# Extra Dimensions & Gravitons FYS4560

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# Introduction



There are 3 different types of spatial extra dimensions (ED) models.

- *Large*
- *Warped*
- *Universal*

Arkani-Hamed, Dimopoulos, and Dvali (1998) propsed the ADD-model[1] (large ED)
Randall and Sundrum (1999) proposed RS1-model[2] (warped ED)

[1] arXiv: hep-ph/9803315
[2] arXiv: hep-ph/9905221

# Kaluza-Klein theory

From linearised gravity,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{1}$$

we can find an interaction term $h_{\mu\nu}T^{\mu\nu}$ and a kinetic term for $h_{\mu\nu}$ in the Lagrangian density. However, a mass term for $h_{\mu\nu}$ can be suggested as

$$ah_{\mu\nu}h^{\mu\nu} + b(\eta_{\mu\nu}h^{\mu\nu})^2 \tag{2}$$

Markus Fierz and Wolfgang Pauli (1939) showed[4] that $a = -b$ in order to avoid unphysical results. The result of this was the Fierz-Pauli Lagrangian for massive gravity.

# Kaluza-Klein theory



Kaluza assumed the metric could be written:

$$\hat{h}_{ab} = V_d^{-1/2} \begin{pmatrix} h_{\mu\nu} + \eta_{\mu\nu}\phi & A_{\mu i} \\ A_{\nu j} & 2\phi_{ij} \end{pmatrix} \tag{3}$$

Kaluza assumed that these fields could be written as expansions, i.e.

$$h_{\mu\nu}(x,y) = \sum_{\substack{n=\{n_1,n_2,\ldots,n_5\}; \\ n_i \in \mathbb{Z}\ \forall\ i}} h_{n,\mu\nu}(x) Y_n(y) \tag{4}$$

and similarly for $A_{\mu i}$and $\phi$. The modes have to satisfy the Fierz-Pauli equations of motion. These, when combined, will show that $h_{\mu\nu}$ satisfies:

$$(\Box + m_n^2)(h_{n,\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h^{\sigma}_{n,\sigma}) = 0 \tag{5}$$

where $m_n^2 = \frac{4\pi^2 n^2}{R^2}$.

# Kaluza-Klein theory

Through some non-trivial steps, one can find a Lagrangian with mass eigenstates $\tilde{h}_{n,\mu\nu}$, $\tilde{A}_{n,\mu i}$, and $\tilde{\phi}_{n,ij}$, defined from the previous fields. From the Hilbert-Einstein action:

$$S_d = \int d^4x \sqrt{-\hat{g}} \mathcal{L}(\hat{g}, S, V, F) \tag{6}$$

where $\hat{g}_{\mu\nu} = \eta_{\mu\nu} + \kappa(h_{\mu\nu} + \eta_{\mu\nu}\phi_{ii})$, we can find Feynman rules:

$$G_n^{\mu\nu\rho\sigma} = i\frac{\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \frac{2}{3}\eta^{\mu\nu}\eta^{\rho\sigma}}{k_G^2 - m_n^2 + i\epsilon} \tag{7}$$

$$-\frac{i\kappa}{8}\left[\gamma_\mu(k_1+k_2)_\nu + \gamma_\nu(k_1+k_2)_\mu - 2\eta_{\mu\nu}(\not{k}_1 + \not{k}_2 - 2m_f)\right] \tag{8}$$

# ADD-model



The ADD model is mainly based on 3 features:

- There exists $d$ new spatial compact dimensions, with compactification volume $V_d$.
- The Planck scale is very low, at the order of one TeV,
- The SM degrees of freedom are localized on a 3D-brane, stretching along 3 non-compact spatial dimensions (i.e. the SM particles move in normal spacetime, not in the new dimension(s)).

