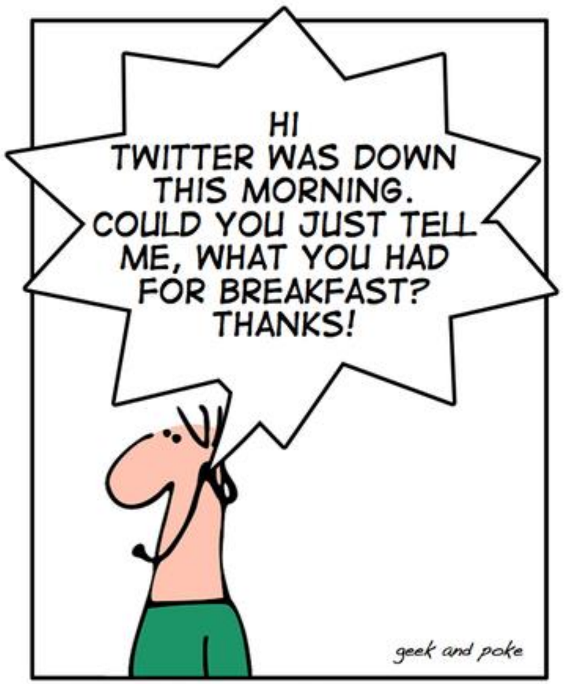
Introduction to Social Network Analysis and Models

Lecture 2



THE REAL FOLLOWERS

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Lecture Outline

- Part I:
  - Social Network Analysis
- Part II:
  - Social Network Models

Social Network Models

Models for Real World Network

	network	type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r	Ref(s).
social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208	20, 416
	company directors	undirected	7 673	55 392	14.44	4.60	–	0.59	0.88	0.276	105, 323
	math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15	0.34	0.120	107, 182
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45	0.56	0.363	311, 313
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088	0.60	0.127	311, 313
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1				8, 9
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0		0.16		136
	email address books	directed	16 881	57 029	3.38	5.22	–	0.17	0.13	0.092	321
	student relationships	undirected	573	477	1.66	16.01	–	0.005	0.001	−0.029	45
	sexual contacts	undirected	2 810				3.2				265, 266
information	WWW `nd.edu`	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	−0.067	14, 34
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7				74
	citation network	directed	783 339	6 716 198	8.57		3.0/–				351
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	–	0.13	0.15	0.157	244
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7		0.44		119, 157
technological	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	−0.189	86, 148
	power grid	undirected	4 941	6 594	2.67	18.99	–	0.10	0.080	−0.003	416
	train routes	undirected	587	19 603	66.79	2.16	–		0.69	−0.033	366
	software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.070	0.082	−0.016	318
	software classes	directed	1 377	2 213	1.61	1.51	–	0.033	0.012	−0.119	395
	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	−0.154	155
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	−0.366	6, 354
biological	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	−0.240	214
	protein interactions	undirected	2 115	2 240	2.12	6.80	2.4	0.072	0.071	−0.156	212
	marine food web	directed	135	598	4.43	2.05	–	0.16	0.23	−0.263	204
	freshwater food web	directed	92	997	10.84	1.90	–	0.20	0.087	−0.326	272
	neural network	directed	307	2 359	7.68	3.97	–	0.18	0.28	−0.226	416, 421

TABLE II Basic statistics for a number of published networks. The properties measured are: type of graph, directed or undirected; total number of vertices n; total number of edges m; mean degree z; mean vertex–vertex distance ℓ; exponent α of degree distribution if the distribution follows a power law (or "–" if not; in/out-degree exponents are given for directed graphs); clustering coefficient $C^{(1)}$ from Eq. (3); clustering coefficient $C^{(2)}$ from Eq. (6); and degree correlation coefficient r, Sec. III.F. The last column gives the citation(s) for the network in the bibliography. Blank entries indicate unavailable data.

Network Properties

- Average Distance between Pairs (small)
- Transitivity (high)
- Degree Distribution (power law)
- Network Resilience (weak under attack)

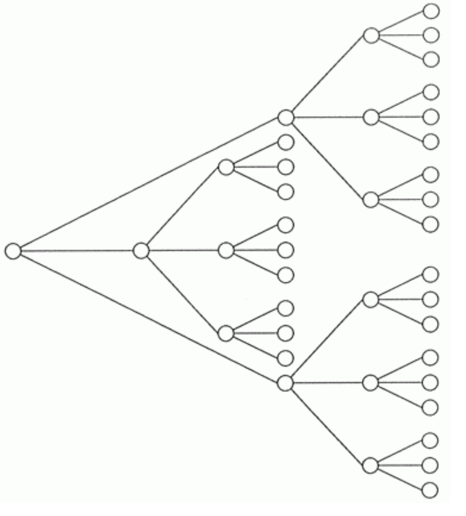
a graph has *k* average degree then

the first neighbours will be *k*

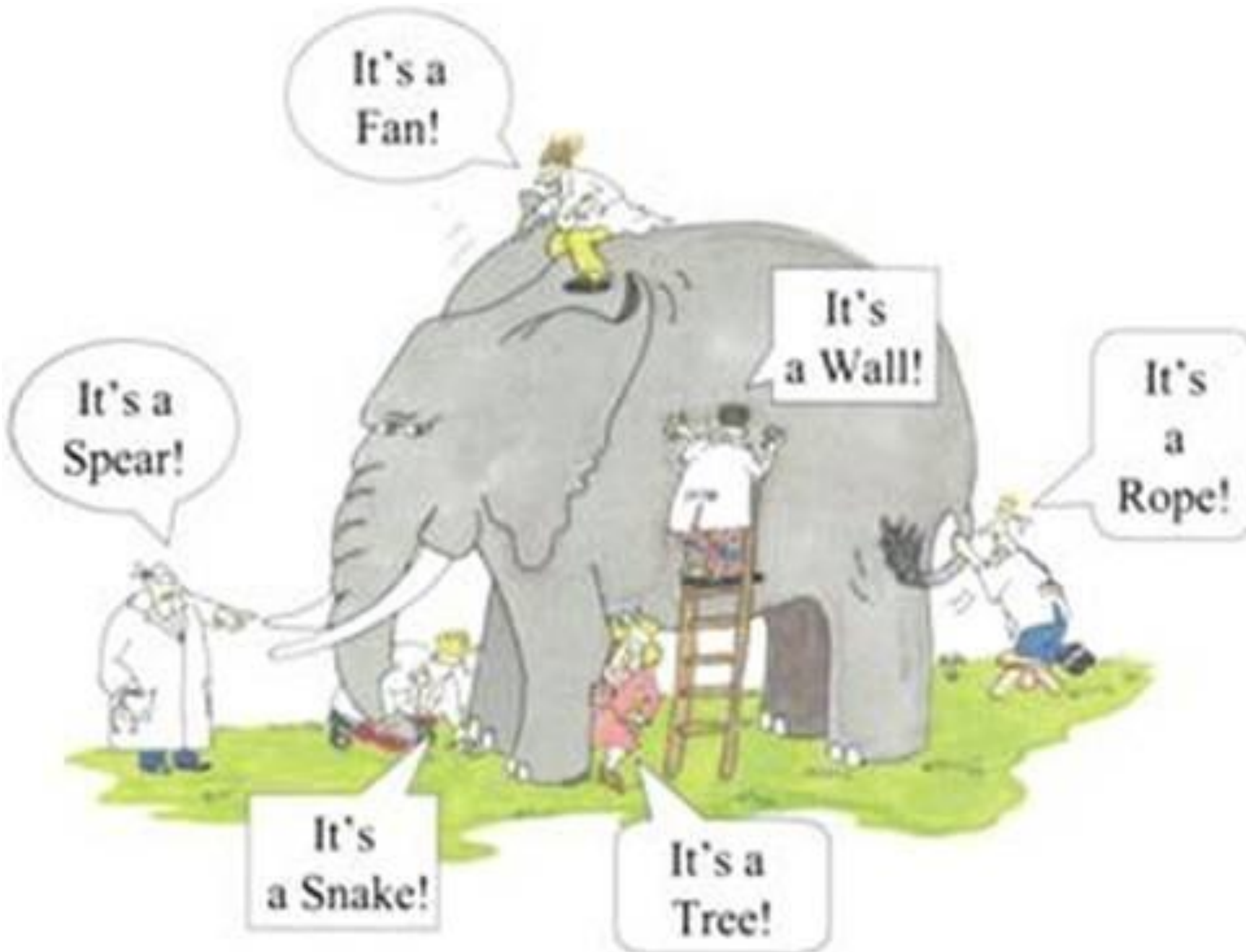
the second neighbours ~ k^2

……………..

the d-th neighbours ~ k^d



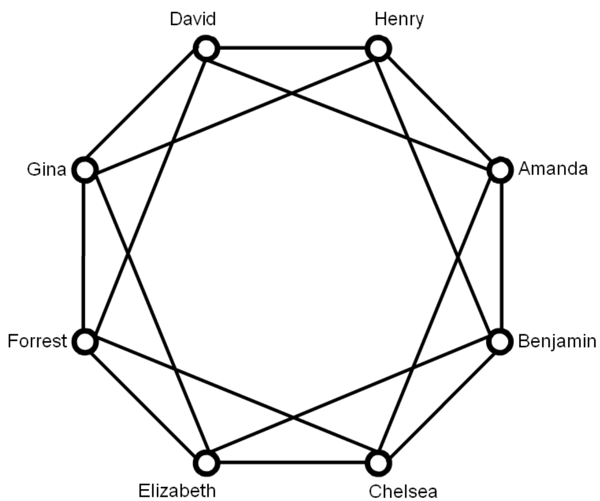
What the structure of a social network look like?

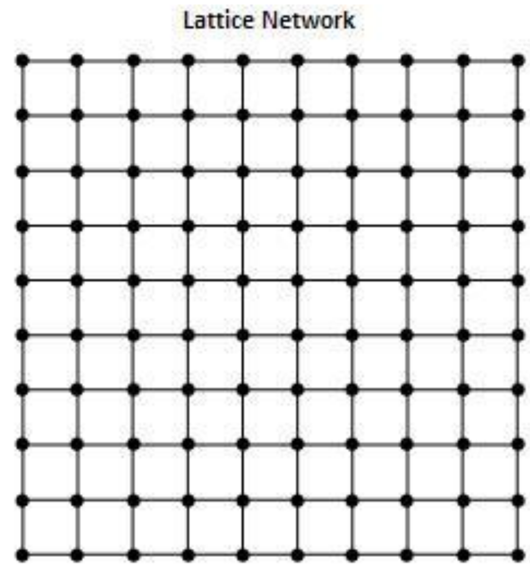


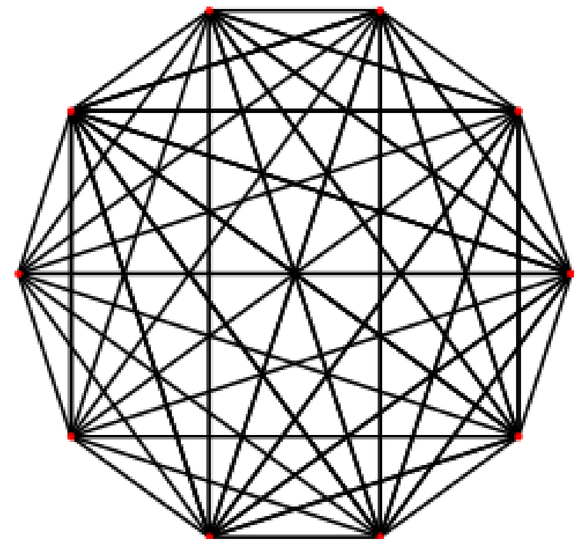
Five Models

- Regular Model
- Random Graph Model
- Small World Model/Watts-Strogatz Model
- Scale Free Model/BA Model
- Geographical Small World Model

Regular Models







$$C_i = \frac{\text{number of triangles connected to vertex } i}{\text{number of triples centered on vertex } i}.$$

Why the regular models are not good ones ?

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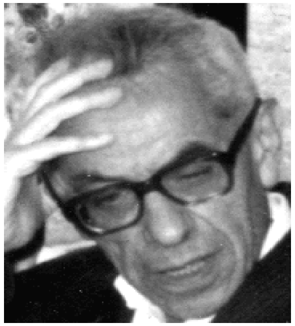
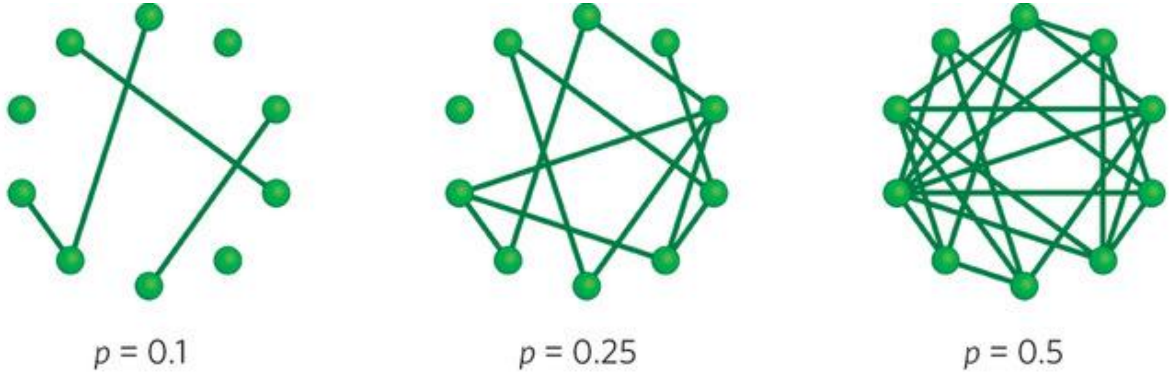
Random Graph Model



Standard Theory of Random Graph
(Erdös and Rényi 1960)

