Momentum and collisions

Lecture Learning Objectives

In this lesson you will learn to:

- Define momentum and impulse.
- Recognise the condition under which momentum is conserved.
- Apply conservation of momentum to solve elastic and inelastic collision problems.

Definition of momentum

In week 4, we covered Newton's second law of motion.

$$\sum \mathbf{F}_i = m\mathbf{a}$$

We can write this as:

$$\sum \mathbf{F}_i = m\frac{d\mathbf{v}}{dt} = \frac{d}{dt}m\mathbf{v}$$

We call $m\mathbf{v}$ the momentum of the system.

$$\mathbf{p} = m\mathbf{v}$$

The units are kg m/s.

Momentum as a vector

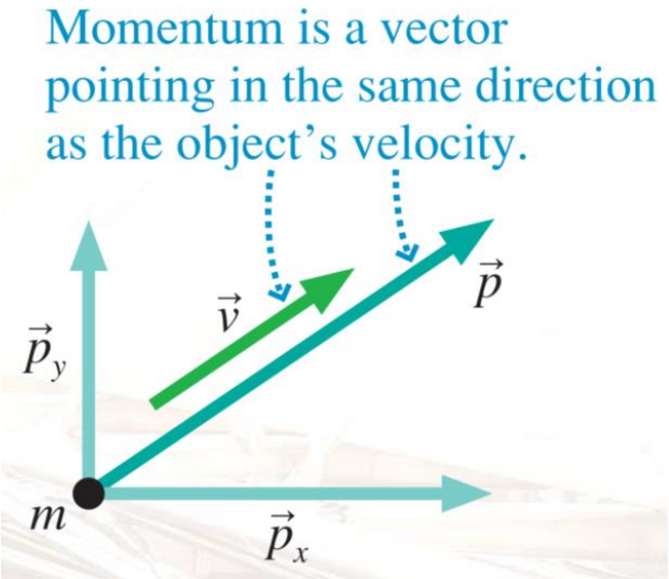
The momentum **p** is a vector in the same direction as the velocity.

We can deal with its components separately.

$$p_x = mv_x;\ p_y = mv_y;\ p_z = mv_z$$

Using the definition of momentum, we can say:

$$\sum \mathbf{F}_i = \frac{d\mathbf{p}}{dt}$$



Impulse

During a collision, a force is applied for a short time:

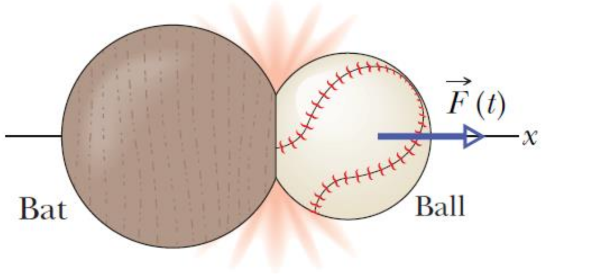
$$\mathbf{F}dt = d\mathbf{p}$$

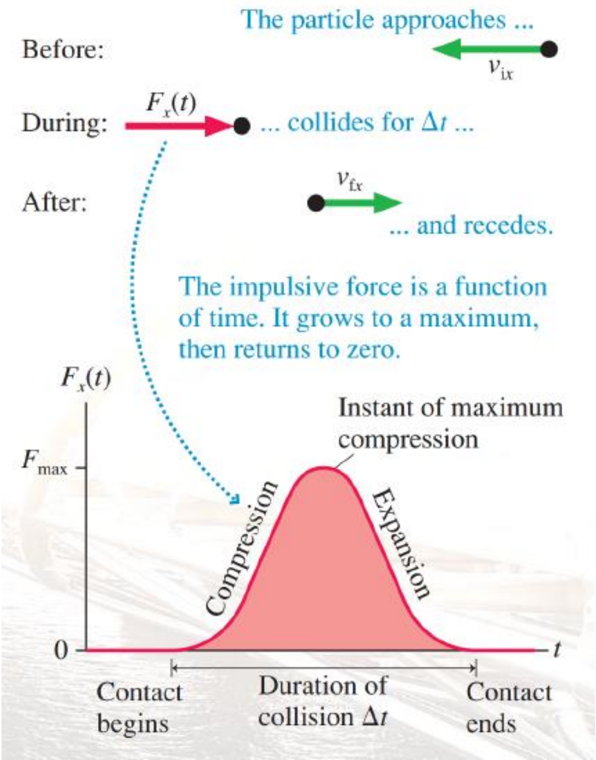
If we integrate:

$$\int_{p_1}^{p_2} d\mathbf{p} = \int_{t_1}^{t_2} \mathbf{F}dt;\ p_2 - p_1 = \int_{t_1}^{t_2} \mathbf{F}dt$$

We define the impulse J as the change in momentum due to a force:

$$\mathbf{J} = \int_{t_1}^{t_2} \mathbf{F}dt = \Delta\mathbf{p}$$





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Impulse

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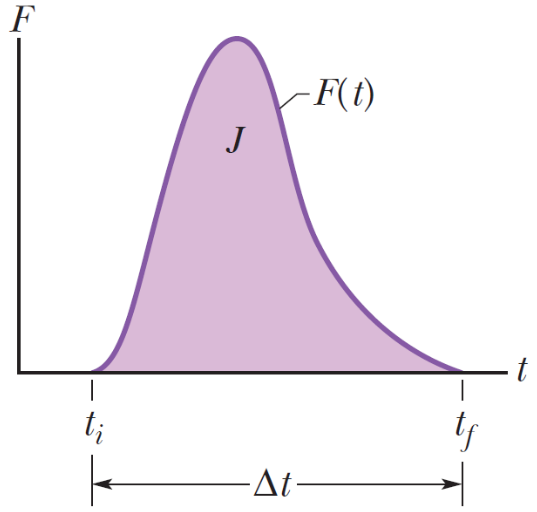
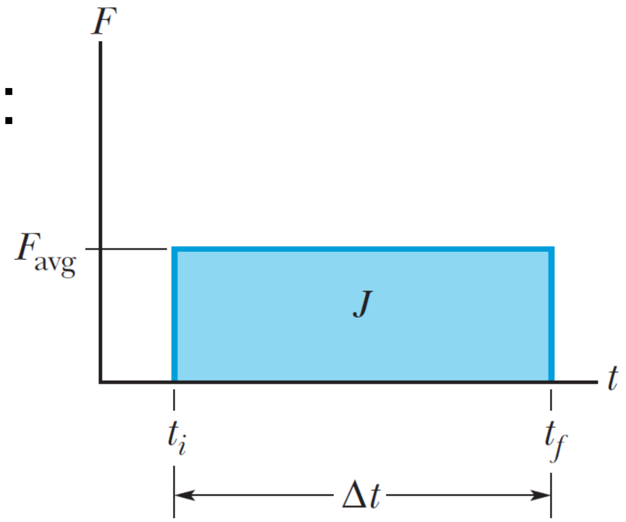
The impulse is the area under the force – time graph.



The impulse is the same if the average force acts over the same time:

$$\mathbf{J} = \int_{t_1}^{t_2} \mathbf{F}dt = \mathbf{F}_{avg}\Delta t$$



This can also be split between the vector components:

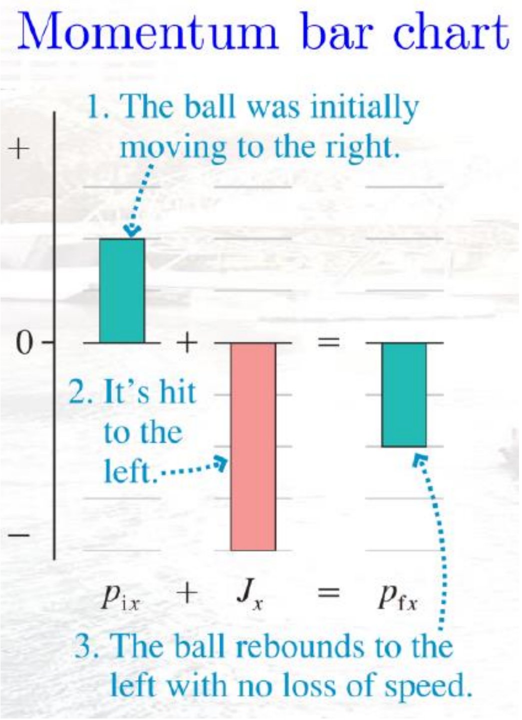
$$J_x = \int_{t_1}^{t_2} F_x dt = F_{avg\ x}\Delta t = p_{2x} - p_{1x} = mv_{2x} - mv_{1x}$$

