nature human behaviour

MATTERS ARISING

https://doi.org/10.1038/s41562-020-00942-4

Check for updates

Reply to: Divisive normalization does influence decisions with multiple alternatives

Sebastian Gluth ✉, Nadja Kern and Cécile L. Vitali

REPLYING TO R. Webb et al. *Nature Human Behaviour* https://doi.org/s41562-020-00941-5 (2020)

The divisive normalization (DN) model assumes that a canonical neural computation exerts a measurable influence on value-based decisions between three or more alternatives. More specifically, it predicts that the relative preference for the best option compared with the second-best option decreases as the sum of all subjective values increases. This constitutes a violation of the axiom of independence of irrelevant alternatives (IIA). Although the study by Louie et al.[1] seemed to provide support for the DN model, our high-powered direct replication of this study together with a separate eye-tracking experiment showed that the proposed violations were neither replicable nor robust[2]. Webb et al.[3] have now reanalysed our direct replication data and claim to find evidence in favour of a more recent version of the DN model when comparing it with standard economic models that are tied to the IIA axiom. Here, we reproduce and extend these analyses. We can confirm the improvement of the model fit of the new DN model. More importantly, however, we show that the fit can be further improved in many more participants by allowing the model to make predictions that go against the rationale of DN. Our results raise general doubts about how behavioural implications of neural computations can be tested convincingly and underscore the need for process models of decision-making.

The recently published new DN model[4] simplifies the normalization equation, so that it differs from standard economic choice models only with respect to one free parameter (that is, ω). This parameter is allowed to be positive in the DN model but is fixed to 0 in the standard economic models. It governs the dependency of choice probability on the sum of the subjective values of all options: a positive ω implies that higher sums make decisions more random. We appreciate this simplification of the original DN model[1], which was overly complex and thus difficult to estimate. When reanalysing our data, Webb et al.[3] combined the new DN equation with different error distributions and compared these new DN variants to their corresponding standard economic models. They found that the increased complexity induced by the parameter ω was justified by the better fits of the DN variants and that ω was significantly positive.

In the present reanalysis, we focused on the error distribution that provided the best account of the data according to Webb et al.[3] (that is, the logit model). We first sought to reproduce the results of Webb et al.[3], using estimation methods that allowed us to estimate parameters on the individual level[5,6]. This approach provided a much better account of the data than pooling the decisions over all participants (compare our results reported in Table 1 with those reported in Table 1 of Webb et al.[3]). We can confirm that the DN model offers a modest yet significant improvement of fit compared to the logit model (Table 1). We can also confirm that the individual parameter estimates of ω are significantly positive ($t(102)=3.30$; $P=0.001$; effect size Cohen's $d=0.33$; 95% confidence interval (CI), [0.016, 0.063]). When looking at the distribution of the individual parameter estimates of ω, however, it is easy to see that the vast majority of participants have estimates that are very close to 0 (the blue bars in Fig. 1a and Extended Data Fig. 1). In line with this observation, there are only 10 out of 103 participants for whom the DN model provided a significantly better account than the logit model (the blue bars in Fig. 1b). Besides showing that the better fit of the DN model is driven by only a handful of participants, these findings further indicate that the decisions of many participants may be better described by assuming a negative rather than positive value of ω (that is, the opposite of what DN assumes).

To explore the possibility that the sum of all values could also influence choice probability in the opposite direction of what DN predicts, we re-estimated the DN model but allowed ω to take on negative values. Remarkably, we found that this extension further improves the fit of the model so that it clearly outperforms the other two models (Table 1). Moreover, ω is estimated to be negative for the majority of participants (the red bars in Fig. 1a and Extended Data Fig. 1), and there are now 36 participants with a significantly improved model fit compared to the logit model (the red bars in Fig. 1b). On average, ω is negative but does not differ significantly from 0 ($t(102)=-1.76$; $P=0.081$; $d=-0.17$; 95% CI, [−0.102, 0.006]). Notably, the same analyses performed on our second dataset from the eye-tracking experiment yielded equivalent results (Extended Data Figs. 2 and 3). To summarize, our reanalysis confirms the notion of Webb et al.[3] that significant IIA violations are present in our data, but it also suggests that most of these violations are incompatible with DN.

