FYS4560;

Elementary Particle Physics

Final Project

Higher Dimensions;

Theoretical and Experimental Aspects

Sean B.S. Miller

Abstract

The postulated graviton will be studied in two seperate models for extra dimensions; The Randall-Sundrum (RS) model of compacted dimensions (warped), and the Arkani-Hamed, Dimopoulos, and Dvali (ADD) model of large extra dimensions. Quark-anti-quark production of the simplest possible massive graviton (1st order excitation of the Kaluza-Klein tower) will be calculated and the angular distribution will be compared with corresponding SM scalar and vector channels. The paper will mostly follow the dr. scient. thesis of E.W. Dvergsnes [1].

June 4, 2016

1 Introduction

Higher dimensions, also called extra dimensions, are theoretical models for the dimensionality of our universe, mostly suggested with the purpose of explaining the hierarchy problem of the standard model. There are *many* theories for higher dimensions. The most recognised are:

- *Large extra dimensions*: The often-heard theory that gravity acts through several dimensions, therefore becoming weaker. The "large" phrase is due to the dimension being finite, yet large compared to particle interactions. It originates from the ADD model as an attempt to solve the hierarchy problem[1].
- *Warped extra dimensions*: Describing our universe as a five-dimensional anti-de Sitter space, and claiming the SM particles are localized on a $(3 + 1)$-dimensional brane(s).
- *Universal extra dimensions*: All particles move universally through the extra dimensions, unlike the two other models where only gravity propagates through them.

Obviously, a thorough description of any of these models is near impossible for such a small paper, let alone all the models together. Therefore, a brief outline of the theory behind the two currently most promising[2] models will be given.
The first is the large extra dimension model by Arkani-Hamed, Dimopoulos, and Dvali. Originally, it was proposed as a model to explain the hierarchy problem (why the weak force is 10^{32} times stronger than gravity). The extra dimensions are then suggested as planes into which gravity, assumed just as strong as the other forces, spreads. Therefore gravity becomes "diluted", while the known SM particles stay in (1,3)-spacetime.
The second model is the warped extra dimension model by Randall and Sundrum, proposed since they were dissatisfied with the current universal extra dimensions models. They assumed that, rather than having universal extra dimensions in which all particles propagate, there is a small extra dimension. They modelled our world as a "brane" in a 5-dimensional anti-de Sitter space[3]. By small, it means the extra dimension has a large curvature, or is *warped.* Our brane is connected to another by a intermediate circle space, through which only gravity may propagate.
A question that then springs to mind is why exactly gravitons and extra dimensions are connected (other than gravitons "carrying" gravity). If the standard model is expanded, but without inclusion of extra dimensions, to include a graviton field, then measuring it would be impossible[4]. However, with extra dimensions, one can perform a Fourier expansion to get a so-called Kaluza-Klein tower whose terms would be gravitons. The lowest-order (mass) gravitons will couple to SM particles at energies well within the reach of today's particle accelerators.
To know how a graviton signal would look in a detector, a detailed calculation of the quark-anti-quark annihilation into a dilepton jet by the graviton channel will be calculated. This will then be compared to the similar SM scalar (Higgs) and vector (Z, γ) channels.

[1] The "hierarchy problem" is the problem in explaining why vast discrepancies "pop-up" in the standard model. For example, why gravity and the weak force are so weak compared to QED and QCD.

[2] "Promising" in the sense that they provide measurable outcomes, and fit very well with what we already know from the standard model. The problem is that so far they have predicted nothing. Finding a graviton would possibly confirm either theory.

[3] This will be explained later on.

[4] Refer to "Can Gravitons be Detected?" by Rothman and Boughn (https://arxiv.org/pdf/gr-qc/0601043v3.pdf) for an impression of the problem.

2 Kaluza-Klein theory

The reason Kaluza-Klein theory is discussed is because one requires knowledge of the Kaluza-Klein tower; massive excitations of an expansion model[5] of the spacetime metric. While RS1 only considers a single extra dimension, ADD works for any number of dimensions. However, for the sake of simplicity, only one extra dimension will be considered in this section.

2.1 Spin-2 gravity

One often says that the graviton is a spin-2 particle, but it would perhaps be interesting to know why this is the consensus.
In linear gravity, the Minkowski metric is expanded on by an additional term $h_{\mu\nu}$[6] such that

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{1}$$

While avoiding the technical details here, the Einstein field equations can be used to find the solution

$$h_{\mu\nu} = C_{\mu\nu} e^{ik_\sigma a^\sigma} \tag{2}$$

which resembles a plane wave equation. In the x_3-direction, one has:

$$C_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & C_{11} & C_{12} & 0 \\ 0 & C_{12} & C_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{3}$$

Adopting the notation $C_{11} = h_+$ and $C_{12} = h_\times$ and defining $h_{L,R} = \frac{h_+ \pm h_\times}{\sqrt{2}}$, one can rotate the matrix and find the fields h_R h_L transform as $h_{R,L} \to h_{R,L} e^{ia\theta}$. Since gravity is massless, chirality is the same as helicity. With a as helcity, the fields are then seen to transform as spin-2 fields.

