Demo

2014/04/01

1

7.13 Bradly-Brassard:

mergesort, triee merege

merge(U[1...n], V[1...m])

i, j ← 1

U[n+1] ← ∞ # infinity for end,

V[m+1] ← ∞ # Python float('inf')

pour k de 1 à m+n

ici

si U[i] < V[i]:

T[k] ← U[i]

i++

Sinon:

T[k] ← V[j]

j++

Return T

Si i>n alors T[k] ← V[j], j++

Sinon si j>m alors T[k] ← V[j] i++

Sinon si U[i] ≤ V[j] alors

2/ Probleme7.14,p.252,livre Brassard-Bratley

Woops...

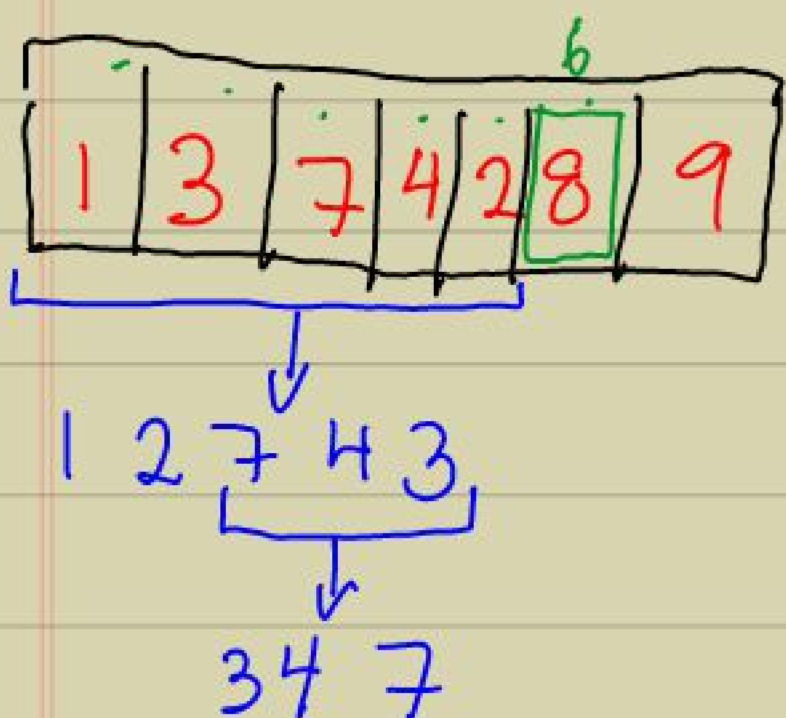
3/ Probleme 7.20 Livre B-B:

a list contains n elements... we want to find the m smallest. Que faire?

a) ordonere T et retronouver les m premir elements (order T and return the smallest) $O(n \log n)$

B) Apply selection(T, i) for i de 1 à m (i in 1 to m)

$O(n)$



C) other choice

selection$(T, \lceil n/2 \rceil)$

$O(n)$

m^e smaller than the element

4/ 7.21 B-B ... we want to find the elements
$\lceil n/2 \rceil, \lceil n/2 \rceil + 1, \ldots, m$

(a) order T and return?

$m > \log(n)$ $\xrightarrow{\text{yes}}$ $O(n \log n) + O(m)$

$\hat{=} O(n \log n)$

$\xrightarrow{\text{no}}$

(B) apply Selection

Total $O(mn)$

(C)

(C) other choice

$k \leftarrow \text{Selection}(T, \lceil n/2 \rceil)$
$l \leftarrow \text{Selection}(T, \lceil n/2 \rceil + m - 1)$

Parcourir et retour entre k et l.

5/ Quicksort & Heapsort.

Show

$$t(n) = n^3 - 3n^2 - n - 8$$

is smooth.

$3n^2 > 3n+1$

$3n^2 > 4n$

$$\frac{d}{dn} t(n) = 3n^2 - 6n - 1$$

if $n > 1$, $6n+1 < 7n$

if $n > 7$, $3n^2 > 21n > 7n > 6n+1$

$\rightarrow 3n^2 - 6n - 1 > 0$

Soit $n_0 = 7n > 6n+1$

$\rightarrow 3n^2 - 6n - 1 > 0$

$t(n)$ est croissante

t(n) est b-lisse

$$t(bn) \in O(t(n))$$

$$\lim_{n\to\infty} = \frac{b^3n^3 - 3b^2n^2 - bn - 8}{n^3 - 3n^2 - n - 8} = b$$

$$= b^3$$

$$b^3 \in R$$

$$.: t(n) \in O(t(n))$$

Prove that Quicksort given "en moyenne" time $O(n \log(n))$ for sorting n elements

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Tout d'abord il existe d et n_0 such that $\forall n \geq n_0$

$k \Rightarrow t(k)$

$$t(n) \leq dn + \frac{2}{n} \sum_{k=0}^{n-1} t(\cancel{V})$$

Where $t(n)$ is the time for Quicksort on average ...