Random Graphs are composed by starting with n vertices. With probability *p* two vertices are connected by an edge



http://ccl.northwestern.edu/netlogo/models/run.cgi?GiantComponent.884.534

Random Graph Model Property

The number m of edges in a Random Graph is a random variable whose expectation value is

$$E(m) = p\,\frac{N(N-1)}{2}$$

The probability to form a particular Graph *G(N,m)* is given by

$$E(G(N,m)) = p^m (1-p)^{\frac{N(N-1)}{2}-m}$$

The degree has expectation value

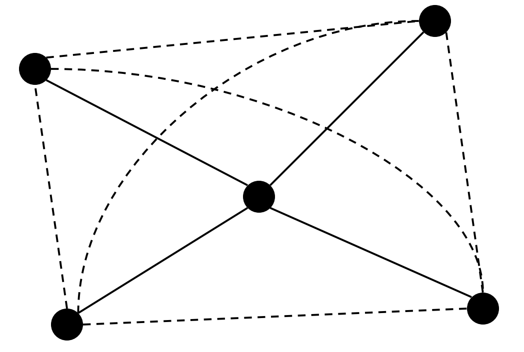
$$E(k) = 2m/N = p(N-1) \cong pN$$

It is easy to check that the degree probability distribution is given by

$$\boxed{P(k) = \binom{N-1}{k} p^k (1-p)^{(N-1)-k} \cong \frac{(pN)^k e^{-pN}}{k!}}$$

Random Graph Model Property

Clustering Coefficient: $E(C) \cong p$



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Average Distance between pairs: $l \leq D \cong \dfrac{\log(N)}{\log(k)}$

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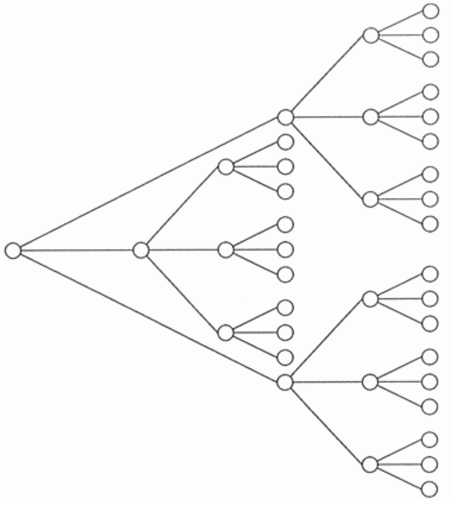

Why the random graph model is inadequate ?

$$E(m) = p\frac{N(N-1)}{2} = C\frac{N(N-1)}{2}$$

$$E(m) = 0.78 \cdot \frac{449913 \cdot 449912}{2} = 78{,}944{,}290{,}485 \approx 79 * 10^9$$

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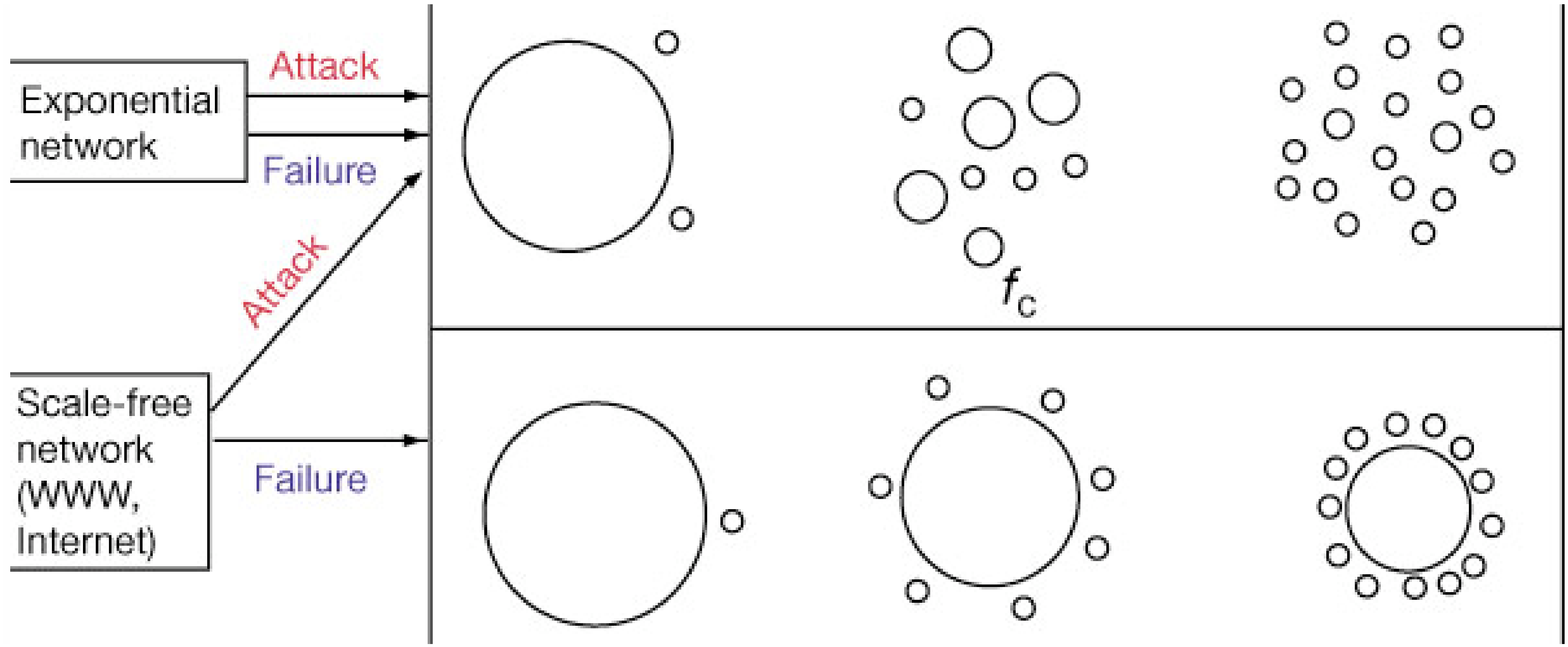
Network Resilience



Source: Error and attack tolerance of complex networks. Réka Albert, Hawoong Jeong and Albert-László Barabási.

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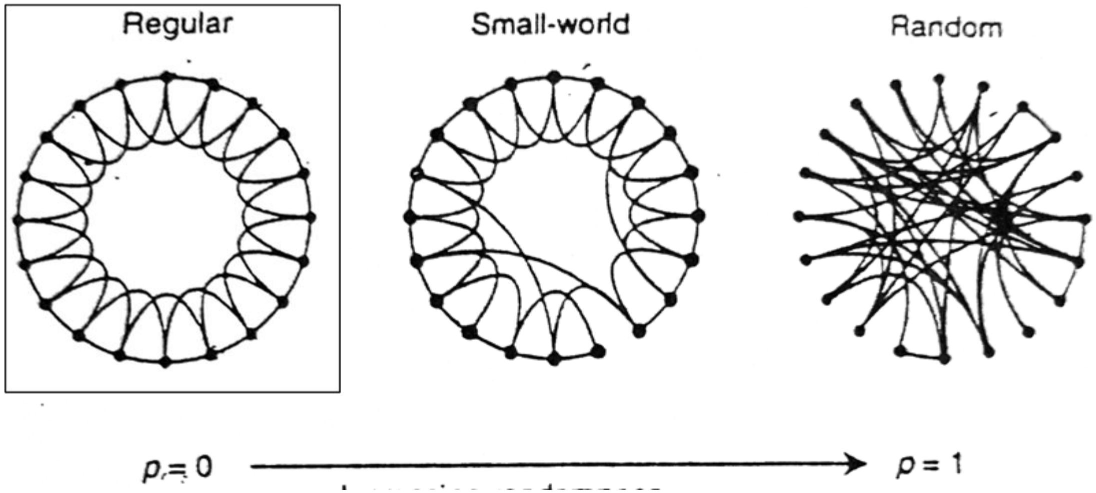
Small World Model

Watts and Strogatz Model (Nature 1998)



The original graph was very clustered: we keep this high clustering.

In the model, we begin with a low-dimension regular graph

For each edge, move one of its ends to another vertex with probablity *p*.

And by creating shortcuts, we decrease the average distance, *i.e.* create a small-world effect.

Watts-Strogatz model:
Generating small world graphs

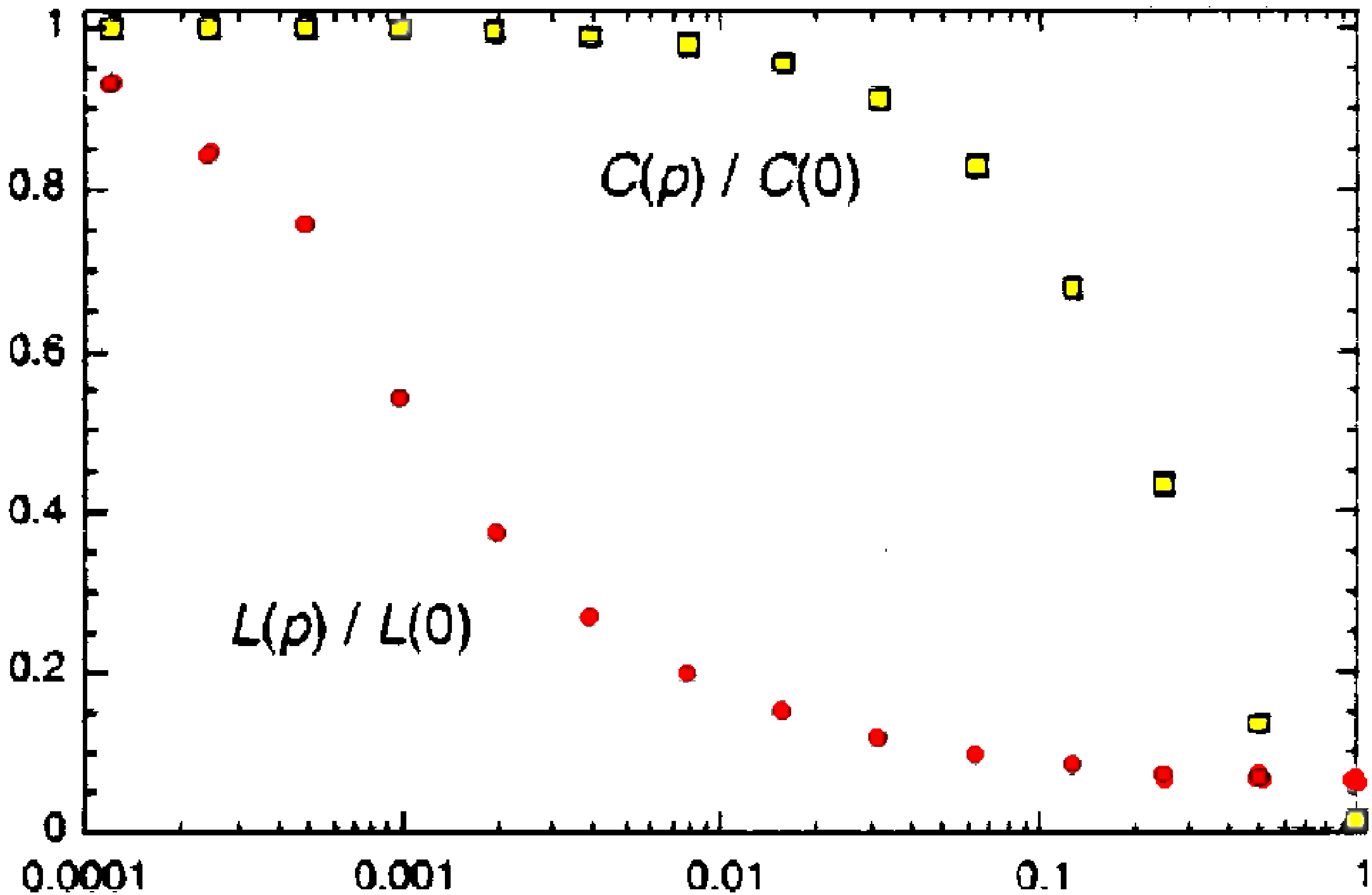
Select a fraction p of edges
Reposition on of their endpoints

Add a fraction p of additional edges leaving underlying lattice intact

- As in many network generating algorithms
  - Disallow self-edges
  - Disallow multiple edges

Source: Watts, D.J., Strogatz, S.H.(1998) Collective dynamics of 'small-world' networks. Nature 393:440-442.

