Work, Energy and Conservation of Energy

Lecture Learning Objectives

In this lesson you will learn to:

- Understand the concepts of work and energy
- Find the change of energy in cases with constant force
- Calculate the work done by a variable force.
- Calculate the changes of potential energy of objects
- Relate the potential energy to other forms of energy
- Use the principle of conservation of energy to solve problems.

Kinetic energy

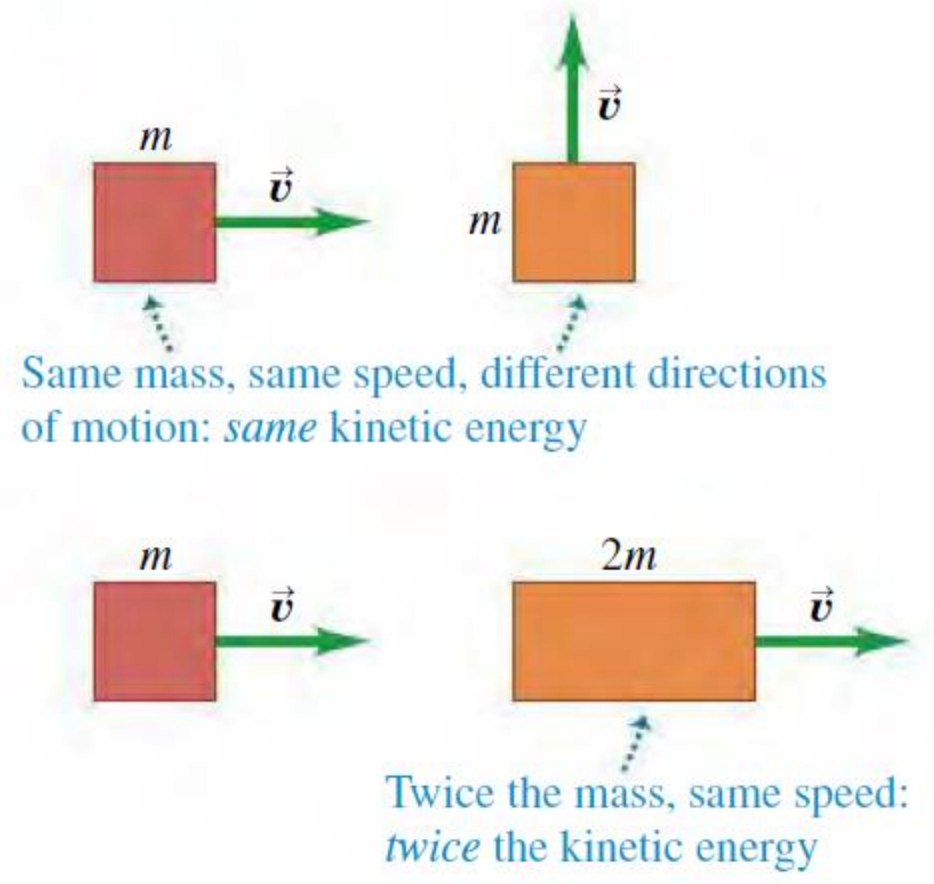
Energy associated with motion.

Defined as:

$$K = \frac{1}{2}mv^2$$

It is a scalar. Depends on the object mass and speed.

Measure in Joules (J) = kg m^2 s^{-2}



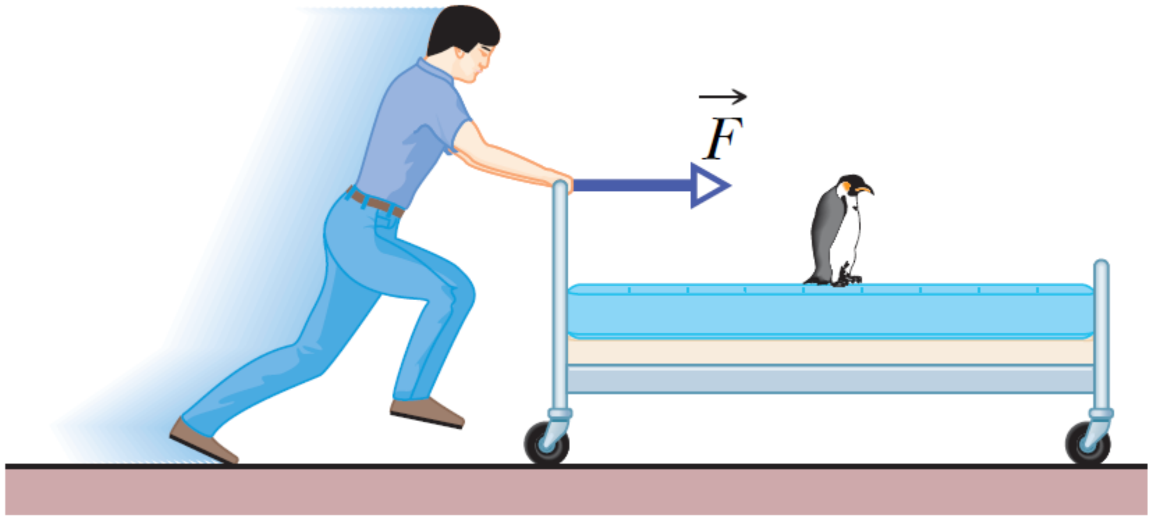
Work

If we apply a force, the object has an acceleration.

Therefore, its kinetic energy changes.

Work is energy transferred to or from an object by means of a force acting on the object.

Energy transferred to the object is positive work and energy transferred from the object is negative work.



Work - kinetic energy

Consider the bead:

