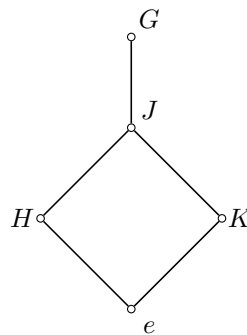


Required Exercises. Chapter 4: 6, 12, 13, 28, 30, 35.

6. Find the order of every element in the symmetry group of the square, D_4 .
12. Find a cyclic group with exactly one generator. Can you find cyclic groups with exactly two generators? Four generators? How about n generators?
13. For $n \leq 20$, which groups $U(n)$ are cyclic? Make a conjecture as to what is true in general. Can you prove your conjecture?
28. If a is an element of a group, what is a generator for the subgroup $\langle a^m \rangle \cap \langle a^n \rangle$?
30. Suppose a and b are elements of a group. Prove that if $|a| = m$ and $|b| = n$ with $\gcd(m, n) = 1$, then $\langle a \rangle \cap \langle b \rangle = \{e\}$.
35. Prove that the subgroups of \mathbb{Z} are exactly $n\mathbb{Z}$ for $n = 0, 1, 2, \dots$

Suggested Exercises. Chapter 4: 38; plus Exercise 47 (stated below).

38. Prove that the order of an element in a cyclic group must divide the order of the group.
47. Explain why the Hasse diagram below cannot be the entire subgroup lattice of a group. (This exercise does not appear in the textbook.)



That is, H , J , and K are supposed to be subgroups of G such that $H \leq J$ and $K \leq J$; moreover, there is supposed to be exactly one maximal subgroup of G , namely J ; finally, $H \cap K = \langle e \rangle$. (*Hint:* Does there exist an element of G that does not belong to J ?)