Rotational motion revision

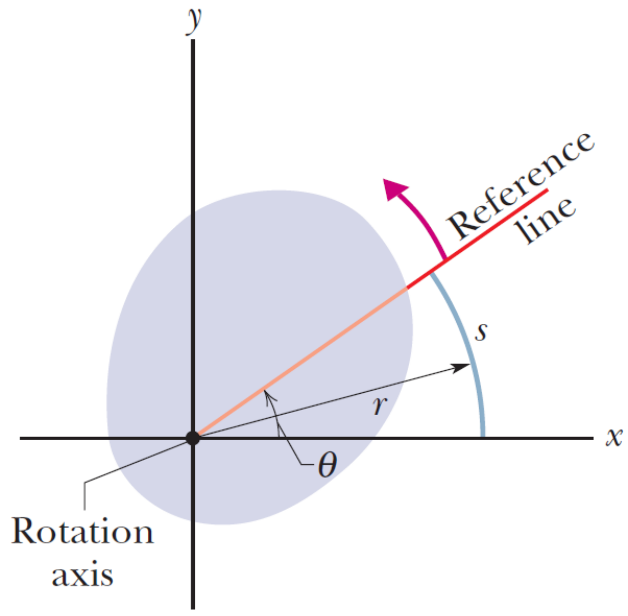
Rigid body rotation

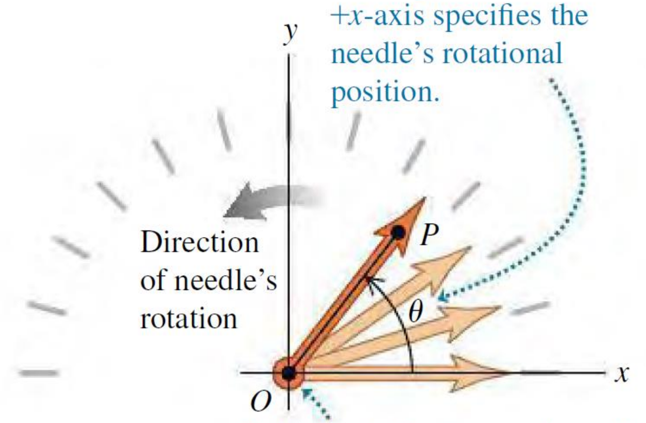
In a rigid body rotation, the relative position of particles does not change as the object rotates.

The period of rotation is the same for all the components of the body. Linear speeds are different.

The angle covered in a given time is the same for the whole body.

Anticlockwise is considered positive.





Radians

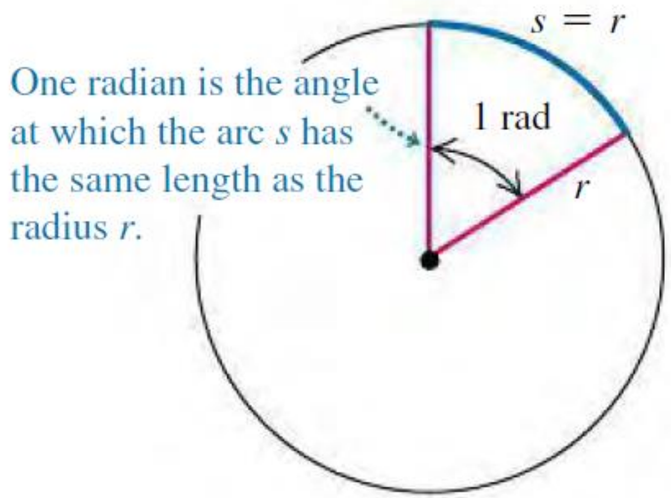
The rotation of rigid bodies is generally measured in radians.

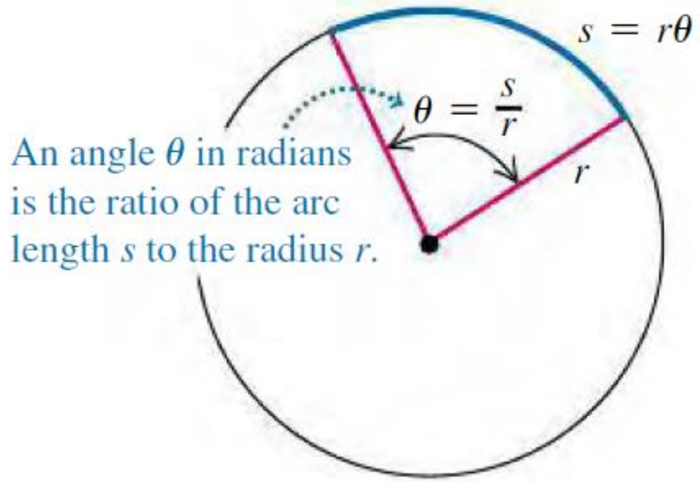
If s is the length of the arc covered by the rotation of a point, and r is the radius, then:

$$\theta = \frac{s}{r}$$

The distance covered is then:

$$s = \theta r$$





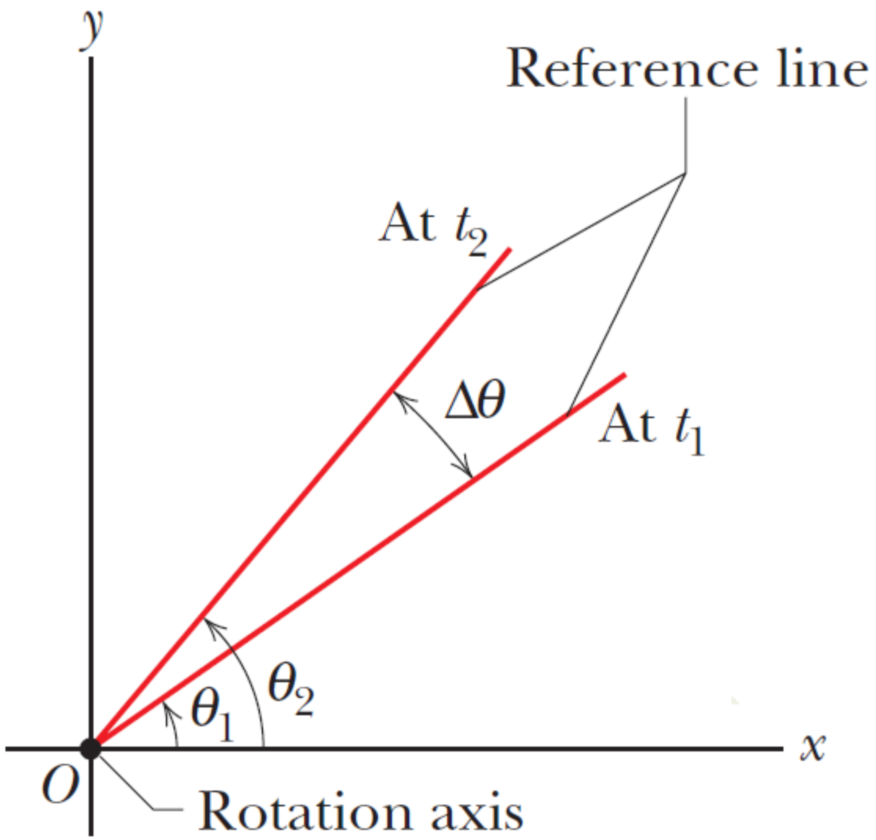
Angular displacement

The rotational displacement can be found using the normal methods used for linear displacements:

$$\Delta\theta = \theta_2 - \theta_1$$

This definition is valid both for the whole rigid body and for each particle which forms the body.

The positive direction is taken as anti-clockwise.



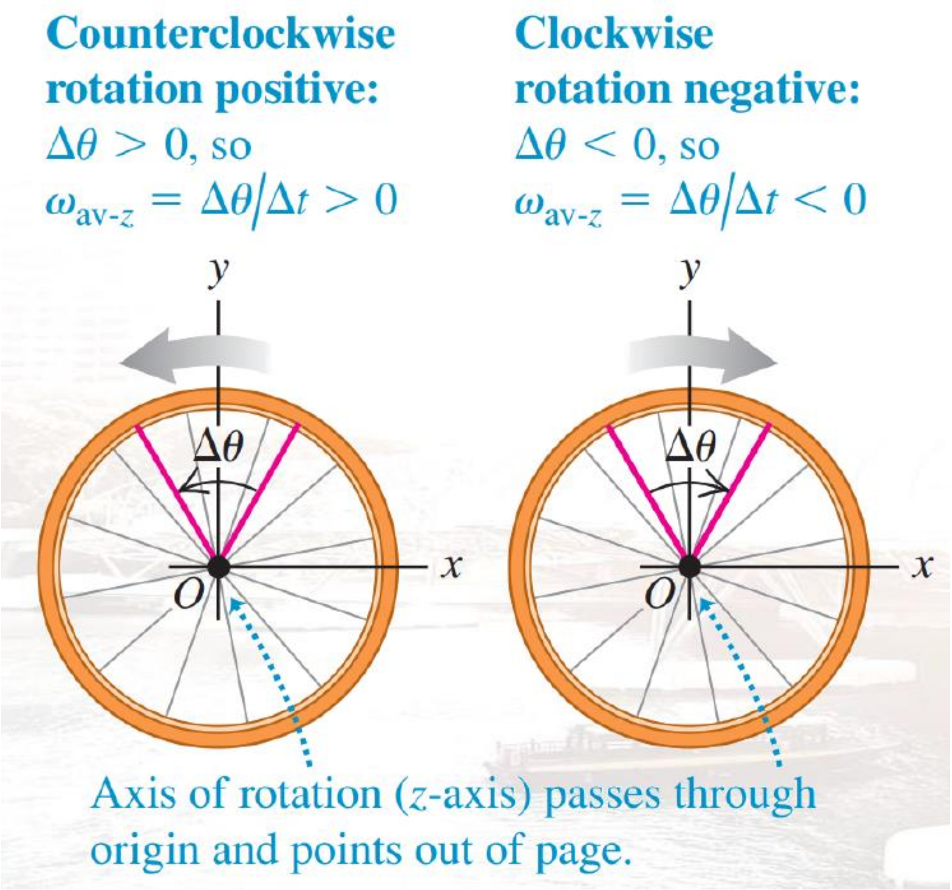
Angular velocity

The angular average velocity is given by:

$$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

As for linear kinematics, the instantaneous velocity can be found with differentiation:

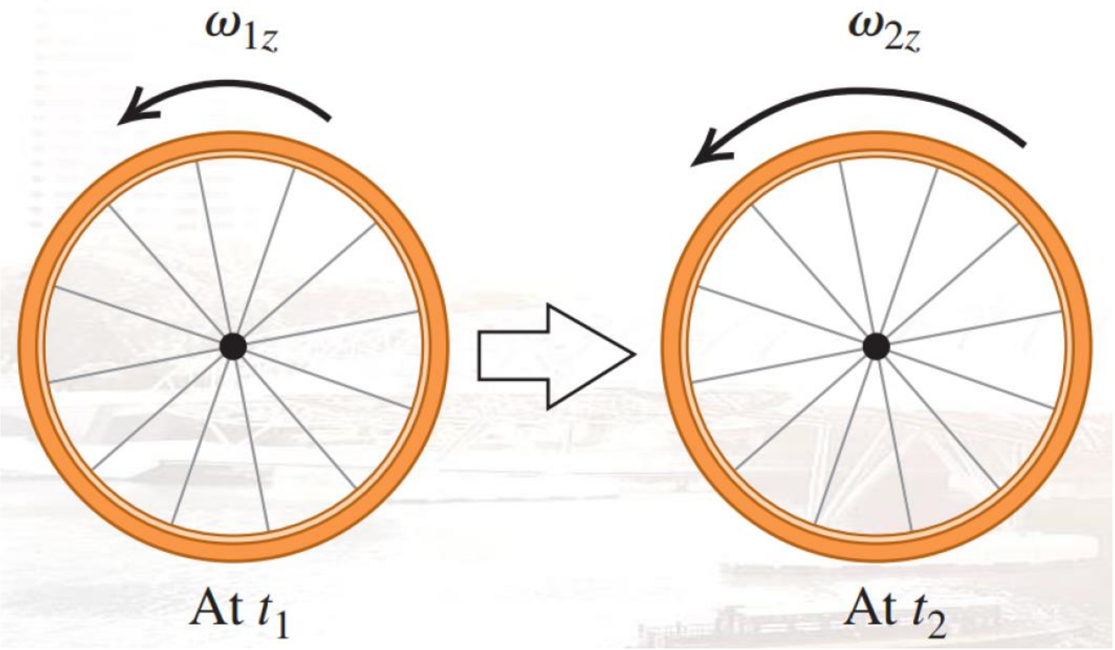
$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$



Angular Acceleration

The angular average acceleration is given by:

$$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$



As for linear kinematics, the instantaneous acceleration can be found with differentiation:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

Constant angular acceleration

Linear Equation	t	$x(t)$ or $\theta(t)$	$v(t)$ or $\omega(t)$	a or α	Angular Equation
$v(t) = v_0 + at$	X		X	X	$\omega(t) = \omega_0 + \alpha t$
$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$	X	X		X	$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$v(t)^2 = v_0{}^2 + 2a\left[x(t) - x_0\right]$		X	X	X	$\omega(t)^2 = \omega_0{}^2 + 2\alpha\left[\theta(t) - \theta_0\right]$
$x(t) - x_0 = \frac{1}{2}\left[v_0 + v(t)\right]t$	X	X	X		$\theta(t) - \theta_0 = \frac{1}{2}\left[\omega_0 + \omega(t)\right]t$

Kinematic example

While you are operating a Rotor (a large, vertical, rotating cylinder found in amusement parks), you spot a passenger in acute distress and decrease the angular velocity of the cylinder from 3.40 rad/s to 2.00 rad/s in 20.0 rev, at constant angular acceleration. (The passenger is obviously more of a “translation person” than a “rotation person.”) (a) What is the constant angular acceleration during this decrease in angular speed? (b) How much time did the speed decrease take?

Kinematic example

Kinematic example

Kinematic example

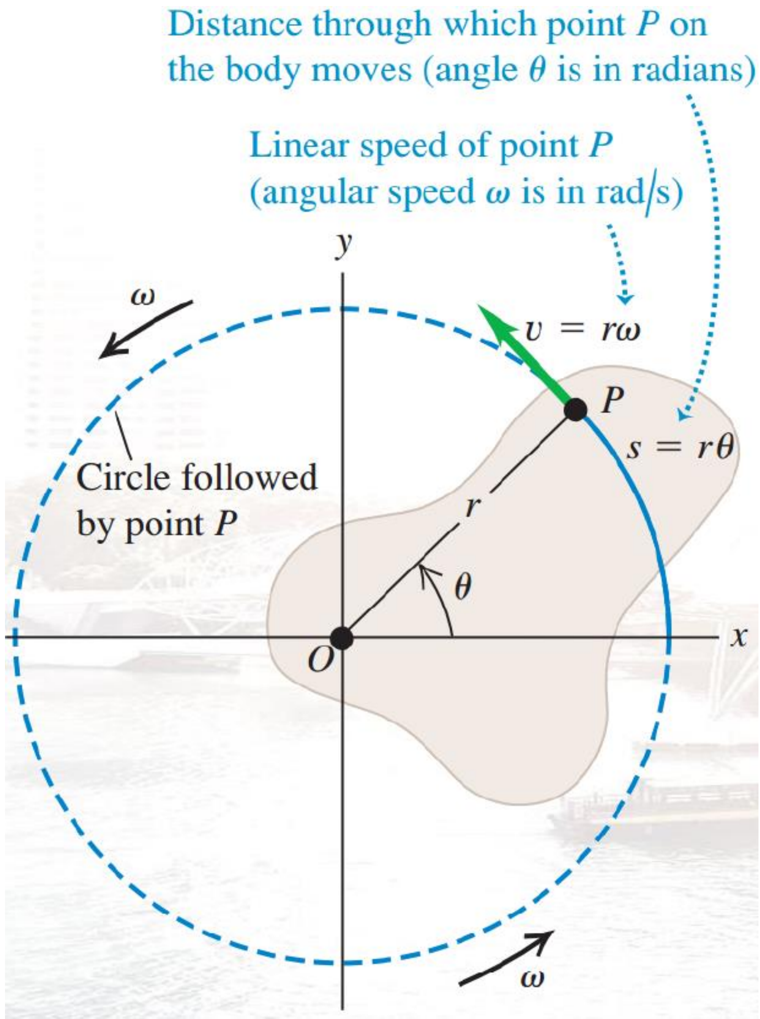
Linear and angular kinematics

The distance *s* covered by a point on the rotating body, *if the angle is indicated in radians*, is given by:

$$s = r\theta$$

We can differentiate this to find the linear velocity of the particle:

$$\frac{ds}{dt} = \frac{d(r\theta)}{dt}; \quad \frac{ds}{dt} = r\frac{d\theta}{dt}; \quad v = r\omega$$



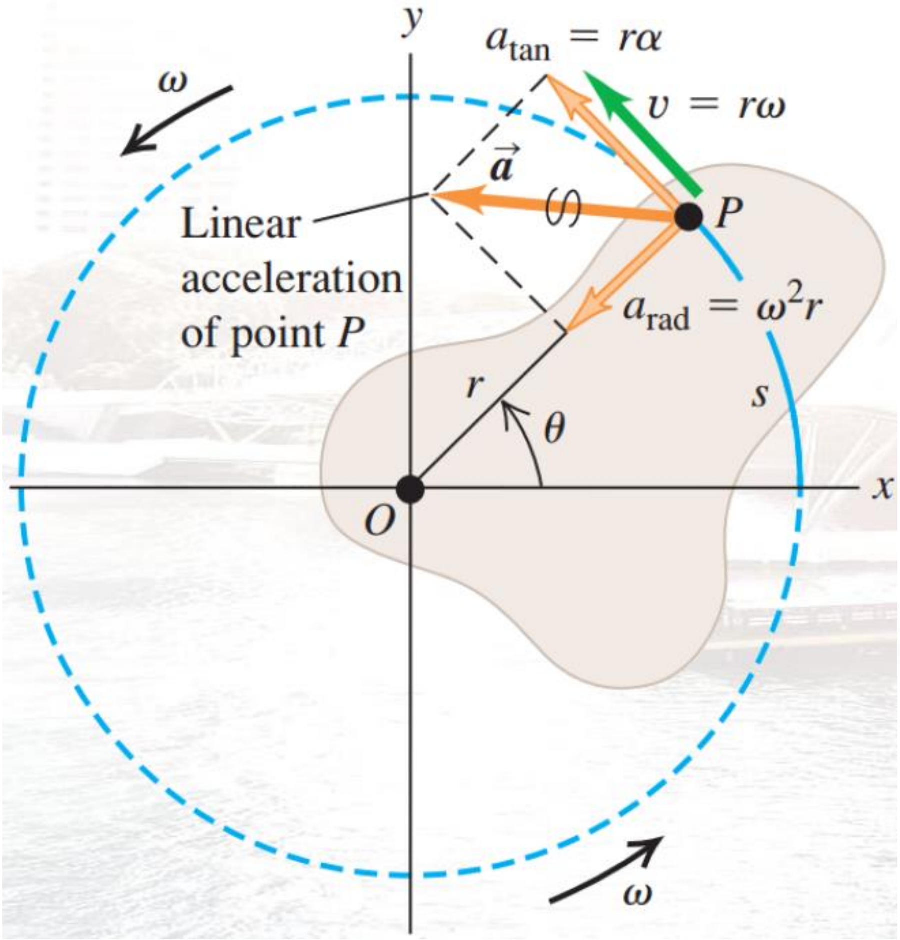
Linear and angular kinematics

The tangential acceleration a_{tan} can also be found through derivation:

$$\frac{dv}{dt} = \frac{d(r\omega)}{dt}; \ \frac{dv}{dt} = r\frac{d\omega}{dt}; \ a_{tan} = r\alpha$$

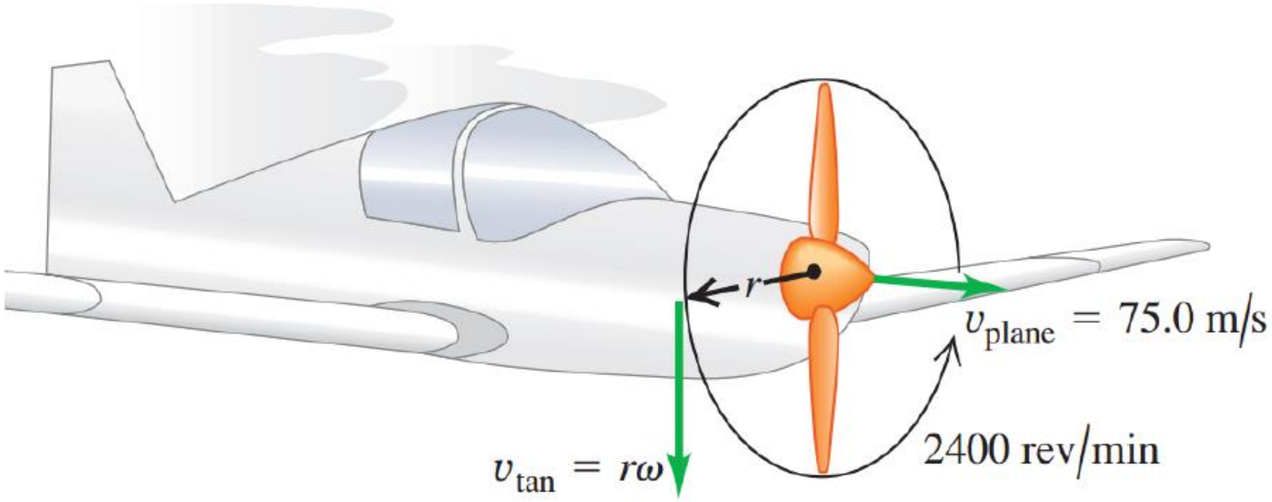
We can also find the centripetal acceleration (a_{rad}) in terms of the angular velocity:

$$a_{rad} = \frac{v^2}{r}; \ a_{rad} = \frac{(\omega r)^2}{r}; \ a_{rad} = \omega^2 r$$

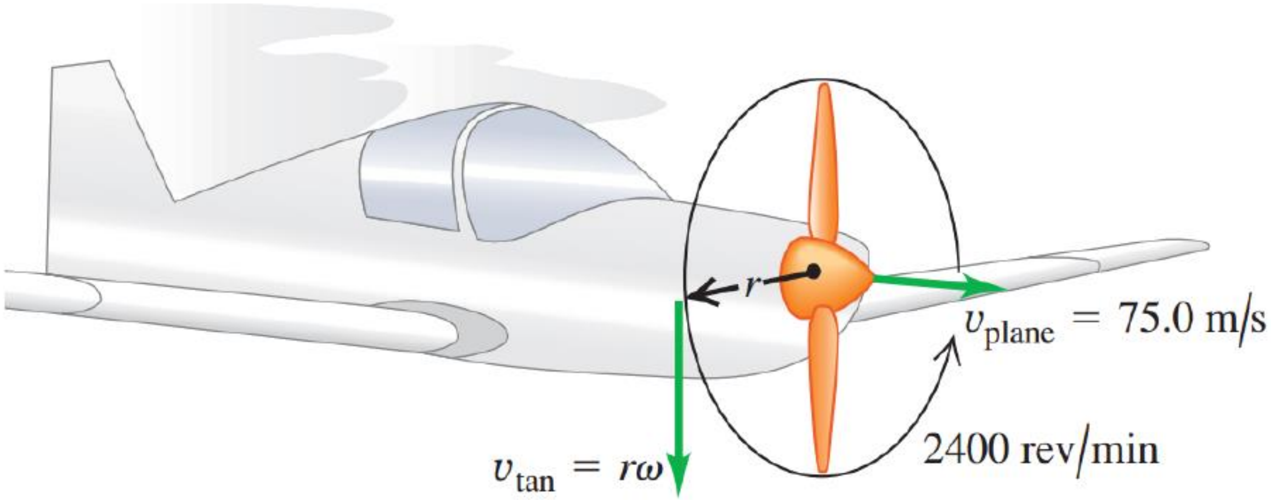


Linear and angular kinematics - Example

You are designing an airplane propeller that is to turn at 2400 rpm. The forward airspeed of the plane is to be 75.0 m/s, and the speed of the propeller tips through the air must not exceed 270 m/s. (This is about 80% of the speed of sound in air. If the propeller tips moved faster, they would produce a lot of noise.) (a) What is the maximum possible propeller radius? (b) With this radius, what is the acceleration of the propeller tip?



