ENG1004 Eng Physics 1

AY2023/24 Trimester 1

Week9: Oscillations (Part 1)

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List of animations (Do not stare too long!)

https://regijs.github.io/simulacoes/pendulo.gif

https://socratic.org/questions/what-are-some-examples-of-simple-harmonic-motion

https://iwant2study.org/ospsg/index.php/interactive-resources/physics/02-newtonian-mechanics/09-oscillations

Content

1. Types of oscillation
2. Simple Harmonic Motion (SHM)
3. Variation with time: x, v, a vs t.
4. Variation with displacement: v, a vs x.
5. Damped Oscillations
6. Forced Oscillations
7. * Oscillation Video Questions (about 20 questions)

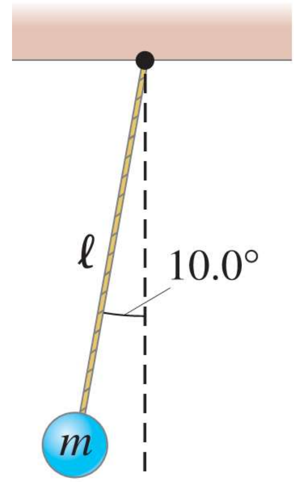
x: displacement
v: velocity
a: acceleration

1. Types of Oscillation

1. Simple Oscillating Pendulum
   https://www.youtube.com/watch?v=fTOuA2Y_IX0
2. Oscillating Spring: Horizontal & Vertical
3. Oscillating cylinder (floating in water)

Oscillations will go on forever if <u>undamped.</u>

<u>Damping:</u> Resistive forces acting on oscillation.



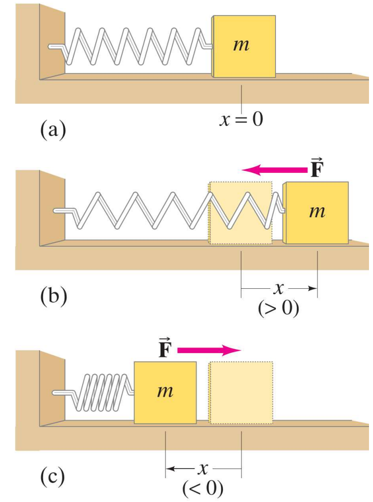


FIGURE 11–1 An object of mass m oscillating at the end of a uniform spring. The force $\vec{\mathbf{F}}$ on the object at the different positions is shown *above* the object.

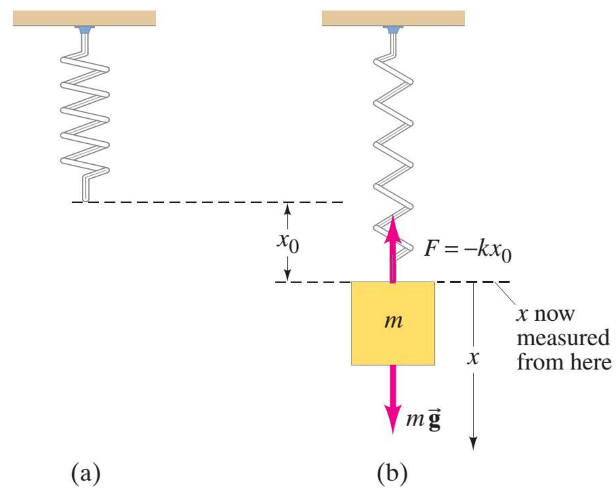
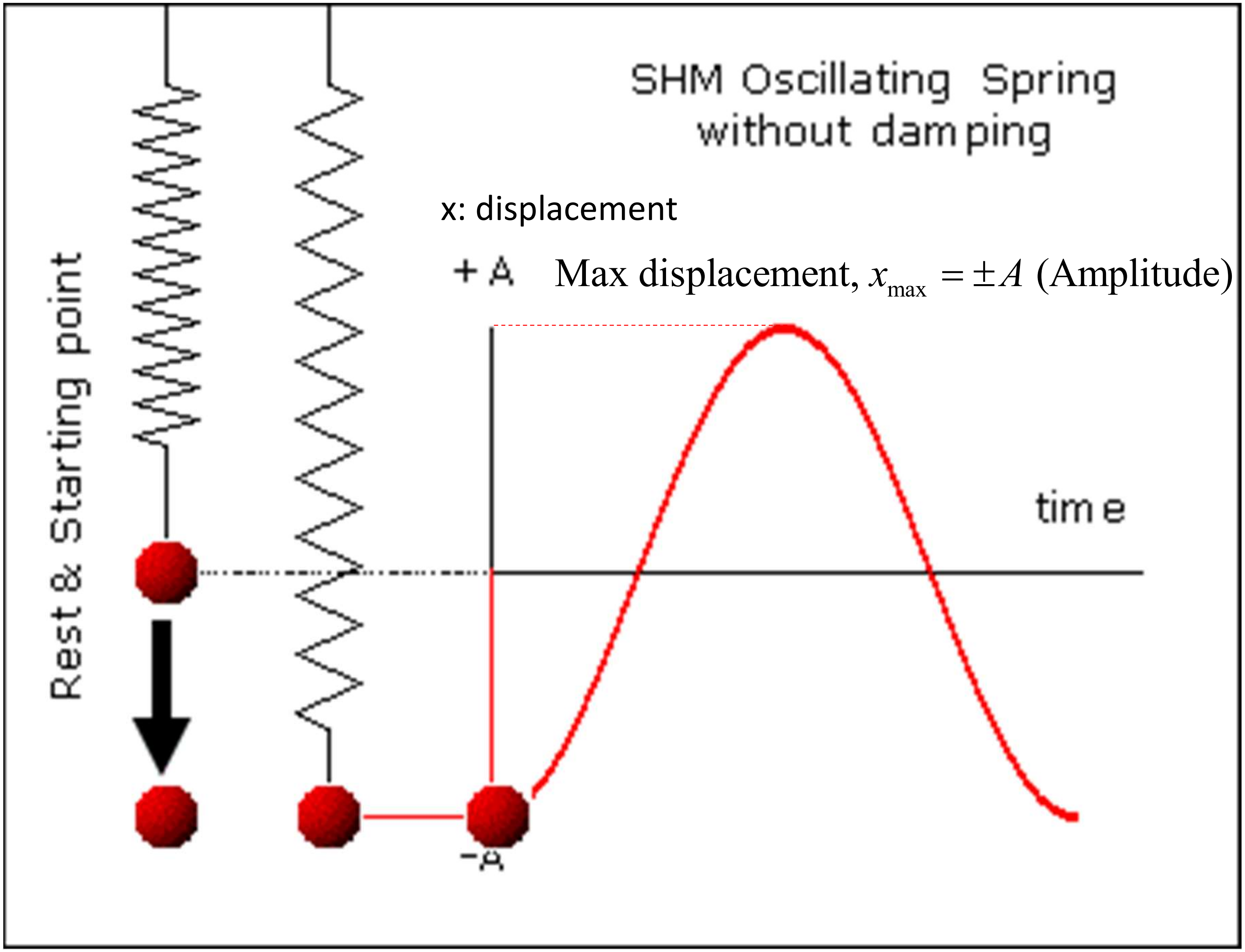
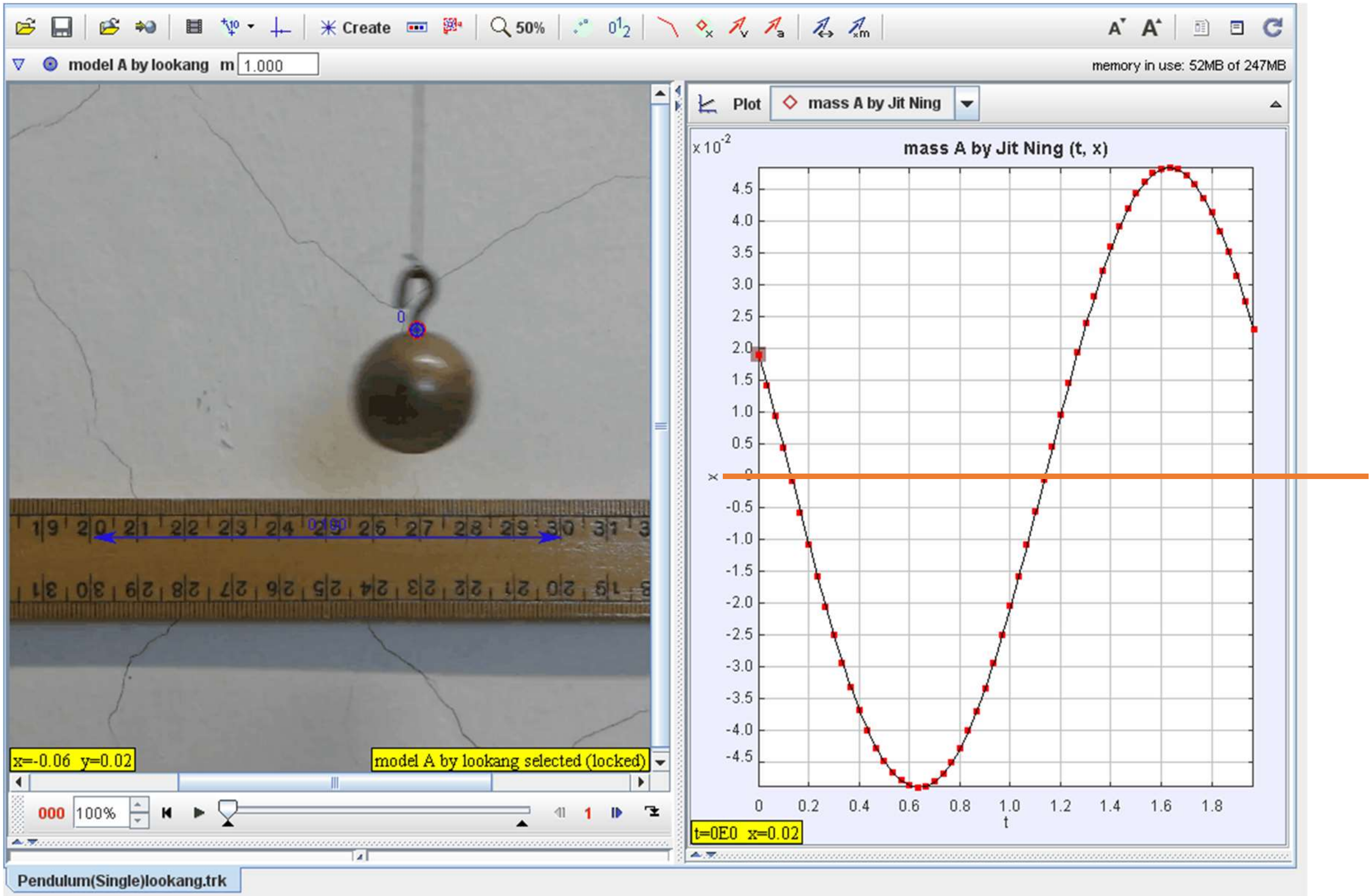


