Summary: The Maxwell's Equations

Gauss's law:

Integral formulation

$$\int_{\partial V} \vec{D} \cdot \mathrm{d}\vec{a} = Q(V) = \sum_{\vec{r}_i \in V} q_i \, .$$

$$\int_{\partial V} \vec{D} \cdot \mathrm{d}\vec{a} = Q(V) = \int_{V} \rho(\vec{r}) \mathrm{d}^3 r \, .$$

Differential formulation

(4.14) $div\vec{D} = \rho$

Gauss's law

Electric fields are generated by electric charges (quasi-static) $\Leftrightarrow$ sources of $\vec{D}$ are electric charges (for conservative electric fields)

Faraday's law of induction:

Integral formulation

$$\int_{\partial A} \vec{E} \cdot \mathrm{d}\vec{r} = -\int_{A} \frac{\partial \vec{B}}{\partial t} \cdot \mathrm{d}\vec{a}$$

(4.8)

Differential formulation

(4.16) $curl\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Faraday's law of induction

Electric fields are generated by rapidly time-varying $\vec{B}$-Fields

These fields are then not conservative ($curl\vec{E} \neq 0$).

Solenoidality of B-field:

Integral formulation

$$\int_{\partial V} \vec{B} \cdot \mathrm{d}\vec{a} = 0 \quad \text{for any control volume } V \, .$$

(3.19)

Differential formulation

(4.15) $div\vec{B} = 0$

Solenoidality of $\vec{B}$-Field

There are no magnetic charges/ magnetic monopoles, at which $\vec{B}$ lines start/end, hence $\Leftrightarrow$ field lines of magnetic fields are always closed

Ampere's law:

Integral formulation

$$\int_{\partial A} \vec{H} \cdot d\vec{r} = I(A) = \int_{A} \vec{j} \cdot d\vec{a}\,.$$

Differential formulation

curl

$$\text{(4.17)} \quad rot\vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$$

Ampère-Maxwell's circuital law

Ampere, Maxwell's circuital law:

$$\int_{\partial A} \vec{H} \cdot d\vec{r} = \int_{A} \left(\vec{j} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{a}$$

Magnetic fields are generated by

- Electric currents (quasi-stationary)
- Rapidly time-variant electric fields (electric displacement current)