SINGAPORE INSTITUTE OF TECHNOLOGY

Engineering Physics

Revision

B.Eng(Hons) in Aerospace Engineering

Constant angular acceleration

The constant acceleration formulae are:

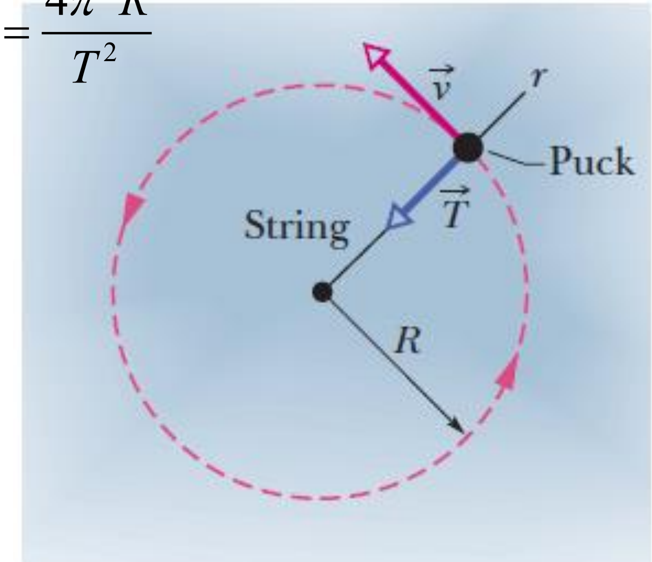
Linear Equation	t	$x(t)$ or $\theta(t)$	$v(t)$ or $\omega(t)$	a or α	Angular Equation
$v(t) = v_0 + at$	X		X	X	$\omega(t) = \omega_0 + \alpha t$
$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$	X	X		X	$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$v(t)^2 = v_0{}^2 + 2a\left[x(t) - x_0\right]$		X	X	X	$\omega(t)^2 = \omega_0{}^2 + 2\alpha\left[\theta(t) - \theta_0\right]$
$x(t) - x_0 = \frac{1}{2}\left[v_0 + v(t)\right]t$	X	X	X		$\theta(t) - \theta_0 = \frac{1}{2}\left[\omega_0 + \omega(t)\right]t$

Rotational motion

- In rotational motion, the rotating objects are subject to an acceleration.
- Therefore, there must be a net force.
- Acceleration magnitude is: $a_{rad} = {v^2}/{r}$ or $a_{rad} = \dfrac{4\pi^2 R}{T^2}$
- Therefore, for uniform motion the magnitude of the **net** force is equal to:

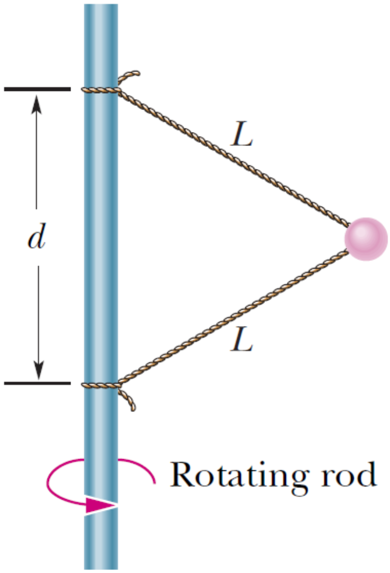
$$F_{net} = {mv^2}/{r} \text{ or } F_{net} = m\frac{4\pi^2 R}{T^2}$$

- The force is directed to the centre, like the acceleration.



Newton's laws example

A 1.34 kg ball is connected by means of two massless strings, each of length L = 1.70 m, to a vertical, rotating rod. The strings are tied to the rod with separation d = 1.70 m and are taut. The tension in the upper string is 35 N. What are the (a) tension in the lower string, (b) magnitude of the net force F_{net} on the ball, and (c) speed of the ball? (d) What is the direction of F_{net} ?



