Proposition Logic

$A(a,b) \Longleftrightarrow a \leftrightarrow b$

i	a	b	A
0	0	0	0
1	0	1	1
2	1	0	1
3	1	1	0

- Value Pattern

$\underline{\hat{W}}[A] = (0,1,1,0)$

- Column A of the Table
- All assignments must be noted as ascending binary numbers
- Must Use Round Brackets

Canonical disjunctive normal form (CDNF) and on set

- These represent the t(true)/1 assignments of A
- In the CDNF's conjunctive terms, variables assigned to f/0 will be negated, variables assigned to t/1 will be noted in positive form

$CDNF[A] \Longleftrightarrow (\neg a \wedge b) \vee (a \wedge \neg b)$

$E[A] \Longleftrightarrow \{(0,1),(1,0)\}$

Canonical Conjunctive Normal form (CCNF) and off-set

- These represent the f(false)/0 - assignments of A
- In the CCNF's disjunctive terms, variables assigned t/1 will be negated, variables assigned to f/0 will be noted in positive form

$CCNF[A] \Longleftrightarrow (a \vee b) \wedge (\neg a \vee \neg b)$

Resolution method to show $CNF[A] \Leftrightarrow f$

RRC : $(\neg x \vee a) \wedge (x \vee b) \Leftrightarrow (\neg x \vee a) \wedge (x \vee b) \wedge (a \vee b)$

$(\neg x \vee a) \wedge (x \vee b) \Rightarrow a \vee b$ (E4)

Deduction Scheme for resolution (Layer algorithm)

CNF[A]	Layer
$\dots \wedge (\neg x \vee a) \wedge (x \vee b) \wedge \dots$	0
$\dots \wedge (a \vee b) \wedge \dots$	1
$\vdots$	
$\wedge f$	n

Generation of Layer k+1

- Compare every term in Layer k with every other term in Layer k
- Compare every term in Layer k with every other term in previous layers
- Absorption to reduce the number of terms
- r := k + 1

$[a \rightarrow (b \rightarrow \neg c)] \wedge a \wedge c \wedge b \Leftrightarrow f$

$\underbrace{(\neg a \vee \neg b \vee \neg c) \wedge a \wedge c \wedge b}_{CNF} \Leftrightarrow f$

$\cancel{(\neg a \vee \neg b \vee \neg c)} \wedge a \wedge c \wedge b$	0
$\wedge \cancel{(\neg b \vee \neg c)} \wedge \cancel{(\neg a \vee \neg b)} \wedge \cancel{(\neg a \vee \neg c)}$	1
$\wedge \neg b \wedge \neg c \wedge \neg a$	2
$\wedge f$	3

Deduction Proof scheme

Task : show $V \Rightarrow S$ with $V \Leftrightarrow V_1 \wedge V_2 \wedge V_3$

Basis : $V \Rightarrow S$ IFF $V \Leftrightarrow V \wedge S$

Approach : Derive further premises V_i from $V_1, V_2, \ldots, V_3$ until conclusion $V_i \Leftrightarrow S$ is obtained

Result : $V_1 \wedge V_2 \Leftrightarrow V_1 \wedge V_2 \wedge V_3$

$$\Leftrightarrow V_1 \wedge V_2 \wedge V_3 \wedge V_4$$

$$\vdots$$

$$\Leftrightarrow V_1 \wedge V_2 \wedge V_3 \wedge \ldots \wedge S$$

$$\Rightarrow S$$

Resolution Rules

	disjunctive form (RRD)	conjunctive form (RRC)
general case	$(x\land \underline{a})\lor(\neg x\land \underline{b}) \iff (x\land a)\lor(\neg x\land b)\lor(a\land b)$	$(x\lor a)\land(\neg x\lor b) \iff (x\lor a)\land(\neg x\lor b)\land(a\lor b)$
special case 1 $a\iff b$	$(x\land \underline{a})\lor(\neg x\land \underline{a})\iff a$	$(x\lor a)\land(\neg x\lor a)\iff a$
special case 2 a is neutral element	$a\iff t$ $x\lor(\neg x\land b)\iff x\lor b$	$a\iff f$ $x\land(\neg x\lor b)\iff x\land b$

Resolution Method

Objective	Disjunctive form	Conjunctive form	Stop Criteria
Proof of Tautology	✓		resolvent: ... $\lor t$
Proof of Contradiction		✓	resolvent: .... $\lor f$
Simplification / Proof of Contingency	✓	✓	no further resolvent are possible

Ring normal Form RNF

(Canonical Ring Normal Form) (Reed Muller expansion)

Base : $(\{t, f\}, \wedge, \leftrightarrow\!\!\!\!\!/\ )$

Form : $RNF[A] \Leftrightarrow M_1 \not\leftrightarrow M_2 \not\leftrightarrow \ldots$

with $M_1 \Leftrightarrow a \wedge b \wedge \ldots$ (Conjunctive term)

Contains only $\wedge$ and $\not\leftrightarrow$

no negation

Calculation by recursive expansion using Law ER4

$$A(\underline{x}) \Leftrightarrow [x_i \wedge (A(x_i \Leftrightarrow t) \not\leftrightarrow A(x_i \Leftrightarrow f))] \not\leftrightarrow A(x_i \Leftrightarrow f)$$

ER4 : $A(\underline{x}) \Leftrightarrow [x_i \wedge (A(x_i \Leftrightarrow t) \not\leftrightarrow A(x_i \Leftrightarrow f))] \not\leftrightarrow A(x_i \Leftrightarrow f)$

$$A(a, b, c, d) \Leftrightarrow a \vee (b \wedge c) \vee \neg d$$

$$A \Leftrightarrow [d \wedge (A(d \Leftrightarrow t) \not\leftrightarrow A(d \Leftrightarrow f))] \not\leftrightarrow A(d \Leftrightarrow f)$$

$$\Leftrightarrow [d \wedge (a \vee (b \vee c)) \not\leftrightarrow t] \not\leftrightarrow t$$

$$[a \vee (b \wedge c)] \overset{B13}{\Leftrightarrow} a \wedge b \wedge c \not\leftrightarrow b \wedge c \not\leftrightarrow a$$

$$RNF[A] \Leftrightarrow \underbrace{a \wedge b \wedge c \wedge d}_{\text{monomes}} \not\leftrightarrow b \wedge c \wedge d \not\leftrightarrow a \wedge d \not\leftrightarrow d \not\leftrightarrow t$$

(product terms containing only possible variables)

- Always canonical
- No further minimization possible

Decision - Trees and Diagrams

Binary **D**ecision

- Nodes represent variables
- each node has exactly two output edges (t- and f- assignment of variable)
  * except terminal node

BDT (SPR3)
- T: Tree each node (except root node) has exactly one input edge

BDD
- D: Diagram equivalent sub-BDTs may be merged into one sub-BDT
  ↳ the first node of the new sub-BDT has multiple input edges

Ordered : **O**BDT / **O**BDD

- Full substitution of variables (SPR1)

- In each path from root to terminal node, each variable appears at most once

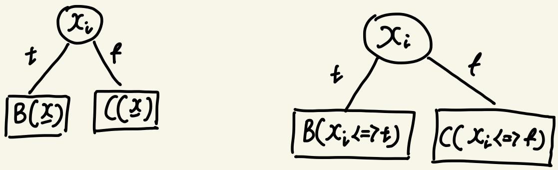
- the variable order is the same in all paths

