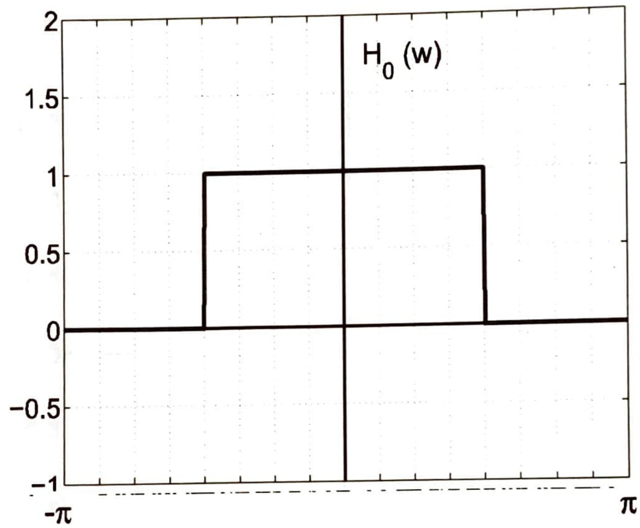
Sean Hwang

Digital Signal Processing I   Exam 3   Fall 2015

Problem 2. In Matlab Homework 2, you used Noble's Identities to convert the Tree-Structured subbander on the previous page to the regular maximally decimated subbander, obtaining products of the form:

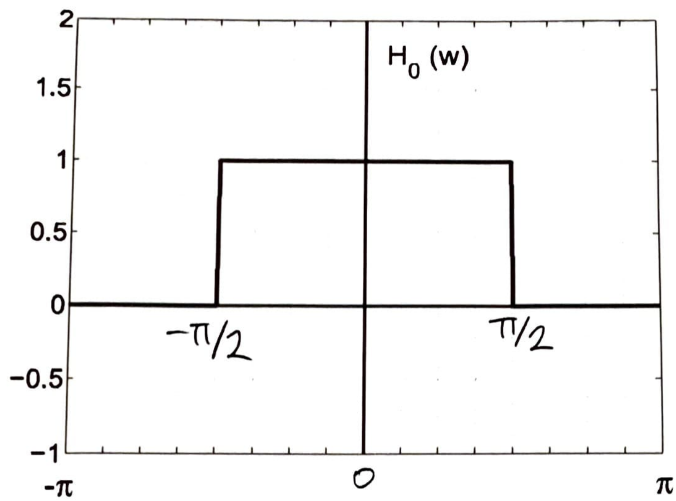
$$H_k(\omega) = H_\ell^{(2)}(\omega)H_m^{(2)}(2\omega)H_n^{(2)}(4\omega) \quad k = 0, 1, ..., 7$$

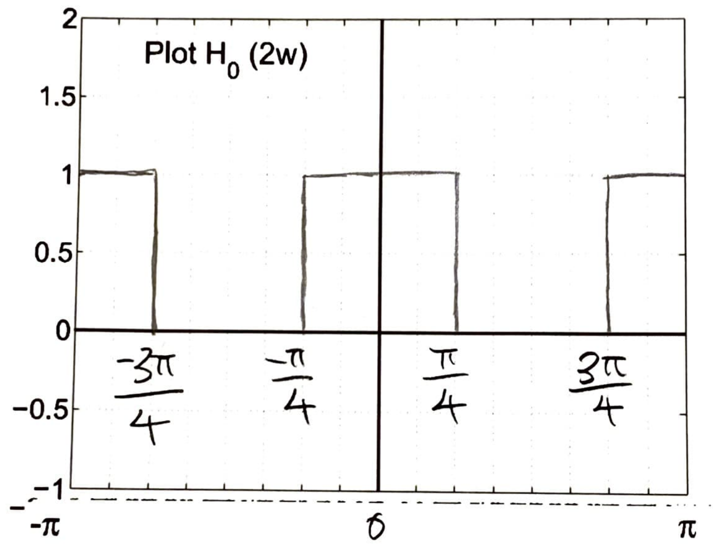
where $\ell \in \{0,1\}$, $m \in \{0,1\}$, and $n \in \{0,1\}$, with $H_0^{(2)}(\omega)$ denoting the lowpass halfband filter below for the two-channel QMF and $H_1^{(2)}(\omega) = H_0^{(2)}(\omega - \pi)$. For this problem, $H_0^{(2)}(\omega)$ is the ideal lowpass filter below (note: a DTFT is always periodic with period 2π.)