2.2 Kaluza-Klein towers

Before adding extra dimensions, the question of how get so-called "massive gravity"; a massive field carries the gravitational force. First, one starts with linearised gravity, i.e. equation 1. The Lagrangian density will then have a interactive term on the form $h_{\mu\nu}T^{\mu\nu}$, and a kinetic term for $h_{\mu\nu}$. The field $h_{\mu\nu}$ can be given mass by saying the Lagrangian density also has a term $ah_{\mu\nu}h^{\mu\nu} + b(\eta_{\mu\nu}h^{\mu\nu})^2$. Markus Fierz and Wolfgang Pauli then, in 1939, wrote an article[4] in which they proved that $a = -b$ in order to avoid unphysical results. The result is the Fierz-Pauli Lagrangian for massive gravity[7] [2]:

$$\frac{1}{\kappa^2}\sqrt{|g|}R = \frac{1}{4}\left(\partial^\mu h^{\nu\rho}\partial_\mu h_{\nu\rho} - \partial^\mu h^\nu_\nu \partial_\mu h^\rho_\rho - 2\partial^\nu h_{\nu\mu}\partial_\rho h^{\rho\mu} + 2\partial_\nu h^{\nu\mu}\partial_\mu h^\rho_\rho\right) + \mathcal{O}(\kappa) \tag{4}$$

Now, additional spatial dimensions may be added. These dimensions will not change the form of equation 4, so one can simply let indices change from greek to latin, i.e. $\mu \to a$, $\nu \to b$, etc..., where $a, b, \ldots \in \{4+d\}$. It is then assumed the perturbative field has a form

$$\hat{h}_{ab} = V_d^{-1/2}\begin{pmatrix} h_{\mu\nu} + \eta_{\mu\nu}\phi & A_{\mu i} \\ A_{\nu j} & 2\phi_{ij} \end{pmatrix} \tag{5}$$

[5]This expansion depends on the model, and is where RS and ADD differ.

[6]This modest assumption is, funnily enough, precisely where all physics concerning gravitons start.

[7]Note that this is *not* the only way to get a massive gravity Lagrangian, and it has some problems as well. It serves well as an introductory example, however.

where V_d is the volume of the compactified space, $\eta_{\mu\nu}\phi$ is a Weyl rescaling and $A_{\mu i}$ is some tensor field. In most phenomenological papers, one simply uses that $V_d \propto R^d$, where R is the "characteristic size" of the extra space. Since the fields are compact, one can claim periodicity of the field, such that a Fourier expansions is possible:

$$h_{\mu\nu}(x, y) = \sum_{\substack{n=\{n_1,n_2,\dots,n_5\}; \\ n_i \in \mathbb{Z}\, \forall\, i}} h_{n,\mu\nu}(x) Y_n(y) \tag{6}$$

where $Y_n(y)$ are orthogonal, normalized eigenfuntions of the Laplace operator on the internal space[8]:

$$\Delta_{K_d} Y_n(y) = \frac{\lambda_n}{R^2} Y_n(y) \tag{7}$$

This is only true for spaces without internal structure. In the case of RS1, Y_n will need to fulfil another equation. The exact form of $Y_n(y)$ depends on the structure and size of the new dimension. This series expansion is called the Kalulza-Klein (KK) tower of modes, and one value of n is called mode n, and the corresponding term the n'th excitation. The field $h_{\mu\nu}(x, y)$ has to satisfy the d'Alembert equation, and in doing this one finds it necessary to redefine the fields $h_{\mu\nu}(x, y)$, $A_{\nu j}$, and ϕ_{ij}, to $\tilde{h}_{n,\mu\nu}$, $\tilde{A}_{n,\mu i}$, and $\tilde{\phi}_{n,ij}$. The detailed steps are omitted, as they are many and non-intuitive. These new fields, when put into the Lagrangian, will give mass eigenstates. It can help memory to think of this redefinition of fields as an analogue to the rotations from the CKM matrix in QCD theory (needed for mass eigenstates in the Lagrangian), but it is not the same (not a rotation). From the equation of motion one can go one to show that the masses are:

$$m_n^2 = \frac{4\pi^2 n^2}{R^2} \tag{8}$$

A relation that will be of importance later is the reduction formula:

$$M_{Pl}^2 = V_d M^{d+2} \tag{9}$$

where $M_{Pl} = G_{N(4)}^{-1/2}$ is the 4-dimensional Planck mass and $M^{d+2} = G_{N(4+d)}^{-1/(4+d)}$ is the fundamental mass scale in the new model. The relation is derived by demanding the Einstein-Hilbert action to be the same with and without the new dimension(s)[9], and performing the integrals by using KK mode expansion on the integrand. I.e one demands:

$$S_{E(4)} = S_{E(d+4)} \tag{10}$$

where

$$S_{E(D)} = \int d^D x \sqrt{-G_D} \frac{1}{16\pi G_{N(D)}} \mathcal{R}^D(G_{ab}) \tag{11}$$

With the principles of the KK tower and massive gravity, one can start to consider theories that are based on extra dimensions.

As is the point with Feynman diagrams, Feynman rules for the graviton would help do calculations. Only the rules necessary for the process mentioned in the introduction will be mentioned. The propagator of massive, spin-2 KK excitations are given by the Fierz-Pauli equation of motion [2], and is

$$G_n^{\mu\nu\rho\sigma} = i \frac{\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \frac{2}{3}\eta^{\mu\nu}\eta^{\rho\sigma}}{k_G^2 - m_n^2 + i\epsilon} \tag{12}$$

The vertex factors with different fields (scalar, spinor, vector-boson) are found by inserting the conserved energy-momentum tensor, for said field, into the Lagrangian, except for the spinor-field. Spinor-graviton couplings, which will be of interest later on, must be found by using the vierbein formalism[10]. See [2], page 10, for a more in-depth discussion. The massive spinor-graviton spin-2 coupling is

[8] The extra dimension/space, denoted K_d.

