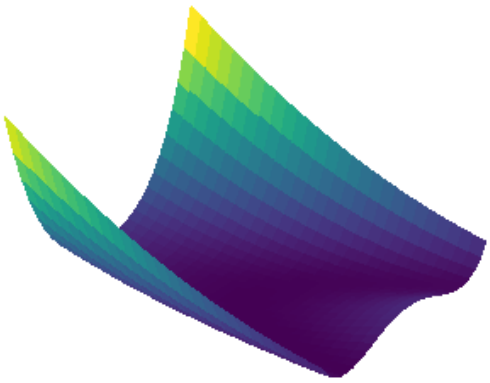
Parallel Particle Swarm Optimisation For Finding Efficient Paths: An Application



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Contents

List of Tables

List of Figures

Declaration

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1 Abstract

This project used a Particle Swarm Optimisation (PSO) approach developed in C programming language to solve a path planning problem and comparing different approaches in parallel. A multiagent swarm was initialised to find the most efficient pathway to a particular minimum or fitness point. This was first tested in serial using benchmark functions with many different local minima and maxima, before being tested using a path planning application that sought to find a destination point for a hypothetical Unmanned Aerial Vehicle (UAV). The algorithm itself was subsequently implemented in parallel using a mix of OpenMP and MPI. Each swarm was divided among multiple processors with the aim of solving the same problems posed by the different benchmark functions and the path planning problem. The results highlighted the benefits of using a parallel approach to this particular algorithm in achieving efficient speedups. All code, datasets, write-ups, source copies and associated materials relating to the project can be found at https://github.com/seancmry/msc_pro.

2 Introduction

Since their development in the 1990s, PSO algorithms have gained credence as an effective solution to large-scale and computationally expensive problems in many different fields. Particle Swarm Optimisation was first described by Kennedy and Eberhart in 1995, who devised an algorithm that relates closely to other types of evolutionary algorithms [9]. Their model was the first to take social behaviour into account and model it based on the principle of "swarming", that is, a population of agents working to devise and actualise a preference for a preferred outcome. The uniqueness of preferences is key to the functioning of this algorithm as they allow the population to change their behaviour based on a number of quality criteria.

Efficient path planning is just one of the many problems that can be addressed through solutions offered by evolutionary algorithms in general and PSO algorithms in particular pathplan. One of the many ways it can be applied in parallel is within the context of a UAV, flying at a given altitude, searching for an optimal route to its destination.

It is precisely this application that this project looks at. It uses C code to implement a particle swarm approach to solve for a series of devised paths in order to find the global optima, by first generating a series of local optima which are then evaluated in accordance with a particular fitness.

The project looks at the efficient implementation of different parallel algorithms using a mix of OpenMP and Message Passing Interface (MPI), specifically evoluationary-derived algorithms, and the potential speedups that could be observed. Many solutions to this problem have been devised, and the project will look at how the use of PSO in parallel contrast with other variations of the algorithm. In particular, the algorithm examined will focus on path selection with the use of waypoints within a defined space. When the space in which vehicles have to navigate is large, and thus the number of waypoints increases and becomes large, algorithm efficiency breaks down, and this is known as the path-planning problem.

In terms of the methodology, the initial implementations that will be examined are standard PSO implementations. These implementations will be examined in parallel through a parallel PSO (PPSO) that will be developed in MPI. With the application of parallel algorithms using MPI architectures, the number of waypoints can be reduced with greater efficiency and the most economic route selected depending on different cost calculations. This computing process can be further parallelised as the number of constraints on navigation increases. These constraints will be tested against controlled variables.

Other algorithms used for the purposes of navigation such as Dijkstra's algorithm can also provide greater efficiencies and speed-ups in light of the quantification of shorter paths to waypoints based on the assumption of

greedy techniques, but this discussion is beyond the scope of this project [17]. For now this project aims to look at the application of PSO in terms of UAV path planning problem i.e. strategies used by UAVs. Aspects of the PSO algorithm itself that will be looked at will include the rate of convergence, the growth of elitism among the generated 'particles', and a look at hybridization techniques which will also form a part of an investigation into possible future research.

Section 1 of this report details how each individual particle is subject to different swarm rules that will dictate the possible approaches each can take. Section 2 details the specifics of the code itself. Section 3 looks at how the testing regime was established and conducted. Section 4 details the results of the investigation and looks at the direction of possible future research in this area.

2.1 Motivation and research questions

In terms of the practical applications of PSO algorithms, and due to the fact that they take much of their dynamic inspiration from natural processes such as flocking or swarming among different animal populations such as within schools of fish, they are numerous. There have been a few prominent examples within the literature of the principles found within processes such as these as they apply to the use of Terrain Relative Navigation (TRN) as demonstrated in different settings, including by the NASA Perseverance Rover to avoid obstacles during its decent and landing on the Martian surface [4]. Likewise, the example used for this project utilises the concept of UAV navigation to avoid obstacles whilst navigating at a given altitude. This prompts the following research questions to be answered over the course of this investigation:

1. Can the parallelisation of the PSO algorithm used to find a given path produce sizeable and meaningful speedups?

2. Can the PSO algorithm be improved to produce more accurate estimates of fitness?

3. Do these fitness estimates converge quicker with parallelisation?

Theory

3 Particle Swarm Optimization

3.1 General specification

For a typical PSO implementation, certain important requirements must be present in the algorithm:

- A search strategy.
- Rules of "swarming" (e.g. move 10 km per day).
- Topological set-up.
- End point that must be reached.
- Other variables i.e. random components which can vary travel distance, denoted by r_n in this study.

In order to calculate the search strategy you will need a number of other important components including personal and team best solutions. The search strategy can be specified in the following manner:

$$\underbrace{\overrightarrow{V_i^{d+1}}}_{\text{Next velocity (tomorrow)}} = \underbrace{w}_{\text{inertia weight}} \underbrace{\overrightarrow{V_i}}_{\text{Current velocity (today)}}$$

$$\ldots + c_1 r_1 \left(\overbrace{\underbrace{\overrightarrow{P_i^d}}_{\text{Personal best solution}} - \overrightarrow{X_i^d}}^{\text{distance to personal best}} \right) + c_2 r_2 \left(\overbrace{\underbrace{\overrightarrow{G^d}}_{\text{Global best solution}} - \overrightarrow{X_i^d}}^{\text{distance to global best}} \right)$$

In addition to the velocity vector, you will also have a position vector which is given as

$$\underbrace{X_i^{d+1}}_{\text{Position in day d+1}} = \underbrace{X_i^d}_{\text{Position in day d}} + \underbrace{V_i^{d+1}}_{\text{Velocity in day d+1}}$$

This variant of PSO is commonly referred to as, quite simply, standard PSO, and is given by the following equations for the updates of particle velocity and position. Respectively, X_i and V_i are the position and the velocity of a particular particle, i. After a certain number of iterations, X will denote the best possible position found by the particle, i. The rest of the swarm, the additional particles generated, will arrive at the coordinate of the best aggregate position and the superscript d is the counter of flights (iterations) of the algorithm. $c1$ and $c2$ are the cognitive and social parameters and w is the inertia weight, while $r1$ and $r2$ are two numbers drawn randomly within a given range.

3.1.1 Pseudo-code

Algorithm 1: Pseudocode of PSO algorithm

```
Initialize controlling parameters( N, c1, c2, Wmin, Wmax, Vmax,
and max iterations);
Initialize the population of N particles;
while end of condition is not satisfied do
    for each particle do
        calculate the objective of the particle;
        update Pbest if required;
        update Gbest if required;
    end
    update the inertia weight;
    for each particle do
        update velocity (v);
        update position (x);
    end
end
return Pbest as the best estimate of the global optimum;
```

In the algorithm implemented, the specification for velocity and position are interlinked as shown. The default values of the user-defined cognition and social parameters c1 and c2 are 1.5 (see code).

3.1.2 Inertia weighting

The inertia weight, ω, governs the rate of convergence. For the algorithm developed here, it is updated in a decreasing manner according to

$$w = w_0 + (w_f - w_0)\frac{N_v}{N_{max} + N_v}$$

This is the same as in Moraes et al. (The general specification and some of the mathematical information come from Moraes et al [13], including the stochastic properties of the algorithm. Refer to Omkar, Venkatesh, and Mudigere for more information [15].) The code provides for a significant degree of user-directed functionality, and so the initial and final weighting parameters can be specified according to the desired specifications.

