Extra Dimensions & Gravitons FYS4560

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Introduction

There are 3 different types of spatial extra dimensions (ED) models.

- *Large*
- *Warped*
- *Universal*

Arkani-Hamed, Dimopoulos, and Dvali (1998) propsed the ADD-model[1] (large ED)
Randall and Sundrum (1999) proposed RS1-model[2] (warped ED)

[1] arXiv: hep-ph/9803315
[2] arXiv: hep-ph/9905221

From linearised gravity,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{1}$$

we can find an interaction term $h_{\mu\nu}T^{\mu\nu}$ and a kinetic term for $h_{\mu\nu}$ in the Lagrangian density. However, a mass term for $h_{\mu\nu}$ can be suggested as

$$ah_{\mu\nu}h^{\mu\nu} + b(\eta_{\mu\nu}h^{\mu\nu})^2 \tag{2}$$

Markus Fierz and Wolfgang Pauli (1939) showed[4] that $a = -b$ in order to avoid unphysical results. The result of this was the Fierz-Pauli (FP) Lagrangian for massive gravity.

Assumed the metric can be written:

$$\hat{h}_{ab} = V_d^{-1/2} \begin{pmatrix} h_{\mu\nu} + \eta_{\mu\nu}\phi & A_{\mu i} \\ A_{\nu j} & 2\phi_{ij} \end{pmatrix} \tag{3}$$

A KK reduction of the FP Lagrangian can be done by assuming the fields can be written as expansions, i.e.

$$h_{\mu\nu}(x,y) = \sum_{\substack{n=\{n_1,n_2,\ldots,n_5\};\\ n_i\in\mathbb{Z}\,\forall\, i}} h_{n,\mu\nu}(x) Y_n(y) \tag{4}$$

and similarly for $A_{\mu i}$and ϕ. The modes have to satisfy the Fierz-Pauli equations of motion. These, when combined, will show that $h_{\mu\nu}$ satisfies:

$$(\Box + m_n^2)(h_{n,\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h^{\sigma}_{n,\sigma}) = 0 \tag{5}$$

where $m_n^2 = \frac{4\pi^2 n^2}{R^2}$.

Through some non-trivial steps, one can find a Lagrangian with mass eigenstates $\tilde{h}_{n,\mu\nu}$, $\tilde{A}_{n,\mu i}$, and $\tilde{\phi}_{n,ij}$, defined from the previous fields. From the Hilbert-Einstein action:

$$S_d = \int d^4x \sqrt{-\hat{g}}\mathcal{L}(\hat{g}, S, V, F) \tag{6}$$

where $\hat{g}_{\mu\nu} = \eta_{\mu\nu} + \kappa(h_{\mu\nu} + \eta_{\mu\nu}\phi_{ii})$, we can find Feynman rules:

$$G_n^{\mu\nu\rho\sigma} = i\frac{\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \frac{2}{3}\eta^{\mu\nu}\eta^{\rho\sigma}}{k_G^2 - m_n^2 + i\epsilon} \tag{7}$$

$$-\frac{i\kappa}{8}\left[\gamma_\mu(k_1+k_2)_\nu + \gamma_\nu(k_1+k_2)_\mu - 2\eta_{\mu\nu}(\not{k}_1 + \not{k}_2 - 2m_f)\right] \tag{8}$$

where $\kappa = \sqrt{16\pi G_N}$

The ADD model is mainly based on 3 assumptions:

- There exists d new spatial compact dimensions, with compactification volume V_d.
- The Planck scale is very low, at the order of one TeV,
- The SM degrees of freedom are localized on a 3D-brane, stretching along 3 non-compact spatial dimensions (i.e. the SM particles move in normal spacetime, not in the new dimension(s)).

The main idea is that $\bar{M} \sim 1\text{TeV}$. By demanding $S_4 = S_{4+d}$, one finds the reduction formula:

$$M_{Pl}^2 = V_d \bar{M}^{d+2} \sim R^d \bar{M}^{d+2} \tag{9}$$

ADD model

Compact dimensions

With $M_{Pl} \approx 1.22 \times 10^{16}$ TeV, one finds:

$$\begin{aligned}
d = 1 \quad &\rightarrow \quad R \sim 10^{11} \text{ m} \\
d = 2 \quad &\rightarrow \quad R \sim 0.1 \text{ mm} \\
d = 3 \quad &\rightarrow \quad R \sim 10^{-7} \text{ mm} \\
\ldots & \\
d = 6 \quad &\rightarrow \quad R \sim 10^{-11} \text{ mm}
\end{aligned}$$

But Newton's law of gravity must still hold for $r >> R$.

