Travelling Waves

Travelling Waves

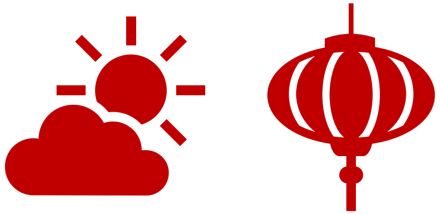
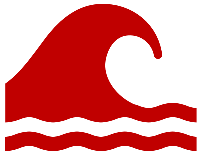
Learning Objectives

By the end of this section you will be able to

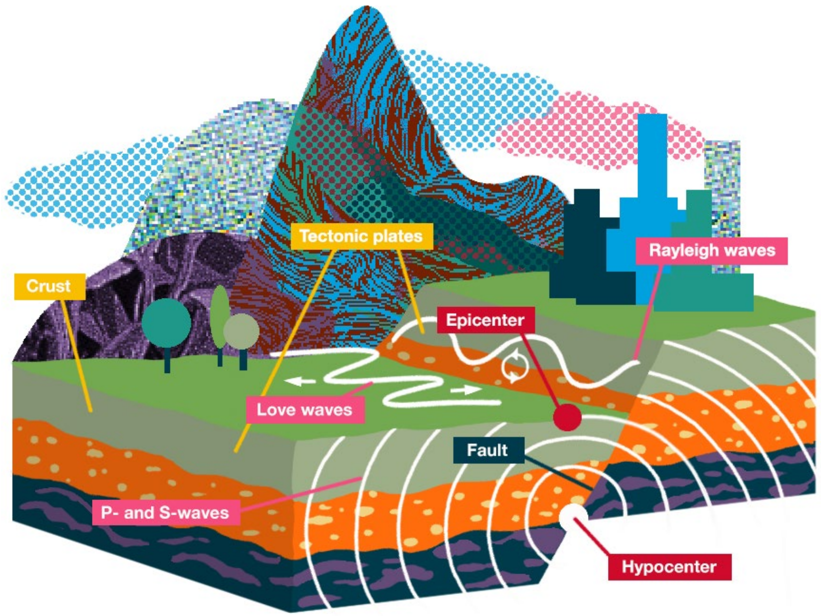
- Describe mathematically the displacement of a particular point in a transverse wave
- Relate speed of a wave to its wavelength, time-period and frequency
- Use the equation for a transverse wave to determine the amplitude, frequency, wavelength, time-period, and speed of the wave

Travelling Waves

Waves around us





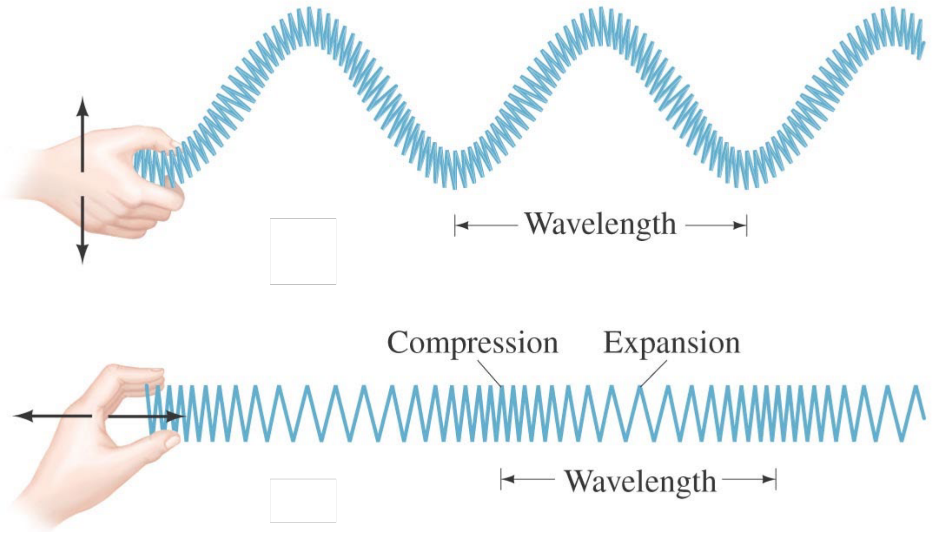
Travelling Waves

Characteristics of a wave

- The motion of particles in a wave can be either

(a) perpendicular to the wave direction (transverse) or

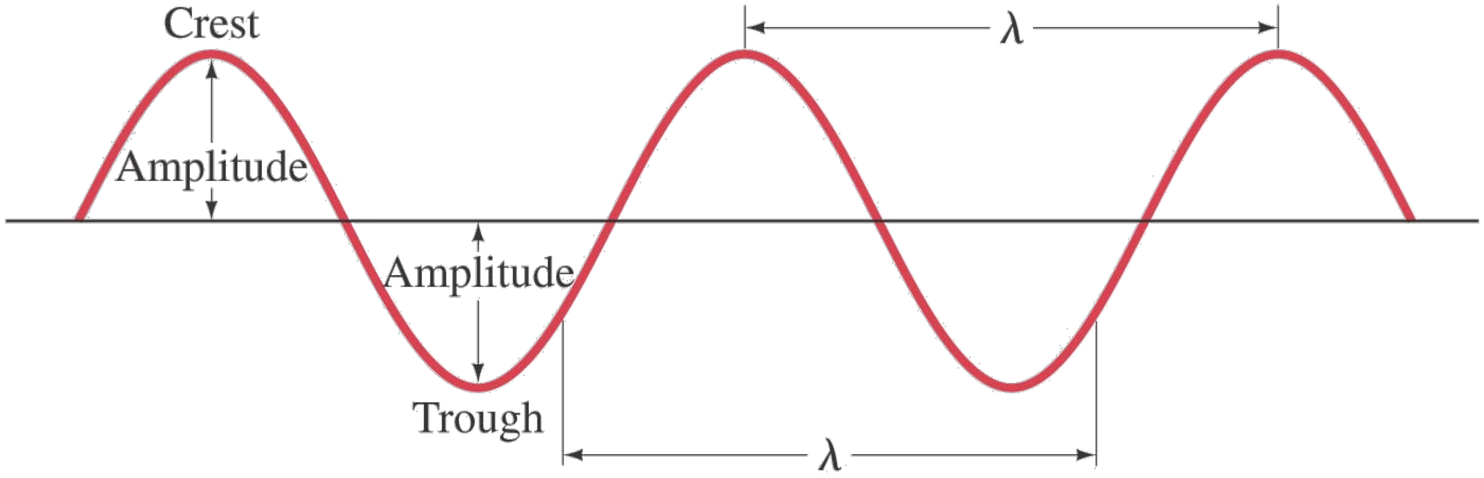
(b) parallel to the direction (longitudinal).



- **Waves transport energy but not matter**

Sinusoidal wave has the following characteristics:

- Amplitude, A (m)
- Wavelength, λ (m)
- Frequency, f (Hz)
- Time period, T (s)
- Wave velocity, v (m/s)



Travelling Waves

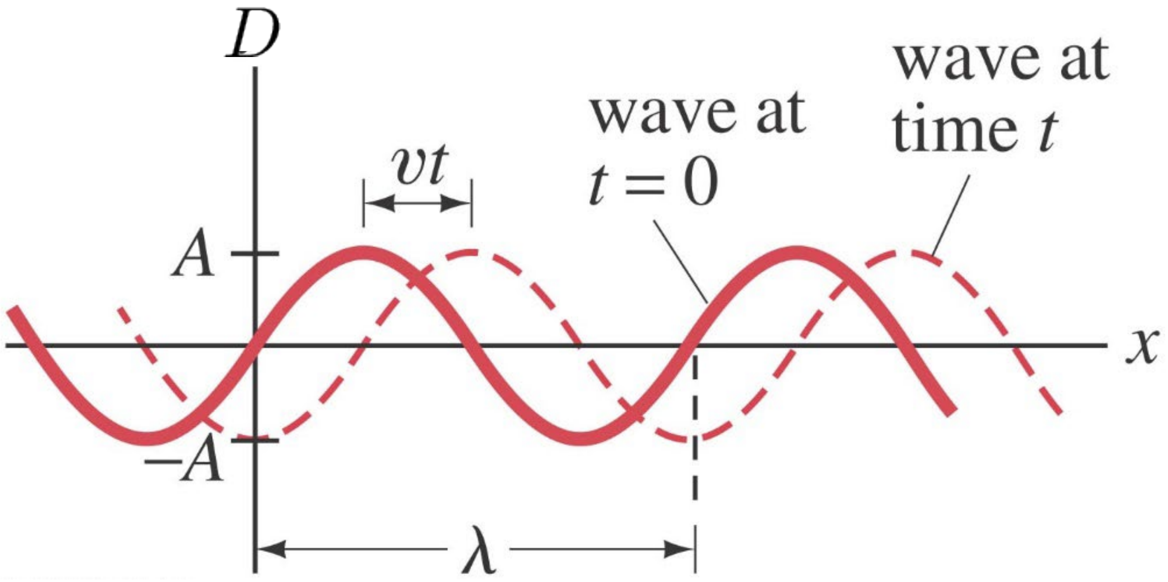
Representation of Traveling wave

- The shape of a wave at t = 0 is mathematically given by: $D(x, t=0) = A\sin\left(\frac{2\pi}{\lambda}x\right)$
- After a time t, the wave has traveled a distance vt,

$$D(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right].$$

or $$D(x,t) = A\sin(kx - \omega t),$$

where $\omega = 2\pi f$ and $k = \frac{2\pi}{\lambda}$.



