Professur Methoden der Signalverarbeitung
School of Computation, Information and Technology
Technical University of Munich

Eexam
Place student sticker here

Note:

- During the attendance check a sticker containing a unique code will be put on this exam.
- This code contains a unique number that associates this exam with your registration number.
- This number is printed both next to the code and to the signature field in the attendance check list.

Circuit Theory

Exam:	1203 / EDE	**Date:**	Tuesday 25th July, 2023
Examiner:	Dr.-Ing. Michael Joham	**Time:**	15:00 – 16:40

	P 1	P 2	P 3	P 4	P 5	P 6	P 7
I							

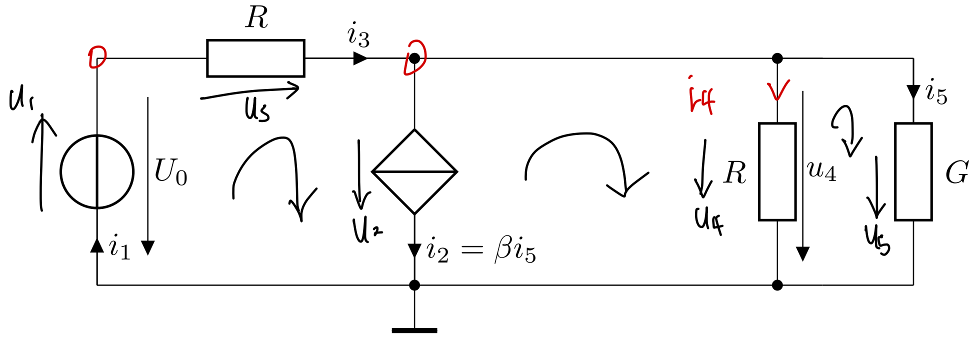
Working instructions

- This exam consists of **18 pages** with a total of **7 problems**.
  Please make sure now that you received a complete copy of the exam.
- The total amount of achievable credits in this exam is 90 credits.
- Detaching pages from the exam is prohibited.
- Allowed resources:
  - 5 double-sided DIN A4 pages of notes
  - one analog dictionary English ↔ native language
- Subproblems marked by * can be solved without results of previous subproblems.
- **Answers are only accepted if the solution approach is documented.** Give a reason for each answer unless explicitly stated otherwise in the respective subproblem.
- Do not write with red or green colors nor use pencils.
- Physically turn off all electronic devices, put them into your bag and close the bag.

Left room from ________ to ________ / Early submission at ________

Problem 1 **Circuit Analysis (17 credits)**

Consider the following circuit with a CCCS, three resistors, and a voltage source.



0 1 2 a)* What is the number of nodes n and the number of branches b of this circuit?

$n = 3$
$b = 5$

0 1 b) Give the number of node voltages for above circuit.

$n - 1 = 2$

0 1 c) What is the number of linearly independent KCL equations in nullspace representation?

$n - 1 = 2$

0 1 d) Give the number of linearly independent KVL equations in nullspace representation.

$b - (n-1) = 3$

0 1 2 e)* Define the branch voltage vector $\boldsymbol{u}$ and the branch current vector $\boldsymbol{i}$.

$\underline{i} = [i_1, i_2, i_3, i_4, i_5]^T$

$\underline{u} = [u_1, u_2, u_3, u_4, u_5]^T$

0 1 f)* Give the number of equations for the KVL in rangespace representation.

$b = 5$

g) Find the loop incidence matrix $\boldsymbol{B}$.

0 1 2 3

$U_1 + U_2 + U_3 = 0$

$-U_2 + U_4 = 0$

$-U_4 + U_5 = 0$

$$\underset{\sim}{B}u = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} u$$

h)* Determine the node incidence matrix $\boldsymbol{A}$.

0 1 2

$-i_1 + i_3 = 0$

$-i_3 + i_2 + i_4 + i_5 = 0$

$$\underset{\sim}{A}\underline{i} = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 \end{bmatrix} \underline{i}$$

i) Based on $\boldsymbol{A}$ and $\boldsymbol{B}$, show that the Tellegen's theorem is fulfilled.

