MATH 3140 Homework 8

Exercises: 1 below and Judson: 6.5bd, 6.11ade, 6.14, 6.16, 6.18
Due date: Friday, 10/26

1. Prove or disprove the following:

(a) There exists a group G of order $|G| = 8$ with an element $g \in G$ of order $|g| = 3$.

(b) If H and K are subgroups of a group G with $|H| = 2$ and $|K| = 3$, then $|G| \geq 6$.

(c) Every subgroup of the integers has finite index.

(d) Every subgroup of the integers has finite order.

6.5. In each case below, list the left cosets of H in G.

b. $G = U(8)$, $H = \langle 3 \rangle$.

c. $G = S_4$, $H = A_4$.

6.11. Let H be a subgroup of a group G and suppose that $g_1, g_2 \in G$. Prove that the following conditions are equivalent:

(a) $g_1 H = g_2 H$

(d) $g_2 \in g_1 H$

(e) $g_1^{-1} g_2 \in H$

6.14 Let G be a group and suppose $g \in G$, $n > 0$, and $g^n = e$. Show that the order of g divides n.

6.16. If $|G| = 2n$, prove that the number of elements of order 2 is odd. Use this result to show that G must contain a subgroup of order 2.

6.18. If $[G : H] = 2$, prove that $gH = Hg$.