Linear and angular kinematics - Example



Angular kinetic energy

We can define a new quantity, the rotational inertia:

$$I = \sum m_i r_i^2$$

The rotational inertia has a similar role as the mass in linear motion.

It is a function of the element mass and its distance from the centre of rotation.

It can be shown that the kinetic energy becomes:

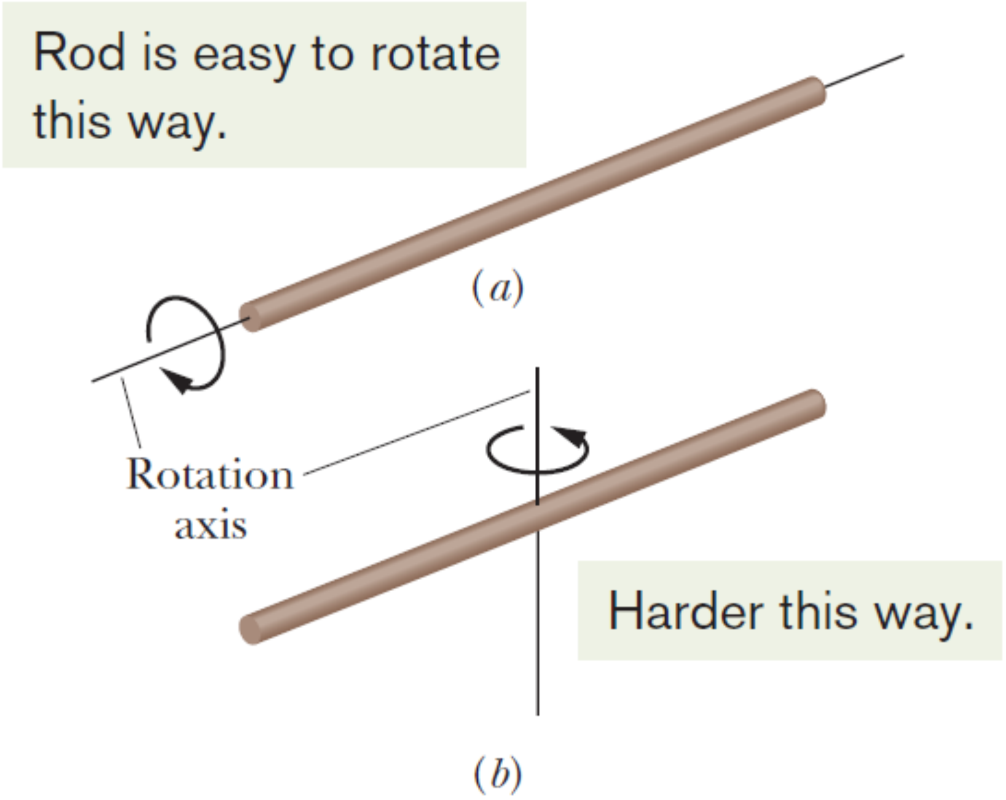
$$K = \frac{1}{2} I \omega^2 \qquad K = \frac{1}{2} m v^2$$

Rotational inertia

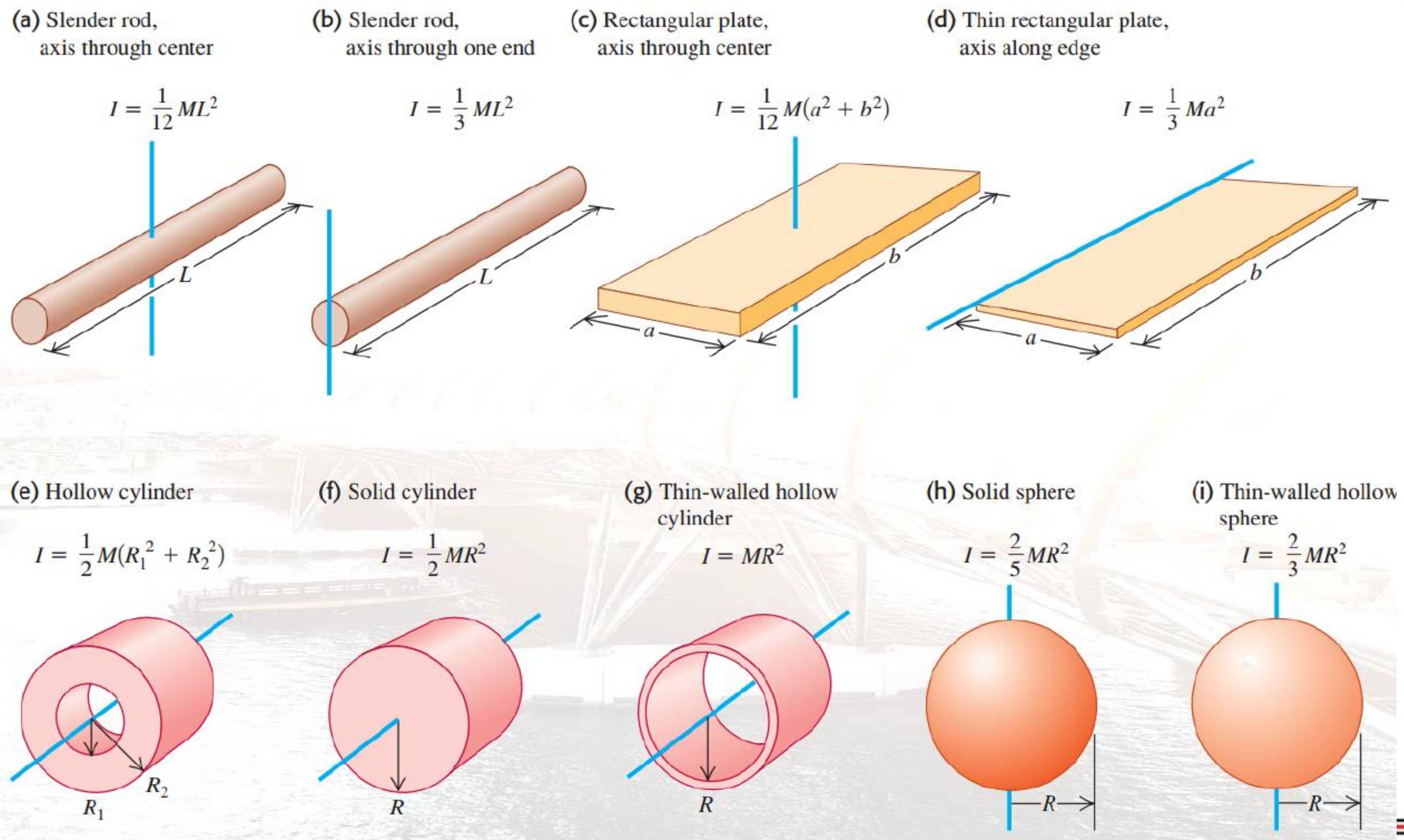
Shapes with more mass away from the rotation axis have a larger rotational inertia. Calculus is used for general shapes:

$$I = \sum m_i r_i^2 = \int r^2 dm$$

A rod rotating around its longitudinal axis has a lower rotational inertia than if it was rotating along a perpendicular axis.



Calculating the rotational inertia



Rotational kinetic energy - example

In 1985,Test Devices, Inc.(www.testdevices.com) was spin testing a sample of a solid steel rotor (a disk) of mass M = 272 kg and radius R = 38.0 cm. When the sample reached an angular speed v of 14 000 rev/min, the test engineers heard a dull thump from the test system, which was located one floor down and one room over from them. Investigating, they found that lead bricks had been thrown out in the hallway leading to the test room, a door to the room had been hurled into the adjacent parking lot, one lead brick had shot from the test site through the wall of a neighbor's kitchen, the structural beams of the test building had been damaged, the concrete floor beneath the spin chamber had been shoved downward by about 0.5 cm, and the 900 kg lid had been blown upward through the ceiling and had then crashed back onto the test equipment. The exploding pieces had not penetrated the room of the test engineers only by luck. How much energy was released in the explosion of the rotor?

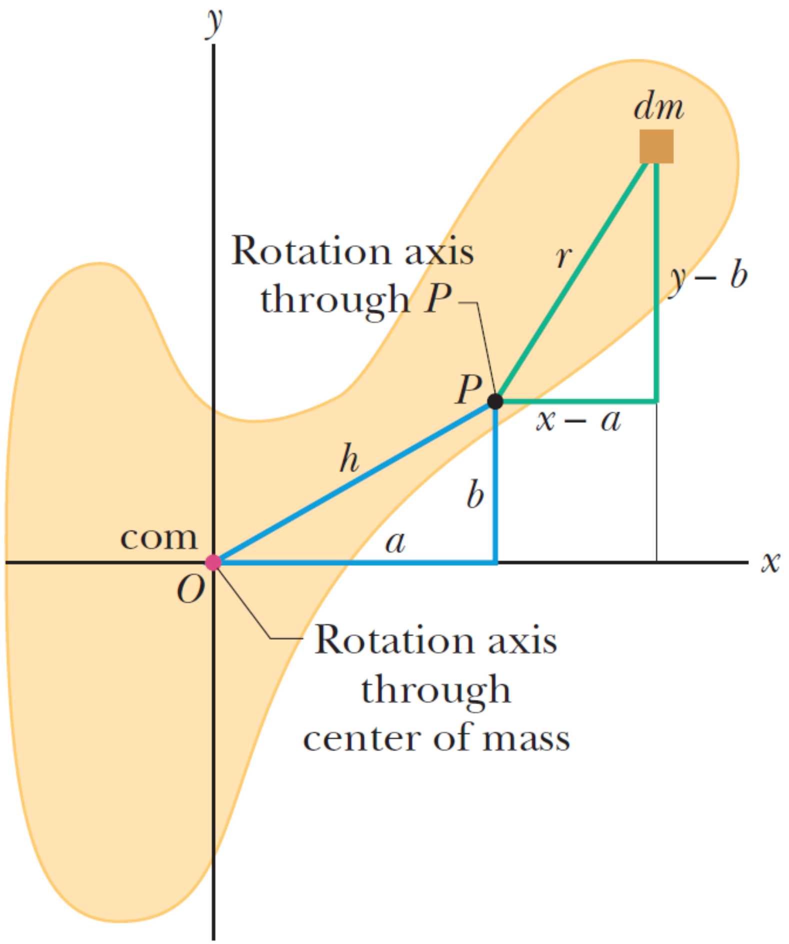
Rotational kinetic energy - example

Parallel axes theorem

If the rotational axis is moved, the rotational inertia also changes. To find the new inertia we can use the parallel axes theorem. If I_{com} is the rotational inertia for an axis through the centre of mass:

$$I = I_{com} + Mh^2$$

Where *M* is the mass of the body and *h* is the distance between the new axis and the original one.



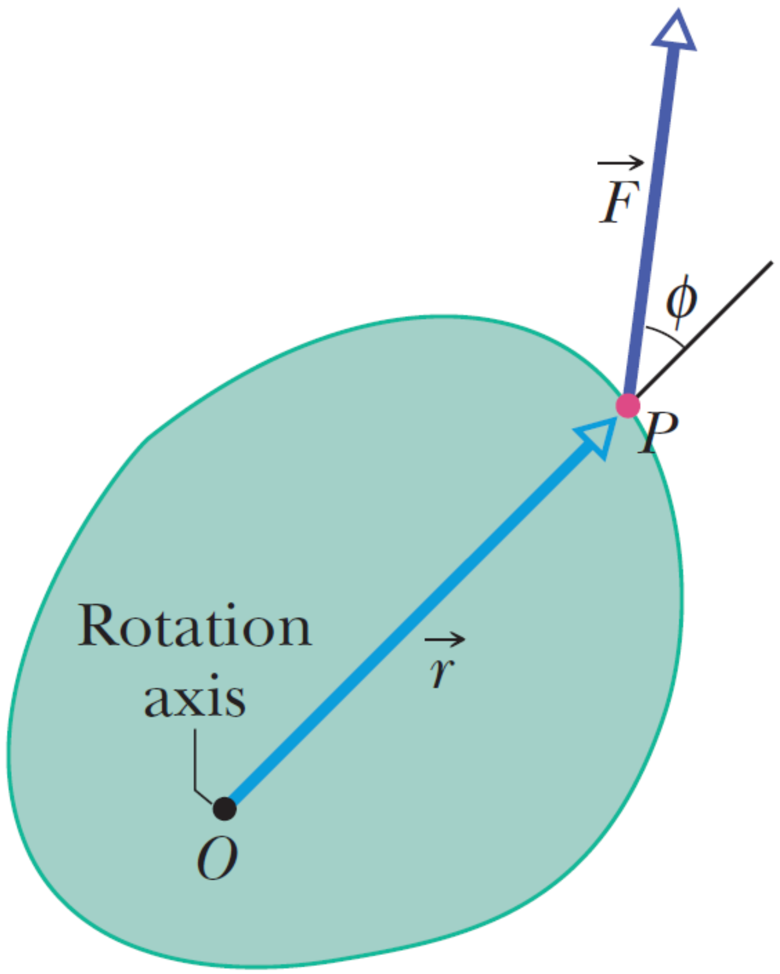
Torque

An object can acquire angular acceleration if a force acts on it.

The effectiveness of the force will be related to its point of application and its direction.

The force can be resolved in two directions, with a radial and a tangential components.

The tangential component will cause an angular acceleration.



Torque

The angular acceleration is determined by the *torque*.

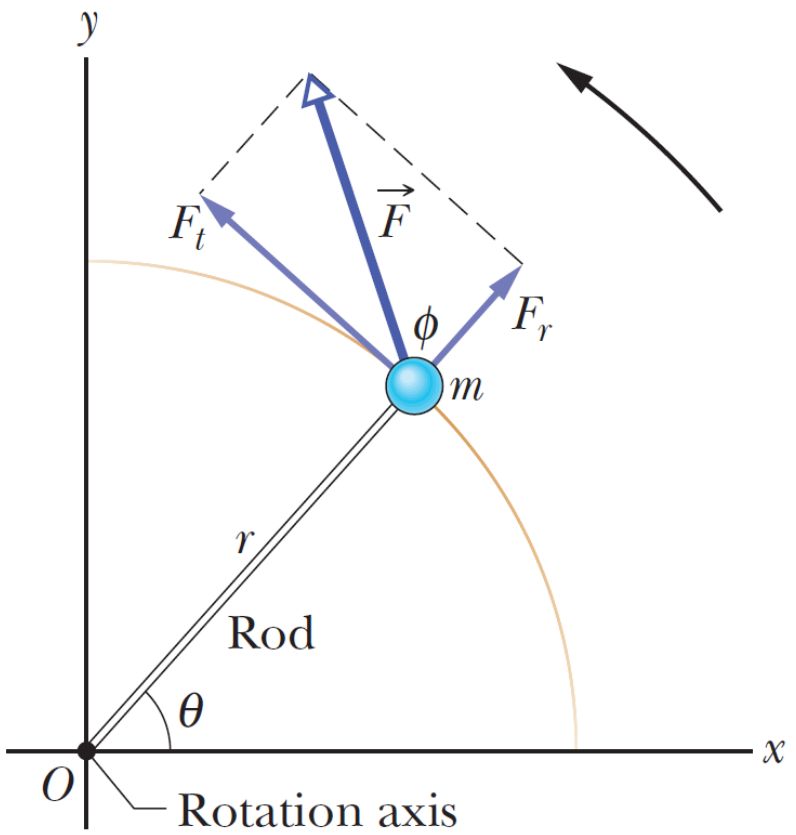
This is the product of the force and the distance between the point of application and the centre of rotation (*r*).

$$\tau = rF_t = rF\sin\phi$$

Newton's second law of motion is also applicable to rotational motion.

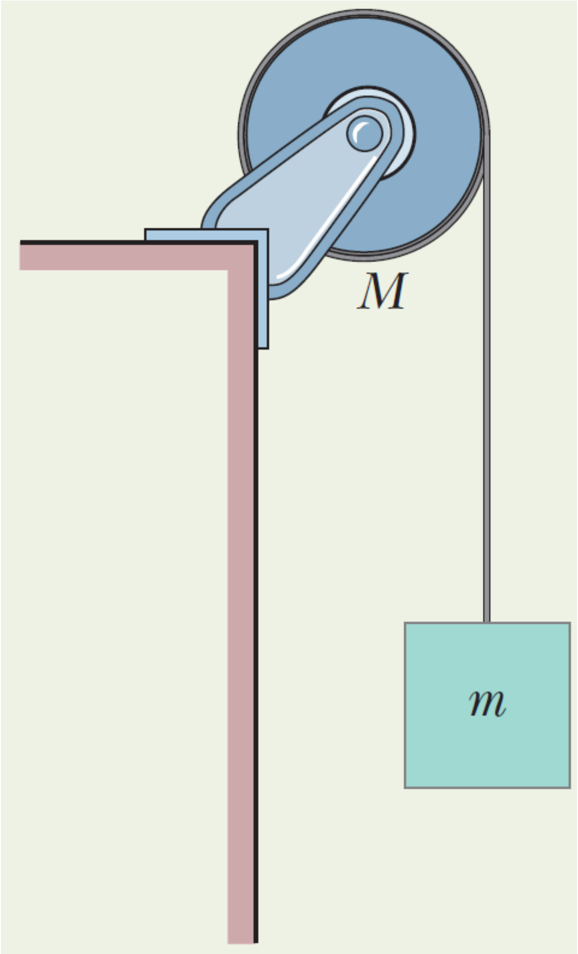
$$\tau_{net} = I\alpha$$

As for the energy, the mass is substituted by the rotational inertia.

