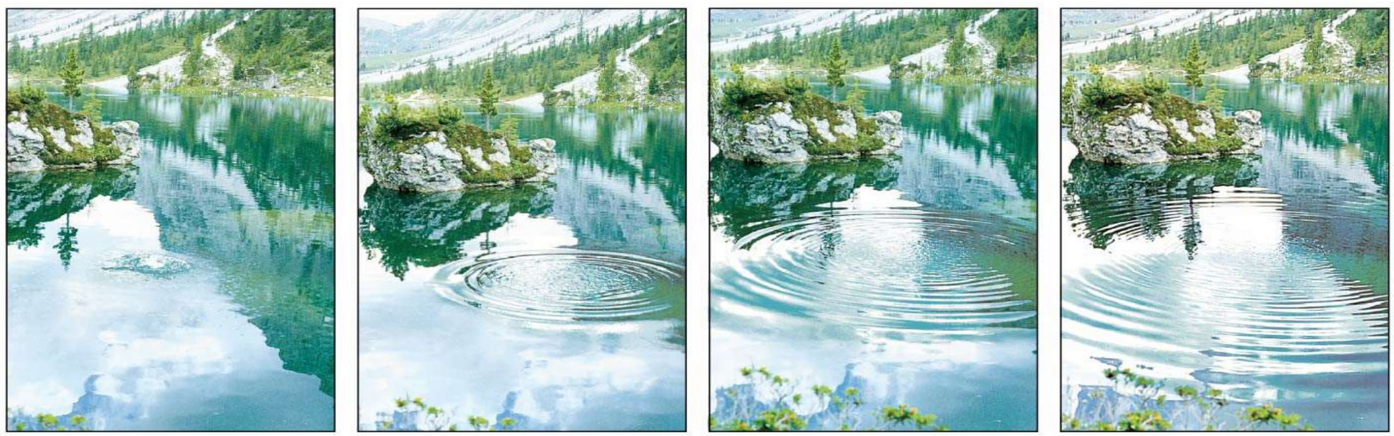
ENG1004 Engineering Physics

AY2023/ 24 Trimester 1, Week 10



Topic: Wave Motion

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Content

1. Types of waves:
   i. Progressive: Sub-types (1) Transverse (2) Longitudinal
   ii. Stationary (Next Topic) **Next week (Prof Venkat)**
2. Graphs of progressive waves
3. Intensity of Waves
4. Polarization (Transverse Waves)

*Stationary Wave: Wave profile does not seem to be travelling/propagating.

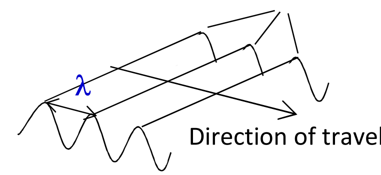
0. Introduction – Wave Motion

Wave characteristics:

1. Crest: Highest point of wave
2. Trough: Lowest point of wave
3. Amplitude, A
4. Wavelength, λ
5. Frequency f (Hz) and period T (s)
6. Wave velocity $v = f\lambda = \dfrac{\lambda}{T}$

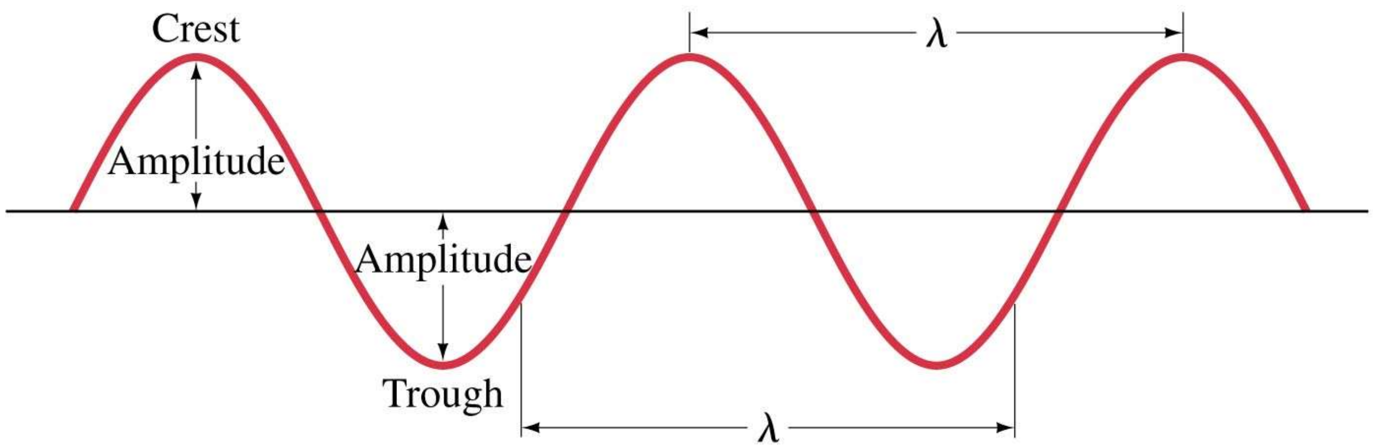
Frequency, $f = 1/T$

Speed of wave, $v = \lambda/T = f\lambda$



NOTE:

Speed of wave particles (in SHM) $\neq f\lambda$!



