Statistics with SpaRRows

Lecture 10

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Outline

- Hypothesis testing in linear models
- Interpretation (a bit)
- Standardizing
- Reporting

Hypothesis testing in linear models

- What do we actually test?

$$y_i = b_0 + b_1 x_i + \varepsilon_i$$

y	i
5	1
3	2
6	3
11	4
0	5

$b_0 = ?$

$b_1 = ?$

x	i
3	1
1	2
4	3
8	4
2	5

	i
?	1
?	2
?	3
?	4
?	5

```
> y<-c(5,3,6,11,0)
> x<-c(3,1,4,8,2)
> mod<-lm(y~x)
> summary(mod)

Call:
lm(formula = y ~ x)

Residuals:
      1       2       3       4       5 
 0.8219  1.5616  0.4521 -0.0274 -2.8082 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)  
(Intercept)  0.06849    1.55075   0.044   0.9675  
x            1.36986    0.35765   3.830   0.0314 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.933 on 3 degrees of freedom
Multiple R-squared:  0.8302,	Adjusted R-squared:  0.7736 
F-statistic: 14.67 on 1 and 3 DF,  p-value: 0.03136
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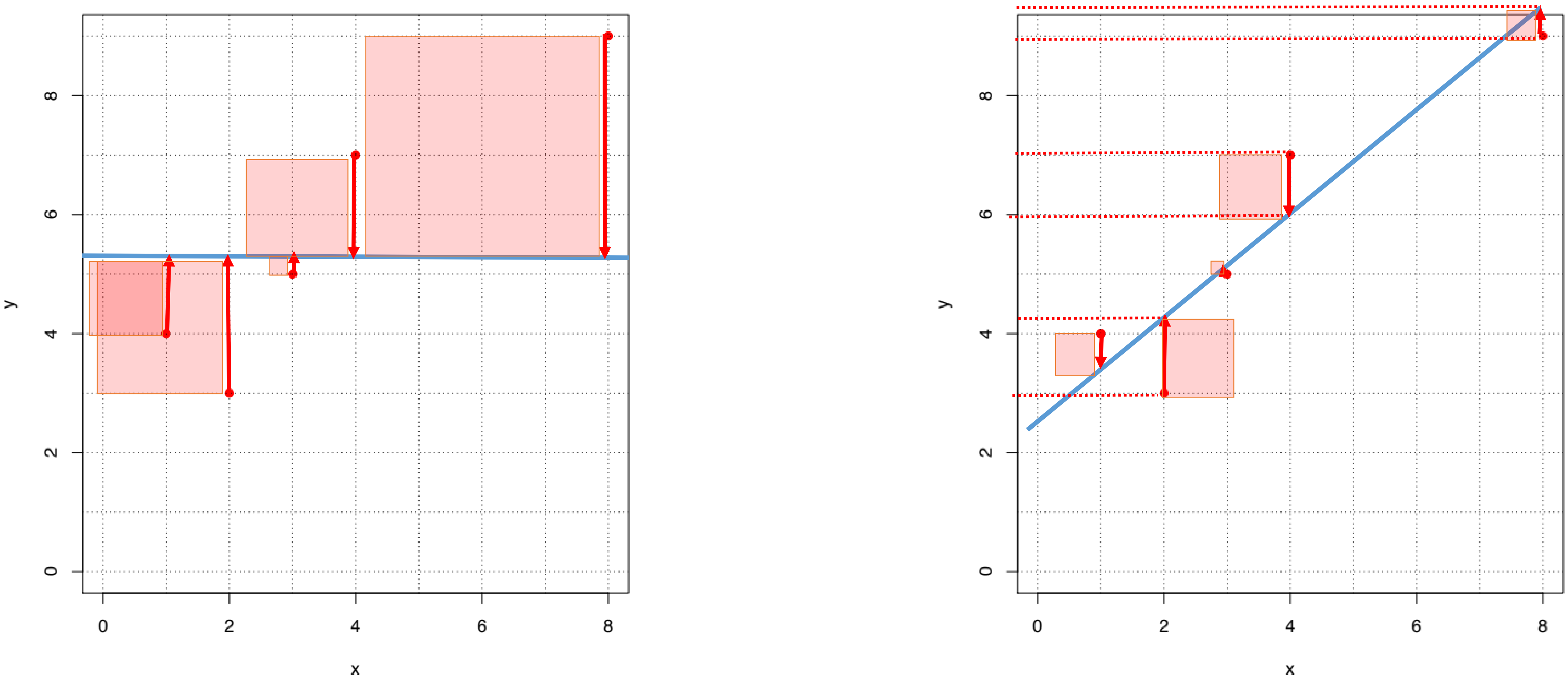
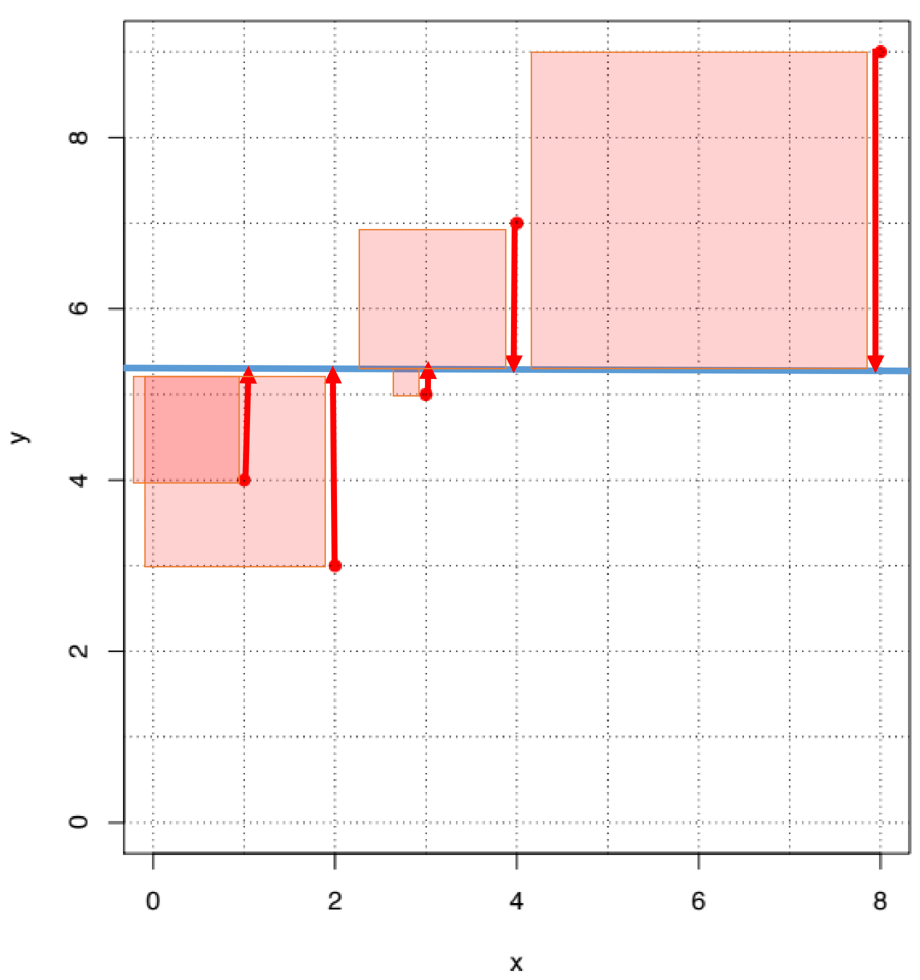
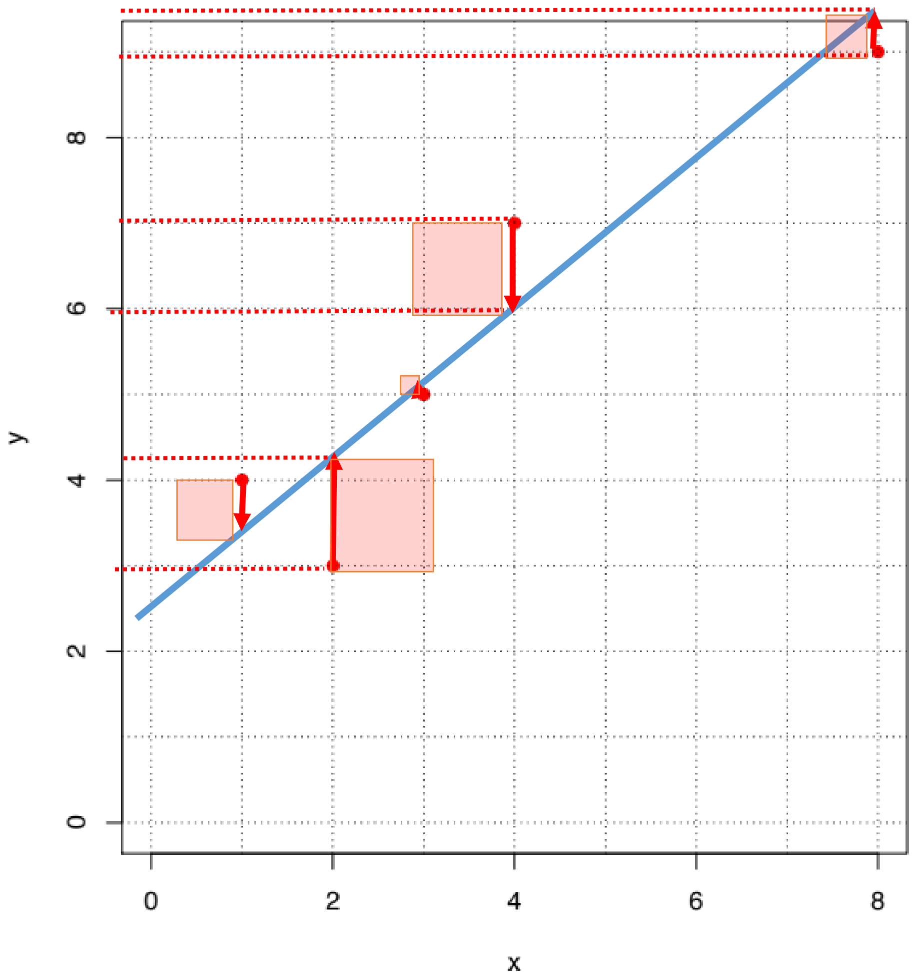


We tested whether the SS of a straight line with slope 0 ($b_1 = 0$) was different from the line we guesstimated, with a different slope





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→ Test if slope (b_1) estimate is different from 0

→ T-test!

