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Exam 1 - written

1-1) Finding the World Transform

a) $\langle 69, 420, 25.807 \rangle$

Translation matrix - Translation

$$\begin{bmatrix} 1 & 0 & 0 & 69 \\ 0 & 1 & 0 & 420 \\ 0 & 0 & 1 & 25.807 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) Rotation matrix

1) Encode the rotation as a quaternion

$\Theta = 90°$ $\quad \vec{a} = \hat{i}\hat{j}\hat{k} = \langle 2, 1, -1 \rangle$

$q = w + (x\hat{i} + y\hat{j} + z\hat{k})$

$\cos(\theta/2) + \sqrt{2}\,\hat{i} + \frac{\sqrt{2}}{2}\hat{j} + \frac{\sqrt{2}}{2}\hat{k}$

$q = \frac{\sqrt{2}}{2} + \sqrt{2}\,\hat{i} + \frac{\sqrt{2}}{2}\hat{j} + \frac{\sqrt{2}}{2}\hat{k}$

2) Find the inverse q^{-1} of the encoded rotation

$q^{-1} = \frac{\sqrt{2}}{2} - \sqrt{2}\,\hat{i} - \frac{\sqrt{2}}{2}\hat{j} - \frac{\sqrt{2}}{2}\hat{k}$

* To rotate $\vec{v}$ by q, do $q\vec{v}q^{-1}$

3) Find local x, y, z vectors

$x_q = \hat{i} = \langle 1, 0, 0 \rangle$

$y_q = \hat{j} = \langle 0, 1, 0 \rangle$

$z_q = \hat{k} = \langle 0, 0, 1 \rangle$

$x_\ell = q x_g q^{-1} = (\frac{\sqrt2}{2} + \sqrt2\hat i + \frac{\sqrt2}{2}\hat j + \frac{\sqrt2}{2}\hat k)(\hat i)(\frac{\sqrt2}{2} - \sqrt2\hat i + \frac{\sqrt2}{2}\hat j - \frac{\sqrt2}{2}\hat k)$

$= (\frac{\sqrt2}{2}\hat i - \sqrt2 + \frac{\sqrt2}{2}\hat k - \frac{\sqrt2}{2}\hat j)(\frac{\sqrt2}{2} - \sqrt2\hat i + \frac{\sqrt2}{2}\hat j - \frac{\sqrt2}{2}\hat k)$

$= \frac12\hat i + \hat i + \frac12\hat k + \cancel{\frac12\hat j} + 1 + 2\hat i \cancel{-\hat j} + \hat k + \frac12\hat k - \hat j - \frac12\hat i + \frac12\hat k$

$\cancel{-\frac12\hat j} + \hat k + \cancel{\frac12\hat j} + \frac12\hat i$

$= 4\hat i + 1\frac12\hat j + 3\frac12\hat k = \langle 4, 1.5, 3.5\rangle$

$y_\ell = q y_g q^{-1} = (\frac{\sqrt2}{2} + \sqrt2\hat i + \frac{\sqrt2}{2}\hat j + \frac{\sqrt2}{2}\hat k)(\hat j)(\frac{\sqrt2}{2} - \sqrt2\hat i - \frac{\sqrt2}{2}\hat j - \frac{\sqrt2}{2}\hat k)$

$= (\frac{\sqrt2}{2}\hat j + \sqrt2\hat k - \frac{\sqrt2}{2}\hat j + \frac{\sqrt2}{2}\hat i)(\frac{\sqrt2}{2} - \sqrt2\hat i - \frac{\sqrt2}{2}\hat j - \frac{\sqrt2}{2}\hat k)$

$= \cancel{\frac12\hat j} + \cancel{\hat k} + \cancel{\frac12\hat j} - \cancel{\frac12\hat i} \cancel{-\hat k} + 2\hat j \cancel{-\hat i} - \hat k - \cancel{\frac12\hat j} - \hat k - \cancel{\frac12\hat j} + \cancel{\frac12\hat i}$

$+ \cancel{\frac12\hat i} + \hat i - \frac12\hat k + \cancel{\frac12\hat j}$

$= \frac12\hat i + 2\frac12\hat j - 2\frac12\hat k = \langle \frac12, 2.5, -2.5\rangle$

$z_\ell = (\frac{\sqrt2}{2} + \sqrt2\hat i + \frac{\sqrt2}{2}\hat j + \frac{\sqrt2}{2}\hat k)(\hat k)(\frac{\sqrt2}{2} - \sqrt2\hat i - \frac{\sqrt2}{2}\hat j - \frac{\sqrt2}{2}\hat k)$

$= \frac{\sqrt2}{2}\hat k + \sqrt2\hat j - \frac{\sqrt2}{2}\hat i - \frac{\sqrt2}{2}\hat k)(\frac{\sqrt2}{2} - \sqrt2\hat i - \frac{\sqrt2}{2}\hat j - \frac{\sqrt2}{2}\hat k)$