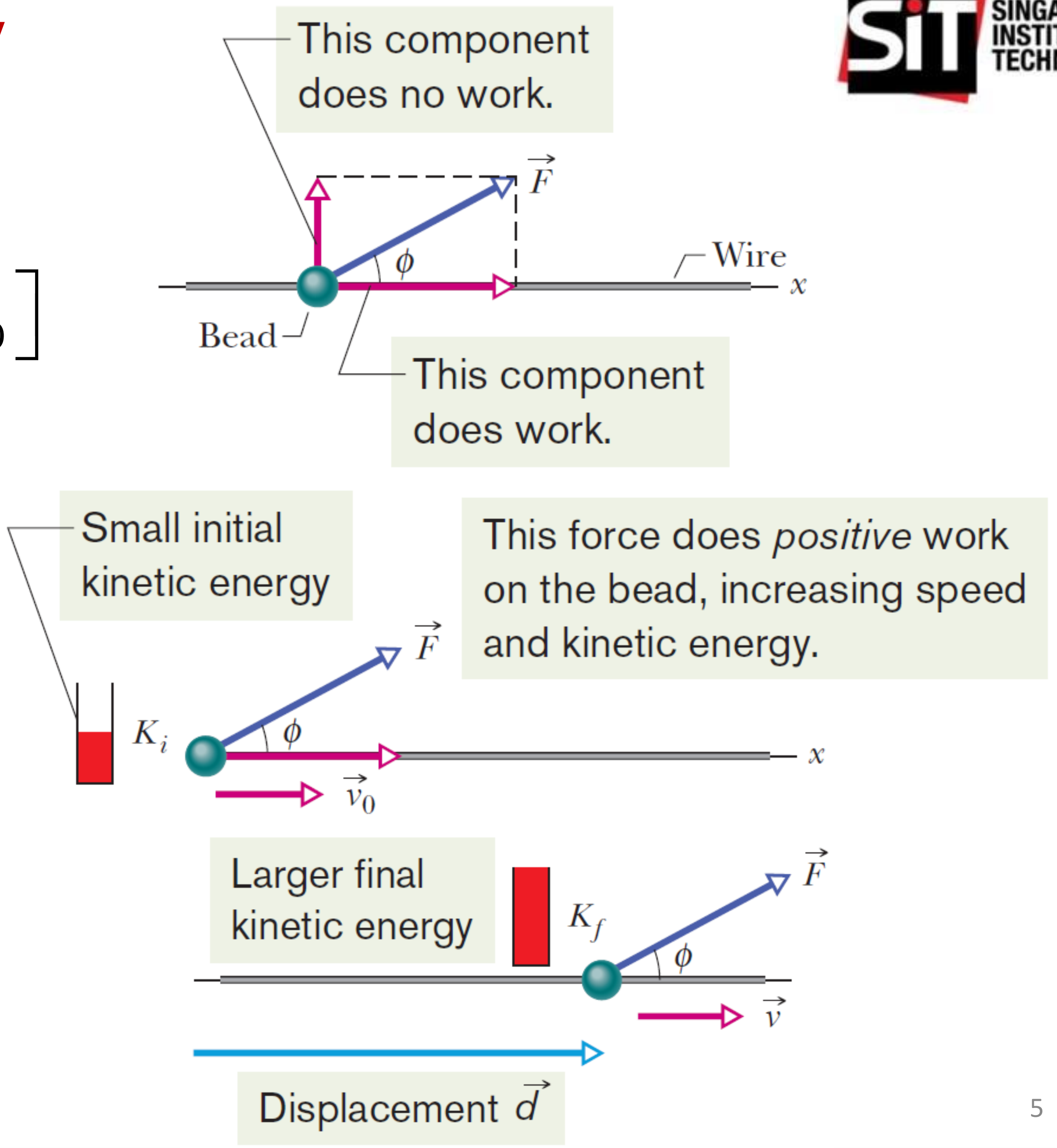
$$F_x = ma_x \qquad v_2^{\ 2} = v_1^{\ 2} + 2a\left[x(t) - x_0\right]$$

$$v_2^{\ 2} = v_1^{\ 2} + 2a_x d$$

$$v_2^{\ 2} = v_1^{\ 2} + 2\frac{F_x}{m}d$$

$$\frac{1}{2}mv_2^{\ 2} - \frac{1}{2}mv_1^{\ 2} = F_x d$$

$$K_2 - K_1 = F_x d$$



Work

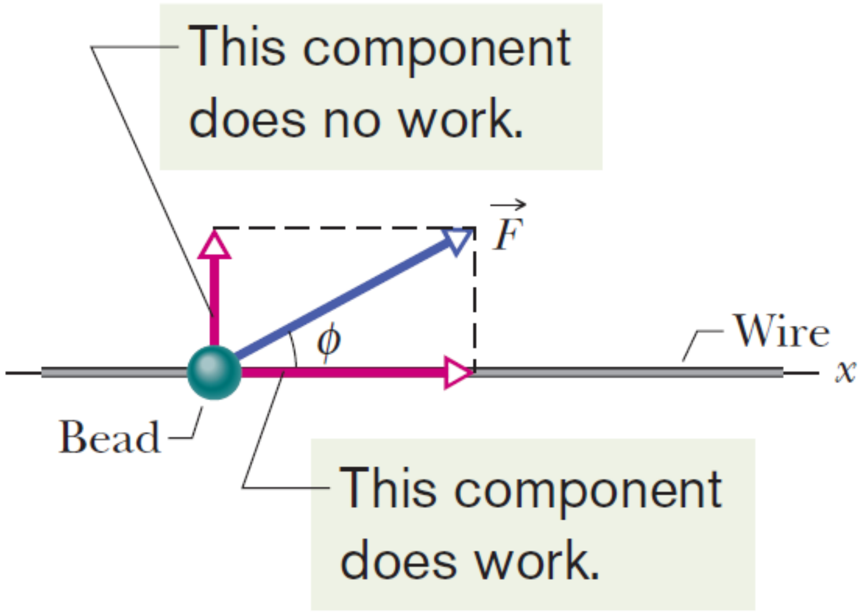
Work is equal to the component of the force in the direction of the displacement times the displacement:

$$F_x d = W$$

Only the component in the direction of the displacement does work.

$$W = |\mathbf{F}||\mathbf{d}|\cos\phi$$

$$W = \mathbf{F} \bullet \mathbf{D}$$



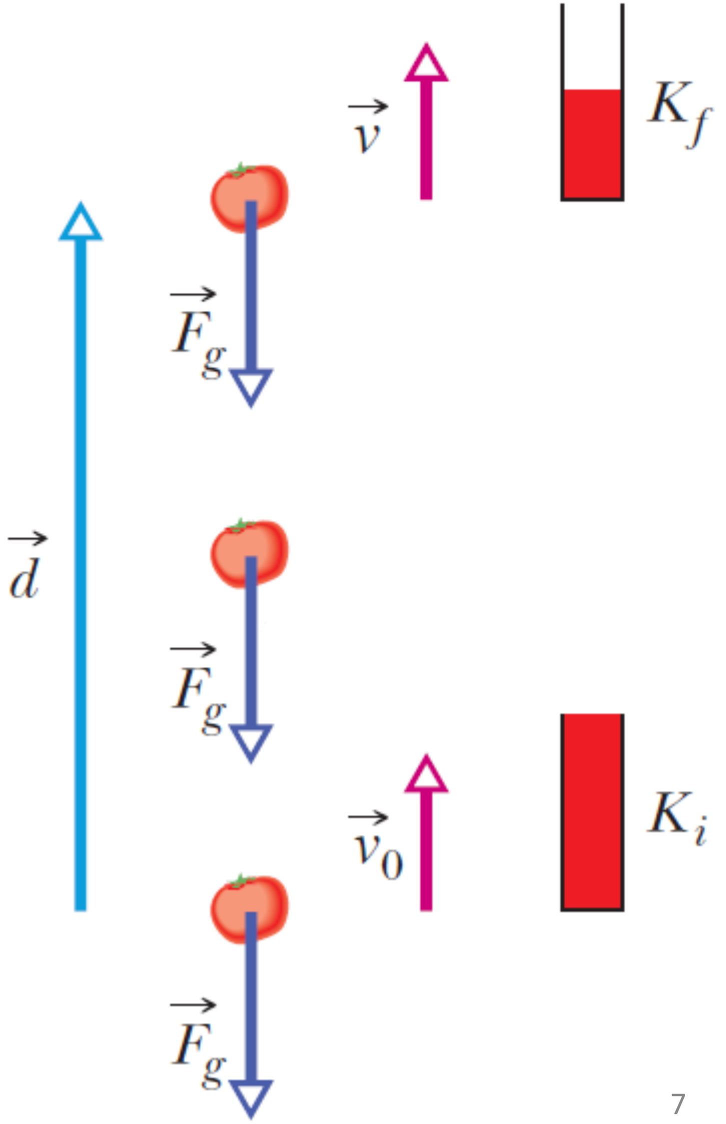
Work done by gravitation

Gravitation exerts a constant force on objects.