In order for the solution to converge to a local optima, all the particles in the swarm must reach to the global optima. However, due to the large computational cost involved, we cannot afford to wait until all particles reach it and instead specify a degree of fitness to be achieved. The basic idea behind the stopping criterion is the definition of a suitable average position of the whole swarm. Due to the possibility of the existence of two or more global optima, that is, optima with the same value for the

best value of the optimal solution but at different x positions, the swarm position must include a separate parameter to denote this coordinate, f. This is given as

$$\tilde{f} = \frac{f_i - min_j(f_j^0)}{max_j(f_j^0) - min_j(f_j^0)}$$

This specification normalises the minimum and maximum values of the objective function among all particles in the swarm, including all of the particles in the initial population of particles.

3.2 Potential drawbacks

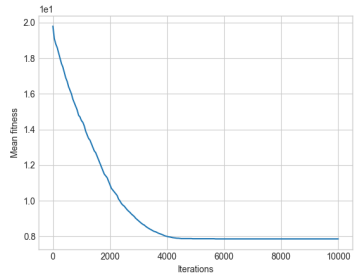
The primary issues with PSO algorithms are (1) getting trapped in local optima, and (2) performance deterioration with increased problem size. One of the objectives of this project is to address the secondary problem head on, but to also highlight some of the issues brought on by the former problem by examining the performance variances as the problem size is increased to a scale that can only be efficiently computed in parallel. As with many different problems that necessitate the use of multi-core optimization, the problem dimension will inevitably affect the performance of the algorithm at larger and larger sizes [1]. A typical implementation would see the execution time increase in a certain proportion to the problem size, N^2. The testing done by Altinoz et al. demonstrated the solution quality-execution time trade-off when it came to how the performance was enhanced using a different PSO variation.

The problem is commonly referred to as premature convergence i.e. some points attaining fitness quicker than others. This arises from too strong a selective procedure towards a best solution (e.g. by recombining them, or mutating them only slightly in the case of genetic algorithms).

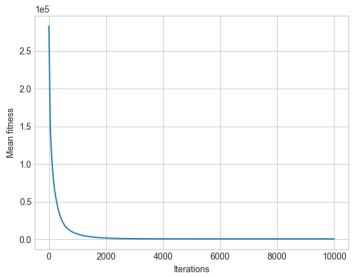
3.3 Evolutionary context

In terms of the topological make-up of the algorithm's application, if this were a master-slave model, the master node would maintain a population and applies genetic operators to the individuals of the population. The master node would then distribute the individuals of a population to slave nodes for fitness evaluations. Since this does away with the genetic approach to evolutionary algorithms, generating a bunch of candidate solutions and check them against an objective function is not required. Then generate subsequent points from the initial generation and see how they evolve.

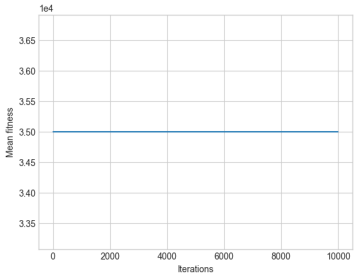
3.4 Premature convergence



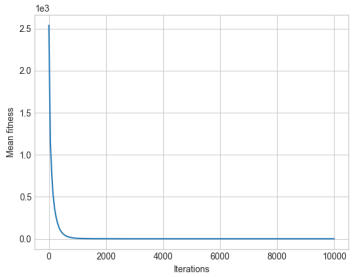
(a) Ackley



(b) Sphere



(c) Rosenbrock



(d) Griewank

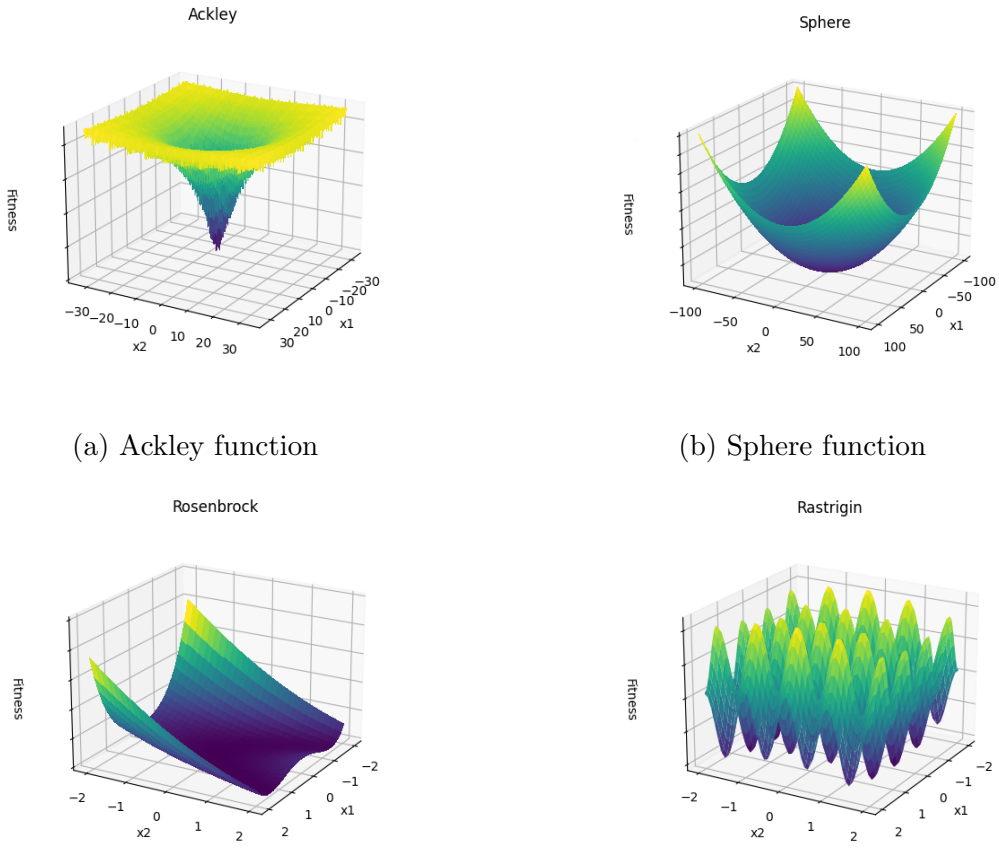
Figure 1: Convergence graphs for each function at $N = 100$ dimensions.

Problems of elitism and premature convergence still need to be discussed, though it will have to be looked at to see if they apply more so in the case of genetic algorithms. If these are not problems that can be picked up on in testing then pick something else to discuss - change the title in that case. The figure above depicts the rate of convergence for the different functions tested. It should be noted that individuals with a higher fitness level have a greater probability of reproducing in the case of genetic algorithms [8]. It might also be worth looking into what algorithms work with multiple populations in a future study.

3.5 Objective functions

The other functions aside from the main function are benchmark functions, in which the PSO is employed as the preferred search strategy for optimization - Ackley, Sphere, Rosenbrock, and Griewank. Note that the figure below depicts the Rastrigin function in place of the Griewank function because the latter is difficult to depict in 3d without developing a program for the purpose of doing so. That being said, the output at the fine grain level is identical for a small search range, and the Rastrigin function is used to test many PSO applications for this reason. These functions

are used to test the performance of the algorithm [20]. The `main.c` file itself contains the basic functions of the algorithm. The parsing of arguments via the command line allows for the use of the benchmark functions, with preassigned values for each contained within the arguments parsed. This could be altered so that these values can be seeded in a randomised fashion.



(a) Ackley function

(b) Sphere function

(c) Rosenbrock function

(d) Rastrigin function

Figure 2: 3d fine grain view for each of the functions and their specifications.

3.6 Function specifications

- **Ackley:**

$$f(x_1 \cdots x_n) = -20exp\left(-0.2\sqrt{\left(\frac{1}{n}\sum_{i=1}^{n} x_i^2\right)}\right)$$

$$-exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos 2\pi x_i\right) + 20 + e$$

- **Sphere:**

$$f(x_1 \cdots x_n) = \sum_{i=1}^{n} {x_i}^2$$

- **Rosenbrock:**

$$f(x_1 \cdots x_n) = \sum_{i=1}^{n}\left[100\left(x_{i+1} - x_i^2\right)^2 + (x_i - 1)^2\right]$$

- **Griewank:**

$$f(x_1 \cdots x_n) = 1 + \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} cos(\frac{x_i}{\sqrt{i}})$$

4 Literature Review

MPI is the most popular method in use across a wide variety of applications that have sought to incorporate parallel methods [11]. There is a wealth of information that currently exists on the types of implementations that can be conducted in order to improve the performance of PSO algorithms and techniques. Will have to search through the articles again and compare several things, including the parallelisation strategies employed and problem definitions.

There is already a multitudinous amount of research and tests conducted using different variations of the PSO algorithm, each applied in different contexts such as robotics, geology, and nuclear physics to name but a few applications [11] [12].