```
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> x<-c(3,1,4,8,2)
> mod<-lm(y~x)
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Hypothesis testing in linear models

- Predicted values:

$$y_i = 0.07 + 1.37\, x_i + \varepsilon_i$$

y		x		i
5	5 = 0.07 + 1.37 * 3 + e	3	?	1
3	3 = 0.07 + 1.37 * 1 + e	1	?	2
6	6 = 0.07 + 1.37 * 4 + e	4	?	3
11	11 = 0.07 + 1.37 * 8 + e	8	?	4
0	0 = 0.07 + 1.37 * 2 + e	2	?	5

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```
> resid(mod)
          1           2           3           4           5 
 0.82191781  1.56164384  0.45205479 -0.02739726 -2.80821918 
> |
```

y		x		i
5	5 = 0.07 + 1.37 * 3 -0.82	3	?	1
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y		x		i
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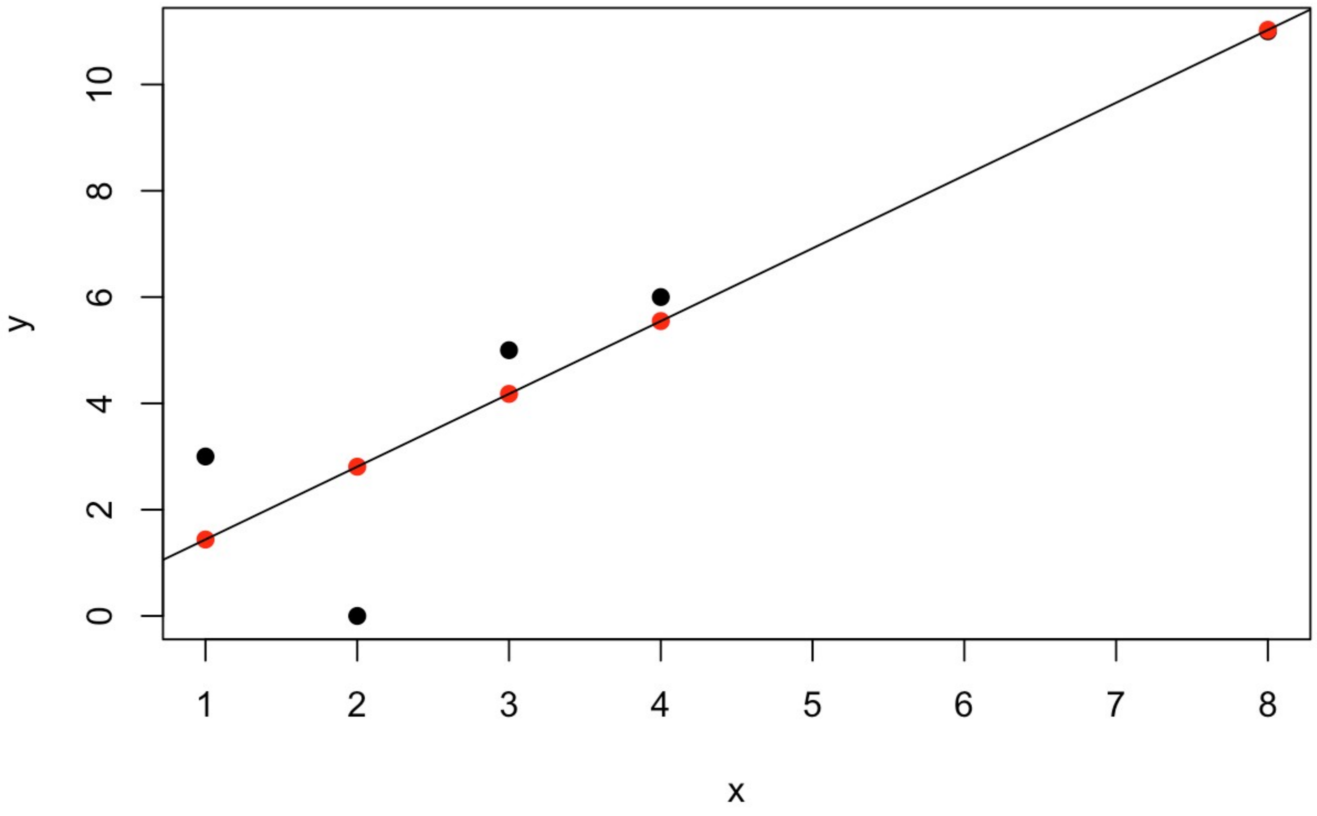
3	3 = 0.07 + 1.37 * 1 + 1.56	1	?	2
6	6 = 0.07 + 1.37 * 4 + 0.45	4	?	3
11	11 = 0.07 + 1.37 * 8 – 0.03	8	?	4
0	0 = 0.07 + 1.37 * 2 – 2.81	2	?	5



y	x	Y predicted
5	3	4.18
3	1	1.44
6	4	5.55
11	8	11.03
0	2	2.81

