God and Math:
Musings and Comic Strips

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NATS487
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Outline for today

- What math is and what it isn't
- The quest for the Theory of Everything
  - Incompleteness
  - Ontological arguments
- Observation ⇒ abstraction ⇒ application
  - Fourier theory: heat, audio, image, electrical
- The joy of exploration, the humility of a student

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- As a math teacher, I apologise!
- This is not math, just memorisation and manipulation of symbols
- Math is both more beautiful and more difficult than this
- Discovery of patterns
- Creation of theory
- Application to new domains
- Quest for elegance

matthen.com

Purity vs. complexity

FIELDS ARRANGED BY PURITY

MORE PURE →

SOCIOLOGY IS JUST APPLIED PSYCHOLOGY

PSYCHOLOGY IS JUST APPLIED BIOLOGY.

BIOLOGY IS JUST APPLIED CHEMISTRY

WHICH IS JUST APPLIED PHYSICS. IT'S NICE TO BE ON TOP.

OH, HEY, I DIDN'T SEE YOU GUYS ALL THE WAY OVER THERE.

SOCIOLOGISTS   PSYCHOLOGISTS   BIOLOGISTS   CHEMISTS   PHYSICISTS   MATHEMATICIANS

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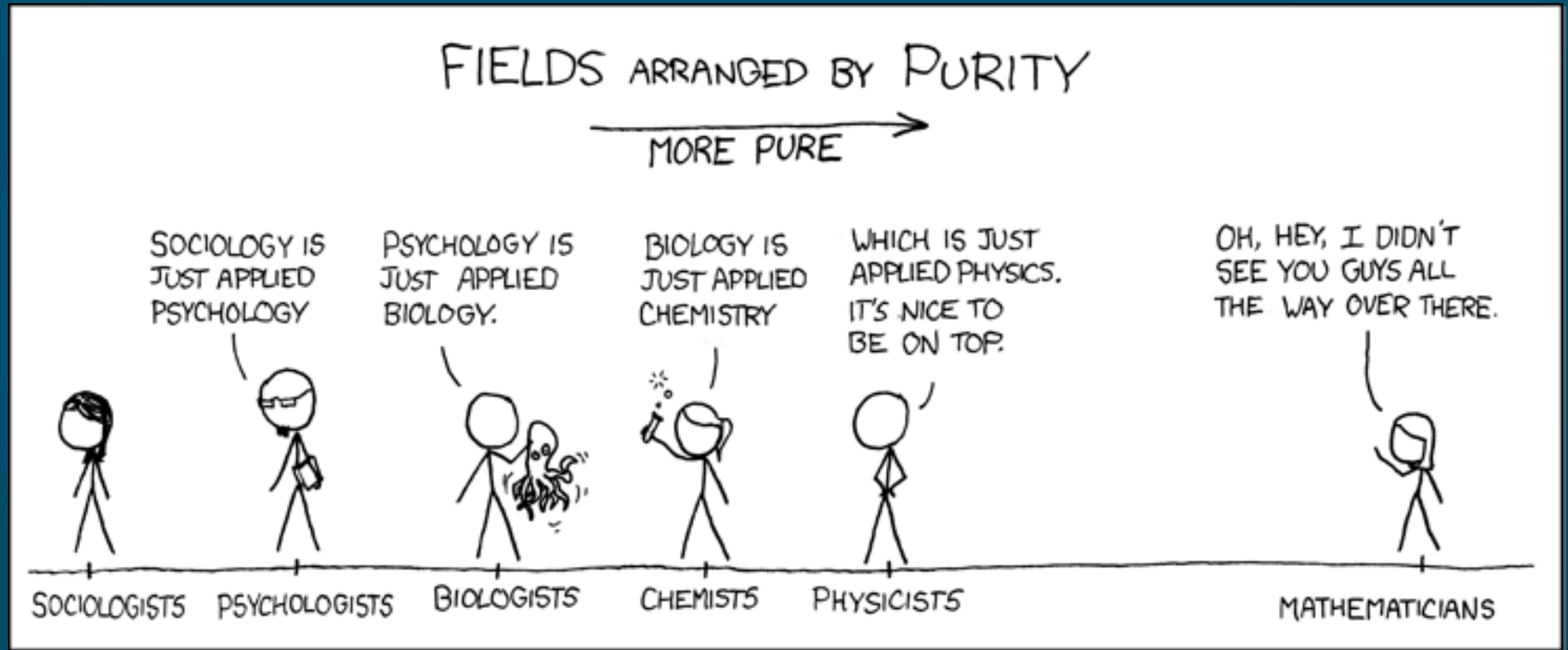
The quest for the bottom

