Motion along a straight line

Topic 1a

A/P Patrick Chua

Learning outcomes for Topic 1a

- How the ideas of displacement and average velocity help us describe straight-line motion.
- The meaning of instantaneous velocity; the difference between velocity and speed.
- How to use average acceleration and instantaneous acceleration to describe changes in velocity.
- How to solve problems in which an object is falling freely under the influence of gravity alone.
- How to analyze straight-line motion when the acceleration is not constant.

Overview of Topic 1a

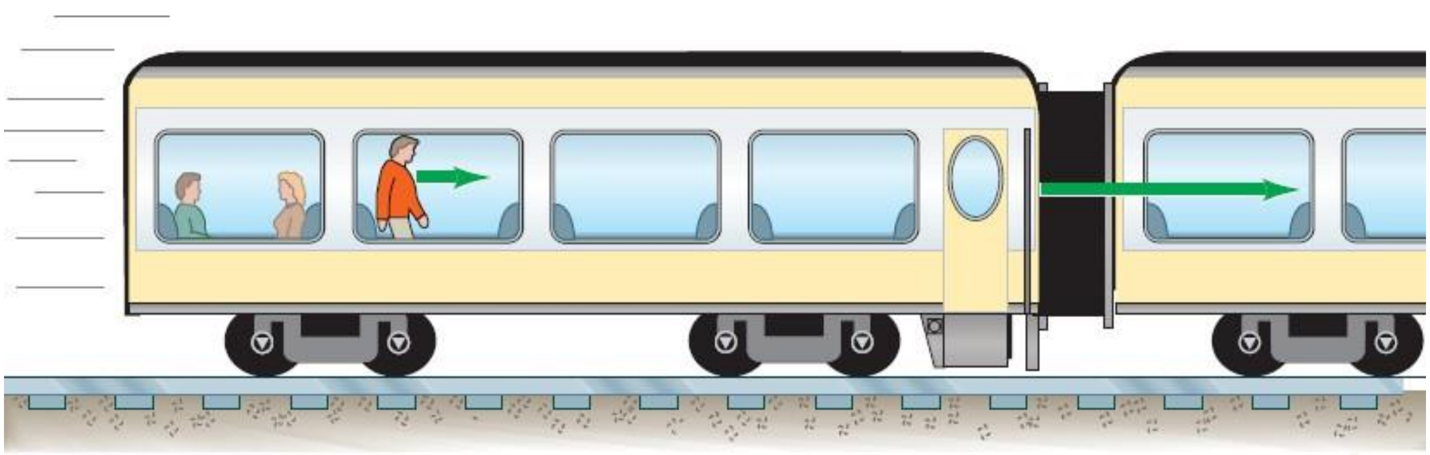
- Kinematics and Dynamics
- Displacement, Velocity and Acceleration
- Instantaneous Velocity and Position Time Graph
- Finding Velocity on a x-t Graph
- Instantaneous and Average Acceleration
- Motion with constant acceleration
- Equations of Motion with constant acceleration
- Freely falling bodies
- Velocity and position by integration

Introduction

- ***Mechanics:*** study of relationship among force, matter and motion
- ***Kinematics*** is the study of motion.
- ***Dynamics***: describes why objects move the way they move
- ***Velocity and acceleration*** are important physical quantities that describe motion along straight line.

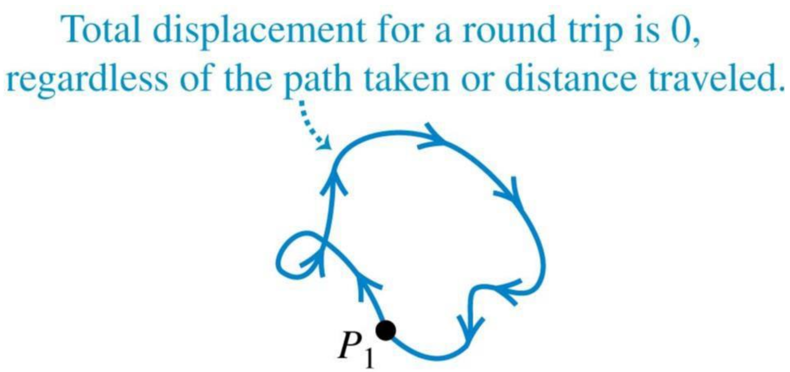
Reference Frames and Displacement

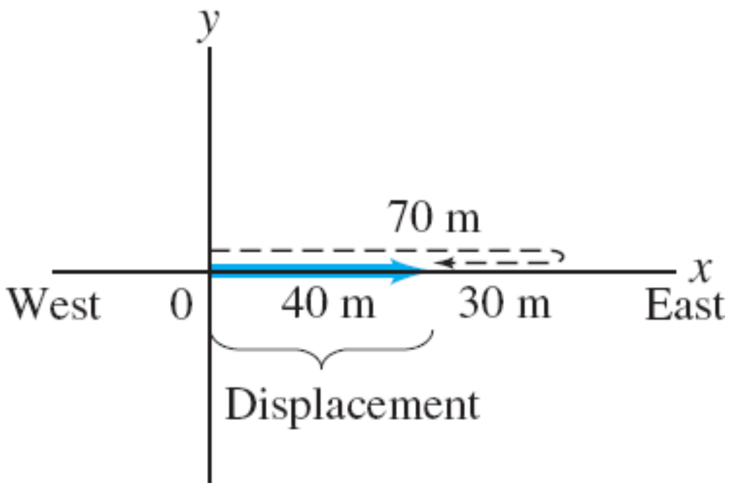
- Measurement of *position*, *distance*, or *speed* must be made with respect to a **reference frame**.
  - Example, if you are sitting on a train and someone walks down the aisle, *the person's speed with respect to the train is a few miles per hour,* at most. *The person's speed with respect to the ground is much higher*.



Reference Frames and Displacement

- We make a distinction between ***distance*** and ***displacement***.
- ***Displacement*** (blue line) is how far the object is from its starting point, *regardless of how it got there*.
- ***Distance traveled*** (dashed line) is measured along the actual path.