0 1 2

$\underset{\sim}{A} \cdot \underset{\sim}{B}^T = \underset{\sim}{0}$

$$\underset{2\times5}{\begin{bmatrix} -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 \end{bmatrix}} \underset{5\times3}{\begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \underset{\sim}{0}$$

j) Find i_3 depending on u_4.

0 1 2

$i_3 = i_2 + i_4 + i_5$

$U_4 = U_5$

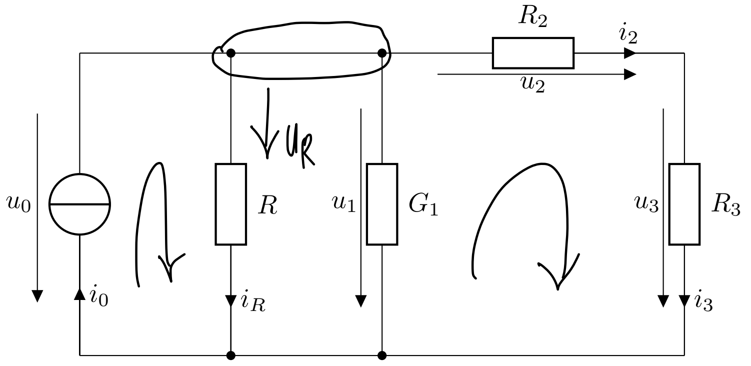
$i_5 = G u_4$

$i_4 = \frac{U_4}{R}$

$i_3 = \beta G U_4 + \frac{U_4}{R} + G U_4$

Problem 2 Resistor Network (10 credits)

The following circuit with four resistors and a current source is given.



0 1 a)* Give the current i_R depending on u_0.

$-U_0 + R i_R = 0 \qquad i_R = \frac{U_0}{R}$

0 1 b)* Determine the overall conductance G_{overall} of the parallel connection of R with G_1.

$G_{overall} = \frac{1}{R} + G_1$

0 1 c)* Find the voltage u_2 depending on i_2.

$U_2 = R_2 i_2$

0 1 d)* What is the overall resistance R_{overall} of the series connection of R_2 and R_3?

$R_2 + R_3$

0 1 2 e) Hence, what are the voltages u_2 and u_3 depending on u_0?

$U_R = U_1 = U_0$

$U_2 = \frac{R_2}{R_2 + R_3} U_0$

$U_3 = \frac{R_3}{R_2 + R_3} U_0$

f) Find i_0 depending on u_0.

$$i_0 = \frac{U_0}{R} + U_0 G_1 + \frac{1}{R_2+R_3} U_0 \qquad i_0 = i_R + i_1 + i_2$$

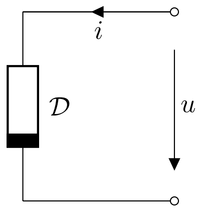
0
1
2

g) Determine i_2 depending on i_0.

$$i_2 = \frac{\frac{\cancel{R_3}}{R_2+R_3}}{\frac{1}{R} + G_1 + \frac{\cancel{R_3}}{R_2+R_3}} i_0 = \frac{\frac{1}{R_2+R_3}}{\frac{1}{R} + G + \frac{1}{R_2+R_3}} i_0$$

0
1
2

Problem 3 **Linearization of a One-Port (7 credits)**



The characteristic of the one-port $\mathcal{D}$ is given by

$$u = r_{\mathcal{D}}(i) = U_0 + U_0 \sin\left(\frac{i - I_0}{I_0}\right)$$

with the constants U_0 and I_0.

a)* Give the port-quantities that control $\mathcal{D}$.

i

In the operating point, the current is $I = 2I_0$.

b)* Find the operating point value U of the voltage across $\mathcal{D}$.

