


1a)

$$\left[\begin{array}{cc|c} 3 & 4 & 1 \\ 2 & 3 & 12 \end{array} \right] \xrightarrow{3R_2 - 2R_1} \left[\begin{array}{cc|c} 3 & 4 & 1 \\ 0 & 1 & 34 \end{array} \right]$$

$$y = 34$$

$$x = \frac{1 - 4(34)}{3}$$

$$= -45$$

b)

$$\left[\begin{array}{cc|c} 3 & -2 & 4 \\ -6 & 4 & 7 \end{array} \right] \xrightarrow{R_2 + 2R_1} \left[\begin{array}{cc|c} 3 & -2 & 4 \\ 0 & 0 & 15 \end{array} \right]$$

\therefore inconsistent

c)

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 4 & 1 & 5 \\ 1 & 3 & 1 & 6 \end{array} \right]$$

$$\begin{aligned} u + v + w &= 6 \\ u + 4v + w &= 5 \\ u + 3v + w &= 6 \end{aligned}$$

$$\begin{aligned} R_2 - R_1 &\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 3 & 0 & -1 \\ 1 & 3 & 1 & 6 \end{array} \right] \\ R_3 - R_1 &\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 3 & 0 & -1 \\ 0 & 2 & 0 & 0 \end{array} \right] \end{aligned}$$

$$\left. \begin{aligned} 3v &= -1 \\ 2v &= 0 \end{aligned} \right\} \text{inconsistent}$$

$$\therefore \text{SLE no soln}$$

$$3R_3 - 2R_2$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 3 & 0 & -1 \\ 0 & 0 & 0 & 2 \end{array} \right) \text{, } D=2, \text{ so SLE is inconsistent}$$

$$^2 \text{ a) } A \underset{2 \times 3}{d} \underset{3 \times 1}{}$$

$$= \begin{bmatrix} 1 & 3 & 5 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$

$$\text{b) } AB + C$$

$$\underset{2 \times 3}{A} \cdot \underset{3 \times 3}{B} + \underset{3 \times 2}{C}$$

$$\underset{2 \times 3}{AB} + \underset{3 \times 2}{C}$$

\therefore Undefined

$$\text{c) } A + C^T$$

$$\begin{bmatrix} 1 & 3 & 5 \\ -1 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 3 & -1 \\ 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 4 \\ 0 & 3 & 4 \end{bmatrix}$$

$$\text{d) } C^T \cdot C$$

$$\begin{bmatrix} 1 & 3 & -1 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 11 & 3 \\ 3 & 21 \end{bmatrix}$$

e) BC

$$\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \times 2 \\ 1 & 1 \\ 3 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ 4 & 8 \\ 5 & -1 \end{bmatrix}$$

f) $d^T \cdot B$

$$\begin{bmatrix} 1 \times 3 \\ 2 & -1 & 1 \end{bmatrix} \times \begin{bmatrix} 3 \times 3 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

g) Cd

$$\begin{bmatrix} 3 \times 2 \\ 1 & 3 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 3 \times 1 \\ 2 \\ -1 \\ 1 \end{bmatrix} \therefore \text{undefined}$$

h) $d^T d$

$$\begin{bmatrix} 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = 6$$

i) dd^T

$$\begin{bmatrix} 3 \times 1 \\ 2 \\ -1 \end{bmatrix} \begin{bmatrix} 1 \times 3 \\ 2 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -1 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}$$

3

San Diego
California
Salt Lake City
Utah

$$SSD + SSL = 90183 - R_1$$

$$SSD - SSL = 5231 - R_2$$

$$(5231 + SSL) + SSL = 90183$$

$$2SSL = 90183 - 5231$$

$$SSL = \$42476$$

$$SSD = 42476 + 5231$$

$$= \$47707$$

$$4 \quad x_1 + x_2 + x_3 = 6$$

$$2x_1 + 4x_2 + x_3 = 5$$

$$2x_1 + 3x_2 + x_3 = 6$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & 4 & 1 & 5 \\ 2 & 3 & 1 & 6 \end{array} \right] \xrightarrow{R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 2 & -1 & -7 \\ 0 & 1 & -1 & -6 \end{array} \right] \xrightarrow{R_3 - \frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 2 & -1 & -7 \\ 0 & -\frac{1}{2} & 0 & 1 \end{array} \right]$$

$$x_2 = -1 \parallel$$

$$R_2 : 2x_2 - x_3 = -7$$

$$x_3 = 7 + 2(-1)$$

$$= 5 \parallel$$

$$R_1 : x_1 + x_2 + x_3 = 6$$

$$x_1 = 6 - 5 - (-1)$$

$$= 2 \parallel$$

5

$$\begin{array}{l} x+y+z=1 \\ 2x-y+z=-1 \\ x+3y-z=7 \end{array}$$

Rank = 3, L.I. is 3 and Consistent

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & -1 & 1 & -1 \\ 1 & 3 & -1 & 7 \end{array} \right] \xrightarrow{R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -3 & -1 & -3 \\ 1 & 3 & -1 & 7 \end{array} \right] \xrightarrow{R_3 - R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -3 & -1 & -3 \\ 0 & 2 & -2 & 6 \end{array} \right] \xrightarrow{3R_3 + 2R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -3 & -1 & -3 \\ 0 & 0 & -8 & 12 \end{array} \right]$$



