ANOVA: Analysis of Variance

- ***Please download:***
  treatment4.sav

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CPSY 501
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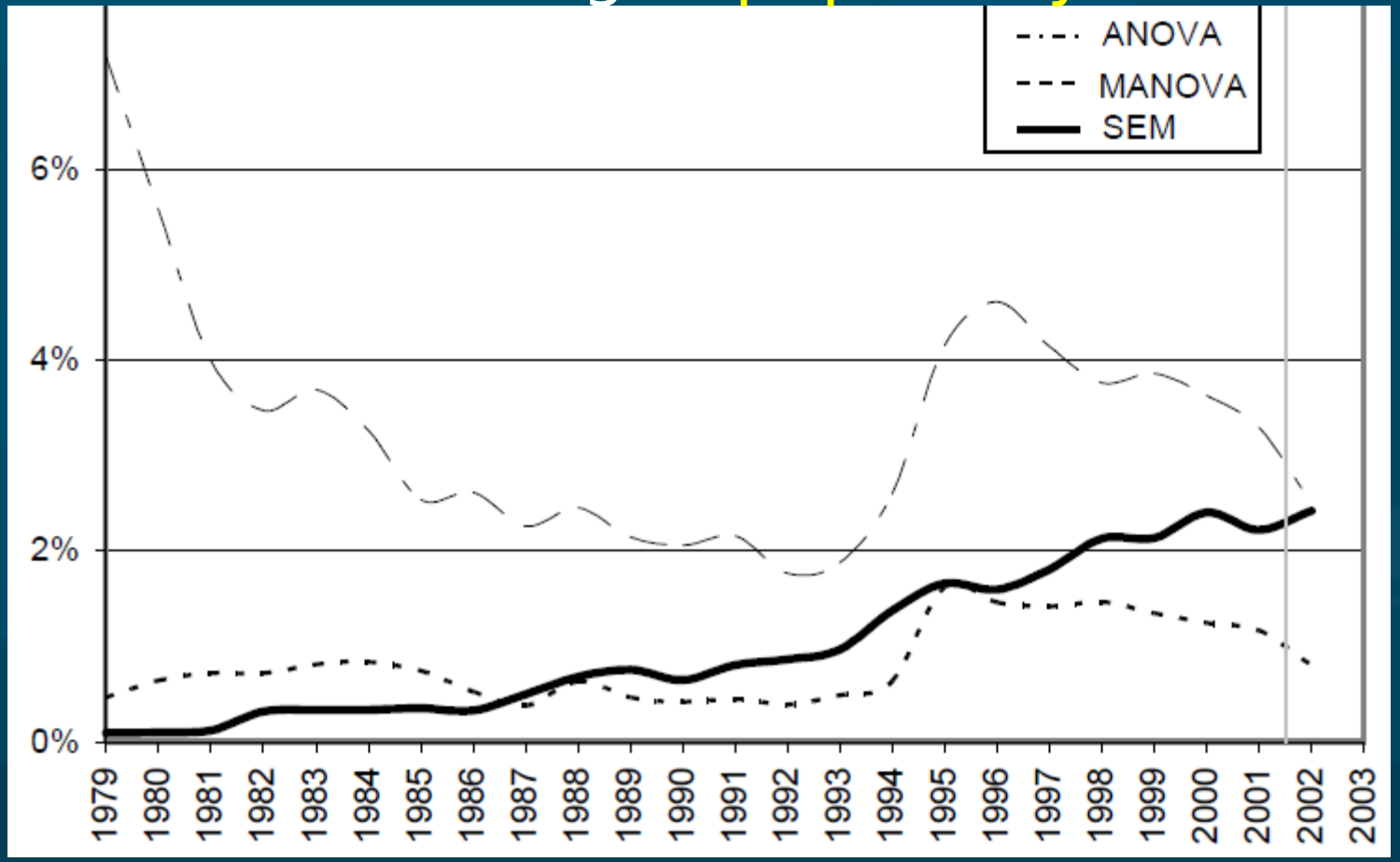
Outline for today

- Core concepts of ANOVA (with pictures!)
  - One-way, factorial, RM, MANOVA, ANCOVA
- Running ANOVA in SPSS
  - Interpreting output
  - Follow-up analysis: post-hoc comparisons
  - Follow-up analysis: planned comparisons
- Assumptions of ANOVA: Parametricity
- Introduction to ANCOVA
  - Use of ANCOVA in therapy research

Trends in Research: ANOVA

Nachtigall, C., Kroehne, U., Funke, F., & Steyer, R. (2003). (Why) Should we use SEM? Pros and cons of structural equation modeling. Methods of Psychological Research - Online, 8(2), 1-22.

- ANOVA is a conceptual framework / foundation
- Other methods like Structured Equation Modelling are also increasing in popularity
- Citations in PsychINFO, 1979-2003



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ANOVA: Core Concepts

- Extension of *t*-test to multiple groups
- Why not just do a bunch of *t*-tests (all pairs)?
  - Multiple comparisons: to control Type-I error, we'd have to spread out our α across all *t*-tests, so each one is a stricter test
  - We will do this in post-hoc analysis!
- ANOVA can be thought of as a form of regression, with categorical predictors
- Global test for group differences (omnibus)
  - Post-hoc tests then locate the differences

Kinds of ANOVA

- All ANOVAs require parametric outcome var!
- One-way ANOVA: One categorical predictor
  - If IV has only two levels, this is a *t*-test!
- Factorial ("between subjects") ANOVA: Two or more predictors, plus interactions
- Repeated Measures ("within subjects") ANOVA: Multiple observations of IV on each participant
  - e.g., time: pre/post/follow-up

Kinds of ANOVA (continued)