$U = r_D(2I_0) = U_0 + U_0 \sin\left(\frac{2I_0 - I_0}{I_0}\right) = U_0 + U_0 \sin(1)$

In following, consider the different operating point $(U,I) = (U_0,I_0)$.

c)* Determine the linearization $r_{\mathcal{D},\mathrm{lin}}(i)$ of $\mathcal{D}$ in the given operating point $(U,I) = (U_0,I_0)$.

$$r_{D\,lin}(i) = \left.\frac{dr_D(i)}{di}\right|_{i=I_0} \cdot (i - I_0) + U_0$$

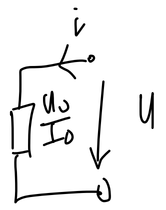
$$= \frac{U_0}{I_0}\cos\left(\cancel{\frac{I_0 - I_0}{I_0}}^{\,0}\right) \cdot (i - I_0) + U_0$$

$$= \frac{U_0}{I_0}(i - I_0) + U_0$$

$$= \frac{U_0}{I_0}\, i$$

d) Draw the equivalent circuit diagram of $r_{\mathcal{D},\mathrm{lin}}(i)$.

0
1



Problem 4 **Linear Two-Port (9 credits)**

The resistance matrix of the two-port $\mathcal{S}$ reads as

$$\boldsymbol{R}_{\mathcal{S}} = \begin{bmatrix} 0 & -\frac{1}{G_{\mathrm{d}}} \\ \frac{1}{G_{\mathrm{d}}} & 0 \end{bmatrix}$$

with finite G_{d}.

0 1 a)* Based on the given resistance matrix $\boldsymbol{R}_{\mathcal{S}}$, show that the two-port $\mathcal{S}$ is not reciprocal.

$r_{12} \neq r_{21} \quad \therefore$ not reciprocal

0 1 b)* What special two-port is $\mathcal{S}$?

gyrator

0 1 c)* Determine the conductance matrix $\boldsymbol{G}_{\mathcal{S}}$ of $\mathcal{S}$.

$$G_s = \frac{1}{\det R}\begin{bmatrix} 0 & \frac{1}{Gd} \\ -\frac{1}{Gd} & 0 \end{bmatrix} = Gd^2\begin{bmatrix} 0 & \frac{1}{Gd} \\ -\frac{1}{Gd} & 0 \end{bmatrix} = \begin{bmatrix} 0 & Gd \\ -Gd & 0 \end{bmatrix}$$

$$\det R = (0)(0) - \left(-\tfrac{1}{Gd}\right)\left(\tfrac{1}{Gd}\right) = \frac{1}{Gd^2}$$

A different two-port $\mathcal{Z}$ has got the conductance matrix

$$\boldsymbol{G}_{\mathcal{Z}} = \begin{bmatrix} 0 & -G_{\mathrm{d}} \\ 2G_{\mathrm{d}} & 0 \end{bmatrix}.$$

0 1 d)* Why is $\mathcal{Z}$ lossy?

$$-G_{\mathcal{Z}}^{T} = \begin{bmatrix} 0 & -2Gd \\ Gd & 0 \end{bmatrix} \neq G_{\mathcal{Z}}$$

The two-ports $\mathcal{S}$ and $\mathcal{Z}$ are connected in parallel to get the two-port $\mathcal{X}$.

0 1 2 e) Find a two-port matrix for the two-port $\mathcal{X}$.

$$\underset{\sim}{G_{\mathcal{X}}} = \underset{\sim}{G_{\mathcal{S}}} + \underset{\sim}{G_{\mathcal{Z}}} = \begin{bmatrix} 0 & 0 \\ Gd & 0 \end{bmatrix}$$

f) Give expressions for u_1 and i_1 of $\mathcal{X}$.

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \underset{\sim}{G_x} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$i_1 = 0$$

$$i_2 = G_d u_i$$

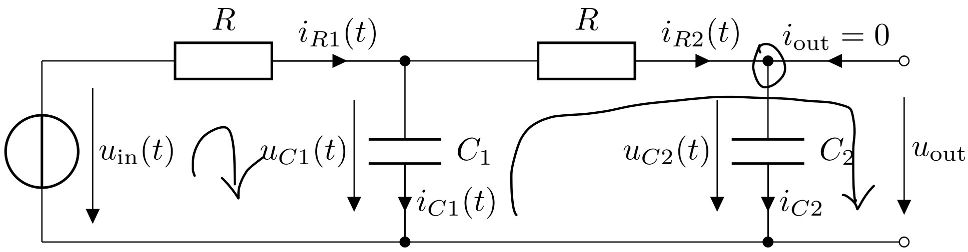
$$u_1 = \frac{1}{G_d} i_2$$

g) What special two-port is $\mathcal{X}$?

