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FINAL PROJECT

Higher Dimensions;
Theoretical and Experimental Aspects

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Abstract

The Randall-Sundrum (RS) model of compacted dimensions will be studied, together with the postulate of the graviton particle. Quark-anti-quark production of the simplest possible massive graviton (1st order tower of Kaluza-Klein excitations) will be calculated.

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1 Introduction

Higher dimensions, also called extra dimensions, are physical models for the dimensionality of our universe, mostly suggested with the goal of explaining the hierarchy problem of the standard model. There are *many* theories for higher dimensions. The most recognised are:

- *Large extra dimensions*: The often-heard theory that gravity acts through several dimensions, therefore becoming weaker. It originates from the ADD model as an attempt to solve the hierarchy problem¹.
- *Warped extra dimensions*: Describing our universe as a five-dimensional anti-de Sitter space, and claiming the SM particles are localized on a $(3 + 1)$ -dimensional brane(s).
- *Universal extra dimensions*: All particles move universally through the extra dimensions, unlike to two other models where only gravity propagates through them.

Obviously, a thorough description of any of these models is near impossible for such a small paper, let alone all the models together. Therefore, a brief outline of the theory behind the two currently most promising² models will be given.

The first is the large extra dimension model by Arkani-Hamed, Dimopoulos, and Dvali (ADD). Originally, it was proposed as a model to explain the hierarchy problem (why the weak force is 10^{32} times stronger than gravity, among other problems). The extra dimensions³ are then suggested as planes into which gravity, assumed just as strong as the other forces, spreads. Therefore gravity becomes "diluted", while the known SM particles stay in $(1,3)$ -spacetime.

The second model is the warped extra dimension model by Randall and Sundrum (RS), made due to disliking the current universal extra dimensions models. They assumed that, rather than having universal extra dimensions in which all particles propagate, there is a small extra dimension. This means they model our world as a 5-dimensional anti-de Sitter space⁴. By small, it means the extra dimension has a large curvature, or is *warped*. From general relativity, gravity and curvature are very much the same thing, and therefore the extra dimension, called the Planckbrane, can easily host gravitons.

A question that then springs to mind is why exactly gravitons and extra dimensions are connected (other than gravitons "carrying" gravity). If the standard model is expanded, but without inclusion of extra dimensions, to include a graviton field, then measuring it would be, at best, very optimistic⁵. Should any extra dimensional model be true, it would certainly be desirable to prove it by measurement. Finding a particle with the properties of the graviton would mean that at least *some* extra dimensions model is true.

2 Kaluza-Klein theory

The reason Kaluza-Klein theory is discussed is because one requires knowledge of the so-called "Kaluza-Klein towers"; massive excitations of an expansion model⁶ of the spacetime metric. While RS1 only considers a single extra dimension, ADD works for different numbers of dimensions. However, for the sake of simplicity, and comparison, only one extra dimension will be considered.

Before adding extra dimensions, the question of how get so-called "massive gravity"; a massive field carries the gravitational force. First, one starts with linearised gravity. Problems in general relativity can be approximated by perturbing Minkowski spacetime;

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{1}$$

¹The "hierarchy problem" is the problem in explaining why gravity and the weak force are so weak compared to QED and QCD.

²"Promising" in the sense that they provide measurable outcomes, and fit very well with what we already know from the standard model. The problem is that so far they have predicted nothing. Finding a graviton would possibly confirm either theory.

³Today, 6 dimensions is the most common expansion.

⁴This will be explained later on.

⁵Refer to "Can Gravitons be Detected?" by Rothman and Boughn (<https://arxiv.org/pdf/gr-qc/0601043v3.pdf>) for an impression of the problem.