- Axioms: assumptions, statements/definitions
  - Formal system: axioms + system of logic
- Is there a simplest formal system that allows us to derive/construct all of known math?
- e.g., exponentiation = repeated multiplication
  - $x^3 = x * x * x$
- multiplication = repeated addition
  - $3*x = x + x + x$
- addition = repeated increment
  - $x + 3 = (\ (\ x{++}\ ){++}\ ){++}$
- Formal system to define numbers?

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Why is 2 + 2 = 4?
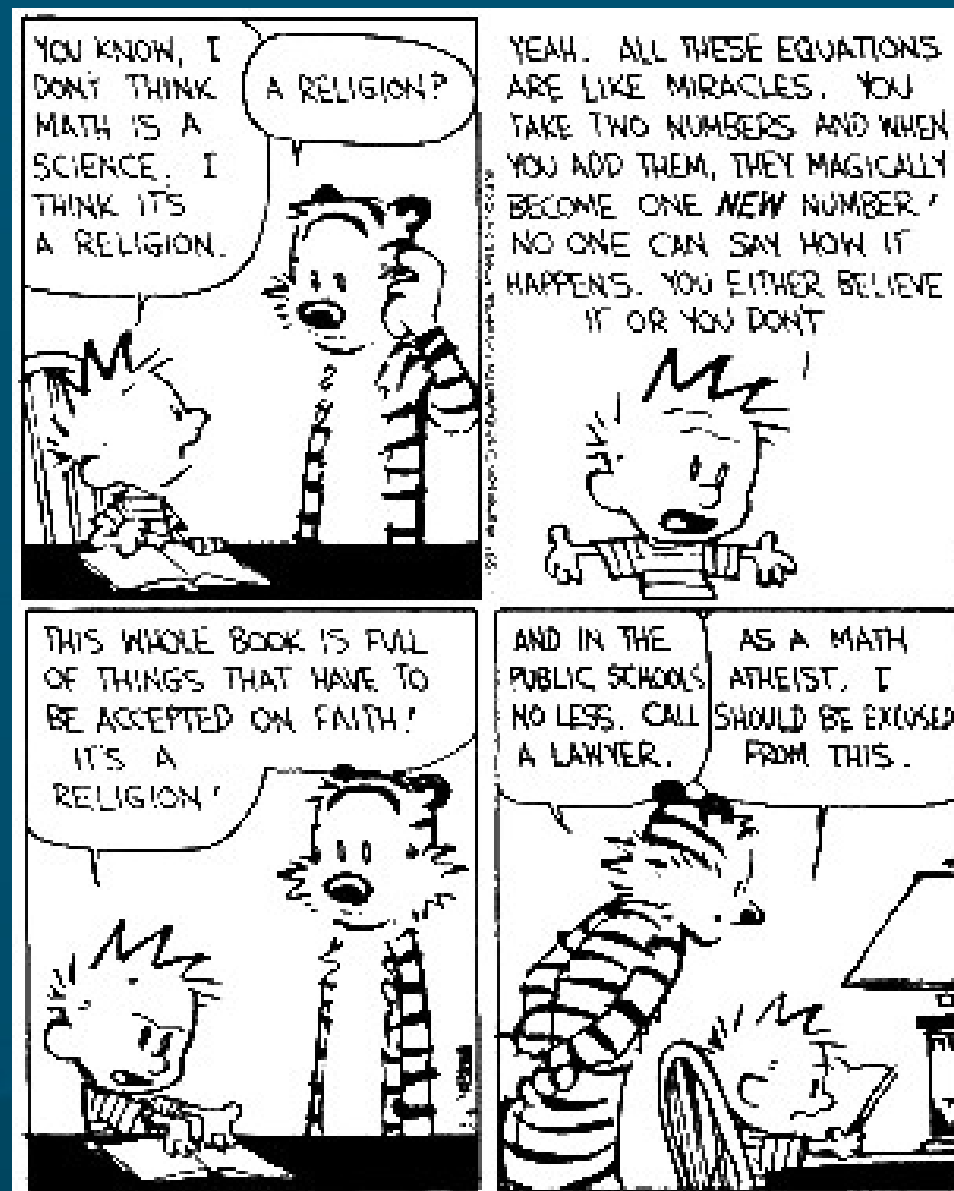
Peano construction of numbers

