Question 1

Consider a horizontal spring-mass system. The object of mass, *m*, is displaced off its equilibrium position by x_o. It is then released and the mass executes simple harmonic motion.

(i) Sketch the displacement-time graph of the object and write down the corresponding equation.

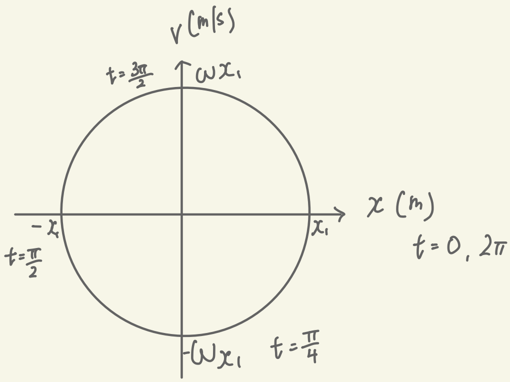
(ii) Sketch a velocity-time graph of the object and write down the corresponding equation.

(iii) Sketch a restoring force-time graph of the object and write down the corresponding equation.

(iv) Sketch a velocity-displacement graph of the object and write down the corresponding equation.

(v) Sketch a graph to show how kinetic energy and potential energy varies with time and write down the corresponding equations.

iii)



$$\cos^2\theta + \sin^2\theta = 1$$

$$x = x_1 \cos \omega t$$

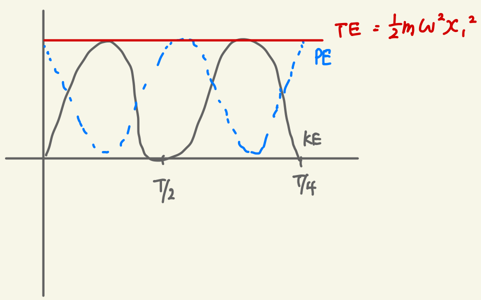
$$v = -\omega x \sin(\omega t)$$

$$\omega^2 x^2 = \omega^2 x_1^2 \cos^2(\omega t)$$

$$v^2 = \omega^2 x_1^2 \sin^2(\omega t)$$

$$\omega^2 x^2 + v^2 = \omega^2 x_1^2$$

$$v = \pm \omega \sqrt{(x_1^2 - x^2)}$$



$$KE = \frac{1}{2} m v^2$$

$$= \frac{1}{2} m \omega^2 x_1^2 \sin^2(\omega t)$$

$$PE = \frac{1}{2} k x^2$$

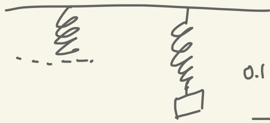
$$= \frac{1}{2} m \omega^2 x_1^2 \cos^2(\omega t)$$

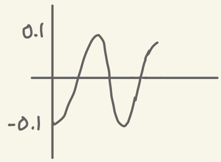
Question 2

A mass hangs in equilibrium from a light helical spring. It is given an initial vertical displacement of 0.1 m and released at time $t = 0$ such that it oscillates with angular frequency of 0.2 rad s^{-1}. Determine the displacement, in m, at time t.

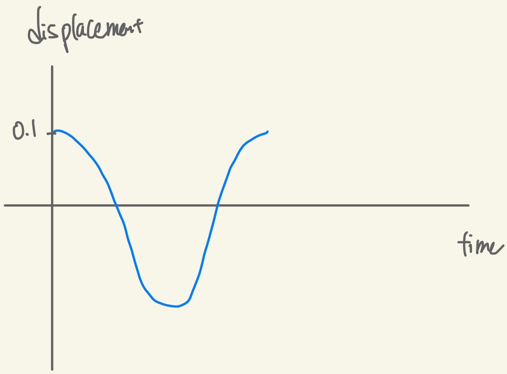
angular frequency $\rightarrow \omega = 0.2$ rad/s