4.1 MPI

The use of MPI along with GPU-based solutions has also been considered in some instances, such as the comparison drawn with greedy modular eigenspaces, a method used in the selection of hyperspectral images [3]. Similarly, coarse-grained asynchronous solutions have also been looked at with regard to improving the time taken between analysis within the points looked at [23].

Nedjah et al. compare the implementation of PSO between MPI, OpenMP, and CUDA architectures for multi-core and many-core optimisation [14]. A predictable speedup was achieved, though the implementations were primitive in terms of their design, akin to a typical master-slave implementation.

Compared to most implementations, a hybrid design is not as common a feature as one many think [18]. Sengupta, Basak and Peters note that whilst some of these implementations have taken hold in recent years, crossovers such as a joint GA-PSO implementation many not offer as much in terms of simplicity; nonetheless, the literature in this area is growing and more research is likely needed.

In terms of setting up the geography of the problem, this work takes inspiration from the parameters outlined in Roberge, Tarbouchi, Labonté which compares a genetic algorithm and PSO algorithm in MATLAB – it does not explicitly use those parameters.

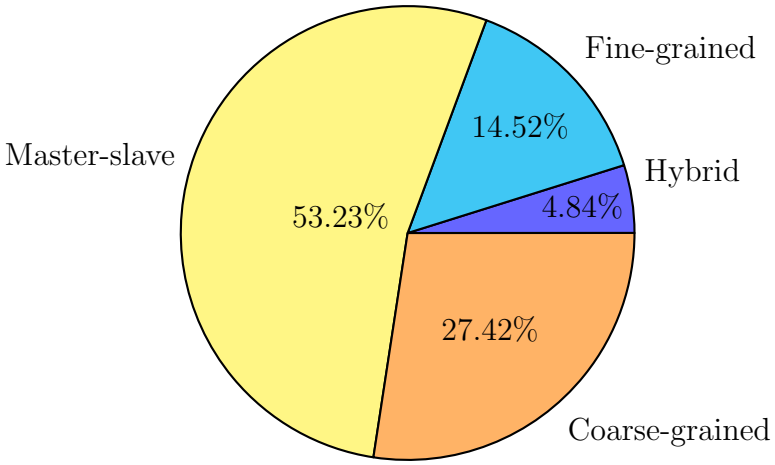


Figure 3: Communication model-based publication analysis on Parallel PSO [11]

Code Design

5 Methodology

5.1 Demo overview

This version demonstrates the efficiency of the PSO algorithm in the context of the 4 different benchmark functions. The sphere function is the default function. The program is adapted to iterate through each function for a given number of steps at a given ω - inertia weight. The original version of this program stops the algorithm once a desired error is reached, and the legacy code from this is given as a comment in the `pso.c` file.

The program will look at how long it takes to get from each step to the next with each inertia weight.

5.2 Code

Figure 4: Swarm initialisation

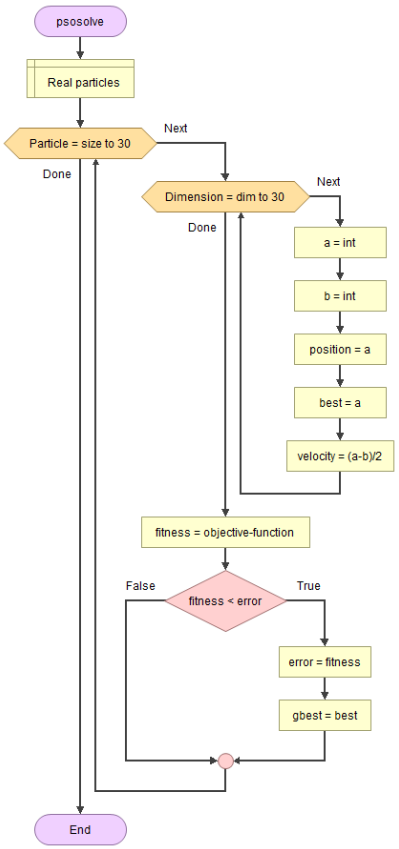
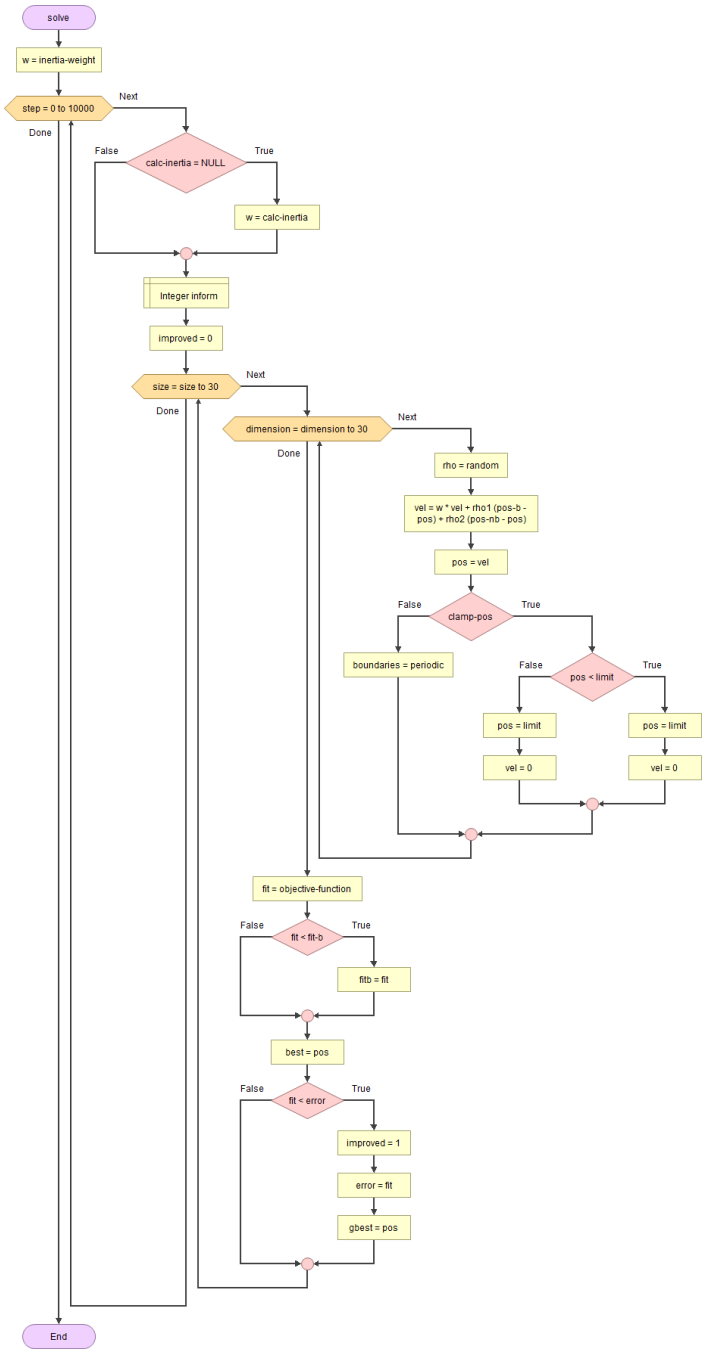


Figure 5: PSO algorithm



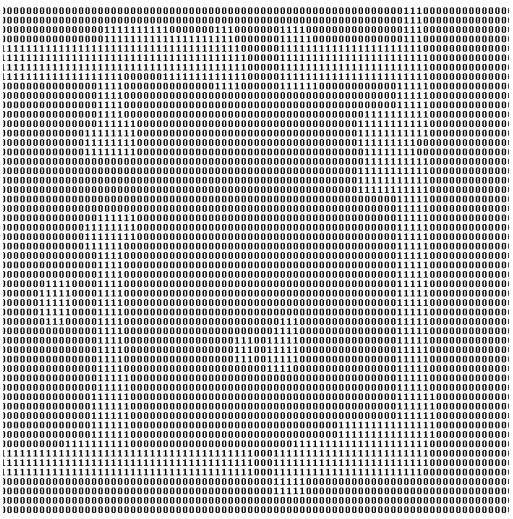
5.3 Inform Functions

The general inform function and associated functions are a bit confusing at first appearances, but they are an integral aspect to the standard algorithm. These are connect to the algorithm with the help of a `struct` of

settings. Declared in a general manner using `void (*inform_fun)()`, it connects with the other inform functions depending on the strategy used; either `inform_global`, `inform_ring`, or `inform_random`. For the purposes of testing and simplicity, the ring function will be set as the default. Bridging the gap between the different disciplines of robotics and computational analysis proves a challenge with terms such as path planning and efficient placement. In computational analysis, these terms do not exist – they are instead defined through abstract variations.