[9] This is because the new dimension(s) must reproduce what we observe, and 4D spacetime fits very well with observations.

[10] A way with which to rewrite the spacetime metric.

$$-\frac{i\kappa}{8}\left[\gamma_\mu(k_1+k_2)_\nu+\gamma_\nu(k_1+k_2)_\mu-2\eta_{\mu\nu}(\not{k}_1+\not{k}_2-2m_f)\right] \tag{13}$$

where momentum follows the fermion lines.

3 The Arkani-Hamed-Dimopoulos-Dvali model

The ADD model is mainly based on 3 features:

- There exists d new spatial compact dimensions, with compactification volume V_d.
- The Planck scale is very low, at the order of one TeV,
- The SM degrees of freedom are localized on a 3D-brane, stretching along 3 non-compact spatial dimensions (i.e. the SM particles move in normal spacetime, not in the new dimension(s)).

The idea is that the electroweak scale is the only fundamental scale in the universe, and that the true Planck scale is actually of the same order. From the reduction formula, one could introduce d new spatial dimensions and find their size, i.e.

$$\frac{M_{Pl}^2}{M^{d+2}}=V_d\sim R^d \quad , \quad \mathcal{O}(M^{d+2})=\mathcal{O}(m_{EW}) \tag{14}$$

where R is the compactification radius as before. With new dimensions, Newton's law for gravitational force would instead be proportional to $r^{-(d+2)}$, which is of course not true. However, since the extra dimensions are compact, only objects with distances $r << R$ feel this new force, while it still goes as r^{-2} for $r >> R$. Therefore, if $d=1$, the radius is of the same order as the earth-moon distance, which greatly violates measurements. $d=2$ should not be possible either, as the radius is then at the order of 0.1 mm and 4 dimensional gravitational effects have been measured to fit at that scale. For $d>2$, the radius is so small that van der Waal forces prevent us from conducting "table-top" experiments, and is where particle physics experiments at CERN become relevant. If ADD is true, then it should be possible to determine the radius by measuring the graviton, which is a direct consequence of extra dimensions. Since the mass m_n is inversely proportional to the size (equation 8), it would be possible to determine the new Planck scale.

A problem with the massive KK spin-2 propagator is that there are infinitely many KK excitations, each with its own mass. When calculating a Feynman diagram, one has to, in principle, perform a sum over all modes, as they are all possible, usually with decreasing probability. In the ADD model, the difference in masses are *very* small due to the size of R by the reduction formula, and evenly spaced. This means one can approximate the sum by an integral over the modes (n). While not too hard, the proof will not be shown, but is clearly presented in Dvergsnes' thesis. The result is presented in the appendix.

4 The Randall-Sundrum model

4.1 The hierarchy problem in RS1

The RS model assumes that there are two points on the $S^1/\mathbb{Z}^2$ ordibfold in which 3-branes are compactified. What this means is that there are two 3-dimensional, non-compact branes that are connected at every point by a "circle".

The two branes, 1 and 2, are located on points $\theta=0$ and $\theta=\pi$, which means the branes are seperated by a distance $2R$, where R is the circle radius.
As mentioned, the model set out to explain the hierarchy problem, and will be briefly explained how below. Firstly, the action of the model is given by the sum of the Hilbert-Einstein action and the "matter part":

$$S_5=S_E+S_M=\int d^4x\int_{-L}^{L}dy\sqrt{-\tilde{g}}(M_5^3\mathcal{R}_5-\Lambda_5) \tag{15}$$

where Λ_5 is the five dimensional cosmological constant, $\tilde{g}$ is the determinant of the (full space) metric tensor, and M_5 is the new mass scale. In order to match real world observations, the new metric must uphold Poinecaré invariance, which can be shown to lead to:

$$ds^2 = e^{-2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \tag{16}$$

where $A(y)$ is called the *warp factor*, hence the name "warped" extra dimensions. Since the orbifold abides $\mathbb{Z}^2$ symmetry, $A(y) = A(-y) \rightarrow A(y) = k|y|$. Solving the Einstein equations, as Randall and Sundrum did in their original article, lets one find that $A(y) = \pm ky$, where $k \equiv \sqrt{\frac{-\Lambda}{24M^3}}$ is a constant ($\Lambda = -24M_5^3k^2$). Because the metric is fundamentally different[11], the reduction formula becomes a little different. Apparently, it now becomes[12];

$$M_{Pl}^2 = \frac{M_5^3}{k}\left[1 - e^{-2kR\pi}\right] \tag{17}$$

where one has set $|y| = \pi R$ (after all, it is a circle). In order to get a a new mass scale which solves the hierarchy problem, in has to require that $kR \sim 12 \rightarrow e^{k\pi R} \sim 10^{15}$. There is a problem with action above, which is that it does not include the energy densities of the two branes, which are:

$$S_1 = \int_{B_1}\int_{S^1/\mathbb{Z}^2} d^4x dy \sqrt{-\tilde{g}(x,y)}\lambda_1\delta(y) \tag{18}$$

$$S_2 = \int_{B_2}\int_{S^1/\mathbb{Z}^2} d^4x dy \sqrt{-\tilde{g}(x,y)}\lambda_2\delta(y-L) \tag{19}$$

where $\tilde{g}(x,0) = g_1$ and $\tilde{g}(x,L) = g_2$ have been used (i.e. $\tilde{g}(x,y)$ contains the regular 4-dimensional metric of each brane). So, the total action for the space is $S = S_E + S_M + S_1 + S_2$.

