Circular and Linear Motion

- Relationship between linear and angular quantities. The radius r, of the circular path is constant and provides the connection between the two sets of quantities.

Quantity	Linear	Angular	Relationship
Displacement	s	θ	$s = r\theta$
Velocity	v	ω	$v = r\omega$
Acceleration	a	α	$a_t = r\alpha$ $a_c = r\omega^2$ $\mathbf{a} = r\alpha\mathbf{t} - r\omega^2\mathbf{r}$

Constant Angular Acceleration

- Kinematical equations for constant angular acceleration are obtained in complete analogy to those for linear motion with constant acceleration:

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$\theta = \theta_0 + \bar{\omega} t$$

$$\omega = \omega_0 + \alpha t$$

$$\bar{\omega} = \tfrac{1}{2}(\omega + \omega_0)$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

Hammer Throw

- An interesting track-and-field event is the hammer throw. The task is to throw the "hammer," a 12-cm diameter iron ball attached to a grip by a steel cable, a maximum distance. The hammer's total length is 121.5 cm, and its total mass is 7.26 kg. The athlete has to accomplish the throw while not leaving a circle of radius 2.13 m. The task is usually accomplished by making the hammer move in a circle several times while accelerating it.

Hammer Throw

- A hammer thrower take 7 turns before releasing hammer. These took 1.52 s, 1.08 s, 0.72 s, 0.56 s, 0.44 s, 0.40 s, and 0.36 s.

PROBLEM 1:

- What is value of the angular acceleration during the seven turns, assuming constant acceleration?

SOLUTION 1:

- Total time:

$$t_{\text{all}} = 1.52\ \text{s} + \ldots + 0.36\ \text{s} = 5.08\ \text{s}$$

- Total angle:

$$\theta_{\text{all}} = 7 \cdot 2\pi = 14\pi \approx 44.0\ \ \text{rad}$$

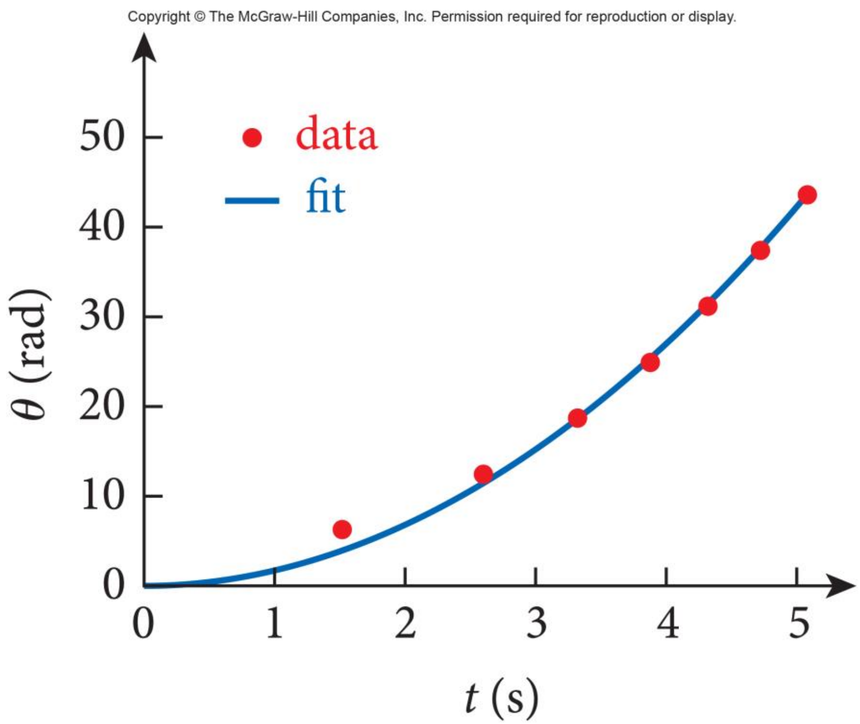
- Constant angular acceleration:

$$\theta = \tfrac{1}{2}\alpha\ t^2 \Rightarrow \alpha = \frac{2\theta}{t^2} = \frac{2 \cdot 44.0}{(5.08\ \text{s})^2} = 3.41\ \text{rad/s}^2$$

Hammer Throw

DISCUSSION:

- We can make a plot of the angle of the hammer as a function of time.
- The line assumes a constant angle acceleration of: $\alpha = 3.41\ \text{rad/s}^2$



PROBLEM 2:

- The radius of the circle on which the hammer moves is: 1.67 m, what is the linear speed with which the hammer is released?