FIGURE 11–3
(a) Free spring, hung vertically.
(b) Mass m attached to spring in new equilibrium position, which occurs when $\Sigma F = 0 = mg - kx_0$.

https://askeyphysics.org/home/shm-spring-amplitude-gif/



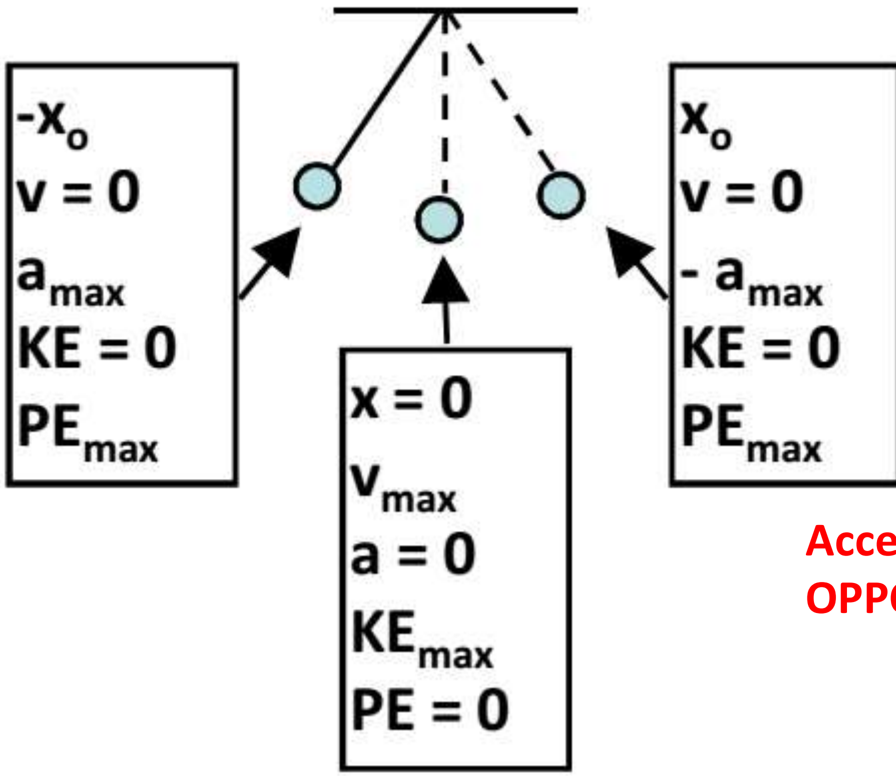
https://weelookang.blogspot.com/2017/04/tracker-animated-gifs-for-oscillations.html



1. Types of Oscillation

$$v = \frac{dx}{dt}$$

Simple Oscillating Pendulum



$$a = \frac{dv}{dt}$$

Amplitudes: x_0, $-x_0$

(maximum displacements

Acceleration & **Velocity** are in **OPPOSITE DIRECTIONS**

Define: Equilibrium position as x = 0

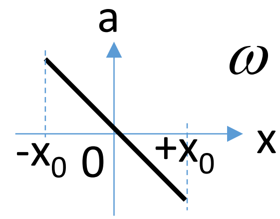
Total Energy, TE= KE+PE

No resistive force (damping): Total energy is constant

2. Simple Harmonic Motion (SHM)

Angular frequency: ω

1. Defining equation $[\ a = -\omega^2 x\ ]$



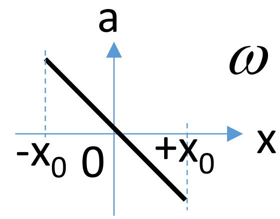
$$\omega = 2\pi f = \frac{2\pi}{T}$$

Unit: rad/ s

2. SHM : An object in an oscillatory motion with acceleration **directly proportional** to displacement from its equilibrium point (x=0) **and** always directed towards that equilibrium point.

2. Simple Harmonic Motion (SHM)

Angular frequency: ω



$$\omega = 2\pi f = \frac{2\pi}{T}$$

1. Defining equation $[\ a = -\omega^2 x\ ]$

2. SHM : An object in an oscillatory motion with acceleration **directly proportional** to displacement from its equilibrium point (x=0) **and** always directed towards that equilibrium point.

