Multiple Regression

Please download from "Example Datasets":
- ***Record2.sav***
- ***Domene.sav***

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CPSY501
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Outline: Multiple Regression

- Requirements on Variables: SampleSize, DV, IVs
- Building a Regression Model
  - Shared vs. Unique Variance
  - Strategies for Entering IVs
  - Interpreting Output
- Diagnostic Tests:
  - Residuals, Outliers, and Influential Cases
- Checking Assumptions:
  - Non-multicollinearity, independence, normality, homoscedasticity, linearity

Encouragement on Research

- Undergrad students: "is this on the test?"
  - "What do I need to do to pass?"
  - Doing the bare minimum: 1 DV, 2 IVs, 1 test
- Graduate students / prep for research:
  - "What structure/effects are in the data?"
  - Do whatever it takes to understand the data
- You may need several RQs
- Your RQs may change as you progress
- Have a theme/goal and aim to tell a story about the effects in the dataset

Regression Modelling Process

(1) RQ: IVs/DVs, metrics, sample size, collect data

(2) Clean: data entry errors, missing data, outliers

(3) Explore: assess requirements, xform if needed

(4) Build model: order & method of entry from RQ

(5) Test model: "diagnostic" issues:

- Multivariate outliers, overly influential cases

(6) Test model: "generalizability" issues:

- Multicollinearity, linearity of residuals

(7) Run final model and interpret results

Selecting Variables

- According to your model or theory, what variables might relate to your outcomes?
  - Does the literature suggest important vars?
- Do the variables meet all the requirements for an OLS multiple regression?
- Record sales example:
  - DV: what is a possible outcome variable?
  - IV: what are possible predictors, and why?

Choosing Good Predictors

- It's tempting to just throw in hundreds of predictors and see which ones contribute most
  - Don't do this! There are requirements on how the predictors interact with each other!
  - Also, more predictors → less power
- Have a theoretical/prior justification for them
- Example: what's a DV you are interested in?
  - Come up with as many possible good IVs as you can – have a justification!
    - Background, internal personal, current external environment

Using Derived Variables

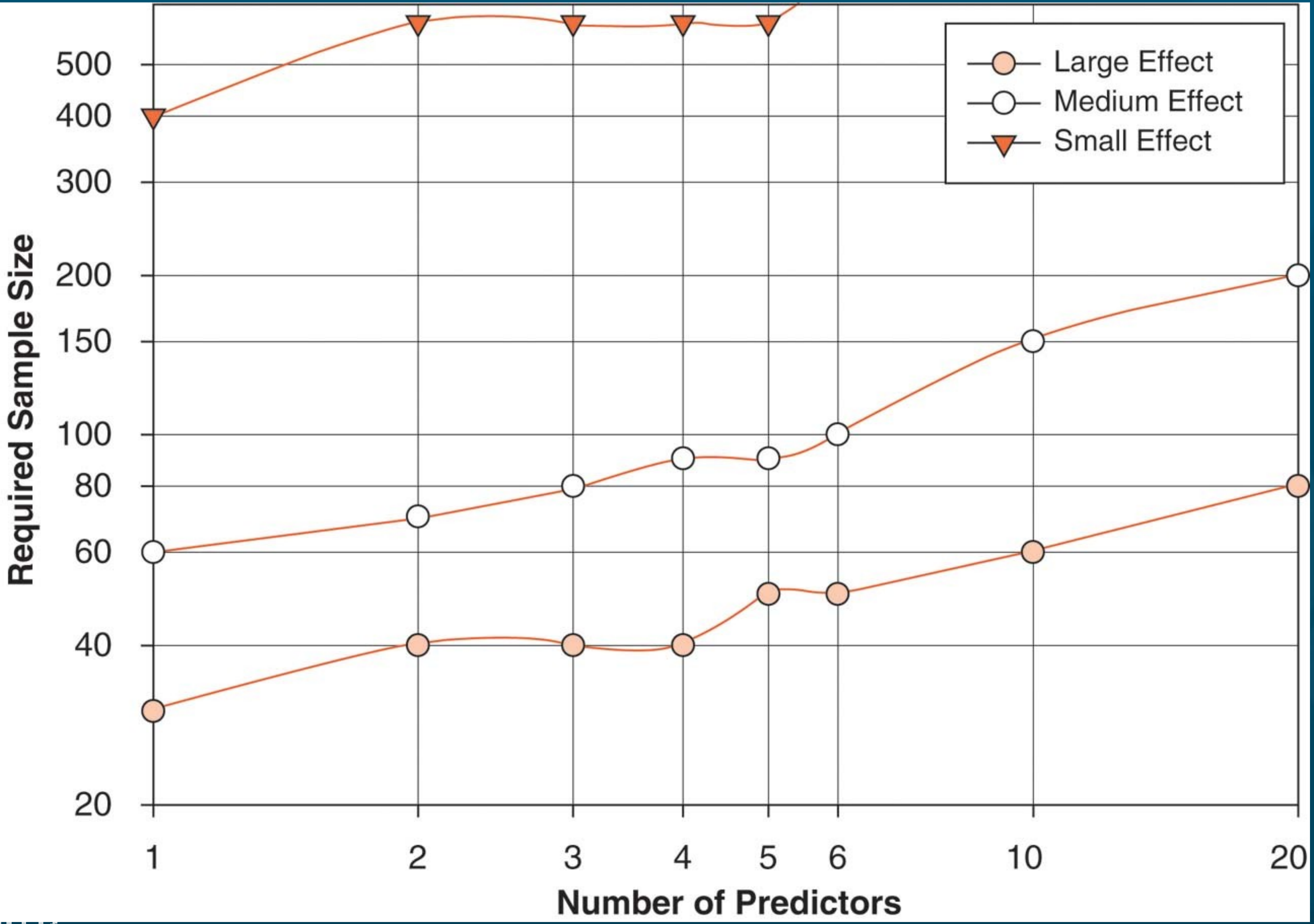
You may want to use derived variables in regression, for example:

