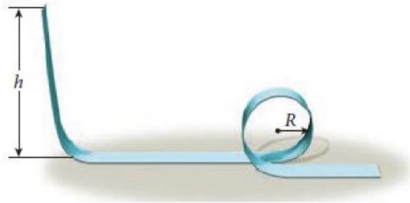
TUTORIAL – CIRCULAR MOTION

WARM UP

1. A point mass *m* starts sliding from a height *h* along the frictionless surface shown in the figure. What is the minimum value of *h* for the mass to complete the loop of radius *R*?



[Ans: 2.5R]

- At the top of the loop, the net force is equal to the centripetal force required to keep the coaster on the track.

$$\vec{F}_c = \vec{F}_{net} = \vec{F}_g + \vec{N}$$

- For weightlessness, the normal force is zero:

$$\vec{F}_c = \vec{F}_{net} \Rightarrow F_c = F_g$$

- The force of gravity is:

$$F_g = mg$$

- The magnitude of the centripetal force is:

$$F_c = ma_c = m\frac{v^2}{r}$$

$$F_c = F_g \Rightarrow m\frac{v_{top}^2}{r} = mg \Rightarrow v_{top} = \sqrt{rg}$$

$$\cancel{m}\cancel{g}h = \cancel{m}\cancel{g}(2R) + \frac{1}{2}\cancel{m}(\cancel{g}R)$$

$$h = 2R + \frac{1}{2}R$$

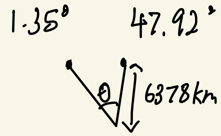
$$= 2.5R$$

Angular Coordinates and Angular Displacement

2. Assuming a perfectly spherical Earth (R_E = 6378 km) how far apart, measured on the Earth's surface, are Singapore (1.35° N latitude), and Ulaanbaatar (47.92° N latitude)? The two cities lie on approximately the same longitude.

[Ans: 5181 km]

$s = r\theta$

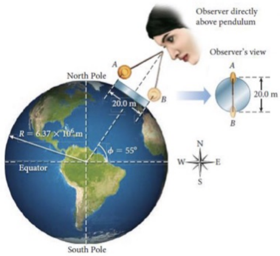


$$\theta = 47.92° - 1.35°$$
$$= \frac{46.57°}{180} \cdot \pi$$

$$s = 6378\left(\frac{46.57}{180} \cdot \pi\right)$$
$$\approx 5184 \text{ km}$$

Angular Velocity, Angular Frequency, and Period

3. Consider a large simple pendulum that is in Glasgow (latitude of 55.0° N) and is swinging in a North-South direction with points A and B being the northernmost and the southernmost points of the swing, respectively. A stationary (with respect to the fixed stars) observer is looking directly down on the pendulum at the moment shown in the figure.



As the Earth (R_E = 6378 km) is rotating once every 23 h and 56 min determine:

a) What are the directions (in terms of N, E, W, and S) and the magnitudes of the velocities of the surface of the Earth at points A and B as seen by the observer? Note: You will need to calculate answers to at least seven significant figures to see a difference.

[Ans: 0.00119 m/s, eastwards]

b) What is the angular speed with which the 20.0-m diameter circle under the pendulum appears to rotate?

[Ans: 5.95×10^{-5} rad/s]

c) What is the period of this rotation?

[Ans: 29.3 hrs]

d) What would happen to a pendulum swinging at the Equator?

a)

$$V_A = \frac{2\pi \times 6.378 \times 10^6}{86160} \cos 55°$$

$$=$$

1 rot = 23h 56min

$= (23 \times 60 + 56) \times 60$

$T = 86160$

$f = \frac{1}{86160}$

$w = 2\pi \left(\frac{1}{86160}\right)$

Angular and Centripetal Acceleration

4. A ring is fitted loosely (with no friction) around a long, smooth rod of length L = 0.50 m. The rod is fixed at one end, and the other end is spun in a horizontal circle at a constant angular velocity of ϖ = 4.0 rad/s. The ring has zero radial velocity at its initial position, a distance of r_0 = 0.30 m from the fixed end. Determine the radial velocity of the ring as it reaches the moving end of the rod.