Reduced : ROBDT / ROBDD

- Complete resolution of variables with SPR 2
  - ↳ no node has two equivalent sub-BDTs

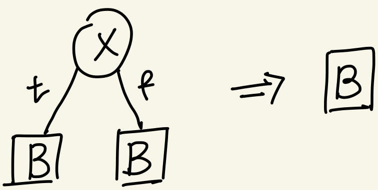
Simplification rule 1

$$B(x_i, B(\underline{x}), C(\underline{x})) \Leftrightarrow B(x_i, B(x_i \Leftrightarrow t), C(x_i \Leftrightarrow f))$$



Simplification rule 2

$$B(X, B, B) \Leftrightarrow B$$



How to read CDNF and CCNF from BDT

CDNF : t-assignments of A , paths direct from BDT

CCNF : f-assignments of A
variables assigned to f will be positives
variables assigned to t will be negated

1.1 You take a programming hands-on training. In 10 minutes is the deadline of the homework. The wheather was good during the last weeks. Thus you are doing the homework at the last moment. There are only few lines of code missing. But now the 'a' of your keyboard is broken. The next line is supposed to be '*If x **and** not(y) then ...*'. What will you do?

X and not (y)

$x \land \neg y$ Formalise – transform to boonlean language

$\neg\neg(x \land \neg y)$ Law A9 Double Negation

$\neg(\neg x \lor y)$ Law A10 DeMorgan → to ensure both lines are equal

not (not (x) or y) De - Formalise - transform back to human language

1.2 Determine the truth tables for the following propositional forms:

1.2.a $A \iff a \land (b \lor c)$

a b c	$(b \lor c)$	$a \land (b \lor c)$
0 0 0	0	0
0 0 1	1	0
0 1 0	1	0
0 1 1	1	0
1 0 0	0	0
1 0 1	1	1
1 1 0	1	1
1 1 1	1	1

1.2.b $B \iff (a \wedge b) \vee (a \wedge c)$

a	b	c	$(a \wedge b)$	$(a \wedge c)$	$A \vee B$
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	1	1
1	1	0	1	0	1
1	1	1	1	1	1

1.2.c $C \iff ((a \vee b) \wedge (b \vee c)) \wedge (c \vee a)$

a	b	c	$(a \vee b)$	$(b \vee c)$	$(A \wedge B)$	$(c \vee a)$	$(A \wedge B) \wedge C$
0	0	0	0	0	0	0	0
0	0	1	0	1	0	1	0
0	1	0	1	1	1	0	0
0	1	1	1	1	1	1	1
1	0	0	1	0	0	1	0
1	0	1	1	1	1	1	1
1	1	0	1	1	1	1	1
1	1	1	1	1	1	1	1

1.2.d $D \iff ((a \wedge b) \vee (b \wedge c)) \vee (c \wedge a)$

a	b	c	$(a \wedge b)$	$(b \wedge c)$	$A \vee B$	$(c \wedge a)$	$(A \vee B) \vee (c \wedge a)$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	1	0	0	0	0	0	0
0	1	1	0	1	1	0	1
1	0	0	0	0	0	0	0
1	0	1	0	0	0	1	1
1	1	0	1	0	1	0	1
1	1	1	1	1	1	1	1

1.2.e $E \iff \neg a \longrightarrow (b \vee c)$

a	b	c	¬a	(b∨c)	¬a → (b∨c)
0	0	0	1	0	0
0	0	1	1	1	1
0	1	0	1	1	1
0	1	1	1	1	1
1	0	0	0	0	1
1	0	1	0	1	1
1	1	0	0	1	1
1	1	1	0	1	1

1.2.h $H \iff a \longleftrightarrow (b \longleftrightarrow c)$

a	b	c	(b ↔ c)	a ↔ B
0	0	0	1	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	1	1
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

1.2.f $F \iff \neg a \longleftrightarrow (b \vee c)$

¬a ↔ (b∨c)
0
1
1
1
1
0
0
0

1.2.g $G \iff (a \longleftrightarrow b) \longleftrightarrow c$

a	b	c	(a ↔ b)	A ↔ c
0	0	0	1	0
0	0	1	1	1
0	1	0	0	1
0	1	1	0	0
1	0	0	0	1
1	0	1	0	0
1	1	0	1	0
1	1	1	1	1

1.3 Consider the following three propositional forms:

I : $(\neg a \wedge a) \vee (a \not\leftrightarrow (b \wedge c))$
II : $[(a \longleftrightarrow b) \longrightarrow b] \wedge (a \not\leftrightarrow a)$
III : $\neg[(c \vee \neg a) \wedge (a \vee \neg b \vee \neg c)]$

1.3.a Are these propositional forms equivalent? Determine their truth tables to check this.

1.3.b Determine the on-sets of propositional forms I to III.

I:

a	b	c	$\neg a$	$\overset{A}{(\neg a \wedge a)}$	$(b \wedge c)$	$\overset{B}{(a \not\leftrightarrow (b \wedge c))}$	$A \vee B$
0	0	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	1	1	1
1	0	0	0	0	0	1	1
1	0	1	0	0	0	1	1
1	1	0	0	0	0	1	1
1	1	1	0	0	1	0	0

II:

$\overset{A}{(a \longleftrightarrow b)}$	$(A \to b)$	$(a \not\leftrightarrow a)$	$(A \to b) \wedge (a \not\leftrightarrow a)$
1	0	0	0
1	0	0	0
0	1	0	0
0	1	0	0
0	0	0	0
0	0	0	0
1	1	0	0
1	1	0	0

III : $\neg[(c \vee \neg a) \wedge (a \vee \neg b \vee \neg c)]$

$(c \vee \neg a)$	$\neg b$	$\neg c$	$\overset{B}{(a \vee \neg b)}$	$(B \vee \neg c)$	$\neg[(c \vee \neg a) \wedge (a \vee \neg b \vee \neg c)]$
1	1	1	1	1	0
1	1	0	1	1	0
1	0	1	0	1	0
1	0	0	0	0	1
0	1	1	1	1	1
1	1	0	1	1	0
0	0	1	1	1	1
1	0	0	1	1	0

On set is true value

1.3.b Determine the on-sets of propositional forms I to III.

$E[I] = \{(0,1,1), (1,0,0), (1,0,1), (1,1,0)\}$

$E[II] = \emptyset$

$E[III] = \{(0,1,1), (1,0,0), (1,1,0)\}$

a b c d

1.4 Mr. Miller is going to have a party and wants to invite Anne, Betty, Charlotte or Dana (no "exclusive OR"!). However, some difficulties occur:

- Anne and Dana can't stand each other, so in no case both of them will appear at the party.
- Charlotte will only come to the party if Dana joins her.
- if Betty comes to the party she will in any case bring along Anne.

What possibilities does Mr. Miller have to avoid every possible conflict?
Choose appropriate logical variables and formalise the given preconditions.

Solve this problem once by setting up the truth table, and once by calculating the CDNF of the resulting propositional form.