- Transformed variables (to satisfy assumptions)
- Interaction terms: (“moderating” variables)
  - e.g., Airplay * Advertising Budget
- Dummy variables:
  - e.g., coding for categorical predictors
- Curvilinear variables (non-linear regression)
  - e.g., looking at X^2 instead of X

Required Sample Size

- Depends on effect size and # of predictors
  - Use G*Power to find exact sample size
  - Rough estimates on pp. 172-174 of Field
- Consequences of insufficient sample size:
  - Regression model may be overly influenced by individual participants (not generalizable)
  - Can't detect “real” effects of moderate size
- Solutions:
  - Collect more data from more participants!
  - Reduce number of predictors in the model

Sample Size Estimates (Field)



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Requirements: Outcome var

- Must be interval/continuous:
  - Solutions:
    - Categorical DV: use Logistic Regression
    - Ordinal DV: use Ordinal Regression,
      or collapse into categories,
      or treat as interval (only if enough ranks)
- Independence of scores (research design):
  - If not: invalid conclusions
  - Solutions: redesign data set, or
    - Multi-level modelling instead of regression

Requirements: Outcome var

- Normal (use normality tests):
  - If not: significance tests may be misleading
  - Solutions: Check for outliers, transform data, use caution in interpreting significance
- Unbounded distribution (obtained range of responses versus possible range of responses):
  - If not: artificially deflated R^2
  - Solutions:
    - Collect more data from missing range
    - Use a more sensitive instrument

Requirements: Predictors

- Interval-level
  - Can be categorical, too (see next page)
  - If ordinal, can collapse into categories or treat as if scale (if have enough ranks)
- Full range of variability
  - Check histogram
  - e.g., if an IV only covers 1-3 on a scale of 1-10, then the regression model will predict poorly for values beyond 3
  - Eliminate/replace poor predictors

Categorical Predictors

- Regression can work for categorical predictors:
- If dichotomous: code as 0 and 1
  - e.g., 1 dichotomous predictor, 1 scale DV:
  - Regression is equivalent to *t*-test!
  - And the slope B_1 is the difference of means
- If *n* categories: use "dummy" variables
  - Choose a base category
  - Create *n-1* dichotomous variables
  - e.g., {BC, AB, SK}: dummys are isAB, isSK