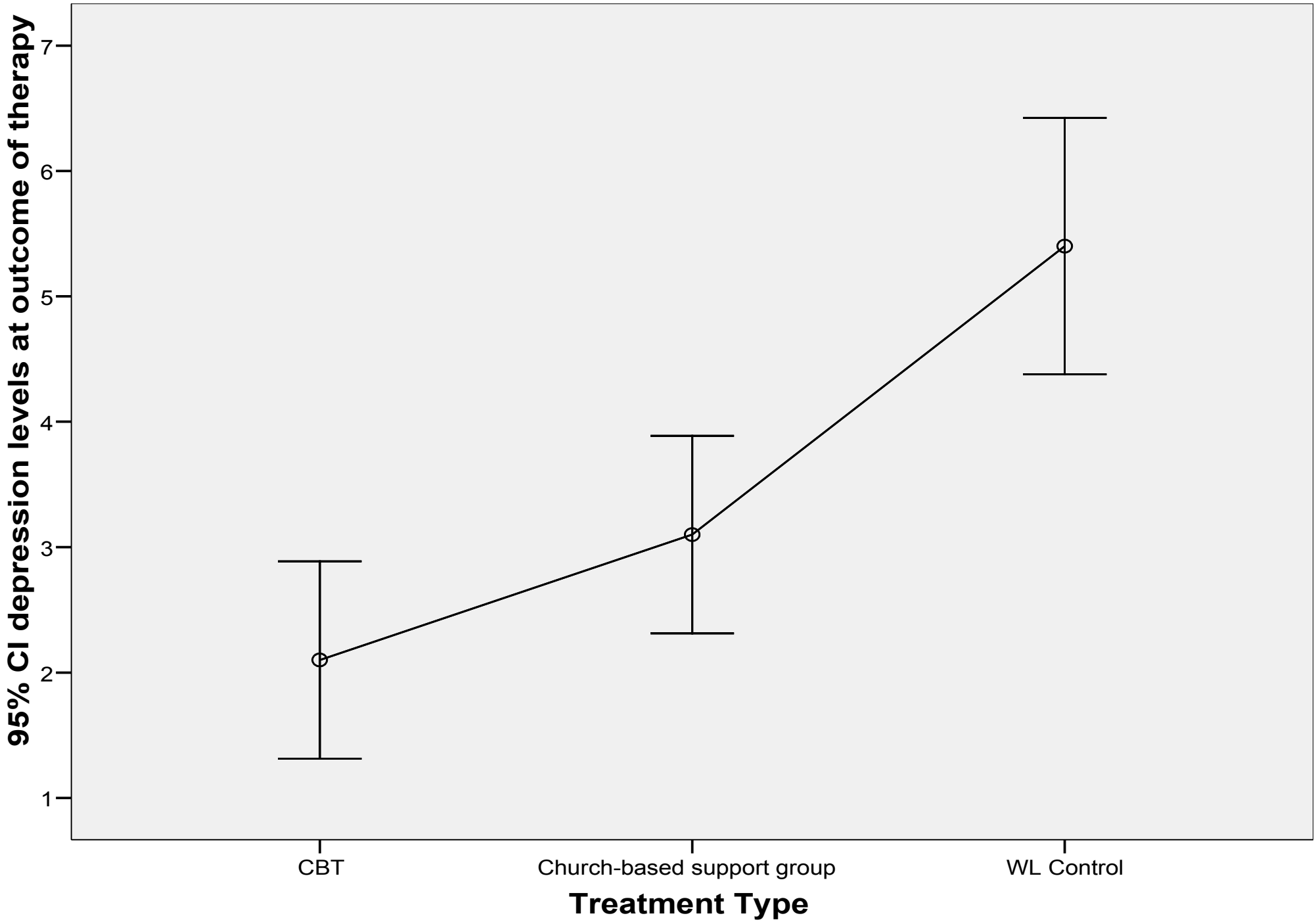
- Mixed (between/within) ANOVA:
  Some predictors are between-groups (factorial), some are within-groups (repeated measures)
- MANOVA: Two or more outcome variables, correlated, and in the same analysis
- ANCOVA: Any of the above designs, trying to control for an extraneous influence (covariate) on the DV

ANOVA: The *F*-ratio

- ANOVA's test statistic is the *F*-ratio:
  $F = MS_{model} / MS_{residual}$
  - "Model": variation/difference among cells
  - "Residual": variation within each cell
  - Same as *F*-ratio in regression: model / error
- Categorical IVs divide the sample into cells: all combinations of groupings from the IVs
- e.g., treatment4.sav:
  - 1 IV (treatment type) with 3 levels: CBT (cog-beh), CSG (church), WL (control)

