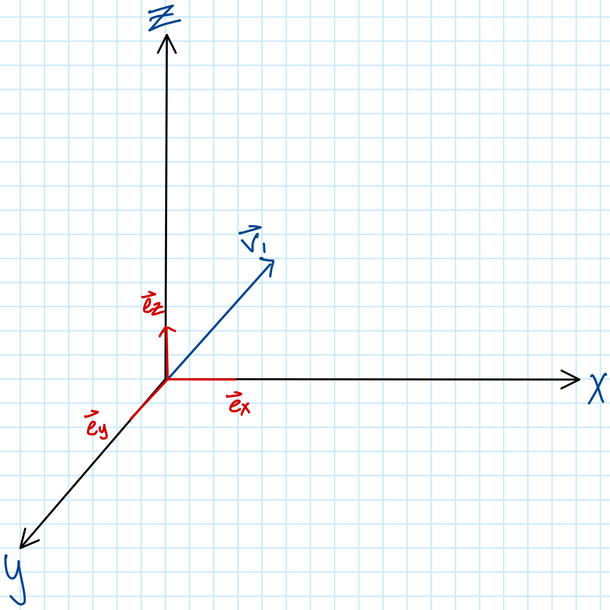
Recall: vectors, vector space representation

- What is a so-called "base-free" representation of quantities in a vector space?

consider the representation of vectors in a coordinate system with defined base vectors, e.g. cartesian coordinate system



$$\vec{v}_1 = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$$

basis vectors

$\vec{e}_x, \vec{e}_y, \vec{e}_z$

$\vec{e}_x \perp \vec{e}_y \perp \vec{e}_z$

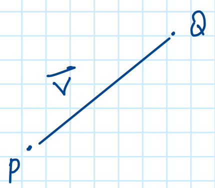
$|\vec{e}_x| = |\vec{e}_y| = |\vec{e}_z| = 1$

$$\vec{v} = x_1 \cdot \vec{e}_x + y_1 \cdot \vec{e}_y + z_1 \cdot \vec{e}_z$$

(i) Vector space; fundamental relations

- Each position can be described by a 3-dimensional vector
- Relations betweens points can be described by vectors as well

What is a vector? = directed connection between two points

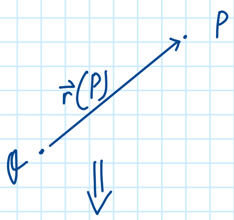


$\vec{v} = \overrightarrow{PQ}$

$Q = P + \vec{v}$

Base-free representation: what is this? Why does it make sense?

Define origin (reference point) $\mathcal{O}$



position of point P is defined with respect to the origin $\mathcal{O}$

$$P(\vec{r}) = \mathcal{O} + \vec{r}(P)$$

with $\vec{r}(P)$ = position vector of P

"base-free representation of P"
(no coordinate system defined)

This is how all equations in this lecture are formulated
For concrete calculations: suitable coordinate system is chosen

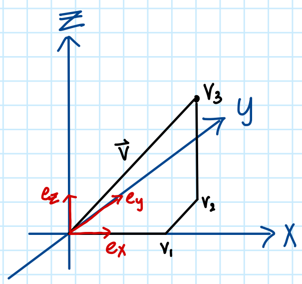
(ii) Representation of vectors in a coordinate system

Introduction of basis vectors,

i.e.: orthonormal system (e.g. cartesian)

- 3 basis vector, $\vec{e}_1$, $\vec{e}_2$, $\vec{e}_3$
- Origin $\mathcal{O}$

$\left.\right\}$ Coordinate System $CS = (\mathcal{O}, \vec{e}_1, \vec{e}_2, \vec{e}_3)$



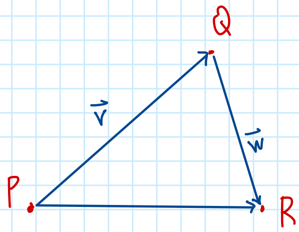
orthonormal: $\vec{b}_i \cdot \vec{b}_j = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{if } i \neq j \end{cases}$

representation of $\vec{v}$ in CS:

$$\vec{v} = v_1 \vec{e}_x + v_2 \cdot \vec{e}_y + v_3 \cdot \vec{e}_z = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

(iii) manipulating vectors

- Vectors can be added:



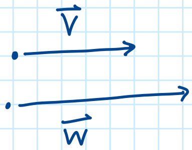
$\vec{v} = \vec{PQ}$

$\vec{w} = \vec{QR}$

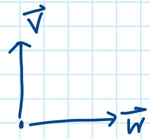
$\vec{PR} = \vec{v} + \vec{w} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} + \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} v_1 + w_1 \\ v_2 + w_2 \\ v_3 + w_3 \end{pmatrix}$

- dot-product/direct product:

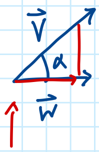
$\vec{v} \cdot \vec{w} = |\vec{v}| \cdot |\vec{w}| \cdot \cos\alpha$



$\vec{v} \cdot \vec{w} = |\vec{v}| \cdot |\vec{w}|$



$\vec{v} \cdot \vec{w} = 0$

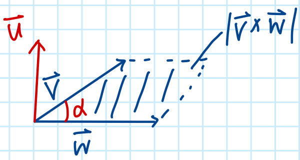


$|\vec{v}| \cos\alpha$

projection of $\vec{v}$ on $\vec{w}$

- vector product, cross product:

$\vec{v} \times \vec{w} = \vec{u}$



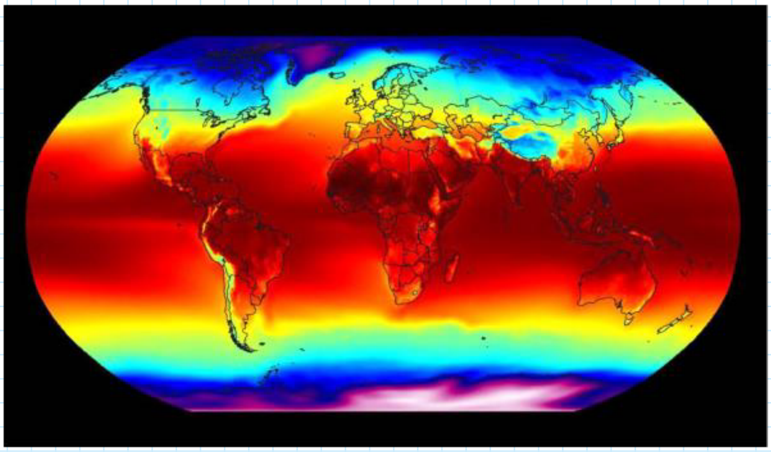
$|\vec{u}| = |\vec{v} \times \vec{w}| = |\vec{v}| \cdot |\vec{w}| \cdot \sin\alpha$

Mathematical Recall: what are fields in a mathematical sense?

- Scalar field
- Vector field
- Representation of vector fields - field lines

Scalar field

Temperature field of earth (=scalar field)



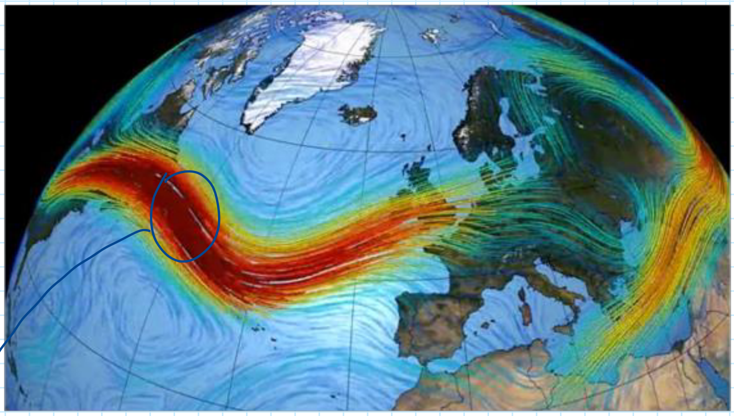
Temperature T

mapped on surface of earth

$$\left.\begin{matrix} T(x,y,z) \\ T(r,\varphi,\theta) \end{matrix}\right\} T(\vec{r})$$

Vectorfield

Flow field (velocity) of air in upper atmosphere = velocity field (= vectorfield)

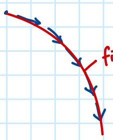


$\vec{v}$ = Velocity of air

$\vec{v}(\vec{r}) = \vec{v}(r,\varphi,\theta)$

Vector fields assign a vectorial quantity to each location in space

in order to enhance the visibility ⇒ introduce field lines = tangents to vectorfield



field line; has a direction

high velocity ⇒ dense field lines