⁶This expansion depends on the model, and is where RS and ADD differ.

where $g_{\mu\nu}$ is the approximated spacetime metric, $\eta_{\mu\nu}$ is the Minkowski metric, and $h_{\mu\nu}$ is the perturbation, and is what eventually was suggested to be gravitons. The Lagrangian density will then have a interactive term on the form $h_{\mu\nu}T^{\mu\nu}$, and a kinetic term for $h_{\mu\nu}$. The field $h_{\mu\nu}$ can be given mass by saying the Lagrangian density also has a term $a h_{\mu\nu}h^{\mu\nu} + b(\eta_{\mu\nu}h^{\mu\nu})^2$. Markus Fierz and Wolfgang Pauli then, in 1939, wrote an article⁷ in which they proved that $a = -b$ in order to avoid unphysical results. The result is the Fierz-Pauli Lagrangian for massive gravity⁸ [SOURCE]:

$$\frac{1}{\kappa^2} \sqrt{|g|} R = \frac{1}{4} (\partial^\mu h^{\nu\rho} \partial_\mu h_{\nu\rho} - \partial^\mu h^\nu_\nu \partial_\mu h^\rho_\rho - 2\partial^\nu h_{\nu\mu} \partial_\rho h^{\rho\mu} + 2\partial_\nu h^{\nu\mu} \partial_\mu h^\rho_\rho) + \mathcal{O}(\kappa) \quad (2)$$

Now, additional spatial dimensions may be added. These dimensions will not change the form of equation 2, so one can simply let indices change from greek to latin, i.e. $\mu \rightarrow a, \nu \rightarrow b$, etc..., where $a, b, \dots \in \{4+d\}$. It is then assumed the perturbative field has a form

$$\hat{h}_{ab} = V_d^{-1/2} \begin{pmatrix} h_{\mu\nu} + \eta_{\mu\nu}\phi & A_{\mu i} \\ A_{\nu j} & 2\phi_{ij} \end{pmatrix} \quad (3)$$

where V_d is the volume of the compactified space, $\eta_{\mu\nu}\phi$ is a Weyl rescaling and $A_{\mu i}$ is some tensor field. Since the fields are compact, one can claim periodicity of the field, such that a Fourier expansions is possible:

$$h_{\mu\nu}(x, y) = \sum_{\substack{n=\{n_1, n_2, \dots, n_5\}; \\ n_i \in \mathbb{Z} \forall i}} h_{n, \mu\nu}(x) Y_n(y) \quad (4)$$

where $Y_n(y)$ are orthogonal, normalized eigenfuntions of the Laplace operator on the internal space⁹:

$$\Delta_{K_d} Y_n(\theta) = \frac{\lambda_n}{R^2} Y_n(\theta) \quad (5)$$

where R is the "characteristic size" of the space K_d (tied to the volume V_d). The exact form of $Y_n(y)$ depends on the structure and size of the new dimension. This series expansion is called the Kalulza-Klein (KK) tower of modes, and one value of n is called mode n , and the corresponding term the n 'th excitation. The field $h_{\mu\nu}(x, y)$ has to satisfy the d'Alembert equation, and in doing this one finds it necessary to redefine the fields $h_{\mu\nu}(x, y)$, $A_{\nu j}$, and ϕ_{ij} , to $\tilde{h}_{n, \mu\nu}$, $\tilde{A}_{n, \mu i}$, and $\tilde{\phi}_{n, ij}$. The detailed steps are omitted, as they are many and non-intuitive. These new fields, when put into the Lagrangian, will give mass eigenstates. This redefinition of fields can be thought of as an analogue to the rotations from the CKM matrix in QCD theory (needed for mass eigenstates in the Lagrangian), but it is not the same (not a rotation). From the equation of motion one can go one to show that the masses are:

$$m_n^2 = \frac{4\pi^2 n^2}{R^2} \quad (6)$$

A relation that will be of importance later is the reduction formula:

$$M_{Pl}^2 = V_d M^{d+2} \quad (7)$$

where $M_{Pl} = G_{N(4)}^{-1/2}$ is the 4-dimensional Planck mass and $M^{d+2} = G_{N(4+d)}^{-1/(4+d)}$ is the fundamental mass scale in the new model. The relation is derived by demanding the Einstein-Hilbert action to be the same with and without the new dimension(s)¹⁰, and performing the integrals by using KK mode expansion on the integrand. I.e one demands:

$$S_{E(4)} = S_{E(d+4)} \quad (8)$$

⁷Fierz, Markus; Pauli, Wolfgang (1939). "On relativistic wave equations for particles of arbitrary spin in an electromagnetic field". *Proc. Roy. Soc. Lond.* A173: 211–232.