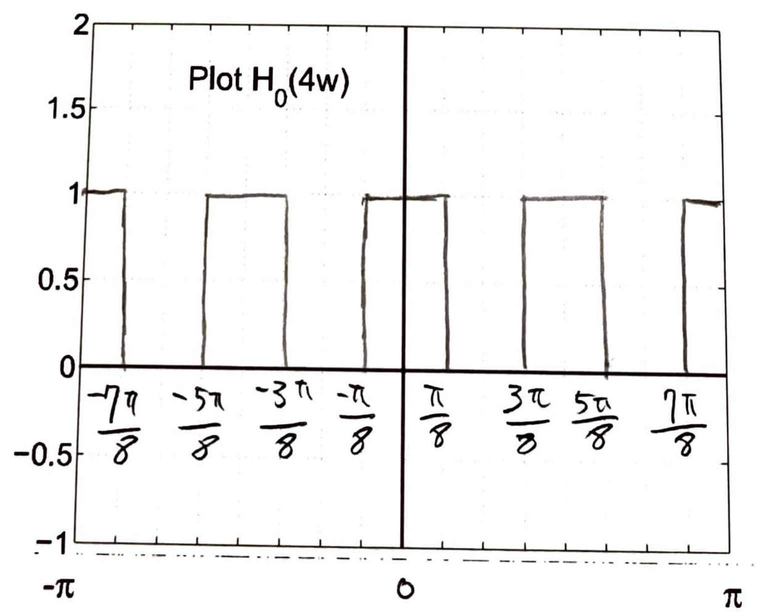


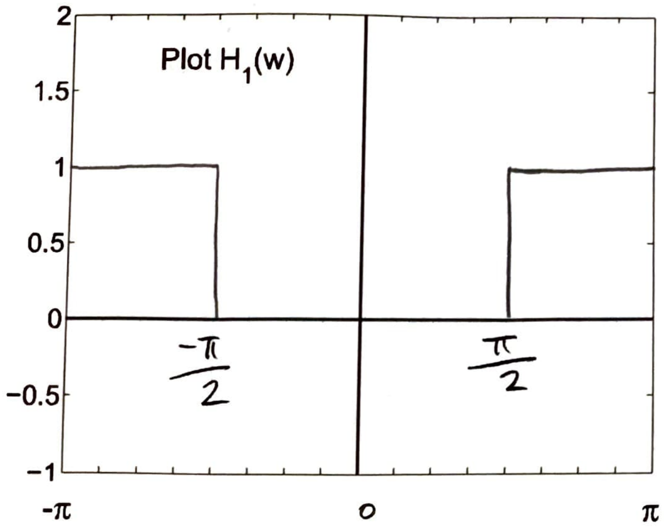
You are required to fill in the table below. **FIRST** you should plot $H_0^{(2)}(2\omega)$ and $H_0^{(2)}(4\omega)$ on the next page, and then plot $H_1^{(2)}(\omega)$, $H_0^{(2)}(2\omega)$ and $H_0^{(2)}(4\omega)$ on the next page after that, in the space provided. In each plot, the abscissa range is from $-\pi$ to $+\pi$ and there is a tic mark and vertical dashed line at every integer multiple of $\pi/8$. These plots will help you fill in the table. I have already filled in the first entry. Only fill in the positive frequency band that is passed; the filters are real-valued and even-symmetric, so their respective frequency responses are real-valued and even-symmetric.

