a) $f(t) = 2e^{-3t} - t^4$

$$F(s) = \frac{2}{s+3} - \frac{24}{s^5}$$

b) $f(t) = 3\cos(5t) - 2\sin(5t)$

$$F(s) = \frac{3s}{s^2+25} - \frac{10}{s^2+25}$$

$$= \frac{3s-10}{s^2+25}$$

c) $f(t) = e^{-t}(t+3)^2$

$$= e^{-t}(t^2+3t+9)$$

$$F(s) = \frac{2}{(s+1)^3} + \frac{3}{(s+1)^2} + \frac{9}{s+1}$$

d) $f(t) = e^t \sin t$

$$F(s) = \frac{1}{(s-1)^2+1}$$

e) $f(t) = \underbrace{t^2}_{g(t)}\delta(t-3)$

$g(t)\delta(t-a)$	$g(a)e^{-as}$

$g(3)$

$= 3^2 = 9$

$$F(s) = 9e^{-3s}$$

2. Using integration, determine the Laplace transform of f(t). Show that it is equivalent to that obtained by shifting in s-domain as well as that using the derivative of the Laplace transform.

$$f(t) = t^2 e^t$$

ANS: $F(s) = \frac{2}{(s-1)^3}$

	u	dv
+	t^2	$e^{-(s-1)t}$
−	$2t$	$\frac{-1}{s-1} e^{\cdots}$
+	2	$\frac{1}{(s-3)^2} e^{\cdots}$
−	0	$\frac{-1}{(s-3)^3} e^{\cdots}$

$$\int_0^\infty t^2 e^t \, dt = \int_0^\infty (t^2 e^t) \cdot e^{-st} \, dt$$

$$= \int t^2 e^{-(s-1)t} \, dt$$

$$= \left\{ e^{-(s-1)t} \left[\frac{-t^2}{s-1} - \frac{2t}{(s-3)^2} - \frac{2}{(s-3)^3} \right] \right\} \Big|_0^\infty$$

$$= 0 - \left(-\frac{2}{(s-3)^3} \right)$$

$$= \frac{2}{(s-3)^3}$$

$$\mathcal{L}\{t^2\} = \frac{2}{s^3} = F(s) \rightarrow F(s-1) = \frac{2}{(s-1)^3} = \mathcal{L}\{t^2 e^t\}$$

$$f(t) = \overset{n=2}{t^2} \underbrace{e^t}_{h(t)}$$

$$\downarrow$$

$$H(s) = \frac{1}{s-1}$$

$$F(s) = (-1)^2 F''(s)$$

$$= \frac{d}{ds} \underbrace{\left[\frac{-1}{(s-1)^2} \right]}_{F'(s)} = \frac{2}{(s+1)^3}$$

3. Determine the Laplace transform of the function below, where k and ω are constants.

$$h(t) = \underbrace{te^{kt}\cos(\omega t)}_{f(t)}$$

ANS: $F(s) = \dfrac{(s-k)^2 - \omega^2}{\left[(s-k)^2 + \omega^2\right]^2}$.

$$F(s) = \frac{(s-k)}{(s-k)^2 + \omega^2}$$

$$H(s) = (-1)\,F'(s)$$

$$= (-1)\frac{d}{ds}\left[\frac{(s-k)}{(s-k)^2 + \omega^2}\right]$$

$$\frac{[(s-k)^2 + \omega^2](1) - (s-k)[2(s-k)(1)]}{[(s-k)^2 + \omega^2]^2}$$

$$= \frac{(s-k)^2 - \omega^2}{[(s-k)^2 + \omega^2]^2}$$

$$= \frac{\omega^2 - (s-k)^2}{[(s-k)^2 + \omega^2]^2}$$

4. Using both integration and the t-domain shifting property., determine the Laplace transform of the following function. Are they equivalent?

$$f(t) = \begin{cases} 0, & t < 2 \\ (t-1)^2, & t \geq 2 \end{cases}$$

ANS: $F(s) = e^{-2s}\left(\frac{1}{s} + \frac{2}{s^2} + \frac{2}{s^3}\right)$. Yes.

$$f(t) = (t-1)^2 u(t-2)$$

$$= (t-2+1)^2 u(t-2)$$

$$= \underbrace{\left[(t-2)^2 + 2(t-2) + 1\right]}_{f(t-2)\ =\ f(t)\ =\ t^2 + 2t + 1} u(t-2)$$

$$F(s) = \left[\frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s}\right] e^{-2s}$$

