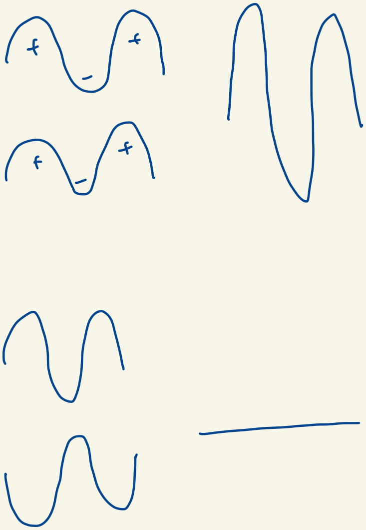
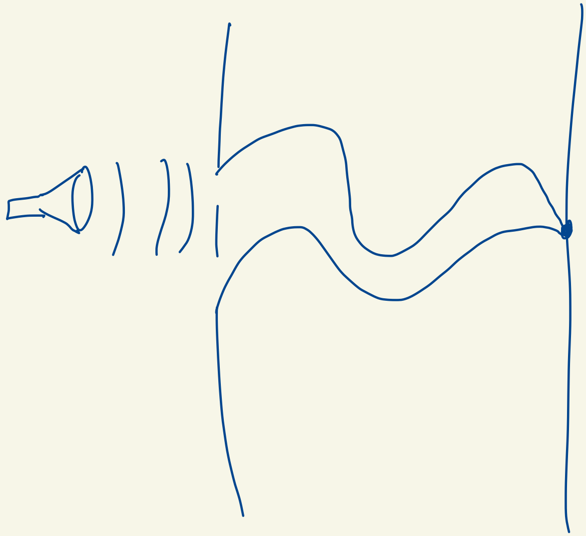




$$d \sin\theta_m = m\lambda$$

Constructive interference

m is the order of fringe

Central maximum, $m=0$

For 1st fring, $d\sin\theta_1 = \lambda$

$$\downarrow$$

$$d\theta_1 = \lambda$$

$$\theta_1 = \frac{\lambda}{d}$$

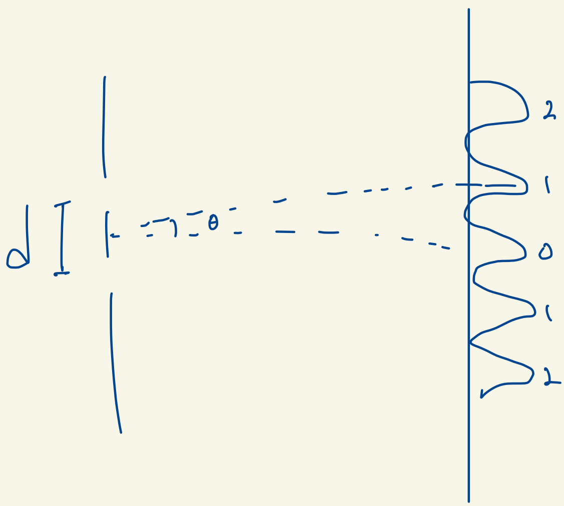
For small angle

$\sin\theta \approx \theta$

$\tan\theta \approx \theta$

$$\tan\theta \approx \theta_1 = \frac{y}{D} = \frac{\lambda}{d}$$

$$\lambda = \frac{dy}{D}$$

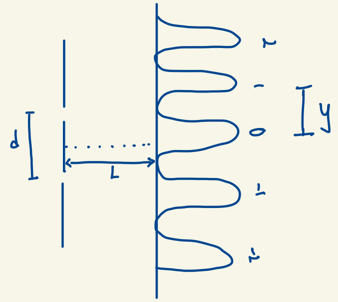


2nd order fringe

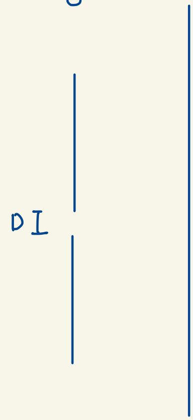
$m=2$

$$d\sin\theta_2 = 2\lambda$$

$$\sin\theta_2 = \frac{2\lambda}{d}$$



Single slit



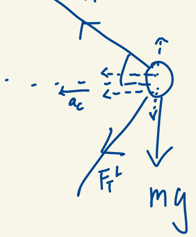
$d\sin\theta = m\lambda$ constructive

$d\sin\theta = (m+\frac{1}{2})\lambda$ destructive

$\tan\theta = \frac{y}{L}$

$\sin\theta = \frac{d/2}{L}$

$F_T^u = 35N$



Vertical

$F_T^u \sin\theta - F_T^L \sin\theta - mg = 0$

Horizontal

$F_{net} = F_T^u \cos\theta + F_T^L \cos\theta = ma_c = \frac{mv^2}{r}$

$v = \sqrt{\frac{F_{net} \cdot r}{m}}$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}M\cancel{R^2}\left(\frac{V^2}{\cancel{R^2}}\right)$$

$$4mgh = (2m + M)v^2$$

$$\omega = \frac{V}{R} = 15.7 \text{ rad/s}$$

$$v = \sqrt{\frac{4mgh}{2m+M}} = 3.13 \text{ m/s}$$

$$mg - F_T = ma$$

$$F_T = m(g-a)$$

$$I\alpha = \tau = F_T R = mR(g-a)$$

$$= \frac{1}{2}MR^2 \alpha$$

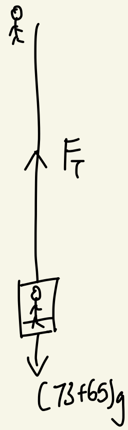
$$= \frac{1}{2}MR^2 \left(\frac{a}{R}\right)$$

$$2m(g-a) = Ma$$

$$2mg = (M+2m)a$$

$$a = \left(\frac{2m}{M+2m}\right)g$$

$$\mu = \frac{31}{61}\ kg/m$$



$$v = \sqrt{\frac{F_T}{\mu}}$$

$$= \sqrt{\frac{(73+65)g}{\mu}}$$

$$t = \frac{d}{v}$$

$$= \frac{61}{v}$$

$$= 1.31s$$

$$L = \frac{3\lambda}{2}$$

$$\lambda = \frac{2L}{3}$$

$$v = f\lambda = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{mg}{\mu}}$$

$$m =$$

2. Light with a wavelength of 650 nm passes through two narrow slits that are 0.05 mm apart. If the screen is 8.5m away from the slits, what is the distance between the third order bright fringe and the central fringe?