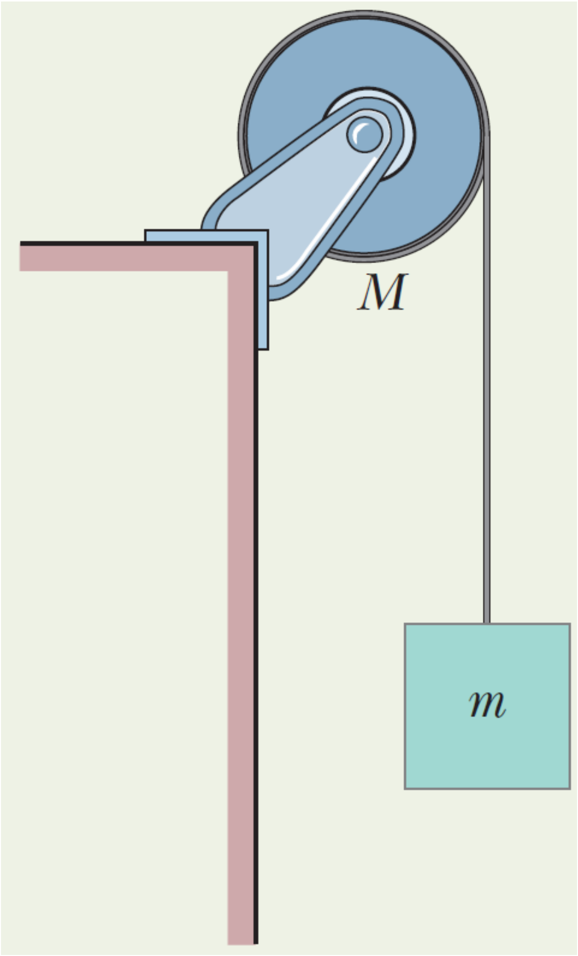


Newton's second law for rotation - example

A uniform disk, with mass M = 2.5 kg and radius R = 20 cm, mounted on a fixed horizontal axle. A block with mass m = 1.2 kg hangs from a massless cord that is wrapped around the rim of the disk. Find the acceleration of the falling block, the angular acceleration of the disk, and the tension in the cord. The cord does not slip, and there is no friction at the axle.



Newton’s second law for rotation - example



Angular momentum

Angular momentum is given by:

Angular momentum of a particle relative to origin O of an inertial frame of reference

Position vector of particle relative to O

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$$

Linear momentum of particle = mass times velocity

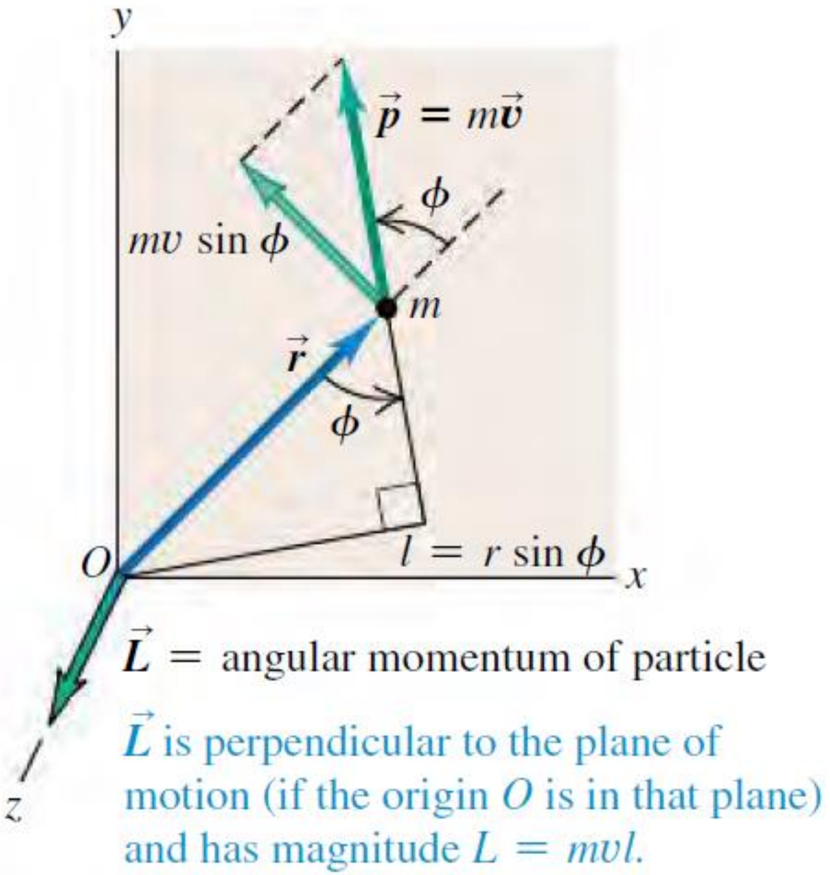
Its direction is perpendicular to the plane of rotation and its magnitude is:

$$L = mvr\sin\phi$$

For rigid body rotation, it can be shown that the total rotational momentum will be:

$$L = I\omega$$

$$P = mv$$

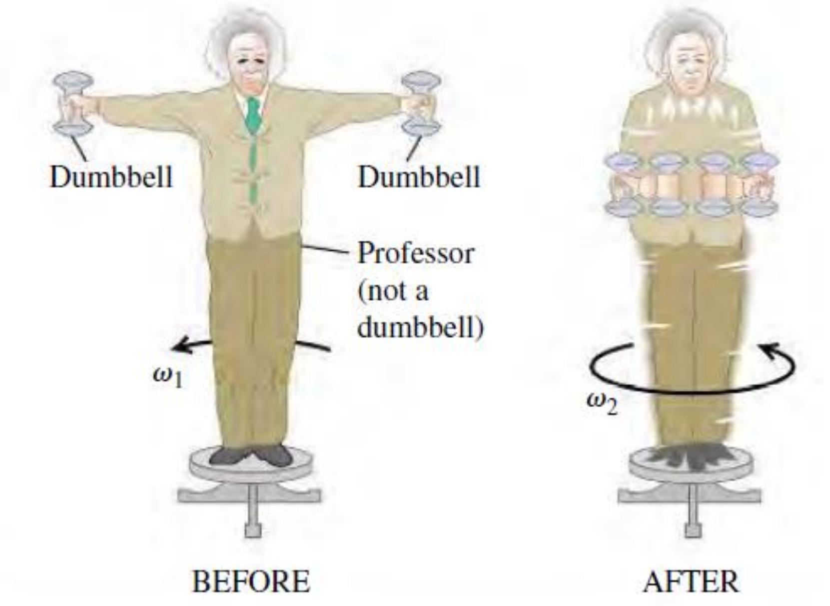


Conservation of angular momentum

Like linear momentum, with no external torque angular momentum is conserved:

$$L_1 = L_2; \quad I_1\omega_1 = I_2\omega_2$$

Note: the linear and rotational momentum are NOT the same. If the rotation is caused by an impact, the original linear momentum has to be converted.



AY 22-23, Trimester 1

SHM

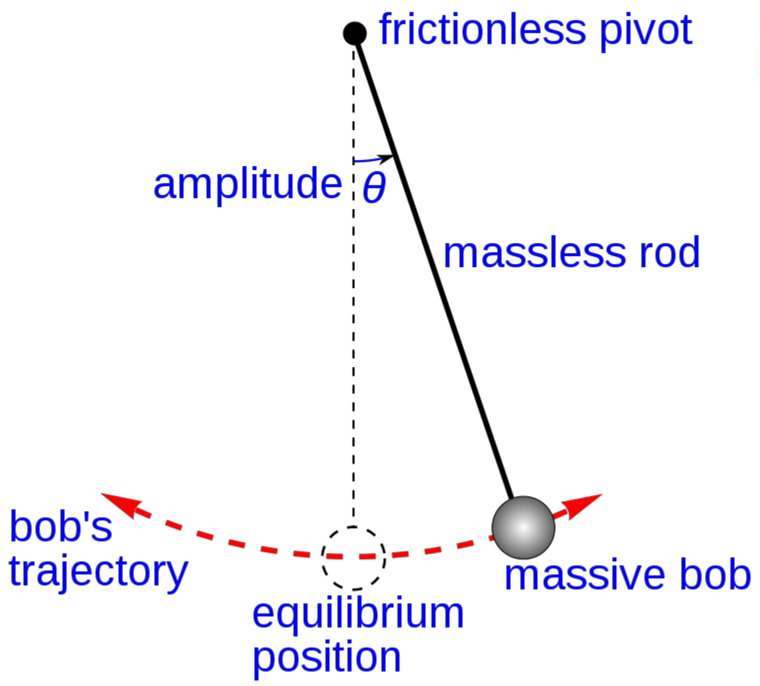
Harmonic motion

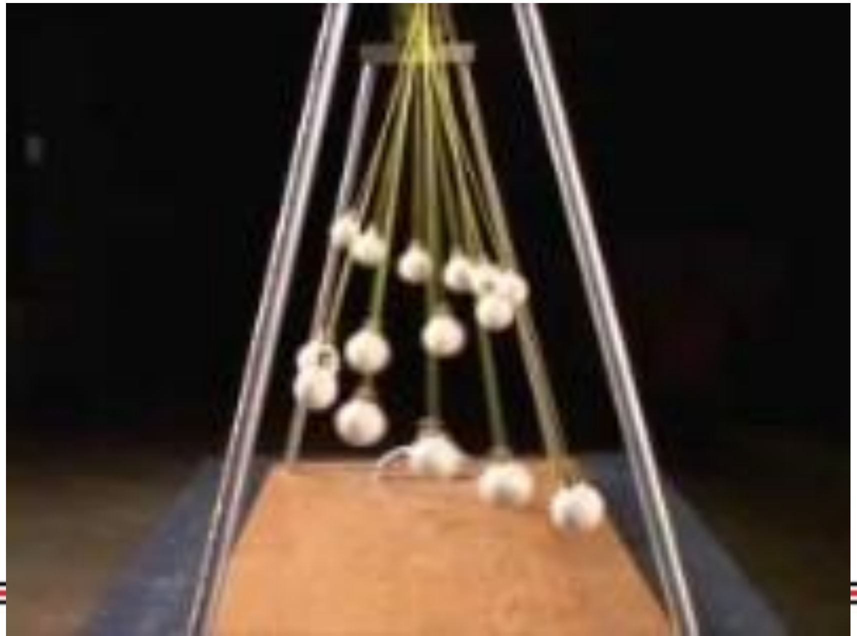
We can see that some systems move in a regular manner

The movements seem to repeat themselves over time, with a more or less simple pattern.

Some of these cases fall under Harmonic Motion.

We can study their properties to predict the movements.





Frequency and periods – circular motion analogy

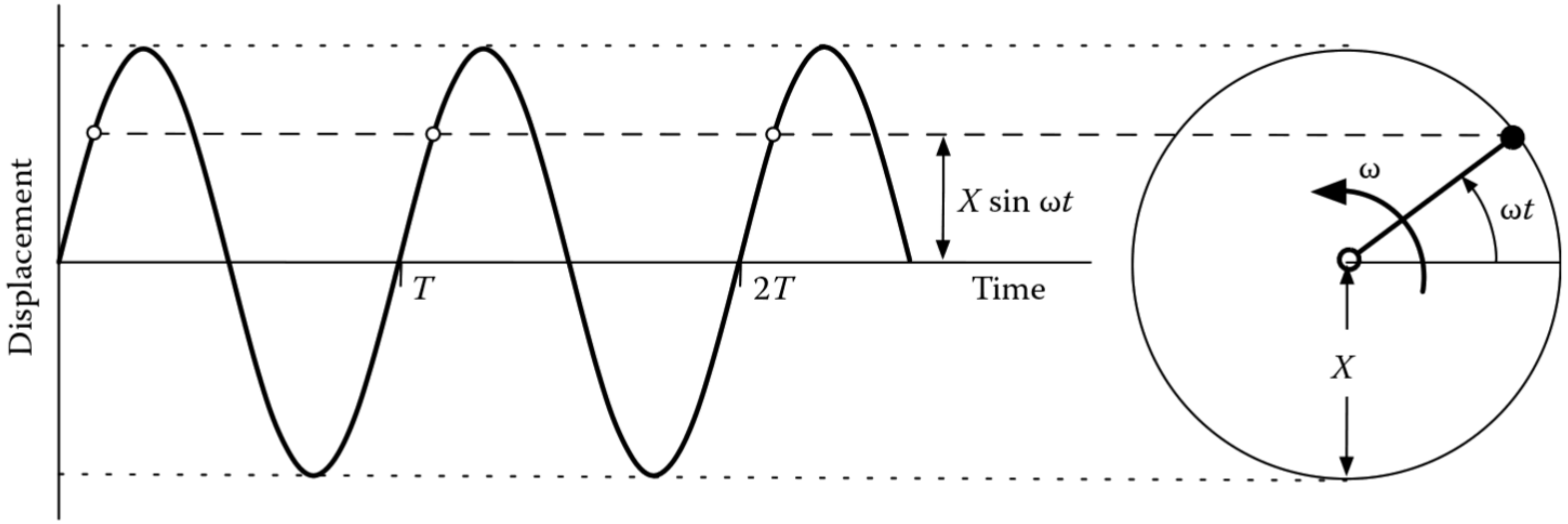
Harmonic motion can be represented as a rotational motion.

The frequency and periods become related to the time to cover a full circle.

Frequency and periods – circular motion analogy

We can see that the harmonic motion can be represented as a rotational motion.

The frequency and periods become related to the time to cover a full circle.



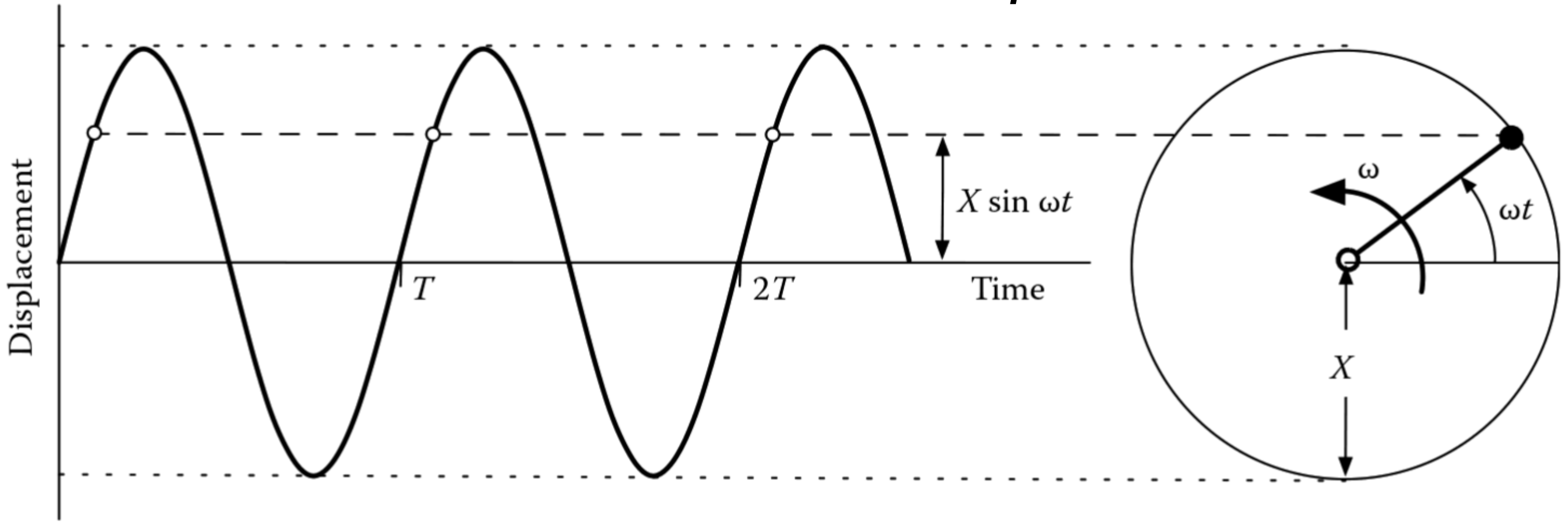
Frequency and periods

Before we used an expression like: $x = A\sin(2\pi f t)$

We can also write $x = A\sin\left(\frac{2\pi}{T}t\right)$, where T is the period, i.e. the time taken for a full cycle and A is the amplitude.

We can express the instead as: $x = A\sin(\omega t)$ where $\omega = 2\pi f = \frac{2\pi}{T}$.

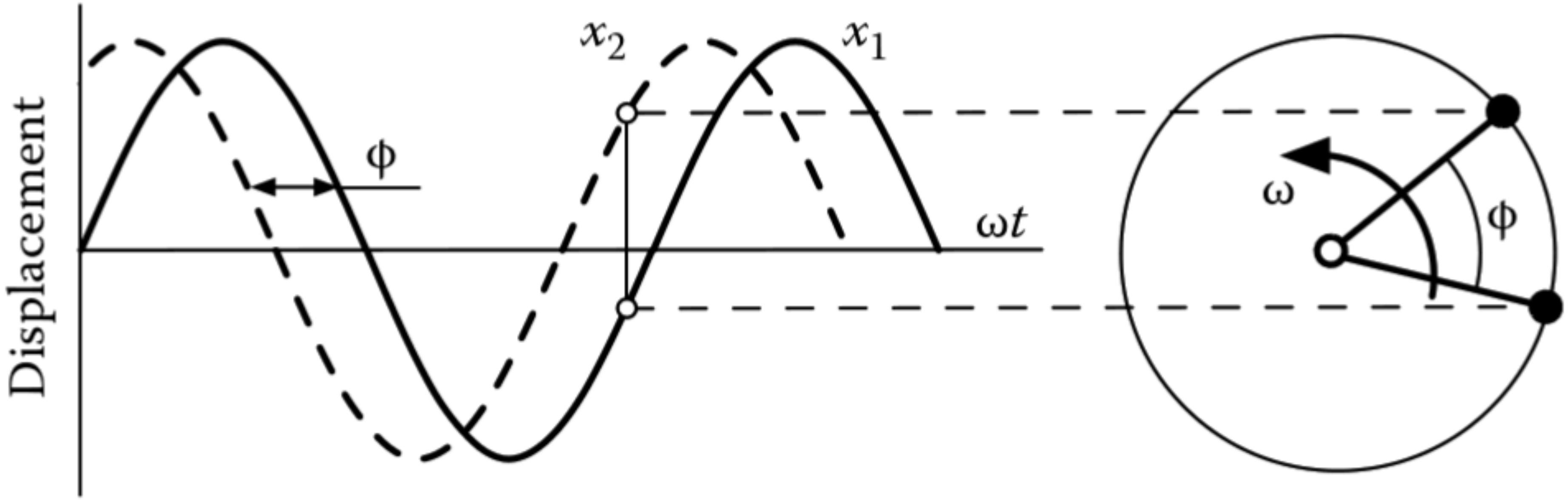
ω is the circular frequency and is measured in rad/s.



Lag / phase

If we have a separate oscillation which has the same frequency, but has a time lag, we can define its movement as: $x = A\sin\left(\omega\left(t+\tau\right)\right)$. Or, more conveniently, $x = A\sin\left(\omega t+\phi\right)$

This can be seen as another point moving along the same circle, and angle ϕ away.



Harmonic motion – velocity and acceleration

We can differentiate the position equation to find the velocity time history.

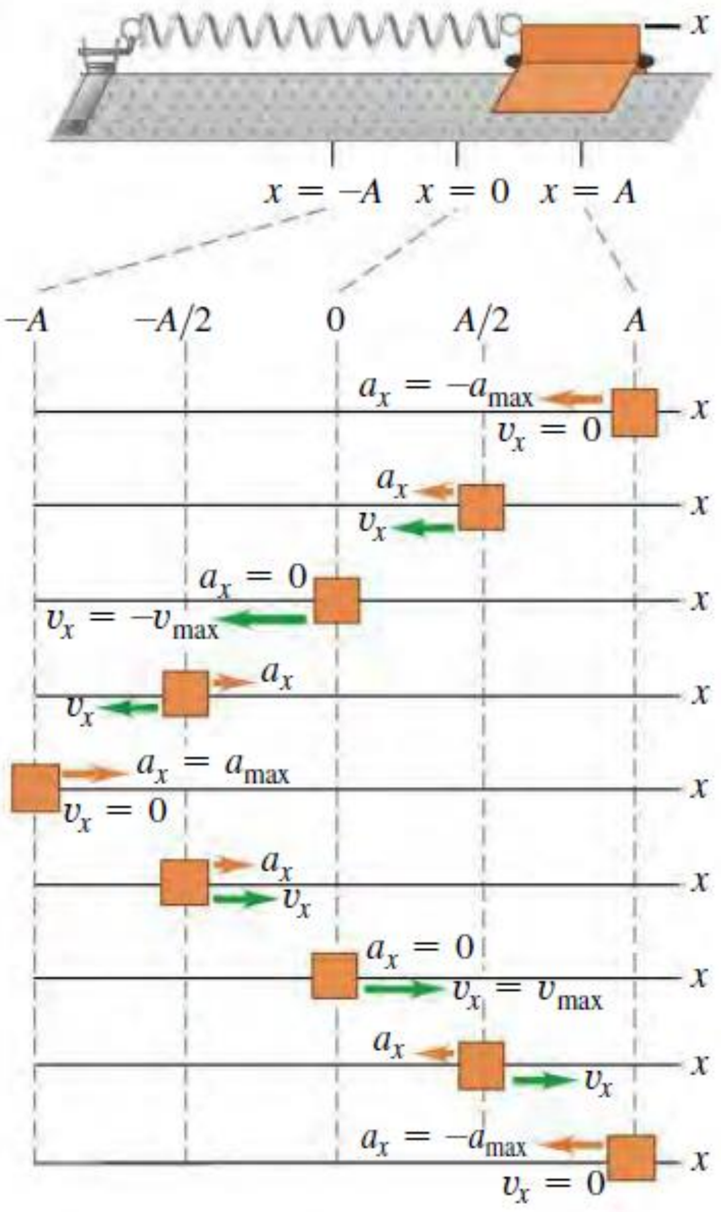
$$x = A\sin(\omega t)$$

$$\dot{x} = \frac{dx}{dt} = A\omega\cos(\omega t)$$

We can differentiate this again to get the acceleration time history:

$$\ddot{x} = \frac{d\dot{x}}{dt} = -A\omega^2\sin(\omega t) = -\omega^2 x$$

These three quantities are $\pi/2$ away from each other.



