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Event-Triggered Optimal Nonlinear Systems Control Based on State Observer and Neural Network*

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Abstract This paper develops a novel event-triggered optimal control approach based on state observer and neural network (NN) for nonlinear continuous-time systems. Firstly, the authors propose an online algorithm with critic and actor NNs to solve the optimal control problem and provide an event-triggered method to reduce communication and computation burdens. Moreover, the authors design weight estimation for critic and actor NNs based on gradient descent method and achieve uniformly ultimate boundednesss (UUB) estimation results. Furthermore, by using bounded NN weight estimation and dead-zone operator, the authors propose a triggering condition, prove the asymptotic stability of closed-loop system from Lyapunov stability perspective, and exclude the Zeno behavior. Finally, the authors provide a numerical example to illustrate the effectiveness of the proposed method.

Keywords Event-triggered control, neural network, optimal control, state observer.

1 Introduction

Over the past few decades, due to the feasibility and simplicity of time-triggered control (TTC), it has attracted widespread attention. TTC is embodied in the time required to periodically execute each T units of time, which causes largely waste of communication and computation resources[1]. In recent years, to reduce the communication and computation burdens,

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researchers modified TTC as event-triggered control (ETC), where the control task is executed only when the system violates some certain conditions. Therefore, the ETC can effectively improve resource utilization and yield better performance[2, 3].

Due to the effectiveness of reducing communication and computation burdens, there are many excellent results on ETC. For linear time-invariant systems, [4] designed triggering conditions as well as self-triggered mechanisms and achieved an input-to-state stability result. Furthermore, [5] proposed a simple self-triggered sample and provided a method to calculate event-triggered interval for the control of continuous nonlinear systems. In [6], the authors proposed a small-gain based dynamic event-triggered strategy and designed sufficient conditions to guarantee the stability of the system. For linear multi-agent systems, [7] studied a consensus problem under a fixed topological structure, designed event-triggered conditions in the cases of concentration as well as dispersion, and given event interval time of positive lower bound to exclude Zeno behavior. Moreover, [8–12] investigated the distributed rendezvous problem of multi-agent systems with several novel event-triggered controllers, proposed combinational measurement approaches, and developed basic ETC algorithms. For the leader-following consensus problem in multi-agent systems, [13] introduced ETC to reduce the number of controllers update. [14] proposed a consensus disturbance rejection protocol based on ETC, which can significantly save communication resources and still achieve a comparable consensus performance given by conventional time triggered consensus protocol. For network control systems, [15] investigated the problem of event-triggered H_∞ control for a networked singular system and demonstrated the capability of the event-triggered approach in reducing the network bandwidth usage.

Most ETC methods assume that full state information is available[2, 4]. However, the full state information is difficult to be achieved in many circumstances. [16] proposed dynamic output-feedback control for linear semi-Markov jump systems. [17, 18] proposed the output-based ETC method, when the difference between the current output of the system and the latest transmission exceeds a certain threshold, the ETC method update the controller and reconstructed the state of the system, which can significantly reduce the number of triggers. In addition, [19–21] studied the ETC based on state observer. For continuous linear time-invariant systems, [22] designed a state observer based on ETC and guaranteed stability of the system.

Since adaptive dynamic programming (ADP) can solve the optimal control problem of nonlinear systems[23–25], it has attracted extensive attention during the past decades. For linear systems, the ETC algorithm adopted offline strategy, used the Riccati equation or Hamilton-Jacobi-Bellman (HJB) equations to solve optimal control problem, and then obtained the optimal controller[26]. However, HJB equations of nonlinear systems are partial differential, whose solution is difficult to be achieved because of the dimension disaste issue. Therefore, one can adopt ADP method to solve HJB equations of nonlinear systems. [27–29] adopted actor neural network (NN) and critic NN to approximate the optimal controller and the optimal cost, respectively. Under ETC, one designs weight tuning laws to ensure the weight estimates being uniformly bounded between triggering instants. To obtain an efficient ETC, [30] designed an adaptive triggering condition based on weight estimates and dead zone, which can avoid unnecessary triggering and reduce computational burdens. [31] proposed an event-triggered receding horizon actor-critic approach for nonlinear continuous-time systems. The method decomposed the infinite optimal control problem into a series of finite horizon optimal control problems, used ADP algorithms to obtain the optimal control law in each horizon, and proposed a novel adaptive triggering condition to reduce the computational burdens.

Inspired from state observer in linear systems, we propose a novel optimal control method for nonlinear systems. The main contributions of this paper are listed as follows. First, we use a state observer to reconstruct unmeasurable system state. Unlike [22] utilizes an observer to estimate the state of the linear system, this paper applies the state observer to nonlinear systems. Moreover, we present a novel critic-actor algorithm to solve HJB equations with state observer. Different from the, which adopted offline strategy to solve the optimal control problem of linear systems in [26], the proposed method employs online strategy to handle the optimal control problem of nonlinear systems. Compared with the optimal control method in [27], this paper presents optimal control based on ETC which can significantly save communication and computation resources. According to the gradient descent method, we design weight tuning laws and a novel triggering condition based on the dead zone operator. Finally, by the Lyapunov stability theory, we prove that the system is uniformly ultimate boundedness (UUB).

The rest of this paper is organized as follows. Section 2 formulates the problem and Section 3 designs an online event-triggered optimal control algorithm with a critic-actor NN framework. In Section 4, we propose a novel triggering condition and prove the stability of the system by Lyapunov theory. Section 5 excludes the Zeno behavior. Then Section 6 provides an illustrative numerical example and Section 7 concludes this paper.

Notations $\mathbb{R}$, $\mathbb{R}^+$, $\mathbb{Z}_0^+$, and $\mathbb{R}^n$ denote the real number set, positive real number set, nonnegative integer set, and real vector set with n-dimension, respectively. $\|\cdot\|$ is the standard Euclid norm for vectors. In addition, $\underline{\lambda}(M)$ $(\overline{\lambda}(M))$ is the smallest (largest) eigenvalue of matrix M.

2 Problem Formulations

Consider the following nonlinear continuous-time system

$$\begin{aligned} &\dot{x}(t) = f(x(t)) + g(x(t))u(t), \quad x(0) = x_0, \quad t \geq 0, \\ &y(t) = Cx(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^q$, and $C \in \mathbb{R}^{q\times n}$ are the system state, control input, output, and output matrix, respectively. $f(x(t)) \in \mathbb{R}^n$ and $g(x(t)) \in \mathbb{R}^{n\times m}$ are drift dynamics.

Remark 2.1 The nonlinear continuous-time system in (1) is a general model and widely used to model nonlinear oscillator systems[28], pendulum systems[30], and single-link robot arm systems[31].

For the convenience of later discussions, we introduce the following mild assumption.

Assumption 2.1 (see [30]) System (1) is controllable and observable, $f(x)$ satisfies $f(0) =$

0, $x = 0$ is an equilibrium point of the system (1), and $f(x) + g(x)u$ is Lipschitz continuous on a compact set $\Omega \subset \mathbb{R}^n$ containing the origin, namely

1) $\|f(x)\| \leq k_f\|x\| + b_f$, where $k_f > 0$ is the Lipschitz constant and $b_f > 0$;

2) $\| g(x) \| \leq g_M$ with $g_M > 0$.