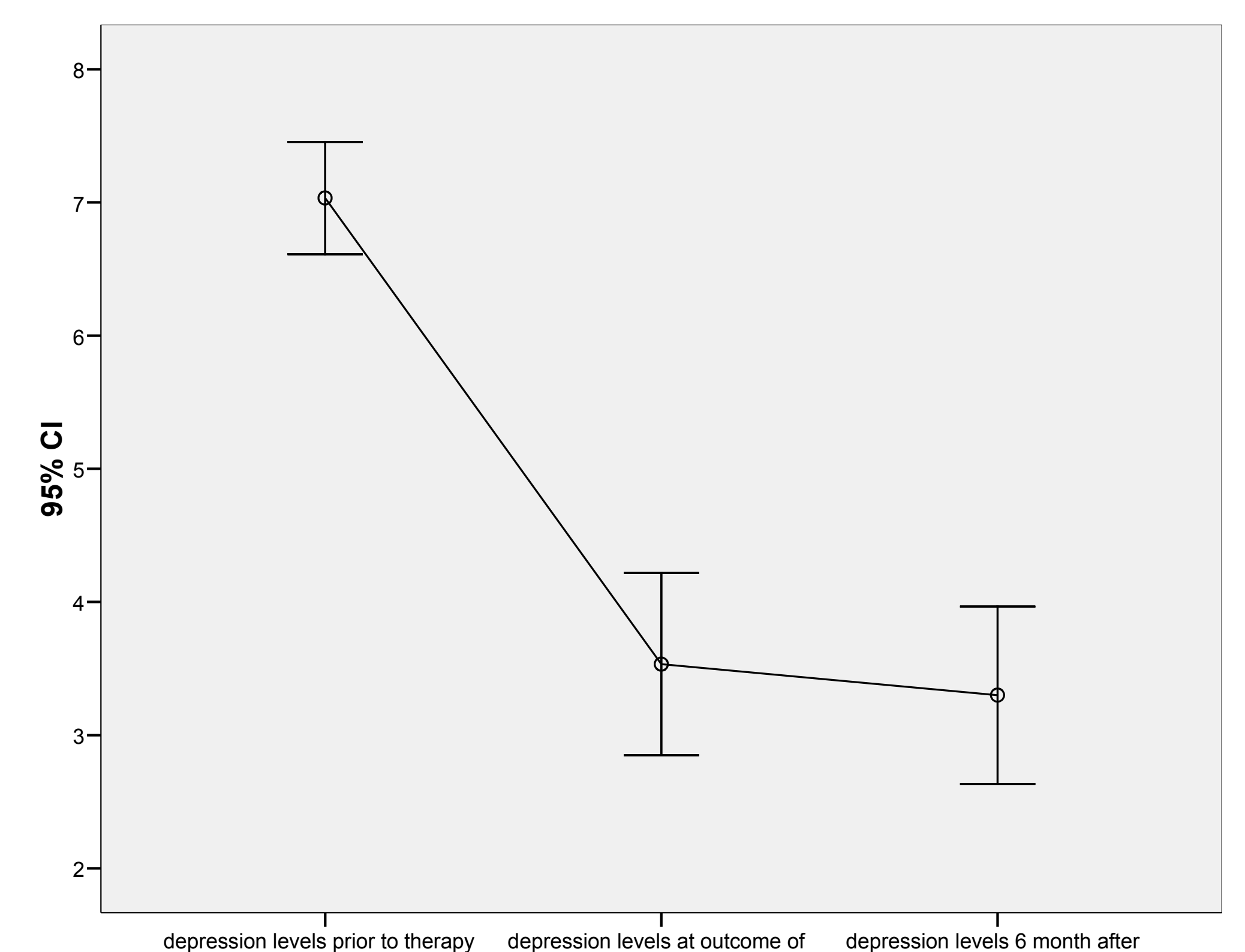
Picture: Within-Cell Variation

- Error bar charts can help visualize both the "between-cell" and "within-cell" variation
  - Confidence intervals around cell means describe within-cell variation (residual)
  - Cell mean differences describe between-cell variation (model effect of IV)
- SPSS: Graphs → Legacy Dialogs → Error bar → Simple and "groups of cases":
  - Variable (DV): depression levels at outcome
  - Category Axis (IV): treatment type
  - (Also try: Line, with Options → "error bars")


95% CI depression levels at outcome of therapy
7
6
5
4
3
2
1
CBT
Church-based support group
WL Control
Treatment Type

Picture: Repeated Measures

- The same DV (depression levels) is measured on the same individual at three different times:
  - Pre-treatment, at outcome of therapy, and at 6-month follow-up
- SPSS: Graphs → Legacy Dialogs → Error bar → Simple and "separate variables":
  - Error Bars (DV): depression at 3 time points
- This graph ignores treatment group; i.e., collapsed across all treatment groups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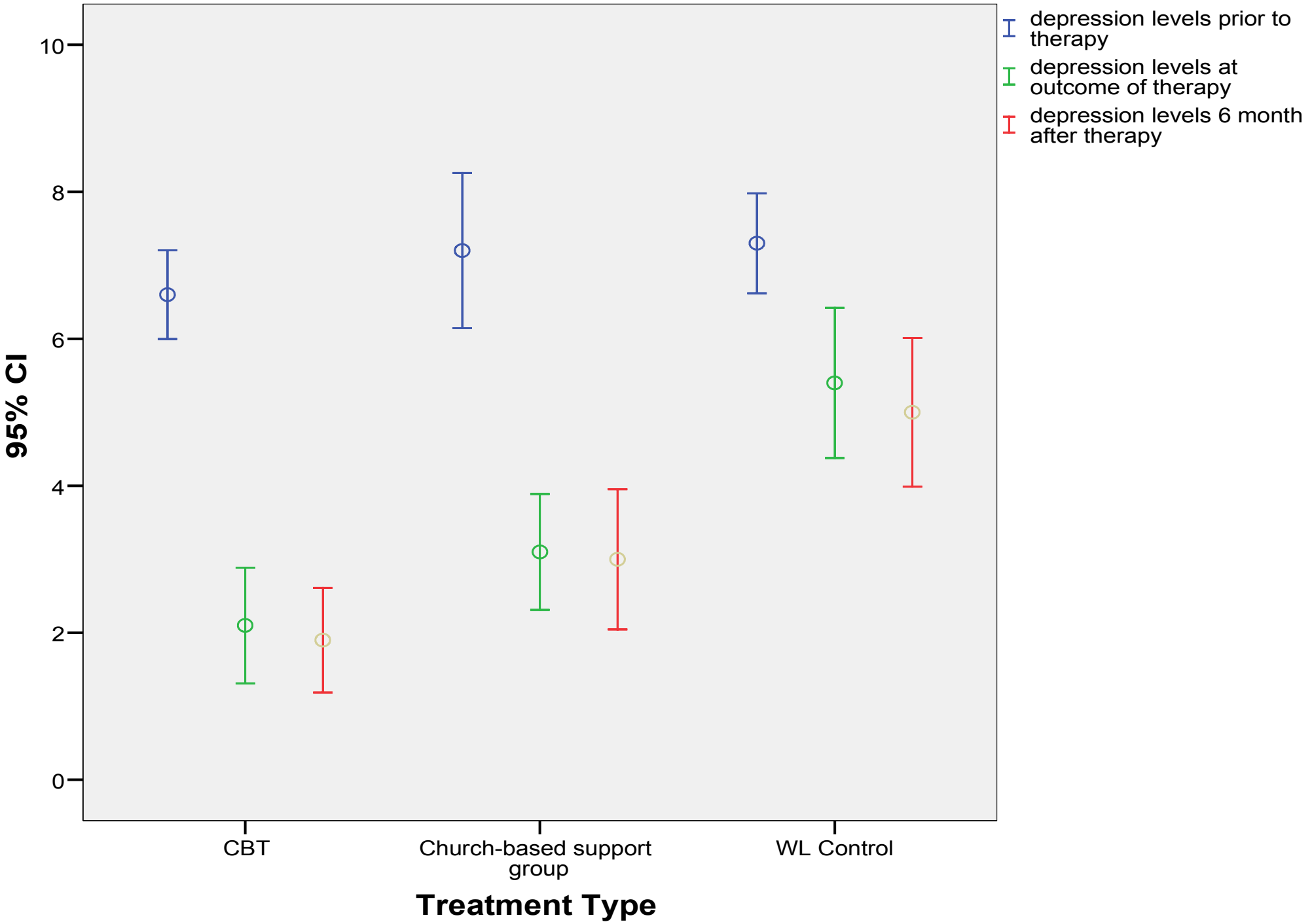
depression levels prior to therapy
depression levels at outcome of
depression levels 6 month after

