Chapter 4: Electromagnetic Induction

We distinguish between two kinds of induction:

motional induction

conductor loop moves in a stationary magnetic field

↓

Lorentz force on moving charges

↓

charge is "deflected"

↓

electric field is built up
results in induced voltage

motionless induction

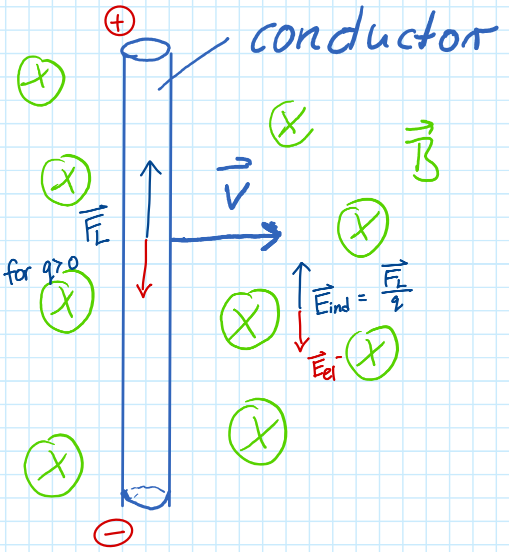
time-varying magnetic field

↓

Electric voltage is induced in resting current loop

4.1 Motional induction

(4.1.1) Electromotive force on a moving conducting material



* Conductor contains movable charges
* it moves at velocity $\vec{v}$ in a $\vec{B}$-field

⇒ $\vec{F}_L$ acting on charges: $\vec{F}_L = q(\vec{v} \times \vec{B})$

* $q>0$ ⇒ $\vec{F}_L$ ↑
* Charge moves along conductor by electromotive force

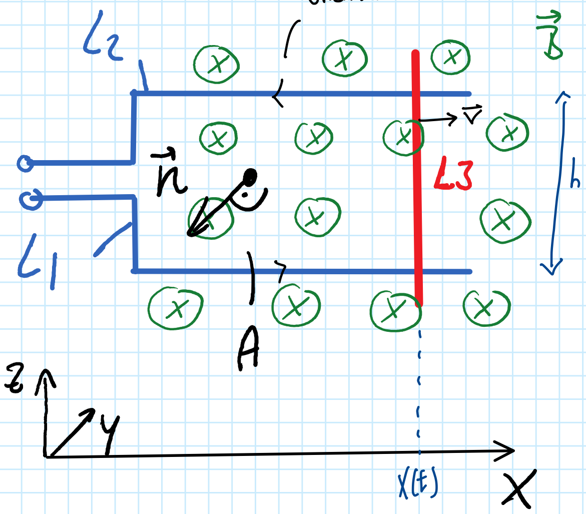
$$\boxed{\vec{E}_{ind} = \frac{\vec{F}_L}{q} = (\vec{v} \times \vec{B})}$$

* electrostatic field is generated by accumulating charges $\vec{E}_{el}$

(4.1.2.) Induced electrical voltage in a time-varying conductor loop

(i) Experiment

Orientation of $C = \partial A$ is Counterclockwise



- Two parallel conductors L1, L2 and a movable wire L3 define a nearly closed conductor loops, open at terminals
- Movable wire 3 connects L1 and L2 electrically
- B-field points into drawing plane : $\vec{B} = B \cdot \vec{e}_y$
- L1, L2, L3 build area A and, at the same time Curve C encloses area A : $C = \partial A$

- Orientation of curve C is counterclockwise: $\Rightarrow$ defines orientation of $\vec{n}$ (right-hand rule) ($\triangleq$ thumb)
- L1 moves at velocity v: $\vec{v} = v \cdot \vec{e}_x = \frac{dx}{dt} \cdot \vec{e}_x$
- Area A changes with time: $A = A(t) = h \cdot x(t)$
- Lorentz force on wire induces electromotive force: $\vec{F}_L = q\,(\vec{v} \times \vec{B})$ along z-axis

Calculate induced voltage from induced electric field:

$$U_{ind} = \int_0^h \vec{E}_{ind}\, d\vec{r} = \int_0^h \frac{\vec{F}_L}{q} \cdot d\vec{r} = \int_0^h (\vec{v} \times \vec{B}) \cdot d\vec{r} = \int_0^h (v \cdot \vec{e}_x \times B \cdot \vec{e}_y) \cdot dz \cdot \vec{e}_z$$

$$= \int_0^h v \cdot B \cdot \vec{e}_z \cdot dz\, \vec{e}_z = \int_0^h v \cdot B \cdot dz = v \cdot B \cdot h = \frac{dx}{dt} \cdot B \cdot h \quad *$$

Define magnetic flux as flux of B-field, which penetrates the area A:

magnetic flux $\boxed{\Phi(A) = \int_A \vec{B} \cdot d\vec{a}}$

for rectangular conductor loop: $\Phi(A) = \int_0^h \int_0^{x(t)} B \cdot \vec{e}_y \cdot -\vec{e}_y\, dx \cdot dz = -B\, h \cdot x(t) \quad *$

Comparison of both calculations results in:

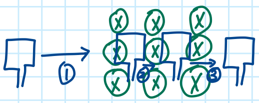
$$\boxed{U_{ind} = -\frac{d\Phi(A)}{dt} = -\frac{d}{dt} \int_{A(t)} B\, d\vec{a}}$$

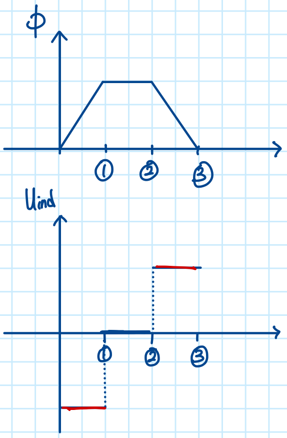
(ii) Generalization to arbitrary conductor geometries:

- (4.3) can be generalized for arbitrary conductor loop geometries
- Or can be also applied for a change in the area, which is penetrated perpendicular by the B-Field (e.g. by deformation or rotation of a conductor loop)

$\Rightarrow$ either curve C can change with time: C(t)

or a part of the loop is moving with velocity v (deformation of the loop)





① Φ is Changing
$U_{ind} \neq 0$

② Φ is not Changing
$U_{ind} = 0$

③ Φ is Changing
$U_{ind} \neq 0$

$U_{ind} = -\frac{d\Phi}{dt}$

$$\boxed{U_{ind} = -\frac{d\Phi(A)}{dt} = -\frac{d}{dt}\int_{A(t)} B\, d\vec{a} = \int_{C(t)=\partial A(t)} \overbrace{(\vec{v}\times\vec{B})}^{\vec{E}_{ind} = \frac{\vec{F}_L}{q}}\, d\vec{r} \qquad 4.4}$$