2. Derive from first principles (using the limit operator) the derivative of each function below.

a) $f(x) = c$ ← constant

b) $f(x) = ax^2 + bx + c$

c) $f(x) = 1/x$

d) $f(x) = \sqrt{ax - b}$

e) $f(x) = \cos x$

ANS: **a)** 0. **b)** 2ax + b. **c)** -1/x^2. **d)** $\frac{a}{2\sqrt{ax-b}}$. **e)** -sin x.

a) $f(x) = c$ $= \lim_{h \to 0} f'(x) = 0$

b) $f(x) = ax^2 + bx + c$

$$= \lim_{h\to 0} f'(x) = \lim_{h\to 0} \frac{[a(x+h)^2 + b(x+h) + c] - [ax^2 + bx + c]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{ax^2} + 2axh + h^2 + \cancel{bx} + bh + \cancel{c} - \cancel{ax^2} - \cancel{bx} - \cancel{c}}{h}$$

$$= \lim_{h\to 0} \frac{2axh + h^2 + bh}{h}$$

$$= \lim_{h\to 0} 2ax + h + b$$

$$= 2ax + b$$

2. Derive from first principles (using the limit operator) the derivative of each function below.

a) $f(x) = c$

b) $f(x) = ax^2 + bx + c$

c) $f(x) = 1/x$

d) $f(x) = \sqrt{ax - b}$

e) $f(x) = \cos x$

ANS: **a)** 0. **b)** 2ax + b. **c)** -1/x^2. **d)** $\frac{a}{2\sqrt{ax-b}}$. **e)** -sin x.

d) $f(x) = \sqrt{ax - b}$

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{a(x+h)-b} - \sqrt{ax-b}}{h}\left(\frac{\sqrt{a(x+h)-b} + \sqrt{ax-b}}{\sqrt{a(x+h)-b} + \sqrt{ax-b}}\right)$$

$$= \lim_{h\to 0} \frac{a(\cancel{x}+h)\cancel{-b} - (\cancel{ax}\cancel{-b})}{h\sqrt{a(x+h)-b} + \sqrt{ax-b}} = \lim_{h\to 0} \frac{ah}{h\sqrt{a(x+h)-b} + \sqrt{ax-b}}$$

$$= \frac{a}{2\sqrt{ax-b}}$$

e) $f(x) = \cos x$

$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$$f'(x) = \lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h\to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

$$= \lim_{h\to 0} \frac{\cos x(\cos h - 1) - \sin x \sin h}{h}$$

$$= \underbrace{\cos x \lim_{h\to 0} \frac{\cos h - 1}{h}}_{=0} - \sin x \underbrace{\lim_{h\to 0} \frac{\sin h}{h}}_{=1}$$

squeeze theorem

$$= -\sin x$$

3. Differentiate the following functions.

a) $f(x) = 3x^4 - \dfrac{1}{\sqrt{1-2x}}$

b) $g(t) = t^3 \ln\left(t^2 - t\right) + \tan(\pi t)$

c) $h(\theta) = \dfrac{e^{\pi\theta}}{\sin(2\theta)}$

d) $z(y) = 7^y \log_7\left(y^{1/3}\right)$

e) $y(x) = \dfrac{2^{\pi x}}{\left(x^2+1\right)^3}$

f) $w(x) = xe^x \cos x$

ANS: **a)** $f'(x) = 12x^3 - \dfrac{1}{(1-2x)^{3/2}}$. **b)** $g'(t) = 3t^2 \ln\left(t^2 - t\right) + \dfrac{t^2(2t-1)}{t-1} + \pi \sec^2(\pi t)$. **c)** $h'(\theta) = \dfrac{e^{\pi\theta}[\pi \sin(2\theta) - 2\cos(2\theta)]}{\sin^2(2\theta)}$. **d)** $z'(y) = \dfrac{7^y}{3\ln 7}\left[\dfrac{1}{y} + \ln 7 \cdot \ln y\right]$. **e)** $y'(x) = \dfrac{2^{\pi x}\left[\pi \ln 2\left(x^2+1\right) - 6x\right]}{\left(x^2+1\right)^4}$. **f)** $w'(x) = e^x(\cos x + x\cos x - x\sin x)$