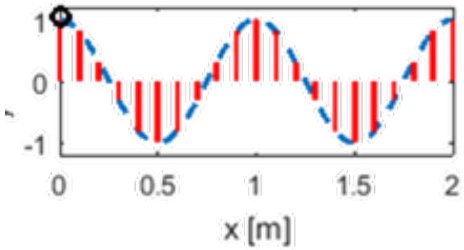
- ω is the **angular frequency** in rad/s or s^{-1}
- k is the **wave number** with unit m^{-1}

Travelling Waves

Representation of Traveling wave

- A wave travelling in the $+x$ direction is then:

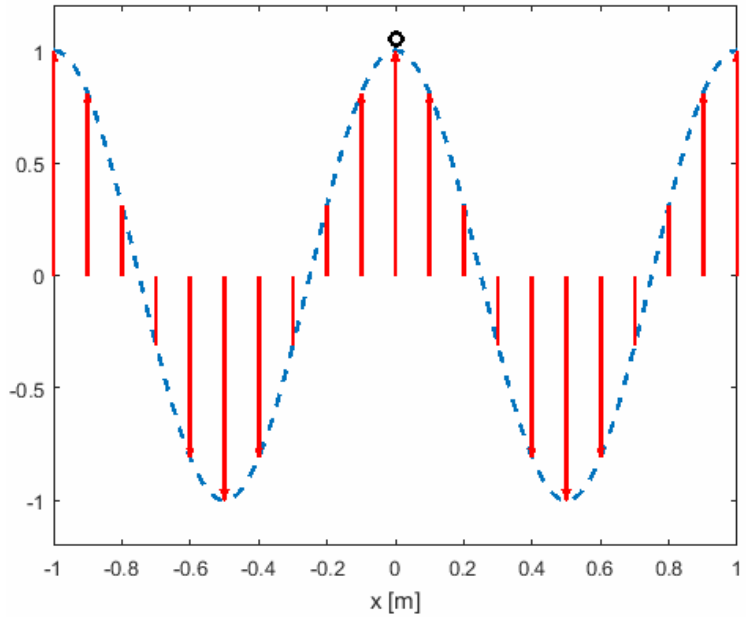
$$D(x,t) = \underbrace{A}_{\text{Amplitude}}\sin(\underbrace{k}_{\text{Wavenumber}}x - \underbrace{\omega}_{\text{Angular Frequency}} t)$$



- The wave velocity or the phase velocity, v is the rate at which the wave “propagates”
- It is the speed at which a constant phase C of the wave, i.e.,

$kx - \omega t = C$ moves, and is therefore

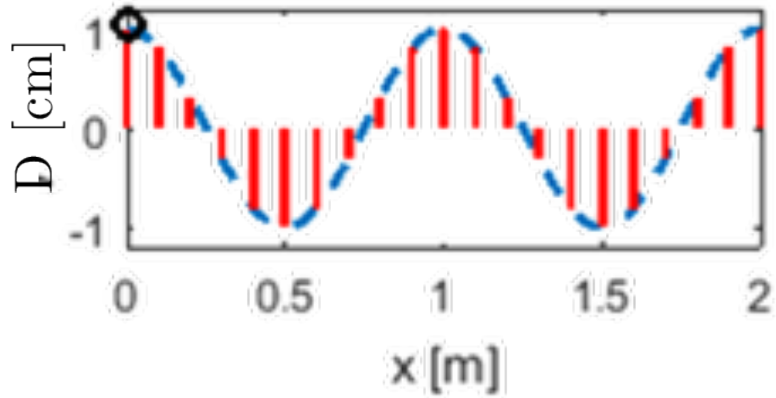
$$v = \frac{dx}{dt} = \frac{\omega}{k} = f\lambda$$



Travelling Waves

Activity

For a travelling wave shown in the animation,



- What is the amplitude of this wave?

Ans: 1.0 cm

- What is the wavelength?

Ans: 1m

- If the speed of the wave is 0.5m/s, then what is the frequency of the wave?

Ans: 0.5Hz

Travelling Waves

Representation of Traveling wave

- For a wave travelling in the $+x$ direction:

$$D(x,t) = A\sin(kx - \omega t)$$

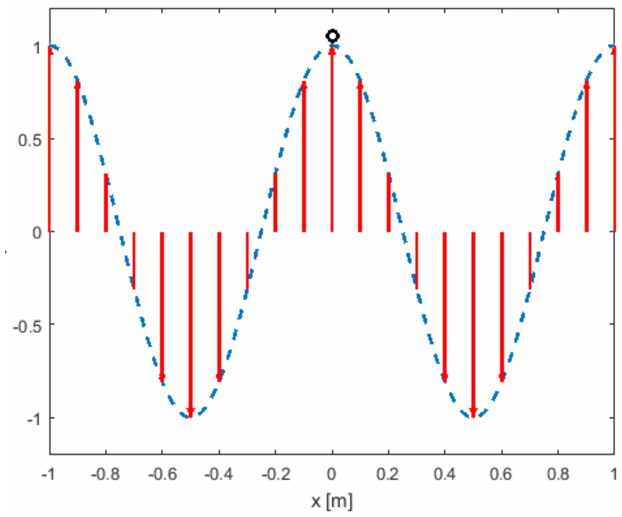
- For the wave travelling to the right ($-x$ direction)

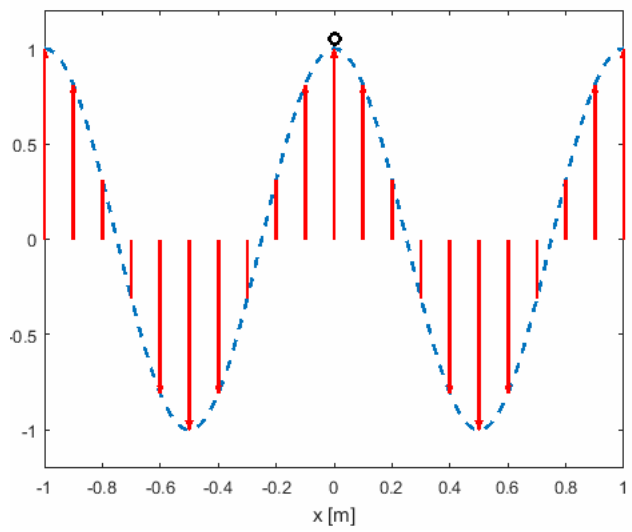
$$D(x,t) = A\sin(kx + \omega t)$$

- In general:

$$D(x,t) = A\sin(kx \pm \omega t + \phi)$$

Phase of the wave at $x = 0$ and $t = 0$



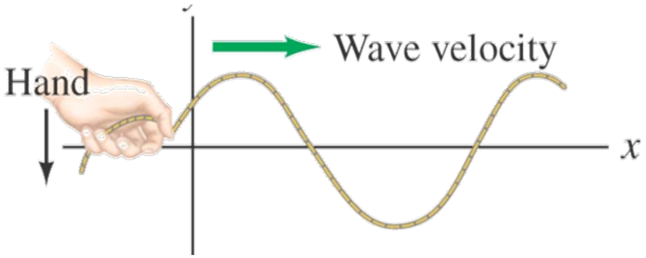


Travelling Waves

Representation of Traveling wave

Example: The left-hand end of a long horizontal stretched cord oscillates transversely in SHM with frequency $f = 250$ Hz and amplitude 2.6 cm. The cord is under a tension of F_t = 140 N and has a linear density $\mu = 0.12$ kg/m.
At $t = 0$, the end of the cord has an upward displacement of 1.6 cm and is falling. Determine
(a) the wavelength of waves produced and
(b) the equation for the traveling wave.

The velocity of the wave in a cord under tension F_t and having a linear density μ is given as $v = \sqrt{\frac{F_T}{\mu}}$



(a).
$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{140}{0.12}} = 34 \text{ m/s}$$
$$\lambda = \frac{v}{f} = \frac{34}{250} = 0.14\text{m} = 14\text{cm}$$

(b).
$$D(x,t) = A\sin(kx - \omega t + \phi)$$
$$D(0,0) = A\sin\phi$$
$$1.6\text{cm} = 2.6\sin\phi \Rightarrow \phi = 38^\circ = 0.66\text{rad}$$
$$\omega = 2\pi f \text{ and } k = \frac{\omega}{v}$$
$$\omega = 1570\text{rad/s}; \quad k = 45\text{m}^{-1}$$
$$D(x,t) = 2.6\sin(45x - 1570t + 0.66)[\text{cm}]$$

Standing Waves

Learning Objectives

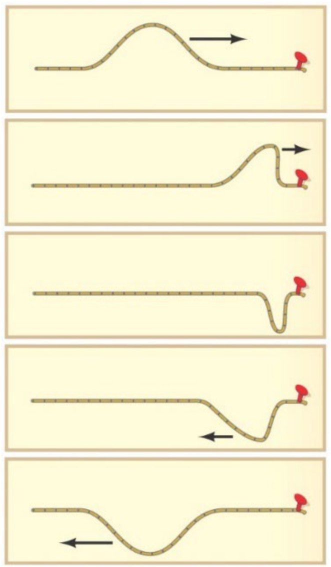
By the end of this section you will be able to

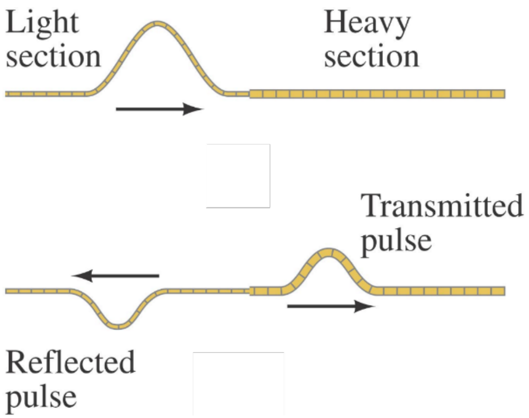
- Describe the phenomenon of reflection and transmission of waves
- Explain the principle of superposition and the phenomena of interference of waves such as constructive and destructive interference
- Use the equation for a travelling wave to describe mathematically a standing wave
- Understand the properties of standing waves, resonant frequencies and how stringed instruments produces sounds of certain frequencies

Standing Waves

Reflection and Transmission

- **Fixed end**: A wave hitting **an obstacle** will be **reflected**, and its reflection will be **inverted**
- **Free end**: A wave reaching the end of its medium, but where the medium is still **free to move**, will be reflected, and its reflection will be **upright**