Hammer Throw

SOLUTION 2:

- Under constant angular acceleration from rest for a period of 5.08 s, the final angular velocity reached is:

$$\omega = \alpha\, t = (3.41\ \mathrm{s}^{-2}) \cdot (5.08\ \mathrm{s}) = 17.3\ \mathrm{rad/s}$$

- The linear velocity is then:

$$v = r\omega = (1.67\ \mathrm{m}) \cdot (17.3\ \mathrm{s}^{-1}) = 28.9\ \mathrm{m/s}$$

PROBLEM 3:

- What is the centripetal acceleration and centripetal force that the hammer thrower has to exert on the hammer right before the hammer gets released?

Hammer Throw

SOLUTION 3:

- The centripetal acceleration right before release is given by:

$$a_c = v\omega = (28.9\ \mathrm{m/s})\cdot(17.3\ \mathrm{s}^{-1}) = 500.\ \mathrm{m/s}^2$$

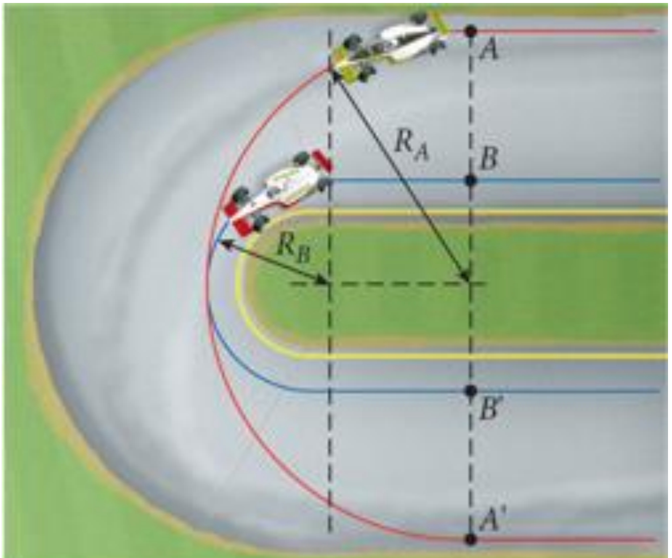
- With a mass of 7.26 kg for the hammer, the centripetal force required is then:

$$F_c = ma_c = (7.26\ \mathrm{kg})\cdot(500\ \mathrm{m/s}^2) = 3630\ \mathrm{N}$$

- Same force as weight of 370 kg object!

Formula 1 Racing

- Formula 1 race cars approach curves from the outside, cut through to the inside, and then drift again to the outside. Suppose that the race cars move through the turn at constant speed. The coefficient of static friction between the tires and the track is $\mu_s = 1.2$. If the inner radius of the curve is $R_B = 10.3$ m and the outer radius of the curve is $R_A = 32.2$ m and the cars move at their maximum speed, ow much time will it take to move from A to A' and from B to B'?



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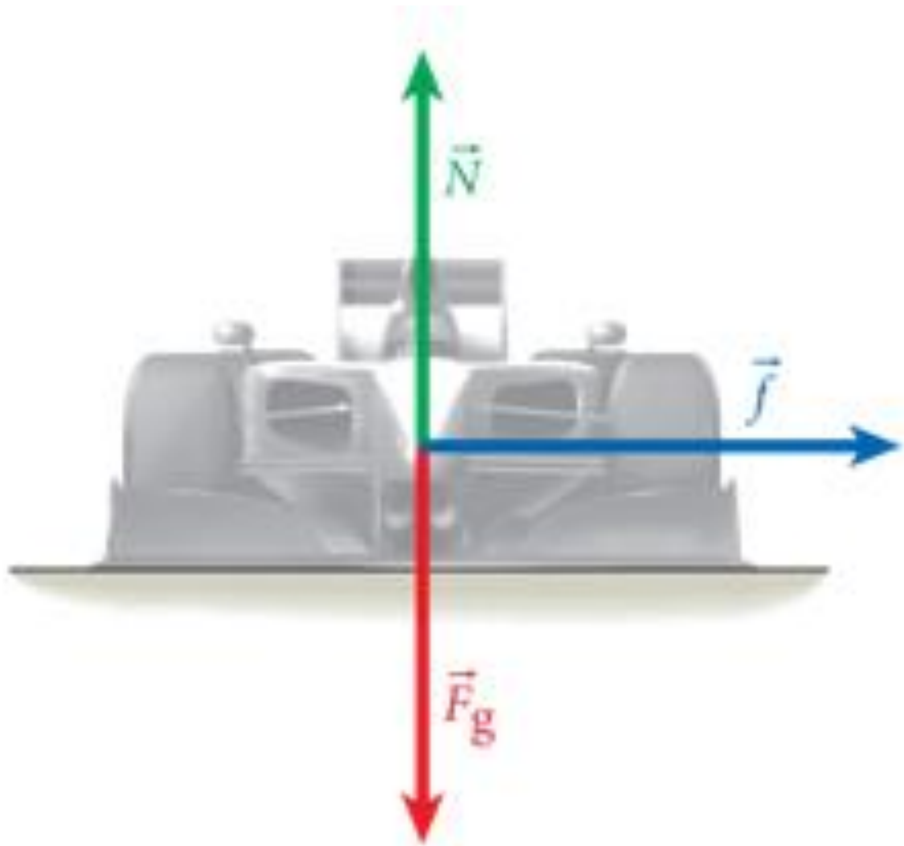
Formula 1 Racing

