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Potential Energy and energy conservation

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Lecture Learning Outcomes

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In this lesson you will learn to:

- Calculate the changes of potential energy of objects
- Relate the potential energy to other forms of energy
- Use the principle of conservation of energy to solve problems.

Potential energy

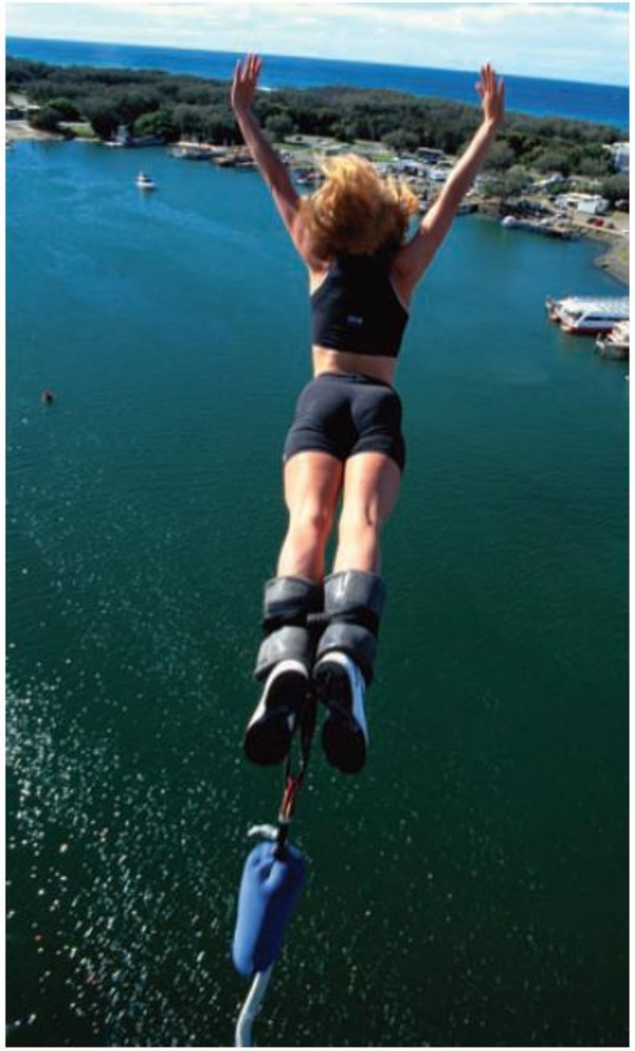
In the previous lesson, we looked at kinetic energy and work.

Potential energy (U) is a further type of energy.

It is the energy associated with the state of a system.

Examples are elastic potential energy and gravitational potential energy.

It can be seen as a “store” of energy.



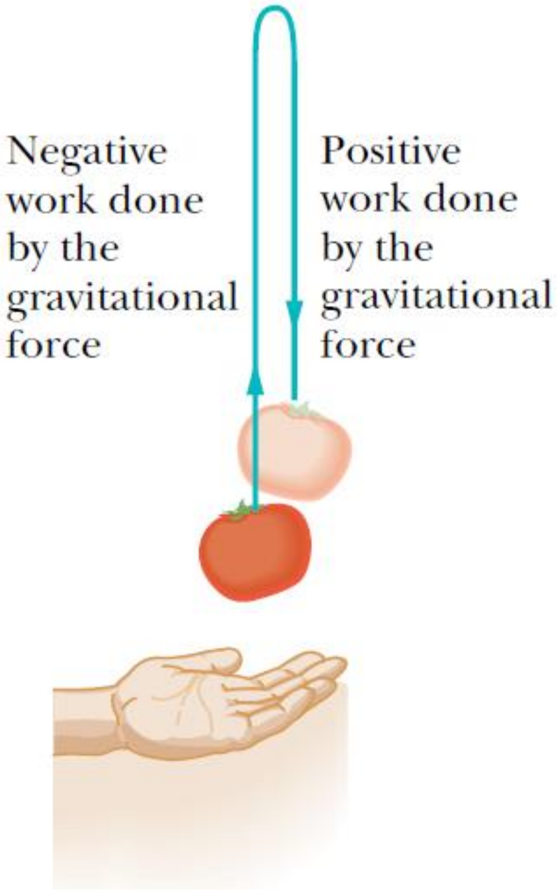
Potential energy and work - gravity

Work can be done to change the potential energy of a system.

For gravity, the higher the object, the higher the potential energy.

When the apple rises, gravity does **negative** work, U_g increases.

When the apple falls, gravity does **positive** work, U_g decreases.