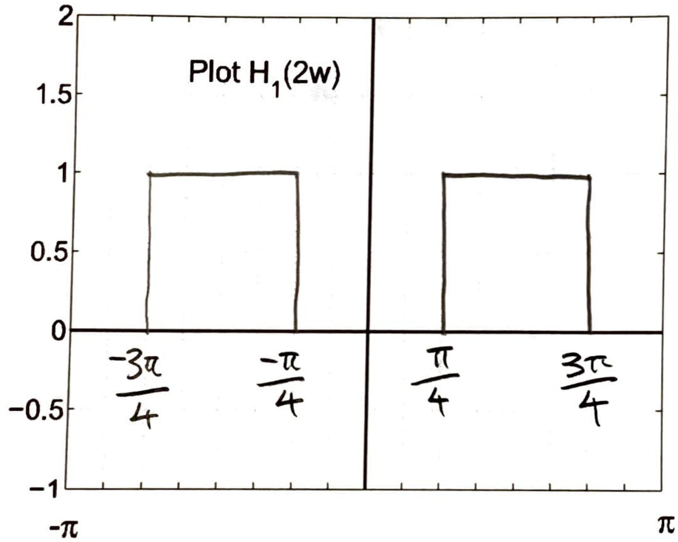
$$\begin{array}{ll}
H_0^{(2)}(\omega)H_0^{(2)}(2\omega)H_0^{(2)}(4\omega) & \text{passes: } 0 < \omega < \pi/8 \\
H_0^{(2)}(\omega)H_0^{(2)}(2\omega)H_1^{(2)}(4\omega) & \text{passes: } \pi/8 < \omega < \pi/4 \\
H_0^{(2)}(\omega)H_1^{(2)}(2\omega)H_0^{(2)}(4\omega) & \text{passes: } 3\pi/8 < \omega < \pi/2 \\
H_0^{(2)}(\omega)H_1^{(2)}(2\omega)H_1^{(2)}(4\omega) & \text{passes: } \pi/4 < \omega < 3\pi/8 \\
H_1^{(2)}(\omega)H_0^{(2)}(2\omega)H_0^{(2)}(4\omega) & \text{passes: } 7\pi/8 < \omega < \pi \\
H_1^{(2)}(\omega)H_0^{(2)}(2\omega)H_1^{(2)}(4\omega) & \text{passes: } 3\pi/4 < \omega < 7\pi/8 \\
H_1^{(2)}(\omega)H_1^{(2)}(2\omega)H_0^{(2)}(4\omega) & \text{passes: } \pi/2 < \omega < 5\pi/8 \\
H_1^{(2)}(\omega)H_1^{(2)}(2\omega)H_1^{(2)}(4\omega) & \text{passes: } 5\pi/8 < \omega < 3\pi/4
\end{array} \quad (1)$$

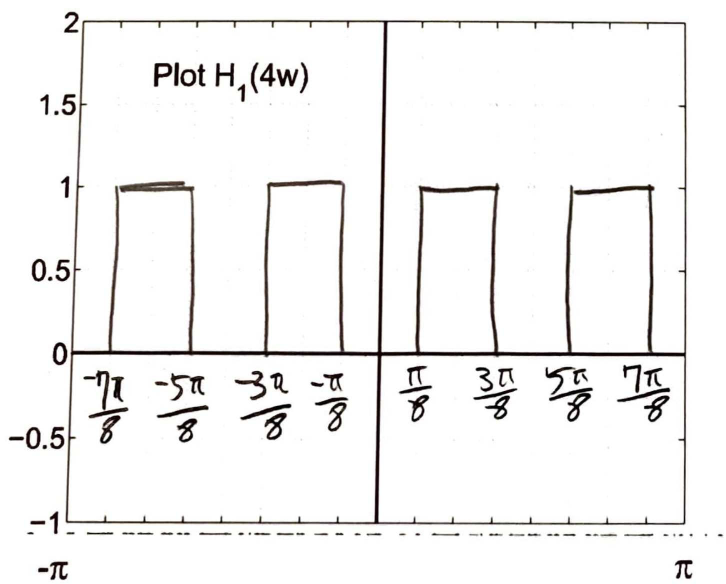

H0 (w)
2
1.5
1
0.5
0
-0.5
-1
−π/2
π/2
-π
0
π


Plot H0 (2w)
2
1.5
1
0.5
0
-0.5
-1
-3π/4
-π/4
π/4
3π/4
-π
0
π


Plot H0(4w)
2
1.5
1
0.5
0
-0.5
-1
-7π/8
-5π/8
-3π/8
-π/8
π/8
3π/8
5π/8
7π/8
-π
0
π


Plot $H_1(w)$
2
1.5
1
0.5
0
−0.5
−1
$\frac{-\pi}{2}$
$\frac{\pi}{2}$
$-\pi$
0
π


Plot $H_1(2w)$
2
1.5
1
0.5
0
−0.5
−1
$\frac{-3\pi}{4}$
$\frac{-\pi}{4}$
$\frac{\pi}{4}$
$\frac{3\pi}{4}$
$-\pi$
π


Plot $H_1(4w)$
2
1.5
1
0.5
0
−0.5
−1
$\frac{-7\pi}{8}$
$\frac{-5\pi}{8}$
$\frac{-3\pi}{8}$
$\frac{-\pi}{8}$
$\frac{\pi}{8}$
$\frac{3\pi}{8}$
$\frac{5\pi}{8}$
$\frac{7\pi}{8}$
$-\pi$
π

Sean Hwang

EE538 Digital Signal Processing I **Final Exam** **Fall 2011**

Problem 1. This problem is about determining whether the four-band analysis filter bank and corresponding four-band synthesis filter bank in Figure 1 achieves Perfect Reconstruction when the respective impulse responses for the four filters

$$f_k[n] = e^{j(k\frac{2\pi}{4})n} h_{LP}[n], \quad k = 0, 1, 2, 3$$

are defined in terms of the lowpass filter with impulse response below, which is of length 4.

$$h_{LP}[n] = u[n] - u[n-4]. \tag{1}$$

This question is more easily answered examining the efficient version of the overall filter bank in Figure 2.