```
> predict(mod)
        1         2         3         4         5
 4.178082  1.438356  5.547945 11.027397  2.808219
>
```

Hypothesis testing in linear models

- Measure precision of estimates:

y_i

```
> summary(mod)

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$_i + \varepsilon_i$

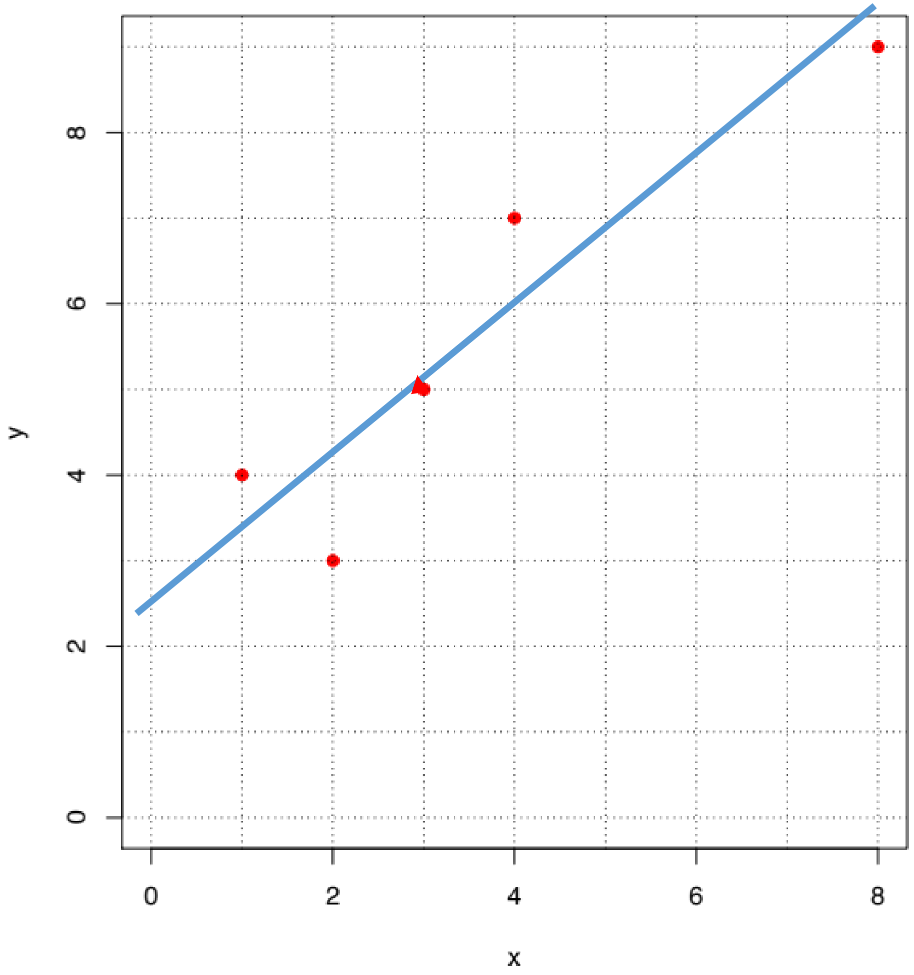
Inference

Tests null hypotheses that

$b_0 = 0$

$b_1 = 0$

What does it mean if the null hypothesis is rejected?



Inference

Tests null hypotheses that
$b_0 = 0$
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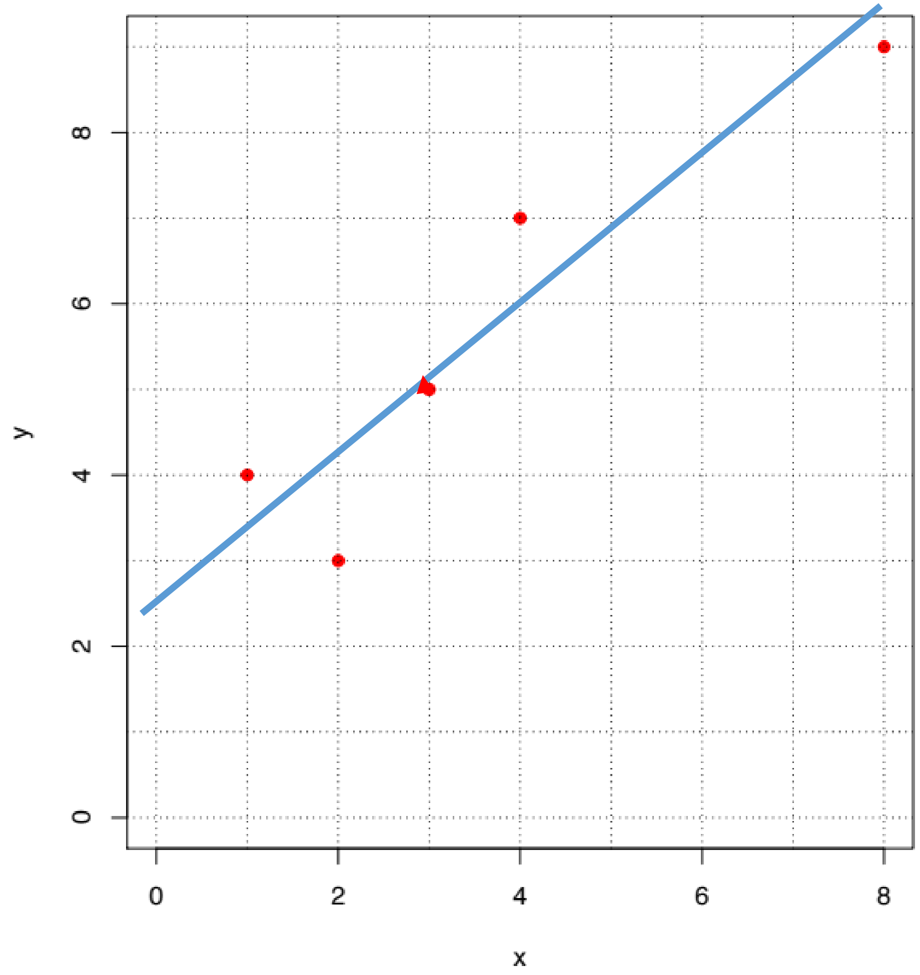
What does it mean if the null hypothesis is rejected?

Usually not much for b_0

The slope is b_1, this gives information about the relationship between y and x.



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Call:
lm(formula = Mass ~ Tarsus, data = d2)

Residuals:
    Min      1Q  Median      3Q     Max 
-7.7271 -1.2202 -0.1302  1.1592  7.5036 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  5.83246    0.98195    5.94 3.48e-09 ***
Tarsus       1.18466    0.05295   22.37  < 2e-16 ***
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Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.841 on 1642 degrees of freedom
Multiple R-squared:  0.2336,    Adjusted R-squared:  0.2332 
F-statistic: 500.6 on 1 and 1642 DF,  p-value: < 2.2e-16
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b_0 = 0, rejected!