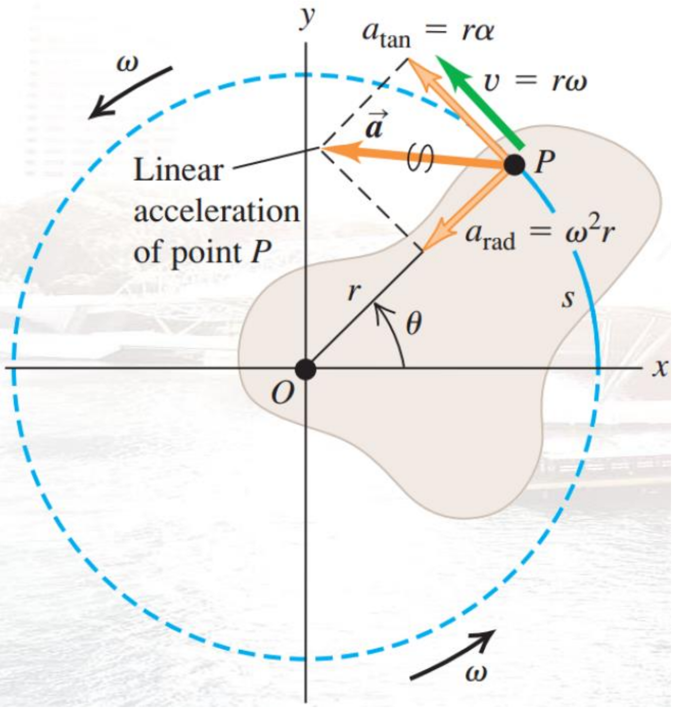
[Ans: T = 8.74 N; F = 37.9 N; v = 6,45 m/s; radially inward]

Linear and angular kinematics

$$\frac{ds}{dt}=\frac{d(r\theta)}{dt};\ \frac{ds}{dt}=r\frac{d\theta}{dt};\ v=r\omega$$

$$\frac{dv}{dt}=\frac{d(r\omega)}{dt};\ \frac{dv}{dt}=r\frac{d\omega}{dt};\ a_{tan}=r\alpha$$

$$a_{rad}=\frac{v^2}{r};\ a_{rad}=\frac{(\omega r)^2}{r};\ a_{rad}=\omega^2 r$$



Rotational inertia and torque

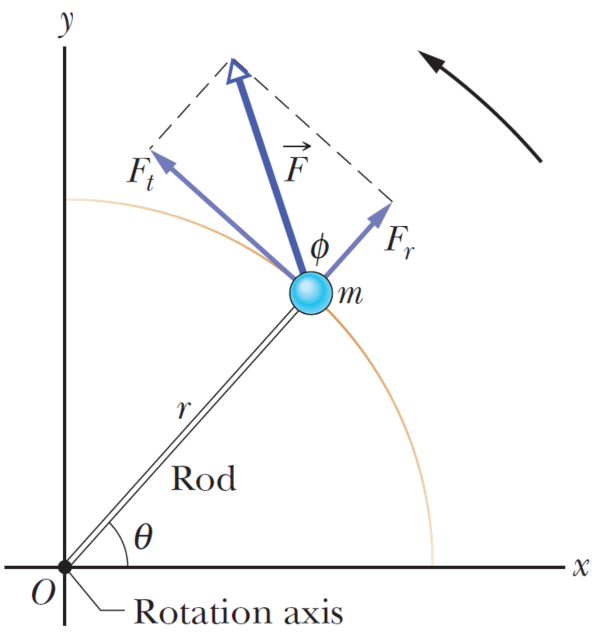
$$K = \frac{1}{2}\left(\sum m_i r_i^2\right)\omega^2$$

We can define a new quantity, the rotational inertia

$I = \sum m_i r_i^2$; Therefore: $K = \frac{1}{2} I\omega^2$

Newton's second law of motion is also applicabl to rotational motion.

