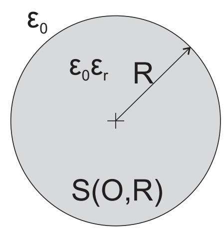
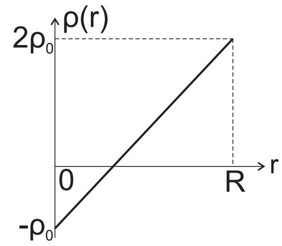
Problem 1 (17 Punkte)

Imagine a sphere S with radius R centered at the origin of the coordinate system O. The permittivity of the sphere is $\epsilon_0\epsilon_R$, the permittivity of the surrounding air is ϵ_0. In the interior of the sphere, there exists a space charge density as follows:

$$\rho(\vec{r}) = \begin{cases} \dfrac{3\rho_0}{R}r - \rho_0 & \text{für } 0 \leq r \leq R \\ 0 & \text{für } r > R \end{cases}$$

where ρ_0 denotes a positive constant and $r = |\vec{r}|$.





Use spherical coordinates for the following calculations.

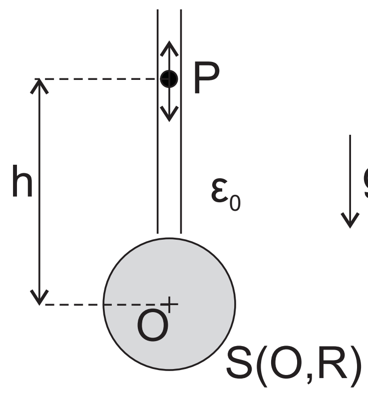
*a) Calculate the electric charge $Q(r)$, which is enclosed in a sphere $S_r(O, r)$ centered around the origin O with radius r in the region $0 \leq r \leq R$.

*b) Calculate the displacement field $\vec{D}(\vec{r})$ everywhere in space (that means for $0 \leq r \leq \infty$).

c) Calculate the electric field $\vec{E}(\vec{r})$ everywhere in space (that means for $0 \leq r \leq \infty$).

d) Draw a qualitative sketch of the behavior of the radial component of the electric field $E_r(r)$ along the radial direction in the region $0 \leq r \leq R$. Label the axis properly.

Above the fixed sphere S there is a point particle P with mass m and electric charge $Q > 0$, which is attached to a guide rail. This allows a vertical displacement of the point particle only. The point particle is exposed to the gravitational field of earth with the gravitational acceleration g.



e) Which equilibrium distance h from the center of the sphere is attained in the case of static mechanical equilibrium, when the point particle is free to move along the guide rail? Assume that the equilibrium distance is larger than the radius of the sphere ($h > R$).

f) The point charge is now displaced from the equilibrium distance h to another distance b. Assume that $b > R$. Calculate the mechanical work W performed by this action.

Problem 2 (14 Punkte)

Consider an infinitely long straight conductor, which has the form of a hollow cylinder with inner radius r_1 and outer radius r_2. The cross-section of such a conductor is displayed in Figure 1. A uniformly distributed, constant current $I > 0$ flows through the conductor. Use cylindrical coordinates for the following calculations.

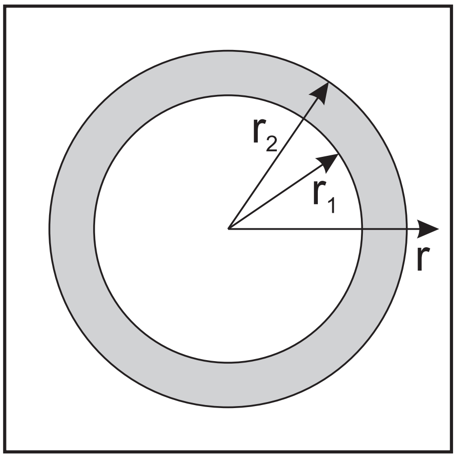


Figure 1

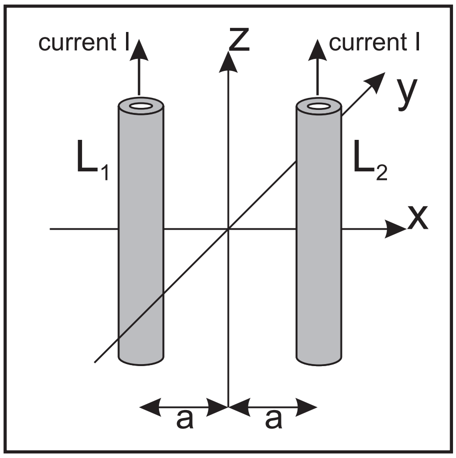


Figure 2

*a) Calculate the current density $\vec{j}(\vec{r})$ in the interior of the conductor ($r_1 \leq r \leq r_2$).

b) Calculate the magnitude and the direction of the magnetic field strength $\vec{H}(\vec{r})$ everywhere in space ($0 \leq r < \infty$).

c) Draw a qualitative sketch of the behavior of the magnitude of the magnetic field strength $|\vec{H}(r)|$ along the r-axis. Label the positions $r = r_1$ and $r = r_2$ on the r-axis!

In the following, we consider a configuration of two identical parallel straight conductors L_1 and L_2 with infinite length (see Figure 2), each having the form of a hollow cylinder. Both conductors carry the same current I. Their central axes are located at the positions $x = -a$, $y = 0$ for L_1 and $x = +a$, $y = 0$ for L_2, respectively.
Hint: If you did not solve subtask b), use the following result instead:

$$\vec{H}(r, \varphi) = I/(2\pi r) \cdot \vec{e}_\varphi(\varphi) \quad \text{for} \quad r > r_2.$$

*d) Draw a qualitative sketch of the magnetic field lines in the xy-plane, which are generated by the two conductors in the region outside of them. Mark clearly the position of the two conductors and the x- and y-axes!

All the best!