Sample

Made with TiKT*

Sean Li

Generated June 11, 2020

1 Problems

1. Let $R = (8, 6)$. The lines whose equations are $8y = 15x$ and $10y = 3x$ contain points P and Q, respectively, such that R is the midpoint of $\overline{PQ}$. The length of PQ equals $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

2. Maryssa, Stephen, and Cynthia played a game. Each of them independently privately chose one of Rock, Paper, and Scissors at random, with all three choices being equally likely. Given that at least one of them chose Rock and at most one of them chose Paper, the probability that exactly one of them chose Scissors can be expressed as $\frac{m}{n}$ for relatively prime positive integers m and n. Compute $100m + n$.

3. Given that $\frac{((3!)!)!}{3!} = k \cdot n!$, where k and n are positive integers and n is as large as possible, find $k + n$.

4. Positive integers a and b satisfy the condition
$$\log_2(\log_{2^a}(\log_{2^b}(2^{1000}))) = 0.$$
Find the sum of all possible values of $a + b$.

5. How many positive integer divisors of 2004^{2004} are divisible by exactly 2004 positive integers?

6. Let $f : \mathbb{N} \to \mathbb{N}$ be a bijection satisfying $f(ab) = f(a)f(b)$ for all $a, b \in \mathbb{N}$. Determine the minimum possible value of $f(n)/n$, taken over all possible f and all $n \leq 2019$.

7. For each $q \in \mathbb{Q}$, let $\pi(q)$ denote the period of the repeating base-16 expansion of q, with the convention of $\pi(q) = 0$ if q has a terminating base-16 expansion. Find the maximum value among
$$\pi\left(\frac{1}{1}\right),\ \pi\left(\frac{1}{2}\right),\ \ldots,\ \pi\left(\frac{1}{70}\right).$$

8. Contessa is taking a random lattice walk in the plane, starting at $(1, 1)$. (In a random lattice walk, one moves up, down, left, or right 1 unit with equal probability at each step.) If she lands on a point of the form $(6m, 6n)$ for $m, n \in \mathbb{Z}$, she ascends to heaven, but if she lands on a point of the form $(6m+3, 6n+3)$ for $m, n \in \mathbb{Z}$, she descends to hell. What is the probability she ascends to heaven?

*By Sean Li. Selected problems belong to their respective authors and oragnizations, as attributed.

9. A triangular array of numbers has a first row consisting of the odd integers $1, 3, 5, \ldots, 99$ in increasing order. Each row below the first has one fewer entry than the row above it, and the bottom row has a single entry. Each entry in any row after the top row equals the sum of the two entries diagonally above it in the row immediately above it. How many entries in the array are multiples of 67?

10. Cyclic quadrilateral $ABCD$ satisfies $\angle ABD = 70^\circ$, $\angle ADB = 50^\circ$, and $BC = CD$. Suppose AB intersects CD at point P, while AD intersects BC at point Q. Compute $\angle APQ - \angle AQP$.

Time limit: 50 minutes.
Each problem is worth one point.

2 Answers

1. 67. Source: AIME 2001 II 4

   Solution: https://artofproblemsolving.com/wiki/index.php/2001_AIME_II_Problems/Problem_4

2. 916. Source: OMO 2019 Fall 4

   Solution: https://artofproblemsolving.com/community/c951538_2019_online_math_open_problems

3. 839. Source: AIME 2003 I 1

   Solution: https://artofproblemsolving.com/wiki/index.php/Positive_integer

4. 881. Source: AIME 2013 II 2

   Solution: https://artofproblemsolving.com/wiki/index.php/2013_AIME_II_Problems/Problem_2

5. 54. Source: AIME 2004 II 8

   Solution: https://artofproblemsolving.com/wiki/index.php/2004_AIME_II_Problems/Problem_8

6. 2/2017. Source: CMIMC 2019 Team 9

   Solution: http://cmimc-official.herokuapp.com/docs/past-tests/2019_Team_S.pdf

7. 33. Source: CMIMC 2018 Number Theory 7

   Solution: http://cmimc-official.herokuapp.com/docs/past-tests/2018_NumberTheory_S.pdf

8. 13/22. Source: HMMT 2019 Combo 5

   Solution: https://hmmt-archive.s3.amazonaws.com/tournaments/2019/feb/comb/solutions.pdf

9. 17. Source: AIME 2008 I 6

   Solution: https://artofproblemsolving.com/wiki/index.php/2008_AIME_I_Problems/Problem_6

10. 40. Source: CMIMC 2017 Geometry 6

    Solution: http://cmimc-official.herokuapp.com/docs/past-tests/2017_Geometry_S.pdf