⁸Note that this is *not* the only way to get a massive gravity Lagrangian, and it has some problems as well. It serves well as an introductory example, however.

⁹The extra dimension/space, denoted K_d .

¹⁰This is because the new dimension(s) must reproduce what we observe, and 4D spacetime fits very well with observations.

where

$$S_{E(D)} = \int d^D x \sqrt{-G_D} \frac{1}{16\pi G_{N(D)}} \mathcal{R}^D(G_{ab}) \quad (9)$$

With the principles of the KK tower and massive gravity, one can start to consider theories that are based on extra dimensions.

As is the point with Feynman diagrams, Feynman rules for the graviton would help do calculations. The propagator of massive, spin-2 KK excitations are given by the Fierz-Pauli equation of motion (see reference BLADIBLA), and is

$$G_n^{\mu\nu\rho\sigma} = i \frac{\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \frac{2}{3}\eta^{\mu\nu}\eta^{\rho\sigma}}{k_G^2 - m_n^2 + i\epsilon} \quad (10)$$

The vertex factors with different fields (scalar, spinor, vector-boson) are found by inserting the conserved energy-momentum tensor, for said field, into the Lagrangian, except for the spinor-field. Spinor-graviton couplings, which will be of interest later on, must be found by using the vierbein formalism¹¹. The massive spinor-graviton spin-2 coupling is

$$-\frac{i\kappa}{8} [\gamma_\mu(k_1 + k_2)_\nu + \gamma_\nu(k_1 + k_2)_\mu - 2\eta_{\mu\nu}(\not{k}_1 + \not{k}_2 - 2m_f)] \quad (11)$$

3 The Arkani-Hamed-Dimopoulos-Dvali model

The ADD model is mainly based on 3 features:

- There exists d new spatial compact dimensions, with compactification volume V_d .
- The Planck scale is very low, at the order of one TeV,
- The SM degrees of freedom are localized on a 3D-brane, stretching along 3 non-compact spatial dimensions (i.e. the SM particles move in normal spacetime, not in the new dimension(s)).

The idea is that the electroweak scale is the only fundamental scale in the universe, and that the true Planck scale is actually of the same order. from the reduction formula, one could introduce d new spatial dimensions and find their size, i.e.

$$\frac{M_{Pl}^2}{M^{d+2}} = V_d \sim R^d \quad , \quad \mathcal{O}(M^{d+2}) = \mathcal{O}(m_{EW}) \quad (12)$$

where R is the compactification radius as before. With new dimensions, Newton's law for gravitational force would instead be proportional to $r^{-(d+2)}$, which is of course not true. However, since the extra dimensions are compact, only objects with distances less $r \ll R$ feel this new force, while it still goes as r^{-2} for $r \gg R$. Therefore, for $d = 1$, the radius is of the same order as the earth-moon distance, and therefore d can't be equal one. $d = 2$ is not possible either. For $d > 2$, the radius is so small that van der Waal forces prevent us from conducting "table-top" experiments, and is where particle physics experiments at CERN become relevant. If ADD is true, then it should be possible to determine the radius by measuring the graviton, which is a direct consequence of extra dimensions. Since the mass m_n is inversely proportional to the size, it would be possible to determine the new Planck scale.

A problem with the massive KK spin-2 propagator is that there are infinitely many KK excitations, each with its own mass. When calculating a Feynman diagram, one has to, in principle, perform a sum over all modes, as one can't know which mode will appear.

