Work and Energy

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Lecture Learning Objectives

In this lesson you will learn to:

- Understand the concepts of work and energy
- Find the change of energy in cases with constant force
- Calculate the work done by a variable force.

Energy

What is energy?

- We call several things "energy"
  - Nuclear energy, electrical energy, chemical energy…
  - They are all associated with different processes. They all can be used to "do" things.
  - What is the overall definition?

Energy conservation

Energy is useful as it is “conserved”

It can change between different forms but does not get destroyed.

Successive changes make it less “useful”

Some forms of energy that we will see: Kinetic energy, gravitational potential energy and elastic potential energy.

Kinetic energy

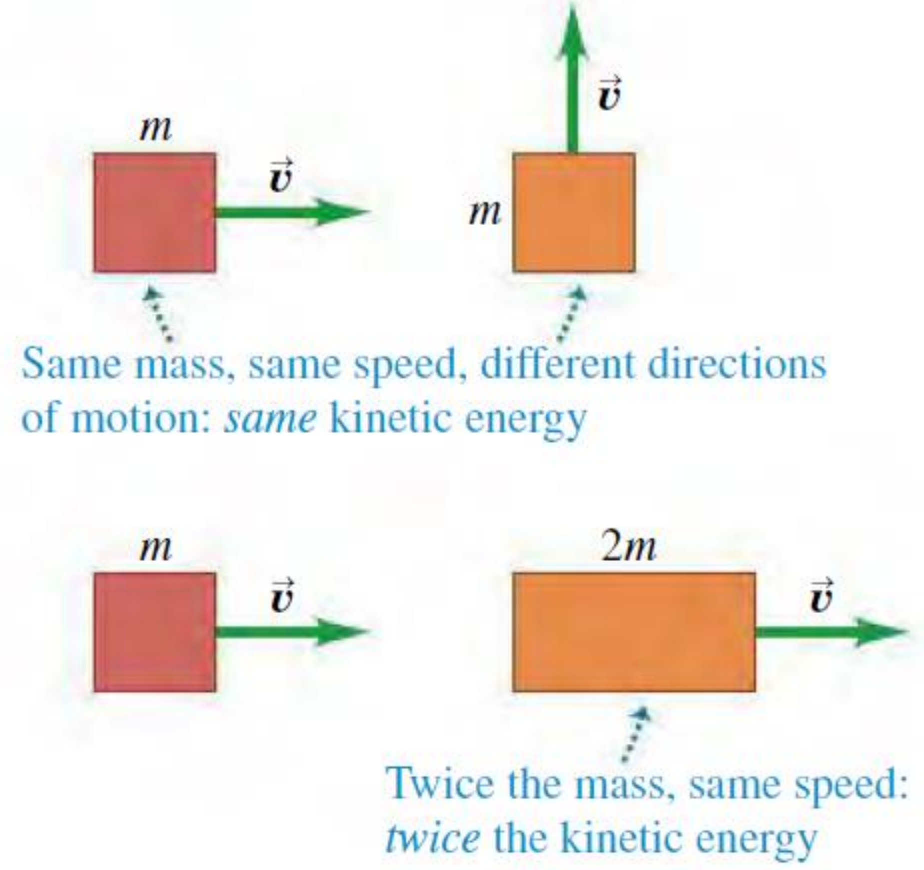
Energy associated with motion.

Defined as:

$$K = \frac{1}{2}mv^2$$

It is a scalar. Depends on the object mass and speed.

Measure in Joules (J) = kg m^2 s^{-2}



Kinetic energy - example

In 1896 in Waco, Texas, William Crush parked two locomotives at opposite ends of a 6.4 km long track, fired them up, tied their throttles open, and then allowed them to crash head-on at full speed in front of 30,000 spectators. Hundreds of people were hurt by flying debris; several were killed. Assuming each locomotive weighed 1.2×10^6 N and its acceleration was a constant $0.26\ m/s^2$, what was the total kinetic energy of the two locomotives just before the collision?

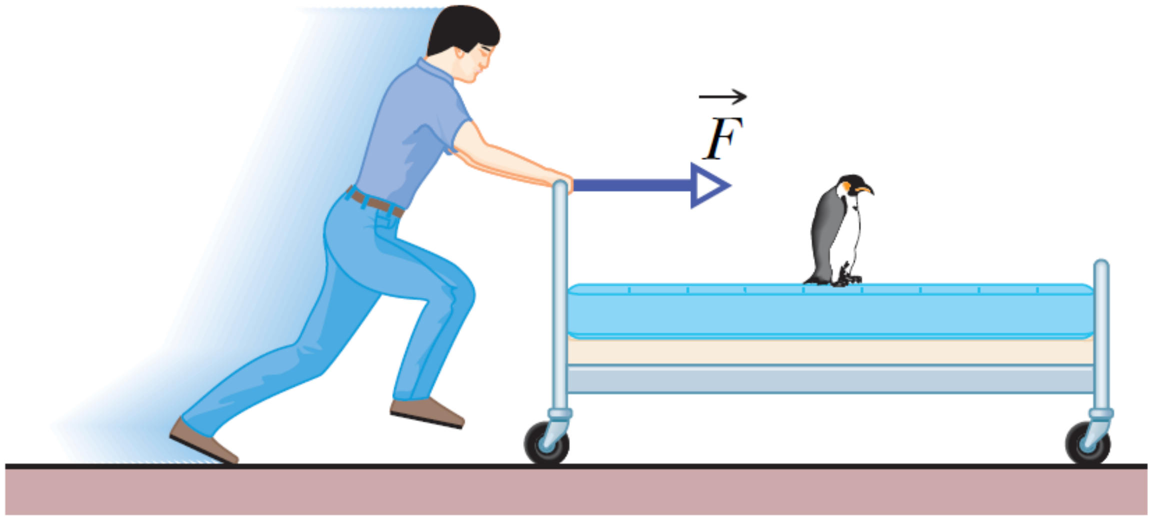
Work

If we apply a force, the object has an acceleration.

Therefore, its kinetic energy changes.

Work is energy transferred to or from an object by means of a force acting on the object.

Energy transferred to the object is positive work and energy transferred from the object is negative work.



