be written as $p = (x_0, \dots, x_{l-1}, y_0, \dots, y_{l-1})$. The parameter domain is defined as $u_0 = t_0 = 0$ and $u_n = t_{l-1} = 1$. For a sequence of points p , a parameter vector $t = (t_i)_i$, is defined as $t_i = t_{i-1} + \Delta_{i-1}$. The task of the PPN is to predict missing values $\Delta = (\Delta_0, \dots, \Delta_{l-2})$ with

$$\Delta_{subi} > 0, i = 0, \dots, l - 2$$

such that $t_0 < t_1$ and $t_{l-2} < t_{l-1}$. We apply a multilayer perceptron (MLP) to the input data p , yielding as output a distribution for parametrization $\Delta^{mlp} = (\Delta_0^{mlp}, \dots, \Delta_{l-2}^{mlp})$ of size $l - 1$.

The PPN further contains additional layers PP1, PP2, and PP3, which will be discussed next.

PP1. Accumulation and Rescaling The output Δ^{mlp} is used to compute a parameter vector t^{mlp} with $t_0^{mlp} = 0$ and

$$t_i^{mlp} = \sum_{j=0}^{i-1} \Delta_j^{mlp}, i = 1, \dots, l-1$$

Since t_{l-1}^{mlp} is usually not 1, rescaling t^{mlp} yields the final parameter vector t with

$$t_i = \frac{t_i^{mlp}}{\max(t^{mlp})}$$

The MLP layer in the PPN uses a softplus activation function defined to be:

$$f(x) = \ln(1 + e^x)$$

PP2. Approximation B-spline curve approximation is included directly into the PPN as a network layer. The input points p and their parameters t are used for an approximation with knot vector $u = (0, 0, 0, 0, 1, 1, 1, 1)$ for $k = 3$. The approximation layer's output $p^{app} = (p_0^{app}, \dots, p_{l-1}^{app})$ is the approximating B-spline curve evaluated at t .

PP3. Euclidean Loss A loss function, which is a Euclidean loss function, is to be used in the PPN. The Euclidean Loss function is defined as

$$\frac{1}{l} \sum_{i=0}^{l-1} \|p_i - p_i^{app}\|_2 \quad (5.2.1)$$

Parametrization Refinement

In some cases, the approximated parametrization of a p^s may have errors. The approximation error of a p^s is computed by using the Hausdorff distance to the input data p .

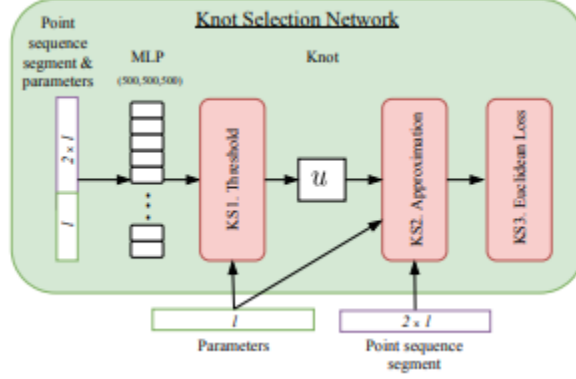


Figure 5.2: The network architecture of the Knot Selection Network (PPN)

The segments p^s that have a large approximation error are computed a new knot using the KSN. For \bar{p}^s and \bar{t}^s , the KSN generates a new estimated knot $\bar{u}_s \in [0, 1]$. The new knot \bar{u}_s is mapped to the actual knot value range $[u_{s-1}, u_s]$ by

$$\tilde{u}_s = u_{s-1} + \bar{u}_s(u_s - u_{s-1})$$

Instead of \tilde{u}_s , the parameter value t_i closest to \tilde{u}_s is inserted into u . u can be further refined until the desired curve approximation error threshold is satisfied.

The KSN Architecture This stage utilizes a KSN, as mentioned earlier. The KSN predicts a new knot u to the interval $(0, 1)$ for a given segment p and parameters t , of which were previously approximated by the PPN. This network uses an MLP, which transforms the input to a single output value u^{mlp} . The RELU function is used as the activation function for the MLP, except for the output layer where the Sigmoid function is used instead.