Repeated Measures, by Group

- If we want to visualize the treatment effect, we can use clustered error bars to see the depression levels of each treatment group at each time point:
- SPSS: Graphs → Legacy Dialogs → Error bar → Clustered and "separate variables":
  - Variables (DV): depression at 3 time points
  - Category Axis (IV): treatment type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Running ANOVA in SPSS

- Analyze → General Linear Model → Univariate:
  - Dependent Variable: our outcome variable
    - e.g., depression levels at outcome of therapy
  - Fixed Factors: our (categorical) predictors
    - e.g., Treatment Type
  - Check: Options → "Estimates of effect size" and "Descriptive statistics"
- Omnibus test: H_0 is that all groups are same, H_A is that there is some group difference somewhere

Interpreting ANOVA Output

- There is a significant effect of treatment type on depression, $F(2,27) = 19.23$, $p < .001$.
- This is a strong / large effect, $\eta^2 = 59\%$

Tests of Between-Subjects Effects

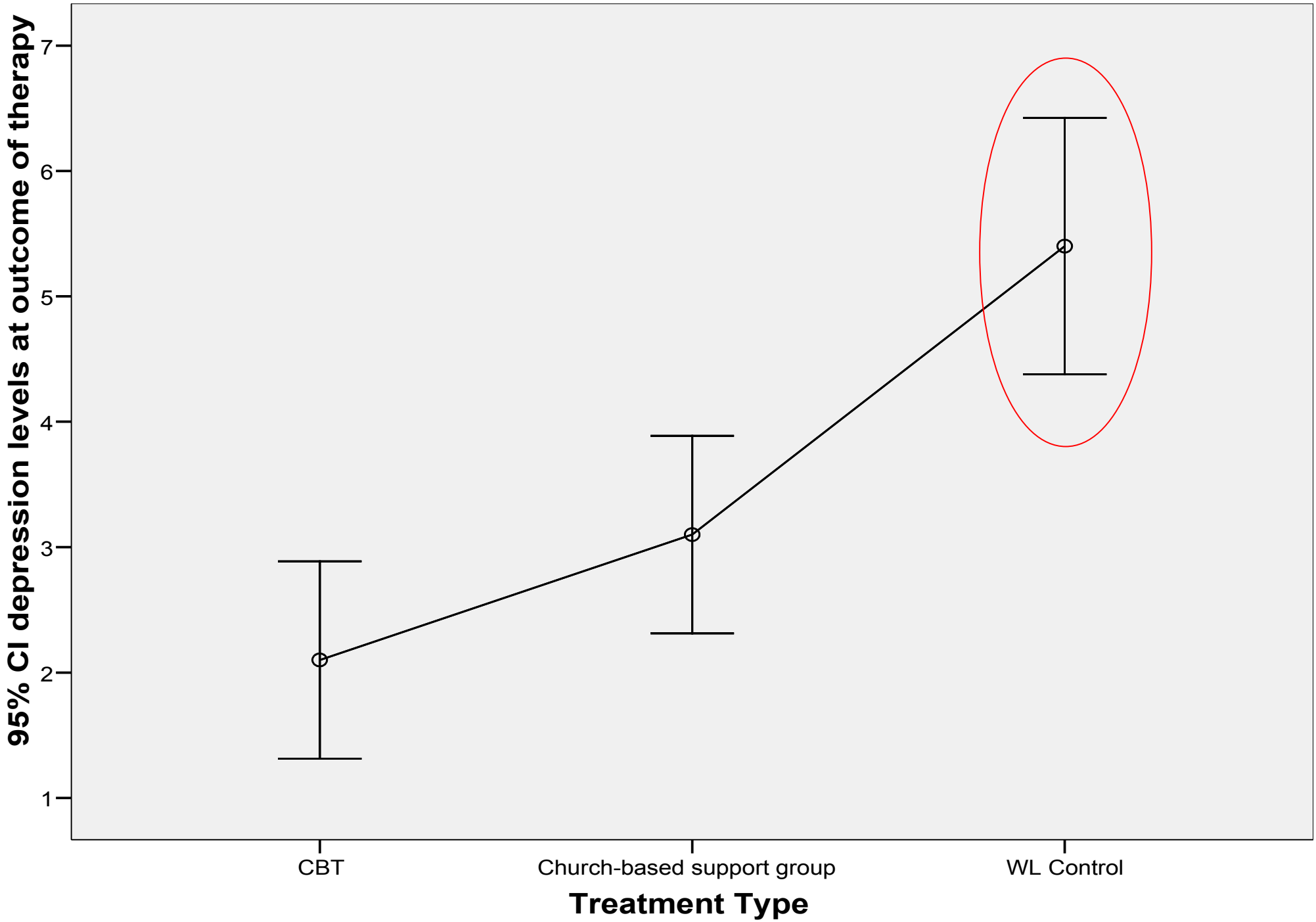
Dependent Variable: Level of trauma symptoms

