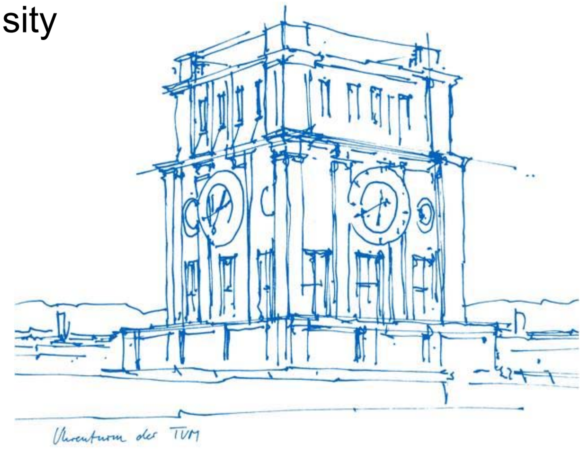
Lecture

Electricity and Magnetism

Chapter 2:

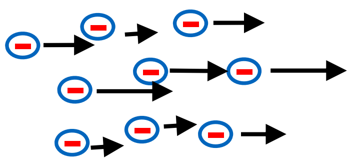
- Stationary Currents – Introduction
- Electric Current and Current Density



2. Stationary Currents – Introduction

Up to now: carrier distribution in stationary equilibrium and in rest (equilibrium of forces in electric field)

Now: charges are moving, at velocity $\vec{v}(\vec{r})$



velocity field $\vec{v}(\vec{r})$

Where relevant?

For example:

- In electrically conducting materials (conductors/semiconductors): $\longrightarrow 10^{15}$-10^{22} mobile carriers per cm^3
- Gas discharging processes, plasma (free carriers – electrons, ions)
- Electron beams (electrodes, hot cathodes)

2. Stationary Currents – Introduction

- Carriers move collectively similar to a fluid
- Driving forces, e.g.:
  - Electrostatic force (electric field): $\boxed{\vec{E} = -\nabla\Phi_{el}}$
  - Gradient of particle density (diffusion): $\sim \nabla n$
  - Temperature gradient (thermo diffusion): $\sim \nabla T$

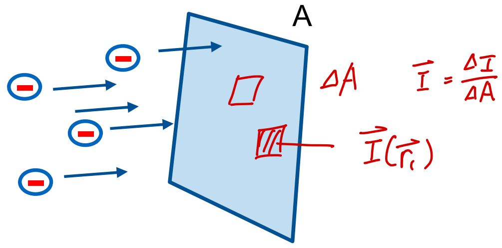
What does stationary mean? (as compared/opposed to static):
Physical quantity (scalar or field quantity) does not depend on time (locally depending distribution is constant)

Example electric charge:

- static = charge is located at fixed position and in rest; there is equilibrium of forces
- Stationary = charges can move at velocity $\vec{v}$, $\vec{v}(\vec{r})$ is in general also position-dependent, but velocity field $\vec{v}(\vec{r})$, i.e. particle flow is constant and not depending on time.

2.1 Electric Current and Current Density

What is electric current?



Definition: charge $dQ(A)$, which transits through area A per time intervall dt:

$$\boxed{I(A) = \frac{dQ(A)}{dt}} \quad (2.1)$$

(Note: A is not closed surface aroung a volume like in Gauss's law, but an arbitrary area A, where the charges pass through)

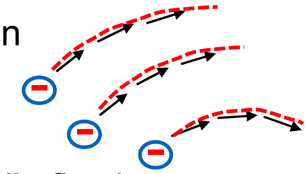
- Physical unit: *dim(I) = [I] = 1 C/s = 1 Ampère = 1 A* (SI unit!)
- *I* is an integral quantity (compare circuit, measured between two terminals)

  $\Longrightarrow$ We introduce a local quantity: current density $\vec{j}(\vec{r})$

2.1 Electric Current and Current Density

Current density $\vec{j}(\vec{r})$ is a vector field

- … that describes a flow field of electric charge
- … whose direction is parallel to the flow field lines of the particles (= tangent to vector field $\vec{v}(\vec{r})$)
- … that depends on position in space (magnitude and direction)



- … that represents the locally flowing current passing through an infinitessimal small control area ΔA:

$$\boxed{\vec{j}(\vec{r}) = \lim_{|\Delta A(\vec{r})| \to 0} \frac{I(\Delta A(\vec{r}))}{|\Delta A(\vec{r})|}}$$

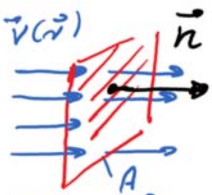
unit: $[|\,\vec{j}(\vec{r})\,|] = 1\frac{A}{m^2}$