For (1), we consider the following cost function

$$J(x(t), u(t)) = \int_0^\infty \psi(x(\tau), u(\tau))d\tau, \tag{2}$$

where $\psi(x(\tau), u(\tau)) = x^{\mathrm{T}}Qx + u^{\mathrm{T}}u$ and Q is a positive definite symmetric matrix. The objective of this paper is to design a feedback control $u(x)$ to minimize the cost function and ensure the asymptotically stable of the closed-loop system. Then we consider the following optimization problem

$$V^*(x(t)) = \min_u(J(x(t), u(t))). \tag{3}$$

We construct a state observer

$$\begin{aligned} &\dot{\widehat{x}}(t) = f(\widehat{x}(t)) + g(\widehat{x}(t))u(\widehat{x}(t)) + \varphi(y, \widehat{y}), \quad t \geq 0, \\ &\varphi(\widehat{y}, y) = F(\widehat{y} - y), \end{aligned} \tag{4}$$

where $\widehat{x}(t) \in \mathbb{R}^n$, $\widehat{u}(t) \in \mathbb{R}^m$, and $\widehat{y}(t) \in \mathbb{R}^q$ are the state, input, and output of the observer, respectively.

Define the Hamiltonian of (4) as follows

$$H\big(x, u, \nabla V^*(\widehat{x})\big) = \nabla V^*(\widehat{x})^{\mathrm{T}}\big(f(\widehat{x}) + g(\widehat{x})u(t) + \varphi(y, \widehat{y})\big) + x^{\mathrm{T}}Qx + u^{\mathrm{T}}u, \quad \forall\, \widehat{x}, u, \tag{5}$$

where $\nabla V^*(\widehat{x}) = \frac{\partial V^*(\widehat{x})}{\partial x}$. According to Bellman's principle, we have

$$\min_u H\big(x, u, \nabla V^*(\widehat{x})\big) = 0. \tag{6}$$

By the stationary condition $\frac{\partial H(x,u,\nabla V^*(\widehat{x}))}{\partial u} = 0$,

$$u^*(\widehat{x}) = -\frac{1}{2}g(\widehat{x})^{\mathrm{T}}\nabla V^*(\widehat{x}). \tag{7}$$

Substituting (7) into (5),

$$\begin{aligned} &H\big(x, u^*(\widehat{x}), \nabla V^*(\widehat{x})\big) \\ =&\nabla V^*(\widehat{x})^{\mathrm{T}}\big(f(\widehat{x}) + \varphi(y, \widehat{y}) - \frac{1}{2}g(\widehat{x})g(\widehat{x})^{\mathrm{T}}\nabla V^*(\widehat{x})\big) + x^{\mathrm{T}}Qx + \frac{1}{4}\nabla V^*(\widehat{x})^{\mathrm{T}}g(\widehat{x})g(\widehat{x})^{\mathrm{T}}\nabla V^*(\widehat{x}) \\ =&\nabla V^*(\widehat{x})^{\mathrm{T}}\big(f(\widehat{x}) + \varphi(y, \widehat{y})\big) + x^{\mathrm{T}}Qx - \frac{1}{4}\nabla V^*(\widehat{x})^{\mathrm{T}}g(\widehat{x})g(\widehat{x})^{\mathrm{T}}\nabla V^*(\widehat{x}) \\ =&0. \end{aligned} \tag{8}$$

To reduce the communication between the controller and the plant, we design event-triggered HJB equation and deduce the event-triggered optimal control strategy according to the TTC strategy

$$u^*(\widehat{x}_j)=-\frac{1}{2}g(\widehat{x}_j)^{\mathrm{T}}\nabla V^*(\widehat{x}_j),\tag{9}$$

where $\widehat{x}_j(t)$ is the sampling of $x_j(t)$. For ETC, the controller is updated if the event is triggered,. Therefore, we have

$$u^*(\widehat{x})=u^*(\widehat{x}_j).\tag{10}$$

Substituting (9) into (5),

$$\begin{aligned}H\big(x,u^*(\widehat{x}_j),\nabla V^*(\widehat{x}_j)\big)=&\nabla V^*(\widehat{x})^{\mathrm{T}}\big(f(\widehat{x})+\varphi(y,\widehat{y})-\frac{1}{2}g(\widehat{x})g(\widehat{x}_j)^{\mathrm{T}}\nabla V^*(\widehat{x}_j)\big)\\&+x^{\mathrm{T}}Qx+\frac{1}{4}\nabla V^*(\widehat{x}_j)^{\mathrm{T}}g(\widehat{x}_j)g(\widehat{x}_j)^{\mathrm{T}}\nabla V^*(\widehat{x}_j).\end{aligned}\tag{11}$$

Define a triggering condition

$$\|e_j(t)\|\le e_{\mathrm{th}},\tag{12}$$

where e_{th} is the triggering threshold and $e_j(t)=\widehat{x}_j(t)-\widehat{x}(t)$. For further proceeding, we present the following assumption.

Assumption 2.2 (see [29]) Optimal control $u^*(\widehat{x})$ is Lipschitz continuous relative to triggering error

$$\|u^*(\widehat{x}_j)-u^*(\widehat{x})\|\le k\|\widehat{x}_j-\widehat{x}\|,\tag{13}$$

where k is a positive Lipschitz constant.

Remark 2.2 Assumption 2.2 is a common assumption that the control input is Lipschitz continuous (see, e.g., [29–31]). It is satisfied in many applications, especially when the controller is affine with respect to e_j.

The corresponding HJB equation equals to 0 when the event is triggered. According to the time-triggered and event-triggered HJB,

$$\begin{aligned}&H\big(x,u^*(\widehat{x}_j),\nabla V^*(\widehat{x}_j)\big)-H\big(x,u^*(\widehat{x}),\nabla V^*(\widehat{x})\big)\\=&-\frac{1}{2}\nabla V^*(\widehat{x})^{\mathrm{T}}g(\widehat{x})g(\widehat{x}_j)^{\mathrm{T}}\nabla V^*(\widehat{x}_j)+\frac{1}{4}\nabla V^*(\widehat{x}_j)^{\mathrm{T}}g(\widehat{x}_j)g(\widehat{x}_j)^{\mathrm{T}}\nabla V^*(\widehat{x}_j)\\&+\frac{1}{4}\nabla V^*(\widehat{x})^{\mathrm{T}}g(\widehat{x})g(\widehat{x})^{\mathrm{T}}\nabla V^*(\widehat{x})\\=&-2u^*(\widehat{x})^{\mathrm{T}}u^*(\widehat{x}_j)+u^*(\widehat{x}_j)^{\mathrm{T}}u^*(\widehat{x}_j)+u^*(\widehat{x})^{\mathrm{T}}u^*(\widehat{x})\\=&\big(u^*(\widehat{x}_j)-u^*(\widehat{x})\big)^{\mathrm{T}}\big(u^*(\widehat{x}_j)-u^*(\widehat{x})\big).\end{aligned}\tag{14}$$

To obtain the optimal controller, it is necessary to solve the event-triggered HJB equation in (11). However, the HJB equation contains the strong nonlinear term, which increase the difficulty to obtain the close form solution. To this end, we propose critic-actor NN to handle this problem in the next section.

