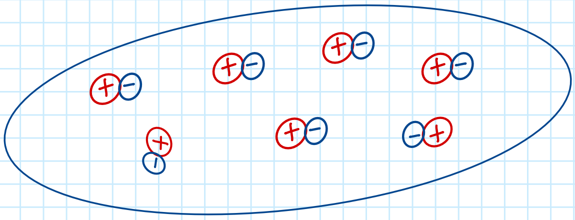
1.7. Electric Fields between Conducting Materials

1.7.1. Electrostatic Induction

Classification of materials according to their electric conductivity

metals   semiconductors   insulators   dielectric materials $\Downarrow$ 1.5, 1.6

Electric conductor (without external electric field)



* Large number of mobile Charge Carriers $10^{22} \dots 10^{23} \frac{1}{cm^3}$
* no external field: if fluctuation of mobile negtive charges occur (e.g. by thermal motion) $\Rightarrow$ they are immediately attracted by positive charge
* negative charge always screens / cancels out the positive charge (= fixed background charge)

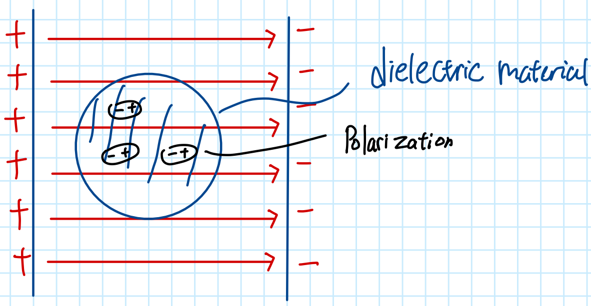
$\Rightarrow$ in total space charge density $\rho(\vec{r}) = 0$ (positive charge = negative charge)

$\operatorname{div} \varepsilon \vec{E} = 0$

$-\operatorname{grad} \phi = 0 \Rightarrow \phi = \text{constant}$; Potential on a conductor is constant

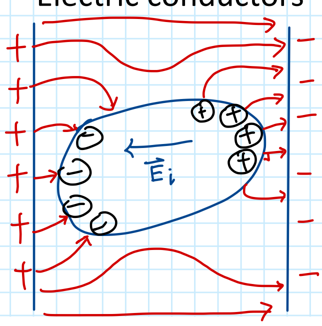
Electric conductor with external electric field

Dielectric materials (insulators, see 1.5)



* dielectric material is inside an $\vec{E}$-field
$\Downarrow$
dipolar form $\Rightarrow$ Polarization P
$\vec{E}$ inside is not zero
$\vec{E} = \frac{1}{\varepsilon_0 \varepsilon_r} \vec{E}$

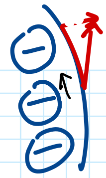
Electric conductors



* inside conducting material, highly mobile negative charges are present
* $\vec{E}$-field lines end at the conductor surface and internal field is compensated / shielded

$\Rightarrow \boxed{\vec{E}_i = 0}$

* At the surface: perpendicular field lines
parallel components of $\vec{E}$ would vanish, because the highly mobile charges would follow and cancel these components



⇒ $\vec{D} \perp$ surface of a conductor

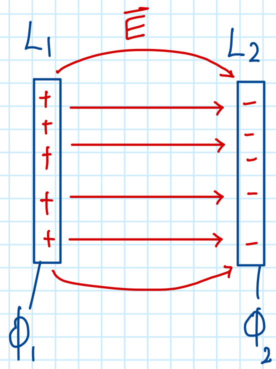
For the surface of a conductor: $\boxed{\vec{D} \cdot \vec{N} = \sigma(\vec{r}) \qquad \sigma |\vec{r}| = |\vec{D}|}$

(result of Gauss's Law ↪ 1.63)

* This phenomenon is called electrostatic induction

1.7.2. Electric Capacitance

(i) Consider:



- Two electric conductors L1 and L2
- Charge located on L1: Q1
  Charge located on L2: Q2
- Electric field inside conductors: $\vec{E} = \vec{E}_2 = 0$ inside conductor
- Charge localised at the surface of the conductors
- Surfaces of L1,L2 are equipotential surfaces

$$\phi_{L_1} = \phi_1 = \text{const.} \qquad \phi_{L_2} = \phi_2 = \text{const.}$$

Between L_1 and L_2 there is a potential difference $\phi_2 - \phi_1 = U_{12}$

- Calculated voltage between L1 und L2 and the charge Q, which is stored on the conductors:

$$U = \int_{L_1}^{L_2} \vec{E}\, d\vec{r} = U_{12}$$

· Define capacitance of a configuration of conducting electrodes:

Capacitance = Capability to store charge Q per voltage U_{12} between electrodes

$$\boxed{C = \frac{Q}{U_{12}}} \qquad (1.42)$$

Consideration on which parameters C depends:

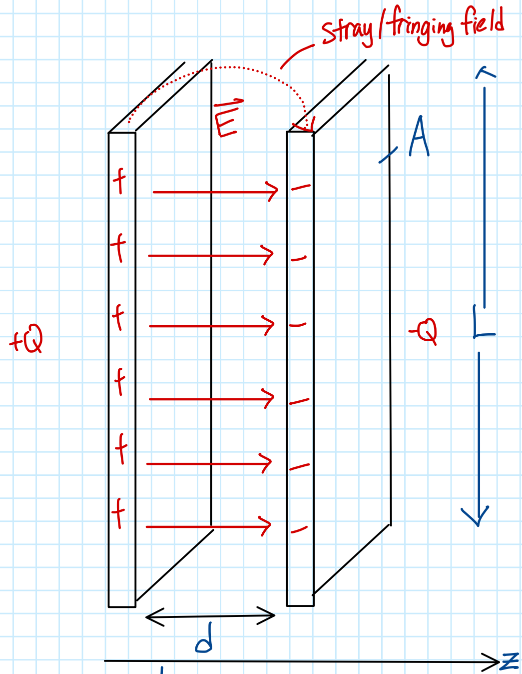
$$U_e = \int_{P_1}^{P_2} \vec{E}\, d\vec{r} \sim \vec{E} \qquad \int_{\partial V} \vec{D} \cdot d\vec{a} = \int_{\partial V} \varepsilon \vec{E} \cdot d\vec{a} = Q$$

depends roughly d $\qquad \sim \vec{E}$ $\qquad$ ⇗ depends roughly on A

⇒ Capacitance C is a function of the geometry and material only!

$$\boxed{C = f(\text{geometry}, \varepsilon)}$$

(ii) Example 1: plate capacitor



- Two parallel conductor plates with area A and at distance d
- Plates are charged with opposite charge : $+Q, -Q$
- Between the plates: dielectric material with permittivity ε
- Assumption: length l of plates is much larger than distance d
  $L \gg d$
  Fringing / Stray fields at the edges can be neglected r $\Rightarrow \vec{E}$ uniform and parallel
- To calculate: voltage U between the plates and hereof the capacitance C: $C = \frac{Q}{U}$

$\vec{E} = E_z \cdot \vec{e}_z$
$d\vec{r} = \vec{e}_z \cdot dz$

$$U_{12} = \int_{L_1}^{L_2} \vec{E}\, d\vec{r} = \int_0^z E_z \cdot \underbrace{\vec{e}_z \cdot \vec{e}_z}_{1} \cdot dz = E_z \cdot d$$

$$Q = \int_V \vec{D} \cdot d\vec{a} = D_z \cdot A = \varepsilon E_z \cdot A$$

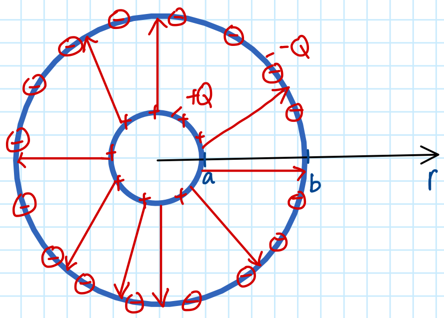
For conductors:

$|\vec{D}| = \varepsilon |\vec{E}| = \sigma$
$Q = \sigma \cdot A$

$$C = \frac{Q}{U} = \frac{\varepsilon E_z \cdot A}{E_z \cdot d} = \frac{\varepsilon A}{d} \qquad \boxed{C = \frac{\varepsilon A}{d}}$$

Circuit symbol : —| |— C   Unit $[C] = 1F$ (Farad)

(iii) example 2: spherical capacitor



- Two concentric, perfectly conducting hollow spheres with radius b and a, respectively (b>a)
- Spheres carry opposite charge $+Q, -Q$
- Between the spheres: dielectric material with permittivity ε
  . spherical symmetry $\Rightarrow$ use spherical coordinates
- To calculate: voltage U between the spheres, and hereof the capacitance:

$$U_{12} = \int_{L_1}^{L_2} \vec{E}\, d\vec{r}$$

no dependance of $\vec{E}$ on θ, φ

$\Rightarrow \vec{E} = E_r(r) \cdot \vec{e_r}$ between $a < r < b$

$$Q = \int_{\partial V} \vec{D} \cdot d\vec{a} = \int_0^{2\pi}\int_0^{\pi} \varepsilon \cdot E_r(r) \cdot \underbrace{\vec{e_r} \cdot \vec{e_r}}_{1}\, r^2 \sin\theta \, d\theta \, d\varphi = \underbrace{4\pi r^2 \varepsilon E_r(r)}$$

$\rightarrow 4\pi r^2 D_r$

$\Downarrow$

$$E_r(r) = \frac{Q}{4\pi r^2 \cdot \varepsilon}$$

$$U_{ab} = \int_a^b \vec{E}\, d\vec{r} = \int_a^b E_r(r) \cdot \vec{e_r} \cdot \vec{e_r}\, dr$$

$$= \int_a^b \frac{Q}{4\pi\varepsilon} \cdot \frac{1}{r^2} \cdot dr = \frac{Q}{4\pi\varepsilon}\left[-\frac{1}{r}\right]_a^b = \frac{Q}{4\pi\varepsilon}\left[-\frac{1}{b} + \frac{1}{a}\right]$$

$$= \frac{Q}{4\pi\varepsilon}\left[\frac{b-a}{ab}\right]$$

$$\Rightarrow C = \frac{Q}{U} = \frac{\cancel{Q} \cdot 4\pi\varepsilon \cdot ab}{\cancel{Q}(b-a)}$$

$$\boxed{C = \frac{4\pi\varepsilon \cdot ab}{b-a}}$$

$\leftarrow$ Only depending on dimension and ε

(i) Relate Q to E-field by Gauss's Law

(ii) insert $\vec{E}$-field into Calculation of $U = \int \vec{E}\, d\vec{r}$

(iii) $C = \frac{Q}{U}$