sample

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1 Problems

1. A point whose coordinates are both integers is called a lattice point. How many lattice points lie on the hyperbola $x^2 - y^2 = 2000^2$?

2. On square $ABCD$, point E lies on side AD and point F lies on side BC, so that $BE = EF = FD = 30$. Find the area of the square $ABCD$.

3. Call a 3-digit number geometric if it has 3 distinct digits which, when read from left to right, form a geometric sequence. Find the difference between the largest and smallest geometric numbers.

4. For each even positive integer x, let $g(x)$ denote the greatest power of 2 that divides x. For example, $g(20) = 4$ and $g(16) = 16$. For each positive integer n, let $S_n = \sum_{k=1}^{2^{n-1}} g(2k)$. Find the greatest integer n less than 1000 such that S_n is a perfect square.

5. A sequence is defined as follows $a_1 = a_2 = a_3 = 1$, and, for all positive integers $n, a_{n+3} = a_{n+2} + a_{n+1} + a_n$. Given that $a_{28} = 6090307, a_{29} = 11201821$, and $a_{30} = 20603361$, find the remainder when $\sum_{k=1}^{28} a_k$ is divided by 1000.

6. Michelle is at the bottom-left corner of a 6×6 lattice grid, at $(0, 0)$. The grid also contains a pair of one-time-use teleportation devices at $(2, 2)$ and $(3, 3)$; the first time Michelle moves to one of these points she is instantly teleported to the other point and the devices disappear. If she can only move up or to the right in unit increments, in how many ways can she reach the point $(5, 5)$?

7. Let $\mathcal{S}$ be a regular 18-gon, and for two vertices in $\mathcal{S}$ define the *distance* between them to be the length of the shortest path along the edges of $\mathcal{S}$ between them (e.g. adjacent vertices have distance 1). Find the number of ways to choose three distinct vertices from $\mathcal{S}$ such that no two of them have distance 1, 8, or 9.

8. For some integer m, the polynomial $x^3 - 2011x + m$ has the three integer roots a, b, and c. Find $|a| + |b| + |c|$.

9. In triangle ABC the medians $\overline{AD}$ and $\overline{CE}$ have lengths 18 and 27, respectively, and $AB = 24$. Extend $\overline{CE}$ to intersect the circumcircle of ABC at F. The area of triangle AFB is $m\sqrt{n}$, where m and n are positive integers and n is not divisible by the square of any prime. Find $m + n$.

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10. A triangular array of numbers has a first row consisting of the odd integers $1, 3, 5, \ldots, 99$ in increasing order. Each row below the first has one fewer entry than the row above it, and the bottom row has a single entry. Each entry in any row after the top row equals the sum of the two entries diagonally above it in the row immediately above it. How many entries in the array are multiples of 67?

Time limit: 50 minutes.
Each problem is worth one point.

2 Answers

1. 98
2. 810
3. 840
4. 899
5. 834
6. 280
7. 240
8. 98
9. 63
10. 17

3 Problem Info

Source: AIME 2000 II 2 Tags: [2020-06, 5C, algebra, manipulate] Solution: https://artofproblemsolving.com/wiki/index.php/2000_AIME_II_Problems/Problem_2Source: AIME 2011 II 2 Tags: [2020-06, 10C, geometry, lengths] Solution: https://artofproblemsolving.com/wiki/index.php/2011_AIME_II_Problems/Problem_2Source: AIME 2009 I 1 Tags: [2020-06, 10C, algebra, careful] Solution: https://artofproblemsolving.com/wiki/index.php/2009_AIME_I_Problems/Problem_1Source: AIME 2006 I 13 Tags: [2020-06, 25C, number-theory, algebra] Solution: https://artofproblemsolving.com/wiki/index.php/2006_AIME_I_Problems/Problem_13Source: AIME 2006 II 11 Tags: [2020-06, 25C, algebra, manipulate] Solution: https://artofproblemsolving.com/wiki/index.php/2006_AIME_II_Problems/Problem_11Source: CMIMC 2018 Combinatorics 3 Tags: [2020-06, 25C, combo, cases, important] Solution: http://cmimc-official.herokuapp.com/docs/past-tests/2018_Combinatorics_S.pdfSource: CMIMC 2016 Combinatorics 5 Tags: [2020-06, 30C, combo, symmetry] Solution: http://cmimc-official.herokuapp.com/docs/past-tests/2016_Combinatorics_S.pdfSource: AIME 2011 I 15 Tags: [2020-06, 30C, algebra, polynomial, structure] Solution: https://artofproblemsolving.com/wiki/index.php/2011_AIME_I_Problems/Problem_15Source: AIME 2002 I 13 Tags: [2020-06, 30C, geometry, lengths] Solution: https://artofproblemsolving.com/wiki/index.php/2002_AIME_I_Problems/Problem_13Source: AIME 2008 I 6 Tags: [2020-06, 35C, number-theory, experiment, courage] Solution: https://artofproblemsolving.com/wiki/index.php/2008_AIME_I_Problems/Problem_6