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The slope is 5.8

For each mm longer tarsus, a sparrow weights 1.18g more.

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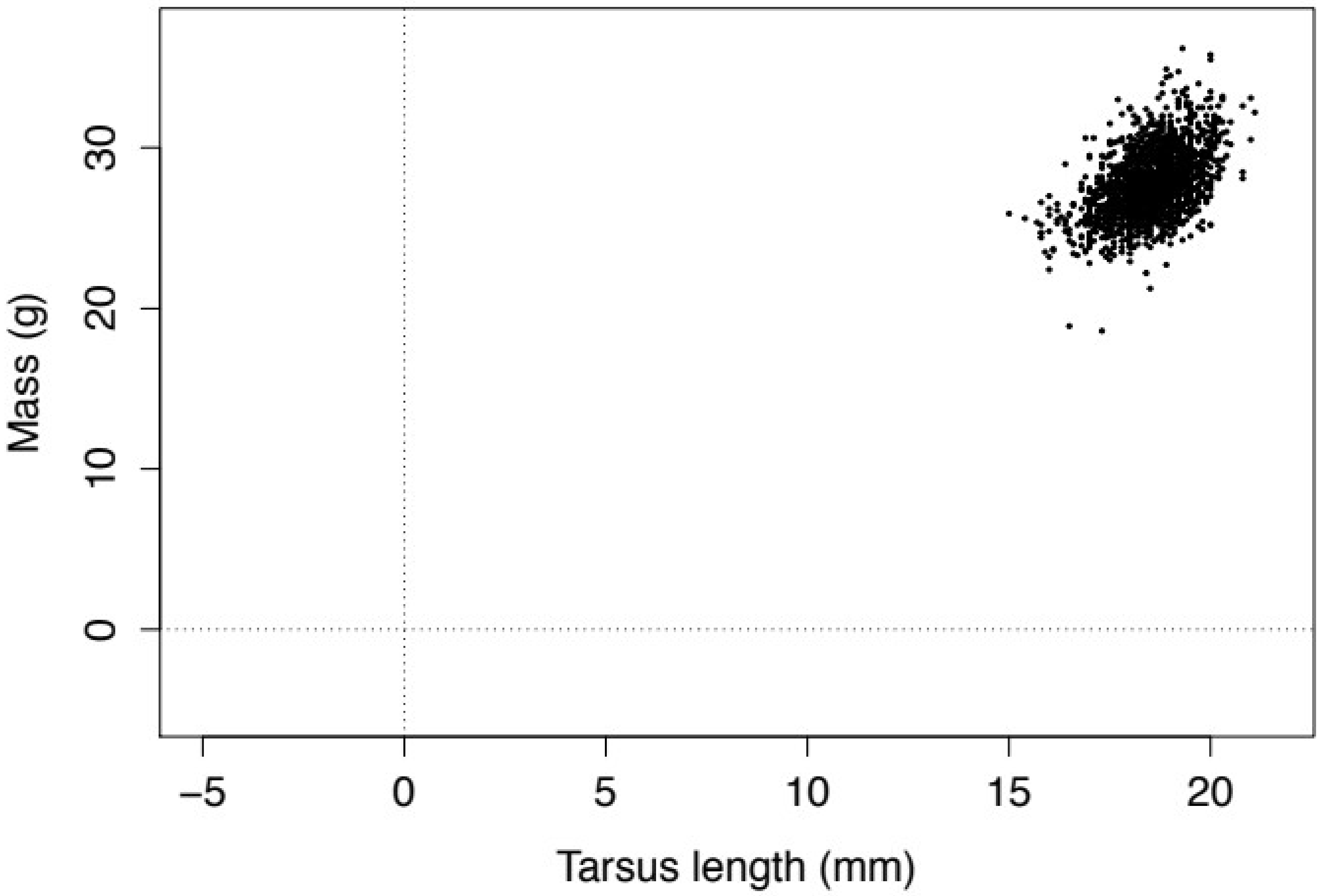