- E.g., Peano construction of the natural numbers
  - ◆ Axioms in language of set theory (ZF+C)
    - Numbers defined via set membership:
      $$0 = \varnothing, \; 1 = \{0\} = \{\varnothing\}, \; 2 = \{1\} = \{\{\varnothing\}\}$$
- Can this describe everything?
  - ◆ Can every true statement within the system be derived from the axioms?
- "God made the natural numbers; the rest is the work of man" (Leopold Kronecker)

KEEP IN MIND THAT A NUMERAL STANDS FOR A CERTAIN NUMBER OF OBJECTS...

NOW, WHEN YOU COUNT, WHAT YOU ARE DOING IS MATCHING ELEMENTS ONE-TO-ONE WITH A SET OF COUNTING NUMBERS...

IN A SET OF NUMBERS, THE LAST NUMERAL MATCHED TO THAT SET IS THE CARDINAL NUMBER...

THERE'S A GOOD PROGRAM ON TV TONIGHT AT SEVEN O'CLOCK

Incompleteness

- Bertrand Russell + Alfred Whitehead, *Principia Mathematica,* 1910s:
  - Attempt to codify all of math in a formal system
  - Admirable, but ultimately impossible!
- Kurt Godel, 2 incompleteness theorems, 1930s:
  - Any formal system that can describe arithmetic cannot be both consistent and complete
    - incomplete: true but unprovable statements
  - Related to Turing's halting problem:
    - Can we build a computer that can detect infinite loops?
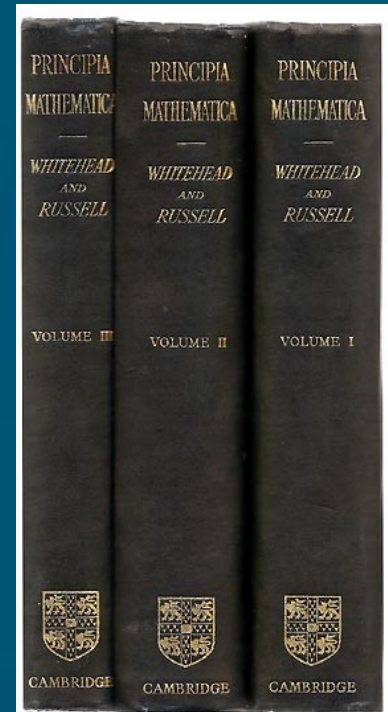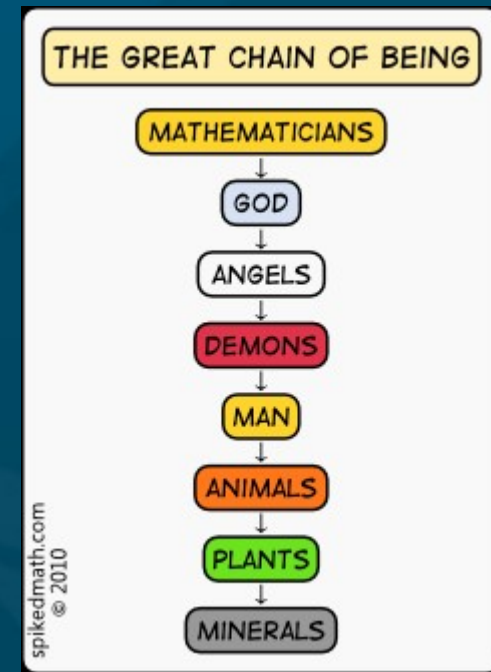
Perfect universal truths

