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- During the attendance check a sticker containing a unique code will be put on this exam.
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Circuit Theory

Exam: 1203 / EDE **Date:** Tuesday 25th July, 2023
Examiner: Dr.-Ing. Michael Joham **Time:** 15:00 – 16:40

	P 1	P 2	P 3	P 4	P 5	P 6	P 7
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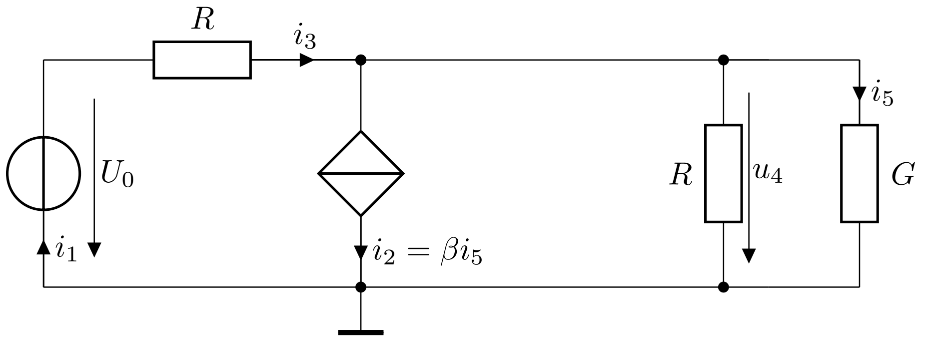
Working instructions

- This exam consists of **18 pages** with a total of **7 problems**.
  Please make sure now that you received a complete copy of the exam.
- The total amount of achievable credits in this exam is 90 credits.
- Detaching pages from the exam is prohibited.
- Allowed resources:
  - 5 double-sided DIN A4 pages of notes
  - one analog dictionary English ↔ native language
- Subproblems marked by * can be solved without results of previous subproblems.
- **Answers are only accepted if the solution approach is documented.** Give a reason for each answer unless explicitly stated otherwise in the respective subproblem.
- Do not write with red or green colors nor use pencils.
- Physically turn off all electronic devices, put them into your bag and close the bag.

Left room from __________ to __________ / Early submission at __________

Problem 1 Circuit Analysis (17 credits)

Consider the following circuit with a CCCS, three resistors, and a voltage source.



0 1 2 a)* What is the number of nodes n and the number of branches b of this circuit?

$n = 3$ √
$b = 5$ √

0 1 b) Give the number of node voltages for above circuit.

$n - 1 = 2$ √

0 1 c) What is the number of linearly independent KCL equations in nullspace representation?

$n - 1 = 2$ √

0 1 d) Give the number of linearly independent KVL equations in nullspace representation.

$b - (n - 1) = 3$ √

0 1 2 e)* Define the branch voltage vector $\boldsymbol{u}$ and the branch current vector $\boldsymbol{i}$.

$\boldsymbol{u} = [u_1, u_2, u_3, u_4, u_5]^{\mathrm{T}}$ √
$\boldsymbol{i} = [i_1, i_2, i_3, i_4, i_5]^{\mathrm{T}}$ √

0 1 f)* Give the number of equations for the KVL in rangespace representation.

$b = 5$ √

g) Find the loop incidence matrix $\boldsymbol{B}$.

0 1 2 3

$u_1 + u_3 + u_2 = 0$
$-u_2 + u_4 = 0$
$-u_4 + u_5 = 0$

$$\boldsymbol{B}\boldsymbol{u} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \boldsymbol{u} \ \checkmark\checkmark\checkmark$$

h)* Determine the node incidence matrix $\boldsymbol{A}$.

0 1 2

$-i_1 + i_3 = 0$
$-i_3 + i_2 + i_4 + i_5 = 0$

$$\boldsymbol{A}\boldsymbol{i} = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 \end{bmatrix} \boldsymbol{i} \ \checkmark\checkmark$$

i) Based on $\boldsymbol{A}$ and $\boldsymbol{B}$, show that the Tellegen's theorem is fulfilled.

