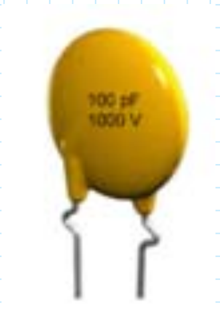
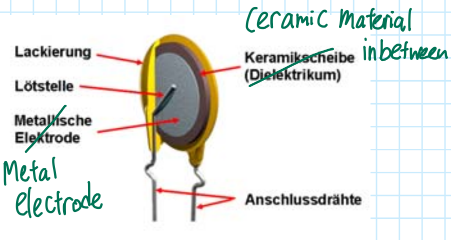
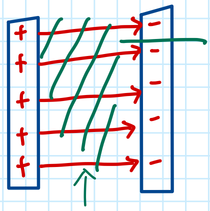
1.5 Electric Fields in Polarizable Materials

Examples - Applications

Ceramic capcitor





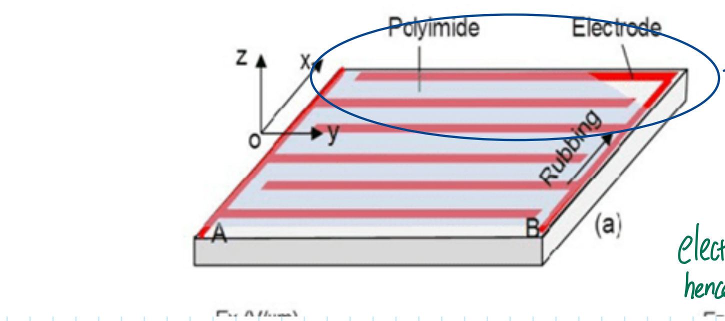


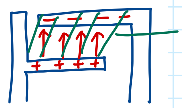
isolating material
(dielectric material)
not conducting

material has an impact on the electric field

Quelle: Wikipedia

Gas sensor based on interdigitated electrodes

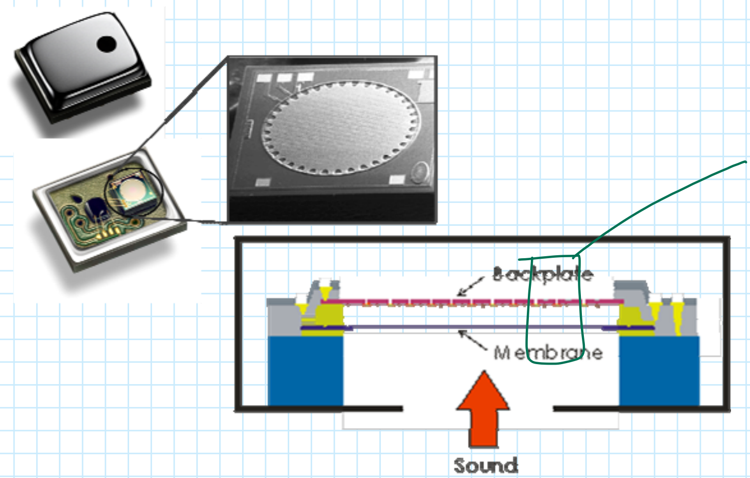


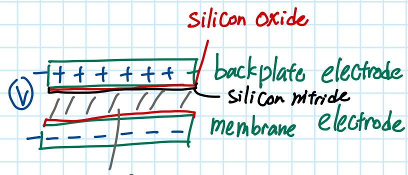


gas sensitive material
(polymide)
not conducting :
dielectric material

this affects the ⇐
electric properties ($\vec{E}$-field) and
hence the measurement Signal

Capacitive Silicon Microphone



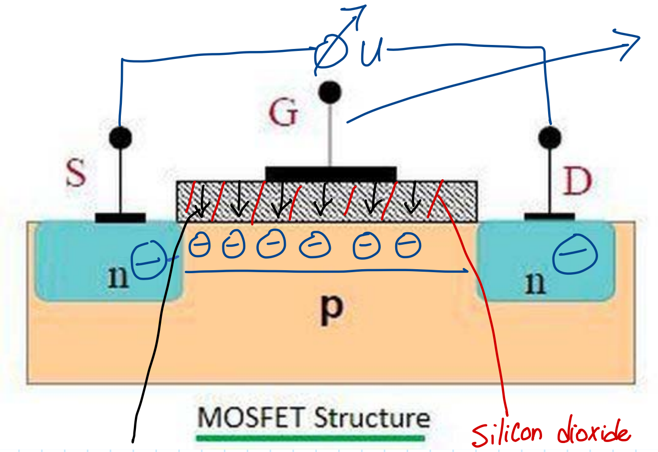


air (not vacuum)

different dielectric materials
between the electrode, which
influence electric field $\vec{E}$