$$\tau_{net} = I\alpha$$

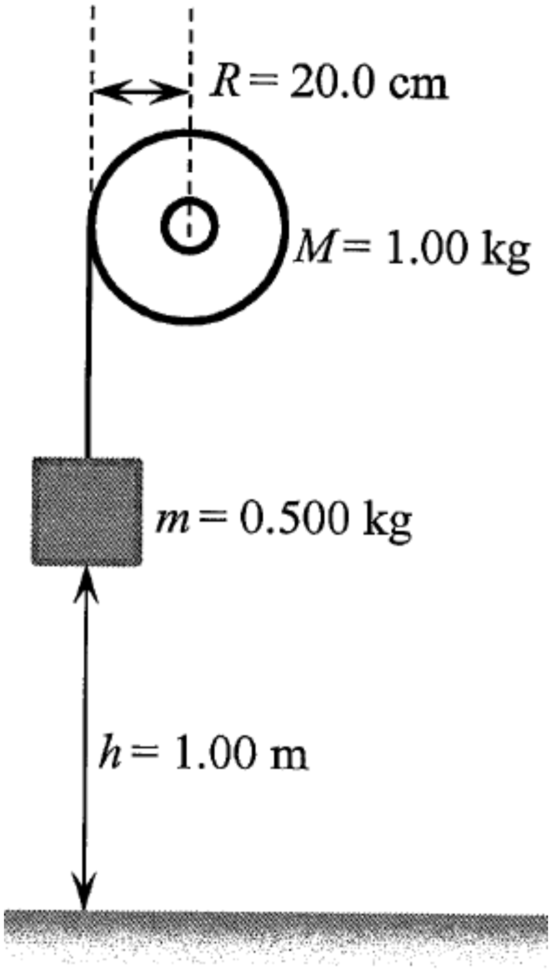


Conservation of momentum example

In a physics experiment, you wrap a light, non-stretching cable around a solid cylinder with mass M = 1.00 kg and radius R = 20.0 cm as shown in the figure. The cylinder rotates with negligible friction about a stationary horizontal axis. You then tie the free end of the cable to a block of mass *m* = 0.500 kg and release the block from rest at a distance *h* = 1.00 m above the floor. As the block falls, the cable unwinds without stretching or slipping, turning the cylinder in the process.

(a) Using energy methods, find the speed of the falling block and the angular speed of the cylinder just before the block strikes the floor.

(b) Using Newton's Second Law for torque, find the acceleration of the falling block.



[Ans: v = 3.13 m/s, ω = 15.7 rad/s; a = 4.9 m/s^2]

Oscillations

Most general solution:

$$x(t) = B\sin(\omega_0 t) + C\cos(\omega_0 t) \quad \text{with} \quad \omega_0 = \sqrt{k/m}$$

Amplitude A

$$A = \sqrt{B^2 + C^2}$$

$$\theta_0 = \tan^{-1}(-C/B)$$

Phase constant θ_0

Alternative form:

$$x(t) = A\sin(\omega_0 t + \theta_0) \quad \text{with} \quad \omega_0 = \sqrt{k/m}$$