$\lambda = 650 \times 10^{-9}$ m $\qquad L = 8.5$

$Y_3 \qquad d = 0.05 \times 10^{-3}$ m

$d\sin\theta = m\lambda$ $\qquad \sin\theta \approx \theta$ $\qquad Y_m = L\tan\theta_m$

$\tan\theta \approx \theta$

$\theta = \frac{Y}{L}$

$d\theta = m\lambda$

$d\frac{Y}{L} = m\lambda$

$dY_m = Lm\lambda \longrightarrow$ only if θ is small

$$Y_3 = \frac{(8.5)(3)(650 \times 10^{-9})}{(0.05 \times 10^{-3})}$$

$= 0.3315$m

3. Light with a wavelength of 550 nm in air passes through two narrow slits in water (n = 1.33). The screen is 3.6m away from the slits. If the fourth order bright fringe is 4.5 mm away from the central fringe, what is the separation distance of the two slits?

$\lambda_0 = 550 \times 10^{-9}$ m $\qquad L = 3.6$m $\qquad Y_4 = 4.5$

$n = 1.33 \qquad m = 4$

$n\uparrow \quad \lambda\downarrow$

$Y_m d = Lm\lambda$

$\lambda_n = \frac{\lambda_0}{n}$

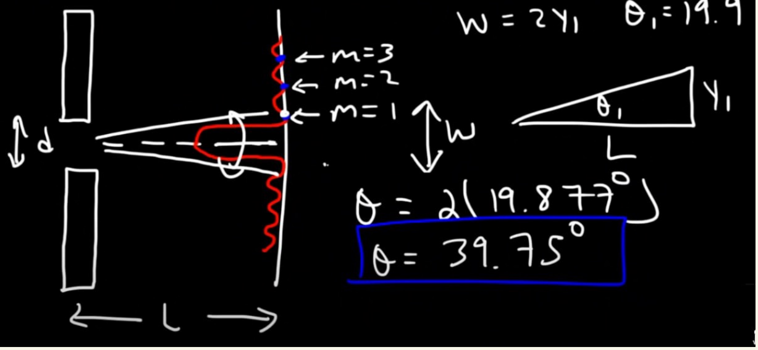
$= \frac{550\text{nm}}{1.33}$

$= 413.5$nm

Single slit

$d\downarrow \ \theta\uparrow \qquad d\uparrow \ \theta\downarrow$

1. Light with a wavelength of 680 nm passes through a single slit that is 2 x 10^-6 m wide. The screen is 1.4m away from the slit. What is the width of the central maximum in degrees and in meters?



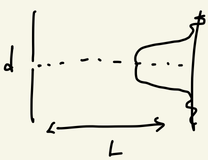
$$\theta = 2(19.877^\circ)$$

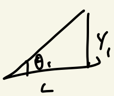
$$\boxed{\theta = 39.75^\circ}$$

2. Light with a wavelength of 570 nm shines on a single slit that is 7.5m away from the screen. The width of the central maximum is 3.2cm. How wide is the slit?

$\lambda = 570\times10^{-9} \qquad L = 7.5m$

$W = 0.032m$





$W = 2Y_1$

$Y_1 = 0.016m$

$$\theta_1 = \tan^{-1}\left(\frac{0.016}{7.5}\right)$$

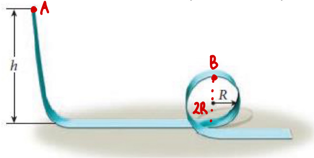
$$= 0.122^\circ$$

$$d\sin(0.122) = 1(570\times10^{-9})$$

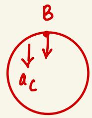
$$d = 2.68\times10^{-4}\,m$$

WARM UP

1. A point mass m starts sliding from a height h along the frictionless surface shown in the figure. What is the minimum value of h for the mass to complete the loop of radius R?



[Ans: 2.5R]



$$TE_A = TE_B$$

$$\cancel{K_A}^{0} + U_A = K_B + U_B$$

$$mgh = \frac{1}{2}mv^2 + mg2R$$

$$gh = \frac{1}{2}v^2 + g2R$$

$$v^2 = 2g(h-2R)$$

$a_c = \frac{v^2}{R}$ Barely touches the track, $F_N = 0$

$$mg = ma_c = \frac{mv^2}{R}$$

$$g = \frac{v^2}{R}$$

$$v^2 = gR$$

$$\cancel{g}R = 2\cancel{g}(h-2R)$$

$$h - 2R = \frac{R}{2}$$

$$h = \frac{5R}{2}$$

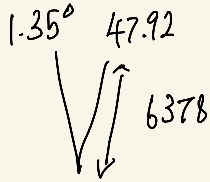
<u>Angular Coordinates and Angular Displacement</u>

2. Assuming a perfectly spherical Earth (R_E = 6378 km) how far apart, measured on the Earth's surface, are Singapore (1.35° N latitude), and Ulaanbaatar (47.92° N latitude)? The two cities lie on approximately the same longitude.

[Ans: 5181 km]

Change Deg to Rad

$$1^{\circ} \times \frac{\pi}{180} = \text{Rad}$$



$$S = r\theta$$

$$\theta = (47.92 - 1.35) \cdot \frac{\pi}{180}$$

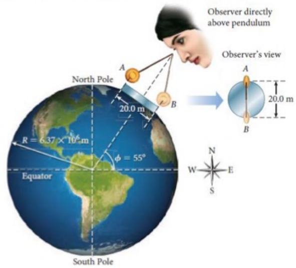
$$= 0.813 \text{ rad}$$

$$S = 6378 \times 0.813$$

$$= 5185 \text{ km}$$

Angular Velocity, Angular Frequency, and Period

3. Consider a large simple pendulum that is in Glasgow (latitude of 55.0° N) and is swinging in a North-South direction with points A and B being the northernmost and the southernmost points of the swing, respectively. A stationary (with respect to the fixed stars) observer is looking directly down on the pendulum at the moment shown in the figure.