3 Critic-Actor algorithm

In this section, we provide critic-actor algorithm for the optimal control of nonlinear systems. When the triggering condition is violated, the event occurs and one samples system state. Critic NN is used to approximate the cost function with the sampled state and actor NN is used to approximate the control law.

3.1 Critic NN

The cost function is approximated by the following NN

$$V^*(x(t)) = W_c^{\mathrm{T}}\phi_c(x(t)) + \varepsilon_c(x(t)), \tag{15}$$

where $W_c \in \mathbb{R}^{N_c}$ is the exact weight, $\phi_c(x(t)) = [\phi_{c1}(x(t)), \phi_{c2}(x(t)), \cdots, \phi_{cN_c}(x(t))]^{\mathrm{T}} \in \mathbb{R}^{N_c}$ is bounded continuous time-varying activation function, N_c is the number of neurons, and $\varepsilon_c(x(t))$ is the residual error. Then

$$\nabla V^*(x) = \nabla\phi_c(x)^{\mathrm{T}}W_c + \nabla\varepsilon_c(x). \tag{16}$$

By using NN approximation,

$$u(\widehat{x}_j) = -\frac{1}{2}g(\widehat{x}_j)^{\mathrm{T}}\big(\nabla\phi_c(x_j)^{\mathrm{T}}W_c + \nabla\varepsilon_c(x_j)\big). \tag{17}$$

Based on (17),

$$\begin{aligned}
&H(x, u(\widehat{x}_j), W_c)\\
=&W_c^{\mathrm{T}}\nabla\phi_c(\widehat{x}_j)\big(f(\widehat{x}) + g(\widehat{x})u(\widehat{x}_j) + \varphi(y,\widehat{y})\big) + x^{\mathrm{T}}Qx + u(\widehat{x}_j)^{\mathrm{T}}u(\widehat{x}_j)\\
&+ \nabla\varepsilon_c(\widehat{x}_j)\big(f(\widehat{x}) + g(\widehat{x})u(\widehat{x}_j) + \varphi(y,\widehat{y})\big)\\
=&W_c^{\mathrm{T}}\nabla\phi_c(\widehat{x}_j)\big(f(\widehat{x}) + g(\widehat{x})u(\widehat{x}_j) + \varphi(y,\widehat{y})\big) + x^{\mathrm{T}}Qx + u(\widehat{x}_j)^{\mathrm{T}}u(\widehat{x}_j) + \varepsilon_{hc},
\end{aligned} \tag{18}$$

where $\varepsilon_{hc} = \nabla\varepsilon_c(\widehat{x}_j)\big(f(\widehat{x}) + g(\widehat{x})u(\widehat{x}_j(t)) + \varphi(y,\widehat{y})\big)$.

In the actual control process, W_c is not available. Under the ETC, we use the estimated weight to approximate the cost function

$$\widehat{V}(\widehat{x}_j) = \widehat{W}_c^{\mathrm{T}}\phi_c(\widehat{x}_j), \tag{19}$$

where $\widehat{W}_c$ is the estimation of the exact W_c. Accordingly, the HJB equation of the estimated cost function is given by

$$H(x, u(\widehat{x}_j), \widehat{W}_c) = \widehat{W}_c^{\mathrm{T}}\nabla\phi_c(\widehat{x}_j)\big(f(\widehat{x}) + g(\widehat{x})u(\widehat{x}_j) + \varphi(y,\widehat{y})\big) + x^{\mathrm{T}}Qx + u(\widehat{x}_j)^{\mathrm{T}}u(\widehat{x}_j). \tag{20}$$

To find the tuning law of the weight, we define the error e_c as

$$\begin{aligned}
e_c =&H(x, u(\widehat{x}_j), \widehat{W}_c)\\
=&\widehat{W}_c^{\mathrm{T}}\nabla\phi_c(\widehat{x}_j)\big(f(\widehat{x}) + g(\widehat{x})u(\widehat{x}_j) + \varphi(y,\widehat{y})\big) + x^{\mathrm{T}}Qx + u(\widehat{x}_j)^{\mathrm{T}}u(\widehat{x}_j)\\
=&\widehat{W}_c^{\mathrm{T}}\nabla\phi_c(\widehat{x}_j)\big(f(\widehat{x}) + g(\widehat{x})u(\widehat{x}_j) + \varphi(y,\widehat{y})\big)\\
&- W_c^{\mathrm{T}}\nabla\phi_c(\widehat{x}_j)\big(f(\widehat{x}) + g(\widehat{x})u(\widehat{x}_j) + \varphi(y,\widehat{y})\big) - \varepsilon_{hc}\\
=&-\widetilde{W}_c^{\mathrm{T}}\nabla\phi_c(\widehat{x}_j)\big(f(\widehat{x}) + g(\widehat{x})u(\widehat{x}_j) + \varphi(y,\widehat{y})\big) - \varepsilon_{hc},
\end{aligned} \tag{21}$$

where $\widetilde{W}_c = W_c - \widehat{W}_c$ is the weight error of critic NN.

To make e_c small enough, we define the following square function

$$K_c = \frac{1}{2} e_c^{\mathrm{T}} e_c. \tag{22}$$

The weight of critic NN is estimated by the following normalized gradient descent method

$$\begin{aligned}\dot{\widehat{W}}_c &= -\alpha_c \frac{1}{(1+\varsigma^{\mathrm{T}}\varsigma)^2} \frac{\partial K_c}{\partial \widehat{W}_c} \\ &= -\alpha_c \frac{\varsigma}{(1+\varsigma^{\mathrm{T}}\varsigma)^2} e_c,\end{aligned} \tag{23}$$

where $\varsigma = \nabla\phi_c(\widehat{x}_j)\big(f(\widehat{x}) + g(\widehat{x})u(\widehat{x}_j(t)) + \varphi(\widehat{y}, y)\big)$.

Because weight $\widehat{W}_c$ is only updated at triggering instants $t = r_j, j \in N$, the gradient descent method was used to train the critic NN.

$$\begin{cases} \dot{\widehat{W}}_c = 0, & t \in (r_j, r_{j+1}), \\ \widehat{W}_c^+ = \widehat{W}_c - \alpha_c \dfrac{\varsigma}{(1+\varsigma^{\mathrm{T}}\varsigma)^2} e_c, & t = r_j, \end{cases} \tag{24}$$

where $\widehat{W}_c^+$ is the updated weight just after the triggering instants, α_c represents the learning rate of critic NN.

The error dynamics for the critic NN weight is expressed as follows

$$\begin{cases} \dot{\widetilde{W}}_c = -\alpha_c \dfrac{\varsigma}{(1+\varsigma^{\mathrm{T}}\varsigma)^2} (\widetilde{W}_c^{\mathrm{T}}\varsigma + \varepsilon_{hc}), \\ \widetilde{W}_c^+ = \widetilde{W}_c - \alpha_c \dfrac{\varsigma}{(1+\varsigma^{\mathrm{T}}\varsigma)^2} (\widetilde{W}_c^{\mathrm{T}}\varsigma + \varepsilon_{hc}), \quad t = r_j. \end{cases} \tag{25}$$

Prior to proceeding further, we present an assumption that has been widely used to analyze the stability of closed-loop systems.