Pendulum

$$\omega_0 = \sqrt{g/\ell}$$

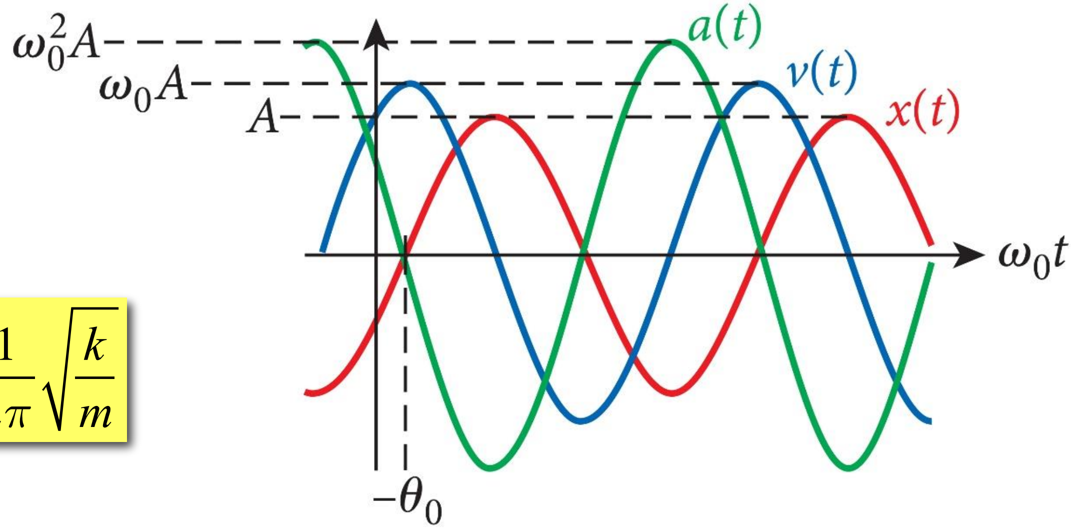
Thin rod

$$\omega_0 = \sqrt{\frac{mgr}{I}}$$

Velocity and acceleration:

$$v(t) = \omega_0 A\cos(\omega_0 t + \theta_0)$$

$$a(t) = -\omega_0^2 A\sin(\omega_0 t + \theta_0)$$



Period and frequency:

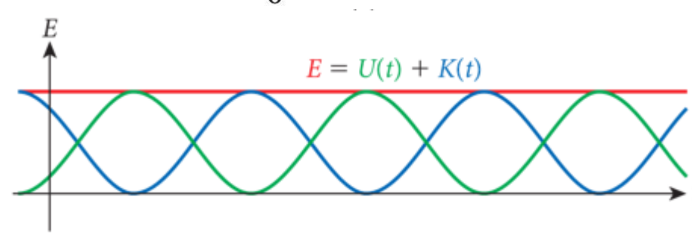
$$T = \frac{2\pi}{\omega_0} = \frac{2\pi}{\sqrt{k/m}} = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Kinetic and potential energy:

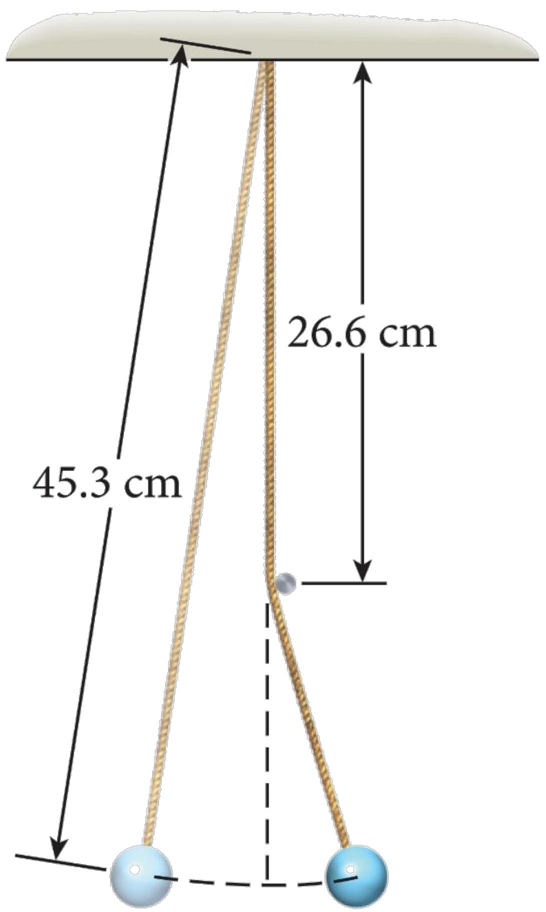
$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega_0^2 A^2 cos^2(\omega_0 t + \theta_0)$$

$$U = \frac{1}{2}mx^2 = \frac{1}{2}mA^2 sin^2(\omega_0 t + \theta_0)$$



Oscillations

A pendulum of length 45.3 cm is hanging from the ceiling and is restricted in its motion by a peg that is sticking out of the wall 26.6 cm directly below its pivot point. What is its period of oscillation?