Field Effect Transistor (MOSFET)

Metal-oxide - Semiconductor field effect transistor



E-field develops

Silicon dioxide insulator, no current is flowing

Gate: to switch transistor off or on

apply a voltage U, electrons (negative charge)

$\Rightarrow$ conducting channel develops

$\Rightarrow$ transistor switched on

Classification of materials according to their conductivity

conductors

e.g. metals

a lot of free electrons

highly mobile

$\approx 10^{22}$ electrons $/cm^3$

↳ Chapter 1.7

semiconductors

silicon, germanium

GaAs $\Rightarrow$ gallium arsenide

⋮

pure silicon

10^{10} electrons $/ cm^3$

for doped silicon

10^{15} - 10^{20} electron $/cm^3$

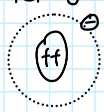
insulators

not electrically conducting

almost no mobile electrons

dielectric materials

Solid matter consists of atoms

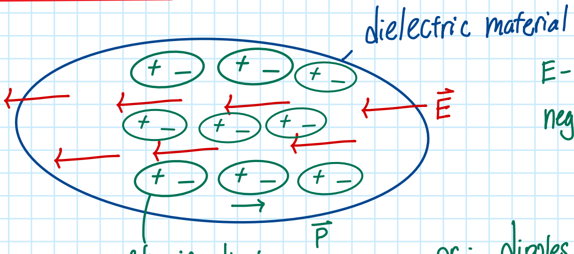


Electric fields in media/material:

- Electric fields influence atomic charges in material
- Dielectrica: contain atomic charges, which can be influenced, but they are not freely movable
- E-Field acting on dielectric material: either atomic dipoles are generated or already existing dipoles are ordered by electric field

$\rightarrow$ $\boxed{\text{Electric polarization}}$

1.5.1. Electric Polarisation



atomic dipoles aligned by $\vec{E}$-field

$\Downarrow$

Polarization $\vec{P}$

$\Downarrow$

$\vec{P}$ is oriented in opposite direction to $\vec{E}$ $\Rightarrow$ inside the material the $\vec{E}$-field is weaker $\Rightarrow$ impact on Coulomb force

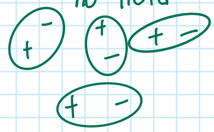
$$\boxed{\vec{F}_q = \frac{1}{\varepsilon_r}\vec{F}_{q\,vac}}$$

E-field shifts positive and negative charge $\Rightarrow$

atomic dipole

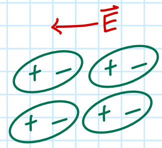
or: dipoles on atomic scale are already present in the crystalline structure

no field



arbitrary orientation

with field



alignment with $\vec{E}$-field

$\Downarrow$

Polarization

ε_r describes impact of material

$$\boxed{\vec{F}_q = \frac{1}{\varepsilon_r}\cdot\frac{q}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{|r-r_i|^3}(\vec{r}-\vec{r}_i)} \quad (1.18)$$

1.5.2. Electric Displacement Field

(i) Introduction of electric displacement field

Remarks:

- Polarization $\vec{P}$ is proportional to $\vec{E}$: $\vec{P} = \varepsilon_r \cdot \vec{E}$

  Simplest material Law we can have
- ε_r = scalar, number, always larger than 1
- (there more complex material laws, where ε_r is not constant)

  in this lecture: ε_r = scalar, constant
- ε_r = relative dielectric constant ≥ 1
- ε_0 = dielectric constant of vacuum $\varepsilon_0 = 8.85 \cdot 10^{-12}\ \mathrm{As/Vm}$
- dielectric constant is also called permittivity
- $\varepsilon_0\varepsilon_r = \varepsilon$ = dielectric constant
- typical values: air, gas $\varepsilon_r = 1.0005$ to $1.001 \approx 1$

  water $\varepsilon_r = 81$

(ii) Introduction of electric displacement field

Intention: universally valid relation between charge distribution in space and electric field and electric displacement field, respectively

→ This can be done by introducing the mathematical/physical quantity "flux of a vector field"