b) $g(t) = t^3 \ln\left(t^2 - t\right) + \tan(\pi t)$

$$g'(t) = 3t^2 \ln(t^2-t) + t^3\left(\frac{1}{t^2-t}\cdot 2t - 1\right) + \pi\sec^2(\pi t)$$

$$= 3t^2\ln(t^2-t) + \frac{t^2(2t-1)}{t^2-t} + \pi\sec^2(\pi t)$$

c) $h(\theta) = \dfrac{e^{\pi\theta}}{\sin(2\theta)}$

$$h'(\theta) = \frac{\sin(2\theta)\cdot\pi e^{\pi\theta} - e^{\pi\theta}\cdot\cos(2\theta)\cdot 2}{(\sin(2\theta))^2}$$

$$= \frac{e^{\pi\theta}[\pi\sin(2\theta) - 2\cos(2\theta)]}{\sin^2(2\theta)}$$

d) $z(y) = 7^y \log_7\left(y^{1/3}\right)$

$$z'(y) = 7^y\Big[\underbrace{\frac{1}{y^{\frac{1}{3}}\ln 7}\cdot\frac{1}{3}y^{-\frac{2}{3}}}_{\text{Chain rule}}\Big] + 7^y\cancel{\ln 7}\,\underbrace{\log_7(y^{\frac{1}{3}})}_{\frac{\ln y^{\frac{1}{3}}}{\cancel{\ln 7}}}$$

$$= 7^y\left[\frac{1}{3y\ln 7} + \ln y^{\frac{1}{3}}\right]$$

e) $$y(x) = \frac{2^{\pi x}}{(x^2+1)^3}$$

$$\ln f(x) = \pi x \ln 2$$

$$\frac{dy}{dx} \to \frac{f'}{f} = \pi$$

$$f'(x) = f(x) \cdot \pi$$

$$= \pi 2^{\pi x}$$

$$y'(x) = \frac{\pi 2^{\pi x} \cdot (x^2+1)^3 - 2^{\pi x} \cdot 3(x^2+1)^2 \cdot 2x}{[(x^2+1)^3]^2}$$

$$= \frac{2^{\pi x}[\pi(x^2+1) - 6x]}{(x^2+1)^4}$$

f) $$w(x) = x\underbrace{e^x \cos x}^{f(x)}$$

$$W'(x) = (1)f(x) + xf'(x) \qquad f'(x) = e^x\cos x - e^x \sin x$$

$$= e^x \cos x + x[e^x\cos x - e^x \sin x]$$

$$= e^x[\cos x + x\cos x - x\sin x]$$

4. Using implicit differentiation, prove the following derivatives.

a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\cos^{-1}x = \dfrac{-1}{\sqrt{1-x^2}}$

b) $\dfrac{\mathrm{d}}{\mathrm{d}x}\csc^{-1}x = \dfrac{-1}{x\sqrt{x^2-1}}$

c) $\dfrac{\mathrm{d}}{\mathrm{d}x}\sec^{-1}x = \dfrac{1}{x\sqrt{x^2-1}}$

d) $\dfrac{\mathrm{d}}{\mathrm{d}x}\cot^{-1}x = \dfrac{-1}{1+x^2}$

b) $\dfrac{\mathrm{d}}{\mathrm{d}x}\csc^{-1}x = \dfrac{-1}{x\sqrt{x^2-1}}$

$y = \csc^{-1} x$

$x = \csc y = \dfrac{1}{\sin y} \rightarrow \sin y = \dfrac{1}{x}$

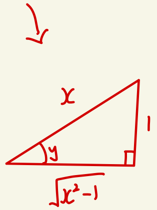
$\dfrac{d}{dx} \rightarrow 1 = -\csc y \cot y \cdot y'$

$\Rightarrow y' = \dfrac{1}{-\csc y \cot y}$

$= -\dfrac{1}{x} \cdot \tan y$

$= \dfrac{-1}{x} \cdot \dfrac{1}{\sqrt{x^2-1}}$

$= \dfrac{-1}{x\sqrt{x^2-1}}$



c) $\dfrac{\mathrm{d}}{\mathrm{d}x}\underbrace{\sec^{-1}x} = \dfrac{1}{x\sqrt{x^2-1}}$

$\hookrightarrow$ Let $y = \sec^{-1} x$

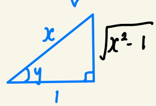
$\hookrightarrow x = \sec y = \dfrac{1}{\cos y} \rightarrow \cos y = \dfrac{1}{x}$

