A Potential for a Three-Field AdS/QCD Model

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Abstract

The Anti-de Sitter Space/Conformal Field Theory (AdS/CFT) correspondence may offer new and useful insights into the non-perturbative regime of strongly coupled gauge theories such as Quantum Chromodynamics (QCD). We present an AdS/CFT-inspired model that describes the spectra of light mesons. The conformal symmetry is broken by a background dilaton field, and chiral symmetry breaking and linear confinement are described by a chiral condensate field. These background fields, along with a background glueball condensate field, are derived from a potential. We describe the construction of the potential, and the calculation of the meson spectra, which match experimental data well. We also argue that the presence of the third background field is necessary to properly describe the meson spectra. The outlook for application of this model to finite temperature systems is also discussed.

1 Introduction

Quantum chromodynamics has been well tested for high-energy collisions, where perturbation theory is applicable. However, at hadronic scales, the interaction is non-perturbative, requiring a new theoretical model. The Anti-de Sitter Space/Conformal Field Theory (AdS/CFT) correspondence establishes a connection between an n-dimensional Super-Yang Mills Theory and a weakly-coupled gravitational theory in $n + 1$ dimensions. Phenomenological models inspired by this correspondence are known as AdS/QCD, and have succeeded in capturing some features of QCD.

Quark confinement in QCD sets a scale that is encoded in a cut-off of the fifth dimension in the AdS theory. Soft-wall models use a dilaton as an effective cut-off to limit the penetration of the meson fields into the bulk. The simplest soft-wall models use a quadratic dilaton to recover the linear Regge trajectories, while models that modify the UV behavior of the dilaton more accurately model the ground state masses.

Previous soft-wall models use parametrizations for the background dilaton and chiral fields that are not derived as the solution to any equations of motion. A well-defined action provides a set of background equations from which these fields can be derived. In addition, this action provides access to the thermal properties of the model through perturbation of the geometry.

In this paper, we review previous attempts to find a suitable potential for the background fields of a soft-wall AdS/QCD model. After demonstrating the limitations of models including a dilaton and chiral field alone, we suggest the inclusion of a background glueball field. We then construct a potential that satisfies the necessary UV and IR limits, and use this potential to generate the background fields and calculate the resulting meson spectra.

2 Review and Motivation

Previous work showed how to construct a potential for a gravity-dilaton-chiral system, assuming that the fields have power-law behavior, which is accurate in both the UV and IR limits. One of the equations of motion is independent of the choice of potential,

$$\dot{\chi}^2 = \frac{\sqrt{6}}{z^2} \frac{d}{dz}(z^2 \dot{\phi}). \tag{1}$$

Examining the IR limit, we know that the dilaton should have quadratic behavior, $\phi(z) = \lambda z^2$. Inserting this into , we find that the chiral field behaves as

$$\chi(z) = 6^{3/4}\sqrt{\lambda}z, \tag{2}$$

which removes one of the independent parameters of the original GKK model. As shown in that paper, the IR coefficient of the chiral field sets the large-n mass splitting between the axial-vector and vector mesons in the model. Using the phenomenological value of λ, which determines the slope of the Regge trajectories, we find a mass splitting that is much too large.

Because this problem arises in the equation that does not involve the potential, this issue cannot be resolved by the choice of potential in the two-field model. Models that derive the field behavior using the superpotential method suffer from the same problem.

To resolve this problem, we suggest to add an additional scalar field to the model, G, representing the glueball condensate. This field must be linear in the IR for linear confinement, and go as z^4 in the UV to match the operator dimension in the AdS/CFT dictionary.

3 Setup

Consider the action in the string frame for three fields: ϕ, χ and G representing the dilaton, a chiral field, and a glueball field with zero mass

$$S_{string} = \frac{1}{16\pi G_5} \int d^5 x \sqrt{-g} e^{-2\Phi} \left(R_s \partial_\mu \Phi \partial^\mu \Phi - \tfrac{1}{2} \partial_\mu \chi \partial^\mu \chi - \tfrac{1}{2} \partial_\mu G \partial^\mu G - e^{-4\Phi/3} V(\phi, \chi, G) \right) \tag{3}$$

The potential in the Einstein frame, where the action has its canonical form, is

$$V(\phi, \chi, G) = e^{2\phi/\sqrt{6}} \tilde{V}(\phi, \chi, G) \tag{4}$$

with

$$\tilde{V} = -12 + 4\sqrt{6}\phi + a_0 \phi^2 - \tfrac{3}{2}\chi^2 + \tilde{U} \tag{5}$$

Here $\tilde{U}$ is more than quadratic in the fields. The dilaton mass is undetermined and is not connected to the dimension of the corresponding operator, as discussed by Kapusta and Springer. It is related to the parameter a_0 by $a_0 = \tfrac{1}{2} \left[(m_\phi L)^2 - 8 \right]$. The potential should be an even function of χ.

