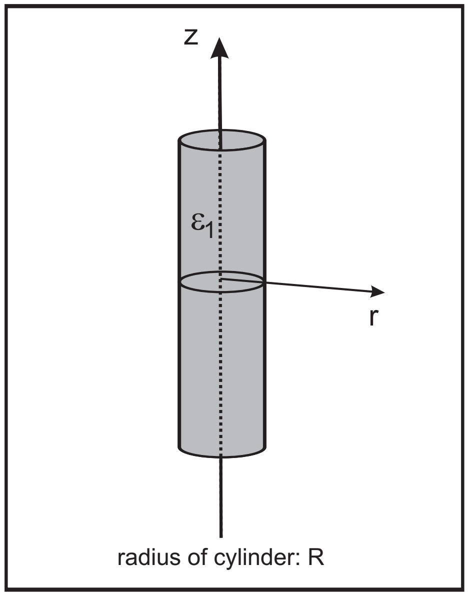
Problem 1 (17 points)

Consider an infinitely long, straight cylinder with radius R, which is filled with a material of permittivity $\varepsilon_1 > \varepsilon_0$ and charged uniformly with a charge q per unit length. The permittivity of the surrounding air is ε_0.



Use cylindrical coordinates (r, φ, z) for the following calculations.

*a) Calculate the electric charge density ρ inside the cylinder.

*b) Calculate the magnitude and the direction of the displacement field $\vec{D}(\vec{r})$, which is generated by this charge distribution everywhere in space (that means for $0 \leq r < \infty$).

c) Calculate the magnitude and the direction of the electric field $\vec{E}(\vec{r})$ everywhere in space (that means for $0 \leq r < \infty$).

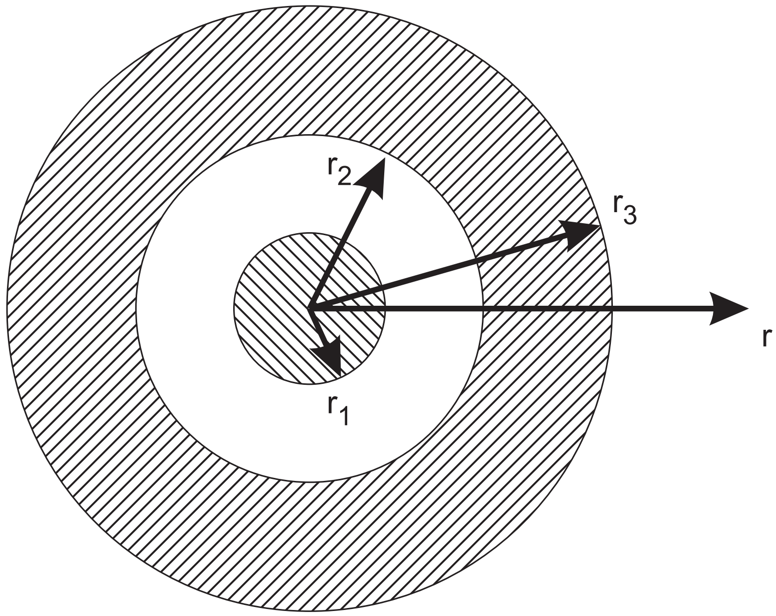
d) Draw a qualitative sketch of the electric field strenght $|\vec{E}(r)|$ as a function of the radius r. Label the axis properly (mark explicitely the position $r = R$ on the r-axis).

e) Calculate the electrostatic potential $\Phi(\vec{r})$ of this configuration everywhere in space (that means for $0 \leq r < \infty$).
The reference point of the potential should be chosen such that $\Phi(r = R) = 0$ on the surface of the cylinder.

f) Calculate the voltage U_{12} between the points $P_1 : (R, 0, 0)$ and $P_2 : (2R, 0, 0)$.
(Note: The two points are given in cylinder coordinates!)

Problem 2 (14 points)

Consider a very long coaxial cable that consists of two concentric conductors (see shaded cross-sections in figure). The inner cylindrical conductor has the radius r_1 and the outer conductive tube has an inner radius r_2 and an outer radius r_3. Assume that the magnetic permeability is $\mu = \mu_0$ in the entire configuration. Use cylindrical coordinates (r, φ, z), where the z-axis points out of the plane of projection.



The electric current I flowing through the inner conductor is directed back through the outer conductive tube. The current density $\vec{j}(r)$ reads:

$$\vec{j}(r) = \begin{cases} I/(r_1^2\pi)\vec{e}_z & \text{for } 0 \le r \le r_1 \\ 0 & \text{for } r_1 < r < r_2 \\ -I/[(r_3^2 - r_2^2)\pi]\vec{e}_z & \text{for } r_2 \le r \le r_3 \\ 0 & \text{for } r > r_3 \end{cases}$$

*a) Calculate the magnetic field $\vec{H}(r, \varphi, z)$ generated by the current density $\vec{j}(r)$ in each of the regions $0 \le r \le r_1$, $r_1 < r < r_2$, $r_2 \le r \le r_3$ and $r > r_3$.

b) Draw the magnitude of the magnetic field $|\vec{H}(r)| = H_\varphi(r)$ as a function of the radius r. Label the positions $r = r_1$, $r = r_2$ and $r = r_3$ on the r-axis and the corresponding values along the H_φ-axis!

*c) The gap region ($r_1 < r < r_2$) between the two conductors is now filled with a material with relative permeability $\mu_r = 3$. Does the magnetic field $\vec{H}$ and the magnetic flux density $\vec{B}$ quantitatively change in the gap, when the current density remains unchanged? Give a reason for your answer.