**Math 3140 – Homework 5** **Due date:** Monday, 10/1

**Required Exercises. Chapter 4: 6, 12, 13, 28, 30, 35**.

**6.** Find the order of every element in the symmetry group of the square, $D_4$.

**12.** Find a cyclic group with exactly one generator. Can you find cyclic groups with exactly two generators? Four generators? How about $n$ generators?

**13.** For $n \leq 20$, which groups $U(n)$ are cyclic? Make a conjecture as to what is true in general. Can you prove your conjecture?

**28.** If $a$ is an element of a group $G$, what is a generator for the subgroup $\langle a^m \rangle \cap \langle a^n \rangle$?

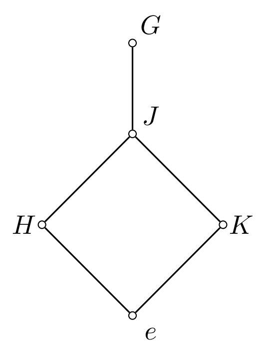
**30.** Suppose $a$ and $b$ are elements of a group. Prove that if $|a| = m$ and $|b| = n$ with $\gcd(m, n) = 1$, then $\langle a \rangle \cap \langle b \rangle = \{e\}$.

**35.** Prove that the subgroups of $\mathbb{Z}$ are exactly $n\mathbb{Z}$ for $n = 0, 1, 2, \ldots$.

**Suggested Exercises**. Chapter 4: 38; plus Exercise 47 (stated below).

**38.** Prove that the order of an element in a cyclic group must divide the order of the group.

**47.** Explain why the Hasse diagram below cannot be the entire subgroup lattice of a group. (This exercise does not appear in the textbook.)




That is, $H$, $J$, and $K$ are supposed to be subgroups of $G$ such that $H \leq J$ and $K \leq J$; moreover, there is supposed to be exactly one maximal subgroup of $G$, namely $J$; finally, $H \cap K = \langle e \rangle$. (*Hint:* Does there exist an element of $G$ that does not belong to $J$?)