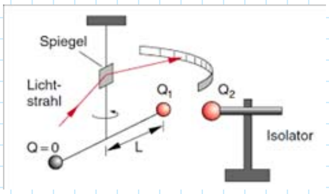
1.2. Forces between Point Charges

1.2.1. Coulomb's Law

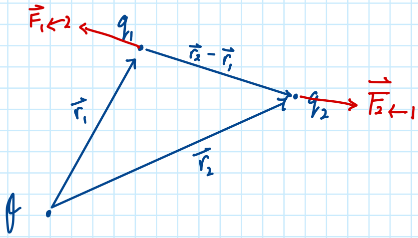
C.A. Coulomb (and Cavendish, Priestly):

Experimental observations:

Coulomb's balance



- Consider two point charges q_1, q_2
- positions of point charges are $\vec r_1, \vec r_2$



(1) force $\vec F_1$ on charge q_1 is equal to force $\vec F_2(\vec r)$ on charge q_2 (in opposite direction): actio = reactio

balance of forces: $|\vec F_{1\leftarrow 2}| = |\vec F_{2\leftarrow 1}|$

magnitude is the same

Direction is parallel to distance vector:

$$\Rightarrow \vec F_{1\leftarrow 2} \overset{\text{anti parallel}}{\uparrow\downarrow} \vec F_{2\leftarrow 1}$$ both are parallel to $\underbrace{\vec r_2 - \vec r_1}_{\text{distance vector}}$

(2) force depends on location and on distance between charges; dependance is inverse proportional to square of magnitude of distance vector:

Coulomb found: $|\vec F_q| \sim \frac{1}{r^2}$

$$|\vec F_{2\leftarrow 1}| = |\vec F_{1\leftarrow 2}| \sim \frac{1}{|\vec r_2 - \vec r_1|^2}$$

Force is proportional to product of charges and a constant γ (= gamma)

$$|\vec F_{2\leftarrow 1}| = |\vec F_{1\leftarrow 2}| = \gamma \cdot \frac{q_1 q_2}{|\vec r_2 - \vec r_1|^2} \qquad \gamma = \frac{1}{4\pi\varepsilon_0} \qquad \varepsilon_0 = 8.85 \cdot 10^{-12}\,\frac{As}{Vm}$$

= dielectric constant of vacuum

permittivity of vacuum

(3) We know, like charges repel each other, unlike charges attract each other:

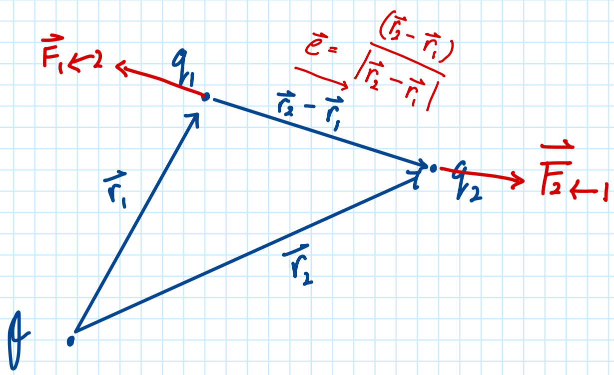
mathematically : $Sgn(q_1) = Sgn(q_2) \Rightarrow$ repulsive

$Sgn(q_1) \neq Sgn(q_2) \Rightarrow$ attractive

(1)-(3) can be summarized in the following equation = Coulomb's law:

$$\vec{F}_{1\leftarrow 2} = -\vec{F}_{2\leftarrow 1} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1\cdot q_2}{|\vec{r}_2-\vec{r}_1|^2}\cdot\frac{(\vec{r}_2-\vec{r}_1)}{|\vec{r}_2-\vec{r}_1|}$$

unit vector parallel to distance vector

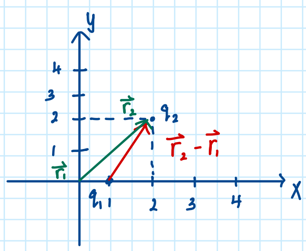


Coulombs' Law : electrostatic force between two point charges q_1 and q_2

$$\boxed{\vec{F}_{1\leftarrow 2} = -\vec{F}_{2\leftarrow 1} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1\cdot q_2(\vec{r}_2-\vec{r}_1)}{|\vec{r}_2-\vec{r}_1|^3}} \qquad (1.1)$$