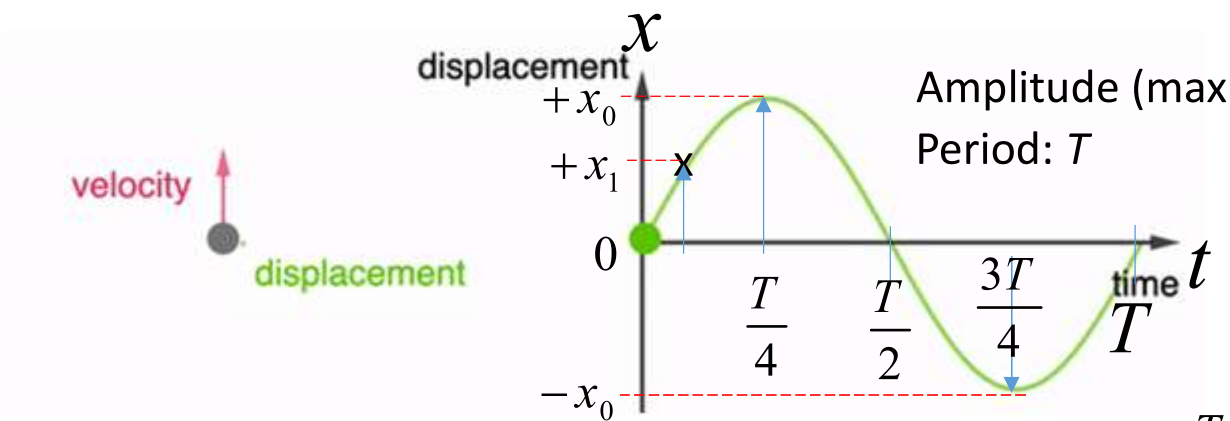
3. Equilibrium Position: Usually x = 0. (a will be ZERO) Newton's 2nd Law (At equilibrium position, resultant force R = 0) because ma = 0 N

4. Other terms:
   i. Period (T) & Frequency (f): Period is fixed regardless of amplitude.
   ii. Displacement x
   iii. Amplitude: Max. displacement $x = x_0$ from equilibrium position.
   iv. Phase: 2 oscillating bodies are in-phase or out-of-phase

f is natural frequency of system that depends on system properties.
E.g. f of **mass-spring system** depends on mass m and spring constant k.

From Newton's 2nd Law

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$$



Amplitude (max displacement): $\pm x_0$

Period: T

$$x = x_0 \sin \omega t$$

$$x = x_0 \sin\left(\frac{2\pi}{T}\right)t$$

At time $\frac{T}{4}$, $x = x_0 \sin\left(\frac{2\pi}{T}\right)\left(\frac{T}{4}\right) = +x_0$

At time $\frac{T}{2}$, $x = x_0 \sin\left(\frac{2\pi}{T}\right)\left(\frac{T}{2}\right) = 0$

At time $\frac{3T}{4}$, $x = x_0 \sin\left(\frac{2\pi}{T}\right)\left(\frac{3T}{4}\right) = -x_0$

At time $0.7T$,

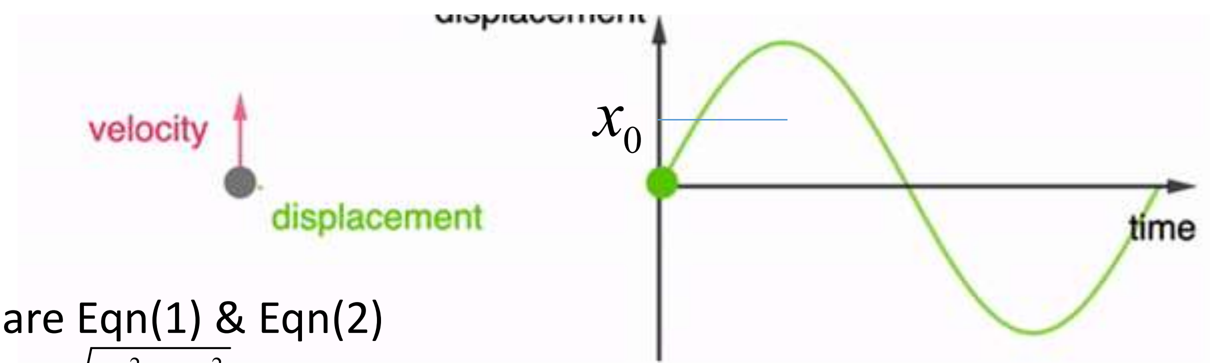
$$x = x_0 \sin\left(\frac{2\pi}{T}\right)(0.7T) = -0.95x_0$$

STARTING CONDITION:

$$x = 0 \text{ at } t = 0$$

***Check:** When using angles in degrees, make sure calculator in **DEGREE** mode
When using radians, make sure calculator in **RADIAN** mode.

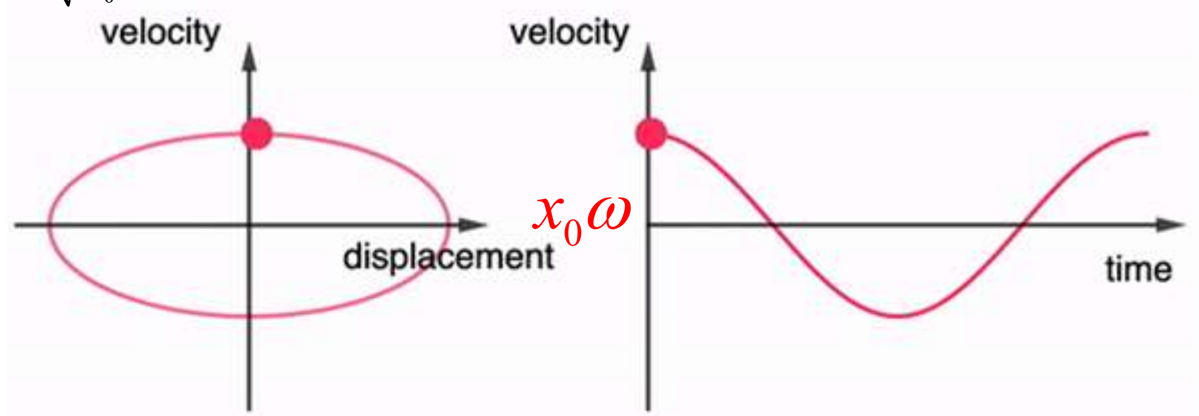
https://www.physicslens.com/category/a-level-topics/10-oscillations/



$$x = x_0 \boxed{\sin \omega t} \quad \text{----(1)}$$

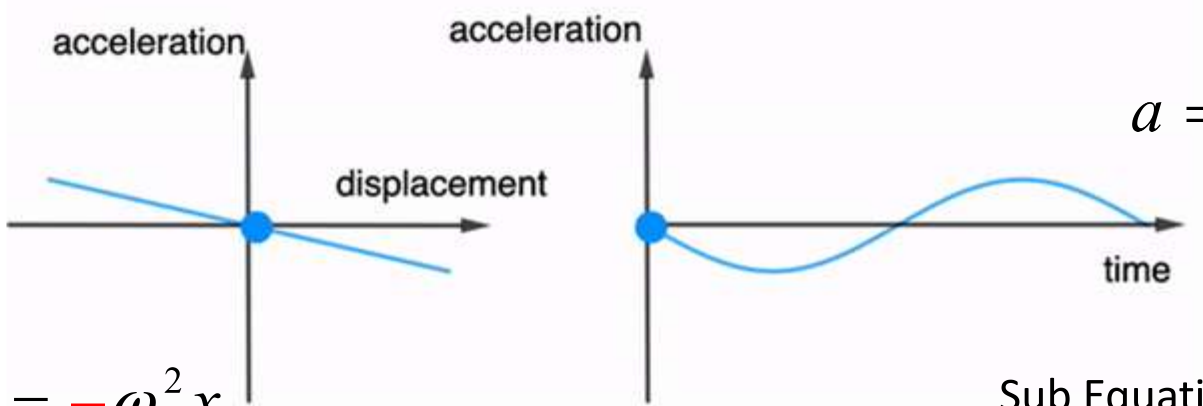
Square Eqn(1) & Eqn(2)

$$v = \pm\omega\sqrt{x_0^{\,2} - x^2}$$



$$v = \frac{dx}{dt} = x_0 \boxed{\omega \cos \omega t}$$

$$v = x_0 \boxed{\omega \cos \omega t} \quad \text{----(2)}$$



$$a = \frac{dv}{dt} = \boxed{-} x_0 \boxed{\omega^2 \sin \omega t}$$

$$a = -\omega^2 \left[x_0 \sin \omega t \right]$$

----(3)