0 1 2

$$\boldsymbol{A}\boldsymbol{B}^{\mathrm{T}} = \boldsymbol{0} \ \checkmark$$

$$\boldsymbol{A}\boldsymbol{B}^{\mathrm{T}} = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \ \checkmark$$

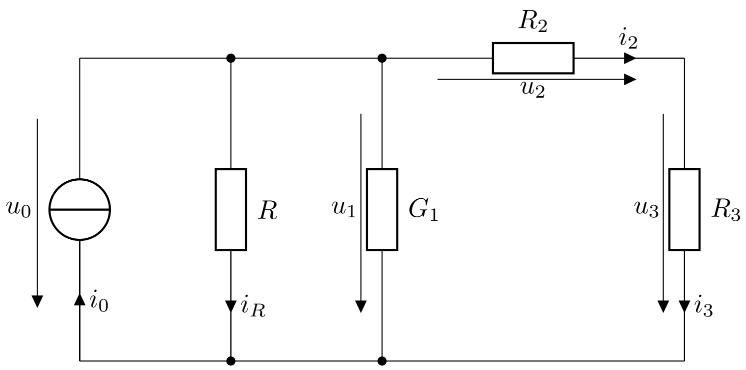
j) Find i_3 depending on u_4.

0 1 2

KCL: $i_3 = i_2 + i_4 + i_5$
KVL: $u_5 = u_4$
Ohm: $i_4 = \frac{u_4}{R}$
$i_5 = Gu_4$ $\checkmark$
$i_3 = \beta G u_4 + \frac{u_4}{R} + Gu_4$ $\checkmark$

Problem 2 Resistor Network (10 credits)

The following circuit with four resistors and a current source is given.



a)* Give the current i_R depending on u_0.

$i_R = \frac{1}{R} u_0$ $\checkmark$

b)* Determine the overall conductance G_{overall} of the parallel connection of R with G_1.

$G_{\text{overall}} = \frac{1}{R} + G_1$ $\checkmark$

c)* Find the voltage u_2 depending on i_2.

$u_2 = R_2 i_2$ $\checkmark$

d)* What is the overall resistance R_{overall} of the series connection of R_2 and R_3?

$R_{\text{overall}} = R_2 + R_3$ $\checkmark$

e) Hence, what are the voltages u_2 and u_3 depending on u_0?

voltage divider: $u_2 = \frac{R_2}{R_{\text{overall}}} u_0 = \frac{R_2}{R_2+R_3} u_0$ $\checkmark$

$u_3 = \frac{R_3}{R_{\text{overall}}} u_0 = \frac{R_3}{R_2+R_3} u_0$ $\checkmark$

f) Find i_0 depending on u_0.

0
1
2

KCL: $i_0 = i_R + G_1 u_1 + i_2$ $\surd$

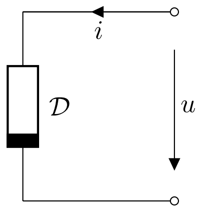
$i_0 = (\frac{1}{R} + G_1 + \frac{1}{R_{\text{overall}}})u_0$ $\surd$

g) Determine i_2 depending on i_0.

0
1
2

current divider: $i_2 = \frac{\frac{1}{R_{\text{overall}}}}{\frac{1}{R}+G_1+\frac{1}{R_{\text{overall}}}} i_0$ $\surd\surd$

Problem 3 Linearization of a One-Port (7 credits)



The characteristic of the one-port $\mathcal{D}$ is given by

$$u = r_{\mathcal{D}}(i) = U_0 + U_0 \sin\left(\frac{i - I_0}{I_0}\right)$$

with the constants U_0 and I_0.

0
1
a)* Give the port-quantities that control $\mathcal{D}$.

i (but not u) √

In the operating point, the current is $I = 2I_0$.

0
1
b)* Find the operating point value U of the voltage across $\mathcal{D}$.

$U = r_{\mathcal{D}}(I_0) = U_0 + U_0 \sin(\frac{2I_0 - I_0}{I_0}) = U_0 + U_0 \sin(1)$ √

In following, consider the different operating point $(U,I) = (U_0,I_0)$.

