Rotation

What We Will Learn

- Kinetic energy due to object's rotation must be accounted for when considering energy conservation
- Parallel axis theorem
- Rotational and translation kinetic energies are related
- Newton's Second Law also applies to rotational motion
- Conservation of angular momentum

Kinetic Energy of Rotation

- We start with some familiar concepts that we introduced to describe circular motion:
  - Angular displacement

    θ

  - Angular velocity

    $$\omega = \frac{d\theta}{dt}$$

  - Angular acceleration

    $$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

Kinetic Energy of Rotation

- We related the variables describing circular motion to the variables describing linear motion:
  - Displacement, velocity, and acceleration:

$$s = r\theta$$

$$v = r\omega$$

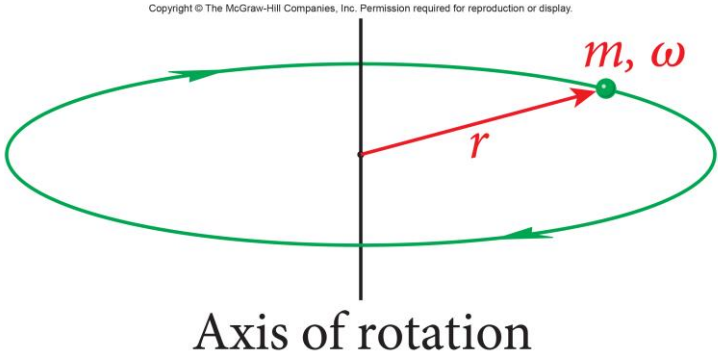
$$a_t = r\alpha \quad a_c = \omega^2 r \quad a = \sqrt{a_c^2 + a_t^2}$$

  - Kinetic energy for linear motion:

$$K = \tfrac{1}{2}mv^2$$

  - Kinetic energy for rotation (point particle):

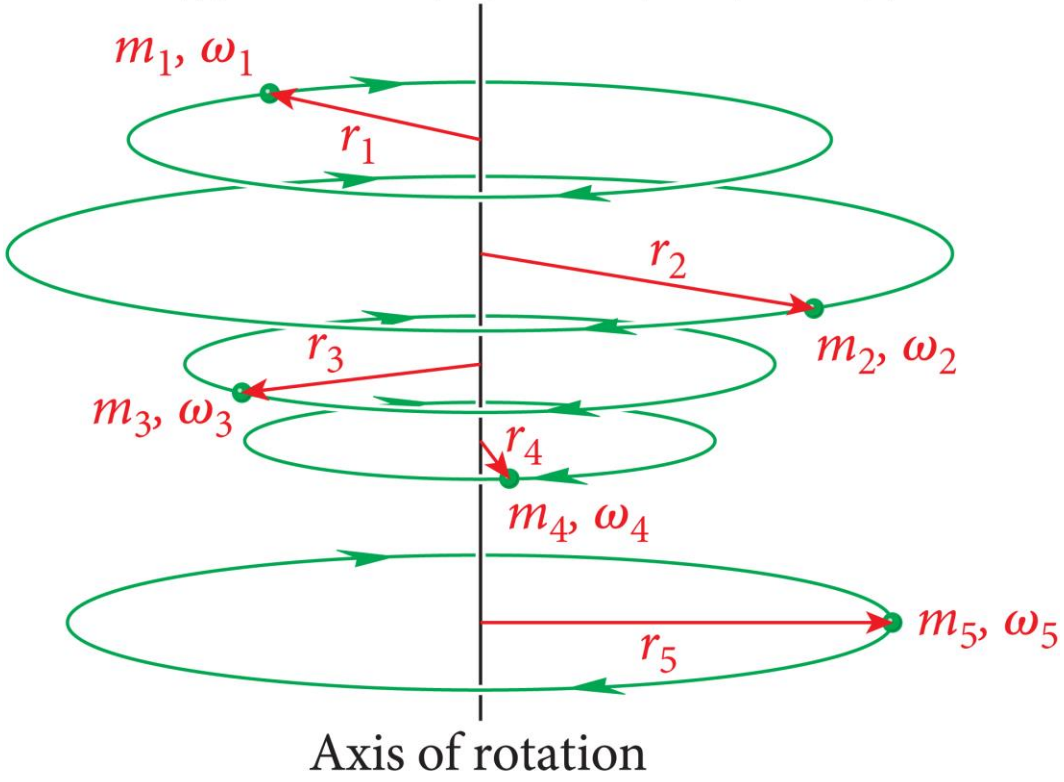
$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m(r\omega)^2 = \tfrac{1}{2}mr^2\omega^2$$



Kinetic Energy of Rotation

- Now let's discuss the kinetic energy of several rotating point particles:

$$K = \sum_{i=1}^{n} K_i = \tfrac{1}{2}\sum_{i=1}^{n} m_i v_i^2 = \tfrac{1}{2}\sum_{i=1}^{n} m_i r_i^2 \omega_i^2$$