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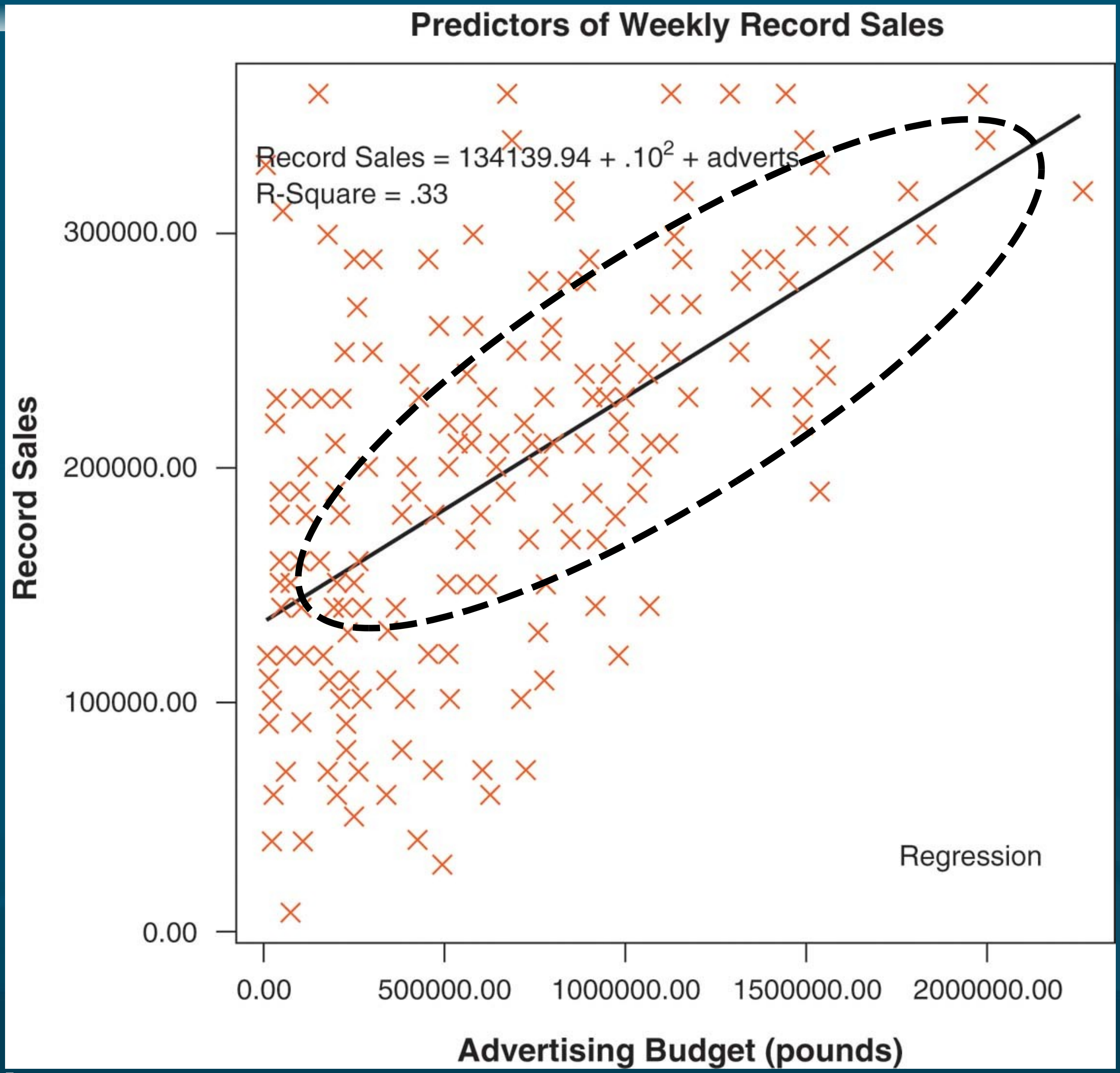