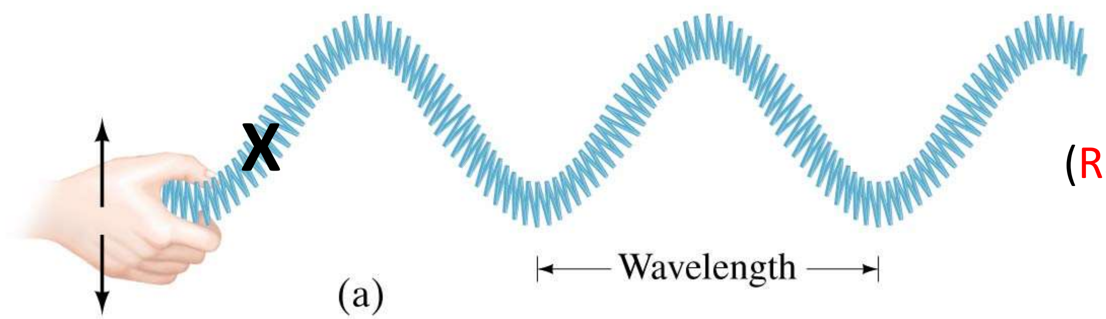
1. Types of Waves: Progressive waves $v = f\lambda$

1. Definition of Progressive Wave:
   Disturbance (vibration) which propagates, carrying energy
   w/o physically transferring wave particles. Means wave particles are NOT transferred
   https://www.youtube.com/watch?v=iT4KAc0Ag1E

2. Oscillation of particles in a progressive wave can either be

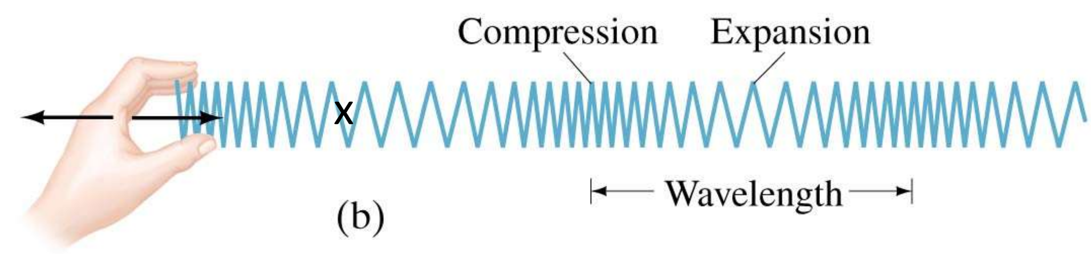
   (i) perpendicular to wave direction (**Transverse**) or

   (ii) parallel to it (**Longitudinal**).



Eg: EM Waves & light

(Radio Wave is an EM wave)

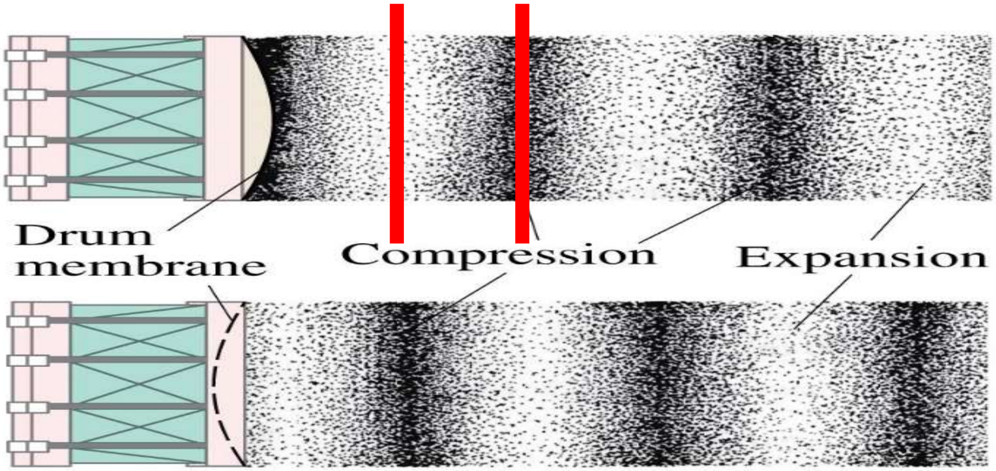


Eg: Sound Waves

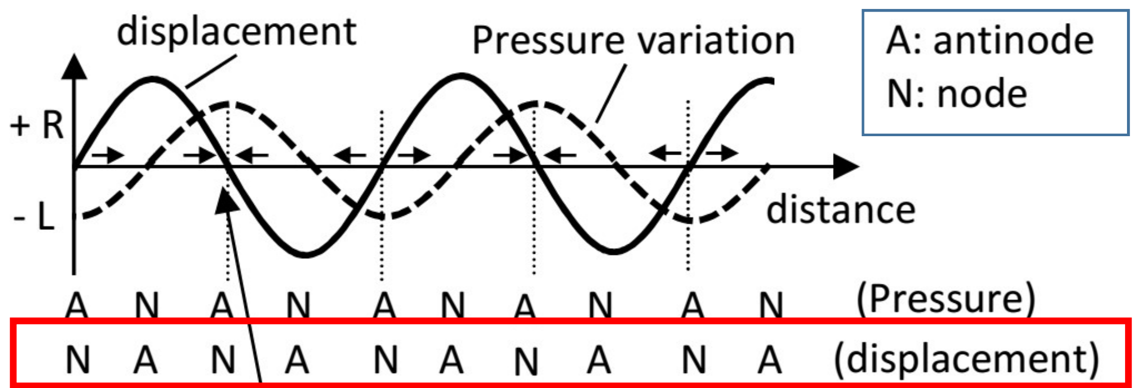
1. Types of Waves: Progressive waves $v = f\lambda$

Longitudinal Wave

Sound waves are longitudinal waves:



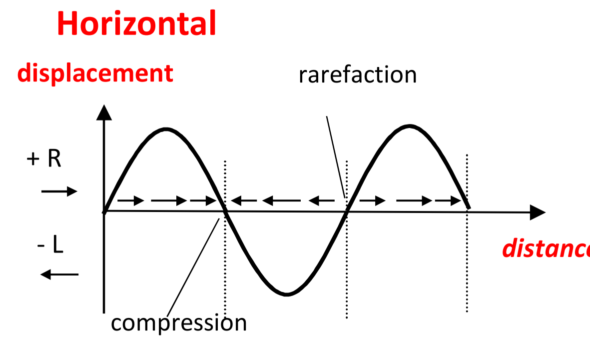
https://www.youtube.com/watch?v=zp1_waJkGcU



Displacement node undergoes max pressure variation: low and high pressure alternate every ½T period; thus it is a pressure antinode.

