Homework 1

The pdf you submit must look exactly like this with the answers and all supporting works shown on the the page with the question.

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1. (5 points) Use Boolean Algebra to prove that

$Y = (\bar{A}*B*\bar{C})+(\bar{A}*B*C)+(A*\bar{B}*C)+(A*B*\bar{C})+(A*B*C)=(A+B)*(B+C) = X$

SOP to POS

$(\bar{A}B\bar{C} + \bar{A}BC) + (A\bar{B}C + AB\bar{C}) + ABC = (A+B)*(B+C)$

A	B	C	Y	X
0	0	0	0	$\bar{C}$
0	0	1	0	C
0	1	0	1	
0	1	1	1	1
1	0	0	0	C
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

minterm

maxterms

taking De morgan's theorem twice get's us

$\bar{Y} = A\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}$

$\bar{Y} = (A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)(\bar{A}+\bar{B}+\bar{C})$

$\bar{\bar{Y}} = (\bar{A}+B+\bar{C})(\bar{A}+B+C)(A+\bar{B}+C)(A+B+\bar{C})(A+B+C)$

Distributive $\Rightarrow$ $(\bar{A}+(B+\bar{C})(B+C))$ $(A+\underbrace{(\bar{B}+C)(B+\bar{C})(B+C)}_{\text{combining}})$

Combining $(\bar{A}+B)\ (A+B+C)$

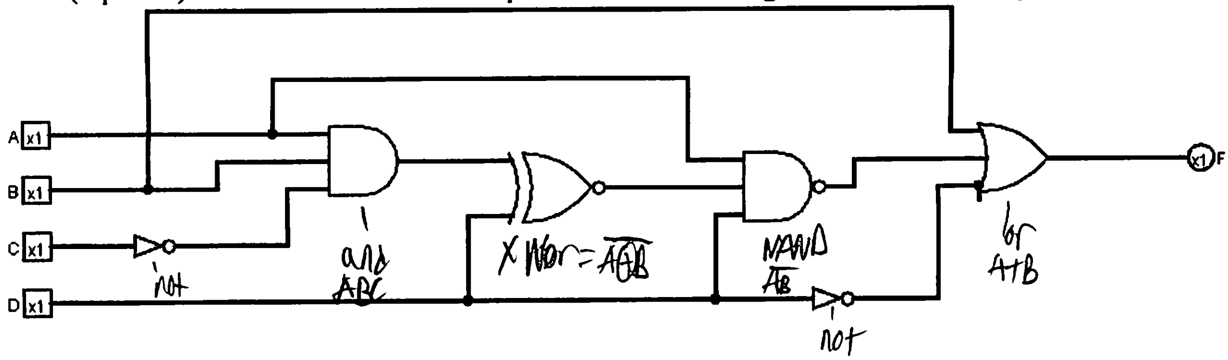
$\boxed{(A+B)*(B+C)}$

2. (3 points) Prove that A XOR B = $A*\bar{B}+\bar{A}*B$

A	B	A XOR B	$A*\bar{B}+\bar{A}*B$
0	0	0	0
0	1	1	1
1	0	1	1
1	1	0	0

These two equations have the same truth tables to each other. Because A XOR B = $A*\bar{B}+\bar{A}*B$ for all cases, through perfect induction we can conclude that the theorem is proved.

3. (3 points) Write the function that represents the following circuit. Do not simplify.



$$\overline{\left(\overline{(AB\bar{C}) \oplus D}\right)(A)(D)} + B + \bar{D}$$

4. Given the following truth table

A	B	C	Output
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

1. (3 points) Write a function in SOP form that behaves according to the truth table. Do not simplify.

$m_0 + m_1 + m_2 + m_5 + m_6$

$\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}C + AB\bar{C}$

2. (3 points) Write a function in POS form that behaves according to the truth table. Do not simplify.

$M_3 + M_4 + M_7$

$(A+\bar{B}+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+\bar{C})$

5. (3 points each) For each of the following problems assume that the variables are $x_0 - x_{N-1}$, with x_0 representing the least significant bit and x_{N-1} the most significant. For example if we had an equation of 3 variables, $m_1 = \bar{x}_2 * \bar{x}_1 * x_0$ and $m_6 = x_2 * x_1 * \bar{x}_0$. For each of the following problems write each function in **both** its most simplified SOP and POS form. There are a total of 5 subquestions
   1. $m_0 + m_1 + m_2$

A	B	$m_0+m_1+m_2$
0	0	1
0	1	1
1	0	1
1	1	0

SOP: $(\bar{A}\bar{B}) + (\bar{A}B) + (A\bar{B})$

$\bar{A} + (\bar{B}B) + (A\bar{B})$

$\boxed{\bar{A} + A\bar{B}}$

POS:

$\boxed{\bar{A} + \bar{B}}$

2. $M_0 * M_3 * M_4 * M_7$

A	B	C	$M_0 M_3 M_4 M_7$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

SOP: $(\bar{A}\bar{B}C)+(\bar{A}B\bar{C})+(A\bar{B}C)+(AB\bar{C})$

$\bar{A}(\bar{B}C+B\bar{C})+A(\bar{B}C+B\bar{C})$

$= \boxed{\bar{B}C+B\bar{C}}$

POS: $(A+B+C)(A+\bar{B}+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+\bar{C})$

$[\cancel{A}+((B+C)(\bar{B}+\bar{C}))]\,[\cancel{\bar{A}}+((B+C)(\bar{B}+\bar{C}))]$

$= \boxed{(B+C)(\bar{B}+\bar{C})}$

3. $m_4 + m_5 + m_7 + m_{12} + m_{13} + m_{15}$

4 digits required

Use K map

CD \ AB	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	0	1	1	0
10	0	0	0	0

SOP

$B\bar{C} + BD$

$\boxed{B(\bar{C}+D)}$ — Distributive

POS

$\boxed{B(\bar{C}+D)}$

4. $m_0+m_3+m_4+m_8+D_2+D_5+D_7+D_{10}+D_{13}+D_{15}$

CD \ AB	00	01	11	10
00	1	1	0	1
01	D	D	D	0
11	1	D	D	0
10	0	0	0	D

SOP

$\overline{AC}+\overline{A}D+A\overline{BD}$

POS

$(\overline{C}+D)(\overline{A}+\overline{B})(\overline{A}+\overline{D})$

5. $m_1+m_3+m_7+m_9+m_{11}+m_{15}+m_{17}+m_{19}+m_{25}+m_{27}+D_4+D_6+D_{12}$
$+D_{14}+D_{16}+D_{18}+D_{20}+D_{22}+D_{24}+D_{26}+D_{28}+D_{30}$

DE \ BC	00	01	11	10
00	0	D	D	0
01	1	0	0	1
11	1	1	1	1
10	0	D	D	0

A=0

DE \ BC	00	01	11	10
00	D	D	D	D
01	1	0	0	1
11	1	0	0	1
10	D	0	D	D

A=1

SOP

$\bar{B}\bar{C}E + DE\bar{A} + B\bar{C}E$

POS

$(\bar{D}+E)(\bar{C}+\bar{A})(D+E)(D+\bar{C}+\bar{A})$