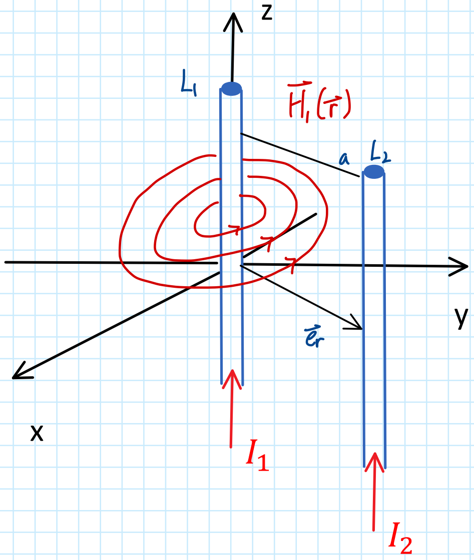
3.6.2 Force between two parallel current-carrying wires



- Wire L_2 carries current I_2 and is located parallel to wire L_1 at distance a

magnetic field generated by L_1:

$$\vec{H}_1(\vec{r}) = \frac{I_1}{2\pi r} \cdot \vec{e}_\varphi$$

$\vec{H}_1$ exerts a Lorentz force $\vec{F}_{12}$ on L_2

distance between L_1 and L_2 is a

distance vector points along $\vec{e}_r$ - direction

Force $d\vec{F}$ on a small wire element $d\vec{s}$ which carries I

$$d\vec{F} = I \cdot d\vec{s} \times \vec{B}$$

Force on L_2 by $\vec{H}_1(\vec{r})$ generated by L_1

$$d\vec{F}_{12} = I_2 \cdot d\vec{s}_2 \times \vec{B}_1 \qquad d\vec{s} = dz \cdot \vec{e}_z \qquad \vec{B}_1 = \mu \cdot \vec{H}_1 = \mu \frac{I_1}{2\pi r} \cdot \vec{e}_\varphi$$

$r(L_2) = a$

$$d\vec{F}_{12} = I_2 \cdot dz \cdot \vec{e}_z \times \mu \frac{I_1}{2\pi a} \cdot \vec{e}_\varphi$$

$$= \frac{I_1 I_2}{2\pi a} dz \, \underbrace{\vec{e}_z \times \vec{e}_\varphi}_{\text{direction of force}} \qquad \vec{e}_\varphi \times \vec{e}_z = \vec{e}_r \qquad \vec{e}_z \times \vec{e}_\varphi = -\vec{e}_r$$

$$\boxed{\frac{d\vec{F}_{12}}{dz} = -\mu \frac{I_1 I_2}{2\pi a} \cdot \vec{e}_r}$$

$\vec{F}_{12}$ is parallel to distance vector

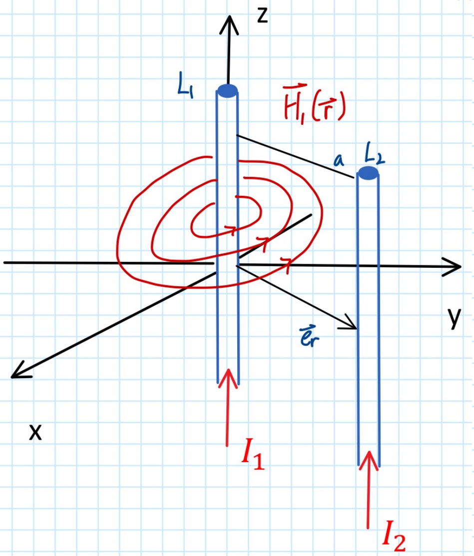
if $I_1 \cdot I_2 > 0$ (both currents are in same direction) $\Rightarrow \frac{d\vec{F}_{12}}{dz} < 0$

pointing in $-\vec{e}_r$

$\Downarrow$

attractive force

- W



if $I_1 \cdot I_2 < 0 \Rightarrow$ opposite direction

$\Downarrow$

$\frac{d\vec{F}_{12}}{dz} > 0$; pointing in $+\vec{e}_r$

$\Downarrow$

repulsive force

Total force on wire with length L

$$\Rightarrow F_{12} = \int_0^L \dots dz = L \dots$$