Work - kinetic energy

Consider the bead:

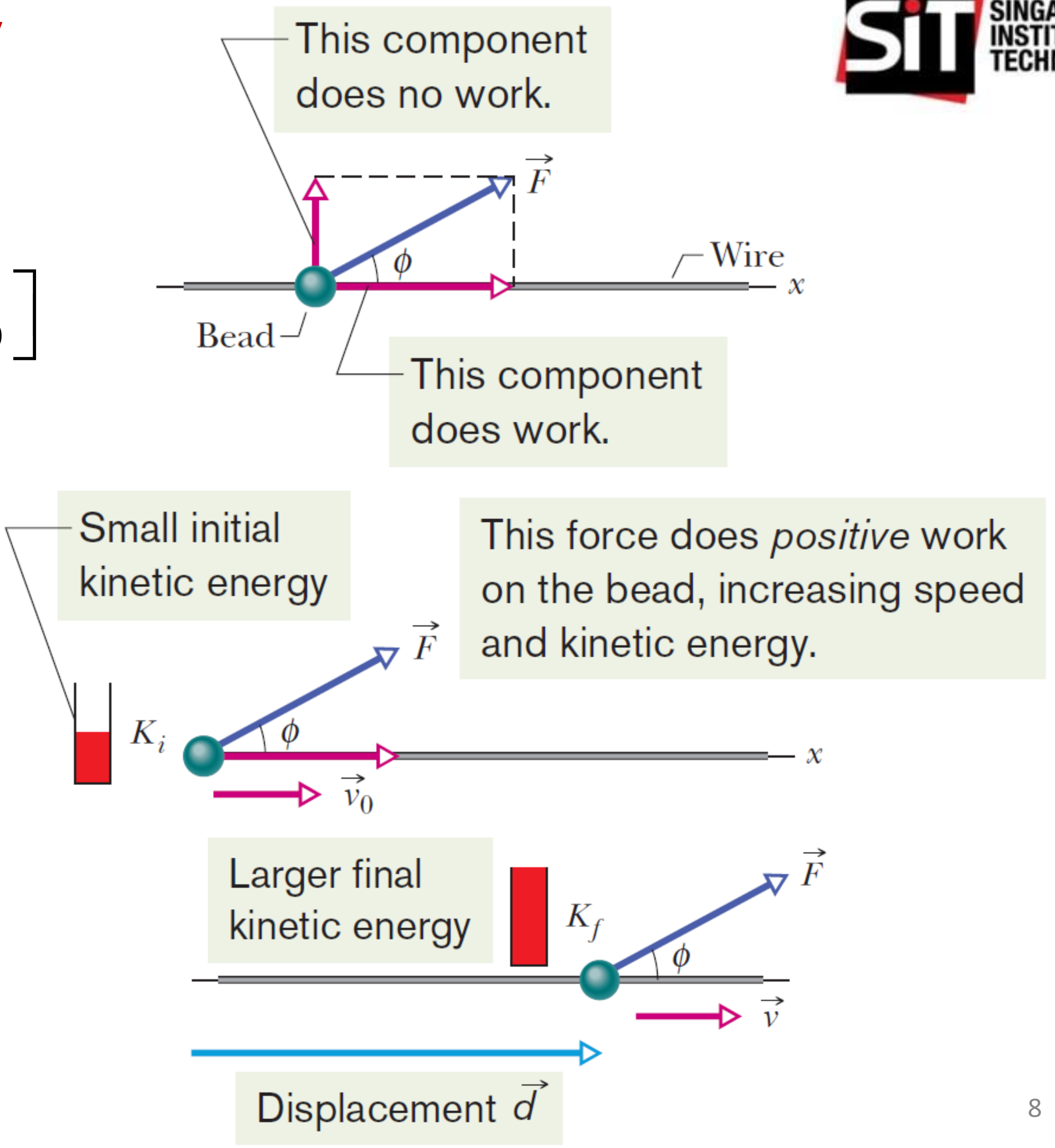
$$F_x = ma_x \qquad v_2^{\,2} = v_1^{\,2} + 2a\left[x(t) - x_0\right]$$

$$v_2^{\,2} = v_1^{\,2} + 2a_x d$$

$$v_2^{\,2} = v_1^{\,2} + 2\frac{F_x}{m}d$$

$$\frac{1}{2}mv_2^{\,2} - \frac{1}{2}mv_1^{\,2} = F_x d$$

$$K_2 - K_1 = F_x d$$



Work - kinetic energy

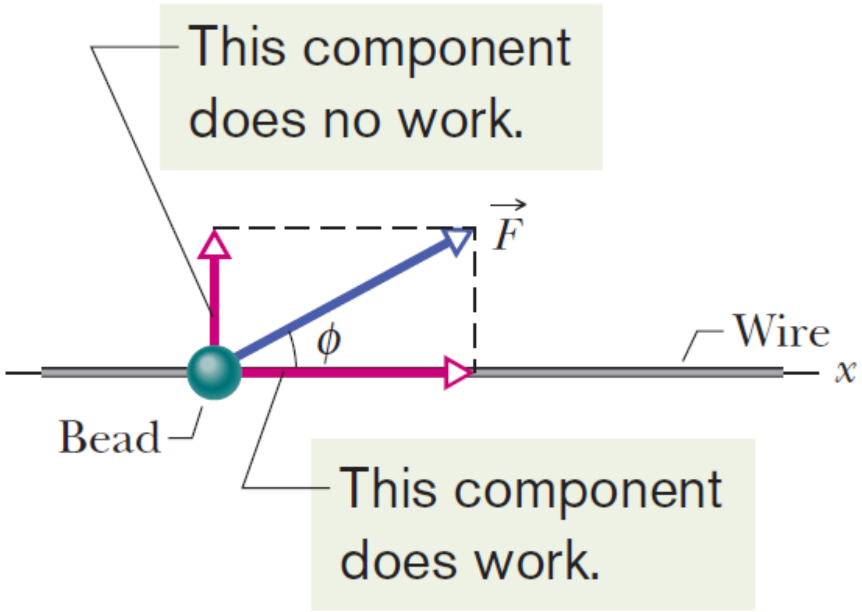
Work is equal to the component of the force in the direction of the displacement times the displacement:

$$F_x d = W$$

Only the component in the direction of the displacement does work.