Displacement, time and average Velocity

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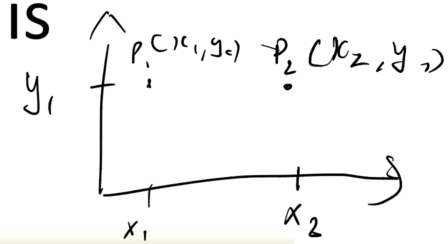
- A particle moving along the *x*-axis has a coordinate *x*.
- ***Displacement***
  - The change in the particle's coordinate, i.e., $\Delta x = x_2 - x_1$.
- The ***average x-velocity*** of the particle is



$$v_{\text{av-}x} = \Delta x / \Delta t.$$

Average *x*-velocity of a particle in **straight-line motion** during time interval from t_1 to t_2

x-component of the particle's displacement

$$v_{\text{av-}x} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

Final *x*-coordinate minus initial *x*-coordinate

Time interval

Final time minus initial time

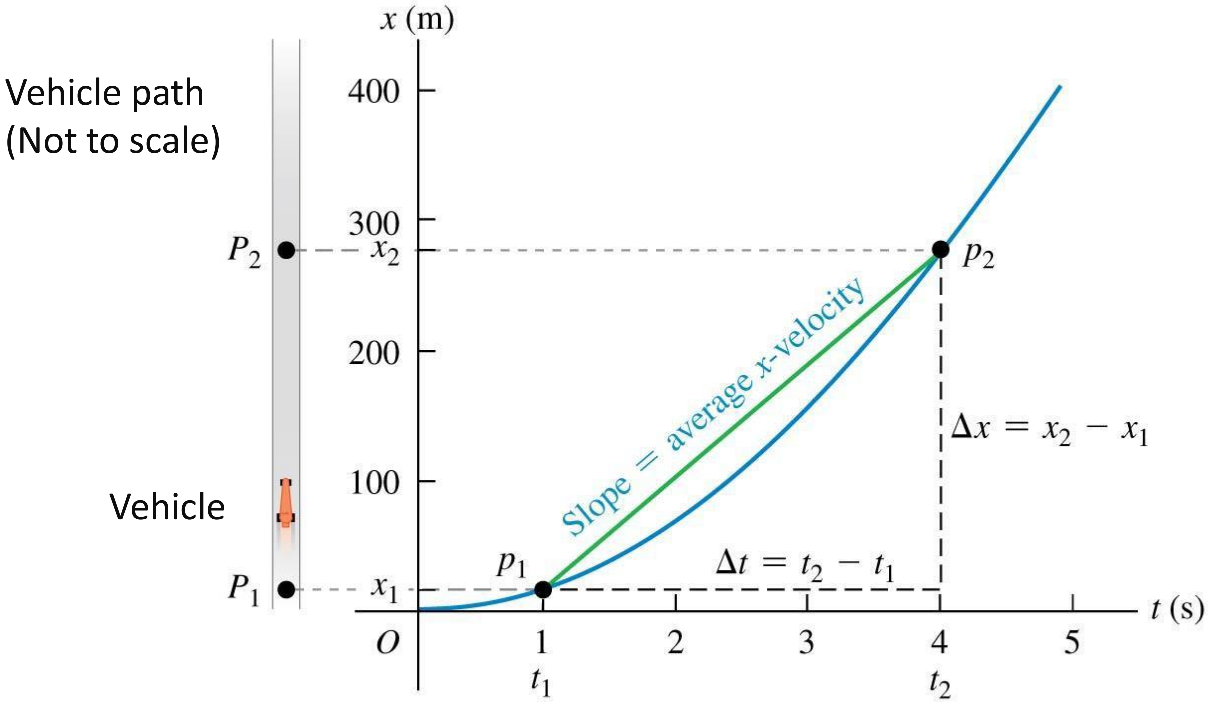
displacement

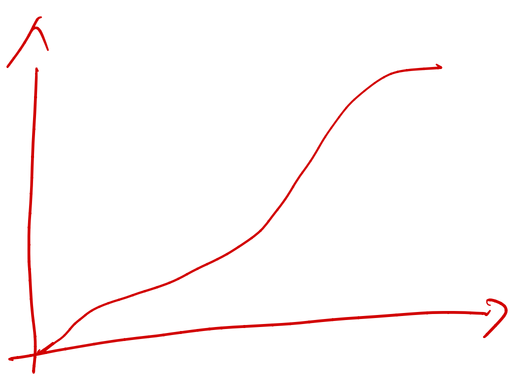
$\Delta x = x_2 - x_1$

- Units: meters/second

A position time graph

- The distance along the curve between points p_1 and p_2 is called path length. The curve represents the changes in the position with time.
- p_1 and p_2 (positions on graph) correspond to points P_1 and P_2 along the path.





Slope = average velocity

Instantaneous velocity

- The ***instantaneous velocity*** is the velocity at a specific instant of time or specific point along the path and is given by $v_x = dx/dt$.

The **instantaneous** **x-velocity** of a particle in **straight-line motion** ...