Ontological arguments

- οντος (being, existing) + λογικος (word, reason)
- "Proof" by logic of the existence (of God)
- Key ingredient: operational definition of God?
  - Anselm: "greatest being" we can imagine
  - Descartes: "supreme perfection"
  - Godel: every "positive property"
  - Plantinga: "maximal excellence" in some world (possibility ⇒ necessity in some universe)

THE GREAT CHAIN OF BEING

MATHEMATICIANS

GOD

ANGELS

DEMONS

MAN

ANIMALS

PLANTS

MINERALS

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Coming up for fresh air...

- Enough theory! I have work to do ....
  - How does math help me get stuff done?

- Common toolset for diverse domains
  - Observation ⇒ abstraction ⇒ application

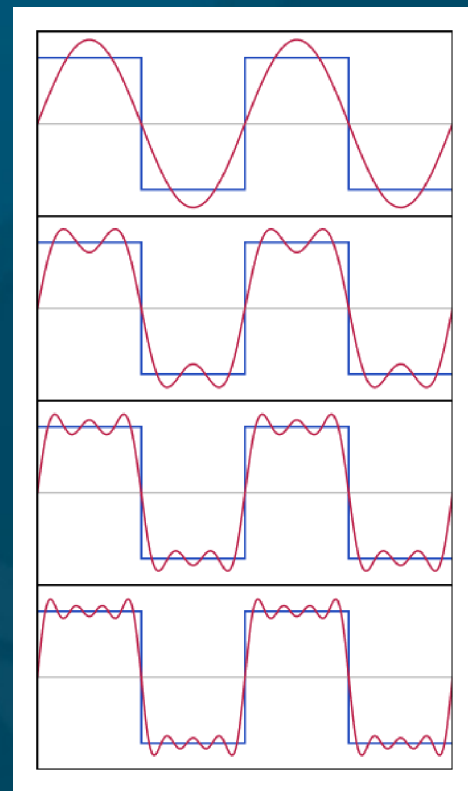# Theory: Fourier transform

- Joseph Fourier, 1807 studying heat transfer in metal plates