$a \Leftrightarrow$ Miller invites Anne

~~$a \Leftrightarrow$ Anne~~ (wrong, name is not a statement)

$b/c/d \Leftrightarrow$ Miller invites Betty / Charlotte / Dana

$V_1(a,d) \Leftrightarrow \neg(a \wedge d)$

$V_2(c,d) \Leftrightarrow c \rightarrow d$

(if charlotte at party, dana will also be at party)

$V_3(b,a) \Leftrightarrow b \rightarrow a$

$V(a,b,c,d) \Leftrightarrow V_1 \wedge V_2 \wedge V_3$

(conditions must be fulfilled, therefore $\wedge$ operator is used)

$X_1 \Leftrightarrow \neg(a \wedge d)$

$X_2 \Leftrightarrow c \rightarrow d$

$X_3 \Leftrightarrow b \rightarrow a$

$X \Leftrightarrow X_1 \wedge X_2 \wedge X_3$

a	b	c	d	$\neg(a \wedge d)$	$c \rightarrow d$	$b \rightarrow a$	X
0	0	0	0	1	1	1	1
0	0	0	1	1	1	1	1
0	0	1	0	1	0	1	0
0	0	1	1	1	1	1	1
0	1	0	0	1	1	0	0
0	1	0	1	1	1	0	0
0	1	1	0	1	0	0	0
0	1	1	1	1	1	0	0
1	0	0	0	1	1	1	1
1	0	0	1	0	1	1	0
1	0	1	0	1	0	1	0
1	0	1	1	0	1	1	0
1	1	0	0	1	1	1	1
1	1	0	1	0	1	1	0
1	1	1	0	1	0	1	0
1	1	1	1	0	1	1	0

on-set $E[X] = \{(0,0,0,0), (0,0,0,1), (0,0,1,1), (1,0,0,0),$
$(1,1,0,0)\}$

Miller can invite :

- nobody
- Dana
- Charlotte, Dana
- Anne
- Anne, Betty

Calculating the Canonical forms

1) Resolve $\leftrightarrow$, $\not\leftrightarrow$, $\rightarrow$ using Laws B9/B10, C9/C10, D1
2) Resolve negated conjunctive terms and disjunctive terms Law A10
3) Apply distributive Laws to get CNF/DNF Law A3
4) Add missing Literals RRD, RRC
5) Eliminate multiple maxterms/minterms Law A4

Goal : CDNF

$$V(a,b,c,d) \Leftrightarrow \neg(a \wedge d) \wedge (c \rightarrow d) \wedge (b \rightarrow a)$$

$$\Leftrightarrow \neg(a \wedge d) \wedge (\neg c \vee d) \wedge (\neg b \vee a) \quad D1$$

$$\Leftrightarrow [(\neg a \vee \neg d) \wedge (\neg c \vee d)] \wedge (\neg b \vee a) \quad A10$$

Apply distributive Law on first 2 terms

A3

$$\Leftrightarrow [(\neg a \wedge \neg c) \vee (\neg a \wedge d) \vee (\neg d \wedge \neg c) \vee \overbrace{(\cancel{\neg d} \wedge d)}^{f}] \wedge (\neg b \vee a)$$

A3

$$\Leftrightarrow (\neg a \wedge \neg b \wedge \neg c) \vee (\neg a \wedge \neg b \wedge d) \vee (\neg b \wedge \neg c \wedge \neg d) \vee$$
$$\cancel{(a \wedge \neg a \wedge \neg c)} \vee (\overbrace{\cancel{a \wedge \neg a}}^{f} \wedge d) \vee (a \wedge \neg c \wedge \neg d) \Leftrightarrow \text{DNF}$$

RRD

$$\Leftrightarrow (\neg a \wedge \neg b \wedge \neg c \wedge d) \vee (\neg a \wedge \neg b \wedge \neg c \wedge \neg d)$$
$$\vee (\neg a \wedge \neg b \wedge c \wedge d) \vee \cancel{(\neg a \wedge \neg b \wedge \neg c \wedge d)}$$
$$\vee (a \wedge \neg b \wedge \neg c \wedge \neg d) \vee \cancel{(\neg a \wedge \neg b \wedge \neg c \wedge \neg d)}$$
$$\vee (a \wedge b \wedge \neg c \wedge \neg d) \vee \cancel{(a \wedge \neg b \wedge \neg c \wedge \neg d)}$$

Cancel like terms

$$\Leftrightarrow (\neg a \wedge \neg b \wedge \neg c \wedge d) \vee (\neg a \wedge \neg b \wedge \neg c \wedge \neg d)$$
$$\vee (\neg a \wedge \neg b \wedge c \wedge d) \vee (a \wedge \neg b \wedge \neg c \wedge \neg d)$$
$$\vee (a \wedge b \wedge \neg c \wedge \neg d)$$

1.6 There has been a murder. Five suspects A, B, C, D and E are being taken into custody. From the circumstances of the crime, the following premises V_1 and V_2 arise:

V_1: only these five persons are to be considered possible murderers.
V_2: the murder was committed by one, two or three persons.
(i.e. at least two suspects are not guilty)

After the first interrogation, the following assertions were made:

B_1: A says "B and C did both not do it."
B_2: B says "There are exactly three committers."
B_3: C says "D is a murderer and if A was involved, then so was E."
B_4: D says "A and B are murderers".
B_5: E says "Either A is a murderer or D, B, and C are murderers".

Use the logic variables a, b, c, d and e according to $a \iff$ "A is a murderer".

Assume that a murderer may lie, whereas an innocent will always tell the truth. According to this assumption, the assertion "A says ..." can be formalized as "$\neg a \to \ldots$".

1.6.a Formalize the premises V_1 and V_2 and the assertions B_1 till B_5 of the first interrogation.

B4: $\neg D \to A \wedge B$

B5: $\neg E \to A \longleftrightarrow (B \wedge C \wedge D)$

B3: $\neg C \to D \wedge (A \to E)$

B1: $\neg A \to \neg B \wedge \neg C$

B2: $\neg B \to (A \wedge B \wedge C \wedge \neg D \wedge \neg E) \vee (A \wedge B \wedge \neg C \wedge D \wedge \neg E)$

$\neg B \to \neg[A \vee B \vee C \vee D \vee E \vee (A \wedge B) \vee (A \wedge C) \vee \ldots\ldots)$

$V_1 \iff A \vee B \vee C \vee D \vee E$

$V_2 \iff$

1.8 Calculate the $CDNF$ as well as the $CCNF$ of the following ternary propositional forms $A_i(a,b,c)$:

1.8.a $A_1 \iff [a \longleftrightarrow (b \longrightarrow c)] \vee \neg(a \longrightarrow \neg b)$

CDNF

D1, B9

$\iff (a \wedge \neg(b \to c)) \vee (\neg a \wedge (b \to c)) \vee \neg(\neg a \vee \neg b)$

B9

$\iff (a \wedge \neg(\neg b \vee c)) \vee (\neg a \wedge (\neg b \vee c)) \vee \neg(\neg a \vee \neg b)$

A9

$\iff (a \wedge b \wedge \neg c) \vee (\neg a \wedge (\neg b \vee c)) \vee (a \wedge b)$

A3

$\iff (a \wedge b \wedge \neg c) \vee (\neg a \wedge \neg b) \vee (\neg a \wedge c) \vee (a \wedge b)$

RRD

$\iff (a \wedge b \wedge \neg c) \vee (\neg a \wedge \neg b \wedge c) \vee (\neg a \wedge \neg b \wedge \neg c) \vee (\neg a \wedge b \wedge c)$

