1)

a) $f(x) = 2 - x^2$

D is $\mathbb{R}$, Range is $(-\infty, 2]$

b) $f(x) = \sqrt{2x+1} - 5$

Domain is $[-\frac{1}{2}, \infty)$, Range is $[-5, \infty)$

c) $f(x) = \frac{1}{3x-5}$

Domain is $\{x \in \mathbb{R} \mid x \neq \frac{5}{3}\}$, Range is $\{f(x) \in \mathbb{R} \mid f(x) \neq 0\}$

d) $f(x) = 1 - e^{-x}$

Domain is $\{x \in \mathbb{R}\}$, Range is $(-\infty, 1)$

e) $f(x) = \ln(x+2)$

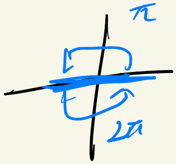
Domain is $(-2, \infty)$ Range is $\mathbb{R}$

1f)

$f(x) = \cot(\pi x)$

To get domain:

$\sin(\underbrace{\pi x}_{\theta}) \neq 0$



$\pi x \neq 0, \pm\pi, \pm 2\pi \dots = n\pi, \ n \in \mathbb{Z}$

$x \neq \frac{n\pi}{\pi} = n$

D is $\{x \in \mathbb{R} \mid x \neq n \ \forall \ n \in \mathbb{Z}\}$

Range is $\mathbb{R}$

2

$y - 0 = -\frac{1}{2}(x - (-3))$

$y = -\frac{1}{2}(x + 3)$

$y = -\frac{x}{2} - \frac{3}{2}$

$2y = -x - 3$

$2y + x = -3$

3.

a) $\frac{20500-12300}{3} = 2733.33$

$V(t) = 20500 - 2733.33t$

b) Equipment depreciate at a rate of \$2733.33 per year

c) when $V(t) = 0$

$t = 7.5$

t and V intercepts of $V(t)$: it will take $t = 7.5$ for the equipments to fully depreciate

d) $V(t) = 3000$

$3000 = 20500 - 2733.33t$

$t \approx 6.4$

5 a)

$f(x) = x - \tan x$

$f(-x) = (-x) - \tan(-x)$

$= (-x) + \tan x$

$= -1(x - \tan x)$

$= -f(x)$ $\therefore$ odd

b) $f(x) = x(2x-1)$

$f(-x) = -x(2(-x)-1)$

$= -x(-2x-1)$

$= (2x^2 + x)$

$= x(2x+1)$

$\neq f(x)$ or $\neq -f(x)$ $\therefore$ neither even or odd

5c)

$$f(x) = \sqrt{2 - \cos(\pi x)}$$

$$f(-x) = \sqrt{2 - \cos(\pi(-x))} \qquad \cos(-\pi) = \cos(\pi)$$

$\therefore f(-x) = f(x)$, even

6a)

$$p(x) = f_e(x) \cdot f_e(x)$$

$$p(-x) = f_e(-x) \cdot f_e(-x)$$

$$= f_e(x) \cdot f_e(x) = p(x)$$

b)

$$q(x) = f_o(x) \cdot f_o(x) = f_o^2(x)$$

$$q(-x) = f_o(-x) \cdot f_o(-x)$$

$$= -f_o(x) \cdot -f_o(x)$$

$$= f_o^2(x) = q(x)$$

c)