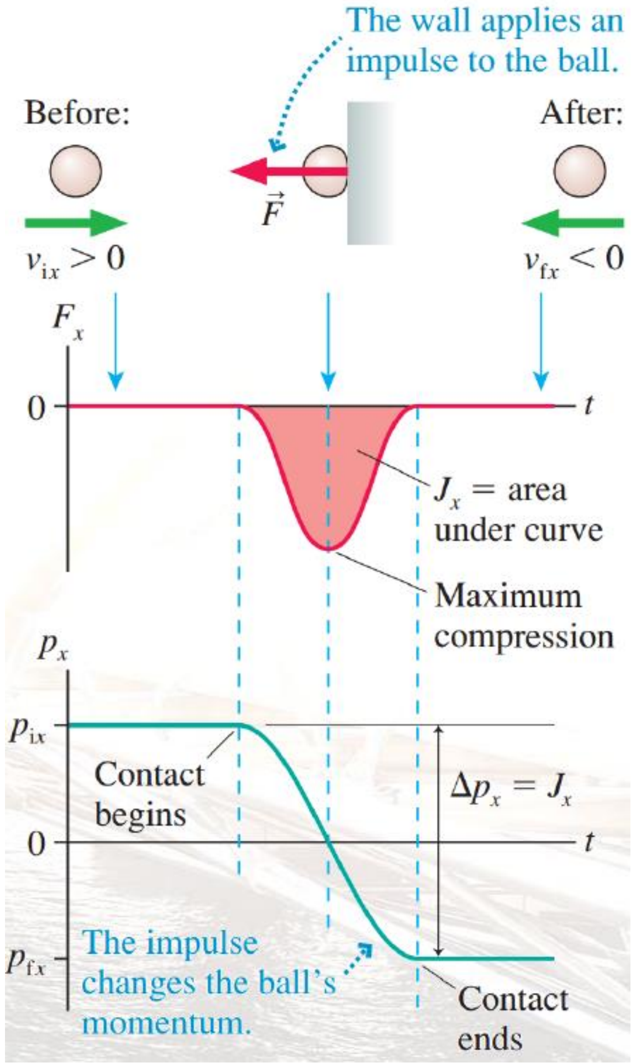
$$J_y = \int_{t_1}^{t_2} F_y dt = F_{avg\ y}\Delta t = p_{2y} - p_{1y} = mv_{2y} - mv_{1y}$$

Momentum - Impulse

We can therefore write the relationship between moment and impulse:

$$\mathbf{J} = \mathbf{p}_2 - \mathbf{p}_1;\ \mathbf{p}_2 = \mathbf{p}_1 + \mathbf{J} = \mathbf{p}_1 + \int_{t_1}^{t_2} \mathbf{F}\,dt$$





Impulse - example

A 10 g shrapnel hits a rigid wall at 2000 m/s. The shrapnel is stopped by the collision in 0.05 ms. What is the average force acting on the projectile? If it the shape can be assumed to be a cube which hits the wall on a flat side, what is the average stress?

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Impulse - example

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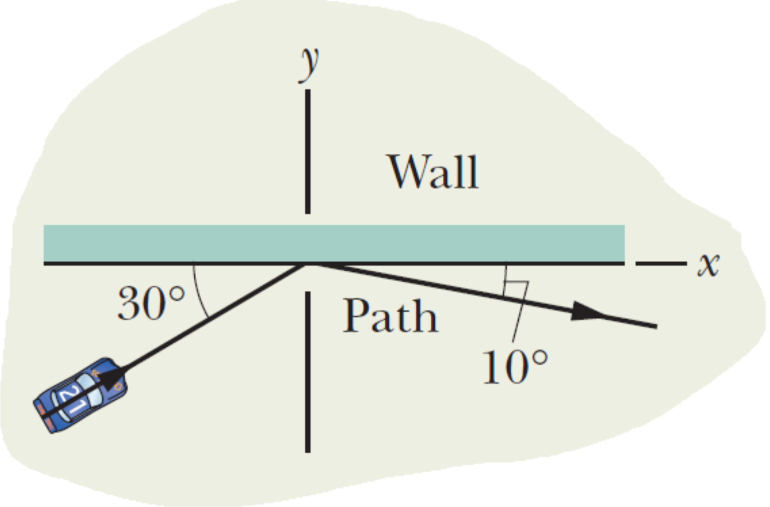
The figure is an overhead view of the path taken by a race car driver as his car collides with the racetrack wall. Just before the collision, he is traveling at speed v_i = 70 m/s along a straight line at 30° from the wall. Just after the collision, he is traveling at speed v_f = 50 m/s along a straight line at 10° from the wall. His mass m is 80 kg. (a) What is the impulse on the driver due to the collision? (b) The collision lasts for 14 ms. What is the magnitude of the average force on the driver during the collision?



a)

$\vec{J} = \vec{P}_2 - \vec{P}_1$

$= M\vec{V}_2 - M\vec{V}_1$

$J_x = MV_{2x} - MV_{1x} = M(V_{2x} - V_{1x})$

$J_y = M(V_{2y} - V_{1y})$

$V_{1x} = 70 \times \cos 30° = 60.6\ m/s$

$V_{1y} = 70 \times \sin 30° = 35\ m/s$

$V_{2x} = 50 \times \cos 10° = 49.2\ m/s$

$V_{2y} = 50 \times \sin 10° = 8.68\ m/s = -8.68\ m/s$

$J_x = 80(49.2 - 60.6)$

$= -912\ kg\,m/s$

$J_y = 80(-8.68 - 35)$

$= -3500\ kg\,m/s$

$\vec{J} = (-912\hat{i} - 3500\hat{j})\ kg\,m/s$

b) $|\vec{J}| = \sqrt{910^2 + (-3500)^2} = 3600\ kg\,m/s$

$\tan\theta = \frac{-3500}{-910}$ $\Delta t = 0.0014 s$

$\theta = 255°$

$F_{Avg} = \frac{J}{\Delta t}$

$= \frac{3600}{0.014}$

$= 2.6 \times 10^5 N$

Impulse and kinetic energy

The momentum and kinetic energy are somehow related, both depend on mass and velocity.

$$\mathbf{p} = m\mathbf{v} \qquad\qquad K = \frac{1}{2}mv^2$$

However, there are key differences:

- Energy is a scalar, momentum is a vector.
- The change in energy is related to the distance. The change in momentum is related to the time.

$$\mathbf{J} = \int_{t_1}^{t_2} \mathbf{F}dt = \mathbf{F}_{avg}\Delta t \qquad\qquad W = \int_{x_1}^{x_2} -Fdx$$

Impulse and kinetic energy - example

Which is easier to catch: a 0.50 kg ball moving at 4.0 m/s or a 0.10 kg ball moving at 20 m/s?

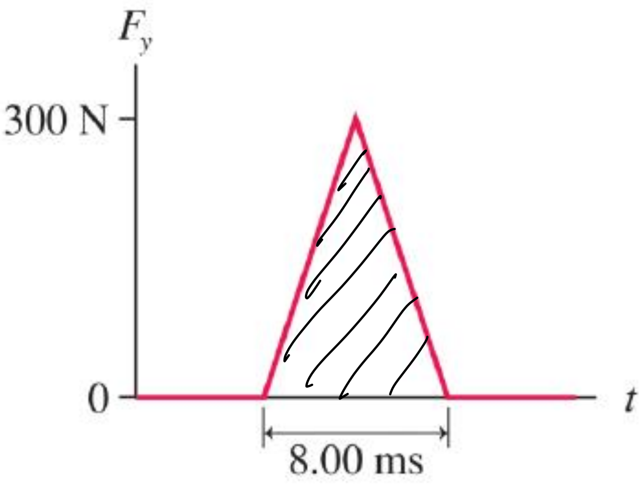
$$W = \int_{x_1}^{x_2} -F dx$$

$$\mathbf{J} = \int_{t_1}^{t_2} \mathbf{F} dt = \mathbf{F}_{avg} \Delta t$$

Impulse - example

EXAMPLE 9.2 A bouncing ball

A 100 g rubber ball is dropped from a height of 2.00 m onto a hard floor. **FIGURE 9.9** shows the force that the floor exerts on the ball. How high does the ball bounce?