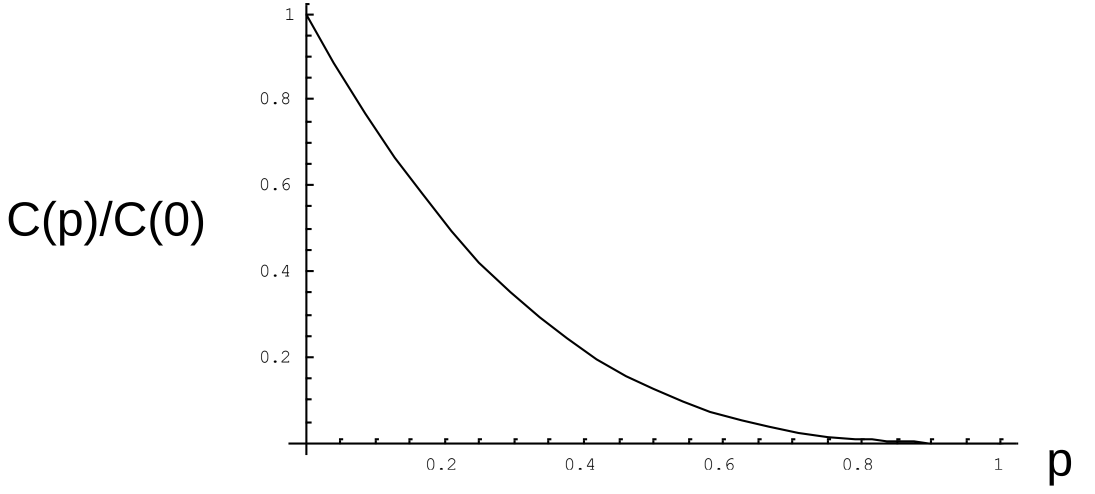
- By varying the probability of rewiring edges



Duncan J. Watts & Steven H. Strogatz, Nature 393, 440-442 (1998)

Watts/Strogatz model: Clustering coefficient

- The probability that a connected triple stays connected after rewiring
  - probability that none of the 4 edges were rewired $(1-p)^4$
- Clustering coefficient = $C(p) = C(p=0)*(1-p)^4$



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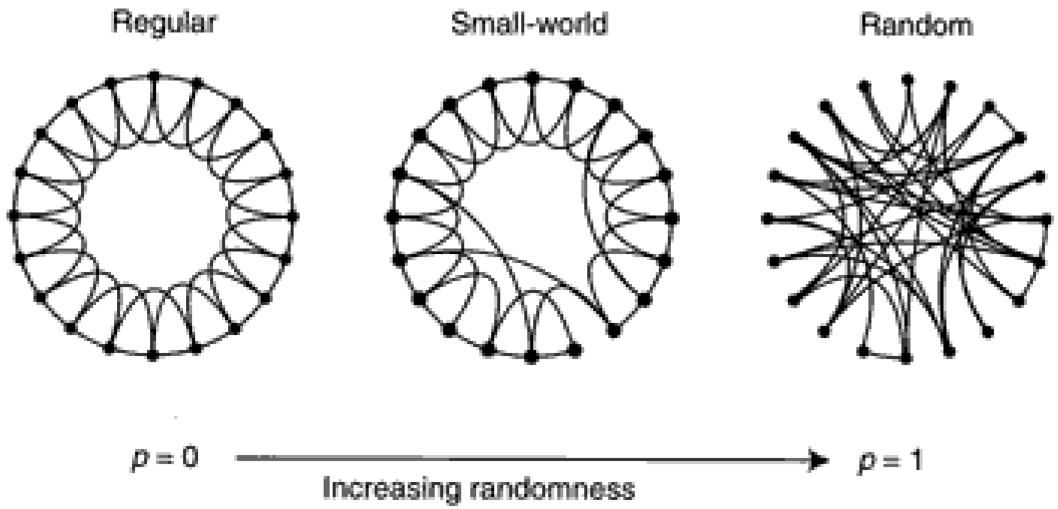
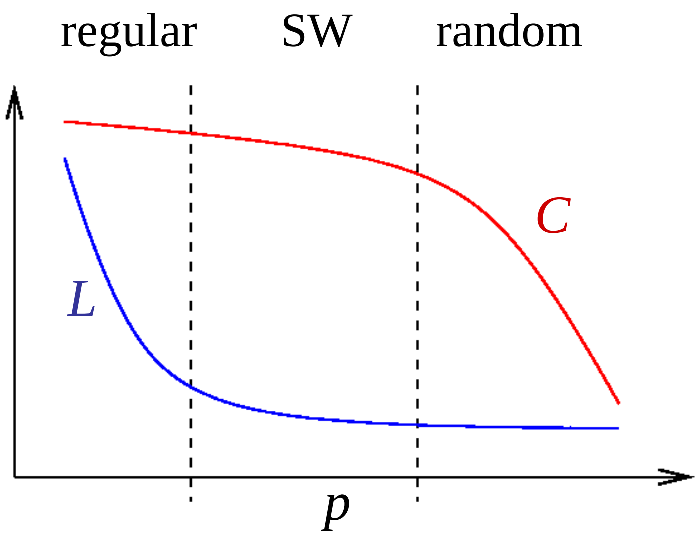
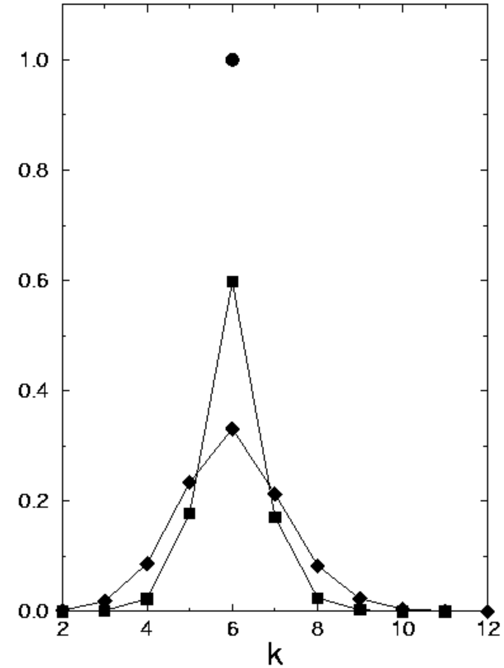
Watts-Strogatz Model

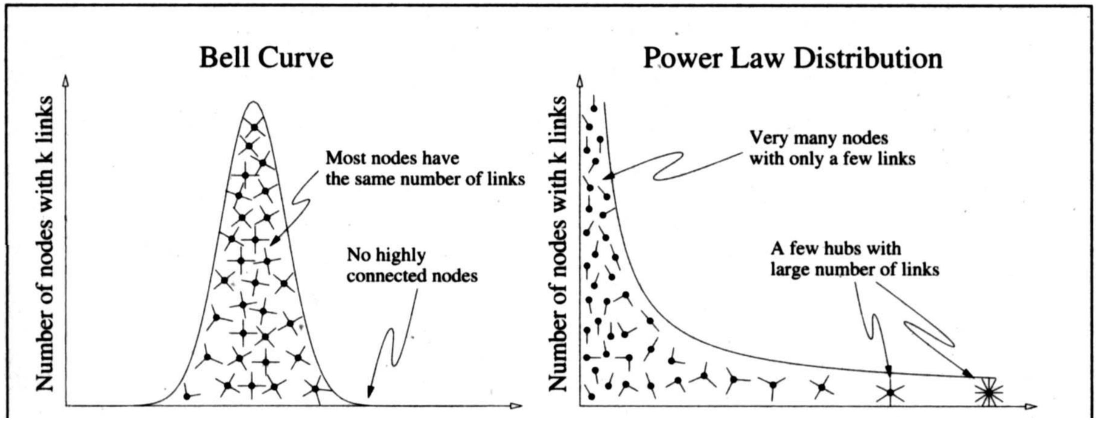




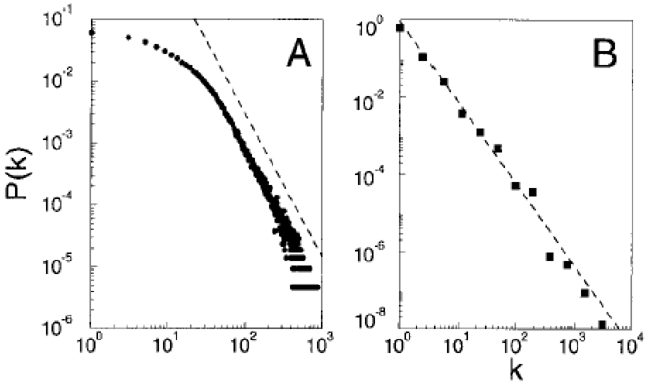


C(p) : clustering coeff.
L(p) : average path length

Did Watts and Strogatz Model Explain All ?



Network	*n*	*k*	*L*	*C*
WWW pages	153127	35.21	3.1	0.1078
Internet AS	6209	4.11	3.76	.3
Math co-authors	70975	3.9	9.5	.59
Power Grid	4941	2.67	18.7	0.08
E-coli reaction	315	28.3	2.62	.59



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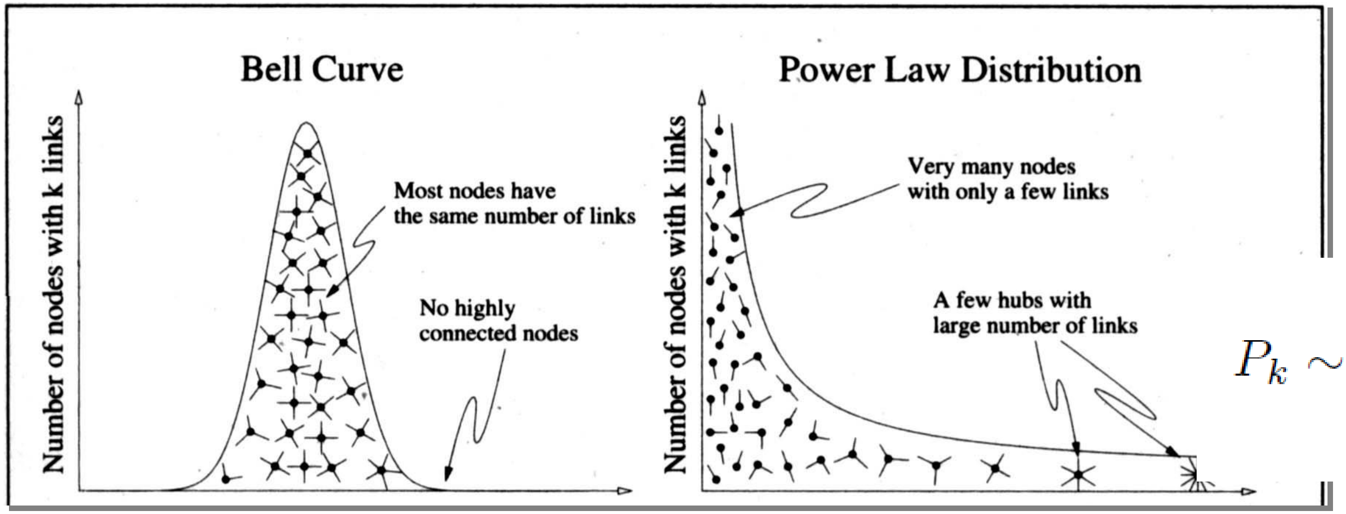
Barabasi-Albert model

Each node connects to other nodes with probability proportional to their degree the process starts with some initial subgraph each node comes with m edges

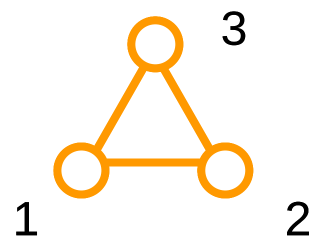
Results in power-law with exponent α = 3



$$P_k \sim \sum_{k'=k}^{\infty} k'^{-\alpha} \sim k^{-(\alpha-1)}.$$

Basic BA-model

- start with an initial set of m_0 fully connected nodes
  - e.g. $m_0 = 3$

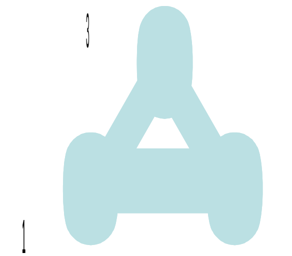


- now add new vertices one by one, each one with exactly *m* edges
- each new edge connects to an existing vertex in proportion to the number of edges that vertex already has → ***preferential attachment***
- easiest if you keep track of edge endpoints in one large array and select an element from this array at random
  - the probability of selecting any one vertex will be proportional to the number of times it appears in the array – which corresponds to its degree

generating BA graphs – cont'd

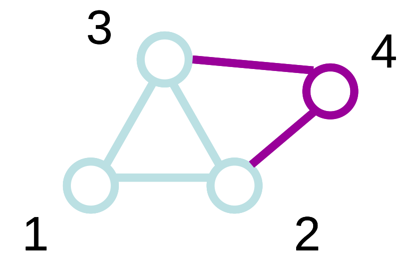
- To start, each vertex has an equal number of edges (2)
  - the probability of choosing any vertex is 1/3

1 1 2 2 3 3



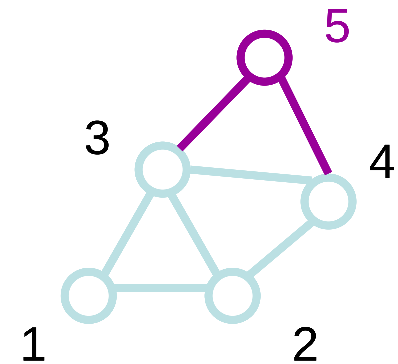
- We add a new vertex, and it will have m edges, here take m=2
  - draw 2 random elements from the array – suppose they are 2 and 3

1 1 2 2 2 3 3 3 4 4



- Now the probabilities of selecting 1,2,3,or 4 are 1/5, 3/10, 3/10, 1/5

1 1 2 2 2 3 3 3 3 4 4 4 5 5



- Add a new vertex, draw a vertex for it to connect from the array
  - etc.

Properties of the BA graph

The distribution is scale free with exponent a = 3

The graph is connected

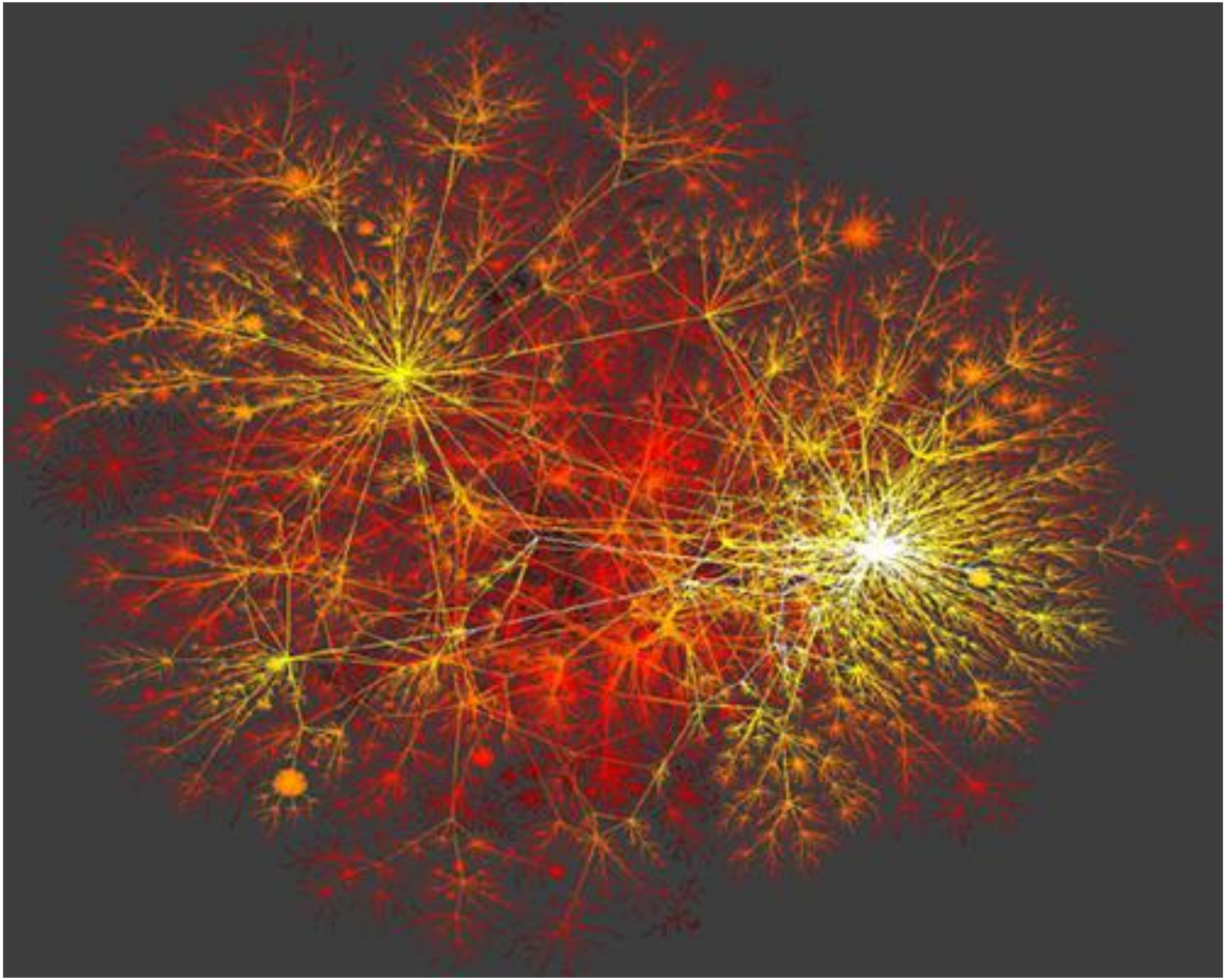
- Every vertex is born with a link (m= 1) or several links (m > 1)
- It connects to older vertices,
  - which are part of the giant component