4.2 RS1 gravitons

Since the metric in RS1 is a bit different from that used in the KK mode expansion, one has to do do it again to get the new current model's graviton masses. One starts with a metric transform, given by:

$$ds^2 = e^{-2k|y|}(\eta_{\mu\nu} + \tilde{h}_{\mu\nu}(x,y))dx^\mu dx^\nu + (1+\phi(x))dy^2 \tag{20}$$

However, this means there will be cross-terms ($\phi\tilde{h}_{\mu\nu}$-terms) in the Lagrangian that prevent mass eigenstates. It can be shown[5] that it can be diagonalized. One will end up with two fields, $h_{\mu\nu}$ and φ, which will be used hereafter.
Then, a KK mode expansion is done on $h_{\mu\nu}(x,y)$, giving

$$h_{\mu\nu}(x,y) = \sum_{n=0}^{\infty} h_{\mu\nu}^{(n)}(x)\frac{\chi_n(y)}{R}, \tag{21}$$

where $\chi_0(y) = 2\sqrt{kR}e^{-2k|y|}$ and

$$\chi_n(y) = N_n\left[C_1Y_2\left(\frac{m_n}{k}e^{k|y|}\right) + C_2J_2\left(\frac{m_n}{k}e^{k|y|}\right)\right] \quad , \quad n \neq 0 \tag{22}$$

While it seems out of the blue, this is the result of diagonalizing the Lagrangian, which is a rather intricate procedure. The functions χ_n are eigenfunction of an equation met during the diagonalising of the Lagrangian. J_2 and Y_2 are the Bessel functions of the first and second kind, respectively. The constants C_1 and C_2 can be determined by the boundary conditions on the vacuum energy terms of the energy-momentum tensor (the delta functions),

$$T_{ab} = \lambda_1\sqrt{g_1}g_{\mu\nu}^{(1)}\delta_a^\mu\delta_b^\nu\delta(y) + \lambda_2\sqrt{g_2}g_{\mu\nu}^{(2)}\delta_a^\mu\delta_b^\nu\delta(y-\pi R), \tag{23}$$

[11] It has actually undergone a so-called Weyl transformation.

[12] Finding the new reduction formula is done by claiming equation 10 to be true and doing the calculation, same as was done to get equation 9.

which give $C_1 = Y_1\left(\frac{m_n}{k}\right)$ and $C_2 = -J_1\left(\frac{m_n}{k}\right)$. Here $\lambda_1 = -\lambda_2 = 24M_5^3 k$, but this will not be discussed further. The tensor leads to an eigenvalue equation, from which one will find that the spin-2 graviton masses are connected to the roots of the Bessel function J_1 by:

$$m_n = \beta_n k e^{-kR\pi}, \quad J_1(\beta_n) = 0 \,\forall\, n \in \mathbb{N} \tag{24}$$

The $h_{\mu\nu}^{(0)}(x)$ field describes the massless graviton, while the $n \geq 1$ states are the massive KK modes, and finally φ describes the massless *radion*.

Now that the masses are known, one can see that they behave very different from those of the ADD model. Here, they are of the order of TeV, and seperated by a few orders of TeV (due to the roots $\beta_n = 3.83,\ 7.02,\ 10.17,\ \ldots$). This means there should be very clear, separate peaks in particle detectors, unlike that of the ADD model where the masses were so close they could be approximated by integration. With a working theory for the RS1 model, one would like to do calculations. Specifically, one would like for the Feynman rules mentioned earlier to still apply. This can be done by comparing the Hilbert-Einstein action for the general KK case, and the RS1 case. One will see that the actions are the same (to first order in κ) should one choose:

$$\kappa = \sqrt{2}\frac{\beta_1}{m_1}\frac{k}{M_{Pl}} \tag{25}$$

As one sees, the RS1 model is completely determined by the m_1 mass and the factor $\frac{k}{M_{Pl}}$, which is believed to lie between 0.01 and 0.1[6].

5 Graviton production at the LHC

With a couple of theories well at hand, it would be interesting to know how they should appear in a particle collider such as the LHC. A quark-anti-quark annihilation in proton-proton collisions can produce a graviton as gravitons and fermion couple. Furthermore, this graviton can then decay to a lepton-pair, such as muons. Obviously there are many channels that give off a dimuon final state. However, there are no spin-2 particles in the standard model, and measuring the angular distribution would be expected to give a characteristic contour. One example of angular distribution is the differential cross section. Here, the angular dependency would be interesting to study and compare to the corresponding scalar and vector channels (H and γ, Z), the two of which have already been studied in previous projects.

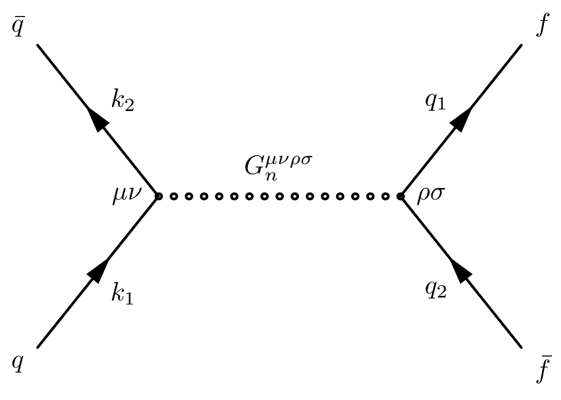

Figure 1: Feynman diagram for $q\bar{q} \to G \to f\bar{f}$ where momentum follows fermion line (i.e. k_1 and q_2 go towards vertices, k_2 and q_1 move away).