0
1
2
3
4
c)* Determine the linearization $r_{\mathcal{D},\text{lin}}(i)$ of $\mathcal{D}$ in the given operating point $(U,I) = (U_0,I_0)$.

$R_{\mathcal{D}} = \frac{\mathrm{d}r_{\mathcal{D}}(i)}{\mathrm{d}i}\Big|_{i=I_0} = \frac{U_0}{I_0}\cos(\frac{i-I_0}{I_0})\Big|_{i=I_0}$ √

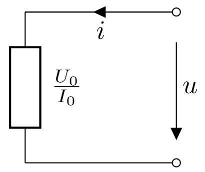
$R_{\mathcal{D}} = \frac{U_0}{I_0}$ √

$r_{\mathcal{D},\text{lin}}(i) = U + R_{\mathcal{D}}(i - I)$ √

$r_{\mathcal{D},\text{lin}}(i) = U_0 + \frac{U_0}{I_0}(i - I_0) = \frac{U_0}{I_0}\, i$ √

d) Draw the equivalent circuit diagram of $r_{\mathcal{D},\text{lin}}(i)$.

0
1



Problem 4 Linear Two-Port (9 credits)

The resistance matrix of the two-port $\mathcal{S}$ reads as

$$\boldsymbol{R}_{\mathcal{S}} = \begin{bmatrix} 0 & -\frac{1}{G_\mathrm{d}} \\ \frac{1}{G_\mathrm{d}} & 0 \end{bmatrix}$$

with finite G_d.

0 1 a)* Based on the given resistance matrix $\boldsymbol{R}_{\mathcal{S}}$, show that the two-port $\mathcal{S}$ is not reciprocal.

$r_{12} = -r_{21}$
Thus, $r_{12} \neq r_{21}$. ✓

0 1 b)* What special two-port is $\mathcal{S}$?

gyrator ✓

0 1 c)* Determine the conductance matrix $\boldsymbol{G}_{\mathcal{S}}$ of $\mathcal{S}$.

$$\boldsymbol{G}_{\mathcal{S}} = \begin{bmatrix} 0 & G_\mathrm{d} \\ -G_\mathrm{d} & 0 \end{bmatrix} \checkmark$$

A different two-port $\mathcal{Z}$ has got the conductance matrix

$$\boldsymbol{G}_{\mathcal{Z}} = \begin{bmatrix} 0 & -G_\mathrm{d} \\ 2G_\mathrm{d} & 0 \end{bmatrix}.$$

0 1 d)* Why is $\mathcal{Z}$ lossy?

$$\boldsymbol{G}_{\mathcal{Z}}^\mathrm{T} = \begin{bmatrix} 0 & 2G_\mathrm{d} \\ -G_\mathrm{d} & 0 \end{bmatrix} \neq -\begin{bmatrix} 0 & -G_\mathrm{d} \\ 2G_\mathrm{d} & 0 \end{bmatrix} = -\boldsymbol{G}_{\mathcal{Z}} \checkmark$$

The two-ports $\mathcal{S}$ and $\mathcal{Z}$ are connected in parallel to get the two-port $\mathcal{X}$.

0 1 2 e) Find a two-port matrix for the two-port $\mathcal{X}$.

$$\boldsymbol{G}_{\mathcal{X}} = \boldsymbol{G}_{\mathcal{S}} + \boldsymbol{G}_{\mathcal{Z}} \checkmark$$

$$\boldsymbol{G}_{\mathcal{X}} = \begin{bmatrix} 0 & 0 \\ G_\mathrm{d} & 0 \end{bmatrix} \checkmark$$

f) Give expressions for u_1 and i_1 of $\mathcal{X}$.

0 1 2

$i_1 = 0$ $\checkmark$
$u_1 = \frac{1}{G_\mathrm{d}} i_2$ $\checkmark$

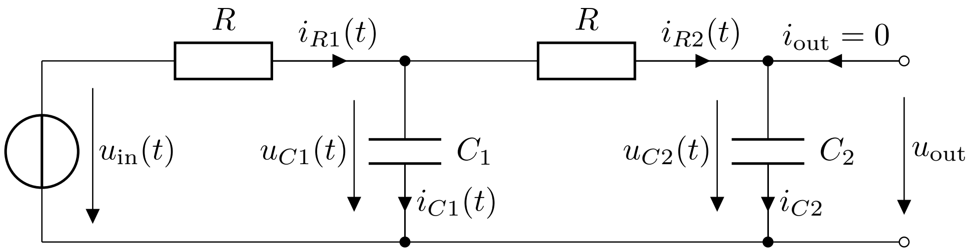
g) What special two-port is $\mathcal{X}$?

