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T3 ________________	Problem Chosen	F3 ________________
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2015

Mathematical Contest in Modeling (MCM/ICM) Summary Sheet

Create a More Sustainable World: A Comprehensive Measurement Model of Sustainability

Summary

Sustainability is a controversial concept, varying over time and between groups. Recognizing and assessing the sustainability fairly and rationally has been attached great significance, considering sustainability is related to the development of a country or a region.

In this paper, we develop a mathematical model to assess the degree of sustainability of different countries and attempt to put forward a 20 year sustainable development plan for the selected Least Developed Country. In addition, we give a methodology to simulate and evaluate the influence of the 20-year sustainability plan and extend this model to more real conditions when additional environmental factors are taken into consideration.

Firstly, we construct the **Comprehensive Measurement Model** of the sustainability of a country, achieved by transforming the meaning of sustainability into mathematical constraints one by one, obtaining the corresponding equations and calculating the scores of the comprehensive sustainability.

Secondly, resources and capitals of each subsystem are evaluated by building the **Comprehensive Evaluation Model**. Based on **linear fitting and interpolation method**, the lack of some part of the data collected is made up. We also introduce a method to normalize the data so as to fairly calculate the influences of all indicators. Then **Principal Component Analysis** method is used to realize a dimensionality reduction. Simultaneously a fewer number of independent variables are obtained, providing the basis of calculating the weight of the various indicators.

Thirdly, the trend of every subsystem is forecasted by the **Linear Prediction Method**. Sine function and exponent function are introduced to simulate the shake of the value and the mutation so as to increase the accuracy.

Fourthly, we improved our model to forecast the resources of every subsystem and **20-year sustainable development plan** for Nepal is specifically proposed, including programs, policies and support ICM to provide help to improve the sustainability of Nepal. The plan is also analyzed and discussed, and the most effective plan aiming at Nepal is attempt to be found.

Finally, sensitivity analysis is implemented to test the robustness of our model and the strengths and weakness of our model are discussed as well.

Contents

1. Introduction and approach

Sustainability is a debatable concept, with theories shaped by different worldviews of different individuals and different organizations, which in turn influences how issues are formulated and actions proposed.[1] In ecology, sustainability is how biological systems remain diverse and productivity, however, from more general perspective, sustainability is the endurance of systems and processes.[2]

Specifically, as a subsystem of a finite global ecosystem, human economy cannot be sustained without preserving natural resources and keeping the social coherence. Thus, sustainability is defined as a part of sustainable development, which presents development without compromising the ability of future generations to meet their own needs.

Sustainability is a new concept of development and is related to the development strategies, and having a fair and rational cognition and assessment of sustainability plays an essential role to help a country or a region to take effective measures to develop healthily. In this paper, we develop a mathematical model to assess the degree of sustainability of different countries and attempt to put forward a 20 year sustainable development plan for a selected Least Developed Country. In addition, we give a methodology to evaluate the influence of the 20-year sustainability plan and extend this model to other conditions when additional environmental factors are taken into consideration.

1.1 Restatement of the Problem

We are required to establish a mathematical model to help ICM use their extensive financial resources and influence to create a more sustainable world. In order to get a clear picture of problem, we divide our tasks into three parts:

- Articulate fair and rational metrics and build the evaluation system to determine the sustainability degree of different countries and policies.
- Create a 20 year sustainable development plan consisting of programs and supports needed for a selected country from Least Developed Countries list.
- Analyze the influences of the 20-year sustainability plan put forward and discuss how our model can be applied with other environment factors taken into account. Then select the most effective strategies that can be implemented to develop sustainably.

1.2 Literature Review

Since World Commission on Environment and Development put forward the concept of sustainable development in 1987, sustainable development is now widely accepted by all countries and regions as the guide for coordinating the relationships between social, commercial events, nature, environment and the resource condition. [3]

Many experiments with forms of sustainability assessment applied for both strategic and operational aspects by governments, private-sector firms, civil society organizations, and so on for the last few years. For instance, Hong Kong applies sustainability assessment in its evaluation of urban infrastructure options (HKSDU, 2002). [4]

In essence, sustainable development is to realize the cooperation of nature and social system in certain time and under certain science &technology. The core is to keep population, resource, economy and environment in a stable and constant condition. So according to the theory of system, we can integrate the 5 subsystems of population, resource, economy, environment and science & technology. [5]

1.3 Model Overview

The process of our model is shown in Figure 1

To determine the sustainability degree of different countries and policies, we have to know what criterion should be used to evaluate sustainability. As is shown in the problem, we search for the predecessors' research and find that many of them are qualitative analysis. What's more, according to the data from all kinds of websites, there exist more than 1000 of the evaluation criterion.

How to select the essential criterion and process scientifically to make them more useful in evaluating is a key problem. The Comprehensive Evaluation Method, which is calculated with scores of five subsystems will be helpful for us to devise our model. For each subsystem, we pick up typical indicators from a large number of indicators.

Combining these indicators with the real data from internet and using Principal Component Analysis, we can get main Components and their Contribution values. Using a simple weighted sum, we can get scores of subsystems. Only under the constraint condition, using another simple weighted sum, can we get a total scores of the sustainability.

To observe more reliable prediction, considering indicators' randomness, we modified the improved model to forecast the scores of each subsystems**.**

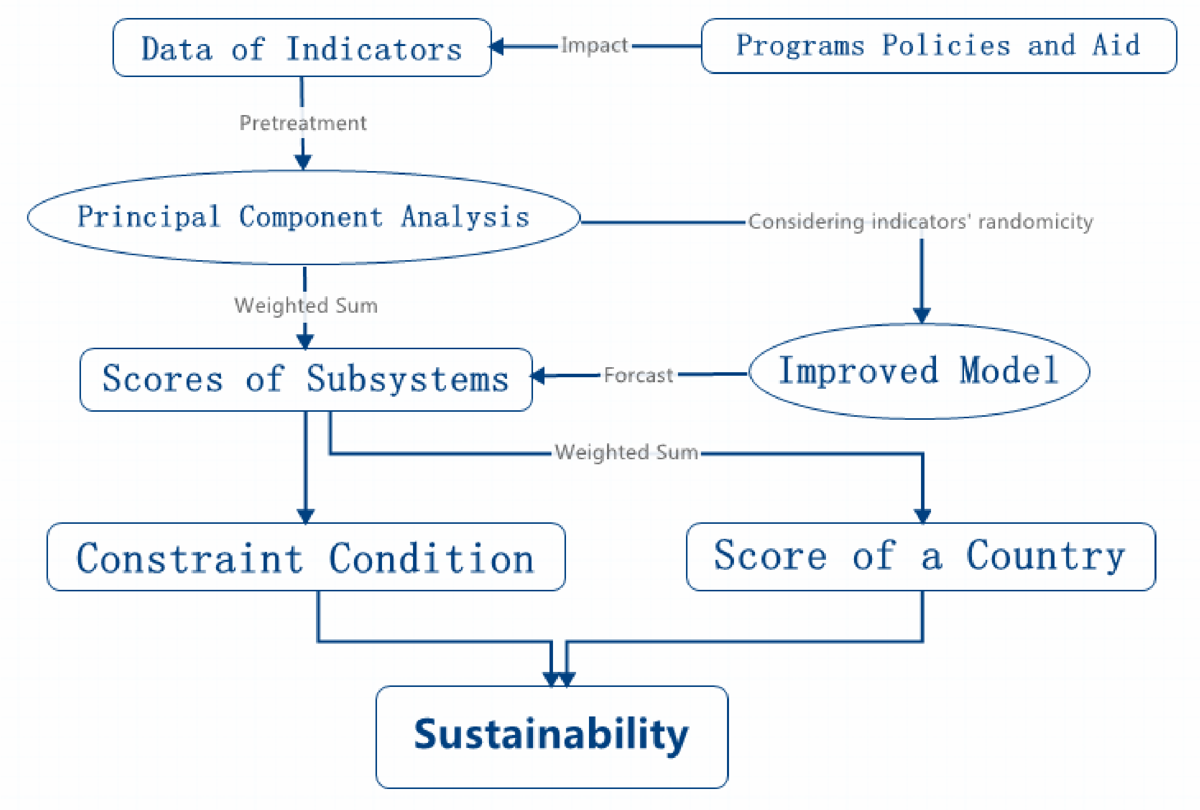


Figure 1 – Model Overview

2. Assumptions and Justifications

- Resources and capitals among the various systems can be optionally replaced by others but the subsystems remain over the lowest state to survive.
- The division of 5 subsystems is reasonable.
- Assume that the factors taken into consideration play an essential role for the evaluation, i.e. reflect sustainability.
- Assume the data that we have collected and used is enough and valid and the quantification is correct.
- The data we used and reference ranks are believable.

