EDE1011 ENGINEERING MATHEMATICS 1

Tutorial 5

Limits & Continuity

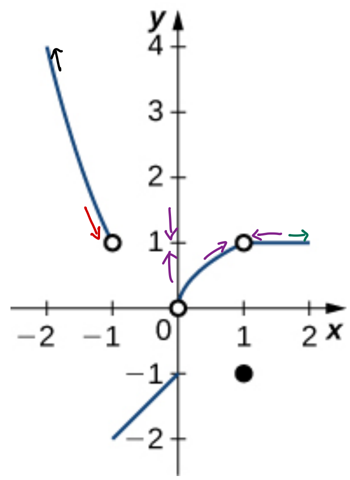
1. (https://openstax.org/books/calculus-volume-1/pages/2-review-exercises)
From the graph below, state the following limits.

4 1 DNE 0

$$\lim_{x\to -2^+} f(x), \quad \lim_{x\to -1^-} f(x), \quad \lim_{x\to -1} f(x), \quad \lim_{x\to 0^+} f(x),$$

$$\lim_{x\to 0} f(x), \quad \lim_{x\to 1} f(x), \quad \lim_{x\to 2^-} f(x)$$

DNE 1



$\lim_{x\to 1} f(x) = 1 \neq f(1) = -1$

Black dot = function value

ANS: 4, 1, DNE, 0, DNE, 1, 1.

2. Explain why the limit below is an indeterminate form. Give examples showing why the limit can be different values dependent on f(x) and g(x).

$$\lim_{x\to a} f(x)^{g(x)} = 0^0$$

3. Determine the horizontal asymptotes of

$$f(x) = \begin{cases} \frac{1-4x^2}{x^2+3x-1}, & x < 0 \\ \frac{x(x^2+x+1)}{5x^3-7}, & x \geq 0 \end{cases}$$

ANS: y = -4, y = ⅕.

4. Evaluate the following limits.

a) $\lim\limits_{x \to 5} \dfrac{x^2 - 25}{x + 5}$

b) $\lim\limits_{x \to -5} \dfrac{x^2 - 25}{x + 5}$

c) $\lim\limits_{x \to 1} (2 - e^x) \cos(\pi x)$

d) $\lim\limits_{x \to \infty} \{e^{-x} - 7\}$

e) $\lim\limits_{x \to 1} x^{\ln x}$

f) $\lim\limits_{x \to \infty} \dfrac{2x^3}{1 - x^3}$

g) $\lim\limits_{x \to 9} \dfrac{\sqrt{x} - 3}{9 - x}$

h) $\lim\limits_{x \to -1} \dfrac{\sqrt{x + 2} - 1}{3 - \sqrt{5 - 4x}}$

i) $\lim\limits_{x \to \infty} \dfrac{\sqrt{x} - 3}{9 - x}$

j) $\lim\limits_{x \to -\infty} x^3 e^{-x}$

k) $\lim\limits_{x \to \infty} \left\{ \sqrt{x^2 + 10} - x \right\}$

ANS: **a)** 0. **b)** -10. **c)** e - 2. **d)** -7. **e)** 1. **f)** -2. **g)** -⅙. **h)** ¾. **i)** 0. **j)** -∞. **k)** 0.

5. Using the squeeze theorem, evaluate the limits below.

a) $\lim\limits_{x \to \infty} e^{-x}(7 \sin x + 4)$

b) $\lim\limits_{x \to -3} \left\{ |f(x)| \sqrt{x + 3} - 1 \right\}$ where $-5 \leq f(x) \leq 5,\ x \neq -3$

$0 \leq |f(x)| \leq 5$

ANS: **a)** 0. **b)** -1.

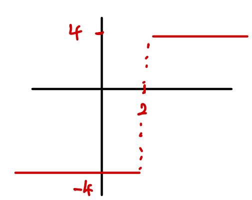
Evaluate the limits at the discontinuities

6. Determine any discontinuity and its type for each of the functions below.

For rational f(x), just look for points where the denominator = 0

a) $f(x) = \dfrac{1}{|x|}$

b) $f(x) = \dfrac{4|x - 2|}{x - 2} = \begin{cases} \dfrac{4(x-2)}{x-2} = 4, & x-2 > 0 \rightarrow x > 2 \\ \dfrac{4[-(x-2)]}{x-2} = -4, & x-2 < 0 \rightarrow x < 2 \end{cases}$



c) $f(x) = \dfrac{\sqrt{x} - 3}{9 - x}$

ANS: **a)** Infinite discontinuity at x = 0. **b)** Jump discontinuity at x = 2.
c) Removable discontinuity at x = 9.