The equations of motion can be written as

$$\dot{\chi}^2 + \dot{G}^2 = \frac{\sqrt{6}}{z^2} \frac{d}{dz} (z^2 \dot{\phi}) \tag{6}$$

$$\tilde{U} = \tfrac{1}{2}\sqrt{6} z^2 \ddot{\phi} - \tfrac{3}{2}(z\dot{\phi})^2 - 3\sqrt{6} z \dot{\phi} - 4\sqrt{6}\phi - a_0 \phi^2 + \tfrac{3}{2}\chi^2 \tag{7}$$

$$\frac{\partial \tilde{U}}{\partial \phi} = 3z\dot{\phi} - 2a_0 \phi \tag{8}$$

$$\frac{\partial \tilde{U}}{\partial \chi} = z^2 \ddot{\chi} - 3z\dot{\chi} \left(1 + \frac{z\dot{\phi}}{\sqrt{6}} \right) + 3\chi \tag{9}$$

$$\frac{\partial \tilde{U}}{\partial G} = z^2 \ddot{G} - 3z\dot{G} \left(1 + \frac{z\dot{\phi}}{\sqrt{6}} \right) \tag{10}$$

We assume that the potential has no explicit dependence on the coordinate z, so the equations 8-10 are not independent, and we can eliminate one.

3.1 IR Limit

The requirement of linear confinement requires a solution in the large z limit of the form

$$\phi = \lambda z^2 \tag{11}$$
$$\chi = Az \tag{12}$$
$$G = Bz. \tag{13}$$

Substitution into (6) gives

$$A^2 + B^2 = 6\sqrt{6}\lambda \tag{14}$$

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The λ is fixed by the slope of the linear trajectory and A is fixed by the axial-vector – vector mass difference. It is useful to write these as

$$A = 6^{3/4}\sqrt{\lambda}\cos\theta$$

$$B = 6^{3/4}\sqrt{\lambda}\sin\theta, \tag{15}$$

where θ now becomes the parameter controlling the axial-vector – vector mass splitting. Inserting (13) into (7-10) suggests the following terms in our ansatz for the potential

$$\tilde{U} = a_1\phi\chi^2 + a_2\phi G^2 + a_3\chi^4 + a_4 G^4 + a_5\chi^2 G^2 + a_6 G^2\tanh(g\phi). \tag{16}$$

We see that there must be a G^2 term in the IR limit, but this is forbidden in the weak-field limit because the glueball condensate field is massless. To circumvent this, we propose the term $G^2\tanh(g\phi)$ with $g > 0$. In the weak field limit this goes to $g\phi G^2$, which is acceptable. The tanh is suggested by (4), and it suggests a rapid exponential transition from the weak field to the strong field limits that is supported by phenomenology. By substitution one finds the following constraints on the parameters:

$$\tilde{U} \to 6 + a_0 + 6\sqrt{6}\left(\cos^2\theta\, a_1 + \sin^2\theta\, a_2\right)$$

$$+6^3\left(\cos^4\theta\, a_3 + \sin^4\theta\, a_4 + \cos^2\theta\sin^2\theta\, a_5\right) = 0 \tag{17}$$

$$\frac{\partial\tilde{U}}{\partial\chi} \to 2a_1 + 24\sqrt{6}\cos^2\theta\, a_3 + 12\sqrt{6}\sin^2\theta\, a_5 + \sqrt{6} = 0 \tag{18}$$

$$\frac{\partial\tilde{U}}{\partial G} \to 2a_2 + 24\sqrt{6}\sin^2\theta\, a_4 + 12\sqrt{6}\cos^2\theta\, a_5 + \sqrt{6} = 0 \tag{19}$$

$$\frac{\partial\tilde{U}}{\partial G} \to a_6 = -\tfrac{3}{2} \tag{20}$$

We have chosen to exclude (8) because it is not independent. The parameter a_6 is determined, and the others will be determined by an examination of the UV limit.

