ENG1004 Engineering Physics 1

Oscillations Tutorial Part 2

Question 1

We can consider a car's suspension system to be a spring under compression with a shock absorber which damps the car's vertical oscillations.

The car is then driven at a steady speed over a rough road on which the surface height varies sinusoidally. Unfortunately, the shock absorber mechanism which normally damps vertical oscillations is not working. Hence, at a certain critical speed, the amplitude of the vertical oscillation becomes very large.

(i) Name the phenomena observed: "amplitude of the vertical oscillation becomes very large"

Resonance [1]

(ii) Given that the spring suspension system obeys Hooke's law, calculate the force constant, k, of the spring suspension system.

$k\Delta y$ $\quad a = 0$ $\quad mg$

$\Delta y k = mg$

$k = \frac{mg}{\Delta y} = \frac{450 \times 9.8}{0.1} = 4.4 \times 10^4 \text{ N/m}$

Important data you can use:

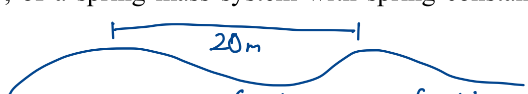
Mass of passengers, m = 450 kg

Total mass of car and passengers, M = 2000 kg

Difference in height of car when passengers alight, Δh = 0.10 m [3]

(iii) Using your answer in (i), determine the critical speed when the amplitude of vertical oscillation is a maximum. The separation of consecutive humps on the road is 20 m.

(Period of oscillation, T, of a spring mass system with spring constant, k and mass, m, is given by $T = 2\pi\sqrt{\frac{m}{k}}$.) [4]



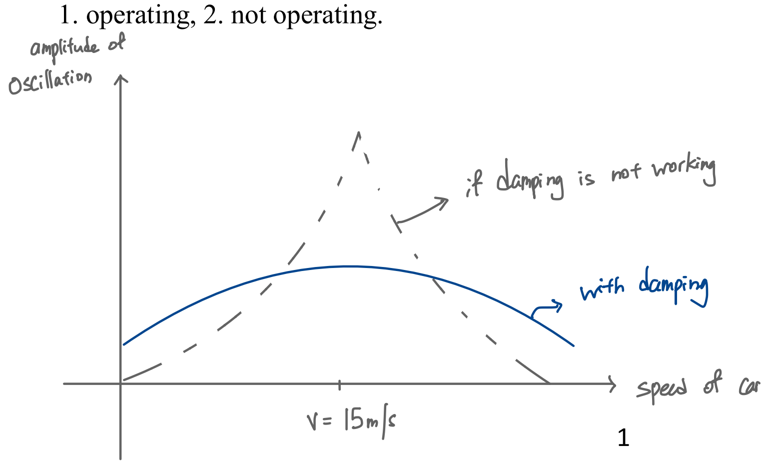
natural frequency, $f = \frac{1}{T}$ $\qquad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{M}}$

Critical speed of Car, $V = f\lambda = \frac{20}{2\pi}\sqrt{\frac{4.4\times10^4}{2000}}$

$= 15 \text{ m/s}$

(iv) Sketch **<u>on the same axes</u>** appropriately labelled graphs to contrast how the amplitude of oscillation would vary at different speeds if the damping mechanism is

1. operating, 2. not operating. [2]



Question 2

On a windy day, a tall building can be set into simple harmonic motion. The horizontal displacement in meters, x, of the top of the building, changes with time in seconds, t, according to the equation:

$x = 1.25\cos(0.209t)$ $[m]$ $\quad x(t) = 1.25\cos(0.209t)$
$= x_0 \cos(\omega t)$

(i) Find the period of the oscillation. $T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi}{0.209} = 30\,s$ [2]

(ii) A man of mass 80 kg stands at the top of the oscillating building. Calculate the maximum kinetic energy of the man. $KE = \frac{1}{2}mv^2$ $\quad v(t) = -x_0\omega\sin(\omega t)$ [2]

$V_{max} = x_0\omega = 1.25 \times 0.209$ $\quad KE = \frac{1}{2}mV_{max}^2 = 2.7J$

(iii) Sketch on the axes below to show how the man's kinetic energy varies with time for **2 complete oscillations**. [2]

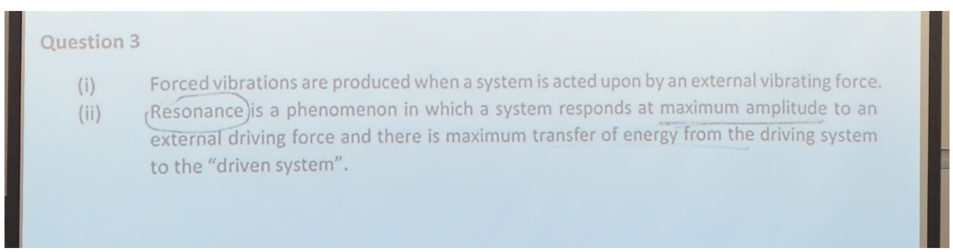
(c) A heavy movable mass, called the mass damper, is placed at the top of the building in part (b) to counter the effect of wind-driven oscillation. It is set into oscillation by the movements of the building. The oscillation of the movable mass is heavily damped. Explain how the damped oscillation of the mass damper reduces the amplitude of oscillation of the building. [2]

Question 3

Any system which can vibrate can be made to perform *forced vibrations* and may show *resonance*.

Explain the meaning of the following terms used in the above sentence:

(i) *forced vibration*
(ii) *resonance*



Answers:

Question 1

(ii) $k = 4.4 \times 10^{-4}$ N m^{-1}

(iii) $v = 15$ m s^{-1}

(iv) Sketch amplitude vs speed graphs on SAME axes.

Question 2

(i) $T = 30$ s
(ii) Max. KE = 2.7 J