Problem 1 (17 points)

a) **(2 points)**

$$\rho = \frac{Q}{V} = \frac{qL}{R^2\pi L} = \frac{q}{R^2\pi}.$$

b) **(5 points)**

$$\int_V \rho(\vec{r})\,\mathrm{d}V = \int_{\partial V} \vec{D}(\vec{r})\,\mathrm{d}\vec{a} = Q_{\text{ein}}(r)$$

Cylindric symmetry: $\vec{D}(\vec{r}) = D_r(r) \cdot \vec{e}_r$.

region 1: $(0 \leq r < R)$

$$Q_{\text{ein}}(r) = \int_0^{2\pi}\int_0^L\int_0^r \rho r'\,\mathrm{d}r'\mathrm{d}z\mathrm{d}\varphi = 2\pi L\rho\frac{r^2}{2}$$

$$Q_{\text{ein}}(r) = \int_0^{2\pi}\int_0^L D_r(r)\vec{e}_r r\,\mathrm{d}z\mathrm{d}\varphi\vec{e}_r = 2\pi L r D_r(r)$$

$$\vec{D}(r) = \frac{qr}{2\pi R^2}\vec{e}_r$$

region 2: $(r \geq R)$

analogue to region 1

$$Q_{\text{ein}}(r) = 2\pi L r D_r(r) = Q$$

$$\vec{D}(r) = \frac{Q}{2\pi L r}\vec{e}_r$$

c) **(2 points)**

applying $D_r(r) = \varepsilon_1 E_r(r)$:

region 1: $(0 \leq r < R)$

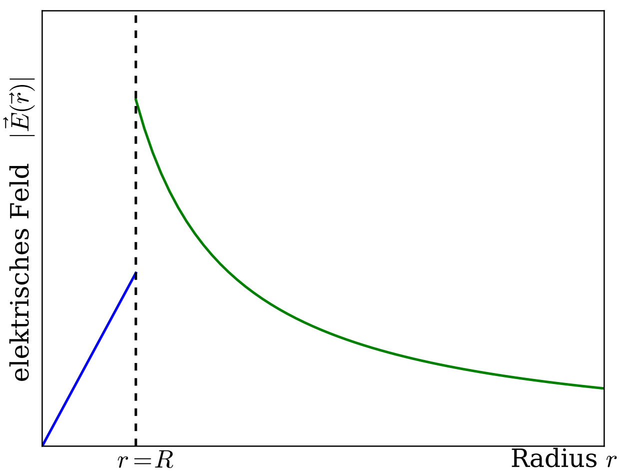
$$\vec{E}(\vec{r}) = \frac{qr}{2\pi R^2\varepsilon_1} \cdot \vec{e}_r.$$

region 2: $(r \geq R)$

analogue to region 1

$$\vec{E}(\vec{r}) = \frac{q}{2\pi\varepsilon_0 r} \cdot \vec{e}_r.$$

d) **(2 points)**



Note: $|\vec{E}(\vec{r})|$ has a discontinuity at $r = R$, since $\varepsilon_1 > \varepsilon_0$.

d) **(4 points)**
Cylindric symmetry: $\Phi(\vec{r}) = \Phi(r)$. Hence:

$$\Phi(r) = \Phi(\vec{r_0}) - \int_{\vec{r_0}}^{\vec{r}} \vec{E}(\vec{s})\ \mathrm{d}\vec{s} = \Phi(R) - \int_R^r E_r(s)\ \mathrm{d}s.$$

applying $\vec{E}(\vec{s}) = E_r(s)\vec{e}_r$ und $\mathrm{d}\vec{s} = \mathrm{d}s\ \vec{e}_r$. reference point of potential: $r_0 = R$.
region 1: $(0 \leq r < R)$

$$\Phi(r) = -\frac{q}{2\pi\varepsilon_1 R^2}\int_R^r s\ \mathrm{d}s = \frac{q}{4\pi\varepsilon_1 R^2}(R^2 - r^2) = \frac{q}{4\pi\varepsilon_1}\left(1 - \frac{r^2}{R^2}\right).$$