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Eta Squared
Corrected Model	57.267[a]	2	28.633	19.231	.000	.588
Intercept	374.533	1	374.533	251.552	.000	.903
TREATMNT	57.267	2	28.633	19.231	.000	.588
Error	40.200	27	1.489			
Total	472.000	30				
Corrected Total	97.467	29				

a. R Squared = .588 (Adjusted R Squared = .557)

Finding Group Differences

- Yay, so the global *F*-test tells us there is a group difference! Now what?
  - Effect size: $\eta^2 = 59\%$: analogous to R^2
- Where do the differences lie? Highest group? Lowest group? Clusters of similar groups?
- Two strategies:
  - Post-hoc: *t*-tests between all pairs of cells
    - Watch out for α of multiple comparisons!
  - Planned comparisons: we tell SPSS to focus on a few pairings that may show differences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Post-Hoc Comparisons

- In post-hoc analysis, we ask SPSS to run *t*-tests between all possible pairings of cells
  - Potentially lots of *t*-tests!
  - Analyze → GLM → Univariate → Post-Hoc
- Useful in exploratory analysis – no prior hypotheses about which groups might differ
- Watch out for Type-I error! Must distribute our $\alpha=0.05$ amongst all comparisons
  - So each *t*-test gets a tiny slice of α
  - Or equivalently, adjust *p*-values up

Types of Post-Hoc Tests

- Tukey or REGW-Q (Ryan, Einot, Gabriel, Welch):
  - Best choice if all groups are of equal size and equal variance
- Gabriel: when sizes are roughly similar (~10%) and variances are equal
- Hochberg's GT2:
  - For different group sizes but equal variances
- Games-Howell: if unequal variances
  - You can select this one anyway and compare it with other methods

Post-Hoc Analysis: Notes

- SPSS's menu system for ANOVA has limited options for post-hoc and planned comparisons.
  - For more complex options, try multiple regression or SPSS syntax
- Pairwise comparisons tables help to show where specific differences lie, but:
- Confidence intervals must be adjusted for multiple comparisons: try Bonferroni or Sidak
  - Otherwise the *p*-values will be smaller than they ought to be

Output: Pairwise Comparisons

Pairwise Comparisons

Dependent Variable: depression levels at outcome of therapy

(I) Treatment Type	(J) Treatment Type	Mean Difference (I-J)	Std. Error	Sig.[a]	95% Confidence Interval for Difference[a]	
					Lower Bound	Upper Bound
CBT	Church-based support group	-1.000	.546	.234	-2.393	.393
	WL Control	-3.300*	.546	**.000**	-4.693	-1.907
Church-based support group	CBT	1.000	.546	.234	-.393	2.393
	WL Control	-2.300*	.546	**.001**	-3.693	-.907
WL Control	CBT	3.300*	.546	**.000**	1.907	4.693
	Church-based support group	2.300*	.546	**.001**	.907	3.693

Based on estimated marginal means

*. The mean difference is significant at the .05 level.

a. Adjustment for multiple comparisons: Bonferroni.

Post-Hoc: Output

Multiple Comparisons

Dependent Variable: depression levels at outcome of therapy

	(I) Treatment Type	(J) Treatment Type	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Tukey HSD	CBT	Church-based support group	-1.00	.546	.178	-2.35	.35
		WL Control	-3.30*	.546	**.000**	-4.65	-1.95
	Church-based support group	CBT	1.00	.546	.178	-.35	2.35
		WL Control	-2.30*	.546	**.001**	-3.65	-.95
	WL Control	CBT	3.30*	.546	**.000**	1.95	4.65
		Church-based support group	2.30*	.546	**.001**	.95	3.65
Bonferroni	CBT	Church-based support group	-1.00	.546	.234	-2.39	.39
		WL Control	-3.30*	.546	**.000**	-4.69	-1.91
	Church-based support group	CBT	1.00	.546	.234	-.39	2.39
		WL Control	-2.30*	.546	**.001**	-3.69	-.91
	WL Control	CBT	3.30*	.546	**.000**	1.91	4.69
		Church-based support group	2.30*	.546	**.001**	.91	3.69
Games-Howell	CBT	Church-based support group	-1.00	.492	.133	-2.26	.26
		WL Control	-3.30*	.571	**.000**	-4.76	-1.84
	Church-based support group	CBT	1.00	.492	.133	-.26	2.26
		WL Control	-2.30*	.571	**.002**	-3.76	-.84
	WL Control	CBT	3.30*	.571	**.000**	1.84	4.76
		Church-based support group	2.30*	.571	**.002**	.84	3.76

Based on observed means.

