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Registration number

Signature

Note:
- Cross your Registration number(with leading zero). It will be evaluated automatically.
- Sign in the corresponding signature field.

Discrete Mathematics for Engineers

Exam:	EDE1202 / Endterm	**Date:**	Tuesday 23rd February, 2021
Examiner:	Prof. Dr. Ing. Ulf Schlichtmann	**Time:**	16:30 – 18:00

Problem 1 **Propositional Logic (8 credits)**

Given is the ring normal form $RNF[A]$ of the propositional form $A(x, y)$.

$$RNF[A(x, y, z)] \iff x \longleftrightarrow y \longleftrightarrow t$$

0 1 2

a)* Give the ring normal form $RNF[dual(A)]$ of the dual form of A.

0 1 2

b)* Give the canonical disjunctive normal form $CDNF[A]$ of A.

0 1

c)* Give the value pattern $\underline{\hat{W}}[A]$ of A.

0 1 2 3

d)* Give all non-trivial conclusions of A.

Problem 2 **Propositional Logic (5 credits)**

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Prove the following implication by means of the deductive proof scheme:

$$(a \longleftrightarrow c) \Longrightarrow (b \longrightarrow a) \longleftrightarrow (b \longrightarrow c)$$

Name all laws you use.

Row		From Row(s)	Law(s)
1	$a \longleftrightarrow c$ $\Longrightarrow (b \longrightarrow a) \longleftrightarrow (b \longrightarrow c)$		
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Problem 3 **Propositional Logic (5 credits)**

a)* Given is the following propositional form $B(a, b, c, d)$:

$$B(a, b, c, d) \iff (a \lor \neg b) \land (c \lor d) \land (\neg a \lor \neg d) \land (b \lor \neg d) \land \neg c.$$

Prove by means of the layer algorithm that $B(a, b, c, d)$ is a contradiction.

$(a \lor \neg b) \land (c \lor d) \land (\neg a \lor \neg d) \land (b \lor \neg d) \land \neg c$	Layer
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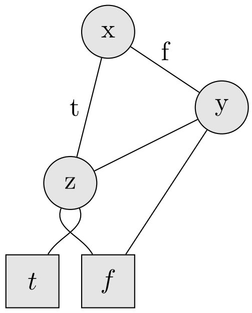
b)* Additionally, the propositional form $C(a, b, c)$ is given:

$$C(a, b, c) \iff c \lor (a \land b)$$

Is the implication $B(a, b, c, d) \Longrightarrow C(a, b, c)$ always valid? Justify your answer.

Problem 4 **Propositional Logic (6 credits)**

Given is the $ROBDD[C]$ of the propositional form $C(x, y, z)$:



a)* Give the premise normal form $PNF[C]$ of the propositional form $C(x, y, z)$.

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1

b)* Give the number of t- and f-assignments in the propositional form $C(x, y, z)$.

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c)* Draw the $ROBDT[C]$ of the propositional from $C(x, y, z)$ with the **variable ordering x,y,z**.

0
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2

d)* Draw the $ROBDT[C]$ of the propositional from $C(x, y, z)$ with the **variable ordering x,z,y**.

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Problem 5 **Propositional Logic (6 credits)**

Given is the propositional form $D(a, b, c)$:

$$D(a, b, c) \iff [a \longleftrightarrow (b \longrightarrow c)] \vee \neg(a \longrightarrow \neg b)$$

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Give the canonical conjunctive normal form $CCNF[D]$ of D.
Name all laws you use.

Problem 6 **Predicate Logic and Sets (8 credits)**

Prove by means of the inductive proof scheme that the following predicate is a tautology for $n \in \mathbb{N}$:

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$$P(n) \iff \sum_{k=1}^{n}(4k-1) = 2n^2 + n$$

Show how the different steps are related by equivalences or implications.