$$W = |\mathbf{F}||\mathbf{d}|\cos\phi$$

$$W = \mathbf{F} \bullet \mathbf{D}$$



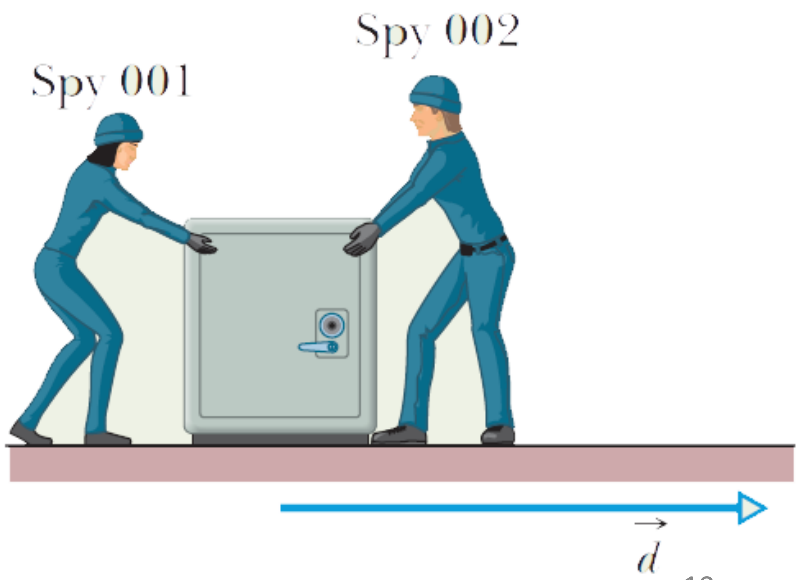
Work - kinetic energy example

Two industrial spies sliding an initially stationary 225 kg floor safe a displacement of magnitude 8.50 m, straight toward their truck. The push of spy 001 is 12.0 N, directed at an angle of 30.0° downward from the horizontal; the pull of spy 002 is 10.0 N, directed at 40.0° above the horizontal. The magnitudes and directions of these forces do not change as the safe moves, and the floor and safe make frictionless contact.

(a) What is the net work done on the safe by forces and during the displacement ?

(b) During the displacement, what is the work W_g done on the safe by the gravitational force and what is the work W_N done on the safe by the normal force from the floor?

(c) The safe is initially stationary. What is its speed v_f at the end of the 8.50 m displacement?

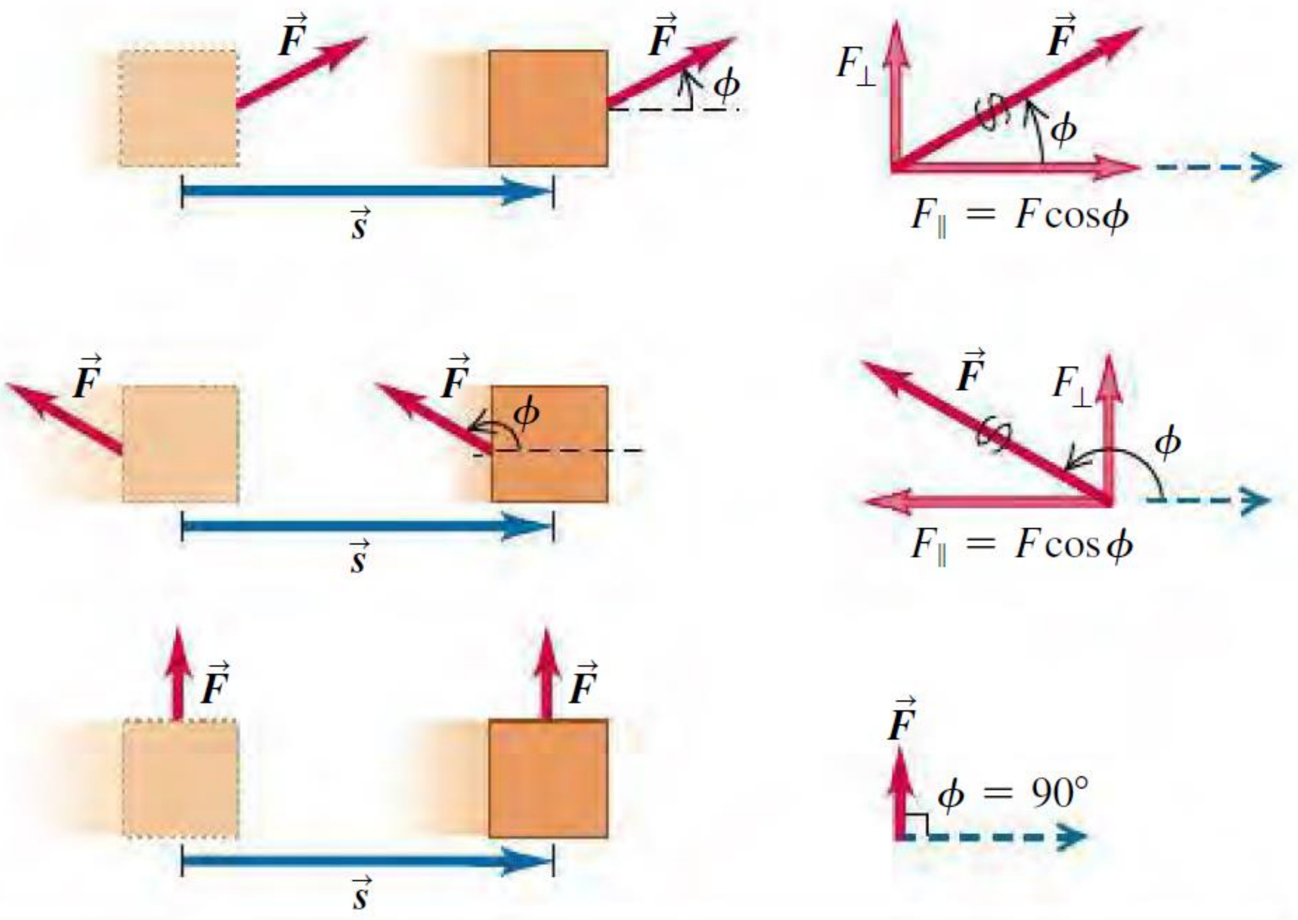


Work - kinetic energy – net work

If the force is in the opposite direction as the displacement, the work is negative.

Positive work can be seen as work done **on** the object. Speed increases.

Negative work can be seen as work done **by** the object. Speed decreases.



