Angular Coordinates and Angular Displacement

- Polar coordinates allow us to describe and analyze circular motion, where the distance to the origin of the object in motion stays constant and the angle θ varies as a function of time.

- The angle θ is measured with respect to the positive x-axis.

- A move in the counterclockwise direction away from the positive x-axis means postive θ.

- The two most commonly used units for angle are:
  - Degrees (°)
  - Radians (rad)
    - $360° = 2\pi$ rad

Angular Coordinates and Angular Displacement

- Like x, θ can be positive or negative, but θ is periodic:
  - A complete turn around the circle (360° or 2π rad) returns the position vector back to the same point.

- We define the angular displacement as:

$$\Delta\theta = \theta_2 - \theta_1$$

- We define the arc length to be the distance traveled along the circular path:

$$s = r\theta$$

where θ is measured in radians.

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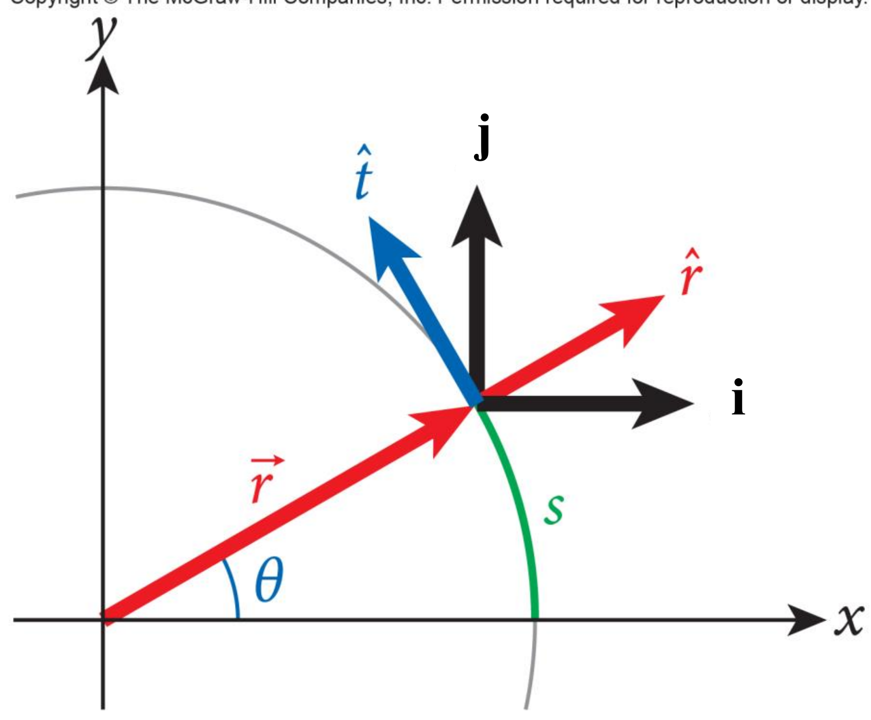
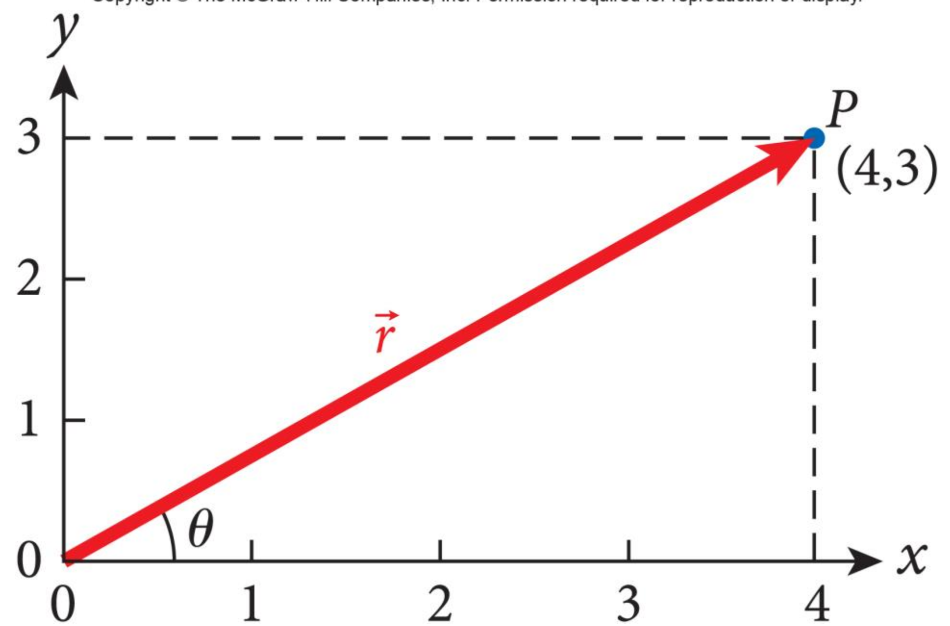


Locating a Point



- A point has a location given in Cartesian coordinates.