Basis step:

Induction step:

Conclusion:

Problem 7 Predicate Logic and Sets (5 credits)

Prove that the following subset relation is valid for arbitrary sets $A, B, C \subseteq A^2$:

0 1 2 3 4 5

$$\overline{A} \setminus (B \cup C) \subseteq \overline{(\overline{B} \cup C) \cap \overline{\overline{B} \setminus A}}$$

Name all laws you use.

Problem 8 Predicate Logic and Sets (6 credits)

Given are the following sets and predicates:

- S: Set of all students.
- L: Set of all lectures.
- Pxy: Student x is present in lecture y.
- Cy: Lecture y is being cancelled.

Formalize the following statements A_1, A_2 and A_3 with the help of predicate logic only. Only use the predicates and sets given above.

0
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2

a)* $A_1 \iff$ For every lecture applies: If there is at least one student present in the lecture, it will not be cancelled.

0
1
2

b) $A_2 \iff$ If statement A_1 is being observed and there is at least one student present in every lecture, then all lectures will take place.

0
1
2

c) $A_3 \iff$ If there is a lecture that has been cancelled, then no student was present in that lecture.

Problem 9 Predicate Logic and Sets (5 credits)

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Transform the following statement given in component free notation into predicate logic notation:

$$[(A \setminus B) \triangle \overline{C} \neq \emptyset] \land (B = \emptyset) \implies A \neq C$$

For this, only use the following predicates, where $A, B, C \subseteq G$:

- $x \in A \iff Px$
- $x \in B \iff Qx$
- $x \in C \iff Sx$

The result must not contain operations from set notation or relation notation.

Problem 10 **Predicate Logic and Sets (6 credits)**

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Prove the following implication by means of the deductive proof scheme:

$$(A \triangle B \subseteq B) \wedge (\overline{C} \cup A = G) \Longrightarrow C \subseteq B$$

G is the universal set. Name all laws you use.

Row		From Row(s)	Law(s)
1	$A \triangle B \subseteq B$		
2	$\overline{C} \cup A = G$ $\quad\quad \Longrightarrow C \subseteq B$		
3			
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Problem 11 Relations (8 credits)

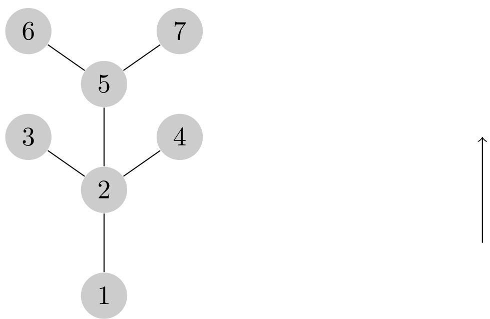
a)* Prove that the following subset relations are logically equivalent:
Name all laws you use.

0 1 2 3 4

$$W^{-1}\overline{X^{-1} \cap Y^{-1}} \subseteq \overline{Z} \iff Z^{-1}W^{-1} \subseteq X \cap Y$$

b)* Given is the Hasse diagram $G_H = (A, H)$ of the partial order relation $R \subseteq A^2$ with $A = \{1, 2, 3, 4, 5, 6, 7\}$:

0 1 2 3 4



Calculate the following bounds and extrema of the set $B = \{2, 3, 4, 5\}$:
$ub(B)$, $max(B)$, $lst(B)$, $glb(B)$

Problem 12 **Relations (5 credits)**

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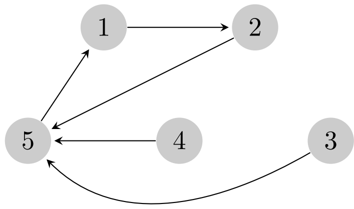
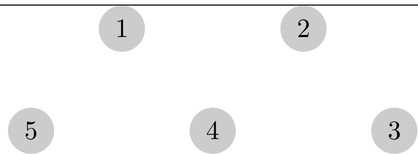
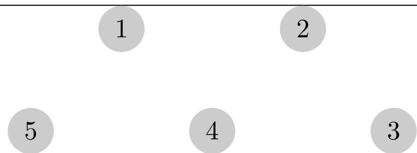
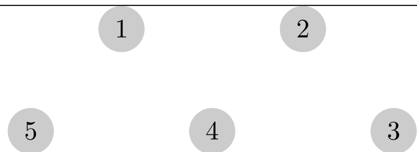
Given is the following graph $G = (A, R)$ of the relation $R \subseteq A^2$ with $A = \{1, 2, 3, 4, 5\}$:



Calculate the transitive closure R^+ by means of the Warshall Algorithm. Use the variable order given in the table below. For each iteration cycle, give $\Gamma^-(i)$, $\Gamma^+(i)$, and $G = (A, R_i)$.

i	$\Gamma^-(i)$	$\Gamma^+(i)$	$G = (A, R_i)$
1			
2			
3			
4			
5			

Problem 13 **Relations (7 credits)**

Given is the relation R by the following successor table, with $R \subseteq A^2$:

x	$\Gamma^+\{x\}$
1	{2,3}
2	{3,4,5,6}
3	{4,6}
4	{1,5}
5	{1,3,6}
6	{1,4,6}

0
1
2
3

a)* Give the adjacency matrices of reflexive closure $r(R)$, the symmetric closure $s(R)$ and the transitive symmetric reflexive closure $tsr(R)$ of R.

$$\underset{\sim}{r(R)} = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & & & & & & \\ 2 & & & & & & \\ 3 & & & & & & \\ 4 & & & & & & \\ 5 & & & & & & \\ 6 & & & & & & \end{array} \qquad \underset{\sim}{s(R)} = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & & & & & & \\ 2 & & & & & & \\ 3 & & & & & & \\ 4 & & & & & & \\ 5 & & & & & & \\ 6 & & & & & & \end{array}$$

$$\underset{\sim}{tsr(R)} = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & & & & & & \\ 2 & & & & & & \\ 3 & & & & & & \\ 4 & & & & & & \\ 5 & & & & & & \\ 6 & & & & & & \end{array}$$

b)* Mark all correct statements in the following.

Mark correct answers with a cross ☒

To undo a cross, completely fill out the answer option ■

To re-mark an option, use a human-readable marking ×■

- [ ] $R = R^+$
- [ ] The nodes 3 and 1 are indirectly accessible.
- [ ] R is antisymmetric.
- [ ] R is semiconnex.
- [ ] $|pred(\{3\})| = 4$
- [ ] R is a homogeneous relation.
- [ ] R is asymmetric.
- [ ] R is connex.

Problem 14 **Relations (10 credits)**

Given are the basic sets $A = \{a, b, c, d\}$ and $B = \{V, W, X, Y, Z\}$, and the following relations:

- $R = \{(a, V), (a, W), (a, X), (b, Z), (c, Z), (d, Y)\}$; $R \subseteq A \times B$
- $S = \{(V, a), (W, c), (X, b), (Y, d), (Z, a)\}$; $S \subseteq B \times A$

a)* Give the relation $R \circ S$ as a set of ordered pairs.

0
1

b)* Is R^{-1} surjective? Justify your answer.

0
1
2

c)* Is S a function? Justify your answer.

0
1
2

d) Draw the equivalence closure $ECL(R \circ S)$ of $R \circ S$.

0
1

e) Give the equivalence class $[a]_{R \circ S}$ of a with respect to $R \circ S$.

0
1

f)* Give the partition Π of A that is defined by the equivalence relation $ECL(R \circ S)$.

0
1

g) Give all partitions Π_i of A that fulfill the following conditions:

0
1
2

- $|\Pi_i| = 2$
- $\underset{K \in \Pi_i}{\exists} |K| = 1$

Additional space for solutions–clearly mark the (sub)problem your answers are related to and strike out invalid solutions.