region 2: $(r \geq R)$

$$\Phi(r) = -\frac{q}{2\pi\varepsilon_0}\int_R^r \frac{1}{s}\ \mathrm{d}s = \frac{q}{2\pi\varepsilon_0}\left(\ln(R) - \ln(r)\right) = \frac{q}{2\pi\varepsilon_0}\ln\left(\frac{R}{r}\right).$$

e) **(2 points)**

$$U_{12} = \Phi\left(\vec{r}_1\right) - \Phi\left(\vec{r}_2\right) = \Phi(R) - \Phi(2R) = -\frac{q}{2\pi\varepsilon_0}\ln\left(\frac{1}{2}\right) = \frac{q}{2\pi\varepsilon_0}\ln(2).$$

Problem 2 (14 points)

a) (8 points)

Ampère's Circuital Law:

$$\int_{\partial A} \vec{H}(\vec{r})\ \mathrm{d}\vec{s} = \int_A \vec{j}(\vec{r})\ \mathrm{d}\vec{a} = I_{\mathrm{encl}}(A)$$

As the coaxial tube is cylindrically symmetrical, the magnetic field reads

$$\vec{H}(\vec{r}) = H_\varphi(r)\vec{e}_\varphi$$

and the path integral is therefore

$$\int_{\partial A} \vec{H}(\vec{r})\ \mathrm{d}\vec{r} = \int_0^{2\pi} H_\varphi(r)\vec{e}_\varphi\vec{e}_\varphi r\ \mathrm{d}\varphi = 2\pi r H_\varphi(r)$$

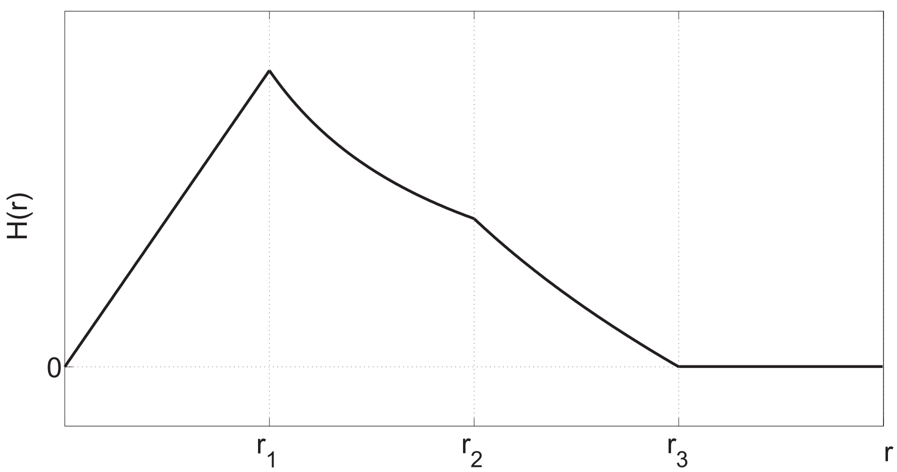
and

$$H_\varphi(r) = \frac{1}{r}\int_0^r j(r')r'\mathrm{d}r'.$$

Consequently, the result in the different is regions is

$$\vec{H}(r) = \begin{cases} \frac{I}{2\pi r_1^2}\cdot r \cdot \vec{e}_\varphi & \text{for} \quad 0 \le r \le r_1 \\ \frac{I}{2\pi r}\cdot \vec{e}_\varphi & \text{for} \quad r_1 < r < r_2 \\ \frac{I}{2\pi r}\cdot\left(1 - \frac{r^2 - r_2^2}{r_3^2 - r_2^2}\right)\cdot \vec{e}_\varphi & \text{for} \quad r_2 \le r \le r_3 \\ 0 & \text{for} \quad r > r_3. \end{cases}$$

b) (4 points)
Drawing



c) (2 points)
Generally: $\vec{B}(r) = \mu\vec{H}(r) = \mu_0\mu_r\vec{H}(r)$.
Here: $\mu_r = 3$

- the magnetic field $\vec{H}(r)$ remains unchanged, since it is not dependent on μ (see subtask a))
- the magnetic flux density $\vec{B}(r)$ will increase by a factor 3 in the region $r_1 < r < r_2$.