$$a = -\omega^2 x$$

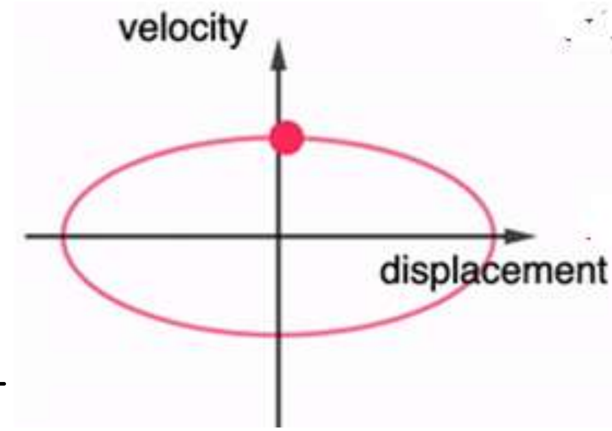
Sub Equation (1) into (3)

$$a = -\omega^2 x$$

$$x = x_0 \sin \omega t \quad \text{----(1)}$$

$$v = x_0 \omega \cos \omega t \quad \text{----(2)}$$

Square Eqn(1) & Eqn(2)



$$(1)^2 : \quad x^2 = x_0{}^2 \sin^2 \omega t$$

$$(2)^2 : \quad v^2 = x_0{}^2 \omega^2 \cos^2 \omega t$$

$$\sin^2 \omega t = \frac{x^2}{x_0{}^2}$$

$$\cos^2 \omega t = \frac{v^2}{x_0{}^2 \omega^2}$$

$$\sin^2 \omega t + \cos^2 \omega t = 1$$

$$\frac{x^2}{x_0{}^2} + \frac{v^2}{x_0{}^2 \omega^2} = 1$$

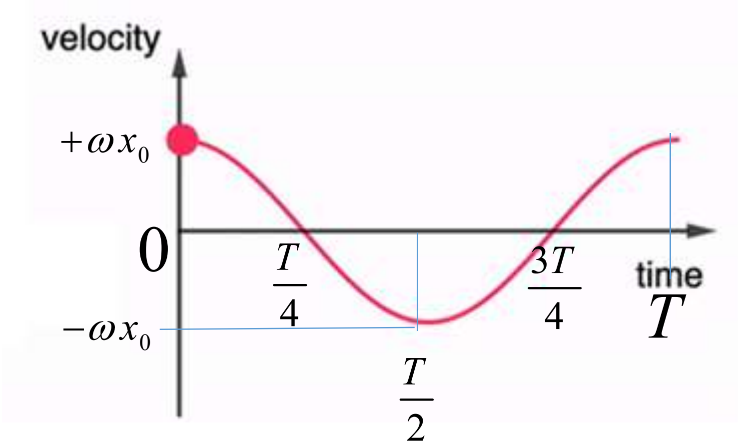
$$\frac{x^2 \omega^2}{x_0{}^2 \omega^2} + \frac{v^2}{x_0{}^2 \omega^2} = 1$$

$$x^2 \omega^2 + v^2 = x_0{}^2 \omega^2$$

$$v^2 = x_0{}^2 \omega^2 - x^2 \omega^2$$

$$v = \pm \sqrt{x_0{}^2 \omega^2 - x^2 \omega^2}$$

$$v = \pm \omega \sqrt{x_0{}^2 - x^2}$$



$$x = x_0 \sin \omega t$$

$$x = x_0 \sin\left(\frac{2\pi}{T}\right)t$$

Constants:

$x_0 \quad \omega$

$$v = \frac{dx}{dt} = \omega x_0 \cos \omega t \Rightarrow \boxed{v = \omega x_0 \cos\left(\frac{2\pi}{T}\right)t}$$

$$v_{\max} = \pm\omega x_0$$

At time t = 0 , cos (0) = 1 $v_{\max} = +\omega x_0$

At time $\frac{T}{2}$, $\cos\pi = -1$ $v_{\min} = -\omega x_0$

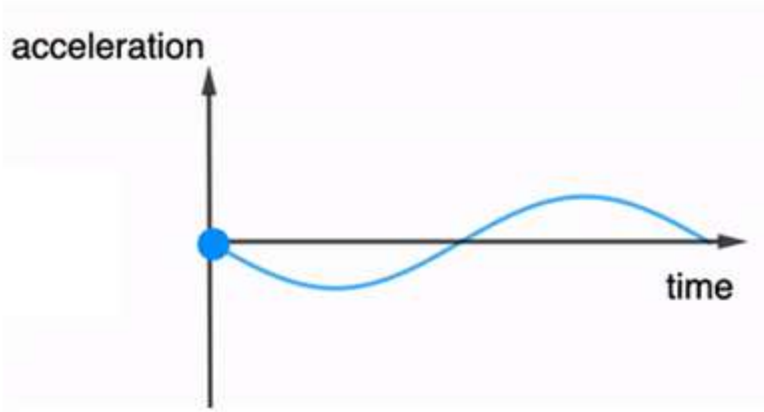
Velocity has magnitude & direction

* Minimum SPEED is = 0

Max **speed** is ωx_0

Min **speed** is 0

Speed has magnitude ONLY
(Magnitude of velocity)



$$v = \frac{\mathrm{d}x}{\mathrm{d}t} = \omega x_0 \cos \omega t \Rightarrow v = \omega x_0 \cos\left(\frac{2\pi}{T}\right)t$$

$$a = \frac{\mathrm{d}v}{\mathrm{d}t}$$

$$v = \omega x_0 \cos \omega t$$

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = (\omega x_0)(\omega)(-\sin \omega t)$$

$$a = -\omega^2 x_0 \sin \omega t \quad \text{-----(1)}$$

$$x = x_0 \sin \omega t \quad \text{-----(2)}$$

Sub Eqn (2) into Eqn (1): $a = -\omega^2 x$

TRY ON YOUR OWN

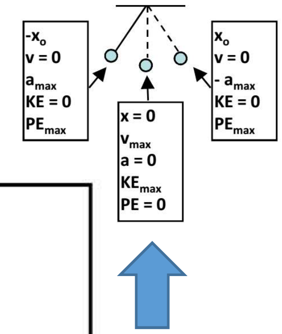
$$x = x_0 \cos \omega t$$

Happens WHEN x = x_0 at t = 0

$$x = -x_0 \cos \omega t$$

Happens WHEN x = -x_0 at t = 0

3. Variation with time: x, v, a **vs** t.

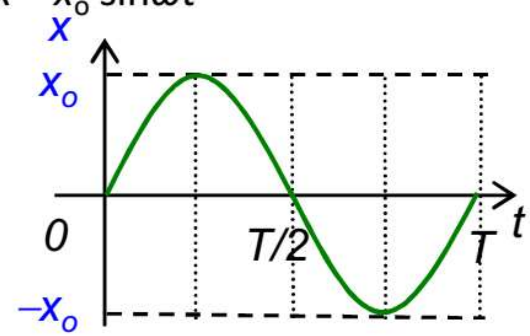


Start here.

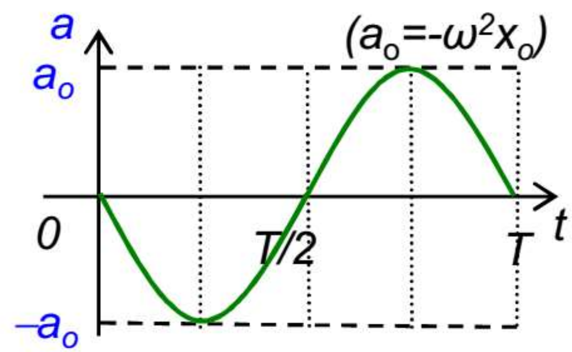
Graphs & equations VARY with initial settings.

