EDE1011 ENGINEERING MATHEMATICS 1

Tutorial 1

Functions

1. Evaluate the domain and range of the following functions.

   a) $f(x) = 2 - x^2$
   b) $f(x) = \sqrt{2x+1} - 5$
   c) $f(x) = \frac{1}{3x-5}$
   d) $f(x) = 1 - e^{-x}$
   e) $f(x) = \ln(x+2)$ since $x+2>0$, D is $(-2, \infty)$, Range is $\mathbb{R}$
   f) $f(x) = \cot(\pi x)$

ANS: **a)** ℝ, (-∞, 2]. **b)** [-½, ∞), [-5, ∞). **c)** {ℝ | x ≠ 5/3}, {ℝ | y ≠ 0}. **d)** ℝ, (-∞, 1). **e)** (-2, ∞), ℝ. **f)** {ℝ | x ≠ nπ, n∈ℤ}, ℝ.

2. Evaluate the equation of a line in all three forms that has a slope of -½ and x-intercept at x = -3.

ANS: y = -½(x+3) ,y = -x/2 - 3/2, 2y + x = -3

3. (https://openstax.org/books/calculus-volume-1/pages/1-2-basic-classes-of-functions)
   A company purchases some computer equipment for $20,500. At the end of a 3-year period, the value of the equipment has decreased linearly to $12,300.

   a) Determine a function V(t) that determines the value V of the equipment at the end of t years.
   b) Interpret the meaning of the slope of V(t). Initial equipment value (when brand new) V(0) = $20.5k
   c) Evaluate and interpret the meaning of the t and V intercepts of V(t).
   d) When will the value of the equipment be $3000?

ANS: **a)** V(t) = -2733.33t + 20500. **b)** Equipment is depreciating at a rate of $2733.33 per year. **c)** t = 7.5 years (when the equipment has zero value), V = $20,500 (initial equipment value). **d)** Approx 6.4 years.

4. By completing the square on the quadratic function below, show that the vertex (max or min point) of the graph is at x = -b/(2a).

$$f(x) = ax^2 + bx + c = \dots = a\left(x+\frac{b}{2a}\right)^2 + k$$

For $a > 0$, ↑∪↑ min

Since $\left(x+\frac{b}{2a}\right)^2 \geq 0$, $f(x) \geq k$

min $f = k$ occurs at $\left(x+\frac{b}{2a}\right)^2 = 0$

$\Rightarrow x = -\frac{b}{2a}$ //

Hence, state the coordinates of the vertex of

$f(x) = -x^2 + 4x$

For $a = -1, b = 4$, $x = -\frac{4}{2(-1)}$

$= 2$

$f(2) = 2(4-2) = 4$

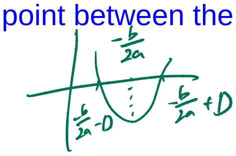
$$f(x) = x(4-x)$$

What do you observe about the vertex of a quadratic function in relation to its roots?

ANS: Vertex at (2, 4). The vertex is always at the midpoint between the roots.

roots $= \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$

$= -\frac{b}{2a} \pm D$



So vertex is always at the midpoint between the roots

5. Evaluate the symmetry of each function below, if any.

↳ Just find $f(-x)$

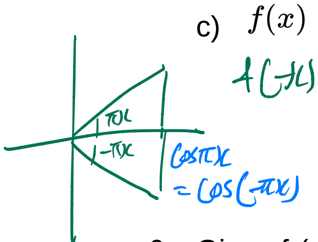
a) $f(x) = x - \tan x$

b) $f(x) = x(2x-1)$

c) $f(x) = \sqrt{2 - \cos(\pi x)}$

$f(-x) = \sqrt{2-\cos(-\pi x)}$

$= \sqrt{2-\cos(\pi x)} = f(x)$, so even



ANS: **a)** Odd. **b)** Neither even nor odd. **c)** Even.

6. Given $f_e(x)$ and $f_o(x)$ are even and odd functions respectively, show that

a) $p(x) = f_e(x) \cdot f_e(x)$ is even.

b) $q(x) = f_o(x) \cdot f_o(x)$ is even.

c) $r(x) = f_e(x) \cdot f_o(x)$ is odd.

d) $u(x) = f_e(x) + f_e(x)$ is even.

e) $v(x) = f_o(x) + f_o(x)$ is odd.

f) $w(x) = f_e(x) + f_o(x)$ is neither even nor odd.

