

## MATH 3140 Homework 8

**Exercises:** 1 below and Judson: 6.5bd, 6.11ade, 6.14, 6.16, 6.18

**Due date:** Friday, 10/26

1. Prove or disprove the following:

- (a) There exists a group  $G$  of order  $|G| = 8$  with an element  $g \in G$  of order  $|g| = 3$ .
- (b) If  $H$  and  $K$  are subgroups of a group  $G$  with  $|H| = 2$  and  $|K| = 3$ , then  $|G| \geq 6$ .
- (c) Every subgroup of the integers has finite index.
- (d) Every subgroup of the integers has finite order.

**6.5.** In each case below, list the left cosets of  $H$  in  $G$ .

**b.**  $G = U(8)$ ,  $H = \langle 3 \rangle$ .

**c.**  $G = S_4$ ,  $H = A_4$ .

**6.11.** Let  $H$  be a subgroup of a group  $G$  and suppose that  $g_1, g_2 \in G$ . Prove that the following conditions are equivalent:

- (a)  $g_1H = g_2H$
- (d)  $g_2 \in g_1H$
- (e)  $g_1^{-1}g_2 \in H$

**6.14** Suppose that  $g^n = e$ . Show that the order of  $g$  divides  $n$ .

**6.16.** If  $|G| = 2n$ , prove that the number of elements of order 2 is odd. Use this result to show that  $G$  must contain a subgroup of order 2.

**6.18.** If  $[G : H] = 2$ , prove that  $gH = Hg$ .