The older are richer

- Nodes accumulate links as time goes on
- preferential attachment will prefer wealthier nodes,
  - who tend to be older and had a head start

http://netlogoweb.org/launch#http://netlogoweb.org/assets/modelslib/Sample%20Models/Networks/Preferential%20Attachment.nlogo

https://www.youtube.com/watch?v=4GDqJVtPEGg



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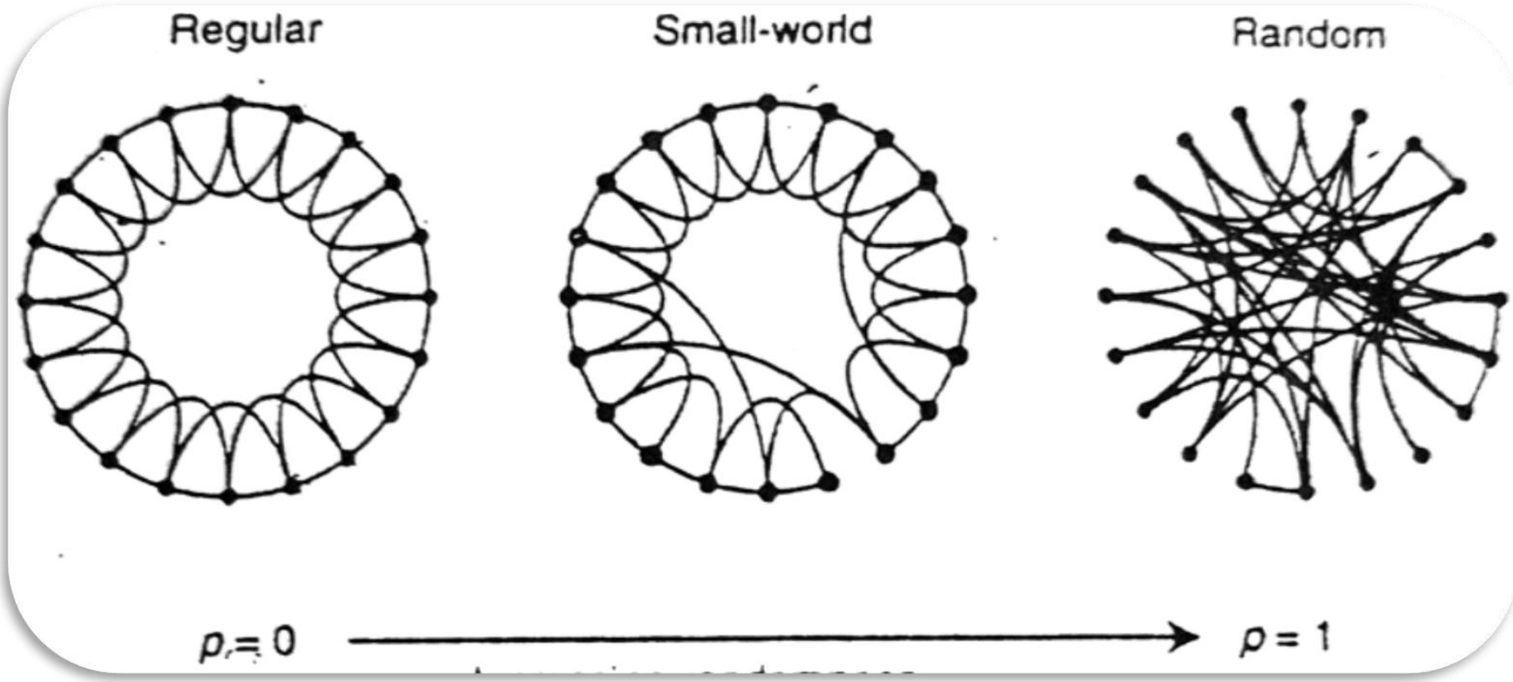
- Network Resilience (weak under attack)

BA Model + Watts/Strogatz Model



Network Properties

- Average Distance between pairs (small)
- Transitivity (high)
- Degree Distribution (power law)
- Network Resilience (weak under attack)

	network	type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r	Ref(s).
social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208	20, 416
	company directors	undirected	7 673	55 392	14.44	4.60	–	0.59	0.88	0.276	105, 323
	math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15	0.34	0.120	107, 182
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45	0.56	0.363	311, 313
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088	0.60	0.127	311, 313
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1				8, 9
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0		0.16		136
	email address books	directed	16 881	57 029	3.38	5.22	–	0.17	0.13	0.092	321
	student relationships	undirected	573	477	1.66	16.01	–	0.005	0.001	−0.029	45
	sexual contacts	undirected	2 810				3.2				265, 266
information	WWW `nd.edu`	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	−0.067	14, 34
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7				74
	citation network	directed	783 339	6 716 198	8.57		3.0/–				351
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	–	0.13	0.15	0.157	244
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7		0.44		119, 157
technological	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	−0.189	86, 148
	power grid	undirected	4 941	6 594	2.67	18.99	–	0.10	0.080	−0.003	416
	train routes	undirected	587	19 603	66.79	2.16	–		0.69	−0.033	366
	software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.070	0.082	−0.016	318
	software classes	directed	1 377	2 213	1.61	1.51	–	0.033	0.012	−0.119	395
	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	−0.154	155
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	−0.366	6, 354
biological	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	−0.240	214
	protein interactions	undirected	2 115	2 240	2.12	6.80	2.4	0.072	0.071	−0.156	212
	marine food web	directed	135	598	4.43	2.05	–	0.16	0.23	−0.263	204
	freshwater food web	directed	92	997	10.84	1.90	–	0.20	0.087	−0.326	272
	neural network	directed	307	2 359	7.68	3.97	–	0.18	0.28	−0.226	416, 421

TABLE II Basic statistics for a number of published networks. The properties measured are: type of graph, directed or undirected; total number of vertices n; total number of edges m; mean degree z; mean vertex–vertex distance ℓ; exponent α of degree distribution if the distribution follows a power law (or "–" if not; in/out-degree exponents are given for directed graphs); clustering coefficient $C^{(1)}$ from Eq. (3); clustering coefficient $C^{(2)}$ from Eq. (6); and degree correlation coefficient r, Sec. III.F. The last column gives the citation(s) for the network in the bibliography. Blank entries indicate unavailable data.

Five Models

- Regular Model
- Random Graph Model
- Small World Model/Watts-Strogatz Model
- Scale Free Model/BA Model
- Geographical Small World Model

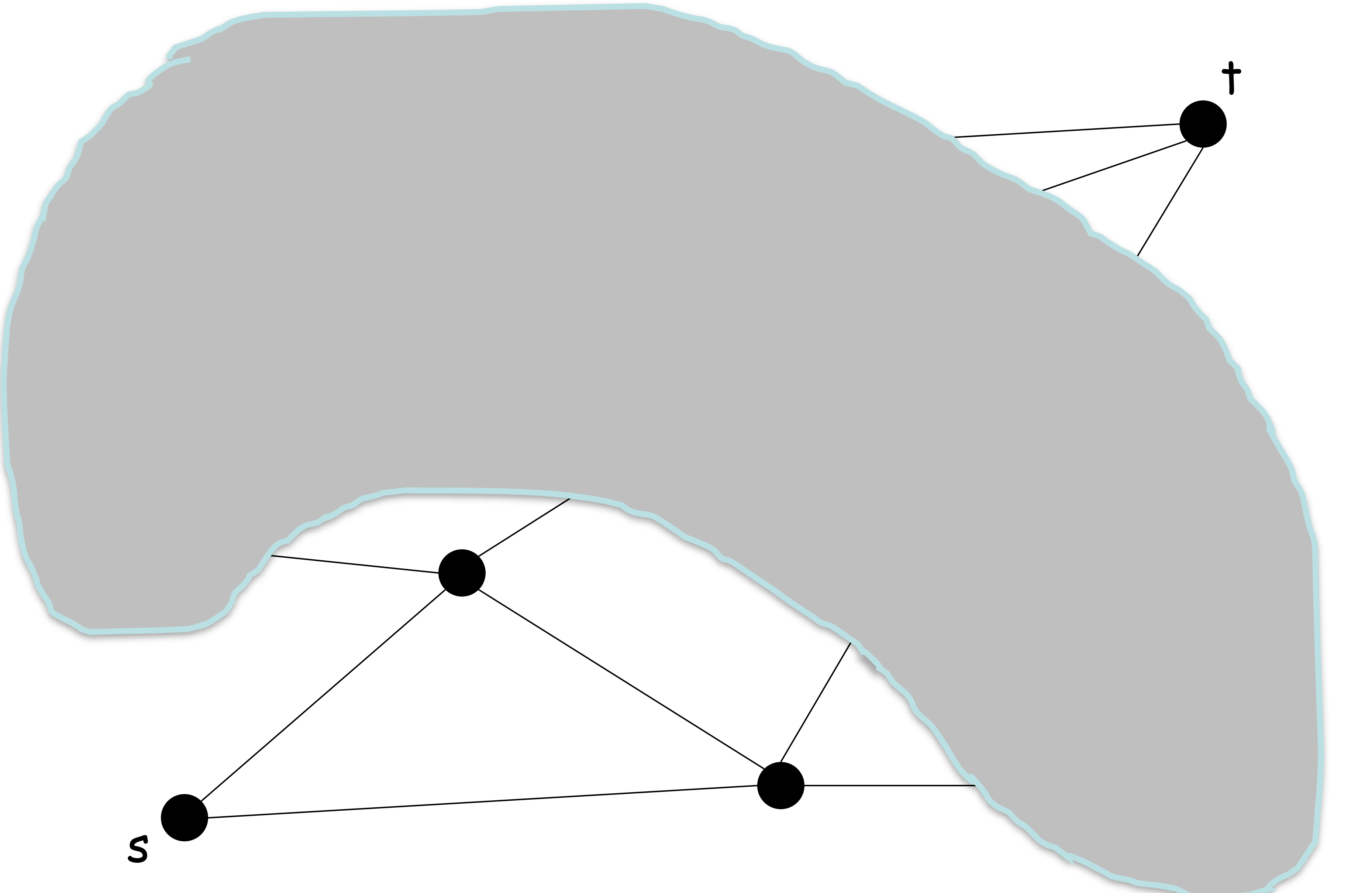
Milgram’s experiment revisited

- What did Milgram’s experiment show?
  - (a) There are short paths in large networks that connect individuals
  - (b) People are able to find these short paths using a simple, greedy, decentralized algorithm
- Small world models take care of (a)

The Algorithmic Side

- Input:
  - Grid G = (V,E)
  - arbitrary nodes *s, t*

- Goal: Transmit a message from *s* to *t* in as few steps as possible using only locally available information



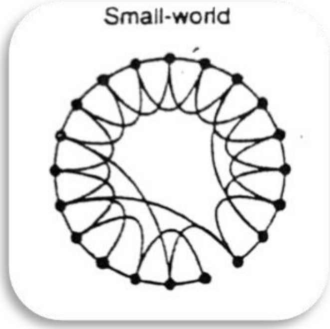
Best Effort Routing

Navigation in a small world network

- Kleinberg (2000)
  - Why should arbitrary pairs of strangers, using only locally available information, be able to ***find*** short chains of acquaintances that link them together?
  - Does this occur in all small-world networks, or are there properties that must exist for this to happen?