In the ADD model, the masses are *very* small, due to the size of R by the reduction formula and the size of M , and evenly spaced. This means one can approximate the sum by an integral over the modes (n).

¹¹A way with which to rewrite the spacetime metric. The details of this are many and unnecessary for purpose of this paper.

4 The Randall-Sundrum model

4.1 The hierarchy problem in RS1

The RS model assumes that there are two points on the S^1/\mathbb{Z}^2 orbifold in which 3-branes are compactified. What this means is that there are two 3-dimensional, non-compact branes that are connected at every point by a "circle", see figure SOME FIG.

The two branes, 1 and 2, are located on points $\theta = 0$ and $\theta = \pi$, which means the branes are separated by a distance $2R$, where R is the circle radius.

As mentioned, the model set out to explain the hierarchy problem, and will be briefly explained how below. Firstly, the action of the model is given by the sum of the Hilbert-Einstein action and the "matter part":

$$S_5 = S_E + S_M = \int d^4x \int_{-L}^L dy \sqrt{-\tilde{g}} (M_5^3 \mathcal{R}_5 - \Lambda_5) \quad (13)$$

where Λ_5 is the five dimensional cosmological constant, \tilde{g} is the determinant of the (full space) metric tensor, and M_5 is the new mass scale. In order to match real world observations, the new metric must uphold Poincaré invariance, which can be shown to lead to:

$$ds^2 = e^{-2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \quad (14)$$

where $A(y)$ is called the *warp factor*, hence the name "warped" extra dimensions. Since the orbifold abides \mathbb{Z}^2 symmetry, $A(y) = A(-y) \rightarrow A(y) = k|y|$. Solving the Einstein equations, as Randall and Sundrum did in their original article, lets one find that $A(y) = \pm ky$, where $k \equiv \sqrt{\frac{-\Lambda}{24M_5^3}}$ is a constant ($\Lambda = -24M_5^3 k^2$). Because the metric is fundamentally different¹², the reduction formula becomes a little different. Apparently, it now becomes¹³;

$$M_{Pl}^2 = \frac{M_5^3}{k} \left[1 - e^{-2kR\pi} \right] \quad (15)$$

where one has set $|y| = \pi R$ (after all, it is a circle). In order to get a new mass scale which solves the hierarchy problem, it has to require that $kR \sim 12 \rightarrow e^{k\pi R} \sim 10^{15}$. There is a problem with action above, which is that it does not include the energy densities of the two branes, which are:

$$S_1 = \int_{B_1} \int_{S^1/\mathbb{Z}^2} d^4x dy \sqrt{-\tilde{g}(x, y)} \lambda_1 \delta(y) \quad (16)$$

$$S_2 = \int_{B_2} \int_{S^1/\mathbb{Z}^2} d^4x dy \sqrt{-\tilde{g}(x, y)} \lambda_2 \delta(y - L) \quad (17)$$

where $\tilde{g}(x, 0) = g_1$ and $\tilde{g}(x, L) = g_2$ have been used (i.e. $\tilde{g}(x, y)$ contains the regular 4-dimensional metric of each brane). So, the total action for the space is $S = S_E + S_M + S_1 + S_2$.

4.2 RS1 gravitons

Since the metric in RS1 is a bit different from that used in the KK mode expansion, one has to do it again to get the new current model's graviton masses. One starts with a metric transform, given by:

$$ds^2 = e^{-2k|y|} (\eta_{\mu\nu} + \tilde{h}_{\mu\nu}(x, y)) dx^\mu dx^\nu + (1 + \phi(x)) dy^2 \quad (18)$$

However, this means there will be cross-terms ($\phi \tilde{h}_{\mu\nu}$ -terms) in the Lagrangian that prevent mass eigenstates. It can be shown¹⁴ that it can be diagonalized. One will end up with two fields, $h_{\mu\nu}$ and φ , which

¹²It has actually undergone a so-called Weyl transformation.