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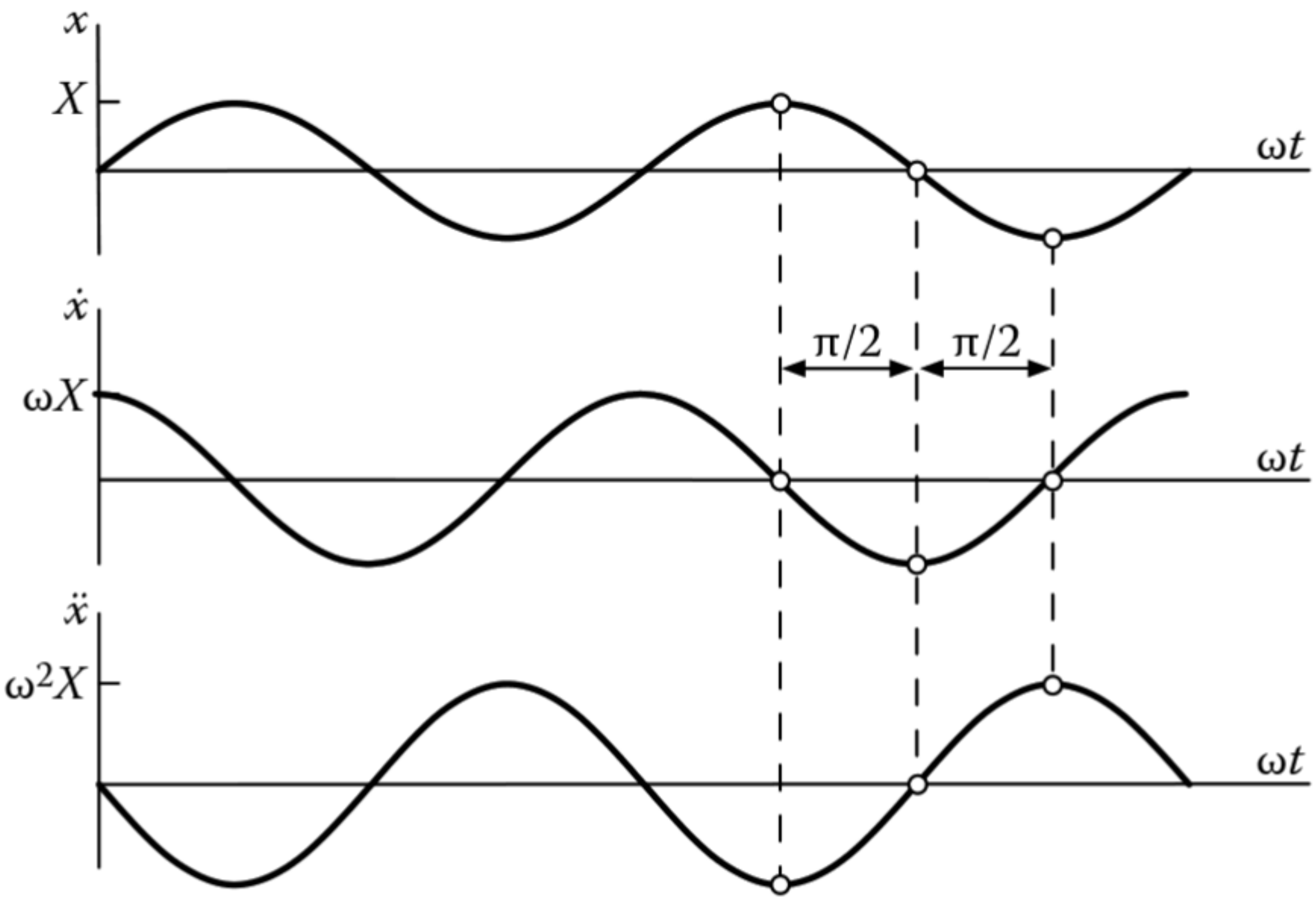


Harmonic motion – velocity and acceleration

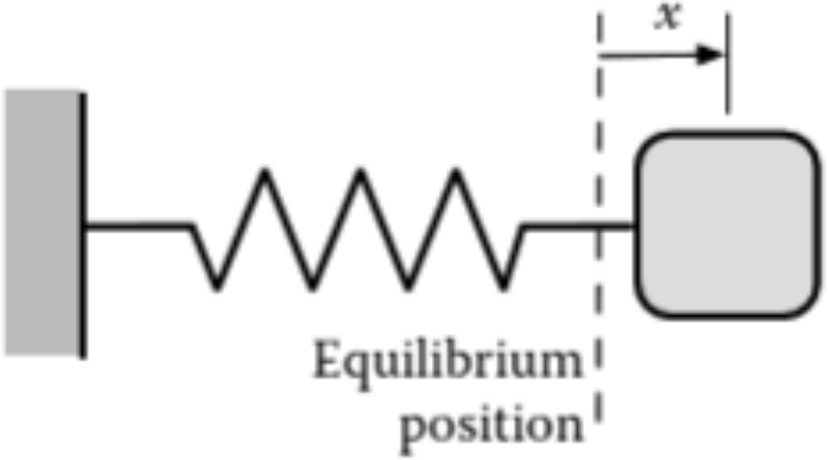
$$\ddot{x} = \frac{d\dot{x}}{dt} = -A\omega^2 \sin(\omega t) = -\omega^2 x$$

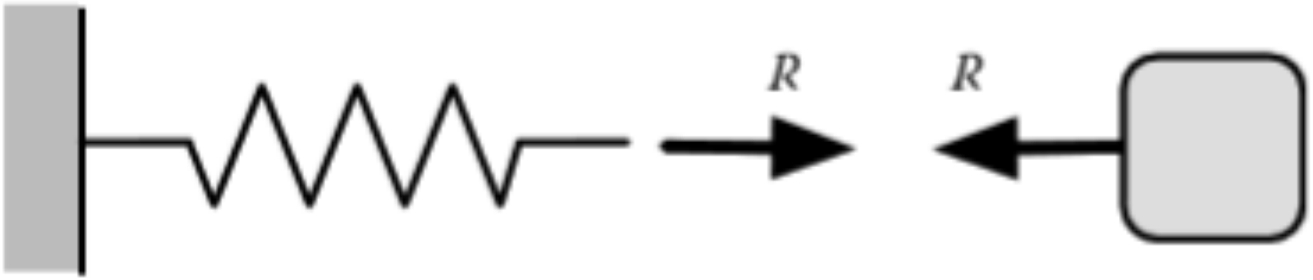
This final result is a general requirement for harmonic motion.

In general, the acceleration, and hence the force applied, needs to oppose the motion.

A typical example of this is a spring system.

We can also use this definition to get some information about the vibration frequency.





Harmonic motion – velocity and acceleration

$$\ddot{x} = -\omega^2 x$$

The force given by a spring is:

$$F = -kx$$

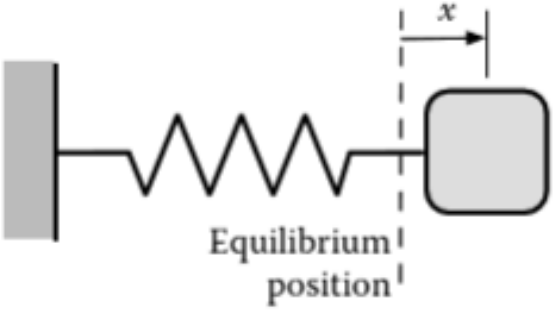
For Newton's second law,

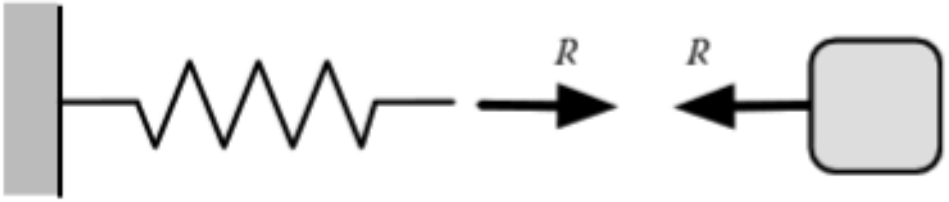
$$F_{net} = ma; \quad -kx = ma; \quad -kx = m\ddot{x}$$

Therefore,

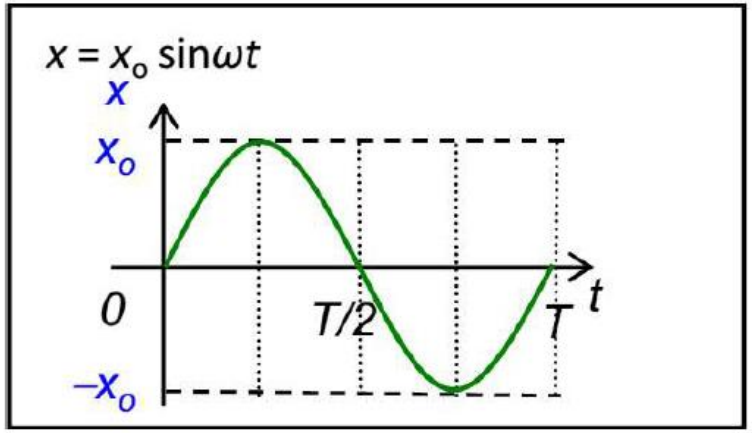
$$\ddot{x} = -\frac{k}{m}x$$

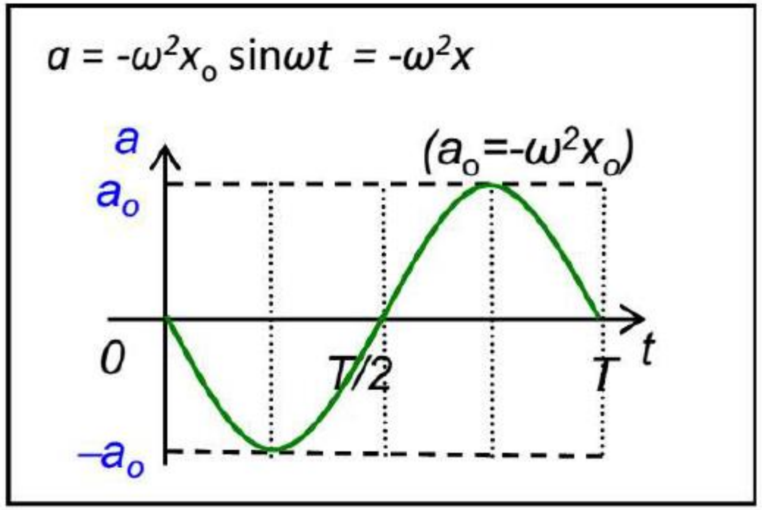
Meaning that: $\omega^2 = \frac{k}{m}; \ \omega = \sqrt{\frac{k}{m}}$

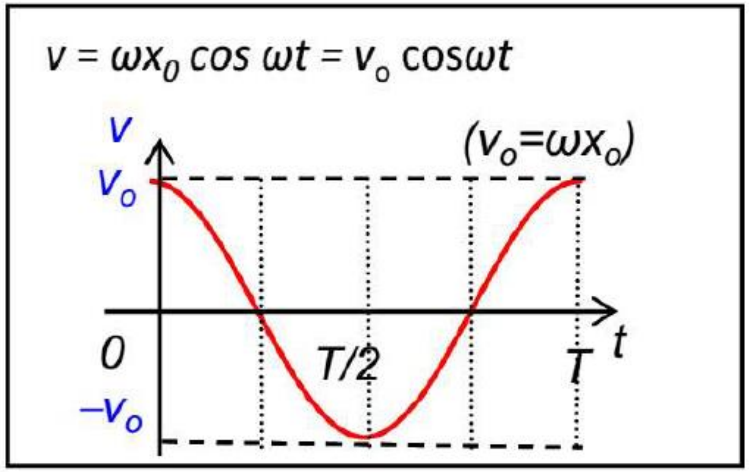


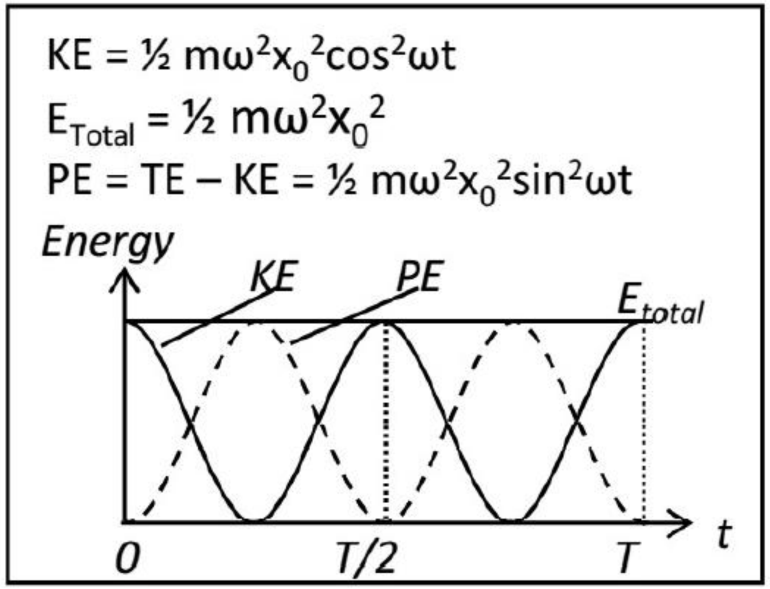


Harmonic motion – variations with time

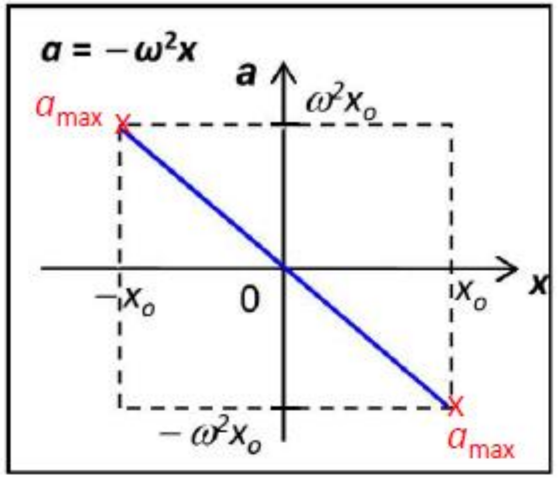


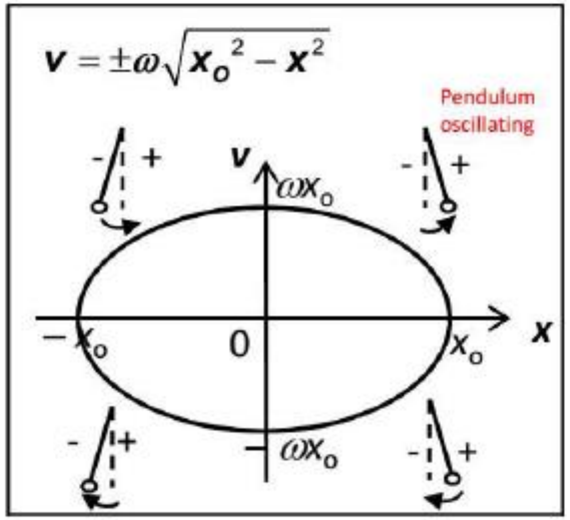






Harmonic motion – variations with displacement

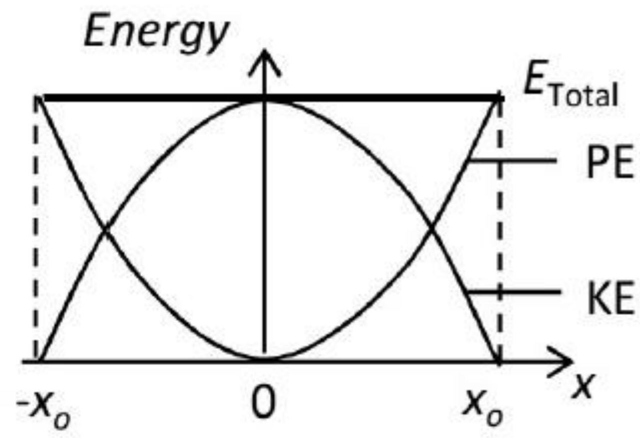




$$E_{Total} = KE + PE = \tfrac{1}{2} mv^2 + \tfrac{1}{2} kx^2$$

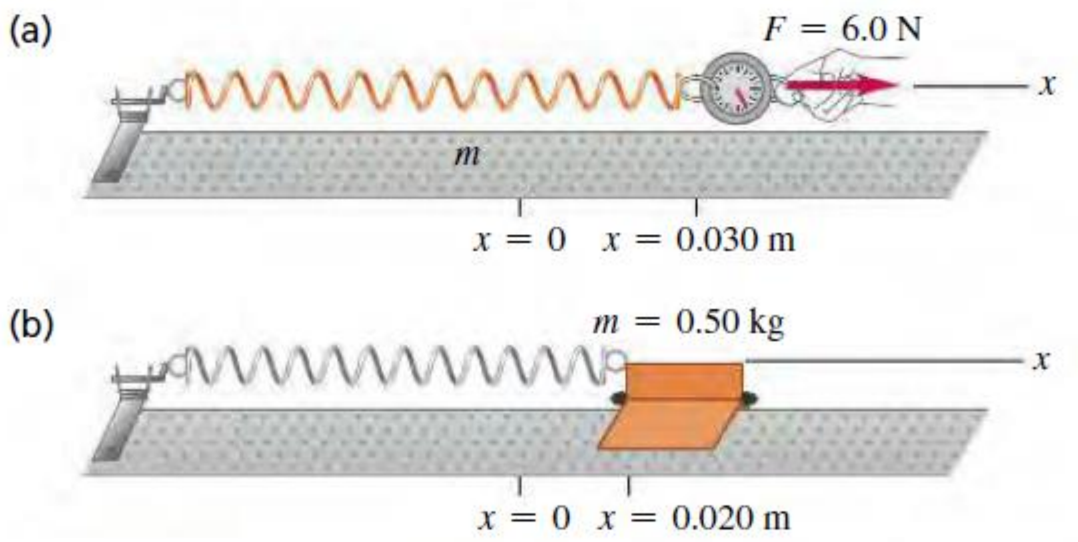
$$= \tfrac{1}{2} m\omega^2(x_o^2 - x^2) + \tfrac{1}{2} m\omega^2 x^2$$

$$= \tfrac{1}{2} m\omega^2 x_o^2$$

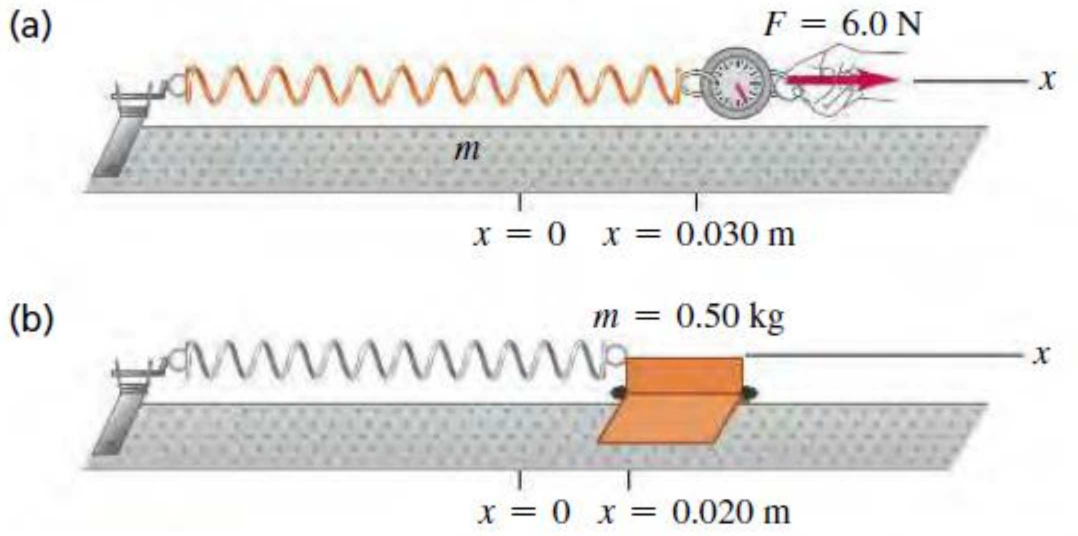


Harmonic motion – example

A spring is mounted horizontally, with its left end fixed. A spring balance attached to the free end and pulled toward the right (**Fig. 14.8a**) indicates that the stretching force is proportional to the displacement, and a force of 6.0 N causes a displacement of 0.030 m. We replace the spring balance with a 0.50-kg glider, pull it 0.020 m to the right along a frictionless air track, and release it from rest (Fig. 14.8b). (a) Find the force constant k of the spring. (b) Find the angular frequency ω, frequency f, and period T of the resulting oscillation.