PROBLEM:

- How do we represent the position of this point in polar coordinates?

SOLUTION:

- The radial coordinate is:

$$r = \sqrt{x^2 + y^2} = \sqrt{4^2 + 3^2} = 5$$

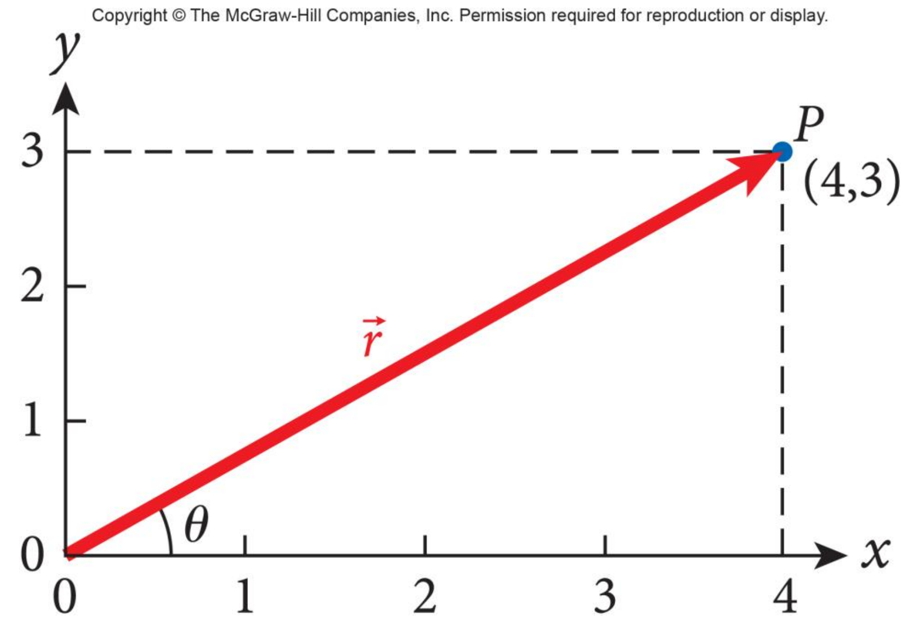
- The angular coordinate is:

$$\theta = \tan^{-1}(y / x) = \tan^{-1}(3 / 4) = 0.64 \text{ rad} = 37^\circ$$

Locating a Point

- So we can write the position of point P in polar coordinates as:

$$(r,\theta) = (5, 0.64 \text{ rad})$$
$$(r,\theta) = (5, 37°)$$



- Note that we can specify the same position by adding any integral multiple of 2π rad or 360° to θ:

$$(r,\theta) = (5, 2\pi + 0.64 \text{ rad})$$
$$(r,\theta) = (5, 360° + 37°)$$

CD Track

- The track on a compact disc (CD) is a spiral originating at an inner radius of 25 mm and finishing at an outer radius of 58 mm.
- The spacing between loops is: 1.6 μm

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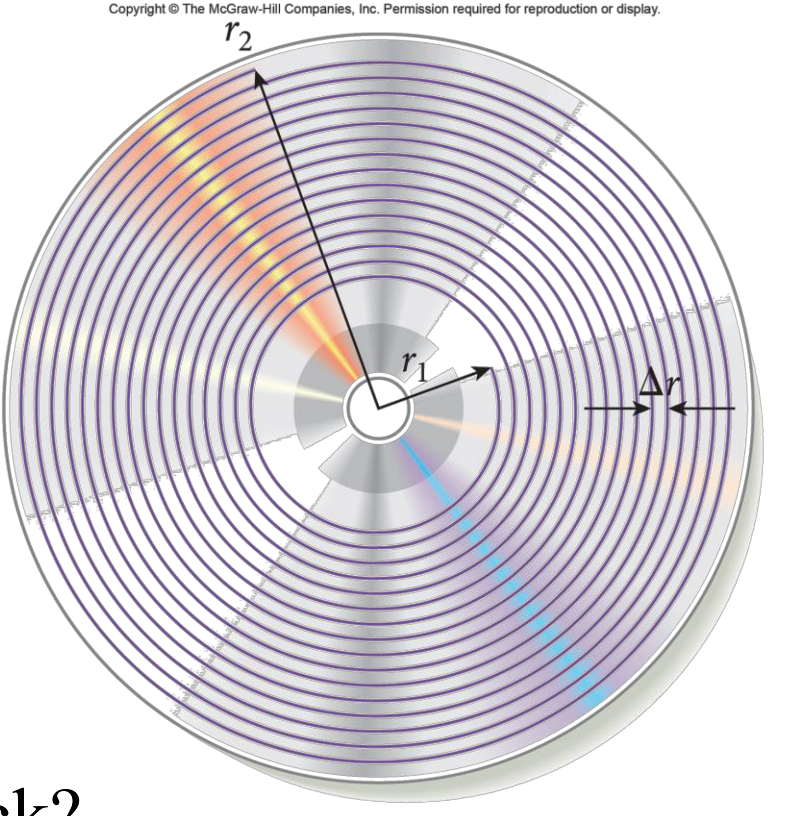

