Math 3140 – Homework 4 **Due date:** Wednesday, 9/26

Chapter 3 (continued)

Required Exercises: Chapter 3. 16, 17, 31, 34, 45, 46, 52.

16. Give a specific example of a group G and elements $g, h \in G$ where $(gh)^n \neq g^n h^n$.

17. Give examples of three different groups with eight elements. Why are the groups different?

31. Show that if $a^2 = e$ for all elements a in a group, then the group must be abelian.

34. Find all the subgroups of $\mathbb{Z}_3 \times \mathbb{Z}_3$. Use this information to show that $\mathbb{Z}_3 \times \mathbb{Z}_3$ is not the same group as $\mathbb{Z}_9$. (See Example 3.38 for a reminder of how one defines the product of groups.)

45. Prove that the intersection of two subgroups of a group G is also a subgroup of G.

46. Prove or disprove: If H and K are subgroups of a group G, then $H \cup K$ is a subgroup of G.

52. Prove or disprove: Every proper, nontrivial subgroup of an nonabelian group is nonabelian.

Additional suggested exercises: Chapter 3. 32, 33, 35, 47, 48, 51, 54.

32. Show that if G is a finite group of even order, then there is an $a \in G$ such that a is not the identity and $a^2 = e$.

33. Show that if $(ab)^2 = a^2b^2$ for all elements a and b in a group, then the group must be abelian.

35. Compute the subgroups of the symmetry group of a square.

47. Prove or disprove: If H and K are subgroups of a group G, then $HK = \{hk : h \in H \text{ and } k \in K\}$ is a subgroup of G. What if G is abelian?

48. Let G be a group and $g \in G$. Show that

$$Z(G) = \{x \in G : gx = xg \text{ for all } g \in G\}$$

is a subgroup of G. This subgroup is called the *center* of G.

51. If $xy = x^{-1}y^{-1}$ for all elements x and y in a group, then the group is abelian.

54. Let $H \leq G$ be a subgroup. If $g \in G$, then $gHg^{-1} = \{g^{-1}hg : h \in H\}$ is also a subgroup of G.