As the Earth (R_E = 6378 km) is rotating once every 23 h and 56 min determine:

a) What are the directions (in terms of N, E, W, and S) and the magnitudes of the velocities of the surface of the Earth at points A and B as seen by the observer? Note: You will need to calculate answers to at least seven significant figures to see a difference.

[Ans: 0.00119 m/s, eastwards]

b) What is the angular speed with which the 20.0-m diameter circle under the pendulum appears to rotate?

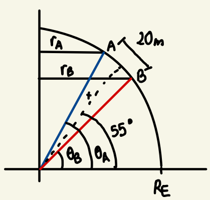
[Ans: 5.95×10^{-5} rad/s]

c) What is the period of this rotation?

[Ans: 29.3 hrs]

d) What would happen to a pendulum swinging at the Equator?

a)



$$\cos 55° = \frac{RE}{x}$$

$$x = \frac{6378}{\cos 55°}$$

$$= 11119.7$$

$$\theta_{AB} = \sin^{-1}\left(\frac{10}{11119.7}\right)$$

$$= 0.051526°$$

$$\cos\theta_A = \frac{RE}{A}$$

$$A = \frac{6378}{\cos(55 + 0.051526)}$$

$$= 11134$$

$$\sin(90 - 55.051526) = \frac{r_A}{A}$$

$$r_A = 6377.995$$

$$23h\,56min = 86160\,s$$

$$\omega = \frac{2\pi}{86160}$$

$$B = \frac{6378}{\cos(55 - 0.051526)}$$

$$= 11105.445$$

$$r_B = B\sin(90 - (55 + 2(0.051526)))$$

$$= 6353.449$$

$$V = r\omega$$

$$V_A = 6377.995\left(\frac{2\pi}{86160}\right)$$

$$= 0.4651128$$

$$V_B = 6353.449\left(\frac{2\pi}{86160}\right)$$

$$= 0.4633228$$

$$\Delta V = V_A - V_B$$

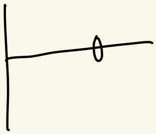
$$=$$

Angular and Centripetal Acceleration

4. A ring is fitted loosely (with no friction) around a long, smooth rod of length L = 0.50 m. The rod is fixed at one end, and the other end is spun in a horizontal circle at a constant angular velocity of ϖ = 4.0 rad/s. The ring has zero radial velocity at its initial position, a distance of r_0 = 0.30 m from the fixed end. Determine the radial velocity of the ring as it reaches the moving end of the rod.

[Ans: 1.60 m/s]

$\omega = 4$ rad/s



$$a_r(t) = \frac{dv(t)}{dt} = \omega^2 r(t)$$

	initial	final
$v(t)$	$v_i = 0$	$v_f = ?$
$r(t)$	$r_i = r_0$	$V_f = L$

$$\frac{dv}{dr} \cdot \frac{dr}{dt} = \omega^2 r$$

$$v \frac{dv}{dr} = \omega^2 r$$

$$v\,dv = \omega^2 r\,dr$$

$$\int_0^{v_f} v\,dv = \omega^2 \int_{r_0}^{L} r\,dr$$

Centripetal Force

5. A speedway turn, with radius of curvature R, is banked at an angle θ above the horizontal.
   a) What is the optimal speed at which to take the turn if the track's surface is iced over (that is, if there is no friction between the tires and the track)?

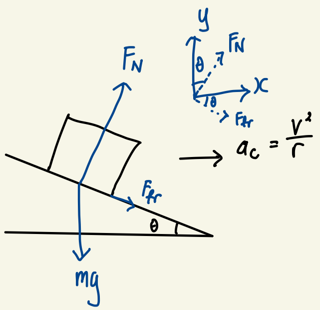
      [Ans: $\sqrt{gR\tan\theta}$]

   b) If the track surface is ice-free and there is a coefficient of friction μ_s between the tires and the track, what are the maximum and minimum speeds at which this turn can be taken?

      [Ans: $v_{MAX} = \sqrt{gR\frac{\sin\theta+\mu_s \sin\theta}{\cos\theta-\mu_s\cos\theta}}$; $v_{min} = \sqrt{gR\frac{\sin\theta-\mu_s \sin\theta}{\cos\theta+\mu_s\cos\theta}}$]

   c) Evaluate the results of parts (a) and (b) for R = 400. m, θ = 45.0°, and μ_s = 0.700.

      [Ans: v = 62.6 m/s; v_{MAX} =149 m/s v_{min} = 26.3 m/s]



x component

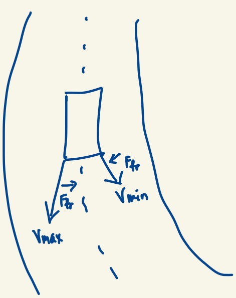
$$F_N \sin\theta + F_{fr}\cos\theta = ma_c = \frac{mv^2}{R} \quad -(1)$$

y component

$$F_N\cos\theta - F_{fr}\sin\theta - mg = 0 \quad -(2)$$

$$mg = F_N\cos\theta - F_{fr}\sin\theta$$

$$m = \frac{F_N\cos\theta - F_{fr}\sin\theta}{g} \quad -(3)$$



a) $F_{fr} = 0$,

$$v = \sqrt{\frac{R(F_N\sin\theta)}{m}}$$

$$= \sqrt{\frac{R(F_N\sin\theta)}{\frac{F_N\cos\theta}{g}}}$$

$$= \sqrt{gR\tan\theta}$$

b) $F_{fr} = \mu_s F_N$

$$V_{max} = \sqrt{\frac{R(F_N \sin\theta + \mu_s F_N \cos\theta)}{\frac{(F_N \cos\theta - \mu_s F_N \sin\theta)}{g}}}$$

$$V_{min} = \sqrt{\frac{R(F_N \sin\theta - \mu_s F_N \cos\theta)}{\frac{(F_N \cos\theta + \mu_s F_N \cos\theta)}{g}}}$$