$x_0 = 0.1$





$= -0.1\cos(0.2t)$



at time t, displacement $= 0.1\cos(0.2t)$

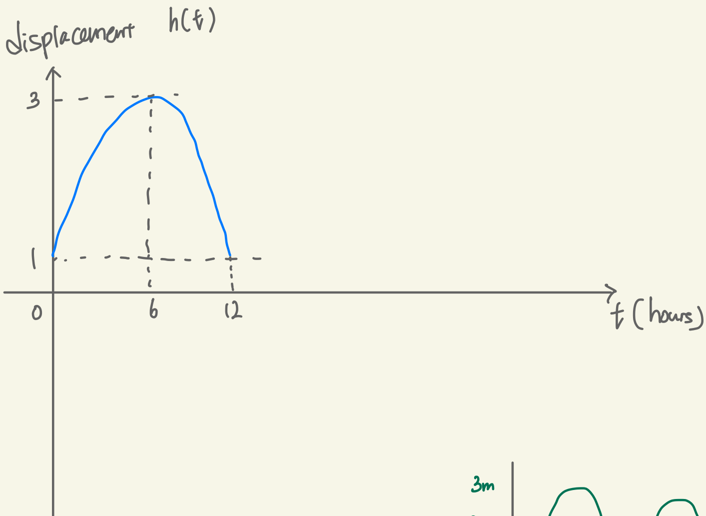
Question 3

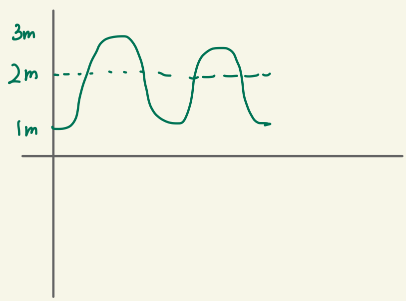
In a harbour, the rise and fall of water is simple harmonic with the time between successive high tides being 12 hours. The depth of the water in the harbour varies from 1.0 m at low tide to 3.0 m at high tide.

A ship which is stranded in the harbour at low tide ($t = 0$) requires a minimum depth of 1.5 m before it can leave the harbour. How long must the ship wait (in hours) before it can leave?

$f = \frac{1}{12}$ $\quad \omega = 2\pi\left(\frac{1}{12}\right)$

$= \frac{\pi}{6}$





$h(t) = -x_0 \cos(\omega t) + 2$

$= -1 \cos\left(\frac{\pi}{6} t\right) + 2$

$1.5 = -\cos\left(\frac{\pi}{6} t\right) + 2$

$t = \dfrac{\cos^{-1}(2-1.5)}{\frac{\pi}{6}}$

$= 2$ hours

Question 4

A mass m at the end of a spring oscillates with a frequency of 0.83 Hz. When an additional 780-g mass is added to *m*, the frequency is 0.60 Hz. What is the value of *m*?

$f_1 = 0.83$

$f_2 = 0.6$

f is natural frequency of system that depends on system properties.
E.g. *f* of **mass-spring system** depends on mass *m* and spring constant *k*.

From Newton's 2nd Law

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$2\pi f = \sqrt{\frac{k}{m}}$$

$$4\pi^2 f^2 = \frac{k}{m}$$

$$m f^2 = \frac{k}{4\pi^2}$$

similar to momentum

$$mf^2 = m_f f^2$$

$$m(0.83)^2 = (m + 0.78)(0.6)^2$$

$$m(0.83)^2 - m(0.6)^2 = (0.78)(0.6)^2$$

$$m = \frac{(0.78)(0.6)^2}{(0.83)^2 - (0.6)^2}$$

$$= 0.854 \text{ kg}$$

Question 1

We can consider a car's suspension system to be a spring under compression with a shock absorber which damps the car's vertical oscillations.

The car is then driven at a steady speed over a rough road on which the surface height varies sinusoidally. Unfortunately, the shock absorber mechanism which normally damps vertical oscillations is not working. Hence, at a certain critical speed, the amplitude of the vertical oscillation becomes very large.

(i) Name the phenomena observed: "amplitude of the vertical oscillation becomes very large" [1]

(ii) Given that the spring suspension system obeys Hooke's law, calculate the force constant, k, of the spring suspension system.

$$K = \frac{450 \times 9.8}{0.1}$$

$$= 44100 = 4.4 \times 10^4 \, N/m$$

Important data you can use:

Mass of passengers, $m = 450$ kg

Total mass of car and passengers, $M = 2000$ kg

Difference in height of car when passengers alight, $\Delta h = 0.10$ m [3]

(iii) Using your answer in (i), determine the critical speed when the amplitude of vertical oscillation is a maximum. The separation of consecutive humps on the road is 20 m.

(Period of oscillation, T, of a spring mass system with spring constant, k and mass, m, is given by $T = 2\pi\sqrt{\frac{m}{k}}$.) [4]

(iv) Sketch **on the same axes** appropriately labelled graphs to contrast how the amplitude of oscillation would vary at different speeds if the damping mechanism is

1. operating, 2. not operating. [2]

Question 2

On a windy day, a tall building can be set into simple harmonic motion. The horizontal displacement in meters, x, of the top of the building, changes with time in seconds, t, according to the equation:

$$x = 1.25\cos(0.209t)$$

(i) Find the period of the oversillation. [2]

$$\omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{0.209}$$

$$= 30\,s$$

(ii) A man of mass 80 kg stands at the top of the oscillating building. Calculate the maximum kinetic energy of the man. [2]

$$\frac{dx}{dt} = -1.25\sin(0.209t) \cdot 0.209$$

KE = max when $\sin t = 1$

max $KE = \frac{1}{2} m \omega^2 x_0^2$

$$= \frac{1}{2}(80)(0.209)^2(1.25)^2$$

$$= 2.7\,J$$

(iii) Sketch on the axes below to show how the man's kinetic energy varies with time for **2 complete oscillations**. [2]