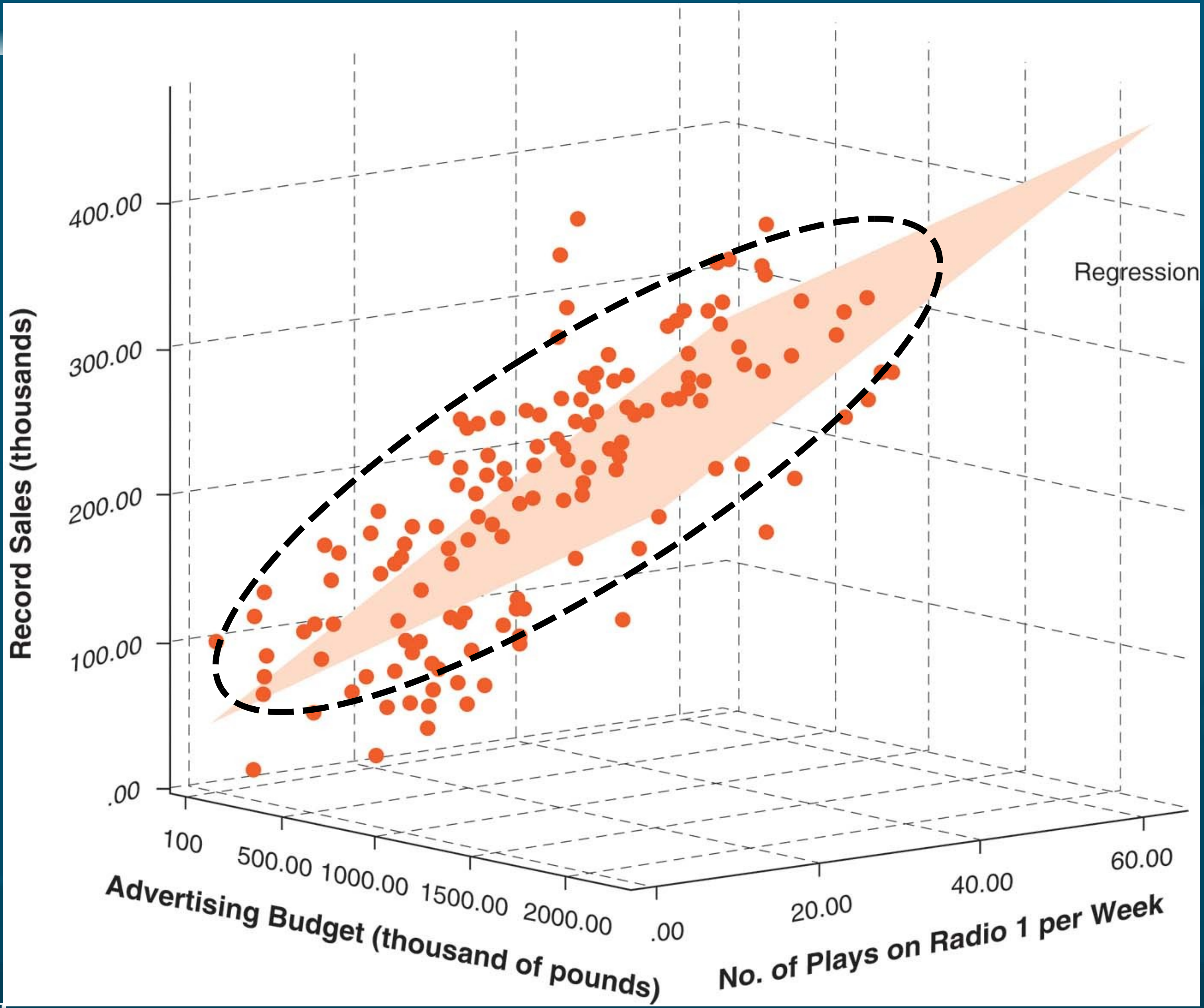
Example: Record Sales data

