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June 7, 2016

Introduction

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Introduction

There are 3 different types of spatial extra dimensions (ED) models.

- Large
- Warped
- Universal

Arkani-Hamed, Dimopoulos, and Dvali (1998) propsed the ADD-model¹ (large ED)

Randall and Sundrum (1999) proposed RS1-model² (warped ED)

²arXiv: hep-ph/9905221



¹arXiv: hep-ph/9803315

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From linearised gravity,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},\tag{1}$$

we can find an interaction term $h_{\mu\nu}T^{\mu\nu}$ and a kinetic term for $h_{\mu\nu}$ in the Lagrangian density. However, a mass term for $h_{\mu\nu}$ can be suggested as

$$ah_{\mu\nu}h^{\mu\nu} + b(\eta_{\mu\nu}h^{\mu\nu})^2 \tag{2}$$

Markus Fierz and Wolfgang Pauli (1939) showed[4] that a=-b in order to avoid unphysical results. The result of this was the Fierz-Pauli (FP) Lagrangian for massive gravity.

Kaluza-Klein theory

Kaluza-Klein towers

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Assumed the metric can be written:

$$\hat{h}_{ab} = V_d^{-1/2} \begin{pmatrix} h_{\mu\nu} + \eta_{\mu\nu}\phi & A_{\mu i} \\ A_{\nu j} & 2\phi_{ij} \end{pmatrix}$$
 (3)

A KK reduction of the FP Lagrangian can be done by assuming the fields can be written as expansions, i.e.

$$h_{\mu\nu}(x,y) = \sum_{\substack{n = \{n_1, n_2, \dots, n_5\}; \\ n_i \in \mathbb{Z} \ \forall \ i}} h_{n,\mu\nu}(x) Y_n(y) \tag{4}$$

and similarly for A_{ui} and ϕ . The modes have to satisfy the Fierz-Pauli equations of motion. These, when combined, will show that $h_{\mu
u}$ satisfies:

$$(\Box + m_n^2)(h_{n,\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h_{n,\sigma}^{\sigma}) = 0$$
 (5)

where $m_n^2 = \frac{4\pi^2 n^2}{R^2}$.

Kaluza-Klein theory

Interactive Lagrangian

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Conclusion

Through some non-trivial steps, one can find a Lagrangian with mass eigenstates $\tilde{h}_{n,\mu\nu}$, $\tilde{A}_{n,\mu i}$, and $\tilde{\phi}_{n,ij}$, defined from the previous fields. From the Hilbert-Einstein action:

$$S_d = \int d^4x \sqrt{-\hat{g}} \mathcal{L}(\hat{g}, S, V, F)$$
 (6)

where $\hat{g}_{\mu\nu}=\eta_{\mu\nu}+\kappa(h_{\mu\nu}+\eta_{\mu\nu}\phi_{ii})$, we can find Feynman rules:

$$G_n^{\mu\nu\rho\sigma} = i \frac{\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \frac{2}{3}\eta^{\mu\nu}\eta^{\rho\sigma}}{k_G^2 - m_n^2 + i\epsilon} \tag{7}$$

$$-\frac{i\kappa}{8} \left[\gamma_{\mu} (k_1 + k_2)_{\nu} + \gamma_{\nu} (k_1 + k_2)_{\mu} - 2\eta_{\mu\nu} (k_1 + k_2 - 2m_f) \right]$$
 (8)

where
$$\kappa = \sqrt{16\pi G_N}$$

ADD model

Theoretical basis

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The ADD model is mainly based on 3 assumptions:

- There exists d new spatial compact dimensions, with compactification volume V_d .
- The Planck scale is very low, at the order of one TeV,
- The SM degrees of freedom are localized on a 3D-brane, stretching along 3 non-compact spatial dimensions (i.e. the SM particles move in normal spacetime, not in the new dimension(s)).

The main idea is that $\bar{M} \sim 1 \text{TeV}$. By demanding $S_4 = S_{4+d}$, one finds the reduction formula:

$$M_{Pl}^2 = V_d \bar{M}^{d+2} \sim R^d \bar{M}^{d+2}$$
 (9)

ADD model

Compact dimensions

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With $M_{Pl} \approx 1.22 \times 10^{16}$ TeV, one finds:

$$d=1 \rightarrow R \sim 10^{11} \, \mathrm{m}$$

$$d=2$$
 $ightarrow$ $R\sim 0.1~\mathrm{mm}$

$$d=3$$
 \rightarrow $R\sim 10^{-7}$ mm

. . .

$$d=6$$
 \rightarrow $R\sim 10^{-11}\,\mathrm{mm}$

But Newton's law of gravity must still hold for r >> R.

