EDE1011 ENGINEERING MATHEMATICS 1

Tutorial 2

Linear Systems & Matrices

1. Solve the following SLEs. Which one is inconsistent?

a) $\begin{aligned} 3x + 4y &= 1 \\ 2x + 3y &= 12 \end{aligned}$

b) $\begin{aligned} 3x - 2y &= 4 \\ -6x + 4y &= 7 \end{aligned}$

c) $\begin{aligned} u + v + w - 6 &= 0 \\ 4v + w + u &= 5 \\ 6 &= w + 3v + u \end{aligned}$

ANS: **a)** x = -45, y = 34. **b)** Inconsistent. **c)** Inconsistent.

2. Given the following matrices, are those below defined? Compute those that are defined.

$$A = \begin{bmatrix} 1 & 3 & 5 \\ -1 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \\ 3 & 2 \\ -1 & 4 \end{bmatrix}, \quad d = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$$\text{(a) } Ad, \quad \text{(b) } AB + C, \quad \text{(c) } A + C^T, \quad \text{(d) } C^T C, \quad \text{(e) } BC,$$
$$\text{(f) } d^T B, \quad \text{(g) } Cd, \quad \text{(h) } d^T d, \quad \text{(i) } dd^T$$

ANS: **a)** $[4, -3]^T$. **b)** Undefined. **c)** $\begin{bmatrix} 2 & 6 & 4 \\ 0 & 3 & 4 \end{bmatrix}$ **d)** $\begin{bmatrix} 11 & 3 \\ 3 & 21 \end{bmatrix}$ **e)** $\begin{bmatrix} 0 & 5 \\ 4 & 8 \\ 5 & -1 \end{bmatrix}$ **f)** $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$ **g)** Undefined. **h)** 6. **i)** $\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}$

3. In 2010, the average salary for all accountants in San Diego, California, and Salt Lake City, Utah, was $45,091.50. The average salary in San Diego alone, however, was $5231 greater than the average salary in Salt Lake City alone. What is the average salary of an accountant in each city, assuming that there are the same number of accountants in each city?

↳ Find S_{SD}, S_{SL}

n

$\frac{n}{2n}(S_{SD} + S_{SL}) = 45091.5$

$S_{SD} + S_{SL} = 90183 \quad - R_1$

$S_{SD} = S_{SL} + 5231$

$S_{SD} - S_{SL} = 5231 \quad - R_2$

ANS: San Diego: $47,707. Salt Lake: $42,476

4. Rewrite the following SLE in matrix form and solve using matrix row operations.

$$\begin{aligned} x_1 + x_2 + x_3 &= 6 \\ 2x_1 + 4x_2 + x_3 &= 5 \\ 2x_1 + 3x_2 + x_3 &= 6 \end{aligned}$$

ANS: x_1=2, x_2=-1, x_3=5.

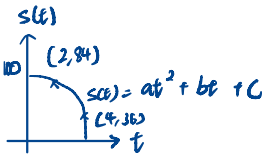
5. Rewrite the following SLEs in matrix form and solve using Gauss-Jordan Elimination. State the type of solution and the rank of the coefficient matrix. How many linearly independent equations does each SLE have? Are the SLEs consistent?

a)
$$\begin{aligned} x + y + z &= 1 \\ 2x - y + z &= -1 \\ x + 3y - z &= 7 \end{aligned}$$

b)
$$\begin{aligned} 3x - 4y &= 8 \\ x + y + z &= 2 \\ 2x - 5y - z &= 6 \end{aligned}$$

ANS: **a)** x = 1, y = 3/2, z = -3/2. Unique solution. Rank = 3.
Linearly independent equations = 3. Consistent.
b) z = ⅓ (-7y-2) = ¼ (-7x+16). Infinite solutions (a line of intersection).
Rank = 2. Linearly independent equations = 2. Consistent.

6. A quadratic function is used to model the displacement-time function of an object being dropped from a 100-metre tall tower. Given the data below, define the quadratic function. How long does it take for the object to reach the ground?



Time, t (s)	0	2	4
Displacement, s (m)	100	100	36

$s(0) = c$
$= 100$

ANS: $s(t) = -4t^2 + 100$. 5 seconds.

7. **Modelling Problem: Preparation of a Solution Mixture.**

A chemist has prepared two acid solutions, one of which is 2% by volume, the other 7%. How many cubic centimetres of each should the chemist mix together to obtain 40cm^3 of a 3.2% acid solution?

ANS: 2%: 30.4 cc, 7%: 9.6 cc

8. **Modelling Problem: Financial Planning**
(https://openstax.org/books/precalculus-2e/pages/9-6-solving-systems-with-gaussian-elimination)

$I_A + I_B = 10000 - R_1$

You have $10,000 to invest into two bonds: bond A (higher risk) pays a 5% coupon annually and bond B (lower risk) pays a 3% coupon. If you want the total coupon payment each year to be $440, how much should you invest in each bond?

Find I_A, I_B

$0.05I_A + 0.03I_B = 440$

$5I_A + 3I_B = 44000 - R_2$

ANS: Bond A: $7000. Bond B: $3000.

9. **Modelling Problem: Winning a Boss Fight in a Game**

In a MMORPG, there are three items that you can equip to increase your attack, defence and dexterity points. The number of each item you can equip are x, y and z respectively. The contributions of each item are shown below:

Item	Attack	Defence	Dexterity
Dragon Scale (x)	-20	40	10
Griffin Claw (y)	50	10	-10
Elven Crystal (z)	10	10	60

$320 = -20x + 50y + 10z$

a) In order to clear a level boss, you need to increase your attack by 320 points and defence by 280 points. If the total number of items you can equip is 16, show that this problem gives the linear system in matrix form:

$$A\mathbf{v} = \mathbf{b} \rightarrow \begin{bmatrix} -2 & 5 & 1 \\ 4 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 32 \\ 28 \\ 16 \end{bmatrix}$$

<- attack criteria
<- defence criteria
<- total items criteria

b) Using row operations on the augmented matrix [A|**b**], solve the matrix equation for the number of units of each item you must equip for the boss fight.

ANS: **b)** Dragon Scale (x) = 4. Griffin Claw (y) = 7. Elven Crystal (z) = 5

For more practice problems (& explanations), check out:

1) https://openstax.org/books/precalculus-2e/pages/9-1-systems-of-linear-equations-two-variables
2) https://openstax.org/books/precalculus-2e/pages/9-2-systems-of-linear-equations-three-variables
3) https://openstax.org/books/precalculus-2e/pages/9-5-matrices-and-matrix-operations
4) https://openstax.org/books/precalculus-2e/pages/9-6-solving-systems-with-gaussian-elimination

End of Tutorial 2

(Email to youliangzheng@gmail.com for assistance.)