5.1 SM backgrounds

The spin-averaged amplitude for $e^+e^- \to \tau^+\tau^-$ in the electroweak channel was in project 2. The formula is quite general for all fermions in the high s limit, and for the current case is:

$$\langle|\mathcal{M}(q\bar{q}\to l^+l^-)|^2\rangle = Q_f^2\left(A'\left(1+\cos^2(\theta)\right)+B'\cos(\theta)\right) \tag{26}$$

where

$$\begin{aligned} A' &= e^4 + \frac{e^2 g_Z^2 P_Z s}{2} c_V^q c_V^l + \frac{|P_Z|^2 g_Z^4 s^2}{16}\left[\left(c_V^q\right)^2 + \left(c_A^q\right)^2\right]\left[\left(c_V^l\right)^2 + \left(c_A^l\right)^2\right] \\ B' &= e^2 g_Z^2 P_Z s c_A^q c_A^l + \frac{1}{2}|P_Z|^2 g_Z^4 s^2 c_V^q c_A^q c_V^l c_A^l \end{aligned} \tag{27}$$

and

- Q_f is the colour factor ($Q_f = 3$).
- $P_Z(s) = \frac{1}{s-m_Z^2+im_Z\Gamma_Z}$, where Γ_Z is the Z-boson decay width.
- c_A^f and c_V^f are the axial and vector coupling coefficients of the Z-boson to Dirac spinor f.
- g_Z is the weak coupling constant.

Furthermore, the corresponding Higgs channel is given by:

$$\langle|\mathcal{M}(q\bar{q}\to l^+l^-)|^2\rangle = \frac{16m_f m_q}{v^2} Q_f^2\left(k_{1,\mu}k_2^\mu - m_q^2\right)\left(q_{1,\mu}q_2^\mu - m_l^2\right) \tag{28}$$

where $v = 246\,\text{GeV}$ is the vacuum expectation value of the Higgs field. As one would expect, there is no angular dependency for a scalar field.

5.2 Angular distributions

In appendix A, a more thorough calculation of the matrix element is performed. Note that one does not consider the more realistic with parton distribution functions. The calculation is mainly just to get an impression of how massive gravitons would behave. Below are the differential cross-sections for the scalar, vector and tensor channels for $q\bar{q} \to l^+l^-$, where in the last case the masses have been neglected. As one can see, each case differs greatly from others.

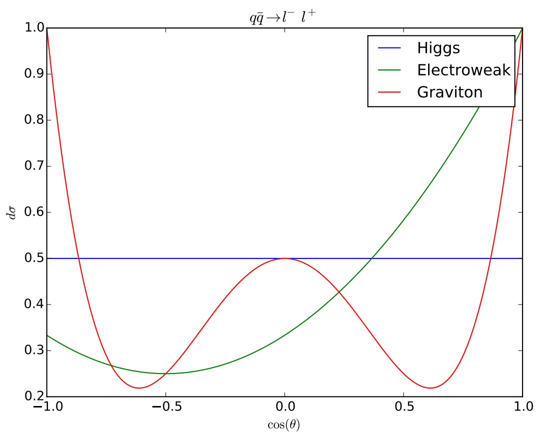

Figure 2: Normalized angular distributions for different channels (except for the scalar case, which was intentionally moved to 0.5). The curve fits nicely with the one shown in [3].

This plot is interesting not only in the sense that it shows a clear way with which to separate the three cases, but it also shows where ought to be looking:

- Spin-0: Shows no favoured angle.

- Spin-1: High probability of products moving orthogonally to particle beams. High p_T physics fits well here.
- Spin-2: Fairly high probability for pure transverse movement, but favours angles around $-\pi$ and π to the collision axis. Little chance for parallel movement.

As one can see, the spin-2 case differs greatly from the scalar and vector cases. Of course, the graphs above will be for different energies, as the different channels becomes favourable at different energies. However, since there are other beyond-the-standard-model models, the cross-section suppression of the scalar and vector channels do not mean a high-energy resonance signifies a graviton. For example, a Z'-boson could be found, or a new Higgs. This is why the angular distribution and spin identification is important.

5.3 Searches at the LHC

At the LHC, there are different channels studied for different models. In the ADD model, the most promising are [7]:

$$\begin{aligned} pp &\rightarrow \text{(jet + missing E)} \\ q\bar{q} &\rightarrow gG^{(n)} \\ gg &\rightarrow gG^{(n)} \end{aligned}$$

While a bit difficult to find current numbers, in 2008 the lower bound on ADD was found, at LHC, to be

d	M[TeV]
2	$4.0 - 7.5$
3	$4.5 - 5.9$
4	$5.0 - 5.3$

with no restraints for higher dimensions.
In the RS1 model, there's:

$$\begin{aligned} q\bar{q} &\rightarrow G^{(1)} \rightarrow l^+l^- \\ gg &\rightarrow G^{(1)} \rightarrow l^+l^- \\ q\bar{q}, gg &\rightarrow G^{(1)} \rightarrow q\bar{q}, gg \end{aligned}$$

The lower bound on M_1 in the RS1 model was found to be at 2080 GeV.