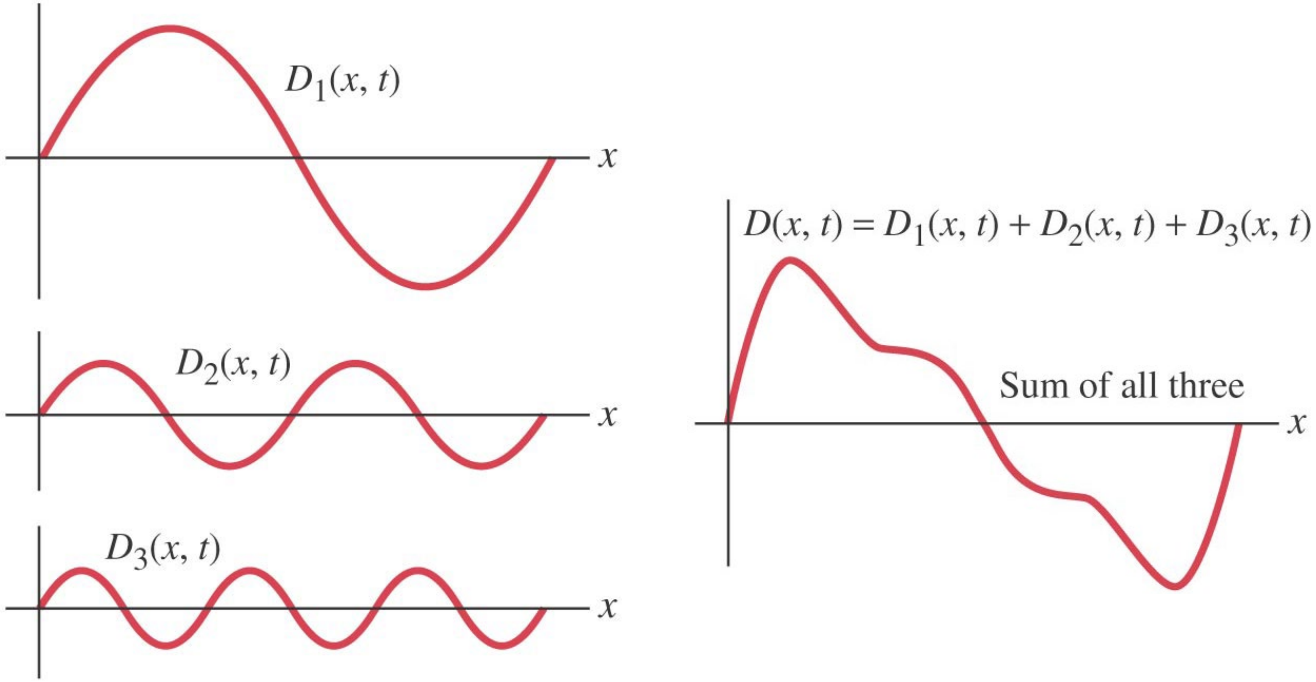


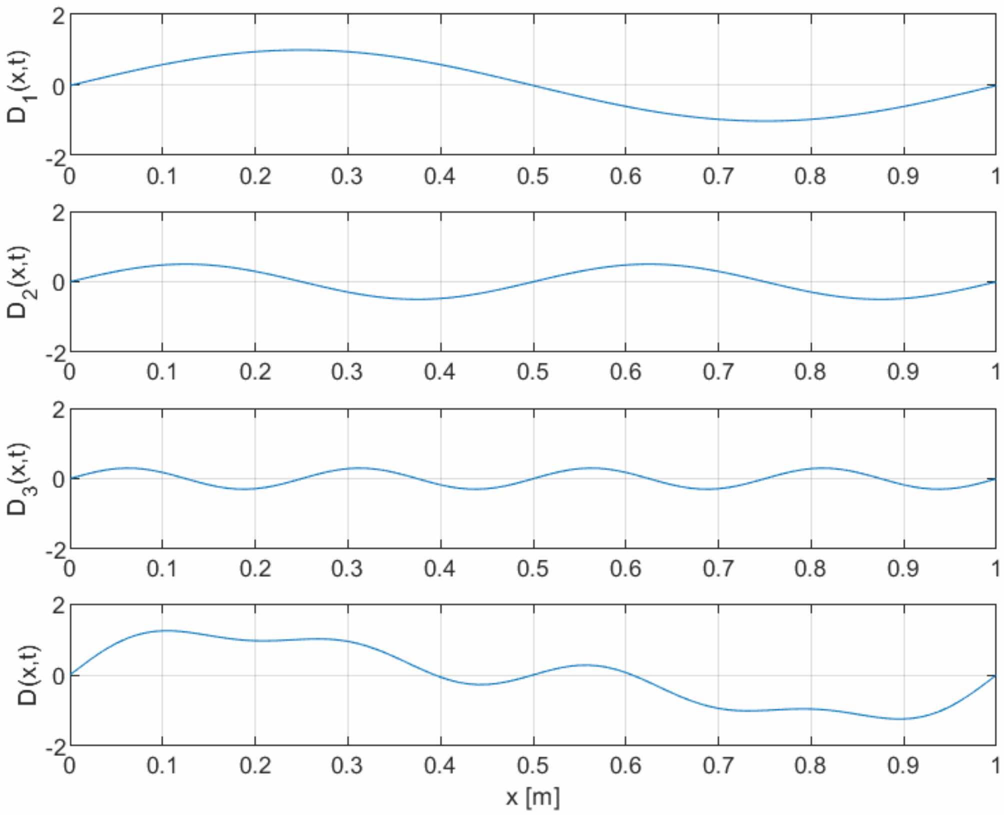
- A wave encountering a denser medium will be partly reflected and partly transmitted; if the wave speed is less in the denser medium, the wavelength will be shorter.

Standing Waves

The principle of Superposition

- **Superposition:** The displacement at any point is **the algebraic sum** of the displacements of **all waves** passing through that point at that instant.

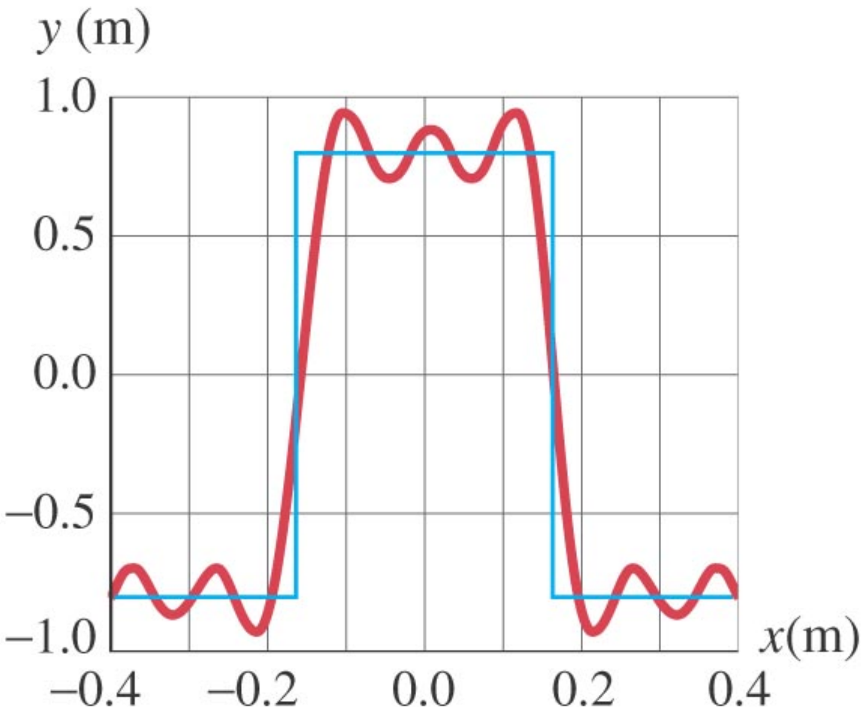


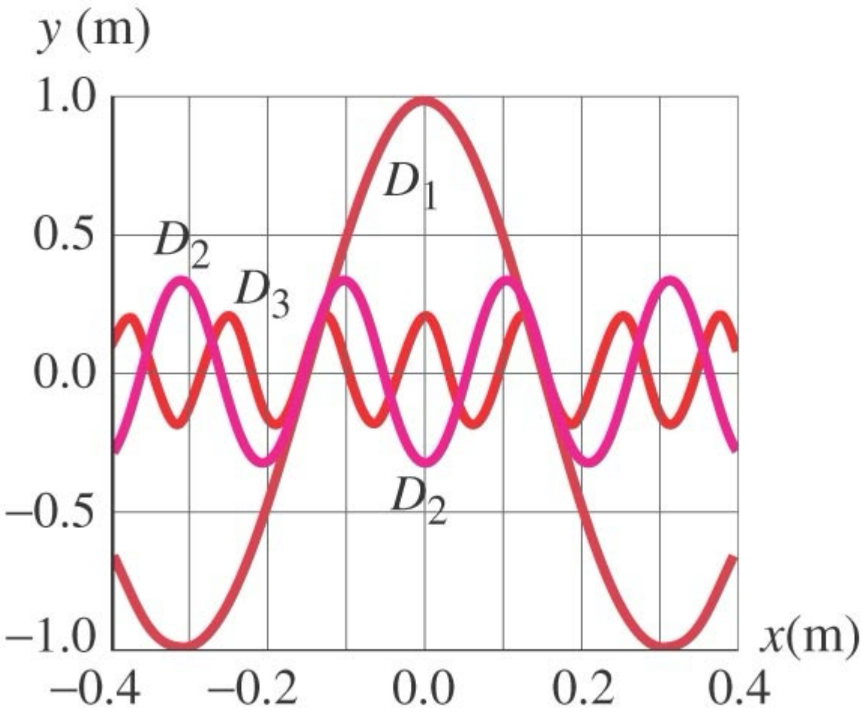


Standing Waves

The principle of Superposition

- **Fourier's theorem:** Any **periodic** wave can be written as the sum of sinusoidal waves of different **amplitudes, frequencies, and phase**.