[Ans: 1.60 m/s]

The ring is free to move and has no centripedal force to hold it in circular path

Radial acceleration a_r is a function of time

$$a_r(t) = \frac{dv_t}{dt} = \omega^2 r = \frac{dv_t}{dr} \cdot \frac{dr}{dt} \; ; \; v_t = \frac{dr}{dt}$$

$$a_r(t) = \frac{dv(t)}{dt} = \omega^2 r(t)$$

	initial	final
$v(t)$	$v_i = 0$	$v_f = ?$
$r(t)$	$r_i = r_0$	$v_f = L$

$$\frac{dv}{dr} \cdot \frac{dr}{dt} = \omega^2 r$$

$$v \frac{dv}{dr} = \omega^2 r$$

$$v\,dv = \omega^2 r\,dr$$

$$\int_0^{v_f} v\,dr = \omega^2 \int_{r_0}^{L} r\,dr$$

$$\frac{dv_t}{dr} v_t = \omega^2 r$$

$$\int_0^{v_f} v_t\,dv_t = \int_{r_0}^{L} \omega^2 r\,dr$$

$$\left[\frac{v_t^2}{2}\right]_0^{v_f} = \left[\omega^2\left(\frac{r^2}{2}\right)\right]_{r_0}^{L}$$

$$\frac{v_f^2}{2} = \omega^2\left(\frac{L^2}{2} - \frac{r_0^2}{2}\right)$$

$$v_f = \omega\sqrt{L^2 - r_0^2}$$

$$= 1.6 \text{ m/s}$$

Centripetal Force

5. A speedway turn, with radius of curvature R, is banked at an angle θ above the horizontal.
   a) What is the optimal speed at which to take the turn if the track's surface is iced over (that is, if there is no friction between the tires and the track)?

      [Ans: $\sqrt{gRtan\theta}$]

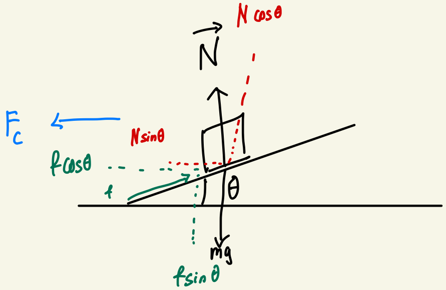
   b) If the track surface is ice-free and there is a coefficient of friction μ_s between the tires and the track, what are the maximum and minimum speeds at which this turn can be taken?

      [Ans: $v_{MAX} = \sqrt{gR\frac{sin\theta+\mu_s sin\theta}{cos\theta-\mu_s cos\theta}}$; $v_{min} = \sqrt{gR\frac{sin\theta-\mu_s sin\theta}{cos\theta+\mu_s cos\theta}}$]

   c) Evaluate the results of parts (a) and (b) for R = 400. m, θ = 45.0°, and μ_s = 0.700.

      [Ans: v = 62.6 m/s; v_{MAX} =149 m/s v_{min} = 26.3 m/s]

a)



$$N\sin\theta + f\cos\theta = F_{net}$$

$$N\cos\theta = mg + f\sin\theta$$

$$f = \mu_s N \;;$$

$$F_c = F_{net} = \frac{mv^2}{R}$$

$$N\sin\theta + \mu_s N\cos\theta = \frac{mv^2}{R}$$

$$N(\sin\theta + \mu_s\cos\theta) = \frac{mv^2}{R}$$

$$N\cos\theta = mg + \mu_s N\sin\theta$$

$$N\cos\theta - \mu_s N\sin\theta = mg$$

$$N(\cos\theta - \mu_s\sin\theta) = mg$$

$$\frac{\cancel{N}(\sin\theta + \mu_s\cos\theta)}{\cancel{N}(\cos\theta - \mu_s\sin\theta)} = \frac{\cancel{m}v^2/R}{\cancel{m}g}$$

$$V = \sqrt{Rg \cdot \frac{(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)}}$$

$$\mu_s = 0 \quad \therefore \quad V = \sqrt{Rg\tan\theta}$$

b)

$$V_{max} = \sqrt{Rg \cdot \frac{(\sin\theta + \mu_s \cos\theta)}{(\cos\theta - \mu_s \sin\theta)}}$$

$$V_{min} = \sqrt{Rg \cdot \frac{(\sin\theta - \mu_s \cos\theta)}{(\cos\theta + \mu_s \sin\theta)}}$$

