A Potential for a Three-Field AdS/QCD Model

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1 Introduction

Quantum chromodynamics has been well tested for high-energy collisions, where perturbation theory is applicable. However, at hadronic scales, the interaction is non-perturbative, requiring a new theoretical model. The Anti-de Sitter Space/Conformal Field Theory (AdS/CFT) correspondence establishes a connection between an n-dimensional Super-Yang Mills Theory and a weakly-coupled gravitational theory in $n+1$ dimensions. Phenomenological models inspired by this correspondence are known as AdS/QCD, and have succeeded in capturing some features of QCD.

Quark confinement in QCD sets a scale that is encoded in a cut-off of the fifth dimension in the AdS theory. Soft-wall models use a dilaton as an effective cut-off to limit the penetration of the meson fields into the bulk. The simplest soft-wall models use a quadratic dilaton to recover the linear Regge trajectories, while models that modify the UV behavior of the dilaton more accurately model the ground state masses.

Previous soft-wall models use parametrizations for the background dilaton and chiral fields that are not derived as the solution to any equations of motion. A well-defined action provides a set of background equations from which these fields can be derived. In addition, this action provides access to the thermal properties of the model through perturbation of the geometry.

In this paper, we review previous attempts to find a suitable potential for the background fields of a soft-wall AdS/QCD model. After demonstrating the limitations of models including a dilaton and chiral field alone, we suggest the inclusion of a background glueball field. We then construct a potential that satisfies the necessary UV and IR limits, and use this potential to generate the background fields and calculate the resulting meson spectra.

2 Review and Motivation

Review the models of Springer/Kapusta and Gherghetta/Batell, and note the phenomenological shortcomings

3 Setup

Consider a potential for three fields: ϕ, χ and G representing the dilaton, a chiral field, and a glueball field with zero mass. The potential in the Einstein frame, where the action has its canonical form, is

$$V(\phi, \chi, G) = \mathrm{e}^{2\phi/\sqrt{6}}\,\tilde{V}(\phi, \chi, G) \tag{1}$$

with

$$\tilde{V} = -12 + 4\sqrt{6}\phi + a_0\phi^2 - \tfrac{3}{2}\chi^2 + \tilde{U} \tag{2}$$

Here $\tilde{U}$ is more than quadratic in the fields. The dilaton mass is undetermined and is not connected to the dimension of the corresponding operator, as discussed by Kapusta and Springer. It is related to the parameter a_0 by $a_0 = \frac{1}{2}\left[(m_\phi L)^2 - 8\right]$. The potential should be an even function of χ.

The equations of motion can be written as

$$\dot{\chi}^2 + \dot{G}^2 = \frac{\sqrt{6}}{z^2}\frac{d}{dz}(z^2\dot{\phi}) \tag{3}$$

$$\tfrac{1}{2}\sqrt{6}z^2\ddot{\phi} - \tfrac{3}{2}(z\dot{\phi})^2 - 3\sqrt{6}z\dot{\phi} - 4\sqrt{6}\phi - a_0\phi^2 + \tfrac{3}{2}\chi^2 = \tilde{U} \tag{4}$$

$$3z\dot{\phi} - 2a_0\phi = \frac{\partial\tilde{U}}{\partial\phi} \tag{5}$$

$$z^2\ddot{\chi} - 3z\dot{\chi}\left(1 + \frac{z\dot{\phi}}{\sqrt{6}}\right) + 3\chi = \frac{\partial\tilde{U}}{\partial\chi} \tag{6}$$

$$z^2\ddot{G} - 3z\dot{G}\left(1 + \frac{z\dot{\phi}}{\sqrt{6}}\right) = \frac{\partial\tilde{U}}{\partial G} \tag{7}$$