3. Notations

Notations	Descriptions
t	Time
$X(t)$	Total sustainability of the country
$X_n(t)$	Total sustainability of the subsystem
$E(t)$	The influence of the external effects on the sustainability

$I(x)$	The import function
$E(x)$	The export function
$Imax(x)$	The maximum import level of the resources
$Emax(x)$	The maximum export level of the resources
X_{ij}	The j-th indicator of the i-th subsystem
b_j	Contribution rate
λ_j	Eigenvalues
Yn	Principal components
$\widetilde{x_n}$	The normalized coefficient of X_n

4. Measurement of the Sustainability of a Country

4.1 Comprehensive Measurement of the Sustainability of a country

4.1.1 Constraints for Defining the Sustainability

The constraints that affect the achievement of sustainability should be necessarily taken into consideration before constructing the sustainable system model, which consist of the basis of the sustainable system. These constraints interact, being influenced by each other and determining the realization degree of sustainability together. In order to assess the sustainability of a country, we divide the sustainable system into five subsystem: Population System, Social System, Economic System, Environment System, Resource System, denoted by $x_1[t], x_2[t], x_3[t], x_4[t], x_5[t]$ respectively.

Evaluation index function is

$$\begin{cases} \mathrm{X}[\mathrm{t}] = \mathrm{f}(x_1[t], x_2[t], x_3[t], x_4[t], x_5[t], \mathrm{E}(t), t) \quad t \in [0, \infty] \\ \mathrm{X}[m] = \mathrm{X}_0 \end{cases}$$

Here ***X*[t]**, the function of time ***t***, indicates the sustainable level of research object in time ***t***, i.e. the condition of the sustainable system, the value of which is determined by the sustainable level and interaction of the 5 subsystems. $x_i[t]$ represents the sustainability of the subsystem. ***E(t)*** refers to the influence of the external effects on the sustainability, also called the error or stochastic. ***f*** is the state function of sustainability of the system to reflect the interaction process within the sustainable system. t_0 is the initial time of the study and x_0 represents the initial sustainable state of study objects.

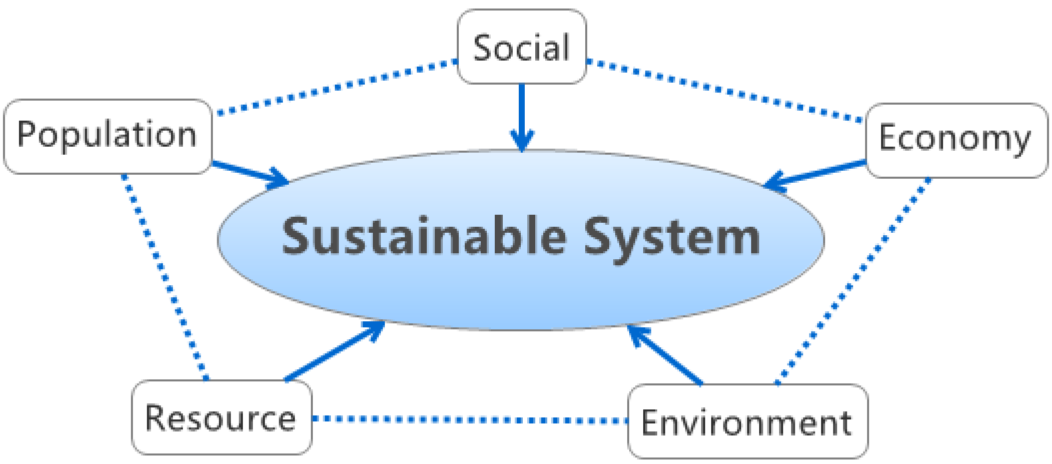


Figure 4

Figure 4 depicts the image of the scope and content of the sustainability need to be considered, which defines the range of study for the sustainable system model. Take the Population System, Social System, Economic System, Environment System, Resource System for examples, sustainable system model will be constructed under different conditions and a variety of models will be compared and analyzed. Finally, we use the equations to describe the interaction of the various subsystems.

1. When it comes to sustainability with constant total, we assume that the total amount of resources or capitals remain constant, simultaneously, resources and capitals among the various systems can be optionally replaced by others. For example, natural capital can be replaced by social capital, and accordingly social capital can be replaced with natural capital. Sustainability of the system can be achieved as long as the total of the resources and capitals remain constant.

   Sustainable System Model with constant total can be expressed as:
   $$\sum \boldsymbol{X_n(t)} \geq \sum \boldsymbol{X_n(t-1)} \quad \boldsymbol{n_i \in [1,5]}$$
   Here, $\sum \boldsymbol{X_n(t)}$ represents the sum of the resources or capitals in $\boldsymbol{t}$ and $\sum \boldsymbol{X_n(t-1)}$ is the sum of the resources or capitals of subsystems in the period of ***(t-1).*** Clearly, the essence of constant sustainability is that the sum of resources or capitals should not be diminished.

2. In addition, the concept of survivability is introduced to enhance the illustration of sustainability, which means that only when the subsystems remain over the lowest state to survive, the sustainability of the whole system can be achieved. Thus, sustainability is a concept based on the perpetual of the all subsystems. If the survivability is considered, the Sustainable System Model should be modified as:
   $$\boldsymbol{X_n \geq MIN(X_n) \quad X_n > 0}$$
   Where $\boldsymbol{X_n}$ is the total resources or capitals of each subsystem**,** $\boldsymbol{MIN(X_n)}$ denotes the minimum amount of resources needed to survive for each subsystem.

3. Meanwhile, the developing characteristic should not be ignored, referring to the fact that every subsystem should be developing constantly and necessarily, rather than merely maintain the minimum level of survival. For instance, with the development of the society, the demand of resources is also increasing accordingly. If the resources supply remain unchanged, there will be a lack of resources for human to develop. Sustainable System Model with considering developing characteristic can be expressed as:
   $$\boldsymbol{X_n(t) \geq X_n(t-1), n \in [1,5]}$$
   Here, $\boldsymbol{X_n(t)}$ represents the size of each subsystem in $\boldsymbol{t}$ and $\boldsymbol{X_n(t-1)}$ is the size of each subsystems in the period of ***(t-1).*** It can be seen from the model 4 that emphasizing the developing characteristic is essentially extension of the sustainability with survivability into consideration. If the size of the system is increasing, the system is developing.

4. However, taking only the survivability and developing characteristic is not enough, considering some constraints should not be ignored. For example, economic development may be restricted by the environment and resources. If the level of economic development is higher than the capacity of the environment and resources, economic development will stagnate.
   $$\mathbf{Imax}(\boldsymbol{X_{11}, X_{12}, \ldots, X_{15}}) \leq \boldsymbol{Emax(X_{11}, X_{12}, \ldots, X_{15})}$$
   Here two new functions, E(x) and I(x), are introduced. E(x) denotes the export function, and the I(x) denotes the import function. When $E_{max}(x)$ is the maximum export level of the resources, $I_{max}(x)$ is the maximum import level of the resources. As illustrated in model 5, only if maximum import resources of the system (Imax（$X_{11}, X_{12}, \ldots, X_{15}$））must be lower than maximum export level of the resources of the system, the sustainability of the whole system can be achieved.