Assumption 3.1 (see [29]) $\varepsilon_c(\cdot)$, $\varepsilon_{hc}(\cdot)$, $\nabla\varepsilon_c(\cdot)$, and $\nabla\phi_c(\cdot)$ are bounded by positive constants ε_{cM}, ε_{hcM}, $\nabla\varepsilon_{cM}$, and $\nabla\phi_{cM}$, respectively.

Remark 3.1 Under the critic NN, we define the square function, take the partial derivative of the function, choose the appropriate weight to make the error e_c small enough, and figure out the tuning law of the weight.

3.2 Actor NN

Recalling NN, the optimal control policy is given by

$$u^*(x(t)) = W_a^{\mathrm{T}}\phi_a(x(t)) + \varepsilon_a(x(t)), \tag{26}$$

where $W_a \in \mathbb{R}^{N_a}$ is the exact weight, $\phi_a(x(t)) = [\phi_{a1}(x(t)), \phi_{a2}(x(t)), \cdots, \phi_{aN_a}(x(t))]^{\mathrm{T}} \in \mathbb{R}^{N_a}$ is bounded continuous time-varying activation function, N_a is the number of neurons, and $\varepsilon_a(x(t))$ is the residual error.

The optimal control strategy obtained from the critic NN cost function, we have

$$W_a^{\mathrm{T}}\phi_a(\widehat{x}_j) + \varepsilon_a(\widehat{x}_j) = -\frac{1}{2}g(\widehat{x}_j)^{\mathrm{T}}\nabla\phi_c(\widehat{x}_j)^{\mathrm{T}}W_c - \frac{1}{2}g(\widehat{x}_j)^{\mathrm{T}}\nabla\varepsilon_c(\widehat{x}_j). \tag{27}$$

Under the ETC, the optimal control strategy approximated by the following actor NN

$$\widehat{u}(\widehat{x}_j) = \widehat{W}_a^{\mathrm{T}} \phi_a(\widehat{x}_j), \tag{28}$$

where $\widehat{W}_a$ is the estimation of W_a. Under the estimated cost function by the critic NN, we rewirte (17) as follows

$$\widetilde{u}(\widehat{x}_j) = -\frac{1}{2} g(\widehat{x}_j)^{\mathrm{T}} \nabla \phi_c(\widehat{x}_j)^{\mathrm{T}} \widehat{W}_c. \tag{29}$$

According to (27), the error function generated by actor NN is

$$\begin{aligned} e_a =& \widehat{u}(\widehat{x}_j) - \widetilde{u}(\widehat{x}_j) \\ =& \widehat{W}_a^{\mathrm{T}} \phi_a(\widehat{x}_j) + \frac{1}{2} g(\widehat{x}_j)^{\mathrm{T}} \nabla \phi_c(\widehat{x}_j)^{\mathrm{T}} \widehat{W}_c \\ =& -\widetilde{W}_a^{\mathrm{T}} \phi_a(\widehat{x}_j) - \frac{1}{2} g(\widehat{x}_j)^{\mathrm{T}} \nabla \phi_c(\widehat{x}_j)^{\mathrm{T}} \widetilde{W}_c - \frac{1}{2} g(\widehat{x}_j)^{\mathrm{T}} \nabla \varepsilon_c(\widehat{x}_j) - \varepsilon_a(\widehat{x}_j). \end{aligned} \tag{30}$$

Define square residual function

$$K_a = \frac{1}{2} e_a^{\mathrm{T}} e_a. \tag{31}$$

Under ETC, the control input with sampled state is updated at triggering instants $t = r_j, j \in N$, and remains unchanged under the action of zero-order holder within the event interval $t \in (r_j, r_{j+1})$. Based on the gradient descent method, the update strategy of actor NN is given by

$$\begin{cases} \dot{\widehat{W}}_a = 0, & t \in (r_j, r_{j+1}), \\ \widehat{W}_a^+ = \widehat{W}_a - \alpha_a \phi_a(\widehat{x}_j) e_a, & t = r_j, \end{cases} \tag{32}$$

where $\widehat{W}_a^+$ is the updated weight just after triggering instants, and α_a is the learning rate of the actor NN. Defining the weight estimation error $\widetilde{W}_a = W_a - \widehat{W}_a$, we rewrite the dynamics of the weight estimation as follows

$$\begin{aligned} \dot{\widetilde{W}}_a =& -\alpha_a \frac{\partial K_a}{\widetilde{W}_a} \\ =& \alpha_a \phi_a \bigg(-\widetilde{W}_a^{\mathrm{T}} \phi_a(\widehat{x}_j) - \varepsilon_a(\widehat{x}_j) - \frac{1}{2} g(\widehat{x}_j)^{\mathrm{T}} \nabla \varepsilon_c(\widehat{x}_j) - \frac{1}{2} g(\widehat{x}_j)^{\mathrm{T}} \nabla \phi_c(\widehat{x}_j)^{\mathrm{T}} \widetilde{W}_c \bigg). \end{aligned} \tag{33}$$

The error function of actor NN weight is expressed as follows

$$\begin{aligned} \widetilde{W}_a^+ =& \widetilde{W}_a - \alpha_a \phi_a \bigg(\widetilde{W}_a^{\mathrm{T}} \phi_a(\widehat{x}_j) + \varepsilon_a(\widehat{x}_j) + \frac{1}{2} g(\widehat{x}_j)^{\mathrm{T}} \nabla \varepsilon_c(\widehat{x}_j) + \frac{1}{2} g(\widehat{x}_j)^{\mathrm{T}} \nabla \phi_c(\widehat{x}_j)^{\mathrm{T}} \widetilde{W}_c \bigg)^{\mathrm{T}} \\ =& \widetilde{W}_a - \alpha_a \phi_a \bigg(\widetilde{W}_a^{\mathrm{T}} \phi_a(\widehat{x}_j) + \chi + \frac{1}{2} g(\widehat{x}_j)^{\mathrm{T}} \nabla \phi_c(\widehat{x}_j)^{\mathrm{T}} \widetilde{W}_c \bigg), \end{aligned} \tag{34}$$

where $\chi = \varepsilon_a(\widehat{x}_j) + \frac{1}{2} g(\widehat{x}_j)^{\mathrm{T}} \nabla \varepsilon_c(\widehat{x}_j)$. Before further proceeding, we present assumption that has been widely used to analyze the stability of closed-loop systems.

Assumption 3.2 (see [31]) $\|\phi_a(\cdot)\|$ and $\|\varepsilon_a(\cdot)\|$ are locally bounded by positive constants ϕ_{aM} and ε_{aM}, respectively.

Remark 3.2 In this paper, we use the actor NN to approximate optimal control law. Since actor NN producing errors, we define the square function, take the partial derivative of the square function, and select appropriate weight estimate value $\widehat{W}_a$ to make the error small enough.

Remark 3.3 Assumptions 3.1 and 3.2 are general assumption that many scholars have used to prove the stability of systems from Lyapunov stability perspective, (see, e.g., [26–30]).

4 Triggering Condition and Stability Analysis

In this section, we design the triggering condition based on the weight and control input of the NN and use Lyapunov method to prove that the system is UUB.

For actor NN, we provide the following assumption.