- Outcome ("criterion"): record sales (RS)
- Predictors: advertising budget (AB), airtime (AT)
  - Both have good 'variability', and n=200
- Research Question: Do AB and AT both show unique effects in explaining Record Sales?
- Research design: Cross-sectional, correlational study with 2 quantitative IVs & 1 quantitative DV (1 year data?)
- Analysis strategy: Multiple regression (MR)

Regression Model with 1 IV



Regression Model with 2 IVs



Asking Precise RQs

- What does literature say about AB and AT in relation to record sales?
  - Previous lit may be theoretical or empirical
  - May focus on these variables or others
  - May be consistent or conflicting results
- Contrast these two seemingly similar RQs:
  - Is AB or AT more important for Sales?
  - Do AB and AT both show unique effects in accounting for the variance of Sales?

Example: Record Sales

- Dataset: Record2.sav
- Analyze → Regression → Linear
- Dependent: Record Sales (RS)
- Independent: Advertising (AB) & Airtime (AT)
  - This is a "simultaneous" regression
- Statistics: check R^2 change and partial correl.
- Review output: *t*–test for each beta coefficient tests significance of unique effects for each predictor

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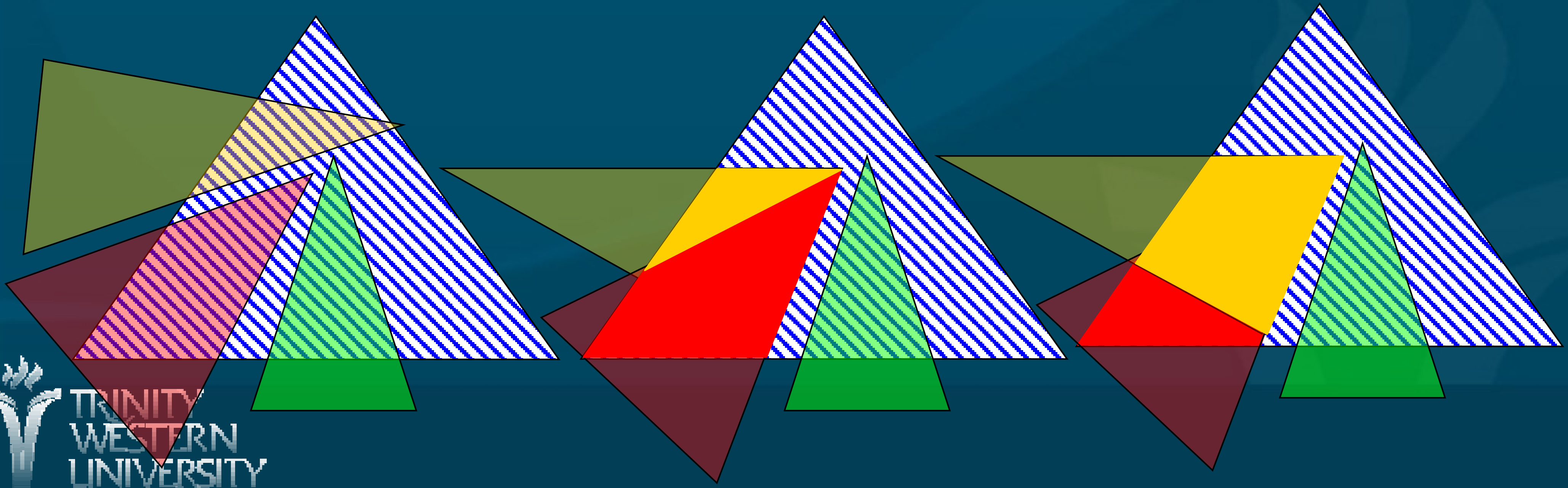
Shared vs. Unique Variance

- When predictors are correlated, they account for overlapping portions of variance in outcome
  - Redundant IVs, mediation, shared background effects, etc.
- Order of entry will help distinguish shared and unique contributions



Order of Entry

- Predictors in same block are entered into model at the same time
- Subsequent blocks only look at remaining variance after previous blocks have been factored out
- To find a predictor's unique contribution, put it last after other predictors are factored out
- Try several runs with different orderings to get each predictor's unique effect
- Order for your final run should reflect theory about relative importance of predictors

Options for Variable Selection

- Within each block, not all IVs need to be used:
  - Manual method: “Enter” (forced entry)
    - All specified IVs will be included
  - “Stepwise” automatic methods:
    - Forward: add significant IVs one-at-a-time
    - Backward: eliminate non-significant IVs
- Best to use “Enter”: manual control
  - You decide order according to theory/lit
- Automatic methods might not show shared effects, interaction effects

Record Sales Example

- Analyze → Regression → Linear
- Dependent: Record Sales
- Statistics: check R^2 change
- Run 1: "simultaneous" regression
  - Both AB and AT in Block 1
- Run 2: AB in Block 1, and AT in Block 2
- Run 3: AT in Block 1, and AB in Block 2

Calculating Shared Variance

- Output from Run 1: Total effect size from both predictors together is 63%
- Run 2: Airtime's unique effect size is 30%
  - Look at last ΔR^2: when airtime is added
- Run 3: Advertising's unique effect size is 27%
- **Shared variance:**
  - = Total minus all unique effects
  - = 63% – 30% – 27% ≈ 6%

Steps for Entering IVs

- First, create a conceptual outline of all IVs and their connections & order of entry.
  - Run a simultaneous regression: look at beta weights & *t*-tests for all unique effects
- Second, create "blocks" of IVs (in order) for any variables that must be in the model
  - Use "Enter" method to force vars into model
  - Covariates may go in these blocks
  - Interaction and curvilinear terms go in last of these blocks

Steps for Entering IVs (cont.)

- Any remaining variables go in a separate block: try all possible combinations to sort out shared & unique variance portions.
  - See record sales example above (no interaction terms were used)
- Summarize the final sequence of entry that clearly presents the predictors & their respective unique and shared effects.
- Interpret the relative sizes of the unique & shared effects for the Research Question

Entering IVs: SPSS tips

- Plan out your order and method on paper
- Each set of variables that should be entered in at the same time should be in a single block.
  - Other vars & interactions go in later blocks
- Usually choose “Enter” method (default)
  - Try automatic (“Backward”) only if needed
- Confirm correct order & method of entry in your SPSS output
  - Usually only need a few blocks of IVs

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Output: “Model Summary”

- R^2: the variance in the outcome accounted for by the model (i.e., combined effect of all IVs)
  - Interpretation is similar to r^2 in correlation
  - Multiply by 100 to convert into a percentage
  - Adjusted R^2: unbiased estimate of the model, always smaller than R^2
- R^2 Change (ΔR^2): Increase in effect size from one block of predictors to the next.
  - *F*-test checks whether this “improvement” is significant.