VCCS with gain in G_d

Problem 5 Complex Phasor Analysis (16 credits)

Given is the following circuit with two capacitors.
The angular frequency of the sinusoidal excitation $u_{in}(t)$ is $\omega > 0$.



0 1 a)* Give $i_{C1}(t)$ depending on $u_{C1}(t)$.

$i_{C1}(t) = C_1 \cdot \dot{U}_{C1}(t)$

Let I_{C1} denote the phasor corresponding to $i_{C1}(t)$ and U_{C1} the phasor for $u_{C1}(t)$.

0 1 b) What is the current phasor I_{C1} depending on the voltage phasor U_{C1}?

$I_{C1} = j\omega \cdot C_1 U_{C1}$

0 1 2 c)* Find the voltage phasor U_{C1} depending on the voltage phasor U_{out} (independent of other current or voltage phasors) taking into account that $I_{out} = 0$.

KVL: $-U_{C1} + RI_{R2} + U_{out} = 0$

$i_{C2} = i_{R2}(t)$

$U_{C2}(t) = U_{out}$

$I_{C2} = j\omega \cdot C_2 U_{out}$

$\therefore U_{C1} = j\omega \cdot RC_2 U_{out} + U_{out}$

0 1 2 d) Determine the current phasor I_{R1} depending on the voltage phasor U_{out}.

KCL: $i_{R1} = i_{R2} + i_{C1}$

$I_{C1} = j\omega \cdot C_1 U_{C1}$

$I_{R2} = j\omega \cdot C_2 U_{out}$

$I_{R1} = j^2\omega^2 \cdot RC_1C_2 U_{out}$
$+ j\omega C_1 U_{out}$
$+ j\omega \cdot C_2 U_{out}$

e)* Express the voltage phasor U_{C1} depending on the voltage phasor U_{in} and the current phasor I_{R1}. 0 1

KVL: $-U_{in} + RI_{R1} + U_{C1} = 0$

$U_{C1} = U_{in} - RI_{R1}$

With the time constant τ, the transfer function of another circuit can be written as

$$H(\mathrm{j}\,\omega) = \frac{U_{\text{out}}(\mathrm{j}\,\omega)}{U_{\text{in}}(\mathrm{j}\,\omega)} = \frac{1}{(\mathrm{j}\,\omega\tau)^2 + \frac{3}{2}\,\mathrm{j}\,\omega\tau + 1}.$$

f)* Investigate $|H(\mathrm{j}\,\omega)|$ at $\omega = 0$, $\omega = \frac{1}{\tau}$, and $\omega \to \infty$. 0 1 2 3

$\omega = 0 : |H(j\omega)| = 1$

$\omega = \frac{1}{\tau} : |H(j\omega)| = \frac{1}{|-1 + \frac{3}{2}j + 1|} = \frac{2}{3}$

$\omega \to \infty : |H(j\omega)| = 0$

g) What filter type (lowpass, highpass, bandpass, bandstop, allpass) is the transfer function $H(\mathrm{j}\,\omega)$? Justify your answer based on above results. 0 1

Lowpass

The input voltage is given by $u_{\text{in}}(t) = 6\,\text{V}\,\cos(\frac{2}{\tau}\,t + \frac{\pi}{4})$.

h)* Give the phasor U_{in} corresponding to the given $u_{\text{in}}(t)$. 0 1

$U_{in} = 6Ve^{j\pi/4}$

i) Find the output phasor U_{out} in polar form, i.e., as the product of magnitude and the exponential depending on the phase. 0 1 2 3

