MATH 3140– Homework 11
Due: Monday, 26 November 2018
Exercises: Ch. 11. 7, 11, 17, 18, 19.

11.7 In the group $\mathbb{Z}_{24}$, let $H = \langle 4 \rangle$ and $N = \langle 6 \rangle$.

(a) List the elements in HN (we usually write $H+N$ for these additive groups) and $H \cap N$.

(b) List the cosets in HN/N, showing the elements in each coset.

(c) List the cosets in $H/(H \cap N)$, showing the elements in each coset.

(d) Give the correspondence between HN/N and $H/(H \cap N)$ described in the proof of the Second Isomorphism Theorem.

11.11 Show that a homomorphism defined on a cyclic group is completely determined by its action on the generator of the group.

11.17 If H and K are normal subgroups of G and $H \cap K = \{e\}$, prove that G is isomorphic to a subgroup of $G/H \times G/K$.

11.18 Let $\varphi : G_1 \to G_2$ be a surjective group homomorphism. Let H_1 be a normal subgroup of G_1 and suppose that $\varphi(H_1) = H_2$. Prove or disprove that $G_1/H_1 \cong G_2/H_2$.

11.19 Let $\phi : G \to H$ be a group homomorphism. Show that ϕ is one-to-one if and only if $\phi^{-1}(e) = \{e\}$.