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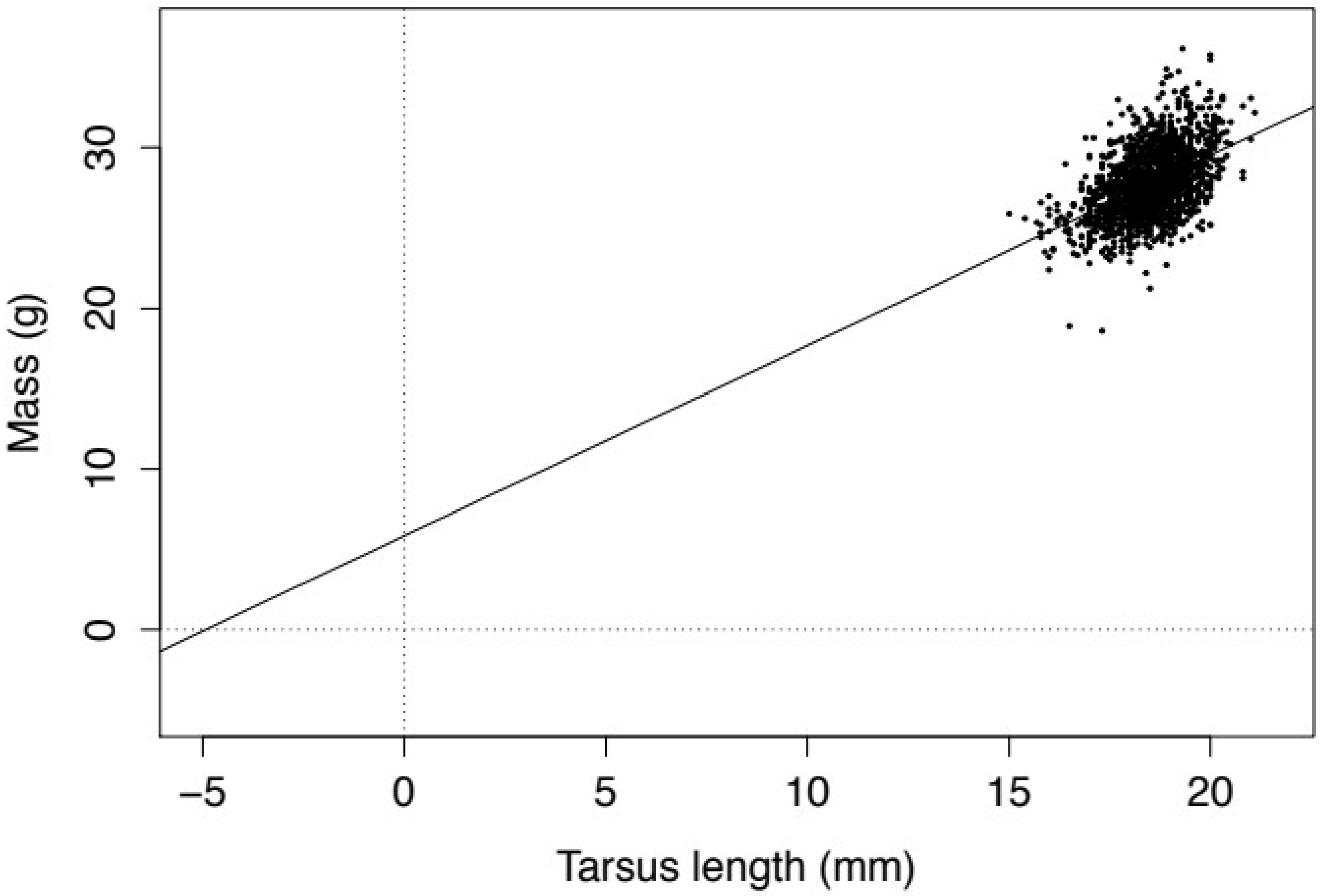
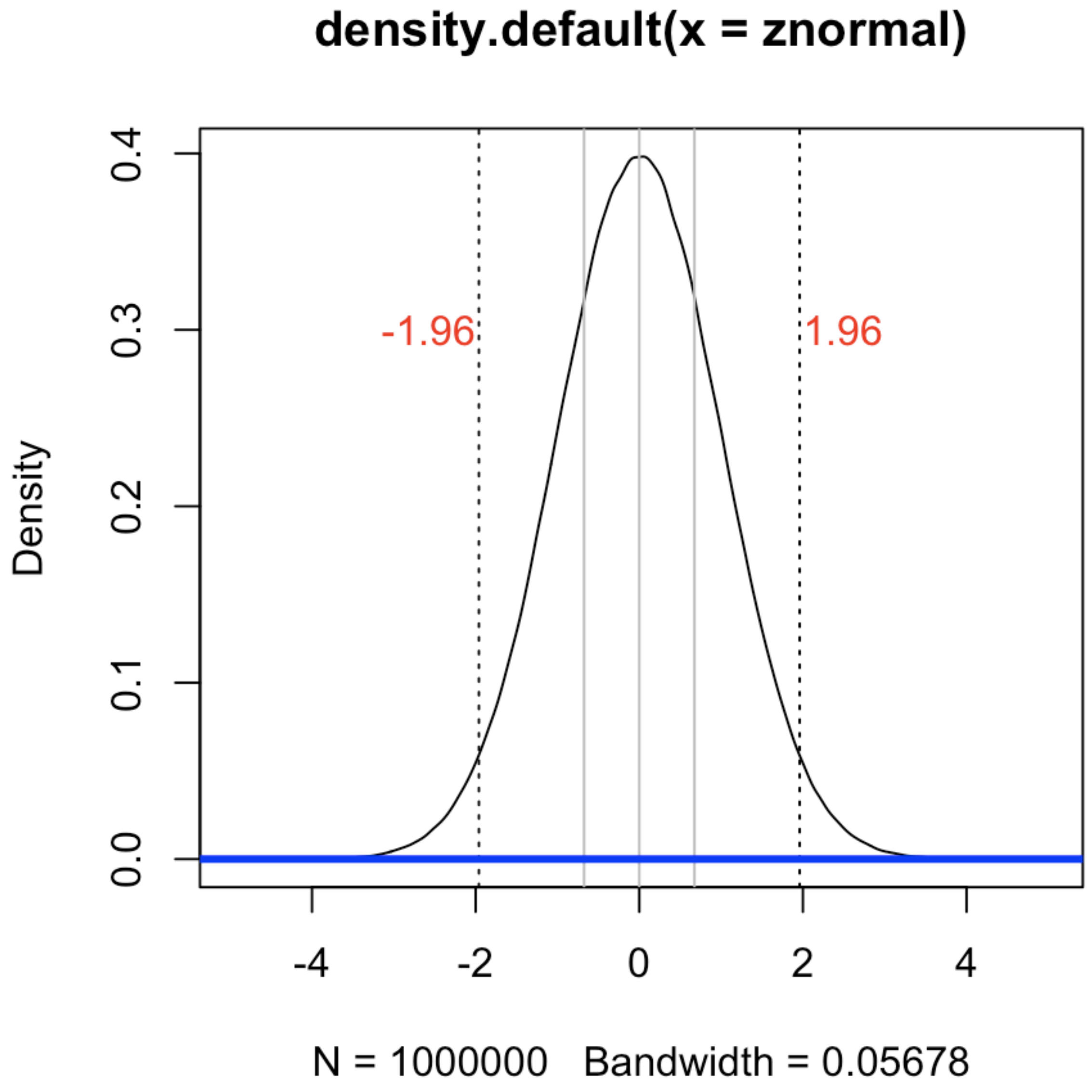


Why standardize da

- z-scores:
- Normal distributed
- Mean of 0
- Sd of 1



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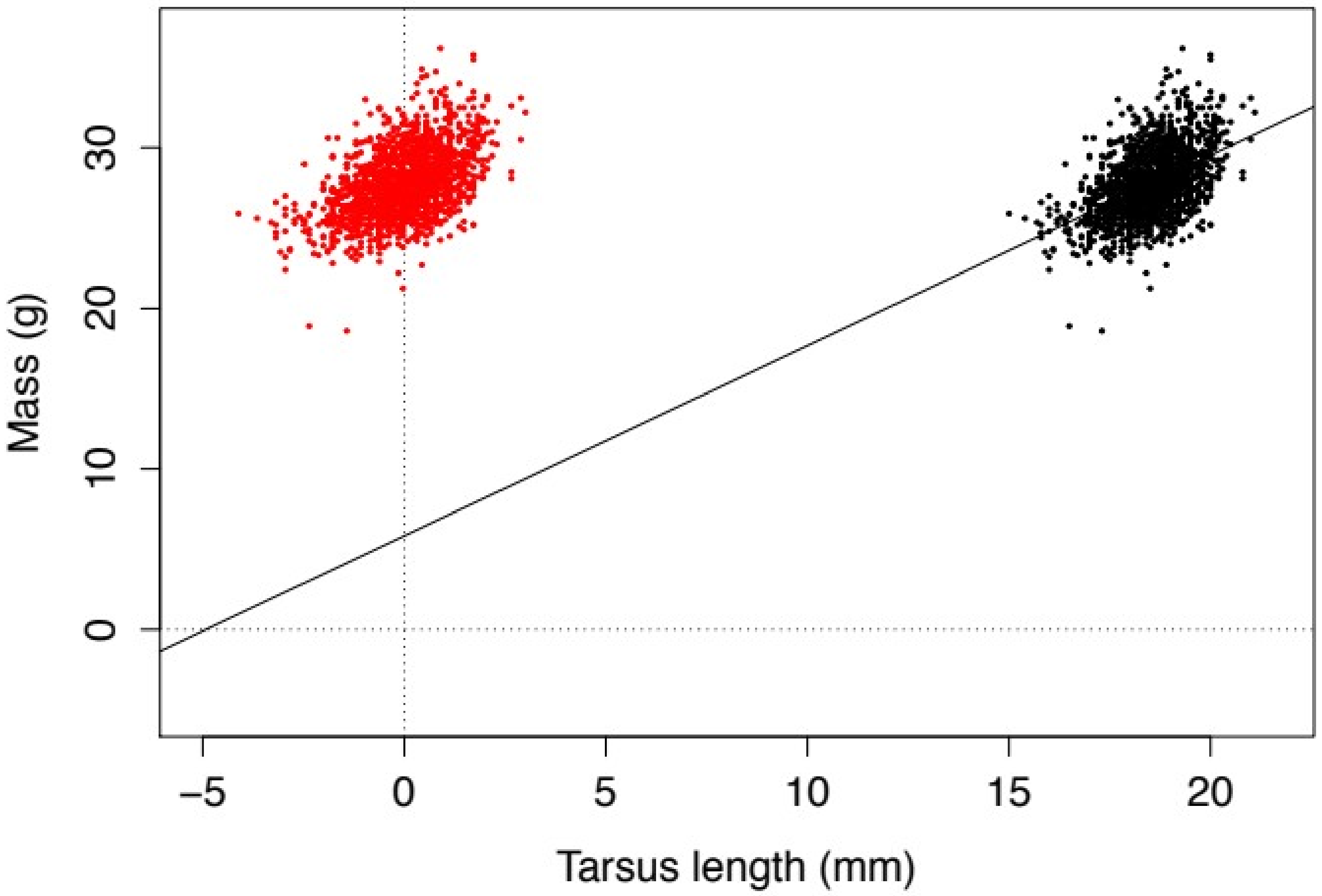