$U_{out} = H(j\frac{2}{\tau})\,U_{in}$

$H(j\frac{2}{\tau}) = \frac{1}{\underbrace{(j2)^2}_{-4} + 3j + 1} = \frac{1}{|3j - 3|} = \frac{1}{\sqrt{9+9}} = \frac{1}{3\sqrt{2}}$

$H(j\frac{2}{\tau}) = \frac{1}{3\sqrt{2}} e^{j\frac{-3\pi}{4}}$

$U_{out} = \sqrt{2}\,Ve^{-j\frac{\pi}{2}}$

Let the output phasor be given by

$$U_{\text{out}} = 2\,\text{V}\ \text{e}^{\text{j}\,\frac{\pi}{3}}\,.$$

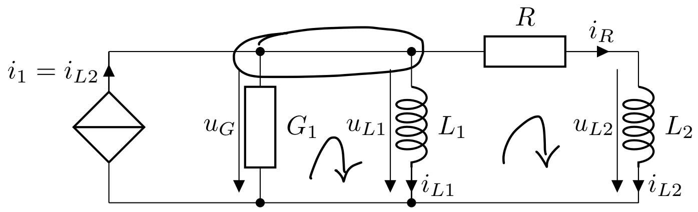
0
1

j)* Give the time-signal $u_{\text{out}}(t)$ corresponding to the phasor U_{out}.

$U_{out}(t) = Re\{2V e^{j\frac{\pi}{3}}\} = 2V\cos(\omega t + \frac{\pi}{3})$

Problem 6 Second-Order Circuit (16 credits)

Given is the following second-order circuit with the two inductors L_1 and L_2 connected to a CCCS and two resistors.



a)* Give the state variables of the circuit.

0 1

$\dot{i}_{L1}$, $\dot{i}_{L2}$

b) Define the state vector $\boldsymbol{x}$.

0 1

$$X = \begin{bmatrix} \dot{i}_{L1} \\ \dot{i}_{L2} \end{bmatrix}$$

c)* Find u_{L1} depending on u_{L2} and i_R.

0 1

$U_{L1} = R i_R + U_{L2}$

d)* Give i_{L2} depending on i_R.

0 1

$i_R = i_{L2}$

e)* Determine u_{L1} depending on u_G.

0 1

$U_G = U_{L1}$

f) Express u_{L1} depending on i_{L1} without using any time derivatives.

0 1 2

KCL: $-i_{L2} + U_G G_1 + i_{L1} + i_R = 0$ $\qquad U_G = U_{L1}$

$U_{L1} = \frac{1}{G_1} i_{L1}$

g) Find u_{L2} depending on i_{L1} and i_{L2}.

$$U_{L2} = U_{L1} - R i_R = \frac{1}{G_1} i_{L1} - R i_{L2}$$

h) Give the state equations for the given circuit.

$$U_L = L \dot{i}_L$$

$$\dot{i}_{L1} = \frac{1}{G_1 L_1} i_{L1} \qquad \dot{i}_{L2} = \frac{1}{G_1 L_1} i_{L1} - \frac{R}{L_1} i_{L2}$$

The state equations for another second-order system read as

$$\dot{x}_1 = -2x_1$$
$$\dot{x}_2 = -3x_1 + x_2$$

with the state vector $\boldsymbol{x} = [x_1, x_2]^{\mathrm{T}}$.

i)* Find the state matrix $\boldsymbol{A}$.

$$\underline{\dot{x}} = \underset{\sim}{A}\,\underline{x}$$

$$A = \begin{bmatrix} -2 & 0 \\ -3 & 1 \end{bmatrix}$$

j) Determine the eigenvalues of $\boldsymbol{A}$.

$$\det(A - \underset{\sim}{I}\lambda) = (-2-\lambda)(1-\lambda) = 0$$

$$\lambda_1 = -2 \quad \text{or} \quad \lambda_2 = 1$$

k) Investigate **both** eigenvalues and explain why the circuit is unstable.