5. Taking all models represented above into consideration, we propose a comprehensive definition of sustainability. The Comprehensive Measurement of the sustainability can be represented as:
   $$\begin{cases} \mathbf{Imax}(\boldsymbol{X_{11}, X_{12}, \ldots, X_{15}}) \leq \boldsymbol{Emax(X_{11}, X_{12}, \ldots, X_{15})} & \mathbf{(5-1)} \\ \boldsymbol{Xn[t] - Xn[t-1] \geq 0, \ n \in [1,6]} & \mathbf{(5-2)} \\ \boldsymbol{X[t] - X[t-1] \geq 0} & \mathbf{(5-3)} \end{cases}$$
   Here, $\mathbf{(5-1)}$ the same as above, $E_{max}(x)$ and $I_{max}(x)$ reach the max at the same time on level of the resources. And only if maximum import resources of the system I(x) must be lower than

maximum export level of the resources of the system $\mathrm{E(x)}$, the total sustainability can be achieved.

The formula $(\mathbf{5-2})$ emphasizes the developing characteristic is essentially extension of the sustainability with survivability into consideration. The formula represents the subsystem function utility rate, when the number≥0, the system is developing.

The formula $(\mathbf{5-3})$ represents the whole system is developing. Sustainability is no only on single subsystem but the whole.

4.1.2 Scores of the Sustainability of the Overall system

Now taking the effect of external factors E(t) and the random factors into account, we define the national development index as follows:

$$\mathrm{X[t]} = \sum x_n[t]$$

$\mathrm{X}[t]$ is the multiplication of $\mathrm{x}_n[t]$.

4.2 Measurement of the Resources of Every Subsystem

As mentioned in the section 4.1, $\mathbf{x_i}$ **[t]** is the score of the sustainability of the subsystem. First, a proper quantity of indicators are selected. Then we introduced Principal Component Analysis (PCA) to realize the reduction of numbers of dimensions so as to abandon the influence of the repetition of the indicators and obtain the relatively objective result.

As for the environment subsystem, owing to the lack of data, so we apply the environmental performance index (EPI) [6] as the score of the environment subsystem, which is based on 25 indicators and indicates how well countries perform on high-priority environmental issues in two broad policy areas: protection of human health from environmental harm and protection of ecosystems. The process of working out environmental performance index (EPI) correspond to the method in Chapter 4, but with more comprehensiveness and accuracy in reflecting the environment subsystem resources.

4.2.1 Indicators of Every Subsystem

The representative part of the indicators are selected from the collected data with the principals, which is concluded from common sense and professional paper, to simplify the process of sustainability measurement.

- ✓ Indicators have to clearly and reasonably represent the quantity of the resources of every subsystem.
- ✓ The number of the chosen indicators should be at least 2 to comprehensively represent every subsystem.

Indicators are shown in Section 4.2.3.

4.2.2 Collection and Pretreatment of the data of the chosen indicators

- ✓ The Data are mainly extracted from the web World Bank Data [7] and EPI [6]. Other data are also used.
- ✓ When we search data on the Internet, we note that some data is missing due to age, for example, GINI coefficients of some countries are missing. Given the fact, Pretreatment has to be introduced to meet the needs of follow-up data processing. The method we adopt is filling the data mainly based on linear fitting and interpolation according to the data in other years.
- ✓ After replenishing the data, the impact of each indicator will be calculated. Then we take the inverse of the data of negative indicators to ensure that the change of indicators will correctly affect the change of the score of the subsystem.

$$\mu_1(\mathrm{Country,k,i,t}) = \frac{1}{\mu_0(\mathrm{Country,k,i,t})}.$$

- ✓ To avoid the bad performance due to different dimension of the indicators, we take the country, whose human development index is the highest among the world, Norway as a standard. μ (NOR,k,i,t) donates the value of the i-th indicator of the subsystem k in year t of Norway.

$$\mu_2(\text{Country}, k, i, t) = \frac{\mu_1(\text{Country}, k, i, t)}{\mu_1(\text{NOR}, k, i, t)}.$$

- ✓ Then we use a special function based on the squire root method aiming at distinguish the divergence of 2 countries.

$$\mu(\text{Country}, k, i, t) = \begin{cases} 10 * \sqrt{|\mu_2(\text{Country}, k, i, t)|}, & \mu_2(\text{Country}, k, i, t) \geq 0 \\ -10 * \sqrt{|\mu_2(\text{Country}, k, i, t)|}, & \mu_2(\text{Country}, k, i, t) < 0 \end{cases}.$$

We can obtain the standardized indicators.

4.2.3 Weight of the Indicators

Based on the principle of least information loss, we use the Principal Component Analysis (PCA) method to transform the indicators, which have relatively greater link to each other, to a fewer number of variables that are independent on others i.e. principal component.

- ✓ PCA method is respectively applied to every subsystem.
- ✓ The data that donates the input of the Principal Component Analysis (PCA) are those of 16 countries (4 countries per class) and from 1990 to 2013. The country class is divided by the Human Development Index(HDI).[8]
- ✓ Countries are Nepal, Kenya, Angola, Pakistan, South Africa, Mongolia, India, Indonesia, Uruguay, China, Mauritius, Ukraine, Norway, USA, Japan and Saudi Arabia.

The data we used in PCA are shown as the following table 4.1.

	Indicator 1	Indicator 2	Other indicators
Country 1, 1990			
Country 1, 1991-2012			
Country 1, 2013			
Country 2, 1990			
Country 2, 1991-2012			
Country 2, 2013			
Other countries, Other years			

Principal Component Analysis (PCA) is a statistical technique that linearly transforms a set of original variables into a substantially smaller set of uncorrelated variables that represents most important information of the original data set, with which a large number of independent variables can be systematically reduced to a smaller, conceptually more coherent set of variables. [9] Principal Component Analysis (PCA) [10] is performed by the following steps:

(1) The raw dataset should be normalized

Assuming that there are $\boldsymbol{m}$ indicator variables: $\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_m}$ and n evaluation objects, then the value of $\boldsymbol{j}$-th indicators of the $\boldsymbol{i}$-th evaluation object is $\boldsymbol{a_{ij}}$. $\boldsymbol{a_{ij}}$ can be transformed as the normalized indicators, that is $\widetilde{\boldsymbol{a_{ij}}}$.

$$\tilde{a}_{ij} = \frac{a_{ij} - \mu_j}{s_j}, (i = 1, 2, \cdots, n;\ j = 1, 2, \cdots, m)$$

Where $\mu_j = \frac{1}{n}\sum_{i=1}^{n} a_{ij}, s_j = \frac{1}{n-1}\sum_{i=1}^{n}(a_{ij} - \mu_j)^2, (j = 1, 2, \cdots, m)$, i.e. $\boldsymbol{u_j}$, $\boldsymbol{s_j}$ denote the sample mean and sample standard deviation of the $\boldsymbol{j}$-th indicator. Respectively, we get the normalized indicators variables, represented as

$$\tilde{x}_i = \frac{x_i - \mu_i}{s_i}, (i = 1, 2, \cdots, m)$$

(2) Calculate the correlation coefficient matrix R, represented as

$$R = (r_{ij})_{m \times m}$$

$$r_{ij} = \frac{\sum_{k=1}^{n} \tilde{a}_{ki} \cdot \tilde{a}_{kj}}{n-1}, (i, j = 1, 2, \cdots, m)$$

In the formula, $r_{ii}=1, r_{ij}=r_{ji}$, r_{ji} denotes the correlation coefficient of the ***i***-th indicator and ***j***-th indicator.