Harmonic motion – example



Damping

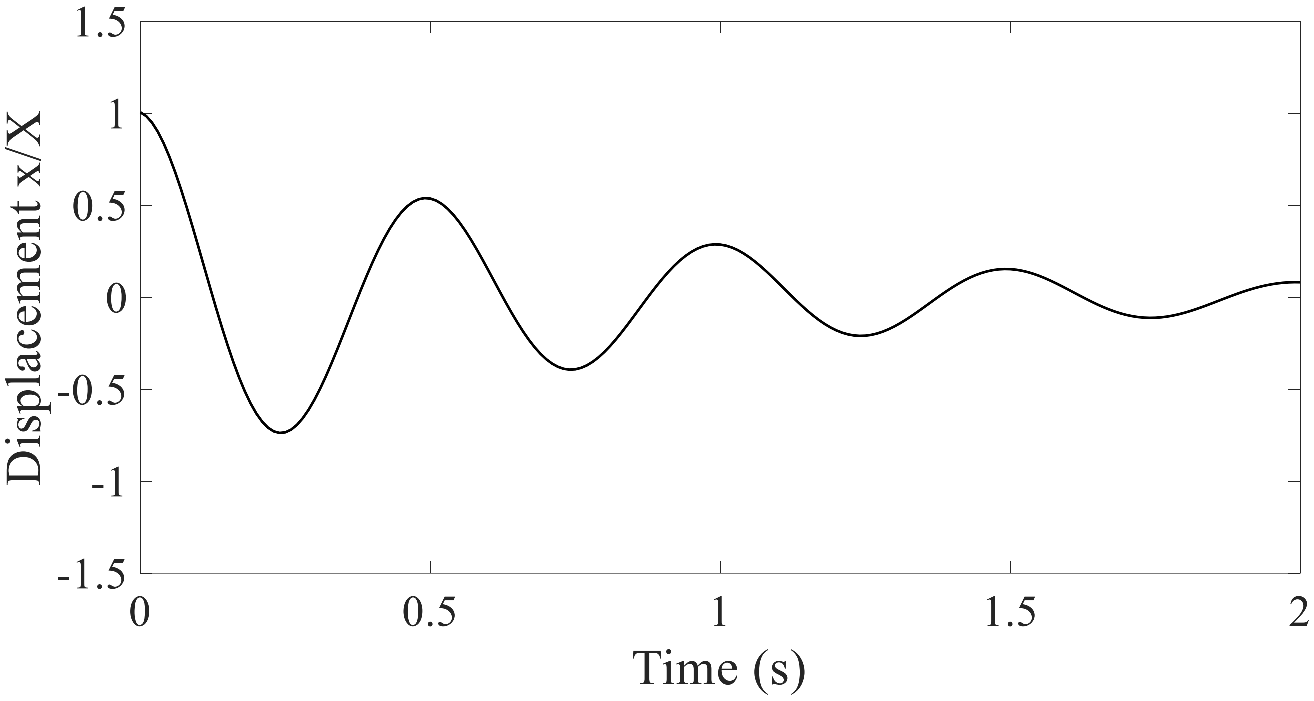
Real systems generally slow down!

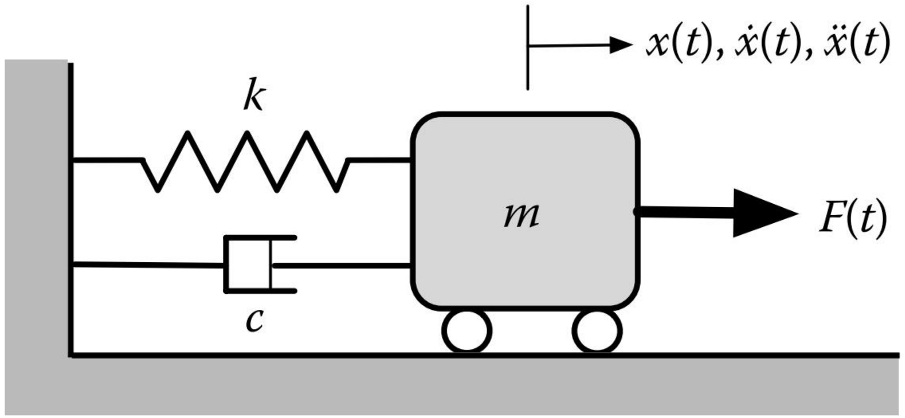
There are many mechanisms which slow down movement, removing energy.

Accounting for these in detail is complex and generally unpractical.

Instead, we generally just group them together and consider them as a single “damping”.

This can be visualized as a damper acting on the moving mass.





(a)

Damping measure: damping ratio

A very convenient way to measure the damping is with the damping ratio. This is defined as:

$$\xi = \frac{c}{2\sqrt{km}}$$

The usefulness mathematically is evident when solving the equation of motion (later!)

The damping ratio is also pretty constant for similar structures, and is often used for design and analysis (at last for earthquake design!)

Serviceability (Damping can be lower for low-level vibration, e.g. for human-comfort consideration)	Welded steel, prestressed concrete, RC with slight cracking	1-3%
	RC with considerable cracking	3-5%
	Bolted/riveted steel	5-7%
At or near yield point (Earth quake normally use 5%)	Welded steel, prestressed concrete without complete loss in prestress	5-7%
	RC structures, prestressed concrete with complete loss in prestress	7-10%
	Bolted/riveted steel, or wood structures with nailed/bolted joints	10-15%

Under damped

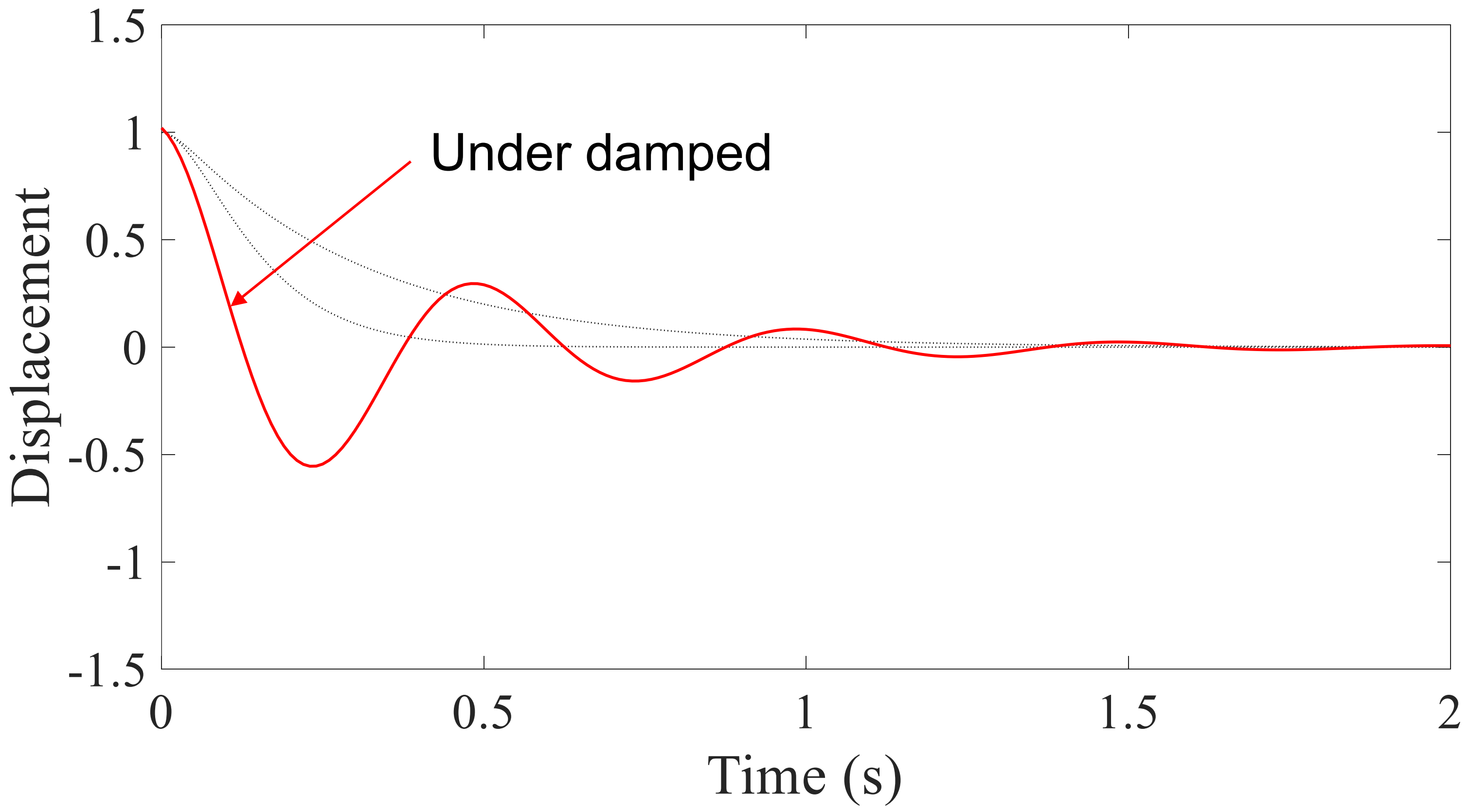
We can find the constants for the usual case of initial displacement, finding:

$$x = \frac{X}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin\left(\omega_d t + \phi\right) \quad \text{Where} \quad \tan\phi = \frac{\sqrt{1-\xi^2}}{\xi}$$

The equation can be reduced to the non damped case seen before.

In this case, we have sustained vibrations, though these reduce due to the exponent.

This is the most common case seen in structures.



Harmonic loading - resonance

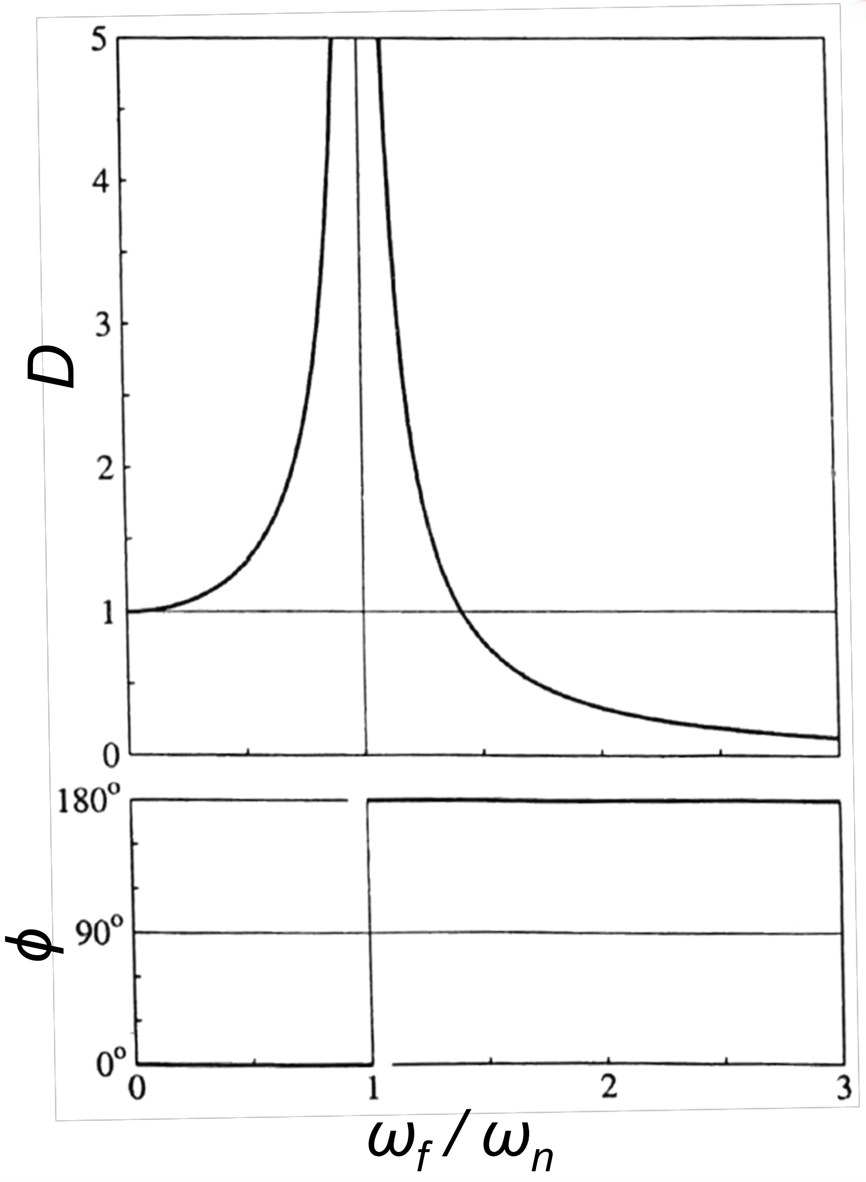
The loading can be periodic as well. In that case, with no damping the deflection time history can be found to be:

$$x = x_0 D \sin\left(\omega_F t - \phi\right)$$

Where:

$$D = \frac{1}{\left|1 - \omega_F{}^2 / \omega_n{}^2\right|}$$

When the load period ω_f and the natural period ω_n match, resonance occurs. D goes to infinity.



Harmonic loading - damping

In the damped case, the deflections are bound (not infinite).

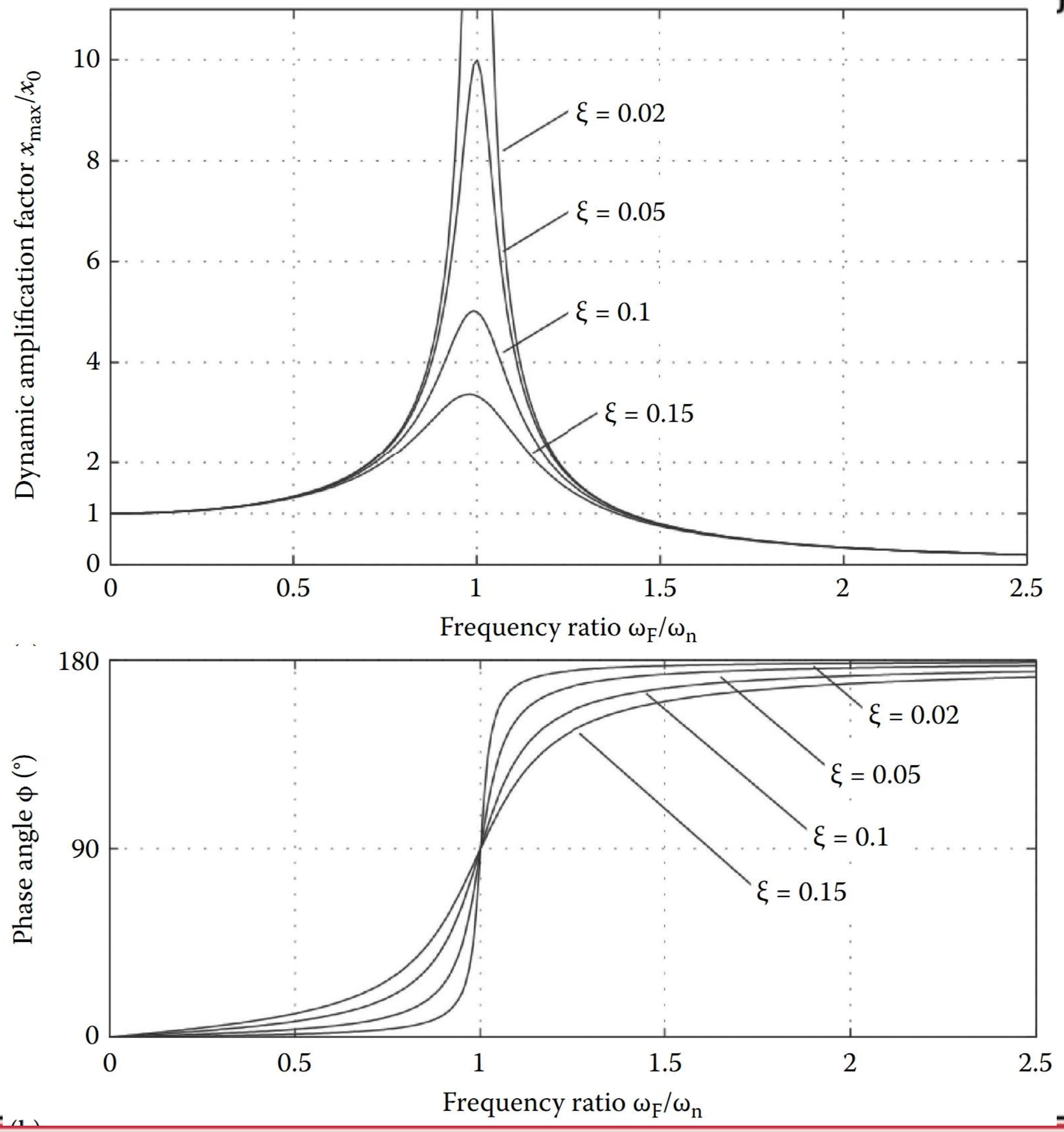
The damping takes some energy away from the structure.

The steady state solution can be again expressed as:

$$x = x_0 D \sin\left(\omega_F t - \phi\right)$$

Where $D = \dfrac{1}{\sqrt{\left(1-\Omega^2\right)^2 + \left(2\xi\Omega\right)^2}}$

and $\tan\phi = \dfrac{2\xi\Omega}{1-\Omega^2}$, with $\Omega = \dfrac{\omega_F}{\omega_n}$

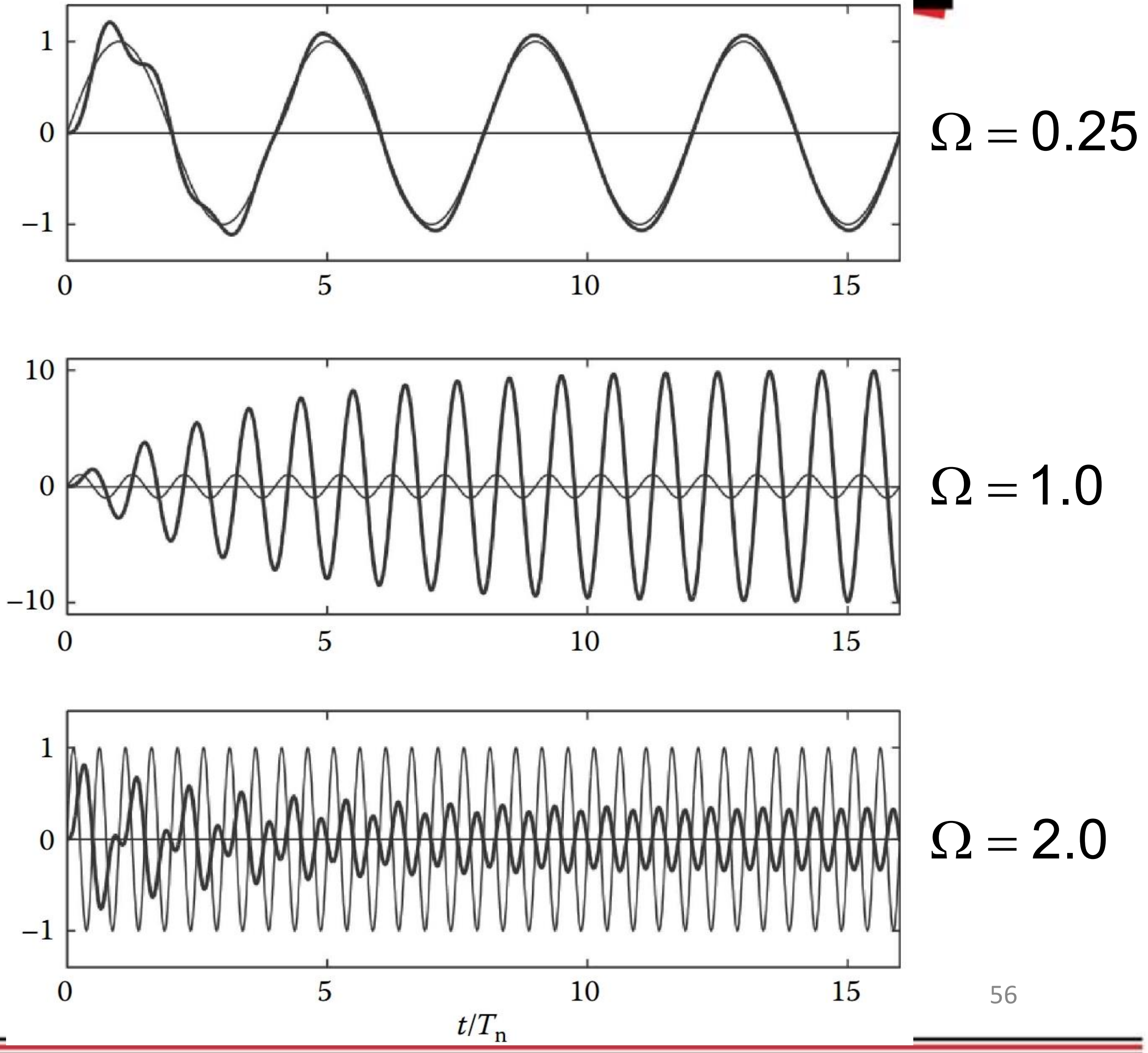


Harmonic loading – response evolution

We can see the evolution of the response of systems at three frequency ratios, below resonance, at resonance and above resonance.

Some time is needed to reach the steady state.

Whilst we tend to not consider this intermediate period, the maximum deflections could happen there away from resonance. However, a non resonant case it is unlikely to be critical for design.