[Ans: 1.11 s]

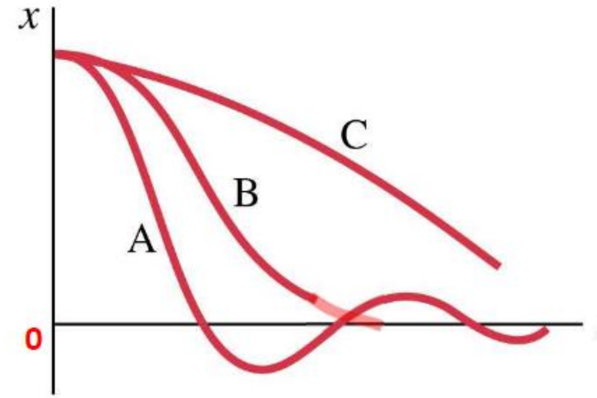
Oscillations

Damped Oscillations - amplitude of oscillation decreases with time due to resistive/dissipative force(s) from the system or environment.

A – underdamping: system will oscillate around equilibrium solution before coming to rest.

B – critical damping: fastest way to reach equilibrium

C – overdamped: system is so damped that it takes a long time (even infinity) to reach equilibrium



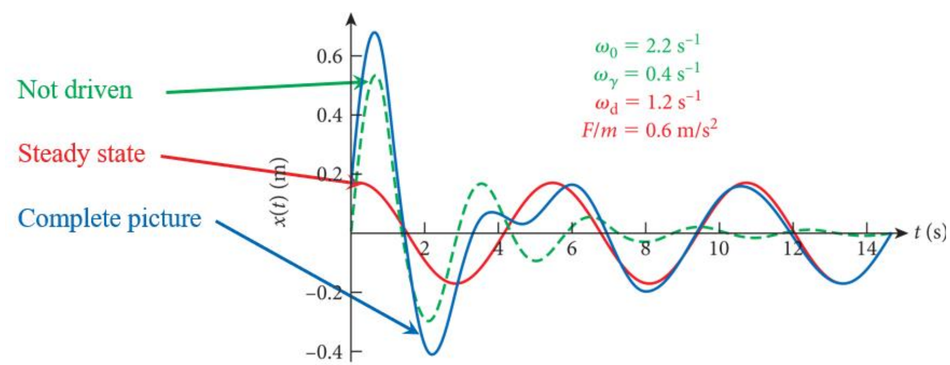
Forced Oscillations - driven by external oscillatory forces (driving force) with varying frequencies. $F(t) = F_d \cos(\omega_d t)$

The solution to the new differential equation becomes:

$$x(t) = B\sin(\omega_0 t) + C\cos(\omega_0 t) + A_d \cos(\omega_d t)$$

$$A_d = \frac{F_d}{m(\omega_0^2 - \omega_d^2)}$$

Driving angular speed



Oscillations

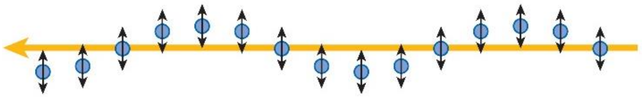
A 70 kg person decides to bungee jump from a 50m tall bridge. A 30m long bungee rope, with damping angular speed of 0.3 /s, is used and stretched 5 m by the weight of the person. Describe the vertical motion of the bungee jumper as a function of time.

Waves

A wave is an excitation that propagates as a function of time but does not generally transport matter with it.

A wave that propagates along the direction in which the oscillators move is called a **longitudinal wave**.

A wave that moves perpendicular to the direction in which the oscillators move is called a **transverse wave**.



Can define the phase shift between successive waves in terms of time or distance: $\Delta\phi = 2\pi\frac{\Delta t}{T} = 2\pi\frac{\Delta x}{\lambda}$

The energy of a wave is given by: $E = \frac{1}{2}m\omega^2A^2$

$$\boxed{E \propto I \propto A^2 \propto \frac{1}{r^2}}$$

The intensity is the power radiated per unit cross-sectional area: $I = \frac{P}{S}$

Unpolarised (infinite planes of vibration) light : $I = \frac{1}{2}I_0$

Polarised (only one plane of vibration) light: $I = I_0 cos\theta^2$

Waves

A repairman (mass 73 kg) sits on top of a cabin of mass 65 kg inside a shaft of a skyscraper. The cabin is suspended by a 61 m long steel cable of mass 38 kg. He sends a signal to his colleague at the top of the shaft by tapping the cable with his hammer. How long will it take for the wave pulse generated by the hammer tap to travel up the cable?

