Centripetal Force

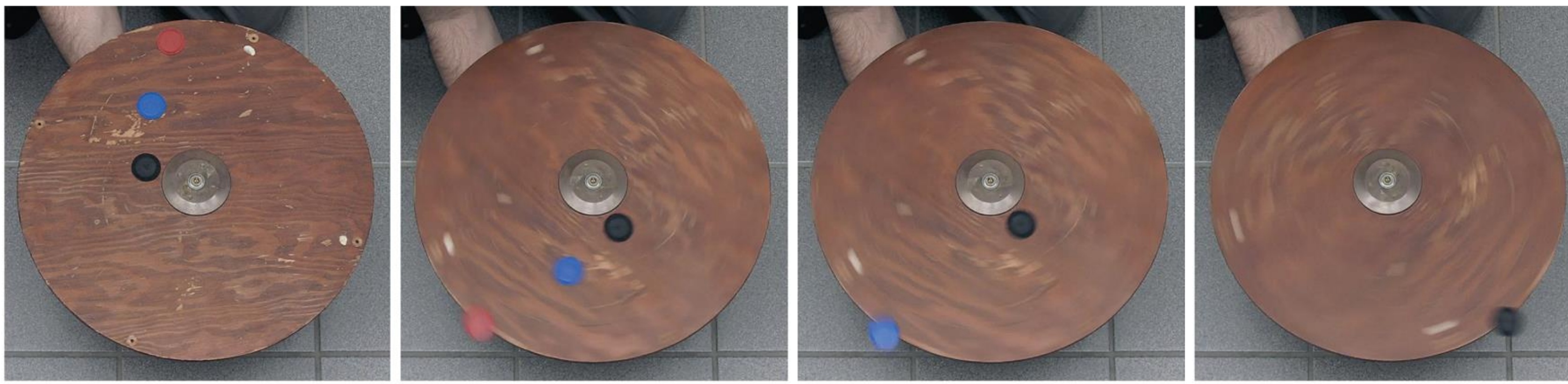
- The centripetal force is not another fundamental force of nature. It should not be drawn on a free body diagram.
- It is the inward force necessary to provide the centripetal acceleration necessary for circular motion.
- It has to point inward toward the circle's center.
- Its magnitude is the product of the mass of the object and the centripetal acceleration required to force the object onto a circular path:

$$F_c = ma_c = mv\omega = m\frac{v^2}{r} = m\omega^2 r$$

$$a_c = \frac{v^2}{r}$$

Centripetal Force

- Consider a spinning table with three poker chips on it.
- We spin the table more and more quickly and observe what happens:



(a) (b) (c) (d)

- Every point on the spinning table has the same angular velocity.
- The centripetal force necessary to keep the poker chip on the table increases with ω^2 and r.

Ultracentrifuge

PROBLEM:

- You want to generate 840,000 *g* of centripetal acceleration in a sample rotating at a distance of 23.5 cm from the rotation axis. What is the frequency you have to enter into the controls? At that frequency, what is the linear speed of the sample?

SOLUTION:

- The centripetal acceleration is:

$$a_c = \omega^2 r$$

- Frequency and angular velocity are related as:

$$\omega = 2\pi f$$

Ultracentrifuge

- So can get the required frequency:

$$a_c = (2\pi f)^2 r \quad \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{a_c}{r}}$$

- Putting in our numbers we get:

$$f = \frac{1}{2\pi}\sqrt{\frac{(840{,}000)(9.81\ \mathrm{m/s^2})}{0.235\ \mathrm{m}}} = 942\ \mathrm{s^{-1}} = 56{,}500\ \mathrm{rpm}$$

- For the linear speed of the sample we have:

$$v = r\omega = 2\pi r f = 2\pi(0.235\ \mathrm{m})(942\ \mathrm{s^{-1}}) = 1.39\ \mathrm{km/s}$$

Conical Pendulum

- The "Wave Swinger" ride at amusement parks has the riders sit in seats suspended from a solid disk by long chains.
- At the beginning of the ride, the chains hang straight down.
- As the ride starts to rotate, the chains form an angle φ with the vertical.
- This angle is independent of the mass of the rider and depends only on the angular velocity of the circular motion.
- What is the value of this angle in terms of the angular velocity?

