1.6 Continuous Charge Distributions

1.6.1. space charge density

- in technical problems: many (!) charges per volume

e.g. Conductors $10^{22} \frac{1}{cm^3}$

- carriers have a certain volume and are distributed in space:

in contrast to Concept of point Charges (fixed location, zero dimension)

→ Take this into account by applying "statistics"; transition to continuous charge distributions

"averaging over a high number of Charges"

Space charge density: $\rho(\vec{r})$ ("rho")

$$\boxed{\rho(\vec{r}) = \frac{\text{amount of Charge inside a volume } \Delta V}{\text{Volume } \Delta V}}$$

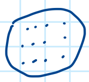
$\Delta V \to$ infinitesimal small

$\frac{C}{m^3}$ = unit of ρ

if $\Delta V \to dv$

total $Q(v)$ inside volume V: $Q(v) = \boxed{\int_V \rho(\vec{r})dv = \int_V \rho(\vec{r})d^3r}$

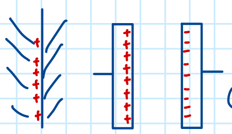
(1.28)

 → ρ = Constant  $\rho(\vec{r})$ = Constant $\rho(\vec{r}) = \rho_0(r)$

1.6.2. Surface Charge Density

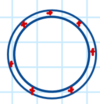
$\sigma(\vec{r})$ ("sigma")

Surface Charge densities occur at boundaries between materials



in a plate Capacitor (Charges are located at the surface of the Conducting electrodes)

in very thin surfaces
⇓
Consider it as 2D ⇒ Surface load



Always if Charged layer is very thin and Control dimensions are large

Surface charge density:

$$\sigma(\vec{r}) = \frac{\text{Charge Contained on area } \Delta A}{\Delta A} = \text{Charge per area}$$

If $\Delta A \to dA$

total Charge Contained in surface A

$$\boxed{Q(A) = \int_A \sigma(\vec{r}) \cdot da}$$

1.6.3. Gauss's law for continuous charge distributions (integral form)

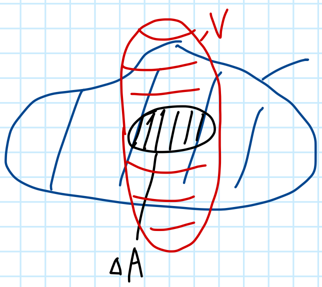
(i) charges distributed inside a certain volume (space charge density)

$$\boxed{\int_{\partial V} \vec{D}\cdot d\vec{a} = Q(V) = \int_V \rho(\vec{r})\cdot dV = \int_V \rho(\vec{r})\, d^3r} \qquad (1.32)$$

V = control volume
∂V = enclosing surface of V

$\Rightarrow$ by applying (1.32), we are able to calculate $\vec{D}\,(\vec{E})$ for a given $\rho(\vec{r})$

(ii) Charges distributed on a surface of a structure (surface charge density)



ΔA is the intersect of A with V: $V\cap S$

surface S, where charges are distributed in
$\Rightarrow$ gives rise to a surface charge density $\sigma(\vec{r})$

$$\int_{\partial V} \vec{D}\cdot d\vec{a} = \underbrace{Q(\Delta A)}_{Q(V)} = Q(V\cap S) = \int_{\Delta A} \sigma(\vec{r})\cdot da$$

$= V\cap S$

$[\sigma] = \frac{C}{m^2}$

(iii) specific case of surface charge density: electric conducting material (conductor)

$\hookrightarrow$ 1.7 $\Rightarrow$ electric conductors