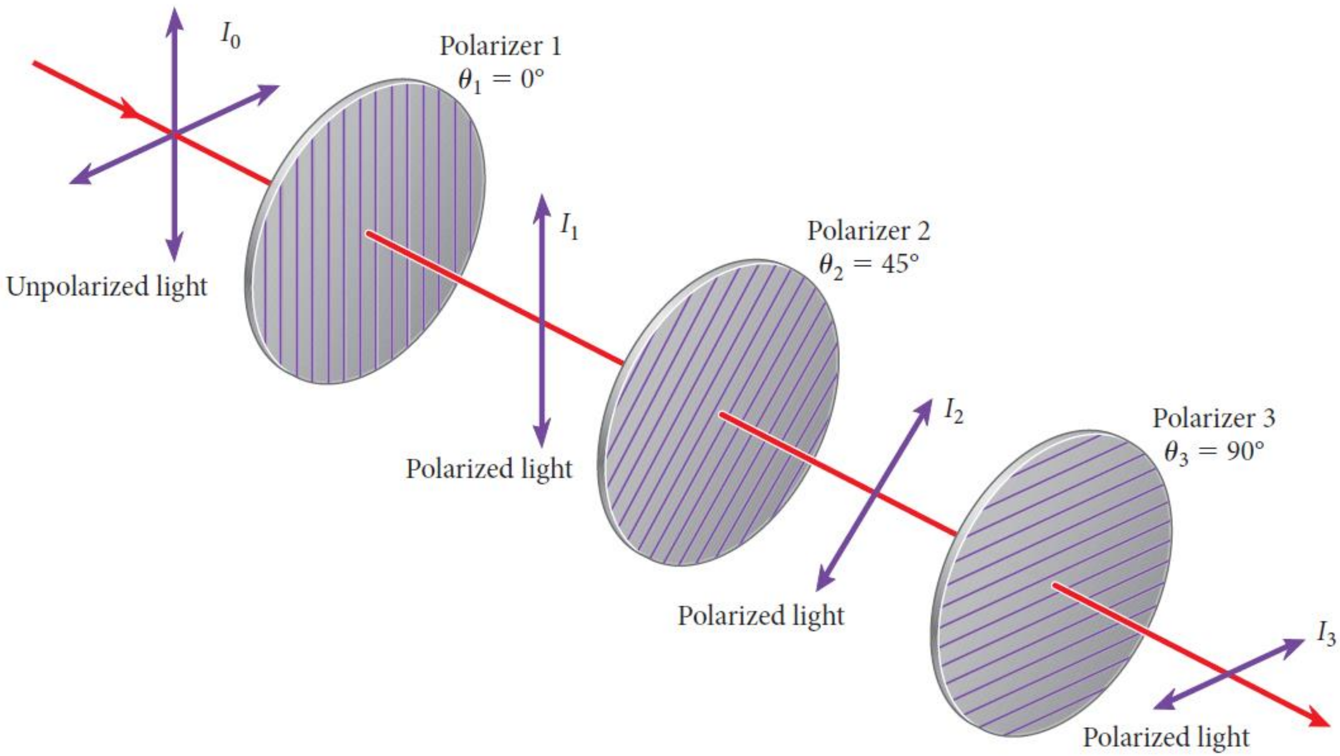
[Ans: 1.31 s]

[Ans: 1.16 s (more accurate but still not exact)]

What causes the difference in the two answers and what could be done if we require an exact solution ?

Waves

Suppose that unpolarized light with intensity I_0 is initially incident on the first of three polarizers in a line. The first polarizer has a polarizing direction that is vertical. The second polarizer has a polarizing angle of 45° with respect to the vertical. The third polarizer has a polarizing angle of 90° with respect to the vertical.