Conclusions

The logic of DN leads to the (one-sided) prediction that higher sums of subjective values should make it more difficult to choose the best option. However, we found this effect to be significant in only very few of our participants (13 out of 140 across both experiments), and we found the opposite effect in many more participants (36 out of 140). Given that DN is a canonical neural computation[7], it is implausible to assume that the brains of these 36 participants exhibit an 'inverted' form of DN. Instead, our results raise general doubts about the rationale of drawing direct links between basic neural computations and overt choice behaviour. We would argue that there are more proximal explanations conceivable. To give just one example, choice accuracy could be driven by confidence in subjective values[8,9]. More specifically, there may be some participants who have more stable value representations of lower-value options (that is, they are very confident about what they do not like). These participants may find it easy to discard low-value options and may thus behave in accordance with the DN model. Other

Department of Psychology, University of Basel, Basel, Switzerland. ✉e-mail: sebastian.gluth@unibas.ch

Table 1 | Model comparison

		Multinomial logit ($\omega = 0$)	DN ($\omega \geq 0$)	Model with ω allowed to be <0
MLE	σ	0.627 / 0.455 (1.100)	0.321 / 0.268 (0.337)	0.996 / 0.490 (1.895)
	ω	0	0.039 / 1.292×10^{-9} (0.121)	−0.048 / −0.022 (0.276)
	LL	−15,615	−15,547	−15,375
	P value on χ^2		0.019	<0.001
	Test for $\omega > 0$		0.001	0.959
	AIC	31,436	31,506	31,161
HBM	σ	0.544 / 0.462 (0.387)	0.417 / 0.416 (0.182)	0.823 / 0.480 (0.975)
	ω	0	0.020 / 0.004 (0.061)	−0.033 / −0.008 (0.119)
	DIC	31,427	31,377	31,068

For the parameters σ and ω, we report the following values: mean / median (standard deviation); note the large difference between the mean and the median of ω in the DN model. MLE, maximum likelihood estimation; HBM, hierarchical Bayesian modelling; LL, log-likelihood; AIC, Akaike's information criterion; DIC, deviance information criterion.