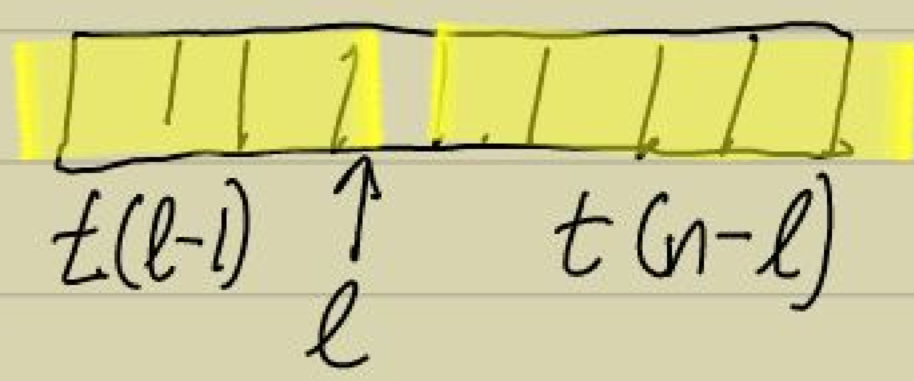
Proof

Proof

- le pivoment lui-meme prende un temps $g(n) \in \Theta(n)$

Quicksort

Pivot : $\Theta(n)$
Quicksort 1^e modalité
Quicksort 2^e moitié



Time to move in pivot pos. l

$$t(n) = g(n) + \sum_{l=i}^{n} \underbrace{\frac{1}{n}}_{\text{Probabilitie}} (t(l-1) + t(n$$

$$t(n) = g(n) + \frac{2}{n} \sum_{K=0}^{n-1} t(\kappa)$$

$$g(n) \in \theta(n) \Rightarrow \exists d, n_0 st. \forall n \geq n_0,$$

$$g(n) \leq \cancel{d(n)}$$
$$dn$$

plus +

$$t(n) \leq dn + \frac{2}{n} \sum_{k=0}^{n-1} t(k)$$

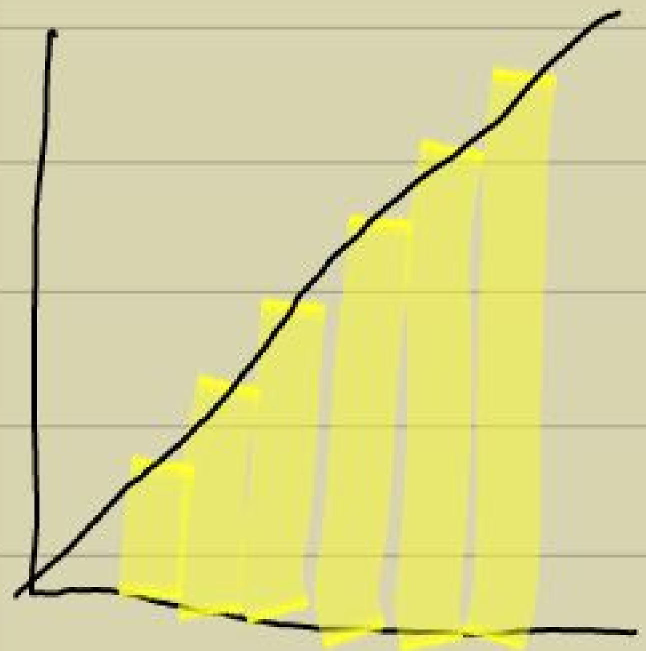
Soit, $n > n_0$ and suppose that $t(k) \leq c\, k \log k$ for $2 \leq k \leq n$

$$t(n) \leq dn + \frac{2}{n} \sum_{k=0}^{n-1} t(k)$$

$$= dn + \frac{2}{n} \left(t(0) + t(1) + \sum_{k=2}^{n-1} t(k) \right)$$

Incorrect.

$$\leq dn + \frac{2a}{n} + \frac{2}{n} \sum_{K=2}^{n-1} K \log K$$



$a = t(0) + t(1)$ H.I.

$$= dn + \frac{2a}{n} + \frac{2c}{n}\left(\left(\frac{n^2 \log n}{2} - \frac{n^2}{4}\right) - \left(\frac{4\log(2) - 1}{2}\right)\right)$$

not positive

$$\leq dn + \frac{2a}{n} + \frac{2c}{n}\left(\frac{n^2 \log n}{2} - \frac{n^2}{4}\right)$$

$$= dn + 2a + cn\log(n) - \frac{cn}{2}$$

$$= cn\log n - \left(\frac{c}{2} - d - \frac{2a}{n^2}\right)n$$

Soit $c + a \quad \frac{c}{2} > d + \frac{2a}{n} \iff$

$$c > d + \frac{4a}{n^2}$$