$U_{G0} = KE_1$ $\quad Mgy_0 = \frac{1}{2}MV_1^2$

$V_1 = \sqrt{2gy_0}$

$= -6.26\ m/s$

Initial momentum $= P_1 = MV_1 = 0.1 \times -6.26$

$= -0.626\ kg.m/s$

Impulse $= J = \frac{0.008 \times 300}{2} = 1.2\ kg\ m/s$

$J = P_2 - P_1$;

$P_2 = P_1 + J = -0.626 + 1.2$

$= 0.574\ kg.m/s$

$P_2 = MV_2$; $\frac{1}{2}MV_2^2 = MgY_3$

$V_2 = \frac{P_2}{M} = \frac{0.574}{0.1} = 5.74\ m/s$

$Y_3 = \frac{V_2^2}{2g}$

$= \frac{5.74^2}{2 \times 9.8}$

$= 1.68\ m$

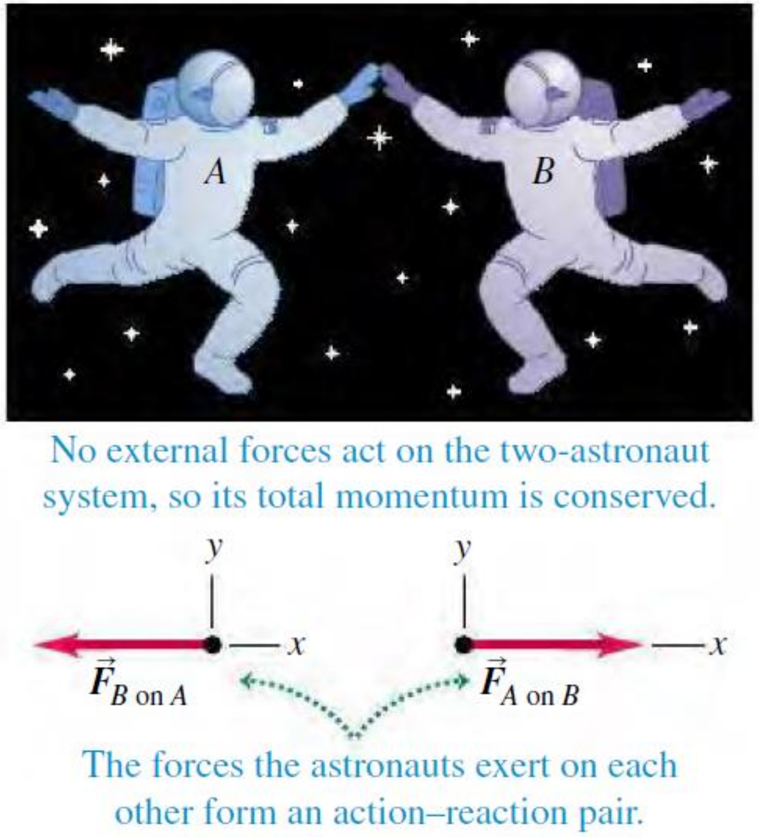
Conservation of momentum

The force exerted on a particle originates an opposite reaction (Newton's third law of motion).

That implies equal and opposite impulses are applied on the two bodies.

Therefore, if there are **no net external** forces (the system is **isolated**) the net impulse is 0, no change in total momentum.

$$\mathbf{J} = \int_{t_1}^{t_2} \mathbf{F} dt = \mathbf{F}_{avg} \Delta t$$



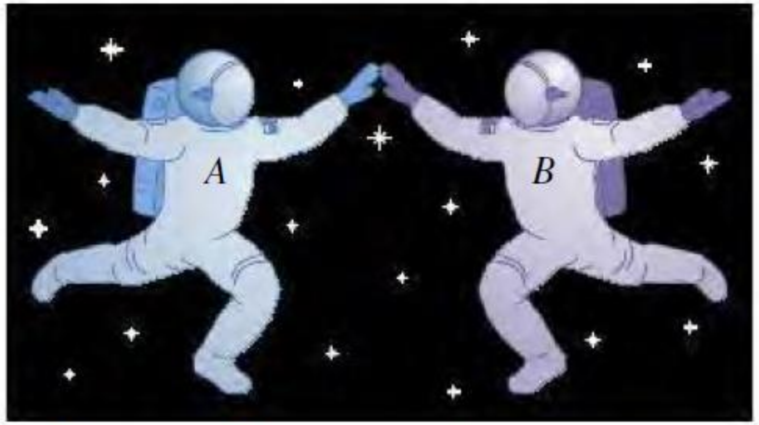
Conservation of momentum

$$\mathbf{F}_{BA} = \frac{d\mathbf{p}_A}{dt}; \; \mathbf{F}_{AB} = \frac{d\mathbf{p}_B}{dt}$$

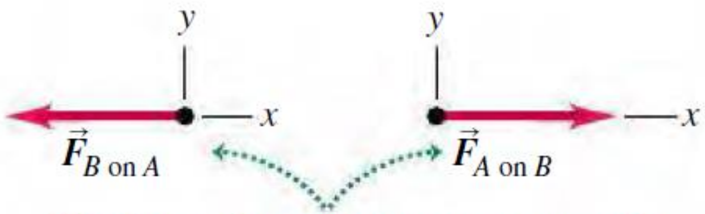
$\mathbf{F}_{BA} = -\mathbf{F}_{AB}$ For Newton's third law of motion

$$\mathbf{F}_{BA} + \mathbf{F}_{AB} = 0$$

$$\mathbf{F}_{BA} + \mathbf{F}_{AB} = \frac{d\mathbf{p}_A}{dt} + \frac{d\mathbf{p}_B}{dt} = \frac{d\left(\mathbf{p}_A + \mathbf{p}_B\right)}{dt} = 0$$



No external forces act on the two-astronaut system, so its total momentum is conserved.



The forces the astronauts exert on each other form an action–reaction pair.

Conservation of momentum

We can find the total momentum of a system:

$$\mathbf{P} = \mathbf{p}_A + \mathbf{p}_B + \mathbf{p}_C + \ldots;$$

$$\mathbf{P} = m\mathbf{v}_A + m\mathbf{v}_B + m\mathbf{v}_C + \ldots$$

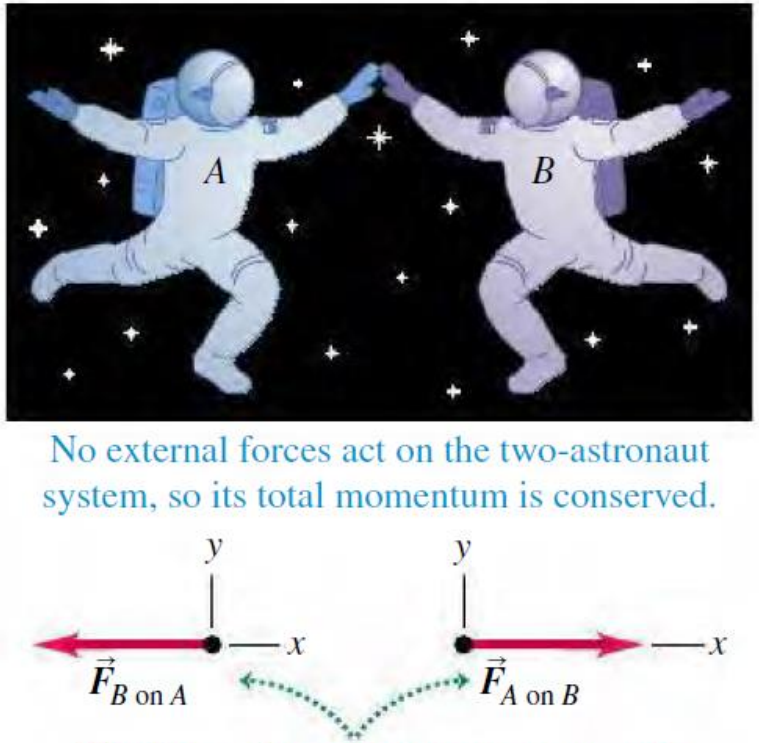
Momentum conservation means:

$$\mathbf{F}_{BA} + \mathbf{F}_{AB} = \frac{d\mathbf{P}}{dt} = 0$$

It can be expressed in components:

$$P_x = p_{Ax} + p_{Bx} + p_{Cx} + \ldots;$$

$$P_y = p_{Ay} + p_{By} + p_{Cy} + \ldots$$



No external forces act on the two-astronaut system, so its total momentum is conserved.

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Conservation of momentum - example

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A marksman holds a rifle of mass m_R = 3.00 kg loosely, so it can recoil freely. He fires a bullet of mass m_B = 5.00 g horizontally with a velocity relative to the ground of v_{Bx} = 300 m/s. What is the recoil velocity v_{Rx} of the rifle? What are the final momentum and kinetic energy of the bullet and rifle?

$P_1 = MV_1 = 0 = P_2$

$P_2 = M_R V_{2R} + M_B V_{2B} = 0$

$3.00 \times V_{2R} + 0.05 \times 3 = 0$

$V_{2R} = \frac{-0.005 \times 300}{3.00} = -0.500\ m/s$

Rifle

$P_{2R} = M_R V_{2R} = 3.00 \times -0.500$

$= -1.50\ kg.m/s$

$KE_R = \frac{1}{2} M V_{2R}^2 = \frac{1}{2} \times 3.00 \times 0.5^2$

$= 0.375J$

Bullet

$P_{2B} = 0.005 \times 3$

$= 1.5\ kg.m/s$

$KE_B = \frac{1}{2} \times 0.005 \times 300^2$

$= 225J$

Conservation of momentum - example

A space hauler and cargo module, of total mass *M* are travelling along an x axis in deep space. They have an initial velocity of magnitude 2100 km/h relative to the Sun. With a small explosion, the hauler ejects the cargo module, of mass 0.20*M*.The hauler then travels 500 km/h faster than the module along the x axis; that is, the relative speed v_{rel} between the hauler and the module is 500 km/h. What then is the velocity of the hauler relative to the Sun?