0 1

voltage-controlled current source with gain factor G_d $\checkmark$

Problem 5 Complex Phasor Analysis (16 credits)

Given is the following circuit with two capacitors.
The angular frequency of the sinusoidal excitation $u_{\text{in}}(t)$ is $\omega > 0$.



0 1 a)* Give $i_{C1}(t)$ depending on $u_{C1}(t)$.

$i_{C1}(t) = C_1 \dot{u}_{C1}(t)$ √

Let I_{C1} denote the phasor corresponding to $i_{C1}(t)$ and U_{C1} the phasor for $u_{C1}(t)$.

0 1 b) What is the current phasor I_{C1} depending on the voltage phasor U_{C1}?

$I_{C1} = \mathrm{j}\,\omega C_1 U_{C1}$ √

0 1 2 c)* Find the voltage phasor U_{C1} depending on the voltage phasor U_{out} (independent of other current or voltage phasors) taking into account that $I_{\text{out}} = 0$.

KVL: $-U_{C1} + R I_{R2} + U_{\text{out}} = 0$ √
$U_{C1} = \mathrm{j}\,\omega R C_2 U_{\text{out}} + U_{\text{out}}$ √

0 1 2 d) Determine the current phasor I_{R1} depending on the voltage phasor U_{out}.

KCL: $I_{R1} = I_{C1} + I_{R2}$ √
$I_{R1} = \mathrm{j}\,\omega C_1 U_{C1} + \mathrm{j}\,\omega C_2 U_{\text{out}}$
$I_{R1} = (\mathrm{j}\,\omega)^2 R C_1 C_2 U_{\text{out}} + \mathrm{j}\,\omega C_1 U_{\text{out}} + \mathrm{j}\,\omega C_2 U_{\text{out}}$ √

e)* Express the voltage phasor U_{C1} depending on the voltage phasor U_in and the current phasor I_{R1}.

0 1

$U_{C1} = U_\text{in} - R I_{R1}$

With the time constant τ, the transfer function of another circuit can be written as

$$H(\mathrm{j}\,\omega) = \frac{U_\text{out}(\mathrm{j}\,\omega)}{U_\text{in}(\mathrm{j}\,\omega)} = \frac{1}{(\mathrm{j}\,\omega\tau)^2 + \frac{3}{2}\,\mathrm{j}\,\omega\tau + 1}.$$

f)* Investigate $|H(\mathrm{j}\,\omega)|$ at $\omega = 0$, $\omega = \frac{1}{\tau}$, and $\omega \to \infty$.

0 1 2 3

$|H(\mathrm{j}\,0)| = 1$

$|H(\mathrm{j}\,\frac{1}{\tau})| = \frac{1}{|-1+\frac{3}{2}\,\mathrm{j}+1|} = \frac{2}{3}$

$\lim_{\omega\to\infty} |H(\mathrm{j}\,\omega)| = 0$

g) What filter type (lowpass, highpass, bandpass, bandstop, allpass) is the transfer function $H(\mathrm{j}\,\omega)$? Justify your answer based on above results.

0 1

Since $|H(\mathrm{j}\,0)| = 1$ and $\lim_{\omega\to\infty} |H(\mathrm{j}\,\omega)| = 0$, it is a lowpass.

The input voltage is given by $u_\text{in}(t) = 6\,\mathrm{V}\,\cos(\frac{2}{\tau}\,t + \frac{\pi}{4})$.

h)* Give the phasor U_in corresponding to the given $u_\text{in}(t)$.

0 1

$U_\text{in} = 6\,\mathrm{V}\,\mathrm{e}^{\mathrm{j}\,\frac{\pi}{4}}$

i) Find the output phasor U_out in polar form, i.e., as the product of magnitude and the exponential depending on the phase.