3.2   UV Limit

Next we look for a solution in the small z limit. The AdS/CFT dictionary dictates that the leading-order UV behavior of the chiral and glueball condensate fields is determined by their dimension. Note also that we are working in the chiral limit where the quark mass is zero. We start by examining only the leading-order terms

$$\chi = \Sigma_0 z^3 \tag{21}$$

$$G = G_0 z^4. \tag{22}$$

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Substitution into (6) and imposing the boundary condition $\phi(0) = 0$ gives

$$\phi = \frac{\sqrt{6}}{28}\Sigma_0^2 z^6 + \frac{\sqrt{6}}{27}G_0^2 z^8 \tag{23}$$

Using only this leading-order behavior in (7-10), the system of equations is inconsistent, as there are more equations from matching powers of z than unknown parameters.

To solve this problem, try adding a term $\Sigma_n z^n$ to χ. Substituting into (6) and keeping only the lowest-order cross-term we find the additional term in ϕ

$$\Delta\phi = \frac{\sqrt{6}n\Sigma_0\Sigma_n}{(n+4)(n+3)}z^{n+3} \tag{24}$$

From (7) we find that

$$\tilde{U} = -\tfrac{3}{2}(z\dot{\phi})^2 - a_0\phi^2 + 3\frac{n^3 - 13n + 12}{(n+4)(n+3)}\Sigma_0\Sigma_n z^{n+3} \tag{25}$$

Since the ϕ^2 terms start out as z^{12}, z^{14}, z^{16}, and so do the terms in the potential, the n can only take the values 9, 11, etc. This term contributes only to the equation for $\partial\tilde{U}/\partial\chi$.

$$\frac{\partial\tilde{U}}{\partial\chi} = -9\Sigma_0\left(\frac{3}{14}\Sigma_0^2 + \frac{8}{27}G_0^2 z^2\right)z^9 + (n-3)(n-1)\Sigma_n z^n \tag{26}$$

By power counting both $n = 9$ and $n = 11$ can contribute.

There could also be higher order terms in G such as $G_m z^m$. This leads to the additional term in ϕ

$$\Delta\phi = \frac{8mG_0G_m}{\sqrt{6}(m+5)(m+4)}z^{m+4} \tag{27}$$

It contributes to the equation for $\partial\tilde{U}/\partial G$ as

$$\frac{\partial\tilde{U}}{\partial G} = -12G_0\left(\frac{3}{14}\Sigma_0^2 + \frac{8}{27}G_0^2 z^2\right)z^{10} + m(m-4)G_n z^m \tag{28}$$

The choice $m = 8$ is not possible as there is no term of the same order to balance it. Terms with $m = 10$ and $m = 12$ are possible. These new terms cannot affect the equation for $\partial\tilde{U}/\partial\phi$ nor can they contribute to the equation for $\partial\tilde{U}/\partial\chi$. Considering higher order terms in both χ and G leads to

$$\tilde{U} = -\tfrac{3}{2}(z\dot{\phi})^2 - a_0\phi^2 + 3\frac{n^3 - 13n + 12}{(n+4)(n+3)}\Sigma_0\Sigma_n z^{n+3} + \frac{4m(m-4)}{m+4}G_0G_m z^{m+4} \tag{29}$$

The appearance of these terms can be understood by writing the following schematic expansions.

$$\chi \sim \Sigma_0 z^3 + \Sigma_0^3 z^9 + G_0^2\Sigma_0 z^{11} + \cdots$$

$$G \sim G_0 z^4 + \Sigma_0^2 G_0 z^{10} + G_0^3 z^{12} + \cdots$$

That is, χ is an odd function of Σ_0 and G is an odd function of G_0. These are the symmetries in the equations of motion. They also follow the spirit of the AdS/CFT correspondence in terms of the dimensionality of the operators and the powers of z.

