# Sample

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Generated June 11, 2020

1 Problems

1. Set A consists of m consecutive integers whose sum is $2m$, and set B consists of $2m$ consecutive integers whose sum is m. The absolute value of the difference between the greatest element of A and the greatest element of B is 99. Find m.

2. Patio blocks that are hexagons 1 unit on a side are used to outline a garden by placing the blocks edge to edge with n on each side. The diagram indicates the path of blocks around the garden when $n = 5$.
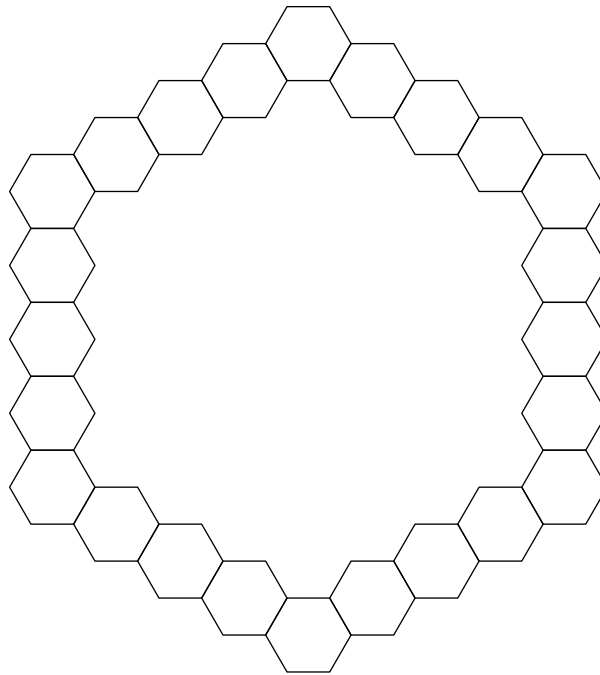
   If $n = 202$, then the area of the garden enclosed by the path, not including the path itself, is $m\left(\sqrt{3}/2\right)$ square units, where m is a positive integer. Find the remainder when m is divided by 1000.

3. Let N be the greatest integer multiple of 36 all of whose digits are even and no two of whose digits are the same. Find the remainder when N is divided by 1000.

4. Let $ABCDEF$ be an equiangular hexagon such that $AB = 6, BC = 8, CD = 10$, and $DE = 12$. Denote by d the diameter of the largest circle that fits inside the hexagon. Find d^2.

5. We say that a binary string s contains another binary string t if there exist indices $i_1, i_2, \ldots, i_{|t|}$ with $i_1 < i_2 < \ldots < i_{|t|}$ such that

$$s_{i_1} s_{i_2} \ldots s_{i_{|t|}} = t.$$

(In other words, t is found as a not necessarily contiguous substring of s.) For example, 110010 contains 111. What is the length of the shortest string s which contains the binary representations of all the positive integers less than or equal to 2048?

6. At a conference, six people place their name badges in a hat, which is shaken up; one badge is then distributed to each person such that each distribution is equally likely. Each turn, every person who does not yet have their own badge finds the person whose badge they have and takes that person's badge. For example, if Alice has Bob's badge and Bob has Charlie's badge, Alice would have Charlie's badge after a turn. Compute the probability that everyone will eventually end up with their own badge.

7. Let $p = 10009$ be a prime number. Determine the number of ordered pairs of integers (x, y) such that $1 \le x, y \le p$ and $x^3 - 3xy + y^3 + 1$ is divisible by p.

8. For $n \ge 1$ call a finite sequence $(a_1, a_2 \ldots a_n)$ of positive integers progressive if $a_i < a_{i+1}$ and a_i divides a_{i+1} for all $1 \le i \le n - 1$. Find the number of progressive sequences such that the sum of the terms in the sequence is equal to 360.

9. Let $P(x)$ be the unique polynomial of degree at most 2020 satisfying $P(k^2) = k$ for $k = 0, 1, 2, \ldots, 2020$. Compute $P(2021^2)$.

10. Fred the Four-Dimensional Fluffy Sheep is walking in 4-dimensional space. He starts at the origin. Each minute, he walks from his current position (a_1, a_2, a_3, a_4) to some position (x_1, x_2, x_3, x_4) with integer coordinates satisfying

$$(x_1 - a_1)^2 + (x_2 - a_2)^2 + (x_3 - a_3)^2 + (x_4 - a_4)^2 = 4 \quad \text{and} \quad |(x_1 + x_2 + x_3 + x_4) - (a_1 + a_2 + a_3 + a_4)| = 2.$$

In how many ways can Fred reach $(10, 10, 10, 10)$ after exactly 40 minutes, if he is allowed to pass through this point during his walk?

Time limit: 50 minutes.
Each problem is worth one point.

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2 Answers

1. 201. Source: AIME 2004 I 2

   Solution: https://artofproblemsolving.com/wiki/index.php/2004_AIME_I_Problems/Problem_2

2. 803. Source: AIME 2002 II 4

   Solution: https://artofproblemsolving.com/wiki/index.php/2002_AIME_II_Problems/Problem_4

3. 640. Source: AIME 2010 II 1

   Solution: https://artofproblemsolving.com/wiki/index.php/2010_AIME_II_Problems/Problem_1

4. 147. Source: AIME 2018 I 8

   Solution: https://artofproblemsolving.com/wiki/index.php/2018_AIME_I_Problems/Problem_8

5. 22. Source: CMIMC 2020 Team 5

   Solution: http://cmimc-official.herokuapp.com/docs/past-tests/2020_Team_S.pdf

6. 16/45. Source: CMIMC 2017 Combinatorics 9

   Solution: http://cmimc-official.herokuapp.com/docs/past-tests/2017_Combinatorics_S.pdf

7. 30024. Source: CMIMC 2020 Algebra/NT 9

   Solution: http://cmimc-official.herokuapp.com/docs/past-tests/2020_ANT_S.pdf

8. 47. Source: AIME 2019 II 12

   Solution: https://artofproblemsolving.com/wiki/index.php/2019_AIME_II_Problems/Problem_12

9. $2021 - \binom{4040}{2020}$. Source: HMMT 2020 Algebra/NT 8

   Solution: https://hmmt-archive.s3.amazonaws.com/tournaments/2020/feb/algnt/solutions.pdf

10. $\binom{40}{10}\binom{40}{20}^3$. Source: HMMT 2019 Combo 10

    Solution: https://hmmt-archive.s3.amazonaws.com/tournaments/2019/feb/comb/solutions.pdf