0 1 2 3

$U_\text{out} = H(\mathrm{j}\,\frac{2}{\tau}) U_\text{in}$

$H(\mathrm{j}\,\frac{2}{\tau}) = \frac{1}{(\mathrm{j}\,2)^2+3\,\mathrm{j}+1} = \frac{1}{-3+3\,\mathrm{j}}$

$H(\mathrm{j}\,\frac{2}{\tau}) = \frac{1}{3\sqrt{2}}\,\mathrm{e}^{\mathrm{j}\,\frac{-3\pi}{4}}$

$U_\text{out} = \sqrt{2}\,\mathrm{V}\,\mathrm{e}^{-\mathrm{j}\,\frac{\pi}{2}}$

Let the output phasor be given by

$$U_{\text{out}} = 2\,\text{V}\ \text{e}^{\text{j}\,\frac{\pi}{3}}\,.$$

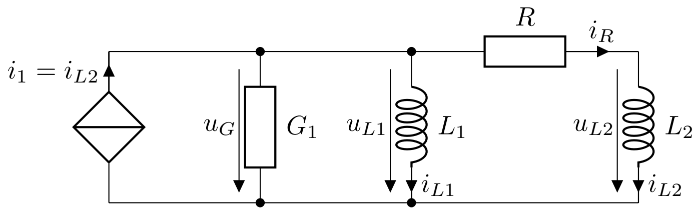
0
1

j)* Give the time-signal $u_{\text{out}}(t)$ corresponding to the phasor U_{out}.

$u_{\text{out}}(t) = \text{Re}\{U_{\text{out}}\ \text{e}^{\text{j}\,\omega t}\} = 2\,\text{V}\cos(\omega t + \frac{\pi}{3})$ ✓

Problem 6 Second-Order Circuit (16 credits)

Given is the following second-order circuit with the two inductors L_1 and L_2 connected to a CCCS and two resistors.



a)* Give the state variables of the circuit.

i_{L1}, i_{L2} √

b) Define the state vector $\boldsymbol{x}$.

$\boldsymbol{x} = \begin{bmatrix} i_{L1} \\ i_{L2} \end{bmatrix}$ √

c)* Find u_{L1} depending on u_{L2} and i_R.

$u_{L1} = u_{L2} + Ri_R$ √

d)* Give i_{L2} depending on i_R.

$i_{L2} = i_R$ √

e)* Determine u_{L1} depending on u_G.

$u_{L1} = u_G$ √

f) Express u_{L1} depending on i_{L1} without using any time derivatives.

KCL: $-i_{L2} + G_1 u_G + i_{L1} + i_R = 0$ √

$u_{L1} = -\frac{1}{G_1} i_{L1}$ √

0 1

g) Find u_{L2} depending on i_{L1} and i_{L2}.

$u_{L2} = -\frac{1}{G_1}\,i_{L1} - Ri_{L2}$ $\surd$

0 1 2

h) Give the state equations for the given circuit.

$\dot{i}_{L1} = -\frac{1}{G_1 L_1}\,i_{L1}$ $\surd$
$\dot{i}_{L2} = -\frac{1}{G_1 L_2}\,i_{L1} - \frac{R}{L_2}\,i_{L2}$ $\surd$

The state equations for another second-order system read as

$$\dot{x}_1 = -2x_1$$
$$\dot{x}_2 = -3x_1 + x_2$$

with the state vector $\boldsymbol{x} = [x_1, x_2]^{\mathrm{T}}$.

0 1 2

i)* Find the state matrix $\boldsymbol{A}$.

$\boldsymbol{A} = \begin{bmatrix} -2 & 0 \\ -3 & 1 \end{bmatrix}$ $\surd\surd$

0 1 2

j) Determine the eigenvalues of $\boldsymbol{A}$.

$\det(\boldsymbol{A} - \lambda\mathbf{1}) = (-2-\lambda)(1-\lambda) = 0$ $\surd$
$\lambda_1 = -2$ and $\lambda_2 = 1$ $\surd$

0 1 2

k) Investigate **both** eigenvalues and explain why the circuit is unstable.

