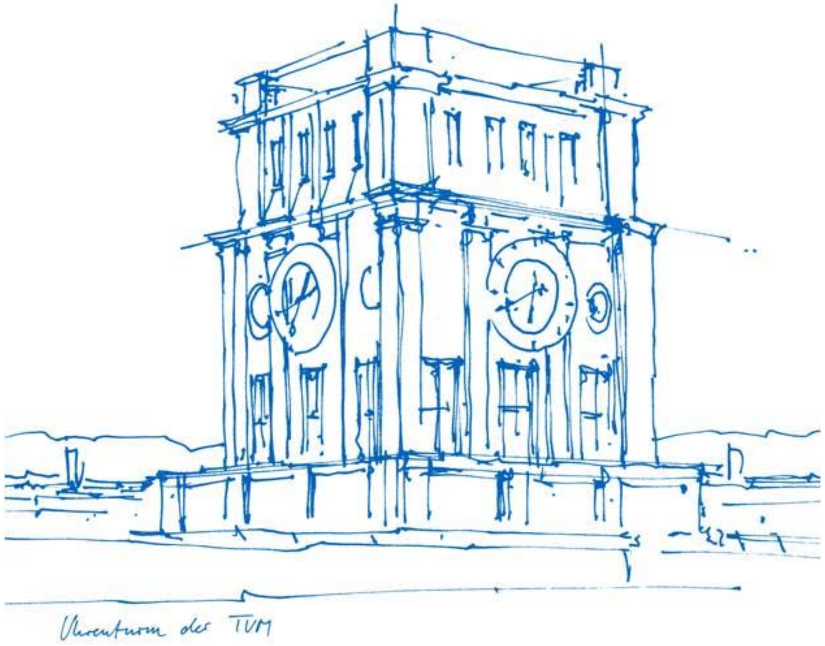
Some questions on electricity and magnetism

(Without any claim to completeness ;-))



- Do two negative charges repel or attract each other?

Repel each other

- Give the force between two point charges q_1 und q_2.

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{q_1 q_2 (\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^3}$$ Coulomb force

- The force between two point charges is
  - is ...smaller...for larger distance
  - is ...smaller... If magnitudes of q_1 /q_2 are smaller
  - Is parallel to ...$(\vec{r}_2 - \vec{r}_1)$ = distance vector...

- How can Coulomb's law for two point charges be generalized to N charges and what is the underlying principle?

superposition principle ⇒ sum up forces of all N charges

$$\vec{F}_q(\vec{r}) = \frac{q}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_i(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3}$$

- What is the definition of the electric field and what is its unit?

$$\vec{E} = \frac{\vec{F}}{q} \qquad \text{unit of } \vec{E} = [|\vec{E}|] = 1\,\frac{V}{m} = 1\,\frac{N}{C}$$

- How do you generally calculate work in a force field, such as electrical work (in an electrostatic field)?

$$W = \int_{P_1}^{P_2} \vec{F}\,d\vec{r} = \int_{P_1}^{P_2} q\cdot\vec{E}\,d\vec{r}$$

- How is the electrical voltage defined?

$$U_{12} = \int_{P_1}^{P_2} \vec{E}\, d\vec{r}$$

- What does it mean for a vector field if the integration path does not play a role?

vector field is conservative (potential field)

- What is the basic law of electrostatics? How do you check it (which criteria apply)?

Electrostatic field is conservative.

(i) $\int_{P_1}^{P_2} \vec{E}\, d\vec{r}$ is independent of path

(ii) $\text{Curl } \vec{E} = 0$

(iii) $\oint \vec{E}\, d\vec{r} = 0$

- Is the electric potential a scalar field or a vector field?

Scalar $\phi(\vec{r}) = \phi_0 - \int_{P_0}^{P} \vec{E}\, d\vec{r} \qquad P = P(\vec{r})$

- Why must a reference potential be defined?