Eg. When <u>started from equilibrium</u> (At $t = 0$, $x = 0$, $v = v_o$)

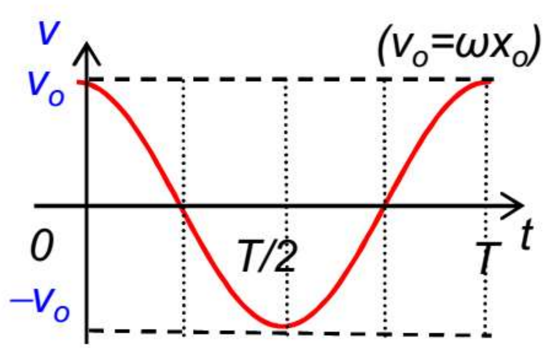
$x = x_o \sin\omega t$



$a = -\omega^2 x_o \sin\omega t = -\omega^2 x$



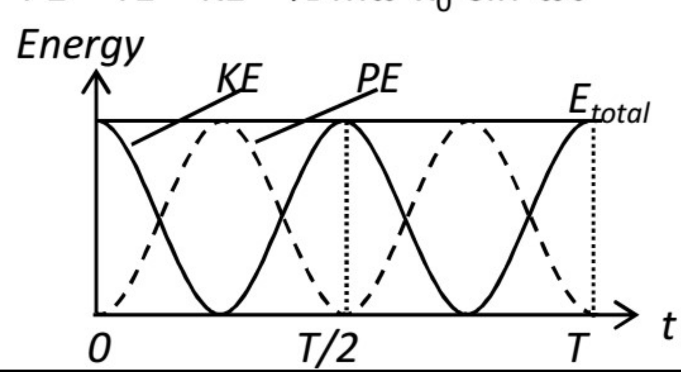
$v = \omega x_0 \cos \omega t = v_o \cos\omega t$



$KE = ½ m\omega^2 x_0^2 \cos^2\omega t$

$E_{Total} = ½ m\omega^2 x_0^2$

$PE = TE – KE = ½ m\omega^2 x_0^2 \sin^2\omega t$



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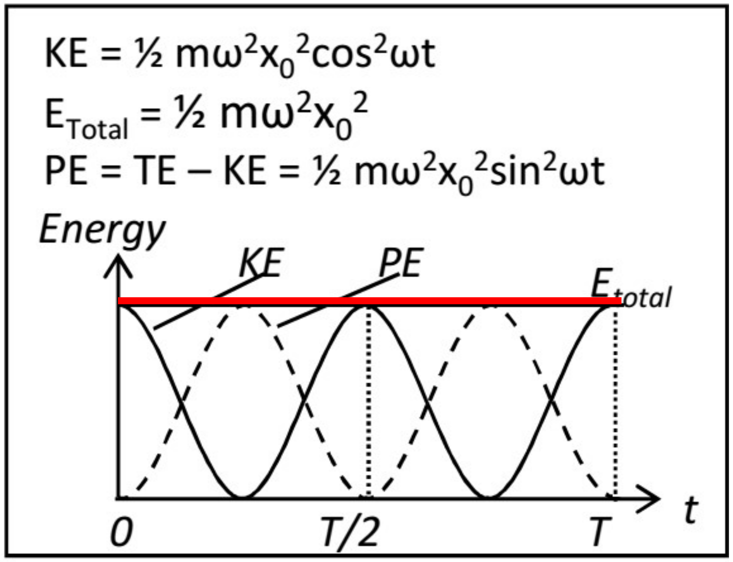


$$x = x_0 \sin \omega t$$

$$v = \omega x_0 \cos \omega t$$

m: Mass of oscillating object

$$KE = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\omega x_0 \cos \omega t\right)^2$$

$$KE = \frac{1}{2} m \omega^2 x_0^2 \cos^2 \omega t$$

$$Total\ E = \frac{1}{2} m \omega^2 x_0^2$$ (Constant with t)

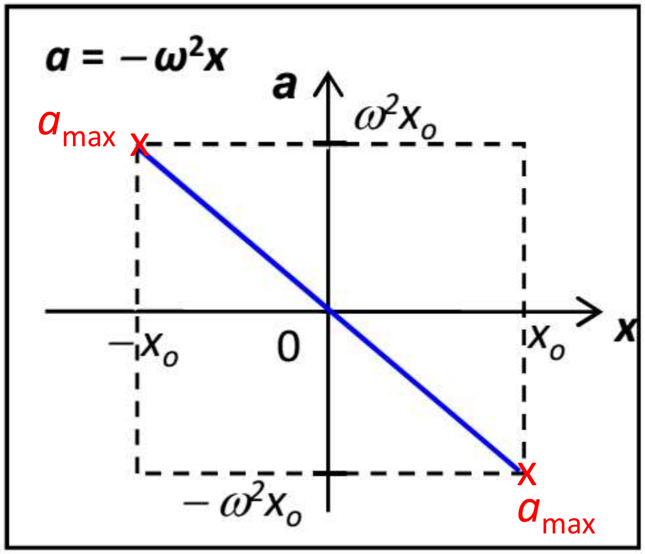
$$PE = \frac{1}{2} m \omega^2 x_0^2 - \frac{1}{2} m \omega^2 x_0^2 \cos^2 \omega t$$

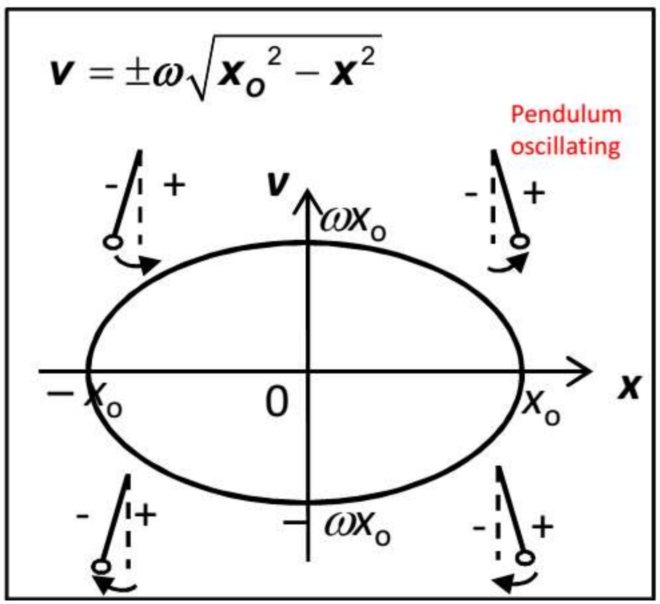
$$= \frac{1}{2} m \omega^2 x_0^2 \left(1 - \cos^2 \omega t\right)$$

$$\Rightarrow PE = \frac{1}{2} m \omega^2 x_0^2 \sin^2 \omega t \qquad \left[PE = \frac{1}{2} m \omega^2 x^2 \right]$$

$$\sin^2 \omega t + \cos^2 \omega t = 1 \Rightarrow \sin^2 \omega t = 1 - \cos^2 \omega t$$