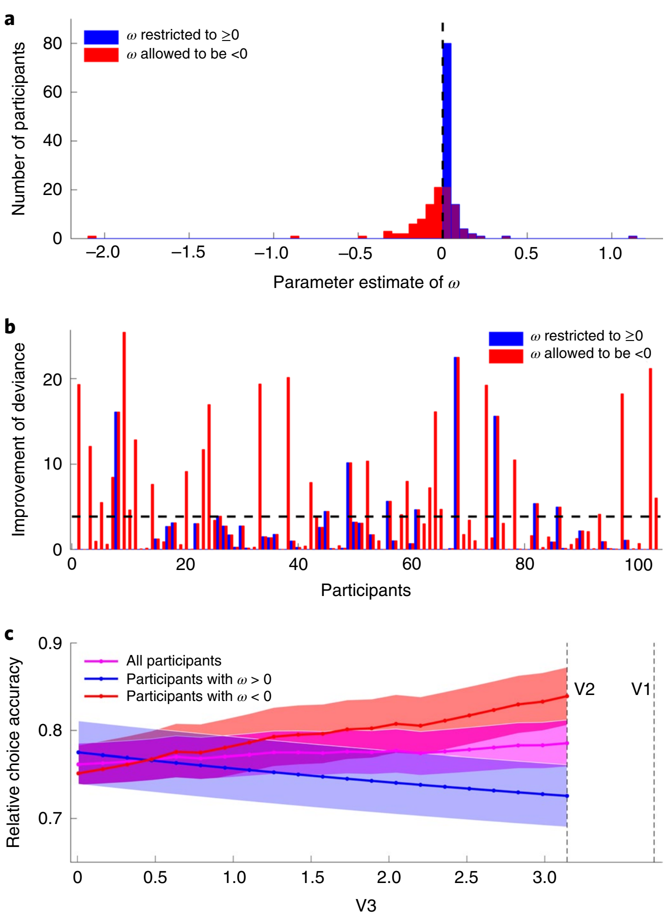


Fig. 1 | Parameter estimates, model fit and predictions. a, Parameter estimates for ω (based on maximum likelihood estimation) when this parameter is forced to be ≥0 (as assumed by DN; blue) or not (red). **b**, Improvement of individual model fits compared to the logit model; the dashed black line indicates the threshold for a significant individual improvement. **c**, Predictions of the dependency of relative choice accuracy (that is, the probability of choosing the better out of the two target options) on the value of the distractor (V3); the shaded areas represent the 95% CI around the mean.

participants, however, may have more stable value representations of higher-value options (that is, they are very confident about what they like) and may thus exhibit choice patterns that are incompatible with DN. Consistent with this idea, we find a small but significantly positive correlation between ω and the relationship between the height and variability of subjective value ratings ($r(101) = 0.196$, $P = 0.047$). Admittedly, this finding provides only correlational evidence, and there are better ways to quantify confidence in subjective value. But it illustrates our general point that a mere choice effect is unlikely to provide strong support for theories of neural computations. Instead, we recommend the development of process models of multi-alternative decision-making that allow more fine-grained predictions (including predictions of response times, confidence ratings and eye movements) in order to narrow the gap between brain and behaviour[10].

Methods

For parameter estimation, we used maximum likelihood estimation and hierarchical Bayesian modelling procedures[5,6]. The procedures and parameter settings followed our original work[2] as closely as possible. Detailed information on our analyses is provided in the Supplementary Information.

Data availability

The data analysed for this work are publicly available on the Open Science Framework (https://osf.io/qrv2e/).

Code availability

The custom code for the analyses reported here is publicly available on the Open Science Framework (https://osf.io/qrv2e/).

Received: 10 April 2020; Accepted: 4 August 2020;
Published online: 14 September 2020

References

1. Louie, K., Khaw, M. W. & Glimcher, P. W. Normalization is a general neural mechanism for context-dependent decision making. *Proc. Natl Acad. Sci. USA* **110**, 6139–6144 (2013).
2. Gluth, S., Kern, N., Kortmann, M. & Vitali, C. L. Value-based attention but not divisive normalization influences decisions with multiple alternatives. *Nat. Hum. Behav.* **4**, 634–645 (2020).
3. Webb, R., Glimcher, P. W. & Louie, K. Divisive normalization does influence decisions with multiple alternatives. *Nat. Hum. Behav.* https://doi.org/s41562-020-00941-5 (2020).
4. Webb, R., Glimcher, P. W. & Louie, K. The normalization of consumer valuations: context-dependent preferences from neurobiological constraints. *Manag. Sci.* https://doi.org/10.1287/mnsc.2019.3536 (2020).
5. Lee, M. D. & Wagenmakers, E.-J. *Bayesian Cognitive Modeling: A Practical Course* (Cambridge Univ. Press, 2013).
6. Farrell, S. & Lewandowsky, S. *Computational Modeling of Cognition and Behavior* (Cambridge Univ. Press, 2018).
7. Carandini, M. & Heeger, D. J. Normalization as a canonical neural computation. *Nat. Rev. Neurosci.* **13**, 51–62 (2012).
8. De Martino, B., Fleming, S. M., Garrett, N. & Dolan, R. J. Confidence in value-based choice. *Nat. Neurosci.* **16**, 105–110 (2013).

9. Lebreton, M., Abitbol, R., Daunizeau, J. & Pessiglione, M. Automatic integration of confidence in the brain valuation signal. *Nat. Neurosci.* **18**, 1159–1167 (2015).
10. Busemeyer, J. R., Gluth, S., Rieskamp, J. & Turner, B. M. Cognitive and neural bases of multi-attribute, multi-alternative, value-based decisions. *Trends Cogn. Sci.* **23**, 251–263 (2019).