5.4 Application

Figure 6: Cropped image of ASCII map illustrating obstacles



The primary difference between the code with the path addition to the algorithm and the basic algorithm set-up is in the use of the settings parameter, which is stored as a singular struct in the basic implementation, but branches out to include parse-friendly values via command arguments in the implementation with the path included. Bridging the gap and seeing which one would be worthy of this is key to the set-up. Outputting the map generated and the different random generation of obstacles and so on is important for the sake of the analysis.

The main function for the path implementation is included in the `path.c` file itself. This application is selected in `main.c` through the use of boolean functionality.

6 Parallelism

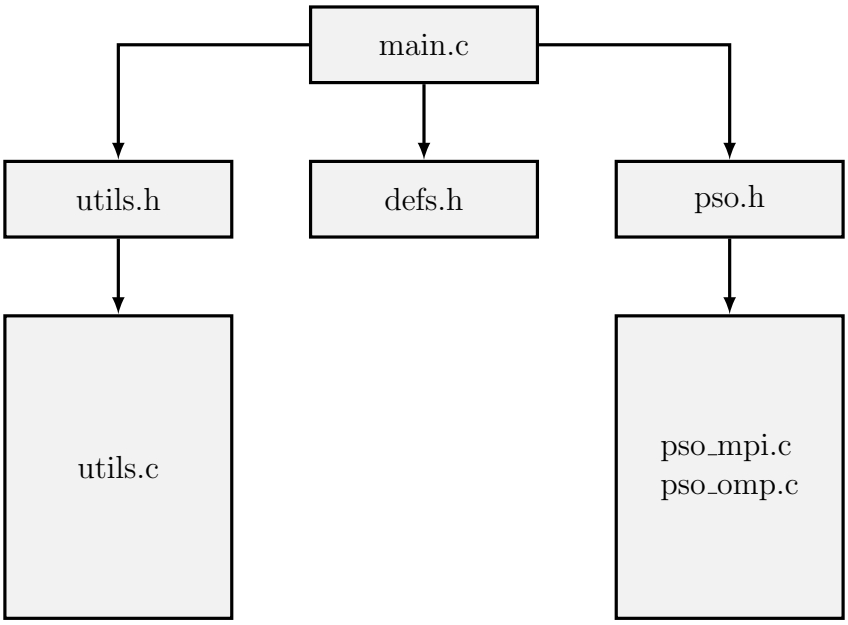


Figure 7: Tree diagram of parallel code for demo implementation (`path.c/.h` files excluded.)

For the sake of simplicity, the parallelisation of the code will primarily consist of the introduction of MPI and OpenMP components to the PSO algorithm only. The parallelisation of the path planning and associated functions is the beyond the scope of this project.

6.1 Hardware

6.1.1 Lonsdale and Kelvin systems

The tests for the both the demo and serial analysis were run on the Trinity Centre for High Performance Computing (TCHPC computer cluster systems 'Lonsdale' and 'Kelvin'. Following upgrades in March 2021 Lonsdale was taken out of service for a prolonged period, and so all testing and analysis on parallel code was conducted on Kelvin. The figures for each system's hardware can be viewed in Appendix B.

6.2 Code

As well as the timing structure and various other instrumentation aspects, a number of other factors were changed in the context of the solving code itself. Chief among these changes was the addition of MPI code and the two primary functions that were selected, namely `MPI_Gather` and `MPI_Bcast`

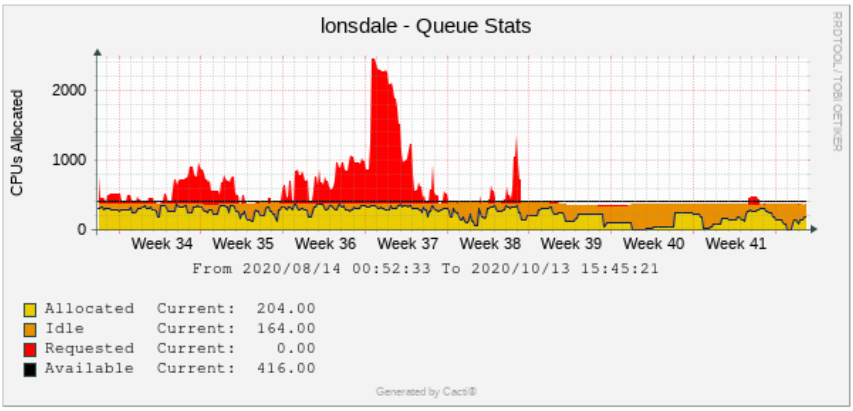


Figure 8: The graph shows the number of Requested, Allocated and Idle CPUs in the Lonsdale cluster over a monthly basis.

6.3 MPI functions

MPI_Gather: MPI_Gather can be defined in the context of the opposing function, namely MPI_Scatter. This routine takes data from different processes and gathers them to one single process, typically a root process. A program that requires a high degree of sorting and searching can benefit from this communicator.

MPI_Bcast: MPI_Bcast broadcasts a message from the root process to all other processes. If you look at the composition of the MPI_Bcast function, you'll note that there is a specification for the root process in the function definition, which means that it is possible to specify whomever you wish to do a send. All processes call the broadcast, including the receives, which don't post a receive.

6.4 OpenMP

The second means of parallelism involved multithreading the programme and using the OpenMP library of functions. Outline steps taken to decide on hybrid model. Give reasons for the hybrid programming approach as well as advantages of it.

For the OpenMP implementations: discuss the type of schedules (static, dynamic, guided, etc.) that have been implemented. Explain why you went with the particular implementation you went with, and how else you could have sectioned up the code with OMP.

OpenMP can generally be used where particles can be updated independently (see [24]). The methodology employed here to update particles in the implementation implies that there is no communication between particles except the step of updating the best model for this particular

implementation.

- `private`: This directive gives a private variable copy to each thread, which is then dispensed with at the end of the parallel region.
- `reduction`: This clause effects a reduction, which is to say that each thread will reduce a local variable where the operator is given an identity.
- `num_threads` and `shared`: In combination with the `shared` directive, the `num_threads` clause allows us to specify the number of threads that should execute a block of non-local variable(s).

6.5 Additional code requirements

6.5.1 `pso_calc_swarm_size`

For the purposes of testing a large enough swarm across multiples processes, modifications were made to the sizing variable that altered how the swarm size is calculated. `settings->size` is assigned a value in `main.c` but it is overwritten by the `pso_calc_swarm_size` function in the settings. The primary function of this function is to generate a size that fits with the maximum possible size for the swarm giving the value for `settings->dim`. The integer for sizing was adjusted for the number of processes and is multiplied by N to give

```
1
2
3 int size = (100. 20. * sqrt(dim))* nproc;
```

Listing 1: Calculation for swarm size

6.5.2 Removal of `inform` function and `pos_nb`?

The primary change that took place during the simple MPI and MPI-OpenMP hybrid runs was the removal of the matrix that informs particles at the local level. This was done for the scalability and for allowing the processes themselves to act as the containers for the particles and how they are informed in future. During this testing phase, the informer functions were not needed and were to be replaced by the Cartesian topology and communicator so that communication between particles can take place. This endeavour was subsequently abandoned due to time contraints.

6.5.3 Velocity update

As part of the removal of the `pos_nb` matrix that informed particles locally, an update was required for the calculation of velocity during the particle

update loop, which consisted of replacing the next best matrix value with the `solution->gbest` value, which then meant that velocity was calculated as

```
1
2 vel[i][d] = w * vel[i][d] +   \
3       rho1 * (pos_b[i][d] - pos[i][d]) +    \
4       rho2 * (solution->gbest[i] - pos[i][d]);
```

Listing 2: New velocity update calculation

Given that the iteration only took place once, the solution result would be arrived at upon the termination of the loop, so this update would not have taken place and was merely for the purpose of executing a clean coded example.

6.6 Modules and evaluation criteria

The table below details the different modules loaded in the cluster system in order to perform the runs.*[0]

Table 1: Loaded modules for testing

Module	Version
gcc	9.3.0
gsl	2.5
openmpi	3.1.6

[0]The updated version of `gsl` that was used required an additional `CFLAGS` and `LDFLAGS` linkage in order to work. These can be viewed in the Makefile in the Github repository.