λ = 2 x internodal distance

Graphical representation



Common mistake:
Note this is a GRAPH & not a transverse wave

Displacement vs Distance

Pressure vs Distance

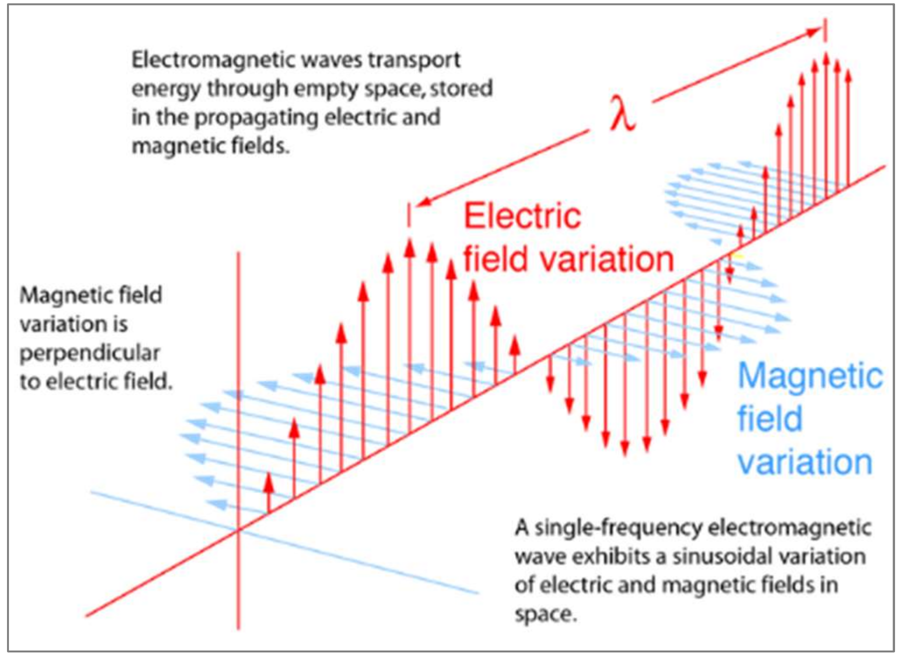
1. Types of Waves: Progressive waves $v = f\lambda$

Speed of EM waves in **vacuum**, c = 3.**0** x 10^8 m/s

Transverse Wave

EM Spectrum (Wavelengths)

1. γ rays $10^{-10} – 10^{-14}$ m
2. X rays $10^{-9} – 10^{-12}$ m
3. UV $10^{-7} – 10^{-10}$ m
4. Visible light **4 x 10^{-7} m – 7 x 10^{-7} m ***
5. IR $10^{-3} – 10^{-7}$ m
6. μ waves $10^{-1} – 10^{-3}$ m
7. **Radio waves** $10^{-2} – 10^{3}$ m



Source: http://hyperphysics.phy-astr.gsu.edu/hbase/Waves/emwavecon.html

EM wave has an oscillating E-field (electric field) & a B-field (magnetic field) that are:

1. Perpendicular to each other
2. Both perpendicular to direction of propagation

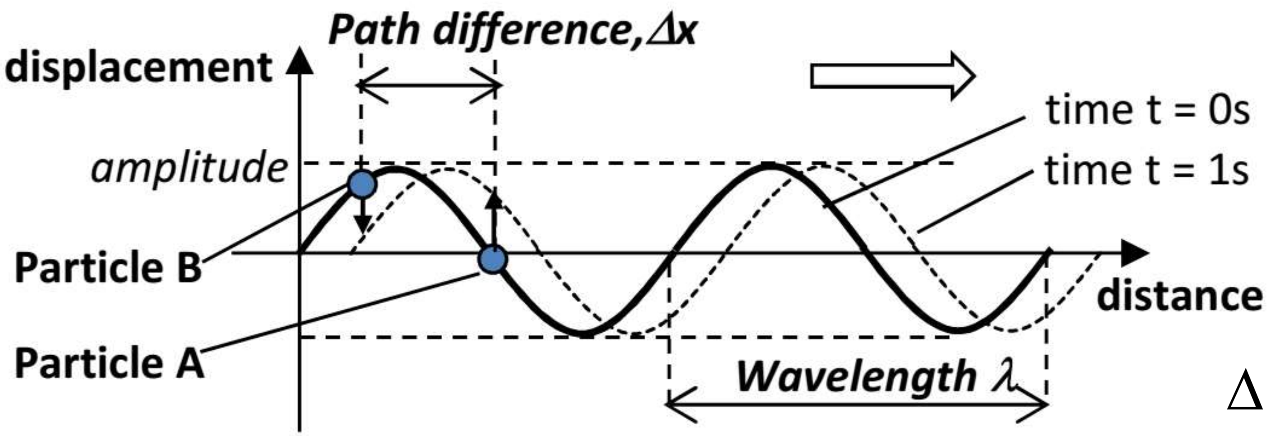
Go find out what are the colors of visible light and their CORRESPONDING WAVELENGTHS

2. Graphs of Progressive waves

Wave travelling to RIGHT (snapshots at different TIMES)

Phase difference between two particles A & B on wave.