Assumption 4.1 (see [31]) $\|\phi_a(\widehat{x}_j) - \phi_a(\widehat{x})\|^2 \leq k_a\|\widehat{x}_j - \widehat{x}\|^2$, where k_a is a positive Lipschitz constant.

Theorem 4.1 *Considering the nonlinear system in* (4)*, we take the weight tuning laws for the critic NN* (24) *and the actor NN* (32)*. Under Assumptions* 2.1, 2.2, 3.1, 3.2*, and* 4.1 *hold, the closed-loop system* (4) *is asymptotically stable as long as the following triggering condition is satisfied*

$$D(\|e_j(t)\|) \leq \sqrt{\frac{(1-\beta^2)\underline{\lambda}(Q)\|x(t)\|^2 - 4\varepsilon_{aM}^2}{2\|\widehat{W}_a\|^2 k_a} + \frac{\|\widehat{u}(\widehat{x}_j)\|^2}{2\|\widehat{W}_a\|^2 k_a}}, \tag{35}$$

where $\beta \in (0,1)$ *and*

$$D(\|e_j(t)\|) = \begin{cases} \|e_j(t)\|, & \|x(t)\| > \sqrt{\dfrac{4\|\widetilde{W}_a\|^2\phi_{aM}^2}{\beta^2\underline{\lambda}(Q)}}, \\ 0, & \text{otherwise}. \end{cases} \tag{36}$$

Proof Consider the following Lyapunov function

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \tag{37}$$

where $V_1(t) = \frac{1}{\alpha_c}\mathrm{tr}(\widetilde{W}_c^{\mathrm{T}}\widetilde{W}_c)$, $V_2(t) = \frac{1}{\alpha_a}\mathrm{tr}(\widetilde{\mathrm{W}}_{\mathrm{a}}^{\mathrm{T}}\widetilde{\mathrm{W}}_{\mathrm{a}})$, $V_3(t) = V^*(\widehat{x})$, and $V_4(t) = V^*(\widehat{x}_j)$. Since the Lyapunov function is discrete at triggering instants and continuous in the event intervals, we discuss the stability in the following two cases.

Case 1. Event is not triggered

Within the event interval, $\dot{V}_4(t) = 0$, $\dot{V}_1(t) = 0$, and $\dot{V}_2(t) = 0$. Then

$$\begin{aligned} \dot{V}_3(t) =& \nabla V^*(t)^{\mathrm{T}}\dot{\widehat{x}} \\ =& \nabla V^*(t)^{\mathrm{T}}\big(f(\widehat{x}) + g(\widehat{x})\widehat{u}(\widehat{x}_j) + \varphi(y,\widehat{y})\big) \\ =& \nabla V^*(t)^{\mathrm{T}}\big(f(\widehat{x}) + g(\widehat{x})u^*(\widehat{x}) + \varphi(y,\widehat{y})\big) - \nabla V^*(t)^{\mathrm{T}}g(\widehat{x})\big(u^*(\widehat{x}(t)) - \widehat{u}(\widehat{x}_j)\big). \end{aligned} \tag{38}$$

Due to $H\big(x, u^*(\widehat{x}(t)), \nabla V^*(\widehat{x}(t))\big) = 0$, we have

$$\nabla V^*(t)^{\mathrm{T}}\big(f(\widehat{x}) + g(\widehat{x}(t))u^*(\widehat{x}) + \varphi(y, \widehat{y})\big) = -x^{\mathrm{T}}Qx - u^*(\widehat{x})^{\mathrm{T}}u^*(\widehat{x}). \tag{39}$$

Substituting (7) and (39) into (38),

$$\begin{aligned}\dot{V}_3(t) =& - x^{\mathrm{T}}Qx - u^*(\widehat{x})^{\mathrm{T}}u^*(\widehat{x}) + 2u^*(\widehat{x})^{\mathrm{T}}u^*(\widehat{x}) - 2u^*(\widehat{x})^{\mathrm{T}}\widehat{u}(\widehat{x}_j)\\ =& - x^{\mathrm{T}}Qx + u^*(\widehat{x}(t))^{\mathrm{T}}u^*(\widehat{x}) - 2u^*(\widehat{x})^{\mathrm{T}}\widehat{u}(\widehat{x}_j).\end{aligned} \tag{40}$$

Recalling Assumption 2.2,

$$\begin{aligned}& u^*(\widehat{x})^{\mathrm{T}}u^*(\widehat{x}) - 2u^*(\widehat{x})^{\mathrm{T}}\widehat{u}(\widehat{x}_j)\\ =& \|u^*(\widehat{x}) - \widehat{u}(\widehat{x}_j)\|^2 - \|\widehat{u}(\widehat{x}_j)\|^2\\ =& \|W_a^{\mathrm{T}}\phi_a(\widehat{x}) + \varepsilon_a(\widehat{x}) - \widehat{W}_a^T\phi_a(\widehat{x}_j)\|^2 - \|\widehat{u}(\widehat{x}_j)\|^2\\ =& \underbrace{\|\widehat{W}_a^{\mathrm{T}}\phi_a(\widehat{x}) - \widehat{W}_a^{\mathrm{T}}\phi_a(\widehat{x}_j) + \varepsilon_a(\widehat{x}) + \widetilde{W}_a^{\mathrm{T}}\phi_a(\widehat{x})\|^2}_{\Phi} - \|\widehat{u}(\widehat{x}_j)\|^2,\end{aligned} \tag{41}$$

where the second equality follows from actor NN. By the Young's inequality $\|a+b\|^2 \le 2\|a\|^2 + 2\|b\|^2$,

$$\begin{aligned}\Phi \le& 2\|\widehat{W}_a^{\mathrm{T}}\phi_a(\widehat{x}) - \widehat{W}_a^{\mathrm{T}}\phi_a(\widehat{x}_j)\|^2 + 2\|\varepsilon_a(\widehat{x}) + \widetilde{W}_a^{\mathrm{T}}\phi_a(\widehat{x})\|^2\\ \le& 2\|\widehat{W}_a^{\mathrm{T}}\phi_a(\widehat{x}) - \widehat{W}_a^{\mathrm{T}}\phi_a(\widehat{x}_j)\|^2 + 4\|\varepsilon_a(\widehat{x})\|^2 + 4\|\widetilde{W}_a^{\mathrm{T}}\phi_a(\widehat{x})\|^2.\end{aligned} \tag{42}$$

Recalling Assumption 4.1, we obtain

$$\begin{aligned}& u^*(\widehat{x})^{\mathrm{T}}u^*(\widehat{x}) - 2u^*(\widehat{x})^{\mathrm{T}}\widehat{u}(\widehat{x}_j)\\ \le& 2\|\widehat{W}_a^{\mathrm{T}}\phi_a(\widehat{x}) - \widehat{W}_a^{\mathrm{T}}\phi_a(\widehat{x}_j)\|^2 + 4\|\varepsilon_a(\widehat{x})\|^2 + 4\|\widetilde{W}_a^{\mathrm{T}}\phi_a(\widehat{x})\|^2 - \|\widehat{u}(\widehat{x}_j)\|^2\\ \le& 2k_a\|\widehat{W}_a\|\|e_j(t)\|^2 + 4\varepsilon_{aM}^2 + 4\|\widetilde{W}_a\|^2\phi_{aM}^2\|\widehat{u}(\widehat{x}_j)\|^2.\end{aligned} \tag{43}$$