Work - kinetic energy – net work

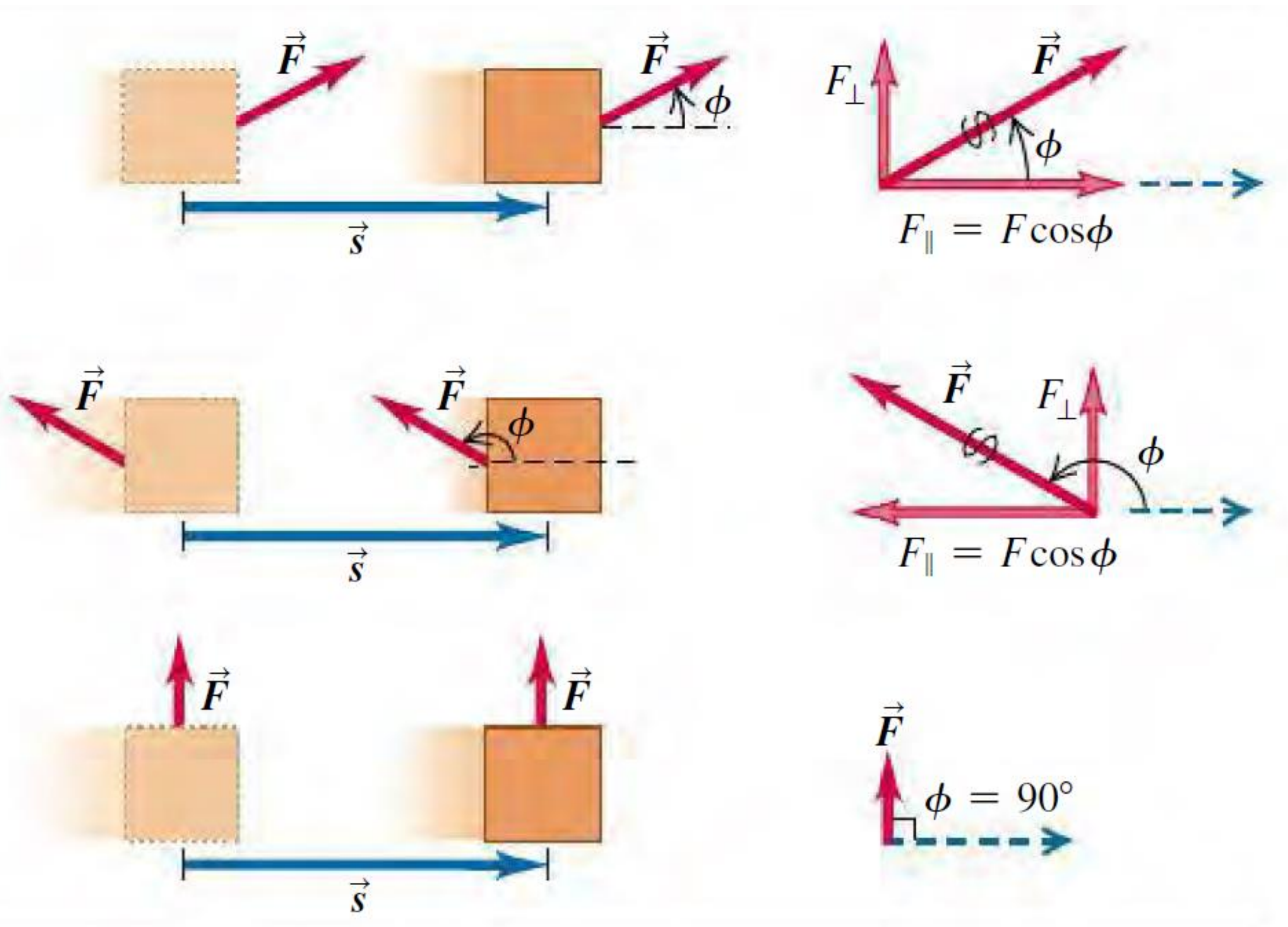
The overall change in energy is given by the **net (or total) work**.

$$\Delta K = W$$

This is also known as the Work-Energy theorem.

The net work can be found finding the work of each force and adding them.

Or the net force can be found first and used to calculate the net work.



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Net work - kinetic energy example

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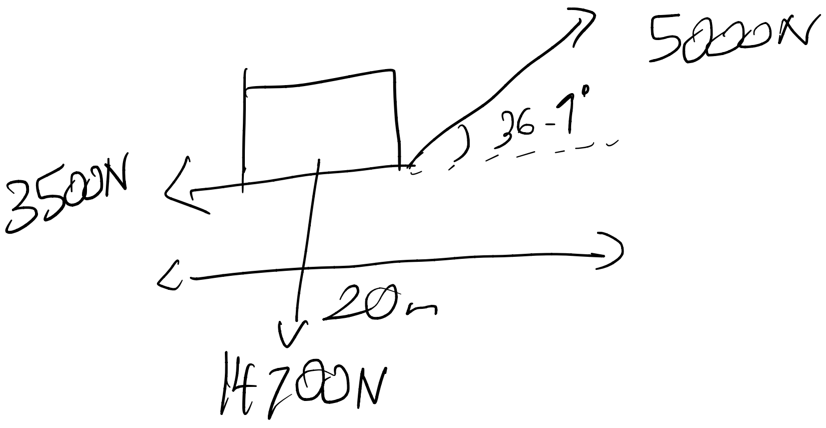
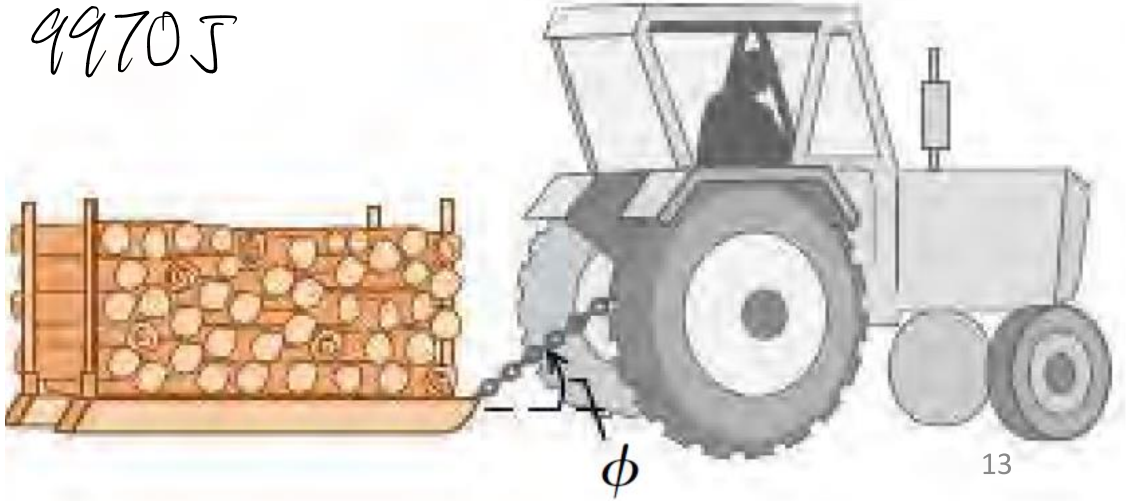