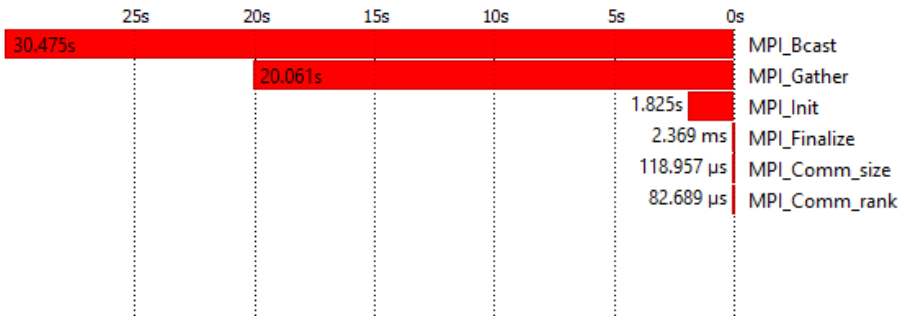
6.7 Preliminary results

6.7.1 Computational imbalances

The preliminary results of the parallel version were indicative of computational imbalances at the higher number of processes for the hybrid implementation. What this suggests is that the benefits of this implementation will produce diminishing marginal returns in terms of speedup achieved at the higher number of processes. In this case, the solution would be to change size of work packages for each process to even out work load across threads, and so the different OpenMP functions will require some work. In terms of the serial optimisation and what should be done next, the solution is clear: it is now necessary to examine a time-sensitive application and see of code modifications can be made accordingly.

6.7.2 Parallel profiling

Figure 9: Function Summary.



As evidenced in the figure developed with Vampir (see next section), the MPI broadcast function is the primary function for the development of the algorithm in parallel. Particles are generated and split between processes, with each process then developing its own global best fitness, which is gathered back to rank 0 once this is found, and another iteration occurs.

6.7.3 L2 Cache count

Some evidence of thrashing was observed with the pure MPI test case, but less so in the case of the hybrid version.

Implementation

7 Testing

7.1 Demo analysis

Table 2: Parameter settings

Parameter	ω_{min}	ω_{max}	$c_1 = c_2$	steps	`clamp_pos`	`nhood_size`
Value	0.7	0.3	1.496	100,000	periodic	5

The present value for ω_{min} is set as `PSO_INERTIA` or 0.7. The strategy implemented is `PSO_W_LIN_DEC`, which decreases the inertia weight. The settings control the degree of descent. `nhood_size` denotes the number of informers for each particle, which in this case is 5.

The demo was primarily analysed to examine the degree of premature convergence present in the algorithm itself. The tests were informed by the work on previous examinations of these specific benchmark functions [2]. The program was executed for the purpose of testing using the `test.sh` bash file. Ten runs of each algorithm were performed for data collection. The `output.dat` files obtained were converted to `*.csv` for analysis.

7.1.1 Search ranges

Table 3: Search ranges for each function

Function	Range
Ackley	$-32.8, 32.8$
Sphere	$-100, 100$
Rosenbrock	$-2.048, 2.048$
Griewank	$-600, 600$

The Ackley function has a global minimum of $f = 0$ where $x = (0, 0, ..., 0)$ though it has many minor local minima.

7.1.2 Analysis

The first element will be to check the error achieved against the number of iterations of the algorithm required to achieve the desired margin of error. The second element of this analysis is to check the desired fitness level against the different levels actually achieved across different dimensions, `dim`. The mean fitness is the figure to look for. Note, mean time is quoted out of 25 runs. The standard deviation is the standard deviation between the mean values for each run, as explained below.

- **Best fitness:** This is the best fitness solution achieved by each benchmark function as represented by the minimum error level across all runs.
- **Mean fitness:** This is the average figure for the fitness achieved over all runs by each benchmark function.
- **Poorest fitness:** The least best fitness level achieved across all runs by each benchmark function.
- **Standard deviation:** As mentioned, this is the standard deviation between the mean levels of fitness achieved in each individual run, aggregated for all runs.
- **Mean time:** The average time taken for each run by each benchmark function.

7.2 Settings

Contained within the settings struct are general PSO settings including the precision goal, the swarm size, the neighbourhood strategy, the neighbourhood size, and the weighting strategy – which assigns the weighting. You then perform the running of the algorithms and free memory.

7.3 Note on objective Functions

The demo code for the objective functions now runs up to the maximum number of specified iterations, without the need to stop at the desired error level. The tests for the for the individual functions were conducted on the basis of the original code, so the results are only presented for 25 runs of each function (during testing).

7.4 Establishing tests

- In `pso.h`, the maximum size a swarm can be is defined along with the default value of the weighting, ω, and determines the balance between global and local search. This is updated in accordance with the boundaries set for the minimum and maximum weights. These are linearly decreasing.
- Matrix strategies: these update the global best, universally the most desirable number for each particle to gravitate towards, and copies it to the next best position.

- There is a general helper function beneath this which performs a similar task, but recurrently updates and informs the different particles as to the location of the global best.
- The different communication topologies are outlined next and include the ring topology, global topology and random topology. Each topology function also contains an informer function in order to inform each as to the directionality of the particles in the swarm.
- Next is the creation of the pso settings, which take the values and definitions given in the .h file. Might be good to list these and explain each in the context of the project report.
- Finally, the algorithm and the different case switches are created. This is the longest function; it is quite self-contained and requires some editing.

7.5 Bugs

Frequently recurring issues included instances of the following:

- Declaring `extern int` variables and their incorrect usage, particularly for `utils.c/.h`. Other definitions need to be looked
- Implicit declarations.
- `struct` pointers and their incorrect initializations.
- Request for members in something that aren't `structs` or `unions`.
- Invalid argument types.

7.6 Modules

Modules loaded included `module load cports` and `module load gsl/2.2.1-gnu` for the random number generation when testing on Lonsdale.

7.7 Outputs

Table 4: Results for benchmark functions with dimensions $N = 100$, size = 30.

Function	Ackley	Sphere	Rosenbrock	Griewank
Best fitness	**6.0542E+00**	**4.2000E+02**	**2.9383E+04**	**1.1134E+00**
Mean fitness	**9.4993E+00**	6.0538E+03	3.5019E+04	**3.2981E+01**
Poorest fitness	**1.9894E+01**	**3.0353E+05**	**4.5821E+04**	**2.7054E+03**
Standard deviation	**4.1190E-01**	4.3564E+02	3.6994E+03	**1.3156E+00**
Mean time (ms)	2908.00	1819.60	2831.20	3194.00

Table 5: Results for benchmark functions with dimensions $N = 500$, size = 54.

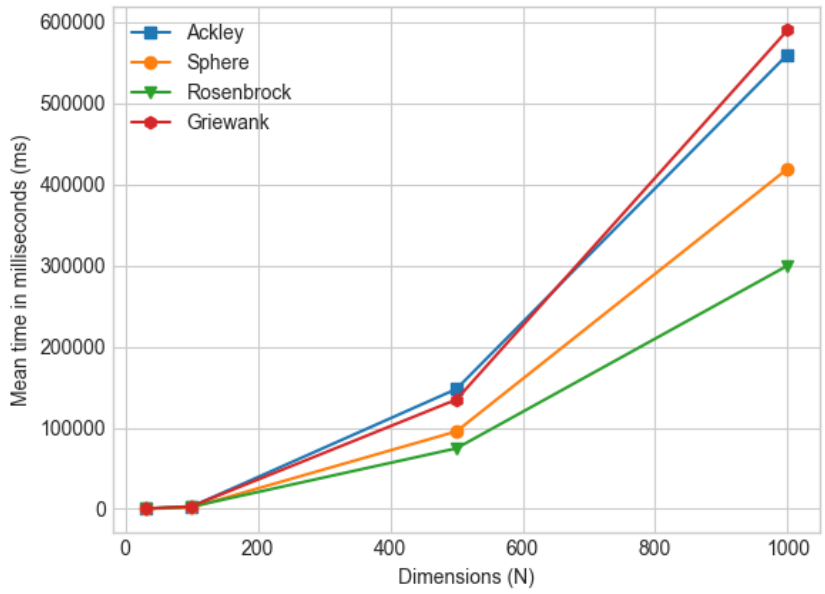
Function	Ackley	Sphere	Rosenbrock	Griewank
Best fitness	**1.4433E+01**	5.7015E+04	1.9829E+05	**4.8773E+01**
Mean fitness	**1.6645E+01**	1.3874E+05	2.1792E+05	**3.0480E+02**
Poorest fitness	**1.9972E+01**	1.5935E+06	2.2998E+05	1.4501E+04
Standard deviation	**6.1797E-01**	2.7717E+04	6.9748E+03	**7.1154E+01**
Mean time (ms)	147818.00	95558.40	74689.60	134804.80

Table 6: Results for benchmark functions with dimensions $N = 1000$, size = 73.

