sample

Made with TiKT*

Sean

Generated June 11, 2020

1 Problems

1. David recently bought a large supply of letter tiles. One day he arrives back to his dorm to find that some of the tiles have been arranged to read CENTRAL MICHIGAN UNIVERSITY. What is the smallest number of tiles David must remove and/or replace so that he can rearrange them to read CARNEGIE MELLON UNIVERSITY?

2. What is the largest factor of 130000 that does not contain the digit 0 or 5?

3. Six congruent circles form a ring with each circle externally tangent to two circles adjacent to it. All circles are internally tangent to a circle C with radius 30. Let K be the area of the region inside circle C and outside of the six circles in the ring. Find $\lfloor K \rfloor$.

4. The members of a distinguished committee were choosing a president, and each member gave one vote to one of the 27 candidates. For each candidate, the exact percentage of votes the candidate got was smaller by at least 1 than the number of votes for that candidate. What was the smallest possible number of members of the committee?

5. Let $x = \frac{4}{(\sqrt{5}+1)(\sqrt[4]{5}+1)(\sqrt[8]{5}+1)(\sqrt[16]{5}+1)}$. Find $(x+1)^{48}$.

6. For every $m \geq 2$, let $Q(m)$ be the least positive integer with the following property: For every $n \geq Q(m)$, there is always a perfect cube k^3 in the range $n < k^3 \leq m \cdot n$. Find the remainder when
$$\sum_{m=2}^{2017} Q(m)$$
is divided by 1000.

7. Centered at each lattice point in the coordinate plane are a circle radius $\frac{1}{10}$ and a square with sides of length $\frac{1}{5}$ whose sides are parallel to the coordinate axes. The line segment from $(0,0)$ to $(1001, 429)$ intersects m of the squares and n of the circles. Find $m+n$.

8. Triangle ABC has side lengths $AB = 7, BC = 8$, and $CA = 9$. Circle ω_1 passes through B and is tangent to line AC at A. Circle ω_2 passes through C and is tangent to line AB at A. Let K be the intersection of circles ω_1 and ω_2 not equal to A. Then $AK = \frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

9. In $\triangle ABC$, the external angle bisector of $\angle BAC$ intersects line BC at D. E is a point on ray AC such that $\angle BDE = 2\angle ADB$. If $AB = 10$, $AC = 12$, and $CE = 33$, compute $\frac{DB}{DE}$.

*By Sean Li. Selected problems belong to their respective authors and oragnizations, as attributed.

10. Consider arrangements of the 9 numbers $1, 2, 3, \ldots, 9$ in a 3×3 array. For each such arrangement, let a_1, a_2, and a_3 be the medians of the numbers in rows 1, 2, and 3 respectively, and let m be the median of $\{a_1, a_2, a_3\}$. Let Q be the number of arrangements for which $m = 5$. Find the remainder when Q is divided by 1000.

Time limit: 50 minutes.
Each problem is worth one point.

2 Answers

1. 5
2. 26
3. 942
4. 134
5. 125
6. 59
7. 574
8. 11
9. 2/3
10. 360

3 Problem Info

1. **Source**: CMIMC 2019 Team 1

   Tags: [2020-06, 5C, combo, experiment]

   Solution: `http://cmimc-official.herokuapp.com/docs/past-tests/2019_Team_S.pdf`

2. **Source**: HMNT 2018 General 1

   Tags: [2020-06, 5C, number-theory, factors]

   Solution: `https://hmmt-archive.s3.amazonaws.com/tournaments/2018/nov/gen/solutions.pdf`

3. **Source**: AIME 2005 I 1

   Tags: [2020-06, 5C, geometry, area]

   Solution: `https://artofproblemsolving.com/wiki/index.php/2005_AIME_I_Problems/Problem_1`

4. **Source**: AIME 2003 II 12

   Tags: [2020-06, 20C, number-theory, algebra, courage]

   Solution: `https://artofproblemsolving.com/wiki/index.php/2003_AIME_II_Problems/Problem_12`

5. **Source**: AIME 2005 II 7

   Tags: [2020-06, 25C, algebra, manipulate]

   Solution: `https://artofproblemsolving.com/wiki/index.php/2005_AIME_II_Problems/Problem_7`

6. **Source**: AIME 2017 I 13

   Tags: [2020-06, 25C, algebra, courage]

   Solution: `https://artofproblemsolving.com/wiki/index.php/2017_AIME_I_Problems/Problem_13`

7. **Source**: AIME 2016 I 14

   Tags: [2020-06, 30C, combo, courage, careful]

   Solution: `https://artofproblemsolving.com/wiki/index.php/2016_AIME_I_Problems/Problem_14`

8. **Source**: AIME 2019 II 11

   Tags: [2020-06, 35C, geometry, lengths, gimmick]

   Solution: `https://artofproblemsolving.com/wiki/index.php/2019_AIME_II_Problems/Problem_11`

9. **Source**: HMNT 2019 Team 8

   Tags: [2020-06, 35C, geometry, lengths]

   Solution: `https://hmmt-archive.s3.amazonaws.com/tournaments/2019/nov/team/problems.pdf`

10. **Source**: AIME 2017 I 11

    Tags: [2020-06, 35C, combo, cases]

    Solution: `https://artofproblemsolving.com/wiki/index.php/2017_AIME_I_Problems/Problem_11`