ADD model
Sum over propagators

Graviton masses are identical to KK mode masses: $m_n^2 = \frac{4\pi^2 n^2}{R^2}$
For $d = 6$, have $R^{-1} \sim$ MeV, and must sum over KK tower. We rewrite:

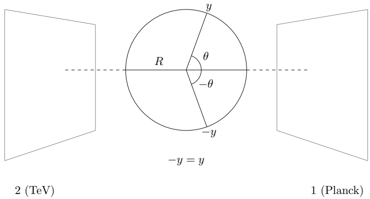
$$-i\kappa^2 D(s) = \sum_n \frac{\kappa^2}{s - m_n^2 + i\epsilon} \rightarrow \int_0^{\mathrm{inf}} dm_n^2 \rho(m_n) \frac{\kappa^2}{s - m_n^2 + i\epsilon} \quad (10)$$

where $\rho(m_n) = \frac{R^d m_n^{d-2}}{(4\pi)^{d/2}\Gamma(d/2)}$. Solve to find

$$-i\kappa^2 D(s) = -i8\pi C_4 \simeq \begin{cases} -\frac{8\pi}{M_S^4} \ln\left(\frac{M_S^2}{s^2}\right) & \text{if } d = 2 \\ -\frac{16\pi}{(d-2)M_S^4} & \text{if } d > 2 \end{cases} \quad (11)$$

M_S is expected to have the same magnitude as $\bar{M}$ (TeV).

RS1-model is quite different. They assumed space had $S^1/\mathbb{Z}^2$ "orbifold" structure[3]. At two points there existed a 3-brane:



From Poincare invariance, it was found:

$$ds^2 = e^{-2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \tag{12}$$

Due to $\mathbb{Z}^2$-symmetry, have $A(y) = A(-y) = A(|y|)$. Solving the Einstein equations let Randall and Sundrum find $A(y) = k|y|$, $k = \sqrt{\frac{-\Lambda}{24M^3}}$.

[3] A 5-dimensional anti-de Sitter space.

The metric forces a new reduction formula:

$$M_{Pl}^2 = \frac{M_5^3}{k}\left[1 - e^{-2kR\pi}\right] \tag{13}$$

For M_5 to solve the hierarchy problem, one has to require $kR \sim 12 \rightarrow e^{k\pi R} \sim 10^{15}$.
With a new metric form, one has to repeat the KK mode expansion:

$$h_{\mu\nu}(x,\theta) = \sum_{n=0}^{\infty} h_{\mu\nu}^{(n)}(x)\frac{\chi_n(\theta)}{R} \tag{14}$$

where $\chi_0(y) = 2\sqrt{kR}e^{-2kR|\theta|}$, and

$$\chi_n(y) = N_n\left[C_1 Y_2\left(\frac{m_n}{k}e^{kR|\theta|}\right) + C_2 J_2\left(\frac{m_n}{k}e^{kR|\theta|}\right)\right] \quad , \quad n \neq 0 \tag{15}$$

RS1 model
Differences

On the TeV brane, KK mode masses are

$$m_n = \beta_n k e^{-kR\pi}, \quad J_1(\beta_n) = 0 \; \forall \; n \in \mathbb{N} \tag{16}$$

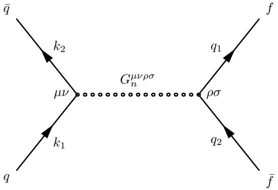
Unlike the ADD-model, the masses are clearly separated. This means the sum over the KK tower is not necessary.
Comparing RS1 Hilbert-Einstein action with the general KK action, one will see that:

$$\kappa = \sqrt{2}\frac{\beta_1}{m_1}\frac{k}{M_{Pl}} \tag{17}$$

meaning RS1 is completely determined by two constants.

Proton-proton collisions
Simplest diagram: angular distribution (1)

One channel for measuring the models is through the $pp \to G \to l^+l^-$ ($q\bar{q} \to G \to l^+l^-$):



Proton-proton collisions
Simplest diagram: angular distribution (2)