ADD model

Sum over propagators

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Graviton masses are identical to KK mode masses: $m_n^2 = \frac{4\pi^2 n^2}{R^2}$ For d=6, have $R^{-1} \sim$ MeV, and must sum over KK tower. We rewrite:

$$-i\kappa^2 D(s) = \sum_n \frac{\kappa^2}{s - m_n^2 + i\epsilon} \rightarrow \int_0^{\inf} dm_n^2 \rho(m_n) \frac{\kappa^2}{s - m_n^2 + i\epsilon}$$
(10)

where $ho(m_n)=rac{R^dm_n^{d-2}}{(4\pi)^{d/2}\Gamma(d/2)}.$ Solve to find

$$-i\kappa^{2}D(s) = -i8\pi C_{4} \simeq \begin{cases} -\frac{8\pi}{M_{5}^{4}} \ln\left(\frac{M_{5}^{2}}{s^{2}}\right) & \text{if } d = 2\\ -\frac{16\pi}{(d-2)M_{5}^{4}} & \text{if } d > 2 \end{cases}$$
 (11)

 M_S is expected to have the same magnitude as \bar{M} (TeV).

RS1 model

Theoretical basis

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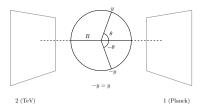
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Conclusion

RS1-model is quite different. They assumed space had S^1/\mathbb{Z}^2 "orbifold" structure³. At two points there existed a 3-brane:



From Poincare invariance, it was found:

$$ds^{2} = e^{-2A(y)} \eta_{\mu\nu} dx^{\mu} dx^{\nu} + dy^{2}$$
 (12)

Due to \mathbb{Z}^2 -symmetry, have A(y) = A(-y) = A(|y|). Solving the Einstein equations let Randall and Sundrum find A(y) = k|y|,

$$k = \sqrt{\frac{-\Lambda}{24M^3}}$$
.

³A 5-dimensional anti-de Sitter space.

RS1 model

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The metric forces a new reduction formula:

$$M_{PI}^2 = \frac{M_5^3}{k} \left[1 - e^{-2kR\pi} \right] \tag{13}$$

For M_5 to solve the hierarchy problem, one has to require $kR\sim 12 \to e^{k\pi R}\sim 10^{15}$.

With a new metric form, one has to repeat the KK mode expansion:

$$h_{\mu\nu}(x,\theta) = \sum_{n=0}^{\infty} h_{\mu\nu}^{(n)}(x) \frac{\chi_n(\theta)}{R}$$
 (14)

where $\chi_0(y) = 2\sqrt{kR}e^{-2kR|\theta|}$, and

$$\chi_n(y) = N_n \left[C_1 Y_2 \left(\frac{m_n}{k} e^{kR|\theta|} \right) + C_2 J_2 \left(\frac{m_n}{k} e^{kR|\theta|} \right) \right] \quad , \quad n \neq 0$$
(15)

RS1 model

Differences

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Conclusio

On the TeV brane, KK mode masses are

$$m_n = \beta_n k e^{-kR\pi}, \quad J_1(\beta_n) = 0 \ \forall \ n \in \mathbb{N}$$
 (16)

Unlike the ADD-model, the masses are clearly separated. This means the sum over the KK tower is not necessary.

Comparing RS1 Hilbert-Einstein action with the general KK action, one will see that:

$$\kappa = \sqrt{2} \frac{\beta_1}{m_1} \frac{k}{M_{Pl}} \tag{17}$$

meaning RS1 is completely determined by two constants.

Simplest diagram: angular distribution (1)

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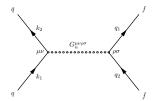
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Conclusions

One channel for measuring the models is through the $pp \to G \to l^+l^-$ ($q\bar{q} \to G \to l^+l^-$):



Simplest diagram: angular distribution (2)