A farmer hitches her tractor to a sled loaded with firewood and pulls it a distance of 20 m along level ground. The total weight of sled and load is 14,700 N. The tractor exerts a constant 5000 N force at an angle of 36.9° above the horizontal. A 3500 N friction force opposes the sled's motion. Find the work done by each force acting on the sled and the total work done by all the forces. What is the final velocity if the initial velocity was 2.0 m/s?

$$(5000 \cos 36.9^\circ \times 20) - (3500 \times 20) = 9970\ J$$

$$\frac{1}{2}\left(\frac{14700}{1500}\right)(v)^2 - \frac{1}{2}(1500)(2)^2 = 9970$$

$$v = 4.15\ m/s$$



Work done by gravitation

Gravitation exerts a constant force on objects.

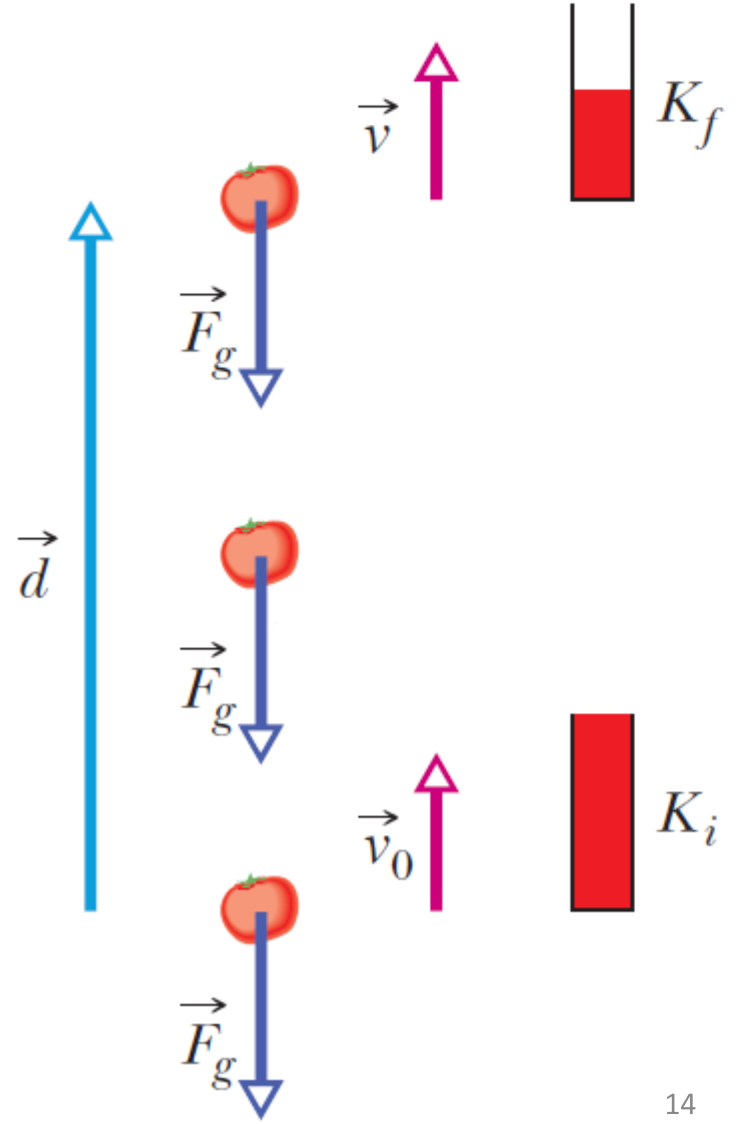
If the objects rises or falls, work is done.