Conical Pendulum

- There are only two forces acting: *gravity* and *string tension*.
- Vertical component of the string tension balances weight:

  $$T\cos\phi = mg$$

- Centripetal force = horizontal component of string tension:

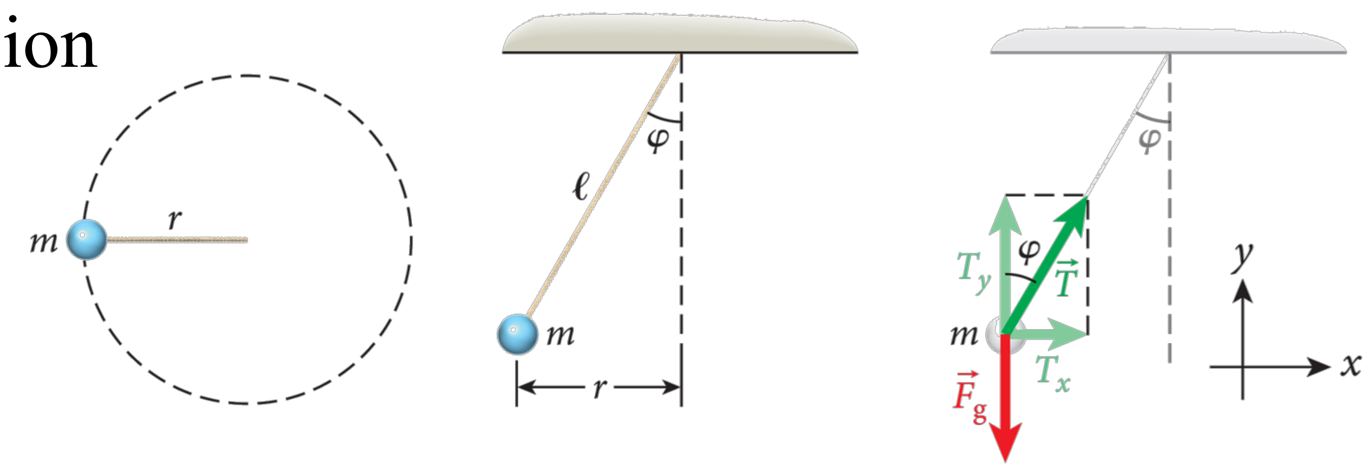
  $$T\sin\phi = mr\omega^2$$

- The radius of the circular motion is:

  $$r = \ell\sin\varphi$$

- The string tension is:

  $$T = m\ell\omega^2$$



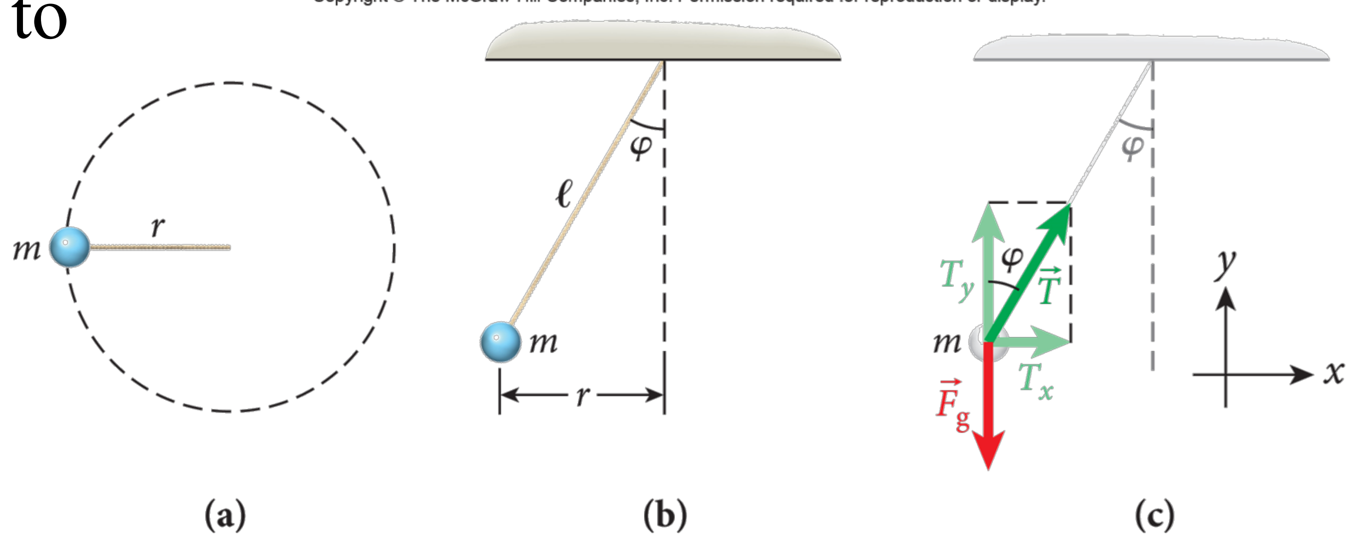
(a) (b) (c)

Conical Pendulum

- Substituting our expression for the tension into the equation for the vertical force components, we get:

$$\left(m\ell\omega^2\right)\cos\varphi = mg \;\Rightarrow\; \omega^2 = \frac{g}{\ell\cos\varphi} \;\Rightarrow\; \omega = \sqrt{\frac{g}{\ell\cos\varphi}}$$

- The mass cancels out, which explains why all the chains have the same angle.
- Note that as φ goes to zero, ω goes to $(g/\ell)^{1/2}$.