[Ans: $I_3 = 0.125I_0$]

Waves

If a wave hits a boundary can have:

- **Fixed**: reflected wave will have phase shifted by 180◦
- **Free**: reflected with no phase shift

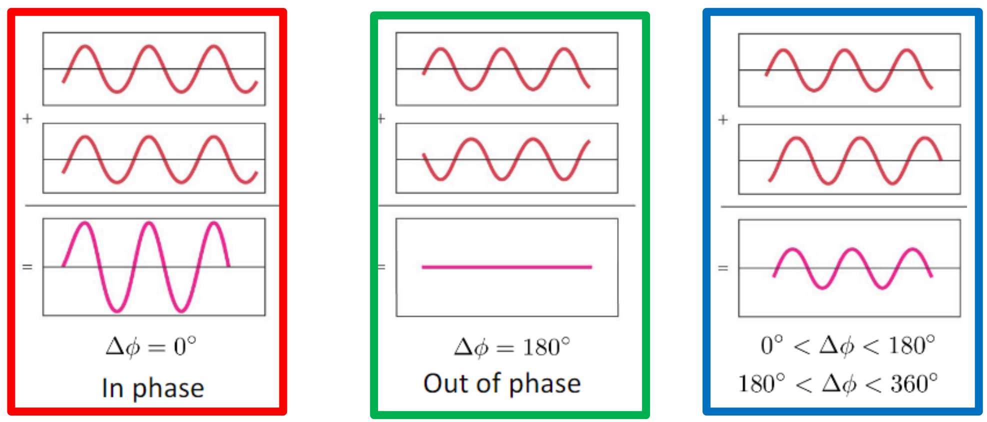
A wave encountering a denser medium will be partly reflected and partly transmitted

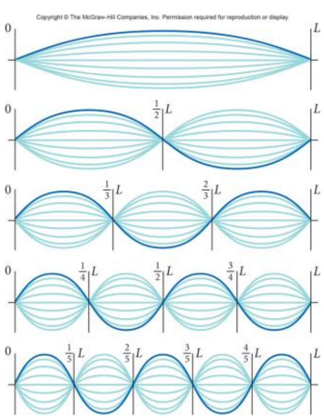
Two or more wave can be added, resulting in another wave - **superposition principle**.

Constructive interference

Destructive interference

Partially destructive interference





$$\lambda_n = \frac{2l}{n} \qquad n = 1, 2, 3, \ldots$$

n: harmonic

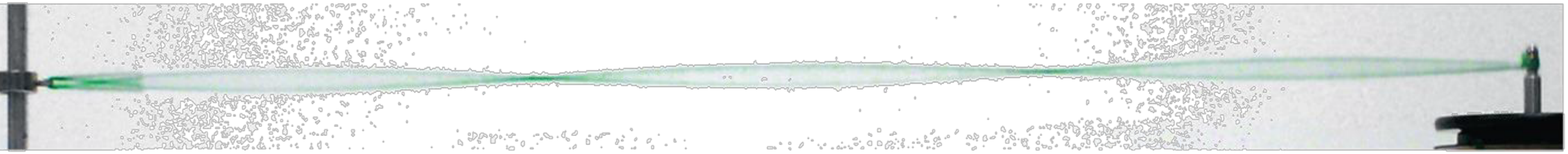
$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2l} = nf_1$$

$$f_n = \frac{v}{\lambda_n} = n\frac{\sqrt{T}}{2L\sqrt{\mu}} = n\sqrt{\frac{T}{4L^2\mu}}.$$

The condition for a standing wave is that an integer multiple, n, of half wavelengths fits exactly into the length of the string L.

