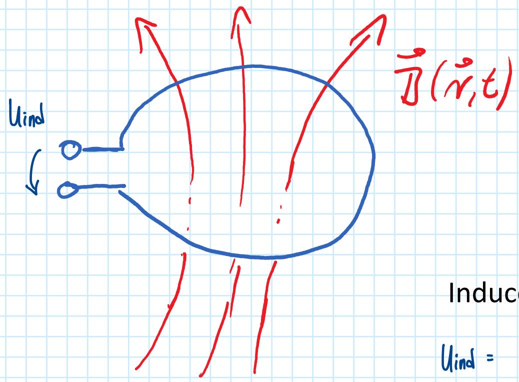
4.2 Motionless induction

Up to now: induction only by moving the conductor loops or change in geometry: A(t)

but: Voltages can be also induced by a time-variant magnetic field → motionless induction

$\frac{\partial \vec{B}}{\partial t}$: $\left(\vec{B}(\vec{r},t) \quad \frac{d\vec{B}}{dt} = \frac{dB(\vec{r},t)}{d\vec{r}} \cdot \frac{d\vec{r}}{dt} + \frac{\partial B}{\partial t} \right)$

4.2.1 Induced Electromotive Force in a motionless conductor loop



Magnetic flux changes due to varying B-field and a voltage is induced

$$U_{ind} = -\frac{d\Phi(A)}{dt} = -\int_{A} \frac{\partial B}{\partial t}(\vec{r},t) \cdot d\vec{a}$$

$A \neq$ not $A(t)$

Induced electric voltage can be corellated with an induced electric field:

$$U_{ind} = \int_{\partial A} E_{ind}\, d\vec{r} = -\int_{A} \frac{\partial B}{\partial t}(\vec{r},t) \cdot d\vec{a} \qquad (4.8)$$

This holds for arbitrary geometries, for arbitrarily shaped conductor loops as well

Please note: The integral in Equation (4.8) is a closed loop integral along the conductor loop and it is not zero!

Induced electric fields are not conservative!

4.2.2 Maxwell's Extension - differential formulation induction law

(Extension to time-variant electromagnetic phenomena)

add el-static field since $\int_{\partial A} \vec{E}_{elst.} d\vec{r} = 0$

$$\int_{\partial A = C} \vec{E} \cdot d\vec{r} = \int_{\partial A = C} \left(\vec{E}_{ind} + \vec{E}_{pot} \right) d\vec{r} = -\int_{A} \frac{\partial B}{\partial t} d\vec{a}$$

It contains also conservative fields, since:

$$\int_{\partial A} \vec{E}_{pot}\, d\vec{r} = 0$$

Total electric field is composed of a potential field (gradient field, conservative field) and an induced electric field (not conservative, not curl-free):

$$\vec{E}_{tot} = \vec{E}_{pot} + \vec{E}_{ind} = -\mathrm{grad}\,\phi(\vec{r}) + \vec{E}_{ind}$$

$$\int_{\partial A} \vec{E}\, d\vec{r} = -\int_A \frac{\partial B}{\partial t} \cdot d\vec{a} \quad \text{with} \quad \int_{\partial A} \vec{E}\, d\vec{r} = \int_A \mathrm{Curl}\, \vec{E}\, d\vec{a}$$

$$\int \mathrm{Curl}\, \vec{E}\, d\vec{a} = -\int_A \frac{\partial B}{\partial t}\, d\vec{a} \Rightarrow \boxed{\mathrm{Curl}\, \vec{E} = -\frac{\partial B}{\partial t}} \quad (4.10) \quad \text{Faraday's law of induction}$$

"Time-varying $\vec{B}$-fields generate $\vec{E}$-fields" (not Conservative, since $\mathrm{Curl}\,\vec{E} \neq 0$)

4.3 General integral form of induction law for conductor loops

4.1 and 4.2 can be summarized in the following equation:

$$\boxed{U_{ind} = -\frac{d}{dt} \int \vec{B}(\vec{r},t) \cdot d\vec{a} = -\frac{\phi(A(t))}{dt}}$$

This means for calculation of induced voltage in a conductor loop:

Determine magnetic flux and calculate the time derivative of it, this amount to the negative induced voltage, if either the area is changing or the B-field is changing with time (or both, of course)