Math 3140 **Homework 6**

Chapter 5: 1bd, 3bd, 4, 6, 17, 18, 27. **Due date:** Friday, 10/05

Additional suggested exercises: 29, 31, 32, 33.

1. Write the following permutations in cycle notation.

(a)
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 1 & 5 & 3 \end{pmatrix}$$

(b)
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 1 & 3 \end{pmatrix}$$

(c)
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 1 & 4 & 2 \end{pmatrix}$$

(d)
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 3 & 2 & 5 \end{pmatrix}$$

3. Express the following permutations as products of transpositions and identify them as even or odd. (Only (b) and (d) are required.)

(a) (14356)

(b) (156)(234)

(c) (1426)(142)

(d) (17254)(1423)(154632)

(e) (142637)

4. Find $(a_1, a_2, \ldots, a_n)^{-1}$.

6. Find all of the subgroups in A_4. What is the order of each subgroup?

17. Prove that S_n is nonabelian for $n \geq 3$.

18. Prove that A_n is nonabelian for $n \geq 4$.

27. Let G be a group and define a map $\lambda_g : G \to G$ by $\lambda_g(a) = ga$. Prove that λ_g is a permutation of G.

Additional suggested exercises: 29, 31, 32, 33.

29. Recall that the *center* of a group G is

$$Z(G) = \{g \in G : gx = xg \text{ for all } x \in G\}.$$

Find the center of D_8. What about the center of D_{10}? What is the center of D_n?

31. For α and β in S_n, define $\alpha \sim \beta$ if there exists an $\sigma \in S_n$ such that $\sigma\alpha\sigma^{-1} = \beta$. Show that $\sim$ is an equivalence relation on S_n.

32. Let $\sigma \in S_X$. If $\sigma^n(x) = y$, we will say that $x \sim y$.

(a) Show that $\sim$ is an equivalence relation on X.

(b) If $\sigma \in A_n$ and $\tau \in S_n$, show that $\tau^{-1}\sigma\tau \in A_n$.

(c) Define the *orbit* of $x \in X$ under $\sigma \in S_X$ to be the set

$$\mathcal{O}_{x,\sigma} = \{y : x \sim y\}.$$

Compute the orbits of α, β, γ where

$$\begin{aligned} \alpha &= (1254) \\ \beta &= (123)(45) \\ \gamma &= (13)(25). \end{aligned}$$

(d) If $\mathcal{O}_{x,\sigma} \cap \mathcal{O}_{y,\sigma} \neq \varnothing$, prove that $\mathcal{O}_{x,\sigma} = \mathcal{O}_{y,\sigma}$. The orbits under a permutation σ are the equivalence classes corresponding to the equivalence relation $\sim$.

(e) A subgroup H of S_X is *transitive* if for every $x, y \in X$, there exists a $\sigma \in H$ such that $\sigma(x) = y$. Prove that $\langle\sigma\rangle$ is transitive if and only if $\mathcal{O}_{x,\sigma} = X$ for some $x \in X$.

33. Let $\alpha \in S_n$ for $n \geq 3$. If $\alpha\beta = \beta\alpha$ for all $\beta \in S_n$, prove that α must be the identity permutation; hence, the center of S_n is the trivial subgroup.