

Instructor: Professor Shelly Harvey **Time:** TR 10:50-12:05PM
Office: 464 Herman Brown **Classroom:** Herman Brown 427
Email: shelly (rest is standard)

Office Hours: M 3-4pm, Tu & Th 12:05 - 12:30pm, F 1:30 - 2:30pm
Course webpage: math.rice.edu/~shelly/564s22/
Course LMS: (Canvas) teaching.rice.edu/canvas
TA: TBA

Ardila's Axioms (borrowed from Prof. Varilly-Alvarado): Mathematics is a human endeavor.

1. Mathematical potential is distributed equally among different groups, irrespective of geographic, demographic, and economic boundaries.
2. Everyone can have joyful, meaningful, and empowering mathematical experiences.
3. Mathematics is a powerful, malleable tool that can be shaped and used differently by various communities to serve their needs.
4. Every student deserves to be treated with dignity and respect.

Texts: *Abstract Algebra*, by David Dummit and Richard Foote (3rd edition).

A secondary text for the course will be *Introduction to Commutative Algebra*, by Michael Atiyah and Ian Macdonald. Other sources might include individual chapters from books, which I will make available to the class.

Course description: The core of the course consists of Chapters 13–18 of Dummit and Foote.

1. Fields and their extensions: Galois Theory. Time permitting, we will cover infinite Galois Theory and rewrite the fundamental theorem of Galois Theory as an equivalence of categories.
2. Algebras: Division algebras over fields (including Frobenius' and Wedderburn's Theorems). Symmetric and exterior algebras, including their use in the construction of differential forms on \mathbb{R}^3 .
3. Homological algebra: tools for (non)commutative algebra. Direct and Inverse limits. Projective, injective, flat modules. Ext and Tor. Derived functors.
4. Introduction to Representation theory: group algebras, character theory. Maschke's Theorem, Frobenius reciprocity.
5. Noncommutative algebra: Chain conditions, (semi-)simplicity, Artin-Wedderburn Theorem.
6. Commutative Algebra and affine algebraic geometry: Advanced ideal theory. Prime spectrum of a ring; the Zariski topology. Coordinate rings of affine algebraic varieties. Noether normalization and Hilbert's Nullstellensatz. Primary decomposition of ideals and its geometric interpretation. Localization, Integral extensions and the Cohen-Seidenberg Theorems.
7. Time permitting: rings of low dimension: Artinian rings and Dedekind domains.

Pre-requisites: MATH 463 or 464 (from fall 2020 or fall 2021)

Learning Outcomes: At the end of this course you should:

1. Understand be able to apply the main theorem of Galois theory, especially around problems involving statements on properties of fields and their extensions. Understand the category-theoretic formulation of the Main Theorem of Galois Theory, in terms of étale algebras.

2. Be familiar with the construction and universal properties that characterize symmetric and exterior algebras. Understand how abstract algebra underpins some other areas of mathematics, e.g., how exterior algebras give a convenient language for differential forms, how projective modules over a ring can give rise to vector bundles, how to properly define a determinant, etc.
3. Be able to apply homological techniques to prove theorems on the structure of modules and groups.
4. Use group representations to probe the internal structure of complicated groups and parse the action of groups on vector spaces.
5. Be able to pass back and forth between affine algebraic varieties and ideals in polynomial rings. You should understand how geometric properties can often be translated into ring-theoretic statements.
6. Be able to use localization as both a simplification tool (zooming in on a prime) and as a way to glean local geometric information on algebraic variety.
7. Understand the equivalence between the category of commutative rings with unit, and the category of affine schemes, setting up the stage for modern algebraic geometry.

Online platforms: This course will combine **three** online platforms:

Canvas, Zoom, and GradeScope.

Discussion: The material in this course can be abstract at times, as well as confusing. These are natural feelings, but they can be seriously amplified by isolation. In order to help each other out, we will be using the discussion platform in Canvas. Participation in Discussion is expected. The first thing you should do is go to the Discussion tab and reply to the **Introductions** thread. I've introduced myself there. I encourage you to do the same.

You can type math on Canvas! It is a little painful, but it can be done. I will post the instructions on Canvas.

Communication Plan: You are welcome to get in touch by email (e.g., requests for disability related accommodations) but that is probably the least efficient way to contact me because of the large numbers of emails I get each day. I am requesting that you ask most of your questions (about logistics, course content, homework problems, etc) get asked in the Discussion board on Canvas. The TA and I will be monitor Canvas frequently and will answer questions, or endorse answers by other students, as quickly as I can.

Grades: Homework will count for 45% of your final grade. The midterm will count for 20% of your grade and the final exam will count for 35% of your grade.

Homework: Due once a week, on **Thursday, at 11:59pm** on GradeScope. No physical homework will be accepted.

The homework is not pledged and you can collaborate with other students in the class. In fact, you are highly *encouraged* to do so! However, you are not allowed to look up solutions in any written form; in particular, you are not allowed to look up solutions online. Students caught violating this rule will be reported to the Honor Council. You should write up your solutions individually.

Undergraduates enrolled in Math 464 will have a reduced homework load. Mathematics graduate students should enroll in Math 564.

Honor Code: This class is not pledged. You can collaborate with other students in the class. In fact, **you are very much encouraged to collaborate**, except in Exams.

Attendance: Attendance is not required. However, you are responsible for all the material and announcements covered in class. While Canvas is a valuable resource, not all announcements will be posted there. You are responsible for reading any emails/announcements I send to the class through Canvas.

Expectations: In my experience as a student, most people do not follow all the details of a class in real time. When you go to a Math lecture you should expect to witness the big picture of what's going on. You should pay attention to the lecturer's advice on what is important and what isn't. A lecturer spends a long time thinking on how to deliver a presentation of an immense amount of material; they do not expect you to follow every step, but they do expect you to go home and fill in the gaps in your understanding.

Statement of Conduct: The Department of Mathematics supports an inclusive learning environment where diversity and individual differences are understood, respected, and recognized as a source of strength. Racism, discrimination, harassment, and bullying will not be tolerated. We expect all participants in mathematics courses (students and faculty alike) to treat each other with courtesy and respect, and to adhere to the mathematics department standards of collegiality, respect, and sensitivity:

mathweb.rice.edu/department-statement-collegiality-respect-and-sensitivity

as well as the Rice Student Code of Conduct. If you think you have experienced or witnessed unprofessional or antagonistic behavior, then the matter should be brought to the attention of the instructor and/or department chair. The Ombudsperson is also available as an intermediate, informal option, and contacting them will not necessarily trigger a formal inquiry. See the above website for details on how to contact the Ombudsperson.

Title IX Statement: Rice University cares about your wellbeing and safety. Rice encourages any student who has experienced an incident of harassment, pregnancy discrimination or gender discrimination or relationship, sexual, or other forms interpersonal violence to seek support through The SAFE Office. Students should be aware when seeking support on campus that most employees, including myself, as the instructor, are required by Title IX to disclose all incidents of non-consensual interpersonal behaviors to Title IX professionals on campus who can act to support that student and meet their needs. For more information, please visit safe.rice.edu or email titleixsupport@rice.edu.

Disability Support: If you have a documented disability that may affect academic performance, you should: (1) make sure this documentation is on file with Disability Resource Center (Allen Center, Room 111 / adarice@rice.edu / x5841) to determine the accommodations you need; and (2) get in touch with me during the first two weeks of class to discuss your accommodation needs. All such discussions will remain as confidential as possible.