Including now $m = 10$ and 12, and $n = 9$ and 11, we have the following set of equations in the small z limit:

$$
\begin{aligned}
\tilde{U}_{\text{LHS}} &= 3\Sigma_0^4 z^{12} \left[4\frac{\Sigma_9}{\Sigma_0^3} - \frac{(54 + a_0)}{2^3 \cdot 7^2} \right] \\
&+ \frac{1}{7}\Sigma_0^2 G_0^2 z^{14} \left[120\frac{G_{10}}{\Sigma_0^2 G_0} + 120\frac{\Sigma_{11}}{\Sigma_0 G_0^2} - \frac{(72 + a_0)}{9} \right] \\
&+ 2G_0^4 z^{16} \left[12\frac{G_{12}}{G_0^3} - \frac{(96 + a_0)}{3^5} \right] \quad (30)
\end{aligned}
$$

$$
\begin{aligned}
\tilde{U}_{\text{RHS}} &= \Sigma_0^4 z^{12} \left[\frac{\sqrt{6}}{28}a_1 + a_3 \right] \\
&+ \Sigma_0^2 G_0^2 z^{14} \left[\frac{\sqrt{6}}{27}a_1 + \frac{\sqrt{6}}{28}(a_2 + ga_6) + a_5 \right] \\
&+ G_0^4 z^{16} \left[\frac{\sqrt{6}}{27}(a_2 + ga_6) + a_4 \right] \quad (31)
\end{aligned}
$$

$$
\left(\frac{\partial \tilde{U}}{\partial \chi} \right)_{\text{LHS}} = 3\Sigma_0^3 z^9 \left[-\frac{9}{14} + 16\frac{\Sigma_9}{\Sigma_0^3} \right] + 8\Sigma_0 G_0^2 z^{11} \left[-\frac{1}{3} + 10\frac{\Sigma_{11}}{\Sigma_0 G_0^2} \right] \quad (32)
$$

$$
\left(\frac{\partial \tilde{U}}{\partial \chi} \right)_{\text{RHS}} = \Sigma_0^3 z^9 \left[\frac{\sqrt{6}}{14}a_1 + 4a_3 \right] + \Sigma_0 G_0^2 z^{11} \left[\frac{2\sqrt{6}}{27}a_1 + 2a_5 \right] \quad (33)
$$

$$(34)$$

$$
\left(\frac{\partial \tilde{U}}{\partial G} \right)_{\text{LHS}} = 6\Sigma_0^2 G_0 z^{10} \left[-\frac{3}{7} + 10\frac{G_{10}}{\Sigma_0^2 G_0} \right] + 32G_0^3 z^{12} \left[-\frac{1}{9} + 3\frac{G_{12}}{G_0^3} \right] \quad (35)
$$

$$
\left(\frac{\partial \tilde{U}}{\partial G} \right)_{\text{RHS}} = \Sigma_0^2 G_0 z^{10} \left[\frac{\sqrt{6}}{14}(a_2 + ga_6) + 2a_5 \right] + G_0^3 z^{12} \left[\frac{2\sqrt{6}}{27}(a_2 + ga_6) + 4a_4 \right] \quad (36)
$$

Altogether, from both the UV and IR limits, there are ten independent equations for the twelve parameters $a_0 - a_6$, Σ_9, Σ_{11}, G_{10}, G_{12}, and g. We take g as the free parameter to use as the rate of transition from small z to large z. The parameters in the potential are found to be

$$
a_0 = \frac{3}{2}\frac{1}{6 + \sin^2\theta} \left[120 + 62\sin^2\theta + 63\sqrt{6}g\sin^2\theta \right] \quad (37)
$$

$$
a_1 = -\frac{3\sqrt{6}}{4}\frac{1}{6 + \sin^2\theta} \left[12 + 8\sin^2\theta + 9\sqrt{6}g\sin^2\theta \right] \quad (38)
$$

$$
a_2 = -\frac{\sqrt{6}}{4}\frac{1}{6 + \sin^2\theta} \left[32 + 24\sin^2\theta + 3\sqrt{6}g(9\sin^2\theta - 2) \right] \quad (39)
$$

$$2a_3 \cos^2\theta + a_5 \sin^2\theta = \frac{1}{24}\frac{1}{6+\sin^2\theta}\left[24 + 22\sin^2\theta + 27\sqrt{6}g\sin^2\theta\right] \quad (40)$$

$$2a_4 \sin^2\theta + a_5 \cos^2\theta = \frac{1}{24}\frac{1}{6+\sin^2\theta}\left[20 + 22\sin^2\theta + 3\sqrt{6}g(9\sin^2\theta - 20)\right] \quad (41)$$

$$a_6 = -\frac{3}{2} \quad (42)$$

The coefficients a_0, a_1, a_2 and a_6 are determined, while there are two equations for the three coefficients a_3, a_4 and a_5. That leaves a_5 as a free parameter, to be fit numerically, along with g, θ, G_0, Σ, and λ.