Output: "ANOVA" Table

- Summarizes results for the model as a whole: Is the "simultaneous" regression a better predictor than simply using the mean score of the outcome?
- Proper APA format for reporting F statistics (see also pp. 136-139 of APA publication manual):
  - $F(3, 379) = 126.43$, $p < .001$

df-regression

df-residual

F-ratio

statistical significance

Output: "Coefficients" Table

- Individual contribution of each predictor, and whether its contribution is significant
- B (b-weight, slope, gradient): Change in outcome, for every unit change of the predictor
- beta (β): Standardized b-weight. Compares the relative strength of the different predictors.
- *t*-test (*p*-value): Tests whether a particular variable contributes a significant unique effect in the outcome variable for that equation.

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Non-significant Predictors

What if the *t*-test shows a predictor's unique effect is non-significant?

- In general, the ΔR^2 will be small. If not, then you have low power for that test & must report that.
- Remove the IV unless there is a theoretical reason for retaining it in the model (e.g., low power, help for interpreting shared effects)
- Re-run the regression after any variables have been removed

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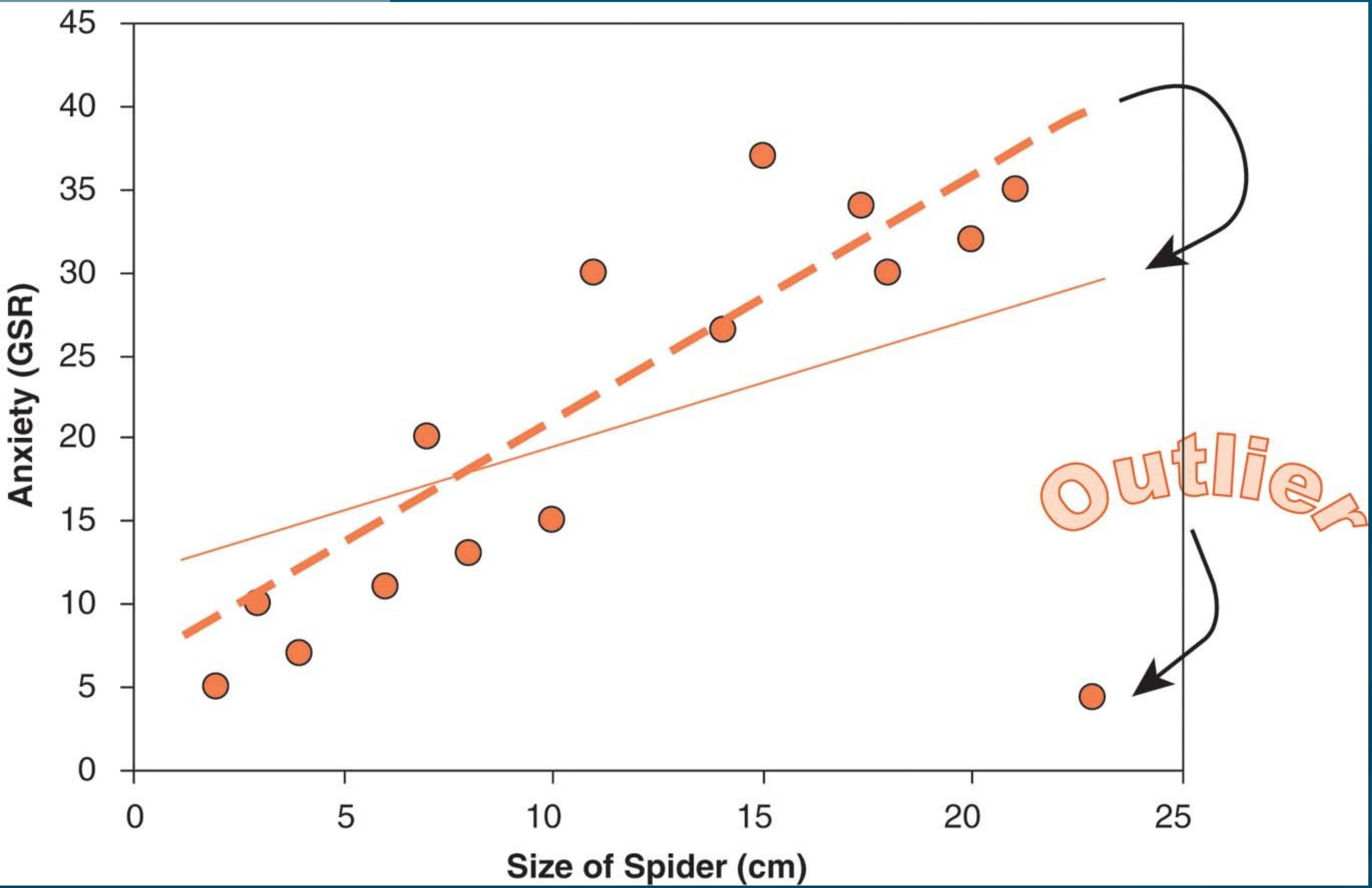
Residuals in Regression