Although hadron colliders are "noisy", they can do a lot for extra dimensions searches. The energies needed, as is seen here, are high. Given the spin-2 particle having such a effect on angular distributions, the signal should nonetheless be characteristic once found.

6 Conclusions

Starting with the simple assumption of an extra dimension, one explained how to, rather simply, expand linear gravity. This then allowed for the mode expansion leading to the Kaluza-Klein tower, which permits massive gravitons to exist. Then, both the idea and workings of the RS1 and ADD models were discussed. These models allow for KK gravitons, but predict different masses, have different interpretations of graviton/gravitional movement, and different SM couplings. The Feynman rules for KK gravitons, however, fitted for both models, except with the linear gravity expansion variable κ differing between models.

While RS1 only requires one extra dimension of complex structure, the ADD allows several. This, in turn, means searches at particle colliders need to deal with several points at once, i.e. which number of extra dimensions and the lower bound the data put on these. Additionally, the size is also adjustable, providing a great many possibilities. In common, however, are is the angular distribution, as shown with the Drell-Yang process $q\bar{q} \to G \to l^+l^-$, as has been calculated. It allows for a distinct signal which will distinguish the graviton from similar vector-boson and Higgs particles form beyond the standard model theories.

A Calculations

A detailed calculation for the channel $q\bar{q} \to l^+l^-$ is as follows.
The n'th resonance spin-2 KK graviton propagator is given equation 12. The $G\bar{\psi}\psi$-coupling is given in equation 13.
The diagram shown in figure 1 therefore has effective amplitudes:

$$
\begin{aligned}
i\mathcal{M}(q\bar{q} \to f\bar{f}) = -Q_f \sum_n \frac{i\kappa^2}{64} &\bar{u}_q \left[\gamma_\rho(q_1+q_2)_\sigma + \gamma_\sigma(q_1+q_2)_\rho - 2\eta_{\rho\sigma}(\not{q}_1 + \not{q}_2 - 2m_f)\right] v_q \\
&\times \left[\frac{\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \frac{2}{3}\eta^{\mu\nu}\eta^{\rho\sigma}}{k_G^2 - m_n^2 + i\epsilon}\right] \\
&\times \bar{v}_k \left[\gamma_\mu(k_1+k_2)_\nu + \gamma_\nu(k_1+k_2)_\mu - 2\eta_{\mu\nu}(\not{k}_1 + \not{k}_2 - 2m_f)\right] u_k \\
= -i\frac{\pi C_4}{4} &\bar{u}_q \left[\gamma_\rho(q_1+q_2)_\sigma + \gamma_\sigma(q_1+q_2)_\rho - 2\eta_{\rho\sigma}(\not{q}_1 + \not{q}_2 - 2m_f)\right] v_q \\
&\times \bar{v}_k \left[\gamma^\rho(k_1+k_2)^\sigma + \gamma^\sigma(k_1+k_2)^\rho + \frac{4}{3}\eta^{\sigma\rho}\right] u_k \\
= -i\frac{\pi C_4}{2} &\bar{u}_q \Big[(q_1+q_2)_\mu(k_1+k_2)^\mu \gamma_\nu v_q \bar{v}_k \gamma^\nu + (\not{k}_1 + \not{k}_2) v_q \bar{v}_k (\not{q}_1 + \not{q}_2) \\
&- 2(\not{q}_1 + \not{q}_2) v_q \bar{v}_k (\not{k}_1 + \not{k}_2) + 4\left(m_q(\not{q}_1 + \not{q}_2) v_q \bar{v}_k + m_f v_q \bar{v}_k (\not{k}_1 + \not{k}_2)\right) \\
&- \frac{32}{3} m_f m_q v_q \bar{v}_k \Big] u_k
\end{aligned}
\tag{29}
$$

where $C_4 \equiv \frac{\kappa^2}{8\pi} D(s)$, $D(s) \equiv \sum_n \frac{1}{k_G^2 - m_n^2 + i\epsilon}$. The above expression can be rewritten as

$$
\begin{aligned}
i\mathcal{M}(q\bar{q} \to f\bar{f}) = -iQ_f \frac{\pi C_4}{2} \bar{u}_q \Big[&(q_1+q_2)_\mu(k_1+k_2)^\mu \gamma_\nu v_q \bar{v}_k \gamma^\nu + (\not{k}_1 + \not{k}_2) v_q \bar{v}_k (\not{q}_1 + \not{q}_2) - \frac{8}{3} m_f m_q v_q \bar{v}_k \\
&- 2(\not{q}_1 + \not{q}_2 - 2m_f) v_q \bar{v}_k (\not{k}_1 + \not{k}_2 - 2m_q)\Big] u_k
\end{aligned}
\tag{30}
$$

