Q1 (3 points)

*a) Consider a point charge q located at position $\vec{r}_0$. State the electric field $\vec{E}(\vec{r})$ which is generated by this point charge.

b) Consider now N point charges q_i, each of them located at a discrete position $\vec{r}_i$ ($i = 1, \ldots, N$), respectively. State the electric field $\vec{E}(\vec{r})$ generated by these N point charges.

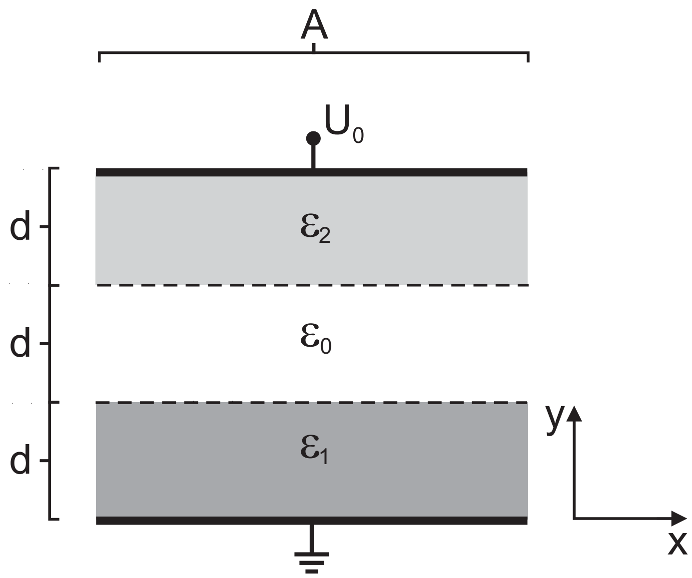
*c) What is the fundamental principle the result in b) is based on?

Q2 (1 point)

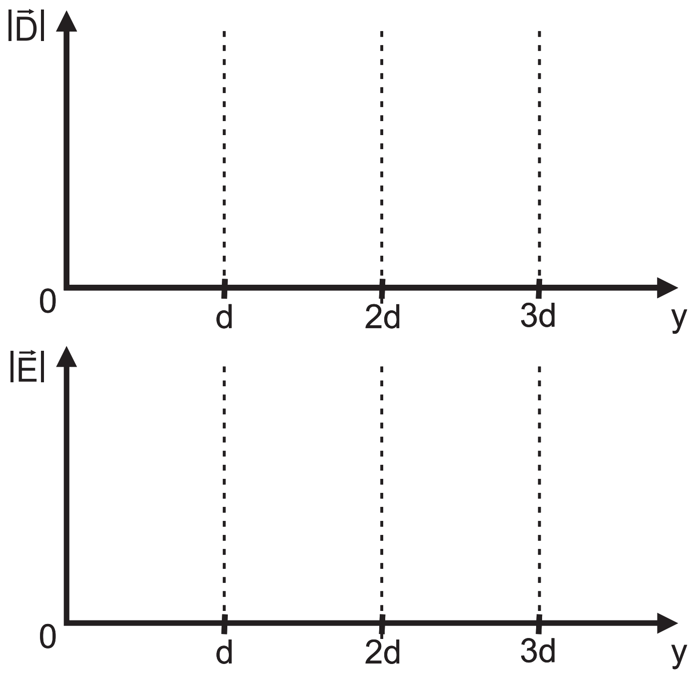
What is the mathematical relation between electrostatic field $\vec{E}(\vec{r})$ and the corresponding electric potential $\Phi(\vec{r})$?

