Circuit Theory – Tutorials

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Contents

Preface

The intention for the tutorial manuscript for the lecture Circuit Theory is to support the students in understanding the contents of the lecture by applying the obtained knownledge to different tutorial problems. The manuscript contains more problems than can be discussed in the tutorials. The additional problems give the opportunity to deepen the understanding by selfstudy.

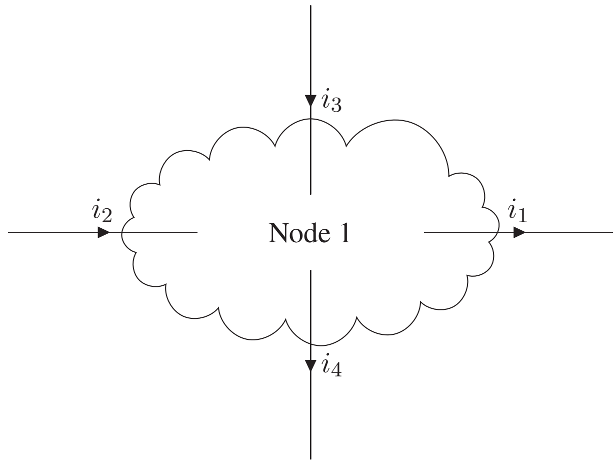
Michael Joham

Exercise 1

Kirchhoff's Laws

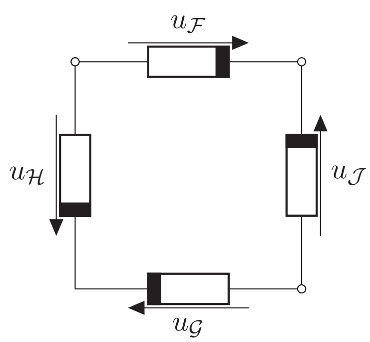
1.1 Kirchhoff's Laws

a) Formulate the Kirchhoff's current law for the depicted node.



b) Give the Kirchhoff's current law in general form.

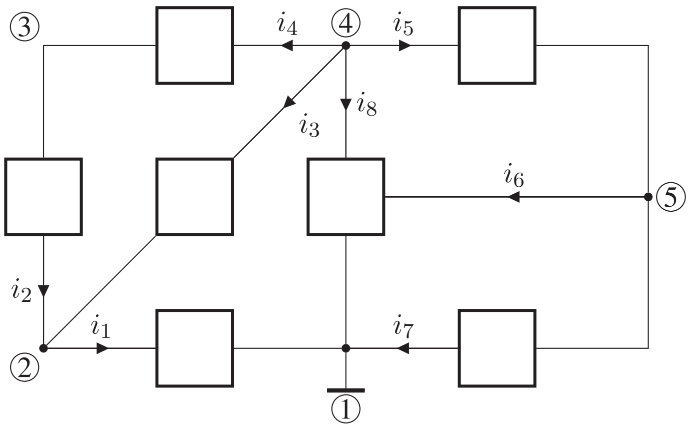
c) Formulate the Kirchhoff's voltage law for the depicted network.



d) Give the Kirchhoff's voltage law in general form.

1.2 Kirchhoff's Laws

Consider the following network.



a) Formulate Kirchhoff's current law (KCL) for all nodes.

b) Find the Kirchhoff's voltage law (KVL) equations for some loops.

c) Give the alternative form of KVL by expressing all branch voltages depending on the node voltages.

Exercise 2

Resistive One-Ports

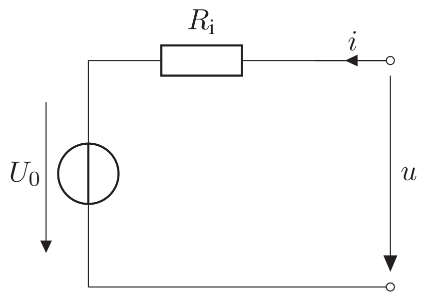
2.1 Linear Sources and Source Transform

a) What is the difference between general linear sources and independent sources? Depict the difference graphically.

b) How are the *internal resistance* and *internal conductance* of linear sources defined?

By source transform, a linear voltage source combined with an internal resistance can be equivalently transformed to a current source combined with an internal conductance.

Hence, consider the following circuit.



c) Draw the equivalent circuit with a current source and internal conductance. Give all parameters depending on U_0 and R_i.

2.2 Resistive Circuits and Duality

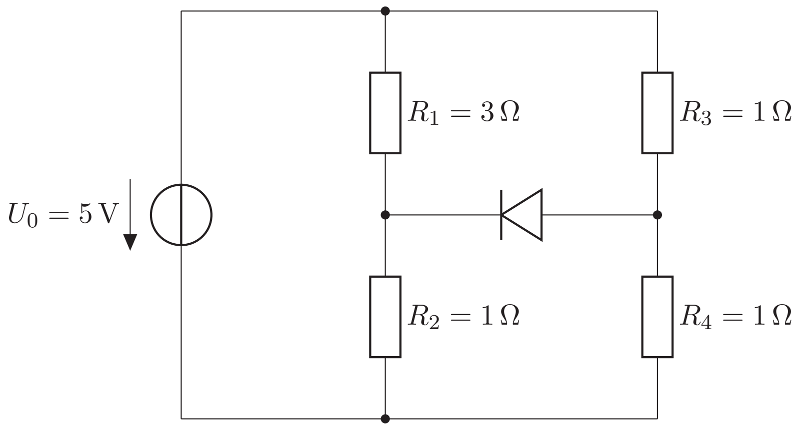
a) Apply the duality transform to an independent current source $I_0 = 1\,\mathrm{A}$ with the duality constant $R_\mathrm{d} = 2\,\Omega$.

Let the duality constant R_d be given.

b) Find the element dual to a strictly linear resistor with conductance G depending on G and R_d.

2.3 Operating Point

Consider the following circuit with an ideal diode.

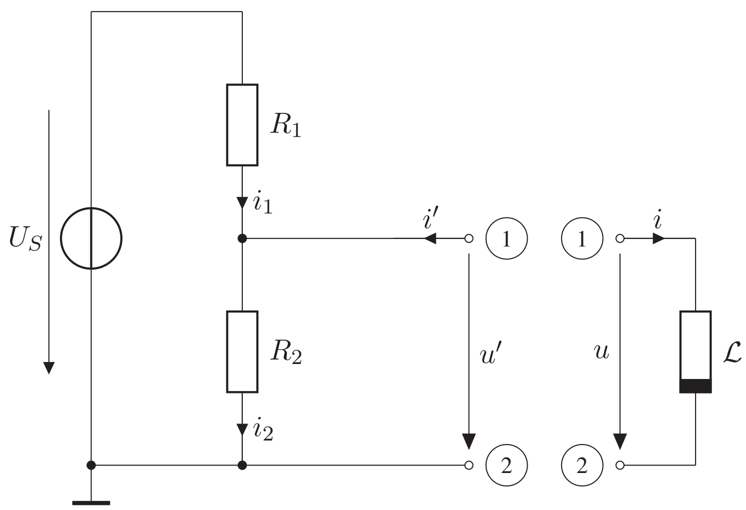


a) Find the operating point of the circuit assuming that the diode is operated in the cut-off region.

b) Determine the operating point of the circuit assuming that the diode is operated in the conductive region.

2.4 Loaded Voltage Divider

Given is the following circuit which is connected to the load $\mathcal{L}$.



For the *voltage divider*, Ohmic resistors are used to get a voltage u' which is smaller than the supply voltage U_S.

a) Determine the output voltage $u' = U_0$ of the voltage divider without a load, i.e., $i' = 0$, depending on U_S, R_1, and R_2. What is the name of U_0?

b) Express U_0 depending on U_S, the total resistance $R_{\text{tot}} = R_1 + R_2$ and the divider ratio $V = \frac{R_2}{R_{\text{tot}}}$.
Let the supply voltage be $U_S = 15\,\text{V}$.

c) Realize a voltage divider with $R_2 = 1\,\text{k}\Omega$ and the appropriate choice for R_1 such that a reference voltage of $3\,\text{V}$ results.

d) Determine the characteristic of the voltage divider if it is interpreted as a linear source between the nodes ① and ②.

The voltage divider is now *loaded*, i.e., connected to a two-terminal load $\mathcal{L}$.

Firstly, $\mathcal{L}$ draws a constant current $i = 1\,\text{mA}$ independent of the voltage u.

e) Model $\mathcal{L}$ by a circuit element. Find the resulting u for this load assuming the previous dimensioning of R_1 and R_2.
Connecting to a load changes the output voltage of the voltage divider.

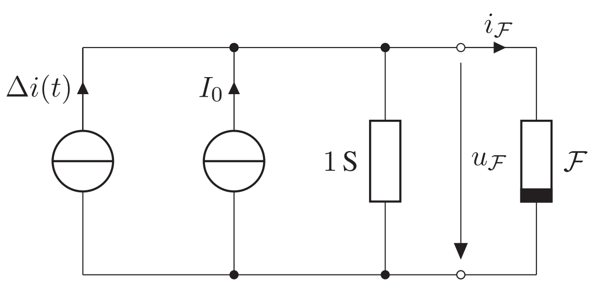
f) What degree of freedom can be used for the design of the voltage divider such that this problem is avoided? What is its disadvantage?

Secondly, the load is an Ohmic resistor with resistance $R_L = 10\,\Omega$.

g) Dimension the voltage divider such that the voltage drop over R_L is exactly $3\,\text{V}$. What is the simplest circuit to reach this goal?

2.5 Linearization in an Operating Point

Given is following connection of non-linear one-port $\mathcal{F}$ and a linear source.



The represenation of $\mathcal{F}$ is given by

$$i_{\mathcal{F}}\left(u_{\mathcal{F}}\right)=\begin{cases} 0\,\mathrm{A} & \text{für } u_{\mathcal{F}}<-2\,\mathrm{V} & \text{(segment I)}\\ 1/2\,\mathrm{S}\cdot u_{\mathcal{F}}+1\,\mathrm{A} & \text{für } -2\,\mathrm{V}\leq u_{\mathcal{F}}<-0.5\,\mathrm{V} & \text{(segment II)}\\ \left(\frac{u_{\mathcal{F}}}{1\,\mathrm{V}}-1\right)^2\cdot 1\,\mathrm{A} & \text{für } u_{\mathcal{F}}>-0.5\,\mathrm{V}. & \text{(segment III)}\end{cases}$$

The circuit is in the operating point $(0\,\mathrm{V};1\,\mathrm{A})$.

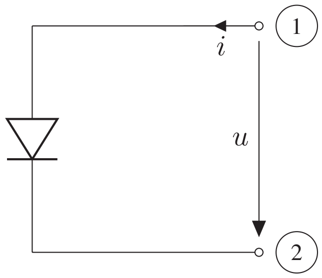
a) Linearize $\mathcal{F}$ in the operating point (the alternating current part of the source $\Delta i(t)$ is initially assumed to be zero, i.e., $\Delta i(t)=0$).

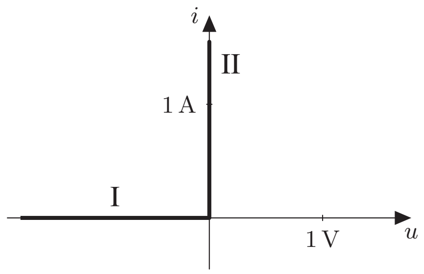
b) Draw the large signal equivalent circuit diagram.

c) Draw the small signal equivalent circuit incorporating the alternating current part $\Delta i(t)$.