$P_1 = M v_1 = M2100$

$P_2 = 0.8 M V_{S2} + 0.2 M V_{C2}$

$P_1 = P_2$

$2100 \cancel{M} = 0.8 \cancel{M} V_{S2} + 0.2 \cancel{M} V_{C2}$

$V_{S2} = V_{C2} + 500$

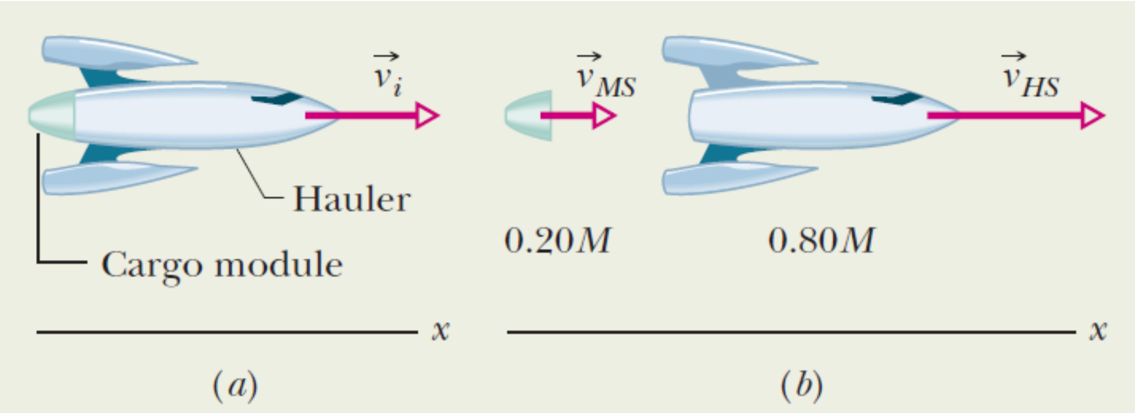
$2100 = 0.8 (V_{C2} + 500) + 0.2 V_{C2}$

$2100 = 0.8 V_{C2} + 400 + 0.2 V_{C2}$

$1700 = 1.0 V_{C2}$; $V_{C2} = 1700$ km/h

$V_{S2} = V_{C2} + 500$

$= 1700 + 500 = 2200$ km/h

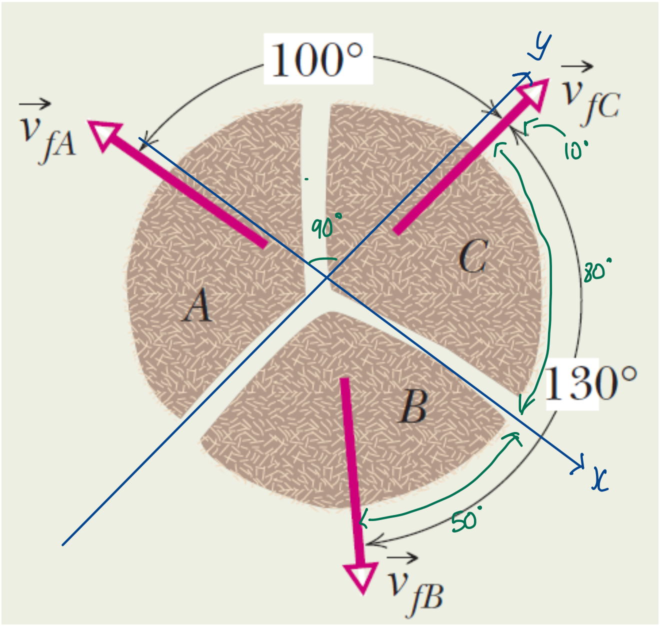


Conservation of momentum - example

$P_o = 0$ kg.m/s

$P_i = 0$ kg.m/s

A firecracker placed inside a coconut of mass *M*, initially at rest on a frictionless floor, blows the coconut into three pieces that slide across the floor. Piece *C*, with mass 0.30*M*, has final speed v_{fC} = 5.0 m/s. (a) What is the speed of piece *B*, with mass 0.20*M*? (b) What is the speed of piece *A*?



P_y : $P_{Ay} = 0$

P_{By} : $M_B V_{By} = 0.2M \times V_B \times \sin(-50^\circ)$

$= -0.153 M V_B$

P_{Cy} : $M_C V_{Cy} = 0.3M \times V_C \times \sin 80^\circ = 0.3M \times 5.0 \times \sin 80^\circ$

$= 1.48M$

$0 = P_{Ay} + P_{By} + P_{Cy} = 0 - 0.153 M V_B + 1.48M = 0$

$0.153 V_B = 1.48$; $V_B = 9.7$ m/s

$M_A = (1 - 0.2 - 0.3)$

$= 0.5M$

$P_{xo} = P_{xi} = 0$

$P_{Ax} = M_A V_{Ax} = 0.5M \times |V_A| \times \cos 180^\circ$

$= -0.5 M V_A$

$P_{Cx} = 0.3M \times V_C \cos 80^\circ$

$= 0.3M \times 5 \times \cos 80^\circ$

$= 0.26M$

$P_{Bx} = 0.20M \times V_B \times \cos(-50^\circ)$

$= 0.2M \times 9.7 \times \cos(-50^\circ)$

$= 1.25M$

$P_{Ax} + P_{Cx} + P_{Bx} = 0$; $-0.5 M V_A + 0.260M + 1.25M = 0$

$-0.5 V_A = -1.51$; $|V_A| = 3.02$ m/s

Centre of Mass

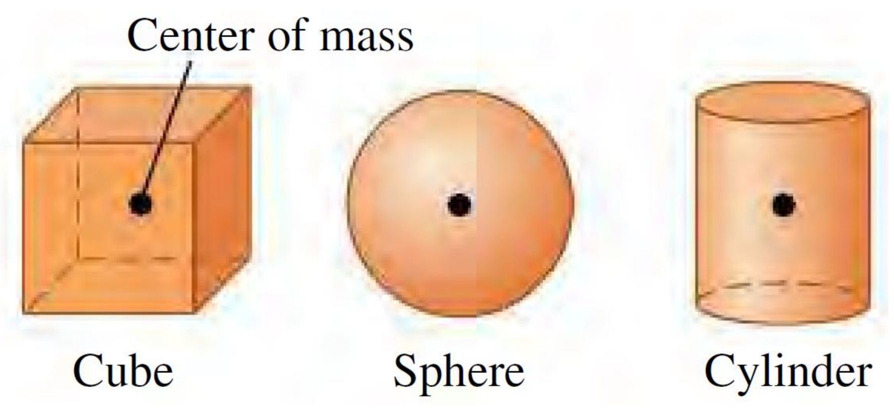
In all the physics topics so far, we considered objects to be “particles”, non dimensional “dots” where all the mass and forces were concentrated.

This is clearly not the case! A car, or a box, is not ALL located at a certain position, say x = 5.5 m.

We need to chose a position we consider to be the position of the object.

This is generally the **centre of mass**.

The centre of mass of an object is defined as the point at which we can imagine all the mass of an object to be concentrated.



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Centre of mass for two particles

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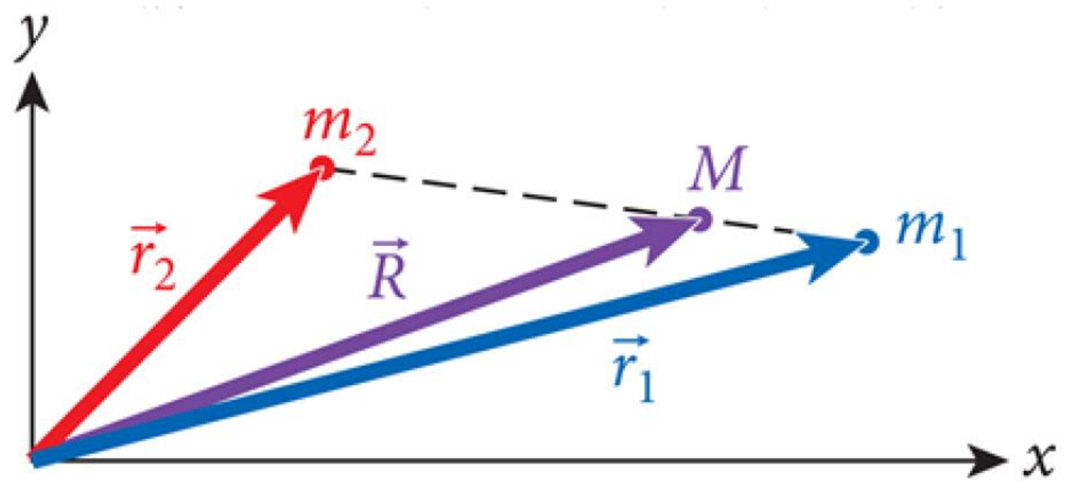
If we have two objects of equal mass, their combined centre of mass lies in the middle between the two.