Displacement-Distance



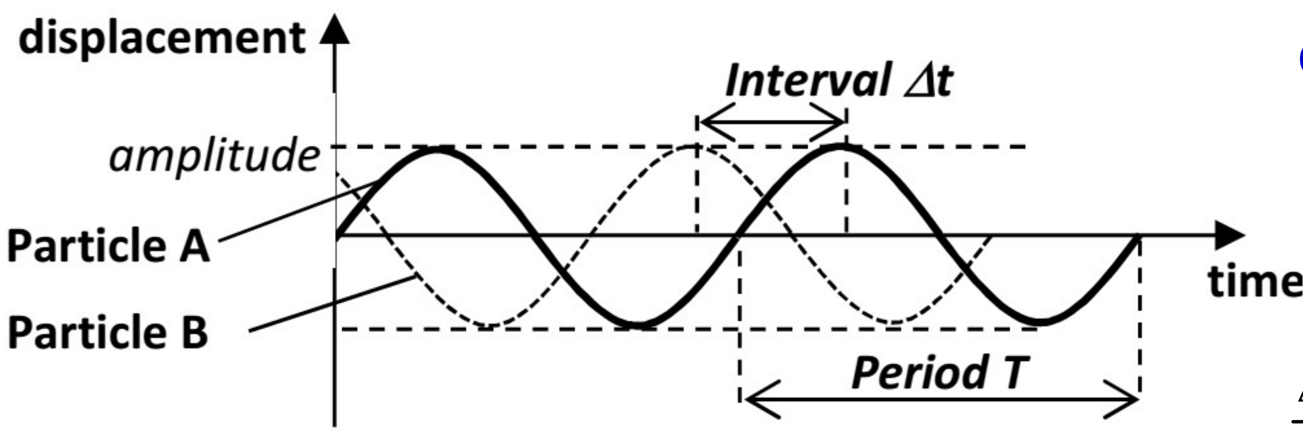
Phase difference:

EITHER displacement-distance graph

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda}$$

$$\Delta\phi = \frac{\Delta x}{\lambda} \times 2\pi \quad \text{rad}$$

Displacement-Time



OR displacement-time graph

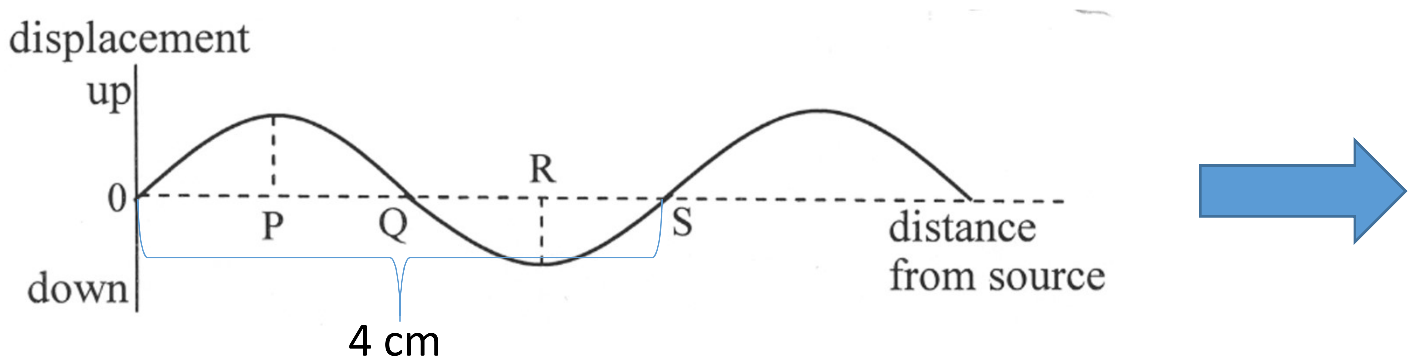
$$\frac{\Delta\phi}{2\pi} = \frac{\Delta t}{T}$$

$$\Delta\phi = \frac{\Delta t}{T} \times 2\pi \quad \text{rad}$$

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda} = \frac{\Delta t}{T}$$

Example

At a particular instant, the variation of the displacement of the particles in a transverse progressive water wave, of wavelength 4 cm, travelling from left to right is shown below.



Which one of the following statements is **incorrect**?

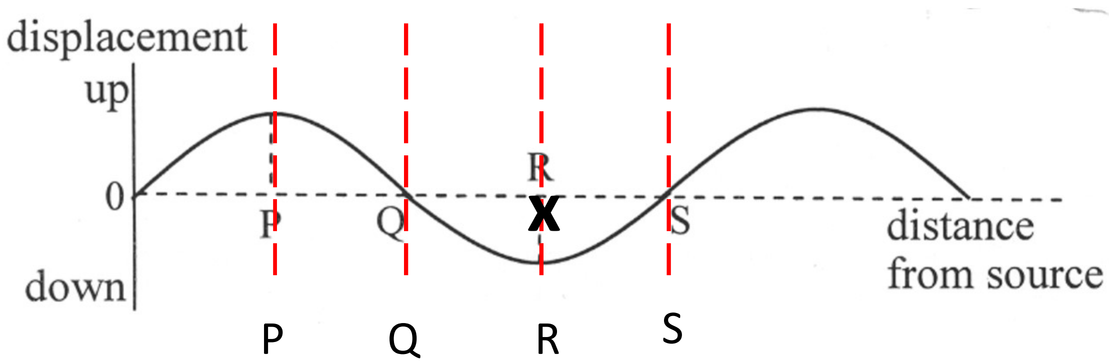
A. Particle at Q has a maximum velocity.

B. Particles at P and R are in phase.

C. Particle at S is moving downwards.

D. Distance PS = 3 cm.

As graph moves towards RIGHT,
particle at P moves **DOWN**
particle at R moves **UP**
Hence P and R are **not in phase**.



As graph moves towards RIGHT,
particle at P moves **DOWN**
particle at R moves **UP**
Hence P and R are **not in phase**.

Particle at S moves **DOWN**

3. Intensity

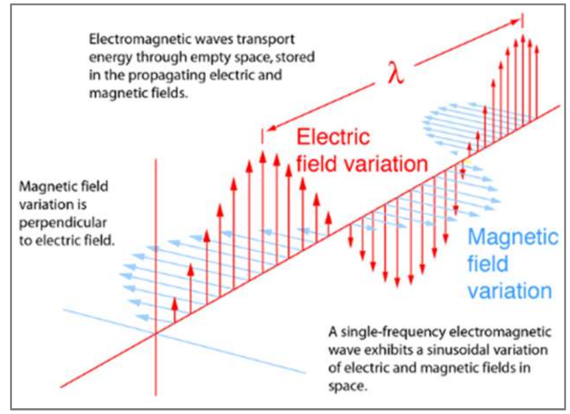
1. Energy possessed by wave particle (& eventually transferred to next particle) :

$$E_{\text{total}} = \frac{1}{2} m\omega^2 {x_0}^2 = \frac{1}{2} kA^2$$

Amplitude: $x_0 = A$

Constant: $k = m\omega^2$ $\quad \omega = \frac{2\pi}{T}$

Total energy, $E = E_{\text{total}}$



Source: http://hyperphysics.phy-astr.gsu.edu/hbase/Waves/emwavecon.html

2. Hence, $E \propto A^2$ $\quad E_{\text{total}} = \frac{1}{2} kA^2$

3. Intensity of wave, I: Rate of energy flow, $\frac{dE}{dt}$, per unit surface area perpendicular to direction of wave propagation (or wave motion). $\quad I = \frac{\frac{dE}{dt}}{S}$

4. Intensity: $I = \frac{\text{Power}}{\perp \text{Area}} = \frac{P}{S}$

Use "***S***" for area because we already used "*A*" for amplitude

5. Hence, $\boxed{I \propto E \propto A^2}$

Power (W or J s^{-1}) $= \frac{dE}{dt}$

3. Intensity

Wave propagation methods

Spherical Wave:

- For wave originating from **point source**, area covered is spherical.
- Every point of impact on spherical surface can approx.. to be perpendicular plane to wave.
- As distance r from point source increases, surface area increases.

$$I = \frac{P}{S} = \frac{P}{4\pi r^2} \Rightarrow I \propto \frac{1}{r^2} \quad \text{Inverse Square Law}$$

Area of spherical surface, $S = 4\pi r^2$

- Comparing intensities from **same wave** at **different distances** r_1 & r_2 from point source: $$\frac{I_1}{I_2} = \frac{r_2^{\,2}}{r_1^{\,2}}$$

- How does amplitude A of spherical wave change with distance r from source?

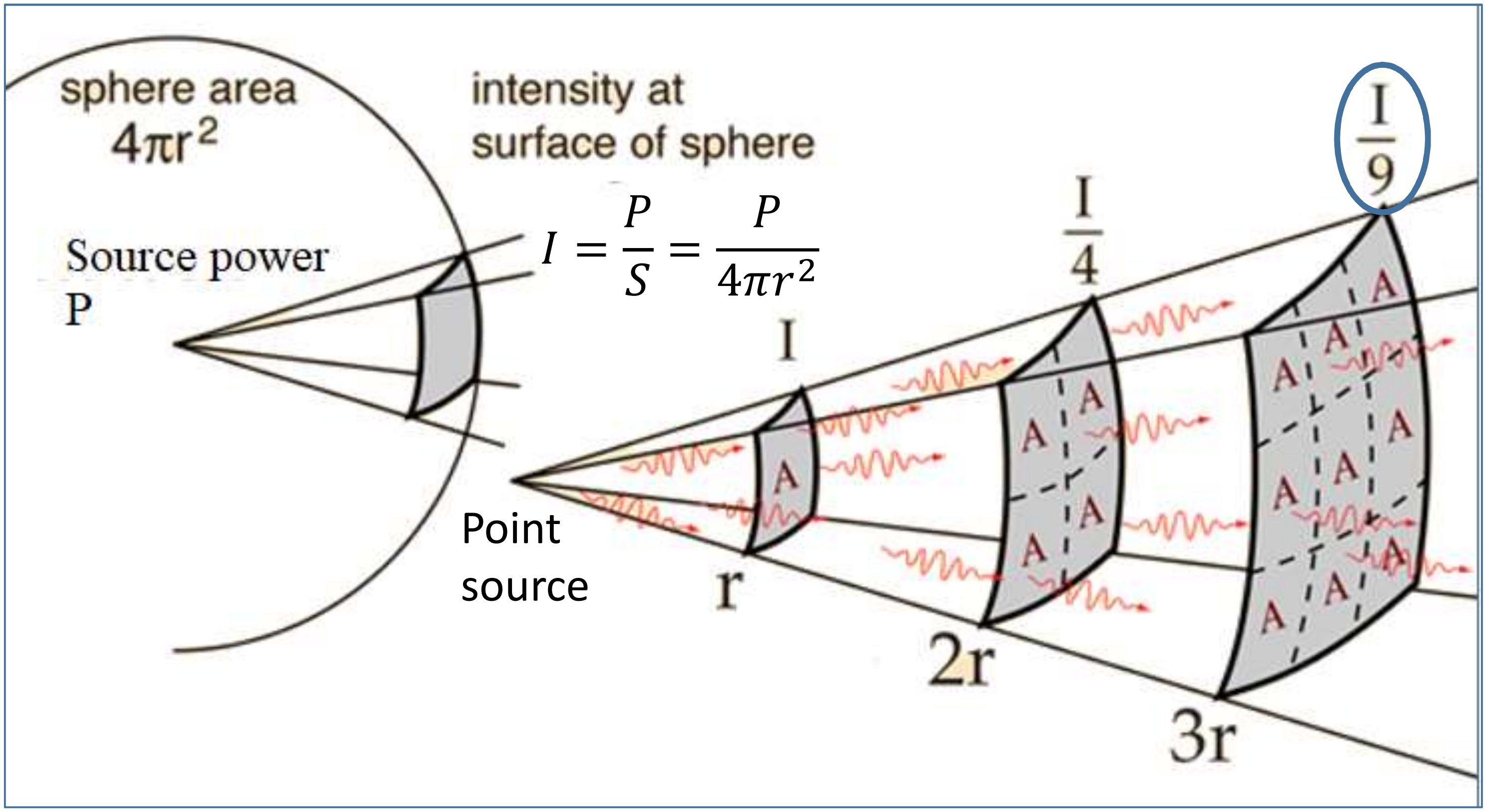
$$I \propto A^2 \propto \frac{1}{r^2} \qquad \Rightarrow A \propto \frac{1}{r}$$

