ENG1004 Eng Physics 1

AY2023/24 Trimester 1

Week9: Oscillations (Part 2)

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Content

$$x = x_0 \sin \omega t$$

$$v = \frac{dx}{dt} = x_0 \omega \cos \omega t$$

1. Types of oscillation

$$a = -\omega^2 x$$

2. Simple Harmonic Motion (SHM)

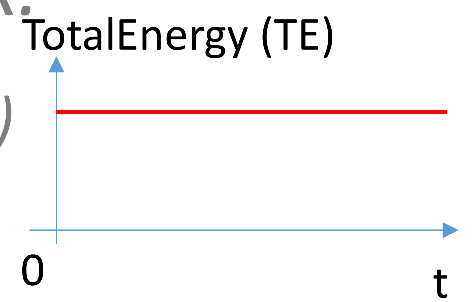
Two types:

(1) Starts at x = 0 at t = 0 $x = x_0 \sin \omega t$

(2) Starts at $x = x_0$ or $x = -x_0$ at t = 0

$x = x_0 \cos \omega t$ $x = -x_0 \cos \omega t$

3. Variation with time: *x*, *v*, *a* vs *t*.
4. Variation with displacement: *v*, *a* vs *x*.
   - *Energy vs time (KE, PE, TE [Total energy] vs t)*



5. Damped Oscillations
6. Forced Oscillations

1. Intro Free, Damped & Forced Oscillations

Oscillations will go on forever if <u>undamped.</u>

<u>Damping:</u> Resistive forces acting on oscillation.

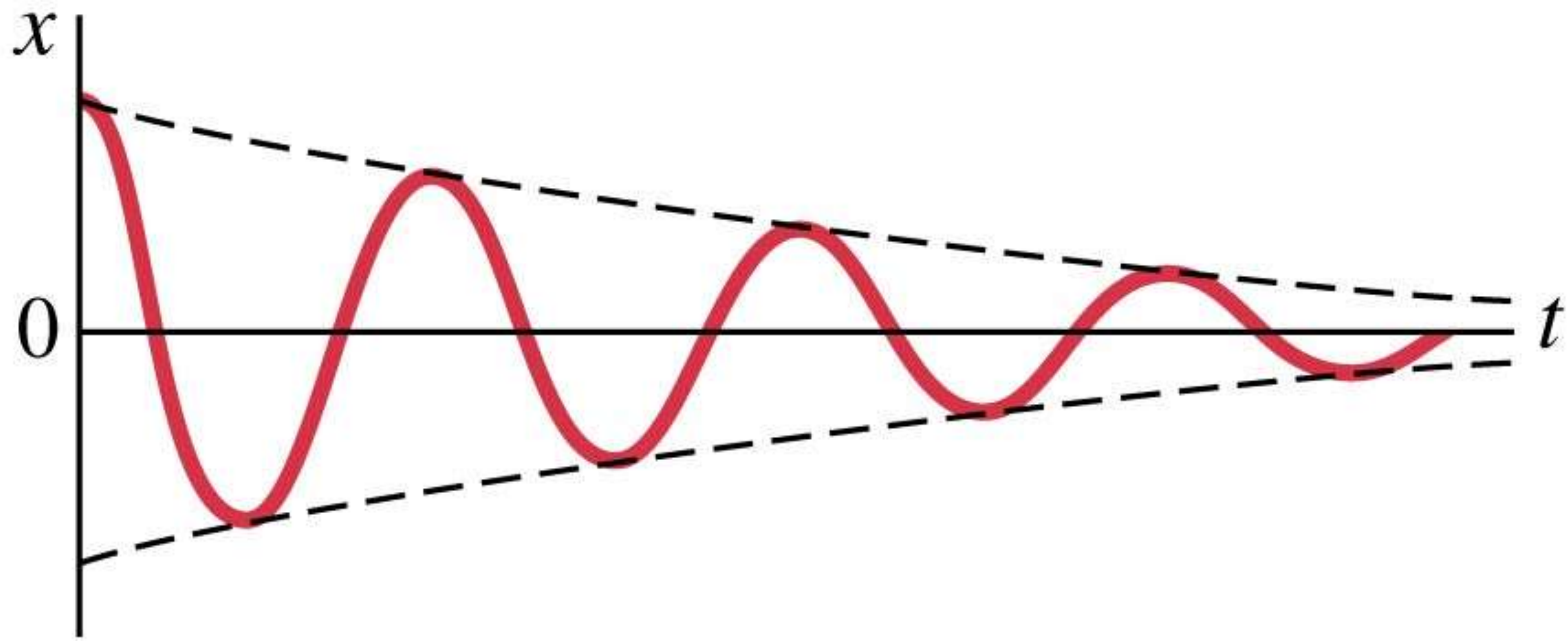
1. Free Oscillations – w/o <u>external</u> stimuli