$$\frac{\partial u}{\partial t} = k \nabla^2 u$$
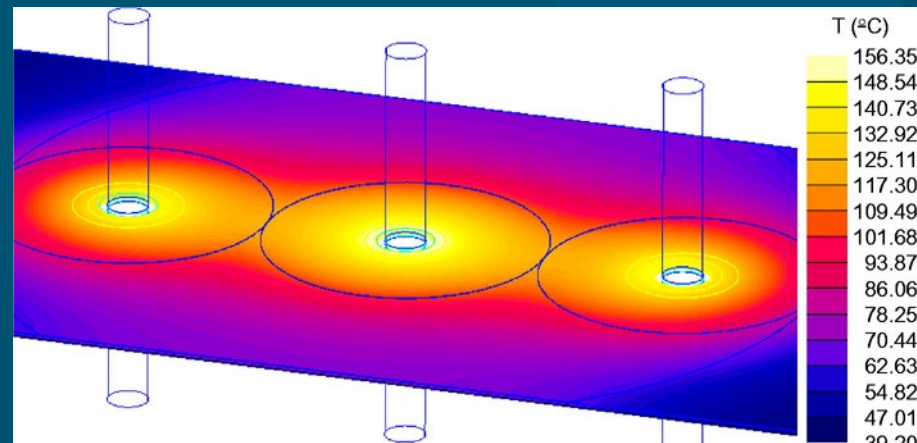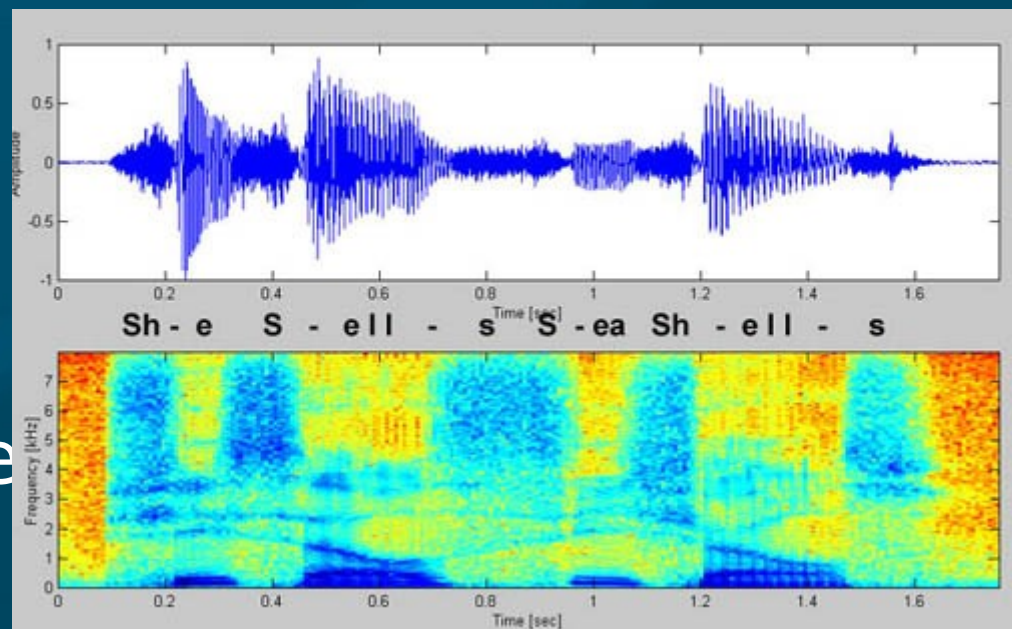

- Decompose a signal into sines+cosines
  - More frequencies ⇒ closer approx
- Spatial domain ↔ frequency domain
  - u(x,y) ↔ $\tilde{\tilde{u}}$(ω)
- Low-pass filter: only use a few sin/cos
  - Attenuate higher-freq components
  - ⇒ this happens with the heat eqn!
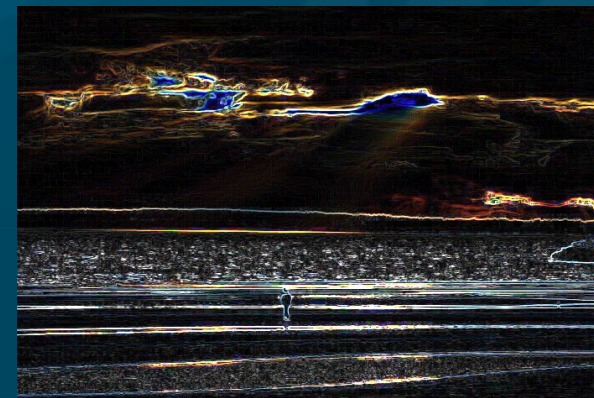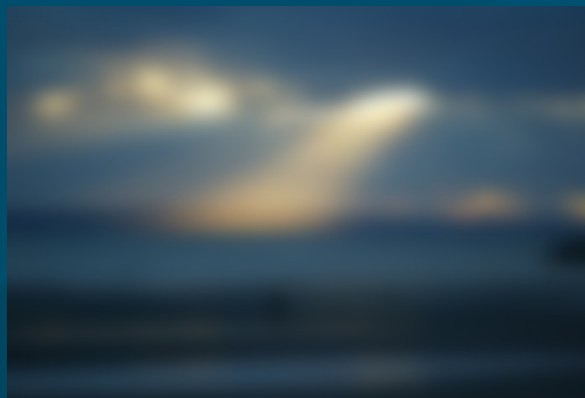
Application: audio signals

- Time-domain waveform: air pressure vs. time
- Frequency-domain signal: volume vs. frequency
- Spectrogram: frequency-domain signal within a sliding window of time: volume vs. freq + time
- What does a low-pass filter do to an audio signal?
  - Crank up the bass!
- The same heat equation can be applied here, too!