The last term in the above equation is actually just the Dirac equation in momentum space,

$$(\not{k} - m_f) u_f(k) = 0, \tag{31}$$

and therefore equals zero. The amplitude is then

$$i\mathcal{M}(q\bar{q} \to f\bar{f}) = -iQ_f \frac{\pi C_4}{2} \bar{u}_q \left[(q_1+q_2)_\mu (k_1+k_2)^\mu \gamma_\nu v_q \bar{v}_k \gamma^\nu + (\not{k}_1 + \not{k}_2) v_q \bar{v}_k (\not{q}_1 + \not{q}_2) - \frac{8}{3} m_f m_q v_q \bar{v}_k \right] u_k \tag{32}$$

for the spin-2 massive KK graviton propagators. In order to talk about measurable quantities, this will have to be squared; an arduous calculation. The complex conjugate of $\mathcal{M}$ is:

$$-i\mathcal{M}^*(q\bar{q} \to f\bar{f}) = iQ_f \frac{\pi C_4^*}{2} \bar{u}_k \left[(q_1+q_2)_\mu (k_1+k_2)^\mu \gamma^\nu v_k \bar{v}_q \gamma_\nu + (\not{q}_1 + \not{q}_2) v_k \bar{v}_q (\not{k}_1 + \not{k}_2) - \frac{8}{3} m_f m_q v_k \bar{v}_q \right] u_q \tag{33}$$

Obviously, $|\mathcal{M}|^2$ is a large expression, but since the KK gravitons do not favour any particular chirality states, the calculation mainly involves application of the trace mechanism when taking the spin average. Before doing any sort of assumption, one will end up with[13]

$$\begin{aligned}
\langle \mathcal{M}|^2\rangle = \pi^2 Q_f^2 |C_4|^2 \Bigg\{ & 2\left[(k_1+k_2)_\mu (q_1+q_2)^\mu\right]^2 \Big[(q_1\cdot k_1)(q_2\cdot k_2) + (q_2\cdot k_1)(q_1\cdot k_2) \\
&\qquad - 2(q_1\cdot q_2)(k_1\cdot k_2 + m_q^2) \\
&\qquad - 2(k_1\cdot k_2)(q_1\cdot q_2 + m_l^2) \\
&\qquad + 4(k_1\cdot k_2 + m_q^2)(q_1\cdot q_2 + m_l^2) \Big] \\
& + 2\left[(k_1+k_2)_\mu (q_1+q_2)^\mu\right] (k_1+k_2)^\nu (q_1+q_2)_\sigma \\
&\qquad \times \Big[(q_1\cdot k_1) q_{2,\nu} k_2^\sigma + (q_1\cdot k_2) q_{2,\nu} k_1^\sigma \\
&\qquad + (q_2\cdot k_1) q_{1,\nu} k_2^\sigma + (q_2\cdot k_2) q_{1,\nu} k_1^\sigma \\
&\qquad - (k_{1,\nu} k_2^\sigma + k_1^\sigma k_{2,\nu})(q_1\cdot q_2 + m_l^2) \\
&\qquad - (q_{1,\nu} q_2^\sigma + q_1^\sigma q_{2,\nu})(k_1\cdot k_2 + m_q^2) \\
&\qquad + g_\nu^\sigma (q_1\cdot q_2 + m_l^2)(k_1\cdot k_2 + m_q^2) \Big] \\
& + \left[2(k_1+k_2)_\mu q_1^\mu (k_1+k_2)_\nu q_2^\nu - (k_1+k_2)^2 (q_1\cdot q_2 + m_l^2)\right] \\
& + \left[2(k_1+k_2)_\mu q_1^\mu (k_1+k_2)_\nu q_2^\nu - (k_1+k_2)^2 (q_1\cdot q_2 + m_l^2)\right] \\
& + \frac{64}{9} m_l^2 m_q^2 (q_1\cdot q_2 + m_l^2)(k_1\cdot k_2 + m_q^2) \Bigg\}
\end{aligned} \tag{34}$$

Now it would be nice to ignore the mass-terms of the SM particles such that

$$\begin{aligned}
k_1 &= (E, 0, 0, E) \\
k_2 &= (E, 0, 0, -E) \\
q_1 &= (E, -\boldsymbol{k}) \\
q_2 &= (E, \boldsymbol{k})
\end{aligned} \tag{35}$$

in the centre of mass (CM) frame. Note that the momentum convention so far has been that it follows the fermion line. For the above to be true, one has to change all k_2 and q_2 to $-k_2$ and $-q_2$, respectively. Since $E = \frac{s}{2}$, neglecting mass-terms requires very high energies[14]. To see that this is a reasonable assumption one only has to look at the propagator factor $|C_4|^2$.

The factor $|C_4|^2$ depends on the model, as one will have to perform the KK tower sum. As was explained in

[13] Where $\langle|\mathcal{M}|^2\rangle = \frac{1}{4}\sum_{\text{all spins}} |\mathcal{M}|^2$.

[14] Especially since the top quark, a possible quark annihilation, has a mass of ~ 173 GeV.

the text, in the ADD model, the sum may approximated by an integral and one gets the solution (assuming $s << M_S$):

$$- i\kappa^2 D_{ADD}(s) = -i8\pi C_4 \simeq \begin{cases} -\frac{8\pi}{M_S^4}\ln\left(\frac{M_S^2}{s^2}\right) & \text{if } d=2 \\ -\frac{16\pi}{(d-2)M_S^4} & \text{if } d>2 \end{cases} \tag{36}$$

In the RS1 model, the KK masses are separated by a few TeV, and for simplicity only the first mode excitation will be considered. Therefore:

$$- i\kappa^2 D_{RS}(s) = -i8\pi C_4 \simeq= -\frac{i\kappa 2}{k_G^2 - m_1^2 + i\epsilon} \quad , \quad \kappa = \sqrt{2}\frac{\beta_1}{m_1}\frac{k}{M_{Pl}} \approx 0.05 \times \sqrt{2}\frac{3.38}{m_1} \tag{37}$$