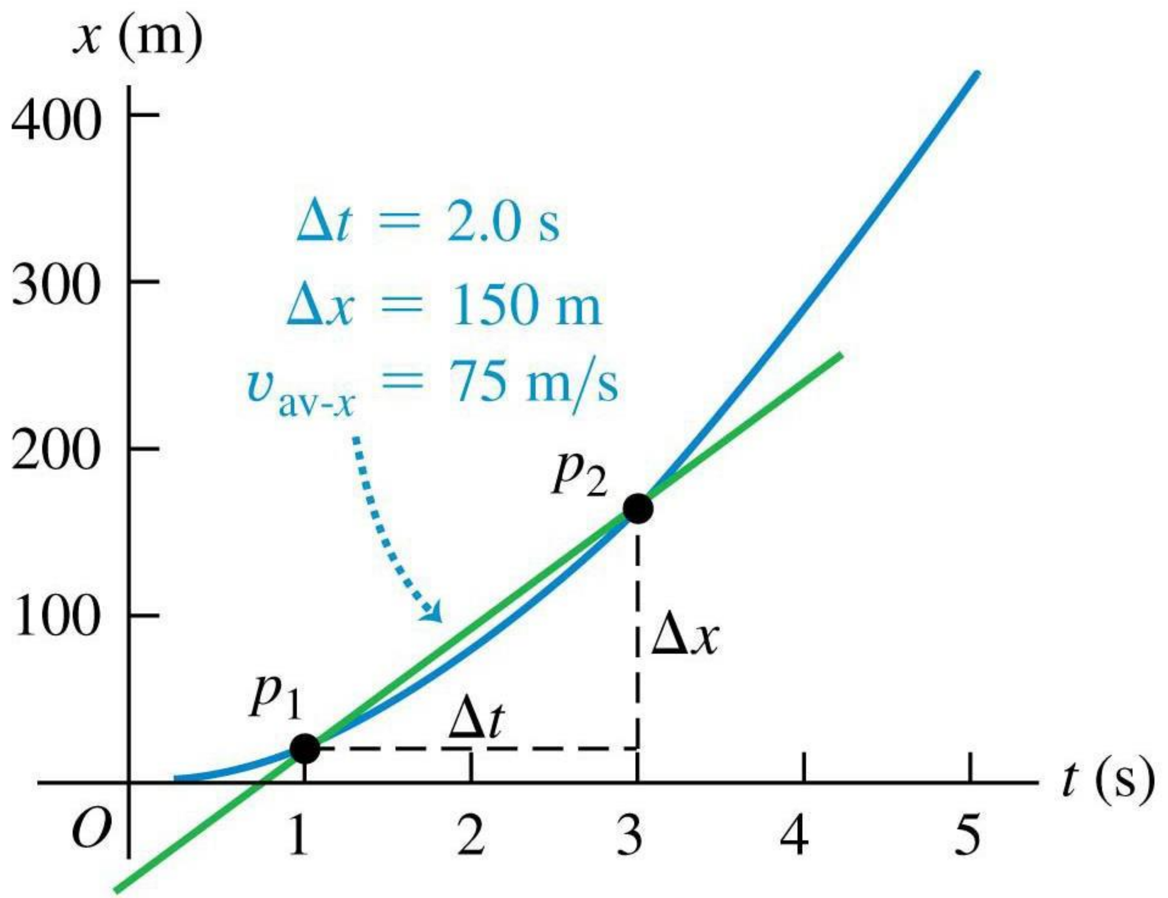
$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

... equals the limit of the particle's average x-velocity as the time interval approaches zero ...

... and equals the instantaneous rate of change of the particle's x-coordinate.

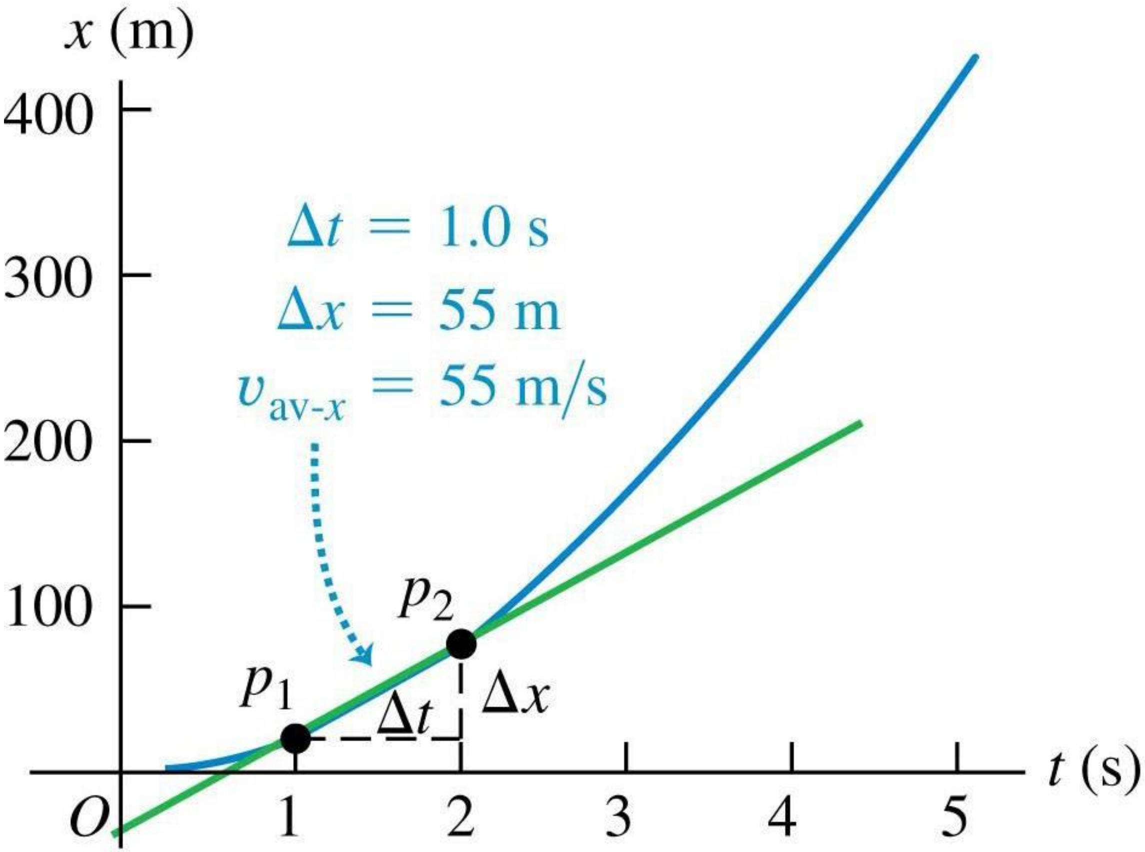
- The average speed is *not* the magnitude of the average velocity!
- Speed is distance (not displacement) divided by time
- Instantaneous speed is the magnitude of instantaneous velocity

Finding instantaneous velocity on x - t graph



As the average x-velocity $v_{av\text{-}x}$ is calculated over shorter and shorter time intervals