2. Damped Oscillations - **amplitude** (x_0) of oscillation decreases with time
   Cause: Resistive forces from surroundings (eg air resistance).

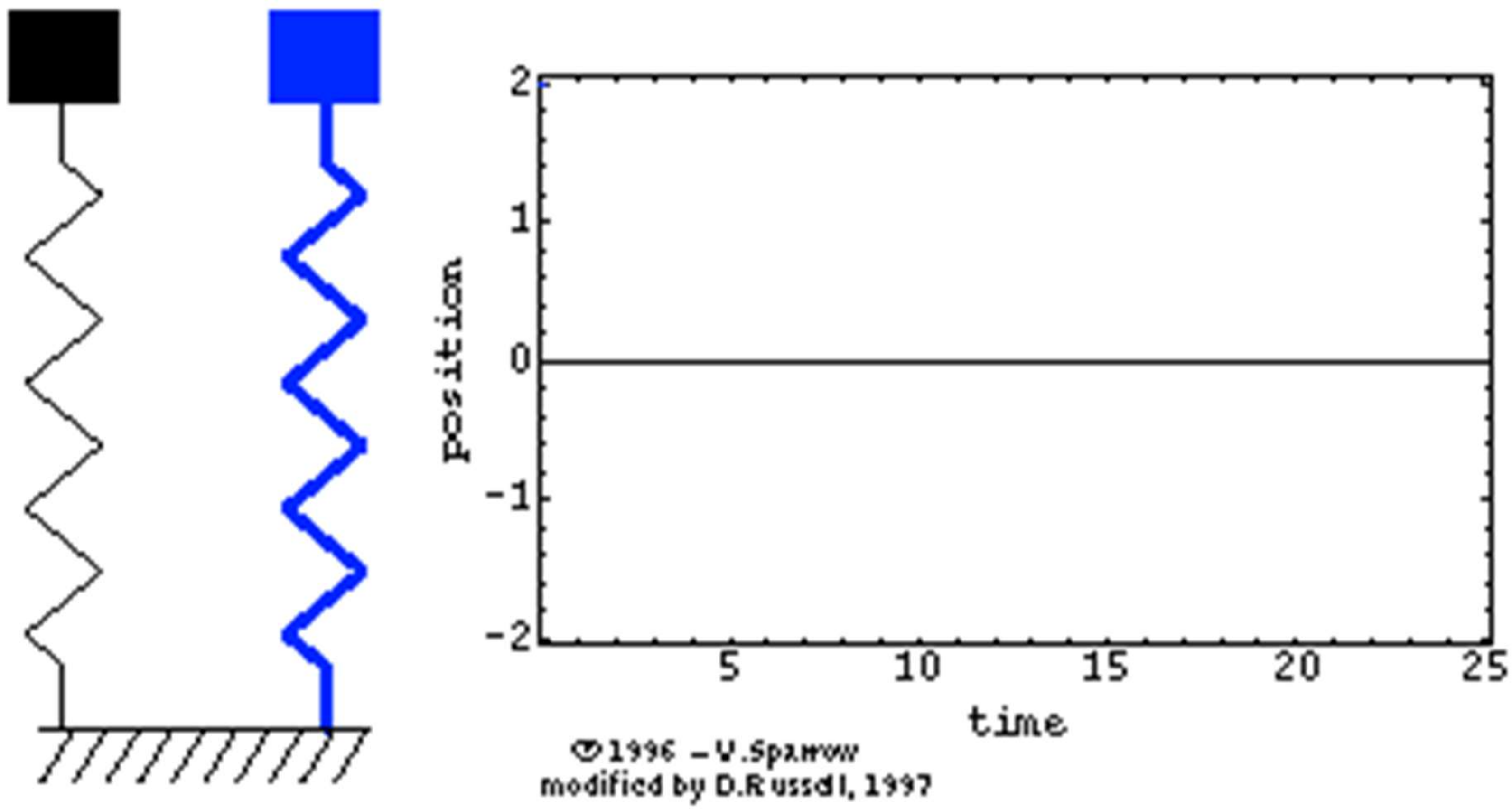
3. Forced Oscillations - Oscillations driven by external oscillatory forces with varying frequencies are known as forced oscillations. (Eg driven by Earthquake)

