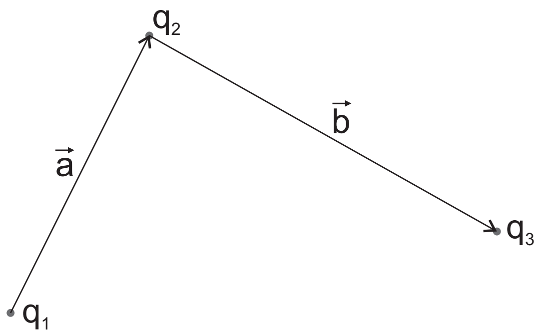
Q1 (6 points)

Consider three point charges q_i $(i = 1, 2, 3)$ in vaccum $(\varepsilon = \varepsilon_0)$ carrying the charge $q_1 = q_2 = -q$ and $q_3 = +2q$, respectively. Their locations are determined by the relative position vectors $\vec{a}$ and $\vec{b}$ as indicated in the figure below.



*a) Calculate the electrostatic force $\vec{F}_{el}$ acting on point charge q_3.

$$\vec{F} = \frac{2q}{4\pi\varepsilon_0} \cdot \left[\frac{-q}{\quad} + \frac{-q}{\quad} \right.$$

*b) Let now the origin of the coordinate system be at the location of point charge q_2. Calculate the electrostatic field $\vec{E}(\vec{r})$, which is generated by the three point charges at an arbitrary point in space $\vec{r}$.

*c) What is the relation between electric energy density $w_{el}(\vec{r})$ and electric field $\vec{E}(\vec{r})$?

Q2 (2 points)

Which of the following equations is a correct representation of Gauss' law?
(Multiple choices/nominations possible).
Here, $\vec{D}$ denotes the dielectric displacement field, ρ the electric space charge density, and V an arbitrary volume with enclosing boundary surface ∂V.

- ☐ $\mathrm{div}\vec{D} = \rho$
- ☐ $\int\limits_{\partial V} \vec{D}\,\mathrm{d}\vec{a} = \int\limits_{V} \rho\,\mathrm{d}^3r$
- ☐ $\mathrm{rot}\vec{D} = \rho$
- ☐ $\int\limits_{V} \mathrm{div}\vec{D}\,\mathrm{d}^3r = \int\limits_{\partial V} \rho\,\mathrm{d}\vec{a}$
- ☐ $\int\limits_{V} \vec{D}\,\mathrm{d}^3r = \int\limits_{V} \rho\,\mathrm{d}^3r$

Q3 (5 points)

Consider an electrostatic potential $\Phi(x, y, z)$ inside a region with constant permittivity ε. $\Phi(x, y, z)$ depends on the x coordinate only ($\Phi(x, y, z) = \Phi(x)$):

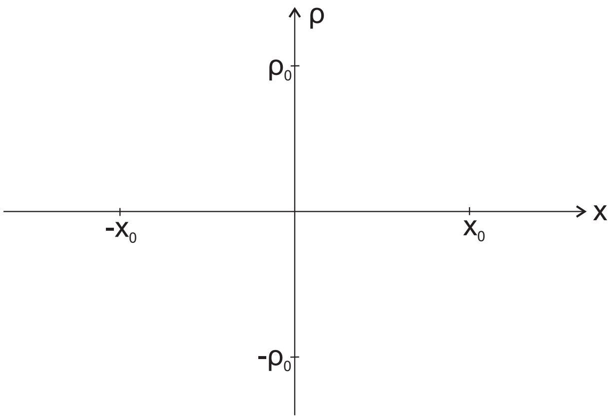
$$\Phi(x) = \begin{cases} +\frac{\rho_0}{2\varepsilon}x_0^2 & \text{for} \quad x < -x_0 \quad (1) \\ -\frac{\rho_0}{2\varepsilon}(x^2 + 2x_0x) & \text{for} \quad -x_0 \leq x \leq 0 \quad (2) \\ +\frac{\rho_0}{2\varepsilon}(x^2 - 2x_0x) & \text{for} \quad 0 < x \leq +x_0 \quad (3) \\ -\frac{\rho_0}{2\varepsilon}x_0^2 & \text{for} \quad +x_0 < x \quad (4) \end{cases}$$

x_0 and ρ_0 are positive constants.

*a) Calculate the electric field $\vec{E}(x)$ for each of the four regions.

*b) Calculate the space charge density $\rho(x)$ for each of the four regions.