$F_{fr} \leftarrow$

x component

$$F_N \sin\theta - F_{fr} \cos\theta = ma_c = \frac{mv^2}{R} \quad -1$$

y component

$$F_N \cos\theta + F_{fr} \sin\theta - mg = 0 \quad -2$$

$$mg = F_N \cos\theta - F_{fr} \sin\theta$$

$$m = \frac{F_N \cos\theta - F_{fr} \sin\theta}{g} \quad -3$$

Circular and Linear Motion

6. Consider a 53-cm-long engine blade rotating about its center at 3400 rpm.
    a) Calculate the linear speed of the tip of the blade.

    [Ans: 93.4 m/s]

    b) If safety regulations require that the blade be stoppable within 3.0 s, what minimum angular acceleration will accomplish this? Assume that the angular acceleration is constant.

    [Ans: −118.7 rad/s²]

a)

$r = \frac{53}{2}$

$v = r\omega$

RPM to rad/s

$3400 \times \frac{2\pi}{60} = 356.05$

$v = \left(\frac{0.53}{2}\right)(356.05)$

$\approx 94.4\ m/s$

b)

$\alpha = \frac{\omega}{t}$

$= \frac{\omega_f - \omega_i}{t} = \frac{0 - 356.05}{3}$

$= -118.7\ rad/s$

Additional Problems

7. A 1 dollar coin is sitting on the edge of a record player that is spinning at 33 rpm and has a diameter of 12 inches. What is the minimum coefficient of static friction between the coin and the surface of the disk to ensure that the penny does not fly off?

[Ans: 0.185]

$$\omega = 33 \times \frac{2\pi}{60} \qquad d = 12\text{inch} = 0.3048\text{m}$$

$$= 3.456$$

$$a_c = r\omega^2$$

$$= \left(\frac{0.3048}{2}\right)(3.456)^2$$

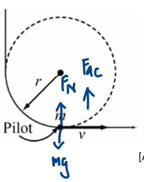
$$\mu_s F_N = m a_c$$

$$\mu_s (\cancel{m}g) = \cancel{m} a_c$$

$$\mu_s = \frac{a_c}{g}$$

$$= 0.185$$

8. A 80 kg pilot in an aircraft moving at a constant speed of 500 m/s pulls out of a vertical dive along an arc of a circle of radius 4 km. Find the centripetal acceleration and the centripetal force acting on the pilot and determine what is the pilot's apparent weight at the bottom of the dive?



[Ans: 62.5 m/s²; 5000 N; 5785 N]

$$v = r\omega \qquad F = m a_c$$

$$\omega = \frac{500}{4000} \qquad = 80 \times 62.5$$

$$= 0.125 \qquad = 5000\text{N}$$

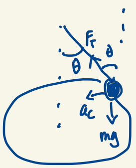
$$a_c = r\omega^2 \qquad F_N = F_{ac} + mg$$

$$= 4000(0.125)^2 \qquad = 5000 + 9.8(80)$$

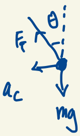
$$= 62.5\text{m/s} \qquad \approx 5784$$

10. You are on a merry-go-round platform holding a pendulum in your hand. The pendulum is 6 m from the rotation axis of the platform. The rotational speed of the platform is 0.02 rev/s. Determine the angle θ at which the pendulum is hanging.

[Ans: 0.553 deg]



$\frac{v^2}{r} = a_c$



$\omega = 0.02 \times 2\pi$

Horizontal component:

$$F_T \sin\theta = ma_c = \frac{mv^2}{R} = m\omega^2 R \quad -①$$

Vertical

$$F_T \cos\theta = mg \quad -②$$

$$\frac{①}{②}: \quad \frac{\cancel{F_T} \sin\theta}{\cancel{F_T} \cos\theta} = \frac{\cancel{m}\omega^2 R}{\cancel{m}g}$$

$$\tan\theta = \frac{\omega^2 R}{g}$$

$$\theta = \tan^{-1}\left[\frac{(0.02 \times 2\pi)^2 (6)}{9.8}\right]$$

$$\approx 0.554 \text{ deg}$$

11. A truck has tires with a diameter of 1.1 m and is traveling at 35.8 m/s. After the brakes are applied, the truck slows uniformly and is brought to rest after the tires rotate through 40.2 turns.

a) What is the angular speed of the tires as the braking manoeuvre starts?

[Ans: $65.1\ /s$]

b) What is the angular acceleration of the tires during the braking manoeuvre?

[Ans: $= -8.39\ /s^2$]

c) What distance does the truck travel before coming to rest?

[Ans: 138.9 m]

a) $\omega = \frac{v}{r} = \frac{35.8}{0.55} = 65.1\ \text{rad/s}$

b) $\Delta\theta = 40.2 \times 2\pi$

$= 80.4\pi$

$\omega^2 = \omega_0^2 + 2\alpha[\Delta\theta]$

$\alpha = \frac{\omega^2 - \omega_0^2}{2(\Delta\theta)}$

$= \frac{0 - (65.1)^2}{2(80.4\pi)}$

$= -8.39\ m/s^2$

c) $S = r\theta$

$= 0.55 \times 80.4\pi$

$= 138.9\ m$

12. A ball of mass 1 kg is attached to a 1 m long string and is whirled in a vertical circle at a constant speed of 10.0 m/s.

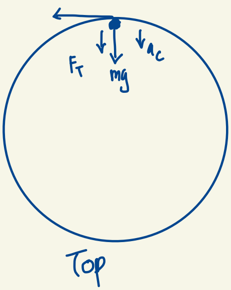
a) Determine the tension in the string when the ball is at the top of the circle.