If the objects are different, we need to find the location in terms of their masses.

$$\mathbf{R} = \frac{\mathbf{r}_1 m_1 + \mathbf{r}_2 m_2}{m_1 + m_2}$$



Centre of mass for two particles - example

The Earth has a mass of 5.97×10^{24} kg and the Moon has a mass of 7.36×10^{22} kg.

The Moon orbits the Earth at a distance of 384,000 km.

How far from the centre of the Earth is the centre of mass of the Earth-Moon system?

$\longmapsto x$

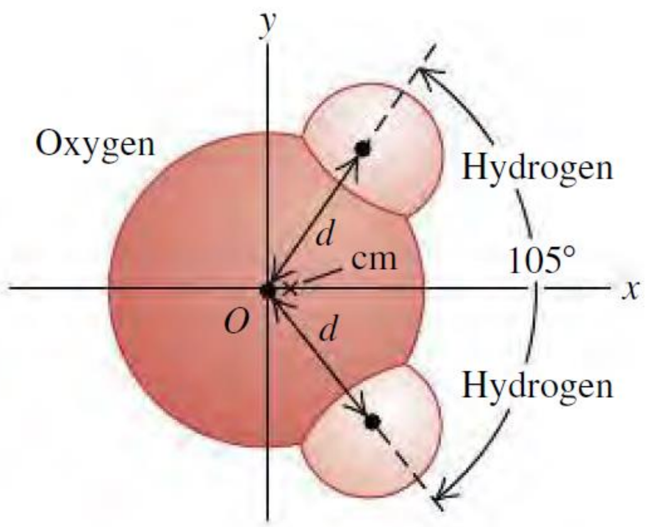
$$R = \frac{M_E r_E + M_m r_E}{M_E + M_m} = \frac{5.97 \times 10^{24} \times 0 + 7.36 \times 10^{22} \times 384000}{5.97 \times 10^{24} + 7.36 \times 10^{22}}$$

$$= 4680 \text{ km}$$

Centre of mass for several particles

We can expand the two objects solution for more objects:

$$\mathbf{R} = \frac{\mathbf{r}_1 m_1 + \mathbf{r}_2 m_2 + \cdots + \mathbf{r}_n m_n}{m_1 + m_2 + \cdots + m_n} = \frac{\sum_{i=1}^{n} \mathbf{r}_i m_i}{\sum_{i=1}^{n} m_i} = \frac{1}{M} \sum_{i=1}^{n} \mathbf{r}_i m_i$$

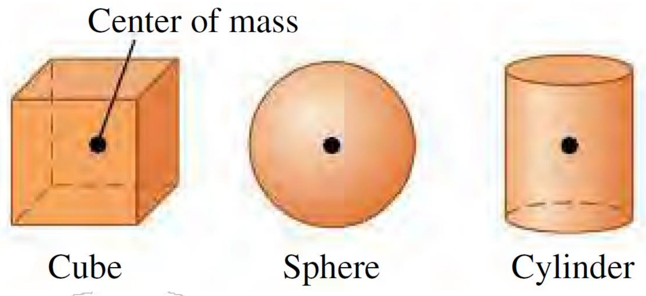


R is a vector, so we can split this for its three components:

$$X = \frac{1}{M} \sum_{i=1}^{n} x_i m_i; \;\; Y = \frac{1}{M} \sum_{i=1}^{n} y_i m_i; \;\; Z = \frac{1}{M} \sum_{i=1}^{n} z_i m_i$$

Centre of mass of objects

If the object we are considering is symmetrical, its centre of mass is the geometrical centre.



If the object is arbitrarily shaped and with different densities, we can split it in small, identical cubes.

We can then find the centre of mass of each cube and use the particle equations to find the overall centre of mass.

Centre of mass of objects

As not all cubes have the same mass, we define their density as:

$$\rho = dm/dV$$

If the density within a cube is constant, this becomes:

$$\rho = M/V$$

Assuming that the density of each cube is different, but that each cube is uniform, the centre of mass becomes:

$$\mathbf{R} = \frac{1}{M}\sum_{i=1}^{n}\mathbf{r}_i dm_i = \frac{1}{M}\sum_{i=1}^{n}\mathbf{r}_i\rho(\mathbf{r}_i)dV$$

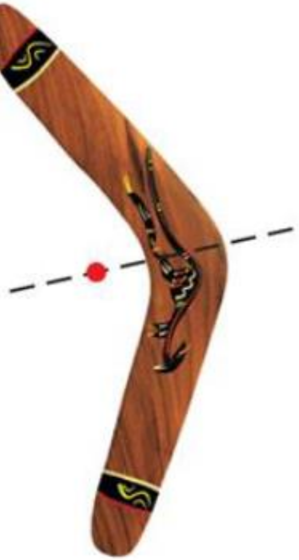
If we reduce the size of the cubes, this becomes:

$$\mathbf{R} = \frac{1}{M}\int_V \mathbf{r}_i\rho(\mathbf{r}_i)dV$$

General considerations

Solving the volume integral is not trivial! A careful choice of coordinate system and mathematical techniques is needed.

Note also that the centre of mass of an object can fall outside, or at an edge of the object itself!



Motion of centre of mass

When a system of particles moves, the centre of mass will also move. We can find its velocity by differentiating according to the time the equation of its position:

$$\mathbf{V} = \frac{\mathbf{v}_1 m_1 + \mathbf{v}_2 m_2 + \cdots + \mathbf{v}_n m_n}{m_1 + m_2 + \cdots + m_n} = \frac{\mathbf{v}_1 m_1 + \mathbf{v}_2 m_2 + \cdots + \mathbf{v}_n m_n}{M}$$

We can rewrite this to get: $M\mathbf{V} = \mathbf{v}_1 m_1 + \mathbf{v}_2 m_2 + \cdots + \mathbf{v}_n m_n$

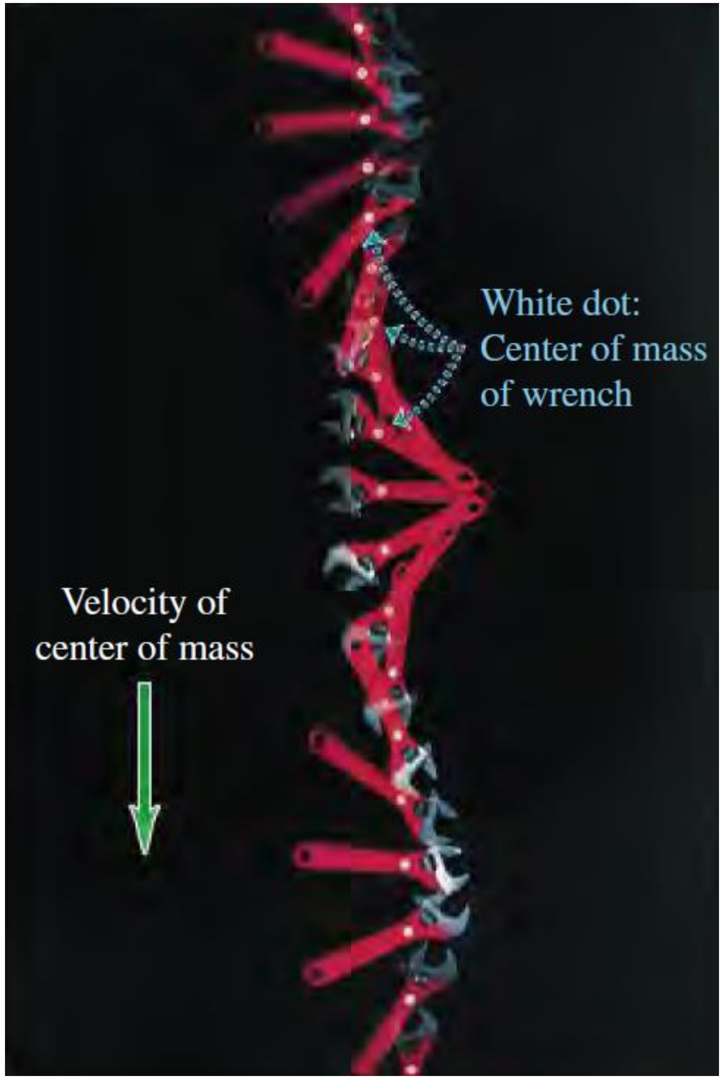
This means that the total momentum of the system is equal to the sum of the momentum of all particles.

Motion of centre of mass

The result above means that if we catch an object, made of a large number of small particles, we will feel the force due to the total momentum, which can be considered to act at the centre of mass.

This helps us justify the use of particles to represent extended objects!

It also means that we can simplify complex solid object motions as the motions of their centre of mass.