If the objects rises or falls, work is done.

$$F_g = mg$$

$$W_g = mgd\cos\phi$$



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Work and Kinetic Energy Example

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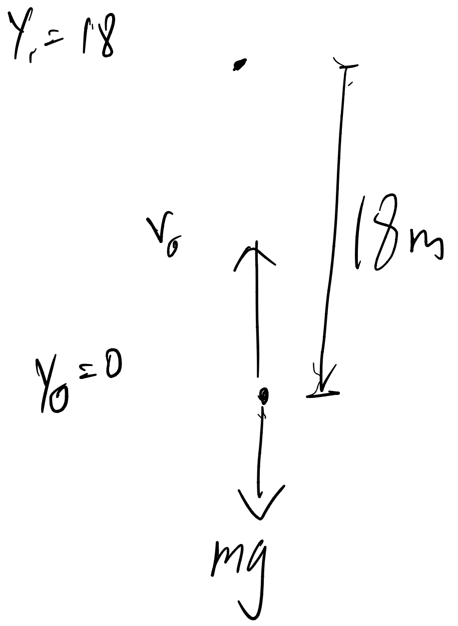
A 200 g ball thrown vertically up with an initial speed of 20 m/s reaches a maximum height of 18 m.

Find: (a) the change in its kinetic energy; (b) the work done by gravity. (c) Explain why the quantities are not equal.



a) $m_B = 200g = 0.2 kg$

$V_0 = 20 m/s$, $V_1 = 0 m/s$

$KE_0 = \frac{1}{2} M_B V_0^2 = \frac{1}{2}(0.2)(20^2)$

$= 40 J$

$KE_1 = \frac{1}{2}(0.2)(0)^2 = 0 J$

$\Delta KE = KE_1 - KE_0 = 0 - 40$

$= -40 J$

b) $W_g = M_B g d \cos\phi$

$= 0.2(9.81)(18-0) \cos 180°$

$= -35 J$

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Work and Kinetic Energy Example

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c) Because gravity is not the only one doing work, gravity accounts for 35J out of 40J, 5J could be because of air resistance

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Work and Kinetic Energy Example 1

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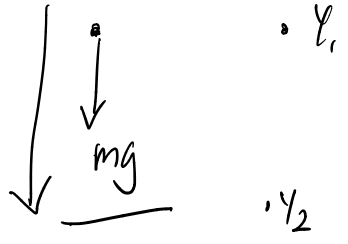
Use the work–energy theorem to solve each of these problems. You can use Newton's laws to check your answers. Neglect air resistance in all cases.

(a) A branch falls from the top of a 95.0 m tall redwood tree, starting from rest. How fast is it moving when it reaches the ground?

(b) A volcano ejects a boulder directly upward 525 m into the air. How fast was the boulder moving just as it left the volcano?

a) $V_1^2 = V_0^2 + 2a\Delta x$

$W_g = M_g |d| \cos\phi = M_g |d|$



$\phi = 0°$

$\Delta KE = W = mg|d|$

$V_1 = 0\,m/s$

$KE_2 - KE_1 = M_g|d|$; $KE_1 = 0$

$KE_2 = M_g|d|$; $\frac{1}{2}\cancel{M}v^2 = \cancel{M}g|d|$; $V_2^2 = 2g|d|$ $\quad V_2 = \sqrt{2 \times 9.80 \times 95}$

$V_2 = \sqrt{2g|d|}$ $\quad = 43.2\ m/s$

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Work and Kinetic Energy Example 1

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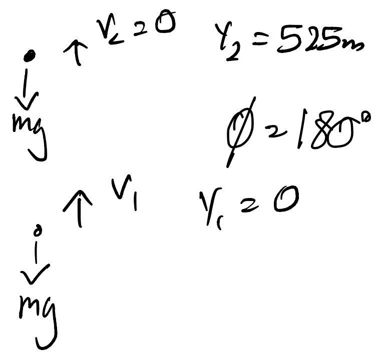

b) $W_g = Mg|d|\cos\phi = -Mg|d|$

$\Delta KE = W_g = -Mg|d| \quad ; \quad KE_2 = 0$

$KE_2 - KE_1 = -Mg|d| \quad ; \quad -\frac{1}{2}\cancel{M}v_1^2 = -\cancel{M}g|d|$

$v_1 = \sqrt{2g|d|}$

$= \sqrt{2 \times 9.80 \times 525} = 101 \text{ m/s}$

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Work done by a non constant force

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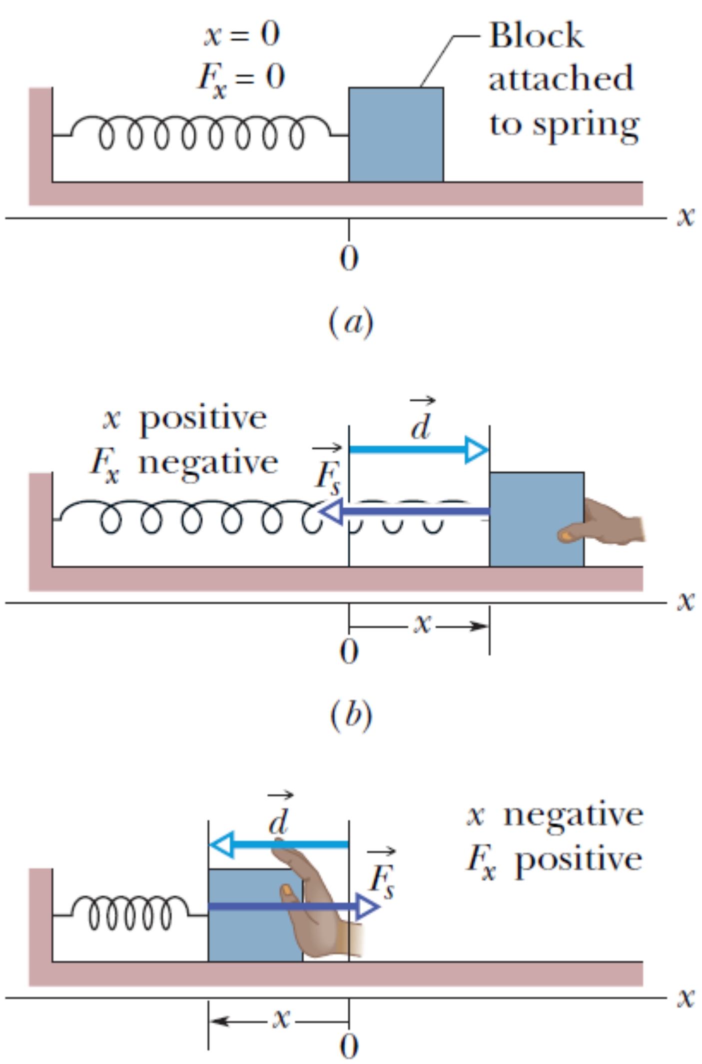
The work done by a variable force can be found with integration.