*. The mean difference is significant at the .05 level.

Equality of variances not assumed

Post-Hoc: Interpretation

- The various options for testing all say:
  - Control group (WL) is significantly different from treatment groups (CSG & CBT), but
  - The treatment groups (CSG & CBT) are not different from one another
- Some choices of post-hoc test are more "conservative" – with lower significance levels reported (e.g., Games-Howell)

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Planned Comparisons

- "A Priori" ("before the fact"):
  planned comparisons / contrasts
- Instead of trying all pairs, use theory and our hypotheses to focus on comparisons of interest
  - "sniper" instead of "shotgun"
  - Only test conceptually relevant contrasts
- Planned comparisons help to overcome the problem of inflated Type-I error due to conducting multiple significance tests
- Also allows for sets of groups to be compared (not just 1:1 pairwise comparisons)

Specifying a Contrast: Weights

- To specify a desired contrast, set weights on each group (each level of the categorical IV)
- All the positive-weight groups will be compared against all the negative-weight groups
- Total sum of weights must balance to 0
- Zero-weighted groups will be omitted
- A group on one side cannot be combined with groups from the other side in subsequent tests
  - So that the contrasts are orthogonal (test non-overlapping portions of variance)

Planned Comparisons: Example

- treatment4.sav: 3 groups: CBT, CSG, WL
  - Contrast 1: control (WL) vs. treatment (both CBT and CSG together)
  - Contrast 2: compare two treatment methods (CBT vs. CSG)
- SPSS: (Contrast1): 1, 1, -2; (Contrast2): 1, -1, 0
- Orthogonal: 2 degrees of freedom, 2 contrasts
- What if we also had anti-anxiety drug treatment (DT) and relaxation-class control (RC)?
  - Possible orthogonal sets of contrasts?

Planned Comparisons: SPSS

- Try this in One-Way ANOVA (also in Univariate):
  Analyze → Compare Means → One-Way ANOVA
  - Set Dependent List (DV) and Factor (IV)
  - Contrasts: enter weightings, in order
    - Contrast 1: (1, 1, -2). Contrast 2: (1, -1, 0)
  - Options: "Descriptive" (group means) and "Homogeneity of variance" (Levene's test)
- Output: "Contrast Tests" gives results for both "Assume equal variances" and "Does not assume equal variances": Use the appropriate one (from Levene's test)

Planned Comparisons: Output

Test of Homogeneity of Variances

depression levels at outcome of therapy

Levene Statistic	df1	df2	Sig.
.795	2	27	.462

Contrast 1: (1, 1, -2)

Contrast 2: (1, -1, 0)

		Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
depression levels at outcome of therapy	Assume equal variances	1	-5.60	.945	-5.925	27	.000
		2	-1.00	.546	-1.833	27	.078
	Does not assume equal variances	1	-5.60	1.030	-5.439	14.486	.000
		2	-1.00	.492	-2.032	18.000	.057

Planned Comp.: Interpretation

- From the results of our planned contrasts, we conclude:
- Contrast 1: The control group is significantly different from the two treatment groups ($p < .001$)
- Contrast 2: The difference between the two treatment groups is not significant ($p = .078$)

Planned Comparisions: Notes

- Plan them out when designing your study, not after you have already run your ANOVA
  - Tied conceptually to your variables
- May need multiple runs to get all your desired comparisons
- SPSS provides tools in One-Way and GLM Univariate ANOVA (less convenient in Factorial)
- More complex designs can also be addressed using Multiple Regression methods
  - Use dummy-coding, include only desired IVs

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Assumption of Parametricity

- Interval level DV: from research design
- Independence of scores: from sampling process
- Normally distributed DV: Check for outliers, run Kolmogorov-Smirnov & Shapiro-Wilks tests:
  - Analyze → Descriptive Statistics → Explore → Plots: "Normality plots with tests"
  - Need DV normal within each cell
- Equality of variances: Levene's test
  - Analyze → General Linear Model → Univariate → Options → "Homogeneity tests"

Robustness of ANOVA

- But! ANOVA is pretty robust to some violations
- Show-stoppers:
  - Non-interval level DV
    (try ordinal or log-linear regr., or non-param)
  - Dependent scores
    (try Repeated-Measures ANOVA or multi-lev)
- ANOVA becomes more robust when:
  - sample sizes are larger
  - the groups are closer to being equal in size
  - violations are minor rather than extreme

Handling Non-Normality

- If DV is not normal (after dealing with outliers):
- Check histogram: if close to normal, proceed
- Otherwise, check histograms for each group: if they are all skewed in a similar way, proceed
  - Graphs → Legacy Dialogs → Histogram → Rows: put IV here
- Consider applying a transform to the DV:
  - e.g., SQRT() undoes a mild right-skew
  - Osborne, Jason (2002). Notes on the use of data transformations. *Practical Assessment, Research & Evaluation*, 8(6)

Handling Unequal Variances

- If Levene's test is significant:
- If sample sizes for each group are close to equal, ANOVA is robust to heteroscedasticity
- Otherwise, try Welch's F instead of regular F
  - Adjusts (lowers) within-group df
  - Only available in One-Way ANOVA in SPSS
  - Analyze → Compare Means → One-Way ANOVA → Options: Welch
  - e.g., "Welch's $F(2, 17.78) = 16.25$, $p < .001$"
- Use appropriate post-hoc tests (Games-Howell)

Assumptions Testing: Practise

- Dataset: Treatment4.sav
- DV: “depression at follow-up”
- IV: “age” (treat as categorical rather than scale)
- Check assumptions of parametricity:
  - What assumptions are violated?
  - For each violation, what should we do?