$= \frac12\hat k \cancel{-\hat j} + \cancel{\frac12\hat i} + \frac12\hat k + \cancel{\hat j} + 2\hat k + \cancel{\hat j} \cancel{-\hat i} - \cancel{\frac12\hat i} \cancel{-\hat i} + \frac12\hat k - \cancel{\frac12\hat j}$

$-\frac12\hat k + \cancel{\hat j} - \cancel{\frac12\hat i} - \frac12\hat k$

$= -1\frac12\hat i + 1\frac12\hat j + 3\frac12\hat k = \langle -1.5, 1.5, 3.5\rangle$

hence: ROTATION MATRIX:

$$\begin{bmatrix} 4 & .5 & -1.5 & 0 \\ 1.5 & 2.5 & 1.5 & 0 \\ 3.5 & -2.5 & 3.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c) It's scale matrix

$p = \langle 69, 420, 25.807 \rangle$

scale by $\langle 2, 1, -1 \rangle$

$$p = \begin{bmatrix} 69 \\ 420 \\ 25.807 \\ 1 \end{bmatrix} \quad S = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S \times p = \begin{bmatrix} 138 \\ 420 \\ -25.807 \\ 1 \end{bmatrix}$$

d) It's world matrix

$T \times R \times S$

$$T = \begin{bmatrix} 1 & 0 & 0 & 69 \\ 0 & 1 & 0 & 420 \\ 0 & 0 & 1 & 25.807 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R = \begin{bmatrix} 4 & .5 & -1.5 & 0 \\ 1.5 & 2.5 & 1.5 & 0 \\ 3.5 & -2.5 & 3.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R \times S = \begin{bmatrix} 8 & .5 & 1.5 & 0 \\ 3 & 2.5 & -1.5 & 0 \\ 7 & -2.5 & -3.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_x(R \times S) = \boxed{\begin{bmatrix} 8 & 0.5 & 1.5 & 69 \\ 3 & 2.5 & -1.5 & 420 \\ 7 & -2.5 & -3.5 & 25.807 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

1.2) Transformation Composition

To combine this sequence of transformations into a single matrix we must multiply the matrices, but we must keep in mind the order of the objects being transformed.

$S \rightarrow T \rightarrow T \rightarrow U \rightarrow B$

$$\boxed{S(T^2\{U(B)\})}$$

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1-3) Applying Quaternions

$q = w + a\hat{i} + b\hat{j} + c\hat{k}$
vector $= \langle x, y, z \rangle$
$\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

Rotation of vector is noted by $q\vec{v}q^{-1}$
so... $q\vec{a}q^{-1}$
$q = w + a\hat{i} + b\hat{j} + c\hat{k}$ algHelp $= a\hat{i} + b\hat{j} + c\hat{k}$
$q^{-1} = w - a\hat{i} - b\hat{j} - c\hat{k} = (w - \text{algHelp})$

$q\vec{a}q^{-1} = (w + \text{algHelp})\,\vec{a}\,(w - \text{algHelp})$
$\neq (w + \text{algHelp})(0 + \vec{a})(w - \text{algHelp})$
$= (w + \text{algHelp})[(0w + \vec{a} \times \text{algHelp}) + (\cancel{0\,\text{algHelp}} + w\vec{a} + \vec{a} \times (-\text{algHelp}))]$
$= (w + \text{algHelp})[\vec{a} \times \text{algHelp} + (w\vec{a} - \vec{a} \times \text{algHelp})]$
$= [w(\vec{a} \times \text{algHelp}) - \text{algHelp} \times (w\vec{a} - \vec{a} \times \text{algHelp})] +$
$[w(w\vec{a} - \vec{a} \times \text{algHelp}) + (\vec{a} \times \text{algHelp})\text{algHelp} + \text{algHelp} \times (w\vec{a} - \vec{a} \times \text{algHelp})]$
$= w(\vec{a} \times \text{algHelp} - \text{algHelp} \times \vec{a}) + \text{algHelp} \times (\vec{a} \times \text{algHelp}) + w^2\,\text{algHelp} -$
$w(\vec{a} \times \text{algHelp}) + (\vec{a} \times \text{algHelp})\text{algHelp} + w(\text{algHelp} \times \vec{a} - \text{algHelp} \times (\vec{a} \times \text{algHelp}))$
$= 0 + 0 + w^2\vec{a} + w(\text{algHelp} \times \vec{a}) + (\vec{a} \times \text{algHelp})\text{algHelp} +$
$w(\text{algHelp} \times \vec{a}) - 0$

PLUG N' CHUG!! algHelp $= a\hat{i} + b\hat{j} + c\hat{k}$
$= w^2(x\hat{i} + y\hat{j} + z\hat{k}) + 2w[(bz - cy)\hat{i} + (cx - az)\hat{j} + (ay - bx)\hat{k}] +$
$(ax + by + cz)(a\hat{i} + b\hat{j} + c\hat{k})$