$$r(x) = f_e(x) \cdot f_o(x)$$

$$r(-x) = f_e(-x) \cdot f_o(-x)$$

$$= f_e(x) \cdot -f_o(x)$$

$$= -r(x) \quad \therefore \text{odd}$$

d) $$u(x) = f_e(x) + f_e(x)$$

$$u(-x) = f_e(-x) + f_e(-x)$$

$$= f_e(x) + f_e(x)$$

$$= u(x) \quad \therefore \text{even}$$

6e) $$v(-x) = f_o(-x) + f_o(-x)$$

$$= -f_o(x) - f_o(x)$$

$$= -(f_o(x) + f_o(x))$$

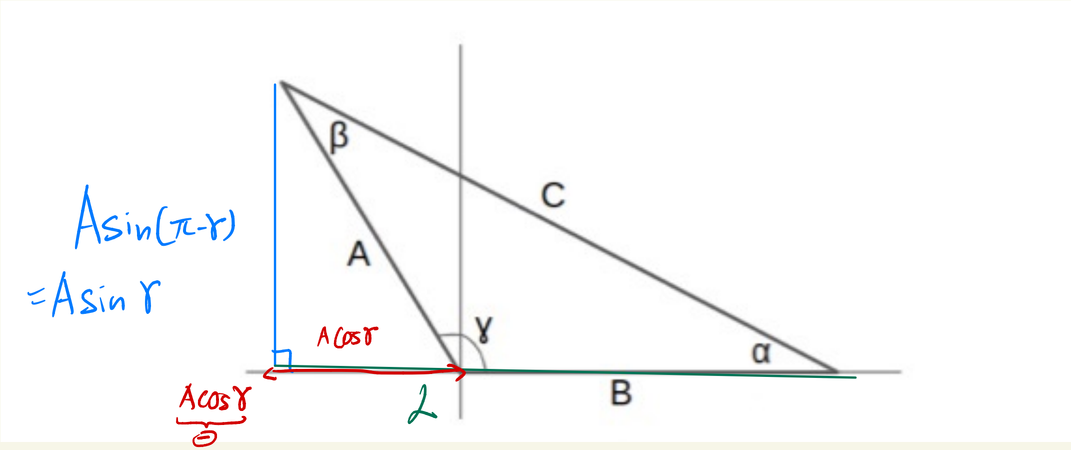
$$= -v(x) \quad \therefore \text{odd}$$

6f) $w(x) = f_e(x) + f_o(x)$

$w(-x) = f_e(-x) + f_o(-x)$

$= f_e(x) - f_o(x) \begin{cases} \neq w(x) \\ \neq -w(x) \end{cases}$ $\therefore$ neither even nor odd

7b



By Pythagoras,

$$C^2 = (A\sin\gamma)^2 + d^2$$

$$= A^2\sin^2\gamma + [B - A\cos\gamma]^2$$

$$= A^2\sin^2\gamma + B^2 - 2AB\cos\gamma + A^2\cos^2\gamma$$

$$= A^2(1) + B^2 - 2AB\cos\gamma \text{ (shown)}$$

8a) $f(x) = \frac{1}{x} + 1 \quad g(x) = \ln(x-2)$

$$h(x) = \frac{g(x)}{f(x)}$$

$$= \frac{\ln(x-2)}{\frac{1}{x} + 1} = \frac{\ln(x-2)}{\frac{1+x}{x}}$$

$$= \frac{x\ln(x-2)}{1+x}$$

D_h is $(2, \infty)$

8b

$$f(x) = \frac{1}{x} + 1$$

$$g(x) = \frac{1}{x-1}$$

$$h(x) = f(g(x)) = \frac{1}{\left(\frac{1}{x-1}\right)} + 1$$

$$= x - 1 + 1$$

$$= x = f(\underbrace{f^{-1}(x)}_{g(x) = f^{-1}(x)})$$

so $f(x)$ & $g(x)$ are inverses of each other

D_g is $\{R \mid x \neq 1\}$, R_g is $\{R \mid g \neq 0\}$

D_f is $\{R \mid x \neq 0\}$,

since $R_g = D_f$, so

D_h is D_g, so $\{R \mid x \neq 1\}$

9a

$$f(x) = 7e^{-3(x+1)} - 1$$

$\rightarrow$ D_f is $\mathbb{R}$

$\rightarrow$ Range_f is $(-1, \infty)$

Let $y = 7e^{-3(x+1)} - 1$

$$7e^{-3(x+1)} = y + 1$$

$$e^{-3(x+1)} = \frac{y+1}{7}$$

$$\ln e^{-3(x+1)} = \ln\left(\frac{y+1}{7}\right)$$

$$-3(x+1) = \ln\left(\frac{y+1}{7}\right)$$

$$x + 1 = -\frac{1}{3}\ln\left(\frac{y+1}{7}\right)$$

$$x = -\frac{1}{3}\ln\left(\frac{y+1}{7}\right) - 1 = f^{-1}(y)$$

$$f^{-1}(x) = -\frac{1}{3}\ln\left(\frac{x+1}{7}\right) - 1$$

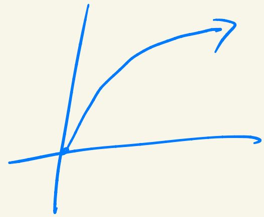
$D_{f^{-1}}$ is $(-1, \infty)$

$\text{Range}_{f^{-1}}$ is $\mathbb{R}$

9b

$$f(x) = \frac{1}{\sqrt{2x-3}} \rightarrow D_f \text{ is } \underbrace{2x-3 > 0}_{x > \frac{3}{2}}, \text{ so } \left(\tfrac{3}{2}, \infty\right)$$

$$\text{Let } y = \frac{1}{\sqrt{2x-3}} \qquad R_f \text{ is } (0, \infty)$$



$$\sqrt{2x-3} = \frac{1}{y}$$

$$2x-3 = \frac{1}{y^2}$$

$$x = \frac{1}{2}\left(\frac{1}{y^2} + 3\right) = f^{-1}(y)$$

$$f^{-1}(x) = \frac{1}{2}\left(\frac{1}{x^2} + 3\right) \rightarrow R_{f^{-1}} \text{ is } \left(\tfrac{3}{2}, \infty\right)$$