$\lambda_1 = -2$: stable eigenvalue

$\lambda_2 = 1$: unstable eigenvalue

Due to λ_2, the circuit is unstable

Problem 7 Non-Linear Two-Port (15 credits)

The current-controlled characteristic of the non-linear two-port $\mathcal{T}$ can be written as

$$\boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \boldsymbol{r}_{\mathcal{T}}(\boldsymbol{i}) = \begin{bmatrix} U_0(\frac{i_1}{I_0} + \frac{i_2^2}{I_0^2}) \\ R_0 i_1 + 2R_0 i_2 \end{bmatrix}$$

$R_0 (i_1 + \frac{i_2^2}{I_0})$

with the constants U_0, I_0, and $R_0 = \frac{U_0}{I_0}$.

$\det(r) = 2R_0^2 - R_0^2 \frac{i_2}{I_0}$

$= R_0^2 (2 - \frac{i_2}{I_0})$

a)* Find u_1 and u_2 of the two-port $\mathcal{T}$ for $i_1 = 0$ and $i_2 = 0$.

0 1

$U_1 = 0 \qquad U_2 = 0$

b) Argue why the two-port is sourcefree.

0 1

The origin is part of the Characteristics as $r_T(0) = 0$

c)* Determine $\boldsymbol{r}_{\mathcal{T}}([i_2, i_1]^{\mathrm{T}})$.

0 1

$$r_T([i_2, i_1]^T) = \begin{bmatrix} R_0 i_2 + R_0 \frac{i_1^2}{I_0} \\ R_0 i_2 + 2R_0 i_1 \end{bmatrix}$$

d) Argue why the two-port $\mathcal{T}$ is not symmetric.

0 1

$$r_T([i_2, i_1]^T) \neq \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} r_T(i)$$

e)* Why does the hybrid-controlled characteristic of $\mathcal{T}$ not exist?

0 1 2

U_2 and i_1 depend on i_2^2

f)* Find the inverse hybrid representation of $\mathcal{T}$.

0 1 2

$$H^{-1} = \frac{1}{r_{11}} \begin{bmatrix} 1 & -r_{12} \\ r_{21} & \det R \end{bmatrix} = \frac{1}{R_0} \begin{bmatrix} 1 & -R_0 \frac{i_2}{I_0} \\ R_0 & R_0^2 (2 - \frac{i_2}{I_0}) \end{bmatrix} = \begin{bmatrix} \frac{1}{R_0} & -\frac{i_2}{I_0} \\ 1 & R_0 (2 - \frac{i_2}{I_0}) \end{bmatrix}$$

$$i_1 = \frac{U_1}{R_0} - \frac{i_2^2}{I_0} \qquad U_2 = U_1 + 2R_0 i_2 - R_0 \frac{i_2^2}{I_0}$$

Assume that the two-port $\mathcal{T}$ is connected to a short circuit at port one and to the current source I_0 at port two.

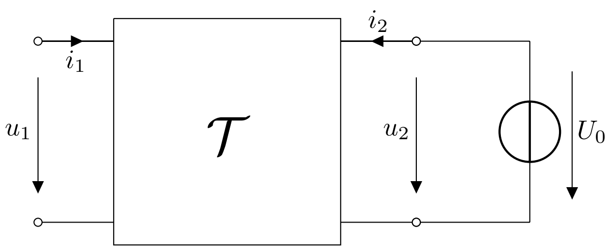
g) Determine the corresponding operating point (U_1,U_2,I_1,I_2).

Short circuit : $U_1 = 0$ $\quad U_2 = 0 + 2U_0 - U_0$

$I_2 = I_0$ $\quad = U_0$

$I_1 = -\frac{I_0^2}{I_0} = -I$

Now the two-port is connected to the voltage source U_0 at port two as illustrated below.



h)* What is u_2 depending on U_0?

$U_2 = U_0$

i) Express i_2 depending on i_1 and I_0.
Remember that $R_0 = \frac{U_0}{I_0}$.

j) What is the power p_2 at port two?

$P_2 = U_2 i_2$

Additional space for solutions–clearly mark the (sub)problem your answers are related to and strike out invalid solutions.