4. Variation with displacement: v, a **vs** x.





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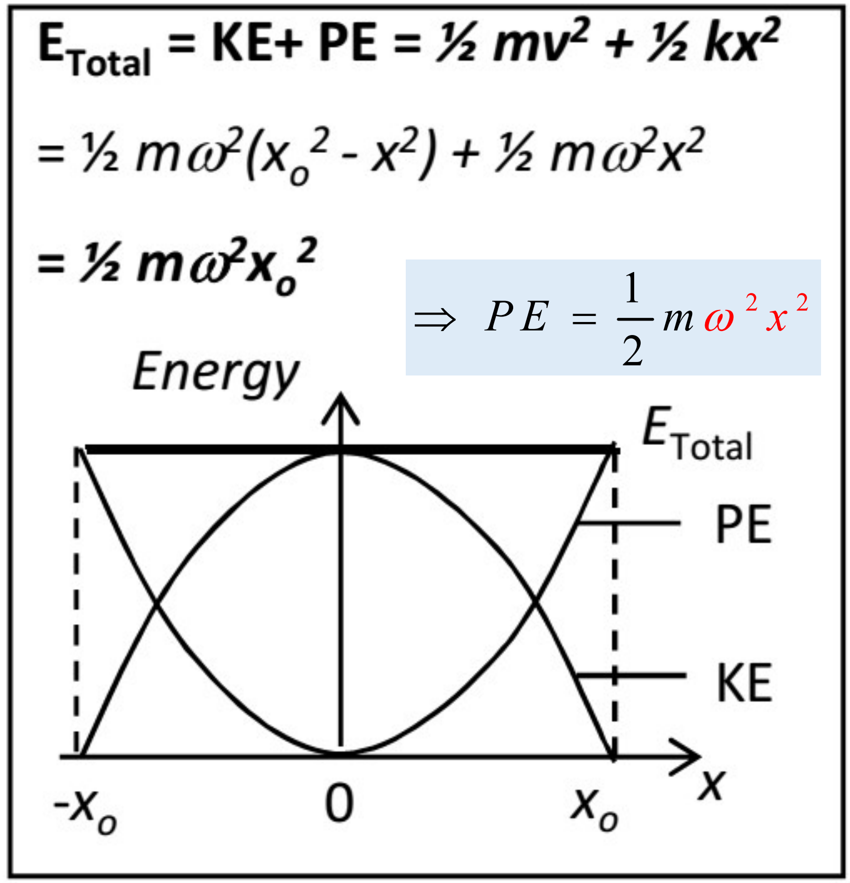
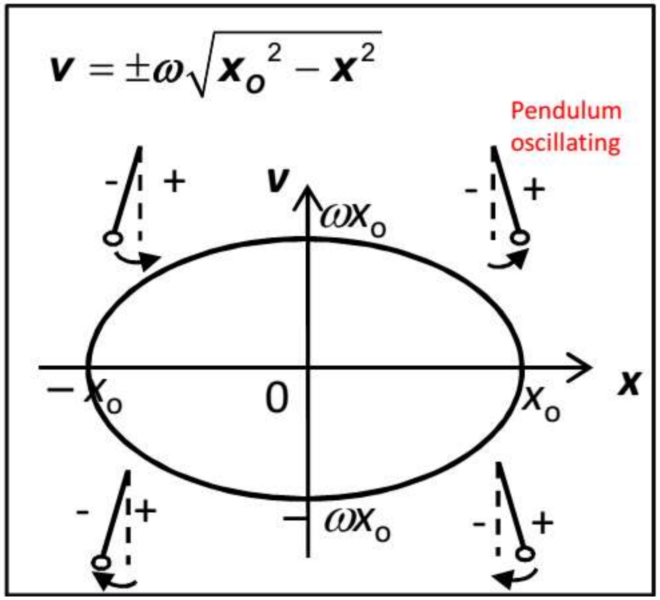




$$x = x_0 \sin \omega t$$

$$\Rightarrow \sin \omega t = \frac{x}{x_0} \quad \text{----(1)}$$

$$\sin^2 \omega t = \left(\frac{x}{x_0}\right)^2 \quad \text{----(1a)}$$

$$v = \omega x_0 \cos \omega t$$

$$\Rightarrow \cos \omega t = \frac{v}{\omega x_0} \quad \text{-------(2)}$$

$$\cos^2 \omega t = \left(\frac{v}{\omega x_0}\right)^2 \quad \text{-------(2a)}$$

$$\sin^2 \omega t + \cos^2 \omega t = 1$$

Eqn(1a) + Eqn(2a):

$$\left(\frac{x}{x_0}\right)^2 + \left(\frac{v}{\omega x_0}\right)^2 = 1 \qquad \Rightarrow \frac{\omega^2 x^2 + v^2}{\omega^2 x_0^2} = 1$$

$$\Rightarrow \omega^2 x^2 + v^2 = \omega^2 x_0^2$$

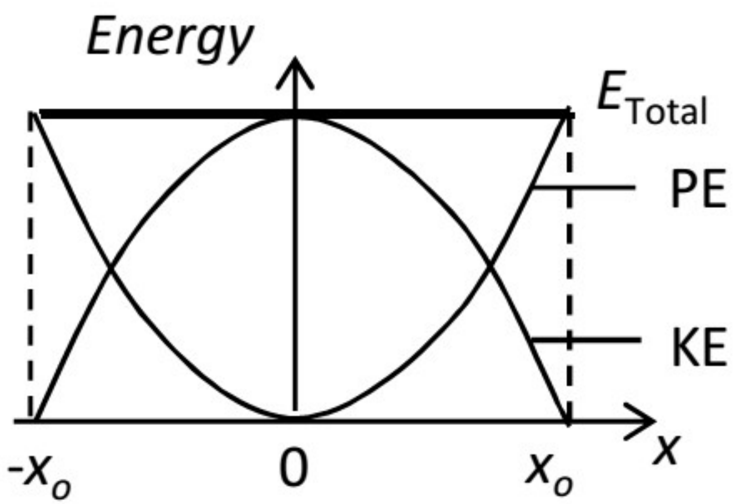
$$\Rightarrow v^2 = \omega^2 x_0^2 - \omega^2 x^2$$

$$\Rightarrow v = \pm\omega\sqrt{x_0^2 - x^2}$$

$$E_{Total} = KE + PE = \tfrac{1}{2} mv^2 + \tfrac{1}{2} kx^2$$

$$= \tfrac{1}{2} m\omega^2(x_o^2 - x^2) + \tfrac{1}{2} m\omega^2 x^2$$

$$= \tfrac{1}{2} m\omega^2 x_o^2$$

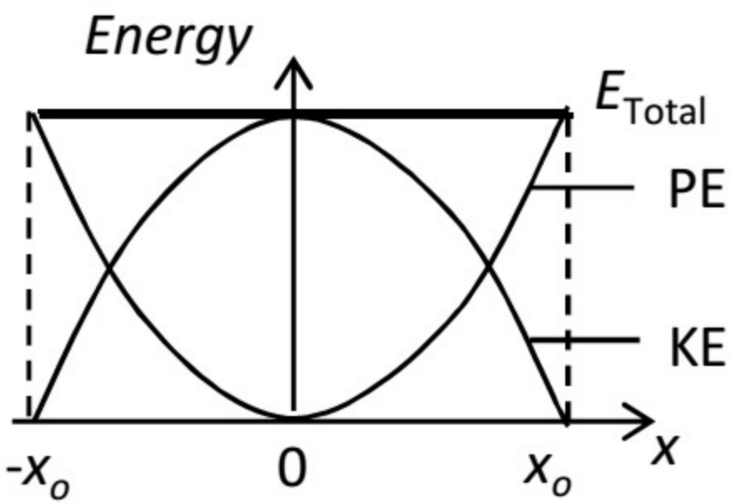


Undamped oscillation: Total E is constant

$$\mathbf{E_{Total} = KE + PE = \tfrac{1}{2} mv^2 + \tfrac{1}{2} kx^2}$$

$$= \tfrac{1}{2}\, m\omega^2(x_o^2 - x^2) + \tfrac{1}{2}\, m\omega^2 x^2$$

$$\mathbf{= \tfrac{1}{2}\, m\omega^2 x_o^2}$$



$$KE = \frac{1}{2} m\omega^2 x_0^2 \cos^2 \omega t$$

$$KE = \frac{1}{2} m\omega^2 x_0^2 \left(1 - \sin^2 \omega t\right)$$

$$KE = \frac{1}{2} m\omega^2 x_0^2 - \frac{1}{2} m\omega^2 x_0^2 \sin^2 \omega t$$

$$KE = \frac{1}{2} m\omega^2 x_0^2 - \frac{1}{2} m\omega^2 x^2$$

$$\text{Total } E = \frac{1}{2} m\omega^2 x_0^2$$ CONSTANT with t

$$PE = \frac{1}{2} m\omega^2 x_0^2 \sin^2 \omega t$$

$$PE = \frac{1}{2} m\omega^2 \left(x_0 \sin \omega t\right)^2$$

$$PE = \frac{1}{2} m\omega^2 x^2$$

Question from class

MUHAMMAD NORAFFIQ BIN MOHD ... to Everyone 10:04 AM

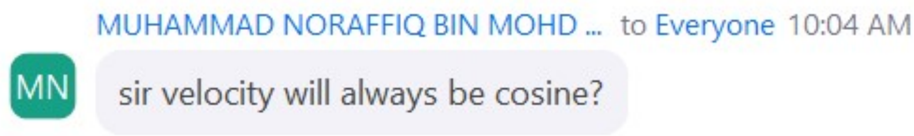
MN sir velocity will always be cosine?