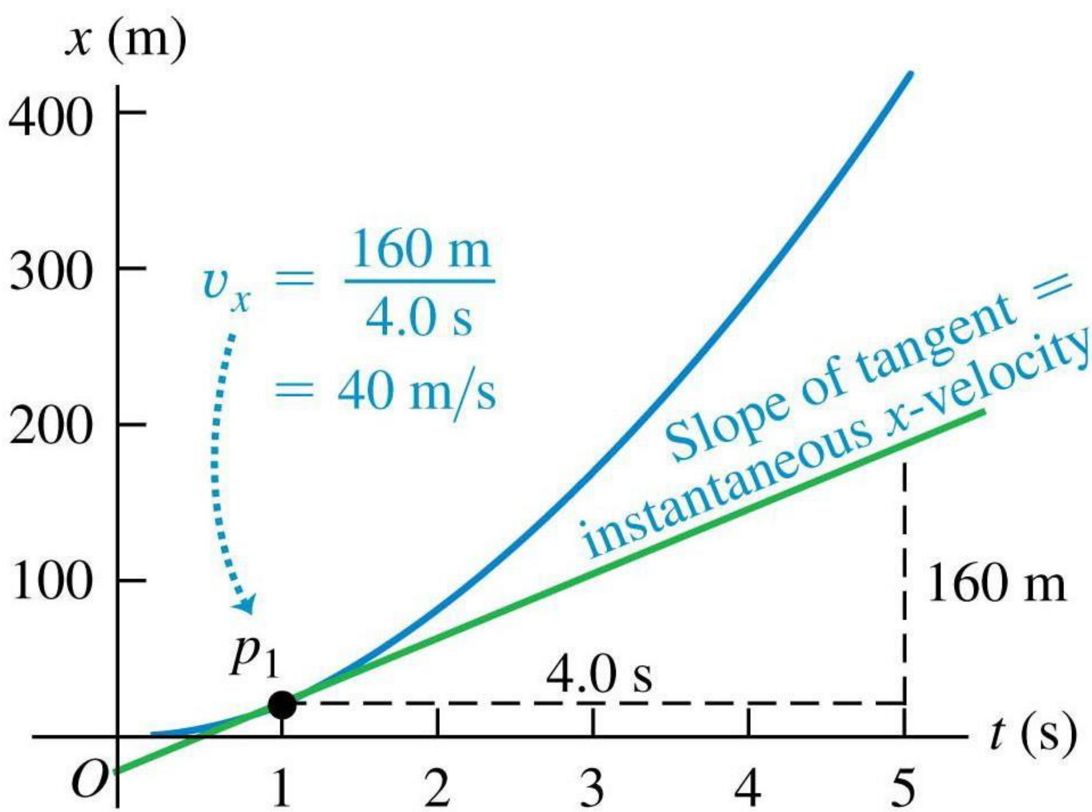
Finding instantaneous velocity of x - t graph



... its value $v_{av\text{-}x} = \Delta x / \Delta t$ approaches the instantaneous x-velocity.

Finding instantaneous velocity of x - t graph

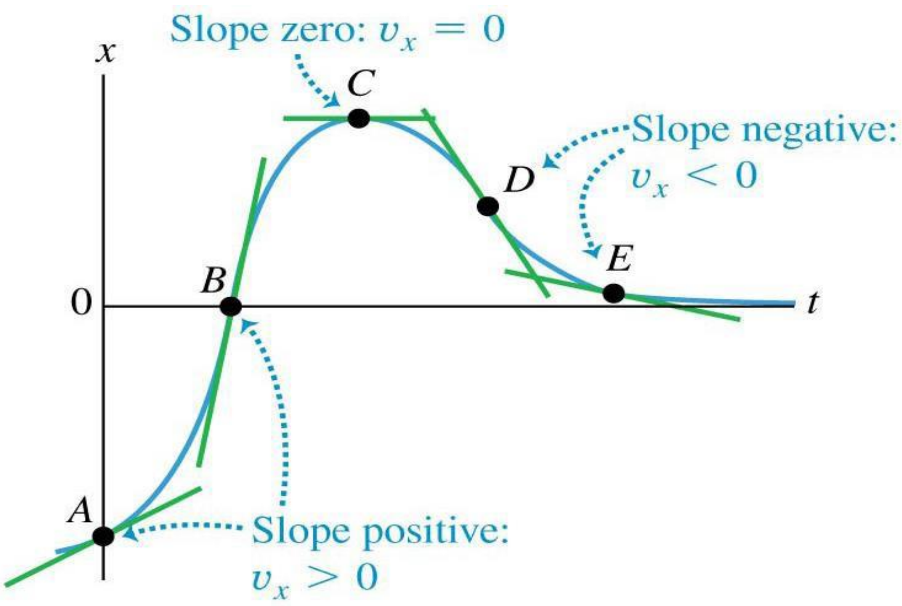
Remember the difference between average velocity and instantaneous velocity



The instantaneous x-velocity v_x at any given point equals the slope of the tangent to the x-t curve at that point.

Finding velocity on x - t graph

- If the tangent slopes upward to the right, the slope is ***+ve*** and velocity is ***+ve***
- If the tangent slopes downward to the right, the slope is ***–ve*** and velocity is ***–ve*** and motion is in the ***-ve x*** direction



Average acceleration

- ***Acceleration*** describes the rate of change of velocity with time.
- The **average x-acceleration** is $a_{\text{av-}x} = \Delta v_x / \Delta t$.

Average *x*-acceleration of a particle in **straight-line motion** during time interval from t_1 to t_2

Change in x-component of the particle's velocity

$$a_{\text{av-}x} = \frac{\Delta v_x}{\Delta t} = \frac{v_{2x} - v_{1x}}{t_2 - t_1}$$

Final x-velocity minus initial x-velocity

Time interval

Final time minus initial time

- Units: meters/sec^2

Instantaneous acceleration

- The ***instantaneous*** acceleration is $a_x = dv_x/dt$.