A special case is a spring.

$$F_x = -kx$$

x = 0 is at the relaxed spring position.

Similar to elastic materials



Work done by a non constant force

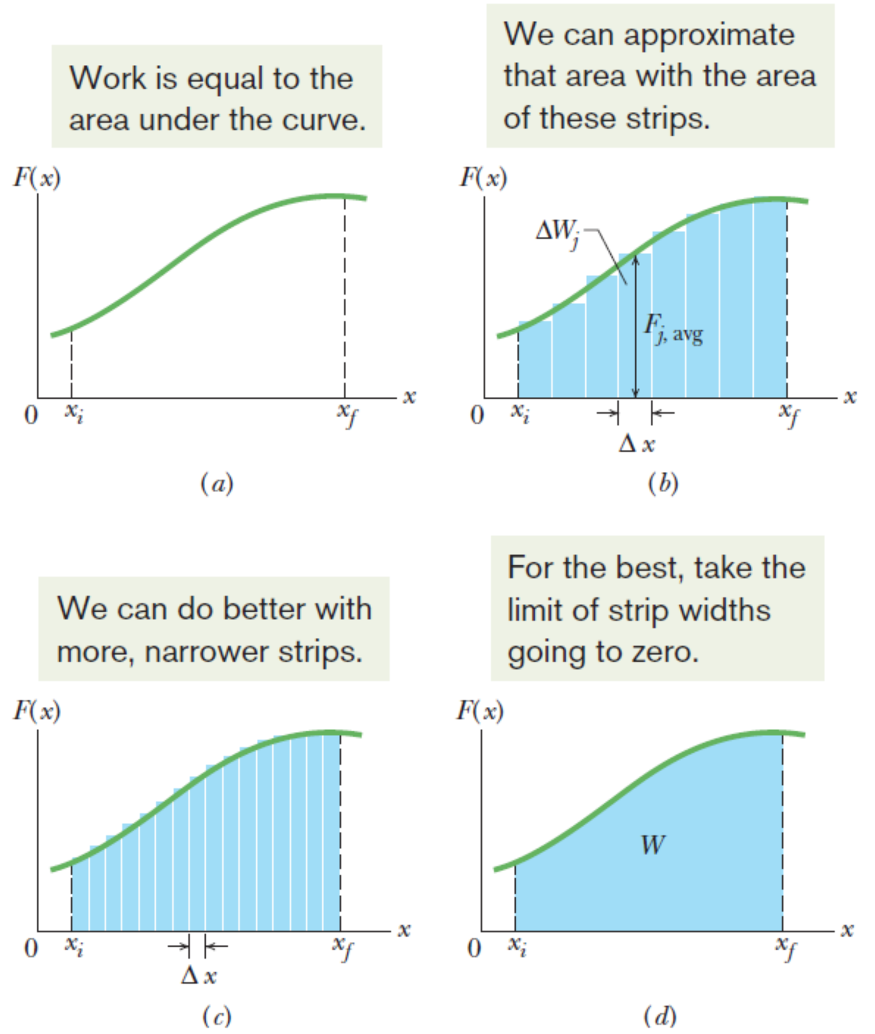
To find the work, we can consider very small intervals, where the force is almost constant, and add them:

$$W_s = \sum -F_{xj}\Delta x$$; As Δx tends to 0:

$$W_s = \int_{x_1}^{x_2} -F_x dx$$

If it is a spring F_x: $W_s = \int_{x_1}^{x_2} -kx dx$

Which gives: $W_s = \frac{1}{2}k\left(x_1^{\,2} - x_2^{\,2}\right)$



Work done Variable Force Example

A spring and block are in the arrangement shown below. When the block is pulled out to x = 4.0 cm, we must apply a force of magnitude 360 N to hold it there.

We pull the block to x = 11 cm and then release it. How much work does the spring do on the block as the block moves from xi = 5.0 cm to (a) x = 3.0 cm, (b) x = -3.0 cm, (c) x = -5.0 cm, and (d) x = -9.0 cm?

$a = 0\ m/s^2$

$x = 0.04\ m$

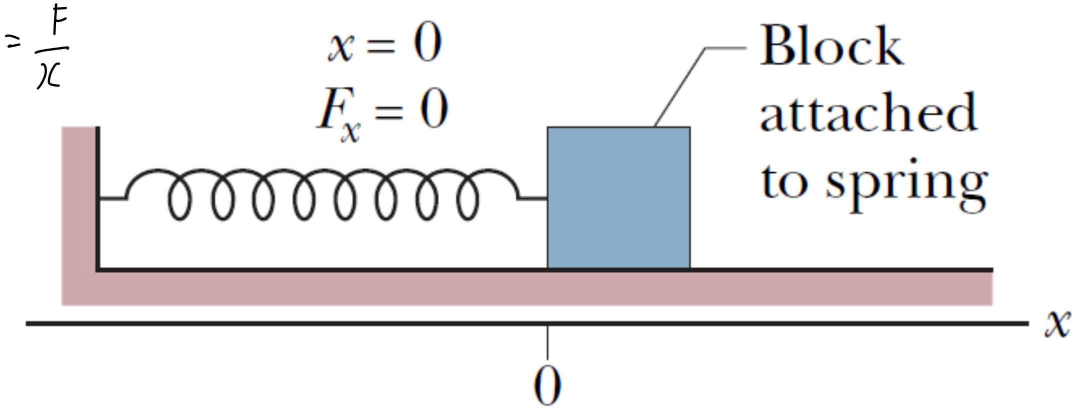
$F_s \longleftarrow \circ \longrightarrow F = 360N$

$k = \frac{360}{0.04} = 9000\ N/m$

$F - F_s = ma = 0$

$F = F_s \qquad F = |-kx| = kx$

$k = \frac{F}{x}$



a) $W_s = \frac{1}{2} k (x_i^2 - x_f^2)$

$= \frac{1}{2} 9000 (0.05^2 - 0.03^2)$

$= 7.2\ J$

Work done Variable Force Example

Potential energy

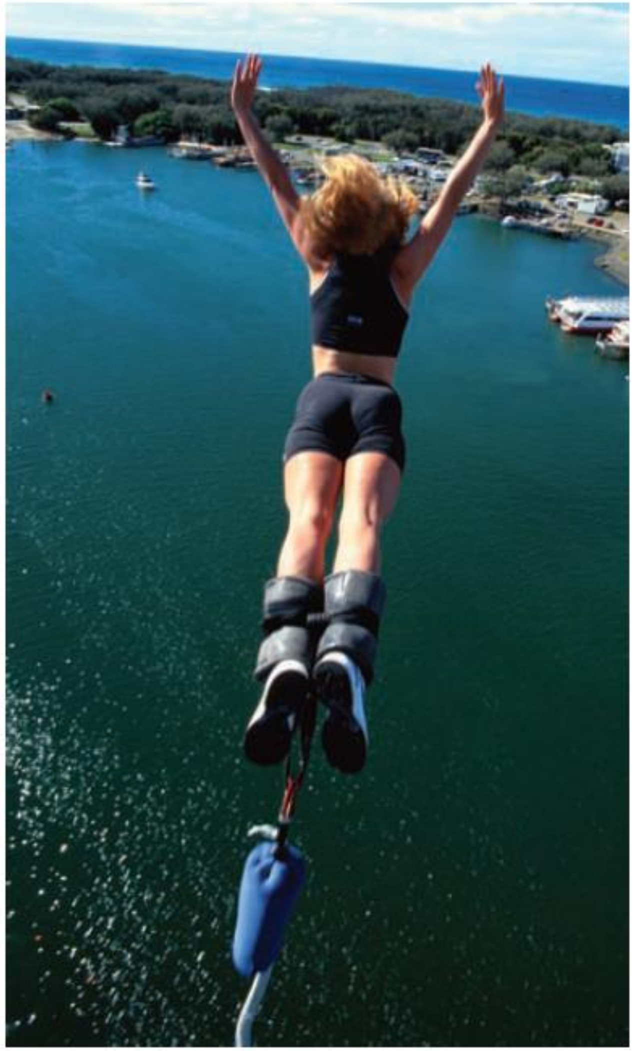
In the previous lesson, we looked at kinetic energy and work.