5. Rewrite the following piecewise function using the unit-step function and evaluate its Laplace transform.

$$g(t) = \begin{cases} e^{-t}, & 1 \le t < 2 \\ t^2, & t \ge 2 \end{cases}$$

ANS: $G(s) = \frac{1}{e(s+1)}e^{-s} + \left[\frac{2}{s^3} + \frac{4}{s^2} + \frac{4}{s} - \frac{1}{e^2(s+1)}\right]e^{-2s}$

$$g(t) = \underbrace{e^{-t}u(t-1)} + (t^2 - e^{-t})u(t-2)$$

$$e^{-(t-1+1)}u(t-1) + \Big[\underbrace{(t-2+2)^2}_{t^2+4t+4} - e^{\overbrace{-(t-2+2)}}\Big]u(t-2)$$

$$\underbrace{\left[e^{-(t-1)} \cdot e^{-1}\right]u(t-1)} \qquad \underbrace{e^{-(t-2)} \cdot e^{-2}}$$

$$\left[\frac{1}{e(s+1)}\right]e^{-s} + \left[\frac{2}{s^3} + \frac{4}{s^2} + \frac{4}{s} - \frac{1}{e^2(s+1)}\right]e^{-2s}$$

6. Determine the Laplace transform of the function below. (Hint: You might need the compound angle formula.)

$$f(t) = \begin{cases} e^{2t}, & 1 \le t < \pi \\ \sin t + e^{2t}, & t \ge \pi \end{cases}$$

ANS: $F(s) = \dfrac{e^2 e^{-s}}{s-2} - \dfrac{e^{-\pi s}}{s^2+1}$

$$f(t) = e^{2t} u(t-1) + (\sin t + \cancel{e^{2t}} - \cancel{e^{2t}}) u(t-\pi)$$

$$= e^{2t} u(t-1) + (\sin t) u(t-\pi)$$

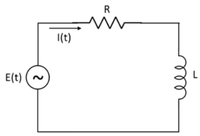
$$F(s) = e^{2(t-1+1)} u(t-1) + [\sin(t-\pi+\pi)] u(t-\pi)$$

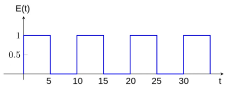
$$= e^{2(t-1)} \cdot e^{2} u(t-1) + [\sin(t-\pi)\underbrace{\cos(\pi)}_{-1} + \cos(t-\pi)\underbrace{\sin(\pi)}_{0}] u(t-\pi)$$

$$= \left(\frac{e^2}{s-2}\right) e^{-s} + [-\sin(t-\pi)] u(t-\pi)$$

$$= \left(\frac{e^2}{s-2}\right) e^{-s} - \left(\frac{1}{s^2+1}\right) e^{-\pi s}$$

7. Determine the Laplace transform of the periodic voltage supply of the resistor-inductor circuit below.





ANS $L\{E(t)\} = \sum_{n=0}^{\infty} \frac{e^{-10n} - e^{-(10n+5)}}{s}$.

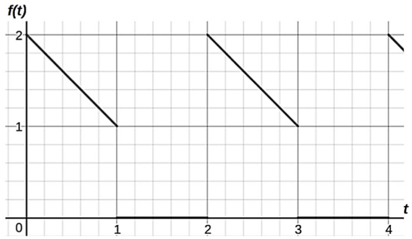
$$E(t) = 1u(t) - u(t-5) + u(t-10) - u(t-15) + \dots$$

$$= \sum_{n=0}^{\infty} \left[u(t-10n) - u(t-10n-5) \right]$$

$$\mathcal{L}\{E\} = \sum_{n=0}^{\infty} \left[u(t-10n) - u(t-(10n-5)) \right]$$

$$= \sum_{n=0}^{\infty} \left[\frac{e^{-10ns}}{s} - \frac{e^{-(10n-5)s}}{s} \right]$$

8. A periodic function f(t) is defined by the following waveform. Given that g(t) represents one cycle of f(t) in [0, 2], define g(t) using the unit-step function. Hence, determine the Laplace transform of f(t).



ANS: $g(t) = (2-t)u(t) - (2-t)u(t-1)$. $F(s) = [\frac{2}{s} - \frac{1}{s^2} + e^{-s}(\frac{1}{s^2} - \frac{1}{s})]\sum_{n=1}^{\infty} e^{-2ns}$.

$$g(t) = \begin{cases} 2-t, & 0 \le t < 1 \\ 0, & 1 \le t < 2 \end{cases}$$

$$= (2-t)u(t) - (2-t)u(t-1)$$

$$f(t) = g(t)u(t) + g(t-2)u(t-2) + g(t-4)u(t-4) + \dots$$

$$= \sum_{n=0}^{\infty} [g(t-2n)u(t-2n)]$$

$$F(s) = \mathcal{L}\{\sum_{n=0}^{\infty}[g(t-2n)u(t-2n)]\}$$

$$= \sum_{n=0}^{\infty} G(s).e^{-2ns}$$

$$G(s) = \mathcal{L}\{(2-t)u(t) - (2-t)u(t-1)\}$$

$$= \mathcal{L}\{2 - (t)u(t) - [2 - (t-1+1)u(t-1)]\}$$

$$\mathcal{L}\{2 - (t)u(t) - [1 - (t-1)u(t-1)]\}$$

$$= \left(\frac{2}{s} - \frac{1}{s^2}\right)(e^{0s}) - \left[\frac{1}{s} - \frac{1}{s^2}\right]e^{-s}$$