SOLUTION:

- Our free-body diagram shows all the forces on the car:
  - Force of gravity
  - Normal force
  - Force of friction



Formula 1 Racing

- For each radius of curvature, the force of friction must provide the necessary centripetal force:

$$f_{\max} = \mu_s mg = m\frac{v^2}{R} \quad \Rightarrow \quad v = \sqrt{\mu_s gR}$$

- For the red and blue curves we get:

$$v_{\text{red}} = \sqrt{\mu_s gR_A} = \sqrt{(1.2)(9.81\text{ m/s}^2)(32.2\text{ m})} = 19.5\text{ m/s}$$

$$v_{\text{blue}} = \sqrt{\mu_s gR_B} = \sqrt{(1.2)(9.81\text{ m/s}^2)(10.3\text{ m})} = 11.0\text{ m/s}$$

- The length of the red curve is:

$$\ell_{\text{red}} = \pi R_A = 101\text{ m}$$

- The length of the blue curve is:

$$\ell_{\text{blue}} = \pi R_B + 2(R_A - R_B) = 76.2\text{ m}$$

Formula 1 Racing

- The blue time is:

$$t_{\text{blue}} = \frac{\ell_{\text{blue}}}{v_{\text{blue}}} = \frac{76.2 \text{ m}}{11.0 \text{ m/s}} = 6.92 \text{ s}$$

- The red time is:

$$t_{\text{red}} = \frac{\ell_{\text{red}}}{v_{\text{red}}} = \frac{101 \text{ m}}{19.5 \text{ m/s}} = 5.02 \text{ s}$$

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NASCAR Racing

- A banked curve helps a NASCAR driver achieve higher speeds:

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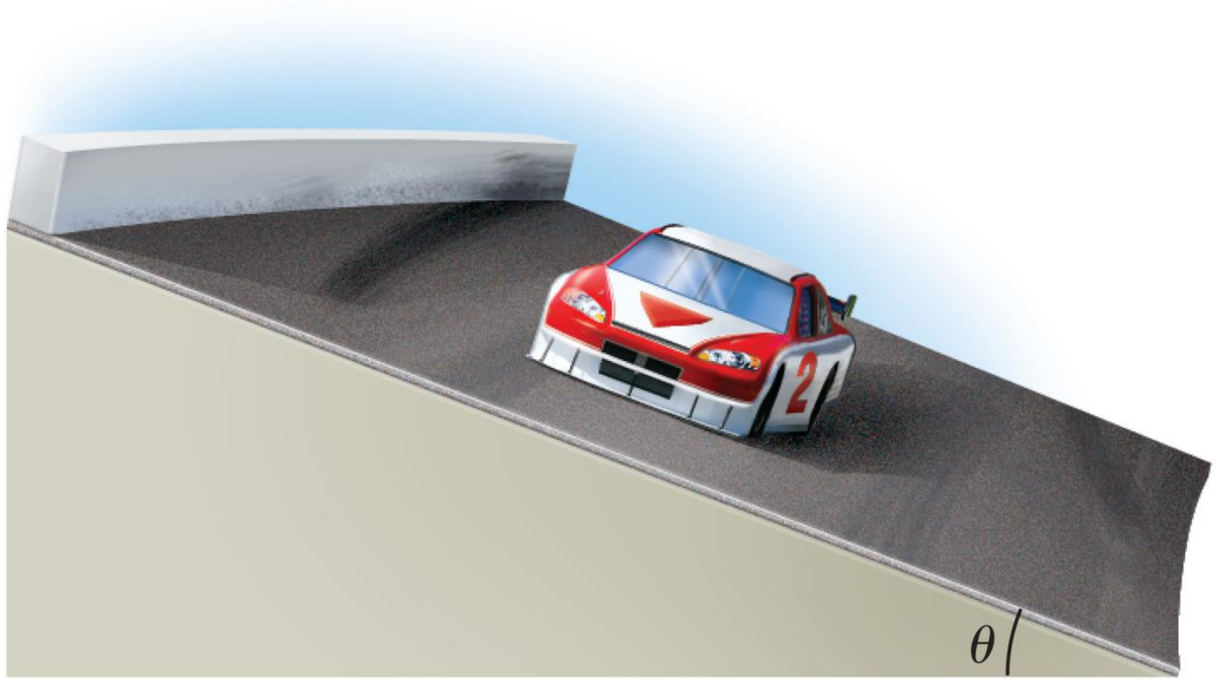