Potential energy (U) is a further type of energy.

It is the energy associated with the state of a system.

Examples are elastic potential energy and gravitational potential energy.

It can be seen as a “store” of energy.



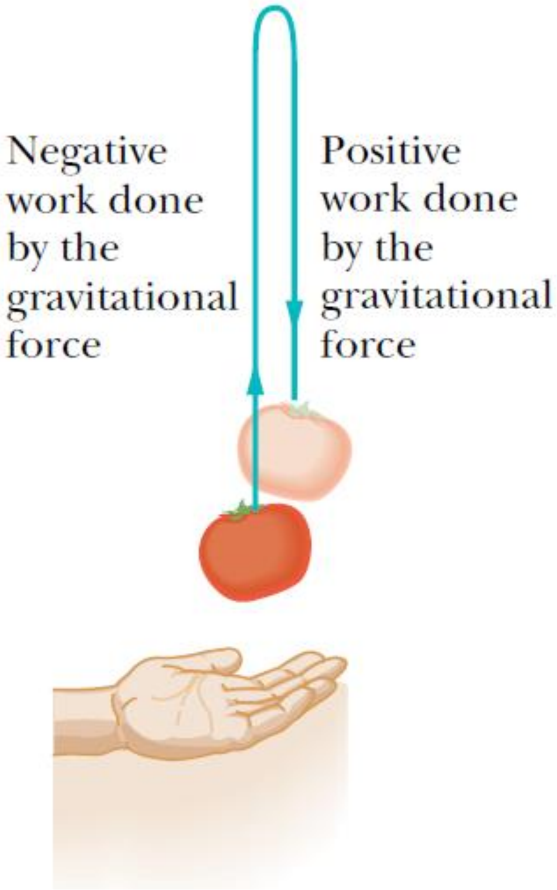
Potential energy and work - gravity

Work can be done to change the potential energy of a system.

For gravity, the higher the object, the higher the potential energy.

When the apple rises, gravity does **negative** work, U_g increases.

When the apple falls, gravity does **positive** work, U_g decreases.



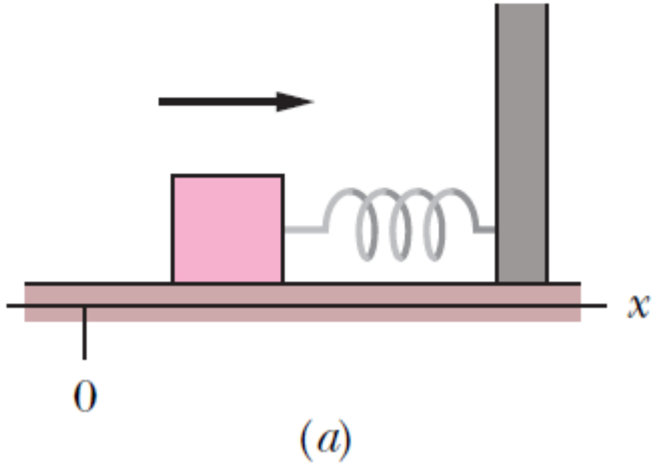
Potential energy and work - springs

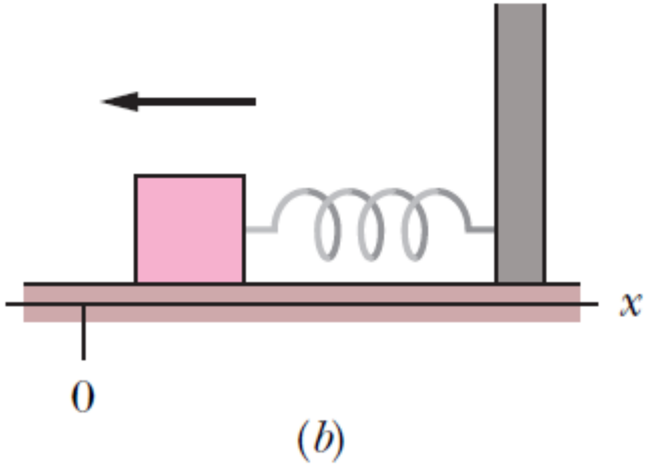
The same applies to springs.

If the spring is compressed, and is pushed further, the spring does **negative** work and U_e increases.

If the spring is allowed to relax, the spring does **positive** work and U_e decreases.

$$\Delta U = -W$$



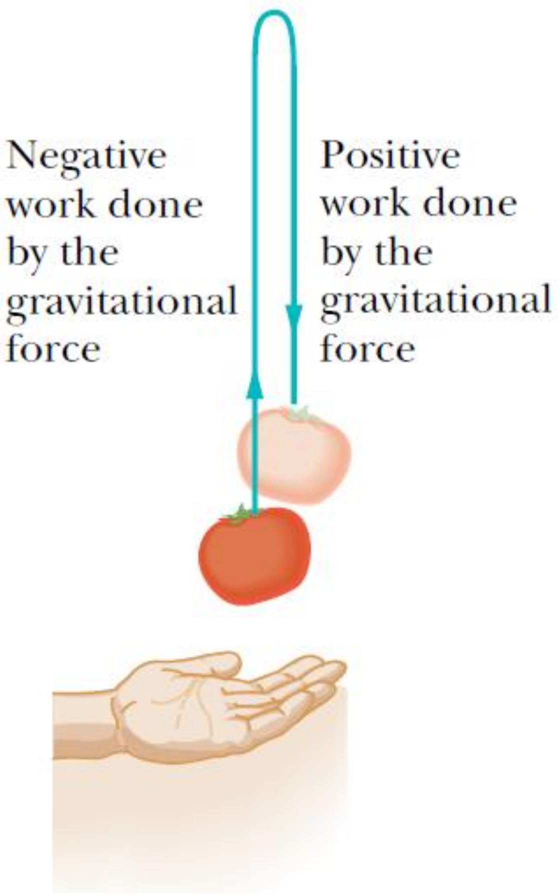


Conservative forces

Both U_e and U_g transform in kinetic energy (K). Once the situation is reversed, the K is changed back into potential energy.