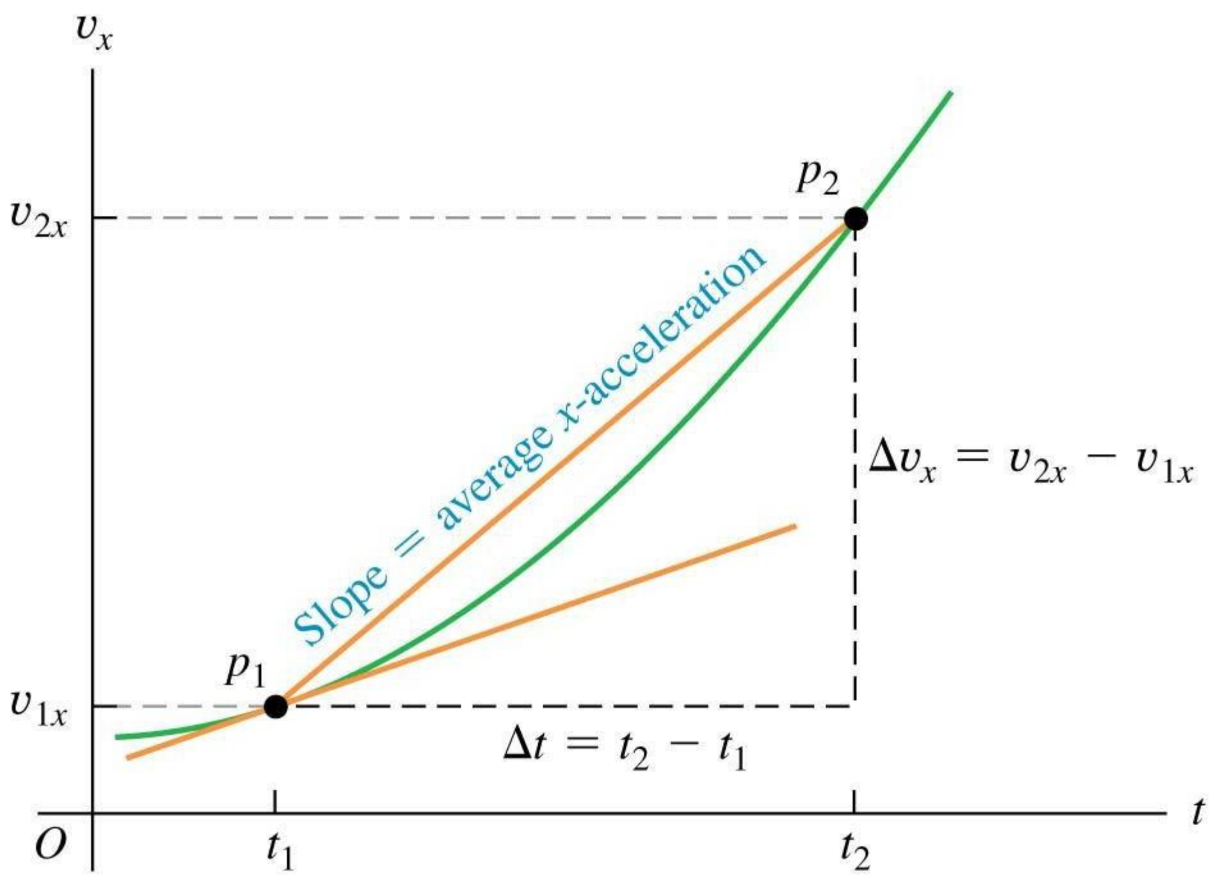
The **instantaneous** ***x*-acceleration** of a particle in **straight-line motion** ...

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

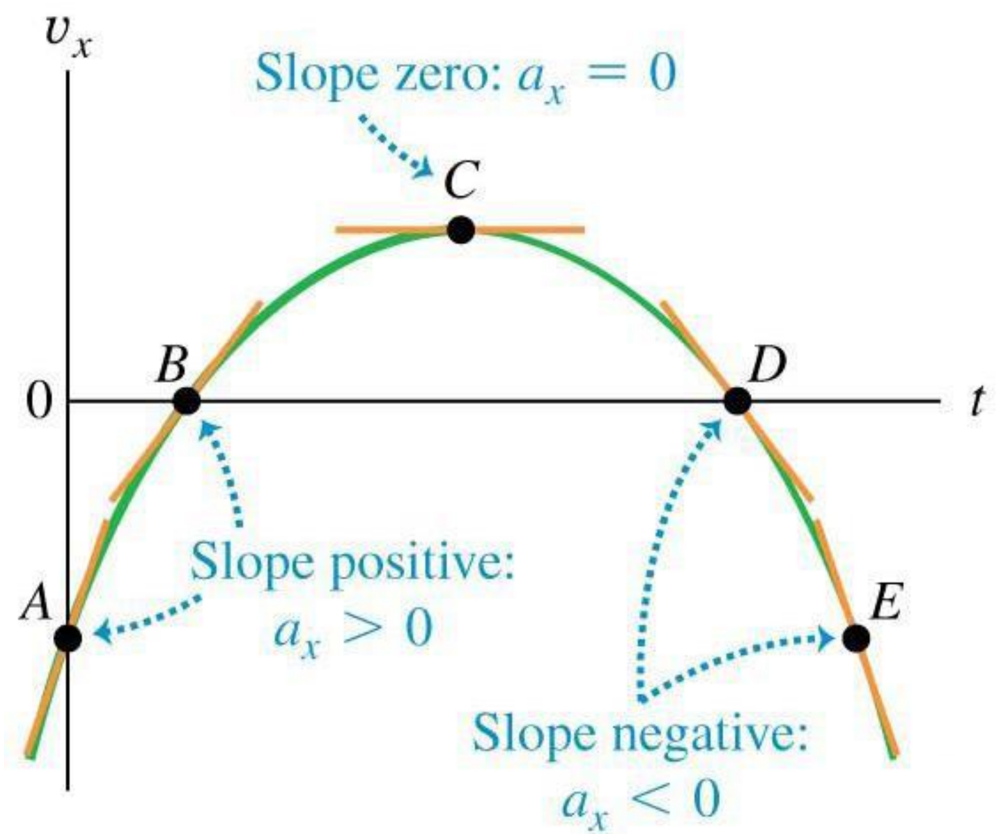
... equals the limit of the particle's average *x*-acceleration as the time interval approaches zero ...

... and equals the instantaneous rate of change of the particle's *x*-velocity.