2. Damped Harmonic Motion

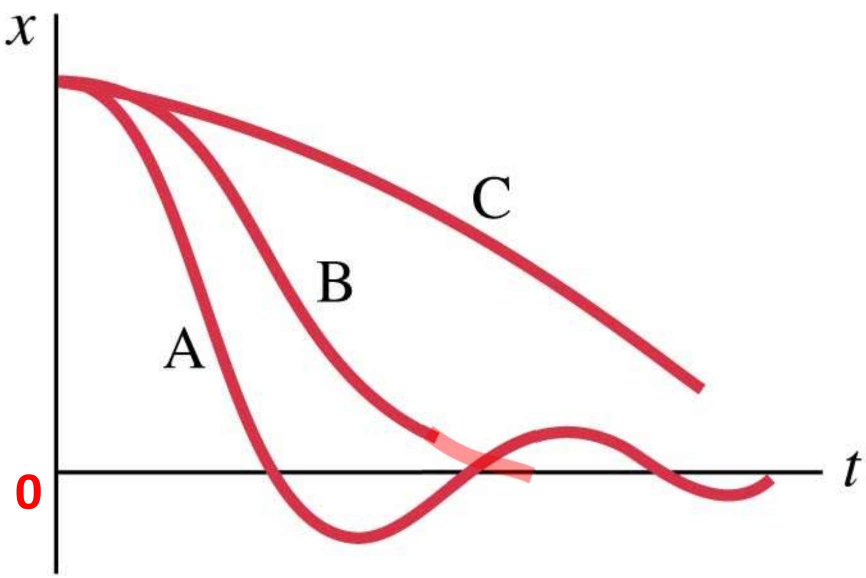
- Damped harmonic motion is harmonic motion with a frictional or drag force.
- If damping is small, we can treat it as an "envelope" that modifies un-damped oscillation. (Oscillation with **DECREASING** amplitudes with time)



https://www.acs.psu.edu/drussell/Demos/SHO/damp.html



2. Damped Harmonic Motion



But, if damping is large, it no longer resembles SHM at all.

A: under-damping (light damping): there are a few small oscillations before oscillator comes to rest.

For B & C, **NO Oscillations**

B: critical damping: fastest way to get to equilibrium ($x = 0$).

C: over-damping (heavy damping): system is slowed so much that it takes a long time (even infinity) to get to equilibrium ($x = 0$).

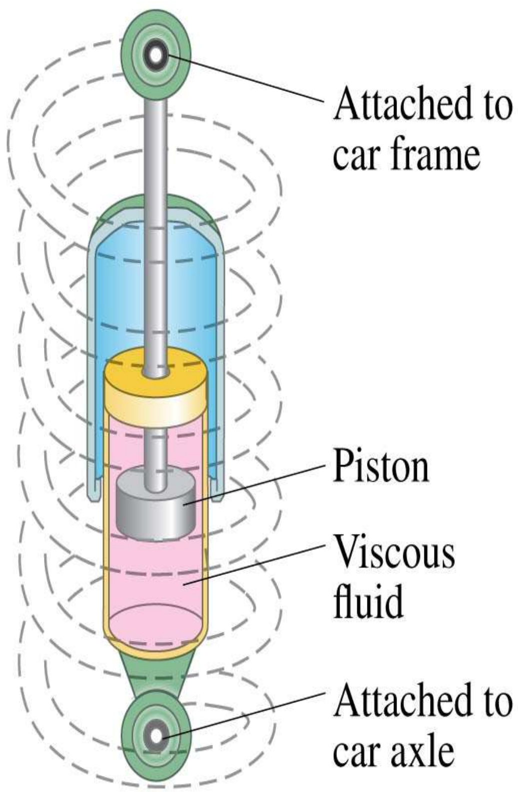
Notice ONLY Graph A has oscillation about $x = 0$.

