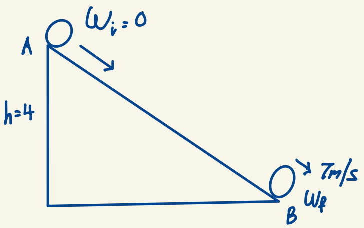
WARM UP

1. A circular object begins from rest and rolls without slipping down an incline, through a vertical distance of 4.0 m. When the object reaches the bottom, its translational velocity is 7.0 m/s. Determine the constant c relating the moment of inertia to the mass and radius of this object.

[Ans: 0.6]



$$TE_A = TE_B$$

$$U_A + \cancel{K_A} = U_B + K_B$$

translational $\frac{1}{2}mv^2$

rotational $\frac{1}{2}I\omega^2$

$I = cmr^2$

$v = r\omega$

$K_{rot} = \frac{1}{2}cmv^2$

$$mgh + 0 = 0 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}cmv^2$$

$$mgh = \frac{1}{2}mv^2(1+c)$$

$$c = \frac{\cancel{m}gh}{\frac{1}{2}\cancel{m}v^2} - 1$$

$$= 0.6$$

Kinetic Energy of Rotation

2. A uniform solid sphere of radius R, mass M, and moment of inertia $I = \frac{2}{5}MR^2$ is rolling without slipping along a horizontal surface. Find the fraction of the sphere's total kinetic energy that is attributable to rotation.

[Ans: 2/7]

$$K_{Tot} = K_{Rot} + K_{trans}$$

$$K_{Tot} = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = \frac{1}{2}CMV^2 + \frac{1}{2}mv^2$$

$$= \frac{1}{2}Mv^2(1+C)$$

$$I = CMR^2$$

$$\omega = \frac{V}{R}$$

$$K_{rot} = \frac{1}{2}Cmv^2$$

$$\frac{K_{rot}}{K_{Tot}} = \frac{\cancel{\frac{1}{2}}C\cancel{M}\cancel{v^2}}{\cancel{\frac{1}{2}}\cancel{M}\cancel{v^2}(1+C)}$$

$$= \frac{\frac{2}{5}}{(1+\frac{2}{5})}$$

$$= \frac{2}{7}$$

Moments of Inertia

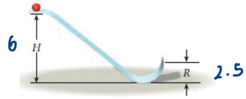
3. Determine the total moment of inertia for three people weighing 60 kg, 45 kg and 80 kg sitting at different points along the edge of a rotating merry-go-round, which has a ~~radius~~ of 12 m.

diameter [Ans: 6660 kgm^2]

$$\sum I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

Rolling without Slipping

4. An object of mass m and radius r has a moment of inertia given by $I = cmr^2$. The object rolls without slipping along a track that ends with a ramp of height R = 2.5 m that launches the object vertically. If the object starts from a height H = 6 m. To what maximum height will it rise after leaving the ramp if c = 0.40?



[Ans: 5 m]

$$mgH = K_{Trans} + K_{Rot} + mgR$$

$$mgH = \frac{1}{2}mv^2 + \frac{1}{2}Cmv^2 + mgR$$

$$mgH - mgR = \frac{1}{2}mv^2(1+C)$$

$$v^2 = \frac{\cancel{m}gH - \cancel{m}gR}{\frac{1}{2}\cancel{m}(1+C)}$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$v^2 = \frac{2g(H-R)}{(1+C)}$$

$$= 49$$

$$0 = 49 + 2(-9.8)(h)$$

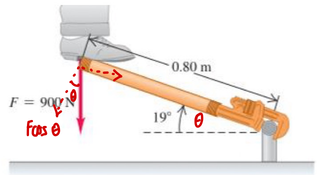
$$h = 2.5$$

$$\text{total height} = 2.5 + 2.5$$

$$= 5m$$

Torque

5. To loosen a pipe fitting, a plumber stands on the end of his wrench, applying his 900 N weight at a point 0.80 m from the centre of the fitting as shown below. The wrench makes an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the centre of the fitting.

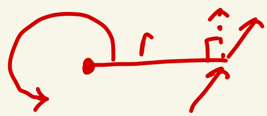


$\tau = rf$

$\tau = F\cos\theta \, r$

$= 900\cos 19^\circ \times 0.8$

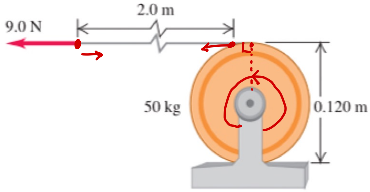
$\approx 680 \text{ Nm}$



$\tau = r f_{tan}$

Newton's Second Law for Rotation

6. A block of mass m is tied to a massless cable around a solid cylinder with mass 50 kg and diameter 12 cm The cylinder rotates with negligible friction about a stationary horizontal axis. You pull the cable with constant force of 9 N for a distance of 2 m. Find the acceleration and velocity of the cable.



[Ans: 0.36 m/s^2; 1.2 m/s]

$\tau_{net} = I\alpha$ $(F = ma)$

$I_{cylinder} = \frac{1}{2}MR^2$

$Fr = I\alpha$ $\quad I = mr^2$

$a = R\alpha$

$9(0.06) = \frac{1}{2}(50)(0.06)^2 \alpha$

$\alpha = 6$

$a = (0.06)(6)$

$= 0.36 m/s^2$

$v^2 = v_0^2 + 2a(x - x_0)$

$v^2 = 2(0.36)(2)$

$v = \sqrt{1.44}$

$= 1.2 m/s$

Work Done by Torque

7. A flywheel is a solid homogeneous metal disk of mass M = 120 kg and radius R = 80.0 cm. The engine rotates the wheel at 500 rpm. In an emergency, to bring the engine to a stop, the flywheel is disengaged from the engine and a brake pad is applied at the edge to provide a radially inward force F = 100 N. If the coefficient of kinetic friction between the pad and the flywheel is μ_k= 0.2, how many revolutions does the flywheel make before coming to rest? How long does it take for the flywheel to come to rest? Calculate the work done by the torque during this time.

[Ans: 524 revs; 126 s; -5.26×10^4 J]