Math Practice 4.0

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Youth EUCLID Association

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1 Problems

1. For breakfast, Mihir always eats a bowl of Lucky Charms cereal, which consists of oat pieces and marshmallow pieces. He defines the *luckiness* of a bowl of cereal to be the ratio of the number of marshmallow pieces to the total number of pieces. One day, Mihir notices that his breakfast cereal has exactly 90 oat pieces and 9 marshmallow pieces, and exclaims, "This is such an unlucky bowl!" How many marshmallow pieces does Mihir need to add to his bowl to double its luckiness?

2. The product $\underline{3}\ \underline{4}\ \underline{5} \cdot \underline{567}$ is equal to a perfect square. However, if we change one of the six underlined digits it is possible to get a product of N^2 for some positive integer N. Compute N.

3. Find the number of ordered pairs of positive integer solutions (m, n) to the equation $20m + 12n = 2012$.

4. Kayla draws three triangles on an infinitely large sheet of paper. What is the maximum possible number of regions, including the exterior region, that the paper can be divided into by the sides of the triangles?

5. Define a search algorithm called `powSearch`. Throughout, assume A is a 1-indexed sorted array of distinct integers. To search for an integer b in this array, we search the indices $2^0, 2^1, \ldots$ until we either reach the end of the array or $A[2^k] > b$. If at any point we get $A[2^k] = b$ we stop and return 2^k. Once we have $A[2^k] > b > A[2^{k-1}]$, we throw away the first 2^{k-1} elements of A, and recursively search in the same fashion. For example, for an integer which is at position 3 we will search the locations $1, 2, 4, 3$.

   Define $g(x)$ to be a function which returns how many (not necessarily distinct) indices we look at when calling `powSearch` with an integer b at position x in A. For example, $g(3) = 4$. If A has length 64, find
   $$g(1) + g(2) + \ldots + g(64).$$

6. An equilateral triangle is inscribed in the ellipse whose equation is $x^2 + 4y^2 = 4$. One vertex of the triangle is $(0, 1)$, one altitude is contained in the y-axis, and the length of each side is $\sqrt{\frac{m}{n}}$, where m and n are relatively prime positive integers. Find $m + n$.

7. A mail carrier delivers mail to the nineteen houses on the east side of Elm Street. The carrier notices that no two adjacent houses ever get mail on the same day, but that there are never more than two houses in a row that get no mail on the same day. How many different patterns of mail delivery are possible?

8. For every subset T of $U = \{1, 2, 3, \ldots, 18\}$, let $s(T)$ be the sum of the elements of T, with $s(\emptyset)$ defined to be 0. If T is chosen at random among all subsets of U, the probability that $s(T)$ is divisible by 3 is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find m.

9. Ms. Math's kindergarten class has 16 registered students. The classroom has a very large number, N, of play blocks which satisfies the conditions:

   (i) If 16, 15, or 14 students are present in the class, then in each case all the blocks can be distributed in equal numbers to each student, and

   (ii) There are three integers $0 < x < y < z < 14$ such that when x, y, or z students are present and the blocks are distributed in equal numbers to each student, there are exactly three blocks left over.

   Find the sum of the distinct prime divisors of the least possible value of N satisfying the above conditions.

10. Rectangle $ABCD$ and a semicircle with diameter AB are coplanar and have nonoverlapping interiors. Let $\mathcal{R}$ denote the region enclosed by the semicircle and the rectangle. Line ℓ meets the semicircle, segment AB, and segment CD at distinct points N, U, and T, respectively. Line ℓ divides region $\mathcal{R}$ into two regions with areas in the ratio $1 : 2$. Suppose that $AU = 84$, $AN = 126$, and $UB = 168$. Then DA can be represented as $m\sqrt{n}$, where m and n are positive integers and n is not divisible by the square of any prime. Find $m + n$.

Time limit: 50 minutes.
Each problem is worth one point.

2 Answers

1. 11. Source: HMNT 2019 Theme 1

   Solution: https://hmmt-archive.s3.amazonaws.com/tournaments/2019/nov/thm/solutions.pdf

2. 441. Source: FARML 2020 Individual 1

   Solution: https://drive.google.com/file/d/1QotkxlmA-pb8c8aQZFIWE7iRSeosnqTu/view

3. 34. Source: AIME 2012 II 1

   Solution: https://artofproblemsolving.com/wiki/index.php/2012_AIME_II_Problems/Problem_1

4. 20. Source: NIMO 2017 C27 1

   Solution: https://artofproblemsolving.com/community/c139h1311402p7026670

5. 808. Source: CMIMC 2019 Combinatorics 4

   Solution: http://cmimc-official.herokuapp.com/docs/past-tests/2019_CCS_S.pdf

6. 937. Source: AIME 2001 I 5

   Solution: https://artofproblemsolving.com/wiki/index.php/2001_AIME_I_Problems/Problem_5

7. 351. Source: AIME 2001 I 14

   Solution: https://artofproblemsolving.com/wiki/index.php/2001_AIME_I_Problems/Problem_14

8. 683. Source: AIME 2018 I 12

   Solution: https://artofproblemsolving.com/wiki/index.php/2018_AIME_I_Problems/Problem_12

9. 148. Source: AIME 2013 I 11

   Solution: https://artofproblemsolving.com/wiki/index.php/2013_AIME_I_Problems/Problem_11

10. 69. Source: AIME 2010 I 13

    Solution: https://artofproblemsolving.com/wiki/index.php/2010_AIME_I_Problems/Problem_13