Finding acceleration on $v_x - t$ graph



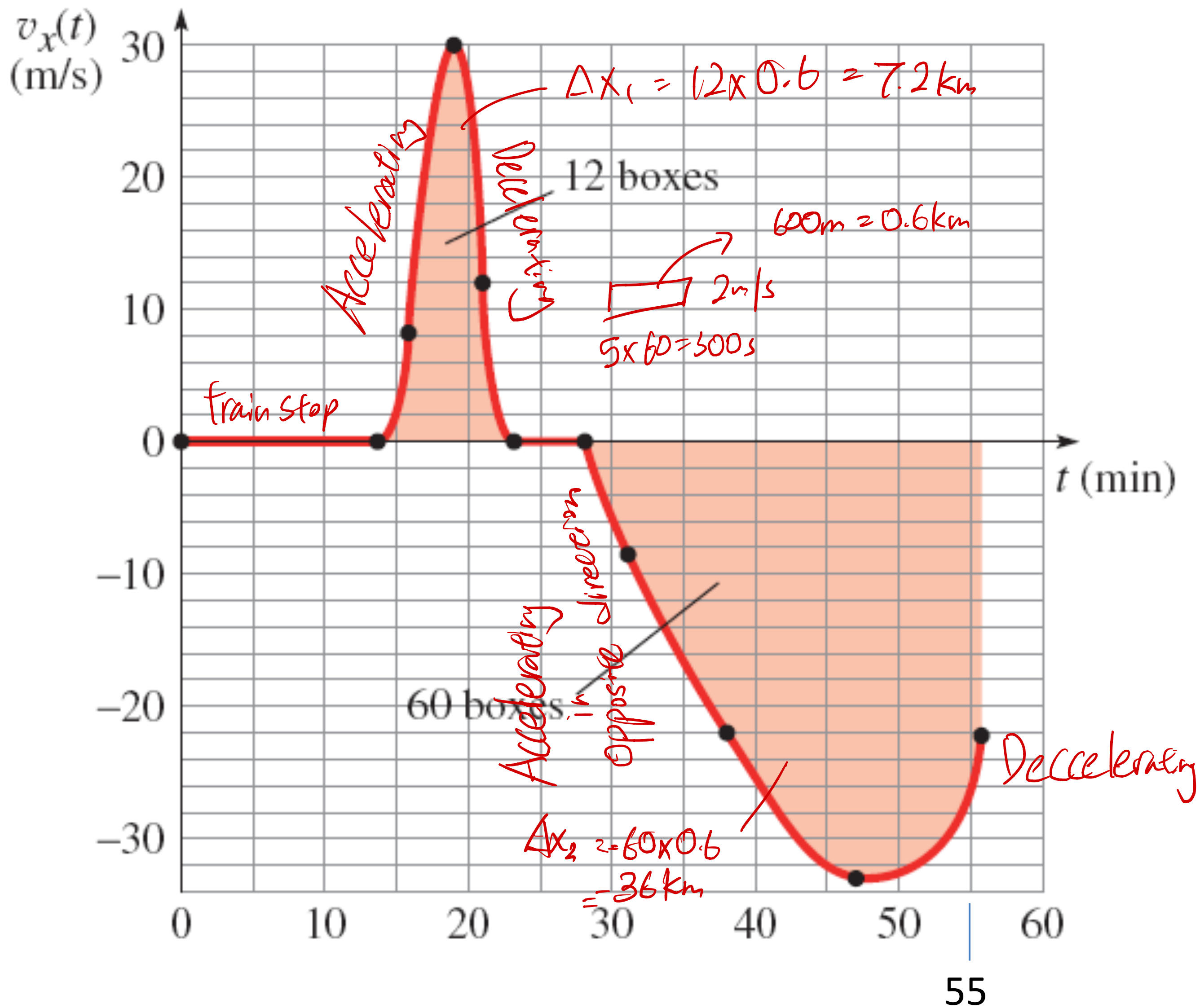
A v_x – t graph


v_x
Slope zero: $a_x = 0$
C
B
D
0
t
A
E
Slope positive:
$a_x > 0$
Slope negative:
$a_x < 0$

Example : Finding Displacement with Changing Velocity

Q: How to interpret this graph of velocity-vs-time plot of a train?

Q2: What is the train's displacement 55 mins after starting from its initial position?

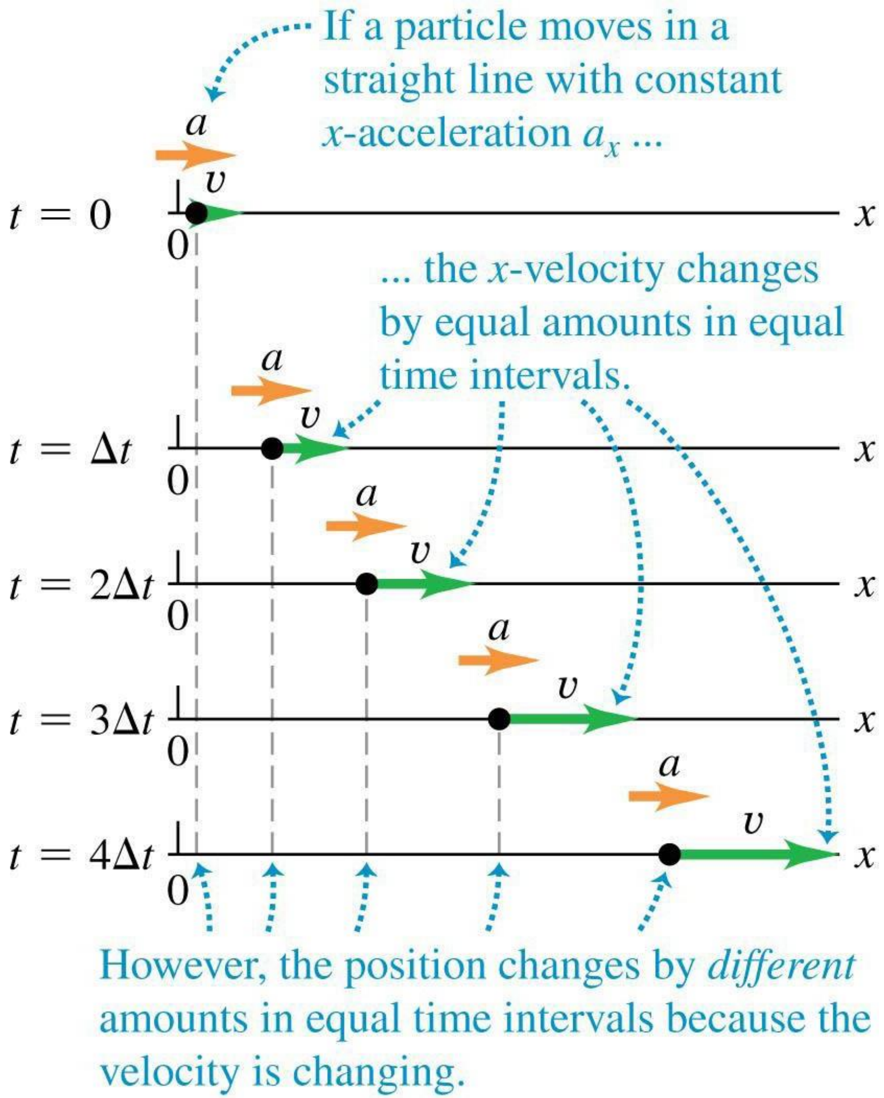


$\Delta x_{final} = \Delta x_1 + \Delta x_2$

$= 7.2 - 36$

$= -28.8$ km

Motion with constant acceleration



Motion with constant acceleration

-- Equations of motion

- With ***x*-acceleration constant**

$$a_x = \frac{v_{2x} - v_{1x}}{t_2 - t_1} \Rightarrow a = \frac{v_2 - v_1}{t_2 - t_1}$$