(3) Compute the eigenvalues and eigenvectors
Compute the eigenvalues ($\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_m \geq 0$) of the correlation coefficient R and the corresponding eigenvectors $u_1, u_2, \ldots, u_m$, where $u_j=(u_1, u_2, \ldots, u_m)^T$, we obtain ***m*** indicator variables consisting of the eigenvectors.

$$\begin{cases} y_1 = u_{11}\tilde{x}_1 + u_{21}\tilde{x}_2 + \cdots + u_{m1}\tilde{x}_m \\ y_2 = u_{12}\tilde{x}_1 + u_{22}\tilde{x}_2 + \cdots + u_{m2}\tilde{x}_m \\ \cdots\cdots\cdots\cdots\cdots \\ y_m = u_{1m}\tilde{x}_1 + u_{2m}\tilde{x}_2 + \cdots + u_{mm}\tilde{x}_m \end{cases}$$

Where y_1 is the first principal component, y_2 is the second principal component, y_m is the first ***m*** principal component

(4) Select ***p*** ($p \leq m$) principal components, and calculate the comprehensive evaluation value.

- ✓ Calculate the information contribution rate and cumulative contribution rate of the eigenvalues $\lambda_j (j=1,2,\ldots,m)$, we obtain the information contribution rate b_j and the cumulative contribution rate α_p, which is accordingly represented as

$$b_j = \frac{\lambda_j}{\sum_{k=1}^{m}\lambda_k}, (j=1,2,\cdots,m) \qquad \text{and} \qquad a_p = \frac{\sum_{k=1}^{p}\lambda_k}{\sum_{k=1}^{m}\lambda_k}$$

From the formulas above, when α_p is close to 1 ($\alpha_p = 0.85, 0.90, 0.95$), we should choose the first p indicator variables as the p principal components and replace them by the original m indicator variables, through which p principal components can be comprehensively analyzed.

- ✓ Calculate the composite score

$$Z = \sum_{j=1}^{p} b_j y_j$$

Where b_j denotes the information contribution rate, and we can assess the weight according to the value of the composite scores.

Table4.2 – Results of Principal Component Analysis method on Economy Subsystem

Eigenvalues λ_j	Contribution rate b_j
3.1186	34.6509
1.4211	15.7905
1.1701	13.0011
1.0207	11.3416
0.9124	10.1379
0.6097	6.7746
0.4166	4.6292
0.3305	3.6723
0.0002	0.0018

It's concluded that the effect of applying the PCA method is great for that the cumulative Contribution Rate α_p of the first 6 principal components can reach 90%. So we select 6 from the 9 principal components to carry on comprehensive measurement.

Table4.3 – the corresponding eigenvectors $u_1, u_2, \ldots, u_m$ of the first 6 principal components

	$\widetilde{x_1}$	$\widetilde{x_2}$	$\widetilde{x_3}$	$\widetilde{x_4}$	$\widetilde{x_5}$	$\widetilde{x_6}$	$\widetilde{x_7}$	$\widetilde{x_8}$	$\widetilde{x_9}$
Y1	-0.1871	0.4488	-0.1316	-0.1053	0.3641	-0.1457	0.5231	0.5241	-0.1820
Y2	-0.3278	0.2520	0.1537	0.3400	0.4383	-0.1659	-0.1771	-0.1782	0.6380
Y3	0.5899	-0.0319	0.7326	0.0554	0.2848	-0.0941	0.0947	0.0943	-0.0579
Y4	-0.2878	0.0955	0.3050	0.2480	-0.0174	0.8535	0.0753	0.0761	-0.1151

Y5	0.0774	-0.1068	-0.1570	0.8943	-0.1359	-0.2219	0.0980	0.1019	-0.2655
Y6	0.5892	0.0172	-0.4719	0.0861	0.1032	0.3803	0.1512	0.1420	0.4734

So we get 6 principal components:

$$\begin{cases} y_1 = -0.1871u_{11} + 0.4488u_{21} + \cdots - 0.1820u_{91} \\ y_2 = -0.3278u_{12} + 0.2520u_{22} + \cdots + 0.6380u_{92} \\ \qquad\qquad \cdots\cdots\cdots\cdots\cdots \\ y_6 = 0.5892u_{16} + 0.0172u_{26} + \cdots + 0.4734u_{96} \end{cases}$$

Put the standardized data on behalf of the original 10 indicators in each region into the expression of four main components, and the contribution rate of four main components can be calculated as the weight. Then we construct the Principal Component Comprehensive Evaluation Model.

Then we get the weight matrix of the indicators: $[W] = [b] \cdot [u]$.
Finally we get the score of every subsystem: $Z = [W] \cdot [\mu]$.

4.3 Forecast Method for the Resources of Every Subsystem

The same method as that pretreatment of the data in Section 4.2.2 is used to preliminarily forecast the change and the future value of the indicators.

- ✓ We apply the linear fitting and interpolation to every single indicator and obtain the change and the value of the indicators.
- ✓ The shake of the value of some indicators cannot be ignored. So we use functions that are acquired by curve fitting as follows to predict the future value of these indicators.
  E.g.
  $$y = -0.03161 * t + 0.3612 * \sin(1.6 * t + 0.887) + 1.228.$$
  is the revised function to predict of the indicator 'Net incurrence of liabilities, domestic (% of GDP)'.
  $$y = 0.7192 * \sin(1.2 * t + 0.2113) + 4.142.$$
  is the revised function to predict of the indicator 'GDP growth (annual %)'.

Then we get the trend of every subsystem. Thanks to Programming using Matlab, the time needed to achieve the result is greatly cut down.

4.4 Results and Analysis of the model

Based on the data that are collected, the model of the measurement of the sustainability of every subsystem are solved and results are as follows.

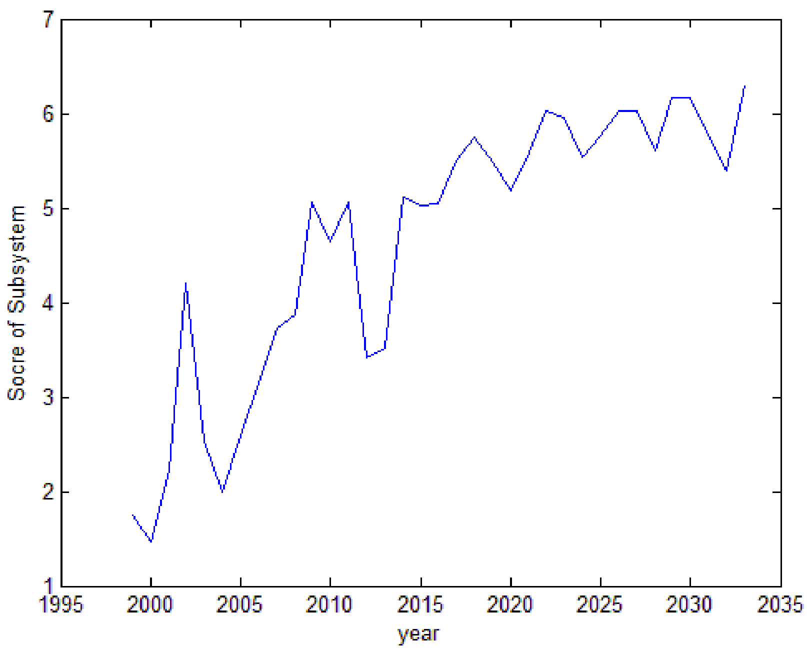
Table4.4 - Indicators and their weight of every subsystem

Subsystem	Indicators	Weight
Economy	Trade in services (% of GDP)	0.0677
Economy	Research and development expenditure (% of GDP)	0.191
Economy	Net incurrence of liabilities, domestic (% of GDP)	0.0613
Economy	Household final consumption expenditure, etc. (% of GDP)	0.0675
Economy	Services, etc., value added (% of GDP)	0.2775
Economy	GDP growth (annual %)	-0.1612
Economy	GDP per capita, PPP (current international $)	0.1726
Economy	GNI per capita, PPP (current international $)	0.1732
Economy	GINI index (World Bank estimate)	0.0389
Social	Public spending on education, total (% of GDP)	0.5078
Social	Employment to population ratio, ages 15-24, total (%) (modeled ILO estimate)	-0.3936