Acceleration of centre of mass

If we differentiate our equation again, we get:

$$M\mathbf{a} = \mathbf{a}_1 m_1 + \mathbf{a}_2 m_2 + \cdots + \mathbf{a}_n m_n$$

$m_1\mathbf{a}_1$ is the force acting on the first particle and so forth, giving:

$$M\mathbf{a} = \mathbf{F}_1 + \mathbf{F}_2 + \cdots + \mathbf{F}_n = \sum \mathbf{F}_i$$

These include all internal and external forces: However, for the third law of Newton, internal forces cancel each other: $M\mathbf{a} = \sum \mathbf{F}_{\text{ext}} + \sum \mathbf{F}_{\text{int}}$

However, for the third law of Newton, internal forces cancel each other:

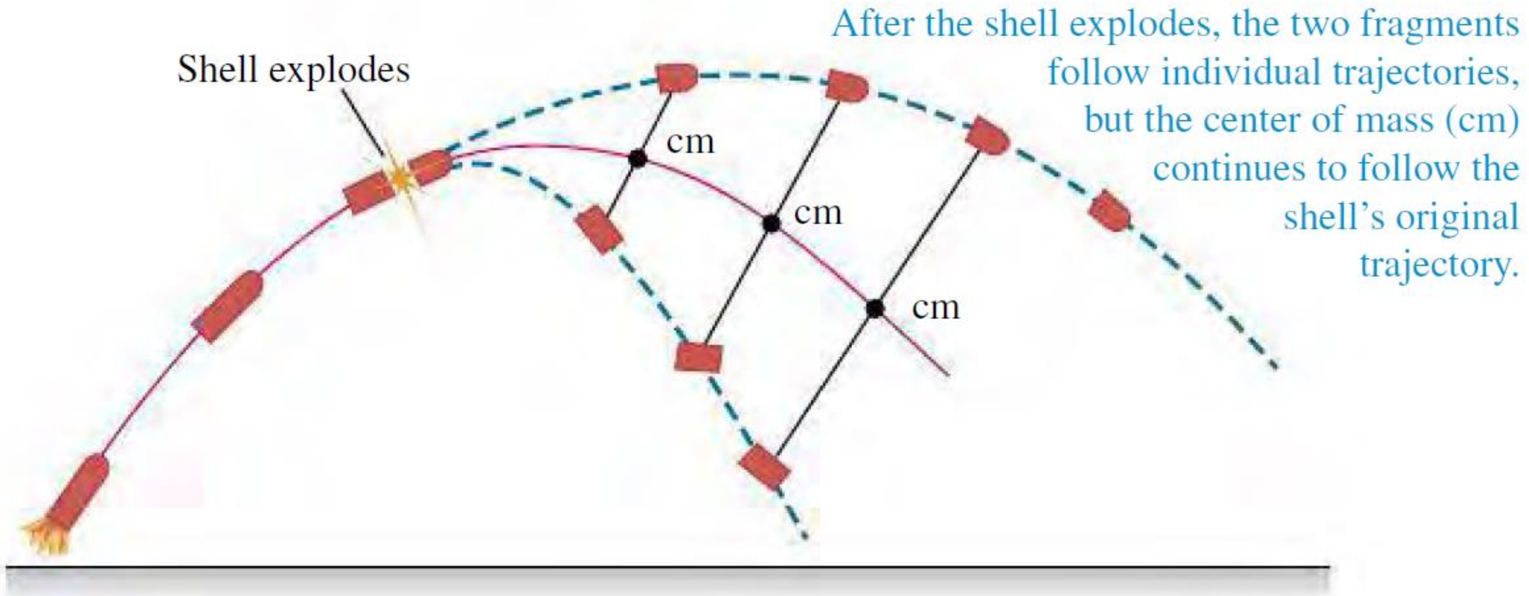
$$M\mathbf{a} = \sum \mathbf{F}_{\text{ext}}$$

Acceleration of centre of mass

This implies that when a collection of particles is subject to an external force, the centre of mass moves as though all the mass were concentrated at that point and it were acted upon by a net force equal to the sum of all the external forces on the system.

It also explains why only external forces affect the motion of a body. This means that if only internal forces are involved, the motion of the centre of mass is unchanged.

$$M\mathbf{a} = \sum \mathbf{F}_{\text{ext}}$$



Collisions

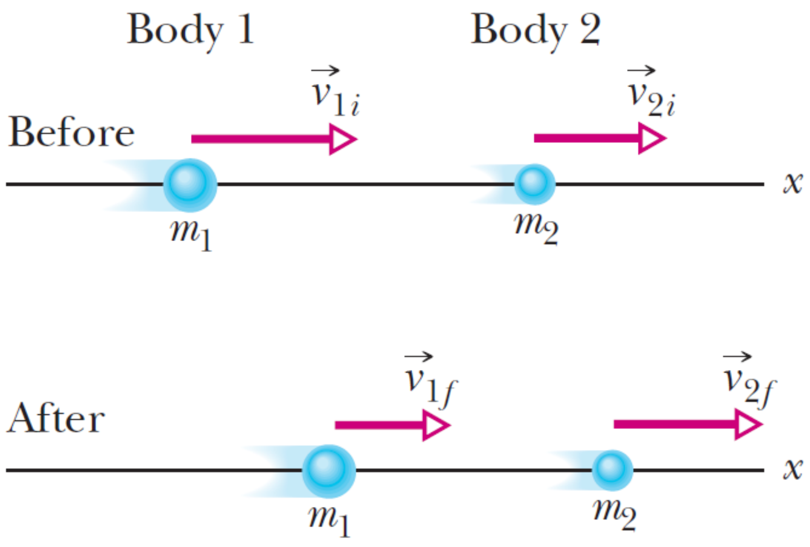
We can use conservation of momentum to consider the effects of collisions.

Momentum is conserved as long as the system is isolated.

Energy is not necessarily conserved (non conservative forces acting).

$$\mathbf{p}_{1i} + \mathbf{p}_{2i} = \mathbf{p}_{1f} + \mathbf{p}_{2f};\ m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$$

$$K_{1i} + K_{2i} \neq K_{1f} + K_{2f}$$



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Completely inelastic collisions

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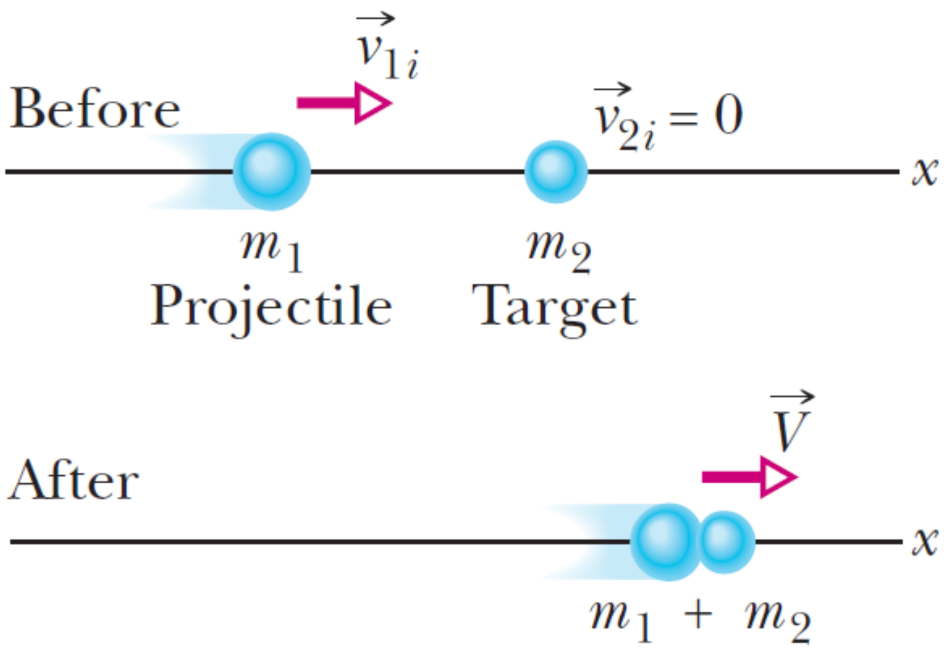
In a completely inelastic collision, the bodies stick together.

