3.6 Calculation of Magnetostatic Fields and Forces

- Only calculation of highly symmetric problems (Cylindric Symmetry)
- Application of Ampère's circuital law

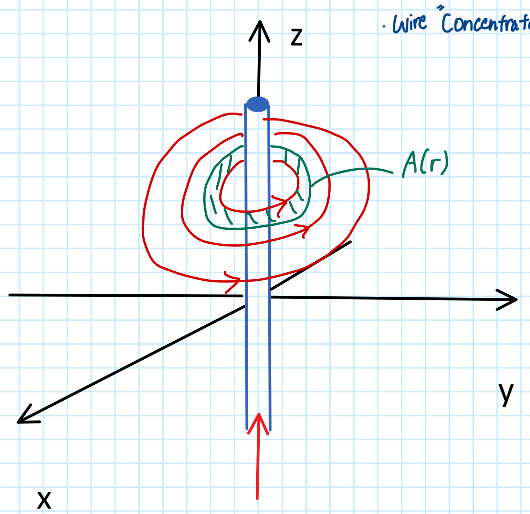
$$\int_{\partial A = C} \vec{H}\, d\vec{r} = I(A) = \int_A \vec{J}\, d\vec{a}$$

A = Control area
∂A = Closed curve around A

- These calculations are one to one exemplary for exam problems

3.6.1 Magnetic field of an infinitely long, straight wire (thin wire)

assumption: $L \gg r \Rightarrow$ wire has almost no extension; thickness can be neglected
· wire "concentrated" on z-axis



- Thin, current-carrying wire placed along z-axis

$\vec{H}/\vec{B}$-field = concentric circles around z-axis

- Cylinder-symmetric -> use cylindric coordinates

(r, φ, z)

$\vec{e}_r,\ \vec{e}_\varphi,\ \vec{e}_z$

$\vec{H}(r, \varphi, z) = H(r)\,\vec{e}_\varphi$

no dependance on φ, z

Control area $A(r)$ with enclosing curve $\partial A(r) = C$

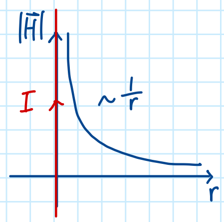
Apply Ampere's law: $\int_{\partial A} \vec{H} \cdot d\vec{r} = I(A(r)) = I$ (I is inside $A(r)$ for any $r > 0$)

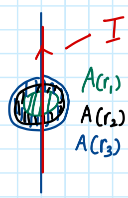
$$\int H_\varphi(r)\vec{e}_\varphi\, \underbrace{r\, d\varphi\, \vec{e}_\varphi}_{d\vec{r} \text{ in } \varphi\text{-direction (follow circle with radius } r)} = \int_0^{2\pi} H_\varphi(r)\, r\, d\varphi = H_\varphi(r) \cdot r \int_0^{2\pi} d\varphi = 2\pi H_\varphi(r) \cdot r$$

$$2\pi H_\varphi \cdot r = I \Rightarrow H_\varphi(r) = \frac{I}{2\pi r}$$

$$\boxed{\vec{H}(\vec{r}) = H_\varphi(r) \cdot \vec{e}_\varphi = \frac{I}{2\pi r} \cdot \vec{e}_\varphi} \qquad (3.28)$$

$|\vec{H}| \sim \frac{1}{r}$

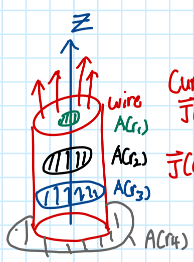




I has no dimension in r direction

$\Downarrow$

for any control volume $A(r)$ $(0 < r < \infty)$
contained current equals to I
(for length of wire $\gg$ diameter)



Current density $\vec{J}(r)$

$\vec{J}(r) = \vec{J}(r) \cdot \vec{e}_z$

$\Downarrow$

Cylindric Symmetric

$\Downarrow$

Ampere's law: $\underbrace{\int_{\partial A = C} \vec{H} \cdot d\vec{r}}_{\Downarrow} = \int_{A(r)} \vec{J} \cdot d\vec{a} = I(A(r))$

$\overrightarrow{H(r)}$ is also Cylindric symmetric

$\Downarrow$

$\overrightarrow{H(\vec{r})} = H_\varphi(r) \cdot \vec{e_\varphi}$

$$\int_0^{2\pi} H_\varphi(r)\, \vec{e_\varphi}\, r\, d\varphi \cdot \vec{e_\varphi} = 2\pi r H_\varphi$$

Vary r form 0 → ∞

$$A(r_1): 2\pi r H_\varphi = \int_{A(r_1)} \vec{J} \cdot d\vec{a} = I(A_1)$$

$$A(r_2): 2\pi r H_\varphi = \int_{A(r_2)} \vec{J} \cdot d\vec{a} = I(A_2)$$

$0 < r < r_{wire}$

$$A(r_3): 2\pi r H_\varphi = \int_{A(r_3)} \vec{J} \cdot d\vec{a} = I(A_3) = I_{total}$$

$r > r_{wire}$

$$A(r_4): 2\pi r H_\varphi = \int \vec{J} \cdot d\vec{a} = I(A_4) = I(A_3) = I_{total}$$

$$H_\varphi = \frac{I(A(r_i))}{2\pi r}$$