Further (larger r) from point source, means **amplitude (A)** is **smaller**.

3. Intensity

Sphere surface area $S = 4\pi r^2$

<u>Spherical waves</u>



At 4r, I/16

Source: http://hyperphysics.phy-astr.gsu.edu/hbase/Forces/isq.html

At infinity, intensity is 0 W/m^2

Question: How about plane waves?

https://www.britannica.com/science/sound-physics/Circular-and-spherical-waves#ref527186

Answer: Plane wave of single frequency propagates forever with no change or loss.

4. Polarization

1. Only transverse waves can be plane-polarised.
2. Unpolarised light has infinite planes of vibration.
3. Polarised light has only a **single plane** of vibration.
4. Polariods:
   - Plastic sheets highly strained to align plastic molecules in ONE direction.
   - Strongly absorbs light in ONE plane (Call it **plane A**) while easily allowing light to pass through in another plane (**plane B**) perpendicular to plane A.
   - Direction of vibration of polarized light: **Polarization axis**.
5. Plane of polarization: Plane which is perpendicular to **E-field** vector's plane of vibration. [Plane containing direction of vibration & propagation of light is called plane of vibration.]
6. **Intensity of unpolarised light** I_0 after passing through polariser: $I_1 = \frac{1}{2} I_0$
7. **Intensity of plane polarized light** is reduced after it passes through another polarizer: $I_2 = I_1 \cos^2 \theta$ Angle θ to plane of polarisation of incident light

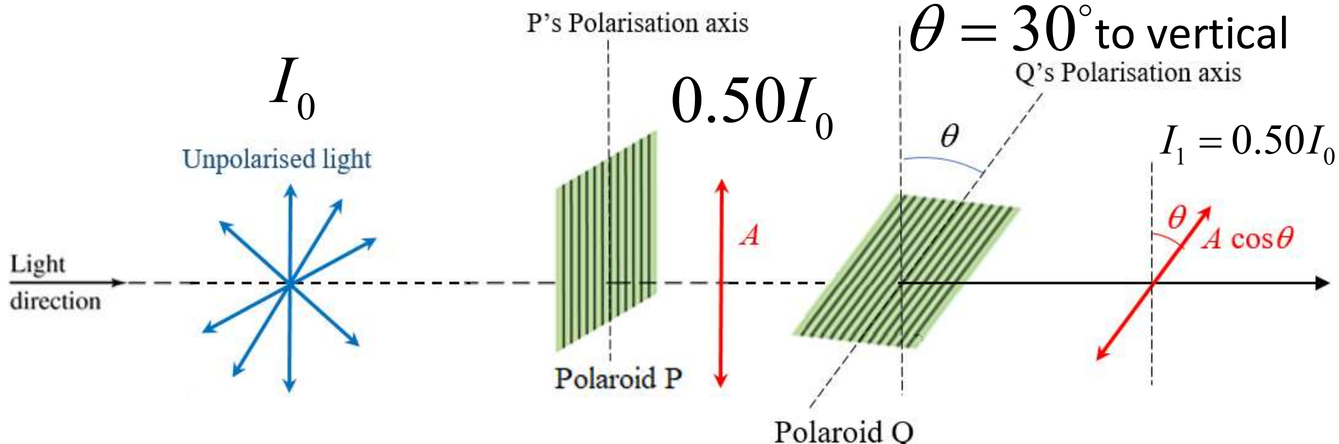
4. Polarization

Polarisation Axis of a polarizing filter: Direction along which filter passes E-field of an EM wave.