Standing Waves

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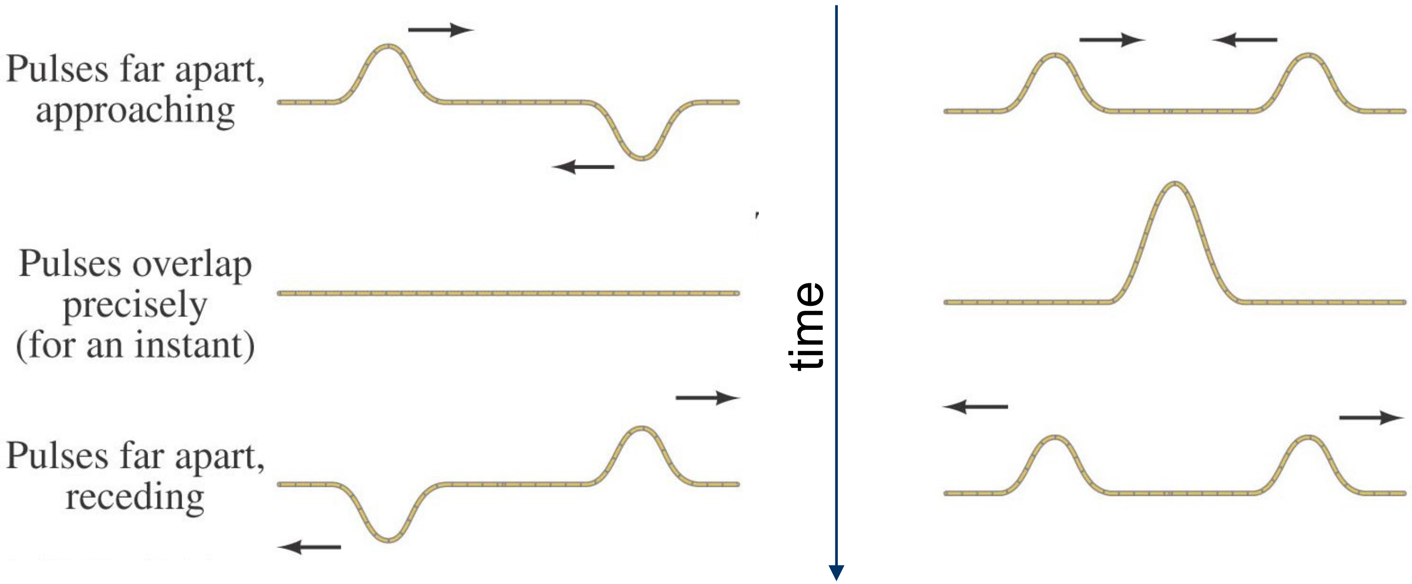
Interference

- The superposition principle says that when two waves pass through the same point, the displacement is the arithmetic sum of the individual displacements.



Destructive interference

Constructive interference

Standing Waves

Interference

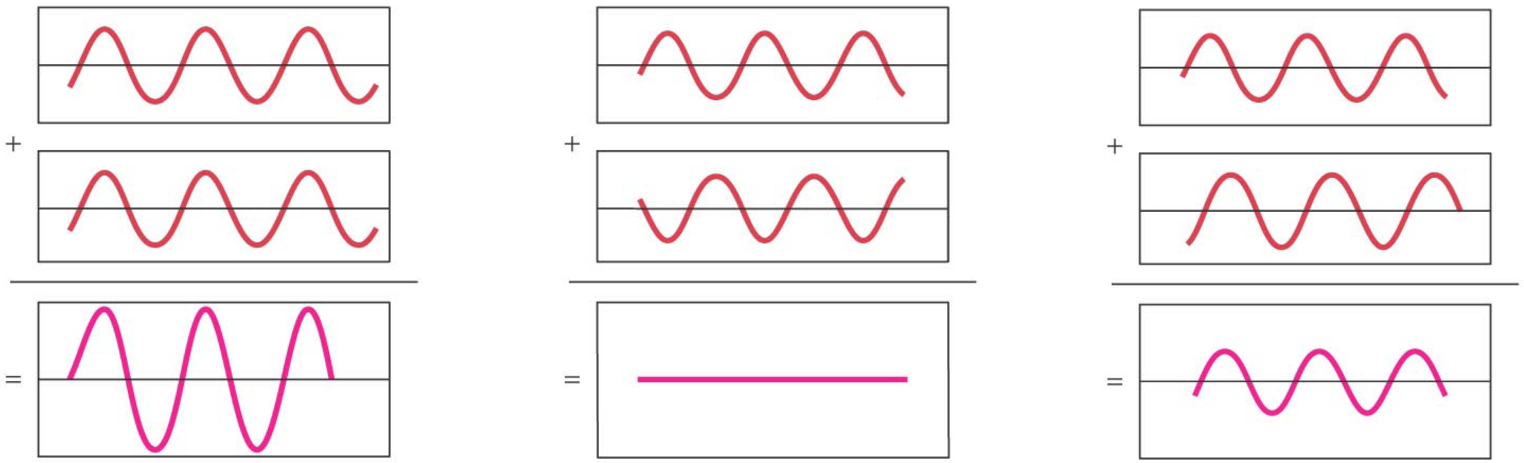
- These graphs show the sum of two waves.

In case (a) they add **constructively**;

In case (b) they add **destructively**; and

In case (c) they **add partially destructively**

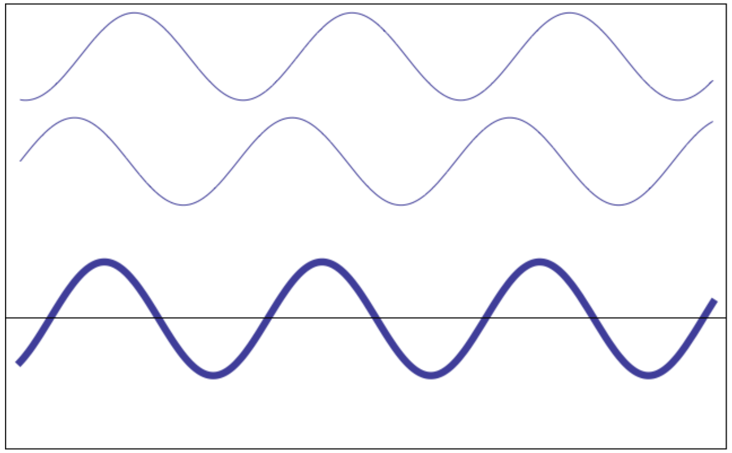
(a) (b) (c)



(a) $\Delta\phi = 0^\circ$ In phase

(b) $\Delta\phi = 180^\circ$ Out of phase

(c) $0^\circ < \Delta\phi < 180^\circ$, $180^\circ < \Delta\phi < 360^\circ$



Travelling Waves

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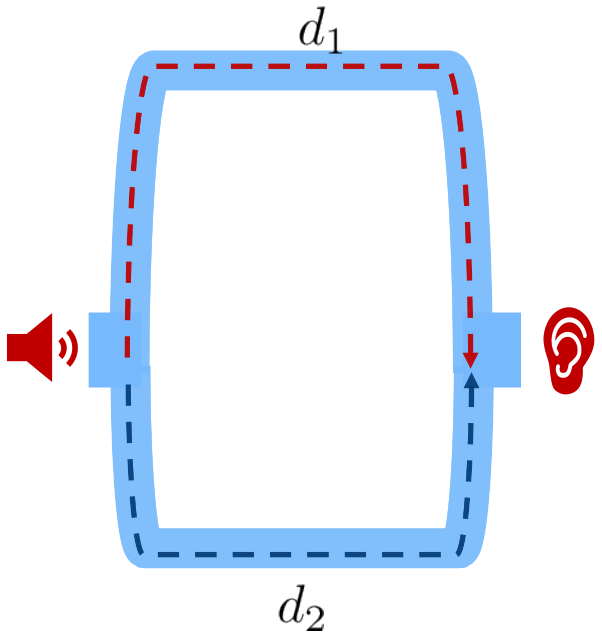
Activity

In an acoustical system shown, sound from the speaker passes and propagates through the tube splitting into two paths as two waves and combine at the opposite side and detected by the receiver (or your ears). For certain lengths of the tubes d_1 and d_2, you observe that no sound is not heard at the receiver. Based on this observation, which of the following could you say about the two distances d_1 and d_2? More than one answer can be correct.
Hint: The two waves are out of phase when they reach the receiver and hence no sound is heard.