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Conclusion:

$$\begin{split} i\mathcal{M}(q\bar{q} \;\to\; f\bar{f}) &= -Q_f \sum_n \frac{i\kappa^2}{64} \bar{u}_q \left[\gamma_\rho (q_1 + q_2)\sigma + \gamma\sigma (q_1 + q_2)\rho - 2\eta_\rho\sigma (\rlap/q_1 + \rlap/q_2 - 2m_f) \right] v_q \\ &\qquad \times \left[\frac{\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \frac{2}{3}\eta^{\mu\nu}\eta^{\rho\sigma}}{k_G^2 - m_n^2 + i\epsilon} \right] \\ &\qquad \times \bar{v}_k \left[\gamma_\mu (k_1 + k_2)\nu + \gamma\nu (k_1 + k_2)\mu - 2\eta_\mu\nu (\rlap/k_1 + \rlap/k_2 - 2m_f) \right] u_k \\ &\qquad \vdots \\ &\qquad \vdots \\ &= i\mathcal{M}(q\bar{q} \;\to\; f\bar{f}) = -iQ_f \frac{\pi C_4}{2} \bar{u}_q \Big[(q_1 + q_2)\mu (k_1 + k_2)^\mu \gamma_\nu v_q \bar{v}_k \gamma^\nu + (\rlap/k_1 + \rlap/k_2)v_q \bar{v}_k (\rlap/q_1 + \rlap/q_2) - \frac{8}{3}m_f m_q v_q \bar{v}_k \Big] u_k \end{split} \label{eq:continuous} \end{split}$$

Simplest diagram: angular distribution (3)

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Conclusions

Square it...

$$\begin{split} \langle |\mathcal{M}|^2 \rangle &= \pi^2 \, Q_I^2 \, |\mathcal{C}_4|^2 \bigg\{ 2 \, \Big[(k_1 + k_2)_\mu (q_1 + q_2)^\mu \Big]^2 \, \Big[(q_1 \cdot k_1) (q_2 \cdot k_2) + (q_2 \cdot k_1) (q_1 \cdot k_2) \\ &\qquad \qquad - 2 (q_1 \cdot q_2) (k_1 \cdot k_2 + m_q^2) \\ &\qquad \qquad - 2 (k_1 \cdot k_2) (q_1 \cdot q_2 + m_I^2) \\ &\qquad \qquad + 4 (k_1 \cdot k_2 + m_q^2) (q_1 \cdot q_2 + m_I^2) \Big] \\ &\qquad \qquad + 2 \, \Big[(k_1 + k_2)_\mu (q_1 + q_2)^\mu \Big] \, (k_1 + k_2)^\nu (q_1 + q_2) \sigma \\ &\qquad \qquad \times \, \Big[(q_1 \cdot k_1) q_{2,\nu} k_2^\sigma + (q_1 \cdot k_2) q_{2,\nu} k_1^\sigma \\ &\qquad \qquad + (q_2 \cdot k_1) q_{1,\nu} k_2^\sigma + (q_2 \cdot k_2) q_{1,\nu} k_1^\sigma \\ &\qquad \qquad - (k_{1,\nu} k_2^\sigma + k_1^\sigma k_{2,\nu}) (q_1 \cdot q_2 + m_I^\sigma) \\ &\qquad \qquad - (q_{1,\nu} k_2^\sigma + q_1^\sigma q_{2,\nu}) (k_1 \cdot k_2 + m_q^2) \Big] \\ &\qquad \qquad + g_\nu^\sigma \, (q_1 \cdot q_2 + m_I^\sigma) (k_1 \cdot k_2 + m_q^2) \Big] \\ &\qquad \qquad + \Big[2 (k_1 + k_2)_\mu \, q_1^\mu (k_1 + k_2)_\nu \, q_2^\nu - (k_1 + k_2)^2 (q_1 \cdot q_2 + m_I^\sigma) \Big] \\ &\qquad \qquad + \Big[2 (k_1 + k_2)_\mu \, q_1^\mu (k_1 + k_2)_\nu \, q_2^\nu - (k_1 + k_2)^2 (q_1 \cdot q_2 + m_I^\sigma) \Big] \\ &\qquad \qquad + \frac{64}{\sigma} \, m_I^\sigma \, q_1^\sigma \, (q_1 \cdot q_2 + m_I^\sigma) (k_1 \cdot k_2 + m_q^\sigma) \Big\} \end{split}$$

(19)

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Conclusions

Would like to neglect SM masses; need to know how large q_1 , q_2 , k_1 , k_2 are in comparison to $m_{q,l}$.

We have $C_4 = \frac{\kappa^2}{8\pi} D(s)$ and $D(s) \equiv \sum_n \frac{1}{k_G^2 - m_n^2 + i\epsilon}$, so must find size of D(s).