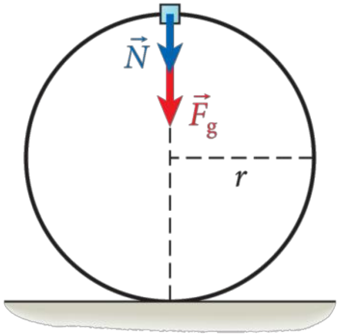
Roller Coaster

PROBLEM:

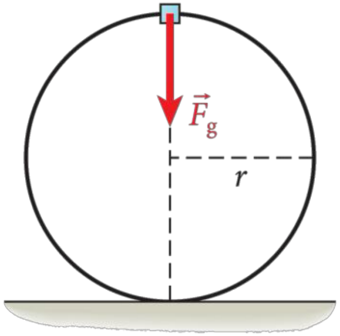
- Suppose the vertical loop of a roller coaster has a radius of 5.00 m.
- What does the linear speed of the roller coaster have to be at the top of the loop for the passengers to feel weightless?

SOLUTION:

- A passenger will feel weightless when there is no supporting force from a seat or a restraint acting to counter his weight.



(a)



(b)

Roller Coaster

- At the top of the loop, the net force is equal to the centripetal force required to keep the coaster on the track.

$$\vec{F}_{\mathrm{c}} = \vec{F}_{\mathrm{net}} = \vec{F}_{\mathrm{g}} + \vec{N}$$

- For weightlessness, the normal force is zero:

$$\vec{F}_{\mathrm{c}} = \vec{F}_{\mathrm{net}} \quad \Rightarrow \quad F_{\mathrm{c}} = F_{\mathrm{g}}$$

- The force of gravity is:

$$F_{\mathrm{g}} = mg$$

- The magnitude of the centripetal force is:

$$F_{\mathrm{c}} = ma_{\mathrm{c}} = m\frac{v^2}{r}$$

$$F_{\mathrm{c}} = F_{\mathrm{g}} \quad \Rightarrow \quad m\frac{v_{\mathrm{top}}^2}{r} = mg \quad \Rightarrow \quad v_{\mathrm{top}} = \sqrt{rg}$$

Roller Coaster

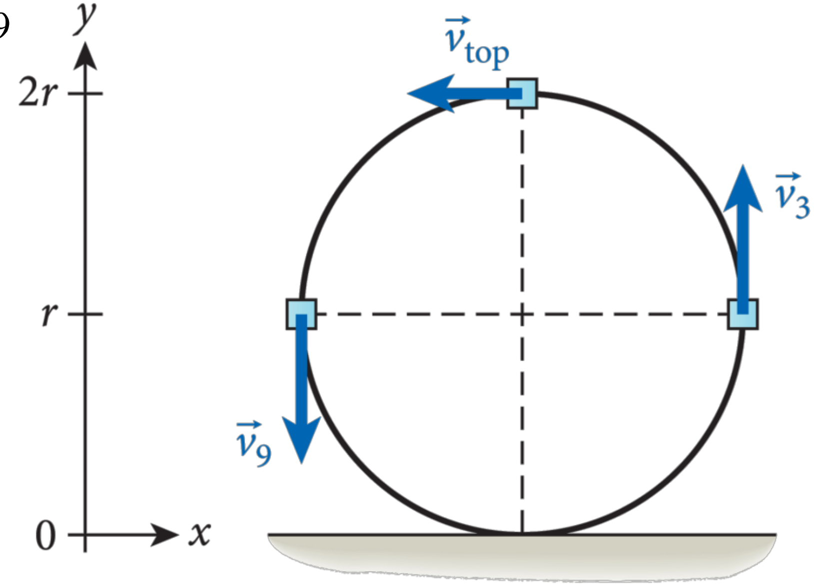
$$v_{\text{top}} = \sqrt{rg} = \sqrt{(5.00\text{ m})(9.81\text{ m/s}^2)} = 7.00357\text{ m/s}$$

- Let's calculate the speed at 3 o'clock and 9 o'clock:

$$E = K_3 + U_3 = K_{\text{top}} + U_{\text{top}} = K_9 + U_9$$

- Clearly the kinetic energies and the speeds will be the same at 3 o'clock and 9 o'clock, so let's do 3 o'clock:

$$\frac{1}{2}mv_3^2 + mgy_3 = \frac{1}{2}mv_{\text{top}}^2 + mgy_{\text{top}}$$



Roller Coaster

- The mass cancels out and we get:

$$v_3 = \sqrt{v_{\text{top}}^2 + 2g\left(y_{\text{top}} - y_3\right)}$$

$$v_3 = \sqrt{\left(7.00\text{ m/s}\right)^2 + 2\left(9.81\text{ m/s}^2\right)\left(5.00\text{ m}\right)} = 12.1\text{ m/s}$$

- The speed is higher at 3 o'clock and 9 o'clock, which lends credence to our result.