2.6 Ideal Diode

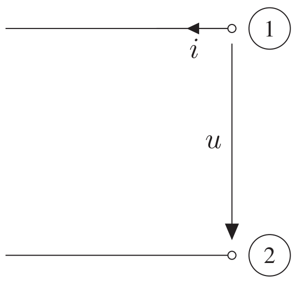
Consider the element symbol and the piecewise linear characteristic of an ideal diode:





The characteristic comrises to segments I and II that allow for different models.The equivalent circuit diagram ECD I for segment I is shown below:

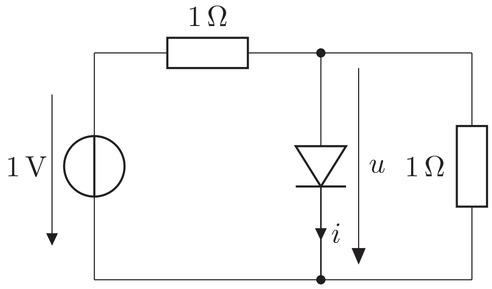
ECD I: $i=0$
$u\leq 0$



The condition $u\leq 0$ is necessary condition for the applicability of the equivalent circuit diagram.

a) Give the corresponding equivalent circuit diagram ECD II for the segment II of the characteristic.

By applying these equivalent circuit diagrams, in which point of the characteristic of the diode the following circuit $\mathcal{S}$ is operated.



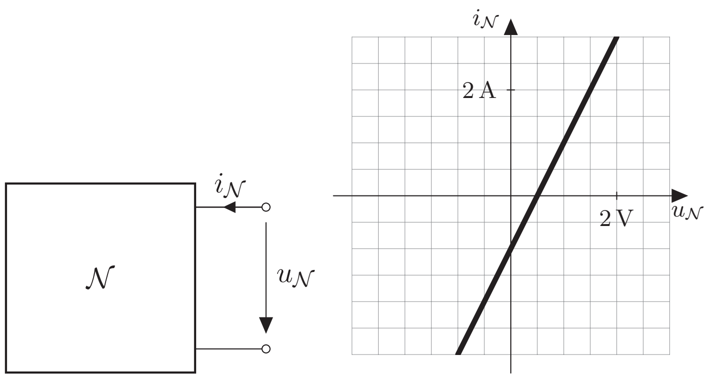
b) Assume that the diode is operated in segment I of its characteristic. Give the corresponding equivalent circuit diagram $\mathcal{S}_I$ of $\mathcal{S}$ and determine the solution $\mathcal{L}_I = (u, i)$ of $\mathcal{S}_I$. Is $\mathcal{L}_I$ a solution of $\mathcal{S}$?

c) Assume that the diode is operated in segment II of its characteristic. Give the corresponding equivalent circuit diagram $\mathcal{S}_{II}$ of $\mathcal{S}$ and find the solution $\mathcal{S}_{II} = (u, i)$ of $\mathcal{S}_{II}$. Is $\mathcal{L}_{II}$ a solution of $\mathcal{S}$?

d) Draw $\mathcal{L}_I$ and $\mathcal{L}_{II}$ into the diagram with the characteristic of the ideal diode.

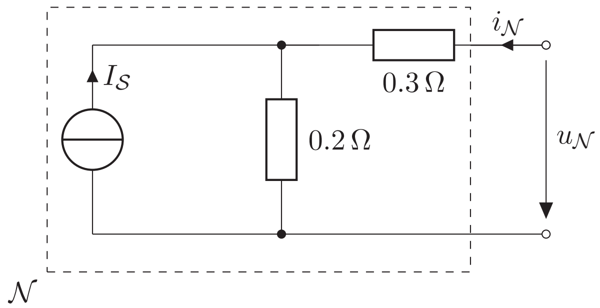
2.7 Source Transform

Given is the linear one-port $\mathcal{N}$ whose characteristic is depicted in the following diagram.



a) The circuit can be represented by two different equivalent circuit diagrams which only contain two network elements. Give the two equivalent circuit diagrams with the element values.

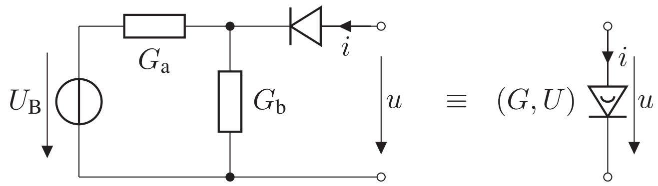
Now the following internal structur of $\mathcal{N}$ is given.



b) Determine the source current $I_{\mathcal{S}}$. (Hint: With the help of the characteristic, find out the behaviour of the circuit if it is connected to an open circuit.)

2.8 Realization of a Piecewise Linear Resistor

The following circuit $\mathcal{S}$ realizes a concave resistor.



Firstly, the parameters G and U of the equivalent concave resistor are determined depending on the parameters U_{B}, G_{a}, and G_{b} of $\mathcal{S}$.

a) Find the breakthrough point of the piecewise linear circuit $\mathcal{S}$. To this end, assume that the diode is also operated in its breakthrough point.

b) Determine the slope G of the right segment of the characteristic of $\mathcal{S}$ if the diode is operate in the conductive region.

c) Draw the characteristic of $\mathcal{S}$ in the u-i-plane and label the axes.

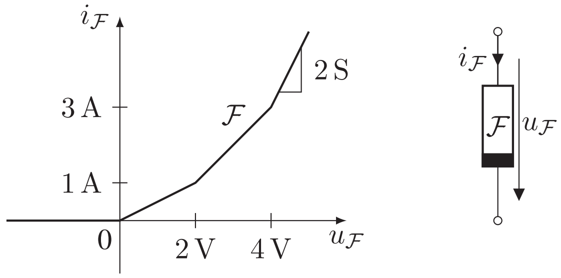
Let $U_{\mathrm{B}} > 0$ be the supply voltage of the circuit.

d) Express G_{a} and G_{b} depending on the parameters U and G. These equations constitute the *design rule* for $\mathcal{S}$.

e) What are the conditions for G and U such that the circuit $\mathcal{S}$ is equivalent to the concave resistor (G, U) can be realized on a chip?

f) Simplify $\mathcal{S}$ and the design rule for the case that $U = 0$.

The piecewise linear resistor $\mathcal{F}$ with following characteristic is now realized.



g) Implement $\mathcal{F}$ as the parallel connection of three concave resistors $(G_j, U_j), j \in \{1, 2, 3\}$ and give the corresponding parameter values.

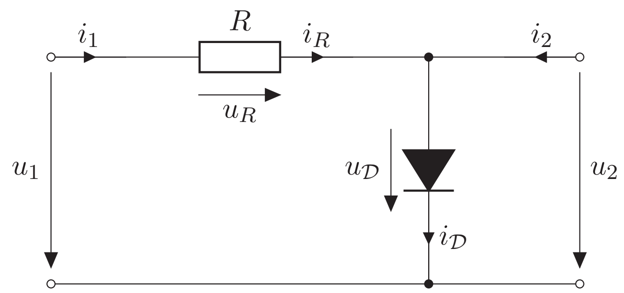
h) Realize $\mathcal{F}$ with ideal diodes, Ohmic resistors, and the voltage source $U_\mathrm{B} = 10\,\mathrm{V}$. Draw the complete circuit with all element values.

Exercise 3

Resistive Two-Ports

3.1 Representation Forms of Two-Ports

Given is the following non-linear two-port consisting of an Ohmic resistor and a pn-diode.



Use the conductance $G = \frac{1}{R}$ in case that the resulting expression can be simplified.

a) Find $u_R, u_\mathcal{D}, i_R$, and $i_\mathcal{D}$ depending on the port quantities.

b) Give the representations for the two one-ports included in the given two-port, both in voltage- and in current-controlled form.

Now derive the following six explicit representations of the two-port.

c) Voltage-controlled representation: $\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \boldsymbol{g}\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right)$

d) Current-controlled representation: $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \boldsymbol{r}\left(\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}\right)$

e) Hybrid representation: $\begin{bmatrix} u_1 \\ i_2 \end{bmatrix} = \boldsymbol{h}\left(\begin{bmatrix} i_1 \\ u_2 \end{bmatrix}\right)$

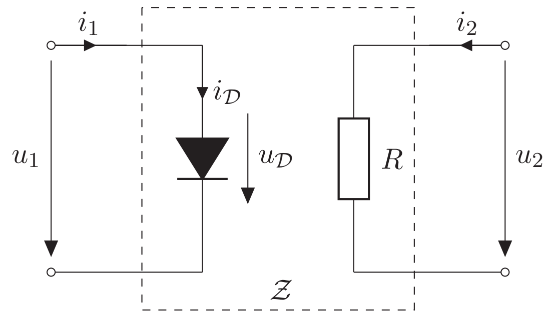
f) Transmission representation: $\begin{bmatrix} u_1 \\ i_1 \end{bmatrix} = \boldsymbol{a}\left(\begin{bmatrix} u_2 \\ -i_2 \end{bmatrix}\right)$

g) Inverse transmission representation: $\begin{bmatrix} u_2 \\ i_2 \end{bmatrix} = \boldsymbol{a}'\left(\begin{bmatrix} u_1 \\ -i_1 \end{bmatrix}\right)$

h) Inverse hybrid representation: $\begin{bmatrix} i_1 \\ u_2 \end{bmatrix} = \boldsymbol{h}' \left(\begin{bmatrix} u_1 \\ i_2 \end{bmatrix} \right)$

3.2 Representation of a Non-Linear Two-Port

Consider the following two-port $\mathcal{Z}$.



a) Can the two-port $\mathcal{Z}$ be represented by a matrix?

b) Give the current-controlled representation of $\mathcal{Z}$.

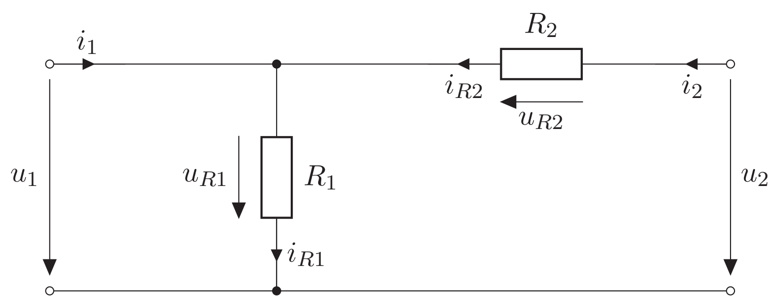
c) Determine the hybrid representation of $\mathcal{Z}$.

d) Find the inverse hybrid representation of $\mathcal{Z}$.

e) Determine the transmission representation of $\mathcal{Z}$.

3.3 Two-Port Matrices

Given is the following strictly linear two-port comprising two Ohmic resistors.



Use in all calculations the conductances $G_1 = \frac{1}{R_1}$ and $G_2 = \frac{1}{R_2}$ if it allows simpler formulas and results.

a) Formulate all elementary relations for the interior of the two-port.

Now determine the six two-port matrices of the two-port.

b) conductance matrix $\boldsymbol{G}$

c) resistance matrix $\boldsymbol{R}$

d) hybrid matrix $\boldsymbol{H}$

e) inverse hybrid matrix $\boldsymbol{H}'$.

f) transmission matrix $\boldsymbol{A}$.

g) inverse transmission matrix $\boldsymbol{A}'$.

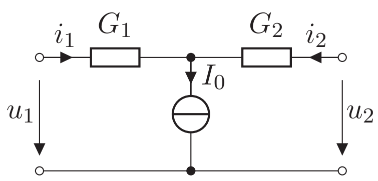
For strictly linear two-ports, the different elements of the two-port matrices can be measured with the help of a particular connection of the two-port. Compare the results with those of above method.

h) Derive the necessary connection to measure the different elements of the hybrid matrix of a strictly linear two-port. Find the hybrid matrix $\boldsymbol{H}$.

i) Derive the connections such that the different elements of the transmission matrix of a strictly linear two-port can be measured. Determine the transmission matrix $\boldsymbol{A}$.