Example: Car/Motorcycle suspensions uses **Critical Damping**

2. Damped Harmonic Motion

<u>Application of damping</u>

There are systems where damping is unwanted, such as clocks and watches.



automobile shock absorbers

earthquake protection for buildings

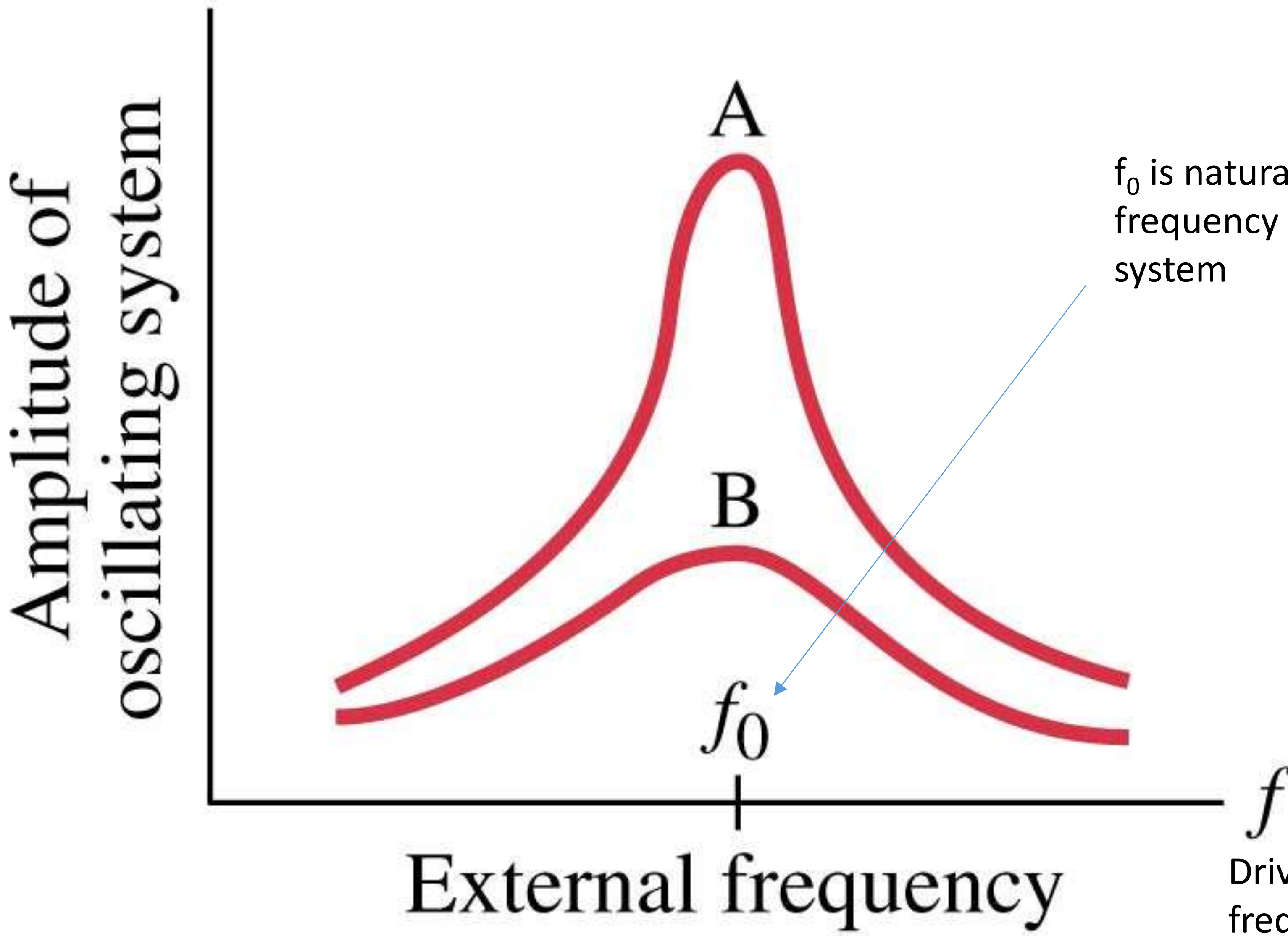
Then there are systems in which it is wanted, and often needs to be as close to critical damping as possible, such
as automobile shock absorbers and earthquake protection for buildings.