The Algorithmic Side

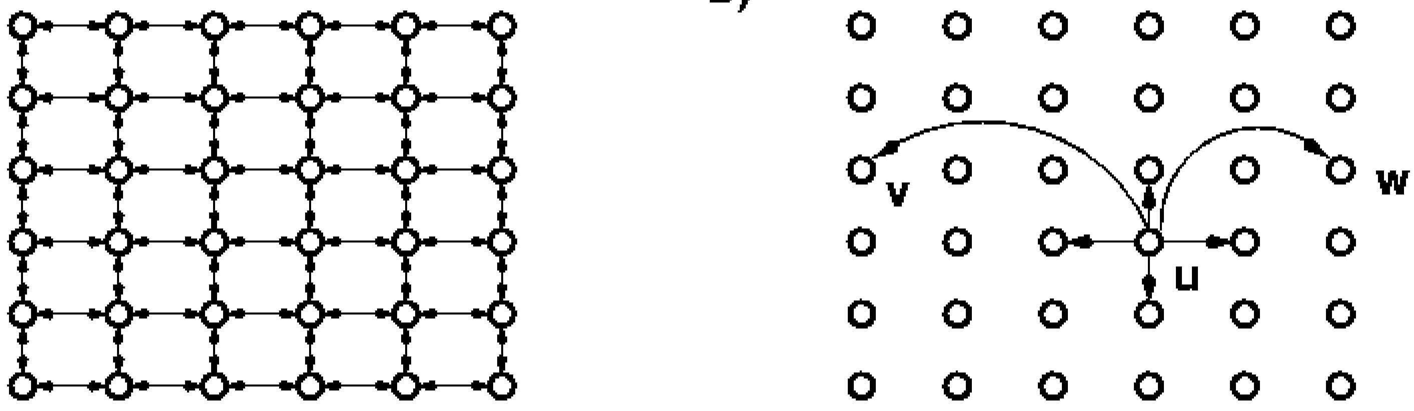
- Assumptions:
  - In any step, the message holder *u* knows
    - The range of <u>local</u> contacts of all nodes
    - The location on the lattice of the target *t*
    - The locations and long-range contacts of all nodes that have previously touched the message
  - *u* does not know
    - the long-range contacts of nodes that have not touched the message



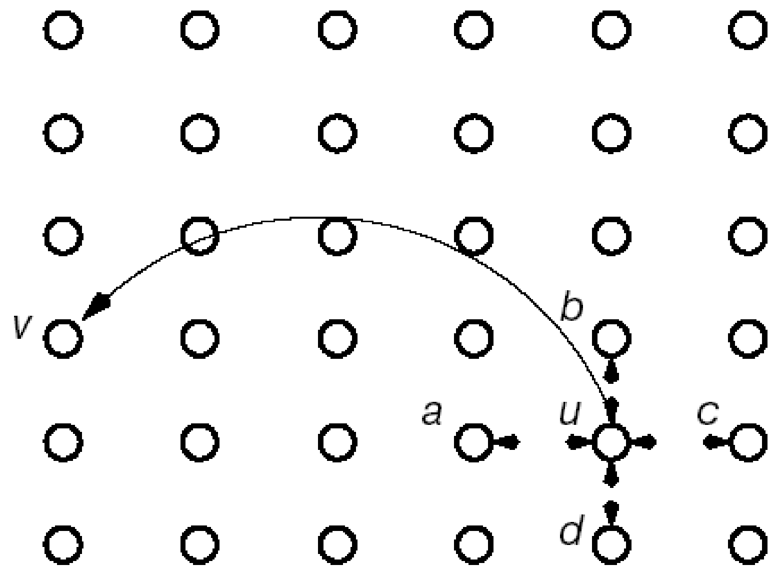
Kleinberg's model

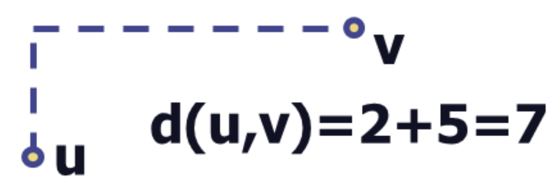
- Consider a directed 2-dimensional lattice
- For each vertex u add q shortcuts
  - choose vertex v as the destination of the shortcut with probability proportional to $[d(u,v)]^{-r}$
  - when $r = 0$, we have uniform probabilities

A) B)



Kleinberg's geographical small world model





nodes are placed on a lattice and connect to nearest neighbors

exponent that will determine navigability

additional links placed with

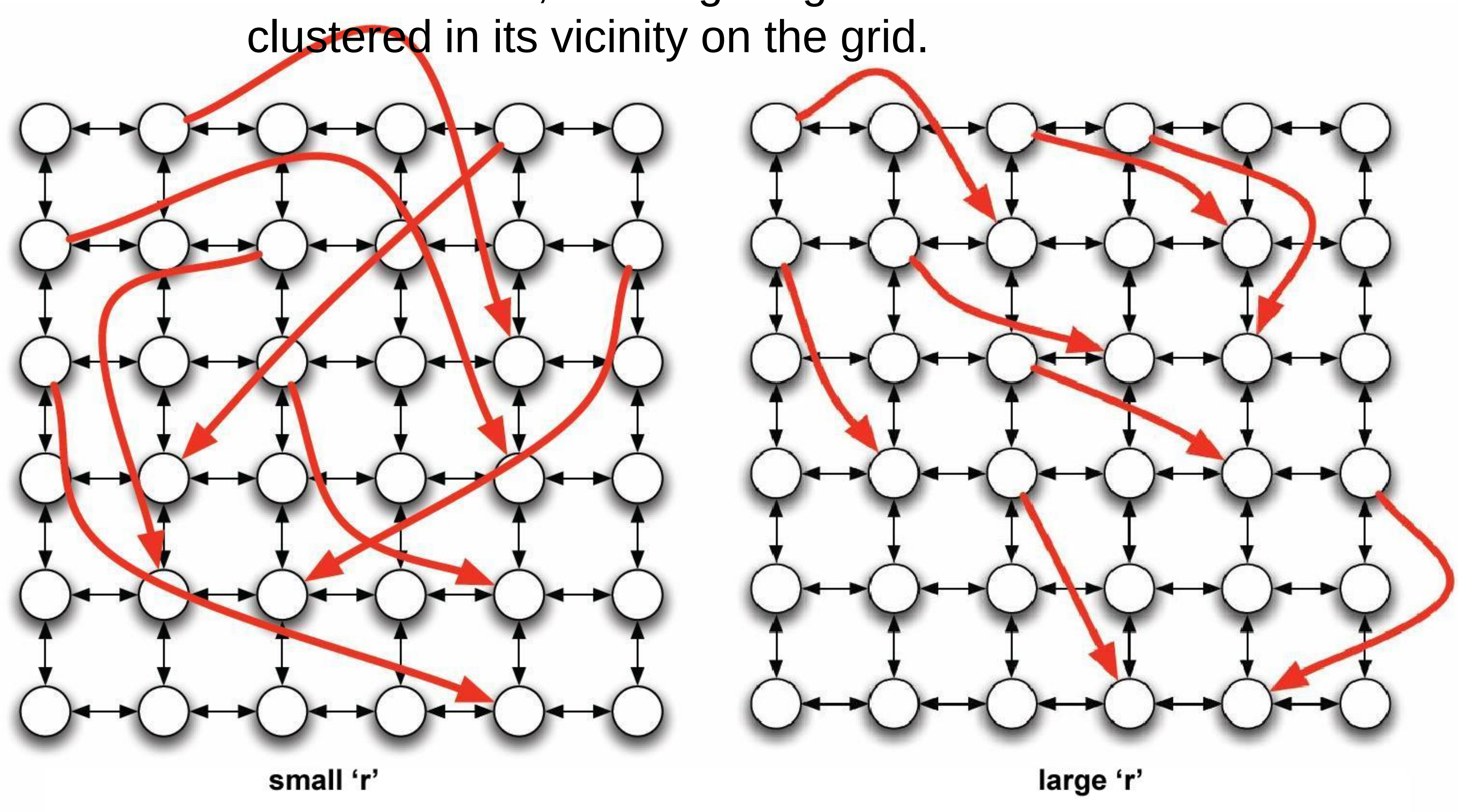
$$p(\text{link between u and v}) = (\text{distance}(u,v))^{-r}$$

Source: Kleinberg, 'Navigation in a small world'

Navigation in a small world network

Infinite family of networks:

- r = 0: each node's long-range contacts are chosen independently of its position on the grid
- As r increases, the long range contacts of a node become clustered in its vicinity on the grid.



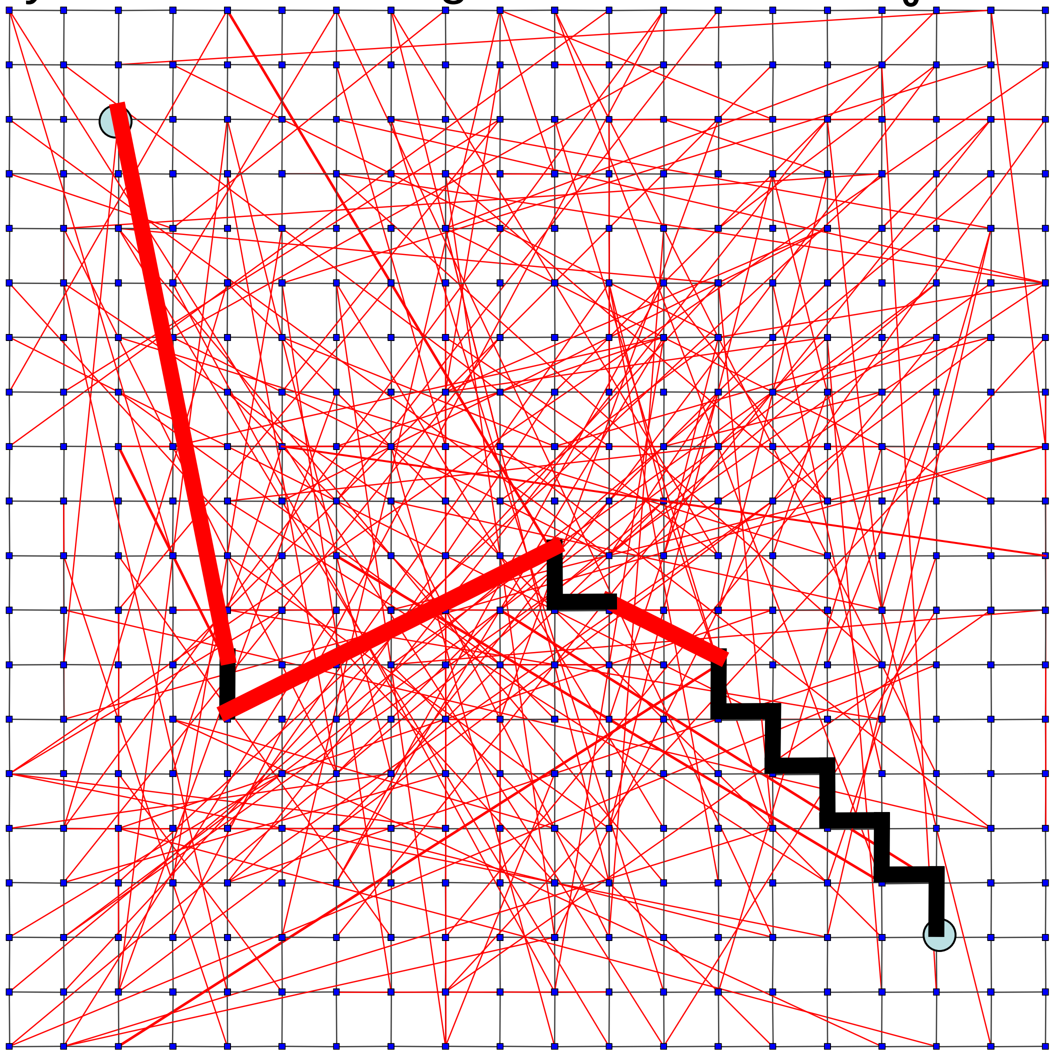
Searching in a small world

- Kleinberg proved the following
  - When $r=2$, an algorithm that uses only local information at each node can reach the destination in expected time $O(\log^2 n)$.
  - When $r<2$ a local greedy algorithm needs expected time$\Omega(n^{(2-r)/3})$.
  - When $r>2$ a local greedy algorithm needs expected time $\Omega(n^{(r-2)/(r-1)})$.
  - Generalizes for a d-dimensional lattice, when $r=d$