¹³Finding the new reduction formula is done by claiming equation 8 to be true and doing the calculation, same as was done to get equation 7.

¹⁴See <http://arxiv.org/abs/hep-th/0105304v3>, section 2, for a rigorous derivation.

will be used hereafter.

Then, a KK mode expansion is done on $h_{\mu\nu}(x, y)$, giving

$$h_{\mu\nu}(x, y) = \sum_{n=0}^{\infty} h_{\mu\nu}^{(n)}(x) \frac{\chi_n(y)}{R}, \quad (19)$$

where $\chi_0(y) = 2\sqrt{kR}e^{-2k|y|}$ and

$$\chi_n(y) = N_n \left[C_1 Y_2 \left(\frac{m_n}{k} e^{k|y|} \right) + C_2 J_2 \left(\frac{m_n}{k} e^{k|y|} \right) \right], \quad n \neq 0 \quad (20)$$

While it seems out of the blue, this is the result of diagonalizing the Lagrangian, which is a rather intricate procedure. The functions χ_n are eigenfunction of an equation met during the diagonalising of the Lagrangian. J_2 and Y_2 are the Bessel functions of the first and second kind, respectively. The constants C_1 and C_2 can be determined by the boundary conditions on the vacuum energy terms of the energy-momentum tensor (the delta functions),

$$T_{ab} = \lambda_1 \sqrt{g_1} g_{\mu\nu}^{(1)} \delta_a^\mu \delta_b^\nu \delta(y) + \lambda_2 \sqrt{g_2} g_{\mu\nu}^{(2)} \delta_a^\mu \delta_b^\nu \delta(y - \pi R), \quad (21)$$

which give $C_1 = Y_1 \left(\frac{m_n}{k} \right)$ and $C_2 = -J_1 \left(\frac{m_n}{k} \right)$. Here $\lambda_1 = -\lambda_2 = 24M_5^3 k$, but this will not be discussed further. The tensor leads to an eigenvalue equation, from which one will find that the spin-2 graviton masses are connected to the roots of the Bessel function J_1 by:

$$m_n = \beta_n k e^{-kR\pi}, \quad J_1(\beta_n) = 0 \quad \forall n \in \mathbb{N} \quad (22)$$

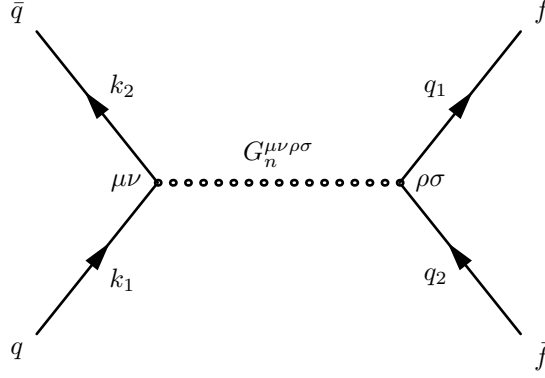
The $h_{\mu\nu}^{(0)}(x)$ field describes the massless graviton, while the $n \geq 1$ states are the massive KK modes, and finally φ describes the massless *radion*.

Now that the masses are known, one can see that they behave very different from those of the ADD model. Here, they are of the order of TeV, and separated by a few orders of TeV (due to the roots $\beta_n = 3.83, 7.02, 10.17, \dots$). This means there should be very clear, separate peaks in particle detectors, unlike that of the ADD model where the masses were so close they could be approximated by integration. With a working theory for the RS1 model, one would like to do calculations. Specifically, one would like for the Feynman rules mentioned earlier to still apply. This can be done by comparing the Hilbert-Einstein action for the general KK case, and the RS1 case. One will see that the actions are the same (to first order in κ) should one choose:

$$\kappa = \sqrt{2} \frac{\beta_1}{m_1} \frac{k}{M_{Pl}} \quad (23)$$

As one sees, the RS1 model is completely determined by the m_1 mass and the factor $\frac{k}{M_{Pl}}$, which is believed to lie between 0.01 and 0.1¹⁵.