Have:

$$-i\kappa^2 D_{ADD}(\mathfrak{s}) = -i8\pi C_4 \simeq \begin{cases} -\frac{8\pi}{M_2^4} \ln\left(\frac{M_2^2}{\mathfrak{s}^2}\right) & \text{if } d = 2\\ -\frac{16\pi}{(d-2)M_2^4} & \text{if } d > 2 \end{cases}$$
 (20)

and

$$-i\kappa^2 D_{RS}(s) = -i8\pi C_4 \simeq -\frac{i\kappa^2}{k_G^2 - m_1^2 + im_1\Gamma_1} \quad , \quad \kappa = \sqrt{2} \frac{\beta_1}{m_1} \frac{k}{M_{Pl}} \simeq 0.1 \times \sqrt{2} \frac{3.83}{m_1}$$
 (21)

where $\Gamma_n=rac{295}{96}rac{eta_n^2}{10\pi}m_n\left(rac{k}{M_{Pl}}
ight)^2~
ightarrow~\Gamma_1pprox 0.0111m_1$

Assuming massless, end up with:

$$\langle |\mathcal{M}|^2 \rangle = 64Q_f^2 \pi^2 |C_4|^2 \left(\frac{s}{2}\right)^8 \left[1 - 3\cos^2(\theta) + 4\cos^4(\theta)\right]$$
 (22)

Simplest diagram: angular distribution (5)

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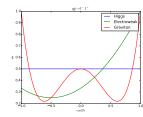
Conclusion

Using

$$\frac{d\sigma}{d\cos(\theta)} = \frac{1}{2E_A 2E_B |v_A - v_B|} \frac{1}{16\pi} \frac{2|\mathbf{p}_1|}{E_{CM}} |\mathcal{M}_{fi}|^2$$
(23)

find

$$\frac{d\sigma_G}{d\cos(\theta)} = \frac{1}{2}\pi Q_f^2 s^6 |C_4|^2 \left[1 - 3\cos^2(\theta) + 4\cos^4(\theta)\right]$$
 (24)



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Process at hadron colliders

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Conclusio

At the LHC, there are 5 possible channels for the ADD model:

Indirect:

$$ightharpoonup pp
ightarrow ext{jet} + \not\!\! E_T$$

$$\blacksquare pp \rightarrow \gamma + \not\!\!E_T$$

Direct:

■
$$gg \rightarrow G \rightarrow I^+I^-$$

$$\blacksquare q\bar{q} \rightarrow G \rightarrow \gamma\gamma$$

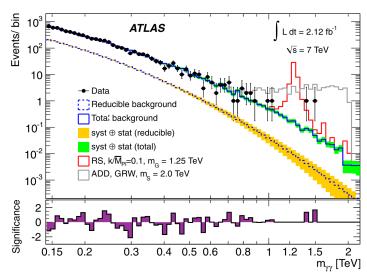
$$\blacksquare q\bar{q} \rightarrow G \rightarrow I^+I^-$$

For the RS1 model, the gravitons have very short lifetimes. the result is mainly a dijet product, and sometimes (a few percent) dilepton. The small width gives large peaks.

Gravitons at the LHC 2012 data from ATLAS

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Table 2 95% CL limits on the value of M_5 (in TeV) for various implementations of the ADD model, using both LO (k-factor = 1) and NLO (k-factor = 1.70) theory cross section calculations.

k-Factor value	GRW	Hewett		HLZ				
		Pos	Neg	n = 3	n = 4	n = 5	n = 6	n = 7
1	2.73	2.44	2.16	3.25	2.73	2.47	2.30	2.17
1.70	2.97	2.66	2.27	3.53	2.97	2.69	2.50	2.36

Table 3 95% CL lower limits on t

95% CL lower limits on the mass (GeV) of the lightest RS graviton, for various values of k/\overline{M}_{pl} . The results are shown for the diphoton channel alone and for the combination of the diphoton channel with the dilepton results of Ref. [12], using both LO (k-factor = 1) and NLO (k-factor = 1.75) theory cross section calculations.

k-Factor value	Channel(s) used	95% CL limit [TeV] $\frac{k/\overline{M}_{Pl} \text{ value}}{}$					
		0.01	0.03	0.05	0.1		
1	$G \rightarrow \gamma \gamma$	0.74	1.26	1.41	1.79		
	$G ightarrow \gamma \gamma / ee/\mu \mu$	0.76	1.32	1.47	1.90		
1.75	$G \rightarrow \gamma \gamma$	0.79	1.30	1.45	1.85		
	$G \rightarrow \gamma \gamma / ee/\mu \mu$	0.80	1.37	1.55	1.95		

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Conclusions

- What KK theory is, and "how" it provides graviton interactions.
- The ADD model provides a new interpretation of the hierarchy problem.
- The RS1 model solves it, giving two new free parameters.
- The graviton is very unique due to spin-2.
- Only lower bounds found at colliders so far.

References

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Conclusions

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Thank you