Sean
Blanchard
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Homework 3-1

3-1

a) Translation

$p = \langle 1,2,3 \rangle$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$p = \langle 5,0,17 \rangle$

$$\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 17 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$p = \langle -1,1,0 \rangle$

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) Scale

$s = \langle 1, 2, 3 \rangle$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$s = \langle 5, 0, -17 \rangle$

$$\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -17 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$s = \langle -1, 1, 0 \rangle$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c) ROTATION

$\hat{k}$

inverse (q^{-1})

$-\hat{k}$

Find local x, y, z vectors:

$x_g = \hat{i} = \langle 1,0,0 \rangle$

$y_g = \hat{j} = \langle 0,1,0 \rangle$

$z_g = \hat{k} = \langle 0,0,1 \rangle$

$x_l = q\, x_g\, q^{-1} = (\hat{k})(\hat{i})(-\hat{k})$

$= (\hat{k})(-\hat{k})$

$= \boxed{-\hat{k}}$

MATH

$y_l = q\, y_g\, q^{-1} = (\hat{k})(\hat{j})(-\hat{k})$

$z_l = q\, z_g\, q^{-1} = (\hat{k})(\hat{k})(-\hat{k})$

$w = 0,\ x = 0,\ y = 0,\ z = 1$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$q = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\hat{i}$

inverse (q^{-1})

$q^{-1} = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\hat{i}$

Find local x, y, z variables

$x_g = \hat{i} = \langle 1,0,0 \rangle$

$y_g = \hat{j} = \langle 0,1,0 \rangle$

$z_g = \hat{k} = \langle 0,0,1 \rangle$

$$x_l = q x_g q^{-1} = \left(\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\hat{i}\right)(\hat{i})\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\hat{i}\right)$$
$$= \left(\frac{\hat{i}}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2}\hat{i}\right)$$
$$= \hat{i} \quad \langle 1,0,0 \rangle$$

$$y_l = q y_g q^{-1} = \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\hat{i}\right)(\hat{j})\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\hat{i}\right)$$
$$= \left(\cancel{\tfrac{1}{2}\hat{j}} + \tfrac{1}{2}\hat{k} + \tfrac{1}{2}\hat{k} - \cancel{\tfrac{1}{2}\hat{j}}\right)$$
$$= \boxed{\hat{k}} \quad \langle 0,0,1 \rangle$$

$$z_l = q z_g q^{-1} = \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\hat{i}\right)(\hat{k})\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\hat{i}\right)$$
$$= \left(\cancel{\tfrac{1}{2}\hat{k}} - \tfrac{1}{2}\hat{j} - \tfrac{1}{2}\hat{j} - \cancel{\tfrac{1}{2}\hat{k}}\right)$$
$$= -\hat{j} \quad \langle 0,-1,0 \rangle$$

$$R = \begin{bmatrix} (x_l) & (y_l) & (z_l) & \begin{matrix}0\\0\\0\end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

c) Rotation

$\frac{\sqrt{2}}{2} + \frac{1}{2}\hat{i} - \frac{1}{2}\hat{j}$

$w = \frac{\sqrt{2}}{2}$, $x = 1/2$, $y = -1/2$, $z = 0$

$$\begin{bmatrix} 1/2 & -1/2 & -1/\sqrt{2} & 0 \\ -1/2 & 1/2 & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3-2) World Transform

a) Translation x Rotation x Scale

$$T = \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad S = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R \times S = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & -6 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T \times (R \times S) = \begin{bmatrix} 2 & 0 & 0 & +5 \\ 0 & 0 & -6 & -4 \\ 0 & 4 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) If you look above you notice the first 3x3 in top left corner stays the same from R x S to T x (R x S). This shows that Translation only effects the last ~~1x3~~ matrix.

3-3 Inverting

a) We frequently want to invert (or undo) transformations

For translation (x_t, y_t, z_t) we apply the negation $(-x_t, -y_t, -z_t)$

$$T = \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T^{-1} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) Inverting Scale Matrix

$$S = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad S^{-1} = \begin{bmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 0 \\ 0 & 0 & 1/6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c) Inverting the Rotation Matrix

$$q\hat{i}q^{-1} = (w + a\hat{i} + b\hat{j} + c\hat{k})\hat{i}(w - a\hat{i} - b\hat{j} c\hat{k})$$

$$q\hat{j}q^{-1} = 0 + \hat{j}(w^2 + a^2 - b^2 - c^2) + \hat{i}(2ab - wc) + \hat{k}(2ac - 2wb)$$

$$q\hat{k}q^{-1} = 0 + \hat{k}(w^2 + a^2 - b^2 - c^2) + \hat{k}(2ab - wc) + \hat{j}(2wc + 2ab)$$

$$\begin{bmatrix} w^2 + a^2 + b^2 - c^2 & w^2 + a^2 + b^2 - c^2 & w^2 + a^2 - b^2 - c^2 & \cancel{w^2 + a^2 + b^2 - c^2}\ 0 \\ 2wc + 2ab & 2ab - wc & 2ab - wc & 0 \\ 2ac - 2wb & 2ac - 2wb & 2wc + 2ab & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

d) Why Invert?

Inverse matrices are really useful for alot of things, but really shines in 3D transforms. With inverse matrix you can undo alot of operations performed on a matrix.

Transforming by the matrix takes you in one direction, transforming by the inverse of the matrix takes you in the opposite direction.

This is probably one of the most useful mental pictures you can have of matrices in general