PROBLEM:

- What is the total length of this track?

SOLUTION WITHOUT CALCULUS:

- At a given radius, the track is almost circular.

CD Track

- The track winds around the CD a total of:

$$n = \frac{r_2 - r_1}{\Delta r} = \frac{58 \text{ mm} - 25 \text{ mm}}{1.6 \ \mu\text{m}} = \frac{3.3 \cdot 10^{-2} \text{ m}}{1.6 \cdot 10^{-6} \text{ m}} = 20{,}625 \text{ times}$$

- The average radius of these 20,625 circles is:

$$\bar{c} = 2\pi\bar{r} = 2\pi \cdot \frac{1}{2}(r_2 + r_1) = 2\pi(41.5) = 0.2608\, m$$

- Multiplying the average circumference by the number of circles gives us the length of the track:

$$L = n\bar{c} = (20{,}625)(0.2608 \text{ m}) = 5.4 \text{ km } (3.3 \text{ mi})$$

SOLUTION WITH CALCULUS:

- The track density is $1/\Delta r = 625{,}000$ lines/m.

- At a given radius r, the track is nearly circular and the length of the track for each circle is $2\pi r$.

CD Track

- We get the overall length of the track by integrating the length of each circle from r_1 to r_2 multiplied by the number of circles per unit length:

$$L = \frac{1}{\Delta r}\int_{r_1}^{r_2} 2\pi r\, dr$$

$$L = \frac{1}{\Delta r}\pi\left(r_2^2 - r_1^2\right) = \pi\left(625{,}000\ \text{m}^{-1}\right)\left(\left(0.058\ \text{m}\right)^2 - \left(0.025\ \text{m}\right)^2\right)$$

$$L = 5.4\ \text{km}$$

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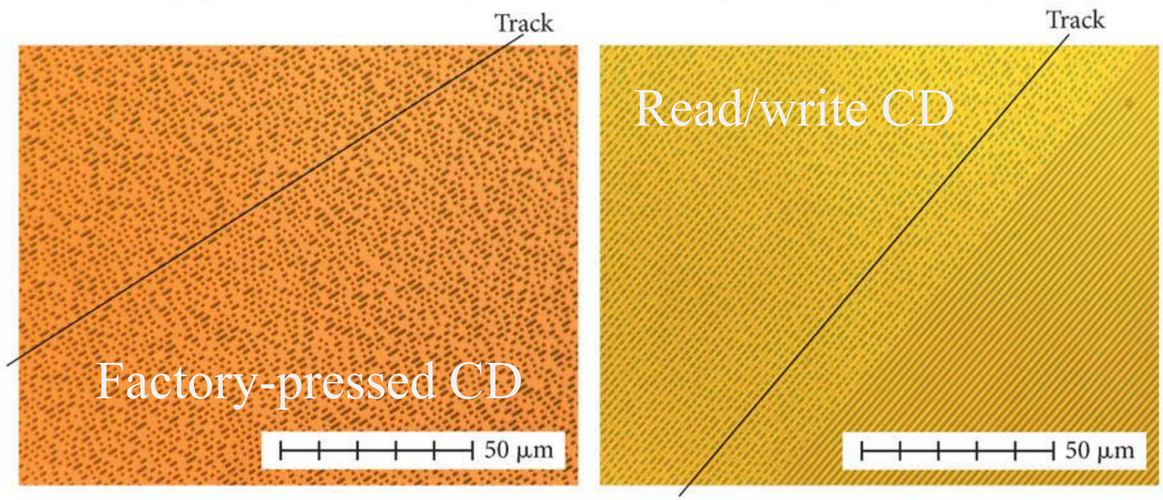

(a) (b)

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