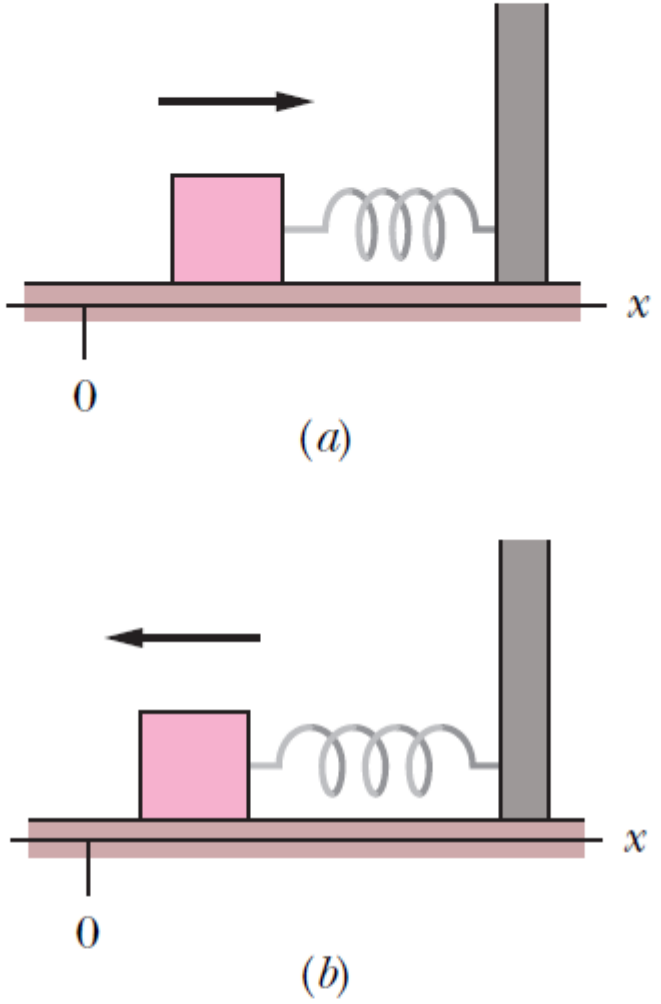
Potential energy and work - springs

The same applies to springs.

If the spring is compressed, and is pushed further, the spring does **negative** work and U_e increases.

If the spring is allowed to relax, the spring does **positive** work and U_e decreases.

$$\Delta U = -W$$



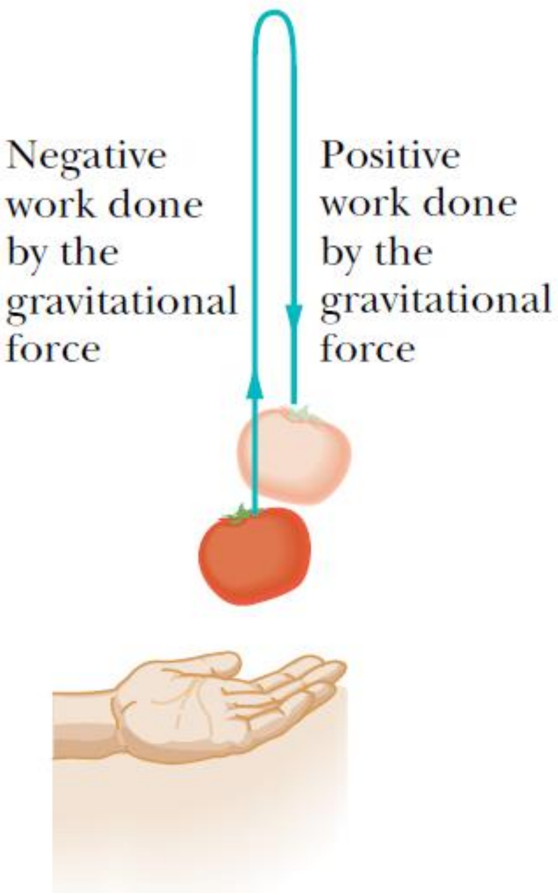
Conservative forces

Both U_e and U_g transform in kinetic energy (K). Once the situation is reversed, the K is changed back into potential energy.

- There is a force applied to an object in the system, and it does work W_1, transferring energy between K and other types of energies.

- If we reverse the situation, the force will do work W_2 and reverse the energy transfer.

$$W_1 = -W_2$$

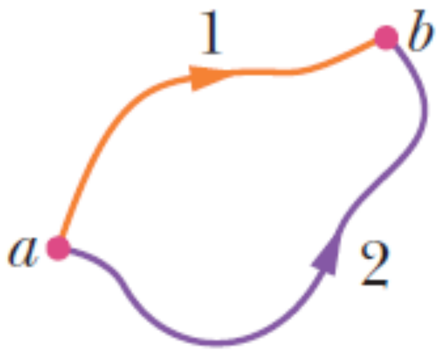


Conservative forces – path independence

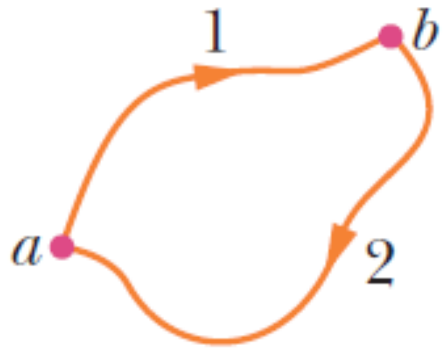
If the force is conservative, the work done is independent of the path taken:

- A closed loop (going back to the initial condition) must give an energy change of 0, i.e. the net work is 0.
- This will happen if we follow the loop in either direction.

$$W_{ab,1} = W_{ab,2}$$



(a)



(b)

Conservative forces – path independence

Proof:

As it is a closed loop of a conservative force,

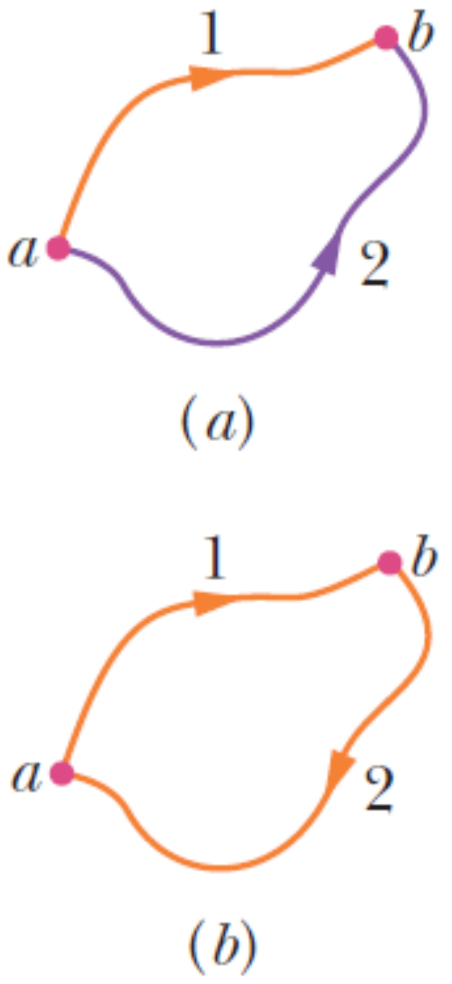
$$W_{ab,1} + W_{ba,2} = 0$$

Therefore:

$W_{ab,1} = -W_{ba,2}$; We know that $W_{ab,2} = -W_{ba,2}$

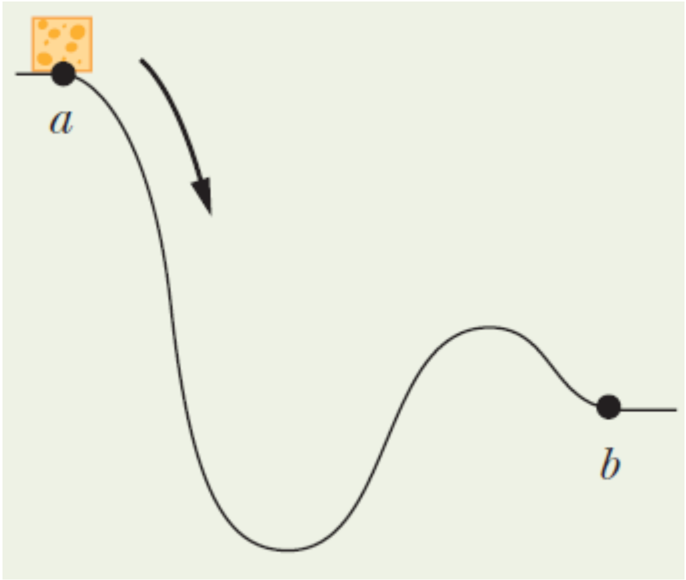
from the definition of conservative forces. Therefore:

$$W_{ab,1} = W_{ab,2}$$



Conservative forces – Example

A 2.0 kg block of slippery cheese slides along a frictionless track from point *a* to point *b*. The cheese travels through a total distance of 2.0 m along the track, and a net vertical distance of 0.80 m. How much work is done on the cheese by the gravitational force during the slide?



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Calculating potential energy - gravity

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It is often hard to define the “total” Potential energy.

However, we can calculate changes in potential energy:

$$\Delta U = -W$$

We know that for any force: $W = \int_{x_1}^{x_2} F(x)\,dx$

Therefore,

$$\Delta U = -\int_{x_1}^{x_2} F(x)\,dx$$

Calculating potential energy - gravity

$$\Delta U = -W$$

The work done by the gravitational force is:

$$W = \int_{y_1}^{y_2} -mgdy$$

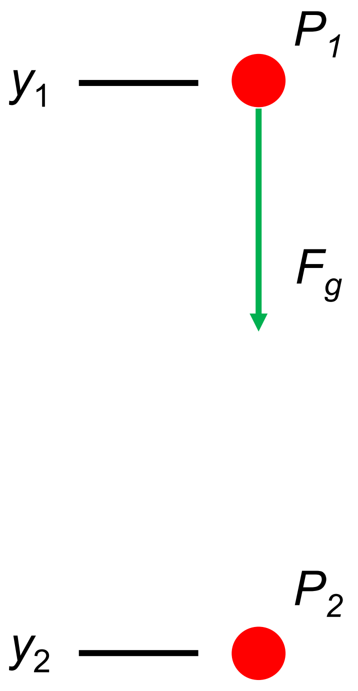
Which gives: $W = -mg\left(y_2 - y_1\right)$; Therefore,

$$\Delta U_g = mg\left(y_2 - y_1\right) = mg\Delta y$$

Therefore if the movement is downwards (Δy is –ve), ΔU is -ve.

If we take U = 0 J at a reference point, say y = 0 m at ground, then:

$$U_g = mg\left(y_2 - 0\right) = mgy$$



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Calculating potential energy – gravity - example

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A 2.0 kg sloth hangs 5.0 m above the ground (Fig. 8-6). (a) What is the gravitational potential energy U of the sloth–Earth system if we take the reference point *y* = 0 to be (1) at the ground, (2) at a balcony floor that is 3.0 m above the ground, (3) at the limb, and (4) 1.0 m above the limb? Take the gravitational potential energy to be zero at *y* = 0.

a) (1) $Mg(y_2 - 0) = 2 \times 9.8 \times 5$

$= 98J$

(2) $Mg(y_2 - 3) = 2 \times 9.8 \times 2$

$= 39.2J$

(3) $Mg(y_2 - 0) = 2 \times 9.8 \times 0$

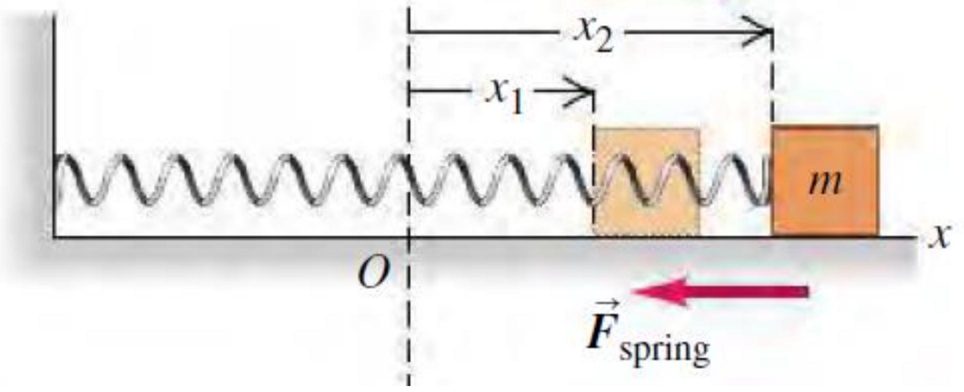
$= 0J$

(4) $Mg(y_2 - 6) = 2 \times 9.8 \times -1$

$= -19.6J$

Calculating potential energy - elastic

$$\Delta U = -W$$



The work done by the elastic force is:

$$W = \int_{x_1}^{x_2} -kx\,dx$$

Which gives: $W = \left(-\frac{1}{2}kx_2^{\ 2}\right) - \left(-\frac{1}{2}kx_1^{\ 2}\right) = \frac{1}{2}kx_1^{\ 2} - \frac{1}{2}kx_2^{\ 2}$

$$\Delta U_e = \frac{1}{2}kx_2^{\ 2} - \frac{1}{2}kx_1^{\ 2}$$

In this case, x = 0 is at relaxed position. At x = 0, U = 0.

$U_e - 0 = \frac{1}{2}kx_2^{\ 2} - 0$; Therefore, $U_e = \frac{1}{2}kx^2$

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Calculating potential energy – elastic - example

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A glider sits on a frictionless, horizontal air track, connected to a spring with force constant k = 5.00 N/m. You pull on the glider, stretching the spring 0.100 m. What is the elastic potential energy?

$$U_e = -\frac{1}{2}kx^2$$

$$= -\frac{1}{2}(5)(0.1)^2$$

$$= -0.025 \text{J}$$

Conservation of energy

If all forces acting are conservative, the total energy of the system does not change. We call this quantity the "Mechanical energy" (E_{mech}).

The energy can transfer from potential to kinetic forms.

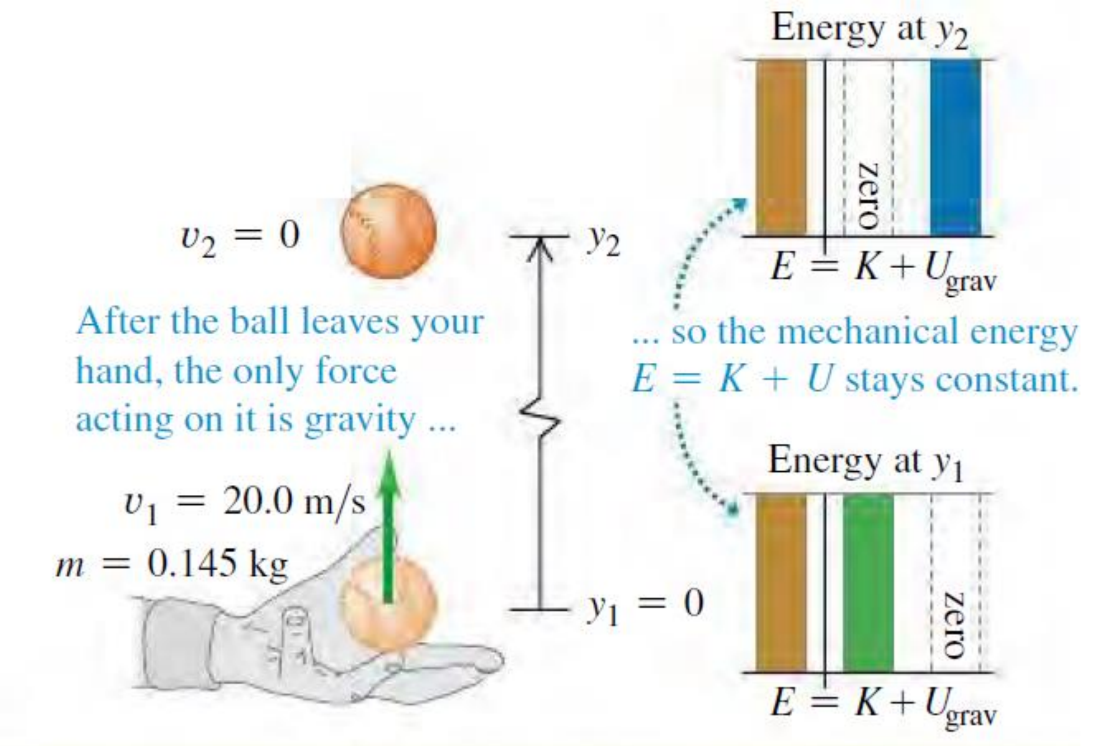
$$E_{mech} = K + U$$

We also know that the changes in *K and U* are given by:

$$\Delta K = W; \qquad \Delta U = -W$$

Which gives:

$$\Delta K = -\Delta U$$



Conservation of energy

$$\Delta K = -\Delta U$$

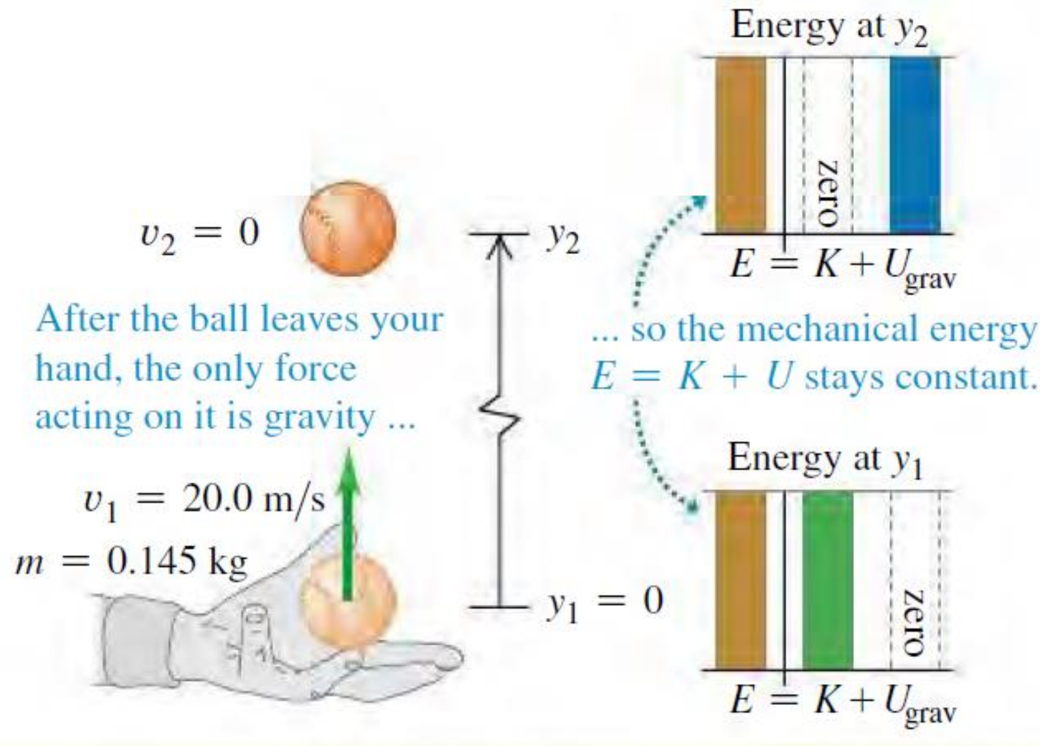
So, if one energy increases, the other decreases.

$K_2 - K_1 = -(U_2 - U_1)$; Or:

$$K_1 + U_1 = K_2 + U_2$$

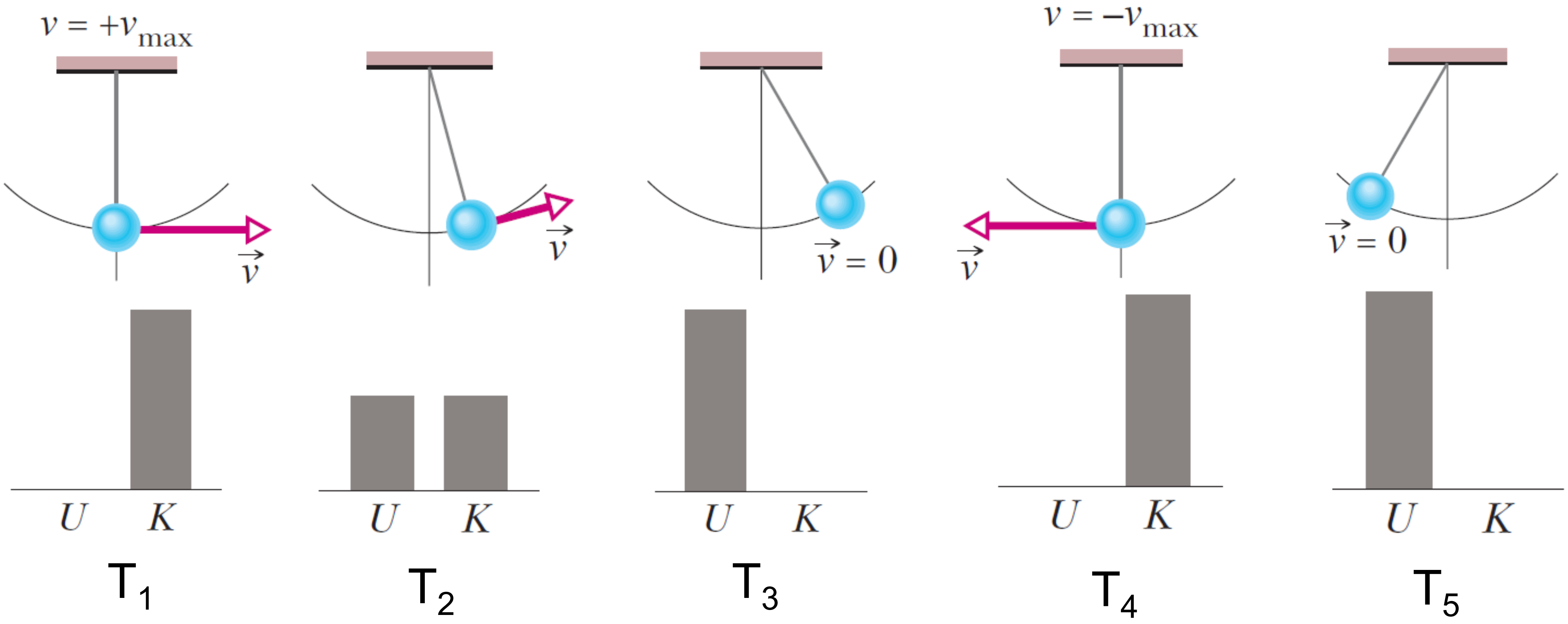
Therefore, in an isolated system, the energy can transfer from kinetic to potential, but the total amount (E_{mech}) stays constant.