Social	Employment to population ratio, 15+, total (%) (modeled ILO estimate)	0.3592
Population	Population density (people per sq. km of land area)	0.3156
Population	Life expectancy at birth, total (years)	0.3336
Population	Population growth (annual %)	-0.298
Population	Population, female (% of total)	0.0077
Population	Urban population (% of total)	0.1103
Resource	Arable land (% of land area)	0.049
Resource	GDP per unit of energy use (constant 2011 PPP $ per kg of oil equivalent)	0.4
Resource	Energy use (kt of oil equivalent)	-0.0305
Resource	Combustible renewables and waste (% of total energy)	-0.1823
Resource	Renewable internal freshwater resources per capita (cubic meters)	0.1748
Environment	Environmental performance index	1

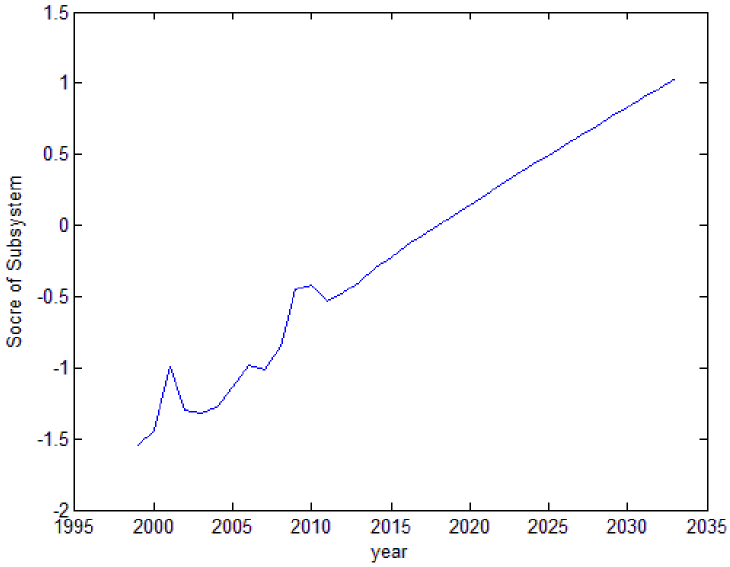
Finally, we can obtain the time series $X_k(t)$ for each subsystem's score.

Figure4.1 - Trends of Scores of Economy Subsystems of Nepal from 1999-2033



Analysis of results, Nepal's economic score is lowest in 2000 and 2004.Compared with statistics data, the actual situation is due to political instability in Nepal delaying the development of the economy. It shows for economic subsystem , we can accurately determine its sustainability.

Figure4.2 - Trends of Score of Social Subsystem of Nepal from 1999-2033



Analysis of results shows that although the coefficient of Nepal's social subsystem in 2002-2004 reduce (due to political instability), the coefficient of the overall volatility rises, which proves subsystem is the sustainable development of Nepal in society.

Figure4.3 - Trends of Score of Population Subsystem of Nepal from 1999-2033

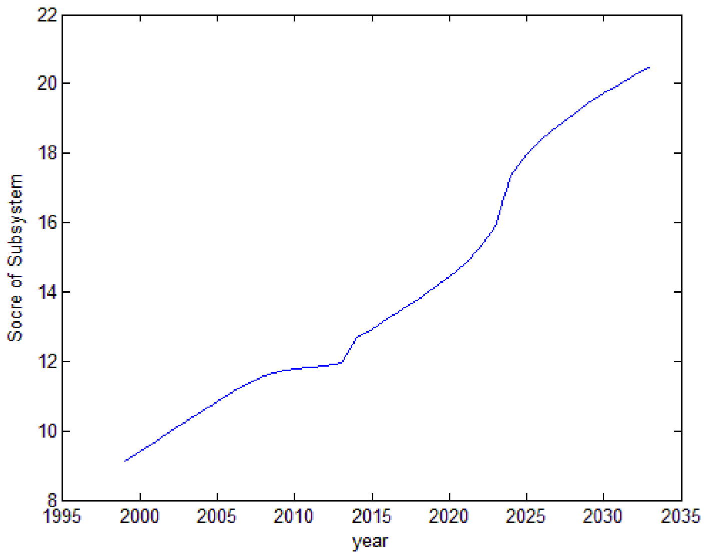


Figure4.4 - Trends of Score of Resource Subsystem of Nepal from 1999-2033

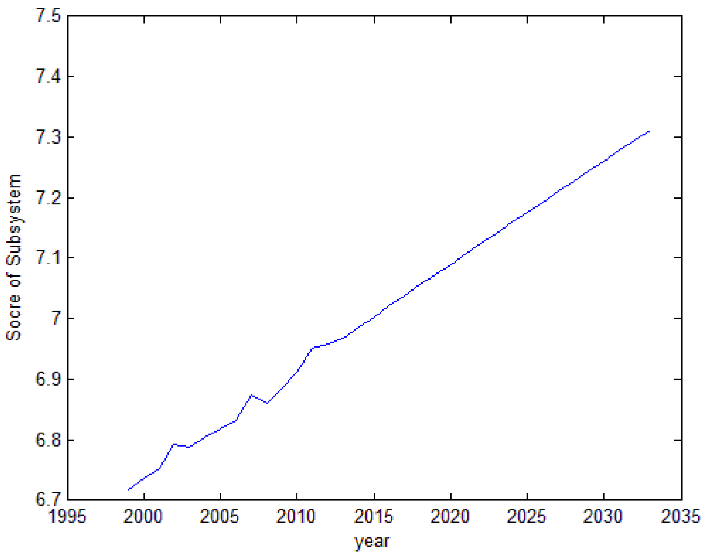


Figure4.5 - Trends of Score of Environment Subsystem of Nepal from 1999-2033

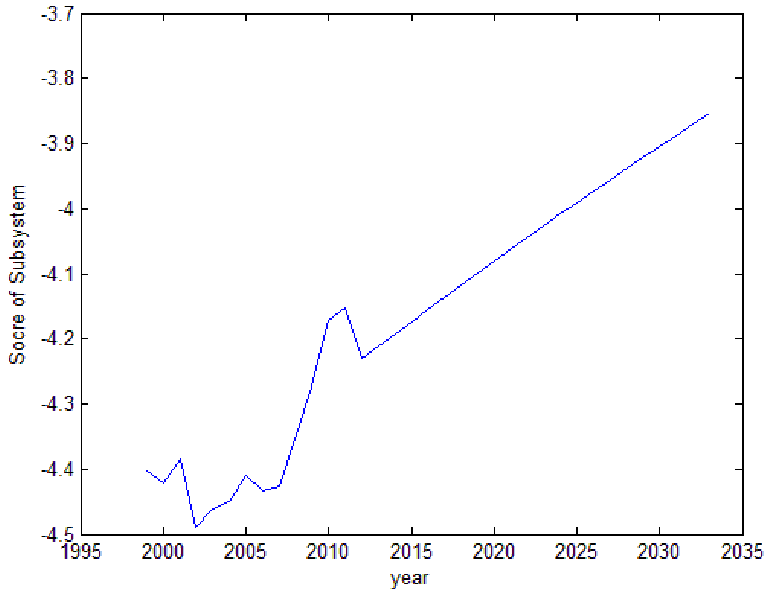
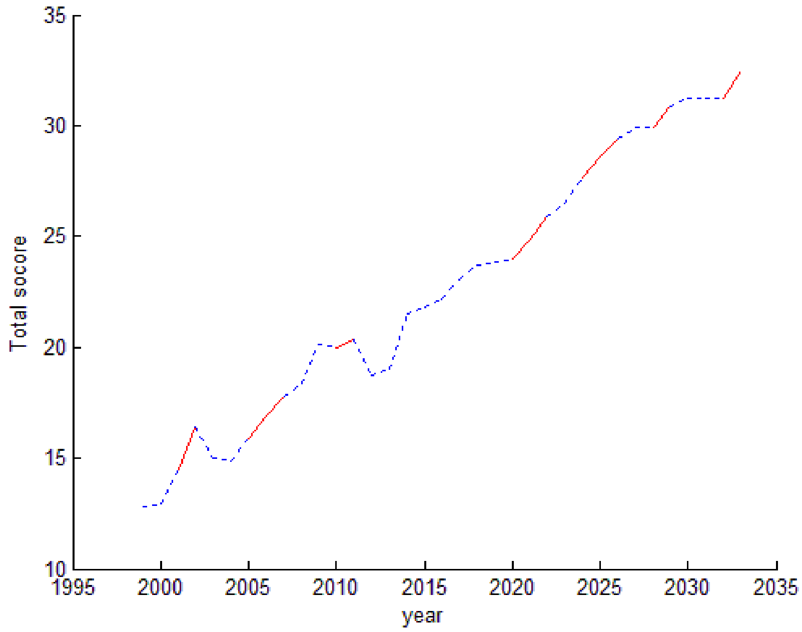


Figure4.6 - Trends of Score of Comprehensive System of Nepal from 1999-2033



We use the different colors and line type to indicate whether Nepal meets the criterion of the sustainability in that year. If the conditions for sustainable development are satisfied, line segments are red full lines.