SHM Formulae

$$x = A\sin(\omega t)$$

$$\dot{x} = \frac{dx}{dt} = A\omega\cos(\omega t)$$

$$\ddot{x} = \frac{d\dot{x}}{dt} = -A\omega^2\sin(\omega t) = -\omega^2 x$$

$$\omega^2 = \frac{k}{m}; \quad \omega = \sqrt{\frac{k}{m}}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$x = A\cos(\omega t)$$

$$\dot{x} = \frac{dx}{dt} = -A\omega\, sin(\omega t)$$

$$\ddot{x} = \frac{d\dot{x}}{dt} = -A\omega^2\cos(\omega t) = -\omega^2 x$$

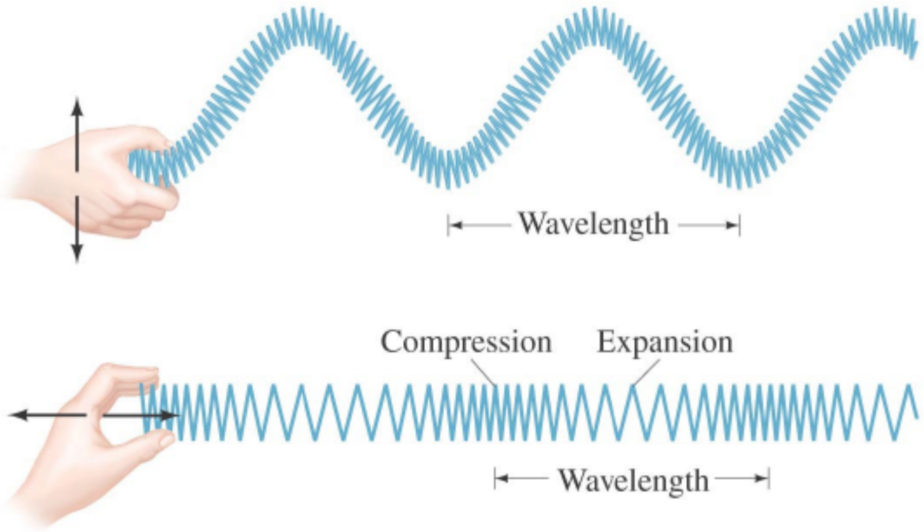
Travelling Waves

Characteristics of a wave

- The motion of particles in a wave can be either

(a) perpendicular to the wave direction (transverse) or

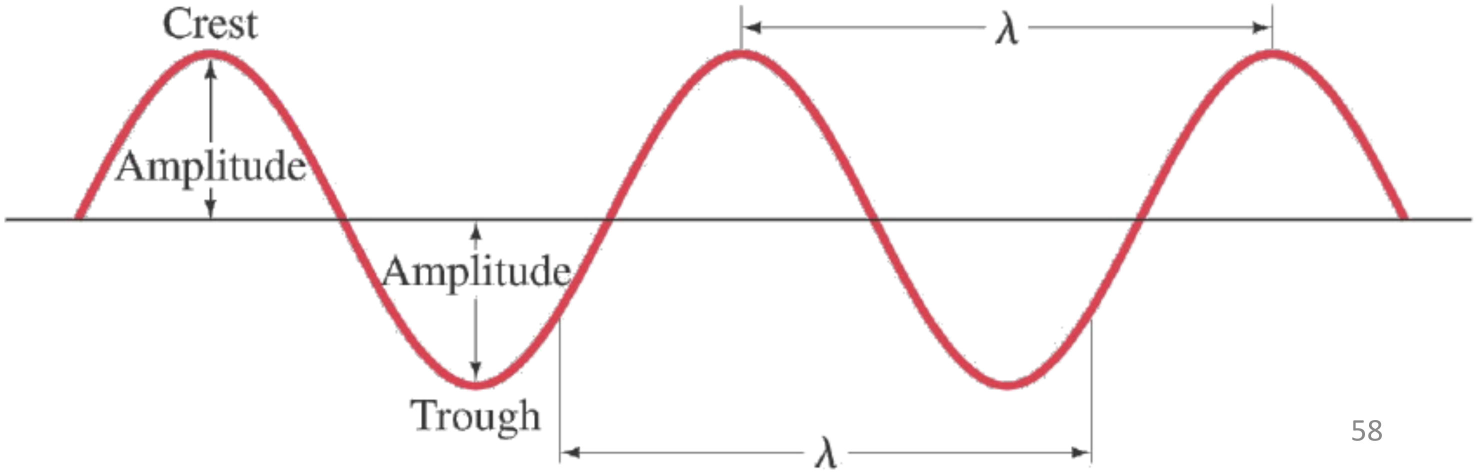
(b) parallel to the direction (longitudinal).



- **Waves transport energy but not matter**

Sinusoidal wave has the following characteristics:

- Amplitude, A (m)
- Wavelength, λ (m)
- Frequency, f (Hz)
- Time period, T (s)
- Wave velocity, v (m/s)



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Wave equation

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Representation of Traveling wave

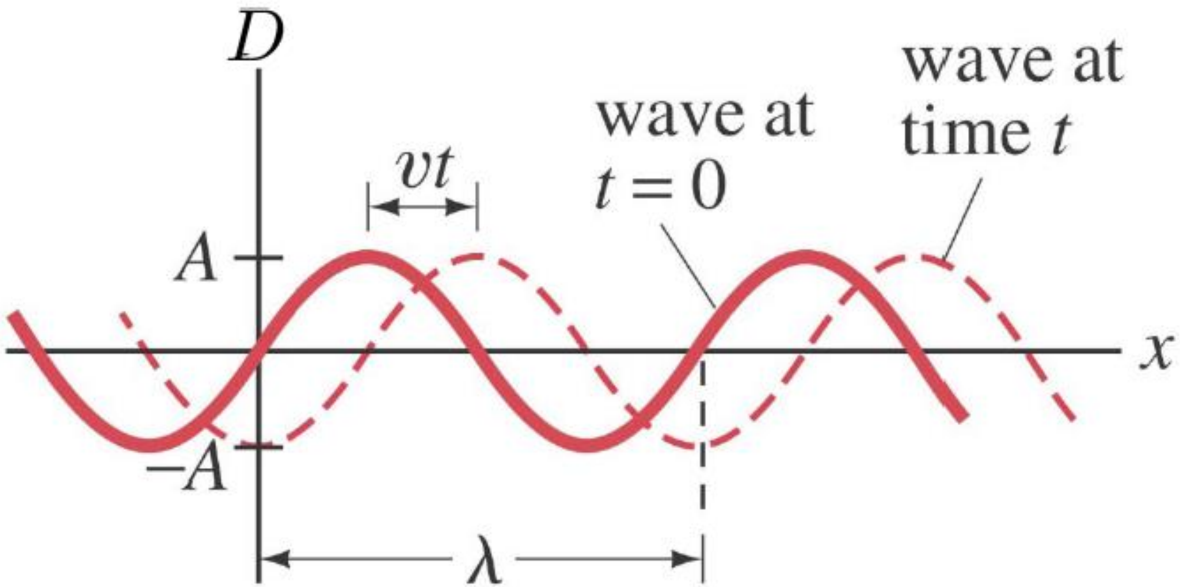
- The shape of a wave at $t = 0$ is mathematically given by: $D(x, t=0) = A\sin\left(\frac{2\pi}{\lambda}x\right)$

- After a time t, the wave has traveled a distance vt,

$$D(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right].$$

or $$D(x,t) = A\sin(kx - \omega t),$$

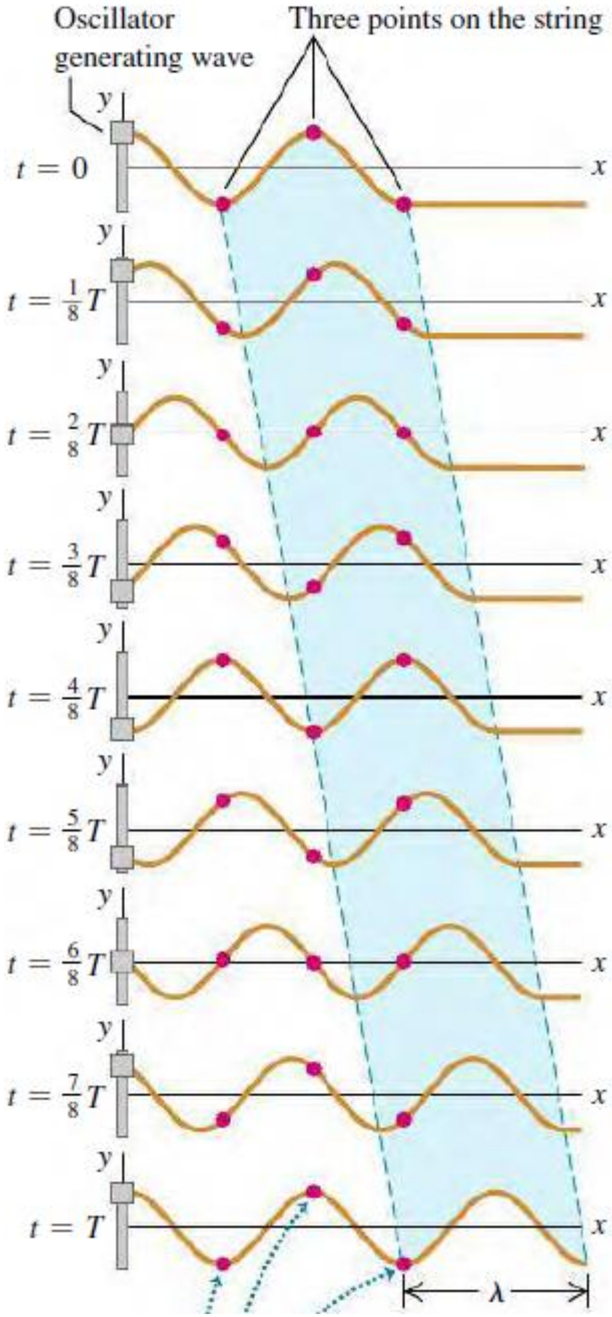
where $\omega = 2\pi f$ and $k = \frac{2\pi}{\lambda}.$



- ω is the **angular frequency** in rad/s or s^{-1}

- k is the **wave number** with unit m^{-1}

Wave speed



Some other useful equations:

- The wave velocity can be found as:

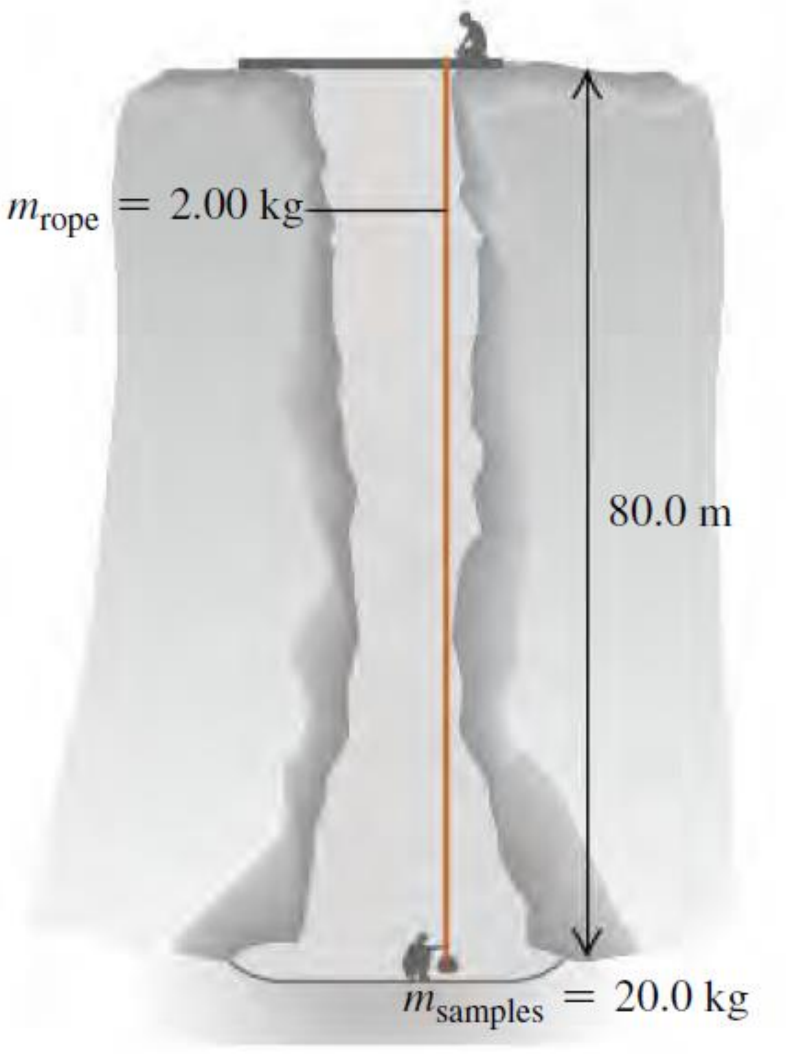
$$v = \frac{dx}{dt} = \frac{\omega}{k} = f\lambda$$

- For string in tension, the wave velocity is:

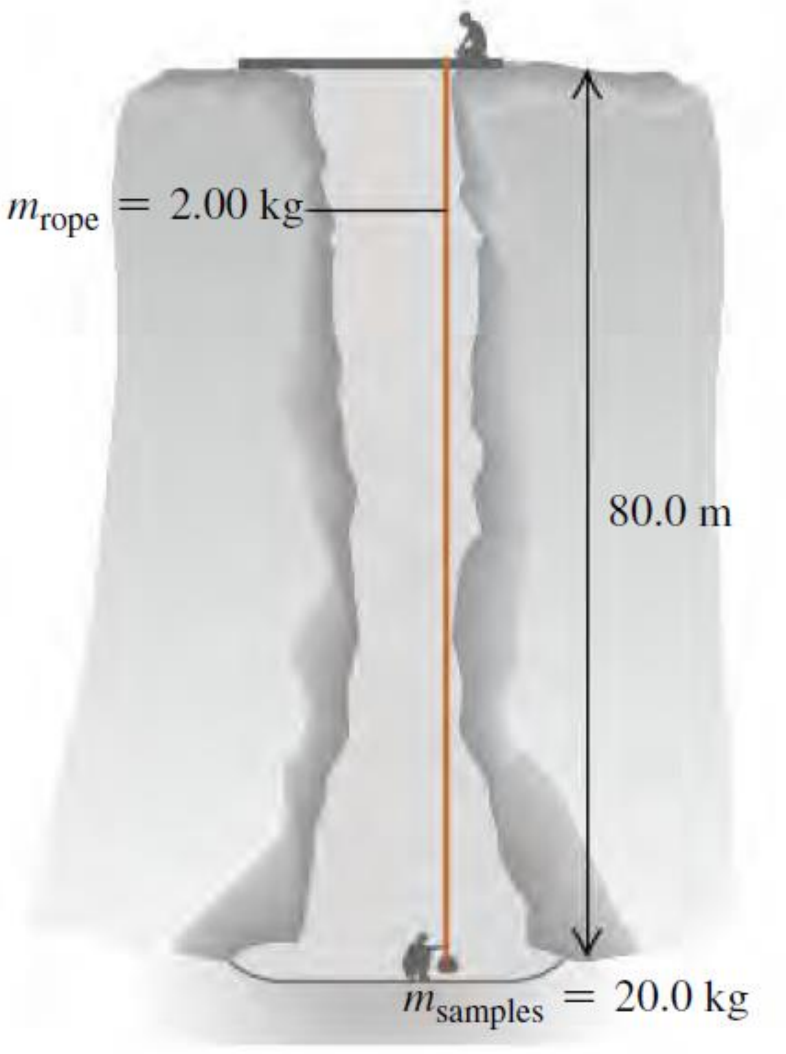
$$v = \sqrt{\frac{F_T}{\mu}}$$

Travelling waves example 2

One end of a 2.00-kg rope is tied to a support at the top of a mine shaft 80.0 m deep (**Fig. 15.14**). The rope is stretched taut by a 20.0-kg box of rocks attached at the bottom. (a) A geologist at the bottom of the shaft signals to a colleague at the top by jerking the rope sideways. What is the speed of a transverse wave on the rope? (b) If a point on the rope is in transverse SHM with $f = 2.00$ Hz, how many cycles of the wave are there in the rope's length?



Travelling waves example



Intensity

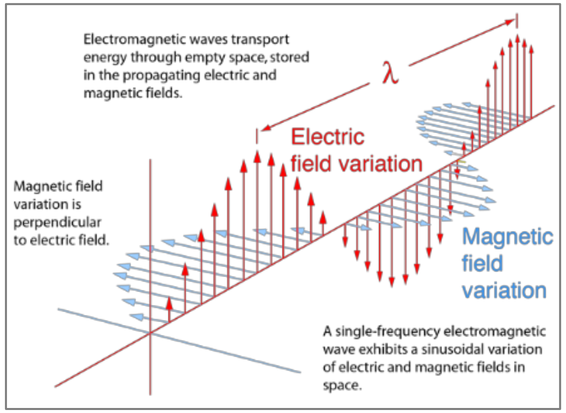
1. Energy possessed by wave particle (& eventually transferred to next particle):

$$E_{\text{total}} = \frac{1}{2} m\omega^2 x_0{}^2 = \frac{1}{2} kA^2$$

Amplitude: $x_0 = A$

Constant: $k = m\omega^2$

Total energy, $E = E_{\text{total}}$



Source: http://hyperphysics.phy-astr.gsu.edu/hbase/Waves/emwavecon.html

2. Hence, $E \propto A^2$

3. Intensity of wave, *I*: Rate of energy flow, $\frac{\mathrm{d}E}{\mathrm{d}t}$, per unit surface area $I = \frac{\frac{\mathrm{d}E}{\mathrm{d}t}}{S}$ perpendicular to direction of wave propagation (or wave motion).

4. Intensity: $I = \frac{\text{Power}}{\perp \text{Area}} = \frac{P}{S}$

5. Hence, $\boxed{I \propto E \propto A^2}$

Use "*S*" for area because we already used "*A*" for amplitude

Intensity

Wave propagation methods

Spherical Wave:

- For wave originating from point source, area covered is spherical.
- Every point of impact on spherical surface can approx.. to be perpendicular plane.
- As distance *r* from point source increases, surface area increases.

$$I = \frac{P}{S} = \frac{P}{4\pi r^2} \Rightarrow I \propto \frac{1}{r^2} \quad \text{Inverse Square Law}$$

Area of spherical surface, $S = 4\pi r^2$

- Comparing intensities from same wave at different distances r_1 & r_2 from point source: $$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$
- How does amplitude *A* of spherical wave change with distance *r* from source?

$$I \propto A^2 \propto \frac{1}{r^2} \qquad \Rightarrow A \propto \frac{1}{r}$$

Further from point source, means amplitude is smaller.

Intensity example

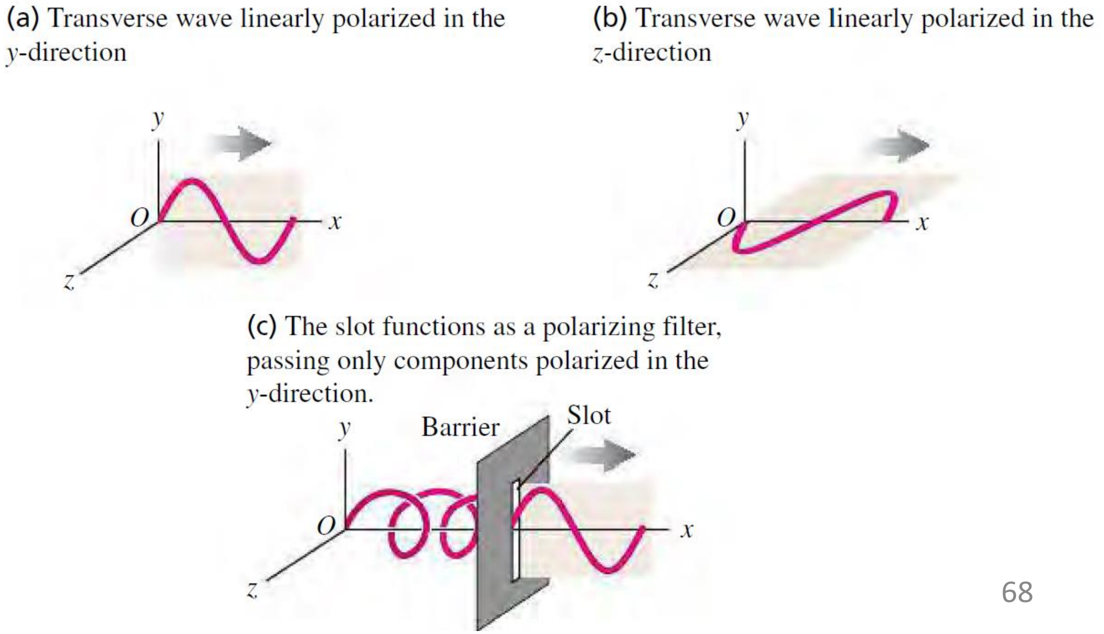
A siren on a tall pole radiates sound waves uniformly in all directions. At a distance of 15.0 m from the siren, the sound intensity is 0.250 W/m^2. At what distance is the intensity 0.010 W/m^2?