Q3 (8 points)

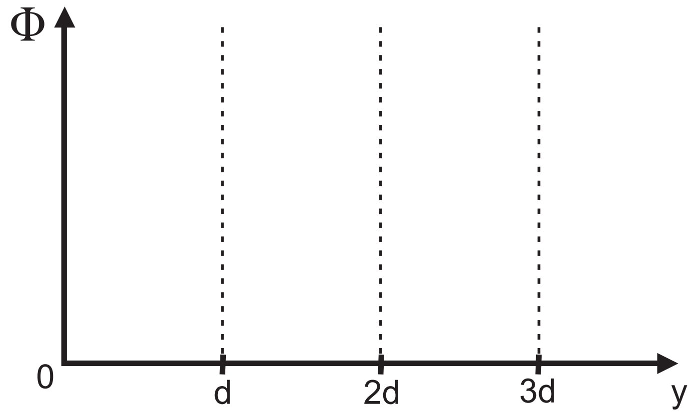
A plate capacitor with area A and plate distance $3d$ consists of three equally-sized regions of thickness d with different permittivities ε_2, ε_0, and ε_1, where $\varepsilon_1 > \varepsilon_2 > \varepsilon_0$. The capacitor is biased with a positive voltage $U_0 > 0$. Stray fields may be neglected.



*a) Sketch the variation of the magnitude of the displacement field $|\vec{D}(y)|$ and the electrostatic field $|\vec{E}(y)|$ along the y-axis.
Use the diagrams given below and consider which of the two fields is continuous at the interfaces of adjacent regions.



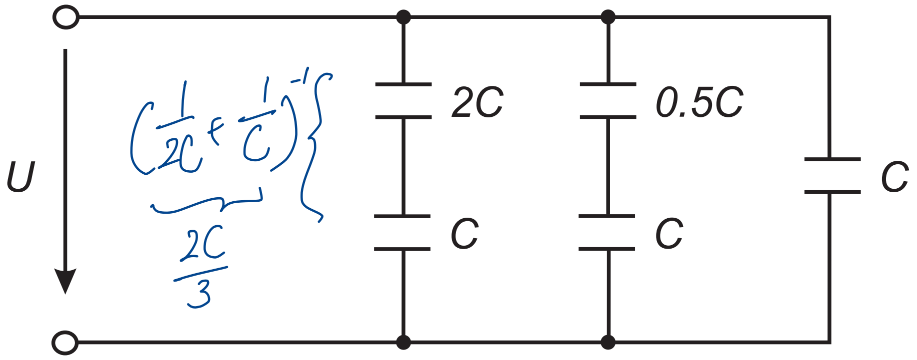
b) Sketch the variation of the electric potential $\Phi(y)$ along the y-direction. Use the diagram given below.



*c) Calculate the electrostatic energy W_{el} which is stored in the interior of the plate capacitor.
(Hint: Describe the given configuration by an equivalent circuit of three single plate capacitors.)

Q4 (3 points)

Calculate the total capacitance C_{tot} of the following capacitor circuit:



$\underbrace{\left(\frac{1}{2C}+\frac{1}{C}\right)^{-1}}_{\frac{2C}{3}}$

$\left(\frac{2}{C}+\frac{1}{C}\right)^{-1}=\frac{C}{3}$

$$\frac{2C}{3}+\frac{C}{3}+C = 4C$$

Q1 (3 points)

*a)

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{q}{|\vec{r} - \vec{r_0}|^3} (\vec{r} - \vec{r_0})$$

b)

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{N} \frac{q_\mathrm{i}}{|\vec{r} - \vec{r_\mathrm{i}}|^3} (\vec{r} - \vec{r_\mathrm{i}})$$