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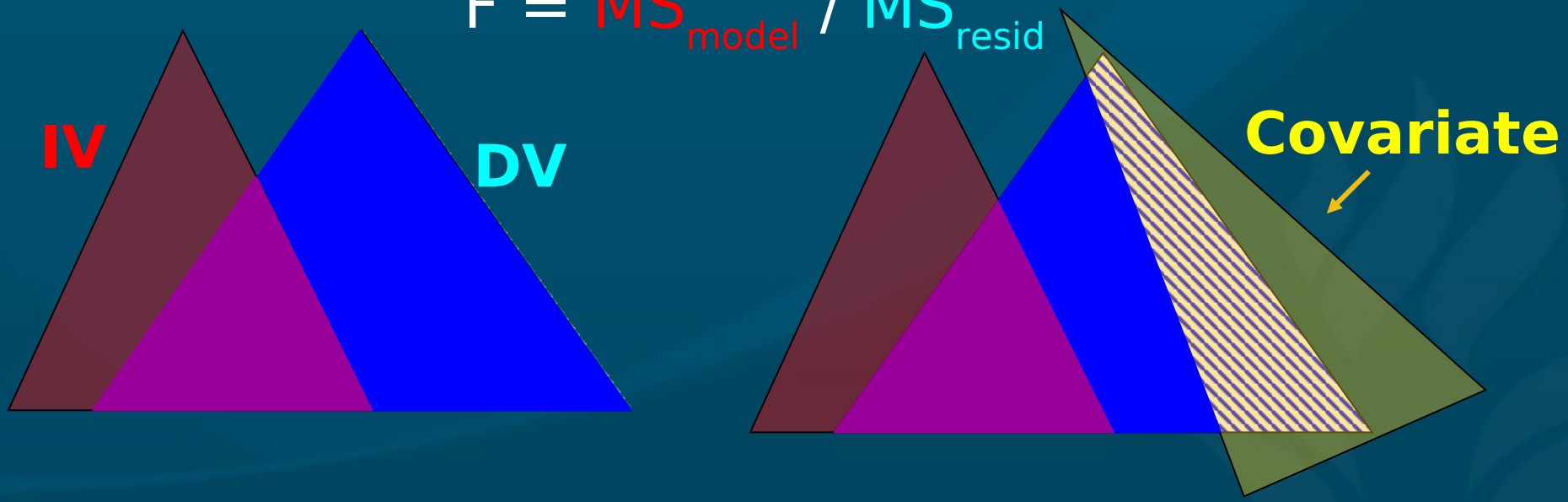
Introduction to ANCOVA

- Covariates are continuous "predictor" variables used as "control" factors to help power
- Covariates may be promoted to IVs if conceptually linked to other IVs or to the DV
- ANCOVA factors out the portion of variance in the DV that is accounted for by the covariates
  - Affects both MS_{model} and $MS_{residual}$
- Caution is required when covariates are correlated with IVs – creating conceptual links

Using ANCOVA in Research

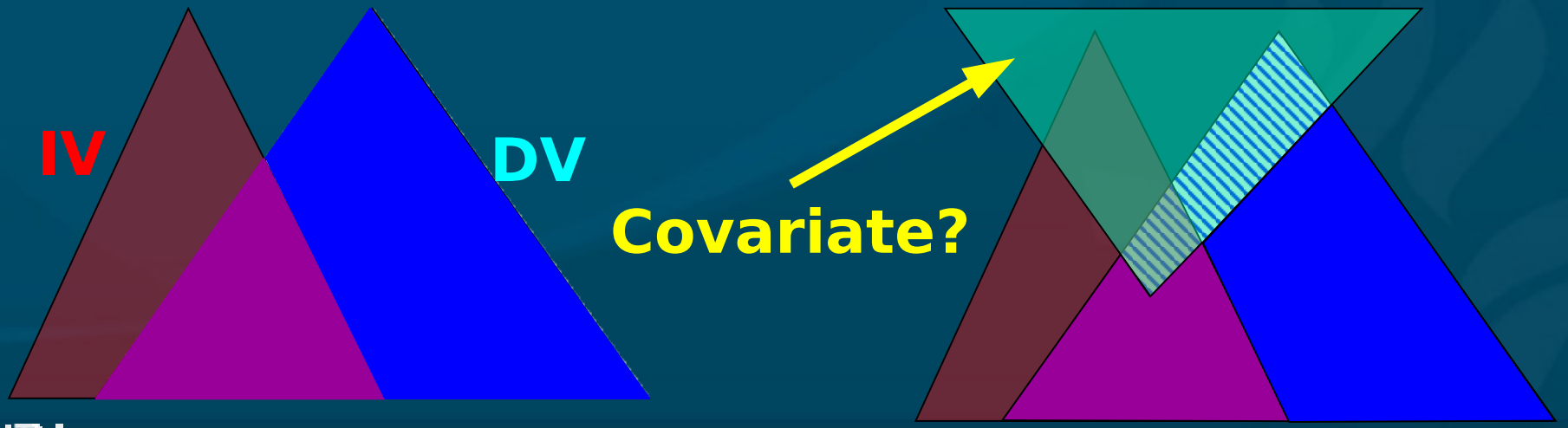
- Reduction of error variance: Including covariate(s) related to the DV in the model accounts for some within-group error variance, thus reducing MS_{resid} and increasing the *F*-ratio.