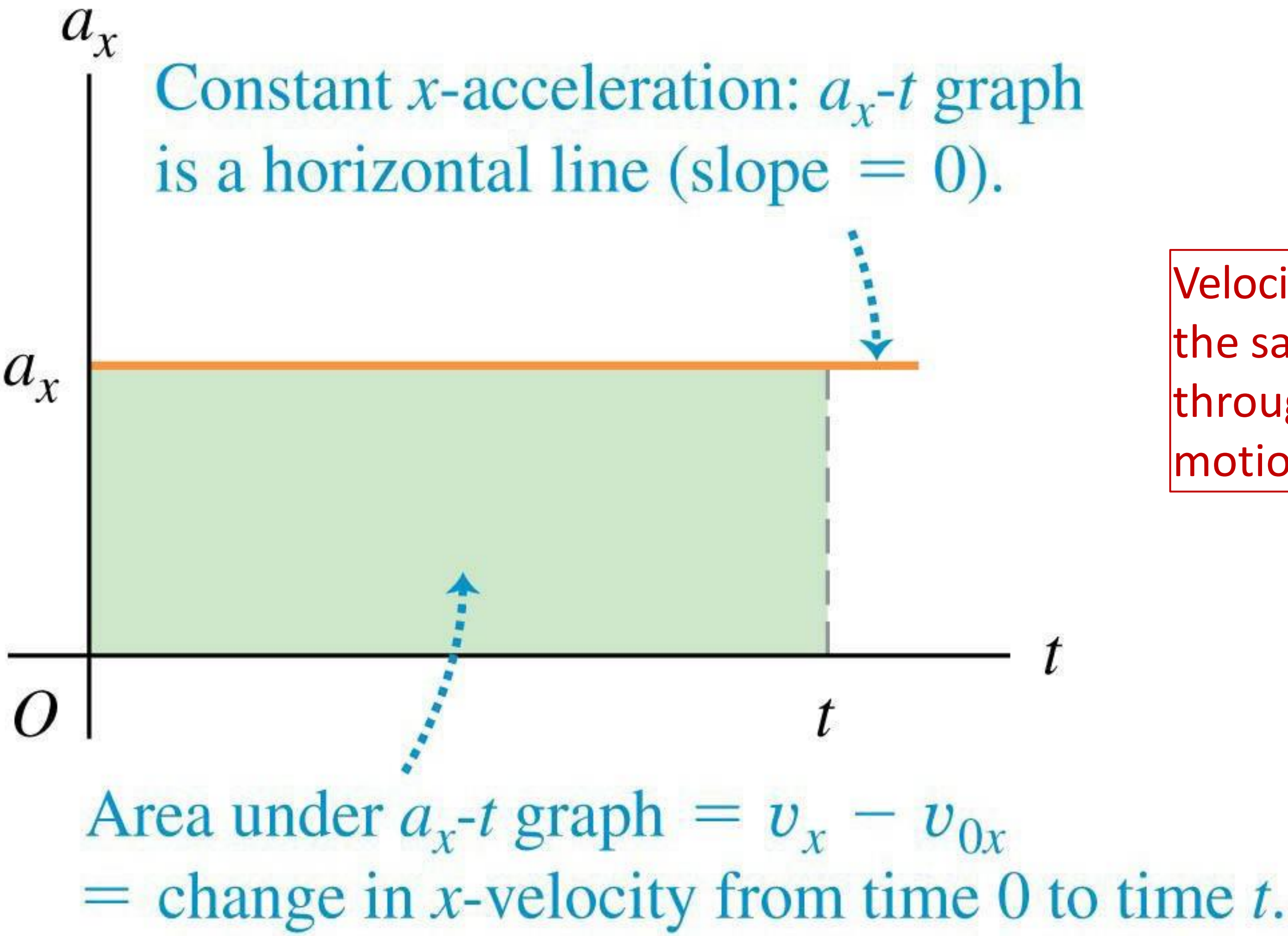
We have omitted the subscript x for convenience,

and if we consider $t_1 = 0, t_2 = t$

$$a = \frac{v_2 - v_1}{t}$$

$$v_2 = v_1 + at \qquad (1)$$ (Equation of motion)

Motion with constant acceleration



Velocity changes at the same rate throughout the motion.

You can obtain this from Eq. 1

Motion with constant acceleration

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- We will derive an expression for the position as a function of time
- The average velocity

$$v_{av} = \frac{x - x_0}{t},$$

We also have for average velocity, $v_{av} = \dfrac{v + v_0}{2}$

Also $v = v_0 + at$, substituting $\dfrac{v_0 + at + v_0}{2} = \dfrac{x - x_0}{t}$

$$\Rightarrow x = x_0 + v_0 t + \frac{1}{2} a t^2 \qquad (2)$$ (Equation of motion)

Motion with constant acceleration

- Further

$$t = \frac{v - v_0}{a} \qquad \text{from Eq. 1}$$

$$x = x_0 + v_0\left(\frac{v - v_0}{a}\right) + \frac{1}{2}a\left(\frac{v - v_0}{a}\right)^2$$

$$\Rightarrow 2a\left(x - x_0\right) = v^2 - v_0^2$$

$$\Rightarrow v^2 = v_0^2 + 2a\left(x - x_0\right) \qquad (3)$$

(Equation of motion)