$\dfrac{d}{dx} \rightarrow 1 = -\dfrac{(-\sin y)}{\cos^2 y} y' = \tan y \sec y \cdot y'$

$= x \tan y \cdot y'$

$\Rightarrow y' = \dfrac{1}{x} \cdot \dfrac{1}{\tan y}$

$= \dfrac{1}{x\sqrt{x^2-1}}$ (shown)



5. Given that x is an implicit function of y in the implicit equation below, determine dx/dy.

$x(y)$

$$x^2 y + \ln x - \sin y = 0$$

$\frac{d}{dy}$

ANS: $x' = \dfrac{\cos y - x^2}{2xy + \frac{1}{x}}$

$$2x \cdot x' y + x^2(1) + \frac{1}{x} x' - \cos y = 0$$

$$x'\left(2xy + \frac{1}{x}\right) = \cos y - x^2$$

$$x' = \frac{\cos y - x^2}{2xy + \frac{1}{x}}$$

6. Determine the equation of the tangent and normal lines on the curves defined below at the given point (0, 1). Verify your answers in Desmos.

a) $y^3 + y^2 - 5y - x^2 = -4.\ (2, 0)$

b) $xy^2 + \dfrac{x}{y} + y^3 = 1.\ (0, 1)$

ANS: **a)** $y = -\frac{4}{5}x + \frac{8}{5}$ (Tangent). $y = \frac{5}{4}x - \frac{5}{2}$ (Normal).

b) $y = -\frac{2}{3}x + 1$ (Tangent). $y = \frac{3}{2}x + 1$ (Normal).

a) $y^3 + y^2 - 5y - x^2 = -4.\ (2, 0)$

$$\frac{d}{dx} \rightarrow 3y^2 \cdot y' + 2y \cdot y - 5 \cdot y' - 2x = 0$$

$$y'(3y^2 + 2y - 5) = 2x$$

$$y' = \frac{2x}{3y^2 + 2y - 5} \quad \text{at } (2, 0), \quad y' = \frac{4}{0 + 0 - 5} = \underbrace{-\frac{4}{5}}_{m}$$

$$m_n = \frac{5}{4}$$

eqn of tangent: $y - 0 = -\frac{4}{5}(x - 2)$

$$y = -\frac{4}{5}x + \frac{8}{5}$$

eqn of normal: $y = \frac{5}{4}(x - 2)$

$$y = \frac{5}{4}x - \frac{5}{2}$$

b) $xy^2 + \dfrac{x}{y} + y^3 = 1.\ (0, 1)$

$$\frac{d}{dx} \rightarrow y^2 + x(2y \cdot y') + \underbrace{\frac{y - y'x}{y^2}}_{\frac{1}{y} - \frac{y'x}{y^2}} + 3y^2 y' = 0$$

$$y'\left(2xy - \frac{x}{y^2} + 3y^2\right) = -y^2 - \frac{1}{y}$$

$$y' = \frac{-y^2 - \frac{1}{y}}{2xy - \frac{x}{y^2} + 3y^2} \rightarrow \text{At } (0, 1),\ y' = \frac{-1 - 1}{0 - 0 + 3} = \underbrace{-\frac{2}{3}}_{m}$$

Eqn of tangent is

$$y - 1 = -\frac{2}{3}(x - 0)$$

$$y = -\frac{2}{3}x + 1$$

Eqn of normal is

$$m_n = -\frac{1}{m} = \frac{3}{2}$$

$$y - 1 = \frac{3}{2}(x - 0)$$

$$y = \frac{3}{2}x + 1$$

7. Determine the first and second derivatives for the implicit equations below, where y is an implicit function of x.

a) $y^2 + y - x = 0$

b) $xye^y = 1$

ANS: **a)** $y' = \frac{1}{2y+1}$, $y'' = \frac{-2}{(2y+1)^3}$. **b)** $y' = \frac{-y}{x(y+1)}$, $y'' = \frac{y}{x^2}\left[\frac{1}{1+y} + \frac{1}{(1+y)^3}\right]$