3.4 Linear Two-Port

We consider the following two-port $\mathcal{T}$ with $G_1 = 1\,\mathrm{S}$, $G_2 = 2\,\mathrm{S}$, and $I_0 = 4\,\mathrm{A}$.



a) Perform the necessary number of measurements to be able to formulate the parametric representation of the two-port $\mathcal{T}$.

b) Rearrange the parametric representation such that you obtain the explicit representation of the form $\boldsymbol{i} = \boldsymbol{G}\boldsymbol{u} + \boldsymbol{i}_G$.

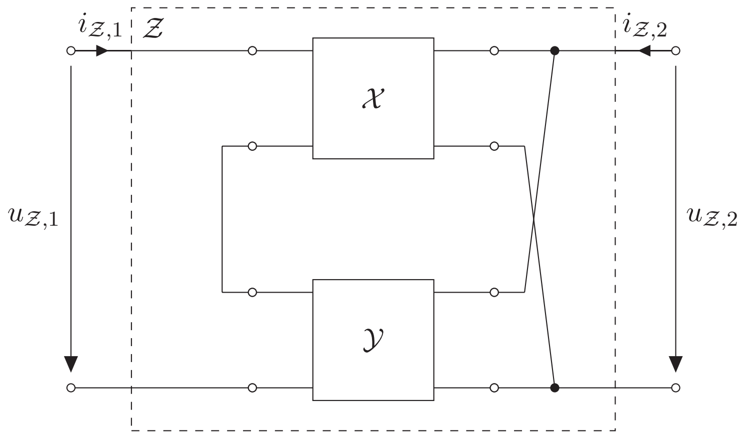
c) Find the conductance matrix $\boldsymbol{G}_0$ for the case that $I_0 = 0\,\mathrm{A}$. What do you observe?

d) Determine the currents i_1 and i_2 for $I_0 = 4\,\mathrm{A}$ if the two ports are connected to short circuits. What do you observe?

e) Draw the equivalent circuit diagram of the circuit.

3.5 Series-Parallel Connection

Consider the following connection of two two-ports.



The two-ports $\mathcal{X}$ and $\mathcal{Y}$ are strictly linear two-ports with the hybrid matrices

$$\boldsymbol{H}_{\mathcal{X}} = \begin{bmatrix} 1\,\Omega & 0 \\ 0.5 & 1\,\mathrm{S} \end{bmatrix} \qquad \boldsymbol{H}_{\mathcal{Y}} = \begin{bmatrix} -1\,\Omega & 0.5 \\ -1.0 & -1\,\mathrm{S} \end{bmatrix}$$

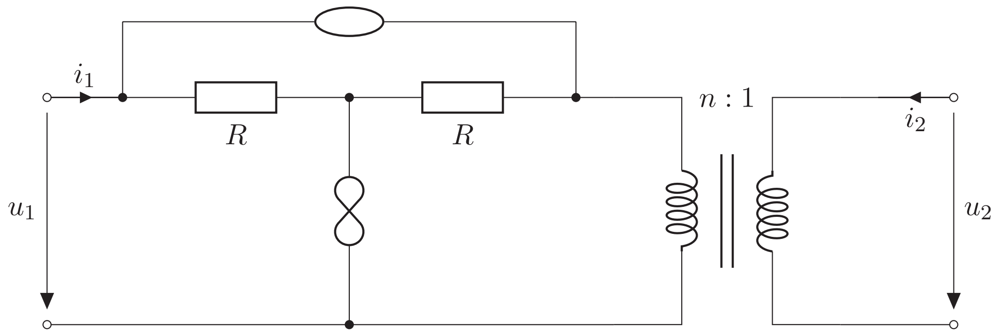
a) What conditions must be fulfilled such that it is possible to obtain the hybrid matrix of $\mathcal{Z}$ from the given hybrid matrices?

b) Assume that the necessary conditions are fulfilled and find the hybrid matrix of $\mathcal{Z}$ depending on $\boldsymbol{H}_{\mathcal{X}}$ and $\boldsymbol{H}_{\mathcal{Y}}$.

c) What type of two-port behaves identically to $\mathcal{Z}$? Give the parameter characterizing that two-port.

3.6 Two-Port with Nullor

The following two-port results from the connection of a NIC and a transformer.



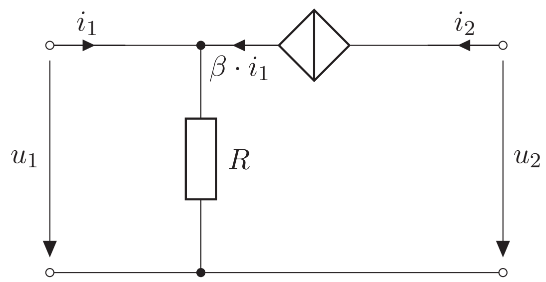
a) What is the connection between the NIC and the transformer?

b) Determine the transmission matrix $\boldsymbol{A}$ of the two-port. What type of two-port is it?

c) Is the two-port reciprocal?

3.7 Properties of Two-Ports

We examine the following circuit.



a) What kind of controlled source contains above circuit? What unit has β?

b) Determine the conductance matrix $\boldsymbol{G}$.

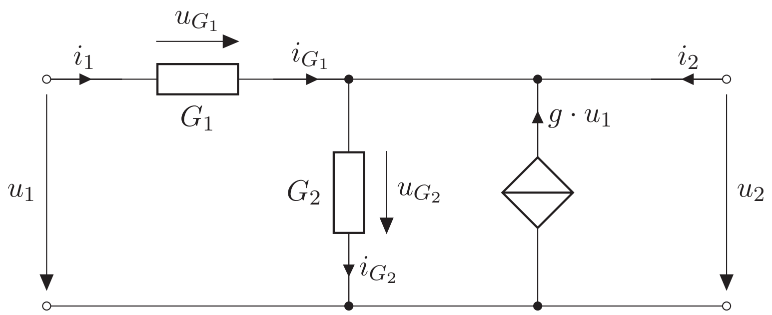
c) Find the value for β such that the two-port is reciprocal.

d) Is the two-port symmetric for a particular β?

e) Does the resistance matrix $\boldsymbol{R}$ exist?

3.8 Two-Port with Controlled Source

Given is the following two-port.



a) Determine u_{G_1} depending on u_1 and u_2.

b) Find i_1 depending on u_1 and u_2.

c) What are u_{G_2} and i_{G_2} depending on u_2.

d) Determine i_2 depending on u_1 and u_2.

e) Give the conductance matrix $\boldsymbol{G}$ of the two-port.

f) Determine g such that the two-port is reciprocal.

g) Is it possible to choose g such that the two-port is symmetric?

h) What condition must be fulfilled such that the resistance matrix $\boldsymbol{R}$ exists?

In the following subproblems, the parametric representation is considered. In the first step, two measurements are performed to obtain the basis matrix $\left[\begin{smallmatrix}\boldsymbol{U}\\ \boldsymbol{I}\end{smallmatrix}\right]$. To this end, two appropriate connections of the two-port must be used.

i) Does the connection to current sources make sense? Justify your answer.

j) Give two connections to determine the basis matrix. Fill out the following table.

	port 1	port 2
measurement 1		
measurement 2		

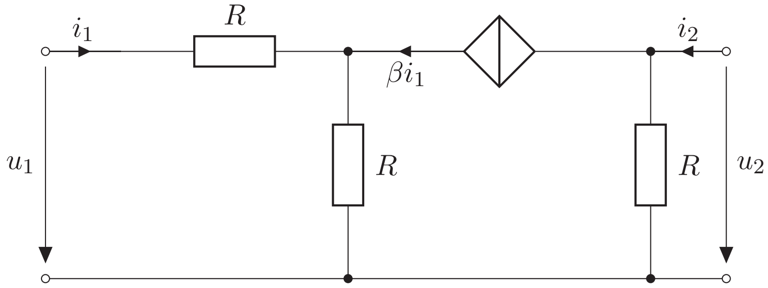
k) Determine the basis matrix $\begin{bmatrix} \boldsymbol{U} \\ \boldsymbol{I} \end{bmatrix}$.

l) Determine the inverse transmission matrix $\boldsymbol{A}'$ with the help of the basis matrix.

m) Use the basis matrix $\begin{bmatrix} \boldsymbol{U} \\ \boldsymbol{I} \end{bmatrix}$ to show that the two-port is lossy.

3.9 Representation of Two-Ports

Given is the two-port $\mathcal{F}$.

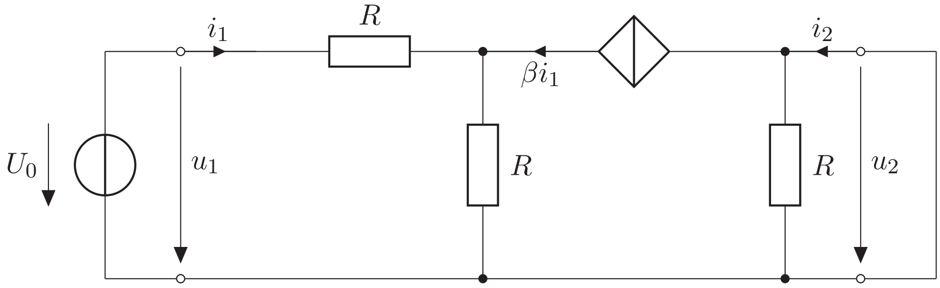


a) Give the general relationship between the vector of port voltage and port currents $\begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{i} \end{bmatrix} \in \mathcal{F}$ of a two-port and its basis matrix $\begin{bmatrix} \boldsymbol{U} \\ \boldsymbol{I} \end{bmatrix}$? (Hint: $\boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ and $\boldsymbol{i} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$)

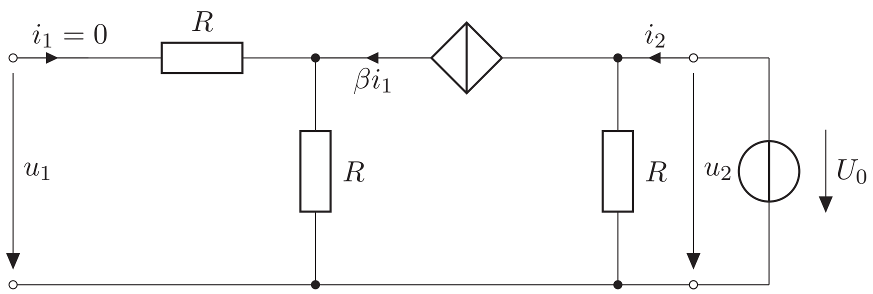
b) What is the general condition for a two-port such that a parametric representation with a basis matrix exists?

c) When does the basis matrix of linear two-ports exist?

d) **Case 1:** Determine all port currents and voltages if the two-port is connected as follows:



e) **Case 2:** Find all port currents and voltages if the two-port is connected as follows:



f) Give the basis matrix of the two-port using the results of the Subproblems d) and e) an such that $\mathcal{F} = \text{Bild}\left[\begin{smallmatrix}\boldsymbol{U}\\ \boldsymbol{I}\end{smallmatrix}\right]$.

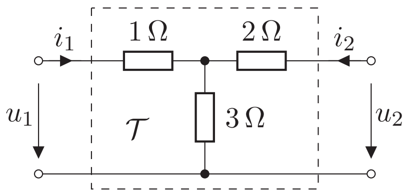
g) Find the hybrid matrix with the help of the basis matrix.