Function	Ackley	Sphere	Rosenbrock	Griewank
Best fitness	**1.7250E+01**	3.2513E+05	4.3102E+05	**7.3685E+02**
Mean fitness	**1.8593E+01**	5.7175E+05	4.5025E+05	1.6362E+03
Poorest fitness	**1.9993E+01**	3.2470E+06	4.6860E+05	2.9311E+04
Standard deviation	**3.2277E-01**	1.1247E+05	1.0289E+04	**3.2838E+02**
Mean time (ms)	560022.80	419196.80	299853.20	590098.00

7.7.1 Timing graphs

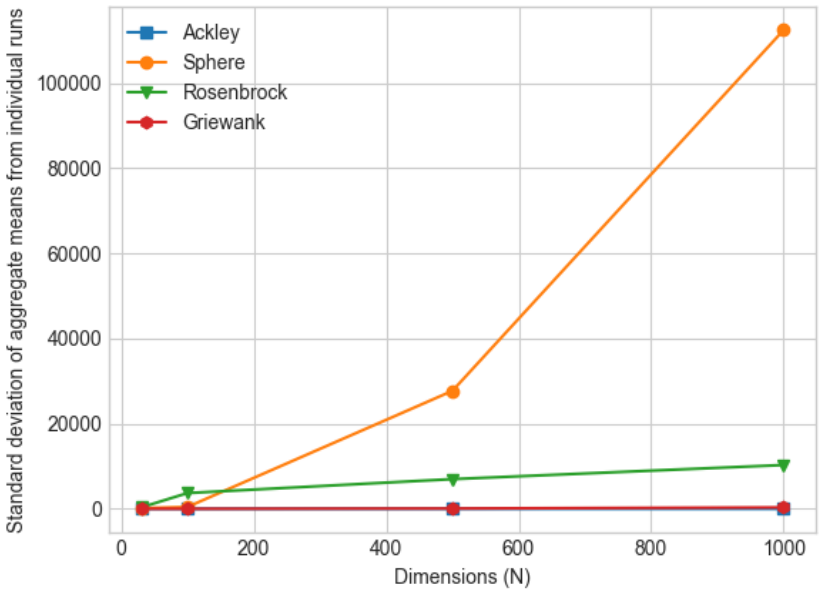
Figure 10: Average time per run for each function.



The above graph highlights how long the different functions takes to complete 100000 iterations. The rate of decay increases quite a bit in 500 dimensions and then accelerates even further.

7.8 Discussion of observations

Figure 11: Standard deviations of the means up to $N = 1000$.



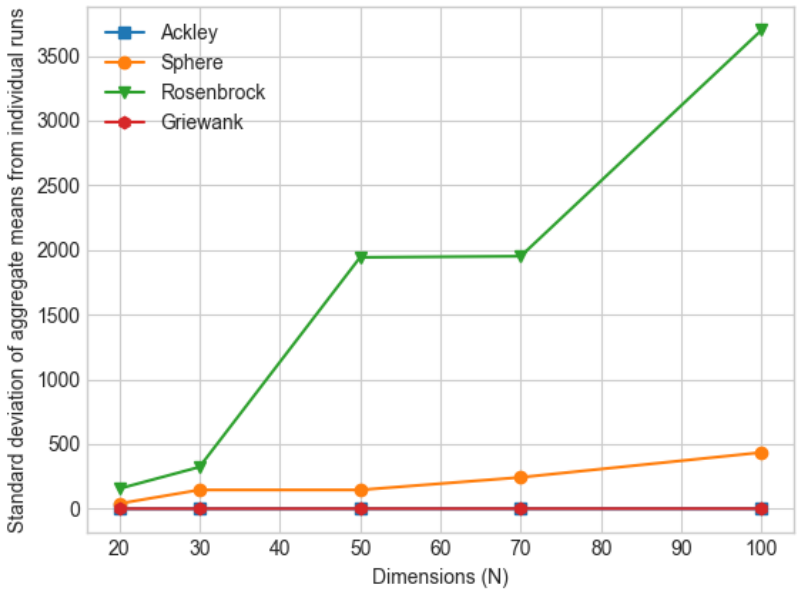
7.8.1 Ackley

In terms of timing, Ackley is fine with 20 and 30 dimensions, but then begins to decay, coming second only to Griewank. That being said, the mean fitness levels achieved are consistently the best out of the benchmark set and even the poorer fitness levels cannot compare with even the better Rosenbrock or Sphere ones in the higher dimensions. The standard deviation between the mean best fitness levels achieved per run (see Figure 1) are also comparatively the lowest.

7.8.2 Sphere

The Sphere function is dependable in lower dimensions and doesn't decay too much in terms of time. This function does however achieve the poorest fitness levels once N is greater than 30. Whilst the means achieved do not deviate as much as those achieved by the Rosenbrock function, it is nonetheless prone to break in terms of deviations earlier.

Figure 12: Standard deviations of the means illustrate the break in consistency with the Rosenbrock benchmark.



7.8.3 Rosenbrock

Rosenbrock is not well behaved in the higher dimensions in spite of its better performance measures at the lower dimensions. A quick median fitness analysis for Rosenbrock at $N = 30$ is 0.77712, so it converges very quickly in lower dimensions. Time decay takes hold when $N = 70$, and so it does not achieve the correct fitness levels afterwards.

7.8.4 Griewank

Griewank decayed the most in terms of timing. Less deviation between the average time of each run for each function due to higher computational load and the sort mechanism in memory is the case here, since the function, whilst able to achieve a consistent fitness level, can also encounter much poorer fitness levels when in the process of evaluating.

Remark. *The next step will be to run these same tests in parallel to see what the performance improvement is by adjusting the number of processors. With the path planning problem, the serial evaluation will look at how increasing the number of particles affects performance, and thus with the parallel evaluation of that application, the same assessment will take place, but with the spread of N particles across multiple processors.*

7.9 Increment and problem dimensionality

```
1 //Calculate swarm size based on dimensionality
2 int pso_calc_swarm_size(int dim) {
3   int size = 10. + 2. * sqrt(dim);
4   return (size > PSO_MAX_SIZE ? PSO_MAX_SIZE : size);
5 }
```

Listing 3: Function for calculating appropriate swarm size

All of the demo runs were performed with swarm sizes that were less than 100, with the highest, $N = 1000$ containing a swarm size of 73. An automatic calculation of the swarm size was performed by the function shown above, which was used in each case. The classical thinking on the population size is that the best performance typically occurs in populations with smaller sizes, hence the restricted allowance for incrementation with the problem dimensions. However, it has been argued recently that the smaller sizes selected to evaluated different optimisation problems may indeed be too small [16].

The authors of this paper have argued that the best results are typically obtained in the 70-500 size range. This is not a problem for the results obtained above as the range of sizes is incremented sufficiently, and evaluation of the impact of higher dimensions versus the increased population size is beyond the scope of this analysis. Nonetheless, it is evident that the impact of higher dimensions likely outweighs any influence a population size beyond 100 would have, as can be seen by the breakdown in performance of the Rosenbrock benchmark.

7.10 Debugging

Debugging can be complicated with the use of multiple `printf` statements, so for the purposes of this project `gdb` was used.

7.10.1 Gdb

`gdb` is a debugger that can be easily added to a program that is compiled with the `gcc` command. It is added to the `Makefile` in the following manner:

```
gcc myprog.c -o myprog -g
```

The `-g` aspect was removed in the instance where runs were used for the accurate measurement of times, as leaving the command in the code would slow down the program. Some faults that were uncovered in the final stages of the coding were common segmentation faults and double free or corruption faults, which were dealt with by adding `set environment MALLOC_CHECK_ 2` and `MALLOC_CHECK_ 1` to the debugger, to make invalid `free` commands visible in the backtrace.

7.11 Profiling

Profile was conducted using a mixture of Vampir, Score-P, and `gprof`, the latter of which was used for both the demo code and serial application. An example of how the profiling was run is given as

```
gprof -b ./prog ackley > analysis.txt
```