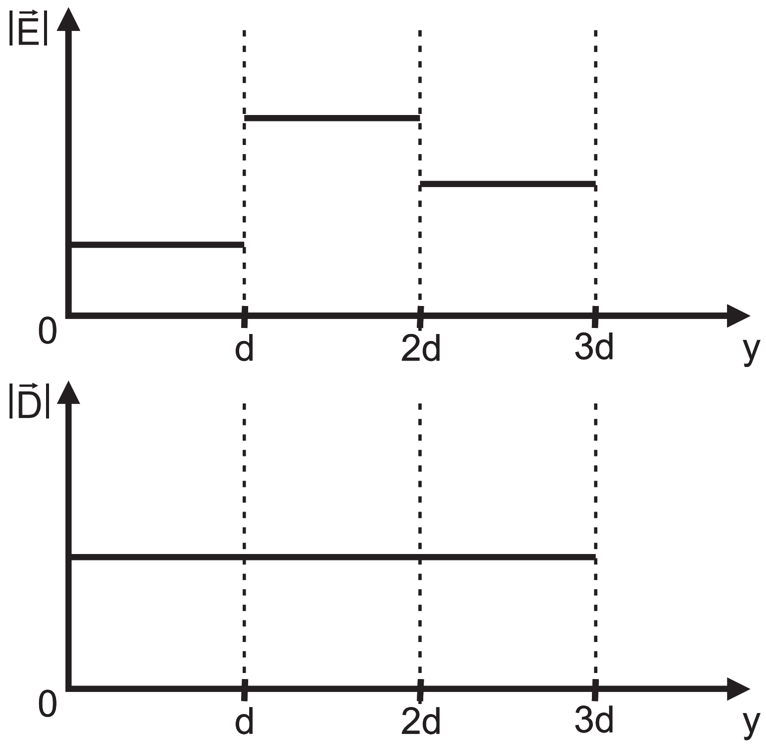
*c) superposition principle

Q2 (1 point)

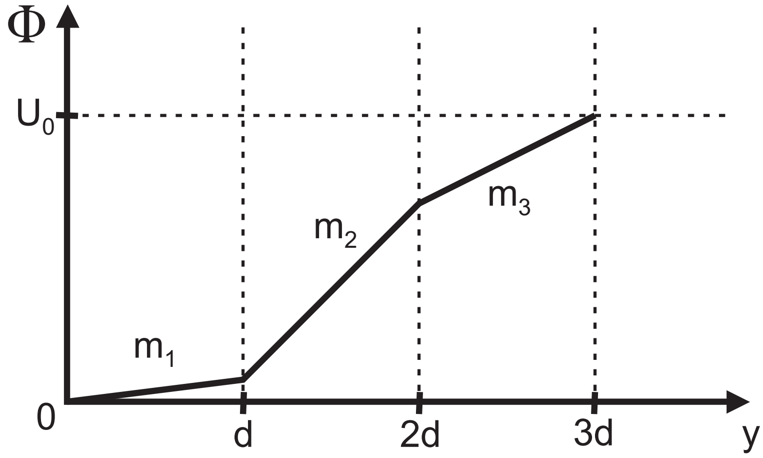
$$\vec{E}(\vec{r}) = -\mathrm{grad}\Phi(\vec{r}); \qquad \text{or: } \Phi(\vec{r}) = \Phi(\vec{r_0}) - \int_{\mathrm{P}_0}^{\mathrm{P}} \vec{E}(\vec{r}\,')\mathrm{d}\vec{r}\,'$$

Q3 (8 points)

*a) Das D-Feld is constant; E-Feld: $|\vec{E}| = \frac{1}{\epsilon}|\vec{D}|$.



b)



For the slopes of the single sections the following relation applies: $m_1 < m_3 < m_2$.

*c)

$$W_{\mathrm{el}} = \frac{1}{2} C_{\mathrm{tot}} U_0^2$$

$$\frac{1}{C_{\mathrm{tot}}} = \frac{1}{C_1} + \frac{1}{C_0} + \frac{1}{C_2}$$

$$= \frac{d}{\epsilon_1 A} + \frac{d}{\epsilon_0 A} + \frac{d}{\epsilon_2 A} = \frac{d}{A}\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_0} + \frac{1}{\epsilon_2}\right)$$

$$W_{\mathrm{el}} = \frac{1}{2}\frac{A}{d}\frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_0} + \frac{1}{\epsilon_2}} U_0^2$$

Q4 (3 points)

$$\frac{1}{C_1} = \frac{1}{2C} + \frac{1}{C} = \frac{3}{2C}$$

$$C_1 = \frac{2}{3} C$$

$$\frac{1}{C_2} = \frac{2}{C} + \frac{1}{C} = \frac{3}{C}$$

$$C_2 = \frac{1}{3} C$$

$$C_{tot} = \frac{2}{3} C + \frac{1}{3} C = 2C$$