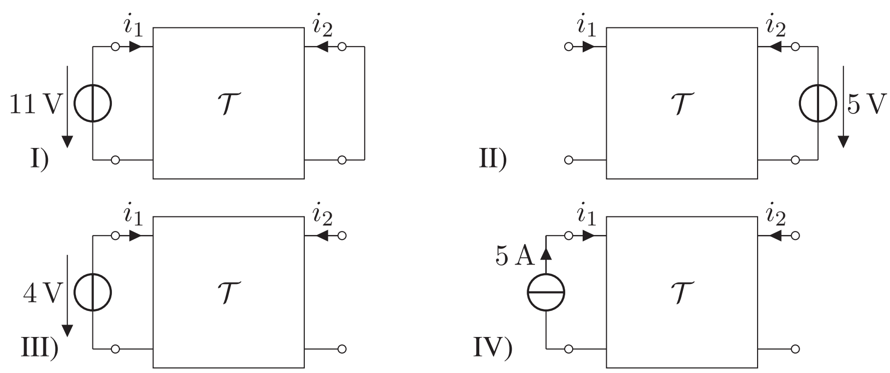
h) When does the hybrid matrix of the two-port exist?

i) With the basis matrix, determine whether the two-port is reciprocal or not.

3.10 Operating Point Vectors of Strictly Linear Two-Ports

Consider the following T-element $\mathcal{T}$ which is connected in four different ways.





a) Discuss the voltage divider for the series connection of two strictly linear resistors.

b) Discuss the current divider for the parallel connection of two strictly linear resistors.

c) Find the operating point vectors $[u_1, u_2, i_1, i_2]^{\mathrm{T}}$ for the different cases I)–IV).

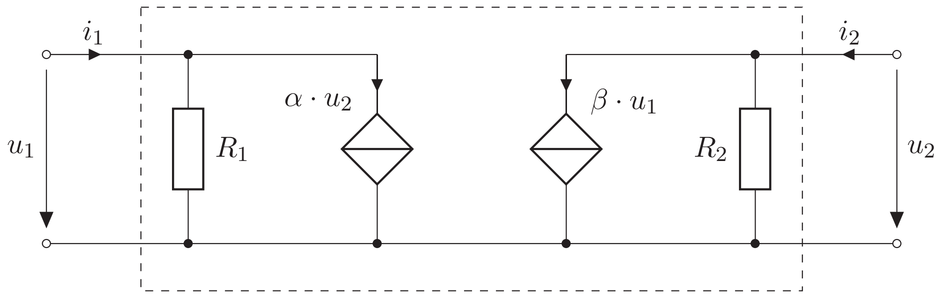
d) Show that the operating point vectors of the circuit III and IV can obtained as the linear combination of the first two measurements.

e) Determine the two-port matrices $\boldsymbol{R}$, $\boldsymbol{G}$, and $\boldsymbol{A}$ for the given two-port.

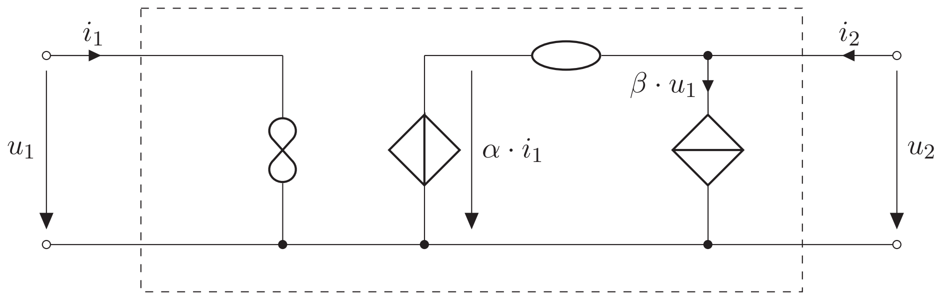
3.11 Representation of Two-Ports

Without calculations, give an explicit two-port representation for following circuits.

a)



b)

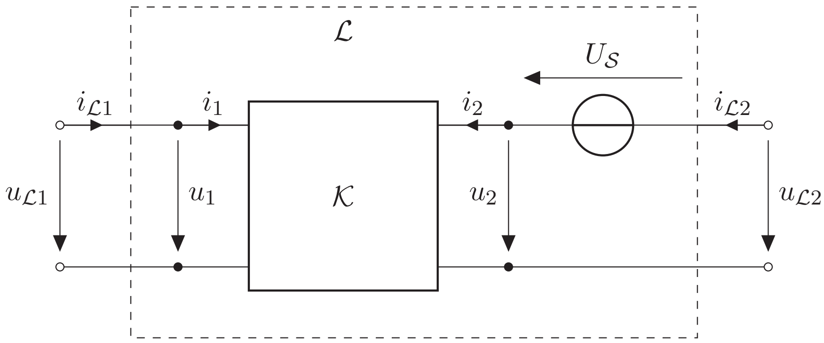


The hybrid representation of the two-port $\mathcal{K}$ is given by

$$\begin{bmatrix} u_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} R_1 & \alpha \\ \beta & \frac{1}{R_2} \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ u_2 \end{bmatrix} = \boldsymbol{H}_{\mathcal{K}} \cdot \begin{bmatrix} i_1 \\ u_2 \end{bmatrix} .$$

c) Find the equivalent circuit diagram for the two-port $\mathcal{K}$ employing only resistors and controlled sources.

The two-port $\mathcal{L}$ results from connecting the two-port $\mathcal{K}$ with the voltage source $U_{\mathcal{S}}$.



d) Determine i_1, i_2, u_1, and u_2 depending on $i_{\mathcal{L}1}$, $i_{\mathcal{L}2}$, $u_{\mathcal{L}1}$, $u_{\mathcal{L}2}$, and $U_{\mathcal{S}}$.

e) Substitute your results into the hybrid representation of $\mathcal{K}$. Formulate the resulting equations in matrix-vector notation.

f) Find the two sources $U_{\mathcal{E}}$ and $I_{\mathcal{E}}$ such that

$$\begin{bmatrix} u_{\mathcal{L}1} \\ i_{\mathcal{L}2} \end{bmatrix} = \boldsymbol{H}_{\mathcal{K}} \begin{bmatrix} i_{\mathcal{L}1} \\ u_{\mathcal{L}2} \end{bmatrix} + \begin{bmatrix} U_{\mathcal{E}} \\ I_{\mathcal{E}} \end{bmatrix}.$$

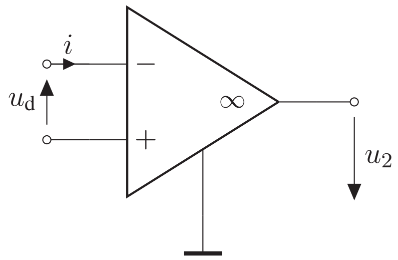
g) Draw the corresponding circuit diagram. (Hint: It is not necessary to include the internal structure of $\mathcal{K}$.)

Exercise 4

Operational Amplifier

4.1 Ideal Op-Amp

Given is the following ideal Op-Amp.



a) What is the assumption for the input current i?

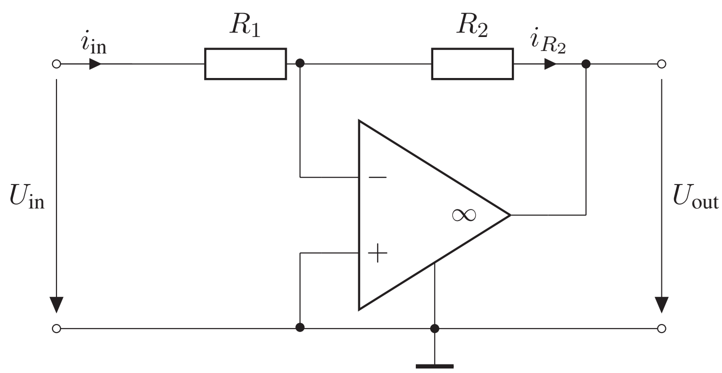
b) Give the mathematical relationship between u_d and u_2 and draw the characteristic in the u_d-u_2-diagram.

The characteristic of the Op-Amp can be divided into three regions.

c) Find the equivalent circuit diagrams for the three regions.

4.2 Amplifier Circuit

Consider the following circuit.



The Op-Amp is operated in the linear region.

a) What is the relationship between i_{in} and i_{R_2}?

b) Express u_{in} depending on i_{in}.

c) Find u_{out} depending on i_{in} aus.

d) Determine u_{in} depending on u_{out}.

e) What is the voltage gain ν of the circuit? What is the purpose of the circuit?

4.3 Controlled Sources 1

a) Give the transmission representation of a VCVS.

b) What Op-Amp circuit can be used to get a negative transfer factor? Draw the circuit.

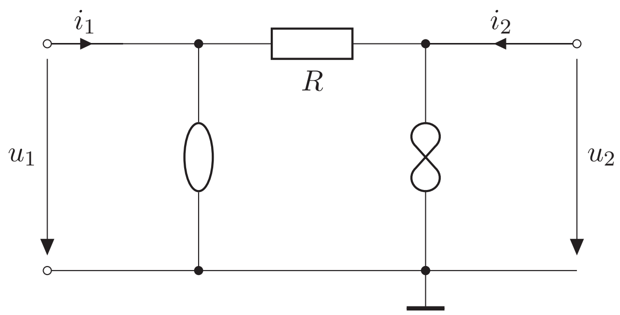
c) What is the disadvantage of this circuit compared to an ideal VCVS?

d) Name the Op-Amp circuit that can be used to remove this issue. Draw the respective circuit.

e) How must the circuits of the two previous subproblems be connected to get an ideal VCVS?

4.4 Controlled Sources 2

Given is following circuit.



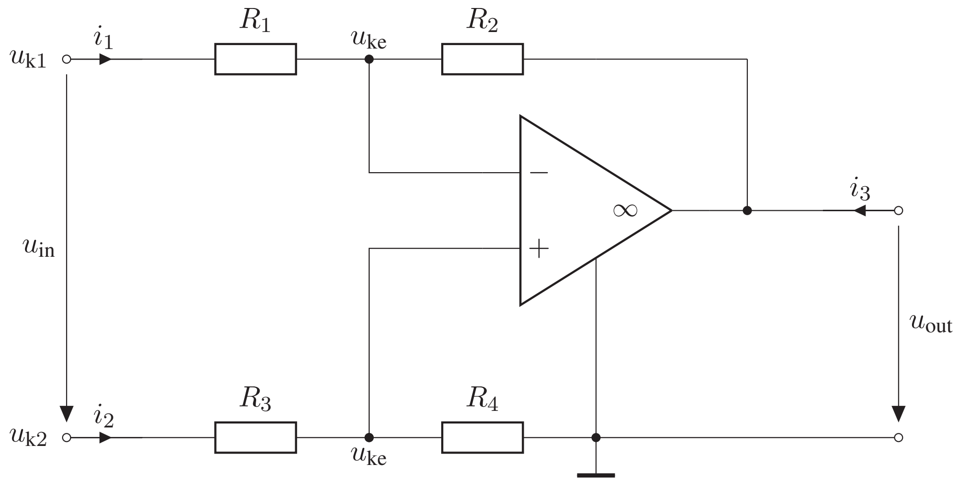
a) Find u_2 depending on i_1.

b) What is the purpose of this circuit?

c) Draw the Op-Amp circuit with the same behaviour.

4.5 Difference Amplifier

The following circuit is a summer with an Op-Amp operated in the linear region.



This circuit is called potential difference amplifier if the output voltage u_{out} only depends on the difference $u_{in} = u_{k1} - u_{k2}$ of the two input potentials, i.e., $u_{out} = v_d u_{in}$.