Substituting (43) into (40),

$$\begin{aligned}\dot{V}(t) =& \dot{V}_3(t)\\ \le& - \underline{\lambda}Q\|x\|^2 + 2k_a\|\widehat{W}_a\|\|e_j(t)\|^2 + 4\varepsilon_{aM}^2 + 4\|\widetilde{W}_a\|^2\phi_{aM}^2 - \|\widehat{u}(\widehat{x}_j(t))\|^2\\ \le& - (1-\beta^2)\underline{\lambda}Q\|x\|^2 - \beta^2\underline{\lambda}Q\|x\|^2 + 2k_a\|\widehat{W}_a\|\|e_j(t)\|^2\\ & + 4\|\widetilde{W}_a\|^2\phi_{aM}^2 - \|\widehat{u}(\widehat{x}_j(t))\|^2 + 4\varepsilon_{aM}^2.\end{aligned} \tag{44}$$

Substituting (35) into (44),

$$\dot{V}(t) \le - \beta^2\underline{\lambda}Q\|x\|^2 + 4\|\widetilde{W}_a\|^2\phi_{aM}^2. \tag{45}$$

The system is asymptotically stable as long as

$$\|x(t)\|^2 \ge \frac{4\|\widetilde{W}_a\|^2\phi_{aM}^2}{\beta^2\underline{\lambda}(Q)}. \tag{46}$$

Case 2. Event is triggered

In this case, we need to discuss the difference of the four indicators separately

$$\Delta V(t) = \Delta V_1(t) + \Delta V_2(t) + \Delta V_3(t) + \Delta V_4(t). \tag{47}$$

The first term of (47) is

$$\begin{aligned}\Delta V_1(t) =&\frac{1}{\alpha_c}\mathrm{tr}((\widetilde{W}_c^+)^{\mathrm{T}}\widetilde{W}_c^+) - \frac{1}{\alpha_c}\mathrm{tr}(\widetilde{W}_c^{\mathrm{T}}\widetilde{W}_c)\\ =&\frac{1}{\alpha_c}\left(\widetilde{W}_c - \alpha_c\frac{\varsigma}{(1+\varsigma^{\mathrm{T}}\varsigma)^2}(\widetilde{W}_c^{\mathrm{T}}\varsigma + \varepsilon_{hc})\right)^{\mathrm{T}}\\ &\times\left(\widetilde{W}_c - \alpha_c\frac{\varsigma}{(1+\varsigma^{\mathrm{T}}\varsigma)^2}(\widetilde{W}_c^{\mathrm{T}}\varsigma + \varepsilon_{hc})\right) - \frac{1}{\alpha_c}\|\widetilde{W}_c\|^2.\end{aligned} \tag{48}$$

Then,

$$\begin{aligned}\Delta V_1(t) \le& -2\frac{\varsigma\varsigma^{\mathrm{T}}}{(1+\varsigma^{\mathrm{T}}\varsigma)^2}\|\widetilde{W}_c\|^2 - 2\frac{\widetilde{W}_c^{\mathrm{T}}\varsigma\varepsilon_{hcM}}{(1+\varsigma^{\mathrm{T}}\varsigma)^2} + \alpha_c\frac{\varsigma\varsigma^{\mathrm{T}}}{(1+\varsigma^{\mathrm{T}}\varsigma)^2}\|\widetilde{W}_c\|^2\\ &+ 2\alpha_c\frac{\varsigma\varsigma^{\mathrm{T}}\widetilde{W}_c^{\mathrm{T}}\varepsilon_{hcM}}{(1+\varsigma^{\mathrm{T}}\varsigma)^2} + \alpha_c\frac{\varsigma\varsigma^{\mathrm{T}}\varepsilon_{hcM}^2}{(1+\varsigma^{\mathrm{T}}\varsigma)^2}\\ \le& -2\frac{\varsigma\varsigma^{\mathrm{T}}}{(1+\varsigma^{\mathrm{T}}\varsigma)^2}\|\widetilde{W}_c\|^2 + 2\alpha_c\frac{\varsigma\varsigma^{\mathrm{T}}}{(1+\varsigma^{\mathrm{T}}\varsigma)^2}\|\widetilde{W}_c\|^2 + 2\alpha_c\frac{\varsigma\varsigma^{\mathrm{T}}\varepsilon_{hcM}^2}{(1+\varsigma^{\mathrm{T}}\varsigma)^2}\\ \le& -(2\underline{\lambda}(\varsigma\varsigma^{\mathrm{T}}) - 2\alpha_c)\|\widetilde{W}_c\|^2 + 2\alpha_c\varepsilon_{hcM}^2.\end{aligned} \tag{49}$$

We find that $\Delta V_2(t)$ yields

$$\begin{aligned}\Delta V_2(t) =&\frac{1}{\alpha_a}\mathrm{tr}((\widetilde{W}_a^+)^{\mathrm{T}}\widetilde{W}_a^+) - \frac{1}{\alpha_a}\mathrm{tr}(\widetilde{W}_a^{\mathrm{T}}\widetilde{W}_a)\\ =&\frac{1}{\alpha_a}\left(\widetilde{W}_a - \alpha_a\phi_a\big(\widetilde{W}_a^{\mathrm{T}}\phi_a(\widehat{x}_j) + \chi + \frac{1}{2}g(\widehat{x}_j)^{\mathrm{T}}\nabla\phi_c(\widehat{x}_j)^{\mathrm{T}}\widetilde{W}_c\big)^{\mathrm{T}}\right)^{\mathrm{T}}\\ &\times\left(\widetilde{W}_a - \alpha_a\phi_a\left(\widetilde{W}_a^{\mathrm{T}}\phi_a(\widehat{x}_j) + \chi + \frac{1}{2}g(\widehat{x}_j)^{\mathrm{T}}\nabla\phi_c(\widehat{x}_j(t)^{\mathrm{T}}\widetilde{W}_c\big)^{\mathrm{T}}\right)\right) - \frac{1}{\alpha_a}\|\widetilde{W}_a\|^2\\ \le&\underbrace{\alpha_a\left\|\phi_a\left(\widetilde{W}_a^{\mathrm{T}}\phi_a(\widehat{x}_j) + \chi + \frac{1}{2}g(\widehat{x}_j)^{\mathrm{T}}\nabla\phi_c(\widehat{x}_j)^{\mathrm{T}}\widetilde{W}_c\right)\right\|^2}_{\varPsi} - 2\phi_{am}^2\|\widetilde{W}_a\|^2,\end{aligned} \tag{50}$$

where the second equality follows from weight error dynamics of actor NN.