Because of the predicted sizes of M_S and m_1, s will have to be large or $\langle|\mathcal{M}|^2\rangle$ is negligible compared to the SM background.
Without mass-terms, one can put the following identities to use (note the sign change on k_2 and q_2):

$$\begin{aligned}
(k_1-k_2)^2 = (q_1-q_2)^2 = (k_1-k_2)_\mu(q_1-q_2)^\mu &= 4E^2 \\
|\boldsymbol{q}_1| = \sqrt{E^2 - m_f^2} &\approx E \\
\boldsymbol{q}_1 \times \boldsymbol{e}_z = |\boldsymbol{q}_1|\cos(\theta) &\approx E\cos(\theta) \\
q_1 \cdot q_2 = k_1 \cdot k_2 &= 2E^2 \\
k_1 \cdot q_2 = k_2 \cdot q_1 &= E^2(1+\cos(\theta)) \\
k_1 \cdot q_1 = k_2 \cdot q_2 &= E^2(1-\cos(\theta)) \\
(q_1-q_2)_\mu k_2^\mu = (k_1-k_2)_\mu q_2^\mu &= 2E^2\cos(\theta) \\
(q_1-q_2)_\mu q_2^\mu = (k_1-k_2)_\mu k_2^\mu &= 2E^2 \\
(q_1-q_2)_\mu k_1^\mu = (k_1-k_2)_\mu q_1^\mu &= -2E^2\cos(\theta) \\
(q_1-q_2)_\mu q_1^\mu = (k_1-k_2)_\mu k_1^\mu &= -2E^2
\end{aligned} \tag{38}$$

where θ is the angle between $\boldsymbol{k}_1$ and $\boldsymbol{q}_1$ in the centre of mass. Having neglected all mass terms, equation 34 then reduces to

$$\langle|\mathcal{M}|^2\rangle = 64Q_f^2\pi^2|C_4|^2\left(\frac{s}{2}\right)^8\left[1 - 3\cos^2(\theta) + 4\cos^4(\theta)\right] \tag{39}$$

which is precisely the result gotten reference
The differential cross section for two-body collision (particle A and B) process is

$$\frac{d\sigma}{d\cos(\theta)} = \frac{1}{2E_A 2E_B|v_A - v_B|}\frac{1}{16\pi}\frac{2|\boldsymbol{p}_1|}{E_{CM}}|\mathcal{M}_{fi}|^2 \tag{40}$$

In the centre of mass frame one has $E_A = E_B = E = \frac{s}{2}$ and $|v_A - v_B| = 2$. Finally, one has:

$$\frac{d\sigma_H}{d\cos(\theta)} = \frac{2}{\pi s^2}\sqrt{1 - 4\frac{m_q^2}{s^2}}\frac{16m_f m_q}{v^2}Q_f^2\left[s^4 - \frac{s^2}{2}(m_q^2 + m_f^2) + m_q^2 m_f^2\right] \tag{41}$$

$$\frac{d\sigma_{\gamma,Z}}{d\cos(\theta)} = \frac{\pi\alpha^2 Q_f^2}{s}\left[A\left(1+\cos^2(\theta)\right) + B\cos(\theta)\right] \quad , \quad \alpha = \alpha_{EM} \simeq \frac{1}{137} \tag{42}$$

$$\frac{d\sigma_G}{d\cos(\theta)} = \frac{1}{2}\pi Q_f^2 s^6|C_4|^2\left[1 - 3\cos^2(\theta) + 4\cos^4(\theta)\right] \tag{43}$$

where

$$\begin{aligned}
A &= 1 + 2\mathrm{Re}(\chi)c_V^e c_V^\tau + |\chi|^2\left[(c_V^e)^2 + (c_A^e)^2\right]\left[(c_V^\tau)^2 + (c_A^\tau)^2\right] \\
B &= 4\mathrm{Re}(\chi)c_A^e c_A^\tau + 8|\chi|^2 c_V^e c_A^e c_V^\tau c_A^\tau
\end{aligned} \tag{44}$$

and $\chi = \frac{g_Z^2 P_Z s}{4e^2}$. As one is mostly interested in comparing the angular distribution of the different cases, the plots will be normalized and one does not really case about the coefficients in front. Note that the vector channel's distribution will change with CM energy. For $s \approx 9.1492$ TeV the coefficients are the same ($A \approx B$).

References

[1] E.W. Dvergsnes, "*Extra Dimensions at Particle Colliders*", 2004, UiB

[2] T. Han, J.D. Lykken, and R.-J. Zhang, *Phys. Rev.* **D 59** (1999) 105006, doi:10.1103/PhysRevD.59.105006 hep-ph/9811350v4

[3] B.C. Allanach, K. Odagiri, M.A. Parker, and B.R. Webber, JHEP 0009:019,2000, doi:10.1088/1126-6708/2000/09/019, hep-ph/0006114v2

[4] M. Fierz, P. Wolfgang, Proc. Roy. Soc. Lond. A173: 211–232, doi: 10.1098/rspa.1939.0140

[5] E.E. Boos, Y.A. Kubyshin, M.N. Smolyakov, and I.P. Volobuev, NPI MSU 2001-21/661, hep-th/0105304

[6] H. Davoudiasl, J.L Hewett, and T. G. Rizzo, Phys. Rev. Lett. **84** 3370 (1999), doi:10.1103/PhysRevLett.84.2080, hep-ph/9911262v2

[7] Y.A. Kubyshin, (2001), hep-ph/0111027v2