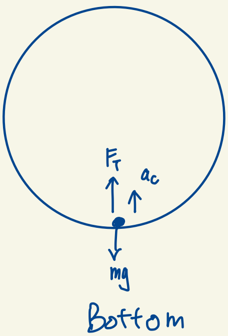
[Ans: 90.2 N]

b) Determine the tension in the string when the ball is at the bottom of the circle.

[Ans: 109.8 N]

c) What can you say about the tension in the string at some point other than the top or bottom





a)

$$F_T + mg = ma_c = m\frac{v^2}{R}$$

$$F_T = 90.2\,N$$

b)

$$F_T - mg = ma_c$$

$$F_T = \frac{mv^2}{R} + mg$$

$$= 109.8\,N$$

WARM UP

1. A circular object begins from rest and rolls without slipping down an incline, through a vertical distance of 4.0 m. When the object reaches the bottom, its translational velocity is 7.0 m/s. Determine the constant c relating the moment of inertia to the mass and radius of this object.

[Ans: 0.6]

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$I = cmr^2$$

$$\omega^2 = \frac{v^2}{r^2}$$

$$\cancel{m}gh = \frac{1}{2}(1+c)\cancel{m}v^2$$

$$\frac{2gh}{v^2} = 1+c$$

$$c = \frac{2gh}{v^2} - 1$$

$$= 0.6$$

Kinetic Energy of Rotation

2. A uniform solid sphere of radius R, mass M, and moment of inertia $I = \frac{2}{5}MR^2$ is rolling without slipping along a horizontal surface. Find the fraction of the sphere's total kinetic energy that is attributable to rotation.

[Ans: 2/7]

$$\frac{K_{rot}}{K_{total}} = \frac{\frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v^2}{R^2}\right)}{\frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v^2}{R^2}\right) + \frac{1}{2}mv^2}$$

$$= \frac{\frac{1}{5}Mv^2}{\frac{7}{10}Mv^2}$$

$$= 2/7$$

Moments of Inertia

3. Determine the total moment of inertia for three people weighing 60 kg, 45 kg and 80 kg sitting at different points along the edge of a rotating merry-go-round, which has a ~~radius~~ of 12 m.

diameter [Ans: 6660 kgm^2]

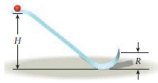
$$I = MR^2$$

$$= (60+45+80)(6)^2$$

$$= 6660 \text{ kgm}^2$$

Rolling without Slipping

4. An object of mass m and radius r has a moment of inertia given by I = cmr^2. The object rolls without slipping along a track that ends with a ramp of height R = 2.5 m that launches the object vertically. If the object starts from a height H = 6 m. To what maximum height will it rise after leaving the ramp if c = 0.40?



[Ans: 5 m]

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}cmv^2 + mgR$$

$$mg(h-R) = \frac{1}{2}m(1+C)v^2$$

$$v^2 = \frac{2g(h-R)}{(1+C)}$$

$$= 49$$

$$v^2 = v_0^2 + 2a(x-x_0)$$

$$\Delta x = \frac{0-v^2}{2(-9.8)}$$

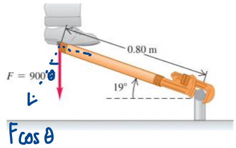
$$= 2.5$$

max height $= 2.5 + R$

$= 5m$

Torque

5. To loosen a pipe fitting, a plumber stands on the end of his wrench, applying his 900 N weight at a point 0.80 m from the centre of the fitting as shown below. The wrench makes an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the centre of the fitting.

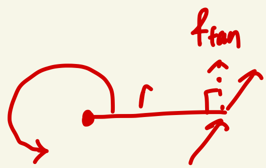


$\tau = rf$

$\tau = F\cos\theta \, r$

$= 900\cos 19° \times 0.8$

$\approx 680 \text{ Nm}$

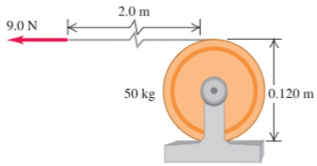


$\tau = r f_{tan}$

[Ans: 680 Nm]

Newton's Second Law for Rotation

6. A block of mass m is tied to a massless cable around a solid cylinder with mass 50 kg and diameter 12 cm The cylinder rotates with negligible friction about a stationary horizontal axis. You pull the cable with constant force of 9 N for a distance of 2 m. Find the acceleration and velocity of the cable.



[Ans: 0.36 m/s^2; 1.2 m/s]

$\tau = I\alpha$

$I_{cylinder} = \frac{1}{2}MR^2$

$F \times R = \frac{1}{2}MR^2 \cdot \alpha$

$\alpha = 6$

$a_{tan} = r\alpha$

$= (0.06)(6)$

$= 0.36 m/s^2$

$v^2 = v_0^2 + 2a(x - x_0)$

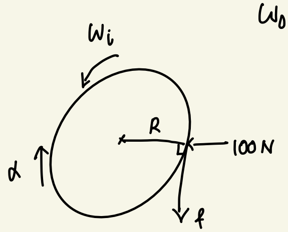
$v^2 = 2(0.36)(2)$

$v = 1.2 m/s$

Work Done by Torque

7. A flywheel is a solid homogeneous metal disk of mass M = 120 kg and radius R = 80.0 cm. The engine rotates the wheel at 500 rpm. In an emergency, to bring the engine to a stop, the flywheel is disengaged from the engine and a brake pad is applied at the edge to provide a radially inward force F = 100 N. If the coefficient of kinetic friction between the pad and the flywheel is μ_k= 0.2, how many revolutions does the flywheel make before coming to rest? How long does it take for the flywheel to come to rest? Calculate the work done by the torque during this time.