PROBLEM:

- If the coefficient of friction between the racetrack surface and the tires of the racecar is $\mu_s = 0.620$ and the radius of the turn is $R = 110.0$ m, what is the maximum speed with which the driver can take this curve banked at 21.1°?

NASCAR Racing

SOLUTION:

- The three forces acting on the race car are:
  - Gravity
  - The normal force
  - Friction

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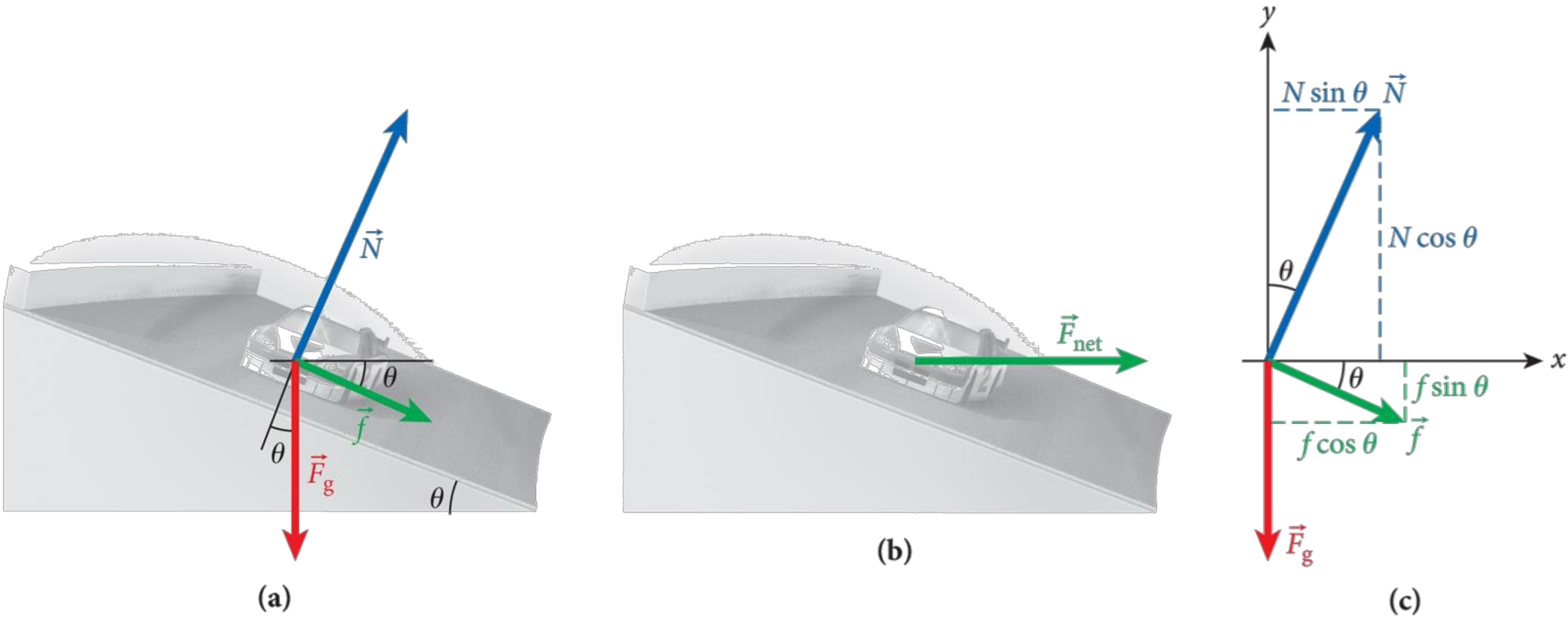