A $d_1 - d_2 = 0$

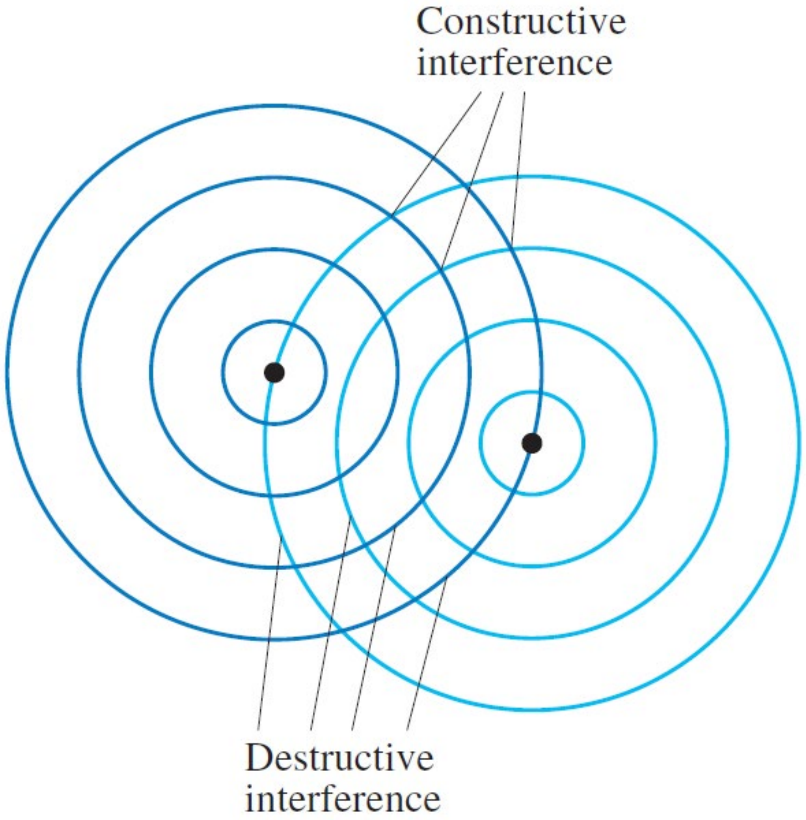
B $d_1 - d_2 = \lambda$

C $d_1 - d_2 = \dfrac{\lambda}{2}$

D $d_1 - d_2 = 3\dfrac{\lambda}{2}$

Standing Waves

Interference



Standing Waves

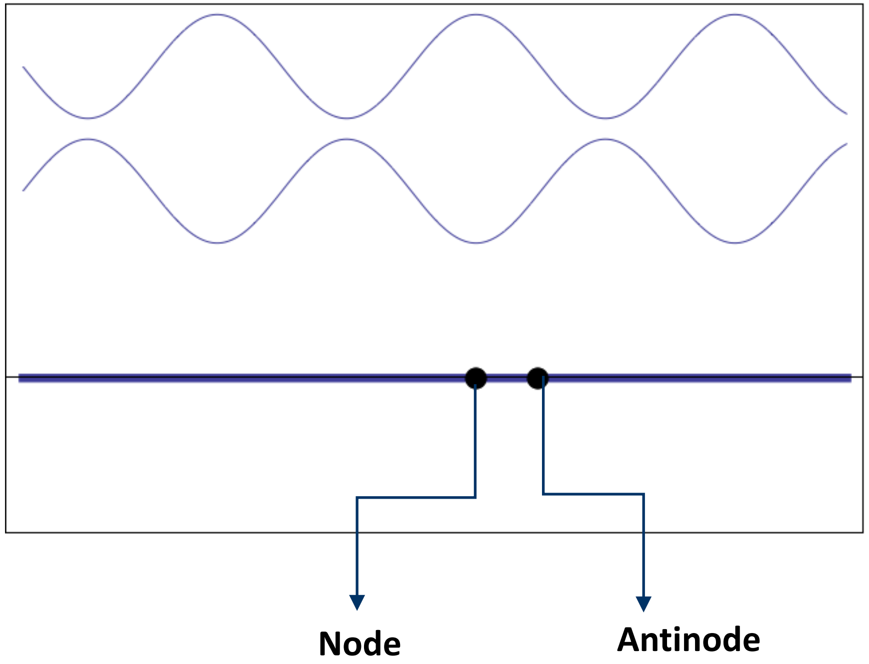
Standing Waves and Resonant Frequencies

- When wave reflected from a fixed end interferes with the incoming wave, a standing wave will be produced.

- Wave travelling along $+x$: $D_1 = A\sin(kx - \omega t)$
- Wave travelling along $-x$: $D_2 = A\sin(kx + \omega t)$

$$D = D_1 + D_2 = A\sin(kx - \omega t) + A\sin(kx + \omega t)$$
$$= 2A\sin kx \cos\omega t$$

$$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$$

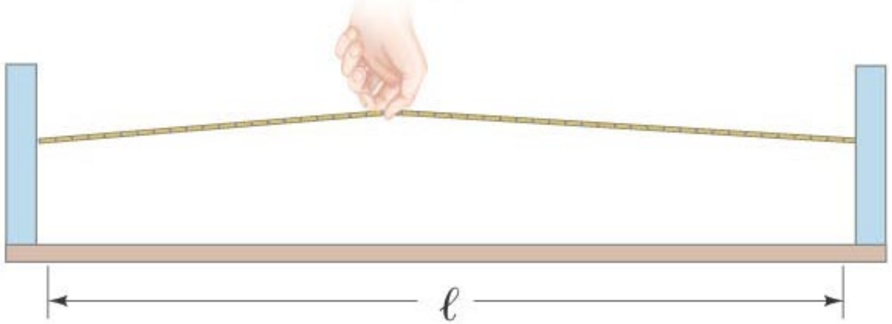


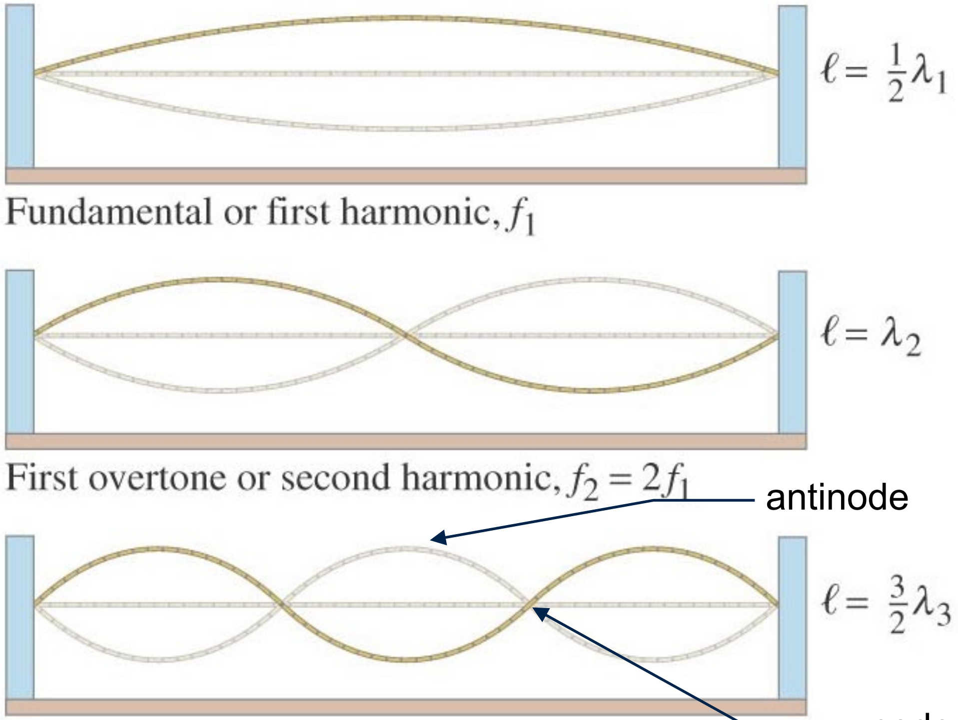
- The wave is a **standing wave**, having a sinusoidal distribution in x but the amplitude changes periodically with time

Standing Waves

Standing Waves and Resonant Frequencies

- The **frequencies** of the standing waves on a particular string are called **resonant frequencies**.
- They are also referred to as the **fundamental and harmonics**.





Fundamental or first harmonic, f_1

First overtone or second harmonic, $f_2 = 2f_1$

Second overtone or third harmonic, $f_3 = 3f_1$

$$\lambda_n = \frac{2l}{n} \qquad n = 1, 2, 3, \ldots$$

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2l} = nf_1$$

Standing Waves

Standing Waves and Resonant Frequencies

Example: A piano string is 1.10 m long and has a mass of 9.00 g.

(a) How much tension must the string be under if it is to vibrate at a fundamental frequency of 131 Hz?

(b) What are the frequencies of the first four harmonics?

Note that the velocity of the wave in a cord under tension F_t and having a linear density μ is given as $v = \sqrt{\frac{F_T}{\mu}}$

$$l = 1.10\text{m} \quad m = 9.00\text{g}$$

$$\text{(a) Mass per unit length, } \mu = \frac{9 \times 10^{-3}}{1.10} = 0.00818\text{kg/m}$$

$$\text{velocity of wave,} v = \sqrt{\frac{F_T}{\mu}} = f\lambda$$

$$f_1 = 131 \text{ Hz}; \quad \lambda_1 = 2l = 2.2\text{m}$$

$$\Rightarrow v = f_1\lambda_1 = 131 \times 2.2 = 288\text{m/s}$$

$$\Rightarrow F_T = \mu v^2 = 0.00818 \times 288^2 = 678\text{N}$$

(b)

$$f_1 = 131\text{Hz}$$

$$f_2 = 2f_1 = 262\text{Hz}$$

$$f_3 = 3f_1 = 393\text{Hz}$$

$$f_4 = 4f_1 = 524\text{Hz}$$

Standing Waves

Standing Waves and Resonant Frequencies

Question: Two waves traveling in opposite directions on a string fixed at $x = 0$ are described by the functions $D_1 = (0.20\ \text{m})\sin(2.0x - 4.0t)$ and $D_2 = (0.20\text{m})\sin(2.0x + 4.0t)$ (where x is in m, t is in s), and they produce a standing wave pattern. Determine

(a) the mathematical representation of the resulting standing wave,

(b) the maximum amplitude at $x = 0.45$ m,

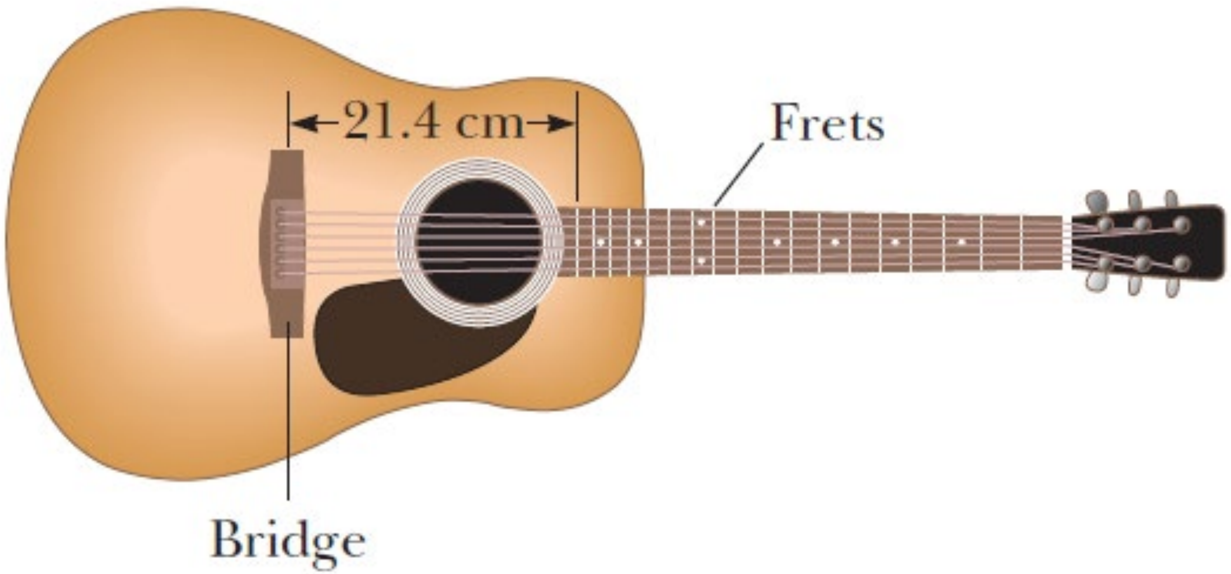
(c) where the other end is fixed ($x > 0$),

(d) the maximum amplitude, and where it occurs.