$$\frac{\partial u}{\partial t} = k \nabla^2 u$$

- Ferrite for RF interference

Sh - e   S - ell -   s   S - ea Sh - ell -   s

FlyingPudding

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Application: image analysis

- Spatial signal: pixel intensity vs. location
  - Treat R, G, B separately
- What does a low-pass filter do to an image?
  - Heat equation ⇒ Gaussian blur! $$\frac{\partial u}{\partial t} = k\,\nabla^2 u$$
- High-pass: detect edges/noise
- Photoshop's "unsharp mask" = high-pass filter only on portions of the image that show edges
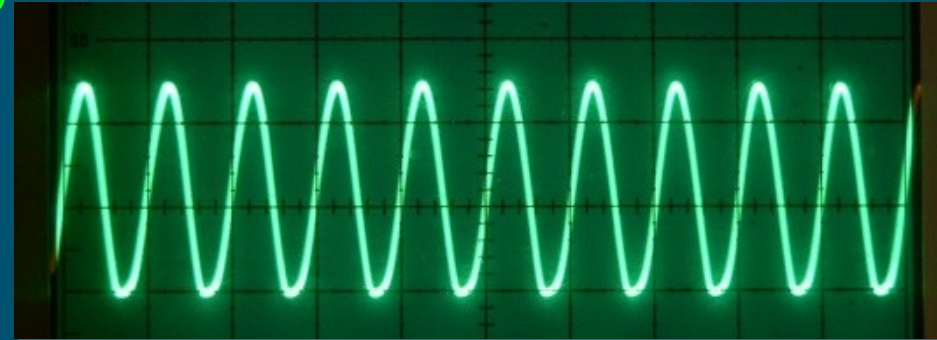
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Application: electrical signals