Therefore, final velocity will be the same:

$$\mathbf{v}_{1f} = \mathbf{v}_{2f} = \mathbf{v}_f$$

$$\mathbf{p}_{1i} + \mathbf{p}_{2i} = \mathbf{p}_{1f} + \mathbf{p}_{2f};$$

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_f + m_2\mathbf{v}_f = \mathbf{v}_f\left(m_1 + m_2\right)$$



If $\mathbf{v}_{2i} = 0$ (a projectile hits a target):

$$\mathbf{v}_f = \frac{m_1}{m_1 + m_2}\mathbf{v}_{1i}$$

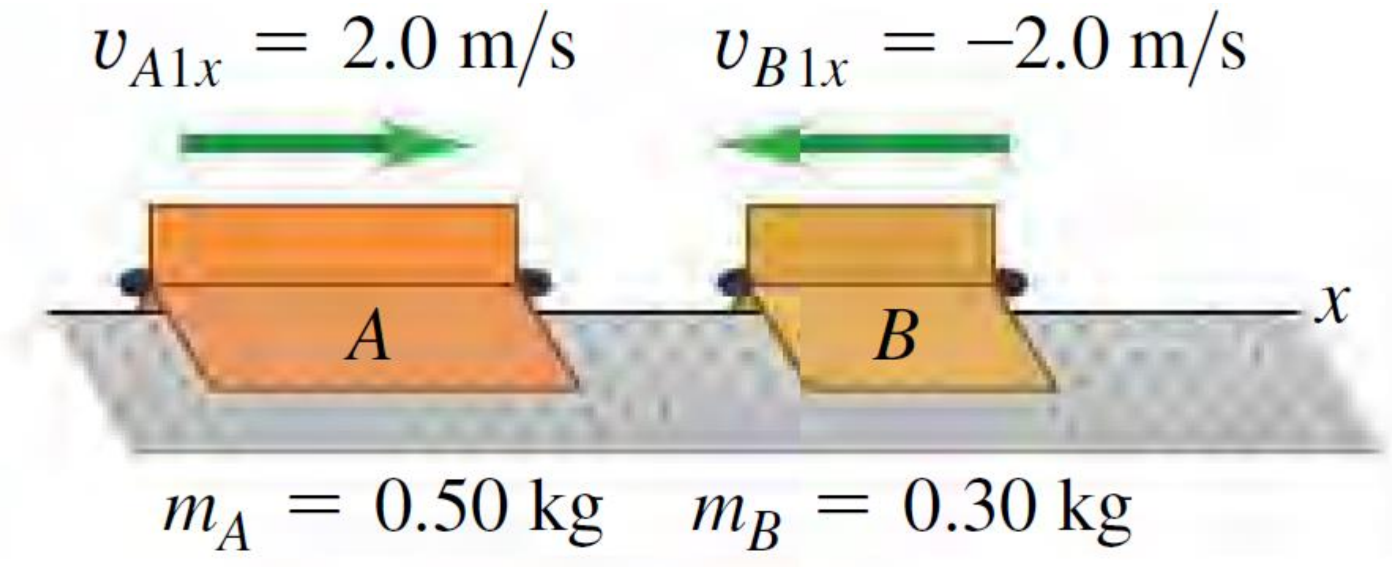
Completely inelastic collisions - example

Two gliders with different masses move toward each other on a frictionless air track. After the collision, they stick together. What is their final speed?

$$P_1 = P_{1A} + P_{1B}$$
$$= 0.5 \times 2 + 0.3 \times (-2)$$
$$= 1 - 0.6$$
$$= 0.4 \ kg \cdot m/s$$

$$V_2 = \frac{0.4}{0.8}$$
$$= 0.5 m/s$$

$$P_1 = P_2$$
$$P_2 = V_2 (M_A + M_B)$$
$$0.40 = V_2 (0.50 + 0.30)$$



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Completely inelastic collisions - example

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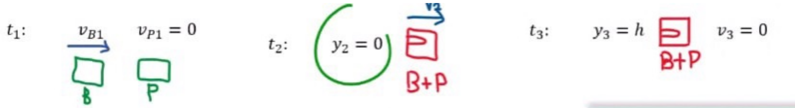
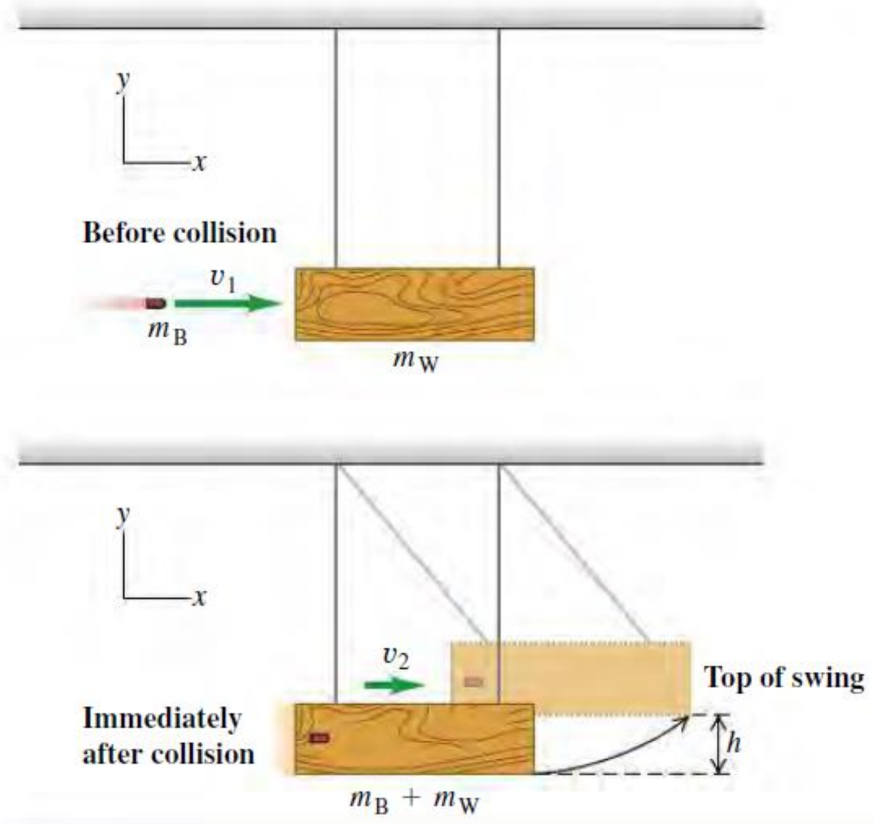
The figure shows a ballistic pendulum, a simple system for measuring the speed of a bullet. A bullet of mass m_B makes a completely inelastic collision with a block of wood of mass m_W, which is suspended like a pendulum. After the impact, the block swings up to a maximum height h. In terms of h, m_B, and m_W, what is the initial speed v_1 of the bullet?





$KE_2 = U_{G3}$

$\frac{1}{2}(\cancel{M_B + M_W})v_2^2 = (\cancel{M_B + M_W})gh$

$V_2 = \sqrt{2gh}$

$P_1 = P_2$, $V_{P1} = 0$ $P_{P1} = 0$

$M_B V_{B1} = V_2 (M_B + M_W)$

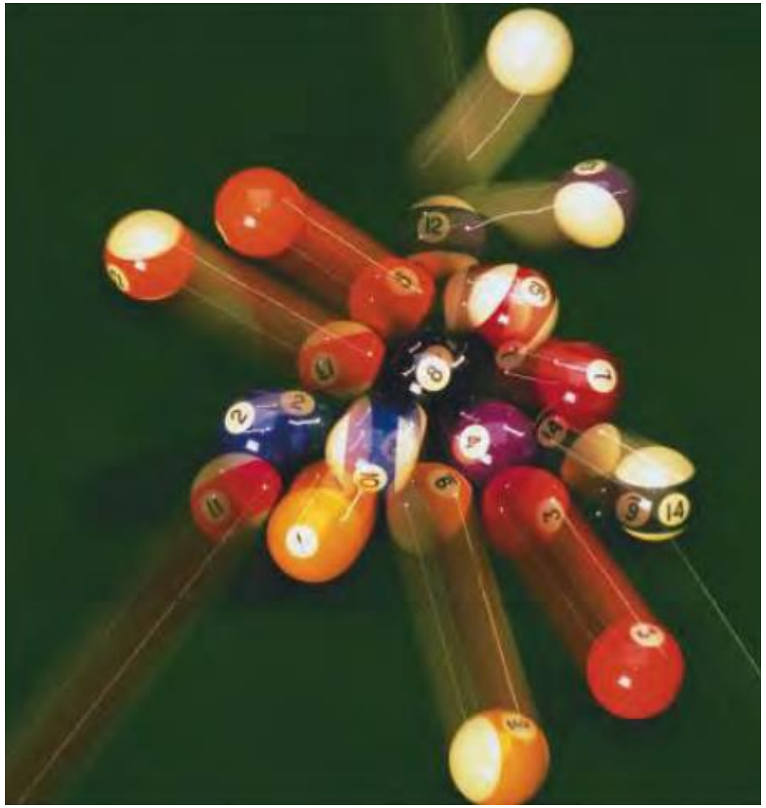
$V_{B1} = V_2 \frac{(M_B + M_W)}{M_B} = \frac{M_B + M_W}{M_B}\sqrt{2gh}$

Elastic collisions

If the collision is completely elastic, the kinetic energy is conserved:

$$K_{1i} + K_{2i} = K_{1f} + K_{2f}$$

$$\frac{1}{2}mv_{1i}^{\ 2} + \frac{1}{2}mv_{2i}^{\ 2} = \frac{1}{2}mv_{1f}^{\ 2} + \frac{1}{2}mv_{2f}^{\ 2}$$



Conservation of momentum:

$$\mathbf{p}_{1i} + \mathbf{p}_{2i} = \mathbf{p}_{1f} + \mathbf{p}_{2f};\ m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$$

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Elastic collisions – stationary target

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$$M_1 V_{1F} = M_1 V_{1i} - M_2 V_{2F}$$

$$V_{1F} = \frac{M_1}{M_1} V_{1i} - \frac{M_2}{M_1} V_{2F}$$

If $v_{2i} = 0$:

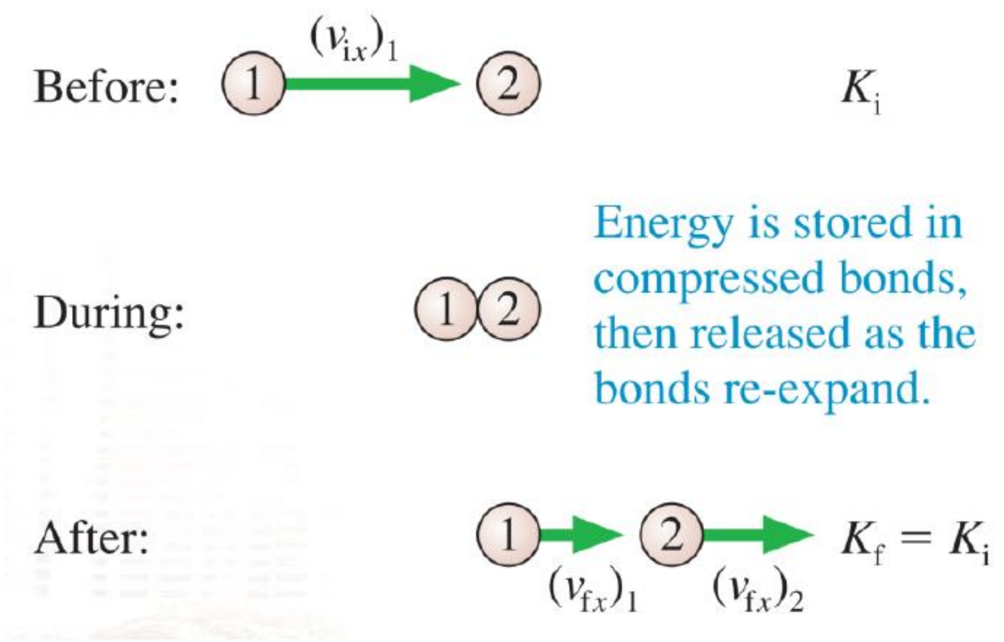
$$\frac{1}{2} m v_{1i}^{\;2} = \frac{1}{2} m v_{1f}^{\;2} + \frac{1}{2} m v_{2f}^{\;2}$$

$$m_1 \mathbf{v}_{1i} = m_1 \mathbf{v}_{1f} + m_2 \mathbf{v}_{2f}$$

From the second equation: $\mathbf{v}_{1f} = \mathbf{v}_{1i} - \dfrac{m_2}{m_1} \mathbf{v}_{2f}$

Putting this in the first equation:

$$\frac{1}{2} m v_{1i}^{\;2} = \frac{1}{2} m \left(\mathbf{v}_{1i} - \frac{m_2}{m_1} \mathbf{v}_{2f} \right)^2 + \frac{1}{2} m v_{2f}^{\;2}$$



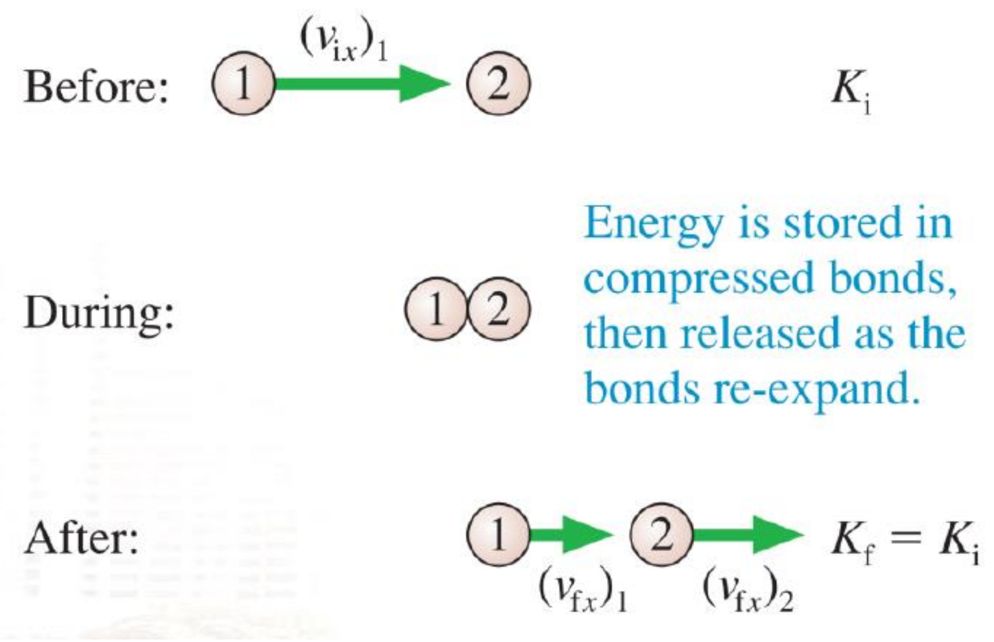
Elastic collisions – stationary target

We can rearrange the equation to find v_{f2}:

$$\frac{1}{2}m_1 {v_{1i}}^2 = \frac{1}{2}m_1\left(v_{1i} - \frac{m_2}{m_1}v_{2f}\right)^2 + \frac{1}{2}m_2 {v_{2f}}^2;$$

$$0 = {m_2}^2 v_{2f} - 2m_2 m_1 v_{1i} + m_2 m_1 v_{2f};$$

$$v_{2f} = v_{1i}\frac{2m_1}{m_2 + m_1}$$



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Completely inelastic collisions - example

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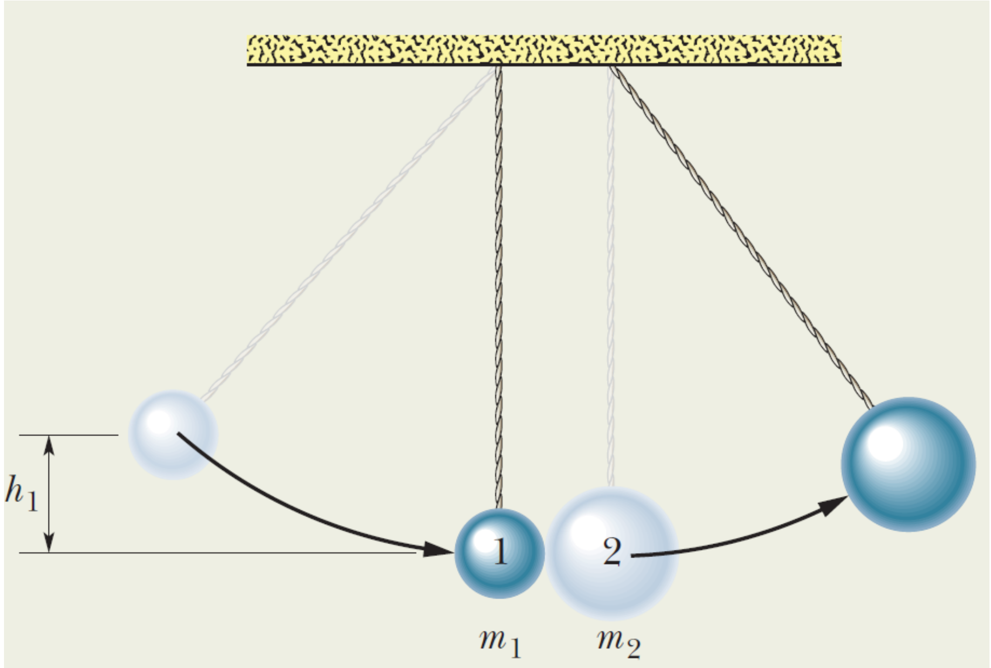
Two metal spheres, suspended by vertical cords, initially just touch. Sphere 1, with mass m_1 = 30 g, is pulled to the left to height h_1 = 8.0 cm, and then released from rest. After swinging down, it undergoes an elastic collision with sphere 2, whose mass m_2 = 75 g. What is the velocity v_{1f} of sphere 1 just after the collision?



V_{1i} :

$U_{G1} = KE_2$;

$Mgh_1 = \frac{1}{2} M v_{1i}^2$

$V_{1i} = \sqrt{2gh} = 1.252\ m/s$

$V_{2i} = 0\ m/s$

$P_i = P_F$; $KE_i = KE_F$

$V_{2F} = V_{1i} \frac{2M_1}{M_1 + M_2}$

$= 1.252 \times 2 \times \frac{0.030}{0.03 + 0.075}$

$= 0.715\ m/s$

$V_{1F} = V_{1i} - \frac{M_2}{M_1} V_{2F}$

$1.252 - \frac{0.075}{0.03} \times 0.715 = -0.54\ m/s$

Check : $0.03 \times 1.252 = 0.03 \times -0.54 + 0.075 \times 0.715$

$0.0376 = 0.0374$

Close enough