$$D_{f^{-1}} \text{ is } (0, \infty)$$

9c

$$f(x) = 2\sin\left(\frac{x}{2}\right) + 5 \rightarrow D_f \text{ is } [0, \pi]$$

since $\sin\left(\frac{x}{2}\right) \in [0, 1]$

$$\underbrace{f(x) \in [5, 7]}_{R_f}$$

$$y = 2\sin\left(\frac{x}{2}\right) + 5$$

$$\sin\left(\frac{x}{2}\right) = \frac{y-5}{2}$$

$$\frac{x}{2} = \sin^{-1}\left(\frac{y-5}{2}\right)$$

$$x = 2\sin^{-1}\left(\frac{y-5}{2}\right) = f^{-1}(y)$$

$$f^{-1}(x) = 2\sin^{-1}\left(\frac{x-5}{2}\right)$$

$$D_{f^{-1}} = R_f = [5, 7]$$

$$R_{f^{-1}} = D_f = [0, \pi]$$

10

$$A(t) = P(1+r)^t$$

$$A(0) = P$$
$$A(1) = P(1+r)$$
$$A(2) = (1+r)A(0)$$
$$= P(1+r)^2 = P(1+r)^t$$

Isolation of t

$$\frac{A}{P} = (1+r)^t$$

$$\log_{1+r}\left(\frac{A}{P}\right) = t(A) \rightarrow A^{-1}(t) = \frac{\ln(t/P)}{\ln(1+r)}$$

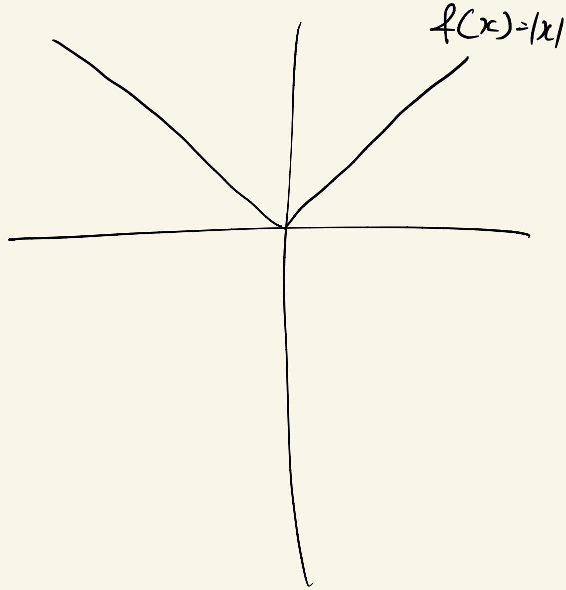
$$t(A) = \frac{\ln(A/P)}{\ln(1+r)}$$ ← Change of base

$t(A)$ calculates the number of years required for the accumulated amount to reach A, given parameters r & P. → r & P are constants

$f(x) = |x|$, $y(x) = -|2x-4|-1$

$\downarrow$

$f_1(x) = |2x|$

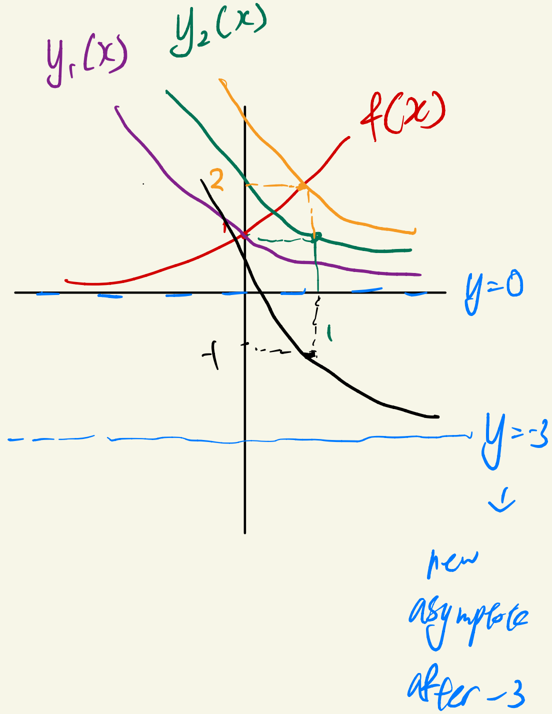


11b) $f(x) = e^x$, $y(x) = 2e^{1-x} - 3$

$y_1(x) = e^{-x}$

$y_2(x) = e^{-x+1} = e^{-(x-1)}$

$y_3(x) = 2e^{-(x-1)}$



D_y is $\mathbb{R}$

R_y is $(-3, \infty)$