Question 2: Object oscillating vertically on spring

Produce equations & graphs when object is displaced downwards from equilibrium and released (i.e. at $t = 0$, $x = + x_o$, $v = 0$)

Means starting graph is **cosine displacement-time graph** (when t = 0, x = x_0)

Question from class



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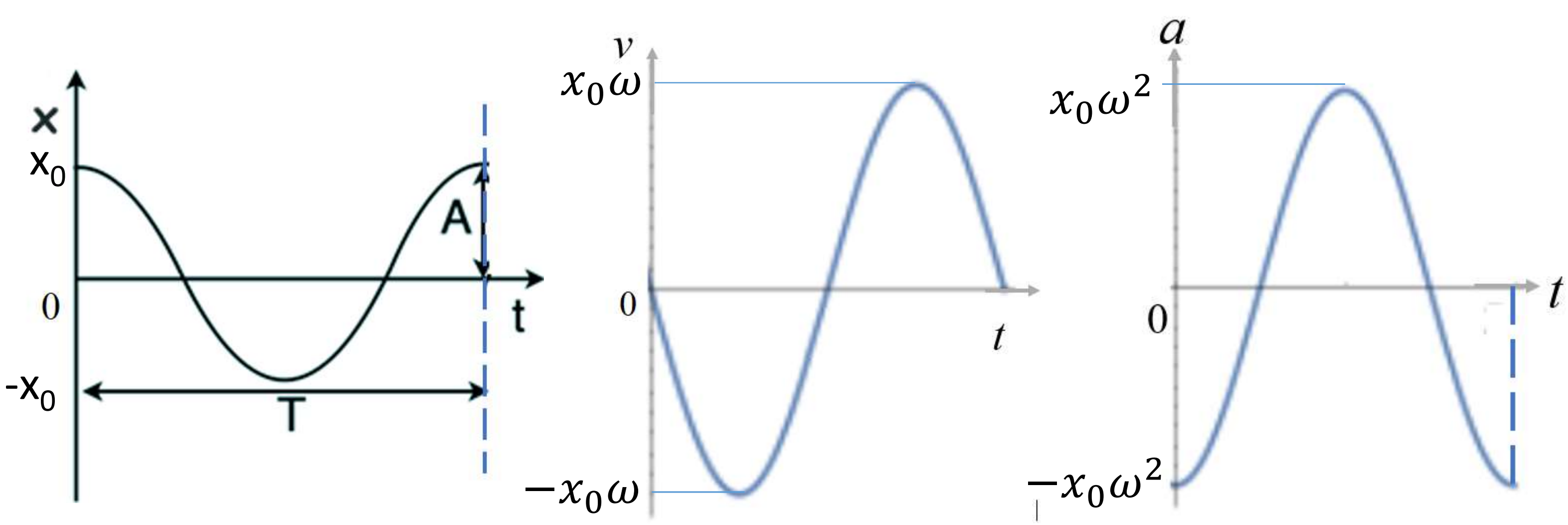
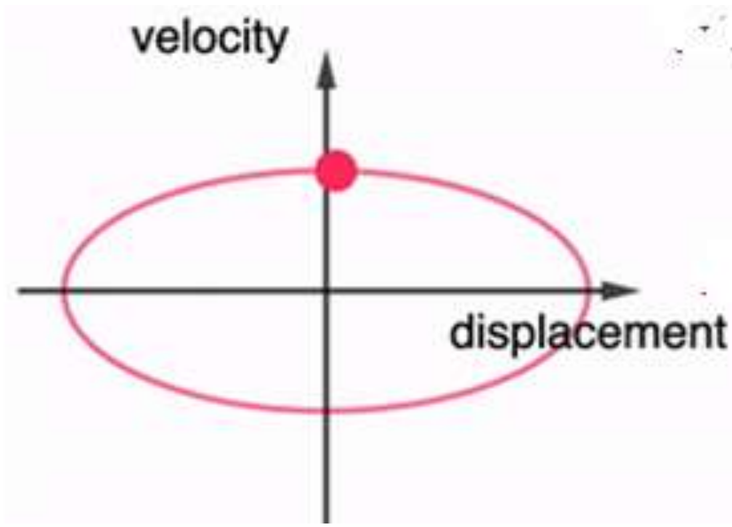


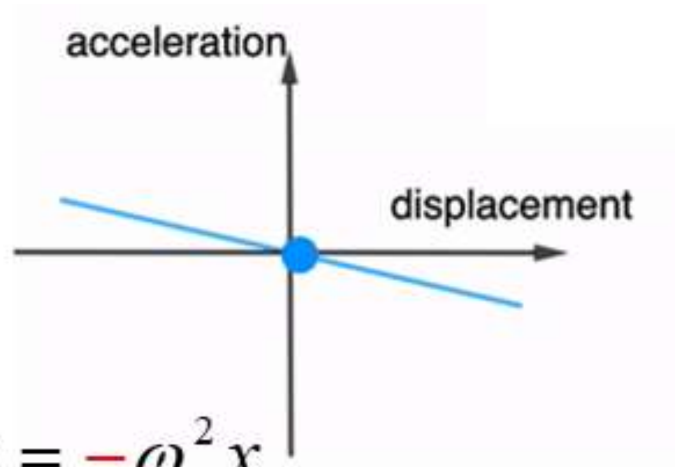
$$x = x_0 \cos \omega t$$

$$v = -x_0 \omega \sin \omega t$$

$$a = -x_0 \omega^2 \cos \omega t$$

$$a = -\omega^2 \left(x_0 \cos \omega t \right) = -\omega^2 x$$





$$a = -\omega^2 x$$

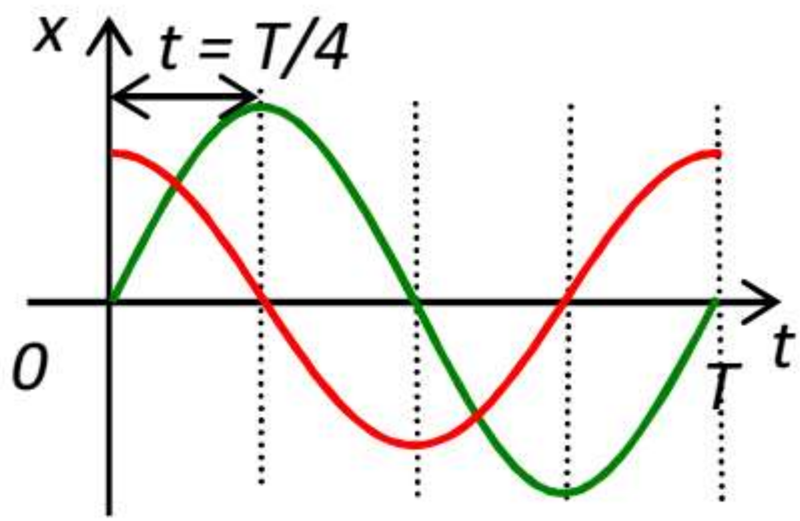
Comparing 2 oscillations with SAME frequency *f*.