5. Improved model of the Forecast for the Resources of Every Subsystem

- Considering the anthropogenic influence seems to be more specific, that is to say, it will obviously affect the value of the indicators, we assess and forecast the essential influence on Nepal by modifying the forecasted indicators.
- Random environmental factors should be introduced to simulate future climate change and the change of the world situation, such as climate change, natural disasters. These changes will bring instability and can simulate the real future, which helps to make better judgments.
- In order to determine the effect of index affected more accurately, we refer to the other countries which influenced by the human factors and random environmental factors, and made the concrete formula of E [t].

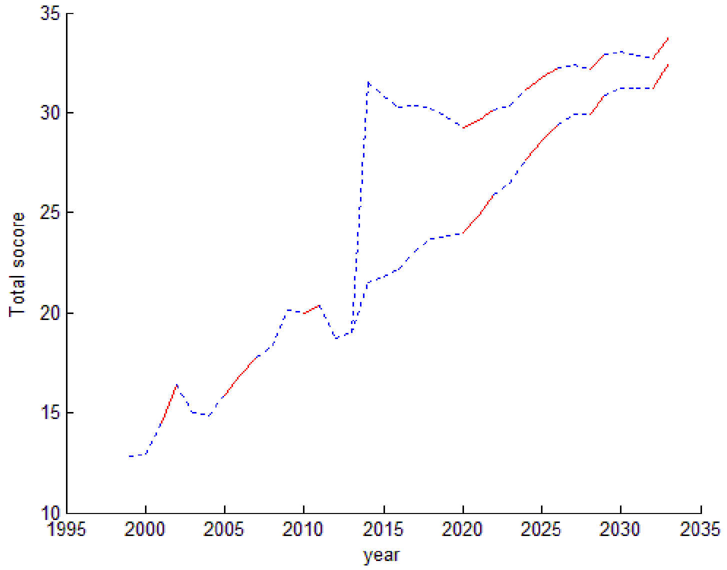
For Nepal, the formula of E[t] is as follow:

$$\mathrm{E[t]} = \begin{cases} 0 & t < t_0 \\ \mathrm{E}_0 \cdot \alpha^{t-t_0} & t \geq t_0 \end{cases}, \quad 0 < a < 1.$$

If the influence is positive, then $\mathrm{E}_0 > 0$. If the influence is negative, then $\mathrm{E}_0 < 0$.

If the influence is oscillating reduction, then $-1 < a < 0$. If the time duration of the influence of E[t] is long, a is set to be close to 1.

Figure5.1 - Trends of Score of Comprehensive System of Nepal with E[t] from 1999-2033



6. 20-year sustainable development plan for Nepal

Take our models and researches mentioned above into practice, we attempt to create a 20 year sustainable development plan for Nepal, a country listed in the 48 Least Developed Countries (LDC) list. According to the statistics of the above indicators, we target to propose specific plans or measures to improve the sustainability of Nepal.

Nepal, is a landlocked country located in South Asia. With an area of 147,181 square kilometres and a population of approximately 27 million. Nepal belong to agricultural country, 80% of people engaged in agricultural production.

According to the above analysis, we can clearly see that the biggest problems which Nepal facing the current on the sustainable development are as follows:

- Social system sustainability is low, referring to the imperfect infrastructure, education and etc.
- In addition to the slow economic development, natural resource utilization and protection is terrible.

6.1 Programs

- **The Clean Energy Exploration Program**

Clean Energy refers to the energy without pollutants emission, including nuclear power and renewable energy. Thus, it is imminent and necessary to explore more clean energy to replace the nonrenewable energy. If Clean Energy Exploration Program is established, the deployment of renewable energy solutions in the world's poorest countries to increase energy access and economic opportunities will be scaled up and respectively, the use of nonrenewable energy, such as coal and oil, will be decreased.

The effect of the Clean Energy Exploration Program, which can improve the whole subsystem, so the modified model can be reflected by the modified weight of a series indicator.

Subsystem	Indicators	Initial weight	Modified weight
Resource	Energy use (kt of oil equivalent)	-0.0305	0.02346
Environment	Environmental performance index (EPI)	1	1.3

Resource	GDP per unit of energy use (constant 2011 PPP $ per kg of oil equivalent)	0.4	0.52

To simplify the problem, the tiny effects to the weight of the indicator are ignored and the interaction of the indicators are not considered between the subsystem.
Our goal is to achieve clean energy exploitation in 5years, and after 10 years of trials and research and development, the method of exploitation of the clean energy will be extended to the daily production and life.

- **Micro-hydro Power Generation program aiming at rural areas in Nepal**

One of the efforts made by ICM should be to promote that energy factors should be incorporates into national development strategies, such as Poverty Reduction Strategy, investment in energy of a country should promote the development in order to achieve the sustainability. Aiming at rural areas in Nepal, Micro-hydro Power Generation program will help Nepal to propose plans to use hydro power to generate the electricity. The program can not only help in energy saving but also in economic development, especially help to narrow the gap between urban and rural areas, represented by indicators as follows:

Subsystem	Indicators	Initial weight	Modified weight
Resource	Energy use (kt of oil equivalent)	-0.0305	-0.02179
Environment	Environmental performance index (EPI)	1	1.4
Resource	GDP per unit of energy use (constant 2011 PPP $ per kg of oil equivalent)	0.4	0.56
Economy	GDP growth (annual %)	-0.1612	-0.22568
Economy	GDP per capita, PPP (current international $)	0.1726	0.24164
Economy	GNI per capita, PPP (current international $)	0.1732	0.24248

We expect to build hydro-power plants in Nepal to help three rural areas and perform trail operation in 5 years, and we expect to reach 40% of hydro-power penetration in Nepal.

- **Program for Climate Resilience**

Program for Climate Resilience will provide a funding window of Climate Investment Funds, assisting Nepal in integrating climate resilience into development planning. By providing grants and concessional financing, ICM can support a wide range of activities, such as monitoring and analyzing weather data, building climate-resilient water supply and sanitation infrastructure and conducting feasibility studies for climate-resilient housing in coastal areas.

Subsystem	Indicators	Initial weight	Modified weight
Resource	Energy use (kt of oil equivalent)	-0.0305	-0.02773
Resource	Combustible renewables and waste (% of total energy)	-0.1823	0.20053
Resource	Renewable internal freshwater resources per capita (cubic meters)	0.1748	0.19228
Environment	Environmental performance index	1	1.1

- **Education and Employment Promotion Program**

Aiming at promote the education and employment, the Education and Employment Promotion Program should be proposed. This program will promote the development of education and employment by increased investment in education and promotion of educational training.