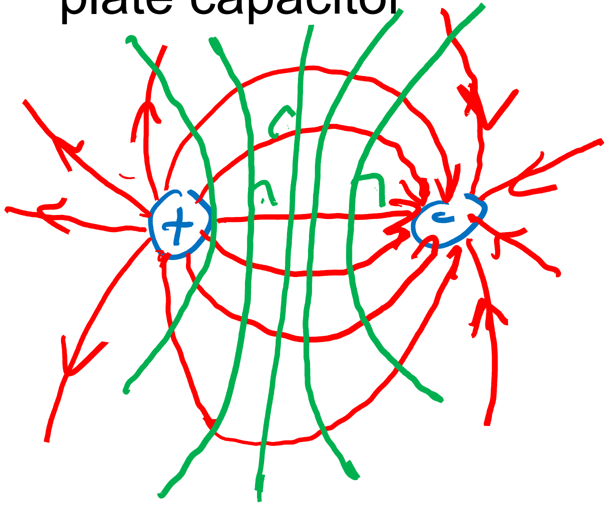
$\vec{E} = -\text{grad}\,\phi$ $\phi \sim \int \vec{E}\,d\vec{r}$

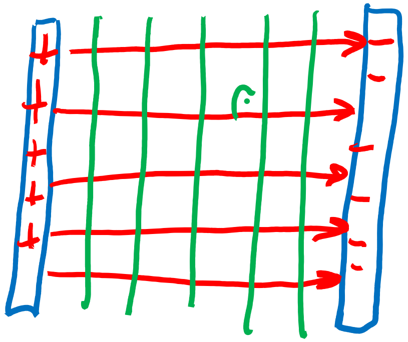
↳ Integration constant; has to be defined

- What do equipotential planes mean in terms of electrical work?

No work is done, when moving along them.

- Draw the equipotential surfaces of an electric dipole field and inside a plate capacitor