b) $xye^y = 1$

$$\frac{d}{dx} \rightarrow ye^y + xy'e^y + xye^y \cdot y' = 0$$

$$y'x\cancel{e^y}(1+y) = -y\cancel{e^y}$$

$$y' = \frac{-y}{x(1+y)}$$

$$\frac{d}{dx}$$

$$y'' = \frac{x(1+y)(-y') + y[1+y+xy']}{x^2(1+y)^2}$$

$$= \frac{y'(-x-\cancel{xy}+\cancel{xy}) + y(1+y)}{x^2(1+y)^2}$$

$$= \frac{-\cancel{x}y'}{\cancel{x^2}(1+y)^2} + \frac{y\cancel{(1+y)}}{x^2\cancel{(1+y)^2}}$$

$$= \frac{y}{x(1+y)} \cdot \frac{1}{x(1+y)^2} + \frac{y}{x^2(1+y)} = \frac{y}{x^2(1+y)}\left[\frac{1}{(1+y)^2} + 1\right]$$

a) $y(x) = x^{\cot x}$

$\ln y = \cot x \ln x$

$\frac{d}{dx} \to \frac{y'}{y} = -\operatorname{cosec}^2 x . \ln x + \frac{\cot x}{x}$

$y'(x) = x^{\cot x}\left[-\operatorname{cosec}^2 x . \ln x + \frac{\cot x}{x}\right]$

b) $f(x) = \left(\frac{x}{1+x}\right)^x$

$\frac{1+x-x}{(1+x)}$

$\ln f = x \ln\left(\frac{x}{1+x}\right)$

$\frac{d}{dx} \to \frac{f'}{f} = \ln\left(\frac{x}{1+x}\right) + \cancel{x}\left[\frac{\cancel{1+x}}{\cancel{x}} \cdot \underbrace{\frac{(1+x)-x}{(1+x)^{\cancel{2}}}}\right]$

$f'(x) = \left(\frac{x}{1+x}\right)^x\left[\ln\left(\frac{x}{1+x}\right) + \frac{1}{1+x}\right]$

d) $h(x) = \frac{x^2 e^{3x}\tan(4x)}{\sqrt{x+1}}$

$\ln(x+1)^{\frac{1}{2}} = \frac{1}{2}\ln(x+1)$

$\ln h = 2\ln x + 3x \overset{1}{\cancel{\ln e}} + \ln \tan(4x) - \underbrace{\ln\sqrt{x+1}}$

$\frac{d}{dx} \to \frac{h'}{h} = \frac{2}{x} + 3 + \left[\frac{1}{\tan(4x)} \cdot \sec^2(4x) . 4\right] - \frac{1}{2}\left(\frac{1}{x+1}\right)$

$h'(x) = \frac{x^2 e^{3x}\tan(4x)}{\sqrt{x+1}}\left[\frac{2}{x} + 3 + \frac{4\sec^2(4x)}{\tan(4x)} - \frac{1}{2(x+1)}\right]$

8. Using logarithmic differentiation, evaluate the derivatives of the functions below.

a) $y(x) = x^{\cot x}$

b) $f(x) = \left(\dfrac{x}{1+x}\right)^x$

c) $g(x) = f\left(x^2\right)^{\sqrt{x}}$

d) $h(x) = \dfrac{x^2 e^{3x} \tan(4x)}{\sqrt{x+1}}$

ANS: **a)** $f'(x) = x^{\cot x}\left(\dfrac{\cot x}{x} - \csc^2 x \ln x\right)$. **b)** $f'(x) = \left(\dfrac{x}{1+x}\right)^x \left[\ln\left(\dfrac{x}{1+x}\right) + \dfrac{1}{1+x}\right]$.

c) $g'(x) = f\left(x^2\right)^{\sqrt{x}}\left[\dfrac{\ln f\left(x^2\right)}{2\sqrt{x}} + \dfrac{2x^{3/2} f'\left(x^2\right)}{f\left(x^2\right)}\right]$.

d) $h'(x) = \dfrac{x^2 e^{3x}\tan(4x)}{\sqrt{x+1}}\left[\dfrac{2}{x} + 3 + \dfrac{8}{\sin(8x)} - \dfrac{1}{2(x+1)}\right]$.