$$F_g = mg$$

$$W_g = mgd\cos\phi$$

$$W_g = mgd\cos 180$$

$$W_g = -mgd$$

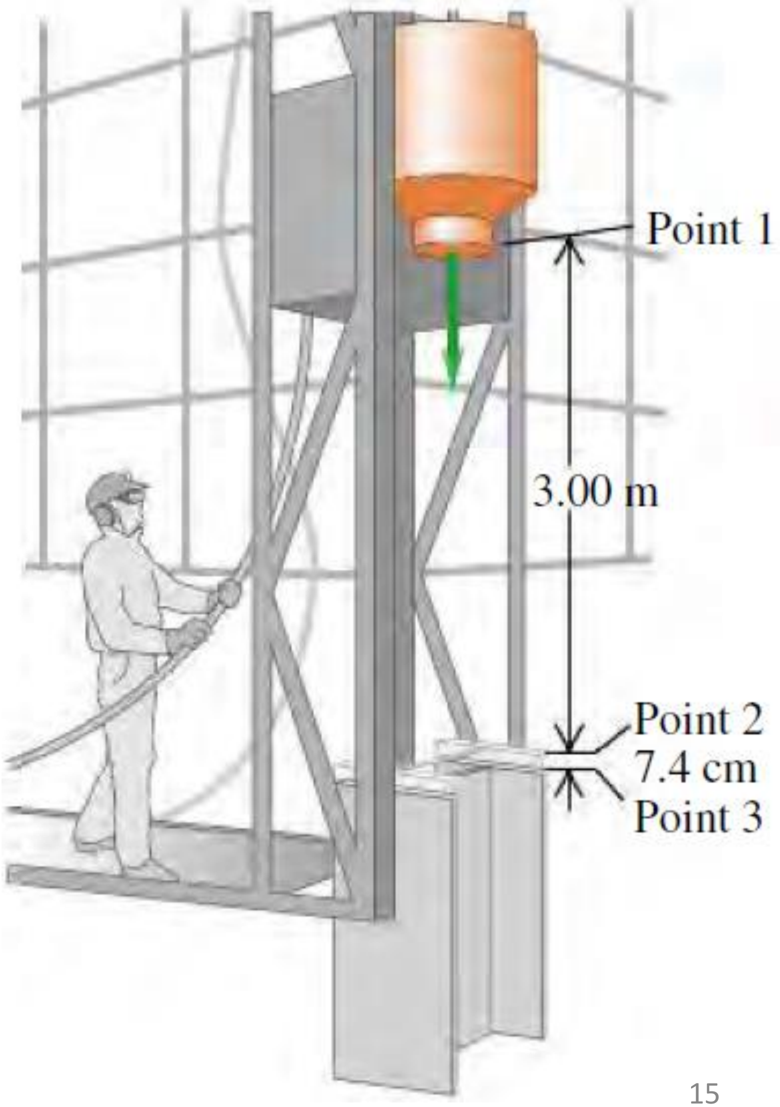


Work done by gravitation example

The 200 kg steel hammerhead of a pile driver is lifted 3.00 m above the top of a vertical I-beam being driven into the ground. The hammerhead is then dropped, driving the I-beam 7.4 cm deeper into the ground. The vertical guide rails exert a constant 60 N friction force on the hammerhead. Use the work–energy theorem to find (a) the speed of the hammerhead just as it hits the I-beam and (b) the average force the hammerhead exerts on the I-beam. Ignore the effects of the air.

3m

7.4cm



a) $200 \times 9.8 \times 3 = 5880$

$5880 - (3 \times 60) = \frac{1}{2}(200)v^2$

$v = \sqrt{57} \approx 7.55 m/s$

b)

Work done by gravitation

Lowering and lifting an object does work.

Forces applied by us and by gravity each do work. The sum of these works (net work) is the change in kinetic energy.

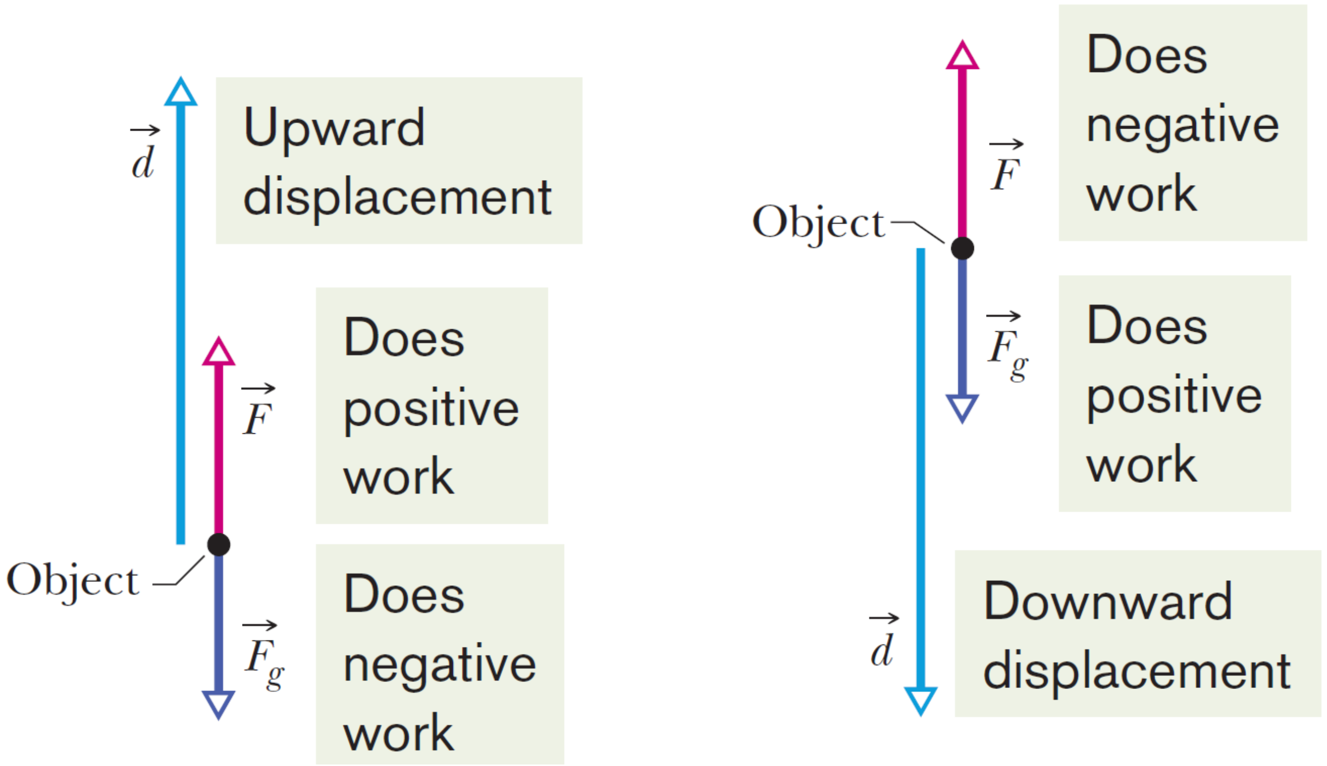
$$\Delta K = K_2 - K_1 = W_f + W_g$$

Often, there is no change in velocity:

$$0 = W_f + W_g$$

$$W_f = -W_g$$

$$mgd\cos\phi = mgd\cos\phi$$



Work done by gravitation - example

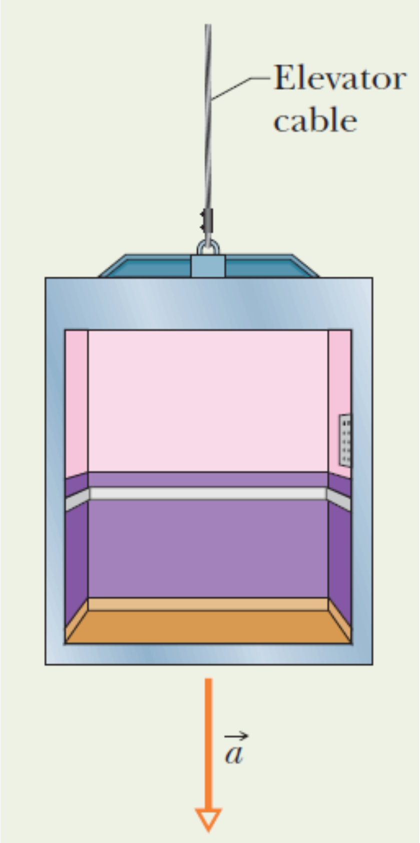
An elevator cab of mass m = 500 kg is descending with speed v_i = 4.0 m/s when its supporting cable begins to slip, allowing it to fall with constant acceleration $a = g/5$.

(a) During the fall through a distance d = 12 m, what is the work W_g done on the cab by the gravitational force ?