[Ans: 524 revs; 126 s; − 5.26×10^4 J]



$\omega_0 = 500 \times \frac{2\pi}{60}$

$= 52.36$

$f = \mu_k F$

$\tau = fR = I\alpha$

$I = \frac{1}{2}MR^2$

$\omega_f^2 = \omega_i^2 - 2\alpha\Delta\theta$

$\alpha = \frac{fR}{I} = \frac{\mu_k FR}{\frac{1}{2}MR^2}$

$\omega_f = \omega_i + \alpha t$

$W = \Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$

$= 0.417$

$0 = \omega_i^2 - 2\alpha\Delta\theta$

$\Delta\theta = \frac{\omega_i^2}{2\alpha}$

$= \frac{(52.36)^2}{2(0.417)}$

$= 3287$

$\therefore \quad n = \frac{\Delta\theta}{2\pi}$

$\approx 524\,\text{rev}$

$$t = \frac{\omega_i}{\alpha} = 126\,s$$

$$W_d = 0 - \frac{1}{2} I \omega_i^2$$

$$= -\frac{1}{2}\left(\frac{1}{2}MR^2\right)(\omega_i)^2$$

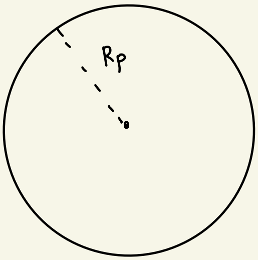
$$\approx -5.26 \times 10^4\ J$$

Angular Momentum

8. A circular platform of radius R_p = 4 m and mass M_p = 400 kg rotates on frictionless air bearings about its vertical axis at 6 rpm. An 80 kg man standing at the very center of the platform starts walking (at t = 0) radially outwards at a speed of 0.5 m/s with respect to the platform. Approximating the man to a vertical cylinder of radius R_m = 0.2 m, determine an equation (specific expression) for the angular velocity of the platform as a function of time. Further determine the angular velocity when the man has reached the outer edge of the platform.

[Ans: 0.449 rad/s]

$L = I\omega$



conservation of angular momentum

$L_i = L_f$

$I_p = \frac{1}{2} M_p R_p^2$ $\quad I_m = \frac{1}{2} m R_m^2$

$d = vt$ ↓ distance travelled by man

Using parallel axis theorem,

$I_m' = I_m + md^2$

$= I_m + m(vt)^2$

$I_{initial} = I_p + I_m$

$I_{final} = I_p + I_m'$

$\therefore$ sub into $L_i = L_f$

$$\left(\frac{1}{2} M_p R_p^2 + \frac{1}{2} m R_m^2\right)\omega_i = \omega_f\left(\frac{1}{2} M_p R_p^2 + I_m + m(vt)^2\right)$$

At Edge, $vt = R_p$

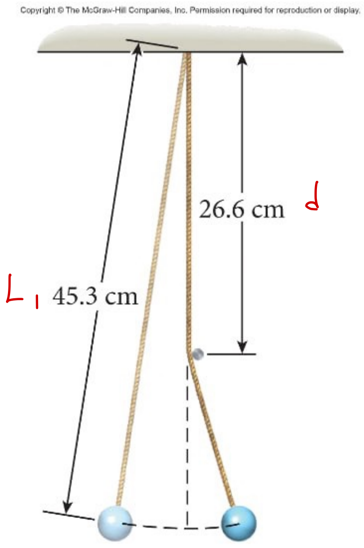
$$\therefore \quad \omega_f = \frac{\left(\frac{1}{2}M_p R_p^2 + \frac{1}{2} m R_m^2\right)}{\left(\frac{1}{2}M_p R_p^2 + \frac{1}{2} m R_m^2 + m R_p^2\right)} \omega_i$$

$$= \frac{\left(\frac{1}{2} \cdot 400 \cdot 4^2 + \frac{1}{2} \cdot 80 \cdot 0.2^2\right)}{\left(\frac{1}{2} \cdot 400 \cdot 4^2 + \frac{1}{2} \cdot 80 \cdot 0.2^2 + 80 \cdot 4^2\right)} \left(6 \times \frac{2\pi}{60}\right)$$

$$= 0.449 \text{ rad/s}$$

A pendulum of length 45.3 cm is hanging from the ceiling and is restricted in its motion by a peg that is sticking out of the wall 26.6 cm directly below its pivot point. What is its period of oscillation?

$\omega_0 = \sqrt{g/L}$



[Ans: 1.11 s]

$L_1 = 45.3 cm$ $\quad L_2 = 0.187 m$

$= 0.453 m$

$$\frac{2\pi}{T} = \sqrt{\frac{g}{L}}$$

$$T = \frac{T_1 + T_2}{2} = \frac{\dfrac{2\pi}{\sqrt{\frac{9.8}{0.453}}} + \dfrac{2\pi}{\sqrt{\frac{9.8}{0.187}}}}{2}$$

$= 1.11 s$

While you are operating a Rotor (a large, vertical, rotating cylinder found in amusement parks), you spot a passenger in acute distress and decrease the angular velocity of the cylinder from 3.40 rad/s to 2.00 rad/s in 20.0 rev, at constant angular acceleration. (The passenger is obviously more of a "translation person" than a "rotation person.") (a) What is the constant angular acceleration during this decrease in angular speed? (b) How much time did the speed decrease take?

$$\omega^2 = \omega_0^2 + 2\alpha[\Delta\theta]$$

$$2^2 = 3.4^2 + 2\alpha[20 \times 2\pi]$$

$$\alpha = \frac{2^2 - 3.4^2}{2[40\pi]}$$

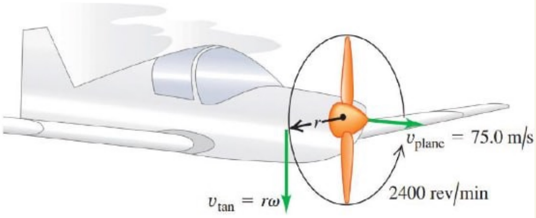
$$= -0.0301 \text{ rad/s}^2$$

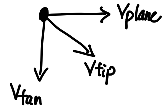
$$\omega = \omega_0 + \alpha t$$

$$2 = 3.4 + (-0.0301)t$$

$$t = 46.5 \text{ s}$$

You are designing an airplane propeller that is to turn at 2400 rpm. The forward airspeed of the plane is to be 75.0 m/s, and the speed of the propeller tips through the air must not exceed 270 m/s. (This is about 80% of the speed of sound in air. If the propeller tips moved faster, they would produce a lot of noise.) (a) What is the maximum possible propeller radius? (b) With this radius, what is the acceleration of the propeller tip?