$\vec{j}(\vec{r})$ describes quantitatively the current flow on continuous field level

2.1 Electric Current and Current Density

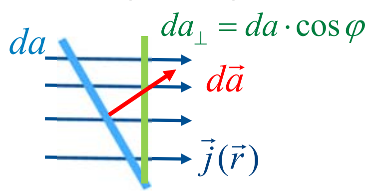
Relation between current density $\vec{j}(\vec{r})$ and current I:

Compare flow of water („river“)
(see inegration in E3):



$\vec{v} \cdot \vec{A} = |\vec{v}| \cdot A \cdot \vec{n}$

Current flowing through an area element da



$$dI = |\vec{j}|\, da_\perp = |\vec{j}| \cos\varphi\, da$$

$$dI = \vec{j} \cdot \vec{N} \cdot da = \vec{j} \cdot d\vec{a}$$

Total current through area S: $\boxed{I = \int_S \vec{j} \cdot d\vec{a}}$ (2.2.)

2.1 Electric Current and Current Density

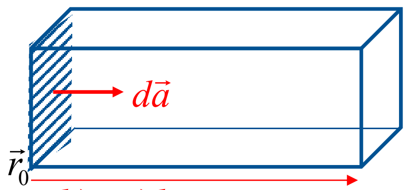
$\rho(\vec{r})$ = continuous space charge density $= qn(\vec{r})$ particle number density

$\vec{j}(\vec{r})$ = continuous flow of charge carriers

➡ Relation between current density $\vec{j}(\vec{r})$ and space charge density $\rho(\vec{r})$

Consider the following situation:

$t=t_0$: charge carriers are loacted at $\vec{r}_0$



They travel the following distance in time intervall dt $\quad d\vec{r} = \vec{v}dt$

Volume dV, which will be coverted during dt: $\quad dV = d\vec{a} \cdot d\vec{r} = d\vec{a} \cdot \vec{v}dt$

Charge dQ, contained in dV: $\quad dQ = \rho(\vec{r})dV = \rho(\vec{r})d\vec{a} \cdot \vec{v}dt = qn(\vec{r})\vec{v} \cdot d\vec{a}dt$

From (2.1) and (2.2.): $\quad dQ = \vec{j} \cdot d\vec{a}dt$

$$\boxed{\vec{j}(\vec{r}) = q\vec{v}(\vec{r})n(\vec{r})} \quad (2.4)$$

Current density expressed by microscopic quantities

2.1 Electric Current and Current Density

Generalization of current densities to multiple species of carriers:

Electric current can be carrierd by different charge carriers (see gas discharging processes, plasma, electric current in differend media, …)

-> generalization:

Space charge density: $$\boxed{\rho(\vec{r}) = \sum_{\alpha=1}^{k} q_\alpha n_\alpha(\vec{r})} \quad (2.5.a)$$

Electric current density: $$\boxed{\vec{j}(\vec{r}) = \sum_{\alpha=1}^{k} q_\alpha n_\alpha(\vec{r})\vec{v}_\alpha(\vec{r})} \quad (2.5.b)$$

n_a = charge carrier concentration of species α (α= 1…..k)

q_a = specific charge of species α (α= 1…..k)

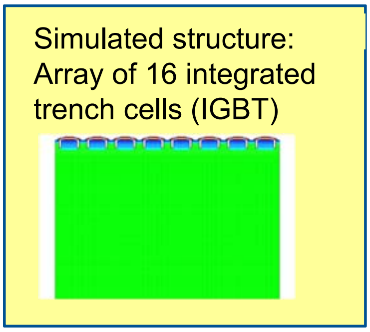
$\vec{v}_a$ = mean drift velocity of species α (α= 1…..k)

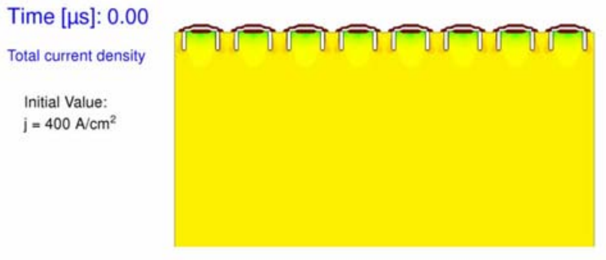
(2.2.), (2.4) und (2.5) = link between microscopic properties and measurable (macroscopic) quantities

2.1 Electric Current and Current Density

Why do we need a local quantity like the electric current density?

- Design of electronic devices and circuits
- Calculation of current carrying capability
- Judge robustness of devices (see also 2.4 Joule's heat)





Up to now: flowing carriers considered and relation between local and integral quantities defined

Chapter 2.2: How can be describe this transport process in an electric field? (relation between charge, electric field and charge transport = electric current)