1.5.2. Electric Displacement Field

(i) Introduction of electric displacement field

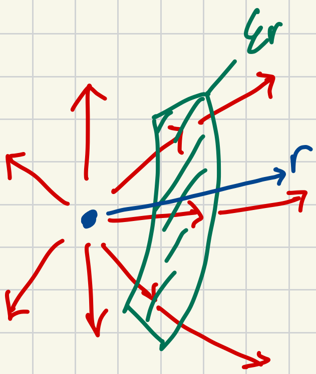
$$\boxed{\vec{F}_q = \frac{1}{\varepsilon_r} \cdot \frac{q}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{|\vec{r}-\vec{r}_i|^3} (\vec{r}-\vec{r}_i)} \quad (1.18)$$

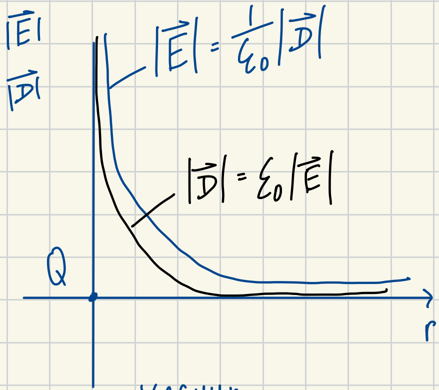
$\varepsilon = \varepsilon_0 \cdot \varepsilon_r$

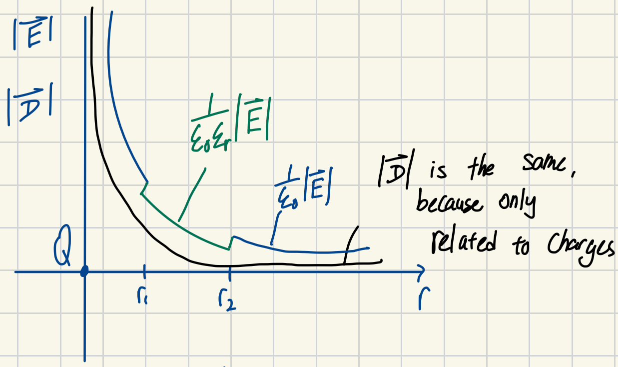
Intention: introduce a quantity, which gives the relation between charges q_i and the generated field, which depends on generating quantities only (= charge)

$\Rightarrow$ electric displacement field $\boxed{\vec{D} = \frac{1}{4\pi} \sum_{i=1}^{n} \frac{q_i}{|\vec{r}-\vec{r}_i|^3} (\vec{r}-\vec{r}_i)}$ (1.21)

$$\boxed{\vec{D} = \varepsilon_r \varepsilon_0 \cdot \vec{E} \qquad \vec{E} = \frac{1}{\varepsilon_0 \varepsilon_r} \vec{D}} \quad (1.20)$$







with dielectric material (s. left)

$\vec{D}$ - field is not affected by the material; depends only on charges.

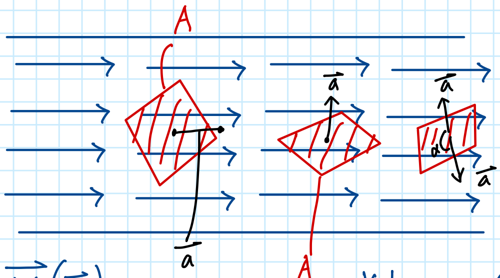
$\vec{E}$ - field is affected by the material (dielectric constant ε_r)

(ii) Introduction of electric displacement field

Intention: universally valid relation between charge distribution in space and electric field and electric displacement field, respectively

→ This can be done by introducing the mathematical/physical quantity "flux of a vector field"

Example: flow field of water (e.g., in a river) flowing with velocity $\vec{v} = \vec{v}(\vec{r})$



$\vec{v}(\vec{r})$ $\quad \vec{a} \perp$ area A $\quad |\vec{n}|=1$ $\quad$ Velocity field $\vec{v}(\vec{r})$

$[\vec{v}] = m/s$

assume a stationary field

$\vec{v}(\vec{r}) \neq f(t)$

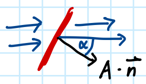
① $A \perp \vec{v}$, $\vec{a} \parallel \vec{v}$ ⇒ flux of water through area A : $|\vec{v}| \cdot A = \vec{v} \cdot \vec{a} = \vec{v} \cdot A \cdot \vec{n}$