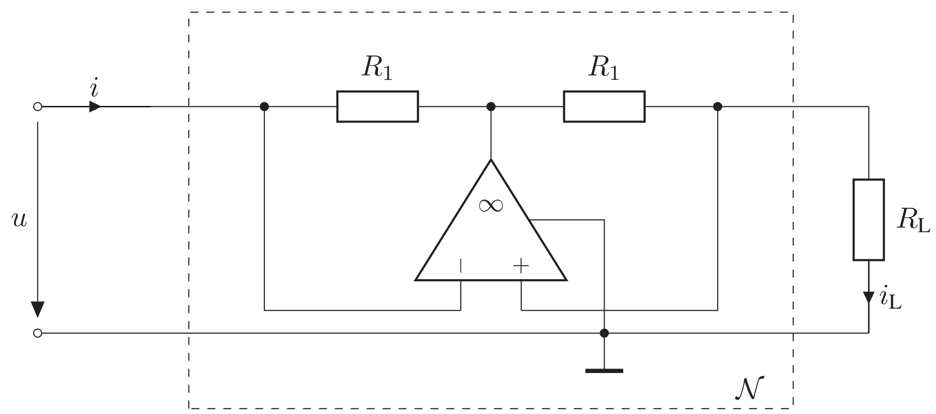
a) Determine u_{out} depending on u_{k1} and u_{k2}. Use u_{ke} for the potential of the Op-Amp inputs.

b) When behaves the circuit like a potential difference amplifer? Give the resulting difference gain v_d.

c) Give the number of degrees freedom available when dimensioning the circuit.

4.6 Negative-Immittance-Converter

Given is following circuit.



a) Replace the Op-Amp by the nullor model and draw the equivalent circuit diagram.

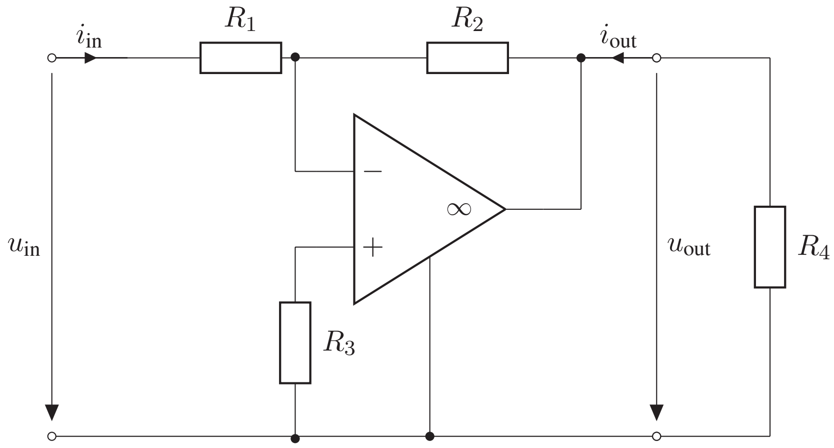
b) Give the relationship between i and i_{L}.

c) Find u depending on i.

d) What is the purpose of the circuit?

4.7 Operating Regions of Op-Amps

Consider the following Op-Amp circuit.



Firstly, the Op-Amp is operated in the **linear region**.

a) Draw the equivalent circuit diagram for this region and give i_{in} depending on u_{in}.

b) Find u_{out} depending on u_{in}. What functionality has the circuit in this region?

c) What is the range for u_{in} such that the Op-Amp is operated in the linear region?

Secondly, the Op-Amp is operated in **positive saturation**.

d) What is the range of u_{in} such that the Op-Amp is operated in positve saturation?

e) Draw the corresponding equivalent circuit diagram for this region.

Lastly, the Op-Amp is operated in **negative saturation**.

f) What is the range for u_{in} such that the Op-Amp is operated in negative saturation?

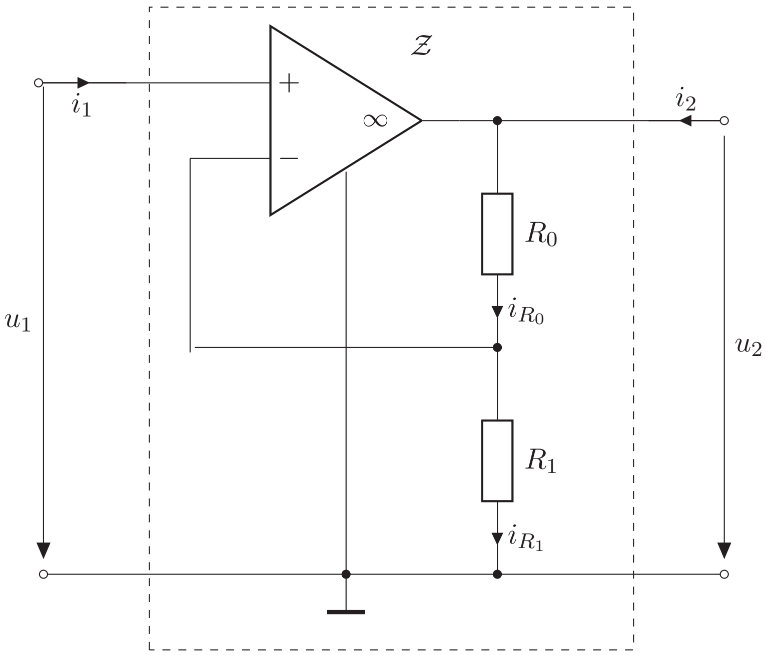
g) Draw the equivalent circuit diagram for this region.

Now the element values are given:
$R_1 = 1\,\Omega$, $R_2 = 2\,\Omega$, $R_3 = R_4 = 5\,\Omega$, and $U_{\text{sat}} = 5\,\text{V}$.

h) Draw the input characteristic of the given circuit.

4.8 Two-Port Amplifier

Given is the two-port $\mathcal{Z}$ which results from the following circuit with an Op-Amp.



Let the saturation voltage of the Op-Amp be denoted as U_{sat}.
Nevertheless, assume initially that the Op-Amp is operated in the linear region.

a) Find i_1 depending on u_1 and the element values.

b) Give the relationship between i_{R_0} and i_{R_1}.

c) Determine u_2 depending on u_1.

d) What is the maximum value of u_2 and what is its minimum value?

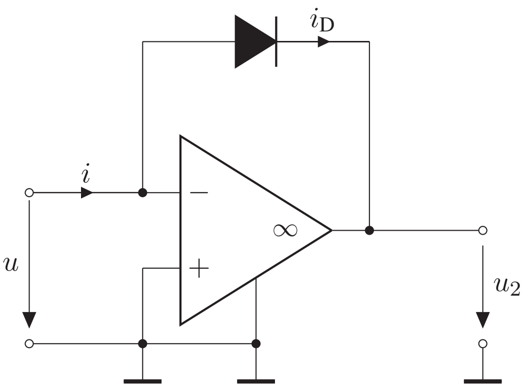
e) Draw the transfer characteristic in the u_1-u_2-plane. Label the axes.

Now the input terminals of the Op-Amp are exchanged.

f) Draw the resulting transfer characteristic in the u_1-u_2-plane. Does this connection of the Op-Amp make sense?

4.9 Ideal Diode

Given is following circuit.



The saturation voltage of the Op-Amp is $U_{sat} = 12.5\,\mathrm{V}$ and the characteristic of the pn-junction diode is

$$i_D = I_s \left(\exp\left(\frac{u_D}{U_T}\right) - 1 \right)$$

with $U_T = 12.5\,\mathrm{mV}$ and $I_s = 10^{-17}\,\mathrm{A}$.

a) Find i depending on u if the Op-Amp is in positive saturation.

b) Determine i depending on u when the Op-Amp is in negative saturation.

c) What is i depending on u if the Op-Amp is in the linear region?

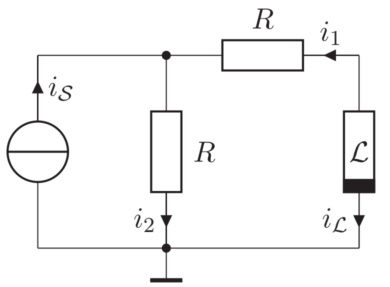
d) Draw the input characteristic in the u-i-diagram.

Exercise 5

General Circuit Analysis

5.1 Kirchhoff's Laws

Consider the following circuit with non-linear load $\mathcal{L}$.



a) What is the number of nodes and what is the number of branches of the circuit?

b) Give the number of linearly independent KCL equations. What is thus the size of the node incidence matrix $\boldsymbol{A}$?

c) Give the number of linearly independent KVL equations. What is therefore the size of the loop incidence matrix $\boldsymbol{B}$?

d) Find the loop incidence matrix $\boldsymbol{B}$ of the given circuit.

e) Determine the node incidence matrix $\boldsymbol{A}$ of the circuit.

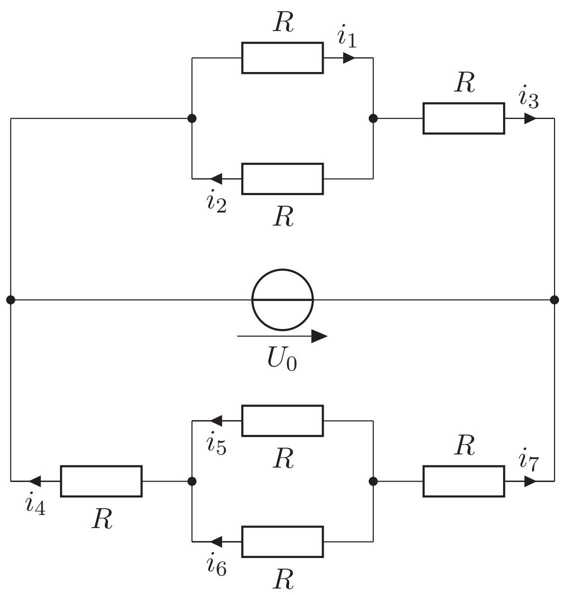
f) What is the rank of $\boldsymbol{A}$, i.e., what is the number of linearly independent row vectors in $\boldsymbol{A}$? Justify your answer.

Based on the node incidence matrix $\boldsymbol{A}$, a different form of the KVL equations can be found.

g) Formulate this alternative form of the KVL.

5.2 Kirchhoff's Laws

Given is the following circuit. The strictly linear resistors all have resistance R.



a) What is the number of nodes? Mark the nodes in the circuit.

b) Determine the node incidence matrix $\boldsymbol{A}$ of the circuit.

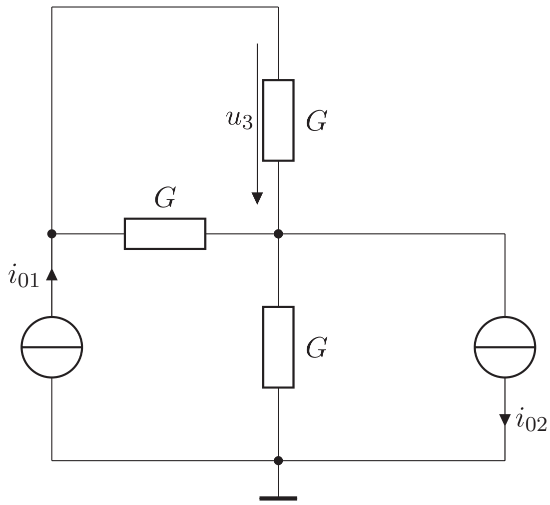
c) What is the number of linearly independent KVL equations?

d) Find the loop incidence matrix $\boldsymbol{B}$.

e) Combine the resistors to a single resistor R_{tot}. Draw the resulting equivalent circuit diagram.

5.3 General Network Analysis

Given is the following resistive circuit. The strictly linear resistors all have conductance G.



a) Label the branches by introducing branch voltages.

b) What is number of linearly independent KVL equations? Find the KVL equations in the form $\boldsymbol{B}\boldsymbol{u} = \mathbf{0}$.

c) What is the number of linearly independent KCL equations? Determine the KCL equations in the form $\boldsymbol{A}\boldsymbol{i} = \mathbf{0}$.

By Tellegen's law, $\boldsymbol{A}\boldsymbol{B}^{\mathrm{T}} = \mathbf{0}$.

d) Verify Tellegen's law.

e) Find a representation of the circuit elements in the form $[\boldsymbol{M}, \boldsymbol{N}] \left[\begin{smallmatrix} \boldsymbol{u} \\ \boldsymbol{i} \end{smallmatrix} \right] = \boldsymbol{e}$.

f) Give the condition for the circuit elements such that $\boldsymbol{N}$ is invertible.

g) Collect the equations of the previous subproblems and formulate the tableau equation system.