Include weighing scales and analog voltmeters/ammeters.

3. Forced Oscillations; Resonance

- Occurs when there is a periodic driving force. Driving force may or may not have same period as **natural frequency** of system.
- Resonance: If driving frequency same as natural frequency, amplitude becomes quite large.

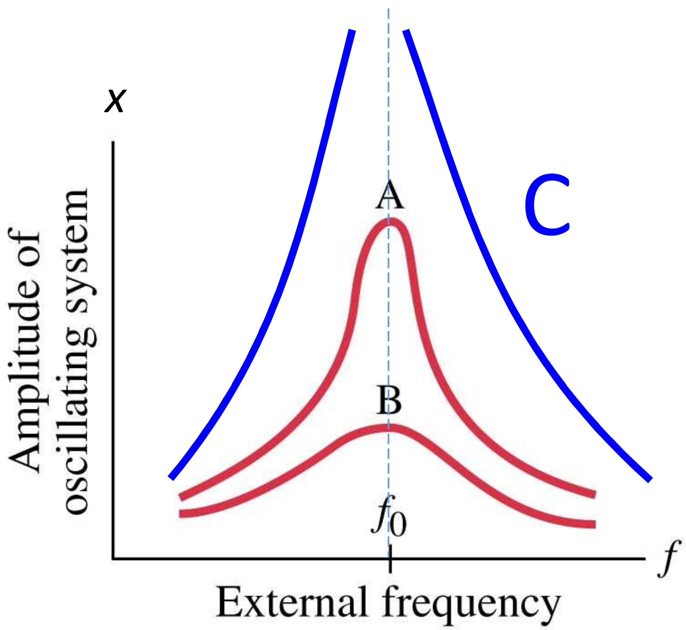
Resonant frequency breaks glass



f_0 is natural frequency of system

Driving frequency

3. Forced Oscillations; Resonance



A: Small damping

B: Heavy damping

C: No damping

*Critical damping between A and B

C: No damping

- Sharpness of resonant peak depends on the damping.
- If damping is **small** (**A**), it can be quite sharp; if damping is larger (B), it is less sharp.
- Like damping, resonance can be wanted or unwanted. Musical instruments and TV/radio receivers depend on it.
- Another application: Microwave Oven.

4. Example

Describe a procedure to measure the spring constant *k* of a car's springs. Assume that the owner's manual gives the car's mass *M* and that the shock absorbers are worn out so that the springs are under-damped. (See Sections 11–3 and 11–5 of Giancoli.)

4. Example

Describe a procedure to measure the spring constant *k* of a car's springs. Assume that the owner's manual gives the car's mass *M* and that the shock absorbers are worn out so that the **springs are underdamped**. (See Sections 11–3 and 11–5 of Giancoli.)

Under-damped oscillation's period is approximately equal to Period of an **un-damped** oscillation.

Period *T* is related to mass & spring constant in equation below.

To measure period of oscillation, you can push down on car's bumpers and time the oscillation period: *T*

Solving equation gives effective spring constant *k* (of 4 springs – one for each wheel).

$$T = 2\pi\sqrt{\frac{m}{k_{\text{effective}}}} \quad \rightarrow \quad k_{\text{effective}} = m\left(\frac{2\pi}{T}\right)^2$$

Effective spring constant is for four springs acting together.

To determine spring constant k_1 of each individual spring, divide by four.