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NASCAR Racing

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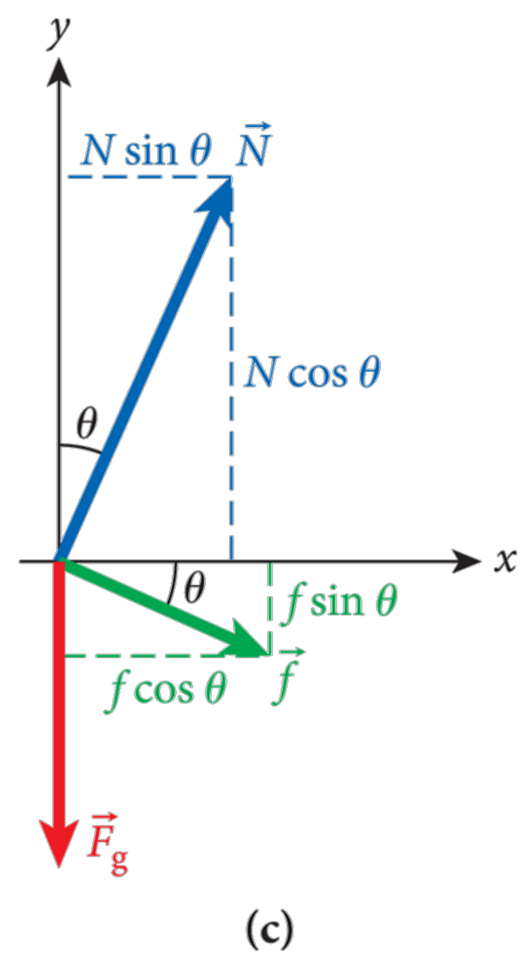

- We start with Newton's Second Law:

$$\sum \vec{F} = m\vec{a}$$

- The x-components are:

$$N \sin\theta + f \cos\theta = F_{\text{net}}$$

- The y-components are:

$$N \cos\theta - F_g - f \sin\theta = F_{\text{net}}$$

- The maximum friction force is:

$$f = \mu_s N$$

- The force of gravity is:

$$F_g = mg$$

- The net force must be equal to the required centripetal force:

$$F_{\text{net}} = F_c = m\frac{v^2}{R}$$

NASCAR Racing

- Combining equations we get:

$$x: \quad N\sin\theta + \mu_s N\cos\theta = N\left(\sin\theta + \mu_s \cos\theta\right) = m\frac{v^2}{R}$$

$$y: \quad N\cos\theta - mg - \mu_s N\sin\theta = 0 \quad \Rightarrow \quad N\left(\cos\theta - \mu_s \sin\theta\right) = mg$$

- Dividing the x equation by the y equation gives us:

$$\frac{N\left(\sin\theta + \mu_s \cos\theta\right)}{N\left(\cos\theta - \mu_s \sin\theta\right)} = \frac{m\frac{v^2}{R}}{mg}$$

- Solve for the speed of the car:

$$v = \sqrt{\frac{Rg\left(\sin\theta + \mu_s \cos\theta\right)}{\cos\theta - \mu_s \sin\theta}}$$

NASCAR Racing

Calculate

- Putting in our numbers:

$$v=\sqrt{\frac{(110.\text{ m})\cdot(9.81\text{ m/s}^2)\cdot(\sin 21.1^\circ+0.620\cdot\cos 21.1^\circ)}{\cos 21.1^\circ-0.620\cdot\sin 21.1^\circ}}=37.7726\text{ m/s}$$

- How fast could the car go around the curve if it were not banked?

$$v=\sqrt{\frac{Rg\left(0+\mu_s\cdot 1\right)}{1-\mu_s\cdot 0}}=\sqrt{\mu_s Rg}=\sqrt{\left(0.620\right)\left(110.\text{ m}\right)\left(9.81\text{ m/s}^2\right)}=25.9\text{ m/s}$$

Summary

Can move between Cartesian and polar coordinate system with appropriate transformation.

$$r = \sqrt{x^2 + y^2} \qquad x = r\cos\theta$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) \qquad y = r\sin\theta$$

For circular motion linear displacements, velocities and accelerations are related to the angular displacements, velocities and accelerations:

Quantity	Linear	Angular	Relationship
Displacement	s	θ	$s = r\theta$
Velocity	v	ω	$v = r\omega$
Acceleration	a	α	$a_t = r\alpha$ $a_c = r\omega^2$ $\mathbf{a} = r\alpha\mathbf{t} - r\omega^2\mathbf{r}$

$$a = \sqrt{a_c^2 + a_t^2} = r\sqrt{\alpha^2 + \omega^4}$$

Summary

- The kinematic equations for circular motion are:

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$\theta = \theta_0 + \overline{\omega} t$$

$$\omega = \omega_0 + \alpha t$$

$$\overline{\omega} = \tfrac{1}{2}(\omega + \omega_0)$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$