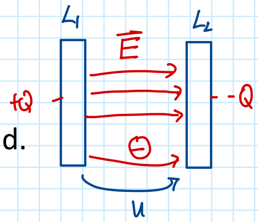
1.7.4. Electrostatic Field Energy

(i) Energy stored in a charged capacitor



- Charging of a capacitor needs energy. This energy is stored inside the electric field.

  Moving negative charge from L_1 to L_2 $\Rightarrow$ $L_1 \to -Q$, $L_2 \to +Q$
- This energy can be calculated from the electrical work which is done in the electric field.

$\to$ work is performed on charge $\int q\vec{E}\cdot d\vec{r}$

Total energy needed to charge a capacitor from Q'=0 to Q'=Q can be expressed as follows:

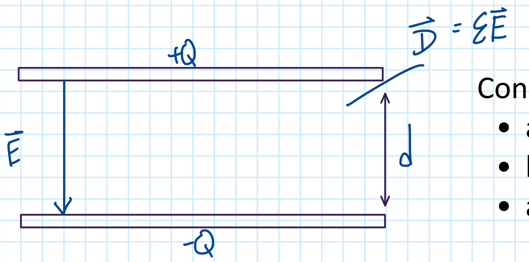
$$W_{el} = \int_0^Q d\,W_{el} \overset{(*)}{\Rightarrow} W_{el} = \int_0^Q \underbrace{\Delta Q}_{dQ} \cdot U_{12}(Q)$$

It is the energy stored in a charged capacitor

$$W_{el} = \int_0^Q dQ \cdot U_{12}(Q) = \int_0^Q \frac{Q}{C}\,dQ = \left[\frac{1}{2}\cdot\frac{Q^2}{C}\right]_0^Q \qquad C = \frac{Q}{U}$$

$$\Rightarrow \boxed{W_{el} = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QU = \frac{1}{2}CU^2} \quad (1.50)$$

(ii) Energy Density of Electric Field



Consider:

- a plate capacitor, charged with +Q/-Q
- having a plate area A and a distance of the plates of d
- and a dielectric material between the plates

Electric energy stored in electric field: $Q = \sigma \cdot A = \vec{D}\cdot\vec{N}\cdot A \qquad |\vec{E}| = \frac{U}{d}$

$$W_{el} = \frac{1}{2}Q\cdot U = \frac{1}{2}|\vec{D}|\cdot A \cdot |\vec{E}|\cdot d = \frac{1}{2}|\vec{D}|\cdot|\vec{E}|\cdot \underbrace{A\cdot d}_{V\,=\,\text{volume}}$$

Energy density of electric field : = electric energy per volume

$$\boxed{w_{el} = \frac{W_{el}}{V} = \frac{1}{2}|\vec{D}|\cdot|\vec{E}| = \frac{\varepsilon}{2}|\vec{E}|^2 = \frac{1}{2\varepsilon}|\vec{D}^2|}$$

$\varepsilon = \varepsilon_r \cdot \varepsilon_0$

$\varepsilon = \varepsilon_0$

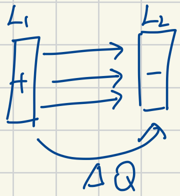
Energy is stored in $\vec{E}$-field / $\vec{D}$-Field

This holds also for vacuum !

Note: it does not matter if the electric field exists in vacuum or in matter. The energy is stored in the electric field itself! This is basis of electromagnetic wave applications (transfer of enery by electromagnetic waves!)

(*) side remarks for Charging a Capacitor

ΔW_{el} work performed on Charge ΔQ



$$\Delta W_{el} = \int_{L_1}^{L_2} \underbrace{\Delta Q \cdot \vec{E}}_{\vec{F}_{el}} \, d\vec{r} = \underset{\downarrow \atop dQ}{\Delta Q} \cdot U_{12}(Q)$$

Voltage depends on the Q present on L_1, L_2,

since $\vec{E}$-field increases during Charging