Angular Acceleration

- The rate of change of angular velocity is the angular acceleration.

- Average angular acceleration:

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t}$$

- Instantaneous angular acceleration:

$$\alpha = \lim_{\Delta t \to 0} \bar{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} \equiv \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

Acceleration Vector

- We can relate linear acceleration to angular acceleration.

- We start with the definition of the linear acceleration vector as the time derivative of the linear velocity vector:

$$\vec{a}(t)=\frac{d}{dt}\vec{v}(t)=\frac{d}{dt}(v\hat{t})=\left(\frac{dv}{dt}\right)\hat{t}+v\left(\frac{d\hat{t}}{dt}\right)$$

- The acceleration in circular motion has two components:
  - *Tangential acceleration* is related to the change in magnitude of the velocity.
  - *Radial acceleration* is related to the change in direction of the velocity.

Acceleration Vector

- Let's look at the time derivative of the linear speed:

$$\frac{dv}{dt} = \frac{d}{dt}(r\omega) = \left(\frac{dr}{dt}\right)\omega + r\left(\frac{d\omega}{dt}\right)$$

- Because r is constant for circular motion, $dr/dt = 0$ and remembering that $d\omega/dt = \alpha$ we have:

$$\frac{dv}{dt} = r\alpha$$

- For the second component, we need to calculate the time derivative of the tangential unit vector:

$$\frac{d}{dt}\hat{t} = \frac{d}{dt}(-\sin\theta, \cos\theta) = \left(-\cos\theta\frac{d\theta}{dt}, -\sin\theta\frac{d\theta}{dt}\right)$$

$$\frac{d}{dt}\hat{t} = -\frac{d\theta}{dt}(\cos\theta, \sin\theta) = -\omega\hat{r}$$

Acceleration Vector

- We then get the linear acceleration vector:

$$\vec{a}(t) = r\alpha\hat{t} - v\omega\hat{r}$$

- Which we can write as:

$$\vec{a}(t) = a_t\hat{t} - a_c\hat{r}$$

a_t: tangential acceleration - change in speed

a_c: centripetal acceleration - change in direction

- The magnitude of the tangential acceleration is:

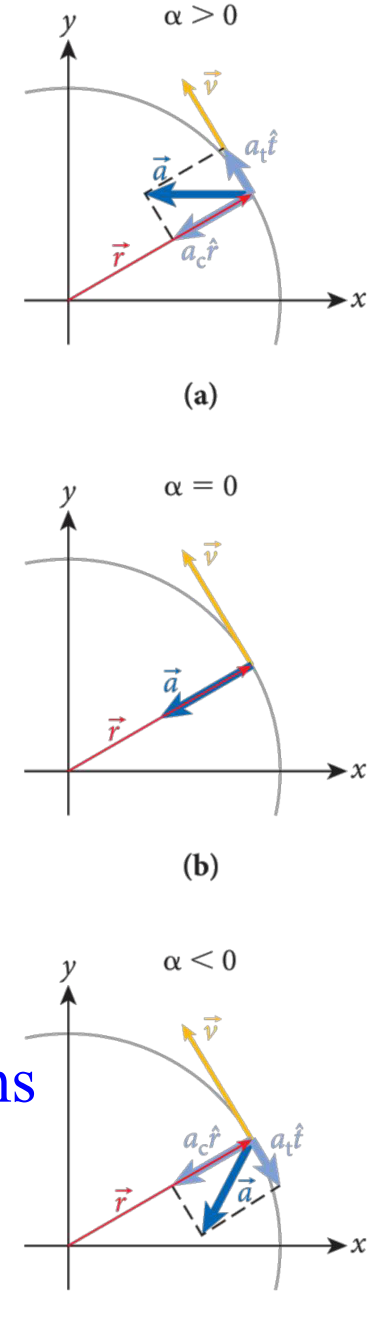
$$a_t = r\alpha$$

- The magnitude of the centripetal acceleration is:

$$a_c = v\omega = \frac{v^2}{r} = \omega^2 r$$ Very important and useful equations

- Magnitude of acceleration in circular motion:

$$a = \sqrt{a_t^2 + a_c^2} = \sqrt{(r\alpha)^2 + (r\omega^2)^2} = r\sqrt{\alpha^2 + \omega^4}$$



(a)
(b)
(c)

Centripetal Acceleration on Earth

- What is the correction to gravity due to the centripetal acceleration on the surface of Earth?

- Answer:

$$
\begin{aligned}
a_c &= \omega^2 r = \omega^2 R_{earth} \cos\theta \\
&= (7.27 \cdot 10^{-5}\ \mathrm{s}^{-1})^2 \cdot (6.37 \cdot 10^{6}\ \mathrm{m}) \cdot \cos\theta \\
&= (0.034\ \mathrm{m/s}^2) \cdot \cos\theta
\end{aligned}
$$

- Correction between between 0.33% (at the equator) and 0% (at the poles).

Concept Check

- The period of rotation of the Earth is 24 hours. At this angular velocity, the centripetal acceleration at the surface of the Earth is small compared with the acceleration of gravity. What would the period of rotation of the Earth be if the magnitude of the centripetal acceleration at the surface of the Earth at the equator due to the rotation of the Earth were equal to the magnitude of the acceleration of gravity?

  A) 0.043 hours

  B) 0.340 hours

  C) 0.841 hours

  D) 1.41 hours

  E) 12.0 hours

Concept Check Solution

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C) 0.841 hours

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E) 12.0 hours

$$a_c = \omega^2 r = \omega^2 R_{earth} = g$$

$$\omega = \sqrt{\frac{g}{R_{earth}}} = \sqrt{\frac{9.81 \text{ m/s}^2}{6.37 \cdot 10^6 \text{ m}}} = 0.00124098 \text{ rad/s}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{0.00124098 \text{ rad/s}} = 5063.08 \text{ s} = 1.41 \text{ hr}$$