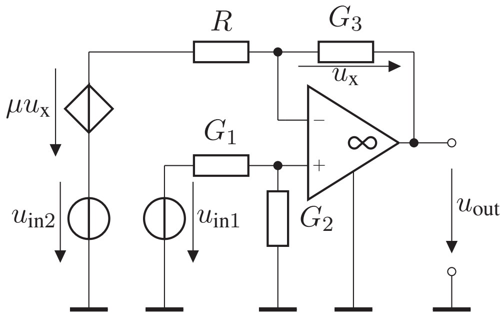
h) Is the equation system in Subproblem g) uniquely solvable?

i) Formulate the nodal analysis for the circuit. To this end, mark the node voltages.

j) Find u_3.

5.4 Nodal Analysis

Consider the following circuit where the Op-Amp is operated in the linear region. With the nodal analysis, the output voltage u_{out} depending on u_{in1} and u_{in2} will be determined.



a) Give the elements which are not voltage-controlled. Discuss how these elements are transformed to voltage-controlled elements.

b) Draw the equivalent circuit diagram where the non-voltage-controlled elements are replaced by voltage-controlled elements. Include the equivalent circuit diagram of the Op-Amp according to the linear region.

c) Number the nodes of the equivalent circuit diagram.

d) Determine u_x and u_{out} depending on the node voltages.

e) Find the node conductance matrix $\boldsymbol{G}'_k$, the node voltage vector $\boldsymbol{u}'_k$, and the node current source vector $\boldsymbol{i}'_q$ ignoring the Op-Amp.

f) Incorporate the Op-Amp to obtain $\boldsymbol{G}_k$, $\boldsymbol{u}_k$, and $\boldsymbol{i}_q$.

g) Solve the resulting equation system.

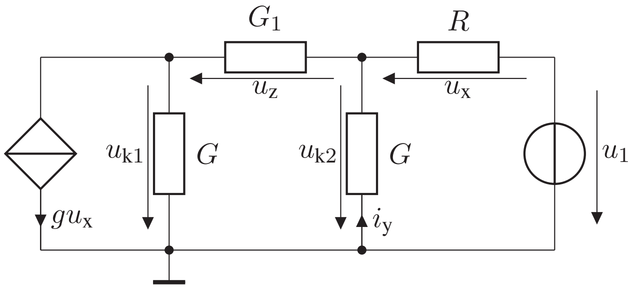
h) Determine u_{out} depending on u_{in1} and u_{in2}.

i) Is it possible to choose a value for the gain factor μ such that u_{out} is independent of u_{in2}? If yes, find the corresponding μ.

j) Is it possible to choose a value for the gain factor μ such that u_{out} is independent of u_{in1}? If yes, find the corresponding μ.

5.5 Nodal Analysis

Given is the following circuit with four strictly linear resistors, a VCCS, and a voltage source.



a) Express u_x depending on the node voltages and u_1.

b) Find i_y depending on the node voltages and u_1.

c) Determine u_z depending on the node voltages and u_1.

d) Change the circuit such that the nodal analysis can be employed.

e) Formulate the nodal analysis.

f) Solve the resulting equation system.

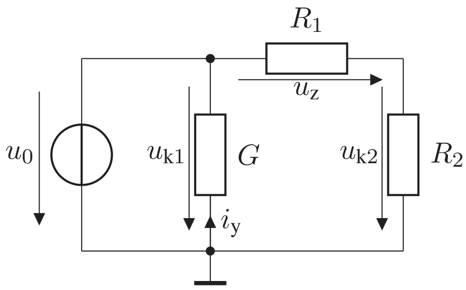
g) What do you obtain for i_y and u_z?

h) Can g chosen such that i_y is zero? If yes, give the according value of g.

i) Can g chosen such that u_z is zero? If yes, give the according value of g.

5.6 Nodal Analysis

Consider the following circuit with three strictly linear resistors and a voltage source.



a) Find i_y depending on u_0.

b) Determine u_z depending on u_0.

c) Which element is the reason that the nodal analysis cannot be employed? Justify your answer.

d) Consider the series connection of the two resistors with conductances G_1 and G_2. What is the total conductance G_{tot}?

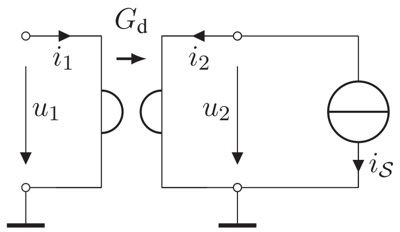
e) Give G_{tot} if $G_2 = -G_1$. Give also an equivalent circuit diagram of G_{tot}.

f) Include G and $-G$ connected in series with the voltage source u_0. Redraw the resulting circuit such that the nodal analysis can be used.

g) Formulate the nodal analysis.

h) Solve the resulting equation system. What do you obtain for i_{y} and u_{z}?

Alternatively, a gyrator with duality constant G_{d} can employed.



i) Find the relationship between u_1 and $i_{\mathcal{S}}$. Give the equivalent circuit diagram for port 1.

j) Replace the voltage source u_0 by a similar gyrator circuit.

k) Formulate the nodal analysis.

l) Solve the resulting equation system. What do you obtain for i_{y} and u_{z}?

Exercise 6

Reactive Elements

6.1 Strictly Linear Reactances

Consider the voltage source $u(t) = U_0 \sin(\omega t)$.

Firstly, the voltage source is connected to a strictly linear capacitor C at time $t = 0$.

a) Find the current $i(t)$ flowing through the capacitor for $t \geq 0$.

b) What is the flux $\Phi(t)$ for $t \geq 0$ under the assumption that $\Phi(0) = \Phi_0$?

c) Determine the charge $q(t)$.

d) What is therefore the initial condition $q(0)$?

Secondly, the voltage source is connected to a strictly linear inductor L at time $t = 0$.

e) Find the current $i(t)$ flowing through the inductor for $t \geq 0$ assuming that $i(t_0 = 0) = -\frac{U_0}{\omega L} \cos(\omega t_0)$.

f) Give the flux $\Phi(t)$ for $t \geq 0$.

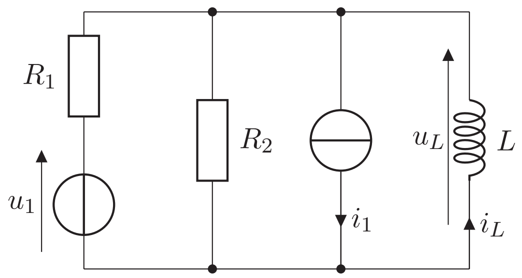
g) Determine the charge $q(t)$ for $t \geq 0$ under the assumption that $q(0) = \frac{U_0}{\omega^2 L}$.

Exercise 7

Linear First-Order Circuits

7.1 Linear First-Order RL-Circuits

Given is the following ciruit with linear elements.



Firstly, assume that the voltage and current sources are switched off ($u_1 = 0$, $i_1 = 0$). The current through the inductor is I_0 at time t_0.

a) Determine the voltage across and the current through the inductor for $t \geq t_0$.

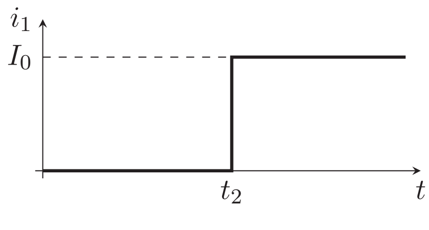
b) Sketch the voltage and the current of the inductor for $R_1 = 1\,\mathrm{k\Omega}$, $R_2 = 2\,\mathrm{k\Omega}$, $L = 100\,\mathrm{mH}$, and $I_0 = 50\,\mathrm{mA}$. Additionally, $t_0 = 0.1\,\mathrm{ms}$.

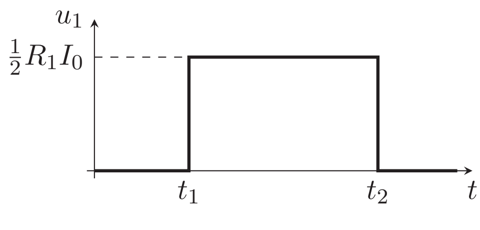
Secondly, assume following excitation: $u_1 = 0.5 R_1 I_0$, $i_L(t_0) = I_0$, and $i_1 = I_0$.

c) Find the voltage and current of the inductor for $t \geq 0$.

d) Sketch the voltage and the current of the inductor for $R_1 = 1\,\mathrm{k\Omega}$, $R_2 = 2\,\mathrm{k\Omega}$, $L = 100\,\mathrm{mH}$, and $I_0 = 50\,\mathrm{mA}$. Additionally, $t_0 = 0.1\,\mathrm{ms}$.

Now the excitations are piecewise linear. Again, $i_L(t_0) = I_0$.





e) Determine the voltage and the current of the inductor for for $t > t_0$ ($t_0 = 0.1$ms).

f) Sketch the voltage and the current of the inductor for $R_1 = 1\,\mathrm{k\Omega}$, $R_2 = 2\,\mathrm{k\Omega}$, $L = 100\,\mathrm{mH}$, and $I_0 = 50\,\mathrm{mA}$. Additionally, $t_0 = 0.1\,\mathrm{ms}$, $t_1 = 0.6\,\mathrm{ms}$, $t_2 = 1.1\,\mathrm{ms}$, and $t_3 = 1.6\,\mathrm{ms}$.

In the following, consider the following excitations u_1 and i_1

$$u_1(t) = 0.5 R_1 I_0 \sin(\omega t), \qquad i_1(t) = I_0$$

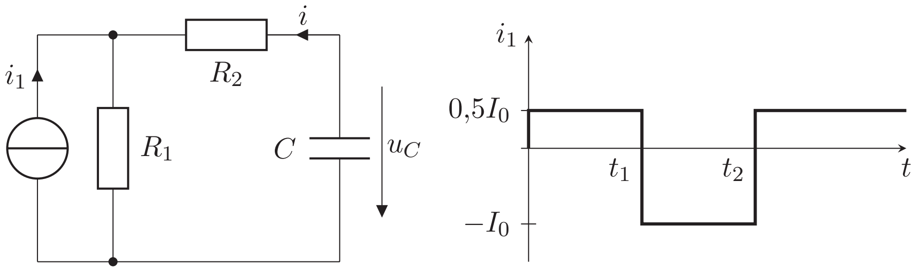
and $i_L(t_0) = I_0$.

g) Find the voltage and current of the inductor for $t > t_0$ ($t_0 = 0\mathrm{s}$).

$$\text{Hint:} \quad \int \exp(ax)\sin(bx)\,\mathrm{d}x = \frac{\exp(ax)}{a^2+b^2}\left(a\sin(bx) - b\cos(bx)\right)$$

7.2 RC-Circuit with Piecewise Linear Excitation

Consider the following circuit with piecewise linear excitation i_1.



a) Find the representation of the linear part as a linear source.

b) What is the time constant τ of the circuit?

c) Determine the voltage $u_C(t)$ for $t \geq 0$ when $u_C(0) = 0$ and where $t_1 = \tau$ and $t_2 = 2\tau$.

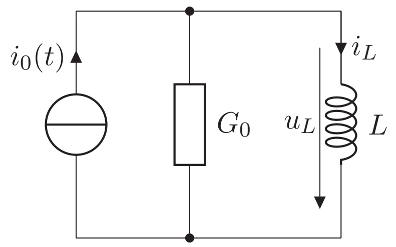
d) Sketch $u_C(t)$.

e) Find $i_C(t)$.

f) Sketch $i_C(t)$.

7.3 General Excitation

Consider the following circuit.