- There is a force applied to an object in the system, and it does work W_1, transferring energy between K and other types of energies.
- If we reverse the situation, the force will do work W_2 and reverse the energy transfer.

$$W_1 = -W_2$$



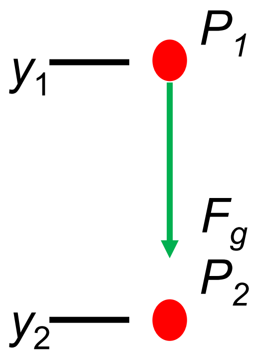
Calculating potential energy

$$\Delta U = -W$$

Gravity: $\Delta U_g = mg(y_2 - y_1) = mg\Delta y$

If the movement is downwards (Δy is –ve), ΔU is -ve.

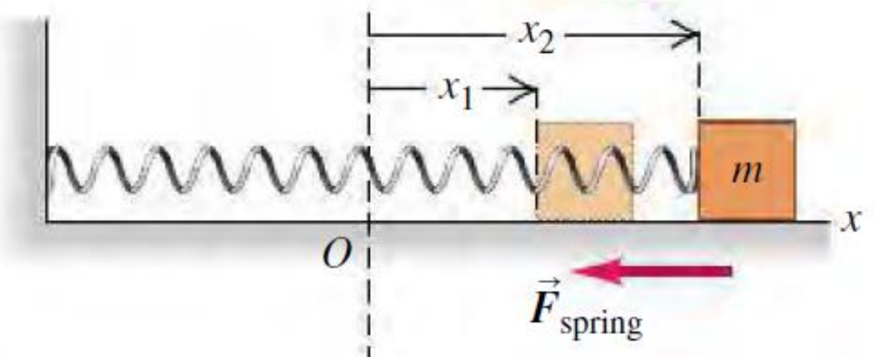
If U = 0 J at a reference point: $U_g = mg(y_2 - 0) = mgy$



Elastic: $\Delta U_e = \frac{1}{2}kx_2{}^2 - \frac{1}{2}kx_1{}^2$

In this case, x = 0 is at relaxed position.
At x = 0, U = 0.

$U_e - 0 = \frac{1}{2}kx_2{}^2 - 0$; Therefore, $U_e = \frac{1}{2}kx^2$



Potential Energy Example

A 2.50 kg mass is pushed against a horizontal spring of force constant 25.0 N/cm on a frictionless air table. The spring is attached to the tabletop, and the mass is not attached to the spring in any way. When the spring has been compressed enough to store 11.5 J of potential energy in it, the mass is suddenly released from rest.

(a) Find the greatest speed the mass reaches. When does this occur?

(b) What is the greatest acceleration of the mass, and when does it occur?

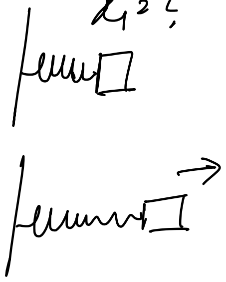
a) $U_{E_1} = KE_{2M}$

$11.5 = \frac{1}{2} M V_2^2$

$11.5 = \frac{1}{2} \times 2.5 \times V_2^2$

$V_2^2 = 9.2 \; ; \; V_2 = 3.03 \text{ m/s}$

t_1 $x_1 = ?$



$V_1 = 0 \text{ m/s}$

t_2 $\rightarrow V_2$

$|F_s| = kx$

$k = 25 \text{ N/cm}$

$= 2500 \text{ N/m}$

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Potential Energy Example

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b) $V_s = \frac{1}{2}kx_1^2 = 11.5$

$$x_1^2 = \frac{2 \times 11.5}{2500} = 9.20 \times 10^{-3}$$

$$x_1 = \sqrt{9.20 \times 10^{-3}} = 0.0959\,m$$

$$|F| = kx = 2500 \times 0.0959 = 240\,N$$

$F = ma$, $240 = 2.5a$

$$a = \frac{240}{2.5}$$

$$= 96\,m/s^2$$

Conservation of energy

If all forces acting are conservative, the total energy of the system does not change. We call this quantity the "Mechanical energy" (E_{mech}).

The energy can transfer from potential to kinetic forms. $E_{mech} = K + U$

We also know that the changes in *K and U* are given by:

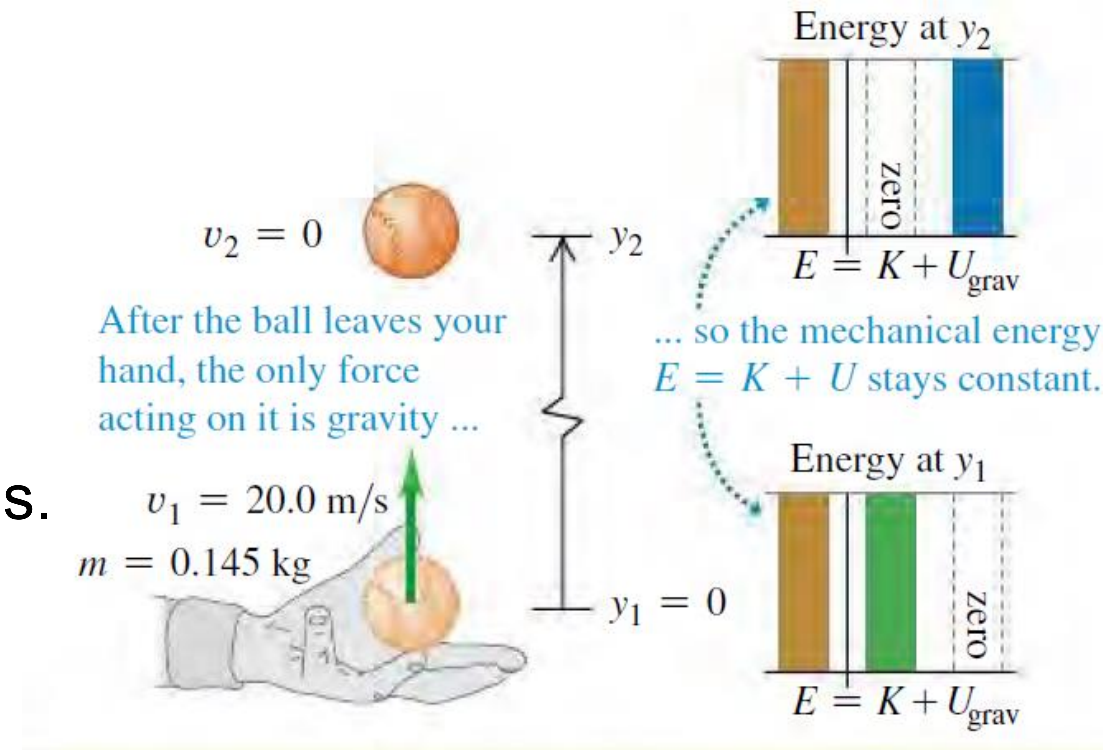
$$\Delta K = W; \qquad \Delta U = -W$$

Which gives: $\Delta K = -\Delta U$

So, if one energy increases, the other decreases.

