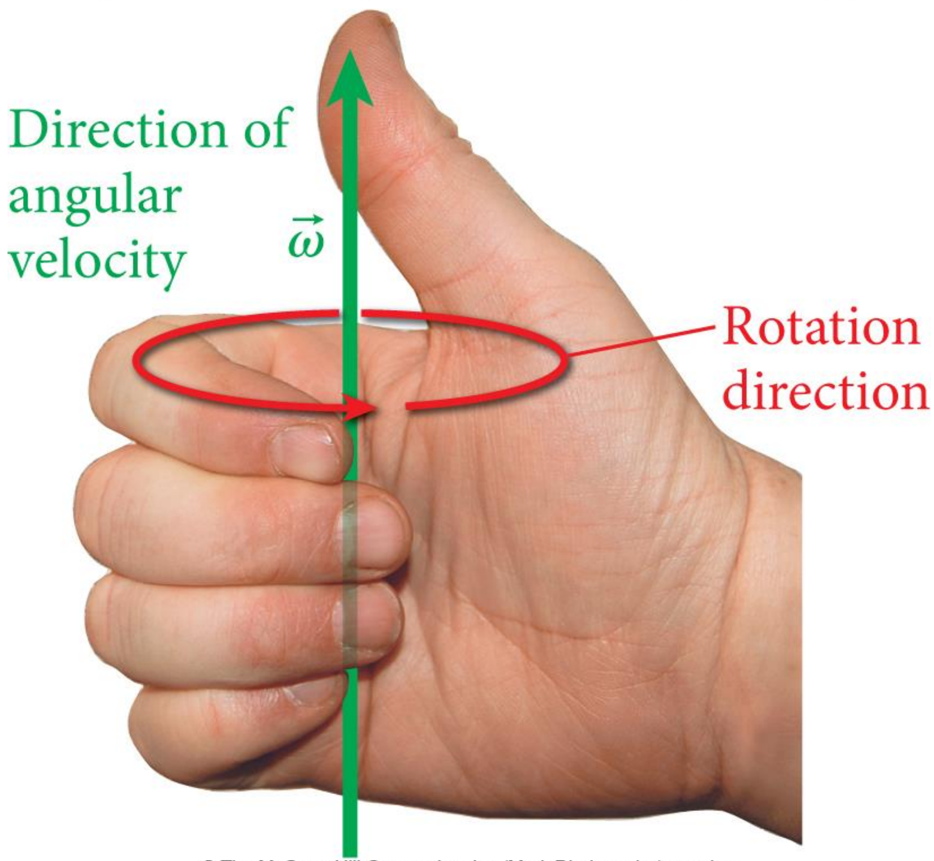
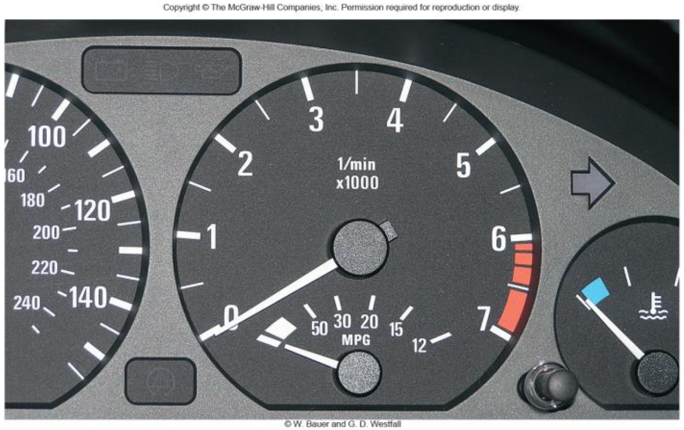
Angular Velocity

- Rate of change of displacement is velocity.
- Rate of change of angular displacement is angular velocity:
  - $$\bar{\omega} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$
  - $$\omega = \lim_{\Delta t \to 0} \bar{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$
- Unit: $[\omega]$ = rad/s
- Direction: right-hand rule