(b) During the 12 m fall, what is the work W_T done on the cab by the upward pull of the elevator cable?

(c) What is the net work W_{done} on the cab during the fall?

(d) What is the cab's kinetic energy at the end of the 12 m fall?



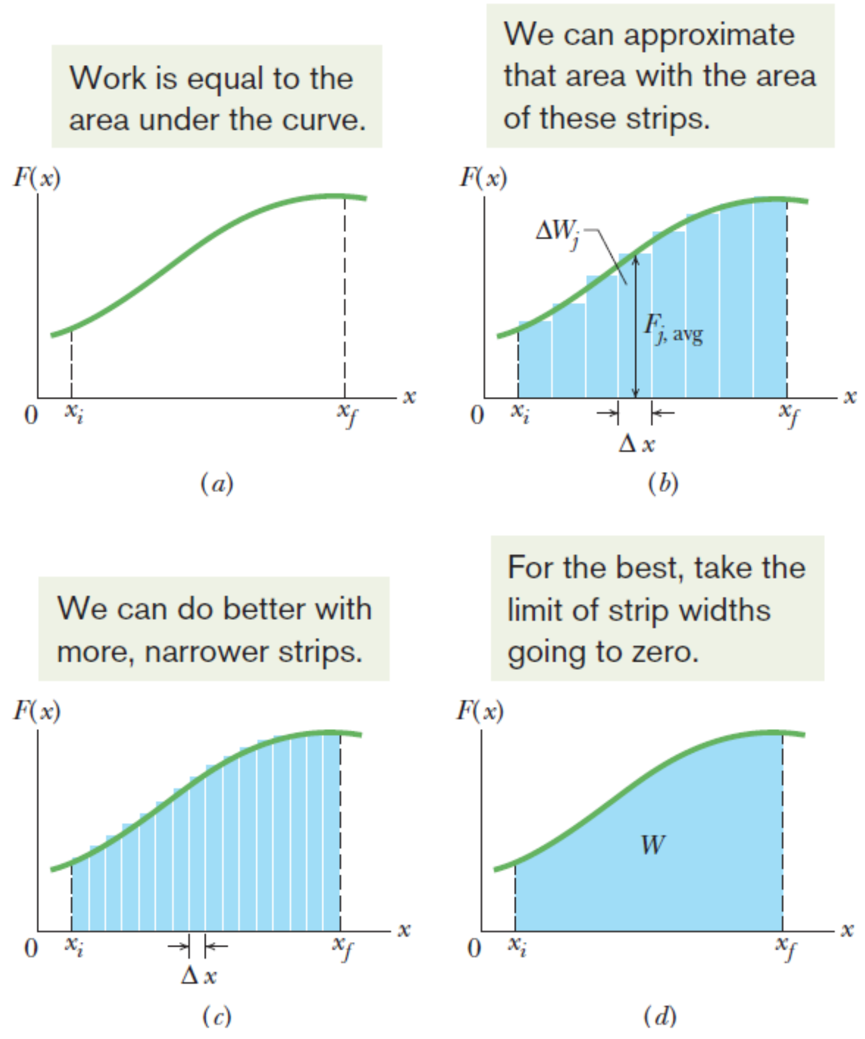
Work done by a generic variable force

In general, the work done is the area under the force-displacement graph.

This can be found through integration if the force equation is available.

For constant force, $F_x = C$; therefore

$$W = F_x\left(x_2 - x_1\right)$$



Work done by a non constant force

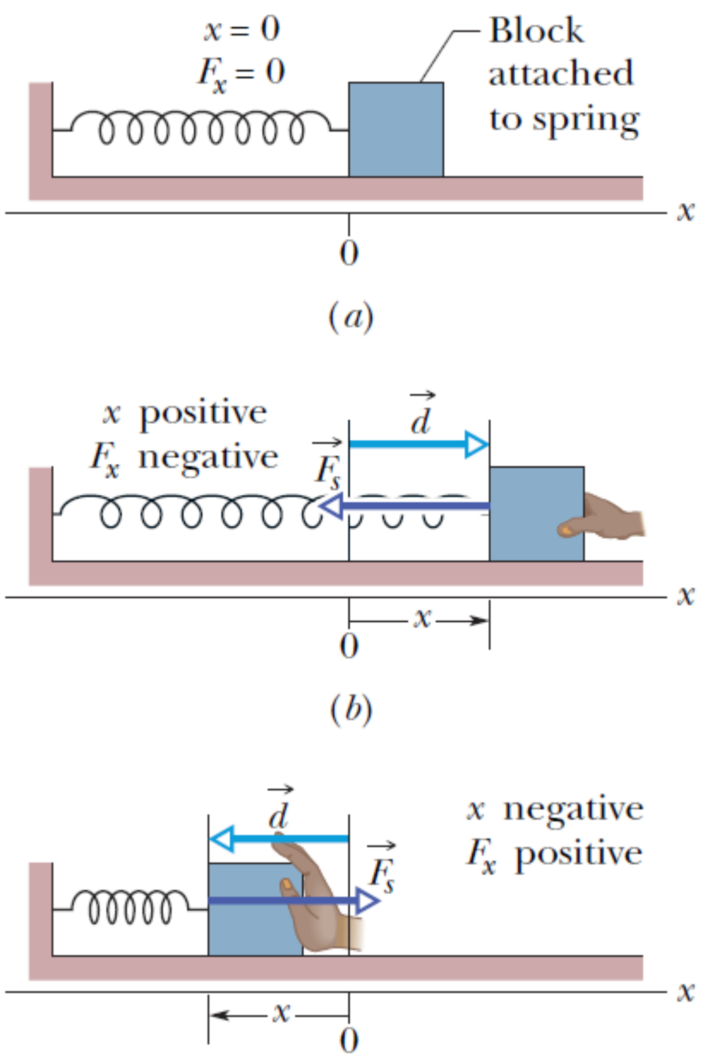
The work done by a variable force can be found with integration.

A special case is a spring.

$$F_x = -kx$$

x = 0 is at the relaxed spring position.

