Circular Motion

What We Will Learn

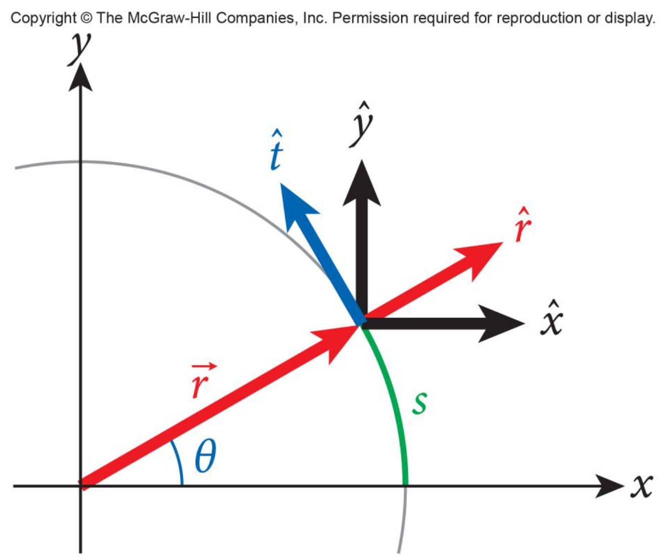
- The motion of objects traveling in a circle rather than in a straight line can be described using coordinates based on radius and angle rather than Cartesian coordinates.
- There is a relationship between linear motion and circular motion.
- Circular motion can be described in terms of the angular coordinate, angular frequency, and period.
- An object undergoing circular motion can have angular velocity and angular acceleration.

Circular Motion

- Circular motion is motion along the perimeter of a circle.
- Circular motion is surprisingly common.
  - CD, DVD, Blu-ray, Indy-car racing, carousel, ferris wheel, etc.

Polar Coordinates

- During an object's circular motion, its x- and y-coordinates change continuously, but the distance from the object to the center of the circular path stays the same.

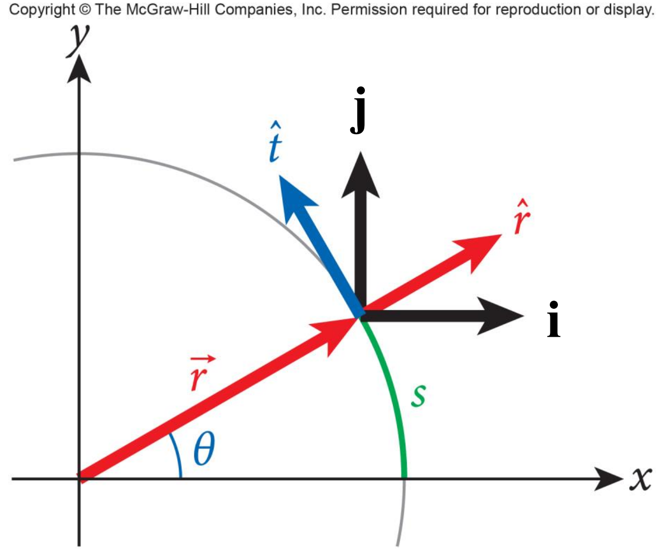


- We can take advantage of this fact by using polar coordinates.

- The position vector of an object in circular motion changes as a function of time, but its tip always moves on the circumference of a circle.

Polar Coordinates

- We can specify the position vector by giving its x- and y-components.
- We can also specify the same vector by giving two other variables: r and θ.
- The relationship between Cartesian coordinates and polar coordinates is:



$$x = r\cos\theta \qquad r = \sqrt{x^2 + y^2}$$

$$y = r\sin\theta \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

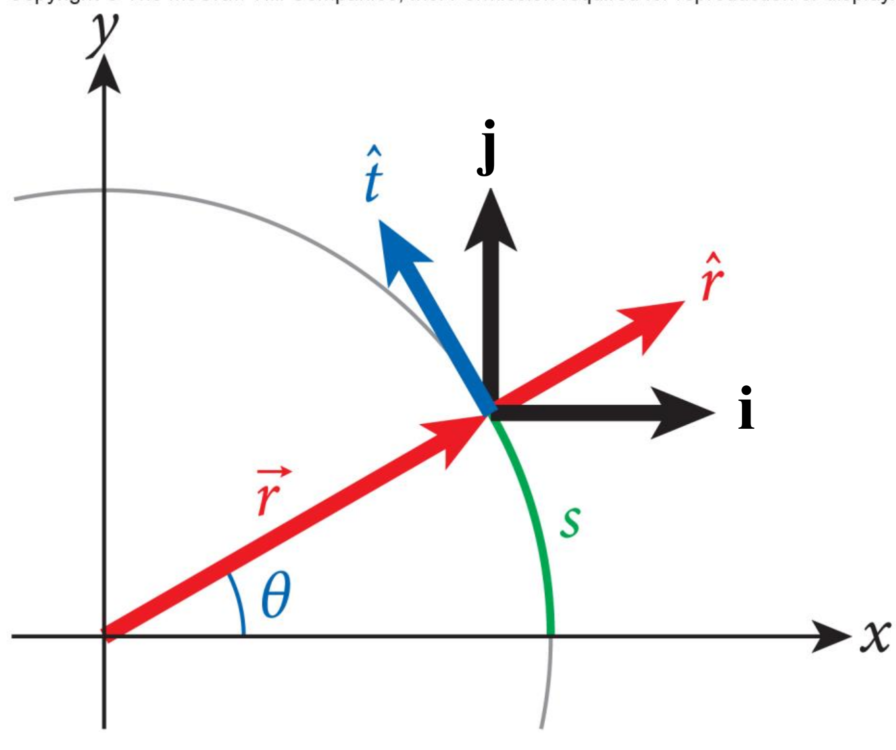
Polar Coordinates

- Using polar coordinates to describe circular motion reduces two-dimension motion on the circumference of a circle to one dimension motion involving θ.

- In the figure, two unit vectors are shown:

  radial unit vector $\hat{r}$

  tangential unit vector $\hat{t}$

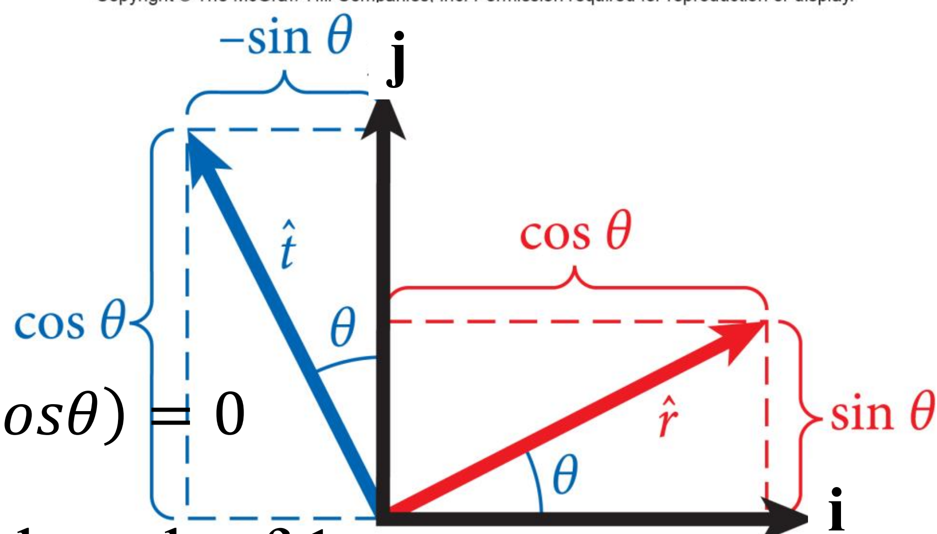


Polar Coordinates

- The radial and tangential unit vectors can be written as:

$$\hat{r} = \frac{x}{r}\mathbf{i} + \frac{y}{r}\mathbf{j} = (cos\theta)\mathbf{i} + (sin\theta)\mathbf{j} = (cos\theta, sin\theta)$$

$$\hat{t} = -\frac{y}{r}\mathbf{i} + \frac{x}{r}\mathbf{j} = (-sin\theta)\mathbf{i} + (cos\theta)\mathbf{j} = (-sin\theta, cos\theta)$$



- The radial and tangential unit vectors are perpendicular

$$\hat{r} \cdot \hat{t} = (cos\theta)(-sin\theta) + (sin\theta)(cos\theta) = 0$$

- These two unit vectors have a length of 1:

$$\hat{r} \bullet \hat{r} = (\cos\theta, \sin\theta) \bullet (\cos\theta, \sin\theta) = \cos^2\theta + \sin^2\theta = 1$$

$$\hat{t} \bullet \hat{t} = (-\sin\theta, \cos\theta) \bullet (-\sin\theta, \cos\theta) = \sin^2\theta + \cos^2\theta = 1$$