Chapter 2 Stationary Currents

2.3. Charge Conservation and Kirchhoff' Current Law

2.3.1 Charge conservation in integral formulation

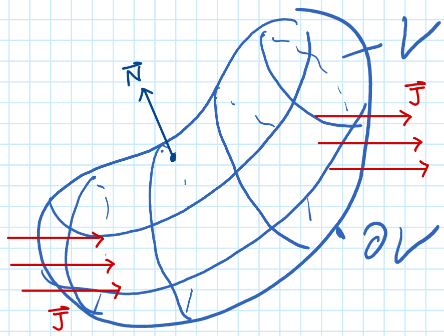
Electric current = transport of charge carriers

$$I = \frac{dQ}{dt} \quad [I] = \frac{C}{s} = A \quad ; \quad \vec{J} = \sigma \vec{E} \Rightarrow I = \int_A \vec{J} \cdot d\vec{a} \quad ; \quad U_{12} = \int_1^2 \vec{E}\, d\vec{r}$$

Generally:

- Transport processes are described by balance equations (How much is flowing into a volumen, how much is flowing out of the volume);
- this is a fundamental procedure, when describing physical phenomena

Balance of charge in volume V:



agreement :

· Outflow of a volume $\int_{\partial V} \vec{J} \cdot d\vec{a} > 0$

$\vec{J} \parallel \vec{N}$

· inflow into a volume $\int_{\partial V} \vec{J} \cdot d\vec{a} < 0$ $\vec{J}$ is not parallel → net inflow

· We look at total charge Q inside volume V ; how Q is changing with time

$\Rightarrow \frac{dQ(V)}{dt} \Rightarrow$ if $Q(V)$ does not change

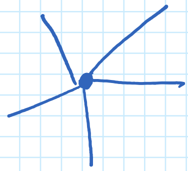
$$\frac{dQ(V)}{dt} = -\int_{\partial V} \vec{J} \cdot d\vec{a}$$

$$\Rightarrow \boxed{\frac{dQ(V)}{dt} + \int_{\partial V} \vec{J} \cdot d\vec{a} = 0} \quad \text{Charge balance equation} \qquad (2.20)$$

if $\frac{dQ(V)}{dt} = 0$ Stationary Current (inflow = outflow)

2.3.2 Kirchhoff's Current Law

- Consider N wires connected in on node:



enlarged

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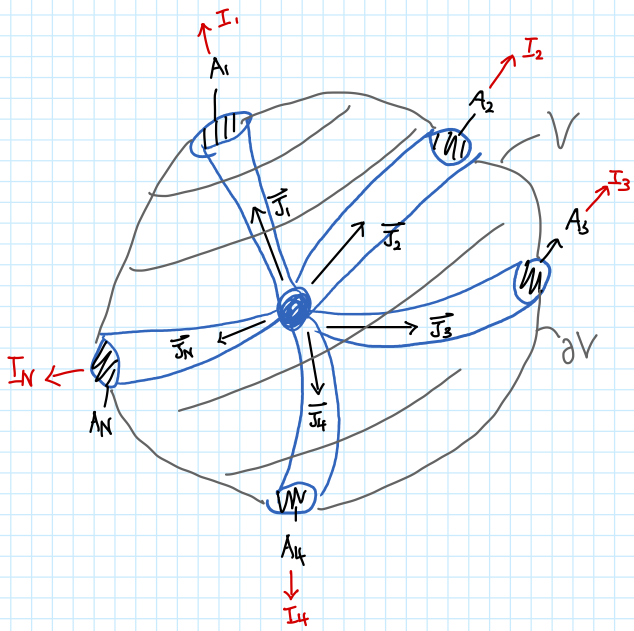

Current $I_1, I_2, I_3, \ldots I_N$ are related to $\vec{J}_1, \vec{J}_2, \vec{J}_3, \ldots \vec{J}_N$ by: $\boxed{I_k = \int_{A_k} \vec{J} \cdot d\vec{a} \; ; \; k = 1 \ldots N}$

agreement : $\vec{J} \uparrow\uparrow \vec{A}_k$ $I_k > 0$

$\vec{J} \uparrow\downarrow \vec{A}_k$ $I_k < 0$

- Balance: Calculate flux integral over closed surface around volume V

$$\int_{\partial V} \vec{J}\cdot d\vec{a} = \sum_{k=1}^{N} \underbrace{\int_{A_k} \vec{J}_k \cdot d\vec{a}}_{I_k} = \sum_{k=1}^{N} I_k$$

Closed surface $\partial V \rightarrow$ sum of integrals at area A_k, which intersect with ∂V

For stationary currents $\Rightarrow$ Q is not changing with time $\Rightarrow \frac{dQ}{dt} = 0$

$$\int_{\partial V} \vec{J}\cdot d\vec{a} + \cancel{\frac{dQ}{dt}} = 0 \quad \Rightarrow \quad \int_{\partial V} \vec{J}\cdot d\vec{a} = 0$$

$$\Rightarrow \boxed{\sum_{k=1}^{N} I_k = 0 \quad (2.22)}$$ Kirchhoff's current Law KCL

(2.22) KCL is a consequence of charge balance equation and charge conservation

It is true, if no source is present ($\frac{dQ}{dt} \neq 0$)

2.3.3 Charge conservation in differential form

(Derivation analog to Gauss's law)

$$\frac{dQ(V)}{dt} = -\int_{\partial V} \vec{J}\cdot d\vec{a} \qquad Q(V) = \int_V \underset{\text{space charge density}}{\rho(\vec{r},t)}\, dV$$

$$\frac{d}{dt}\int \rho(\vec{r},t)\, dV \underset{\substack{\text{if Volume V}\\ \text{does not change with}\\ \text{time}}}{=} \int \frac{\partial}{\partial t}\rho(\vec{r},t)\, dV$$

$$\int_V \frac{\partial}{\partial t}\rho(\vec{r},t)\, dV = -\int_{\partial V} \vec{J}\cdot d\vec{a} = -\int_V \operatorname{div}\vec{J}\cdot d\vec{a}$$

$$\frac{\partial \rho(\vec{r},t)}{\partial t} = -\operatorname{div}\vec{J}$$

Charge balance equation in different form:

$$\boxed{\frac{\partial \rho(\vec{r},t)}{\partial t} + \operatorname{div}\vec{J} = 0} \quad (2.23)$$