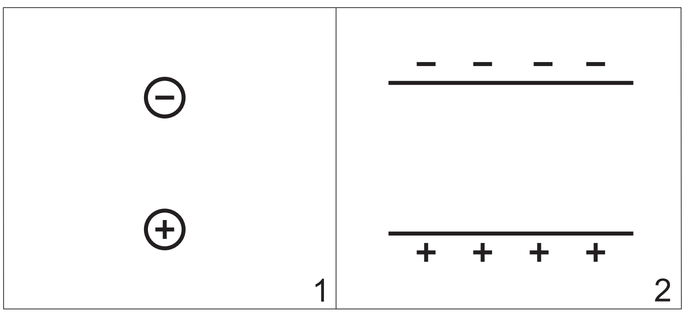
c) Draw the profile of the space charge density $\rho(x)$ in the diagram sketched below.



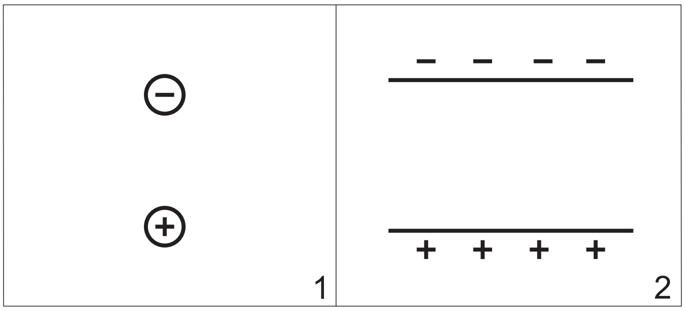
Q4 (4 points)

Consider the two different charge configurations sketched below. Configuration 1 consists of two opposite point charges, configuration 2 is made up of a charged plate capacitor.

*a) Sketch the field lines of the electric field $\vec{E}$, generated by configuration 1 and configuration 2, respectively.

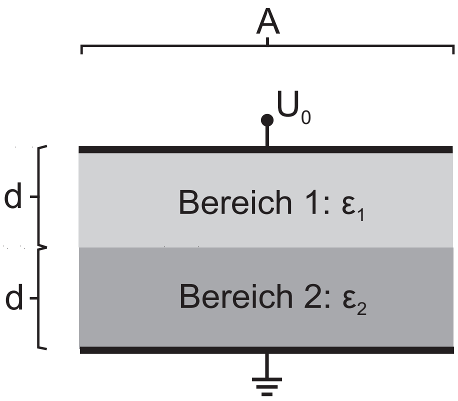


*b) Sketch the equipotential lines of the electric potential Φ for both configurations.



Q5 (6 points)

Consider a plate capacitor with area A. Two equally-sized regions of thickness d inside the plate capacitor are filled with dielectric materials of permittivity ε_1 in region 1 and permittivity ε_2 in region 2, respectively (see figure below). Assume $\varepsilon_1 > \varepsilon_2$. A positive bias voltage $U_0 > 0$ is applied to the plate capacitor. Stray fields may be neglected.



*a) Calculate the charge Q on the upper plate of the capacitor.
Hint: Calculate first the total capacitance of the given set-up.

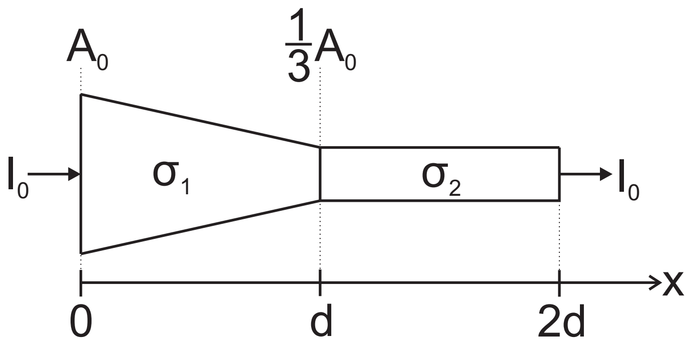
$C = \frac{\varepsilon A}{d}$

$C_{tot} = \frac{1}{C_1} + \frac{1}{C_2}$

*b) In which region is the magnitude of the electric field $|\vec{E}|$ larger? Give reasons for your answer!

Q6 (7 points)

An electric current I_0 is flowing through two regions of different conductivity σ_1 and σ_2, where the cross section $A(x)$ depends on the position x. The current is distributed uniformly over the cross section $A(x)$, and enters and leaves the conductor at the front- and the rearside only.