$K_2 - K_1 = -(U_2 - U_1)$; Or:

$$K_1 + U_1 = K_2 + U_2$$



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Conservation of Energy Example 2

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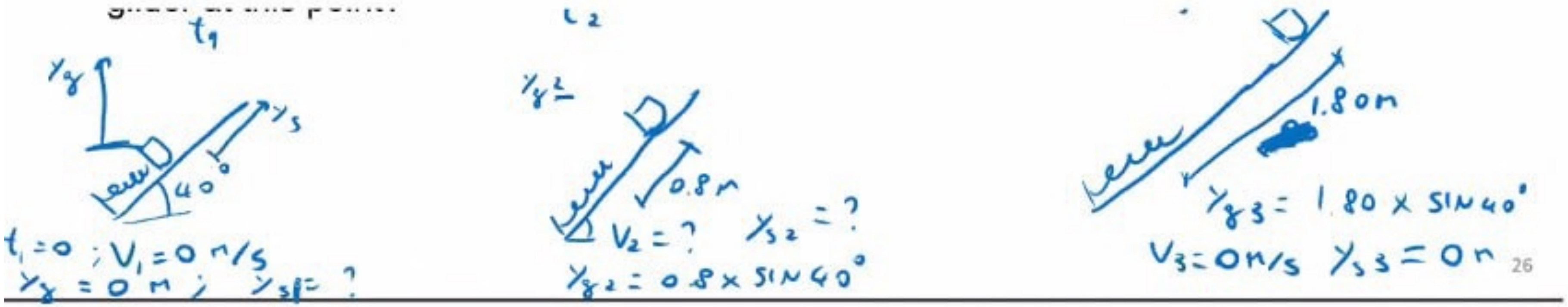
A small glider is placed against a compressed spring at the bottom of an air track that slopes upward at an angle of 40.0º above the horizontal. The glider has mass 0.0900 kg. The spring has k = 640 N/m and negligible mass. When the spring is released, the glider travels a maximum distance of 1.80 m along the air track before sliding back down. Before reaching this maximum distance, the glider loses contact with the spring.

(a) What distance was the spring originally compressed?

(b) When the glider has traveled along the air track 0.80 m from its initial position against the compressed spring, is it still in contact with the spring? What is the kinetic energy of the glider at this point?



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Conservation of Energy Example 2

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a) $U_{s_1} + \cancel{V_{G_1}}^{0} + \cancel{KE_1}^{0} = \cancel{V_{s_3}}^{0} + \cancel{KE_3}^{0}$

$\frac{1}{2} k Y_{s_1}^2 = mg Y_{g_3}$; $\frac{1}{2} \times 640 \times Y_{s_1}^2 = 0.09 \times 9.8 \times 1.8 \sin 40$

$320 Y_{s_1}^2 = 1.02$; $Y_{s_1}^2 = 3.188 \times 10^{-3}$; $Y_{s_1} = 0.0565\,m$

b) $U_{s_1} = \cancel{V_{s_2}}^{0} + V_{G_2} + KE_2$; $\frac{1}{2} k Y_{s_1}^2 = mg Y_{g_2} + KE_2$

$\frac{1}{2} \times 640 \times 0.0565^2 = 0.09 \times 9.8 \times 0.8 \times \sin 40^\circ + KE_2$

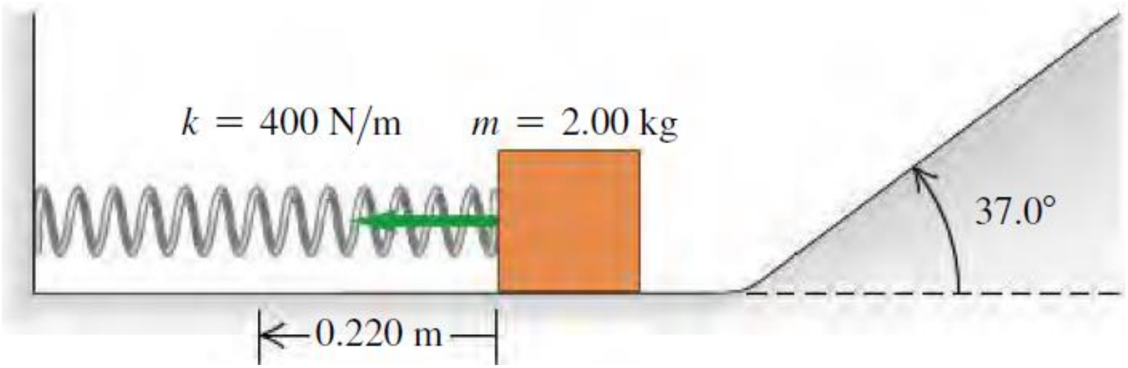
$1.02J = 0.454J + KE_2 = 0.566J = \frac{1}{2} M V^2$

Conservation of Energy Example 4

A 2.00 kg block is pushed against a spring with negligible mass and force constant k = 400 N/m, compressing it 0.220 m. When the block is released, it moves along a frictionless, horizontal surface and then up a frictionless incline with slope 37.0º.

(a) What is the speed of the block as it slides along the horizontal surface after having left the spring?

(b) How far does the block travel up the incline before starting to slide back down?



Conservation of Energy Example 4

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Work done by external forces

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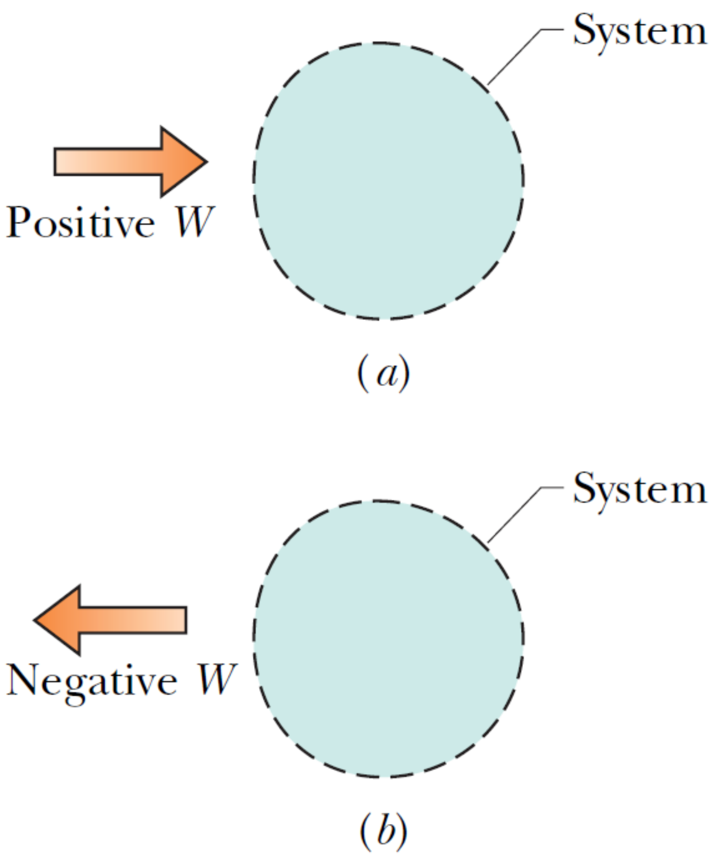
The total energy of the system can be changed by external forces.

Positive work (work done on the system) will increase the energy.