Standing Waves

Standing Waves and Resonant Frequencies

- **Example**: The fret closest to the bridge on a guitar is 21.4 cm from the bridge as shown. When the thinnest string is pressed down at this first fret, the string produces the highest frequency that can be played on that guitar, 2349 Hz. The next lower note that is produced on the string has frequency 2217 Hz. How far away from the first fret should the next fret be?



For the 2349Hz note, the length of the vibrating string be L.

For the 2217Hz note, the length of the vibrating string be $L+x$, where x is the distance to the next fret which is to be determined

$$f_1 = 2349\text{Hz} = \frac{v}{2L}$$

$$f_2 = 2217\text{Hz} = \frac{v}{2(L+x)}$$

Solving for x, the next fret would be 1.27 cm away.

Standing Waves

Summary

- Vibrating objects are sources of waves, which may be either pulses or continuous.
- Wave velocity: $v = \lambda f$
- Transverse wave: oscillations perpendicular to direction of wave motion
- Longitudinal wave: oscillations parallel to direction of wave motion

- When two waves pass through the same region of space, they interfere. Interference may be either constructive or destructive.
- Standing waves can be produced on a string with both ends fixed. The waves that persist are at the resonant frequencies.
- Nodes occur where there is no motion; antinodes where the amplitude is maximum

Standing Waves

Explore Further: Standing Waves in air-columns

Air-columns (like a flute) can be either (a) open-ended on both ends or (b) closed at one end and open at other. Sketch the standing wave pattern of the first 3 harmonics. Hint: Sound waves are longitudinal and standing waves occur due to interference of longitudinal waves. The open end of the air-column is a pressure anitnode and the closed end is a pressure node.

1st Harmonic

2nd Harmonic

3rd Harmonic

Online Resource: https://www.acs.psu.edu/drussell/Demos/StandingWaves/StandingWaves.html

Pitch in music reference to the frequency of sound. For a same length, which of the two air-column instruments, open-ended (flute) or closed ended (clarinet) has a lower pitch.

Interference of Light

Interference of Light

Learning Objectives

By the end of this section you will learn

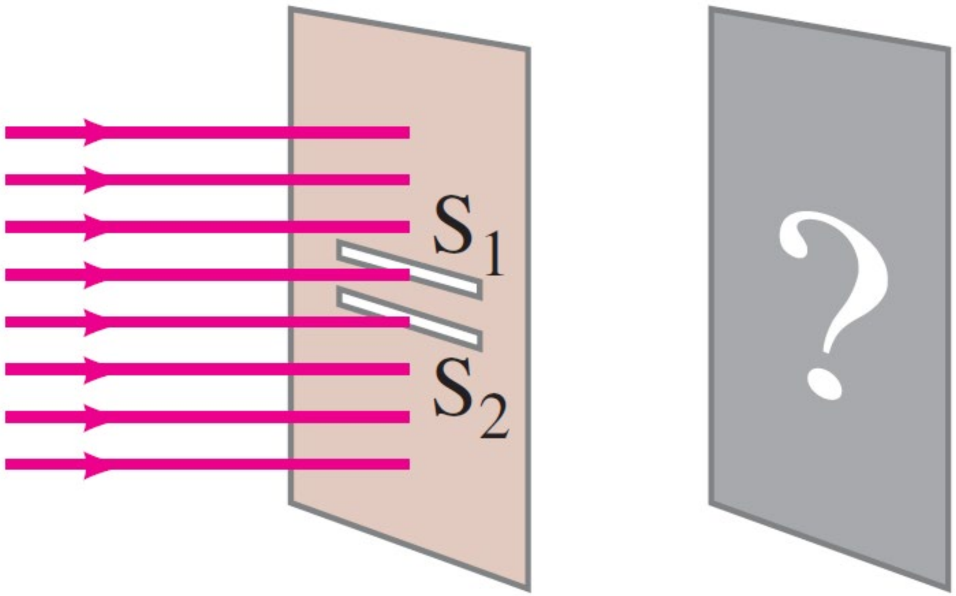
- what happens when two waves combine, or interfere, in space.
- how the interference pattern formed by the interference of two coherent light waves

Interference of Light

Interference of Light

Young's Double-Slit Experiment

- When a monochromatic light is passed through two slits., what will be the pattern in appearing in the screen?



Particle theory prediction

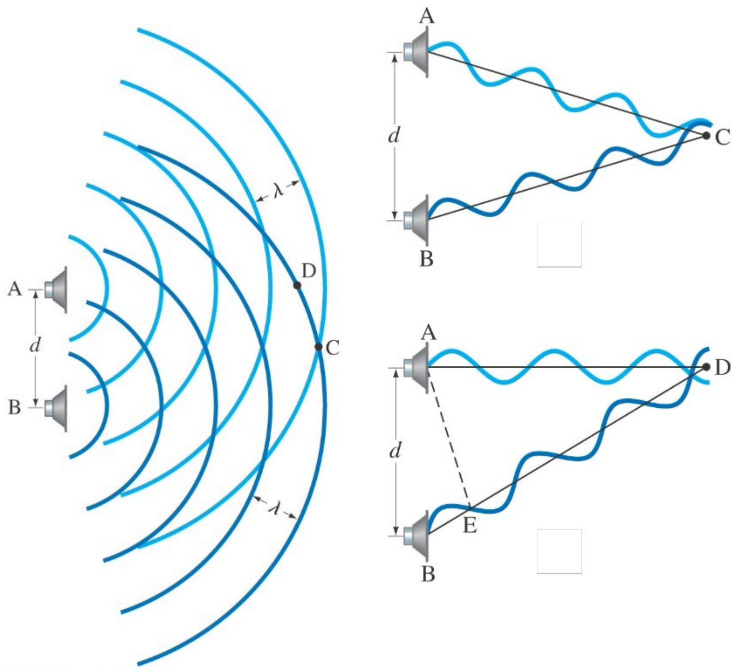
Wave theory prediction

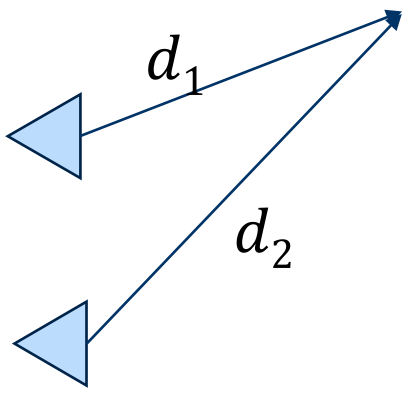
- If light consists of particles, we would expect to see two bright lines on the screen behind the slits.
- In fact, many lines are observed.

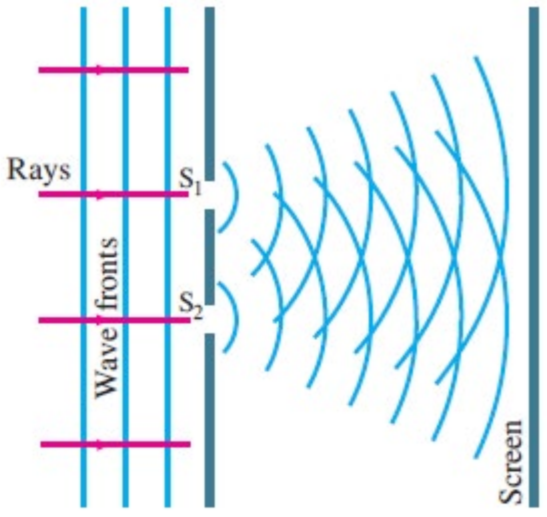
Interference of Light

Young's Double-Slit Experiment

- Sound waves interfere in the same way that other waves do **in space**.





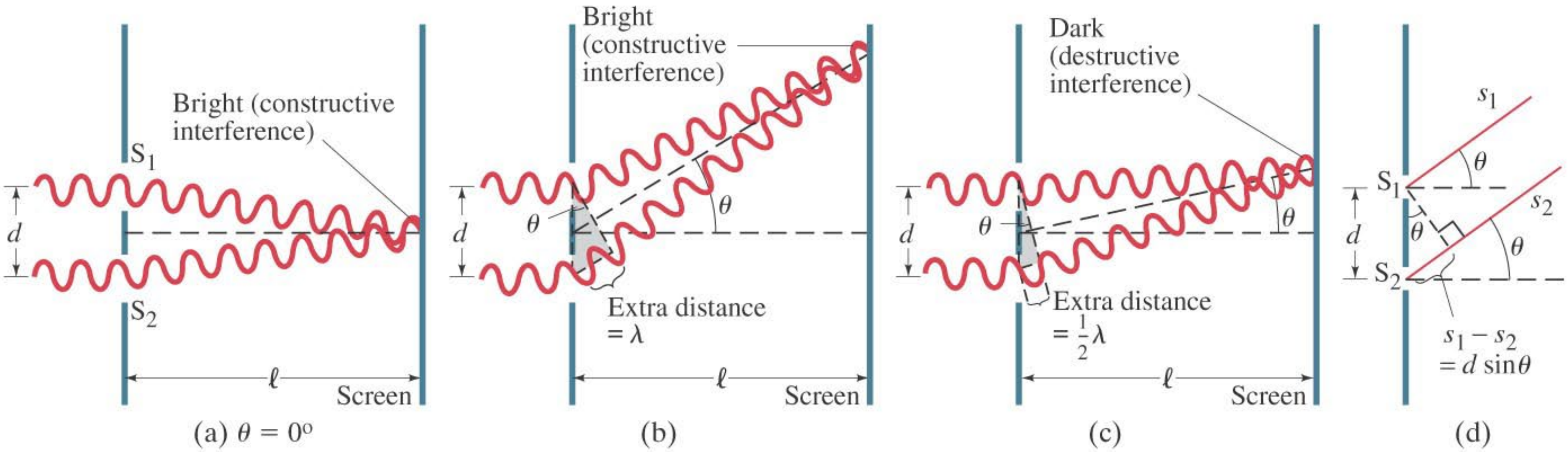


- Constructive Interference: $|d_1 - d_2| = n\lambda, \qquad n = 0, 1, 2, 3...$

- Destructive Interference: $|d_1 - d_2| = \left(n + \frac{1}{2}\right)\lambda, \quad n = 1, 2, 3, \ldots \qquad \text{or} \qquad \frac{n\lambda}{2}, \quad n = 1, 3, 5, \ldots$