*a) The cross section $A(x)$ decreases linearly with x in region 1 and remains constant in region 2. Calculate the area $A(x)$ for $0 < x < 2d$.

b) Calculate the electric current density $j_x(x)$ for $0 < x < 2d$. Components of the current density in y- and z-direction may be neglected.

c) Calculate the x-component of the electric field $E_x(x)$ for $0 < x < 2d$.

d) Which statement holds true for the electric field at the position $x = d$, if $\sigma_1 > \sigma_2$? Please tick the correct anwer(s).

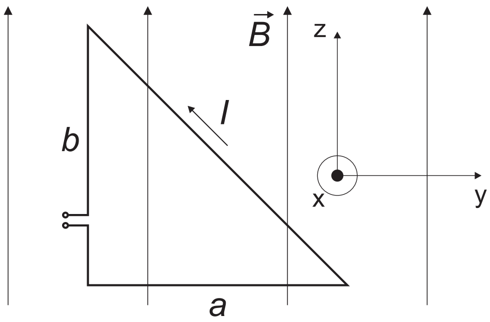
O The electric field has a continuous first derivative.

O The electric field is continuous.

O The electric field is discontinuous.

Q7 (5 points)

Consider a triangular, nearly closed conductor loop with dimensions a and b as displayed in the sketch. A current I flows through the conductor loop, which is exposed to a uniform magnetic field $\vec{B} = B_0\vec{e}_z$ (see figure to the right).

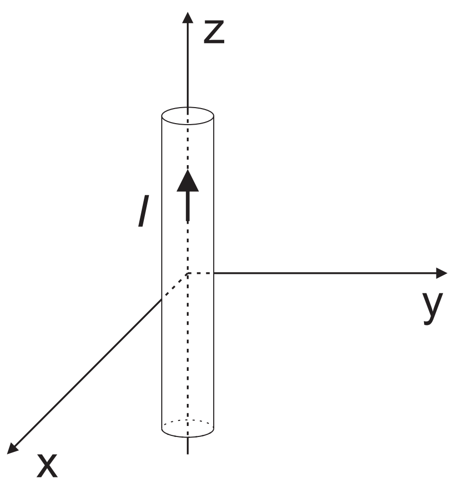


*a) Calculate the magnetic moment $\vec{m}$ of the conductor loop and state the unit of $\vec{m}$ in SI units.

*b) Calculate the torque $\vec{M}$ acting on the conductor loop.

Q8 (4 points)

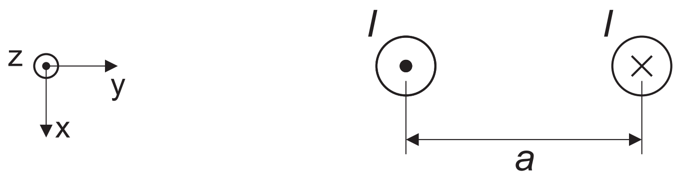
A constant current I flows uniformly through a long, straight cylindric wire located parallel to the z-axis of the coordinate system. It generates a magnetic field $\vec{H}(\vec{r})$.



*a) Sketch the field lines of the magnetic field $\vec{H}(\vec{r})$ in the xy-plane outside the wire. To this end, complete the figure to the right.

Now, a second wire is mounted parallel to first one at a distance a. The current I is flowing in antiparallel direction (i.e., $-\vec{e}_z$-direction) through this wire.

*b) Sketch field lines of the total magnetic field $\vec{H}(\vec{r})$ outside the two wires in the figure below.



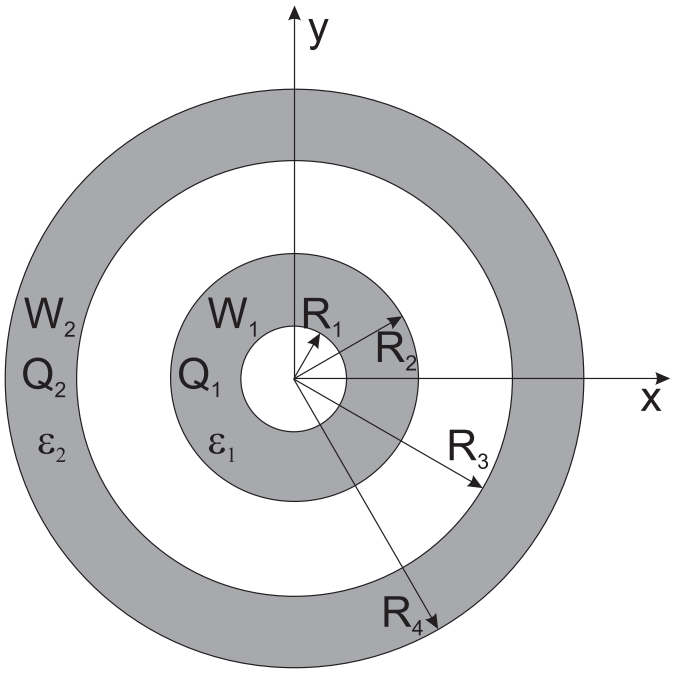
*c) What is the direction of the forces which act on the two wires due to the magnetic field.