The excitation is given by

$$i_0(t) = \begin{cases} 0 & \text{for } t < 0, \\ I_0\,\mathrm{e}^{-\frac{t}{G_0 L}}\cos(\omega t) & \text{for } t \geq 0. \end{cases}$$

a) Determine $i_L(t)$ for $t > 0$ assuming that $i_L(0) = I_0$. Use the abbreviation $\tau = G_0 L$.

7.4 Parasitic Effect

Consider a linear capacitor C which is connected to a voltage source U. The voltage of the voltage source jumps at time $t_0 = 0\,\mathrm{s}$ from $U = 0\,\mathrm{V}$ to $U = U_0$.

a) Give the current assuming an ideal capacitor. Justify your answer.

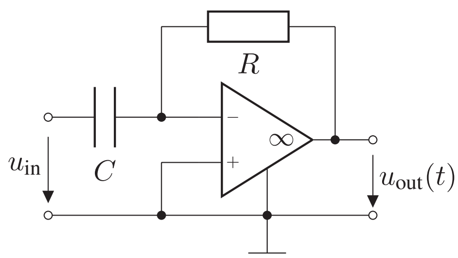
b) Describe the behavior for a real capacitor.

c) Develop a model for the real capacitor such that this behavior is represented. Find the resulting current for $u_C(t < 1\,\mathrm{s} = 0\,\mathrm{V}$.

d) Sketch the current and voltage of the capacitor. Skizzieren Sie alle Ströme und Spannungen.

7.5 Integrator and Differentiator

Given is following circuit. The Op-Amp is operated in the linear region.



a) Find the output voltage $u_{\text{out}}(t)$ depending on the input voltage u_{in} for $t > t_0$.

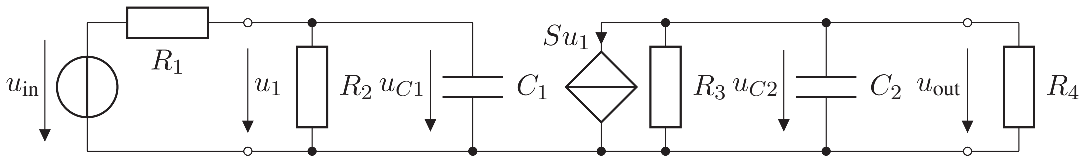
b) Exchange the capacitor and the resistor. Determine the output voltage $u_{\text{out}}(t)$ for $t > t_0$ and $u_C(t_0) = 0$.

Exercise 8

Linear Second-Order Circuits

8.1 Formulation of State Equations (1)

Given is following circuit.

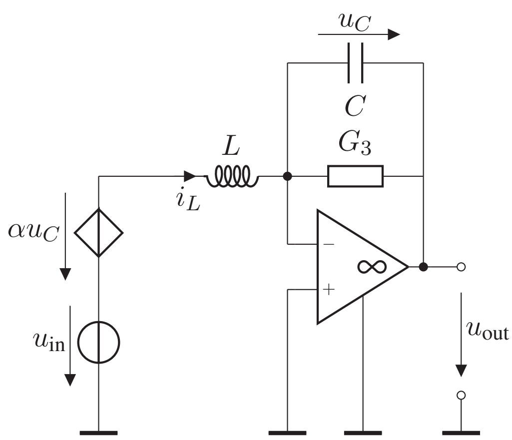


a) Formulate the state equations. Give $\boldsymbol{x}$, $\boldsymbol{A}$, $\boldsymbol{b}$, and v.

b) Find the output equation. Give $\boldsymbol{c}^{\mathrm{T}}$ and d.

8.2 Formulation of State Equations (2)

Given is following circuit with an Op-Amp operated in the linear region.

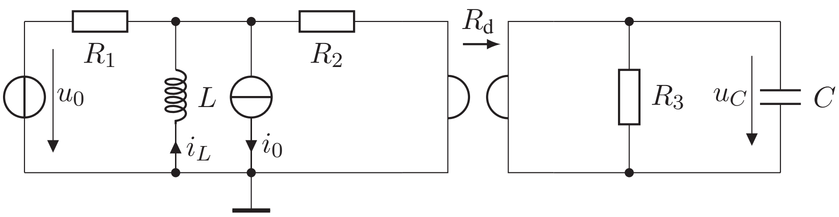


a) Formulate the state equations. Give $\boldsymbol{x}$, $\boldsymbol{A}$, $\boldsymbol{b}$, and v an.

b) Find the output equation. Give $\boldsymbol{c}^{\mathrm{T}}$ and d.

8.3 Formulation of State Equations (3)

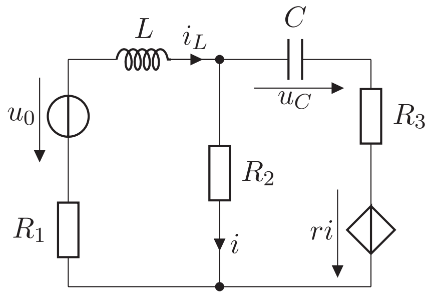
Given is the following circuit with a gyrator.



a) Find the parameters $\boldsymbol{x}$, $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{v}$.

8.4 Formulation of State Equations (4)

Consider the following circuit with $R_1 = R_2 = R_3 = r = R$.



a) Formulate the state equations.

b) Give $\boldsymbol{x}$, $\boldsymbol{A}$, $\boldsymbol{b}$, and v.

8.5 Solution of Homogeneous State Equations (1)

Consider the circuit in Problem 8.1.

Given are the following element values: $R_1 = 1/3\,\Omega$, $R_2 = 1\,\Omega$, $R_3 = 2\,\Omega$, $R_4 = 2\,\Omega$, $C_1 = 1\,\mathrm{F}$, $C_2 = 1\,\mathrm{F}$, $S = -3\,\mathrm{S}$, and $u_\mathrm{in} = 0\,\mathrm{V}$.

a) Find the eigenvalues of the state matrix.

b) Determine two linearly independent eigenvectors whose second component ist $-3\,\frac{1}{\mathrm{s}}$.

c) Give the resulting general solution and the general solution for $u_{C1}(t_0) = 1\,\mathrm{V}$ and $u_{C2} = -1.5\,\mathrm{V}$.

d) Draw $u_{C1}(t)$ and $u_{C2}(t)$.

e) Sketch the resulting phase portrait in the u_{C1}-u_{C2}-plane.

8.6 Solution of Homogeneous State Equations (2)

Consider the circuit in Problem 8.1.

Given are the following element values: $R_1 = 1/3\,\Omega$, $R_2 = 1\,\Omega$, $R_3 = 2\,\Omega$, $R_4 = 2\,\Omega$, $C_1 = -1\,\mathrm{F}$, $C_2 = -1\,\mathrm{F}$, $S = 3\,\mathrm{S}$, and $u_{\text{in}} = 0\,\mathrm{V}$.

a) Find the eigenvalues of the state matrix.

b) Determine two linearly independent eigenvectors whose second component ist $-3\,\frac{1}{\mathrm{s}}$.

c) Give the resulting general solution and the general solution for $u_{C1}(t_0) = 1\,\mathrm{V}$ and $u_{C2} = -1\,\mathrm{V}$.

d) Draw $u_{C1}(t)$ and $u_{C2}(t)$.

e) Sketch the resulting phase portrait in the u_{C1}-u_{C2}-plane.

8.7 Solution of Homogeneous State Equations (3)

Consider the circuit in Problem 8.1.

Given are the following element values: $R_1 = 2\,\Omega$, $R_2 = 2\,\Omega$, $R_3 = 2\,\Omega$, $R_4 = 2\,\Omega$, $C_1 = 1\,\mathrm{F}$, $C_2 = -1\,\mathrm{F}$, $S = 3\,\mathrm{S}$, and $u_{\text{in}} = 0\,\mathrm{V}$.

a) Find the eigenvalues of the state matrix.

b) Determine two linearly independent eigenvectors whose second component is $-3\,\frac{1}{\mathrm{s}}$.

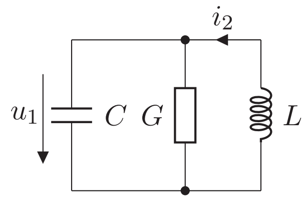
c) Give the resulting general solution and the general solution for $u_{C1}(t_0) = 1\,\mathrm{V}$ and $u_{C2} = -1\,\mathrm{V}$.

d) Draw $u_{C1}(t)$ and $u_{C2}(t)$.

e) Sketch the resulting phase portrait in the u_{C1}-u_{C2}-plane.

8.8 Solution of Homogeneous State Equations (4)

Consider the following circuit.



a) Formulate the state equation. Determine the state matrix $\boldsymbol{A}$ for $G = 4\,\mathrm{S}$, $C = 1\,\mathrm{F}$, and $L = 1/3\,\mathrm{H}$.

b) Find and sketch the solution for the initial conditions $u_1(t_0) = 1\,\mathrm{V}$ and $i_2(t_0) = -1\,\mathrm{A}$.

c) Sketch corresponding phase portrait in the x_1-x_2-plane.

d) Determine and sketch the solution for $G = -4\,\mathrm{S}$, $C = 1\,\mathrm{F}$, and $L = 1/3\,\mathrm{H}$. The initial conditions are $u_1(t_0) = 1\,\mathrm{V}$ and $i_2(t_0) = -1\,\mathrm{A}$.

e) Sketch the corresponding phase portrait in the x_1-x_2-plane.

f) Find and sketch the solution for $G = 1\,\mathrm{S}$, $C = 1\,\mathrm{F}$, and $L = -1/2\,\mathrm{H}$. The initial conditions are $u_1(t_0) = -1\,\mathrm{V}$ and $i_2(t_0) = -1.5\,\mathrm{A}$.

g) Sketch the corresponding phase portrait in the x_1-x_2-plane.

h) Determine and sketch the solution for $G = 0\,\mathrm{S}$, $C = 1\,\mathrm{F}$, and $L = 1\,\mathrm{H}$. The initial conditions are $u_1(t_0) = 1\,\mathrm{V}$ and $i_2(t_0) = 0\,\mathrm{A}$.

i) Sketch the corresponding phase portrait in the ξ_{real1}-ξ_{real2}-plane.

j) Find and sketch the solution for $G = 2\,\mathrm{S}$, $C = 1\,\mathrm{F}$, and $L = 1/2\,\mathrm{H}$. The initial conditions are $u_1(t_0) = -2\,\mathrm{V}$ and $i_2(t_0) = -2\,\mathrm{A}$.

k) Sketch the corresponding phase portrait in the ξ_{real1}-ξ_{real2}-plane.

Gegeben seien nun die Elementwerte $G = -2$ S, $C = 1$ F, and $L = 1/2$ H.

l) Determine and sketch the solution for $G = -2\,\mathrm{S}$, $C = 1\,\mathrm{F}$, and $L = 1/2\,\mathrm{H}$. The initial conditions are $u_1(t_0) = -2\,\mathrm{V}$ and $i_2(t_0) = 2\,\mathrm{A}$.

m) Sketch the phase portrait in the ξ_{real1}-ξ_{real2}-plane.

8.9 Solving Autonomous State Equations

Consider the circuit of Problem 8.1.

Given are the following element values: $R_1 = 1\,\Omega$, $R_2 = 1\,\Omega$, $R_3 = 2\,\Omega$, $R_4 = 2\,\Omega$, $C_1 = 1\,\mathrm{F}$, $C_2 = 1\,\mathrm{F}$, $S = -3\,\mathrm{S}$, and $u_{\text{in}} = 3\,\mathrm{V}$.

a) Transform the system into a homogeneous system with the state variables x_1' and x_2'. Give the resulting state equation.

b) Decouple the system of differential equations from Subproblem a) and give the decoupled general solution.

c) Transform back to the representation with x_1' and x_2'. Give the corresponding general solution. Employ eigenvectors whose second components are -3V. Sketch the phase portrait in the x_1'-x_2'-plane.

d) Find the general solutions for x_1 and x_2. Sketch the phase portrait in the x_1-x_2-plane.

e) Perform the same analysis for $C_1 = -1\,\mathrm{F}$ and $C_2 = 1\,\mathrm{F}$.

f) Perform the same analysis for $C_1 = -1\,\mathrm{F}$ and $C_2 = -1\,\mathrm{F}$.

g) Perform the same analysis for $C_1 = 2\,\mathrm{F}$ and $C_2 = 1\,\mathrm{F}$.