Subsystem	Indicators	Initial weight	Modified weight
Social	Employment to population ratio, ages 15-24, total (%) (modeled ILO estimate)	-0.3936	-0.47232
Social	Employment to population ratio, 15+, total (%) (modeled ILO estimate)	0.3592	0.43104
Social	Public spending on education, total (% of GDP)	0.5078	0.60936
Economy	GDP growth (annual %)	-0.1612	-0.19344
Economy	GDP per capita, PPP (current international $)	0.1726	0.20712
Economy	GNI per capita, PPP (current international $)	0.1732	0.20784

6.2 Policies

- **Irrigation Act**

Irrigation Act is proposed to assist Nepal in the growing of agricultural crops, maintenance of landscapes, and revegetation of disturbed soils in dry areas and during periods of inadequate rainfall. For Nepal is an agricultural country, meanwhile given that productive agriculture is a crucial element of inclusive growth, enhancing the efficiency of irrigation systems will continue to be critical to increase agricultural productivity, incomes, and rural livelihoods.
Irrigation Act include using a new, more scientific and effective irrigation equipment and method and recycling old equipment before, for the loss caused by changing equipment subsidies, unified management by the government to maintain the irrigation facilities. Of course implementation is divided into areas period of time, completely update need ten to fifteen years or more.

Subsystem	Indicators	Initial weight	Modified weight
Economy	GDP growth (annual %)	-0.1612	-0.20956
Economy	GNI per capita, PPP (current international $)	0.1732	0.22516
Economy	Research and development expenditure (% of GDP)	0.0675	0.08775
Population	Life expectancy at birth, total (years)	0.3336	0.43368
Population	Population growth (annual %)	-0.298	-0.3874
Environment	Environmental performance index	1	1.3
Resource	Arable land (% of land area)	0.049	0.0637

- **Wildlife protection policy**

Wildlife protection policy will be proposed and be urged to implement as soon as possible, in order to conserve the wild species and improve species diversity.

- **Water Initiative**
- **Food and Health Security Initiative**

In addition, we plan to enact two initiatives, Water Initiative and Food and Health Security Initiative to increase water resources and improve food security issues respectively.
The effect of the three policies is illustrated as follows:

Subsystem	Indicators	Initial weight	Modified weight
Resource	Renewable internal freshwater resources per capita (cubic meters)	0.1748	0.1923
Population	Population density (people per sq. km of land area)	0.3156	0.3472
Population	Life expectancy at birth, total (years)	0.3336	0.3670
Population	Population growth (annual %)	-0.298	-0.3278
Population	Population, female (% of total)	0.0077	0.0085
Population	Urban population (% of total)	0.1103	0.1213

6.3 Aids

Here, we use the same model as that in Section 5.1.

$$E[t] = \begin{cases} 0 & t < t_0 \\ E_0 \cdot \alpha^{t-t_0} & t \geq t_0 \end{cases}, \quad 0 < \alpha < 1$$

- **Provide the analytical and advisory services**

International Conglomerate of Money (ICM) builds the bridge between countries and regions, providing information, experiences and statistics resources for various countries and aiming at helping to create a more sustainable world. Thus, ICM should take advantage of statistics to provide the analytical and advisory services as long as it is useful for the development of the country. For example, ICM can provide the regular economic updates and advises the Nepalese authorities on key economic policies. In addition, it can also provide core data on poverty trends and access to the services of the other international organizations.

More importantly, ICM is supposed to provide relative information to promote the implementation of the projects and policies mentioned above.

Subsystem	E_0	α	t_0
Economy	2	0.9	15
Environment	0.2	0.9	15

- **Provide Capacity-building assistance**

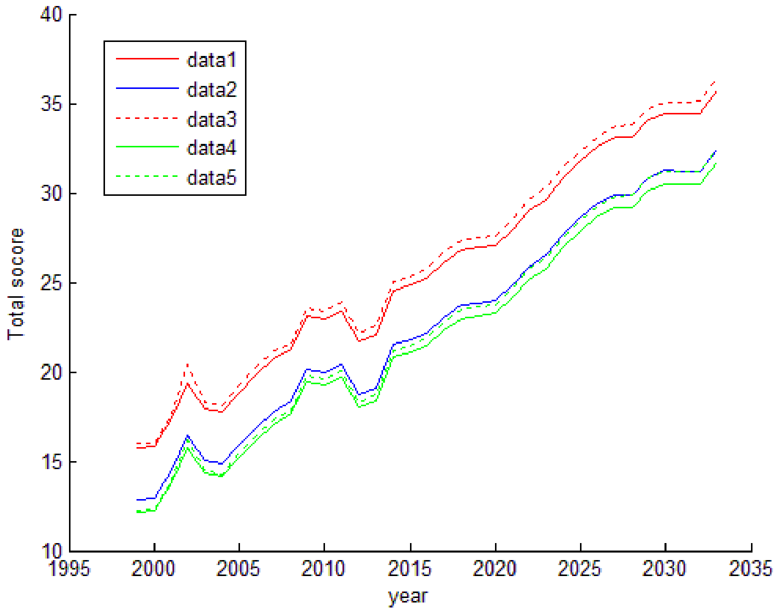
ICM can also take advantage of its core resources and influence within global to raise funds or provide technical support so that Nepal can get enough support to develop sustainably.

1) Provide financial support
   Firstly, Nepal should be encouraged to take active participate in regional and multilateral global initiatives. In this aspect, introducing foreign investment and funding may give Nepal confidence to conform to the relative initiatives and policies. In the meanwhile, the employment and economic development can also be promoted.
2) Provide technical support
   In addition, technical support should also be provided to enhance the integration through investment activities, such as trade finance facilities for local banks and advisory activities, such as investment climate strengthening and trade facilitation, considering the confidence of investors may be enhanced.

Subsystem	E_0	α	t_0
Economy	2	0.9	15
Social	0.1	0.9	15

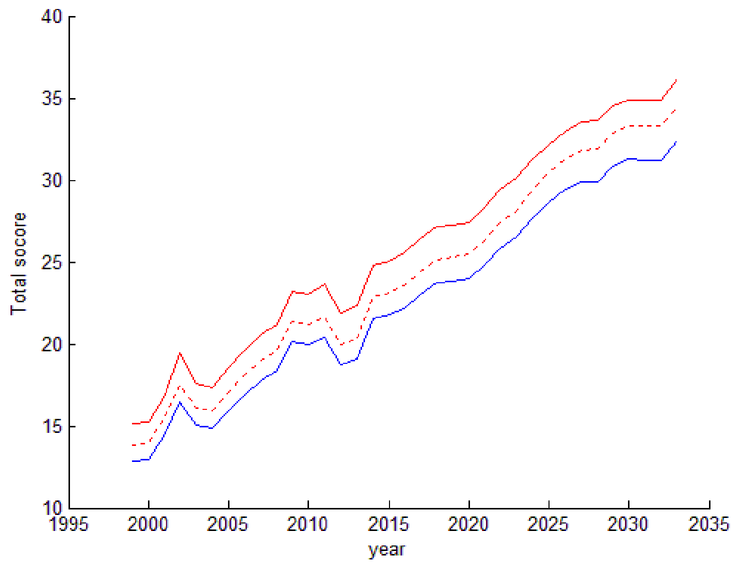
6.4 Result of the effect of the 20-year sustainability plan for Nepal

Figure 6.1 – Different result of different programs



- The blue curve shows the original score and the red dotted curve brings the best result. It means **Micro-hydro Power Generation program aiming at rural areas in Nepal** is the best program.

Figure 6.2 – Different result of different policies



- The blue curve shows the original score and the red full curve brings the best result. It means **Irrigation Act** is the best policy.

Figure 6.3 – Different result of different aids.

- The blue curve shows the original score and the red full curve is a little better. It means **Provide the analytical and advisory services** is the best aid.

7. Sensitive Analysis

Considering some inputs of our model may be hard to obtain or there might be some uncertainty in our inputs, all these kinds of deviation might influence the result of our model. Thus, sensitivity analysis is implemented to test the robustness of our model.

We test our model in three aspects：

✓ Modify the weight of the indicators, the effect of the subsystem resources score is analyzed.