By Assumption 3.1 and the Young's inequality $\|a+b\|^2 \le 2\|a\|^2 + 2\|b\|^2$,

$$\begin{aligned}\varPsi \le& 2\alpha_a\left\|\phi_a\left(\widetilde{W}_a^{\mathrm{T}}\phi_a(\widehat{x}_j) + \chi\|^2 + \frac{1}{2}\alpha_a\|g(\widehat{x}_j)^{\mathrm{T}}\nabla\phi_c(\widehat{x}_j)^{\mathrm{T}}\widetilde{W}_c\right)\right\|^2\\ \le& 4\alpha_a\phi_{aM}^4\|\widetilde{W}_a\|^2 + 4\alpha_a\chi^2 + \frac{1}{2}\alpha_a g_M^2\nabla\phi_{cM}^2\|\widetilde{W}_c\|^2.\end{aligned} \tag{51}$$

Substituting (51) into (50),

$$\Delta V_2(t) \le (-2\phi_{am}^2 + 4\alpha_a\phi_{aM}^4)\|\widetilde{W}_a\|^2 + 4\alpha_a\chi^2 + \frac{1}{2}\alpha_a g_M^2\nabla\phi_{cM}^2\|\widetilde{W}_c\|^2. \tag{52}$$

Combining (48) and (52),

$$\begin{aligned}\Delta V_W(t) =&\Delta V_1(t) + \Delta V_2(t)\\ \leq&(-2\phi_{am}^2 + 4\alpha_a\phi_{aM}^4)\|\widetilde{W}_a\|^2 + 4\alpha_a\chi^2 + \frac{1}{2}\alpha_a g_M^2 \nabla\phi_{cM}^2\|\widetilde{W}_c\|^2\\ &+ 2\alpha_c\varepsilon_{hcM}^2 - (2\underline{\lambda}(\varsigma\varsigma^{\mathrm{T}}) - 2\alpha_c)\|\widetilde{W}_c\|^2\\ \leq& -\left(2\underline{\lambda}(\varsigma\varsigma^{\mathrm{T}}) - 2\alpha_c - \frac{1}{2}\alpha_a g_M^2\nabla\phi_{cM}^2\right)\|\widetilde{W}_c\|^2 - (2\phi_{am}^2 - 4\alpha_a\phi_{aM}^4)\|\widetilde{W}_a\|^2 + \varepsilon, \end{aligned} \tag{53}$$

where $0 < \phi_{am} \leq \|\phi_a\| \leq \phi_{aM}$ and $\varepsilon = 2\alpha_c\varepsilon_{hcM}^2 + 4\alpha_a\chi^2$. Therefore, $\Delta V_w(t) < 0$, as long as

$$\|\widetilde{W}_a\|^2 > \frac{\varepsilon}{2\phi_{am}^2 - 4\alpha_a\phi_{aM}^4} \tag{54}$$

or

$$\|\widetilde{W}_c\|^2 > \frac{\varepsilon}{2\underline{\lambda}(\varsigma\varsigma^T) - 2\alpha_c - \frac{1}{2}\alpha_a g_M^2\nabla\phi_{cM}^2}. \tag{55}$$

This indicates the critic and actor weight estimates errors are UUB at triggering instants.

Then, $\Delta V_J(t)$ is given by

$$\Delta V_J(t) =\Delta V_3(t) + \Delta V_4(t) = V^*(x^+) - V^*(x) + V^*(\widehat{x}_{j+1}) - V^*(\widehat{x}_j). \tag{56}$$

Due to the cost function is continuous and $\dot{V}(t) < 0$, there exists $V^*(x^+) < V^*(\widehat{x})$. Then,

$$V^*(x^+) \leq V^*(x). \tag{57}$$

Because the state $x(t)$ is UUB in Case 1,

$$V^*(\widehat{x}_{j+1}) \leq V^*(\widehat{x}_j). \tag{58}$$

Therefore,

$$\Delta V_J(t) \leq 0. \tag{59}$$

Combining (53) and (59), one has $\Delta V(t) < 0$. Furthermore, we conclude that the system state is asymptotically stable and the NN estimates errors are UUB at trigger instants $t = r_j, j \in N$. The proof is finished. ∎

Remark 4.2 In order to ensure the asymptotic stability of the closed-loop system, Theorem 4.1 proposes the triggering condition based on the dead zone operator and NN. The dead zone operator is designed to avoid unnecessary triggering caused by the reconstruction error of NN and achieve a more adaptive triggering condition.

Remark 4.3 The defined Lyapunov function is composed of four parts. $V_1(t)$ and $V_2(t)$ are related to the actor NN and the critic NN weights. When the event is triggered, we conclude that the weights of the NN error is UUB. $V_3(t)$ and $V_4(t)$ are the optimal value function with respect to the observer state and the sampled state, which is designed need to ensure that observer state and sampled state are UUB.

5 Zeno Behavior Analysis

In this section, we analyze and exclude the Zeno behavior.

Theorem 5.1 *Consider the nonlinear system with the observer* (4). *Let Assumptions* 2.1, 2.2, 3.1, 3.2, *and* 4.1 *hold. If the event-triggering condition given by* (35), *then the Zeno behavior can be avoided.*

Proof Based on Assumption 2.1,

$$\begin{aligned}\|\dot{\widehat{x}}\| &= \|f(\widehat{x}) + g(\widehat{x})\widehat{u}(\widehat{x}_j) + \varphi(y, \widehat{y})\| \\ &\leq k_f\|\widehat{x}\| + g_M\|\widehat{u}(\widehat{x}_j)\| + FC\|\widehat{x} - \widehat{x}_j\| + b_f. \end{aligned} \tag{60}$$

Recalling Assumption 3.2 and (28),

$$\|\widehat{u}(\widehat{x}_j)\| \leq \nabla\phi_{aM}\|\widehat{W}_a\|. \tag{61}$$

Then

$$\|\dot{\widehat{x}}\| \leq k_f\|\widehat{x}\| + g_M\nabla\phi_{aM}\|\widehat{W}_a\| + FC\|\widehat{x} - \widehat{x}_j\| + b_f. \tag{62}$$

Recalling $\widetilde{W}_a$ is UUB, $\widehat{W}_a$ is UUB. Let $\|\widehat{W}_a\| \leq \delta_{\widehat{W}_a}$, we have

$$\|\dot{\widehat{x}}\| \leq k_f\|\widehat{x}\| + g_M\nabla\phi_{aM}\delta_{\widehat{W}_a} + FC\|\widehat{x} - \widehat{x}_j\| + b_f. \tag{63}$$

Since $\dot{\widehat{x}} = -e_j(t)$ in (12), we rewrite (63) as follows

$$\|\dot{e_j}(t)\| \leq (k_f + FC)\|e_j(t)\| + k_f\|\widehat{x}_j\| + g_M\nabla\phi_{aM}\delta_{\widehat{W}_a} + b_f. \tag{64}$$

Defining

$$K = \max\{(k_f + FC), k_f, g_M\nabla\phi_{aM}\delta_{\widehat{W}_a}\} > 0, \tag{65}$$

we conclude

$$\|\dot{e_j}(t)\| \leq K\|e_j(t)\| + K\|\widehat{x}_j\| + K + b_f. \tag{66}$$

Applying the Comparison Lemma (see Lemma 3.4 in [32])

$$\|e_j(t)\| \leq \frac{K\|\widehat{x}_j\| + K + b_f}{K}(\mathrm{e}^{K(t-t_j)} - 1). \tag{67}$$

From Theorem 4.1, we derive the triggering threshold at the next triggering instant t_{j+1}

$$\frac{K\|\widehat{x}_j\| + K + b_f}{K}(\mathrm{e}^{K(t_{j+1}-t_j)} - 1) > \|e_j(t_{j+1})\|. \tag{68}$$

From Assumption 2.1, we know that the solution of the system (1) is nonzero. Thus, we have obtain $\|e_j(t_{j+1})\| > 0$.