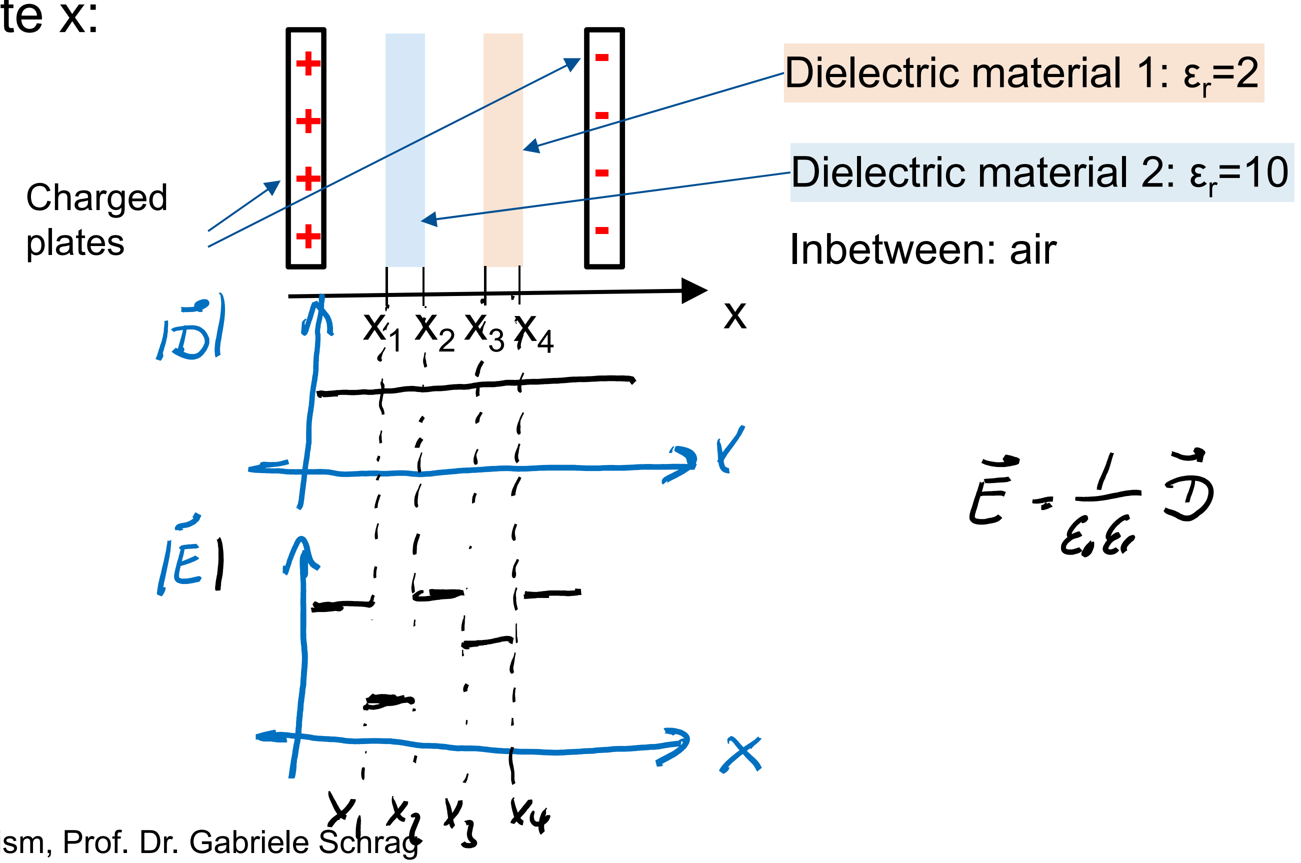




- How is the electric displacement field defined?

$$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$$

- Sketch the course of the magnitude of the electric field and the dielectric displacement field for the following arrangement as a function of the coordinate x:



Charged plates

Dielectric material 1: $\varepsilon_r=2$

Dielectric material 2: $\varepsilon_r=10$

Inbetween: air

$$\vec{E} = \frac{1}{\varepsilon_0 \varepsilon_r} \vec{D}$$

- How is the space charge density defined?

$$\rho(\vec{r}) = \lim_{\Delta V \to 0} \frac{\Delta Q}{\Delta V} \quad (\text{charge per Volume}) \quad \text{3D}$$

- How is the surface charge density defined?

$$\sigma(\vec{r}) = \lim_{\Delta \to 0} \frac{\Delta Q}{\Delta A} \quad (\text{charge per area/Surface}) \quad \text{2D}$$

- What is Gauss's law in general form and how is it related to space charge density?

$$\int_{\partial V} \vec{D} \cdot d\vec{a} = Q(V(\vec{r})) = \int_{V} \rho(\vec{r})\, dV$$

- How is the dielectric displacement field at the surface of an electrical conductor calculated?

$$\vec{D} \cdot \vec{N} = \sigma \qquad |\vec{D}| = \sigma$$

- What is the charge distribution in an electrical conductor that is in an electric field? And what is this phenomenon called?

electric induction

charges are attracted to surface $\Rightarrow$ σ at surface of the conductor ; σ = homogeneous

- How large is the electric field in an electric conductor? What does this mean for the electric potential in electrically conductive media?

$\vec{E}_i$ (conductor) $= 0$

ϕ is constant on electrically conducting surfaces (equipotential)

- How is the electric current I defined and what is the physical unit of I?

$$I = \frac{dQ}{dt}\bigg|_A$$

$$[I] = 1\,A$$

- What is the relationship between electric current density and electric current I ? Explain qualitatively the difference between the two quantities.

$$I = \int_A \vec{j}\, d\vec{a}$$

$\vec{j}(\vec{r})$ = " local " quantity, magnitude and direction depends on $\vec{r}$ (vector field)

I = integral quantity ; total current, scalar value

- How is the electric current density related to microscopic properties such as space charge density, particle number density and charge?

$$\vec{j} = q \cdot n \cdot \vec{v}$$

- What parameters does electrical conductivity depend on?