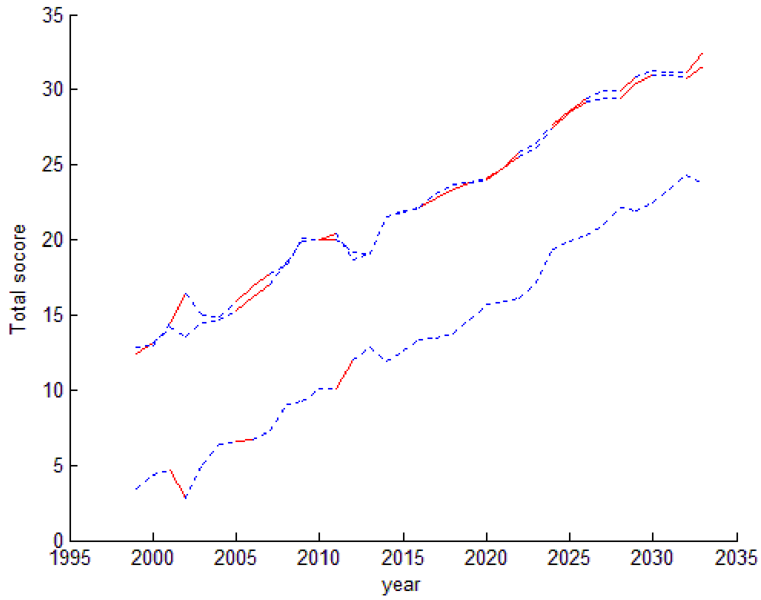


Figure 7.1 – the effect of different weights of indicators

The upper two curves are respectively calculated by the PCA method with different data of 16 countries from 1990 to 2013 and data of 16 countries from 1999 to 2013. And we can learn that the trends of the total scores are similar to each other.

However, the curve below is calculated with the data of 4 least developed countries from 1999 to 2013. And we can conclude that more countries contribute to a more precise result.

✓ Modify the inputs of the indicators, and determine whether a single index will affect the score of the entire subsystem.

In this time, we change the values of GPD and GNI very greatly compared to its dimension.

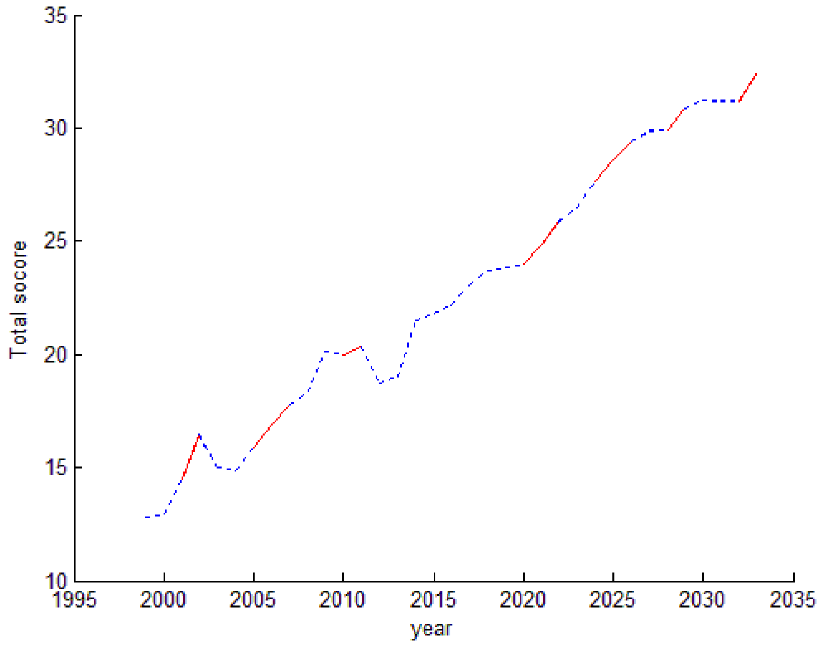


Figure 7.2 – the effect of different values of indicators

As illustrated in the figure, two curves with only one different value of indicator are almost in the same position. So we can get to the conclusion that the model we designed can comprehensively measure the sustainability of a country without being influenced by only differences of few values of indicators.

✓ Modify the formula to take more factors into consideration, and test the effect to the sustainability of the systems.

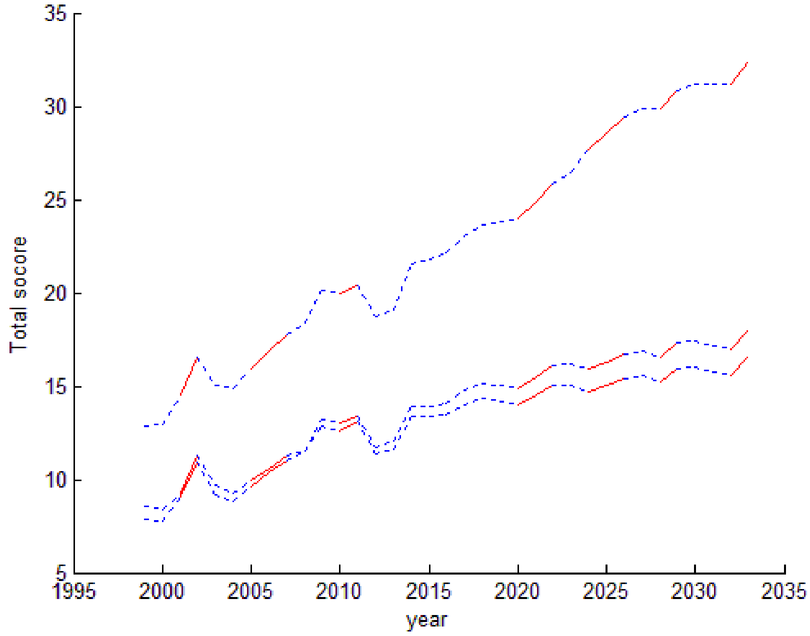


Figure 7.3 – the effect of different indicators

As is shown in the figure, we can see curves calculated with the data of 23,18,13 indicators. And we can be sure that the number of indicators can affect the trends and the values of the sustainability of a country, which in fact is the simple sign of whether we select enough and overall indicators.

The analysis proves that our model does not demonstrate a chaotic behavior, showing a good sensitivity.

8. Strengths and Weaknesses

8.1 Strengths

- **Avoidance of the duplication of the indicators**
  Principal Component Analysis (PCA) method is used when determining the weight of the indicators, which avoid the influence of the duplicated indicators and the subjectivity of AHP method.
- **We choose 23 indicators to give the comprehensive index an overall and convincing credit. What's more, we improve our model by considering shake along the time line horizon and different dimensions of the indicators.**
- **We respectively plot every indicator and find the transitional rates of most indicators of a country are almost constant, that is to say linear, so the model is much simpler.**
- **The Constraints clearly and exactly represent the meaning of sustainability of a country.**
  A set of equation is set for restricting the scores of every subsystem.
- **Our data comes from the website of some authority international organization such as World Bank, so it's convincing.**

8.2 Weaknesses

While our approaches and models are simplified and based on some consumptions, there remain several kinds of model weaknesses:

- **Not objective and accurate enough**

We use a large number of indicators but without the separate accurate analysis of every indicator, so the result maybe subjective and not accurate enough.

- **Smaller applying scale of countries**

Some indicators we used may be not applicable for some countries, which leads to Smaller applying scale of countries.

- **Too much simplicity on the determination of the Restricting function equation**

In order to simplify the problem, we propose the Restricting function equation with only limited factors into consideration, however, it might be more complicated in the real world.

- **A lack of part of the data greatly limits the process of selecting proper indicators and acquiring the weight, while the measurement of sustainability is very complex.**

9. Future Work (Modification or improvements)

- Expert scoring process of AHP to modify the determination of the weight of the indicators can be introduced.
- More rational restricting function equation to reflect the actual situation better should be proposed.
- Models of some countries to which the selected indicators are not applicable can be modified.
- The effect of prior knowledge and experience of the indicators can be considered when determining the weight of the indicators.

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