When comparing oscillations of same frequency,

Phase difference (in rad): $\Delta\phi = \left(\dfrac{2\pi}{T}\right)t$

1. In phase: $\Delta\phi$ = 0 or 2π rad
2. In anti-phase: $\Delta\phi = \pi$ rad out of phase

***Example: What is phase difference ($\Delta\phi$) for oscillations below?**



Answer:

$$\Delta\phi = \left(\frac{2\pi}{T}\right)\left(\frac{T}{4}\right) = \frac{\pi}{2}\ \text{rad.}$$

Question 3

If the frequency of a system undergoing simple harmonic motion doubles, by what factor does the maximum value of acceleration change?

Answer: 4 times

Hints: $\omega = 2\pi f$

$a = -\omega^2 x$ where you can take $x = x_0 \sin \omega t$ or $x = x_0 \cos \omega t$

Question 3

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$a = -\omega^2 x$ where you can take $x = x_0 \sin \omega t$ or $x = x_0 \cos \omega t$

$$a = -\omega^2 x$$

$$a_1 = -\omega_1{}^2 x \qquad \omega_1 = 2\pi(2f)$$

$$a_1 = -\left(2\pi(2f)\right)^2 x = -4\left[(2\pi f)^2 x\right] = -4\omega^2 x = -4a$$

Question 3

If the frequency of a system undergoing simple harmonic motion doubles, by what factor does the maximum value of acceleration change?

Answer: 4 times

$$\omega = 2\pi f$$

$$a = -\omega^2 x$$

$$a = -4\pi^2 f^2 x$$

$$\omega_1 = 2\pi(2f) = 4\pi f$$

$$a_1 = -\omega_1^2 x$$

$$a_1 = -16\pi^2 f^2 x = 4\left(-4\pi^2 f^2 x\right)$$

$$a_1 = 4a$$

Question 4

A point on the string of a violin moves up and down in simple harmonic motion with an amplitude of 1.24 mm and a frequency of 875 Hz.
(a) What is the maximum **speed** of that point in SI units?
(b) What is the maximum acceleration of the point in SI units?

Answer: (a) 6.82 m/s (b) -3.75×10^4 m/s^2

Question 4

$$1\ \text{mm} = 10^{-3}\ \text{m}$$

A point on the string of a violin moves up and down in simple harmonic motion with an amplitude of 1.24 mm and a frequency of 875 Hz.

(a) What is the maximum speed of that point in SI units?

(b) What is the maximum acceleration of the point in SI units?

Answer: (a) 6.82 m/s (b) -3.75 x 10^4 m/s^2

$$x = x_0 \sin \omega t$$

$$x_0 = 1.24 \times 10^{-3}\ \text{m} \quad f = 875\ \text{Hz}$$

$$\omega = 2\pi(875)$$

$$v = \frac{\mathrm{d}x}{\mathrm{d}t} = x_0 \omega \cos \omega t \qquad v_{\max} = x_0 \omega \quad \text{when } \cos \omega t = 1$$

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -x_0 \omega^2 \sin \omega t$$

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$$a_{\max} = -x_0 \omega^2 \quad \text{when } \sin \omega t = 1$$

$$a_{\max} = -\left(1.24 \times 10^{-3}\right)\left[2\pi(875)\right]^2 = -3.75 \times 10^4 \text{ m s}^{-2}$$

Question 5

A mass, suspended from the end of a spring, is oscillating with SHM.
If the angular frequency is 2.0 rad s^{-1}, what is the period of the oscillation?

Answer: 3.1 s

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$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2.0} = 3.1\ \mathrm{s}$$

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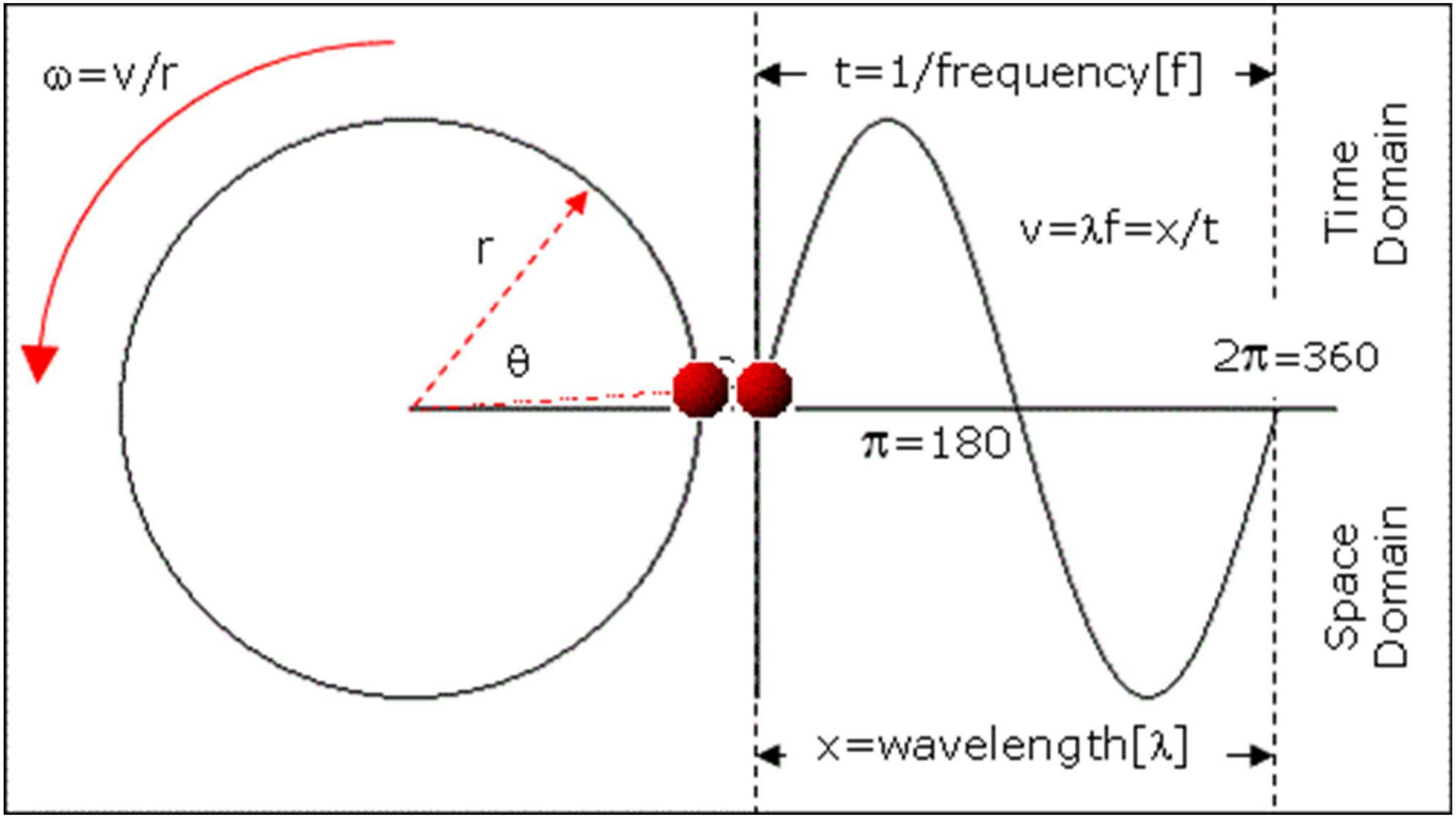

Compare with circular motion

Summary

1. Equations of motion of SHM (x, v, a versus t) when oscillation starts at (a) $x = 0$ at $t = 0$ s **(b)** $\boldsymbol{x = \pm x_0}$ **at** $\boldsymbol{t = 0}$ **s.**

2. Corresponding graphs for No. 1 above.

3. Derive equations relating v & x. $\Rightarrow v = \pm\omega\sqrt{x_0^{\,2} - x^2}$

4. Derive equations relating KE, PE and TE (Total energy) with t & x.

$$a = -\omega^2 x$$