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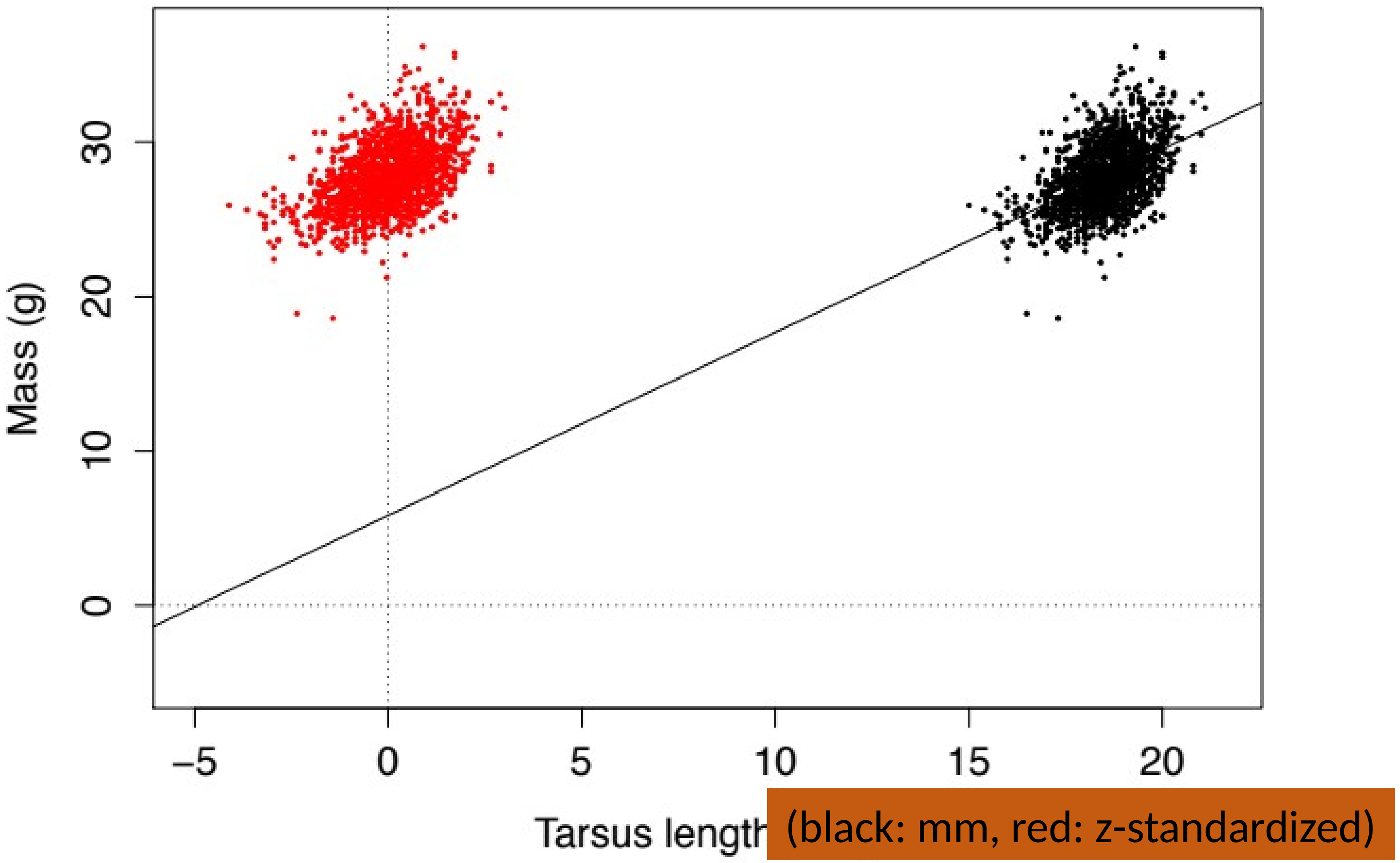

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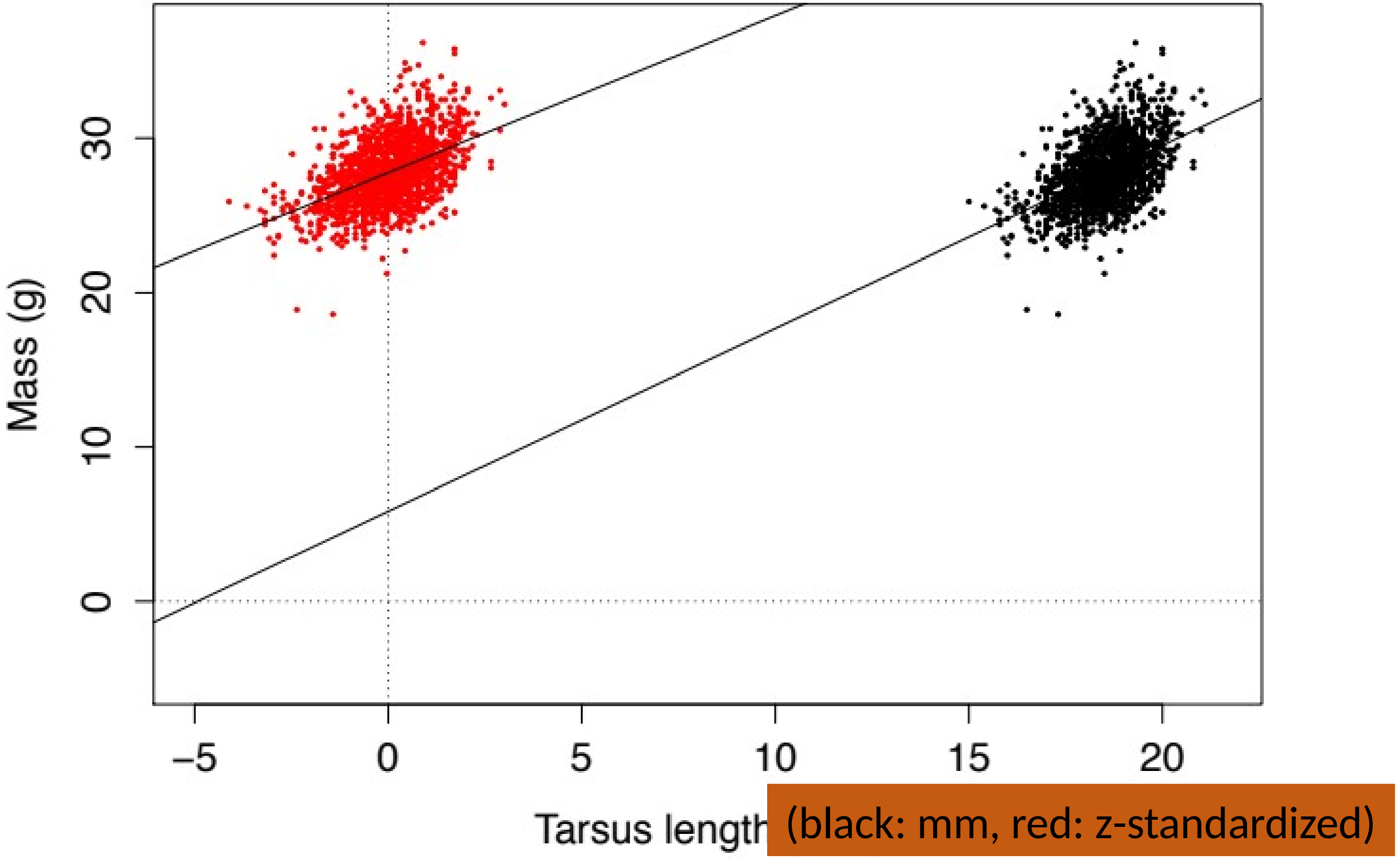