(reversing direction of frictional force)

c)

$$V_a = \sqrt{400 \times 9.8 \times \tan 45^\circ}$$

$$\approx 62.6 \, m/s$$

$$V_{max} = \sqrt{400 \times 9.8 \cdot \frac{\sin 45^\circ + 0.7\cos 45^\circ}{\cos 45^\circ - 0.7\sin 45^\circ}}$$

$$= 149 \, m/s$$

$$V_{min} = \sqrt{400 \times 9.8 \cdot \frac{\sin 45^\circ - 0.7\cos 45^\circ}{\cos 45^\circ + 0.7\sin 45^\circ}}$$

$$= 26.3 \, m/s$$

Circular and Linear Motion

6. Consider a 53-cm-long engine blade rotating about its center at 3400 rpm.
   a) Calculate the linear speed of the tip of the blade.

   [Ans: 93.4 m/s]

   b) If safety regulations require that the blade be stoppable within 3.0 s, what minimum angular acceleration will accomplish this? Assume that the angular acceleration is constant.

   [Ans: −118.7 rad/s²]

a)

$$\omega = 3400\,\text{rpm} = 3400 \times \frac{2\pi\,\text{rad}}{60\,\text{s}}$$

$$= 356.04\,\text{rad/s}$$

$$r = \frac{53}{2}$$

$$v = r\omega$$

$$= \left(\frac{0.53}{2}\right)(356.04)$$

$$\approx 94.3\,\text{m/s}$$

b)

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{0 - 356.04}{3}$$

$$= -118.68\,\text{rad/s}^2$$

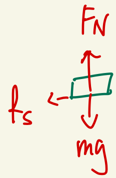
7. A 1 dollar coin is sitting on the edge of a record player that is spinning at 33 rpm and has a diameter of 12 inches. What is the minimum coefficient of static friction between the coin and the surface of the disk to ensure that the penny does not fly off?

[Ans: 0.185]

$$\omega = 33 \text{ rpm} = 33 \times \frac{2\pi}{60}$$

$$= 3.46 \text{ rad/s}$$

$$r = \frac{12}{2} \text{inch} = 15.24 \text{cm} = 0.1524 \text{m}$$



$$\Rightarrow \quad F_N = mg$$

$$f_s = \mu_s mg = m\omega^2 r$$

$$\mu_s = \frac{\cancel{m}\omega^2 r}{\cancel{m}g}$$

$$= \frac{3.46^2 \times 0.1524}{9.8}$$

$$\approx 0.186$$

8. A 80 kg pilot in an aircraft moving at a constant speed of 500 m/s pulls out of a vertical dive along an arc of a circle of radius 4 km. Find the centripetal acceleration and the centripetal force acting on the pilot and determine what is the pilot's apparent weight at the bottom of the dive?

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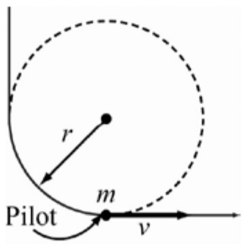
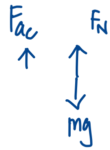




[Ans: 62.5 m/s²; 5000 N; 5785 N]

$$F_c = m\frac{v^2}{r}$$

$$= 80\left(\frac{500^2}{4\times10^3}\right)$$

$$= 5000\,N$$

$$F_c = m a_c$$

$$a_c = \frac{5000}{80}$$

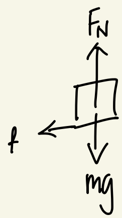
$$= 62.5\,m/s^2$$

Bottom of dive : $F_c + F_g = 5000 + (80 \cdot 9.8)$

$$\approx 5784\,N$$

9. A car starts from rest and accelerates around a flat curve of radius R = 36 m. The tangential component of the car's acceleration remains constant at a_t = 3.3 m/s², while the centripetal acceleration increases to keep the car on the curve as long as possible. The coefficient of friction between the tires and the road is μ = 0.95. What distance does the car travel around the curve before it starts to skid?