Interference of Light

Young's Double-Slit Experiment



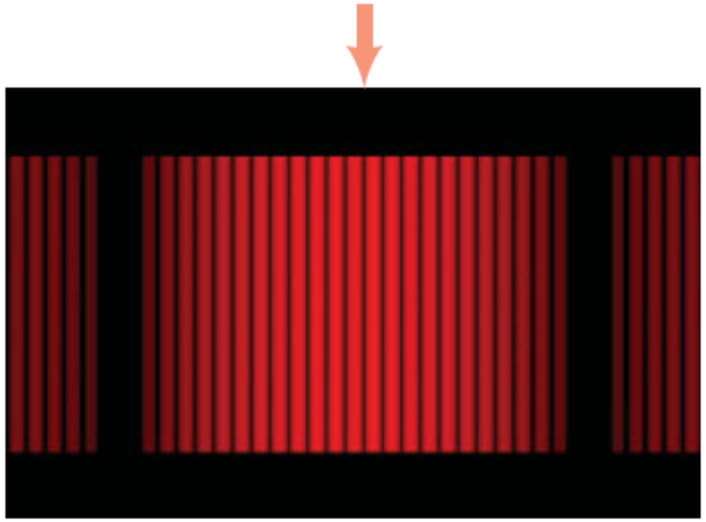
- Constructive Interference: $|d_1 - d_2| \approx d\sin\theta = m\lambda, \qquad m = 0, 1, 2, 3...$

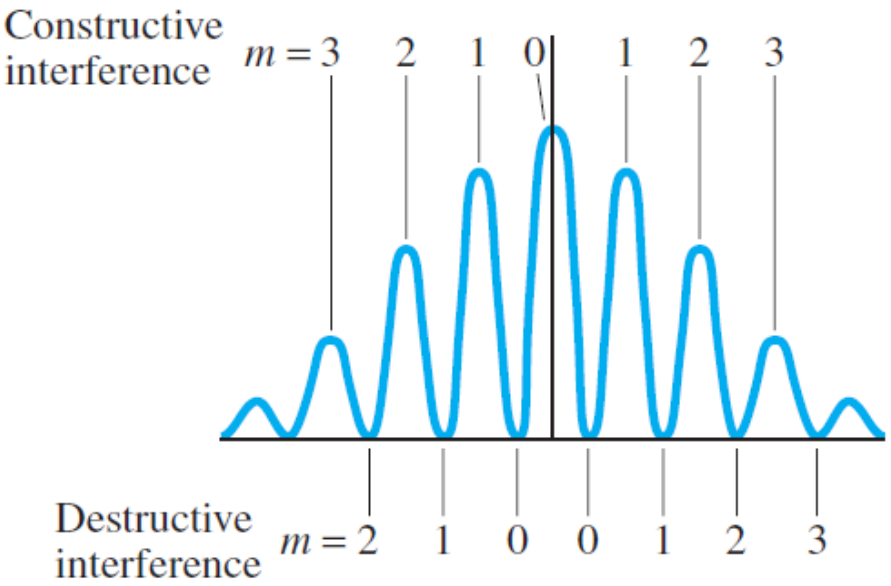
- Destructive Interference: $|d_1 - d_2| \approx d\sin\theta = \left(m + \frac{1}{2}\right)\lambda, \quad m = 1, 2, 3, ... \qquad \text{or} \qquad \frac{m\lambda}{2}, \quad m = 1, 3, 5, ...$

Interference of Light

Young's Double-Slit Experiment

- Interference pattern is in the form of "fringes"





- Intensity of the bright fringes is greatest for the central fringe (*m*=0) and decreases for higher orders
- The decrease in intensity of the higher order fringes depends on the width of each of the two slits

Interference of Light

Young's Double-Slit Experiment

Example: A screen containing two slits 0.100 mm apart is 1.20 m from the viewing screen. Light of wavelength λ = 500 nm falls on the slits from a distant source. Approximately how far apart will adjacent bright interference fringes be on the screen? *Hint*: For small angle of θ, $\sin\theta \approx \tan\theta \approx \theta$

$$d = 0.100 \text{ mm} = 1.00 \times 10^{-4} \text{ m}, \lambda = 500 \times 10^{9} \text{ m}, l = 1.20 \text{ m}$$

m^{th} order fringe occurs at an angle of θ_m

$$\sin\theta_m = \frac{m\lambda}{d}$$

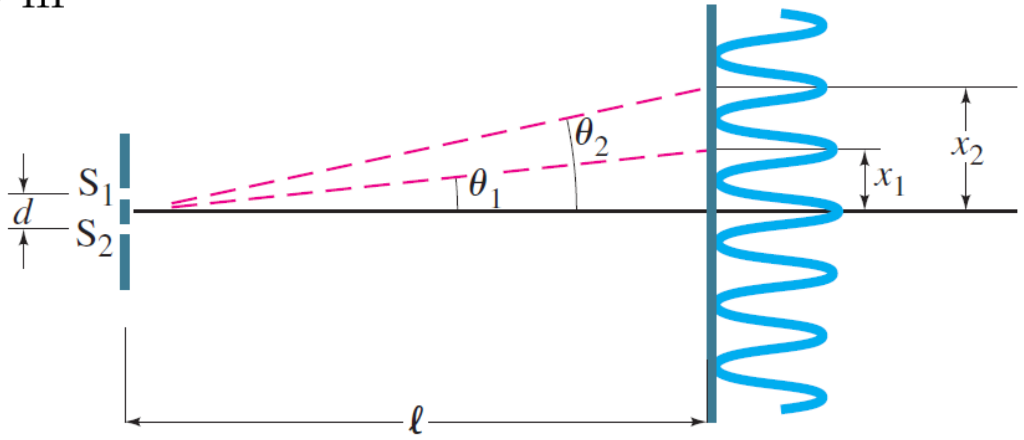
The first order fringe (m=1) occurs at an angle θ_1 such that

$$\sin\theta_1 = \frac{m\lambda}{d} = \frac{(1)(500 \times 10^{-9})}{1.00 \times 10^{-4}} = 500 \times 10^{-3} \approx \theta_1$$

$$x_1 = l\theta_1 = 6.00 \text{ mm}$$

$$x_2 \approx l\theta_2 = l\frac{2\lambda}{d} = 12.00 \text{ mm}$$

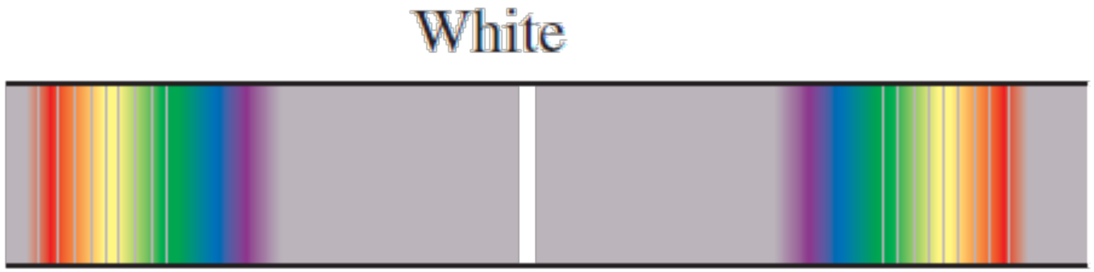
The lower order fringes are 6 mm apart.



$$\frac{x_1}{l} = \tan\theta_1 \approx \theta_1 \Rightarrow x_1 = l\theta_1$$

Interference of Light

Young's Double-Slit Experiment



- Interference pattern for white light
- First-order fringes for a double slit are a full spectrum, like a rainbow.

$$d\sin\theta_m \approx d\theta_m = m\lambda, \qquad m = 0, 1, 2, 3\ldots$$

- Zeroth order fringe is independent of wavelength

$$\theta_1 \approx \frac{\lambda}{d}$$

- Position of first and higher order fringe depends on the wavelength

- θ_1 is smaller for violet compared to red $\Rightarrow$ red has higher wavelength (lower frequency) than violet