$$\omega = 2400 \times \frac{2\pi}{60}$$

$$= 80\pi$$

$$V_{tip}^2 = V_{tan}^2 + V_{plane}^2$$

$$V_{tan} = \sqrt{V_{tip}^2 - V_{plane}^2}$$

$$r\omega = \sqrt{270^2 - 75^2}$$

$$r = \frac{\sqrt{270^2 - 75^2}}{80\pi}$$

$$= 1.03\text{m}$$

b) $a_{tan} = 0$

$$a_{rad} = \omega^2 r$$

$$= (80\pi)^2 (1.03)$$

$$= 65060 \text{ rad/s}$$

$$\approx 6.5 \times 10^4 \text{ m/s}$$

In 1985,Test Devices, Inc.(www.testdevices.com) was spin testing a sample of a solid steel rotor (a disk) of mass M = 272 kg and radius R = 38.0 cm. When the sample reached an angular speed v of 14 000 rev/min, the test engineers heard a dull thump from the test system, which was located one floor down and one room over from them. Investigating, they found that lead bricks had been thrown out in the hallway leading to the test room, a door to the room had been hurled into the adjacent parking lot, one lead brick had shot from the test site through the wall of a neighbor's kitchen, the structural beams of the test building had been damaged, the concrete floor beneath the spin chamber had been shoved downward by about 0.5 cm, and the 900 kg lid had been blown upward through the ceiling and had then crashed back onto the test equipment. The exploding pieces had not penetrated the room of the test engineers only by luck. How much energy was released in the explosion of the rotor?

$$\omega = 14000 \times \frac{2\pi}{60}$$

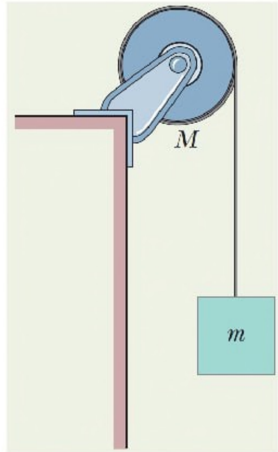
$$= 1466$$

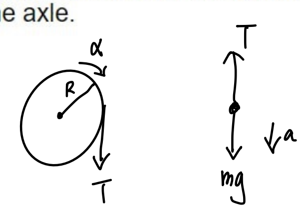
$$\frac{1}{2} I \omega^2 = \frac{1}{2}\left(\frac{1}{2} M R^2\right)\omega^2$$

$$= \frac{1}{4} M R^2 \omega^2 = \frac{1}{4}(272)(0.38)^2(1466)^2$$

$$= 21.1 \times 10^6 \text{ J}$$

A uniform disk, with mass $M = 2.5$ kg and radius $R = 20$ cm, mounted on a fixed horizontal axle. A block with mass $m = 1.2$ kg hangs from a massless cord that is wrapped around the rim of the disk. Find the acceleration of the falling block, the angular acceleration of the disk, and the tension in the cord. The cord does not slip, and there is no friction at the axle.





$-TR = -I\alpha$ $\quad$ $T - mg = -ma$ - ② $\quad$ $a = \alpha R$

$TR = I\alpha$ - ① $\quad$ $\alpha = \frac{a}{R}$

$$TR = \frac{1}{2}MR^2 \cdot \frac{a}{R}$$

$$TR = \frac{1}{2}MRa$$

$$T = \frac{1}{2}Ma$$

sub T into ② :

$$\frac{1}{2}Ma - mg = -ma$$

$$Ma + 2ma = 2mg$$

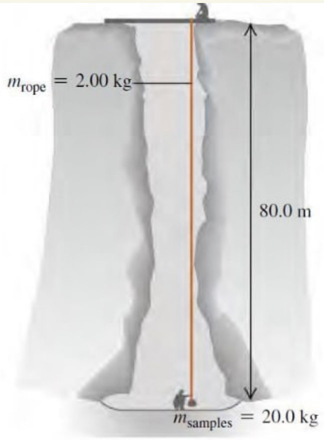
$$a = \frac{2mg}{M+2m} = \underline{\underline{4.8 m/s^2}}$$

$$\alpha = \frac{4.8}{0.2} = \underline{\underline{24 rad/s^2}}$$

$$T = \frac{1}{2}(2.5)(4.8)$$

$$= \underline{\underline{6N}}$$

One end of a 2.00-kg rope is tied to a support at the top of a mine shaft 80.0 m deep (**Fig. 15.14**). The rope is stretched taut by a 20.0-kg box of rocks attached at the bottom. (a) A geologist at the bottom of the shaft signals to a colleague at the top by jerking the rope sideways. What is the speed of a transverse wave on the rope? (b) If a point on the rope is in transverse SHM with $f = 2.00$ Hz, how many cycles of the wave are there in the rope's length?



a) $v = \sqrt{\frac{F_T}{\mu}}$ $\mu = \frac{m_{rope}}{l}$

$= \frac{2}{80} = 0.025$

$= \sqrt{\frac{20 \times 9.8}{0.025}}$

$\approx 88.5 m/s$

b) $v = f\lambda$

$88.5 = 2\lambda$

$\lambda = 44.25 m$

A siren on a tall pole radiates sound waves uniformly in all directions. At a distance of 15.0 m from the siren, the sound intensity is 0.250 W/m^2. At what distance is the intensity 0.010 W/m^2?

At 15m, $I = 0.25 W/m^2$

$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$

$r_2 = r_1 \sqrt{\frac{I_1}{I_2}}$

$= 75m$

A mechanical driver is used to set up a standing wave on an elastic string. Tension is put on the string by running it over a frictionless pulley and hanging a metal block from it. The length of string from the pulley to the driver is 1.25 m. The linear mass density of the string is 5.00 g/m. The frequency of the driver is 45.0 Hz. What is the mass of the metal block?

