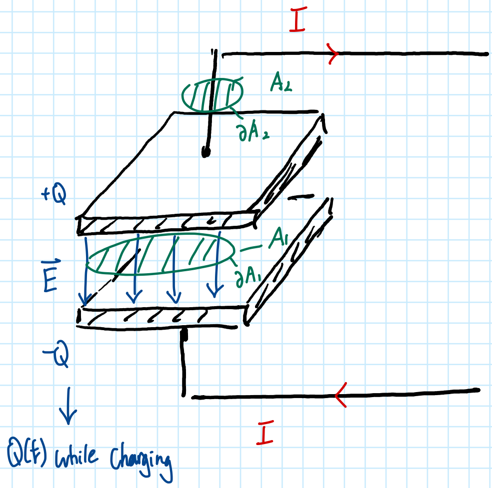
3.7 Extension of Ampère's Circuital Law to fast time-variant phenomena

Ampère'sches Gesetz: $\oint_{\partial A} \vec{H}\, d\vec{r} = I(A) = \int_A \vec{j}(\vec{r})\, d\vec{a}$

Example: Charging of a capacitor



* Current I charging capacitor
  ⇒ ±Q ; Q(t) during charging
* $\vec{J}$ is stationary current density ; $\frac{dQ}{dt} \neq 0$
* $\vec{E}$-field inside capacitor increases as long as $\frac{dQ}{dt} \neq 0$
* Ampere's law:

$$\oint_{\partial A = C} \vec{H}\, d\vec{r} = \begin{cases} 0 & \text{for } A_1,\ C = \partial A_1 \\ I & \text{for } A_2,\ C = \partial A_2 \end{cases}$$

⇓

Contradiction; because results depends on how we choose control area A

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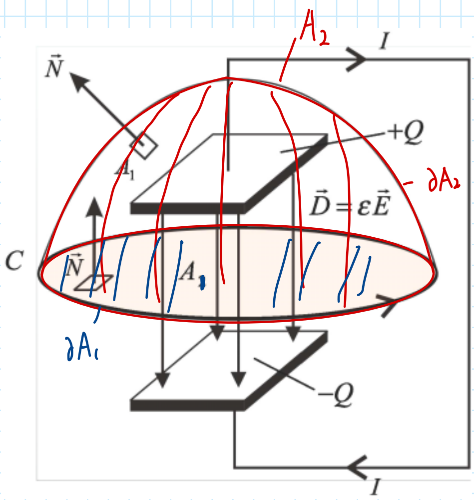
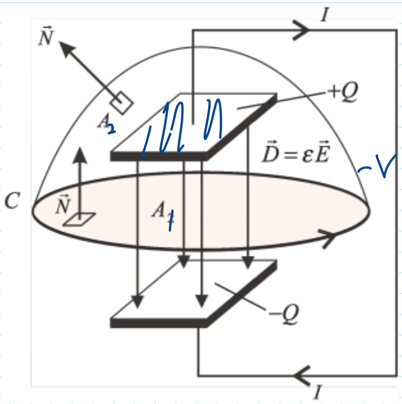
We have to extend Ampere's law for time-varying process



3.7.1 Extension of Ampère's Circuital Law

Consider a closed control volume, which is enclosed by area A1 and are A2 ("cap") both sharing the same curve C as enclosing curve



Gauss' law

$$+Q = \int_{\partial A} \vec{D}\cdot d\vec{a} = \int_{A_1 \cup A_2 = \partial A} \vec{D}\cdot d\vec{a}$$

on the other hand: Q is changing with time

$$I(A_2) = -\frac{dQ}{dt}\Big|_{A_2}$$

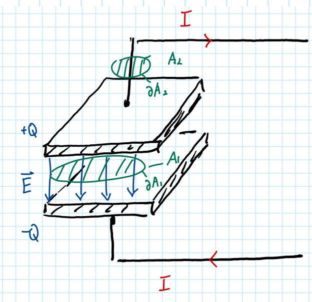
$$-I(A_2) = \frac{dQ}{dt} = \int_{A_2} \vec{J}\cdot d\vec{a}$$

$$-\frac{dQ}{dt} = -\int \frac{\partial \vec{D}}{\partial t} d\vec{a} = -\int \vec{J}\cdot d\vec{a}$$

for time-varying processes: $\frac{\partial \vec{D}}{\partial t}$ can be seen as a "current density"

and is called "electric displacement current"

$\frac{\partial \vec{D}}{\partial t}$ also generate a $\vec{H}$-field!



$$\int_C \vec{H}(\vec{r}) = \begin{cases} \int_{A_1} \frac{\partial \vec{D}}{\partial t} & \text{for } C = \partial A_1 \text{ (inside capacitor)} \\ \int_{A_2} \vec{J}\cdot d\vec{a} & \text{for } C = \partial A_2 \text{ (outside capacitor)} \end{cases}$$

$\Rightarrow$ Maxwell's extension of Ampere's Circuital law

$$\boxed{\int_{\partial A = C} \vec{H}\, d\vec{r} = \int_A \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) d\vec{a}} \qquad (3.31)$$

Magnetic fields are generated by electric currents and time-varying electric fields

3.7.1 Ampère-Maxwell's Circuital Law in Differential Formulation

We apply: $\int_A \text{Curl}\, \vec{H}\, d\vec{a} = \int_{\partial A = C} \vec{H}\, d\vec{r}$ Stoke's Theorem

applied to (3.31)

$$\int_{\partial A} \vec{H}\, d\vec{r} = \int_A \text{Curl}\, \vec{H}\, d\vec{a} = \int_A \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)\cdot d\vec{a}$$

$$\Rightarrow \boxed{\text{Curl}\, \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad (3.32)} \qquad 3^{rd} \text{ Maxwell's eqn}$$

Couples $\vec{E}, \vec{D}$ with $\vec{B}, \vec{H}$