Polarization

1. Only transverse waves can be plane-polarised.
2. Unpolarised light has infinite planes of vibration.
3. Polarised light has only a single plane of vibration.
4. Plane of polarization: Infraction of E-field due to dominant effects of E-field when EM waves interact with matter.
5. **Intensity of unpolarised light** after passing through polariser: $I_1 = \frac{1}{2} I_0$
6. **Intensity of plane polarized light** is reduced after it passes through another polarizer:

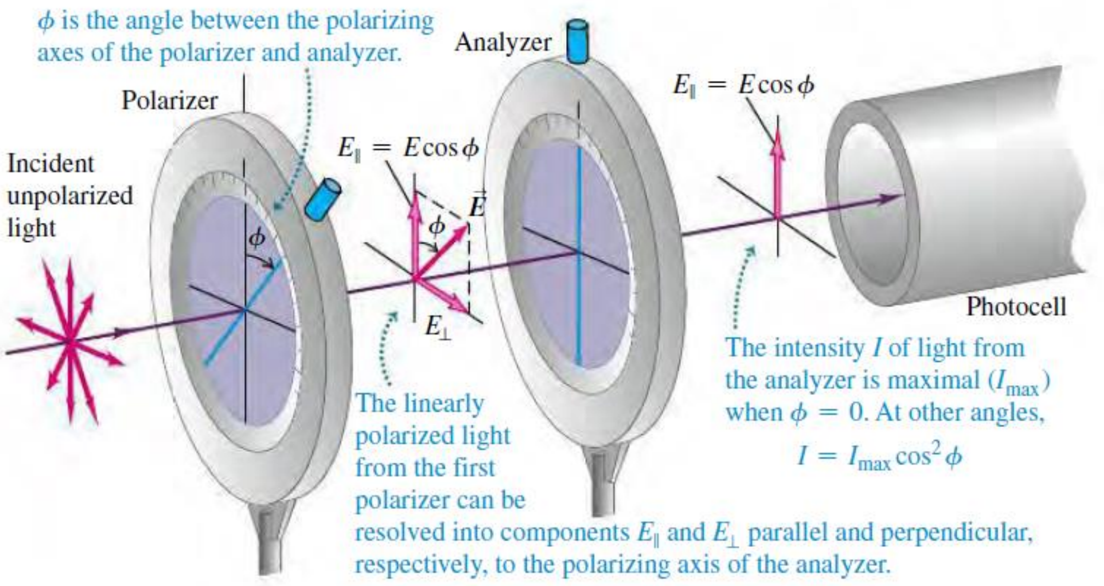
$$I_2 = I_1 \cos^2 \theta$$

Angle θ to plane of polarisation of incident light



Polarization example

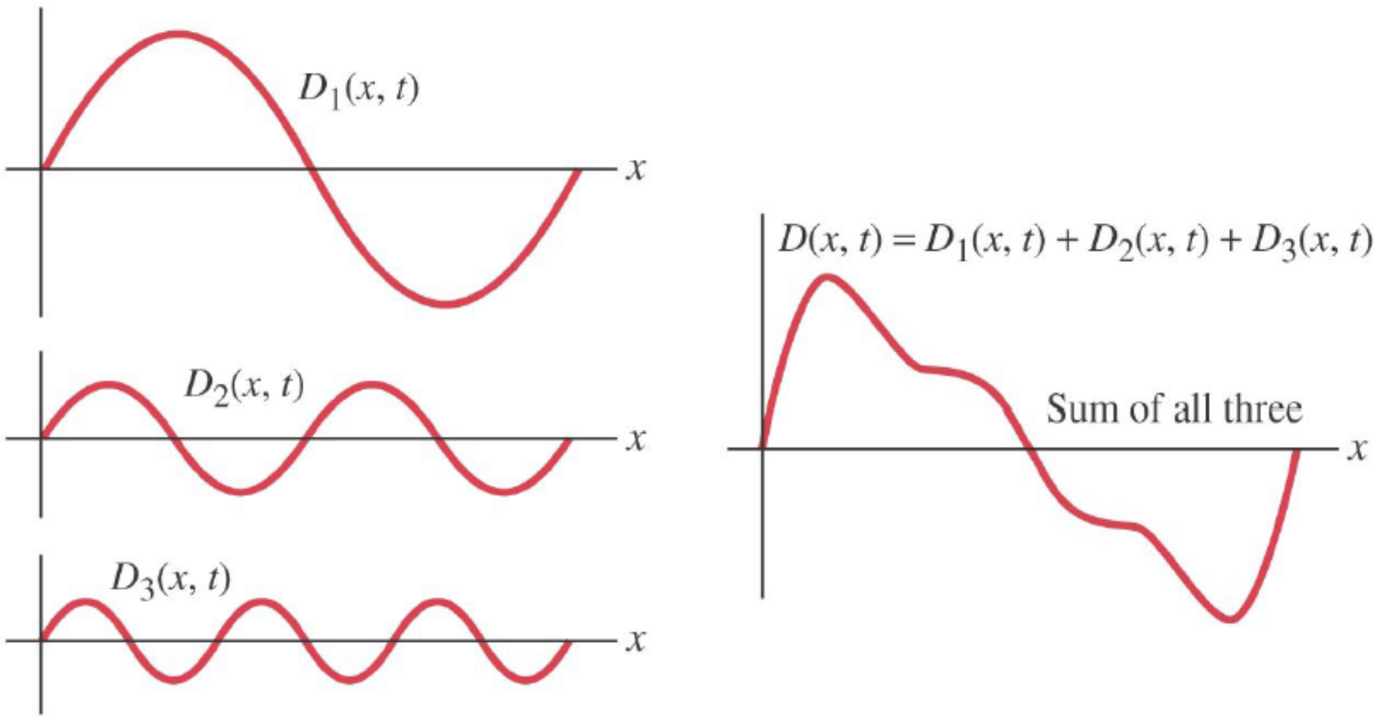
In Fig. 33.24 the incident unpolarized light has intensity I_0. Find the intensities transmitted by the first and second polarizers if the angle between the axes of the two filters is 30°.

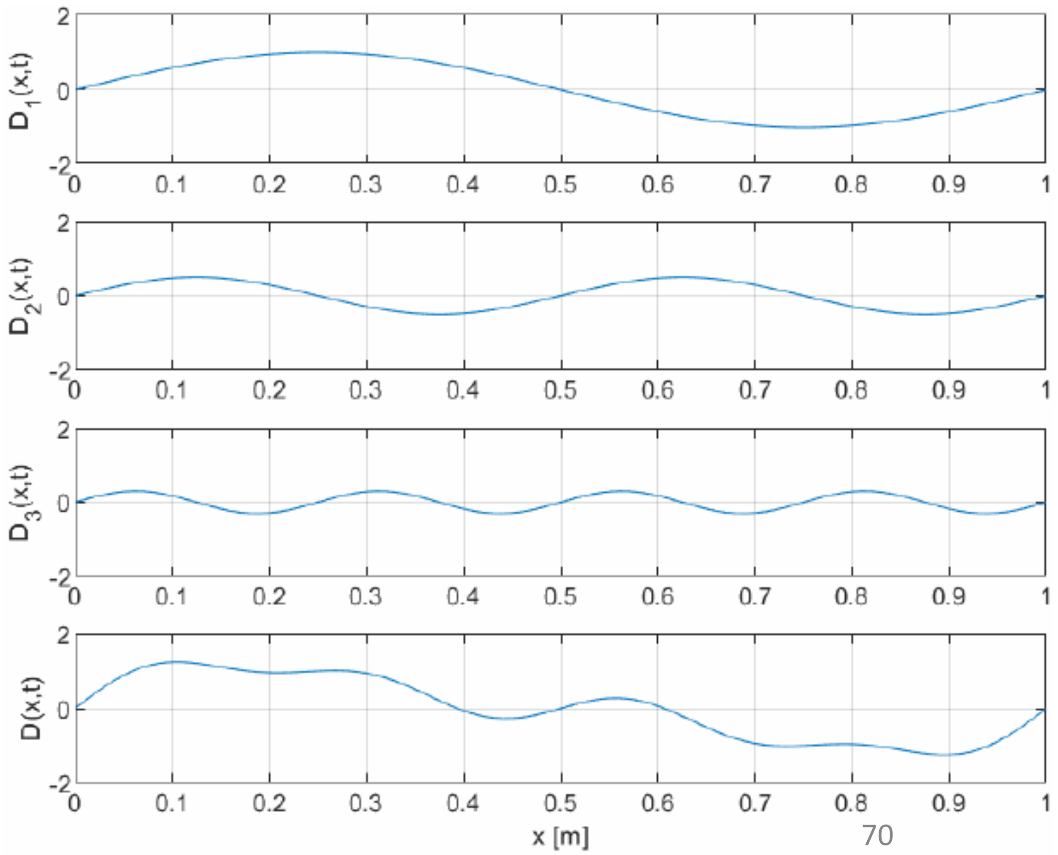


Standing waves – waves superposition

The principle of Superposition

- **Superposition:** The displacement at any point is **the algebraic sum** of the displacements of **all waves** passing through that point at that instant.

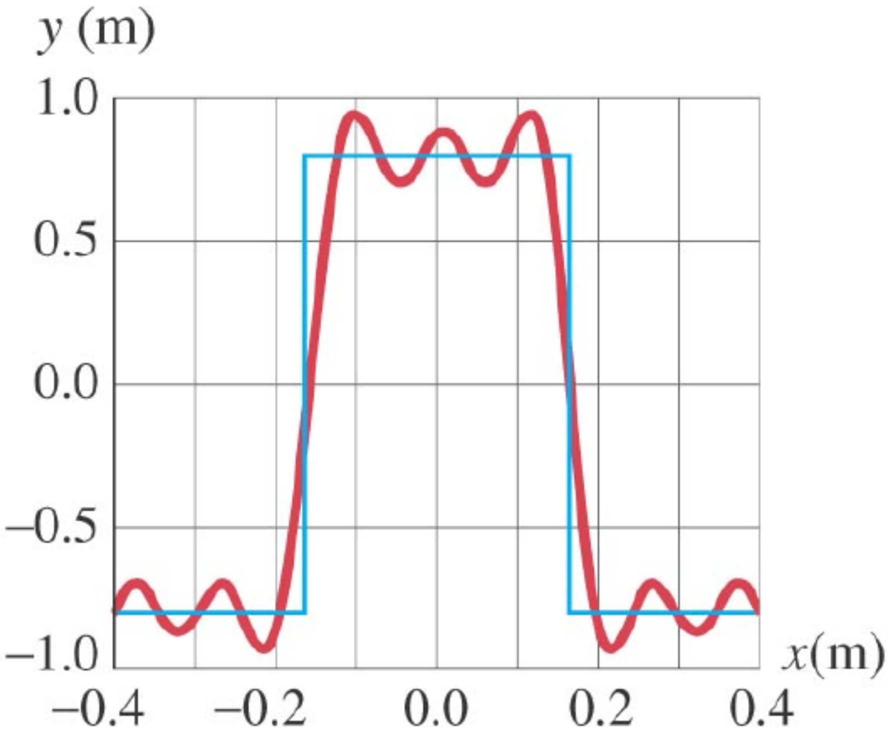


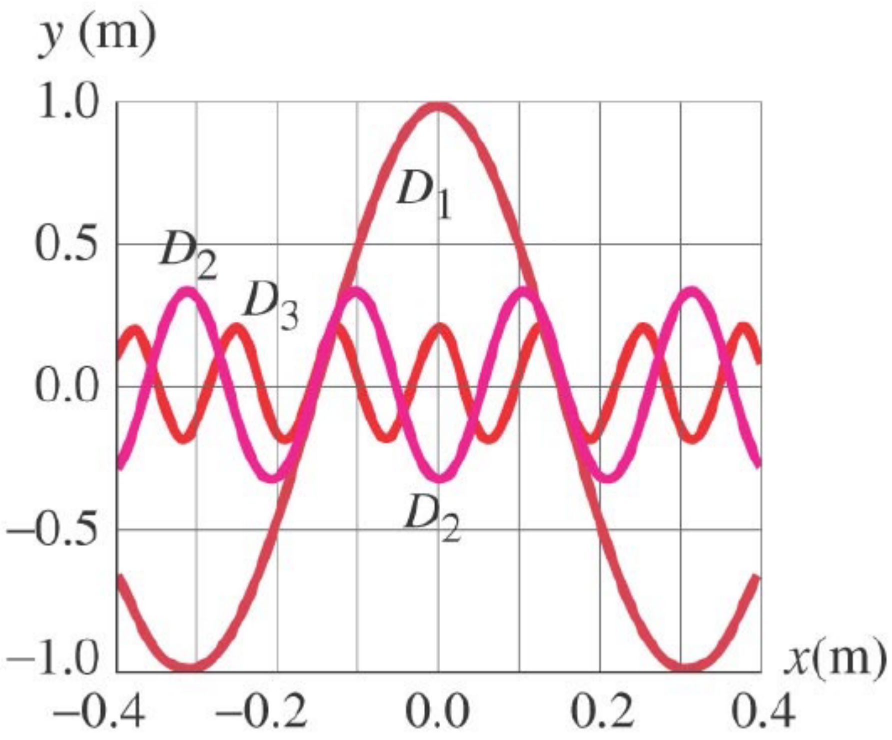


Standing waves – waves superposition

The principle of Superposition

- **Fourier's theorem:** Any **periodic** wave can be written as the sum of sinusoidal waves of different **amplitudes, frequencies, and phase**.





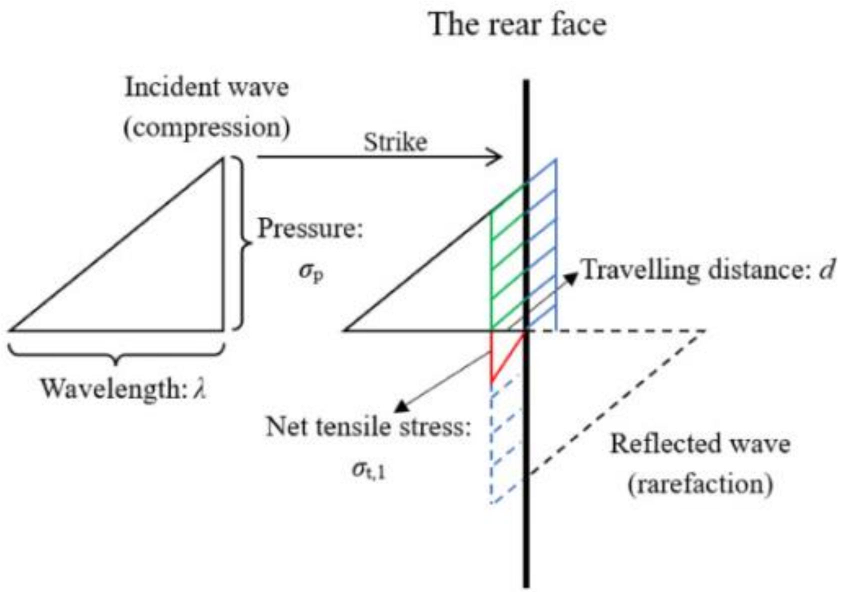
Waves superposition – concrete spalling

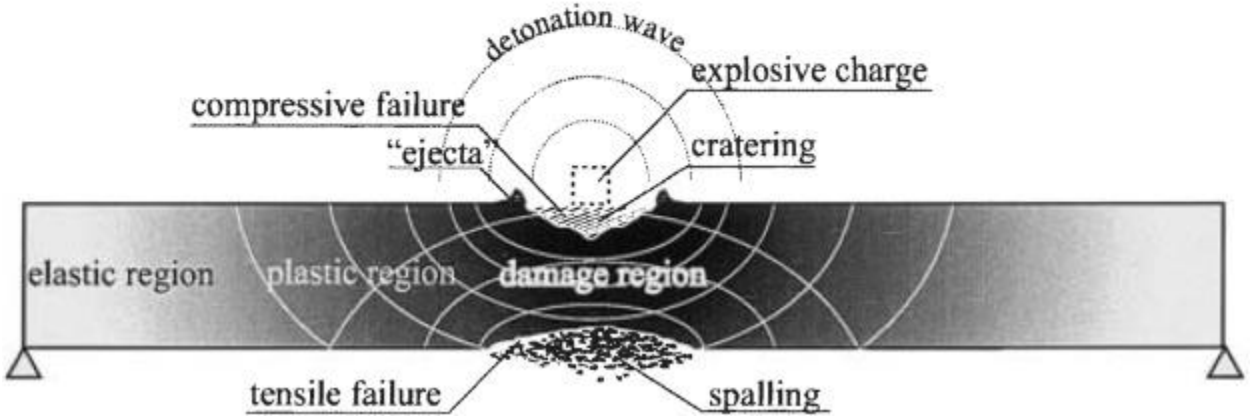
If concrete is subject to shock waves (strong, sudden loading) on one face, often the failure happens on the opposite side.

This is due to the complex interaction of compressive waves and their reflections.

Tensile waves are created, failing the concrete at specific locations.

These can be predicted quite accurately from basic physics principles.





Standing waves – waves superposition

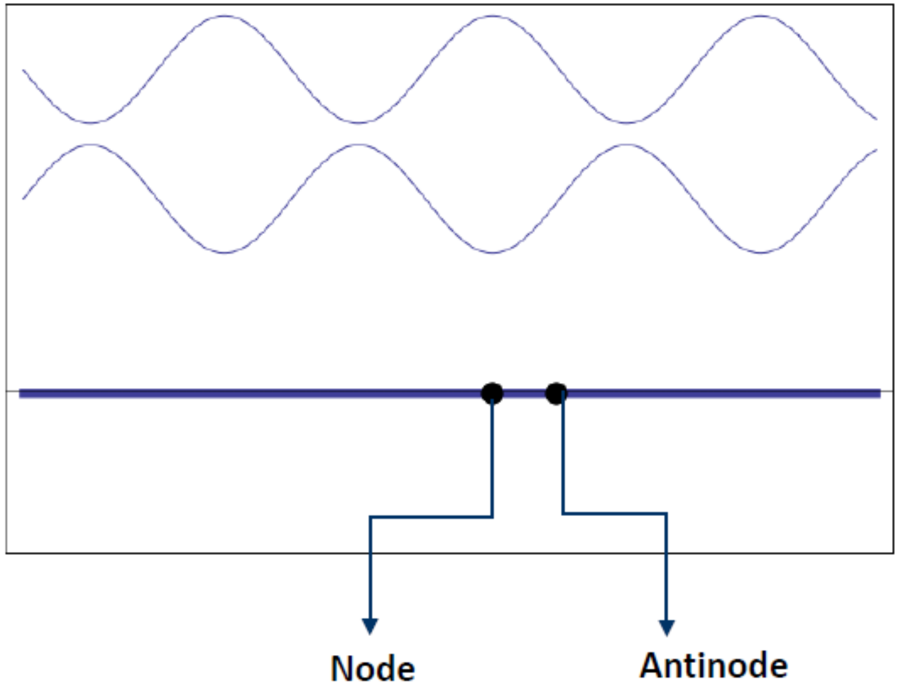
Standing Waves and Resonant Frequencies

- When wave reflected from a fixed end interferes with the incoming wave, a standing wave will be produced.

- Wave travelling along $+x$: $D_1 = A\sin(kx - \omega t)$
- Wave travelling along $-x$: $D_2 = A\sin(kx + \omega t)$

$$D = D_1 + D_2 = A\sin(kx - \omega t) + A\sin(kx + \omega t)$$
$$= 2A\sin kx \cos \omega t$$

$$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$$

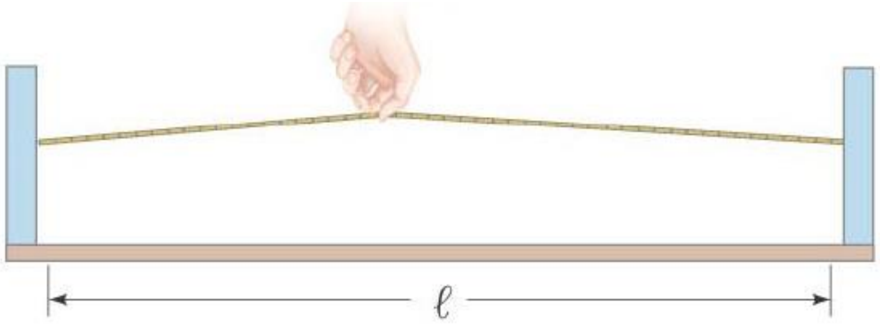


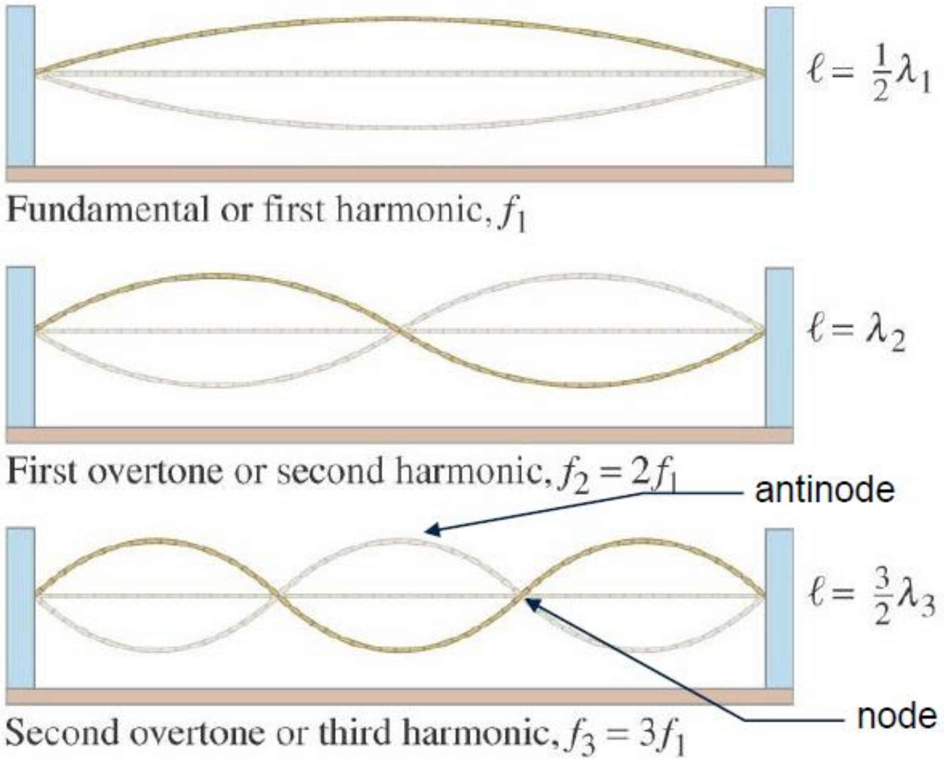
- The wave is a **standing wave**, having a sinusoidal distribution in x but the amplitude changes periodically with time

Standing waves – waves superposition

Standing Waves and Resonant Frequencies

- The **frequencies** of the standing waves on a particular string are called **resonant frequencies**.
- They are also referred to as the **fundamental and harmonics**.





$$\lambda_n = \frac{2l}{n} \quad n = 1, 2, 3, \ldots$$

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2l} = nf_1$$

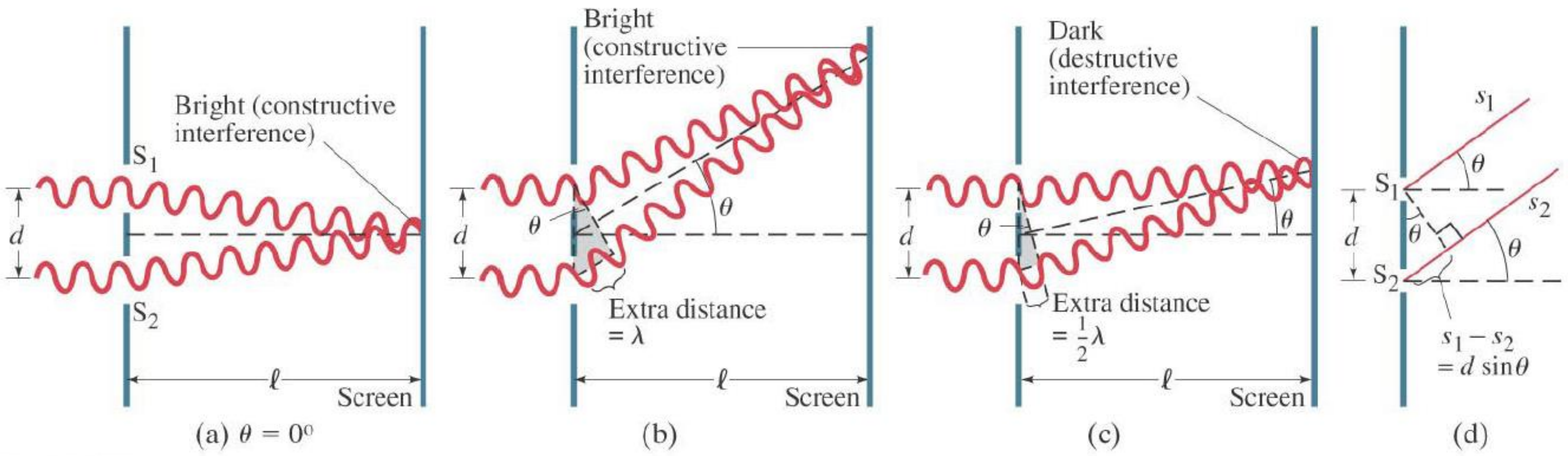
Standing wave example 2

In an attempt to get your name in *Guinness World Records*, you build a bass viol with strings of length 5.00 m between fixed points. One string, with linear mass density 40.0 g/m, is tuned to a 20.0-Hz fundamental frequency (the lowest frequency that the human ear can hear). Calculate (a) the tension of this string, (b) the frequency and wavelength on the string of the second harmonic, and (c) the frequency and wavelength on the string of the second overtone.