$$R_2 : -3y - z = -3$$

$$\begin{aligned} -8z &= 12 \\ z &= -\frac{3}{2} \end{aligned}$$

$$y = \frac{3-z}{3}$$

$$\begin{aligned} &= \frac{3 - (-\frac{3}{2})}{3} \\ &= \frac{3}{2} \end{aligned}$$

$$R_1 : x + y + z = 1$$

$$x = 1 - \frac{3}{2} + \frac{3}{2}$$

$$= 1$$

Unique Solⁿ

b)

$$3x - 4y = 8$$

$$x + y + z = 2$$

$$2x + 5y - z = 6$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 3 & -4 & 0 & 8 \\ 2 & -5 & -1 & 6 \end{array} \right] \xrightarrow{\substack{R_2 - 3R_1 \\ R_3 - 2R_1}} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -7 & -3 & 2 \\ 0 & -7 & -3 & 2 \end{array} \right]$$

make zero

$$\xrightarrow{R_3 - R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -7 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right] \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{2 L.I. eqns} \quad \leftarrow \text{SLB is not L.I.}$$

& of rank 2.

If has ∞ solutions since
it is not full rank

$$\xrightarrow{7R_1 + R_2} \left[\begin{array}{ccc|c} 7 & 0 & 4 & 16 \\ 0 & -7 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad \begin{array}{l} \text{From row 2: } -7y - 3z = 2 \\ z = \frac{-7y - 2}{3} \end{array}$$

$$\text{From row 1: } 7x + 4z = 16$$

$$z = \frac{16 - 7x}{4}$$

6

$$S(0) = C \\ = 100$$

$$S(2) = a(2^2) + b(2) + 100 = 84$$

$$4a + 2b = -16$$

$$2a + b = -8 \quad -R_1$$

$$S(4) = a(4^2) + b(4) + 100 = 36$$

$$16a + 4b = -64$$

$$4a + b = -16 - R_2$$

$$R_2 - R_1 \rightarrow 2a = -8$$

$$a = -4$$

$$b = 0$$

$$\therefore S(t) = -4t^2 + 100$$

$$S(t) = 0$$

$$-4t^2 + 100 = 0$$

$$t^2 - 25 = 0$$

$$(t+5)(t-5) = 0$$

$$t = 5 \quad \text{or} \quad t = -5$$

(reject)

$$t = 5$$

7.

Let 2% solⁿ be x

Let 7% solⁿ be y

$$x + y = 40 - R_1$$

$$0.02x + 0.07y = 40 \times 0.032$$

$$0.02x + 0.07y = 1.28 \quad - R_2$$

$$\text{From } R_1 : \quad y = 40 - x$$

Sub. into R_2 :

$$0.02x + 0.07(40-x) = 1.28$$

$$0.02x + 2.8 - 0.07x = 1.28$$

$$-0.05x = -1.52$$

$$x = 30.4 \text{ cc}$$

$$y = 40 - 30.4 = 9.6 \text{ cc}$$

8.

$$I_A + I_B = 10000 - R_1$$

$$0.05I_A + 0.03I_B = 440$$

$$5I_A + 3I_B = 44000 - R_2$$

From R_1 : $I_B = 10000 - I_A$

sub to R_2 : $5I_A + 3(10000 - I_A) = 44000$

$$5I_A - 3I_A = 44000 - 30000$$

$$2I_A = 14000$$

$$I_A = \$7000$$

$$I_B = 10000 - 7000$$

$$= \$3000$$

$$9 \left[\begin{array}{ccc|c} -2 & 5 & 1 & 32 \\ 4 & 1 & 1 & 28 \\ 1 & 1 & 1 & 16 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 16 \\ -2 & 5 & 1 & 32 \\ 4 & 1 & 1 & 28 \end{array} \right] \xrightarrow{\substack{R_2 + 2R_1 \\ R_3 - 4R_1}} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 16 \\ 0 & 7 & 3 & 64 \\ 0 & -3 & -3 & -36 \end{array} \right]$$

$$\xrightarrow{R_3 + R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 16 \\ 0 & 7 & 3 & 64 \\ 0 & 4 & 0 & 28 \end{array} \right]$$

$$\text{From } R_3 : 4y = 28$$

$$y = \underline{\underline{7}}$$

$$\text{From } R_2 : 7y + 3z = 64$$

$$3z = 64 - 7(7)$$

$$z = \underline{\underline{5}}$$

$$\text{From } R_1 : x + y + z = 16$$

$$x = 16 - 7 - 5$$

$$= \underline{\underline{4}}$$