- A residual is the difference between the actual score and the score predicted by the model
  - I.e., the amount of error for each case
- Examine the residuals in a trial run
  - Include all IVs: simultaneous regression
  - Save the residuals in a new variable:
- Analyze → Regression → Linear → Save: “standardized” and/or “unstandardized”

Multivariate Outliers

- Definition: Cases from a different population than what we want to study
  - Combination of scores across predictors is substantially different from rest of sample
- Consequence: distortion of regression line, reduced generalizability
- Screening: Standardized residual $\geq \pm 3$, and Cook's distance > 1
- Solution: remove outliers from from sample (if they exert too much influence on the model)

Effect of Multivariate Outliers



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Overly-Influential Cases

- Definition: A case that has a substantially greater effect on the regression "line" than the majority of other cases in the sample
- Consequence: reduced generalizability
- Screening & Solution:
  - if max. leverage value ≤ 0.20 then safe;
  - if leverage > 0.50 then remove;
  - if in between,
    remove if max. Cook's distance > 1

Outliers & Influential cases

- Outliers and influential cases should be examined and removed together
  - Unlike other aspects of MR, screen only once
  - Why shouldn't you repeat this screening?
- SPSS: Analyze → Regression → Linear:
  - Save: Standardized Resid, Cook's, Leverage
  - Will be saved as additional vars in dataset
- Examine the Residual Statistics table
- Examine the saved scores in the data set
  - Try sorting: Data → Sort

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Multicollinearity

- Definition: Predictors covary too highly; i.e., too much overlap of shared variance
- Consequences: deflated R^2; may interfere with evaluation of β (depending on RQ & design)
- In "Statistics": check "Collinearity Diagnostics"
- Indicators of possible problems: any of:
  - Any VIF (Variance Inflation Factor) score > 10
  - Average VIF is NOT approximately = 1
  - Tolerance < 0.2
- Solution: delete, combine, or transform some of the multicollinear variables

Independence of Residuals

- Definition: Residuals for different cases should not be systematically related
- Consequence: Can interfere with α and power, although effect size is unaffected
- Screening: Durbin-Watson scores that are relatively far away from 2 (on possible range of 0 to 4) indicate a problem with independence.
  - D-W sensitive to case ordering, so ensure cases aren't inherently ordered in dataset
- Solution: No easily implemented solutions. Possibly use multi-level modelling techniques.

