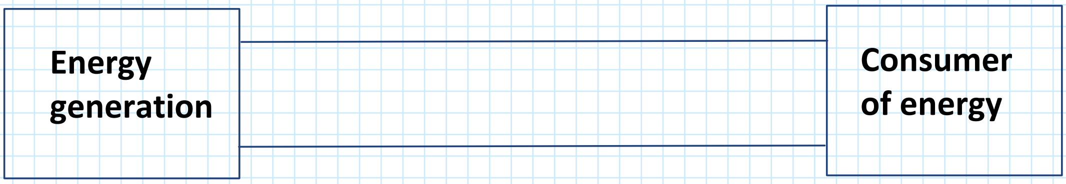
Chapter 2 Stationary Currents

2.4. Electric Power and Power Transmission



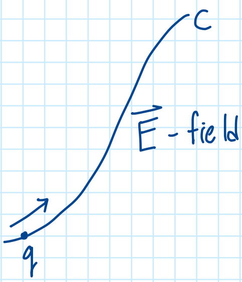
e.g. energy plant,
provides electric energy

e.g. households
Needs electric
energy

- What is electric power? How do we calculate it?
- Which part of the generated power is delivered to the consumer? How can we minimize the losses and maximize the power available at the consumer side?

2.4.1 Electric Power of a Point Charge in an Electric Field

We know from chapter 1: differential work of a point charge in an electric field along a curve



$$dW_{el} = \vec{F}_q d\vec{r} = q \cdot \vec{E} d\vec{r} \qquad \left(W = q \int_{P_1}^{P_2} \vec{E} \cdot d\vec{r} \right)$$

$\Rightarrow$ q moves at velocity $d\vec{v} = \frac{d\vec{r}}{dt}$

Power $P = \frac{\text{work}}{\text{time}} = \frac{dW}{dt}$

electric power : $P_{el} : \frac{dW_{el}}{dt} = \frac{q \cdot \vec{E} \cdot d\vec{r}}{dt} = q \cdot \vec{E} \cdot \vec{v}$

Power of one point charge q moving at velocity $\vec{v}$ in an electric field $\vec{E}$

$$P_{el} = q \cdot \vec{E} \cdot \vec{v}$$

2.4.2 Electric Power of a Flow Field (current density)

$\vec{J} = q \cdot n \cdot \vec{v}$ $\qquad$ $n = \frac{N}{V} = \frac{\text{number of particles (electrons)}}{\text{Volume}}$

one particle : $P_{el} = q \cdot \vec{E} \cdot \vec{v}$

multiply by N : $N \cdot P_{el} = N \cdot q \cdot \vec{E} \cdot \vec{v}$

↓

divide by Volume V $\quad \frac{N}{V} \cdot P_{el} = \overbrace{\left(\frac{N}{V}\right)}^{n} q \cdot \vec{E} \cdot \overbrace{\vec{v}}^{\vec{J}}$

$N \cdot P_{el}$ = total power for all particles

$\frac{N \cdot P_{el}}{V} = \frac{\text{total power}}{\text{Volume}}$

= power density p_{el}

$\boxed{p_{el} = \vec{J} \cdot \vec{E}}$ (2.27)

power density of an electric current density

2.4.3 Electric Power Losses

$p_{el} = \vec{J} \cdot \vec{E}$, if we consider Ohmic transport : $\vec{J} = \sigma \vec{E}$ $\quad \sigma$ = conductivity

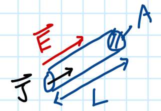
⇓

$p_{el} = \sigma \vec{E}^2 = \frac{1}{\sigma} |\vec{J}|^2 > 0$ $\qquad$ for ohmic transport : p_{el} is always a power loss

p_{el} is dissipated as heat $\qquad$ Ohmic transport means $U \sim I$

$\vec{J} \sim \vec{E}$

Power loss in an Ohmic resistor:



Cylindric wire : Cross section A, length L

$p_{el} = \vec{J}\vec{E} \quad \left(\frac{\text{power}}{\text{Volume}}\right)$

Total power dissipation: ⇒ integrate over volume V of wire

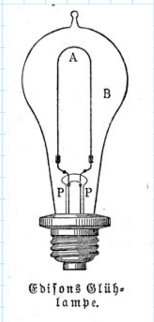
$$P_{el} = \int_V p_{el}\, dv = \int_V \vec{J} \cdot \vec{E} \cdot dV = |\vec{J}| \cdot |\vec{E}| \cdot \overbrace{L \cdot A}^{V} = \underbrace{|\vec{J}| \cdot A}_{I} \cdot \underbrace{|\vec{E}| \cdot L}_{U}$$

$$\boxed{P_{el} = I \cdot U = \frac{U^2}{R} = R \cdot I^2} \quad (2.29)$$

Examples:

Bulb:

Sources: Wikipedia



Edison-bulb (1888)

bulb 230V,
40 W power consumption
E14

wire :

$R = \rho \cdot \frac{L}{A}$

~~Brightness vs. Life time~~

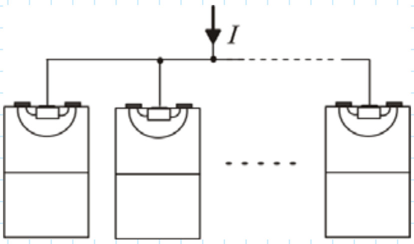
$$P = U \cdot I = \frac{U^2}{R} = I^2 \cdot R$$