Author contributions

S.G. analysed the data and wrote the manuscript. N.K. and C.L.V. revised the manuscript.

Competing interests

The authors declare no competing interests.

Additional information

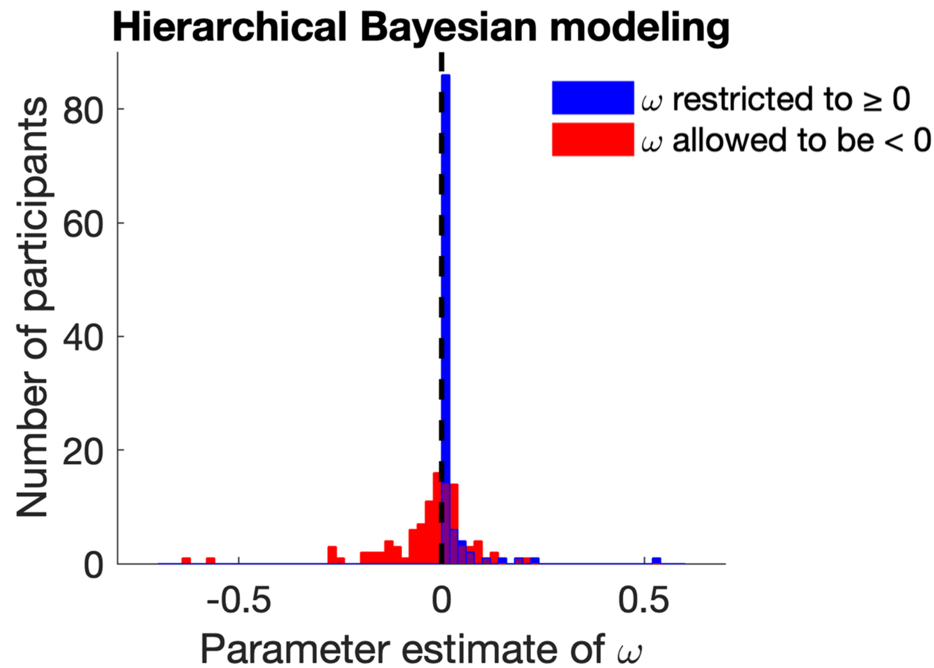
Extended data is available for this paper at https://doi.org/10.1038/s41562-020-00942-4.

Supplementary information is available for this paper at https://doi.org/10.1038/s41562-020-00942-4.