$\theta = 30^{\circ}$ to vertical

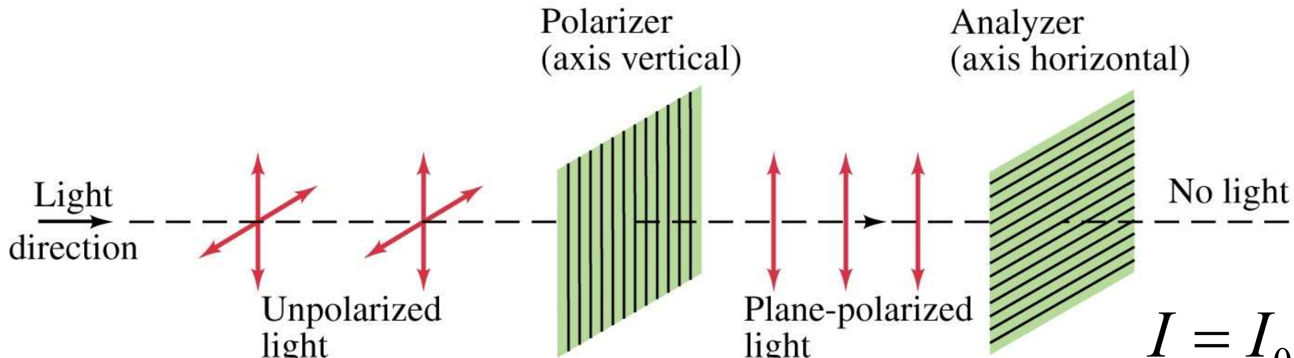
I_0

$0.50 I_0$



$I_1 = 0.50 I_0 \cos^2 30^{\circ} = 0.375 I_0$

$I_1 = 0.5 I_0$



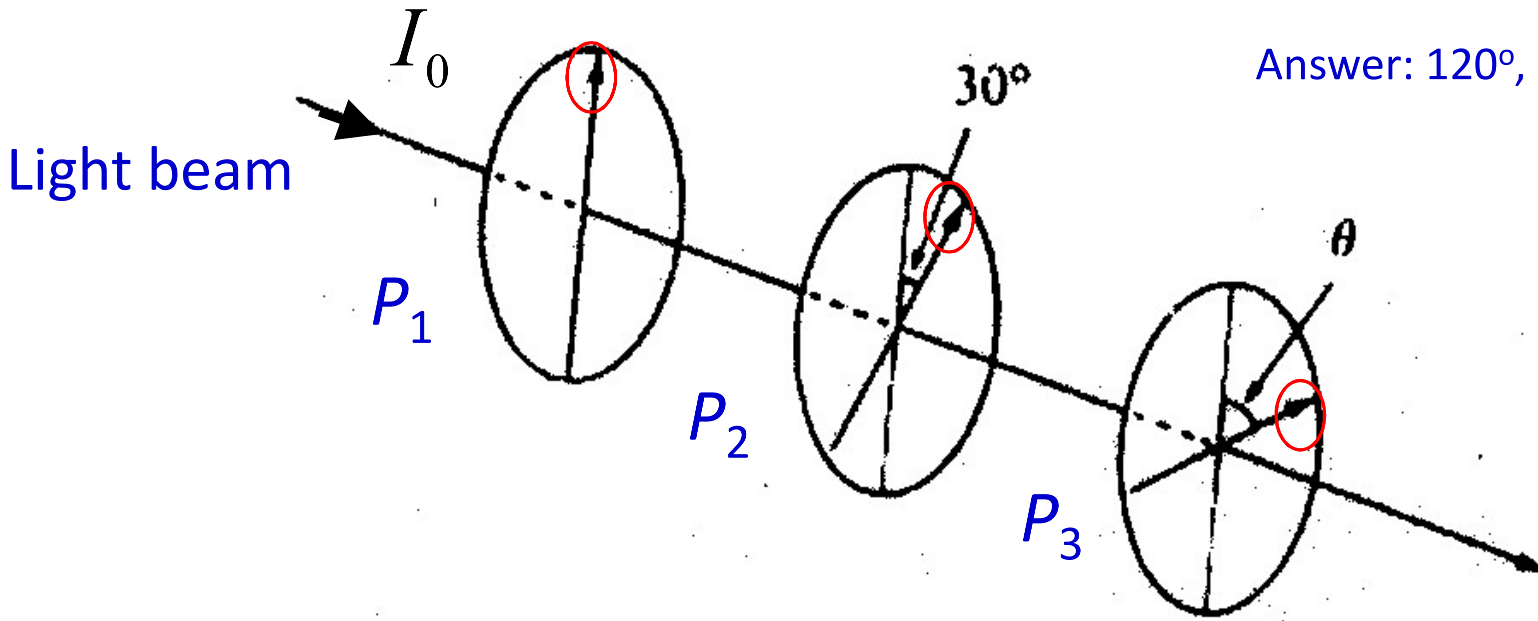
$I = I_0 \cos^2 90° = 0$

Example

The figure below shows a beam of initially unpolarised light passing through three polarisers P_1, P_2 and P_3.

The polarizing axis of each polariser is shown by an arrow. Polarisers P_1 and P_2 are fixed, with their polarizing axes at 30° to one another and P_3 can be set with its polarising axis at a variable angle θ to that of P_1.

Determine for what values of θ do intensity minima of the emergent light occur?