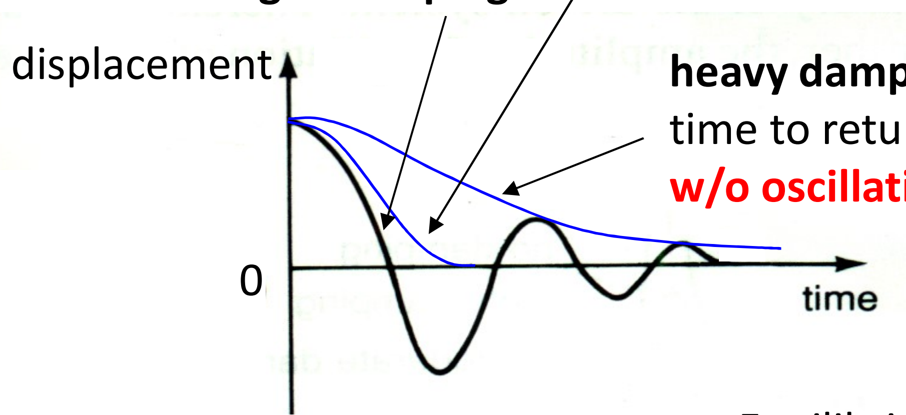
$$k_1 = \frac{1}{4}k_{\text{effective}} = \frac{m}{4}\left(\frac{2\pi}{T}\right)^2 = \frac{m\pi^2}{T^2}$$

Summary

1. Damping:

critical damping – return to equilibrium point (x=0) in shortest time **w/o oscillation**. e.g. car suspension system

light damping

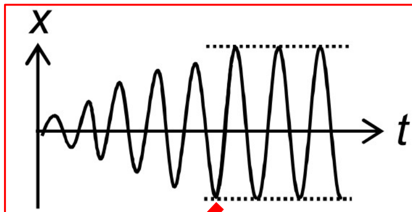


heavy damping – Needs infinitely long time to return to equilibrium point (x=0) **w/o oscillation.**