$\lambda_1 = -2$: stable eigenvalue
$\lambda_2 = 1$: unstable eigenvalue
Due to λ_2, the circuit is unstable. $\surd\surd$

Problem 7 Non-Linear Two-Port (15 credits)

The current-controlled characteristic of the non-linear two-port $\mathcal{T}$ can be written as

$$\boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \boldsymbol{r}_{\mathcal{T}}(\boldsymbol{i}) = \begin{bmatrix} U_0(\frac{i_1}{I_0} + \frac{i_2^2}{I_0^2}) \\ R_0 i_1 + 2R_0 i_2 \end{bmatrix}$$

with the constants U_0, I_0, and $R_0 = \frac{U_0}{I_0}$.

a)* Find u_1 and u_2 of the two-port $\mathcal{T}$ for $i_1 = 0$ and $i_2 = 0$. 0 1

$u_1 = 0$ and $u_2 = 0$ $\surd$

b) Argue why the two-port is sourcefree. 0 1

The origin is part of the characteristic as $\boldsymbol{r}_{\mathcal{T}}(\boldsymbol{0}) = \boldsymbol{0}$. $\surd$

c)* Determine $\boldsymbol{r}_{\mathcal{T}}([i_2, i_1]^{\mathrm{T}})$. 0 1

$$\boldsymbol{r}_{\mathcal{T}}([i_2, i_1]^{\mathrm{T}}) = \begin{bmatrix} R_0 i_2 + R_0 \frac{i_1^2}{I_0} \\ R_0 i_2 + 2R_0 i_1 \end{bmatrix} \surd$$

d) Argue why the two-port $\mathcal{T}$ is not symmetric. 0 1

$$\boldsymbol{r}_{\mathcal{T}}([i_2, i_1]^{\mathrm{T}}) \neq \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \boldsymbol{r}_{\mathcal{T}}(\boldsymbol{i}) \surd$$

e)* Why does the hybrid-controlled characteristic of $\mathcal{T}$ not exist? 0 1 2

u_1 and i_1 depend on i_2^2. $\surd$
It is impossible to find a unique expression for i_2 depending on the i_1 and u_2. $\surd$

f)* Find the inverse hybrid representation of $\mathcal{T}$. 0 1 2

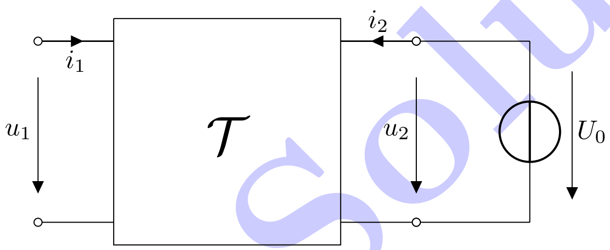
$i_1 = \frac{u_1}{R_0} - \frac{i_2^2}{I_0}$ $\surd$
$u_2 = u_1 + 2R_0 i_2 - U_0 \frac{i_2^2}{I_0^2}$ $\surd$

Assume that the two-port $\mathcal{T}$ is connected to a short circuit at port one and to the current source I_0 at port two.

g) Determine the corresponding operating point (U_1,U_2,I_1,I_2).

short circuit: $U_1 = 0$ $\checkmark$
current source: $I_2 = I_0$ $\checkmark$
$I_1 = -\frac{I_0^2}{I_0} = -I_0$ $\checkmark$
$U_2 = U_0$ $\checkmark$

Now the two-port is connected to the voltage source U_0 at port two as illustrated below.



h)* What is u_2 depending on U_0?

$u_2 = U_0$ $\checkmark$

i) Express i_2 depending on i_1 and I_0.
Remember that $R_0 = \frac{U_0}{I_0}$.

$i_2 = -\frac{1}{2}i_1 + \frac{I_0}{2}$ $\checkmark$

j) What is the power p_2 at port two?

$p_2 = u_2 i_2 = -\frac{1}{2}U_0 i_1 + \frac{U_0 I_0}{2}$ $\checkmark$

Additional space for solutions–clearly mark the (sub)problem your answers are related to and strike out invalid solutions.