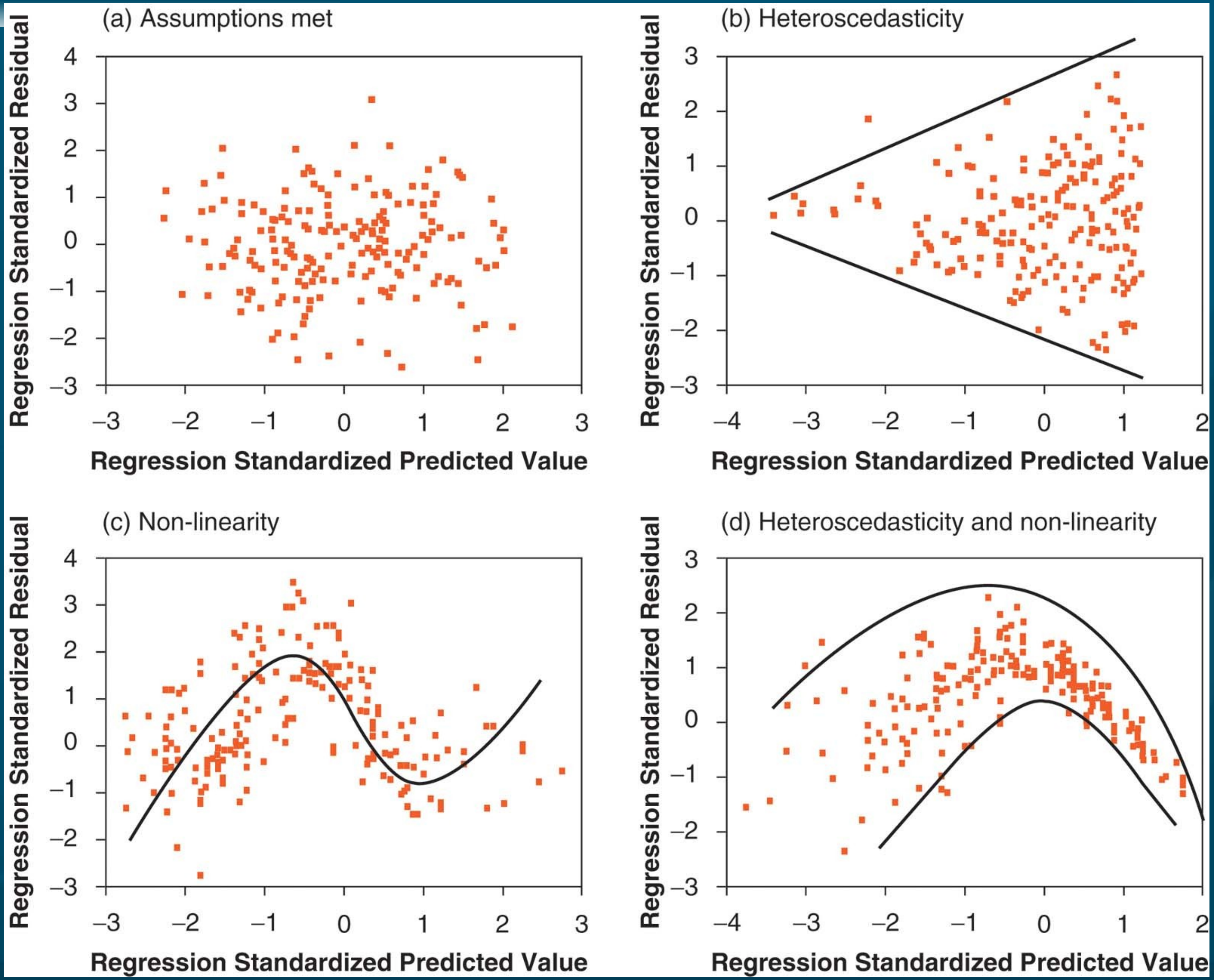
Normally Distributed Residuals

- Definition: Residuals normally distributed
  - Predictors don't have to be normal!
- Consequence: reduced generalizability (predictive value of the model is distorted)
- Screening: normality tests/plots on residuals
  - save standardized residuals
  - Analyze → Descriptives → Explore → "Normality tests with plots"
- Solution: screen dataset for problems with the predictor variables (non-normal, or based on ordinal measurements), and deal with them

Homoscedastic Residuals

- Definition: Residuals should have similar variances at every point on the regression line
    - Generalisation of homogeneity of variance
- Consequence: the model is less accurate for some people than others
- Screening: fan-shaped residual scatterplots:
    - Analyze → Regression → Linear → Plots: X: “ZPRED” Y: “ZRESID”
- Solution: identify the moderating variable and incorporate it; use weighted OLS regression; or accept it and acknowledge the drop in accuracy

Heteroscedascticity



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Non-linear Relationships

- Definition: Relationship between predictor and outcome is not linear (i.e., a straight line).
- Consequences: sub-optimal fit for the model (R^2 is lower than it should be)
- Screening: examine residual scatterplots OR use curve estimation:
  - Analyze → Regression → Curve estimation
- Solutions: Model the non-linear relationship by entering a polynomial term into the regression equation (e.g., IV^2, IV^3)
  - Or just accept the poorer fit

Exercise: Regression with SPSS

- Dataset: Domene.sav
- You try it! Build a regression model with:
  - DV: “educational attainment”
  - IV: Block 1: “academic performance”
  - IV: Block 2: “educational aspirations” and “occupational aspirations”
  - Use “Enter” method (force entry)
- Ask SPSS for ΔR^2 and partial correlation scores

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