$$\Delta E_{mech} = \Delta K + \Delta U = 0$$



Conservation of energy

$$\Delta E_{mech} = \Delta K + \Delta U = 0$$

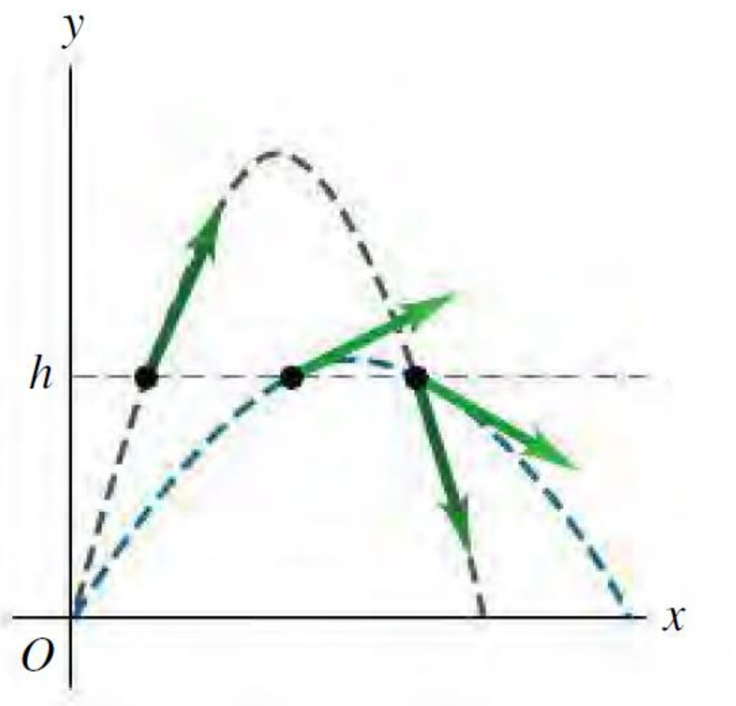


Conservation of energy - example

You throw a 0.145 kg baseball straight up, giving it an initial velocity of magnitude 20.0 m/s. Find how high it goes, ignoring air resistance.

Conservation of energy - example

A batter hits two identical baseballs with the same initial speed and from the same initial height but at different initial angles. Prove that both balls have the same speed at any height *h* if air resistance can be ignored.



Work done by external forces

The total energy of the system can be changed by external forces.

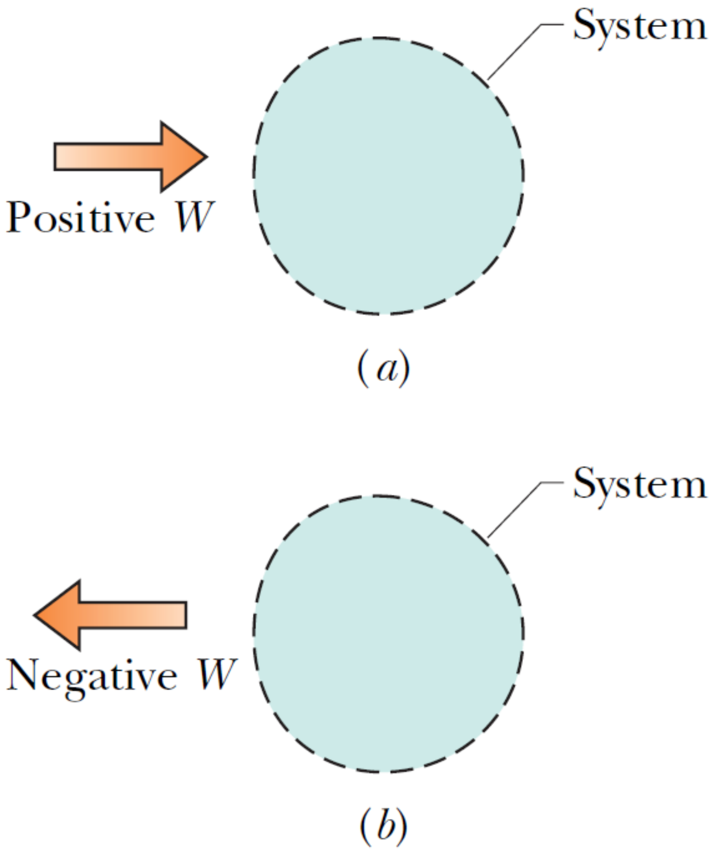
Positive work (work done on the system) will increase the energy.

Negative work, (work done by the system), will decrease the energy.

$$W = \Delta K + \Delta U = \Delta E_{mech}$$

Rearranging: $W = K_2 - K_1 + U_2 - U_1$

$$K_1 + U_1 + W = K_2 + U_2$$



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Work done by external forces - example

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In the previous ball example, suppose your hand moves upward by 0.50 m while you are throwing the ball. The ball leaves your hand with an upward velocity of 20.0 m/s. (a) Find the magnitude of the force (assumed constant) that your hand exerts on the ball. (b) Find the speed of the ball at a point 15.0 m above the point where it leaves your hand. Ignore air resistance.

a) $W = \frac{1}{2}mv^2 + mgh$

$= \frac{1}{2}(0.145)(20^2) + (0.145)(9.8)(0.5)$

$= 29.7J$

$W = Fd = 29.7$

$F = \frac{29.7}{0.5} = 59N$

b) $29.7 = \frac{1}{2}(0.145)(v^2) + (0.145)(9.8)(15.5)$

$v = 10.3 m/s$

Work done by external forces

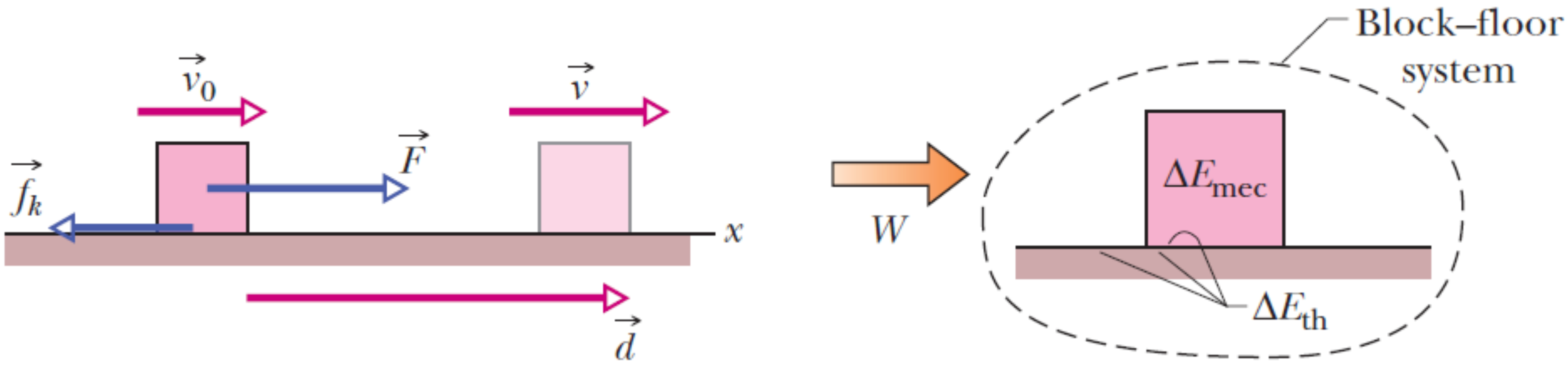
Friction will do negative work on the system as it opposes the motion.

It can be calculated starting from Newton: $F - f_k = ma$

As the forces are constant, *a* is constant: $v^2 = v_0{}^2 + 2ad$

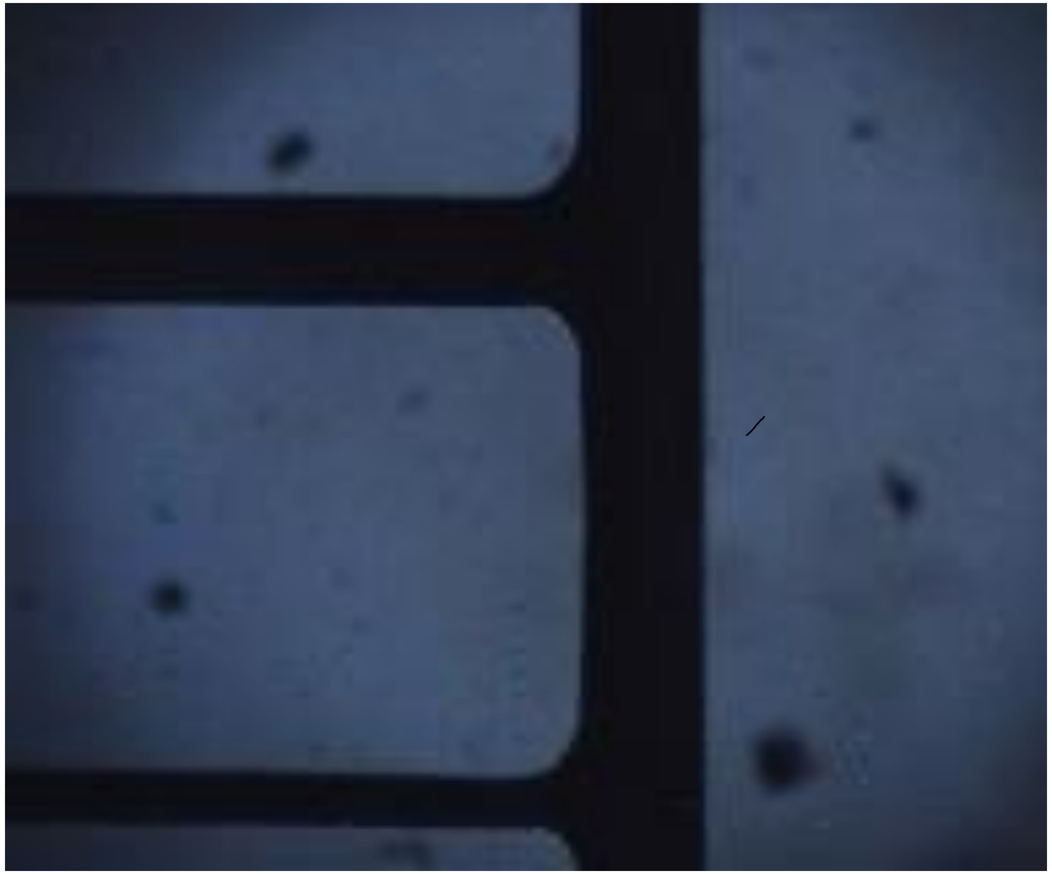
Substituting *a* and rearranging: $Fd = \frac{1}{2}mv^2 - \frac{1}{2}mv_0{}^2 + f_k d$

Or: $Fd = \Delta K + f_k d$



Work done by external forces

What happens to the energy removed from the system with friction, material failure and other non conservative events?



Work done by external forces

In general, we can say that:

$$\Delta E_{mech} = W_{tot} = W_{FE} + W_{FF} = Fd + \left(-f_k d\right)$$ Therefore: $$Fd = \Delta E_{mech} + f_k d$$

Experience tells us that frictions heats the parts in contact. So we can say that the energy is converted to *thermal energy* by the friction.