0.005 kg/m

[Ans: 0.72 kg]

$$\lambda = \frac{2l}{3}$$

$$= 0.833$$

$$v = \sqrt{\frac{F_T}{\mu}}$$

$$F_T = v^2 \mu$$

$$= (0.833 \times 45)^2 (0.005)$$

$$= 7.03$$

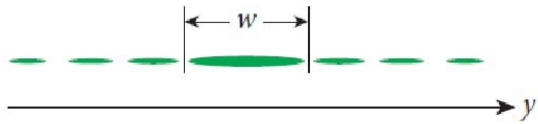
$$F_T = mg$$

$$m = \frac{F_T}{g}$$

$$= \frac{7.03}{9.8}$$

$$\approx 0.72\text{ kg}$$

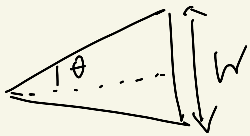
The single-slit diffraction pattern shown in Figure 34.29 was produced with light of wavelength λ =510.0 nm. The screen on which the pattern was projected was located a distance d =1.40 m from the slit. The slit had a width of D = 7.00 mm.



$\lambda = 510 \times 10^{-9}$ m $\qquad$ $d = 1.4$ m

$D = 7 \times 10^{-3}$ m

[Ans: 0.2 mm]



$$D \sin\theta = m\lambda$$

$$\theta = \sin^{-1}\left(\frac{(1)(510 \times 10^{-9})}{7 \times 10^{-3}}\right)$$

$$= 0.00417^\circ$$

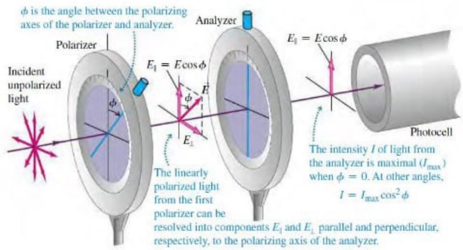
$$\frac{W}{2} = d\tan\theta$$

$$W = 2d\tan\theta$$

$$= 0.0002$$

$$\approx 0.2\,\text{mm}$$

In Fig. 33.24 the incident unpolarized light has intensity I_0. Find the intensities transmitted by the first and second polarizers if the angle between the axes of the two filters is 30°.



$$I_2 = \frac{I_0}{2}\cos^2(30)$$

$$= \frac{3I_0}{8}$$

In an attempt to get your name in *Guinness World Records*, you build a bass viol with strings of length 5.00 m between fixed points. One string, with linear mass density 40.0 g/m, is tuned to a 20.0-Hz fundamental frequency (the lowest frequency that the human ear can hear). Calculate (a) the tension of this string, (b) the frequency and wavelength on the string of the second harmonic, and (c) the frequency and wavelength on the string of the second overtone.

$l = 5m$

$\mu = 0.04 kg/m$ $\quad f = 20Hz$ $\quad$ fundamental = 1

$$\lambda_1 = \frac{2l}{1}$$

$$= 10m$$

$$V = \sqrt{\frac{F_T}{\mu}}, \quad F_T = v^2\mu$$

$$= (10\times20)^2(0.04)$$

$$= 1600N$$

b) 2nd harmonic, $\quad v = 200\ m/s$

$$\lambda_2 = \frac{2l}{2}$$

$$= l = 5\ m$$

$$f = \frac{v}{\lambda} = 40\ Hz$$

c) 3rd harmonic,

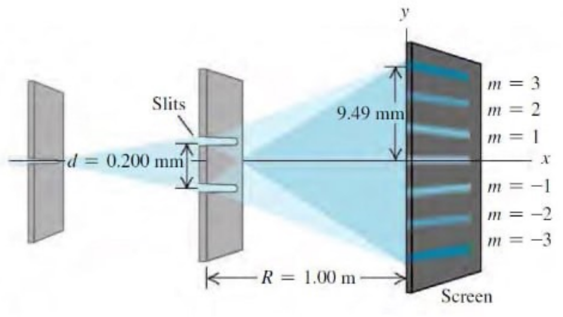
$$\lambda_3 = \frac{2l}{3}$$

$$= \frac{10}{3} = 3.33\ m$$

$$f = \frac{200}{\frac{10}{3}} = 60\ Hz$$

Figure 35.7 shows a two-slit interference experiment in which the slits are 0.200 mm apart and the screen is 1.00 m from the slits. The $m = 3$ bright fringe in the figure is 9.49 mm from the central fringe. Find the wavelength of the light.

35.7 Using a two-slit interference experiment to measure the wavelength of light.



$$\theta = \tan^{-1}\left(\frac{9.49 \times 10^{-3}}{1}\right)$$

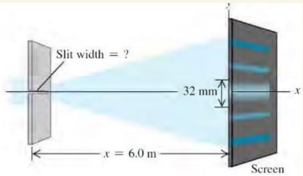
$$= 0.5437^\circ$$

$$(0.2 \times 10^{-3}) \sin\theta = 3\lambda$$

$$\lambda = \frac{(0.2 \times 10^{-3}) \sin\theta}{3}$$

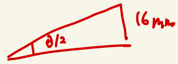
$$= 0.6 \times 10^{-6}\ m$$

You pass 633-nm laser light through a narrow slit and observe the diffraction pattern on a screen 6.0 m away. The distance on the screen between the centers of the first minima on either side of the central bright fringe is 32 mm (**Fig. 36.7**). How wide is the slit?



$D\sin\theta = m\lambda$, $m=1$

find θ



$D\sin\theta = \lambda$

$$\theta = \tan^{-1}\left(\frac{16\times10^{-3}}{6}\right)$$

$$D = \frac{\lambda}{\sin\theta}$$

$= 0.153°$

$$D = \frac{633\times10^{-9}}{\sin(0.153°)}$$

$= 2.4\times10^{-4}$ m