↳ $w(-x) = f_e(-x) + f_o(-x)$

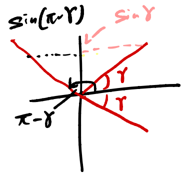
$= f_e(x) - f_o(x)$ $\begin{cases} \neq w(x) \\ \neq -w(x) \end{cases}$

For even functions, $f_e(-x) = f_e(x)$

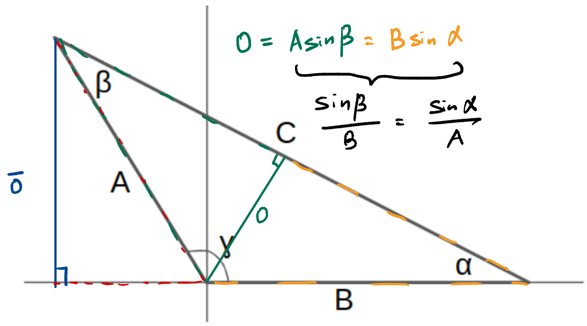
For odd functions, $f_o(-x) = -f_o(x)$

7. From the triangle shown below, derive the following triangle laws.

$\bar{o} = C\sin\alpha = \underbrace{A\sin(\pi-\gamma)} = A\sin\gamma$



$\frac{\sin\alpha}{A} = \frac{\sin\gamma}{C}$



a) Law of sines.

$$\frac{\sin\alpha}{A} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$$

b) Law of cosines. (Hint: Use the Pythagoras theorem to find length C.)

$$C^2 = A^2 + B^2 - 2AB\cos\gamma$$

8. Evaluate each composition / combination of functions below and its domain. What is the relation between f(x) and g(x) in (b)?

a) $f(x) = \frac{1}{x} + 1, \quad g(x) = \ln(x-2), \quad h(x) = \frac{g(x)}{f(x)}$

b) $f(x) = \frac{1}{x} + 1, \quad g(x) = \frac{1}{x-1}, \quad h(x) = f(g(x))$

ANS: **a)** $h(x) = \frac{x\ln(x-2)}{1+x}$, (2, ∞). **b)** $h(x) = x$, {ℝ | x ≠ 1}. f(x) and g(x) are inverse functions of each other.

9. Evaluate the inverse of each function below and its domain and range.

a) $f(x) = 7e^{-3(x+1)} - 1$

b) $f(x) = \frac{1}{\sqrt{2x-3}}$

c) $f(x) = 2\sin(x/2) + 5, \quad 0 \le x \le \pi$

for f(x) to be 1-to-1

ANS: **a)** $f^{-1}(x) = -\frac{1}{3}\ln\left(\frac{x+1}{7}\right) - 1$, (-1, ∞), ℝ. **b)** $f^{-1}(x) = \frac{1}{2x^2} + \frac{3}{2}$, (0, ∞), (3/2, ∞). **c)** $f^{-1}(x) = 2\sin^{-1}\left(\frac{x-5}{2}\right)$, [5, 7], [0, π].

10. When an initial investment P is compounded at an annual interest of r percent, the accumulated amount A at the end of year t is

$A(0) = P$

$$A(t) = P(1+r)^t$$

$A(1) = P(1+r)$

$A(2) = (1+r)A(1)$

$= P(1+r)^2$

Determine the inverse function of A(t) and describe its use in layman.

ANS: $t(A) = \log_{1+r}\left(\frac{A}{P}\right) = \frac{\ln(A/P)}{\ln(1+r)}$. t(A) calculates the number of years required for the accumulated amount to reach A, given parameters r & P.

11. Obtain the graph of y(x) by transformations from f(x) for each function below and state its domain and range.

a) $f(x) = |x|\,,\ \ y(x) = -|2x-4|-1$

b) $f(x) = e^x,\ \ y(x) = 2e^{1-x}-3$

ANS: **a)** ℝ, (-∞, -1]. **b)** ℝ, (-3, ∞).

For more practice problems (& explanations), check out:

1) https://openstax.org/books/calculus-volume-1/pages/1-1-review-of-functions
2) https://openstax.org/books/calculus-volume-1/pages/1-2-basic-classes-of-functions
3) https://openstax.org/books/calculus-volume-1/pages/1-3-trigonometric-functions
4) https://openstax.org/books/calculus-volume-1/pages/1-4-inverse-functions
5) https://openstax.org/books/calculus-volume-1/pages/1-5-exponential-and-logarithmic-functions
6) https://openstax.org/books/calculus-volume-1/pages/1-review-exercises

End of Tutorial 1

(Email to youliangzheng@gmail.com for assistance.)