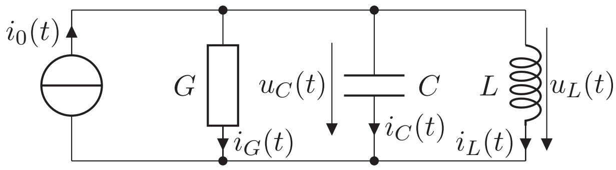
h) Can this circuit have complex eigenvalues? Justify your answer.

Exercise 9

Complex Phasor Analysis

9.1 RLC-Resonant Circuit

Consider the following parallel RLC-resonant circuit with a sinusoidal $i_0(t) = I_{0\text{m}} \cos(\omega t + \pi)$.



a) Formulate the state equation for this circuit. Give $\boldsymbol{A}$.

b) What are the eigenvalues of $\boldsymbol{A}$?

c) Give the relationship between $i_L(t)$ and $u_C(t)$.

d) Express the phasor I_L depending on the phasor U_C corresponding to $u_C(t)$.

e) Determine the phasor I_G depending on U_C.

f) Give I_0 corresponding to $i_0(t)$.

g) Find I_0 depending on U_C.

h) What is, therefore, U_C depending on I_0?

i) Determine the angular frequencies ω such that U_C is real-valued.

j) Give the values of U_C for these angular frequencies.

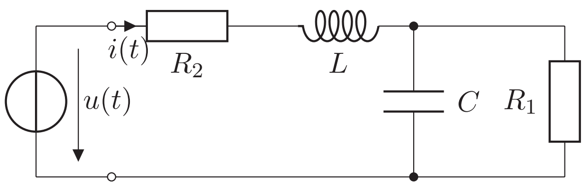
Thus, this circuit is a bandbass and one of these values for U_C corresponds to the maximum $U_{C,\text{max}}$ of $|U_C|$.

k) Perform a dual transform of the given circuit with the duality constant R_d. Draw the resulting circuit.

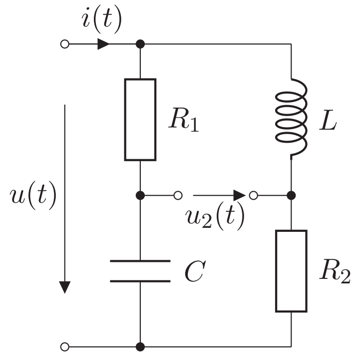
9.2 RLC-Circuits

Given are the following two circuits with an inductor $L = 318\,\mathrm{mH}$, a capacitor $C = 3.18\,\mu\mathrm{F}$, and two resistors $R_1 = 1000\,\Omega$ and $R_2 = 300\,\Omega$.

Note that $u(t)$ is sinusoidal with angular frequency $\omega = 2\pi f$ and that $3.18\pi = 10$.



Circuit A



Circuit B

a) Find $I(\mathrm{j}\,\omega)$ for circuit A and $U_2(\mathrm{j}\,\omega)$ for circuit B depending on $U(\mathrm{j}\,\omega)$.

b) Show that for $f = 50\,\mathrm{Hz}$, $I = \frac{1}{894\,\Omega}\,\mathrm{e}^{\mathrm{j}\,0.464}\,U$ for circuit A and $U_2 = (-0.4 - \mathrm{j}\,0.2)U$ for circuit B.
Note that $\arctan(0.5) = 0.464$, $\frac{4000}{\sqrt{20}} = 894$, and $\sqrt{0.2} = 0.447$.

Let $u(t) = 1\,\mathrm{V}\,\cos(\omega t + \frac{\pi}{4})$.

c) Give the corresponding phasor U.

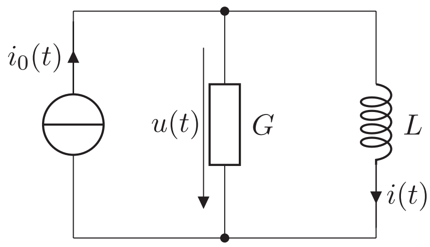
d) Determine the steady-state response of the current $i(t)$ for circuit A and of the voltage $u_2(t)$ for circuit B.

e) Determine the complex power for the circuit A.

f) What is the average power P_{W}?

9.3 GL-Stage

The following circuit is investigated with the complex phasor analysis.



a) Give the phasor I_0 for $i_0(t) = \hat{I}_0 \cos(\omega t)$ and also for $i_0(t) = \hat{I}_0 \sin(\omega t)$.

b) What ist the relationship between $i(t)$ and $u(t)$?

c) Express this relationship for the phasors I and U.

d) Based on the Kirchhoff's laws and Ohm's law, give I depending on U and I_0.

e) Find the phasor I depending on I_0.

In the following, $i_0(t)$ is the input to the circuit and $i(t)$ its output.

f) Show that the circuit is a lowpass, i.e., signals $i_0(t)$ with low frequency can pass and portions with high frequency are suppressed. To this end, investigate $|\frac{I}{I_0}|$ for $\omega = 0$, $\omega = \frac{1}{GL}$, and $\omega \to \infty$.

g) Determine the phasor I for $I_0 = \frac{1}{\sqrt{2}}(1 + \mathrm{j})\hat{I}_1$ and $\omega = \frac{1}{GL}$.

h) Find I for $I_0 = (1 - 0.5\,\mathrm{j})\hat{I}_2$ and $\omega = \frac{2}{GL}$.

i) Give $i(t)$ for both cases.

Now the input current is given by

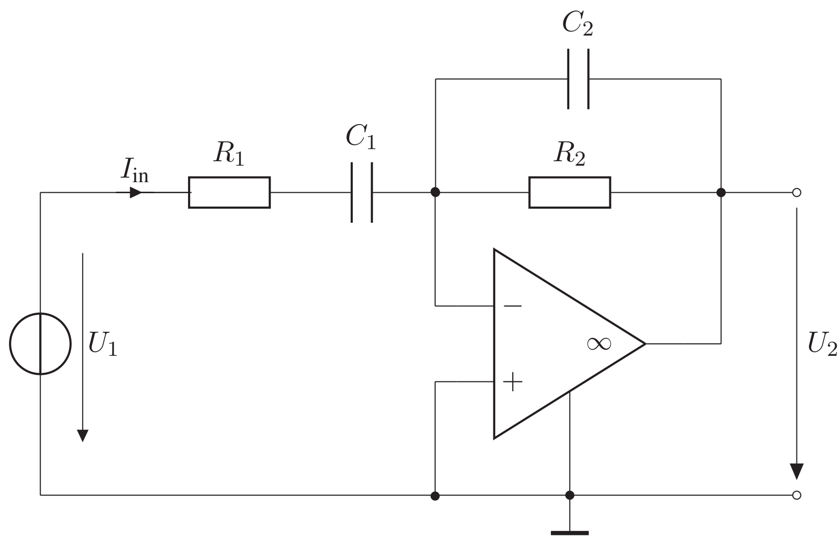
$$i_0(t) = \hat{I}_1 \cos\left(\frac{1}{GL}t + \frac{\pi}{4}\right) + \sqrt{1.25}\hat{I}_2 \cos\left(\frac{2}{GL}t + \arctan(-0.5)\right).$$

j) Find $i(t)$ resulting from this particular $i_0(t)$.

k) Assuming that $i_G(t) = Gu(t)$, investigate $|\frac{I_G}{I_0}|$ for $\omega = 0$, $\omega = \frac{1}{GL}$, and $\omega \to \infty$.

9.4 Transfer Function

Consider the following circuit with ideal Op-Amp, the resistors R_1 and R_2, and the capacitors C_1 and C_2.
The Op-Omp is operated in the linear region.



a) Determine the transfer function $H(\mathrm{j}\,\omega) = U_2/U_1$ by formulating the KVL and KCL equations taking into account the properties of the ideal Op-Amp (nullor).

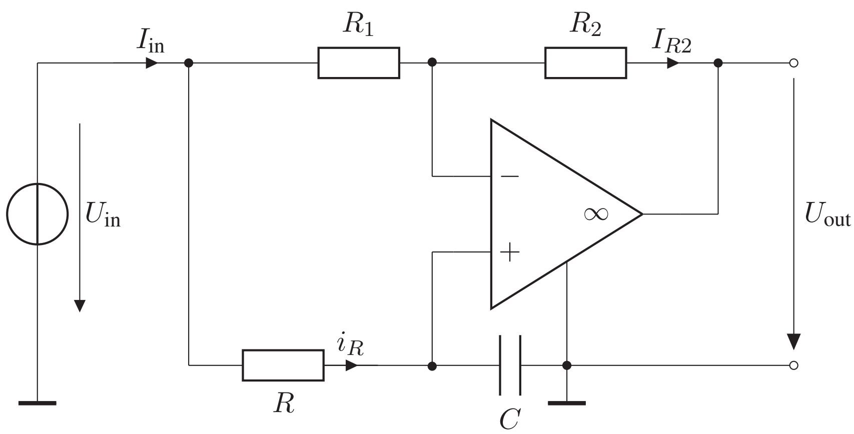
Set $R_1 = R$, $R_2 = 10R$, $C_1 = C$, and $C_2 = 10C$. Also use $RC = 1$.

b) Investigate $H(\mathrm{j}\,\omega)$ at $\omega = 0$, $\omega = \frac{1}{10}$, and $\omega \to \infty$.

c) What can the circuit be used for?

9.5 Transfer Function, Allpass

Consider the circuit below. The Op-Omp is operated in the linear region.

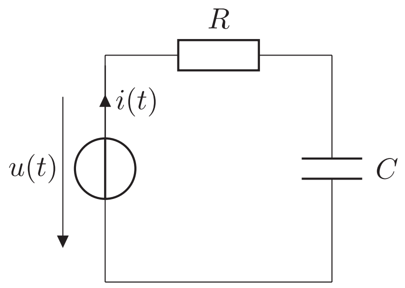


a) Determine the transfer function $H(\mathrm{j}\,\omega) = U_{out}/U_{in}$.

b) Find R_1 and R_2 such that the circuit is an allpass ($|H(\mathrm{j}\,\omega)| = 1$).

9.6 Complex Power

Consider the following circuit with $u(t) = \hat{U}\cos(\omega t)$.



a) Give the complex phasor U corresponding to $u(t)$ depending on R, C, and the phasor I.

b) Determine the complex power P that is consumed by the circuit depending on U.

c) Give the average power P_W and the blind power P_B depending on U.

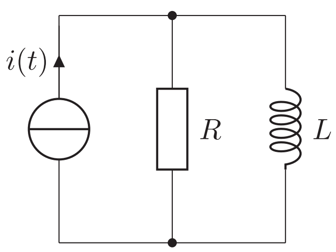
d) Find the apparent power $S = |P|$.

e) What is the energy that is delivered by the source in every period?

f) Find the angular frequency ω such that this energy is maximized. Give the maximum energy E_max.

9.7 Complex Power

Consider the following circuit with $i(t) = \hat{I}\cos(\omega t + \varphi)$.



a) Give the current phasor I corresponding to $i(t)$.

b) Determine the impedance Z of the parallel connection of R and L.

c) Find the complex power of the circuit.

d) How much energy is dissipated by the circuit in every period?

e) Find the angular frequency ω to maximize the energy. What is the value of the maximum energy?