By simplifying (68),

$$T_j = t_{j+1} - t_j > \frac{1}{K}\ln(1 + \varpi), \quad j \in \{0, 1, \cdots\}, \tag{69}$$

where $\varpi = \frac{K\|e_j(t_{j+1})\|}{K\|\widehat{x}_j\|+K+b_f}$. Based on Assumption 2.1, $k_f > 0$, $b_f > 0$, and $g_M > 0$, we have $\varpi > 0$. Taking the minimum of ϖ, $\varpi_{\min} = \min\{\varpi\} > 0$, we have

$$\min\{T_j\} > \frac{1}{K}\ln(1+\varpi_{\min}) > 0. \tag{70}$$

Through the above analysis, we prove that the proposed method avoids Zeno behavior. ∎

6 Numerical Example

In this section, in order to illustrate the effectiveness of the proposed method, we consider the following nonlinear system

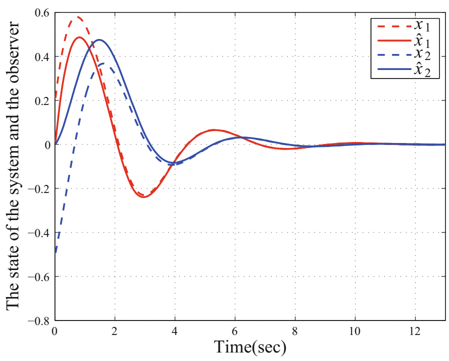
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -x_1^3 - 2x_2^2 \\ x_1 - x_2 + 0.5\cos(x_1^2)\sin(x_2^3) \end{bmatrix} + \begin{bmatrix} 1 \\ \sin x_1 \end{bmatrix} u, \tag{71}$$

$$\varphi(\widehat{y}, y) = F(\widehat{y} - y), \tag{72}$$

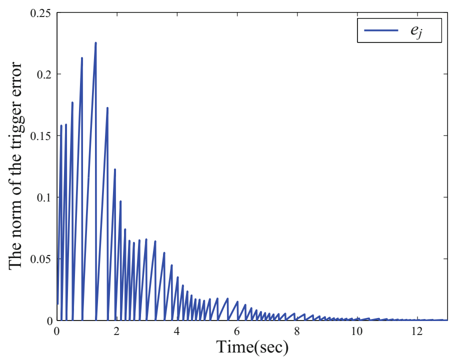
where $y = Cx$, x is the state of the system with $x(0) = [0.2, -0.5]^{\mathrm{T}}$. Initialize $\widehat{W}_c$ and $\widehat{W}_a$ as $\widehat{W}_{c0} = [0.7, 0.73, 0.8]^{\mathrm{T}}$ and $\widehat{W}_{a0} = [0.8, 0.6, 0.7]^{\mathrm{T}}$, respectively. Choose $F = [0.8, 0.6]^{\mathrm{T}}$, $C = [-0.5, -1]$, $k_a = 1$, $\alpha_a = 0.15$, and $\alpha_c = 0.15$.

The simulation results are shown in Figures 1–3. Figure 1(a) presents that the observer state gradually approaches the actual state of the system. Figure 1(b) describes that the event-triggered error $\|e_j\|$ converges to 0 with a small error bound when the threshold is reached. Figure 2 illustrates the convergence of the tuning laws of the actor and critic NN weight estimates. Figure 3 indicates the trigger time interval under ETC critic-actor NN and the critic-actor NN proposed in [26]. According to Figure 3(a), we know that $\min\{T_j\} = 0.12$ s, which excludes the Zeno behavior.

We provide the cumulative numbers of sampling under the proposed ETC critic-actor NN and the conventional TTC method with the critic-actor NN in [26] in Table 1, which illustrates that the proposed ETC can decrease the computational and communication burdens.

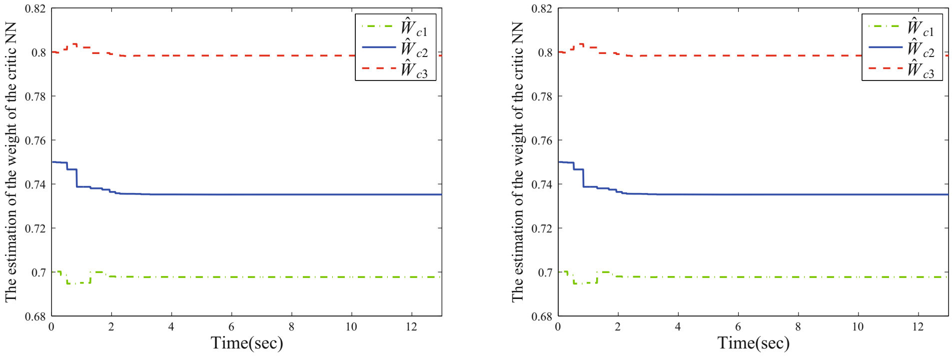


(a) System state and observer state under ETC



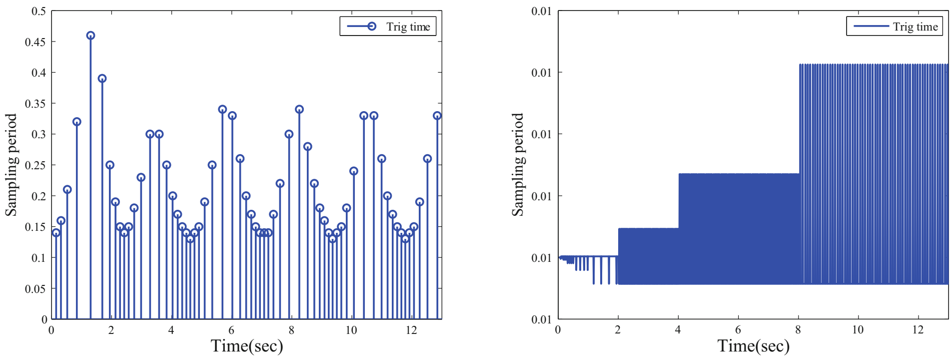
(b) Error norms of ETC

Figure 1 System state, observer state, and error norms under ETC



(a) Weight tuning law of critic NN (b) Weight tuning law of actor NN

Figure 2 Weight tuning laws of critic NN and actor NN



(a) Trigger time interval under ETC critic-actor NN (b) Trigger time interval under the critic-actor NN proposed in [26]

Figure 3 Trigger time interval under ETC critic-actor NN and the critic-actor NN proposed in [26]

Table 1 Cumulative number of sample states of the methods in this paper and [26]

Time (sec)	5	7	9	11	13	15
Proposed ETC	23	32	41	51	61	75
TTC method	500	700	900	1100	1300	1500

7 Conclusions

In this paper, event-triggered optimal control based on state observer and neural network was investigated for nonlinear systems. A critic-actor algorithm was proposed to solve the optimal control problems, where a critic NN was utilized to approximate the optimal cost and an actor NN was developed to approximate the optimal controller. Based on NN weight estimates and dead zone operator, a novel triggering condition was proposed to guarantee the UUB of the closed-loop system and reduce computational and communication costs. It is proved that the closed-loop system is asymptotically stable with NN weight estimation being UUB. Future work will be concentrated on extending receding horizon and infinite horizon.

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