Frequency

- Frequency, f, measures numbers of turns around the circle.
- Example: rpm on tachometer
- Since 1 turn = 2π radians:



$$f = \frac{\omega}{2\pi} \Leftrightarrow \omega = 2\pi f$$

- Unit: [f]=1/s
- In honor of Heinrich Rudolf Hertz (1857-1894): 1/s = 1 Hz
- Period, T:

$$T = \frac{1}{f}$$

- Relationship with angular velocity:

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Concept Check

- The angular speed of the hour hand of a clock in radians per second is:
   A. π/21600
   B. π/7200
   C. π/3600
   D. π/1800
   E. π/60

the hour hand goes around once every 12 hours

$$\omega = 2\pi f = 2\pi \frac{1}{(3600\ \text{s/h})(12\ \text{h})} = \pi / 21600\ \text{rad/s}$$

Linear and Angular Velocity

- Write coordinate vector in Cartesian coordinates, then use transformation to polar coordinates and take derivatives:

$\vec{r} = rcos\theta\mathbf{i} + \mathrm{rsin}\theta\mathbf{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(rcos\theta)\mathbf{i} + \frac{d}{dt}(rsin\theta)\mathbf{j}$$

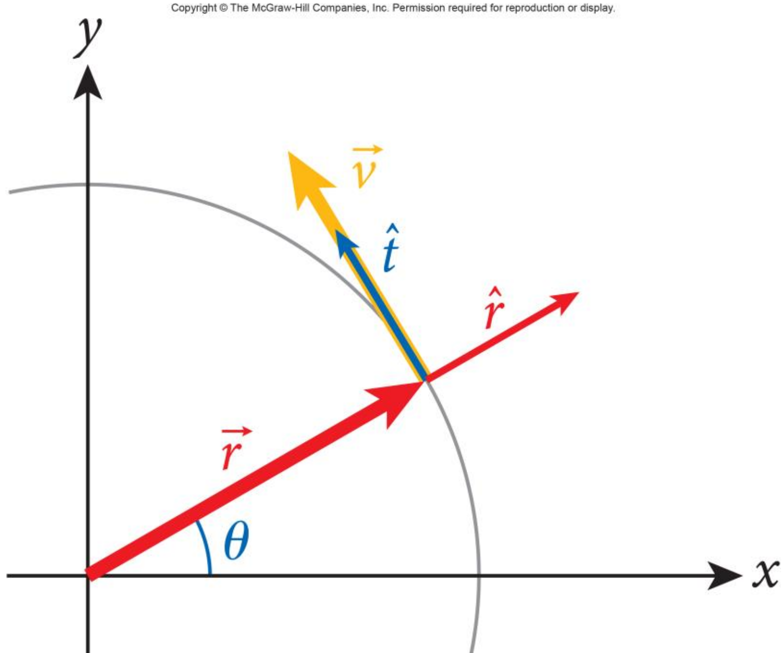
$$= -rsin\theta\frac{d\theta}{dt}\mathbf{i} + rcos\theta\frac{d\theta}{dt}\mathbf{j} = \mathrm{r}\frac{d\theta}{dt}(-sin\theta\mathbf{i} + cos\theta\mathbf{j}) = r\omega\hat{\mathbf{t}}$$

- Result: linear velocity vector points in tangential direction:

$$\boxed{\vec{v} = r\omega\hat{t}}$$

- Magnitude:

$$\boxed{v = r\omega}$$



Linear Velocity and Coordinate Vector

- Finally, let us take the scalar product of the linear velocity vector and radius vector for circular motion:

$$
\begin{aligned}
\vec{r} \bullet \vec{v} &= (r\cos\theta)(-r\omega\sin\theta) + (r\sin\theta)(r\omega\cos\theta) \\
&= (r\cos\theta, r\sin\theta) \bullet (-r\omega\sin\theta, r\omega\cos\theta) \\
&= -r^2\omega\cos\theta\sin\theta + r^2\omega\sin\theta\cos\theta \\
&= 0
\end{aligned}
$$

- Scalar product always vanishes for circular motion.

- Coordinate vector and velocity vector are perpendicular to each other at every point in time for circular motion.

Concept Check

- A bicycle has wheels with a radius of 33.0 cm. The bicycle is traveling with a speed of 6.5 m/s. What is the angular speed of the front tire?

A) 0.197 s^{-1}
B) 1.24 s^{-1}
C) 5.08 s^{-1}
D) 19.7 s^{-1}
E) 215 s^{-1}

$$v = r\omega$$

$$\omega = \frac{v}{r} = \frac{6.5\ \text{m/s}}{0.33\ \text{m}} = 19.7\ \text{s}^{-1}$$

Revolution and Rotation of the Earth

- The Earth orbits the Sun once per year and rotates on it pole-to-pole axis once per day.

PROBLEM:

- What are the angular velocities, frequencies and linear speeds of these motions?

SOLUTION:

- Periods:

$$T_{\text{Earth}} = (1\text{ day})\frac{24\text{ h}}{1\text{ day}}\frac{3600\text{ s}}{\text{h}} = 8.64\cdot 10^{4}\text{ s}$$

$$T_{\text{Sun}} = (1\text{ year})\frac{365\text{ days}}{1\text{ year}}\frac{24\text{ h}}{1\text{ day}}\frac{3600\text{ s}}{\text{h}} = 3.15\cdot 10^{7}\text{ s}$$

Revolution and Rotation of the Earth

- Frequencies:

$$f_{\text{Earth}} = \frac{1}{T_{\text{Earth}}} = 1.16 \cdot 10^{-5} \text{ Hz}; \ \omega_{\text{Earth}} = 2\pi f_{\text{Earth}} = 7.27 \cdot 10^{-5} \text{ rad/s}$$

$$f_{\text{Sun}} = \frac{1}{T_{\text{Sun}}} = 3.17 \cdot 10^{-8} \text{ Hz}; \ \omega_{\text{Sun}} = 2\pi f_{\text{Sun}} = 1.99 \cdot 10^{-7} \text{ rad/s}$$

- The 24-hour day we normally use represents how long it takes for the Sun to reach the same position in the sky.
- If we want to specify f_{Earth} and ω_{Earth} to greater precision, we use the sidereal day, which is the time it takes for the fixed stars in the night sky to reach the same position:
- Linear velocity of Earth orbiting the Sun:

66,000 mph!

$$v_{\text{Sun}} = r_{\text{orbit}}\omega_{\text{Sun}} = \left(1.49 \cdot 10^{11} \text{ m}\right)\left(1.99 \cdot 10^{-7} \text{ rad/s}\right) = 2.97 \cdot 10^{4} \text{ m/s}$$

Revolution and Rotation of the Earth

- Linear speed of point on the surface of the rotating Earth depends on the latitude:

$$r = R = 6380 \text{ km}$$

- At the equator, the radius of rotation is:

$$r = R\cos\vartheta$$

- Away from the equator, the radius of rotation is:

$$v_{\text{Earth}} = R\omega_{\text{Earth}}\cos\vartheta$$

$$v_{\text{Earth}} = \left(6{,}38\cdot 10^{6} \text{ m}\right)\left(7.27\cdot 10^{-5} \text{ s}^{-1}\right)\cos\vartheta$$

$$v_{\text{Earth}} = \left(464 \text{ m/s}\right)\cos\vartheta$$

- The linear speed is:

East Lansing: $\vartheta = 42.7^\circ$ $v_{\text{Earth}} = 341 \text{ m/s}$

Miami: $\vartheta = 25.7^\circ$ $v_{\text{Earth}} = 418 \text{ m/s}$