$\vee \cancel{(\neg a \wedge \neg b \wedge c)} \vee (a \wedge b \wedge c) \vee \cancel{(a \wedge b \wedge \neg c)}$

$\iff (a \wedge b \wedge \neg c) \vee (\neg a \wedge \neg b \wedge c) \vee (\neg a \wedge \neg b \wedge \neg c)$

$\vee (\neg a \wedge b \wedge c) \vee (a \wedge b \wedge c)$

1.8.a $A_1 \iff [a \longleftrightarrow (b \longrightarrow c)] \vee \neg(a \longrightarrow \neg b)$

CCNF

B10, D1

$\iff [(\neg a \vee \neg(b \to c)) \wedge (a \vee (b \to c))] \vee \neg(\neg a \vee \neg b)$

D1

$\iff [(\neg a \vee \neg(\neg b \vee c)) \wedge (a \vee (\neg b \vee c))] \vee \neg(\neg a \vee \neg b)$

A10

$\iff [(\neg a \vee (b \vee \neg c)) \wedge (a \vee (\neg b \vee c))] \vee (a \wedge b)$

A3

$\iff [(\neg a \vee b) \wedge (\neg a \vee \neg c) \wedge (a \vee \neg b \vee c)] \vee (a \wedge b)$

A3

$\iff \cancel{(\neg a \vee a \vee b)} \wedge \cancel{(\neg a \vee a \vee \neg c)} \wedge (\cancel{a} \vee a \vee \neg b \vee c)$

$\wedge (\neg a \vee b \vee \cancel{b}) \wedge (\neg a \vee b \vee \neg c) \wedge \cancel{(a \vee b \vee \neg b \vee c)}$

RRC

$\iff (a \vee \neg b \vee c) \wedge (\neg a \vee b \vee c) \wedge (\neg a \vee b \vee \neg c)$

$\wedge \cancel{(\neg a \vee b \vee \neg c)}$

$\iff (a \vee \neg b \vee c) \wedge (\neg a \vee b \vee c) \wedge (\neg a \vee b \vee \neg c)$

CDNF → CCNF

$CDNF[A] \to E[A] \to \overline{E}[A] \to CCNF[A]$

$CDNF[A]: \quad A_1 \Leftrightarrow (a \wedge b \wedge \neg c) \vee (\neg a \wedge \neg b \wedge \neg c) \vee (\neg a \wedge \neg b \wedge c) \vee (\neg a \wedge b \wedge c) \vee (a \wedge b \wedge c)$

$E[A] = \{ \overset{6}{(1,1,0)}, \overset{0}{(0,0,0)}, \overset{1}{(0,0,1)}, \overset{3}{(0,1,1)}, \overset{7}{(1,1,1)} \}$

$\overline{E}[A] = \{ \overset{2}{(0,1,0)}, \overset{4}{(1,0,0)}, \overset{5}{(1,0,1)} \}$

$CCNF[A]: \quad A_1 \Leftrightarrow \overset{2}{(a \vee \neg b \vee c)} \wedge \overset{4}{(\neg a \vee b \vee c)} \wedge \overset{5}{(\neg a \vee b \vee \neg c)}$

1.8.c $A_3 \iff [a \longrightarrow (b \longleftrightarrow c)] \wedge (\neg a \longrightarrow b)$

Work from outers layars to inner

CCNF

$$\overset{D1}{\iff} [\neg a \vee (b \longleftrightarrow c)] \wedge (a \vee b)$$

$$\overset{C10}{\iff} [\neg a \vee ((b \vee \neg c) \wedge (\neg b \vee c))] \wedge (a \vee b)$$

$$\overset{A3}{\iff} (\neg a \vee b \vee \neg c) \wedge (\neg a \vee \neg b \vee c) \wedge (a \vee b) \iff CNF$$

$$\overset{RRC}{\iff} (\neg a \vee b \vee \neg c) \wedge (\neg a \vee \neg b \vee c) \wedge (a \vee b \vee c) \wedge (a \vee b \vee \neg c)$$

CDNF

$$\bar{E}[A_3] = \{\overset{5}{(1,0,1)}, \overset{6}{(1,1,0)}, \overset{0}{(0,0,0)}, \overset{1}{(0,0,1)}\}$$

$$E[A_3] = \{\overset{2}{(0,1,0)}, \overset{3}{(0,1,1)}, \overset{4}{(1,0,0)}, \overset{7}{(1,1,1)}\}$$

$CDNF[A_3]$: $A_3 \iff (\neg a \wedge b \wedge \neg c) \vee (\neg a \wedge b \wedge c) \vee (a \wedge \neg b \wedge \neg c) \vee (a \wedge b \wedge c)$

1.11 The propositional variables a, b and c

a : Kevin loves Suzy.
b : Kevin dates Suzy.
c : Kevin goes dancing with Linda.

are used to formulate the following five assertions:

I : $(b \longrightarrow a) \wedge (b \longrightarrow \neg c) \iff \neg b \vee (a \wedge \neg c)$
II : $(b \longrightarrow a) \wedge (b \longrightarrow \neg c) \iff (a \longrightarrow c) \longrightarrow \neg b$
III : $b \wedge (b \longrightarrow a) \wedge (b \longrightarrow \neg c) \Longrightarrow a \wedge \neg c$
IV : $c \wedge (b \longrightarrow a) \wedge (b \longrightarrow \neg c) \Longrightarrow \neg b$
V : $c \wedge (b \longrightarrow a) \wedge (b \longrightarrow \neg c) \Longrightarrow \neg a$

1.11.a Prove the validity of assertions I and II using equivalence transformations.

1.11.b Verify the validity of assertions III and IV using the deductive proof scheme.

1.11.c Set up the on-set $E[c \wedge (b \longrightarrow a) \wedge (b \longrightarrow \neg c)]$

1.11.d How many non-trivial conclusions can be drawn from $c \wedge (b \longrightarrow a) \wedge (b \longrightarrow \neg c)$?

I : $(b \to a) \wedge (b \to \neg c) \iff \neg b \vee (a \wedge \neg c)$

$LHS \overset{D1}{\iff} (\neg b \vee a) \wedge (\neg b \vee \neg c)$

$\overset{A3}{\iff} \neg b \vee (a \wedge \neg c)$

$\overset{A1}{\iff} (a \wedge \neg c) \vee \neg b$

$\overset{D2}{\iff} \neg(a \to c) \vee \neg b$

$\overset{D1}{\iff} (a \to c) \to \neg b$

I: $(b \to a) \wedge (b \to \neg c) \iff \neg b \vee (a \wedge \neg c)$

idea $(\overset{D1}{\iff} b \to (a \wedge \neg c))$

$\overset{D7}{\iff} b \to (a \wedge \neg c)$

$\overset{D1}{\iff} \neg b \vee (a \wedge \neg c)$

II : $(b \to a) \wedge (b \to \neg c) \iff (a \to c) \to \neg b$ idea $(\iff b \to \neg(a \to c))$

$\overset{D7}{\iff} b \to (a \wedge \neg c)$

$\overset{D2}{\iff} b \to \neg(a \to c)$

$\overset{D3}{\iff} (a \to c) \to \neg b$

III : $b \wedge (b \longrightarrow a) \wedge (b \longrightarrow \neg c) \Longrightarrow a \wedge \neg c$
IV : $c \wedge (b \longrightarrow a) \wedge (b \longrightarrow \neg c) \Longrightarrow \neg b$