Answer: 120°, 300°

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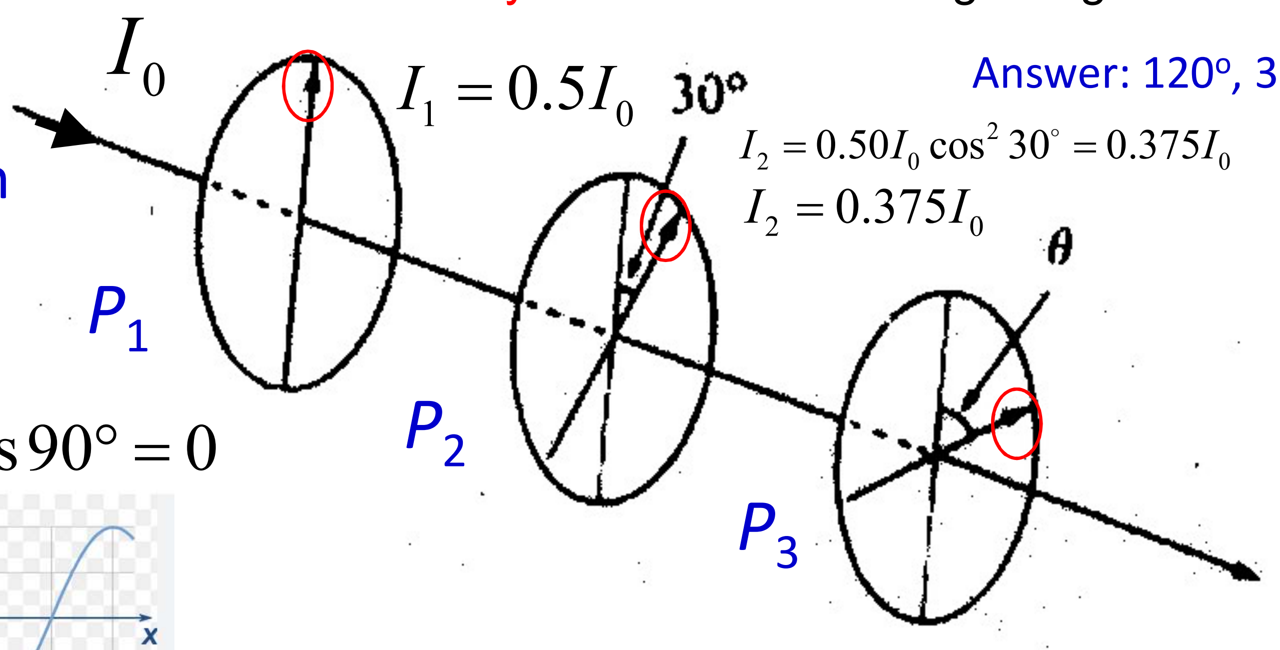
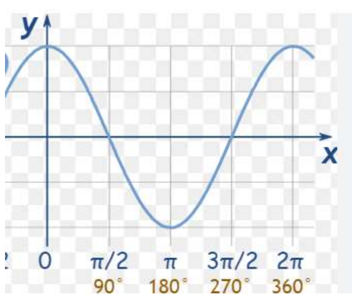
Answer: 120°, 300°



We want $\cos 90^\circ = 0$

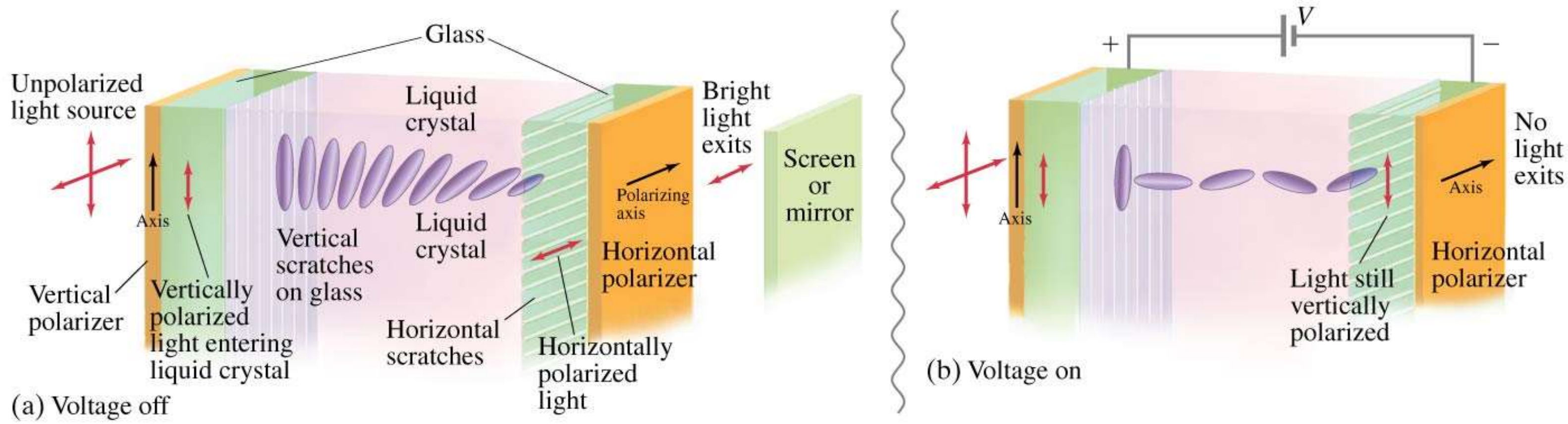


https://www.mathsisfun.com/algebra/trig-sin-cos-tan-graphs.html

4. Polarization

Application: Liquid Crystal Displays (LCD)

1. Liquid crystals are unpolarized in absence of an external voltage, and will easily transmit light.
2. When an external voltage is applied, crystals become polarized and no longer transmit light; they appear dark.



<u>**Topic worth exploring for your videos:**</u>
Colour liquid crystal displays – How they work?

Summary

1. Progressive Waves; Transverse & Longitudinal
2. Do not confuse velocity of wave **particles** in SHM with $v=f\lambda$
3. Phase difference between 2 particles on a travelling wave:

EITHER displacement-distance graph

$$\Delta\phi = \frac{\Delta x}{\lambda} \times 2\pi \quad \text{rad}$$

OR displacement-time graph

$$\Delta\phi = \frac{\Delta t}{T} \times 2\pi \quad \text{rad}$$

4. Spherical wave: $I \propto \frac{1}{r^2}$ *Inverse Square Law*

$I \propto E \propto A^2$

5. Only **transverse waves** can be polarized. (CONCLUSIVE PROOF)
6. Intensity of plane polarized light (original intensity I_0) is reduced after it passes through another polarizer: $I = I_0 \cos^2\theta$

Intensity of unpolarised light after passing through polariser: $I = 0.50 I_0$