The main idea is that $\bar{M} \sim 1\text{TeV}$. By demanding $S_4 = S_{4+d}$, one finds the reduction formula:

$$M_{Pl}^2 = V_d \bar{M}^{d+2} \sim R^d \bar{M}^{d+2} \tag{9}$$

# ADD-model



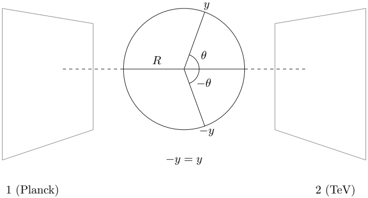
With $M_{Pl} \approx 1.22 \times 10^{16}$ TeV, one finds:

$$\begin{aligned}
d = 1 \quad &\rightarrow \quad R \sim 10^{11} \text{ m} \\
d = 2 \quad &\rightarrow \quad R \sim 0.1 \text{ mm} \\
d = 3 \quad &\rightarrow \quad R \sim 10^{-7} \text{ mm} \\
\ldots \\
d = 6 \quad &\rightarrow \quad R \sim 10^{-11} \text{ mm}
\end{aligned}$$

But Newton's law of gravity must still hold for $r >> R$.

# RS1-Model

RS1-model is quite different. They assumed space had $S^1/\mathbb{Z}^2$ "orbifold" structure. At two points there existed a 3-brane:




From Poincare invariance, it was found:

$$ds^2 = e^{-2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \tag{10}$$

Due to $\mathbb{Z}^2$-symmetry, have $A(y) = A(-y) = A(|y|)$. Solving the Einstein equations let Randall and Sundrum find $A(y) = k|y|$, $k = \sqrt{\frac{-\Lambda}{24M^3}}$.

# RS1-Model

The metric forces a new reduction formula:

$$M_{Pl}^2 = \frac{M_5^3}{k}\left[1 - e^{-2kR\pi}\right] \tag{11}$$

For $M_5$ to solve the hierarchy problem, one has to require $kR \sim 12 \rightarrow e^{k\pi R} \sim 10^{15}$.
With a new metric form, one has to repeat the KK mode expansion:

$$h_{\mu\nu}(x,\theta) = \sum_{n=0}^{\infty} h_{\mu\nu}^{(n)}(x)\frac{\chi_n(\theta)}{R} \tag{12}$$

where $\chi_0(y) = 2\sqrt{kR}e^{-2kR|\theta|}$, and

$$\chi_n(y) = N_n\left[C_1 Y_2\left(\frac{m_n}{k}e^{kR|\theta|}\right) + C_2 J_2\left(\frac{m_n}{k}e^{kR|\theta|}\right)\right] \quad , \quad n \neq 0 \tag{13}$$

# RS1-model



On TeV brane, KK mode masses are

$$m_n = \beta_n k e^{-kR\pi}, \quad J_1(\beta_n) = 0 \, \forall \, n \in \mathbb{N} \tag{14}$$

Unlike the ADD-model, the masses are clearly separated. This means the sum over the KK tower is not necessary.
Comparing RS1 Hilbert-Einstein action with the general KK action, one will see that:

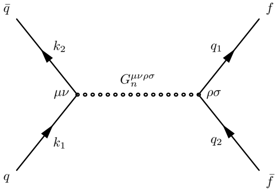
$$\kappa = \sqrt{2} \frac{\beta_1}{m_1} \frac{k}{M_{Pl}} \tag{15}$$

meaning RS1 is completely determined by to constants.

# Proton-proton collisions



One channel for measuring the models is through the $pp \to G \to l^+l^-$ ($q\bar{q} \to G \to l^+l^-$):



# Proton-proton collisions



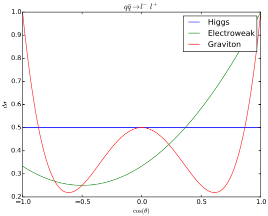
After a little math...

# Proton-proton collisions



$$\frac{d\sigma_G}{d\cos(\theta)} = \frac{1}{2}\pi Q_f^2 s^6 |C_4|^2 \left[1 - 3\cos^2(\theta) + 4\cos^4(\theta)\right] \tag{16}$$

where $C_4 = \frac{\kappa^2}{8\pi} D(s)$ and $D(s) \equiv \sum_n \frac{1}{k_G^2 - m_n^2 + i\epsilon}$.



# Gravitons at the LHC

# Conclusions

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