geographical search when network lacks locality

When **r=2**, links are randomly distributed, ASP ~ **log(n)**, n size of grid

When **r<2**, any decentralized algorithm is at least $\mathbf{a_0 n^{2/3}}$

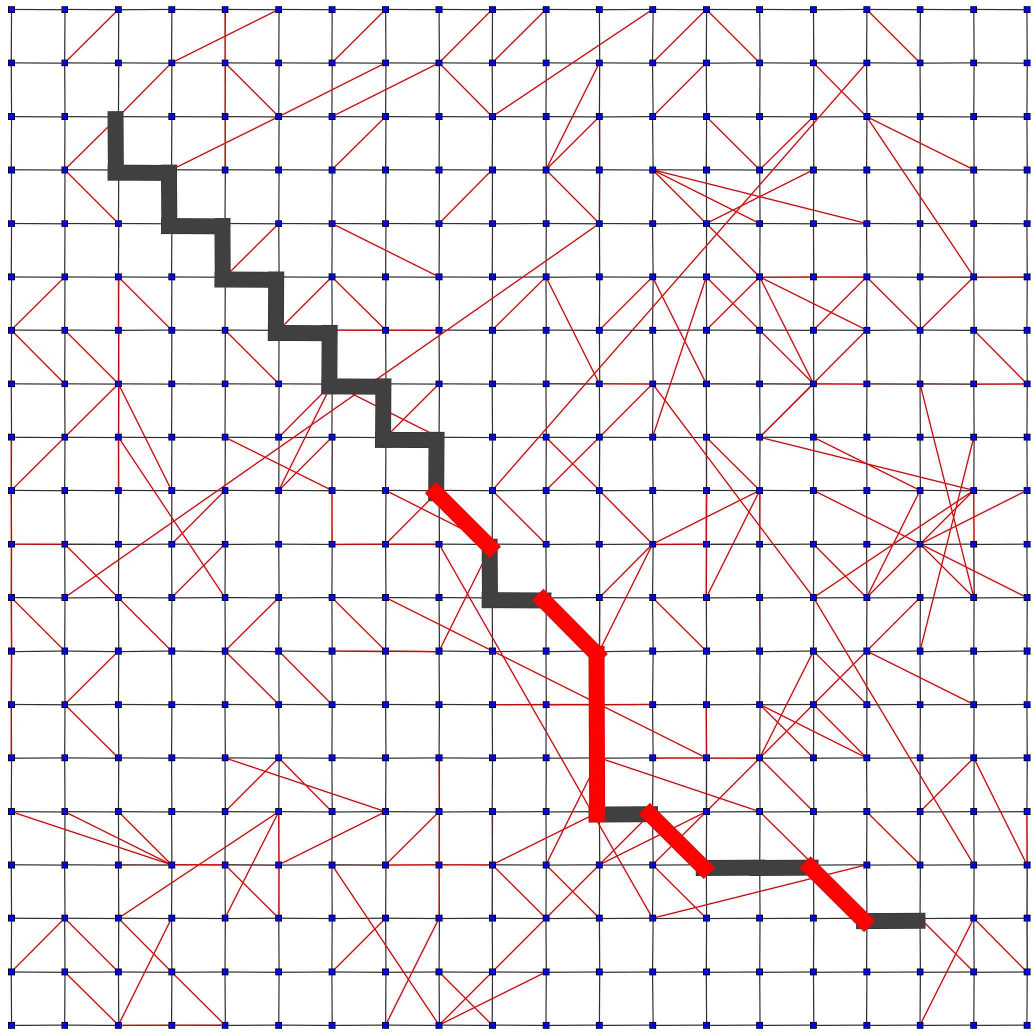


When **r<2**, expected time at least $\alpha_r n^{(2-r)/3}$

Overly localized links on a lattice

When **r>2** expected search time ~ $\mathbf{N}^{\mathbf{(r-2)/(r-1)}}$

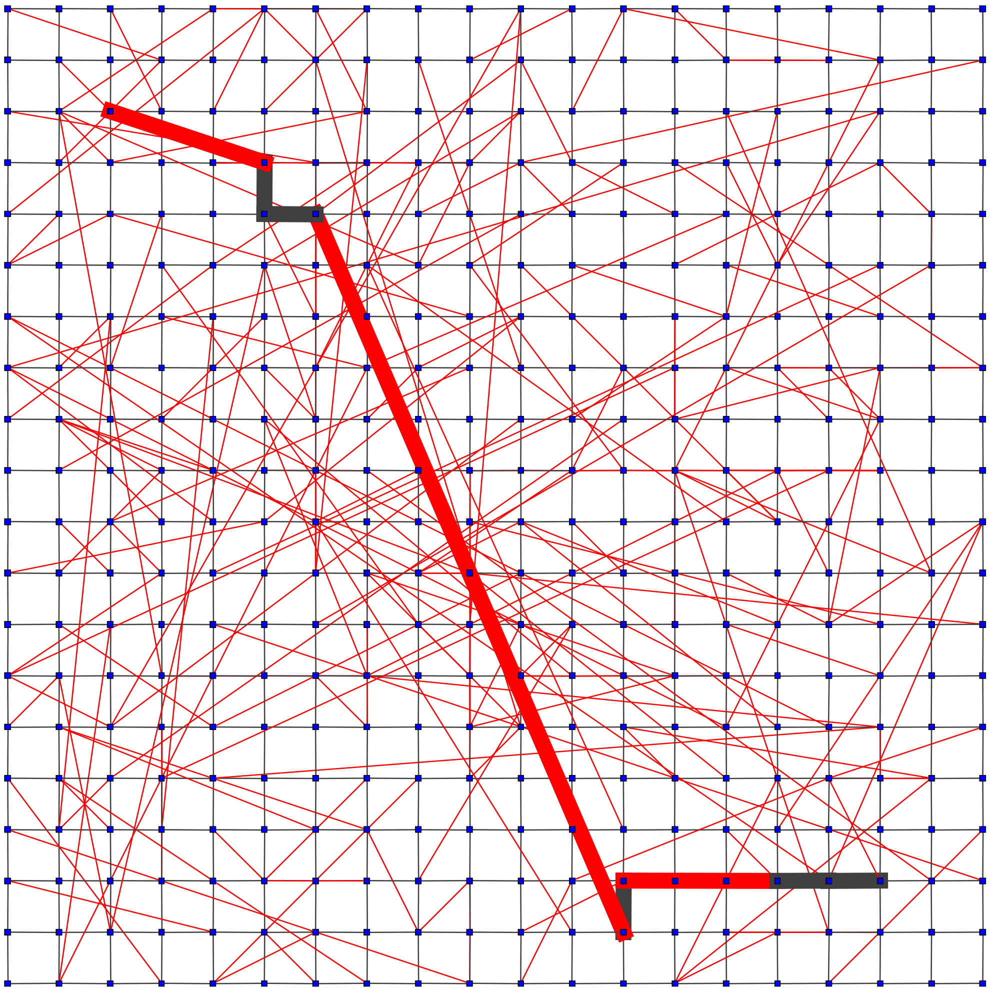
$$p \sim \frac{1}{d^4}$$



geographical small world model
Links balanced between long and short range

When **r=2,** expected time of a DA is at most **C (log N)^2**

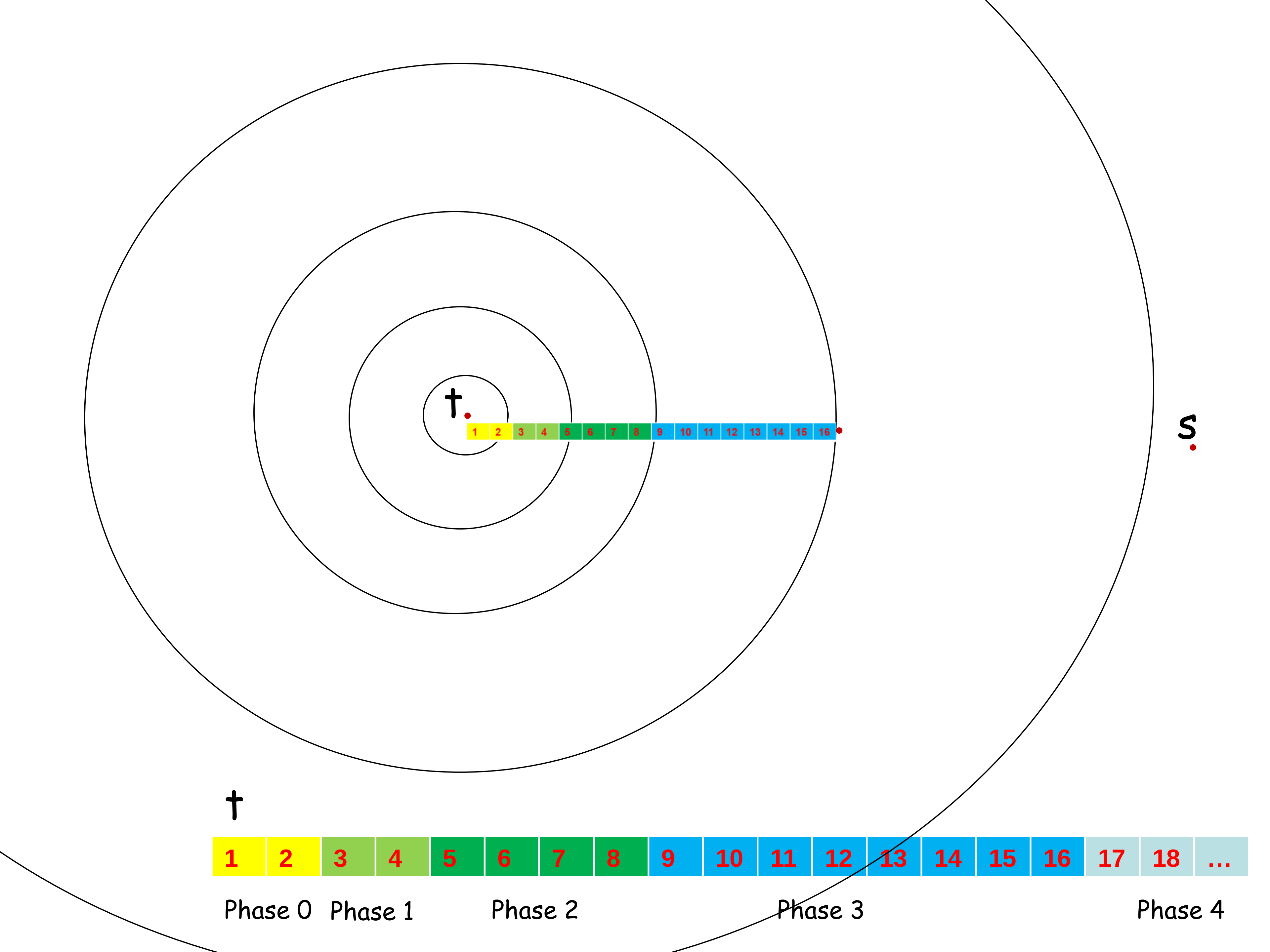
$$p \sim \frac{1}{d^2}$$



$$R = 2$$

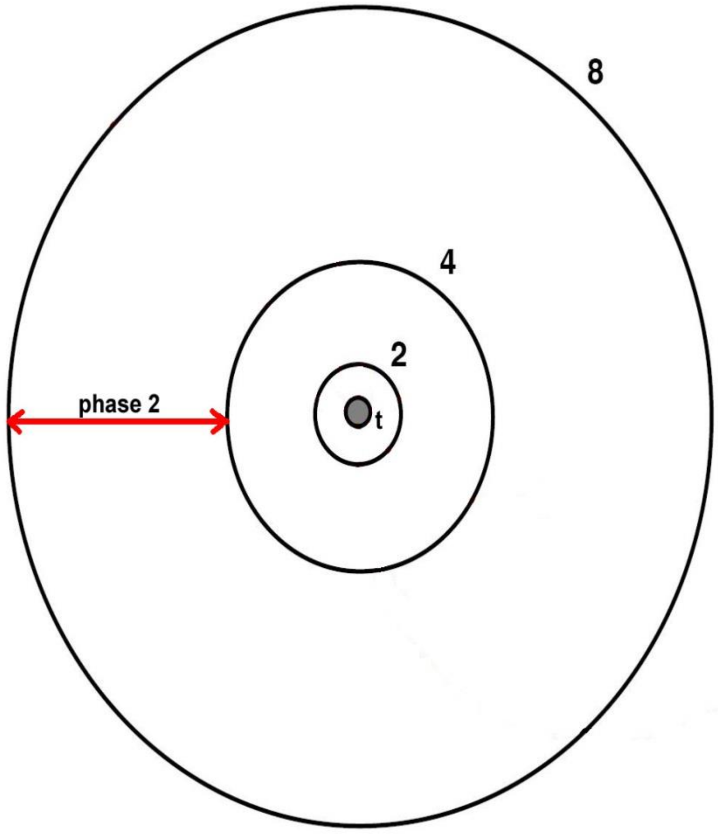
The Algorithm

- In each step the current message holder passes the message to the contact that is as close to the target as possible.


t
s
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
t
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 …
Phase 0
Phase 1
Phase 2
Phase 3
Phase 4

Analysis

- Algorithm in phase j:
  - At a given step,
    $2^j < d(u,t) \leq 2^{j+1}$
  - Alg. is in phase 0 :
    - message is no more than 2 lattice steps away from the target t.
  - $j \leq \log_2 n$.



Analysis

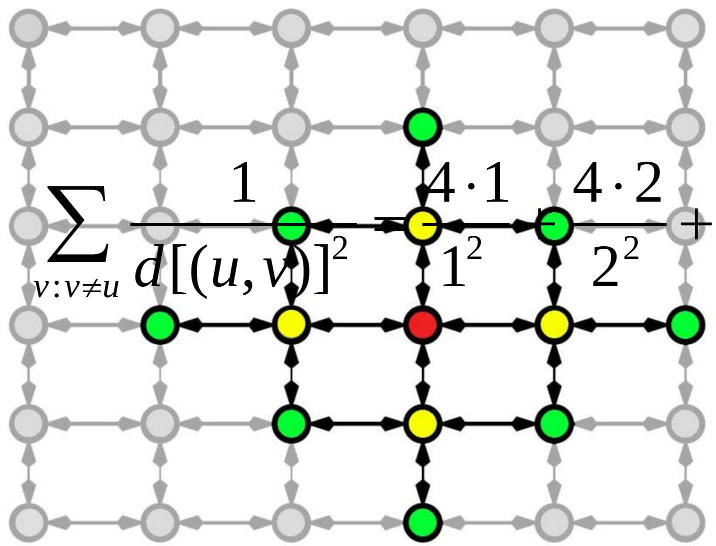
Questions:

- How many steps will the algorithm take?
- How many steps will we spend in phase j?
- In a given step, with what probability will phase j end in this step?
- What is the probability that node u has a node v in the next phase as its long range contact?

Analysis

- Pr [*u* has *v* as its long range contact] ?

$$= \frac{[d(u,v)]^{-2}}{\sum [d(u,v)]^{-2}}$$



$$\sum_{v:v\neq u} \frac{1}{d[(u,v)]^2} = \frac{4\cdot 1}{1^2} + \frac{4\cdot 2}{2^2} + \frac{4\cdot 3}{3^2} \ldots \le \sum_{j=1}^{2n-2} \frac{4j}{j^2}$$

Questions:
- How many steps will the algorithm take?
- How many steps will we spend in phase *j*?
- In a given step, with what probability will phase *j* end in this step?
- What is the probability that node *u* has a node *v* as its long range contact?

Analysis

- Pr[*u* has *v* as its long range contact]?

$$\sum_{v:v\neq u}[d(u,v)]^{-2} \leq \sum_{j=1}^{2n-2}\frac{4j}{j^2} = 4\sum_{j=1}^{2n-2} 1/j \leq 4[1+\ln(2n-2)] \leq 4\ln(6n)$$

$$\geq \frac{[d(u,v)]^{-2}}{4\ln(6n)}$$

- Thus *u* has *v* as its long-range contact with probability

$$\geq \frac{1}{4\ln(6n)\cdot[d(u,v)]^2}$$

Questions:

- How many steps will the algorithm take?
- How many steps will we spend in phase *j*?
- In a given step, with what probability will phase *j* end in this step?
- What is the probability that node *u* has a node *v* as its long range contact?