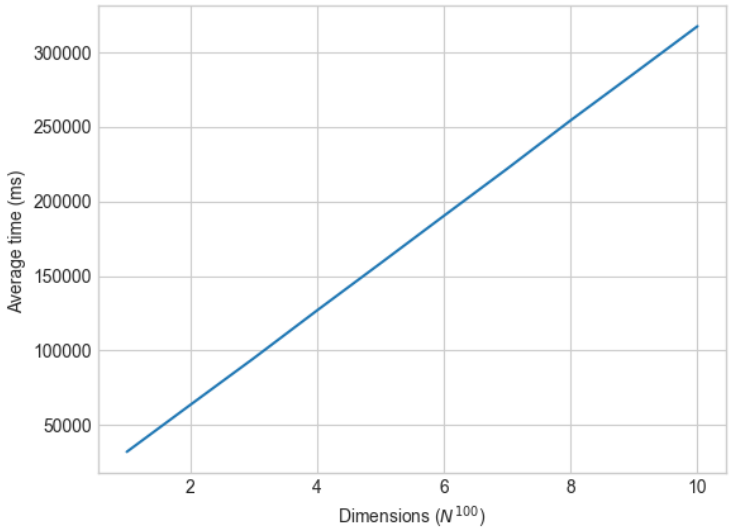
```
Flat profile:

Each sample counts as 0.01 seconds.
  %   cumulative   self              self     total
 time   seconds   seconds    calls   s/call   s/call  name
 80.75      0.92     0.92        1     0.92     1.14  pso_solve
 12.29      1.06     0.14   200040     0.00     0.00  pso_ackley
  7.02      1.14     0.08    10001     0.00     0.00  inform
  0.00      1.14     0.00    10001     0.00     0.00  calc_inertia_lin_dec
  0.00      1.14     0.00    10001     0.00     0.00  inform_ring
  0.00      1.14     0.00        4     0.00     0.00  pso_matrix_new
  0.00      1.14     0.00        1     0.00     0.00  elapsed_time
  0.00      1.14     0.00        1     0.00     0.00  end_timer
  0.00      1.14     0.00        1     0.00     0.00  init_comm_ring
  0.00      1.14     0.00        1     0.00     0.00  parse_arguments
  0.00      1.14     0.00        1     0.00     0.00  print_elapsed_time
  0.00      1.14     0.00        1     0.00     0.00  pso_calc_swarm_size
  0.00      1.14     0.00        1     0.00     1.14  pso_demo
  0.00      1.14     0.00        1     0.00     0.00  pso_settings_free
  0.00      1.14     0.00        1     0.00     0.00  pso_settings_new
  0.00      1.14     0.00        1     0.00     0.00  start_timer
```

As evidenced by the `gprof` profiling of the demo code, the key sections that the parallel implementation will seek to improve will include the `pso_solve` algorithm elements, the topological functions and related inform functions (`inform_ring`, `inform_random`, and `inform_global`), and the elements that deal with the allocation of matrices.

As well as these functions, the evidence from the profiling suggests that the calculation of the inertia weight should be sped up in light of the call count on that function (`calc_inertia_lin_dec`).

7.12 Serial path analysis

Figure 13: Timing across runs.

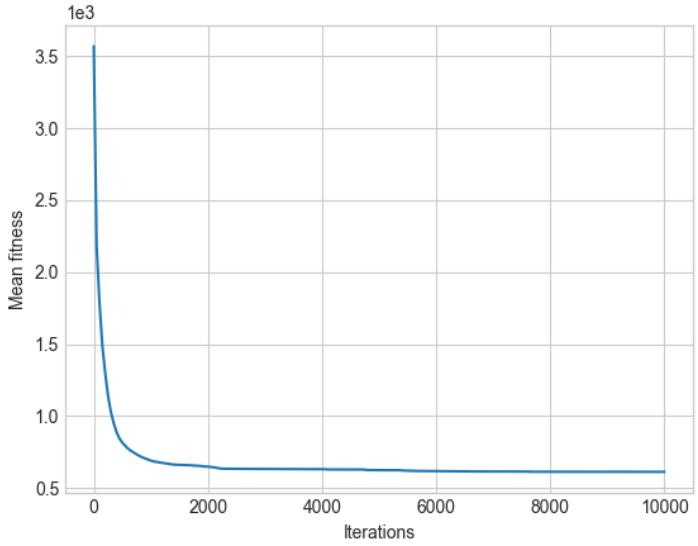


7.13 Timing

The timing for the serial path application was measured across different sizes of N, and can be seen to be more linear than some of the benchmarking functions. This would suggest that a similar outcomes is likely when the parallel version is tested.

7.14 Convergence

Figure 14: Convergence graph.



As shown in the graph, the rate of convergence is stable across many iterations, in line with the highly linear nature of the timing across different dimensions.

7.14.1 Number of obstacles per run

This merits a discussion as the informed observer would think that a random generation of obstacles would give different results no matter how many dimensions each run was conducted in. This is not the case as tests were carried out with a map containing pre-existing obstacles and there were no measurable effects on timing found.

7.14.2 Testing

Testing was conducted on the provided map `sample_map_Doorways.txt` to check for the influence on obstacles on the timing of the initial runs and it was found that they had no real measurable effect. The same can be said for the effect on the error. This is likely due to the low number of obstacles included with the map; this number would have to be increased if there are to be any noticeable effects, but this is beyond the scope of this report.

Table 7: Results across different N^{100}.

N	1	2	3	4	5	6	7	8	9	10
Obstacles (avg)	0	19	99.6	264.8	548	804.4	1284	2058	2034.8	2812.4
Solution distance (avg)	611.78	1854.10	3628.08	6699.95	9435.24	12243.68	15541.26	19564.87	21802.40	25707.28
Avg error	6.6852E+02	2.0838E+03	3.9828E+03	7.1107E+03	9.8900E+03	1.2818E+04	1.6163E+04	2.0074E+04	2.2412E+04	2.6416E+04
Best	4.3074E+02	1.3755E+03	2.1111E+03	5.0095E+03	7.2606E+03	8.1969E+03	1.2458E+04	1.6955E+04	1.9115E+04	2.1504E+04
Worst	4.2178E+03	8.866E+03	1.4031E+04	1.9283E+04	2.3186E+04	2.9204E+04	3.3699E+04	3.8305E+04	4.3142E+04	4.7813E+04
Standard deviation	1.1201E+02	3.3027E+02	6.8822E+02	8.2608E+02	1.0447E+03	1.2427E+03	1.5363E+03	1.1984E+03	1.4494E+03	1.5479E+03
Time (average,ms)	31956	63414.8	94690	126811.2	158425.2	190279.6	221928	254361.2	285816.4	317517.2

7.15 Parallel Implementation

7.15.1 Pseudocode

Algorithm 2: Parallel PSO algorithm pseudocode

```
for each step do
    Communicate particle data for boundary cells;
    update current step;
    update inertia weight if required;
    //Check optimization goal;
    update pos_nb matrix;
    reset improved value;
    //Update all;
    for each particle do
        find the location (i,j) within cell the current particle is in;
        for each dimension do
            calculate ρ1 and ρ2;
            update velocity;
            for each cell i, j do
                update position;
            end
        end
    end
    for each particle do
        for each dimension do
            update particle fitness;
            update personal best position;
            copy pos i to pos_b i (migration step)
        end
    end
    //update gbest?;
    for each particle do
        for each dimension do
            copy to gbest if required;
        end
    end
end
```

7.16 Instrumentation

7.16.1 Timing

There are a couple of changes that needed to be made to the algorithmic setup prior to testing. A new timer was implemented using the `sys/time.h` header library, and placed within the solving function itself so that an accurate performance for the MPI process could be gauged.

7.16.2 Score-P

Score-P is a joint performance measurement run-time infrastructure set-up that can be used in conjunction with other profilers such as Vampir (see next subsection). It is an instrumentation tool that contains several run-time libraries and helper tools, and uses common output formats (`OTF2, CUBE4`). It is useful for the generation of call-path profiles and event traces, for using direct instrument and sampling, flexible measurement without need for re-compilation, recording time, visits, communication data, and hardware counters. In order to use it, simply prefix all compile/link commands with `scorep` and affix the advanced instrumentation (detailed in the next subsection).

7.16.3 Profiling

Vampir version 9.9 was selected for the purposes of profiling the code, with Score-P enabled as the primary code instrumentation and run-time measurement framework for Vampir. This has a wide use base including instrumentation at both source level and at compile/link time.

Results

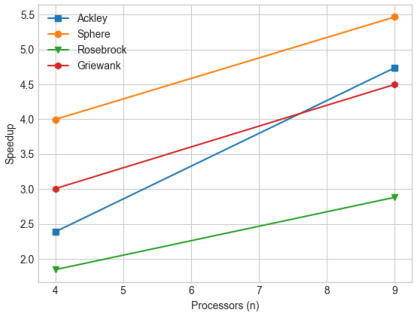
8 Discussion of Results and Future Research

8.1 Scaled improvement

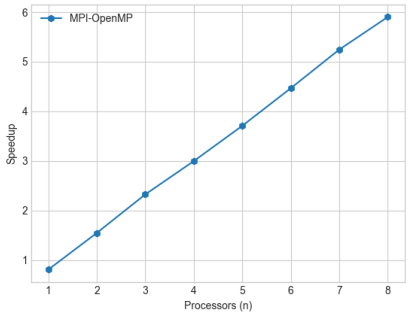
For the purposes of calculating the speedup for each of the different functions and later on for the application, we use the standard formula for calculating improvements in latency scaled for the number of nodes known as Gustafson's law:

$$Speedup \leq p + (1 - p)s$$

As with the related Amdahl's Law, p is the number of processes and s is the serial execution time. This being true, a programme that runs on 4 cores should perform a speedup factor of 4x of the same programme running on 1 core and so on. The numerical and graphical analysis point to this law being true for only one of the functions, namely the Sphere function, for the purely MPI version of the code for 4 processes. Though superscaling is always possible - where the speedup exceeds the number of processors in use - this does not appear to have occurred here. Speedup graphs show the improvements for 1000 dimensions, the largest size tested, for 4 and 9 processes respectively.