1) $b \wedge (b \to a)$

2) $b \to \neg c$ $\Rightarrow a \wedge \neg c$

3) $b \wedge (\neg b \vee a)$ 1) D1

4) $\underbrace{(b \wedge \neg b)}_{f} \vee (b \wedge a)$ 3) A3

5) $(b \wedge a)$ 4) A6

6) $\neg b \vee \neg c$ 2) D1

III : $b \wedge (b \to a) \wedge (b \to \neg c) \Rightarrow a \wedge \neg c$

1)	b	
2)	$\wedge\ b \to a$	
3)	$\wedge\ b \to \neg c$	$\Rightarrow a \wedge \neg c$
4)	$\wedge\ a$	1, 2, E9
5)	$\wedge\ \neg c$	1, 3, E9
6)	$a \wedge \neg c$	4, 5

IV : $c \wedge (b \longrightarrow a) \wedge (b \longrightarrow \neg c) \Longrightarrow \neg b$

1) c

2) $b \to a$

3) $b \to \neg c$ $\quad\quad \Rightarrow \neg b$

---

4) $c \to \neg b$ $\quad\quad$ 3, D3

5) $\neg b$ $\quad\quad$ 1, 4, E9

1.11.c Set up the on-set $E[c \wedge (b \longrightarrow a) \wedge (b \longrightarrow \neg c)]$

a b c	$(b \to a)$ $\neg b \vee a$	$(b \to \neg c)$ $\neg b \to \neg c$	$c \wedge (b \to a) \wedge (b \to \neg c)$	$\neg a$
0 0 0	1	1	0	1
0 0 1	1	1	1	1
0 1 0	0	1	0	1
0 1 1	0	0	0	1
1 0 0	1	1	0	0
1 0 1	1	1	1	0
1 1 0	1	1	0	0
1 1 1	1	0	0	0

$$E[c \wedge (b \to a) \wedge (b \to \neg c)] = \{(0,0,1),\ (1,0,1)\}$$

1.11.d How many non-trivial conclusions can be drawn from $c \wedge (b \longrightarrow a) \wedge (b \longrightarrow \neg c)$?

number of non-trivial conclusions : $2^m - 1$

m : number of false-assignments (0 on the value pattern)

$m = 6$

$2^6 - 1 = 63$

1.11.e Verify or falsify the validity of the assertion V using the on-sets $E[c \wedge (b \longrightarrow a) \wedge (b \longrightarrow \neg c)]$ and $E[\neg a]$.

$E[\neg a] = \{(0,0,0),\ (0,0,1),\ (0,1,0),\ (0,1,1)\}$ no (1,0,1)

$V: \quad c \wedge (b \to a) \wedge (b \to \neg c) \Rightarrow \neg a$ IFF

$E[c \wedge (b \to a) \wedge (b \to \neg c)] \subseteq E[\neg a]$

But $E[c \wedge (b \to a) \wedge (b \to \neg c)] \nsubseteq E[\neg a]$

implication V is not valid

1.26 Given is the propositional form $A(a, b, c) \iff a \longrightarrow (b \vee c)$.

1.26.a Give the dual form $\text{dual}(A(a, b, c))$ of $A(a, b, c)$.

1.26.b Calculate the number of non-trivial conclusions that can be drawn from $\text{dual}(A(a, b, c))$.

1.26.c Name two non-trivial conclusions that can be drawn from $\text{dual}(A(a, b, c))$.

a) $A \iff a \to (b \vee c)$

$\text{dual}(A(a,b,c)) \iff \neg A(\neg a, \neg b, \neg c)$

$\iff \neg(\neg a \to (\neg b \vee \neg c))$

$\overset{DI}{\iff} \neg(a \vee (\neg b \vee \neg c))$

$\overset{AIO}{\iff} \neg a \vee b \vee c \iff$ CDNF (one minterm) $(\neg a \vee b \vee c) \vee$

b) non-trival conclusions: $2^m - 1$

1 minterm $\triangleq$ 1 true-assignment

$2^3 - 1$ Maxterms $\triangleq$ 7 false-assignments

$\therefore m = 7, \quad 2^7 - 1 = 127$

c) non-trivial conclusions

- must not be tautology (non-trivial)
- must evaluate to true for $(0,1,1)$

e.g.

$S_1 \iff \neg a \wedge b \wedge c$

$S_2 \iff (\neg a \wedge b \wedge c) \vee (a \wedge b \wedge c)$

1.27 Let the number of non-trivial conclusions that can be drawn from an arbitrary k-ary contingency D, be denoted with n.
The number of value assignments that falsify the dual form of D equals the number of value assignments that verify D.
Calculate the number of non-trivial conclusions l that can be drawn from $dual(D)$, dependent on k und n.

Given:

- k: # Variables in D
- n: # non-trivial Conclusions

Find l: # non-trivial conclusions from dual (D)

$n = 2^m - 1$ (m: # false assignments of D)

D has $2^k - m$ t assignments

$\hookrightarrow$ dual (D) has $m_{\text{dual}(D)} = 2^k - m$ f-assignments

$m_{\text{dual}(D)} = 2^k - \log_2(n+1)$ — appears from $n = 2^m - 1$ and making m the subject

$$l = 2^{m_{\text{dual}(D)}} - 1 = 2^{2^k - \log_2(n+1)} - 1 = \frac{2^{2^k}}{n+1} - 1$$

1.13.b $[\neg a \longrightarrow (b \vee d)] \wedge (a \longleftrightarrow c) \wedge (b \longrightarrow c) \wedge (d \longrightarrow c) \Longrightarrow c$

idea E12
$(a \to c) \wedge (\neg a \to c) \Rightarrow c$

1) $\neg a \to (b \vee d)$
2) $a \leftrightarrow c$
3) $b \to c$
4) $a \to c \quad \Rightarrow c$

---

5) $a \to c$ 2, C11, E4

E4 : $a \wedge b \Rightarrow a$
C11 done:
$a \leftrightarrow c \Leftrightarrow (a \to c) \wedge (c \to a)$

6) $c \leftrightarrow a$ 2, C11, E4
7) $(b \vee d) \to c$ 3, 4, D8
8) $\neg a \to c$ 1, 7, E11
9) c 3, 8, E12

1.13.e $[(\neg a \longrightarrow \neg b) \longrightarrow c] \wedge [b \longrightarrow ((a \longrightarrow c) \longrightarrow \neg c)] \quad \Longrightarrow b \not\leftrightarrow c$

idea
$\Leftrightarrow \neg(b \leftrightarrow c)$
$\Leftrightarrow \neg b \leftrightarrow c$
$\Leftrightarrow (\neg b \to c) \wedge (c \to \neg b)$

1) $(\neg a \to \neg b) \to c$
2) $b \to ((a \to c) \to \neg c) \quad \Rightarrow b \not\leftrightarrow c$

---

3) $(a \vee \neg b) \to c$ 1, D1
4) $a \to c$ 3, D8
5) $\neg b \to c$ 3, D8, E4
6) $(a \to c) \to (b \to \neg c)$ 2, D9
7) $b \to \neg c$ 4, 6, E9
8) $c \to \neg b$ 7, D3
9) $\neg b \leftrightarrow c$ 5, 8, C11
10) $\neg(b \leftrightarrow c)$ 9, C5
11) $b \not\leftrightarrow c$ 10, B11