Analysis

Questions:

- How many steps will the algorithm take?
- How many steps will we spend in phase *j*?
- In a given step, with what probability will phase *j* end in this step?
- What is the probability that node *u* has a node *v* as its long range contact?

$$\geq \frac{1}{4\ln(6n)\cdot[d(u,v)]^2}$$

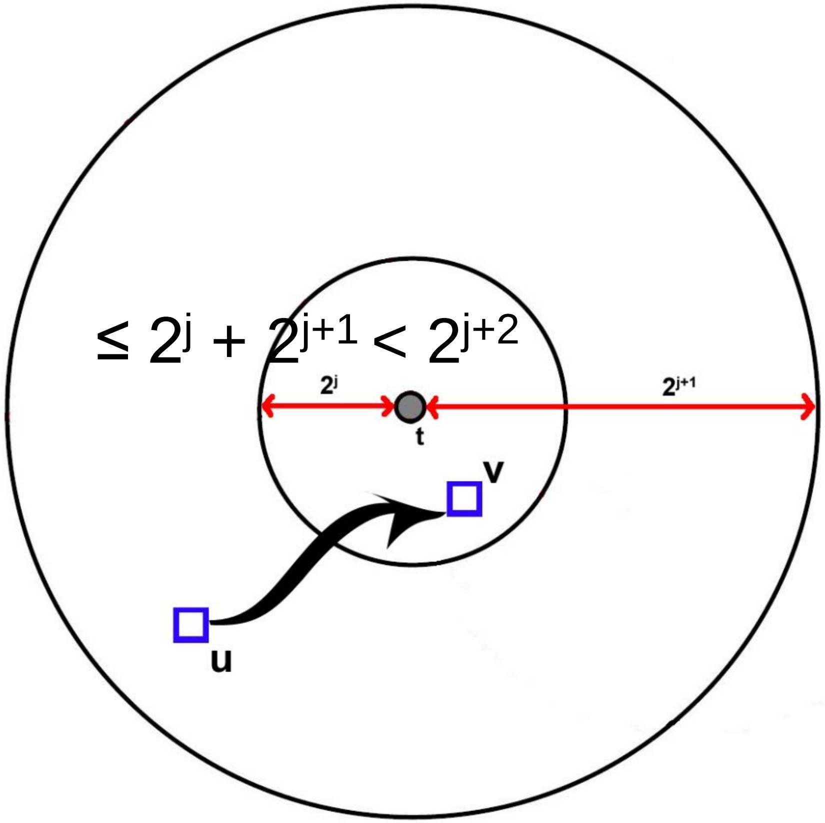
- In any given step, Pr[phase j ends in this step]?
  - Phase j ends in this step if the message enters the set B_j of nodes within distance 2^j of t. Let v_f be the node in B_j that is farthest from *u*.

$$\Pr[\text{phase } j \text{ ends in this step}] = \sum_{v \in B_j} \Pr\left[u \text{ is friends with } v \in B_j\right]$$

$$\geq |B_j| \cdot \left(\frac{1}{4\ln(6n)\cdot[d(u,v_f)]^2}\right)$$

Analysis

- $\Pr[\text{phase } j \text{ ends in this step}] \geq |B_j| \cdot \left(\frac{1}{4\ln(6n)\cdot[d(u,v_f)]^2} \right)$
  - What is d[(u,v_f)]?



Questions:

- How many steps will the algorithm take?
- How many steps will we spend in phase j?
- In a given step, with what probability will phase j end in this step?
- What is the probability that node u has a node v as its long range contact?

$$\geq \frac{1}{4\ln(6n)\cdot[d(u,v)]^2}$$

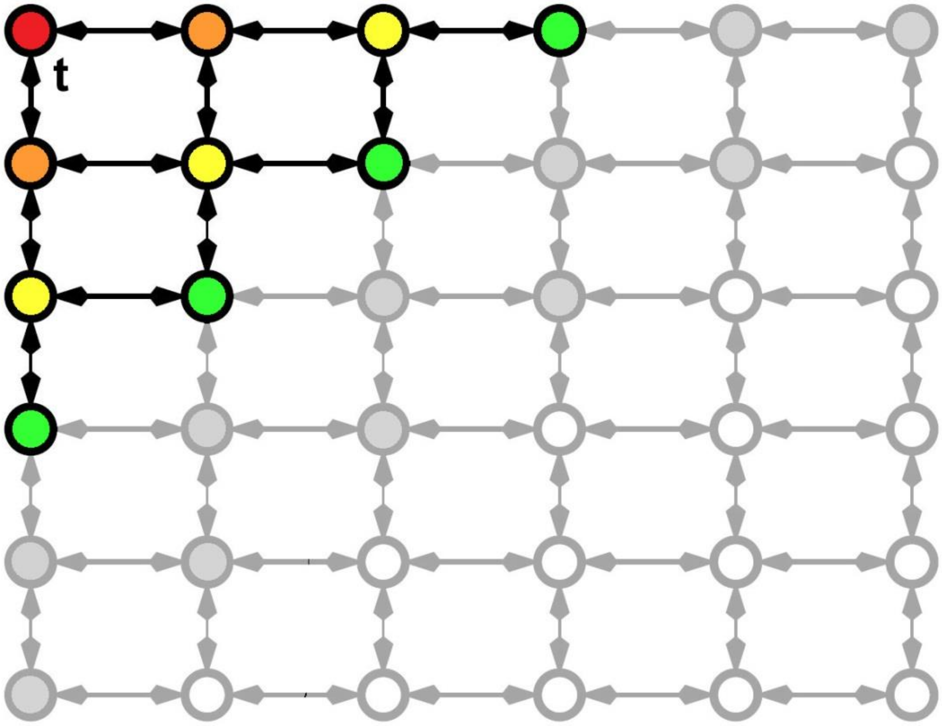
Analysis

- $\Pr[\text{phase } j \text{ ends in this step}] \geq |B_j| \cdot \left(\frac{1}{4\ln(6n) \cdot 2^{2j+4}} \right)$
- How many nodes are in B_j?

Questions:

- How many steps will the algorithm take?
- How many steps will we spend in phase *j*?
- In a given step, with what probability will phase *j* end in this step?
- What is the probability that node *u* has a node *v* as its long range contact?

$$\geq \frac{1}{4\ln(6n) \cdot [d(u,v)]^2}$$



Analysis

Questions:

- How many steps will the algorithm take?
- How many steps will we spend in phase *j*?
- In a given step, with what probability will phase *j* end in this step?
- What is the probability that node *u* has a node *v* as its long range contact?

$$\geq \frac{1}{4\ln(6n)\cdot[d(u,v)]^2}$$

- In any given step, Pr[phase j ends in this step]?
  - Pr[*u* has a long-range contact in B_j]?

$$\geq \#\ of\ nodes\ in\ B_j \cdot (probability\ u\ is\ friends\ with\ farthest\ v \in B_j)$$

$$\geq 2^{2j-1}\left(\frac{1}{4\ln(6n)\cdot 2^{2j+4}}\right) = \frac{2^{2j-1}}{4\ln(6n)\cdot 2^{2j+4}} = \frac{1}{128\ln(6n)}$$

Analysis

Questions:

- How many steps will the algorithm take?
- How many steps will we spend in phase *j*?
- In a given step, with what probability will phase *j* end in this step?

$$\geq \frac{1}{128 \ln(6n)}$$

- What is the probability that node *u* has a node *v* as its long range contact?

$$\geq \frac{1}{4 \ln(6n) \cdot [d(u,v)]^2}$$

- How many steps will we spend in phase *j*?
  - Let X_j be a random variable denoting the number of steps spent in phase *j*.
    - X_j is a random variable with a probability of success at least

Analysis

Questions:

- How many steps will the algorithm take?
- How many steps will we spend in phase *j*?
- In a given step, with what probability will phase *j* end in this step?

$$\geq \frac{1}{128\ln(6n)}$$

- What is the probability that node *u* has a node *v* as its long range contact?

$$\geq \frac{1}{4\ln(6n)\cdot[d(u,v)]^2}$$

- How many steps will we spend in phase *j*?
  - Since X_j is a geometric random variable, we know that

$$E[X_j] = \frac{1}{p} \leq \frac{1}{1/128\ln(6n)} = 128\ln(6n)$$

Analysis

Questions:

- How many steps will the algorithm take?
- How many steps will we spend in phase *j*?

$$\leq 128 \ln(6n)$$

- In a given step, with what probability will phase *j* end in this step?

$$\geq \frac{1}{128 \ln(6n)}$$

- What is the probability that node *u* has a node *v* as its long range contact?

$$\geq \frac{1}{4 \ln(6n) \cdot [d(u,v)]^2}$$

- How many steps does the algorithm take?
  - Let X be a random variable denoting the number of steps taken by the algorithm.
  - By Linearity of Expectation we have

$$E[X] \leq (1 + \log n)(128 \ln(6n)) = O(\log n)^2$$

Analysis

Questions:

- How many steps will the algorithm take?
- How many steps will we spend in phase *j*?

$$\leq 128\ln(6n)$$

- In a given step, with what probability will phase *j* end in this step?

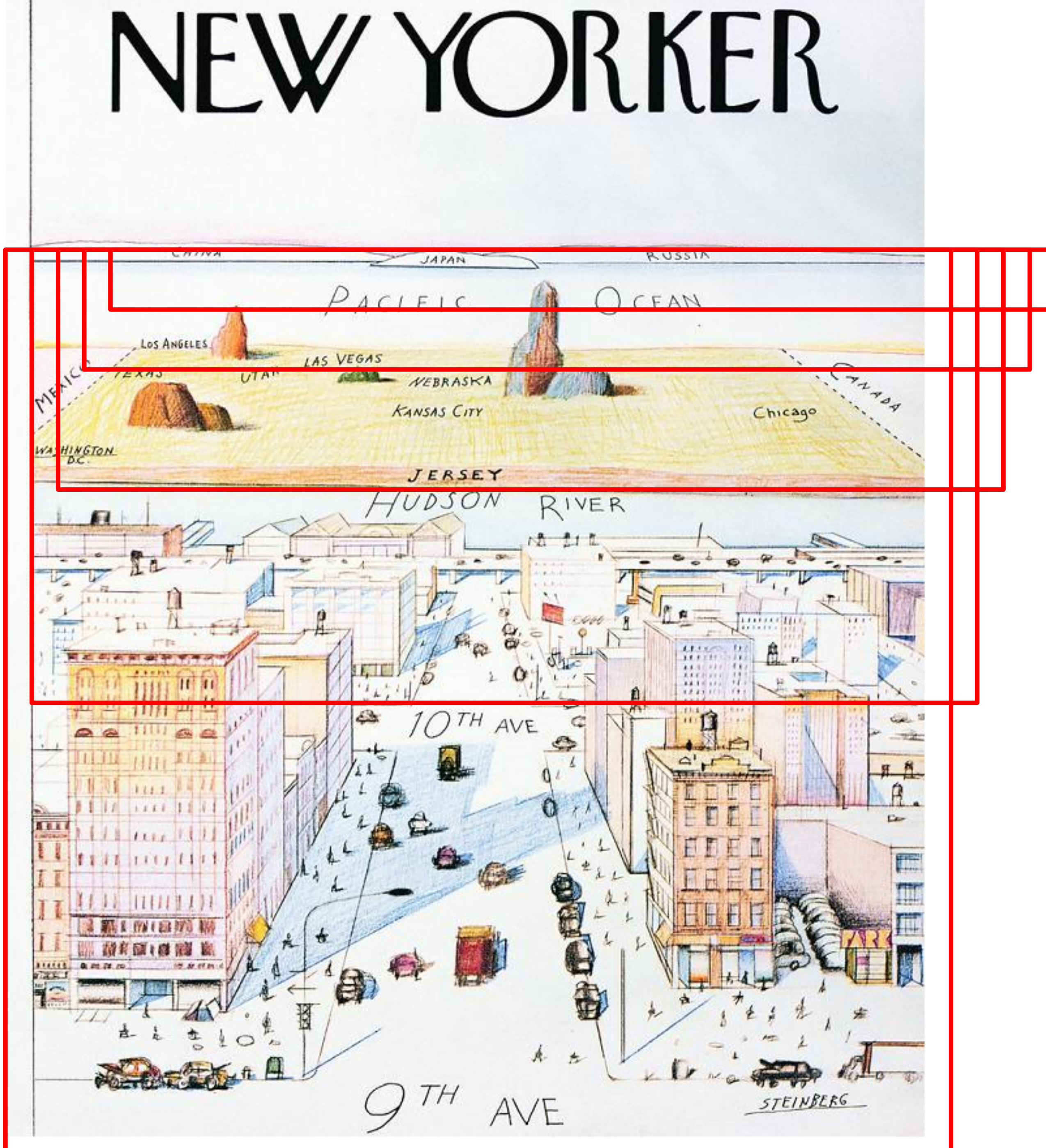
$$\geq \frac{1}{128\ln(6n)}$$

- What is the probability that node *u* has a node *v* as its long range contact?

$$\geq \frac{1}{4\ln(6n)\cdot[d(u,v)]^2}$$

- When r = 2, expected delivery time is

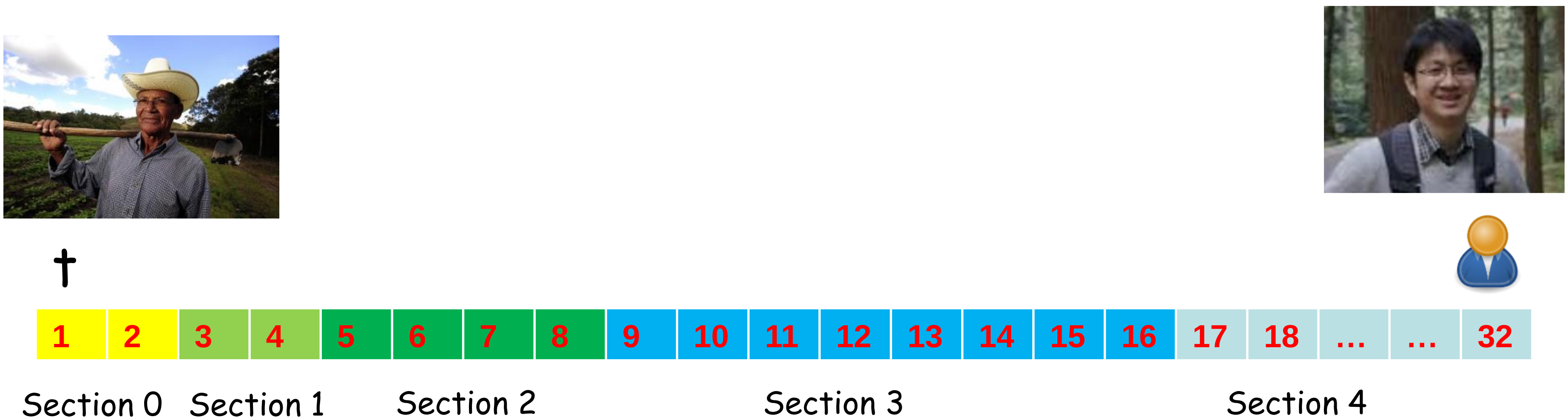
$$O(\log n)^2$$


NEW YORKER
JAPAN
RUSSIA
PACIFIC OCEAN
LOS ANGELES
MEXICO
TEXAS
UTAH
LAS VEGAS
NEBRASKA
KANSAS CITY
CANADA
Chicago
WASHINGTON D.C.
JERSEY
HUDSON RIVER
10TH AVE
9TH AVE
STEINBERG