¹⁵See H. Davoudiasl, J.L Hewett and T. G. Rizzo, Phys. Rev. Lett. **84**, 3370 (1999) [arXiv:hep-ph/9905221]

Figure 1: $q\bar{q} \rightarrow f\bar{f}$

5 Graviton production at the LHC

With a couple of theories well at hand, it would be interesting to know how they should appear in a particle collider such as the LHC. A quark-anti-quark annihilation in proton-proton collisions can produce a graviton as gravitons and fermion couple. Furthermore, this graviton can then decay to a lepton-pair, such as muons. Obviously there are many channels that give off a dimuon final state. However, there are no spin-2 particles in the standard model, and measuring the angular distribution would be expected to give a characteristic contour. One example of angular distribution is the differential cross section. Here, the forward-backward asymmetry would be interesting to study and compare to the corresponding scalar and vector channels (H and γ, Z), the two of which have already been studied in previous projects.

5.1 SM backgrounds

The spin-averaged amplitude for $e^+e^- \rightarrow \tau^+\tau^-$ in the electroweak channel was then found. The formula is quite general for all fermions in the high s limit, in the current case will be:

$$\langle |\mathcal{M}(q\bar{q} \rightarrow l^+l^-)|^2 \rangle = Q_f (A' (1 + \cos^2(\theta)) + B' \cos(\theta)) \quad (24)$$

where

$$\begin{aligned} A' &= e^4 + \frac{e^2 g_Z^2 P_Z s}{2} c_V^q c_V^l + \frac{|P_Z|^2 g_Z^4 s^2}{16} \left[(c_V^q)^2 + (c_A^q)^2 \right] \left[(c_V^l)^2 + (c_A^l)^2 \right] \\ B' &= e^2 g_Z^2 P_Z s c_A^q c_A^l + \frac{1}{2} |P_Z|^2 g_Z^4 s^2 c_V^q c_A^q c_V^l c_A^l \end{aligned} \quad (25)$$

and

- Q_f is the colour factor ($Q_f = 3$).
- $P_Z(s) = \frac{1}{s - m_Z^2 + i m_Z \Gamma_Z}$, where Γ_Z is the Z -boson decay width.
- c_A^f and c_V^f are the axial and vector coupling coefficients of the Z -boson to Dirac spinor f .
- g_Z is the weak coupling constant.

Furthermore, the corresponding Higgs channel is given by:

$$\langle |\mathcal{M}(q\bar{q} \rightarrow l^+l^-)|^2 \rangle = \frac{16 m_f m_q}{v^2} Q_f (k_{1,\mu} k_2^\mu - m_q^2) (q_{1,\mu} q_2^\mu - m_l^2) \quad (26)$$

where $v = 246$ GeV is the vacuum expectation value of the Higgs field. As one would expect, there is no angular dependency for a scalar field.

6 Conclusions

A Calculations

A detailed calculation for the channel $q\bar{q} \rightarrow l^+l^-$ is as follows.

The n 'th resonance spin-2 KK graviton propagator is given equation 10. The $G\bar{\psi}\psi$ -coupling is given in equation 11.

The diagram shown in figure 1 therefore has effective amplitudes:

$$\begin{aligned}
i\mathcal{M}(q\bar{q} \rightarrow f\bar{f}) &= -\sum_n \frac{i\kappa^2}{64} \bar{u}_q \left[\gamma_\rho(q_1 + q_2)_\sigma + \gamma_\sigma(q_1 + q_2)_\rho - 2\eta_{\rho\sigma}(\not{q}_1 + \not{q}_2 - 2m_f) \right] v_q \\
&\quad \times \left[\frac{\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \frac{2}{3}\eta^{\mu\nu}\eta^{\rho\sigma}}{k_G^2 - m_n^2 + i\epsilon} \right] \\
&\quad \times \bar{v}_k \left[\gamma_\mu(k_1 + k_2)_\nu + \gamma_\nu(k_1 + k_2)_\mu - 2\eta_{\mu\nu}(\not{k}_1 + \not{k}_2 - 2m_f) \right] u_k \\
&= -i\frac{\pi C_4}{4} \bar{u}_q \left[\gamma_\rho(q_1 + q_2)_\sigma + \gamma_\sigma(q_1 + q_2)_\rho - 2\eta_{\rho\sigma}(\not{q}_1 + \not{q}_2 - 2m_f) \right] v_q \\
&\quad \times \bar{v}_k \left[\gamma^\rho(k_1 + k_2)_\sigma + \gamma^\sigma(k_1 + k_2)_\rho + \frac{4}{3}\eta^{\sigma\rho} \right] u_k \\
&= -i\frac{\pi C_4}{2} \bar{u}_q \left[(q_1 + q_2)_\mu(k_1 + k_2)^\mu \gamma_\nu v_q \bar{v}_k \gamma^\nu + (\not{k}_1 + \not{k}_2) v_q \bar{v}_k (\not{q}_1 + \not{q}_2) \right. \\
&\quad \left. - 2(\not{q}_1 + \not{q}_2) v_q \bar{v}_k (\not{k}_1 + \not{k}_2) + 4 \left(m_q(\not{q}_1 + \not{q}_2) v_q \bar{v}_k + m_f v_q \bar{v}_k (\not{k}_1 + \not{k}_2) \right) \right. \\
&\quad \left. - \frac{32}{3} m_f m_q v_q \bar{v}_k \right] u_k
\end{aligned} \tag{27}$$

where $C_4 \equiv \frac{\kappa^2}{8\pi} D(s)$, $D(s) \equiv \sum_n \frac{1}{k_G^2 - m_n^2 + i\epsilon}$. The above expression can be rewritten as

$$\begin{aligned}
i\mathcal{M}(q\bar{q} \rightarrow f\bar{f}) &= -i\frac{\pi C_4}{2} \bar{u}_q \left[(q_1 + q_2)_\mu(k_1 + k_2)^\mu \gamma_\nu v_q \bar{v}_k \gamma^\nu + (\not{k}_1 + \not{k}_2) v_q \bar{v}_k (\not{q}_1 + \not{q}_2) - \frac{8}{3} m_f m_q v_q \bar{v}_k \right. \\
&\quad \left. - 2(\not{q}_1 + \not{q}_2 - 2m_f) v_q \bar{v}_k (\not{k}_1 + \not{k}_2 - 2m_q) \right] u_k
\end{aligned} \tag{28}$$

The last term in the above equation is actually just the Dirac equation in momentum space,

$$(\not{k} - m_f)u_f(k) = 0, \tag{29}$$

and therefore equals zero. The amplitude is then

$$i\mathcal{M}(q\bar{q} \rightarrow f\bar{f}) = -i\frac{\pi C_4}{2} \bar{u}_q \left[(q_1 + q_2)_\mu(k_1 + k_2)^\mu \gamma_\nu v_q \bar{v}_k \gamma^\nu + (\not{k}_1 + \not{k}_2) v_q \bar{v}_k (\not{q}_1 + \not{q}_2) - \frac{8}{3} m_f m_q v_q \bar{v}_k \right] u_k \tag{30}$$

for the spin-2 massive KK graviton propagators. The next step depends on the model, as one will have to perform the KK tower sum. As was explained in the text, in the ADD model, the sum may be approximated by an integral and one gets the solution (assuming $s \ll M_S^2$):

$$-i\kappa^2 D(s) = -i8\pi C_4 \simeq \begin{cases} -\frac{8\pi}{M_S^4} \ln\left(\frac{M_S^2}{s}\right) & \text{if } d = 2 \\ -\frac{16\pi}{(d-2)M_S^4} & \text{if } d > 2 \end{cases} \tag{31}$$