$$
\begin{aligned}
i\mathcal{M}(q\bar{q} \to f\bar{f}) = -Q_f \sum_n \frac{i\kappa^2}{64} \bar{u}_q & \left[\gamma_\rho (q_1+q_2)_\sigma + \gamma_\sigma (q_1+q_2)_\rho - 2\eta_{\rho\sigma}(\not{q}_1 + \not{q}_2 - 2m_f)\right] v_q \\
& \times \left[\frac{\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \frac{2}{3}\eta^{\mu\nu}\eta^{\rho\sigma}}{k_G^2 - m_n^2 + i\epsilon}\right] \\
& \times \bar{v}_k \left[\gamma_\mu (k_1+k_2)_\nu + \gamma_\nu (k_1+k_2)_\mu - 2\eta_{\mu\nu}(\not{k}_1 + \not{k}_2 - 2m_f)\right] u_k \\
& \vdots \\
= i\mathcal{M}(q\bar{q} \to f\bar{f}) = -iQ_f \frac{\pi C_4}{2} \bar{u}_q & \left[(q_1+q_2)_\mu (k_1+k_2)^\mu \gamma_\nu v_q \bar{v}_k \gamma^\nu + (\not{k}_1 + \not{k}_2) v_q \bar{v}_k (\not{q}_1 + \not{q}_2) - \frac{8}{3} m_f m_q v_q \bar{v}_k\right] u_k
\end{aligned} \tag{18}
$$

Square it...

$$
\begin{aligned}
\langle|\mathcal{M}|^2\rangle = \pi^2 Q_f^2 |C_4|^2 \Big\{ & 2\left[(k_1+k_2)_\mu (q_1+q_2)^\mu\right]^2 \Big[(q_1\cdot k_1)(q_2\cdot k_2) + (q_2\cdot k_1)(q_1\cdot k_2) \\
& \qquad - 2(q_1\cdot q_2)(k_1\cdot k_2 + m_q^2) \\
& \qquad - 2(k_1\cdot k_2)(q_1\cdot q_2 + m_l^2) \\
& \qquad + 4(k_1\cdot k_2 + m_q^2)(q_1\cdot q_2 + m_l^2)\Big] \\
& + 2\left[(k_1+k_2)_\mu (q_1+q_2)^\mu\right](k_1+k_2)^\nu (q_1+q_2)_\sigma \\
& \qquad \times \Big[(q_1\cdot k_1) q_{2,\nu} k_2^\sigma + (q_1\cdot k_2) q_{2,\nu} k_1^\sigma \\
& \qquad + (q_2\cdot k_1) q_{1,\nu} k_2^\sigma + (q_2\cdot k_2) q_{1,\nu} k_1^\sigma \\
& \qquad - (k_{1,\nu} k_2^\sigma + k_1^\sigma k_{2,\nu})(q_1\cdot q_2 + m_l^2) \\
& \qquad - (q_{1,\nu} q_2^\sigma + q_1^\sigma q_{2,\nu})(k_1\cdot k_2 + m_q^2) \\
& \qquad + g_\nu^\sigma (q_1\cdot q_2 + m_l^2)(k_1\cdot k_2 + m_q^2)\Big] \\
& + \left[2(k_1+k_2)_\mu q_1^\mu (k_1+k_2)_\nu q_2^\nu - (k_1+k_2)^2 (q_1\cdot q_2 + m_l^2)\right] \\
& + \left[2(k_1+k_2)_\mu q_1^\mu (k_1+k_2)_\nu q_2^\nu - (k_1+k_2)^2 (q_1\cdot q_2 + m_l^2)\right] \\
& + \frac{64}{9} m_l^2 m_q^2 (q_1\cdot q_2 + m_l^2)(k_1\cdot k_2 + m_q^2)\Big\}
\end{aligned}
\tag{19}
$$

Proton-proton collisions
Simplest diagram: angular distribution (4)

Would like to neglect SM masses; need to know how large q_1, q_2, k_1, k_2 are in comparison to $m_{q,l}$.
We have $C_4 = \frac{\kappa^2}{8\pi} D(s)$ and $D(s) \equiv \sum_n \frac{1}{k_G^2 - m_n^2 + i\epsilon}$, so must find size of $D(s)$.
Have:

$$-i\kappa^2 D_{ADD}(s) = -i8\pi C_4 \simeq \begin{cases} -\frac{8\pi}{M_S^4} \ln\left(\frac{M_S^2}{s^2}\right) & \text{if } d = 2 \\ -\frac{16\pi}{(d-2)M_S^4} & \text{if } d > 2 \end{cases} \tag{20}$$

and

$$-i\kappa^2 D_{RS}(s) = -i8\pi C_4 \simeq= -\frac{i\kappa^2}{k_G^2 - m_1^2 + im_1\Gamma_1} \quad , \quad \kappa = \sqrt{2}\frac{\beta_1}{m_1}\frac{k}{M_{Pl}} \simeq 0.1 \times \sqrt{2}\frac{3.83}{m_1} \tag{21}$$

where $\Gamma_n = \frac{295}{96}\frac{\beta_n^2}{10\pi} m_n \left(\frac{k}{M_{Pl}}\right)^2 \rightarrow \Gamma_1 \approx 0.0111 m_1$
Assuming massless, end up with:

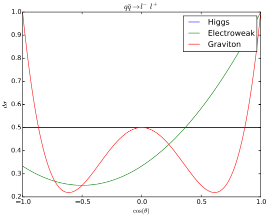
$$\langle|\mathcal{M}|^2\rangle = 64 Q_f^2 \pi^2 |C_4|^2 \left(\frac{s}{2}\right)^8 \left[1 - 3\cos^2(\theta) + 4\cos^4(\theta)\right] \tag{22}$$

Using

$$\frac{d\sigma}{d\cos(\theta)} = \frac{1}{2E_A 2E_B |v_A - v_B|} \frac{1}{16\pi} \frac{2|\mathbf{p}_1|}{E_{CM}} |\mathcal{M}_{fi}|^2 \tag{23}$$

find

$$\frac{d\sigma_G}{d\cos(\theta)} = \frac{1}{2}\pi Q_f^2 s^6 |C_4|^2 \left[1 - 3\cos^2(\theta) + 4\cos^4(\theta)\right] \tag{24}$$



Gravitons at the LHC

Process at hadron colliders

At the LHC, there are 5 possible channels for the ADD model:

- Indirect:
  - $pp \rightarrow \text{jet} + \not{E}_T$
  - $pp \rightarrow \gamma + \not{E}_T$
- Direct:
  - $gg \rightarrow G \rightarrow l^+l^-$
  - $q\bar{q} \rightarrow G \rightarrow \gamma\gamma$
  - $q\bar{q} \rightarrow G \rightarrow l^+l^-$

For the RS1 model, the gravitons have very short lifetimes. the result is mainly a dijet product, and sometimes (a few percent) dilepton. The small width gives large peaks.

Gravitons at the LHC

2012 data from ATLAS

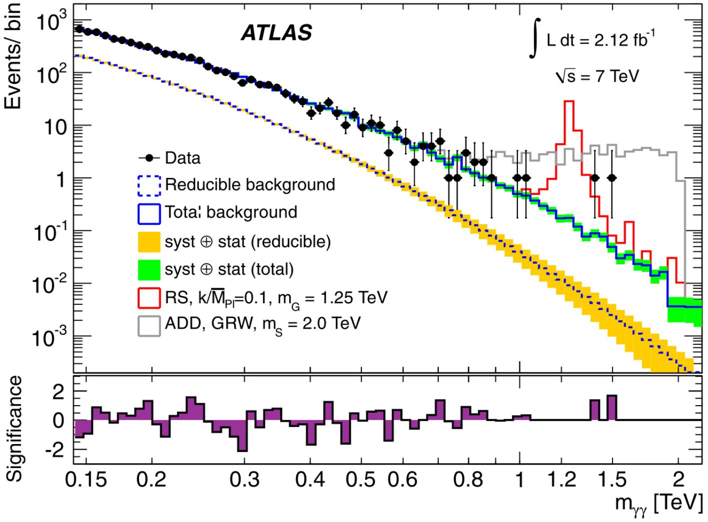


Table 2
95% CL limits on the value of M_S (in TeV) for various implementations of the ADD model, using both LO (k-factor = 1) and NLO (k-factor = 1.70) theory cross section calculations.

k-Factor value	GRW	Hewett		HLZ				
		Pos	Neg	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$
1	2.73	2.44	2.16	3.25	2.73	2.47	2.30	2.17
1.70	2.97	2.66	2.27	3.53	2.97	2.69	2.50	2.36

Table 3
95% CL lower limits on the mass (GeV) of the lightest RS graviton, for various values of $k/\overline{M}_{Pl}$. The results are shown for the diphoton channel alone and for the combination of the diphoton channel with the dilepton results of Ref. [12], using both LO (k-factor = 1) and NLO (k-factor = 1.75) theory cross section calculations.

k-Factor value	Channel(s) used	95% CL limit [TeV]			
		$k/\overline{M}_{Pl}$ value			
		0.01	0.03	0.05	0.1
1	$G \to \gamma\gamma$	0.74	1.26	1.41	1.79
	$G \to \gamma\gamma/ee/\mu\mu$	0.76	1.32	1.47	1.90
1.75	$G \to \gamma\gamma$	0.79	1.30	1.45	1.85
	$G \to \gamma\gamma/ee/\mu\mu$	0.80	1.37	1.55	1.95

Conclusions

- What KK theory is, and "how" it provides graviton interactions.
- The ADD model provides a new interpretation of the hierarchy problem.
- The RS1 model solves it, giving two new free parameters.
- The graviton is very unique due to spin-2.
- Only lower bounds found at colliders so far.

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Thank you