512

1024

512

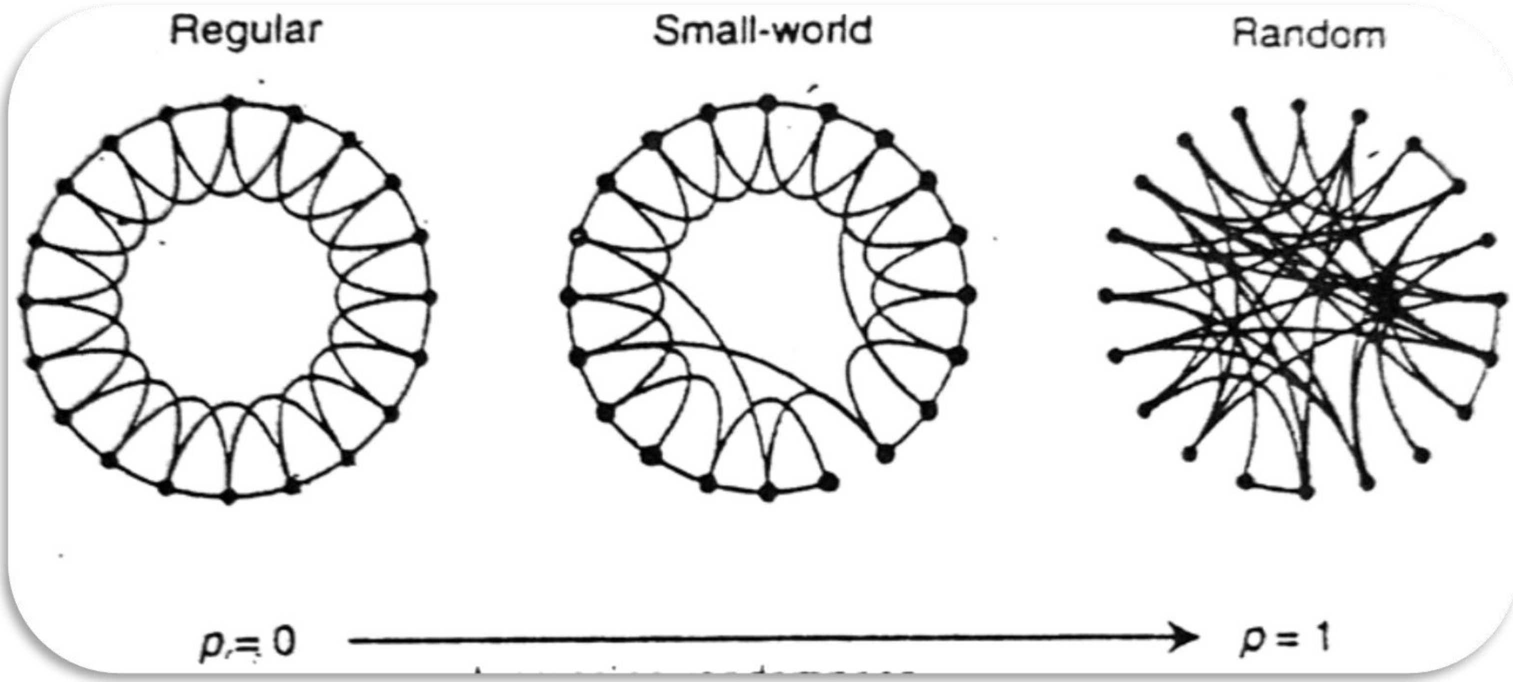


Probability of having a long distance friend at Section 4

=

Probability of having a long distance friend at Section 0, 1, 2, 3

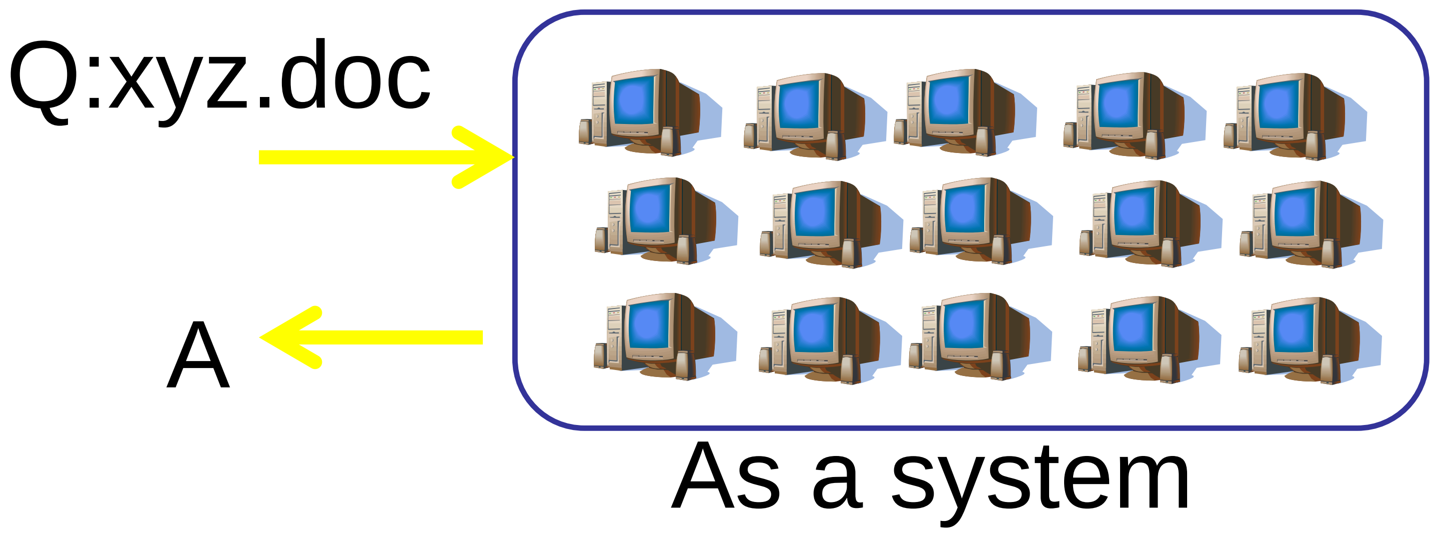
BA Model + Watts/Strogatz Model+ Geographical Model



Mimic a small world system to design a data storage system

Given a huge set of computers, how to utilize them to have a data storage system

- P1: how data are stored
- P2: how data are efficiently retrieved



P1: How data are stored ?

Content Addressable

Hash(abc.doc **) = 14**

Hash(xyz.doc **) = 0**

Q(abc.doc **)**

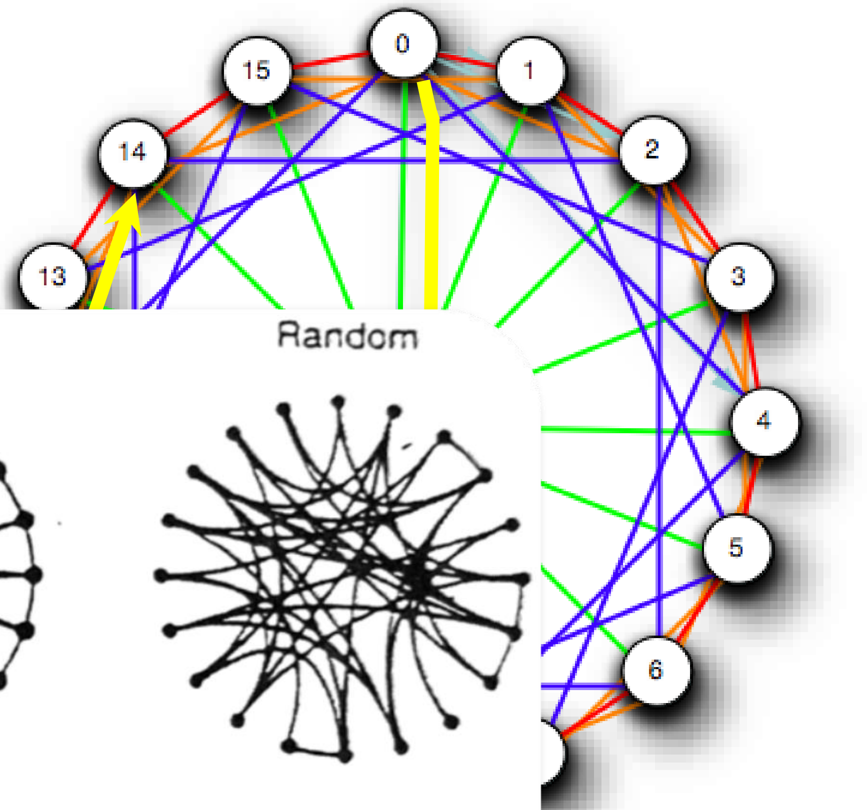
= Hash (abc.doc **)**

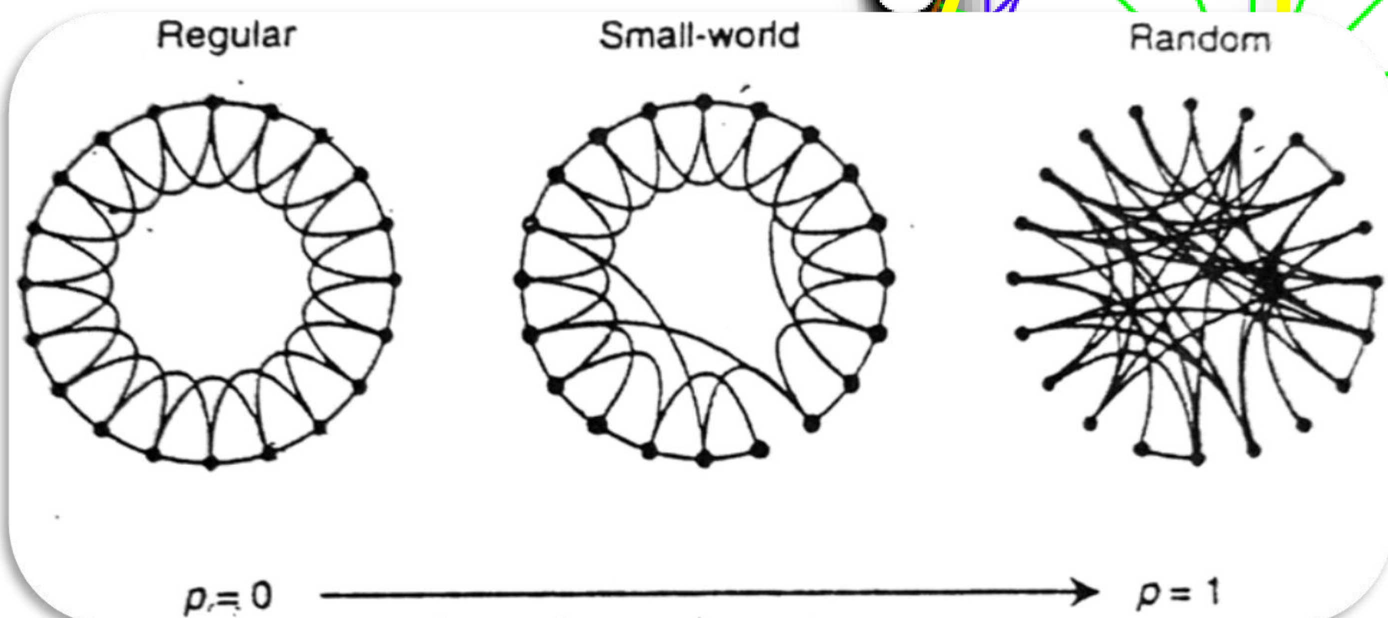
= 14

P2: How data are retrieved ?

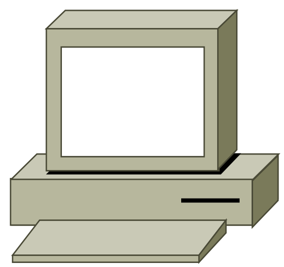
Small World Like Topology

Q (abc.doc)





• Hash (data) =14



• 2
• 6
• 12
•

Thoughts

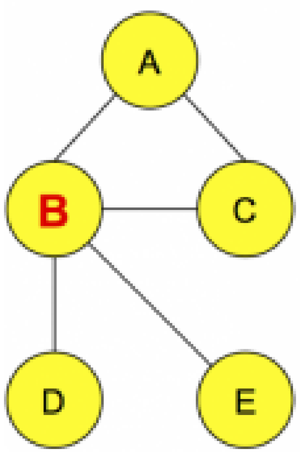
- What about a well-structured network?
- What about a p2p like network?

Closeness Centrality

In a connected graph, closeness centrality (or closeness) of a node is a measure of centrality in a network, calculated as the sum of the length of the shortest paths between the node and all other nodes in the graph. Thus the more central a node is, the closer it is to all other nodes.

$$CC(i) = \frac{N-1}{\sum_j d(i,j)}$$

where
$i \neq j$,
d_{ij} is the length of the shortest path between nodes i and j in the network,
N is the number of nodes.



	A	B	C	D	E	farness $\sum_{j=1}^{n} d(i,j)$	$CC(i) = \frac{N-1}{\sum_j d(i,j)}$
A	0	1	1	2	2	6	(5-1)/6= 0.67
B	1	0	1	1	1	4	1.00
C	1	1	0	2	2	6	0.67
D	2	1	2	0	2	7	0.57
E	2	1	2	2	0	7	0.57

N = 5 (# of nodes)