c) $g(x) = f\left(x^2\right)^{\sqrt{x}}$

$$\ln g(x) = \sqrt{x}\,\underline{\ln f(x^2)} \xrightarrow{\frac{d}{dx}}$$

$$\frac{d}{dx} \rightarrow \frac{g'(x)}{g(x)} = \frac{1}{2\sqrt{x}} \ln f(x^2) + \sqrt{x}\left[\underbrace{\frac{1}{f(x^2)}}_{\frac{d\ln f}{\cancel{df}}} \cdot \underbrace{f'(x^2)}_{\frac{\cancel{df}}{\cancel{dx^2}}} \cdot \underbrace{2x}_{\frac{\cancel{dx^2}}{dx}}\right]$$

$$\frac{g'(x)}{g(x)} = f(x^2)^{\sqrt{x}} \left\{\frac{\ln f(x^2)}{2\sqrt{x}} + \frac{2x^{\frac{3}{2}} f'(x^2)}{f(x^2)}\right\} //$$

d) $h(x) = \underbrace{\dfrac{x^2 e^{3x} \tan(4x)}{\sqrt{x+1}}}$

$$\ln h(x) = \ln\{\quad\} = \ln x^2 + \ln e^{3x} + \ln \tan(4x) - \ln\sqrt{x+1}$$

$$\frac{d}{dx} \rightarrow \frac{h'(x)}{h(x)} = \frac{1}{x^2}(2x) + 3 + \frac{1}{\tan(4x)} \cdot \sec^2(4x) \cdot 4 - \frac{1}{\sqrt{x+1}}\left(\frac{1}{2\sqrt{x+1}}\right)$$

$$h'(x) = h(x)\left[\frac{2}{x} + 3 + \underbrace{\frac{4(2)}{(2)\sin(4x)\cos(4x)}}_{\frac{8}{\sin(8x)}} - \frac{1}{2(x+1)}\right] //$$

9. Given the following information, determine the derivatives below.

$$f(-4)=3,\quad f(1)=0,\quad f(2)=1,\quad f(3)=2,$$
$$f'(-4)=1,\quad f'(1)=0,\quad f'(2)=3,\quad f'(3)=-1$$

$$g(-4)=9,\quad g(1)=3,\quad g(2)=-2,\quad g(3)=0,$$
$$g'(-4)=-3,\quad g'(1)=1/2,\quad g'(2)=6,\quad g'(3)=-4$$

a) $p'(3)$ where $p(x)=3f(x)-2g(x)$

b) $q'(2)$ where $q(x)=f(x)/g(x)$

c) $r'(2)$ where $r(x)=g(3f(x))$

ANS: **a)** 5. **b)** -3. **c)** -36.

a) $p'(x) = 3f'(x) - 2g'(x)$

$p'(3) = 3f'(3) - 2g'(3)$

$= 3(-1) - 2(-4)$

$= -3 + 8$

$= 5$

c) $r'(2)$ where $r(x)=g(3f(x))$ $\longrightarrow \frac{d}{dx} \rightarrow$ $r'(x) = g'(3f(x)).(3).f'(x)$

$r'(2) = 3\, g'(3f(2)) . f'(2)$

$= 3g'(3) . (3)$

$= 9(-4)$

$= -36$

b) $q'(2)$ where $q(x)=f(x)/g(x)$ $\frac{d}{dx} \rightarrow$ $q'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

$q'(2) = \dfrac{g(2)f'(2) - f(2)g'(2)}{(g(2))^2}$

$= \dfrac{(-2)(3) - 6}{(-2)^2}$

$= \dfrac{-12}{4}$

$= -3$

10. Determine if each function below is differentiable in $\mathbb{R}$. If not, specify where it is not differentiable.

a) $f(x) = x|x|$

b) $f(x) = e^{|2x+1|}$

c) $f(x) = \begin{cases} x^2 + 1, & x < 0 \\ x^3 \sin x + 1, & x \geq 0 \end{cases}$

ANS: **a)** Yes. **b)** No. At x = -½. **c)** Yes.

b) $f(x) = e^{|2x+1|} = \begin{cases} e^{2x+1}, & 2x+1 \geq 0 \to x \geq -\frac{1}{2} \\ e^{-(2x+1)}, & 2x+1 < 0 \to x < -\frac{1}{2} \end{cases}$

$$f'(x) = \begin{cases} \overset{f_1'(x)}{2e^{2x+1}}, & x > -\frac{1}{2} \\ \underset{f_2'(x)}{-2e^{-(2x+1)}}, & x < -\frac{1}{2} \end{cases}$$