Kinetic Energy of Rotation

- If we assume that these particles keep their distances fixed with respect to each other (solid object, all moving with the same angular velocity) we can write:

$$K = \tfrac{1}{2}\sum_{i=1}^{n} m_i r_i^2 \omega^2 = \tfrac{1}{2}\left(\sum_{i=1}^{n} m_i r_i^2\right)\omega^2 = \tfrac{1}{2} I\omega^2$$

- Where I is the moment of inertia given by:

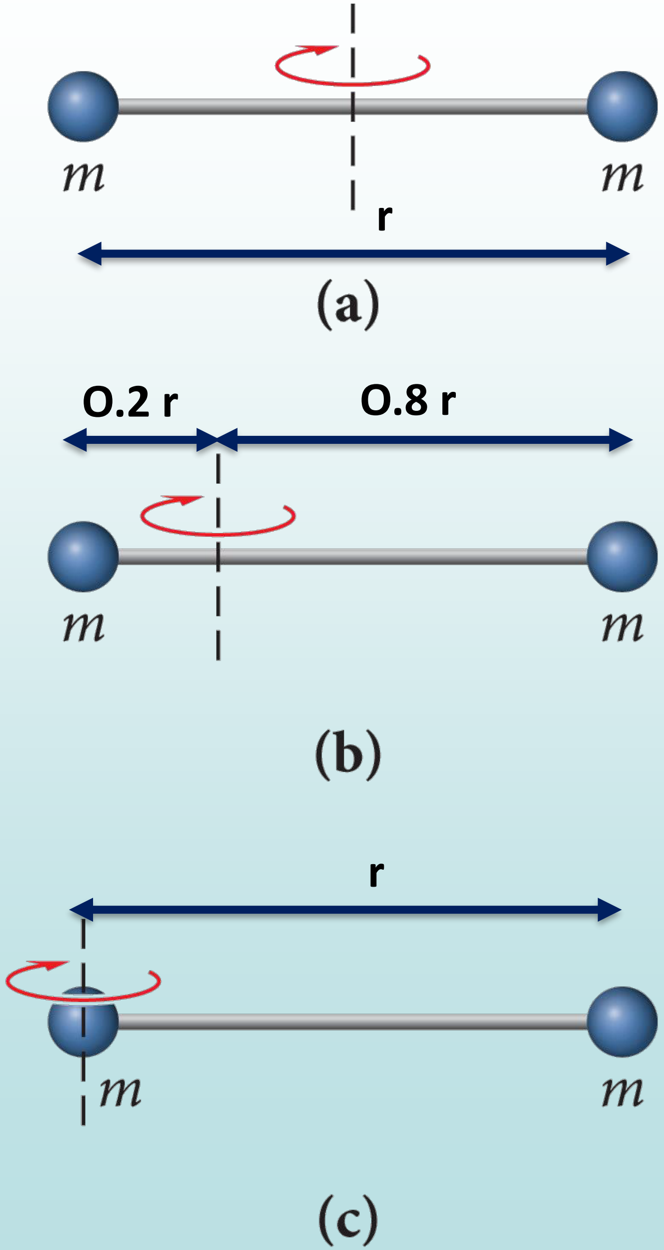
$$I = \sum_{i=1}^{n} m_i r_i^2$$

- We saw that all quantities associated with circular motion have equivalents in linear motion:

$$\text{Compare} \quad K_{\text{linear}} = \frac{1}{2} m v^2 \quad \Leftrightarrow \quad \boxed{K_{\text{rotation}} = \frac{1}{2} I\omega^2}$$

Kinetic Energy of Rotation

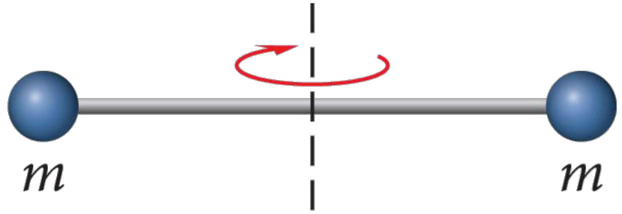
- Consider two masses each of mass *m*.
- They are connected by a thin, massless rod.
- In the three drawings, the two masses spin in a horizontal plane around a vertical axis represented by a dashed line.
- Which of the systems has the highest rotational inertia?



Kinetic Energy of Rotation

$$I = m\left(\frac{r}{2}\right)^2 + m\left(\frac{r}{2}\right)^2$$

$$I = \frac{1}{2}mr^2$$



(a)

$$I = m\left(\frac{r}{5}\right)^2 + m\left(\frac{4r}{5}\right)^2$$

$$I = \frac{17}{25}mr^2$$

(b)

$$I = m(0)^2 + m(r)^2$$

$$I = mr^2$$

(c)