$f_k d = \Delta E_{th}$; and: $Fd = \Delta E_{mech} + \Delta E_{th}$

In general: $W = \Delta E_{mech} + \Delta E_{th}$

And: $W = \Delta E_{mech} + \Delta E_{th} + \Delta E_{int}$

In an isolated system:

$$\Delta E_{mech} + \Delta E_{th} + \Delta E_{int} = 0$$

Work done by external forces - example

A food shipper pushes a wood crate of cabbage heads (total mass m = 14 kg) across a concrete floor with a constant horizontal force of magnitude 40 N. In a straight line displacement of magnitude d = 0.50 m, the speed of the crate decreases from v_0 = 0.60 m/s to v = 0.20 m/s. (a) How much work is done by force , and on what system does it do the work? (b) What is the increase ΔE_{th} in the thermal energy of the crate and floor?

a) $W = Fd\cos\theta$

$= 40 \times 0.5 \times \cos 0^\circ$

$= 20J$

b) $W = \Delta E_M + \Delta E_{th} = KE_2 - KE_1 + \Delta E_{TH}$

$\Delta KE = \frac{1}{2}MV_1^2 - \frac{1}{2}MV_2^2$

$= \frac{1}{2}M(V_1^2 - V_0^2)$

$= \frac{1}{2}(14)(0.2^2 - 0.6^2)$

$= -2.2J$

$\rightarrow 20 = -2.2 + \Delta E_{TH}$

$\Delta E_{TH} = 22J$

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Work done by external forces - example

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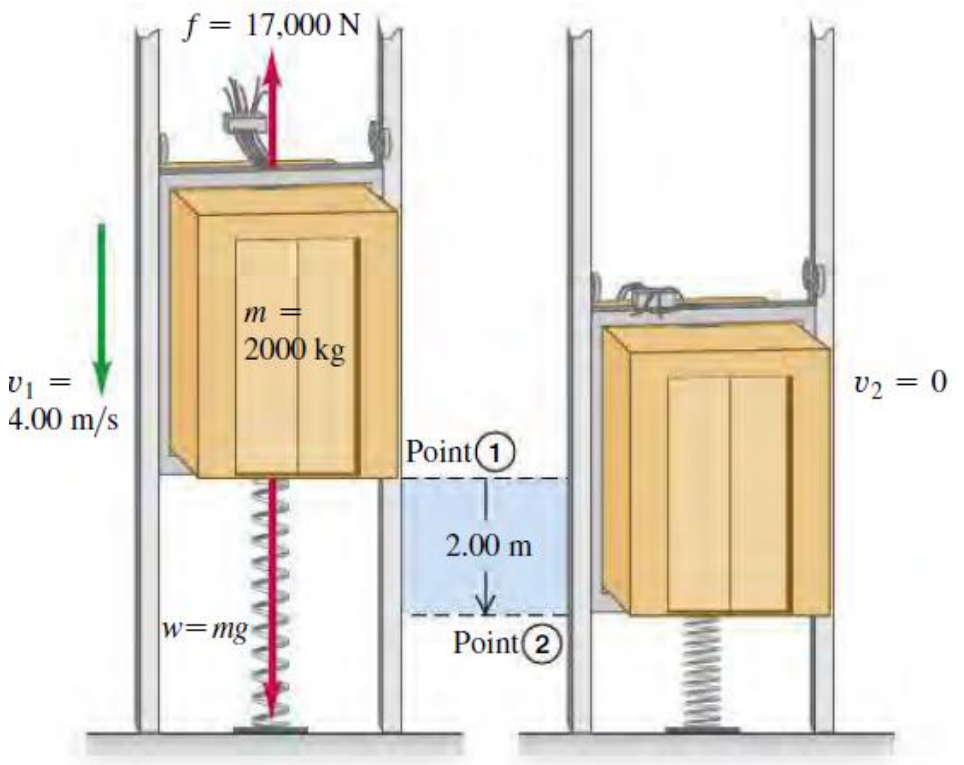
A 2000 kg (19,600 N) elevator with broken cables in a test rig is falling at 4.00 m/s when it contacts a cushioning spring at the bottom of the shaft. The spring is intended to stop the elevator, compressing 2.00 m as it does so. During the motion a safety clamp applies a constant 17,000 N friction force to the elevator. What is the necessary force constant k for the spring?



$$U_{G_1} + \cancel{U_{S_1}} + K_{E_1} + W_F = \cancel{K_{E_2}} + \cancel{U_{G_2}} + U_{S_2}$$

$$\frac{1}{2}Mv_1^2 + MgY_1 + (Y_2 - Y_1)F = \frac{1}{2}k(Y_2 - Y_1)^2$$

$$\frac{1}{2}(2000)(4)^2 + (19600)(2) + (-2)(17000) = \frac{1}{2}k(-2)^2$$

$$K = 10\,600 \text{ N/m}$$

$Y_1 = 2$

$Y_2 = 0$

Power

The rate at which work is being done is called the *power*:

$$P = \frac{dW}{dt}$$

Measured in watts: 1 watt = 1 W = 1J/s

It is used commonly to quantify energy use, i.e. electrical consumption. For a force at an angle φ to the displacement, it can be expressed as:

$$P = \frac{F \cos\phi dx}{dt} = Fv\cos\phi = \mathbf{F} \bullet \mathbf{v}$$

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Power- example

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Each of the four jet engines on an Airbus A380 airliner develops a thrust (a forward force on the airliner) of 322,000 N. When the airplane is flying at 250 m/s (900 km/h), what power does each engine develop?

$$P = FV\cos\theta$$

$$P = 322\,000 \times 250 \times \cos 0^{\circ}$$

$$= 8.05 \times 10^{7}\,W$$

322 000

V

θ

F