Negative work, (work done by the system), will decrease the energy.



$$W = \Delta K + \Delta U = \Delta E_{mech}$$

Rearranging: $W = K_2 - K_1 + U_2 - U_1$

Emech, Emech$_2$

$$K_1 + U_1 + W = K_2 + U_2$$

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Work done by external forces

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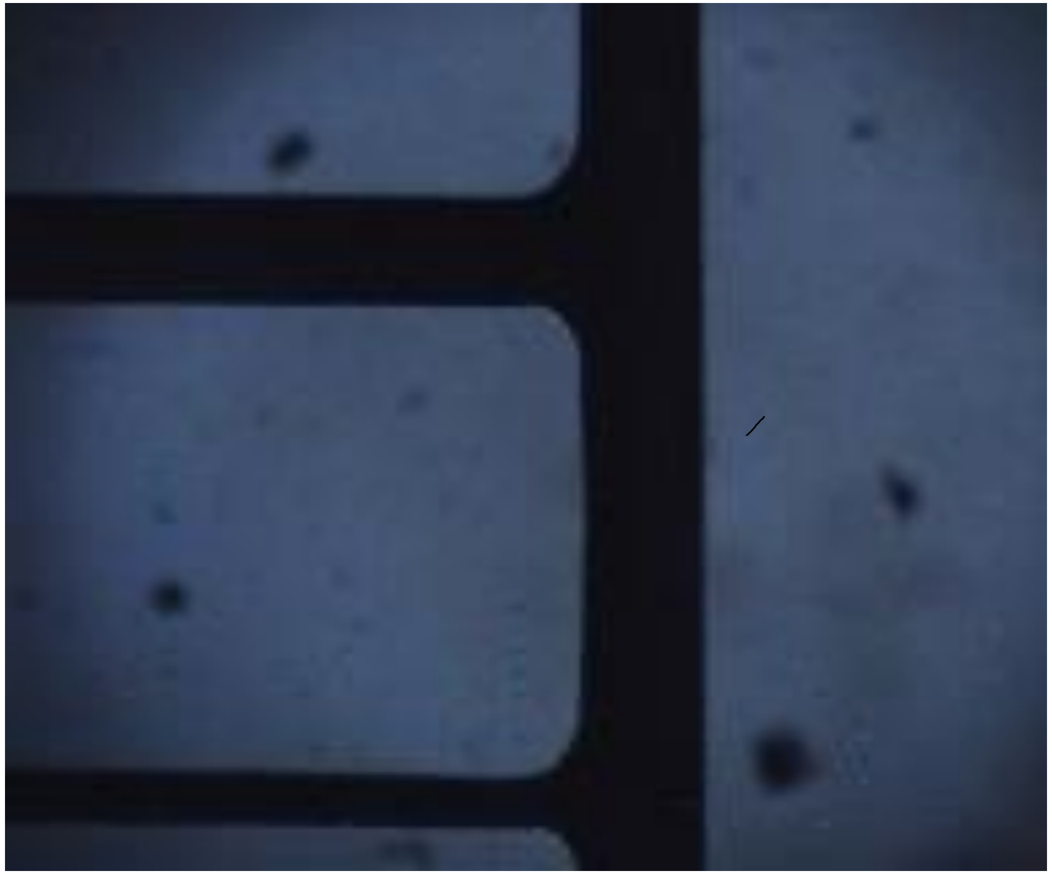
What happens to the energy removed from the system with friction, material failure and other non conservative events?



Work done by external forces

In general, we can say that:

$$\Delta E_{mech} = W_{tot} = W_{FE} + W_{FF} = Fd + \left(-f_k d\right)$$ Therefore: $$Fd = \Delta E_{mech} + f_k d$$

Experience tells us that frictions heats the parts in contact. So we can say that the energy is converted to *thermal energy* by the friction.

$f_k d = \Delta E_{th}$; and: $Fd = \Delta E_{mech} + \Delta E_{th}$

In general: $W = \Delta E_{mech} + \Delta E_{th}$

And: $W = \Delta E_{mech} + \Delta E_{th} + \Delta E_{int}$

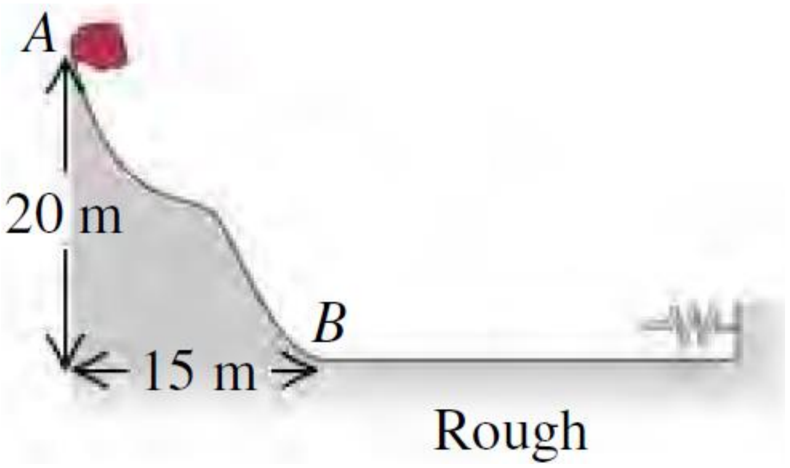
In an isolated system:

$$\Delta E_{mech} + \Delta E_{th} + \Delta E_{int} = 0$$

External Forces Example 2

A 15.0 kg stone slides down a snow-covered hill, leaving point A at a speed of 10.0 m/s. There is no friction on the hill between points A and B, but there is friction on the level ground at the bottom of the hill, between B and the wall. After entering the rough horizontal region, the stone travels 100 m and then runs into a very long, light spring with force constant 2.00 N/m. The coefficient of kinetic friction between the stone and the horizontal ground is 0.20.

(a) How far will the stone compress the spring?



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External Forces Example 2

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$$U_{G_1} + \cancel{U_{s_1}}^{0} + KE_1 - W_F = \cancel{U_{G_2}}^{0} + U_{s_2} + \cancel{KE_2}^{0}$$

$$mgY_1 + \frac{1}{2}MV_1^2 - F_f d = \frac{1}{2}kx_2^2 \quad ; \quad F_F = mg\mu_k$$

$$d = 100 + x_2$$

$$mgY_1 + \frac{1}{2}MV_1^2 - mg\mu_k(100 + x_2) = \frac{1}{2}kx_2^2$$

Power

The rate at which work is being done is called the *power*:

$$P = \frac{dW}{dt}$$

Measured in watts: 1 watt = 1 W = 1J/s

It is used commonly to quantify energy use, i.e. electrical consumption. For a force at an angle φ to the displacement, it can be expressed as:

$$P = \frac{F\cos\phi dx}{dt} = Fv\cos\phi = \mathbf{F}\bullet\mathbf{v}$$

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Power Example

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Your job is to lift 30 kg crates a vertical distance of 0.90 m from the ground onto the bed of a truck.

How many crates would you have to load onto the truck in 1 minute for an average power output of 100 W .

$W = 30 \times 0.9 \times 9.8 = 264.6$

$P = 100$

$P \times 1 min = 100 \times 60 = 6000$ J/min

Crates/min $= \frac{6000}{264.6}$

$= 22.7$ crates/min