$$\sigma = q \cdot n \cdot \mu$$

$$\rho = \frac{1}{\sigma}$$

Charge

particle density

mobility

N/V

- State local Ohm's law. Name the quantities contained in it.

$$\vec{j} = \sigma \vec{E}$$

$\vec{j}$ Current density

$\vec{E}$ electric field

σ electric conductivity $[\sigma] = \Omega m$

- State Ohm's law in integral form. Name the quantities contained in it.

$$U = R \cdot I$$

$$[R] = \Omega$$

Voltage

resistance R

Current

$\vec{j}$

- How do you calculate the electric power density of a current density in an electric field?

$\vec{E}$

$$P_{el} = \text{Power}/\text{Volume} = \vec{j}\cdot\vec{E}$$

- Why is the power dissipated in Ohmic transport always positive? Explain by giving an equation.

Ohmic transport: $\vec{j} = \sigma\vec{E}$

$$P_{el} = \vec{j}\cdot\vec{E} = \sigma\cdot\vec{E}^2 > 0$$

- How do you calculate the power dissipated at an ohmic resistor R?

$$P = U\cdot I = R\cdot I^2$$

- How is electrical capacitance generally defined?

$$C = \frac{Q}{U}$$

- How do you calculate the capacitance of an parallel plate capacitor?

$$C = \varepsilon_0 \cdot \varepsilon_r \frac{A}{d}$$

- How do you calculate the equivalent capacitance value of N capacitors connected in parallel ?

$$C_{tot} = \sum_{i=1}^{N} C_i$$

- How do you calculate the equivalent capacitance value of N capacitors connected in series?

$$\frac{1}{C_{tot}} = \sum_{i=1}^{N} \frac{1}{C_i} \Rightarrow C_{tot} = \left(\sum_{i=1}^{N} \frac{1}{C_i} \right)^{-1}$$

- How can we see that an external magnetic field is present?

Moving charge is deflected.

- How is this force termed and how is it calculated?

Lorentz force $\vec{F}_L = q(\vec{v} \times \vec{B})$

- How is the physical quantity termed that causes this force?

$\vec{B}$-field (magnetic induction, magnetic flux density)

Circulation

- Which of the following statements for the gyration of a particle charged with q are correct?
  - ○ If the magnitude of the B-field is larger, the particle rotates at a lower frequency.
  - ☒ If the mass of the particle is larger, it rotates at a lower frequency.
  - ○ If the mass of the particle is larger, the radius of the gyration is smaller.
  - ☒ If the magnetic field is larger, the radius of the particle's gyration is smaller.
  - ○ The trajectories of particles with a higher speed have a smaller radius.
  - ○ The gyrational (no) frequency and the radius (yes) of the trajectory both depend on the speed of the particle.

- Give the differential force on an element ds of a conductor through which a current I flows calculated differentially in a constant B-field?

  How do you calculate from this the force on a conductor of length l and cross-section A?

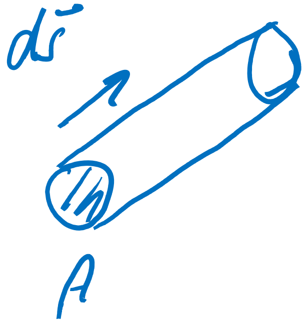
$d\vec{F} = I\, d\vec{s} \times \vec{B}$ (derived from Lorentz force density $\vec{f}_L = \vec{j} \times \vec{B}$)

↓ unit Force: N

↳ unit: $\frac{N}{m^3}$

⇓

Area A already accounted for in current I ($\vec{j} \cdot A$)



$\Rightarrow \vec{F}_{wire} = \int_0^l I\, d\vec{s} \times \vec{B}$

if $d\vec{s} \uparrow\uparrow \vec{e}_x$, $\vec{B} \uparrow\uparrow \vec{e}_y$ $\Rightarrow$ $\vec{F}_{wire} = l \cdot I\, B \cdot \vec{e}_z$