1.13.f $\neg a \wedge [b \longrightarrow (c \longrightarrow a)] \wedge [\neg(c \longrightarrow b) \longrightarrow a] \qquad \Longrightarrow \quad a \longleftrightarrow c$

1) $\neg a$

2) $b \rightarrow (c \rightarrow a)$ Simplify these conditions

3) $\neg(c \rightarrow b) \rightarrow a$ $\qquad \Rightarrow a \leftrightarrow c \qquad \Leftrightarrow (a \rightarrow c) \wedge (c \rightarrow a)$

---

4)	$a \rightarrow c$	1, E7
5)	$(c \wedge \neg b) \rightarrow a$	3, D2
6)	$(c \wedge b) \rightarrow a$	2, D9
7)	$[((c \wedge \neg b) \vee ((c \wedge b)] \rightarrow a$	5, 6, D8
8)	$[c \wedge (\overbrace{\neg b \vee b}^{t})] \rightarrow a$	7, A3
9)	$c \wedge \rightarrow a$	8, A8, A6
10	$a \leftrightarrow c$	4, 9, C11

1.18 Determine if the following propositional forms are contradictions, tautologies or partly valid using the resolution method:

1.18.a $A \iff (a \vee b) \wedge (\neg b \vee \neg c) \wedge (c \vee \neg d)$

Proof for ~~Tautology~~ ; Contradiction ; Contingency

$A \iff (a \vee b) \wedge (\neg b \vee \neg c) \wedge (c \vee \neg d)$ 0

$(a \vee \neg c) \wedge (\neg b \vee \neg d)$ 1

$(a \vee \neg d)$ 2

$\iff$ Contingency

Proof for tautology ; ~~Contradiction~~ ; Contingency

1.18.c $C \iff a \vee (\neg a \wedge b) \vee (\neg a \wedge \neg c) \vee (\neg b \wedge c)$ 0

$b \vee \neg c \vee (\neg a \wedge c) \vee (\neg a \wedge \neg b)$ 1

$\neg a \vee c \vee \neg b$ 2

3

$\vee t$

$\Rightarrow$ tautology

Proof for ~~tautology~~ ; contradiction ; contingency

1.18.d $D \iff \neg a \wedge (a \vee \neg b) \wedge (a \vee c) \wedge (b \vee \neg c)$ 0

$\neg b \wedge c \wedge (a \vee \neg c) \wedge (a \vee b)$ 1

$\wedge\, a \wedge \neg c \wedge b$ 2

$\wedge\, f$ 3

$\Rightarrow$ Contradiction

$\boxed{1.19}$ Prove by only using the resolution rule in condition clause form (RRP) that it holds:

$$A(a, b, c) \iff (a \longrightarrow \neg b) \wedge (\neg a \longrightarrow \neg b) \wedge (\neg b \longrightarrow f) \iff f.$$

RRP: $(\underline{x} \to a) \wedge (\underline{\neg x} \to b) \iff (x \to a) \wedge (\neg x \to b) \wedge (\neg a \to b)$

$A(a,b,c) \iff (\underline{a} \to \neg b) \wedge (\underline{\neg a} \to \neg b) \wedge (\underline{\neg b} \to f)$ $(\iff f)$

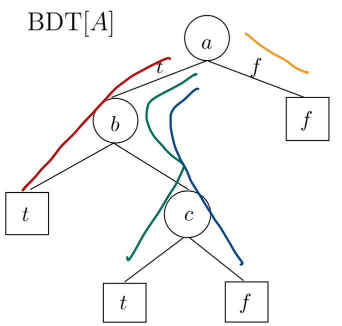
$\wedge\, (\underline{b} \to \neg b)$

$\wedge\, (\underbrace{\neg(\neg b)}_{b} \to f)$

$\wedge\, (\neg f \to f)$

$\iff \dots \wedge (t \to f) \iff \dots \wedge f \iff f$

$\boxed{\textbf{1.21}}$ Consider the following BDT of a propositional form $A(a, b, c)$.



1.21.a How many t- and f-assignments does A own?
Set up the CDNF and the CCNF by reading off the BDT.

t/f assignments all paths to t/f nodes

→ all n variables on path : 1 assignment

→ n-1 variables on path : 2 assignments

→ n-2 variables on path : 4 assignments

⋮

→ n-x variables on path : 2^x assignments

1 + 2 = 3 true assignments

1 + 4 = 5 false assignments

1.21.b Determine the premise normal form $PNF[A]$ by reading off the BDT.

$$CDNF[A] \Leftrightarrow \underline{(a \land \neg b \land c)} \lor \underline{(a \land b \land c)} \lor \underline{(a \land b \land \neg c)}$$

$$CCNF[A] \Leftrightarrow \underline{(\neg a \lor b \lor c)} \land \underline{(a \lor b \lor c) \land (a \lor b \lor \neg c) \land (a \lor \neg b \lor c) \land (a \lor \neg b \lor c)}$$

$$PNF[A] \Leftrightarrow \beta(a,\ \beta(b, t, \beta(c, t, f)), f)$$

1.21.c Calculate the CDNF from the PNF.

$$PNF[A] \Leftrightarrow \beta(a,\ \beta(b, t, \underbrace{\beta(c, t, f)}), f)$$

BB1(a)

$$\Leftrightarrow \beta(a,\ \beta(b, t,\ c),\ f)$$

BB3(a∨b)

$$\Leftrightarrow \beta(a,\ b \lor c,\ f)$$

BB3(a∧b)

$$\Leftrightarrow a \land (b \lor c) \qquad \Leftrightarrow CNF$$

A3

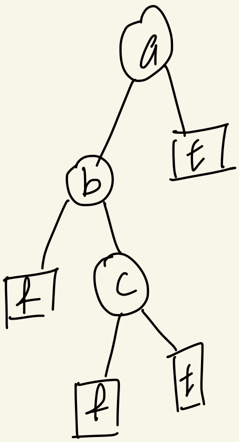
$$\Leftrightarrow (a \land b) \lor (a \land c) \qquad \Leftrightarrow DNF$$

$$CDNF[A] \Leftrightarrow \underline{(a \land b \land c)} \lor (a \land b \land \neg c) \lor \underline{\cancel{(a \land b \land c)}} \lor (a \land \neg b \land c)$$

$$\Leftrightarrow (a \land b \land c) \lor (a \land b \land \neg c) \lor (a \land \neg b \land c)$$

1.21.d How to obtain the BDT[$\neg A$]?

Interchange terminal nodes

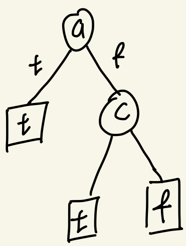


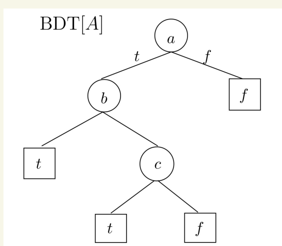
1.21.e Set up the ROBDD[$A \vee B$] with $B \iff a \vee c$.