Check continuity : $\lim_{x \to -\frac{1}{2}^-} f(x) = \lim_{x \to -\frac{1}{2}^-} e^{-(2x+1)} = 1 = \lim_{x \to -\frac{1}{2}^+} e^{2x+1} = 1 = f(-\frac{1}{2})$

so $\lim_{x \to -\frac{1}{2}^-} f(x) = f(-\frac{1}{2})$, $\therefore$ f(x) is continuous in $\mathbb{R}$

Check smoothness : f(x) is differentiable in $x > -\frac{1}{2}$ & $x < -\frac{1}{2}$, so check at $x = -\frac{1}{2}$

$$\lim_{x \to -\frac{1}{2}^-} f_2'(x) = \lim_{x \to -\frac{1}{2}^-} -2e^{-(2x+1)} = -2$$

$$\lim_{x \to -\frac{1}{2}^+} f_1'(x) = \lim_{x \to -\frac{1}{2}^+} 2e^{2x+1} = 2 \neq \lim_{x \to -\frac{1}{2}^-} f_2'(x)$$

so f(x) is not differentiable at $x = -\frac{1}{2}$

11. Determine the values of parameters of a, b & c in order for the function below to be differentiable in $\mathbb{R}$.

$$f(x) = \begin{cases} ae^{-x} + b, & x < 1 \\ 2, & x = 1 \\ c/x & x > 1 \end{cases} \xrightarrow{\frac{d}{dx}} f'(x) = \begin{cases} -ae^{-x}, & x < 1 \\ -\frac{c}{x^2}, & x > 1 \end{cases}$$

$f_1(x)$ (for $ae^{-x} + b$), $f_2(x)$ (for c/x)

ANS: a = 2e, b = 0, c = 2.

Observe that $f_1(x)$ and $f_2(x)$ are differentiable in their own intervals so enforce conditions of differentiability at $x = 1$

(Continuity) $\lim_{x\to a} f(x) = f(a)$

(Collinearity of tangents) $\underbrace{\lim_{x\to a^-} f'(x) = \lim_{x\to a^+} f'(x)}$

$$\lim_{x\to 1^-} ae^{-x} + b = \frac{a}{e} + b = f(1) = 2$$

$$\lim_{x\to 1^+} \frac{c}{x} = c = f(1) = 2$$

$$\lim_{x\to 1^-} -ae^{-x} = \frac{-a}{e} = \lim_{x\to 1^+} -\frac{c}{x^2} = -c$$

solve SLE :

$$\frac{a}{e} + b = 2 \longrightarrow b = 2 - \frac{a}{e} = 2 - 2 = 0$$

$$c = 2$$

$$\left.\begin{matrix} c = 2 \\ \frac{a}{e} = c \end{matrix}\right\} \frac{a}{e} = 2 \rightarrow a = 2e$$

ANS: **b)** a = 40, b = -150. **c)** $2.1 per yard. Average increase in cost per yard when production increases from 81 to 101 yards of fabric. **d)** $ per yard. Additional cost of producing the next yard when production is at x yards. C'(100) = $2 per yard.

12. **Mathematical Modelling**: Marginal Cost of Production

A cloth manufacturer, Muthu, produces bolts of a fabric with a fixed width. From his experience, he models the cost ($) of producing x yards of this fabric to be

$$C(x) = \begin{cases} \overbrace{2x + 50}^{C_1(x)}, & 0 \le x \le 100 \\ \underbrace{a\sqrt{x} + b}_{C_2(x)}, & x > 100 \end{cases}$$

a) From an economic perspective, explain why possibly the cost function is linear when x is smaller and transits to a root function when x is bigger.

b) Muthu thinks that his cost model should be continuous and smooth. Determine parameters a & b to fulfil this criteria.

gradient of secant line
(Avg slope between 2 points)

c) Determine $\dfrac{C(101) - C(81)}{101 - 81}$ and state its unit. What does it represent in layman?

$ / yard

d) State the unit of C'(x) and explain its meaning. What is C'(100) and what does it represent in layman? $/yard

increase in cost when producing the 101st yard

e) Without calculation, is C'(100) or C'(1000) bigger and why? Do you think this trend is always true? Explain. C'(100) > C'(1000) due to economy of scale

Might not be true if demand outstrips supply