[Ans: 47.5 m]



at rest,

$$F_N = mg$$

$$f = \mu_s F_N = ma$$

$$a = \mu_s g = 0.95 \times 9.8$$
$$= 9.31$$

$$a = \sqrt{a_t^2 + a_c^2}$$

$$a_c = \sqrt{a^2 - a_t^2}$$
$$= 8.71$$

$$a_c = \frac{v^2}{r}$$

$$v^2 = 313.56$$

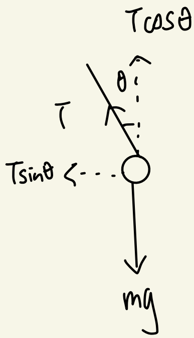
$$v^2 - \cancel{v_0^2}^{\,0} = 2a_t s$$

$$s = \frac{v^2}{2a_t}$$

$$= 47.5\text{m}$$

10. You are on a merry-go-round platform holding a pendulum in your hand. The pendulum is 6 m from the rotation axis of the platform. The rotational speed of the platform is 0.02 rev/s. Determine the angle θ at which the pendulum is hanging.

[Ans: 0.553 deg]



$$T\sin\theta = m\omega^2 r \qquad \omega = 0.02\,\text{rev/s}$$

$$T\cos\theta = mg \qquad = 0.02 \times 2\pi$$

$$= 0.04\pi$$

$$\frac{\cancel{T}\sin\theta}{\cancel{T}\cos\theta} = \frac{\cancel{m}\omega^2 r}{\cancel{m}g}$$

$$\tan\theta = \frac{\omega^2 r}{g}$$

$$\theta = \tan^{-1}\left(\frac{(0.04\pi)^2 \times 6}{9.8}\right)$$

$$\approx 0.554^\circ$$

11. A truck has tires with a diameter of 1.1 m and is traveling at 35.8 m/s. After the brakes are applied, the truck slows uniformly and is brought to rest after the tires rotate through 40.2 turns.
    a) What is the angular speed of the tires as the braking manoeuvre starts?
    [Ans: $65.1\ /s$]
    b) What is the angular acceleration of the tires during the braking manoeuvre?
    [Ans: $= -8.39\ /s^2$]
    c) What distance does the truck travel before coming to rest?
    [Ans: 138.9 m]

a) $\omega = \frac{v}{r} = \frac{35.8}{0.55}$

$\approx 65.1 m/s$

b) $\Delta\theta = 40.2 \times 2\pi$

$= 80.4\pi$

$\cancel{\omega^2}^{\,0} = \omega_0^2 + 2\alpha\Delta\theta$

$\alpha = -\frac{\omega_0^2}{2\Delta\theta}$

$= -\frac{65.1^2}{2(80.4\pi)}$

$\approx -8.39/s^2$

c) $s = r\theta$

$= 0.55 \times 80.4\pi$

$\approx 138.9m$

12. A ball of mass 1 kg is attached to a 1 m long string and is whirled in a vertical circle at a constant speed of 10.0 m/s.

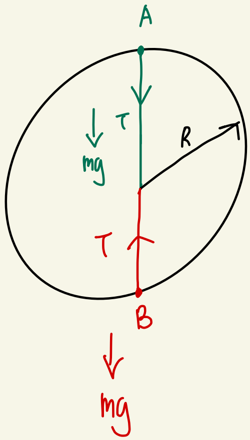
a) Determine the tension in the string when the ball is at the top of the circle.

[Ans: 90.2 N]

b) Determine the tension in the string when the ball is at the bottom of the circle.

[Ans: 109.8 N]

c) What can you say about the tension in the string at some point other than the top or bottom



a) $$T + mg = \frac{mv^2}{R}$$

$$T = m\left(\frac{v^2}{R} - g\right)$$

$$T_A = 1\left(\frac{10^2}{1} - 9.8\right)$$

$$= 90.2N$$

b) $$T - mg = \frac{mv^2}{R}$$

$$T = m\left(\frac{v^2}{R} + g\right)$$

$$= 109.8N$$