$$F = MS_{model} / MS_{resid}$$



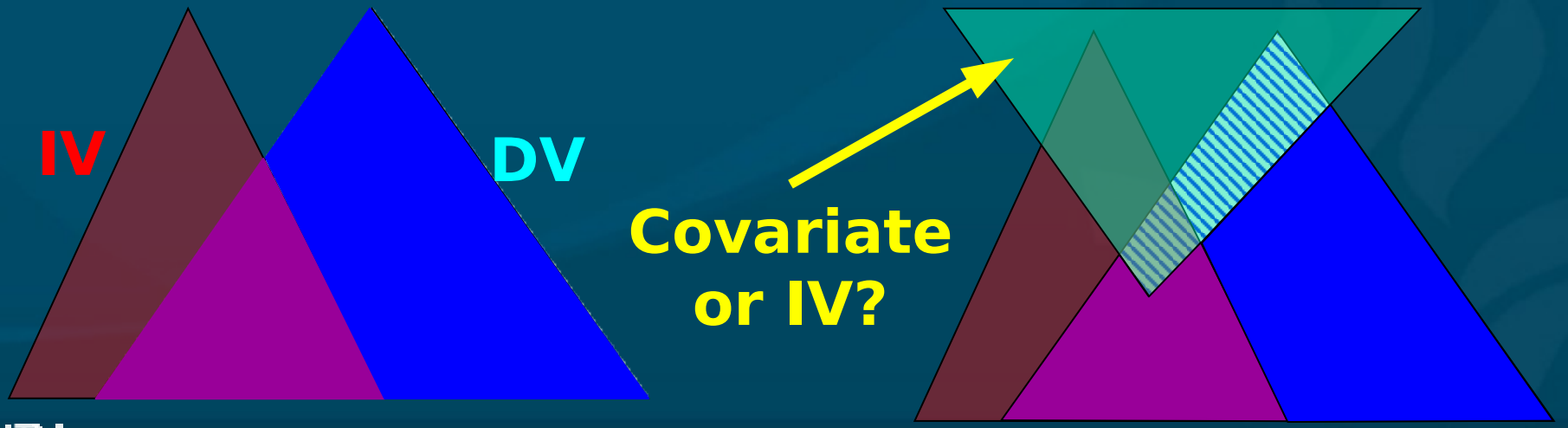
ANCOVA: Confounding Vars

- "Confounding" variables:
  External variables that may systematically influence an experimental manipulation.
- They can be identified through theory
- Control for them by entering them as covariates (though this may or may not improve *F*-ratio)



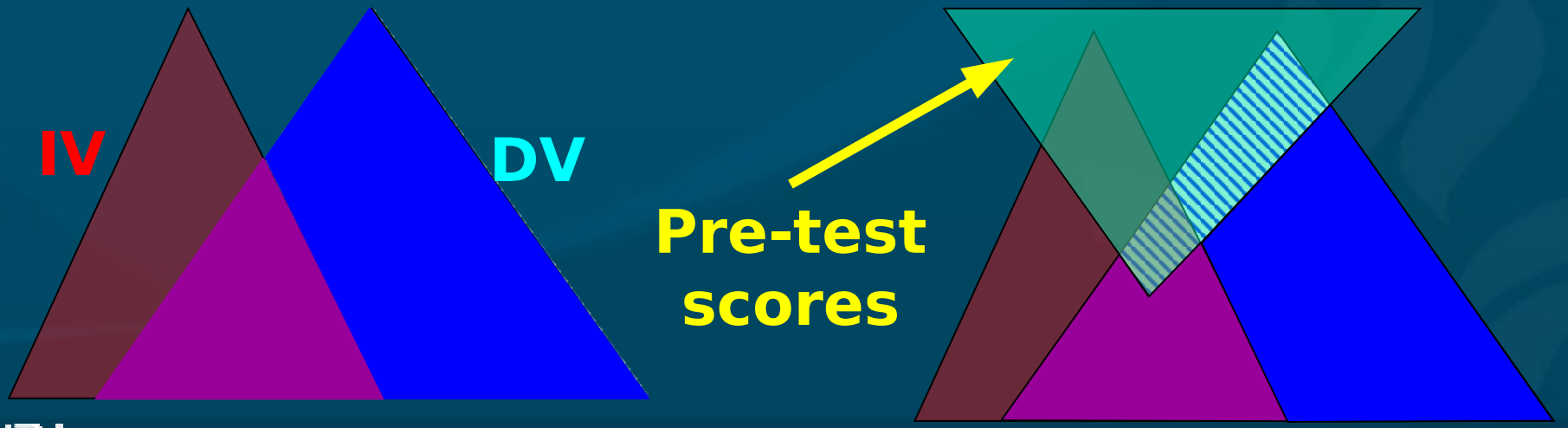
Covariates vs. Predictors

- If a covariate is 'linked' conceptually / theoretically with another IV or with the DV, then treat the covariate as an IV.
  - It could potentially be a moderator
- Any interactions or interpretable IV-Cov correlations then become part of the analysis.



ANCOVA in Therapy Research

- In therapy studies, different treatment groups often have different pre-treatment scores
  - How to compare treatments when the starting points are different?
- Solution: When in doubt, treat pre-treatment scores as another IV, not as a covariate.



Assumptions of ANCOVA

- Parametricity of DV (as with regular ANOVA)
- Homogeneity of regression slopes:
  - Regression of the DV on the Cov is the same for all groups
  - i.e., Cov is not a moderator
  - Test for interactions between IVs & Cov
- Conceptual independence of Cov & IVs
  - So that the shared variance is “external” to our RQ

ANCOVA: SPSS

- Analyze → General Linear Model → Univariate
  - Add variables to "Covariates" box
- Reporting: "Controlling/accounting for the influence of <Cov>, the effect of <IV> on <DV> is / is not significant, $F(df_{IV}, df_{error}) = \_\_\_, p = \_\_\_$."