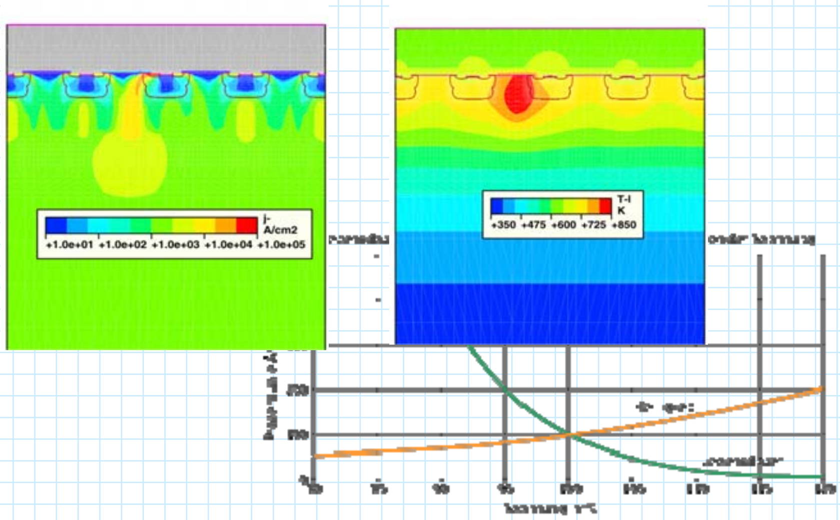
~~At~~ at location, where diameter is slightly smaller ⇒ A smaller ⇒ R is Larger

$P = I^2 \cdot R \Rightarrow I = \text{const.}$, but $R\uparrow \Rightarrow P\uparrow$ ⇒ this point warms up / heated ; for metals : R increases with temperatu

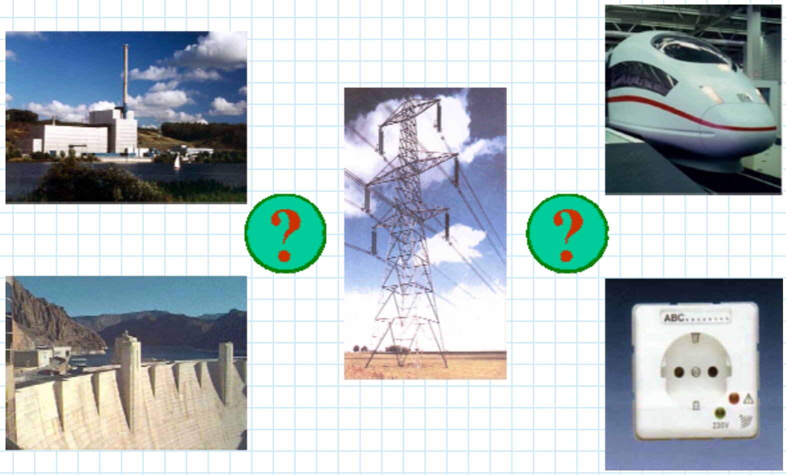
Heat dissipation in an array of power transistors:

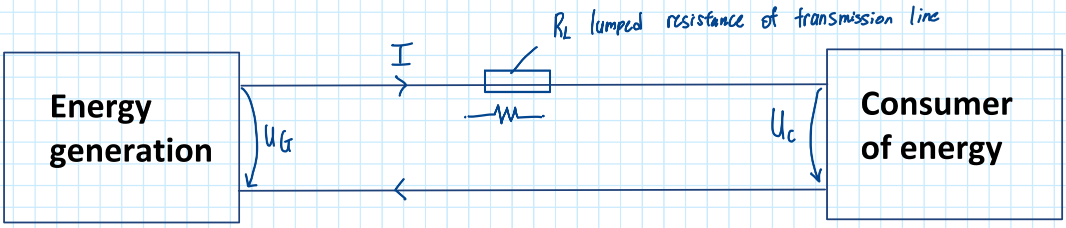
Soure: dissertation at TEP@TUM





2.4.4 Electric Energy Transmission





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(i) Energy generation plantvoltage and power at ~~consumer~~ generator side: U_G ; $P_G = U_G \cdot I$

(ii) voltage and power at consumer side: U_c ; $P_c = U_c \cdot I$

(iii) resistance (lumped) of transmission lines (ohmic losses) : $R_L \Rightarrow U_c = U_G - R_L \cdot I$

(iv) efficiency of energy transmission: $\eta = \frac{P_c}{P_G}$ (η = "eta")

Insert (i) -(iii) in (iv) leads:

efficiency of power transmisson :

$$\eta = \frac{P_c}{P_G} = \frac{U_c \cdot I}{U_G \cdot I} = \frac{U_c}{U_G} = \frac{U_G - R_L \cdot I}{U_G} = 1 - \frac{R_L \cdot I}{U_G} = 1 - \frac{R_L \cdot U_G \cdot I}{U_G^2}$$

$$\boxed{\eta = 1 - \frac{R_L \cdot P_G}{U_G^2}} \quad (2.34)$$

$\Rightarrow$ R_L should be low
+ U_G should be high
} to minimise the losses

HVDC transmission = High Voltage Direct Current transmission