Standing wave example 2

Interference of light

Young's Double-Slit Experiment



- Constructive Interference: $|d_1 - d_2| \approx d \sin\theta = m\lambda, \qquad m = 0, 1, 2, 3...$
- Destructive Interference: $|d_1 - d_2| \approx d \sin\theta = \left(m + \frac{1}{2}\right)\lambda, \quad m = 1, 2, 3, ... \qquad \text{or} \qquad \frac{m\lambda}{2}, \quad m = 1, 3, 5, ...$

Interference of light

The distance between constructive interference bands can be found with trigonometry.

The angle at which a bright band happens is:

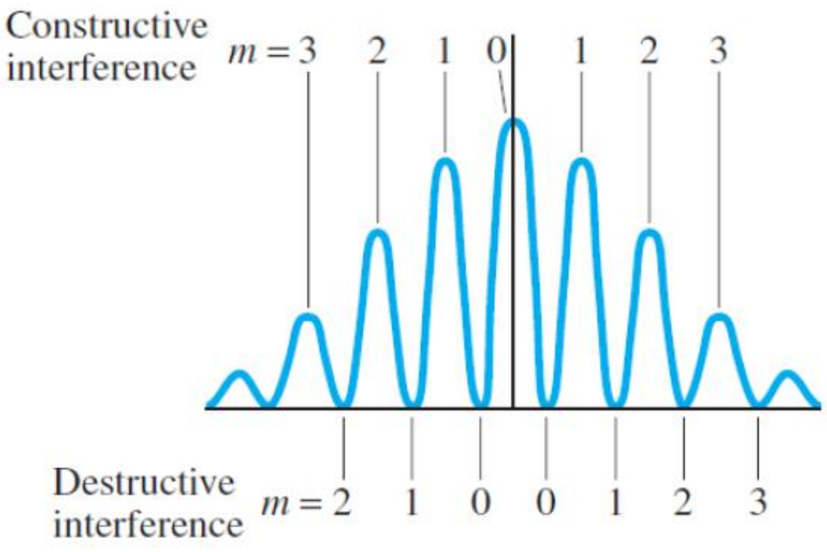
$$d\sin\theta = m\lambda \qquad \sin\theta_m = \frac{m\lambda}{d}$$

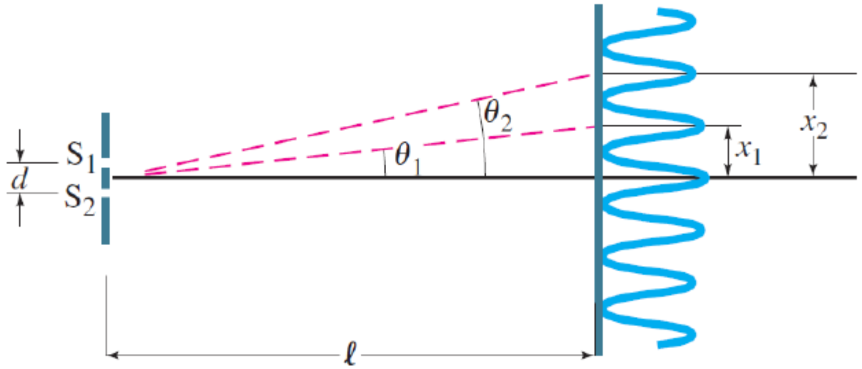
The distance between bands on the screen is:

$$\frac{x_1}{l} = \tan\theta_1 \approx \theta_1 \Rightarrow x_1 = l\theta_1$$

With the final result assuming the angle is small, i.e.:

$$\sin\theta = \theta; \;\; \tan\theta = \theta$$





$$\frac{x_1}{l} = \tan\theta_1 \approx \theta_1 \Rightarrow x_1 = l\theta_1$$

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Interferometry

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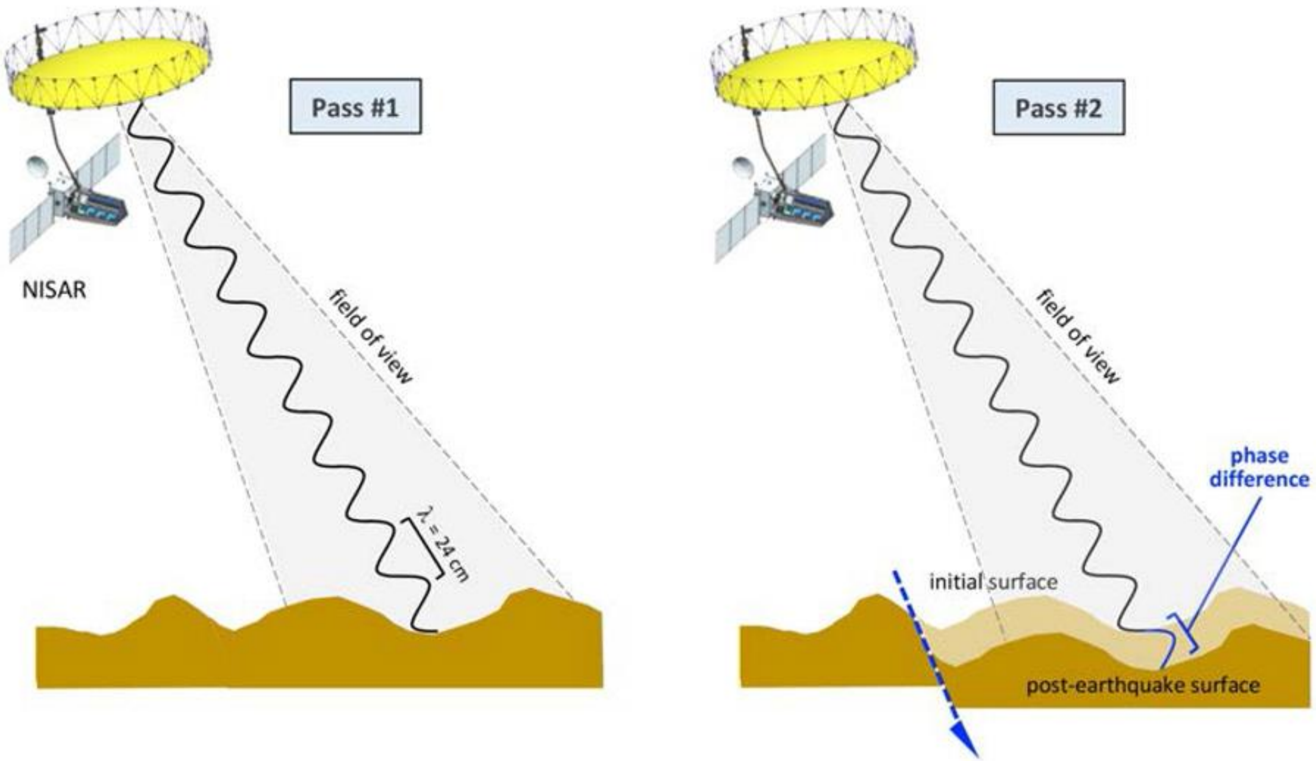
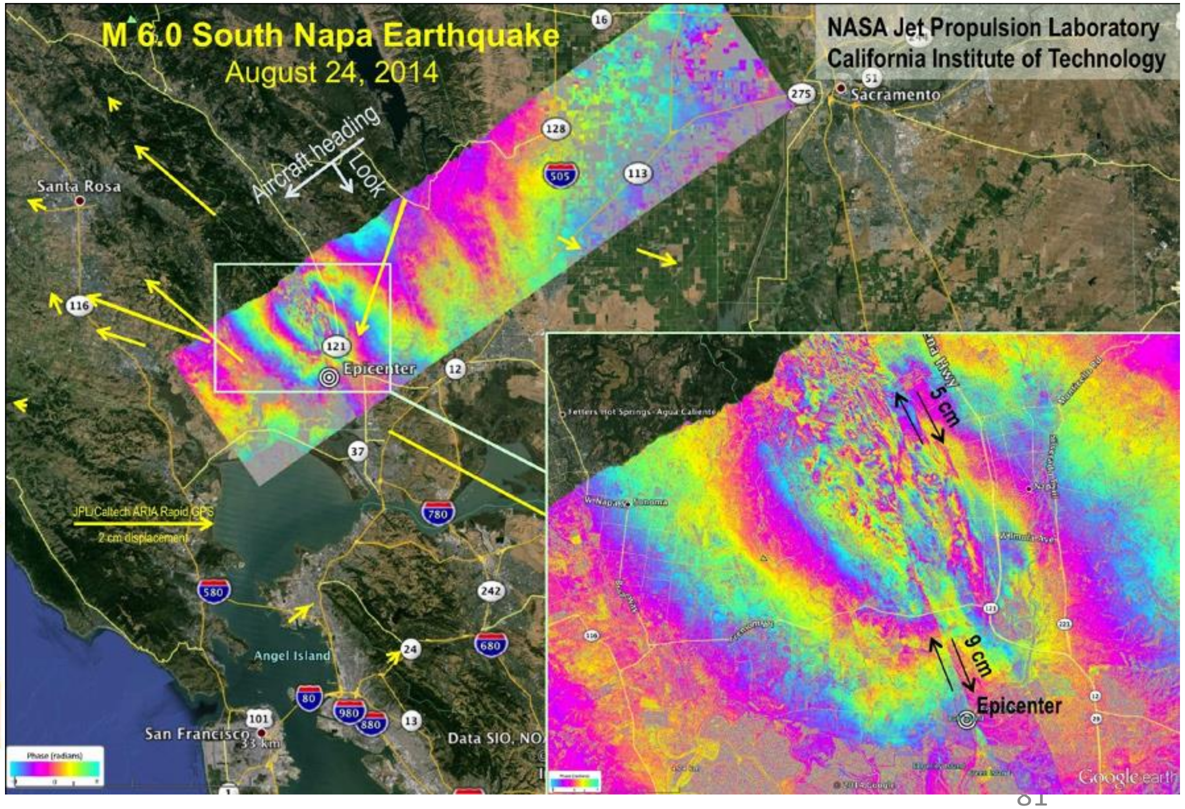
A common use for this technique is to measure distances and movements.

The phase difference between signals, sometimes weeks apart, can be used to detect movements.

The results can be used to monitor slope stability, structural movements and other issues.

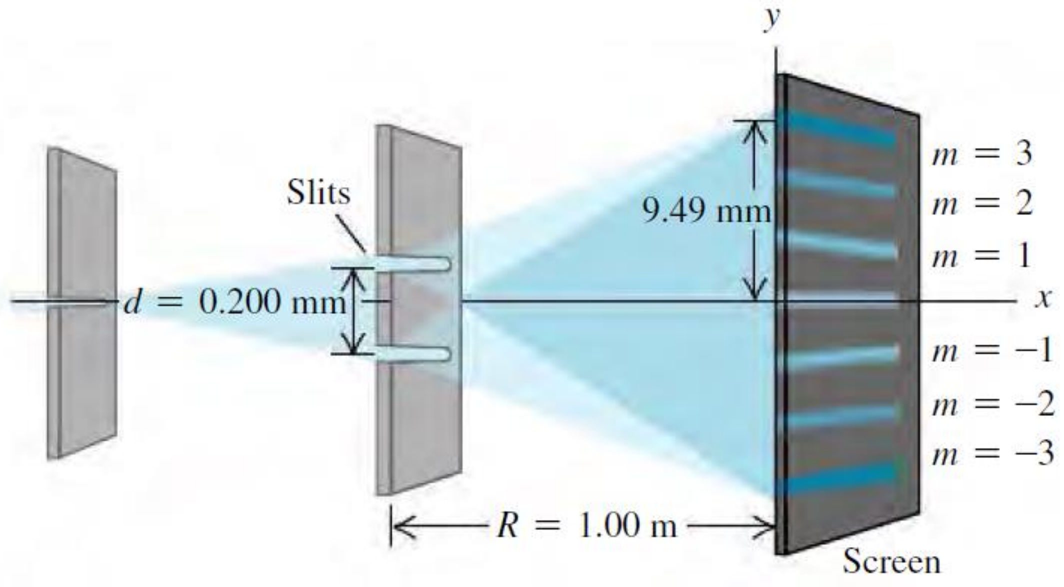




Interference example

Figure 35.7 shows a two-slit interference experiment in which the slits are 0.200 mm apart and the screen is 1.00 m from the slits. The $m = 3$ bright fringe in the figure is 9.49 mm from the central fringe. Find the wavelength of the light.

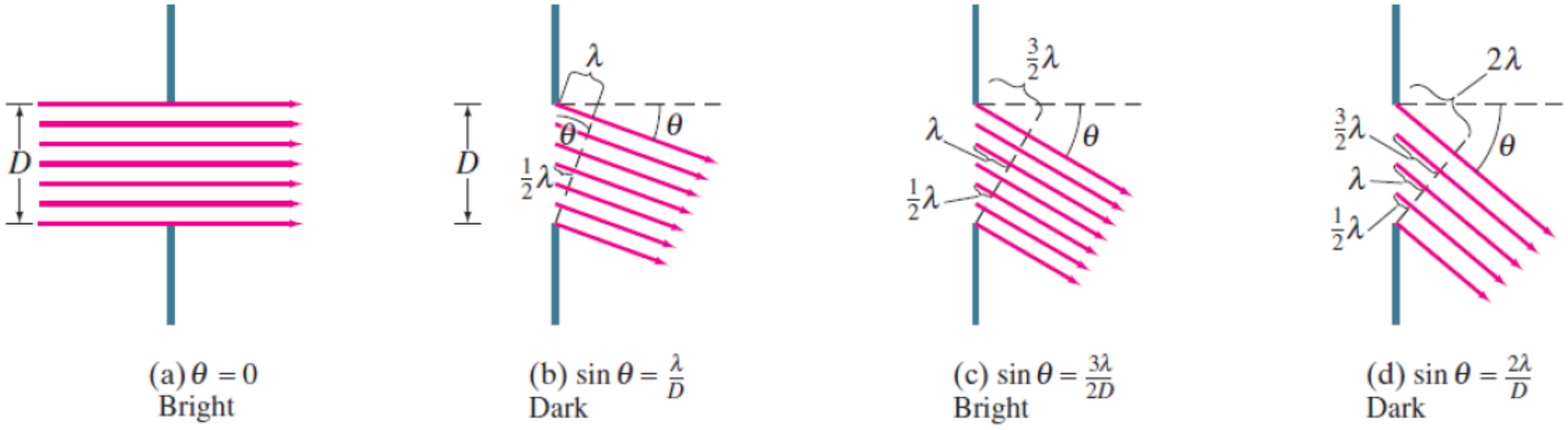
35.7 Using a two-slit interference experiment to measure the wavelength of light.

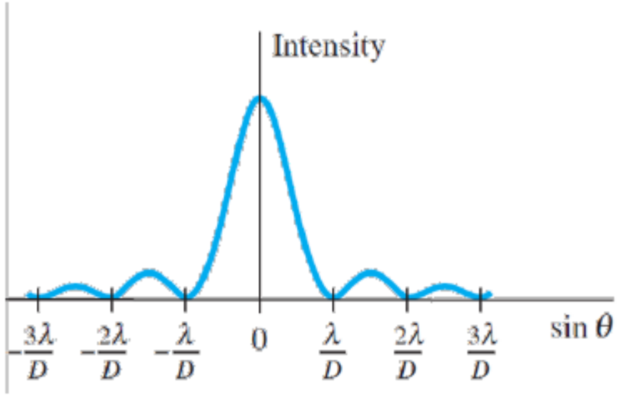


Diffraction

Single-Slit Experiment

- Monochromatic light passing through a narrow slit creates a diffraction patten due to the bending of the light waves





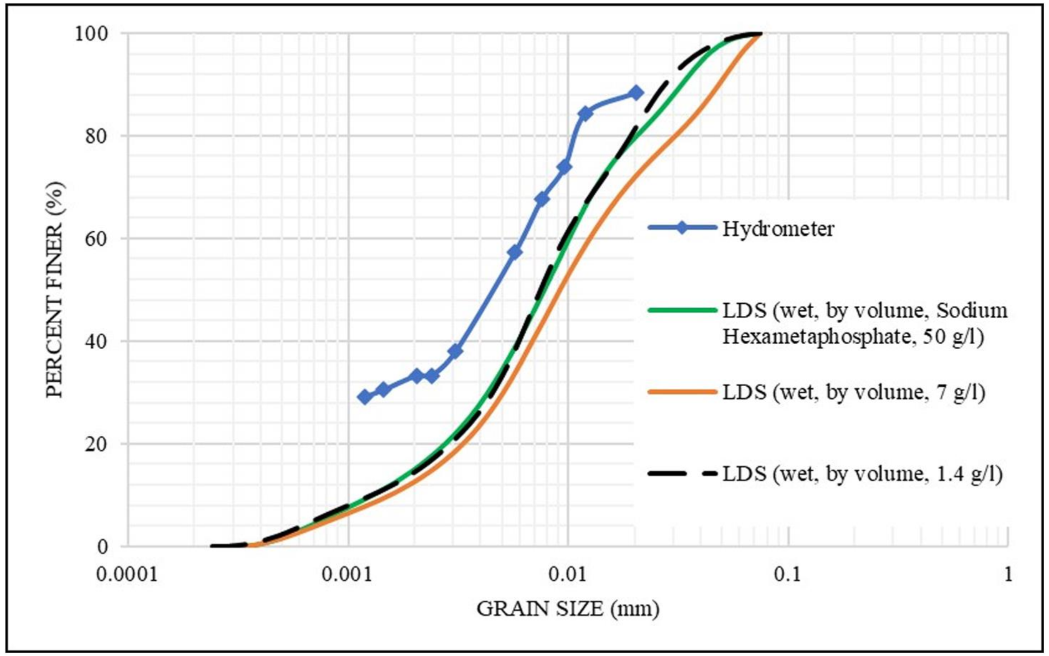
- Minima in intensity when $D\sin\theta = m\lambda,\qquad m = \pm1, \pm2, \pm3, \ldots$ and peak when $m = 0$

- Between the minima, smaller intensity when, $m \approx \frac{3}{2}, \frac{5}{2}, \ldots$

Particle size measurements

Diffraction can be used to determine the size of minute particles, such as those in fine soils.

The results are reasonable compared to existing techniques.



Diffraction example

You pass 633-nm laser light through a narrow slit and observe the diffraction pattern on a screen 6.0 m away. The distance on the screen between the centers of the first minima on either side of the central bright fringe is 32 mm (**Fig. 36.7**). How wide is the slit?