EDE1012 MATHEMATICS 2

Tutorial 7

Introduction to Laplace Transform

1. Determine the Laplace transform of the following functions:

a) $f(t) = 2e^{-3t} - t^4$

b) $f(t) = 3\cos(5t) - 2\sin(5t)$

c) $f(t) = e^{-t}(t+3)^2$

d) $f(t) = e^t \sin t$

e) $f(t) = t^2\delta(t-3)$

f) $f(t) = \begin{cases} t-\pi, & \pi < t < 2\pi \\ 0, & \text{otherwise} \end{cases}$

g) $f(t) = \sin(\omega t) - \omega t\cos(\omega t)$

ANS: **a)** $F(s) = \frac{2}{s+3} - \frac{24}{s^5}$. **b)** $F(s) = \frac{3s-10}{s^2+25}$. **c)** $F(s) = \frac{2}{(s+1)^3} + \frac{6}{(s+1)^2} + \frac{9}{s+1}$. **d)** $F(s) = \frac{1}{(s-1)^2+1}$. **e)** $F(s) = 9e^{-3s}$. **f)** $F(s) = \frac{e^{-\pi s}}{s^2} - e^{-2\pi s}(\frac{1}{s^2} + \frac{\pi}{s})$. **g)** $F(s) = \frac{2\omega^3}{(s^2+\omega^2)^2}$.

2. Using integration, determine the Laplace transform of f(t). Show that it is equivalent to that obtained by shifting in s-domain as well as that using the derivative of the Laplace transform.

$$f(t) = t^2e^t$$

ANS: $F(s) = \frac{2}{(s-1)^3}$

3. Determine the Laplace transform of the function below, where k and ω are constants.

$$h(t) = te^{kt}\cos(\omega t)$$

ANS: $F(s) = \frac{(s-k)^2 - \omega^2}{\left[(s-k)^2+\omega^2\right]^2}$.

4. Using both integration and the t-domain shifting property., determine the Laplace transform of the following function. Are they equivalent?

$$f(t) = \begin{cases} 0, & t < 2 \\ (t-1)^2, & t \geq 2 \end{cases}$$

ANS: $F(s) = e^{-2s}\left(\frac{1}{s} + \frac{2}{s^2} + \frac{2}{s^3}\right)$. Yes.

5. Rewrite the following piecewise function using the unit-step function and evaluate its Laplace transform.

$$g(t) = \begin{cases} e^{-t}, & 1 \leq t < 2 \\ t^2, & t \geq 2 \end{cases}$$

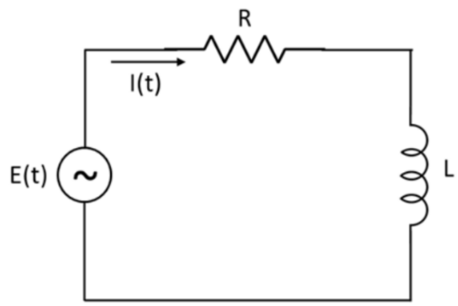
ANS: $G(s) = \frac{1}{e(s+1)}e^{-s} + \left[\frac{2}{s^3} + \frac{4}{s^2} + \frac{4}{s} - \frac{1}{e^2(s+1)}\right]e^{-2s}$

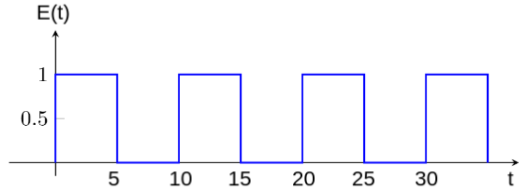
6. Determine the Laplace transform of the function below. (Hint: You might need the compound angle formula.)

$$f(t) = \begin{cases} e^{2t}, & 1 \leq t < \pi \\ \sin t + e^{2t}, & t \geq \pi \end{cases}$$

ANS: $F(s) = \frac{e^2 e^{-s}}{s-2} - \frac{e^{-\pi s}}{s^2+1}$

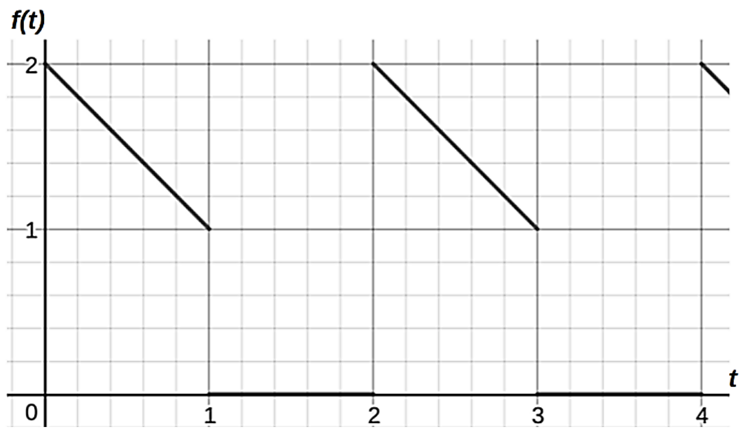
7. Determine the Laplace transform of the periodic voltage supply of the resistor-inductor circuit below.





ANS $L\{E(t)\} = \sum_{n=0}^{\infty} \frac{e^{-10n} - e^{-(10n+5)}}{s}$.

8. A periodic function f(t) is defined by the following waveform. Given that g(t) represents one cycle of f(t) in [0, 2], define g(t) using the unit-step function. Hence, determine the Laplace transform of f(t).



ANS: $g(t) = (2-t)u(t) - (2-t)u(t-1)$. $F(s) = [\frac{2}{s} - \frac{1}{s^2} + e^{-s}(\frac{1}{s^2} - \frac{1}{s})] \sum_{n=1}^{\infty} e^{-2ns}$.

For more practice problems (& explanations), check out:

1) https://math.libretexts.org/Bookshelves/Differential_Equations/Differential_Equations_for_Engineers_(Lebl)/6%3A_The_Laplace_Transform/6.E%3A_The_Laplace_Transform_(Exercises)

End of Tutorial 7
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