Diffraction

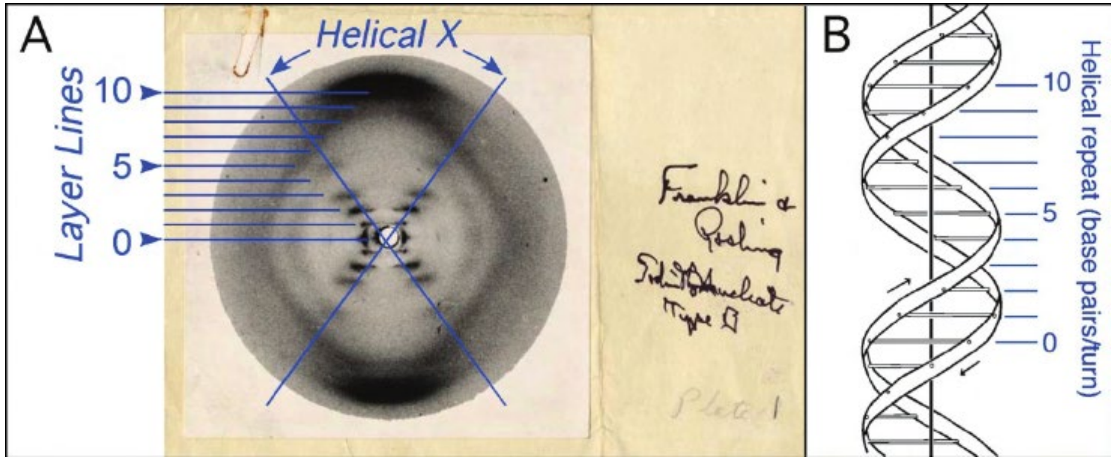
Diffraction

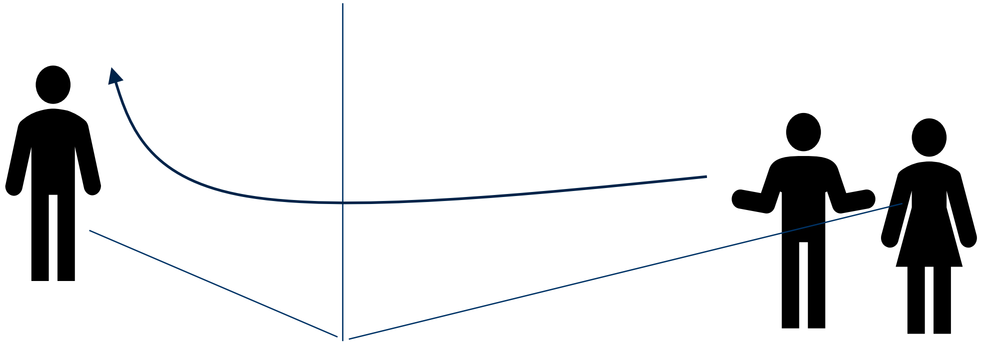
Learning Objectives

By the end of this section, you will learn

- What happens when coherent light shines on an object with an edge or aperture.
- How to understand the diffraction pattern formed when coherent light passes through a narrow slit

Diffraction

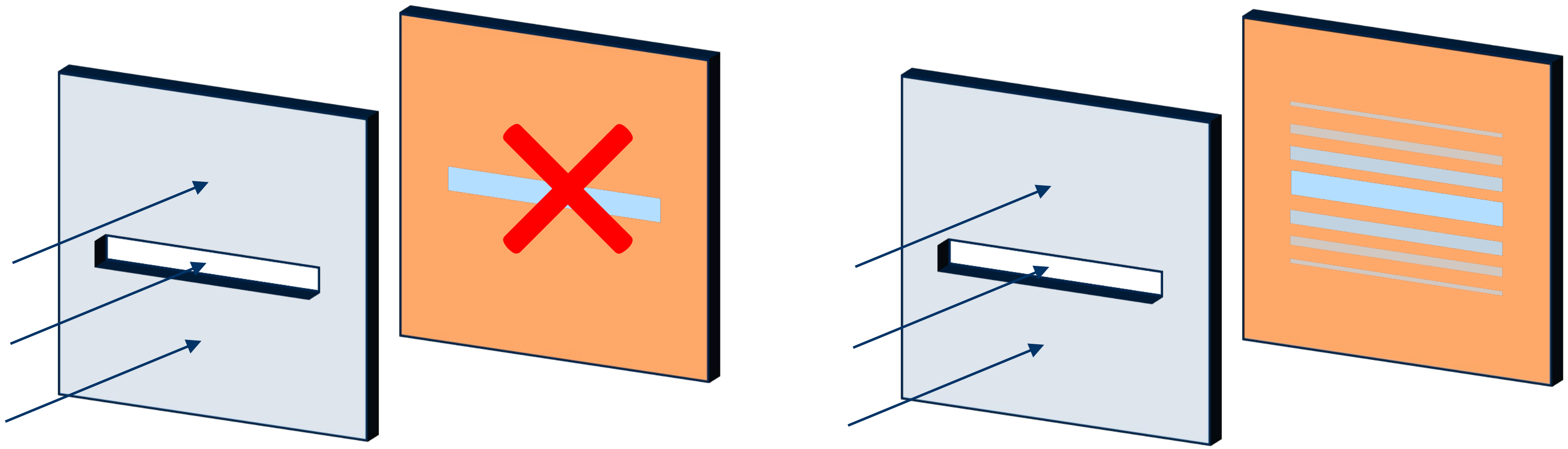


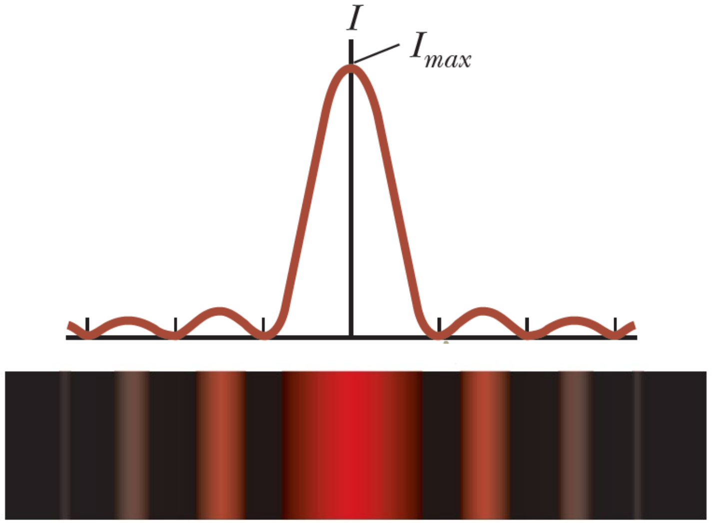


Diffraction

Single-Slit Experiment

- Diffraction, like interference, is an easily observable phenomenon that gives tangible evidence of the wave nature of light.

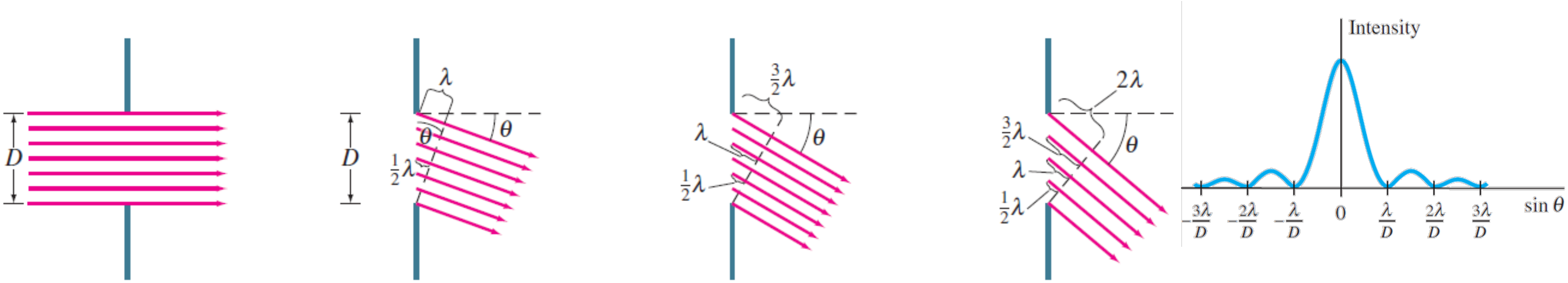




Diffraction

Single-Slit Experiment

- Monochromatic light passing through a narrow slit creates a diffraction patten due to the bending of the light waves



(a) $\theta = 0$
Bright

(b) $\sin\theta = \frac{\lambda}{D}$
Dark

(c) $\sin\theta = \frac{3\lambda}{2D}$
Bright

(d) $\sin\theta = \frac{2\lambda}{D}$
Dark

- Minima in intensity when $D\sin\theta = m\lambda, \qquad m = \pm1, \pm2, \pm3, \ldots$ and peak when $m = 0$

- Between the minima, smaller intensity when, $m \approx \frac{3}{2}, \frac{5}{2}, \ldots$

Diffraction

Single-Slit Experiment

Example: Light of wavelength 750 nm passes through a slit 1.0 × 10^{-3} mm wide. How wide is the central maximum (a) in degrees, and (b) in centimeters, on a screen 20 cm away?

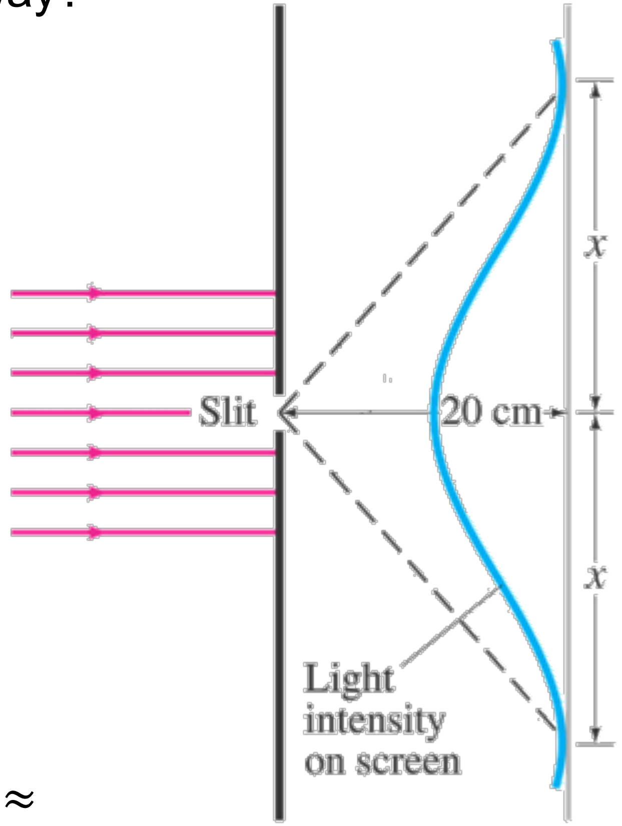
- First minima occurs when $\sin\theta = \frac{\lambda}{D} = \frac{7.5 \times 10^{-7}}{1.0 \times 10^{-6}} = 0.75$

i.e., when $\theta = 49°$

(a) Angle subtended by whole central maximum is then 98°

(b) Width of the central maximum is $2x$ where $\tan\theta = x/20$cm

$2x$ = 46cm



- Noted that angle being higher, we ***cannot*** use the small angle approximation of $\theta \approx \sin\theta \approx \tan\theta$