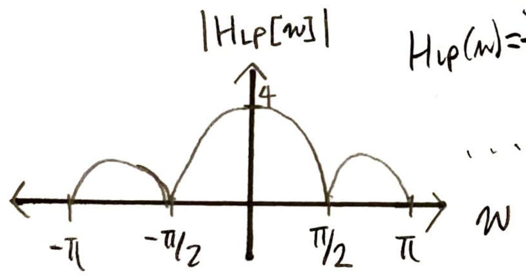
(a) Plot the magnitude of the DTFT $H_{LP}(\omega)$ of $h_{LP}[n]$ over $-\pi < \omega < \pi$. Will there be aliasing present in each of the outputs $z_k[n]$?

(b) The four polyphase components of $h_{LP}[n]$ are defined as

$$h_\ell^+[n] = h_{LP}[4n + \ell], \quad \ell = 0, 1, 2, 3. \tag{2}$$

Do a separate stem plot (time-domain) of each of the four impulse responses $h_\ell^+[n]$, $\ell = 0, 1, 2, 3$. Are they all the same?

a)



$$H_{LP}(w) = \frac{\sin 2w}{\sin \frac{1}{2} w} e^{-j(\frac{3}{2})w}$$

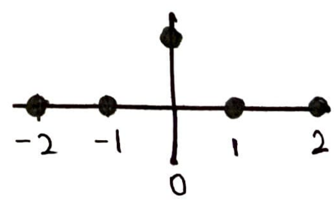
There will be substantial alising as $H_{LP}(w)$ passes evergy outside the band $|w| < \frac{\pi}{4}$.

b). $h_0^+[n] = h_{LP}[4n] = \delta[n]$

$h_1^+[n] = h_{LP}[4n+1] = \delta[n] = h_{LP}[1]$

$h_2^+[n] = h_{LP}[4n+2] = \delta[n] = h_{LP}[2]$

$h_3^+[n] = h_{LP}[4n+3] = \delta[n] = h_{LP}[3]$



(c) The digital subbanding on the left hand side of Figure 1 is alternatively effected more efficiently with the polyphase filters via the block diagram on the left side of Figure 2, where the respective input signals are defined as below. Fill in the values of $\beta_{k\ell}$ in the table below so that the outputs, $z_k[n]$, $k = 0, 1, 2, 3$, at the output of the left hand side in Figure 2 are exactly the same as the outputs $z_k[n]$ of the left hand side in Figure 1. Show all work.

$$\begin{bmatrix} z_0[n] \\ z_1[n] \\ z_2[n] \\ z_3[n] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \beta_{11} & \beta_{12} & \beta_{13} \\ 1 & \beta_{21} & \beta_{22} & \beta_{23} \\ 1 & \beta_{31} & \beta_{32} & \beta_{33} \end{bmatrix} \begin{bmatrix} s_0[n] \\ s_1[n] \\ s_2[n] \\ s_3[n] \end{bmatrix} \quad \Rightarrow \quad \mathbf{B} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \beta_{11} & \beta_{12} & \beta_{13} \\ 1 & \beta_{21} & \beta_{22} & \beta_{23} \\ 1 & \beta_{31} & \beta_{32} & \beta_{33} \end{bmatrix} \tag{3}$$

$\beta_{k\ell}$	$\ell = 1$	$\ell = 2$	$\ell = 3$
$k = 1$	j	-1	$-j$
$k = 2$	-1	1	-1
$k = 3$	$-j$	-1	j

$$f_k[n] = e^{j(k \times \frac{\pi}{2})n} \, h_{LP}[n] \quad , \quad k = 0 \ldots 3$$

$$n = 4n + \ell \Rightarrow \underline{e^{j2\pi kn} \, e^{j\frac{\pi}{2}k\ell} \, h_\ell^+[n]}$$

gives $j^{k\ell} h_\ell^+[n]$.