Equations of Motion with constant acceleration

- The four equations below apply to any straight-line motion with constant acceleration *a* *(all in the x direction).*

$$v_2 = v_1 + at \qquad (1)$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \qquad (2)$$

$$v^2 = v_0^2 + 2a\left(x - x_0\right) \qquad (3)$$

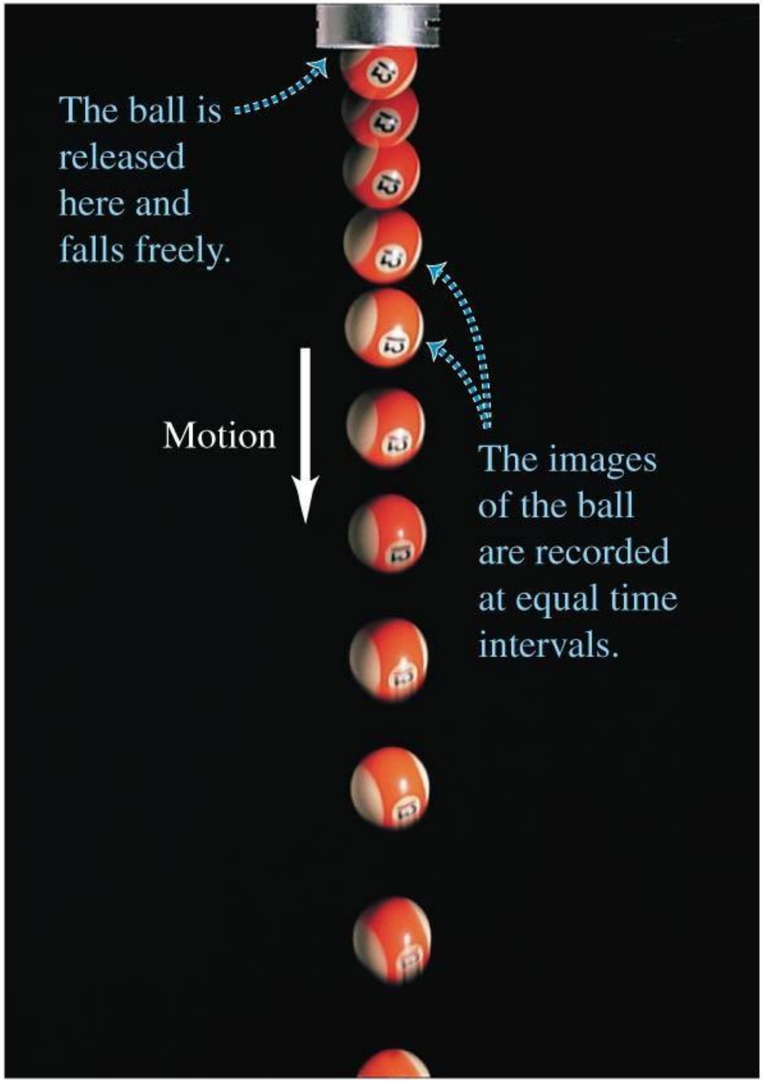
$$v_{av} = \frac{v + v_0}{2} \qquad (4)$$

The 1st EOM is referred to as the velocity-time relation.

The 2nd EOM is referred to as the position-time relation.

The 3rd EOM is referred to as the position – velocity relation.

Freely falling bodies



- ***Free fall*** is the motion of an object under the ***influence of only gravity***.
- In the figure, a strobe light flashes with equal time intervals between flashes.
- The velocity change is the same in each time interval, so the acceleration is constant.

A freely falling coin

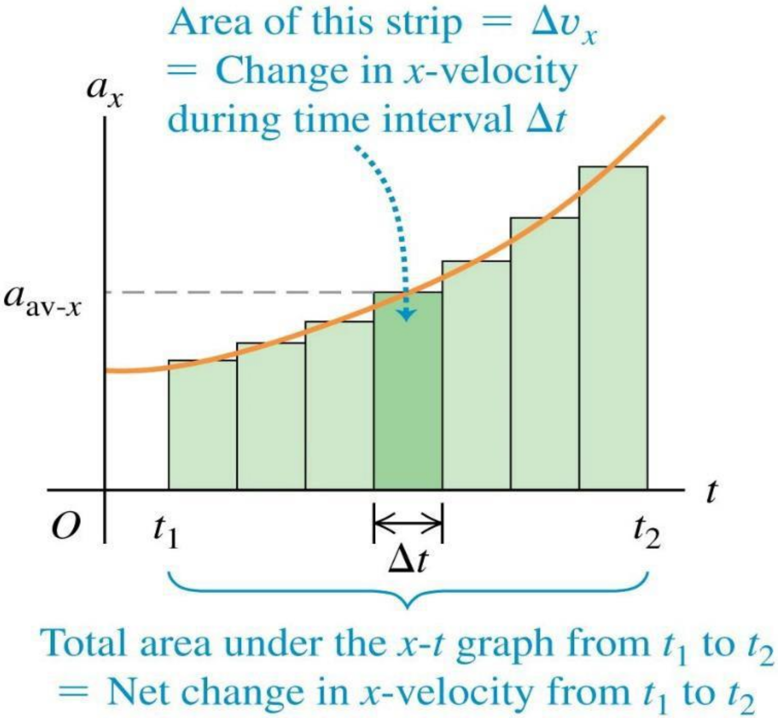
- If there is no air resistance, the downward acceleration of any freely falling object is $g = 9.81\ m/s^2$ $(32\ ft/s^2)$

Velocity and position by integration

- Given the acceleration, we can determine velocity and position at any time by integrating over many small time intervals.

$$v_x = v_{0x} + \int_0^t a_x dt$$

$$x = x_0 + \int_0^t v_x dt.$$



Summary

Average x-velocity of a particle in **straight-line motion** during time interval from t_1 to t_2

x-component of the particle's displacement

$$v_{\text{av-}x} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

Final x-coordinate minus initial x-coordinate

Time interval

Final time minus initial time

The **instantaneous x-velocity** of a particle in **straight-line motion** ...

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \quad (2.3$$

... equals the limit of the particle's average x-velocity as the time interval approaches zero ...

... and equals the instantaneous rate of change of the particle's x-coordinate.

Average x-acceleration of a particle in **straight-line motion** during time interval from t_1 to t_2

Change in x-component of the particle's velocity

$$a_{\text{av-}x} = \frac{\Delta v_x}{\Delta t} = \frac{v_{2x} - v_{1x}}{t_2 - t_1}$$

Final x-velocity minus initial x-velocity

Time interval

Final time minus initial time

The **instantaneous x-acceleration** of a particle in **straight-line motion** ...

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \quad (2.5$$

... equals the limit of the particle's average x-acceleration as the time interval approaches zero ...

... and equals the instantaneous rate of change of the particle's x-velocity.

Velocity and Acceleration

Summary

$$v_2 = v_1 + at \qquad (1)$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \qquad (2)$$

$$v^2 = v_0^2 + 2a\left(x - x_0\right) \qquad (3)$$

$$v_{av} = \frac{\mathrm{v} + v_o}{2} \qquad (4)$$

Straight line motion with <u>constant</u> acceleration

$$a_y = -g = -9.81 \text{ m/s}^2$$

Freely falling body

End