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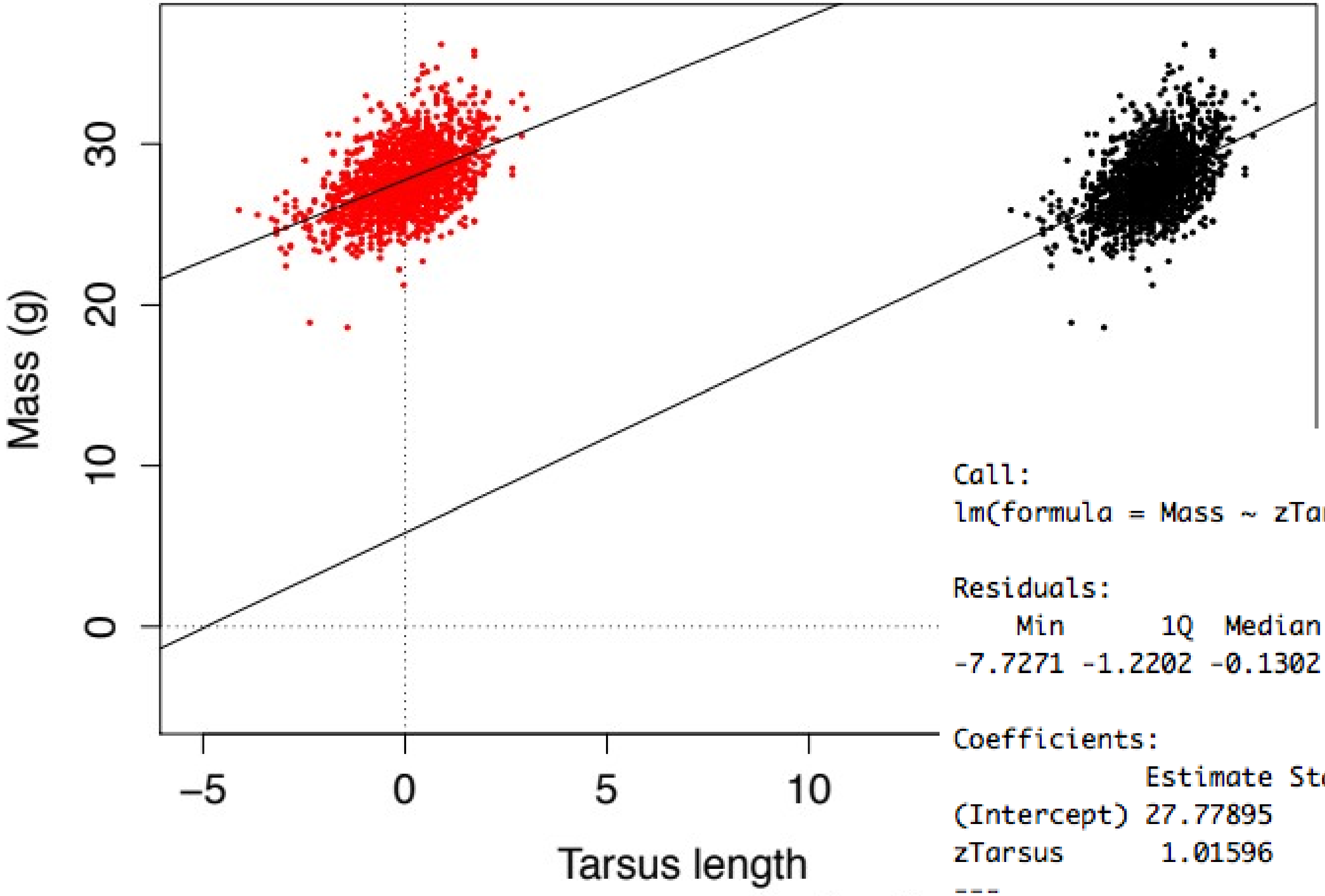

(black: mm, red: z-standardized)

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Call:
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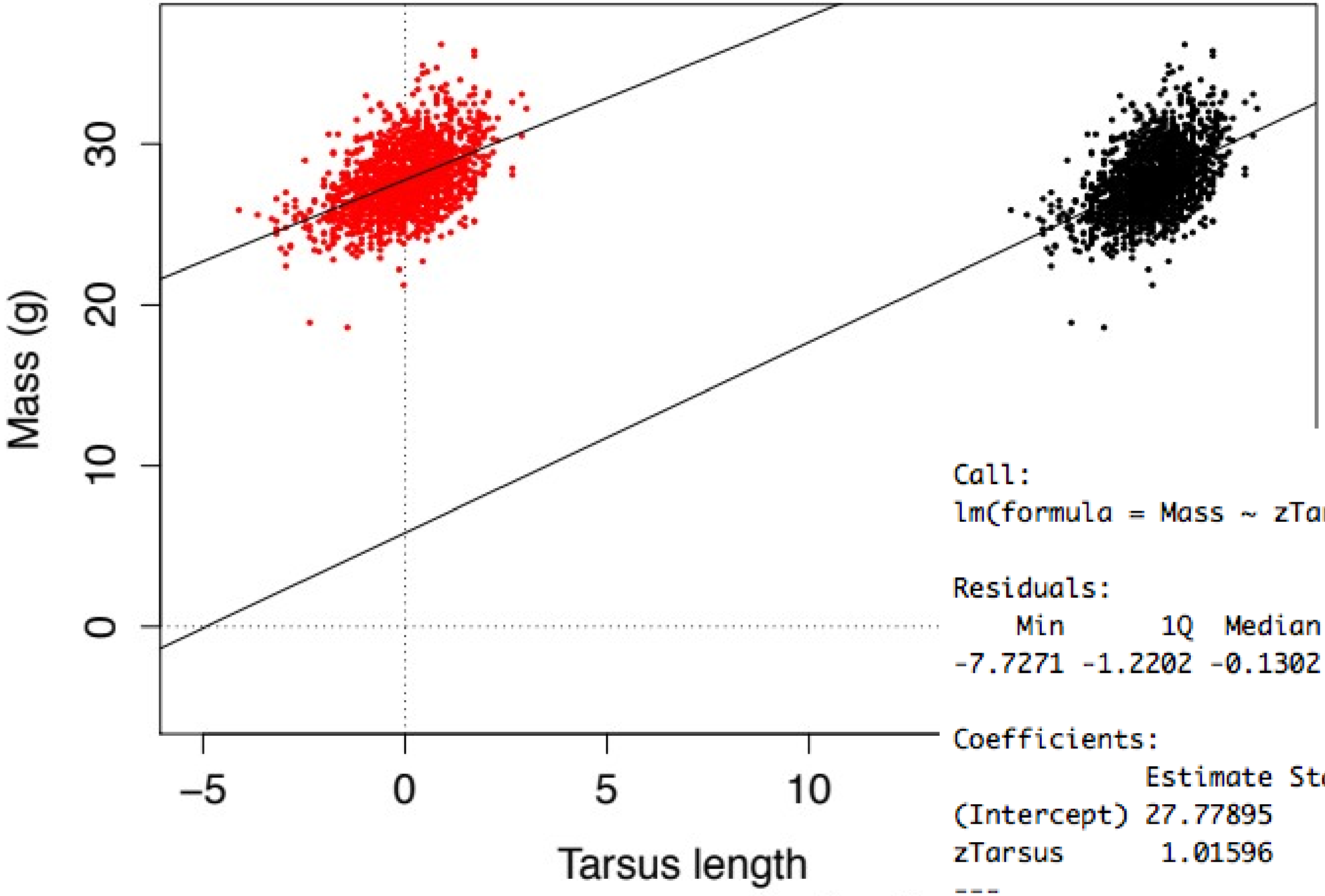
It’s the mean of y!