$k=1, \ell=1 = j^1 = j$

$k=3, \ell=3 = (j^3)^\ell = (-j)^3 = j$

(d) The zero inserts followed by filtering is done more efficiently in a collective manner. First, note, in order to match the inputs with the outputs, we effect the efficient implementation of "L zero inserts followed by filtering" in an alternative fashion relative to the way it was originally derived in class. In this case, we interleave in a **clockwise** fashion the following outputs below for $\ell = 0, 1, 2, 3$.

$$y[Ln - \ell] = x[n] * h[Ln - \ell] \quad (4)$$
$$y[Ln - \ell] = x[n] * h_\ell^-[n] \quad (5)$$
$$(6)$$

where $h_\ell^-[n] = h[Ln - \ell]$, $\ell = 0, 1, ..., L - 1$.

Thus, the polyphase components of interest for this part of the problem are

$$h_\ell^-[n] = g_0[4n - \ell], \quad \ell = 0, 1, 2, 3, \quad (7)$$

where $g_0[n] = h_{LP}[-n]$ with $h_{LP}[n] = u[n] - u[n-4]$ as before. It follows that

$$h_\ell^-[n] = h_{LP}[\ell - 4n], \quad \ell = 0, 1, 2, 3, \quad (8)$$

where, again, $h_{LP}[n] = u[n] - u[n-4]$ is only of length 4.

With this in mind, determine the value of each of the amplitude coefficients $\alpha_{\ell k}$, $k = 1, 2, 3$, $\ell = 1, 2, 3$ so that the interleaved output $y[n]$ in Figure 2 is the SAME as the overall output in Figure 1. Show all work on the next page but input your final answers in the table below.

$$\begin{bmatrix} y_0[n] \\ y_1[n] \\ y_2[n] \\ y_3[n] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \alpha_{11} & \alpha_{12} & \alpha_{13} \\ 1 & \alpha_{21} & \alpha_{22} & \alpha_{23} \\ 1 & \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix} \begin{bmatrix} z_0[n] \\ z_1[n] \\ z_2[n] \\ z_3[n] \end{bmatrix} \Rightarrow \mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \alpha_{11} & \alpha_{12} & \alpha_{13} \\ 1 & \alpha_{21} & \alpha_{22} & \alpha_{23} \\ 1 & \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix} \quad (9)$$

$\alpha_{\ell k}$	$k=1$	$k=2$	$k=3$
$\ell = 1$	$-j$	-1	j
$\ell = 2$	-1	1	-1
$\ell = 3$	j	-1	$-j$

$$g_k[n] = e^{-j\frac{2\pi}{4}k(-n)} h_{LP}[-n]$$
$$= e^{j\frac{2\pi}{4}kn} h_{LP}[-n]$$

$$g_k[4n+\ell] = e^{j\frac{2\pi}{4}(4n-\ell)} h_{LP}[-(4n-\ell)]$$
$$= e^{-j\frac{\pi}{2}k\ell} h_{LP}[\ell-4n]$$

(e) For part (c), we defined the following four polyphase components of $h_{LP}[n]$ (10)

$$h_\ell^-[n] = g_0[4n - \ell], \quad \ell = 0, 1, 2, 3.$$

where $g_0[n] = h_{LP}[-n]$ with $h_{LP}[n] = u[n] - u[n-4]$. Do a separate stem plot (time-domain) of each of the four impulse responses $h_\ell^-[n]$, $\ell = 0, 1, 2, 3$. Are they all the same?

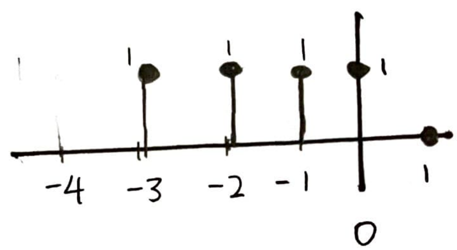
(f) Does $y[n] = 4x[n]$? You need to either prove or disprove $y[n] = 4x[n]$ with all that you've developed so far for parts (a) through (e).
Hint: assess if $y[4n - \ell] = 4x[4n - \ell]$, for $\ell = 0, 1, 2, 3$.

e) $h_0^-[n] = \delta[n]$

$h_1^-[n] = \delta[n]$

$h_2^-[n] = g_0[4n-2] = \delta[n] = g_0[-2]$

$h_3^-[n] = g_0[4n-3] = \delta[n] = g_0[-3$



f) Yes, $y[n] = 4x[n]$; By Perfect reconstruction.

$h_\ell^+[n] * h_\ell^-[n] = \delta[n] * \delta[n] = \delta[n]$

Here, h_ℓ^+ and h_ℓ^- are in Series, and if a inverse DFT matrix is cascaded w/ DFT matrix, we get a Identity matrix.

Therefore, using previous answers I can say

$y[4n-\ell] = 4x[4n-\ell]$ for $\ell = 0, 1, 2, 3$.

which is $y[n] = 4x[n]$