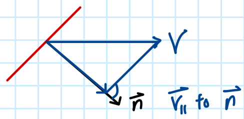
Units : $\left[\frac{m}{s} \cdot m^2\right] = \frac{m^3}{s}$

② $A \parallel \vec{v}$; $\vec{a} = |A| \cdot \vec{n} \perp \vec{v}$ ⇒ flux of water through area A: $\vec{v} \cdot \vec{a} = \vec{v} \cdot A \cdot \vec{n} = 0$

③ Angle α between $\vec{a} = \vec{A} \cdot \vec{n}$ and $\vec{v}$: flux of water: $\vec{v} \cdot \vec{a} = |\vec{v}| \cdot A \cdot |\vec{n}| \cdot \cos\alpha$

=

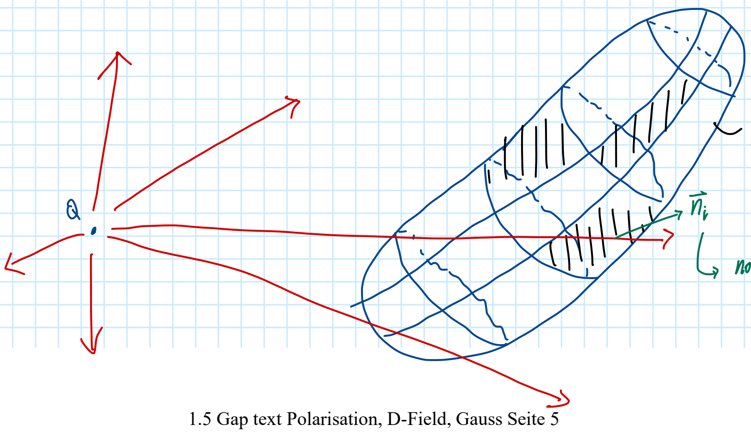




➢ Dielectric Displacement Flux

Dielectric displacement current

- Consider an arbitrary volume V in space (" control/test volume") enclosed by boundary surface ∂V



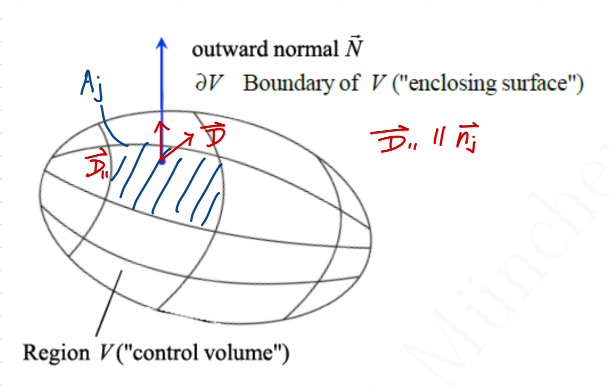
Control volume V

$A_i \in \partial V$ (small parts of ∂V)

enclosed by a surface ∂V (= closed surface around V)

↳ normal vector to A_i

- Determine Flux of D through the enveloping surface:



  - divide ∂V in small pieces A_j

    normal vector $\vec{n}_j$ pointing outward

    $\Rightarrow \vec{A}_j = A_j \cdot \vec{n}_j$

  - determine flux of $\vec{D}$ through $\vec{A}_j$

    for each element A_j of ∂V we determine flux of $\vec{D}$-field: $\vec{D}_j \cdot \vec{A}_j = |\vec{D}_j| \cdot \underbrace{|\vec{A}_j|}_{A_j \vec{n}} \cdot \cos\alpha$

    $= |\vec{D}_j| \cdot A_j \cdot \cos\alpha$

  - Total flux through enveloping surface of V:

    $\Rightarrow$ summing up fluxes through all A_j: $\sum_{j=1}^{n} \vec{D}_j \cdot \vec{A}_j$

  - for infinitesimal small areas A:

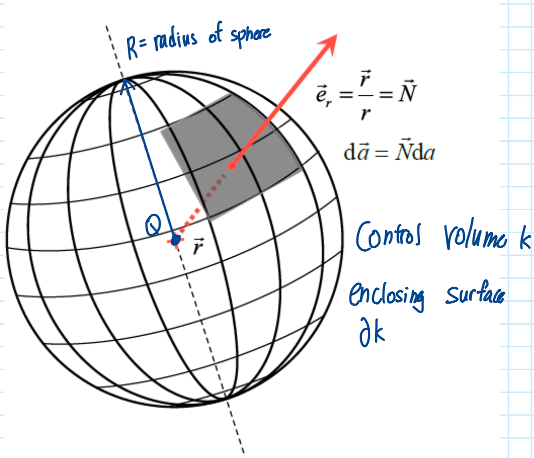
    $\vec{A}_j = A_j \cdot \vec{n}_j \Rightarrow \underset{\text{differential surface element}}{d\vec{a}} \Rightarrow$ integration along closed surface ∂V

$$\boxed{\int_{\partial V} \vec{D} \cdot d\vec{a} = \int_{\partial V} \vec{D} \cdot \vec{n} \cdot d\vec{a}} \quad (1.22)$$

flux of electric displacement field

1.5.2. Gauss's law (= central law of electrostatics)

(i) For a point charge Q in spherical control volume K(0,R)



- point charge Q located in the center of a spherical control volume K
- location of point charge= origin of coordinate system

  Q = located at $\vec{r}_k$, $\vec{r}_k = \begin{pmatrix}0\\0\\0\end{pmatrix}$

  $|\vec{r} - \vec{r}_k| = |\vec{r}| = r$

- field of point charge:

$$\vec{D} = \frac{1}{4\pi} \frac{Q}{|\vec{r}-\vec{r}_k|^3} (\vec{r} - \vec{r}_k) \qquad \vec{e}_r = \frac{\vec{r}}{r}$$

$$= \frac{1}{4\pi} \frac{Q}{r^2} \cdot \vec{e}_r = \frac{1}{4\pi} \frac{Q}{r^3} \cdot \frac{\vec{r}}{r}$$

- electric displacment flux:

r is fixed to r = R

$$\int_{\partial k} \vec{D} \cdot d\vec{a} = \int_{\partial k} \overbrace{\frac{1}{4\pi} \frac{Q}{r^2} \cdot \frac{\vec{r}}{r}}^{\vec{D}} \cdot \underbrace{\overbrace{\frac{\vec{r}}{r}}^{\vec{e}_r} \cdot da}_{d\vec{a}} = \frac{Q}{4\pi} \int_{\partial k|_{r=R}} \frac{r^2}{r^4} \cdot da = \frac{Q}{\cancel{4\pi}} \frac{1}{\cancel{R^2}} \overbrace{\int_{\partial k} da}^{\cancel{4\pi R^2}}$$

$$= \boxed{\int_{\partial k} \vec{D} \cdot d\vec{a} = Q} \quad (1.23)$$

does not depend on R

(ii) Generalization to arbitrary control volume

$\Rightarrow$ Gauss's Law : $\boxed{\begin{array}{ll} \displaystyle\int_{\partial V} \vec{D} \cdot d\vec{a} = Q & \text{if } \vec{r}_Q \in V \\ \displaystyle\int_{\partial V} \vec{D} \cdot d\vec{a} = 0 & \text{if } \vec{r}_Q \notin V \end{array}}$ (1.25)

(1.25) can be applied to calculate $\vec{D}/\vec{E}$ - fields from given charge distribution

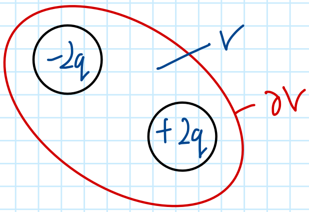
(iii) Gauss's law for a system of point charges (superposition principle)

We consider many point charge q_i inside the control volume V

Total charge $Q(V)$ inside V : $Q(V) = \sum_{\vec{r}_i \in V} q_i$ $\qquad$ $\vec{r}_i \in V$ $\uparrow$ located inside V

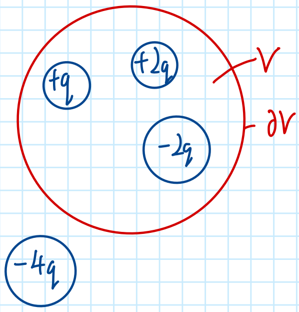
$\Rightarrow$ (1.25) : $\boxed{\int_{\partial V} \vec{D} \cdot d\vec{a} = \sum_{\vec{r}_i \in V} q_i}$ (1.27)

(i)



$\int_{\partial V} \vec{D} d\vec{a} = 0$

(ii)



$\int_{\partial V} \vec{D} \cdot d\vec{a} = +q$