B023 $A \vee B \iff B(A, t, B)$

BDT[$A \vee B$] $\iff$ BDT[A (set f node to B)]

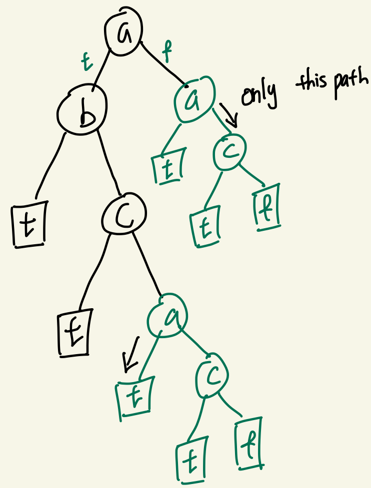
BDT[B]



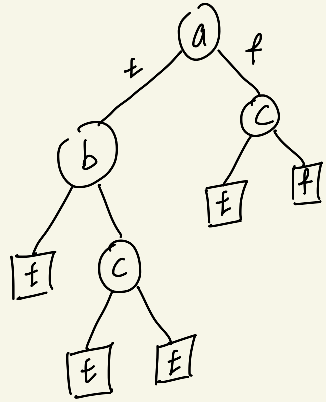


Combine to get $(A \vee B)$

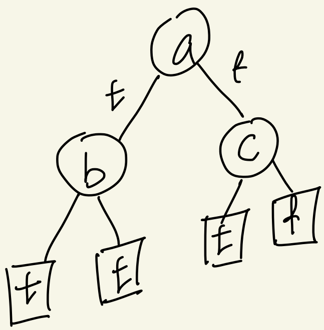
BDT[A v B]

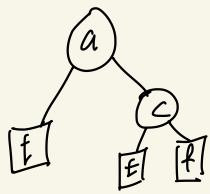


i) Goal : OBDT use SPR1

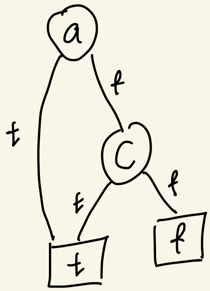


ii) Goal : ROBDT use SPR2





iii) Goal : ROBDD : Use SPR3 (Diagram only 1 t and 1 f)



$\boxed{1.22}$ Consider the following propositional form $C(a,b,c)$:

$$C(a,b,c) \iff [a \not\longleftrightarrow (b \wedge \neg c)] \longleftrightarrow [b \longrightarrow (a \wedge c)]$$

1.22.a Calculate the $PNF[C]$.
Choose $a,\ b,\ c$ as processing order of the variables.

$$C(a,b,c) \iff [a \not\leftrightarrow (b \wedge \neg c)] \leftrightarrow [b \rightarrow (a \wedge c)]$$

$$\overset{\text{BO17}}{\iff} \beta(a, [t \not\leftrightarrow (b \wedge \neg c)] \leftrightarrow [b \rightarrow (t \wedge c)],\ [f \not\leftrightarrow (b \wedge \neg c)] \leftrightarrow [b \rightarrow (f \wedge c)])$$

$$\overset{\text{B5, A6}}{\iff} \beta(a,\ \neg(b \wedge \neg c) \leftrightarrow (b \rightarrow c),\ (b \wedge \neg c) \leftrightarrow (b \rightarrow f))$$

$$\overset{\text{D5}}{\iff} \beta(a,\ \neg(b \wedge \neg c) \leftrightarrow (b \rightarrow c),\ (b \wedge \neg c) \leftrightarrow \neg b)$$

$$\overset{\text{BO17}}{\iff} \beta(a,\ \beta(b,\ \neg(t \wedge \neg c) \leftrightarrow (t \rightarrow c),\ \neg(f \wedge \neg c) \leftrightarrow (f \rightarrow c)),\ \beta(b,\ (t \wedge \neg c) \leftrightarrow \neg t,\ (f \wedge \neg c) \leftrightarrow \neg f))$$

$$\iff \beta(a,\ \beta(b,\ \neg(\neg c) \leftrightarrow c,\ \neg f \leftrightarrow t),\ \beta(b,\ \neg c \leftrightarrow f,\ f \leftrightarrow t))$$

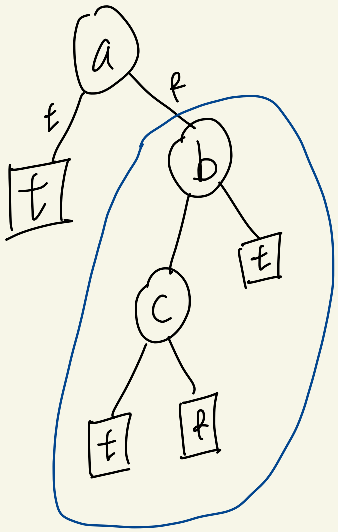
has to be t ∴ replace with another beta

$$\overset{\text{BO1}}{\iff} \beta(a,\ \underbrace{\beta(b,t,t)}_{t},\ \beta(b,c,f))$$

$$c \rightarrow \beta(c,t,f)$$

$$\overset{\text{BB1}}{\iff} \beta(a,\ t,\ \beta(b,\ \beta(c,t,f),\ f)) \iff PNF[C]$$

1.22.b Draw the BDT of C.



$(b \wedge c)$

1.22.c Calculate the ring normal form RNF of C.

Approach use BDT + ER4

$$\text{ER4}: A(\underline{x}) \Leftrightarrow [x_i \wedge (\underbrace{A(x_i \Leftrightarrow t)}_{C(a \Leftrightarrow t) \Leftrightarrow t} \not\Leftrightarrow \overbrace{A(x_i \Leftrightarrow f)}^{C(a \Leftrightarrow f) \Leftrightarrow b \wedge c})] \not\Leftrightarrow A(x_i \Leftrightarrow f)$$

set $x_i \Leftrightarrow a$

then we get : $C(a,b,c) \Leftrightarrow [a \wedge (t \not\Leftrightarrow (b \wedge c))] \not\Leftrightarrow (b \wedge c)$

$\Leftrightarrow [\ a \not\Leftrightarrow a \wedge b \wedge c] \not\Leftrightarrow (b \wedge c)$

$\Leftrightarrow \quad a \not\Leftrightarrow a \wedge b \wedge c \not\Leftrightarrow b \wedge c \Leftrightarrow RNF[C]$

1.25 Prove: $A \iff (a \vee \neg b \vee c) \vee (\neg a \wedge b \wedge \neg c) \iff t$

1.25.a by setting up a truth table.

a	b	c	masterm $a \vee \neg b \vee c$	$\neg a \wedge b \wedge \neg c$	A
0	0	0	1	0	1
0	0	1	1	0	1
0	1	0	0	1	1
0	1	1	1	0	1
1	0	0	1	0	1
1	0	1	1	0	1
1	1	0	1	0	1
1	1	1	1	0	

1.25.b by use of transformations.

$$A \iff (a \lor \neg b \lor c) \lor (\neg a \land b \land \neg c) \iff t$$

$\overset{A3}{\iff} [(a \lor \neg b \lor c) \lor \neg a]$

$\land [(a \lor \neg b \lor c) \lor b]$

$\land [(a \lor \neg b \lor c) \lor \neg c]$

$\iff t \land t \land t \iff t$

1.25.c by replacement (e.g., $E \iff a \lor \neg b \lor c$).