Similar to that of the PPN, the KSN has three additional layers: KS1, KS2, and KS3.

KS1. Threshold Layer The new knot u computed has to satisfy the following conditions: $u \in (0, 1)$, $t \cap [0, u] \neq \emptyset$, and $t \cap [u, 1] \neq \emptyset$. Satisfying these conditions will require us to use a threshold layer which maps u^{mlp} to

$$u = \begin{cases} \epsilon & , \text{ if } u^{mlp} \leq 0 \\ 1 - \epsilon & , \text{ if } u^{mlp} \geq 1 \\ u^{mlp} & , \text{ otherwise} \end{cases}$$

Introducing a small $\epsilon = 1e - 5$ makes sure that the knot multiplicity at the end knots stays equal to k .

KS2. Approximation Approximation in the KSN is generally similar to that of PPN. The only different is that of the knot vector. The knot vector in the KSN is defined to be $u = (0, 0, 0, 0, u, 1, 1, 1, 1)$. For backpropagation, the derivative of the B-Spline basis functions with respect to u is required.

KS3. Euclidean Loss The loss function for the KSN is the same as that of the PPN. See 5.2.1.

Network Training

The training of the PPN and KSN will be based from the work of Laube, P., et. al. [?]. The input size of the network will be $l = 100$.

The data set generated by the original authors consists of 150,000 curves. This data was synthesized from B-spline curves. Random control points c_i were generated using a normal distribution μ and variance δ to define cubic ($k = 3$) B-spline curves with $(k + 1)$ -fold end knots and no interior knots. The y -coordinates are given the configuration: $\delta = 2$ and $\mu = 10$. For the x -coordinates, $\delta = 1$ and $\mu = 10$ are used for

the first control point. All consecutive points have μ increased by $\Delta\mu = 1$. Curves with self-intersections are discarded, because the sequential order of their sampled points is not unique, and point sequences are usually split into subsets at the self-intersections. Smaller δ for the x-coordinates of control points reduces the number of curves with self-intersections. To closely match the target input as much as possible, we also include curves that have been manually fitted to raster images. These curves can be obtained from vector images available online.

For each curve, l points $p = (p_0, \dots, p_{l-1})$ are sampled. These curves then to have increasing x-coordinates from left to right. As such, index-flipped versions of the point sequences of the dataset are added, resulting in 300,000 point sequences. 20% of the sequences are used as test data in the training process.

The PPN is trained first since the KSN requires point parametrizations t , which is obtained from the PPN. After training, the PP2 and PP3 layers are discarded and PP1 becomes the output layer of the PPN. The parametric values t are computed for the training dataset by applying the PPN and train the KSN on the combined input. After training, KS2 and KS3 are discarded, with KS1 becoming the network output layer. The MLPs of the PPN and KSN will consist of three hidden layers with sizes (1000, 1000, 1000) and (500, 500, 500) respectively. Dropout is applied to the MLP layers. The network is trained using the Adam optimizer.

5.3 Region Colouring and Merging

Once the curves have been approximated, the vectorization of the region will be filled with the colour prominent in the raster version of the region. The regions will be plotted unto their locations in the original raster input. Once all the regions have

been plotted, they will be grouped into a single vectorization. This will now be the vectorization output of the raster image.

Chapter 6

Describing How You Validated Your Approach.

Chapter 7

Stating Your Results and Drawing Insights From Them.

Chapter 8

Summarizing Your Thesis and Drawing Your Conclusions.

Appendix A

What should be in the Appendix

What goes in the appendices? Any material which impedes the smooth development of your presentation, but which is important to justify the results of a thesis. Generally it is material that is of too nitty-gritty a level of detail for inclusion in the main body of the thesis, but which should be available for perusal by the examiners to convince them sufficiently. Examples include program listings, immense tables of data, lengthy mathematical proofs or derivations, etc.

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