Example : q_1 at location $\vec{r}_1 = \begin{pmatrix}1\\0\end{pmatrix} = 1\cdot\vec{e}_x$

q_2 at location $\vec{r}_2 = \begin{pmatrix}2\\2\end{pmatrix} = 2\cdot\vec{e}_x + 2\cdot\vec{e}_y$



Calculate Coulomb's force :

$(\vec{r}_2-\vec{r}_2) = \begin{pmatrix}2\\2\end{pmatrix} - \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\2\end{pmatrix} = 1\cdot\vec{e}_x + 2\cdot\vec{e}_y$

$|\vec{r}_2-\vec{r}_1| = \sqrt{x^2+y^2} = \sqrt{1^2+2^2} = \sqrt{5}$

$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1\cdot q_2}{(\sqrt{5})^3}\cdot\begin{pmatrix}1\\2\end{pmatrix}$

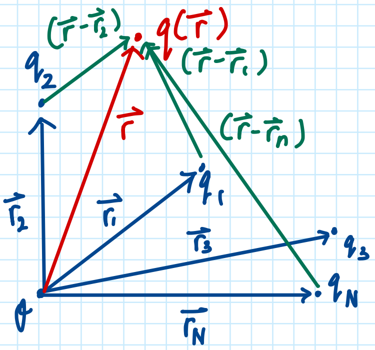
1.2.2. Generalization of (1.1.): Superposition Principle of Electrostatics

Generalization of (1.1.) to N charges:

A test charge q experiences the following force by N charges q_i (i=1,2,……N), localized at locations r_i (i= 1,2,….N):

$q(\vec{r})$ is a "test Charge"

What is the total force on $q(\vec{r})$ exerted by all fixed charges $q_i(\vec{r}_i)$



$$\vec{F}_q(\vec{r}) = \underbrace{\frac{1}{4\pi\varepsilon_0}\frac{q(\vec{r})\cdot q_1(\vec{r}_1)\cdot(\vec{r}-\vec{r}_1)}{|\vec{r}-\vec{r}_1|^3}}_{\vec{F}_{qq_1}} + \underbrace{\frac{1}{4\pi\varepsilon_0}\frac{q(\vec{r})\cdot q_1(\vec{r}_2)\cdot(\vec{r}-\vec{r}_2)}{|\vec{r}-\vec{r}_2|^3}}_{\vec{F}_{qq_2}} \underbrace{+ \cdots + \cdots}_{\vec{F}_{qq_i}} + \frac{1}{4\pi\varepsilon_0}\frac{q(\vec{r})\cdot q_1(\vec{r}_n)\cdot(\vec{r}-\vec{r}_n)}{|\vec{r}-\vec{r}_n|^3}$$

$$\Rightarrow \boxed{\vec{F}_q(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q(\vec{r})\cdot q(\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3}(\vec{r}-\vec{r}_i)} \qquad (1.2)$$

Generalization of Coulomb's Law for N fixed point Charges

= <u>Superposition principle</u>

$$\vec{F}_q(\vec{r}) = q(\vec{r})\sum_{i=1}^{N}\frac{1}{4\pi\varepsilon_0}\frac{q(\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3}(\vec{r}-\vec{r}_i)$$

1.3. Electric field

From 1.1 and 1.2. we know: charges exhibit a long-range impact/effect on other charges.

This is described by Coulomb's force:

- Between two charges:

$$\vec{F}_{2\leftarrow 1} = -\vec{F}_{1\leftarrow 2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_2-\vec{r}_1|^3}(\vec{r}_2-\vec{r}_1)$$

- Force acting on a charge q at position r in a field of N fixed charges q_i

$$\vec{F}(\vec{r}) = \frac{q}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{|\vec{r}-\vec{r}_i|^3}(\vec{r}-\vec{r}_i)$$

superposition principle!

↳ Can be described by a force field

This can be described by a force field, which acts on charge q and, hence describes the action of N fixed charges on a (test) charge q at an arbitrary location r

Division of (1.2) by q yields:

$$\frac{\vec{F}(\vec{r})}{q(\vec{r})} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i(\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3}(\vec{r}-\vec{r}_i) = \vec{E}(\vec{r}) \quad (1.3)$$

electric field generated by N point charges at locations $\vec{r}_i$

$$\text{Definition of electric field}: \vec{E}(\vec{r}) = \frac{\vec{F}(\vec{r})}{q}$$

Electric field is a force field.

dim $(|\vec{E}|)$ = physical unit = $1\frac{N}{C} = 1\frac{V}{m}$

dimension

Coulomb C = unit of the charge = A·S = Ampere · seconds

N = Newton

V = Volt

m = meter

Example: parallel plate capacitor