So...

$$\boxed{\begin{aligned} q\,\vec{a}\,q^{-1} = {} & [w^2\hat{k} + 2w(b\hat{z} - c\hat{y}) + a(a\hat{k} + b\hat{j} + c\hat{z})]\,\hat{i} + \\ & [w^2 y + 2w(c\hat{h} - a\hat{z}) + b(a\hat{x} + b\hat{j} + c\hat{z})]\,\hat{j} + \\ & [w^2 z + 2w(a\hat{y} - b\hat{x}) + c(a\hat{x} + b\hat{j} + c\hat{z})]\,\hat{k} \end{aligned}}$$

1-4) Vector mechanics

a) we know:

light source is @ $\vec{l}$

point on surface located @ $\vec{s}$

To find out how far the point is from the light source we can subtract $(\vec{l} - \vec{s})$

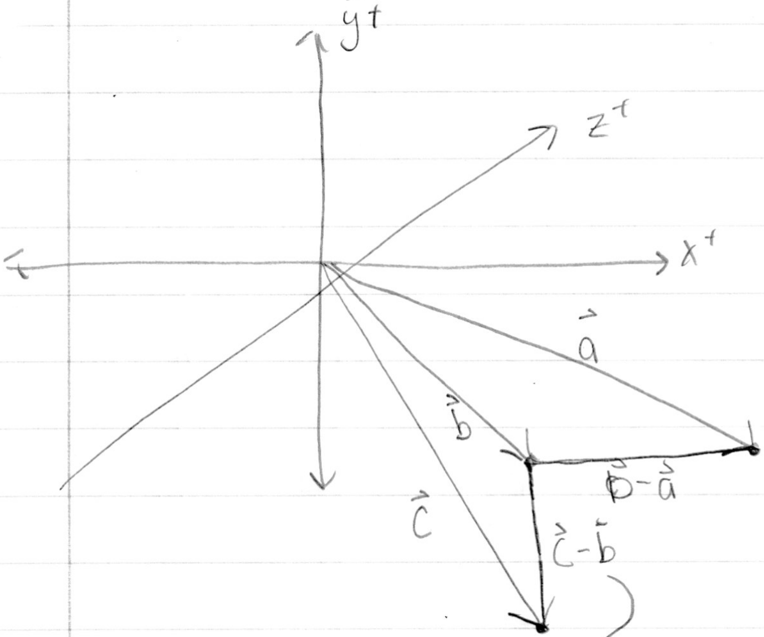
If we want to find out the direction of the normal vector, we can see its going from $\vec{s}$ to $\vec{l}$, thus it would be

if $\vec{l} - \vec{s}$ is greater than 0 it will be facing the light and lit

if $\vec{l} - \vec{s}$ is less than 0 it will be facing below and not lit.

b) Use the dot product to find the angle between two vectors

$$U \cdot V = \|U\|\,\|V\| \cos\Theta$$



$$\vec{b}\cdot\vec{c} = \|\vec{b}\|\,\|\vec{c}\|\cos\theta$$

$$\|\vec{c}-\vec{b}\|^2 = \|b\|^2 + \|\vec{c}\|^2 - 2\|\vec{b}\|\|\vec{c}\|\cos\theta$$

Dot product

$$\|\vec{c}-\vec{b}\|^2 = (\vec{c}-\vec{b})\cdot(\vec{c}-\vec{b})$$
$$= (\vec{c}-\vec{b})\cdot\vec{c} - (\vec{c}-\vec{b})\cdot\vec{b}$$
$$= \vec{c}\cdot\vec{c} - \vec{b}\cdot\vec{c} - \vec{c}\cdot\vec{b} + \vec{b}\cdot\vec{b}$$
$$= \vec{c}\cdot\vec{c} - \vec{b}\cdot\vec{c} - \vec{b}\cdot\vec{c} + \vec{b}\cdot\vec{b}$$
$$= \|c\|^2 - 2\vec{b}\cdot\vec{c} + \|b\|^2$$

$$\|c\|^2 - 2\vec{b}\cdot\vec{c} + \|\vec{b}\|^2 = \|b\|^2 + \|\vec{c}\|^2 - 2\|\vec{b}\|\|\vec{c}\|\cos\theta$$
$$-2\vec{b}\cdot\vec{c} = -2\|\vec{b}\|\|\vec{c}\|\cos\theta$$
$$\vec{b}\cdot\vec{c} = \|\vec{b}\|\|\vec{c}\|\cos\theta$$

$$\cos\theta = \frac{\vec{b}\cdot\vec{c}}{\|\vec{b}\|\|\vec{c}\|}$$

We can use this to find the cosine of the two nonzero vectors, thus giving us the angle to find the slope. VII