Similar to elastic materials



Work done by a non constant force

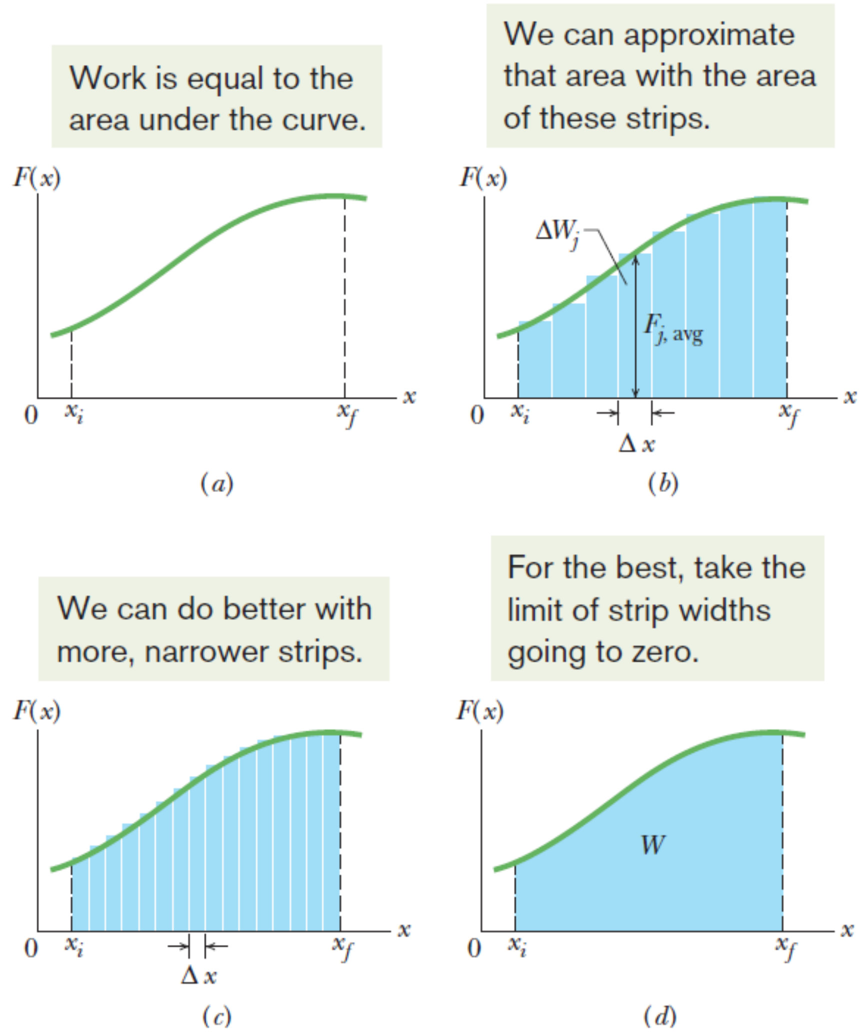
To find the work, we can consider very small intervals, where the force is almost constant, and add them:

$W_s = \sum -F_{xj}\Delta x$; As Δx tends to 0:

$$W_s = \int_{x_1}^{x_2} -F_x dx \quad \text{substituting } F_x\text{:} \quad W_s = \int_{x_1}^{x_2} -kx dx$$

$$W_s = \left[-\frac{1}{2}kx^2\right]_{x_1}^{x_2} = -\frac{1}{2}k\left(x_2^{\,2} - x_1^{\,2}\right)$$

$$W_s = \frac{1}{2}k\left(x_1^{\,2} - x_2^{\,2}\right)$$



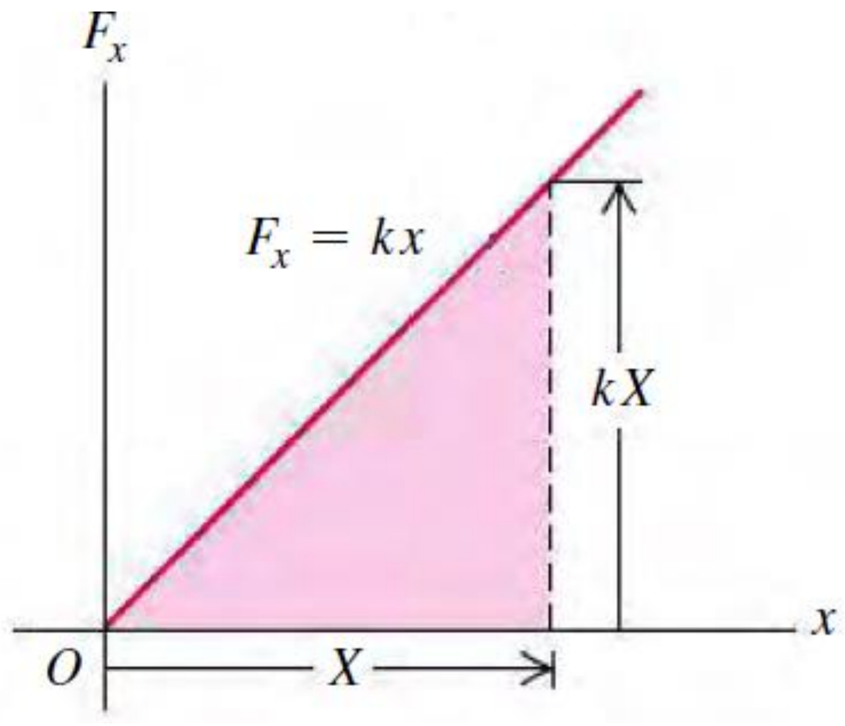
Work done by an external force on a spring

As per the gravitational force, if other forces are applied the total change in K is the sum of the works:

$$\Delta K = K_2 - K_1 = W_f + W_s$$

If there is no speed at the end:

$$0 = W_f + W_s;\; W_f = -W_s$$



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Work done by springs example

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A cumin canister of mass m = 0.40 kg slides across a horizontal frictionless counter with speed v = 0.50 m/s. It then runs into and compresses a spring of spring constant k = 750 N/m. When the canister is momentarily stopped by the spring, by what distance d is the spring compressed?

$x_2 = -d$, $x_1 = 0$

$KE_2 - KE_1 = W_s$

$-KE_1 = W_s$

$-KE_1 = \frac{1}{2}k(x_1^2 - x_2^2)$; $-KE = \frac{1}{2}k(0 - d^2)$

$KE_1 = \cancel{\frac{1}{2}}kd^2$; $\cancel{\frac{1}{2}}Mv_1^2 = \frac{1}{2}kd^2$; $d^2 = \frac{Mv_1^2}{k}$

$d = \sqrt{\frac{Mv_1^2}{k}} = \sqrt{\frac{0.4 \times 0.5^2}{750}} = 0.012m$

$= 1.2cm$