(a) Speedup for each benchmark function.



(b) Speedup for the path application

Figure 15: Speedups measured across 25 runs for each number of processes

8.1.1 Function speedups

In terms of the benchmark functions, the Ackley function once again demonstrates its ability to scale well with an increased number of processes. A speedup of just over 4.5, though not perfect, was only slightly less than the sphere function, which produced a speedup of around 5.5x for 9 processes. The worst performing function was once again the Rosenbrock function,

which achieved a speedup of just under 3.0 at 9 processes. The problem with this function lies in its multimodality, which is to say that the computation of the global optima is difficult due to the sparse nature of the local optima. The Ackley function could therefore be considered the easier problem given the numerous local optima in the one basin.

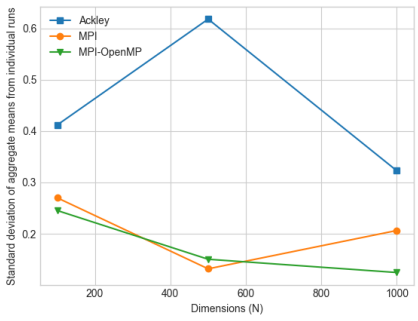
8.1.2 Path speedups

The highest speedup achieved for the path application was just under 6.0 at 8 processes. This is better than the best performance achieved by the sphere function, which achieved the highest speedup. Given the simple nature of the path laid out, this is not surprising, and this is benefited by the fact that the obstacles, along with the particles, were dispersed across different processes and scaled. What cannot be explained is why, at 1 process, the performance of the algorithm actually decreased relative to what the serial version had produced for the same dimensionality. This will require further investigation.

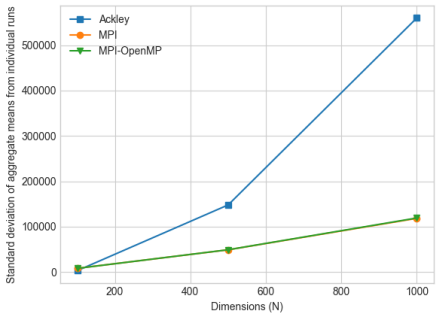
Table 8: Average time per run for each number of processes (path timings)

Processes (N)	1	2	3	4	5	6	7	8
Time (ms)	393.4014	205.3003	136.6032	105.9651	85.6234	71.0863	60.5233	53.7862

8.2 Observations



(a) Standard deviation of the average fitness achieved across 25 runs for the Ackley function.



(b) Timing for both serial and parallel functions (Ackley).

Figure 16: Speedups measured across 25 runs for each number of processes

The converged fitness is typically achieved quicker with the hybrid version. This is to be expected given the quicker performance achieved in the lower dimensions and the efficient multithreading architecture at this performance level. However, the standard deviation is variable when it comes to the parallel function applications, and this is evident if we look at the Griewank and Ackley functions in particular. The standard deviation of the means is worse in the Griewank function and better with the Ackley function evaluated in parallel. This is due to the more numerous minima present in the Griewank function, which leads to less efficient speedups.

8.3 Research questions

8.3.1 Speedups

In terms of whether an efficient speedup was produced as posed at the beginning of this report, the speedup achieved by the path application in parallel could be considered efficient in light of how it compares to Amdahl's law, though imperfect. The speedups achieved by each of the different benchmark functions, though less efficient than those achieved by the path application, were less satisfactory and would not be considered efficient in context (see above figure).

8.3.2 Improvements to the algorithm

The algorithm itself demonstrated no sizeable improvements in terms of the standard deviation of means, the best fitness achieved, the worst fitness achieved, and the average fitness level when examined in parallel. This analysis was not conducted for the path application based on the results achieved for the benchmark functions.

8.3.3 Convergence

Likewise, as questioned at the beginning of the report, there was no demonstrable quickening in the rate of convergence for the parallel application or benchmark functions in parallel.

8.4 Future directions

8.4.1 Program design

The composition of testing and the large nature of the program that was tested meant that a lot of functionality was available to be tested, but much of this was not relevant to the study at hand. It would be best to eliminate redundant functionality and to simplify the requirements of the program so that better quality testing could be carried out. This would allow for analysis to be conducted on the part of the program instead of manually assessing raw data produced.

8.4.2 Cartesian topology

The use of 4 and 9 processes to test the benchmarking functions was deliberate in that it was hoped that development could be done on a virtual topology setup for the algorithm. The problem with this was that the decomposition of a 2D matrix couldn't be done with the setup developed. In the serial version, the rows inform the columns, with the calculation done within the columns, and this would necessitate two collective operations: a broadcast inside the columns and a reduction in the rows. To do this in MPI, a communicator group would have to be developed with a virtual topology. This is easier to do if the particles were stored within a matrix in 2 dimensions, but this was never conducted and time constraints along with the challenges of developing a topology to match meant that this endeavour was ultimately abandoned.

8.4.3 CUDA

Compute Unified Device Architecture (CUDA) is another popular way of implementing a parallel program, utilising the GPU power of a machine. As mentioned previously in the literature review, MPI is still the most effective way of incoroporating parallelism into PSO applications on multiple cores [11]. That being said, CUDA implementations are equally as useful where GPU processing power and the limitations of CPUs are considered, particularly in the case of PSO [25]. The research by Zou et al. demonstrates a unique approach to the problem of developing intelligent optimization algorithms (IOA) with the use of a OpenMP and CUDA utilization with GA-PSO hybrid implementations. Their results demonstrated particularly usefulness for industrial applications, and a sample of the PSO code can be seen below.

Appendix A

Acronyms

ASCII American Standard Code for Information Interchange

CPU Central Processing Unit

CUDA Compute Unified Device Architecture

GA Genetic Algorithm

GDB GNU Debugger

GPU Graphical Processing Unit

PPSO Parallel PSO

IOA Intelligent Optimization Algorithms

MPI Message Processing Interface

OMP OpenMP

PSO Particle Swarm Optimisation

TRN Terrain Relative Navigation

UAV Unmanned Aerial Vehicle

Citations

Citations were generated using: https://www.citationmachine.net/apa/cite-a-website.

Code

The implementations used for this project are available at: https://github.com/seancmry/msc_pro. If for any reason the code is not available at the URL, email: murras11@tcd.ie.

Flowcharts

Flowcharts were created using Flowgorithm: http://www.flowgorithm.org/index.htm. These files were first saved as `.fprg` files and are available in the Github repository as `.png` files.

Descriptive statistics

All descriptive statistics were filtered and calculated through Microsoft Excel. The datasets used contains any formulae used in these calculations and are available in the Github repository.

Version Control

A Git repository was established on the TCHPC system. For the purposes of running version control on Lonsdale and Kelvin, a remote set-up was established in the manner outlined below.

SSH keys

- An SSH key was first created using `ssh-keygen -t rsa`
- A folder called `ids` with the new keys were generated in the `.ssh` folder with an executable permission added using `chmod +x ~/.ssh/`, followed by `chmod 400 ~/.ssh/*`, adjusting permissions of the files of the public keys and private keys, which should only be readable by the user and not anything by any other user.
- Finally, the key was copied to Lonsdale using `ssh-copy-id -i ~/.ssh/ids murras11@lonsdale.tchpc.tcd.ie`.
- The key was then tested using `ssh -i ~/.ssh/ids.pub murras11@lonsdale.tchpc.tcd.ie`.

We then have set up a server which can be used to push/pull to and from Lonsdale. Using `git clone` then allowed for the existing repository, which is available on both my own machine and at https://github.com/seancmry/msc_pro, to be visible on Lonsdale.

Appendix B

	Lonsdale	Kelvin
Vendor:	ClusterVision	Dell
Available to:	TCD Researchers	Irish researchers
Processor Type:	Opteron	Intel
Architecture:	64-bit	64-bit
Number of Nodes:	154	100
RAM per node:	16	24
RAM:	3.2TB (128x16GB, 24x32GB, 2x64GB)	2.4TB
Clock Speed:	2.30GHz	2.66GHz
Interconnect:	Infiniband DDR	Qlogic Infiniband QDR
Theoretical Peak Performance:	11.33TF	12.76TF
Total number of cores:	1232	1200
Number of sockets per Node:	2	2
Number of Cores per Socket:	4	6
Linpack Score:	8.9TF	10.8TF

Table 9: TCHPC Cluster System Specifications

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