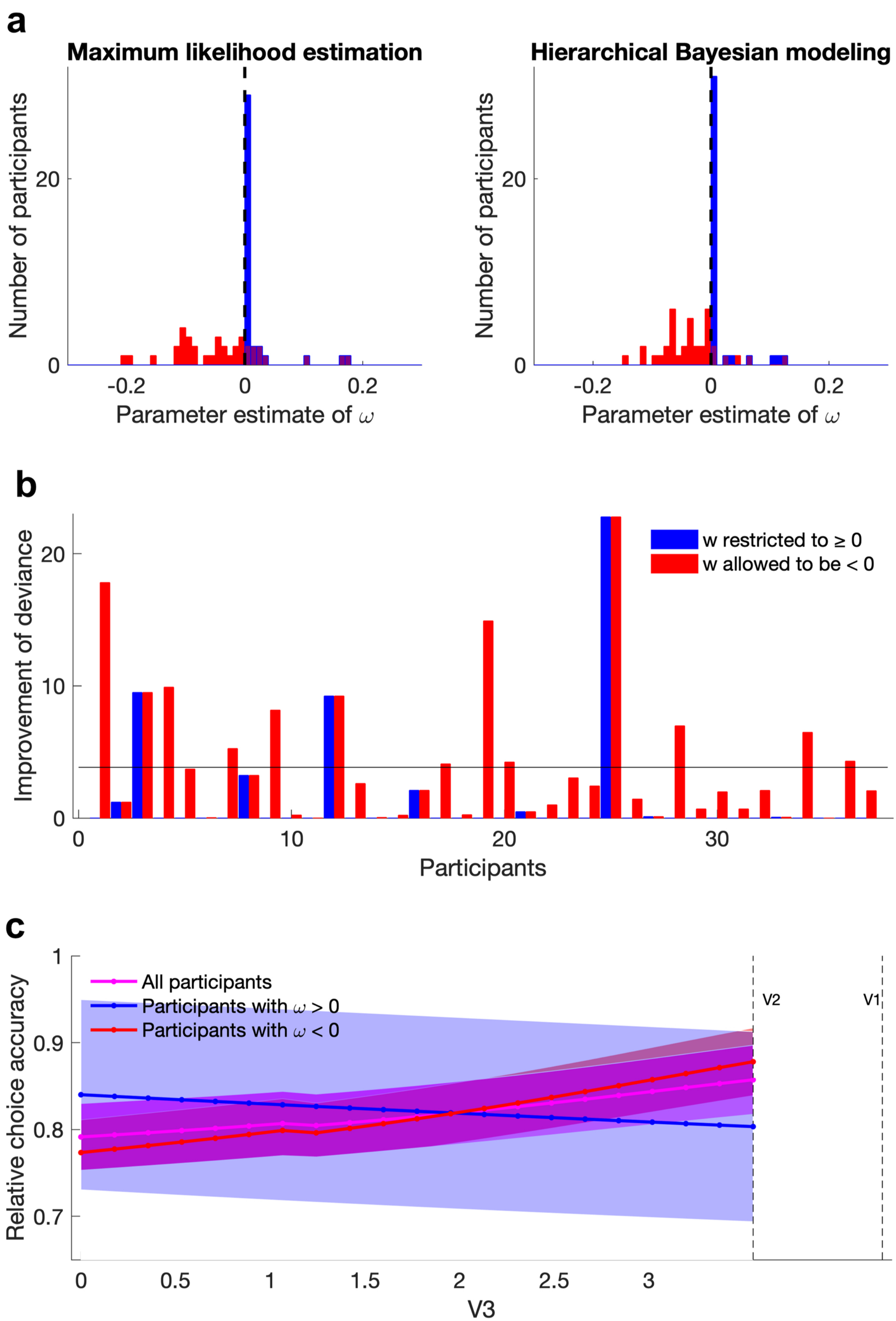
Correspondence and requests for materials should be addressed to S.G.

Reprints and permissions information is available at www.nature.com/reprints.

Publisher's note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.



Extended Data Fig. 1 | Parameter estimates based on hierarchical Bayesian modeling. Parameter estimates for ω (based on hierarchical Bayesian modeling) when this parameter is forced to be ≥0 (as assumed by DN; blue) or not (red). Note the shrinkage of individual estimates as compared to the estimates based on maximum likelihood presented in Fig. 1a.



Extended Data Fig. 2 | Modeling results of the eye-tracking experiment dataset. Note that the very large 95% CI for participants with $\omega > 0$ in **c** is due to the fact that there were only very few participants with a positive estimate of this parameter.

		Multinomial Logit ($\omega = 0$)	Divisive Normalization ($\omega \geq 0$)	Model with ω allowed to be < 0
MLE	σ	0.521 / 0.470 (0.311)	0.414 / 0.410 (0.296)	0.934 / 0.848 (0.735)
	ω	0	0.015 / 6.32e-10 (0.042)	-0.041 / -0.041 (0.083)
	LL	-5948	-5924	-5871
	p-value on χ^2		.094	< .001
	test for $\omega > 0$		.015	.998
	AIC	11970	11996	11890
HBM	σ	0.516 / 0.475 (0.279)	0.411 / 0.419 (0.128)	0.860 / 0.783 (0.493)
	ω	0	0.015 / 0.002 (0.033)	-0.034 / -0.031 (0.052)
	DIC	11963	11945	11867

Extended Data Fig. 3 | Model comparison of the eye-tracking experiment dataset. Definitions of the abbreviations are provided in Table 1.