Q9 (3 points)

State Faraday's law of induction in its most comprehensive (integral) formulation.
Which kinds of induction can be derived from this general formulation? Refer in your answer to the individual terms in the equation.

Problem 1 (15 points)

Consider two uniformly charged concentric spherical regions W_1 (total charge $Q_1 = +Q$, permittivity ε_1) and W_2 (total charge $Q_2 = -Q$, permittivity ε_2) in vacuum (see figure). The region inbetween is vacuum as well.



Please use spherical coordinates (r, ϑ, φ) for solving the problem.

*a) Calculate the space charge density ρ_1 and ρ_2 within the region W_1 and W_2, respectively.

*b) Determine the magnitude and the direction of the electric field $\vec{E}(\vec{r})$ generated by these space charge densities in the region $\boldsymbol{R_1 \leq r \leq R_3}$ as function of the radial coordinate radius r.

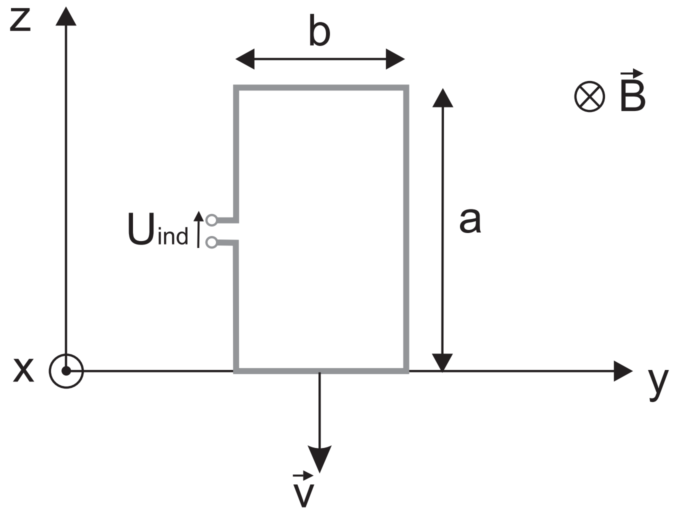
*c) Determine the magnitude of the electric field $\left|\vec{E}(\vec{r})\right|$ outside the charged regions, i.e. for $r > R_4$. Give a reason for your answer!

d) Calculate the electrical voltage U between R_2 and R_3, i.e., the outer surface of region W_1 and the inner surface of region W_2.

e) Calculate the electric energy density $\mathrm{w_{el}}(\vec{r})$ of the electric field within the region between W_1 and W_2, i.e. for $R_2 < r < R_3$

Problem 2 (14 points)

Consider a nearly closed, rectangular conductor loop of lenght a and width b with ohmic resistance R. At the time $t = 0$, the loop is positioned as depicted in the figure below. It moves with constant velocity $\vec{v} = -v_0\vec{e}_z$ along a straight line parallel to the z-axis. It is exposed to an inhomogeneous magnetic field $\vec{B}(z) = (B_o + k_0 z)(-\vec{e}_x)$, where B_0 and k_0 are constants.



*a) Calculate the magnetic flux $\Phi(t)$ penetrating the conductor loop as a function of time. Pay attention to the orientation of the magnetic field and the conductor loop (see figure).

b) Calculate the voltage U_{ind} induced in the open-circuit configuration..

The terminals of the conductor loop are now shortened.

c) Calculate the current I_{ind} flowing through the loop. Indicate the direction of the induced current in the figure above.

*d) Calculate the magnitude and direction of the total magnetic force $\vec{F}_m$ acting on the conductor loop. Discuss first, which sections of the conductor loop contribute to the total magnetic force $\vec{F}_m$. The current generated by the additionally generated magnetic field may be neglected.
Hint: If you didn't solve subproblem c), please use $I_{ind} = \frac{A_0 k_0 v_0}{R}$ with $A_0 = const.$ for your further calculations (assume that the current flows in clockwise direction).