- Which equation describes the solenoidality of the B-field? What does this mean physically? Name three equivalent physical statements/consequences.

$$\mathrm{div}\,\vec{B} = 0 \quad \left(\text{or} \int_{\partial V} \vec{B} \cdot d\vec{a} = 0 \right)$$

- no magnetic monopoles exist
- no magnetic charges exist
- $\vec{B}$-field lines are always closed

- How are magnetostatic fields generated and which law is used to describe this? State this law.

generated by electric curren (densities)

Ampère's circuital law: $\int_{\partial A} \vec{H} \, d\vec{r} = \int_{A} \vec{j} \cdot d\vec{a} = I(A(r))$

- What is the relationship between B-field and H-field?

$B = \mu_0 \mu_r \vec{H}$

- What is the relationship between H-field and magnetization in the simplest case and which material constant describes this?

$\vec{M} = \chi_m \cdot \vec{H}$ χ_m = magnetic susceptibility

- Name the three forms of magnetism that occur in magnetizable materials What results for the $\vec{B}$ field inside the three types of material?

Diamagnetism $\Downarrow$ $\vec{B}$-field is weakened

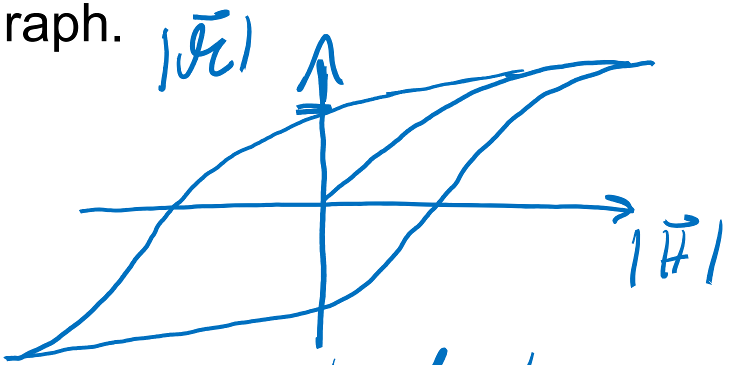
Paramagnetism $\Downarrow$ $\vec{B}$-field is larger than outside

Ferromagnetism $\Downarrow$ $\vec{B}$-field much large than external field

- What is hysteresis in context of magnetization and what does remanence magnetization mean? Draw a graph.

Occurs in ferromagnetic materials;

remanent magnetization:
$\vec{M}$ is still present although field is switched off (strong interaction between magnetic dipoles.



- What happens if you heat a permanent magnet above the Curie temperature?

$\vec{M}$ vanishes

- What is Maxwell's extension of Ampére's circuital law and what is its physical significance?

Magnetic fields are generated by time-varying $\vec{D}/\vec{E}$-fields as well.

$$\int_{\partial A} \vec{H}\, d\vec{r} = \int_A \left(\vec{j} + \frac{\partial \vec{D}}{\partial t}\right) d\vec{a}$$

- What are the two types of induction? How can electric fields be induced?

- motional induction ⇒ change of magnetic flux in conductor loop by moving/rotating the loop in an $\vec{B}$-field or by changing the area $A(t)$
- motionless induction ⇒ time-varying $\vec{B}$-field

- How is the magnetic flux Φ defined and how is it calculated?

$$\Phi = \int_{A(t)} \vec{B}(\vec{r},t) \cdot d\vec{a}$$

- How is the induced voltage calculated from the magnetic flux? Does this formula apply to motion induction, rest induction or in general?

$$U_{ind} = -\frac{d\phi}{dt} \Rightarrow \text{applies in general}$$

- Are induced electric fields conservative? Give a reason to your answer by stating the according Maxwell's equation.

No, since $\text{curl}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t} \neq 0$

or: $\oint \vec{E}\,d\vec{r} = \frac{d}{dt} \int_{A(V)} \vec{B}(\vec{r},t)\,d\vec{a} \neq 0$