Waves

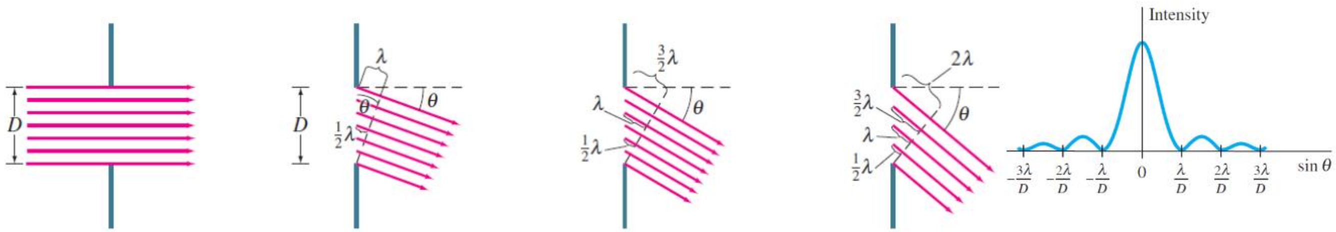
A mechanical driver is used to set up a standing wave on an elastic string. Tension is put on the string by running it over a frictionless pulley and hanging a metal block from it. The length of string from the pulley to the driver is 1.25 m. The linear mass density of the string is 5.00 g/m. The frequency of the driver is 45.0 Hz. What is the mass of the metal block?



[Ans: 0.72 kg]

Waves

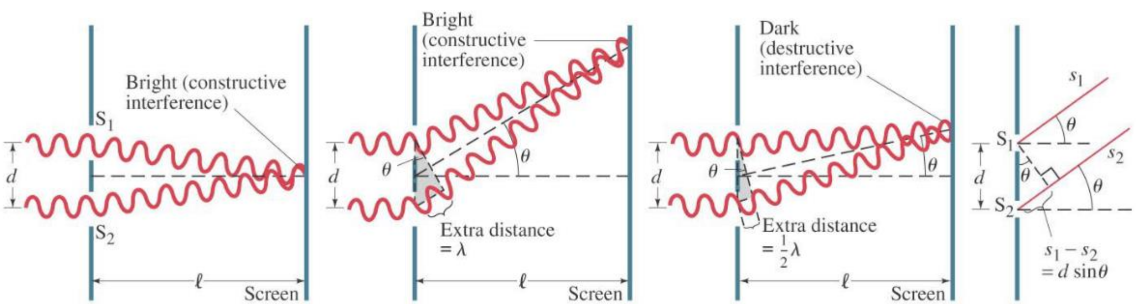
Any wave passing through an opening experiences **diffraction -** the interference or bending of waves through an aperture into the region of geometrical shadow of the aperture.



(a) $\theta = 0$ Bright (b) $\sin\theta = \frac{\lambda}{D}$ Dark (c) $\sin\theta = \frac{3\lambda}{2D}$ Bright (d) $\sin\theta = \frac{2\lambda}{D}$ Dark

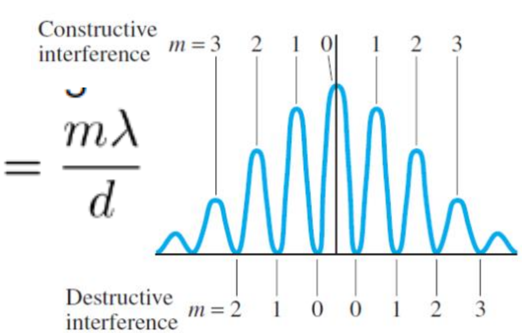
$$D \sin\theta = m\lambda$$

The wave nature of light causes the light waves passing through the two slits to interfere, producing bright and dark bands on the screen



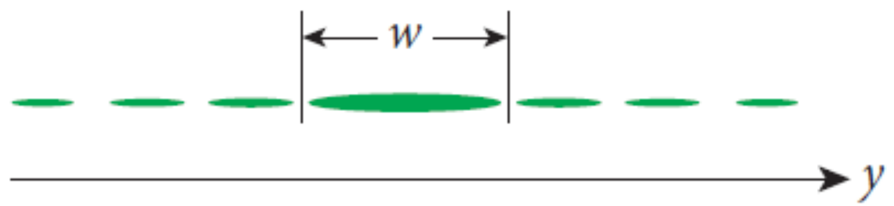
(a) $\theta = 0°$ (b) (c) (d)

$$\sin\theta_m = \frac{m\lambda}{d}$$



Waves

The single-slit diffraction pattern shown in Figure 34.29 was produced with light of wavelength λ =510.0 nm. The screen on which the pattern was projected was located a distance d =1.40 m from the slit. The slit had a width of D = 7.00 mm.



[Ans: 0.2 mm]

Questions ?