$E \iff a \lor \neg b \lor c$, $\neg E \iff \neg a \lor b \lor \neg c$

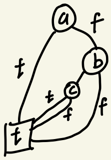
$A \iff E \lor \neg E \iff t$

$t \lor \ldots \iff t$

$\neg f \lor \ldots$

1.25.d by setting up a binary decision diagram (BDD).

BDD[A]



$A(a \iff f) \iff (\neg b \lor c) \lor (b \land \neg c)$

$A(a \iff f, b \iff t) \iff c \lor \neg c \iff t$

1.25.e by using the resolution method.

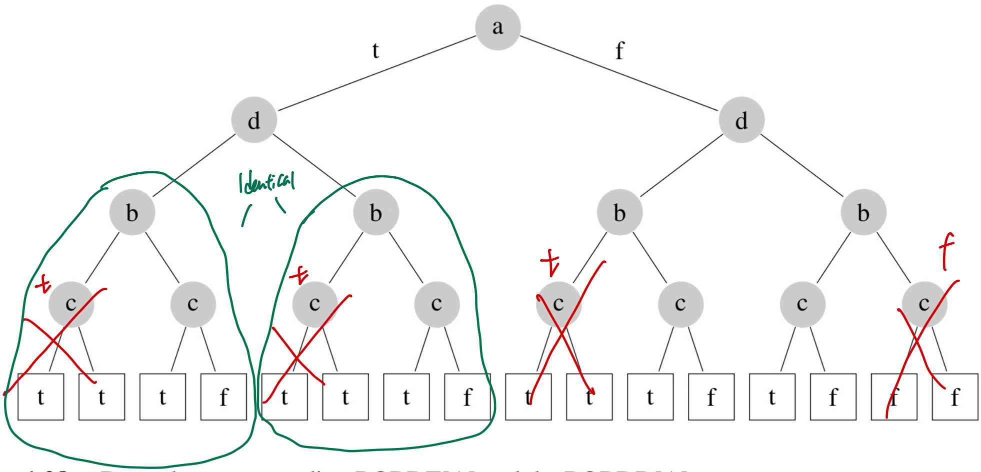
$a \lor \neg b \lor c \lor (\neg a \land b \land \neg c)$ 0

$\lor (b \land \neg c) \lor (\neg a \land \neg c) \lor (\neg a \land b)$ 1

$\lor \neg c \lor b \lor a$ 2

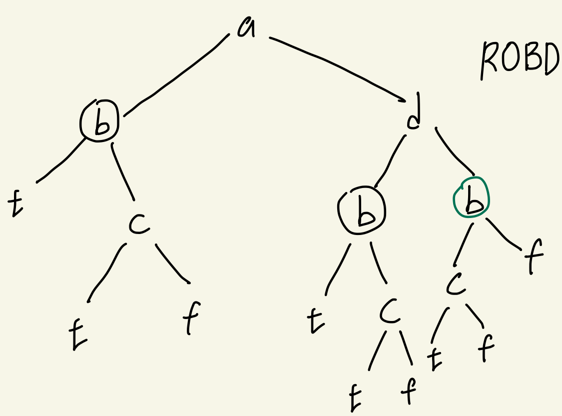
$\lor t$ 3

1.28 Given is the following Ordered Binary Decision Tree (OBDT) of the propositional form A:



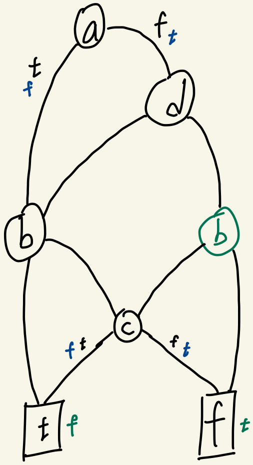
1.28.a Draw the corresponding ROBDT[A] and the ROBDD[A].

ROBDT



ROBDD

start from bottom to top



1.28.b Draw the ROBDD[dual(A)].

$dual(A(x)) \Leftrightarrow \neg A(\neg x)$

old exam question

Calculate RNF of $A(x,y,z) \Leftrightarrow z \leftrightarrow (x \rightarrow y)$ using ER4

ER4: $A(\underline{x}) \Leftrightarrow [x_i \wedge (A(x_i \Leftrightarrow t) \overset{+}{\leftrightarrow} A(x_i \Leftrightarrow f))] \overset{+}{\leftrightarrow} A(x_i \Leftrightarrow f)$

$$\mathrm{RNF}[A(x \Leftrightarrow f,\ y \Leftrightarrow f,\ z)] \Leftrightarrow z \leftrightarrow (f \rightarrow f) \Leftrightarrow z \leftrightarrow t \Leftrightarrow z$$

$$\mathrm{RNF}[A(x \Leftrightarrow f,\ y \Leftrightarrow t,\ z)] \Leftrightarrow z \leftrightarrow (f \rightarrow t) \Leftrightarrow z$$

RNF can't contain $\neg$

$\downarrow$

B5

$$\mathrm{RNF}[A(x \Leftrightarrow t,\ y \Leftrightarrow f,\ z)] \Leftrightarrow z \leftrightarrow (t \rightarrow f) \Leftrightarrow z \leftrightarrow f \Leftrightarrow \neg z \Leftrightarrow z \overset{+}{\leftrightarrow} t$$

$$\mathrm{RNF}[A(x \Leftrightarrow t,\ y \Leftrightarrow t,\ z)] \Leftrightarrow z \leftrightarrow (t \rightarrow t) \Leftrightarrow z$$

$$\mathrm{RNF}[A(x \Leftrightarrow f,\ y,\ z)] \underset{x_i \Leftrightarrow y}{\overset{ER4}{\Leftrightarrow}} [y \wedge (z \overset{+}{\leftrightarrow} z)] \overset{+}{\leftrightarrow} z$$

$$\Leftrightarrow [y \wedge f] \overset{+}{\leftrightarrow} z \Leftrightarrow f \overset{+}{\leftrightarrow} z \Leftrightarrow z$$

$$\mathrm{RNF}[A(x \Leftrightarrow t,\ y,\ z)] \underset{x_i \Leftrightarrow y}{\overset{ER4}{\Leftrightarrow}} [y \wedge (z \overset{+}{\leftrightarrow} t \overset{+}{\leftrightarrow} z)] \overset{+}{\leftrightarrow} z \overset{+}{\leftrightarrow} t$$

$$\Leftrightarrow [y \wedge t] \overset{+}{\leftrightarrow} z \overset{+}{\leftrightarrow} t \Leftrightarrow y \overset{+}{\leftrightarrow} z \overset{+}{\leftrightarrow} t$$

$$\mathrm{RNF}[A(x,y,z)] \underset{x_i = x}{\overset{ER4}{\Leftrightarrow}} [x \wedge (y \overset{+}{\leftrightarrow} z \overset{+}{\leftrightarrow} t \overset{+}{\leftrightarrow} z)] \overset{+}{\leftrightarrow} z$$

$$\Leftrightarrow [x \wedge (y \overset{+}{\leftrightarrow} t)] \overset{+}{\leftrightarrow} z \Leftrightarrow x \wedge y \overset{+}{\leftrightarrow} x \wedge t \overset{+}{\leftrightarrow} z$$

$$\Leftrightarrow x \wedge y \overset{+}{\leftrightarrow} x \overset{+}{\leftrightarrow} z$$