CSCI 432 - Project Deliverable 3

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Written by Timothy A. Chan the Optimal Output-Sensitive Convex Hull Algorithm in two and three dimension is meant to help optimize the process of finding a convex hull. A convex hull is finding the smallest enclosing polygon for a set of points on a Euclidean plane that encompasses all points in the set within the polygon as well as using some of the points to define the outline. This works for Euclidean planes or Euclidean spaces.

This specific algorithm created by Chan is an improvement upon previous convex hull algorithms due to its nature of being output-sensitive. This means that the runtime depends on the output of the algorithm as well as the input. In this case the output of the algorithm is the number of vertexes in the polygon. Chan's algorithm helps to solve the problem of runtime being completely dependent on the input and saying that the size of the output or the number of vertexes in the convex has an impact on the how fast the convex can be calculated. A lot of other convex hull algorithm rely solely on the number of points in the set to determine the runtime. Chan managed to do this by elaborating and improving previous convex hull algorithms. For two dimensional Jarvis's March and Graham's Scan and for three dimensional the gift-wrapping method.

Pseudo Code

Algorithm 1 Hull2D

```
Input: $P$, $m$, $H$, where $P \subset E^2$, $3 \le m \le n$, and $H \ge 1$
Output: either $\textit{incomplete}$ or list of points

partition $P$ into subsets $P_1, \ldots, P_{\lceil n/m \rceil}$ each of size at most $m$
for $i, \ldots \lceil n/m \rceil$ do
    compute conv($P_i$) by Graham's scan and store its vertices in an array in ccw order
end for
$p_0 \leftarrow (0, -\infty)$
$p_1 \leftarrow$ the rightmost point of $P$
for $k = 1, \ldots, H$ do
    for $i = 1, \ldots, \lceil n/m \rceil$ do
        compute the point $q_i \in P_i$ that maximizes $\angle p_{k-1} p_k q_i$ ($q_i \neq p_k$) by performing a binary search on the vertices of conv($P_i$)
    end for
    $p_{k+1} \leftarrow$ the point $q$ from $\{q_1, \ldots, q_{\lceil n/m \rceil}\}$ that maximizes $\angle p_{k-1} p_k q$
    if $p_{k+1} = p_1$ then
        return the list $\langle p_1, \ldots, p_k \rangle$
    end if
end for
return $\textit{incomplete}$
```

Algorithm 2 Hull2D

```
Input: $P$, where $P \subset E^2$
Output: $L$, list of vertices in the hull
for $t = 1, 2, \ldots$ do
    $L \leftarrow$ Hull2D$(P, m, H)$. where $m = H = min\{2^{2^t}, n\}$
    if $L \neq \textit{incomplete}$ then
        return $L$
    end if
end for
```