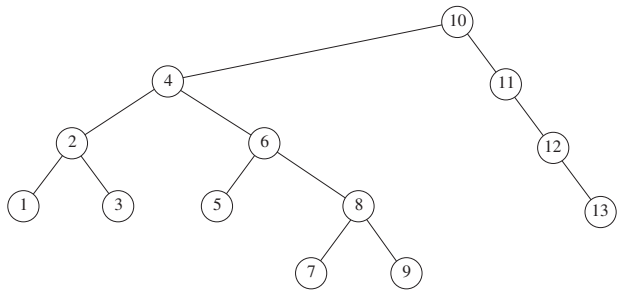
Data Structures and Algorithms (CS 130A) **Subhash Suri**

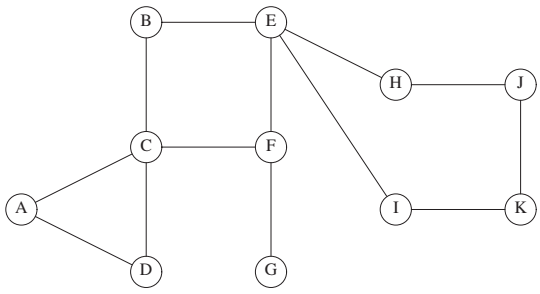
Homework Assignment 3

Handed Out: Feb 26 **Due:** *Mar 7*

1. (25 pts) Show the sequence of splay operations performed, along with the final results, of **accessing** keys 3 followed by 9 in the following splay tree.



2. (25 pts) Find all the articulation points in the graph below. Show the Depth-First Search tree, along with the values of Num and low for each vertex, used to compute the articulation points.



3. (25 pts) Given an undirected graph $G = (V, E)$, we want to find the *shortest cycle*, where length of the cycle is the number of edges in it. Consider the following strategy for this problem:

   - Perform a depth-first-search (DFS), starting from any vertex r. Keep track of the *level* of each node v, where $level(v)$ is the distance in the DFS tree from root vertex to v.
   - When we encounter a *back edge* (v, w), we observe that this edges forms a cycle of length $level(v) - level(w) + 1$. We keep track of the shortest cycle found during the DFS, and output it as the answer.

   Prove or disprove whether this strategy always finds the shortest cycle in a graph. (If true, give a proof. Otherwise, show a counterexample and explain what goes wrong.)

4. (25 pts) Show the result of the following sequence of instructions:
union(1,2), union(3,4), union(3,5), union(1,6), union(3,7), union(8,9), union(1,8), union(3,10), union(3,11), union(3,12), union(3,13), union(14,15), union(16,0), union(14,16), union(1,3), union(1,14), when the unions are

   - Performed by height (always making the shallower tree the child),
   - Performed by size (always making the smaller tree the child).

   For each of these (final) trees, do a find with path compression on the deepest node, and show the resulting tree.