b) 1) There are no discontinuities each piecewise function since they are constants

2) Check for discontinuities at interval transitions

$\lim\limits_{x \to 2^-} f(x) = -4$, $\lim\limits_{x \to 2^+} f(x) = 4 \neq \lim\limits_{x \to 2^-} f(x) = -4$

so f(x) is discontinuous at x=2 with a jump discontinuity

7. Evaluate the interval where each function below is continuous.

a) $$f(x) = \begin{cases} x^2 - 3, & -3 \le x < 2 \\ \frac{5}{3+x}, & x \ge 2 \end{cases}$$

b) $$f(x) = \begin{cases} \frac{x^2-4}{x+2}, & x < 0 \\ 2, & x = 0 \\ \frac{3}{4-x}, & x > 0 \end{cases}$$

c) $$f(x) = \frac{x^2 + 3x + 2}{x^3 - x^2 - 2x}$$

ANS: **a)** [-3, ~~2~~∞). **b)** (-∞, -2) U (-2, 0) U (0, 4) U (4, ∞).
c) (-∞, -1) U (-1, 0) U (0, 2) U (2, ∞).

8. Determine the value of c if f(x) is to be continuous in ℝ.

$$f(x) = \begin{cases} cx^2 + 2x, & x < 2 \\ x^3 - cx, & x \ge 2 \end{cases}$$

ANS: c = ⅔.

9. Determine the value of a & b if f(x) is to be continuous in ℝ.

$$f(x) = \begin{cases} ax + b, & x < 1 \\ 4, & x = 1 \\ 2ax - b, & x > 1 \end{cases}$$

ANS: a = 8/3, b = 4/3.

10. Show that a solution exists for each equation below and determine the interval where the solution lies.

a) $x^7 + x^5 + x^3 + x = 7$

b) $x^3 e^x - 99 = 0$

c) $\ln x = \dfrac{1}{\sqrt{x}}$

11. For each equation in the previous question, employ the bisection method to approximate the solution x accurate up to 5 decimal places.

ANS: **a)** 1.13828. **b)** 2.21261. **c)** 2.02075.

12. **Mathematical Modelling**: Growth of a Tree

A botanist studying the growth of a tree over 10 years tabulates the tree's height from its base to the top of its crown as shown below. Due to an accident In year 5, the botanist did not manage to conduct a measurement leading to a missing data record for that year.

Year, t	Height, h (m)	Year, t	Height, h (m)
0	2	6	9
1	3.5	7	9.4
2	4.2	8	9.5
3	5.1	9	9.6
4	6.6	10	9.7

a) Enter the above dataset into Desmos and perform a polynomial regression of degree 4 to obtain the height function of year as

$$h(t) = a_4 t^4 + a_3 t^3 + a_2 t^2 + a_1 t + a_0$$

That is, find the coefficients of the polynomial using Desmos. This is known in data analytics as a predictive model. (To be explored in depth in Math 3.)

b) The botanist wants to know the $\overbrace{\text{tree's height in year 5}}^{h(5)}$. Using the predictive model, estimate the tree's height. (This is known as regression imputation in data analytics.)

c) The botanist wants to know the year t when the tree's height is at 8.5 m. How can he evaluate it and what is the value? ↳ $t(h)$ but cannot inverse $h(t)$!

ANS: **a)** $h(t) = 0.00116591t^4 - 0.0339437t^3 + 0.229555t^2 + 0.672769t + 2.19511$.

↳ $h(t) = 8.5 \rightarrow f(t) = h(t) - 8.5 = 0 \rightarrow$ since $f(t)$ is cont. IVT applies for any interval. Since $h = 8.5$ is in $[7.784, 9]$ so the sol^{n} (t) is in $[5, 6]$ ← use bisection program

b) 7.784 m. **c)** Year 5.78.

For more practice problems (& explanations), check out:

1) https://openstax.org/books/calculus-volume-1/pages/2-2-the-limit-of-a-function
2) https://openstax.org/books/calculus-volume-1/pages/2-3-the-limit-laws
3) https://openstax.org/books/calculus-volume-1/pages/2-4-continuity
4) https://openstax.org/books/calculus-volume-1/pages/2-review-exercises

End of Tutorial 5

(Email to youliangzheng@gmail.com for assistance.)