YEAR 1 ANNUAL PROGRESS REVIEW REPORT

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1. Introduction

Here, I will outline my PhD research progress since my program began in October 2024. I will begin by outlining a relevant literature review, where I will assume an understanding of the definitions of Coxeter and Artin groups, as well as basic related constructions. See [PS21] for a relevant introduction to Coxeter and Artin groups.

1.1. **Literature review.**

Let (W, S) denote a Coxeter system where W is the Coxeter group and S is its generating reflections. Associated to this system is an edge labelled graph Γ. The vertices of Γ correspond to elements of S. There is an edge labelled m connecting the vertices corresponding to $s, t \in S$ if there is a relation between s and t in W of the form $(st)^m = 1$. We use W_Γ and A_Γ as a shorthand for the Coxeter and Artin groups associated to a Coxeter system with graph Γ. Note that Γ also defines a Coxeter system, so we will use this notation in place of (W, S).

Let $|\Gamma|$ denote the number of vertices in Γ. Associated to a Coxeter system Γ, there is the so-called *Tits cone*, denoted T, which is a subset of $\mathbb{R}^{|\Gamma|}$. The Tits cone sees a canonical way of realising W_Γ as a linear group. The action of W_Γ on T is by reflections (corresponding to a possibly non-standard inner product) through hyperplanes intersecting T. These hyperplanes define a tiling of T where each tile is a simplical cone. If we take There is a conical, simplicial tiling of the Tits cone where each cell cell corresponds to a fundamental domain of W_Γ acting on $\mathbb{R}^{|\Gamma|}$.

References

[PS21] Giovanni Paolini and Mario Salvetti. Proof of the K(pi,1) conjecture for affine Artin groups. *Inventiones mathematicae*, 224(2):487–572, May 2021.

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