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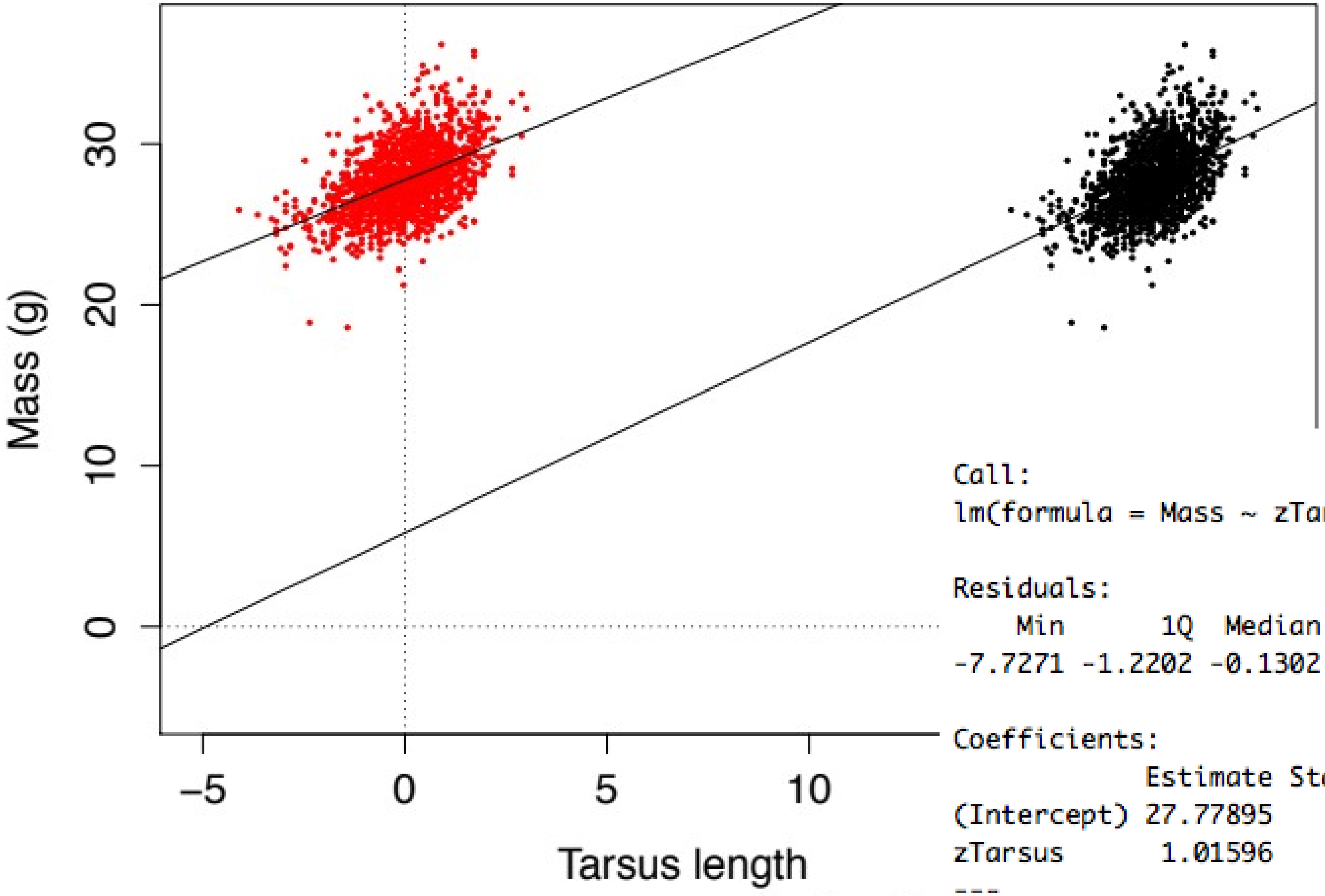

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- Also, units. What if we’d measured tarsus in cm?
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Interpretation of statistics – take home

- Always consider units!
- Standardize to make intercept meaningful
- T-tests test for null hypothesis that parameter estimates equal –
- Always think of the biological meaning in units!

How to report?

Methods:

To test whether heavier birds also had longer tarsi, I used a linear model, where body mass (g) was the response variable, and tarsus length (mm) the explanatory variable.

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Table 1: Results from a linear model explaining body mass of Lundy sparrows with tarsus length. Tarsus length was z-standardized. N = 1646.

Variable	*b*	*SE*	*t*	*p*
Intercept	27.78	0.05	611.94	<0.001
Tarsus	1.02	0.05	22.37	<0.001

- Clear headers!
- Headers of columns center aligned!

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How to report?

- Clear headers!
- Headers of columns center aligned!
- Always report all variables in the model. Don't be selective!
- Right align first column!

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Tarsus	1.02	0.05	22.37	<0.001

How to report?

Results:

I used data from 1646 observations. The sparrows weighed on average 27.78 g (SD 2.10, range: 18.60–36.20). The tarsi of the sparrows were on average 18.5mm long (SD 0.86, range: 15.00–21.10). I found a positive, statistically significant association between mass and tarsus (Table 1). An increase in 1SD Tarsus length meant an increase of 1.02g in body mass.

- Clear headers!
- Headers of columns center aligned!
- Always report all variables in the model. Don't be selective!
- Right align first column!
- Left align numbers!
- Round to two digits (or what else is biologically reasonable)!
- Always report parameter estimates (b)

Table 1: Results from a linear model explaining body mass of Lundy sparrows with tarsus length. Tarsus length was z-standardized. N = 1646.

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- Table legend goes on top of tables (below figures)
- Table legend should be self-explanatory without refering to text!

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How to report – take home

- Methods:
- Always describe *all analyses you present in results. Be precise and specific. Describe what’s the response variable. Say what is the explanatory variable, and say WHY you used those. Give the units. Say when you standardize, and WHY.
- Justify, justify, justify.

How to report – take home

- Results:
- Start with describing the dataset.
- Sample size, mean, range, missing values ect.
- Explain results of each analysis
- Make nice tables and think before copy/pasting values from R!
- Legends need be self-explanatory!