Equilibrium position, x = 0

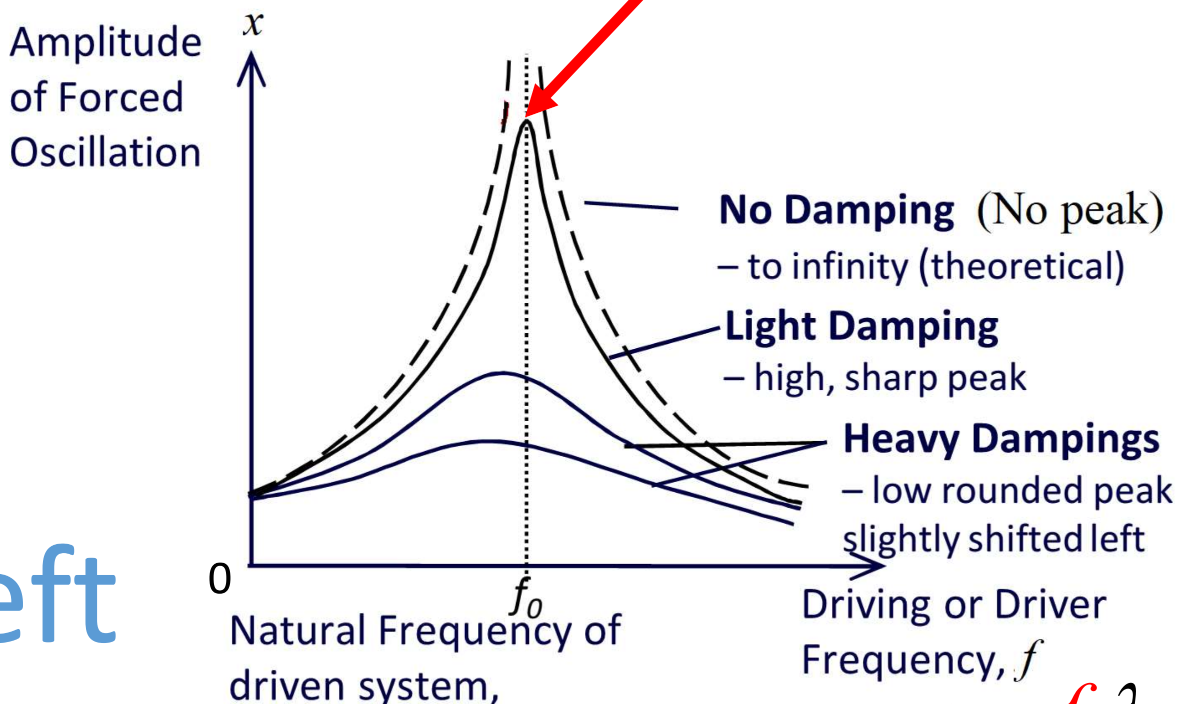
Summary

2. Forced Oscillation:



Resonance *occurs when* $f = f_o$,

Efficient transfer of energy,
max. amplitude



Left

Right

$$v = f\lambda$$

SPRINGS: SERIES VERSUS PARALLEL

Spring constant k: Larger k means harder to extend spring

PARALLEL SPRINGS

- One spring requires F to extend to x.
- Two identical such springs in PARALLEL requires a force LARGER than F to extend to x
- **Means effective spring constant of // springs is HIGHER**.

$$\text{Parallel springs (4 springs): } k_{\text{effective}} = k_1 + k_2 + k_3 + k_4$$

SERIES SPRINGS

- One spring requires F to extend to x.
- Two identical such springs in SERIES requires a force SMALLER than F to extend to x
- **Means effective spring constant of series springs is SMALLER**.

$$\text{Series springs (4 springs): } \frac{1}{k_{\text{effective}}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \frac{1}{k_4}$$

FORCE: (HOOKE'S LAW): $F = k\ x$

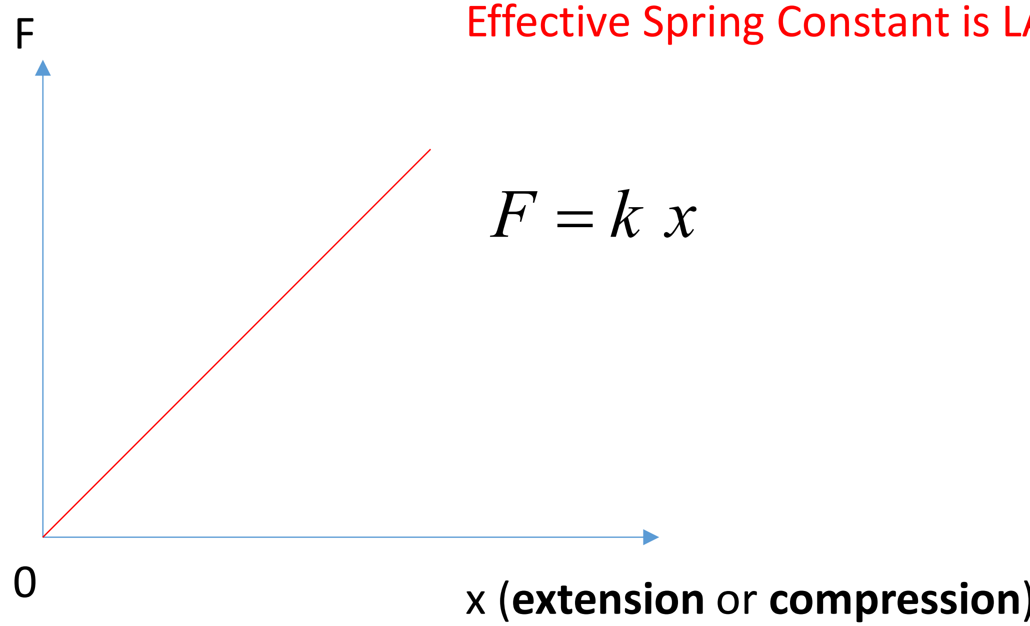
SPRINGS: SERIES VERSUS PARALLEL

PARALLEL SPRINGS

Parallel springs (4 identical springs): $k_{\text{effective}} = k + k + k + k$

Parallel springs (4 identical springs): $k_{\text{effective}} = 4k$

Effective Spring Constant is LARGER



$$F = k\ x$$