- Time-domain signal: voltage vs. time

- Alternating current (AC): 60 Hz sine wave

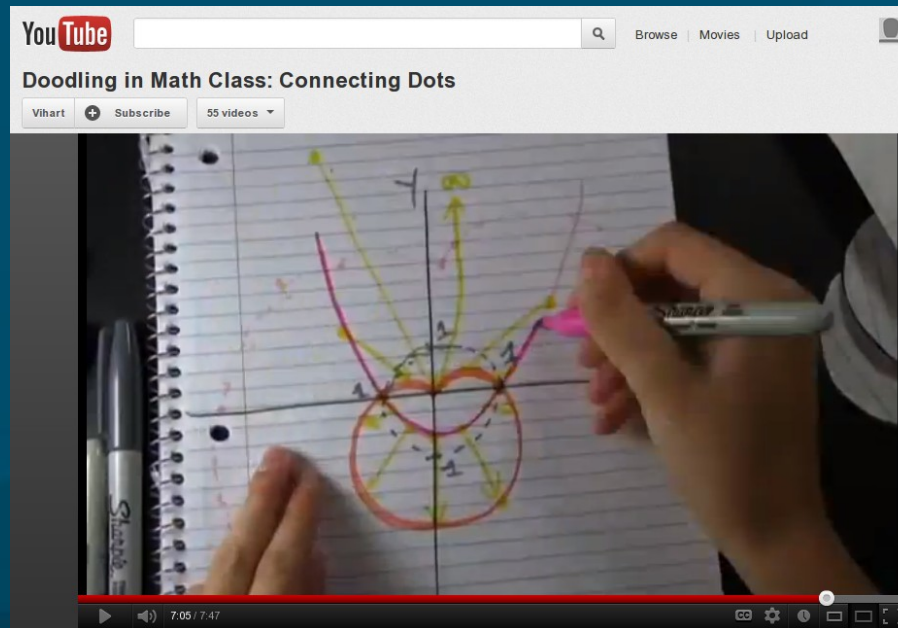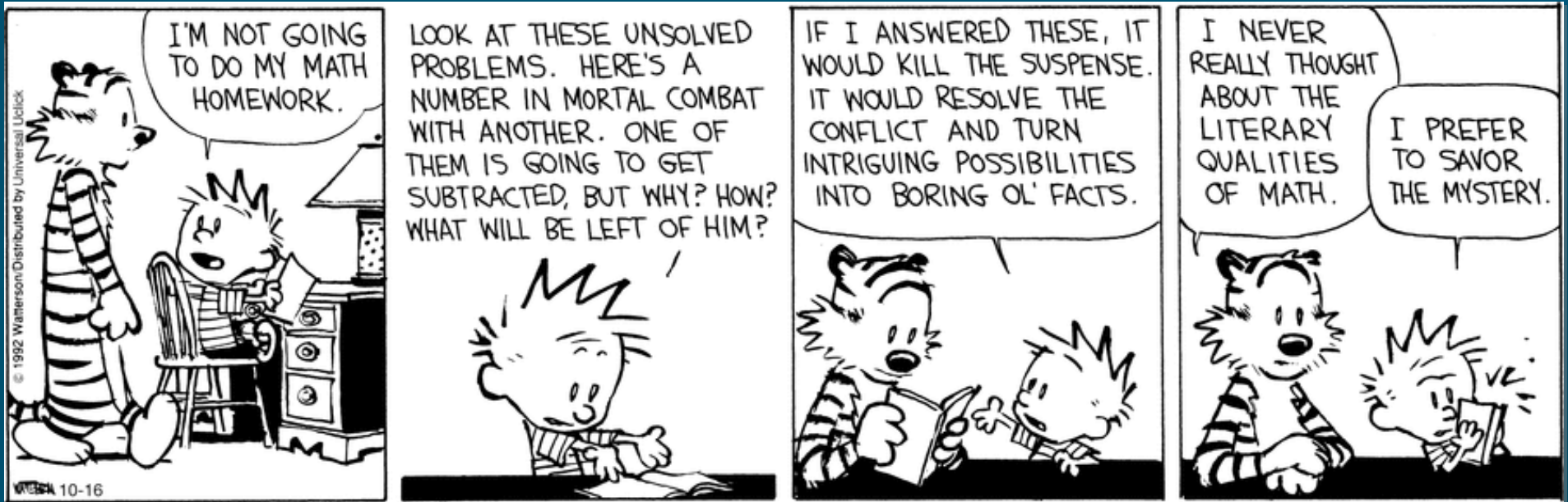 - "Dirty" power: disturbances on top of the basic AC signal

- VGA cable: image data, serialised and converted to electrical signal

  - RF interference: very high freq unwanted extra disturbances

  - Ferrite core: low-pass filter (heat eqn again!) $\dfrac{\partial u}{\partial t} = k \, \nabla^2 u$

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The joy of exploration in math

Go check out
Vi Hart's site!

Job 38:1-7

Then the Lord answered Job out of the storm. He said: "Who is this that darkens my counsel with words without knowledge? Brace yourself like a man; I will question you, and you shall answer me."

"Where were you when I laid the earth's
  foundation? Tell me, if you understand.

Who marked off its dimensions? Surely you know!
  Who stretched a measuring line across it?

On what were its footings set,
  or who laid its cornerstone —

While the morning stars sang together
  and all the angels shouted for joy?

Job 38:31-36

"Can you bind the beautiful Pleiades?
    Can you loose the cords of Orion?

Can you bring forth the constellations in their
    seasons or lead out the Bear with its cubs?

Do you know the laws of the heavens?
    Can you set up God's dominion over the earth?

"Can you raise your voice to the clouds
    and cover yourself with a flood of water?

Do you send the lightning bolts on their way?
    Do they report to you, 'Here we are'?

Who endowed the heart with wisdom
    or gave understanding to the mind?

Math as servanthood

- Observation ⇒ abstraction ⇒ application
- We study math in order to help others and to reveal the beauty & elegance in God's design,
  - Not in order to prove God or to elevate our own logic so as to judge God's words

- "Where is the wise man? Where is the scribe? Where is the debater of this age? Has not God made foolish the wisdom of the world?"
- "The foolishness of God is wiser than men, and the weakness of God is stronger than men."
    - → (1 Cor 1:20, 25)

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