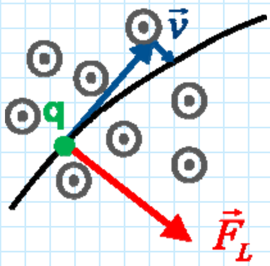
3.1. Forces on Moving Charges in Magnetic Fields

3.1.2 Motion of a charged particle in a constant magnetic field

(here: only sketch of solution; for detailed calculation explicite solution procedures see lecture notes and problems/exercises in tutorial notes)

- Charge moves ~~on~~ in magnetic field with velocity v



We may assume (without losing general character of considerations):

- B field oriented parallel to z-axis

$\vec{B} = B \cdot \vec{e_z}$

- Newton's equation of motion: $\vec{F_a} = m \cdot \vec{a} = \vec{F_L}$ $\quad \vec{a} = \frac{d\vec{v}}{dt}$

$$m \cdot \frac{d\vec{v}}{dt} = q \cdot (\vec{v} \times \vec{B}) \Leftarrow 3 \text{ Components} \qquad (3.3)$$

$$\vec{v} \times \vec{B} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ B \end{pmatrix} = \begin{pmatrix} Bv_y \\ -v_x \cdot B \\ 0 \end{pmatrix}$$

insert this in equation of motion:

$$m \cdot \frac{dv_x}{dt} = q \cdot B \cdot v_y \qquad (3.4)$$

$$m \cdot \frac{dv_y}{dt} = -q \cdot B \cdot v_x \qquad (3.5)$$

$$m \cdot \frac{dv_z}{dt} = 0 \qquad (3.6)$$

(i) Look at (3.6): first order differential equation

$\Rightarrow$ define a condition for $t = t_0$ to determine integration constant

$\Rightarrow$ initial velocity in z-direction: $v_z(t=0) = v_{z0}$

$\frac{dv_z}{dt} = 0 \Rightarrow v_z(t) = \text{Constant} = v_{z0}$

$\Rightarrow v_z \triangleq$ velocity component parallel to $\vec{B} = B \cdot \vec{e_z}$ is not affected; constant

(ii) motion in xy-plane $\perp$ z-axis ($\perp \vec{B}$)

$\vec{v_\perp} = \begin{pmatrix} v_x(t) \\ v_y(t) \\ 0 \end{pmatrix}$

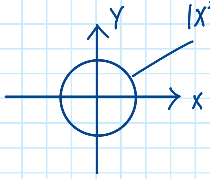
We know: no work is performed by $\vec{B}$ on moving charge;

$\Rightarrow$ magnitude of $\vec{v}$ will not be change

$$|\vec{v}| = v_x^2 + v_y^2 + v_z^2 = \text{Const.} = v_0^2 \text{ (velocity for } t=0)$$

$$|v_z| = v_{z0} = \text{Const.}$$

$$|v_x^2 + v_y^2| = \underbrace{v_0^2 - v_{z0}^2}_{\text{Const. value}}$$



$\Downarrow$

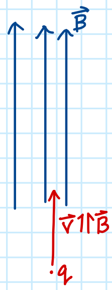
Circular motion in XY-plane (Perpendicular to $\vec{B}$)

insert this info (3.4), (3.5) & solve $\Rightarrow v_x, v_y$

integrate $v_x, v_y, v_z \Rightarrow \vec{r}(t) = (x(t), y(t), z(t))$ } in detail in lecture notes

Trajectory of Charge

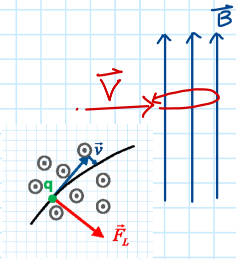
Qualitative interpretation: particle moving in a $\vec{B}$-field



$\vec{v} = v_0 \cdot \vec{e}_z$

$\downarrow$

q will move at constant velocity v_{0z} parallel to $\vec{B}$-field lines $\Rightarrow$ no $\vec{F}_L$



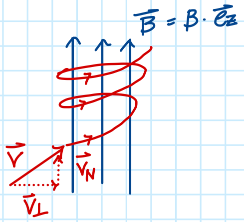
$\vec{v} \perp \vec{B}$

particles experience $\vec{F}_L$

$\vec{F}_L \perp \vec{v} \perp \vec{B}$

$\Downarrow$

Circle in XY-Plane; gyration around $\vec{B}$ (circular)



angle α between $\vec{v}$ & $\vec{B}$

helical motion

$\vec{v}_\parallel$ is not affected;

$\vec{v}_\perp \Rightarrow \vec{F}_L = q \cdot (\vec{v}_\perp \times \vec{B})$

From equation of motion:

gyration / circulation frequency: $\Omega = \frac{q \cdot B}{m}$ (Ω = Omega)

radius of the motion: $R = \frac{v_\perp}{\Omega} = \frac{v_\perp \cdot m}{q \cdot B}$

3.1.3 Lorentz force acting on a current distribution

Current density (general formulation):

Lorentz force on one carrier species:

Lorentz force density: