



The University of Manchester

Department of Computer Science
Project Report 2020

**Reinforcement learning for a learnable agent
in classic arcade games**

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Abstract

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Acknowledgements

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Chapter 1

Introduction

My project aims to replicate some of the reinforcement learning algorithms that can be used to play classic Atari 2600 games. It also compares the results of different tests with these algorithms such as varying the hyperparameters of the network. By observing the effects on the trained agents¹ when the hyperparameters are changed we can deduce a set of optimal values such that the networks can play three different Atari 2600 games. Overall, the main features of the project are the following:

- Agents with raw pixel game data as input, outputting a set of values for the best action.
- Agents attempt to find an optimal model of the environment without any prior knowledge.
- Visualization of the agents “brain” to provide insight into what information the agent is learning.

1.1 Impact of COVID-19

COVID-19

1.2 Motivation

Over the past 10 years, there has been significant improvement in the RL (reinforcement learning) algorithms. One reason is that computing power has become cheaply available by using discrete graphics cards. For example, for my project, I used an Nvidia GeForce GTX 1070 that provides 1920 CUDA cores that can be used to accelerate training of neural networks. Despite this, RL algorithms are massively computationally expensive and hence take a long time to train.

Over recent years one of the pioneers in this area is DeepMind, which was acquired by Google in 2014, and they developed the DQN (deep q-network) algorithm in 2013 which they demonstrated could learn directly from the raw pixel data of games to achieve either human-level or super-human level performance.

¹Agent. In this case, agent refers to a trained neural network that takes actions in a chosen environment.

This research was expanded upon by DeepMind and OpenAI which is based on the original DQN by DeepMind. This research focused on trying to approximate a Q-function and thereby infer the optimal policy. On the other hand, there has recently been a focus on other methods such as A3C and PPO which instead seek to directly optimise in the policy space of the environment.

1.3 Objectives

Further to what was described in section 1, there were a few main objectives of the project. Firstly, I chose three games on which I decided to train the agents, Pong, Breakout, and Space Invaders which are shown below in Figure 1.1.

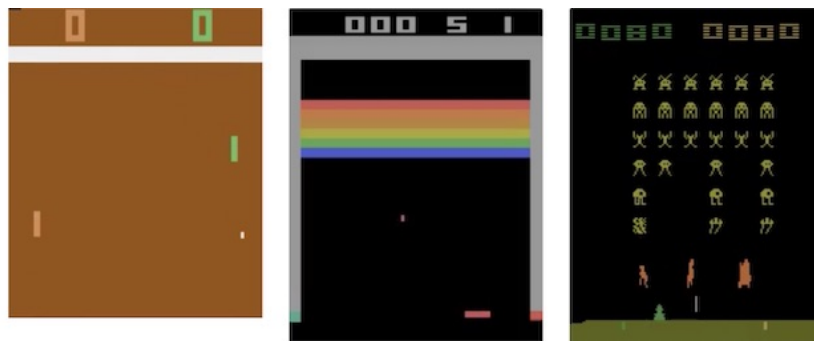


Figure 1.1: Screenshots of Pong, Breakout, Space Invaders (left to right).

Secondly, I wanted to find a way to explore the internals of a trained agent, to give further insight into what the agent is trying to learn. The reason for this is a researcher could use this information to determine, for example, where the agent has learnt to focus on the frame. Additionally, it provides an insight into how to optimally tune the hyperparameters which are described in section TODO: include section.

TODO: include gantt chart

1.4 Report structure

My report is divided into three main sections. First, describing the background of the problem, then going onto giving details of my implementation and finally project evaluations/conclusions.

Chapter 2

Background

This chapter will cover some of the background material required for the following sections, it will cover the history of reinforcement learning (RL) and the field's evolution to the current state-of-the-art. Additionally, it will cover the related work to this project and also cover some details of the past research papers for which this project has been based upon.

2.1 Reinforcement learning

Reinforcement learning is an area of machine learning that has been under active research since the late 1980s [17]. The first defining algorithm of RL was called ‘*temporal-difference learning*’ often referred to as TD-Learning. This algorithm learns by bootstrapping from the current value function in order to iteratively converge towards an optimal policy (i.e. the agent’s strategy for taking actions in the environment).

Further work by R. Sutton led to the development of TD-Lambda, an algorithm that was applied to the game of Backgammon, in 1992, by Gerald Tesauro to create TD-Gammon [13]. It was a computer program that was shown to compete at expert-human level. The program also found novel strategies that were either unexplored, or dismissed in error as poor strategies. This was the first example of RL aiding in discovery or reconsideration of board game strategies. This trend of RL algorithms helping to improve human play would prove to only continue with DeepMind’s AlphaZero program mastering the games of Chess (beating the strongest Chess programs such as Stockfish¹) and Go.

2.1.1 Deep reinforcement learning

Following from section 2.1 on RL, this section talks about the combination of two areas, deep learning and reinforcement learning methods, called deep reinforcement learning (DRL). Deep learning (DL) is a common class of machine learning methods that has been of much research focus over the past decade and can deal with high-dimensional sensory input; for the case of Atari this is 84x84 greyscale images after pre-processing of the raw Atari frames. On the other hand, reinforcement learning allows us to create an agent which can learn an optimal policy to navigate some environment in order to optimise its reward.

¹[Stockfish](#). One of the strongest Chess programs based on the CCRL ratings list.

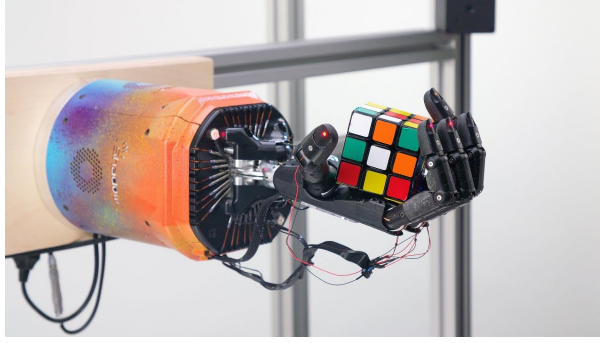


Figure 2.1: OpenAI Robot solving a Rubik’s cube

Through the combination of these methods, it has proven to provide solutions to previously intractable problems [1] in areas such as robotics, computer vision and healthcare. For example, in 2019 OpenAI developed a robotic hand that could solve a Rubik’s cube 2.1, trained using deep reinforcement learning. As an extension to DRL, end-to-end reinforcement learning is a method for single layered neural network, trained by reinforcement learning. Figure 2.2 shows, diagrammatically, how DL and RL are used together in order to produce a single end-to-end model. In this simple architecture, there are two main components, the agent and the environment. This is a key feature of all DRL methods, an agent observes some state and reward from the environment after taking an action.

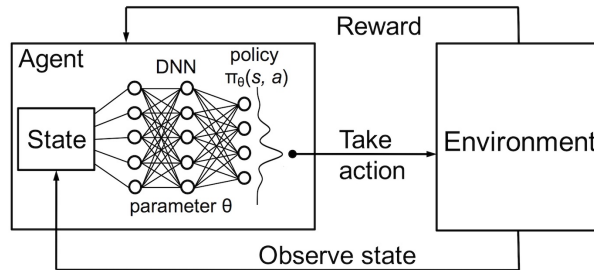


Figure 2.2: Representation of end-to-end RL architectures

As was mentioned in Section 2.1 there has been a renewed focus on the area of DRL. This research trend originated in 2013 when DeepMind showed that using a combination of Q-Learning and deep learning, it can produce agents that can compete with expert-humans in Atari.

2.2 DQN on Atari 2600

As mentioned in previous sections, one of the pioneering companies in the area was DeepMind. In 2013, V.Mnih et al. while working at DeepMind, released a paper titled “*Playing Atari with Deep Reinforcement Learning*” [8]. This paper combined DRL, convolutional neural networks and a novel strategy called **experience replay**. DeepMind, in 2014, patented Q-Learning and its application with deep learning on Atari games. Further, they also published further papers expanding on the idea in prestigious journals such as NIPS and Nature. Figure 2.3 shows the structure of a Deep Q-Network as used to play Atari 2600 games.

Experience replay was an important improvement that helped in stabilising the Q-Learning algorithm. It does this by removing the correlations between a observation sequence, i.e. we don't learn from a chronological series of frames, rather we store all the experience and learn using random samples of this memory.

Over the following few years, this area would see rapid progress with many different advancements by both DeepMind and OpenAI. As was noted with the introduction of Deep Q-Learning (often referred to as ‘*Deep Q-Networks*’ or *DQN*) the algorithm suffers from overestimating the value of some actions. This can lead to a poor performance on more complex Atari games such as Space Invaders. Further detailed descriptions of the Q-Learning algorithm is provided in Section 3.3.

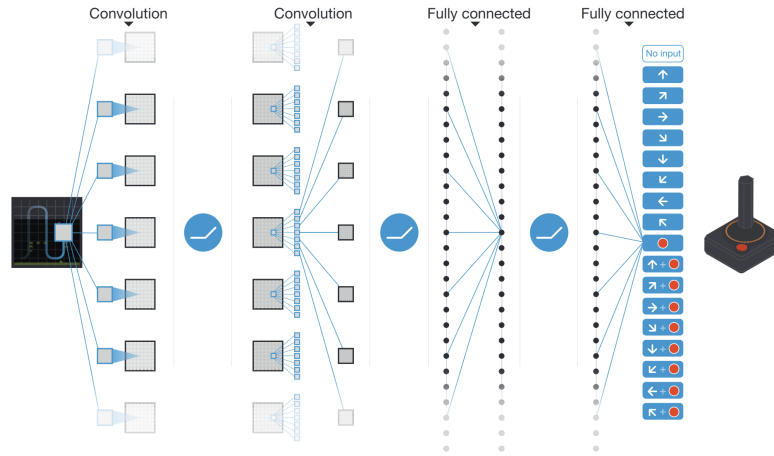


Figure 2.3: Deep Q-Network architecture that from left-to-right is showing how a single frame is processed. First passing through convolutional layers, before being flattened to a single tensor which is connected to a fully connected network. This MLP acts as the Q-function approximator which will produce an output of predicted Q-values for each action.

2.3 CNN Visualisation

A common criticism of Neural Networks is that what the network learns is difficult to understand and interpret, therefore, over recent years some methods have been developed in order to visualise how the network learns. This section will describe some of the most common methods and those that produce the most visually pleasing results.

There are some considerations we need to make when visualising the CNN, for example, if we include a ReLU layer, the ReLU neurons don't have a semantic meaning by themselves. On the otherhand, in the individual filters in a CONV layer, we can observe the layer activations which indicate whether or not the network is converging.



Figure 2.4: Visualisation of the first CONV layer of a CNN to play Pong, there are many filters completely black, in this agent, it means the learning rate was too high. Red highlighted filters are those filters with zero activation.

Based on Figure 2.4, some of the filters show zero activation which can be a symptom of a high learning rate; these are often called “dead” filters. Also, in terms of Atari games, during training, we can see what features the network is focusing on in attempt to maximise the rewards. Based on this information, one can tune both the learning rate during fine-tuning in order to extract more performance from the model.

In addition to visualising the individual layers of the CNN, there are some more advanced visualisation techniques. A common approach is to use Principle Component Analysis (PCA) which can be used to reduce the number of dimensions of the fully-connected layers down to 2 dimensions, showing the variance of each principal component which can allow the researcher to construct a new dataset that will improve the performance using the components with a high variance.

Another, more complex algorithm is called t-SNE [14] which is a dimensionality reduction algorithm for high-dimensional datasets. It has been known to continuously produce visually pleasing results and was used in some of the papers referenced during the development of this project.

Chapter 3

Design

This section will cover the details of the methods and algorithms that were used in the implementation of the project. The previous section covered the history of some of these techniques which were imperative for building the foundation of the methods described in this chapter.

Deep Q-Networks and its enhancements have the majority of focus, as it is the basis of the project. During the development of the project, the major focus was having an understanding of both reinforcement learning mathematically, and also how it is implemented in practise. It was noted in David Silver UCL reinforcement learning lecture series [11] that some of the requirements for policy convergence are not as strict in practise (making the actual implementation simpler).

3.1 Markov decision process

This section will describe some of the basics of Markov decision processes (MDP), they are the foundation of the reinforcement learning methods that will be described later in this chapter. First, we need some mathematical definitions of MDPs, these are from David Silver's excellent lecture series on Reinforcement Learning.

Markov decision processes are just markov reward processes with decisions, i.e. At each state S_t , we have a finite set of actions to choose from in order to get to a new state S_{t+1} .

Definition 3.1. A Markov decision process is a tuple $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma \rangle$.

- \mathcal{S} , finite set of states
- \mathcal{A} , finite set of actions
- \mathcal{P} , state transition probability matrix,
 $\mathcal{P}_{ss'}^a = \mathbb{P}[S_{t+1} = s' | S_t = s, A_t = a]$
- \mathcal{R} , reward function, $\mathcal{R}_s^a = \mathbb{E}[R_{t+1} | S_t = s, A_t = a]$
- γ discount factor, $\gamma \in [0, 1]$

The definition above defines a Markov decision process, which we use as a basis for describing the methods in reinforcement learning. In order to illustrate the idea, let us

consider the game of Pong. For simplicity, assume we can encode each frame of the game into the set of states \mathcal{S} . In order to play the game, we need to know what the best action to take would be at each frame of the game to move the paddle under the ball, hitting the ball back and possibly scoring a point.

Given a frame of the game S_t and the set of actions we can choose from A_t , we are going to try and maximise our future (expected) reward using the reward function $\mathcal{R}_{S_t}^{A_t}$. We want to choose the best action a , that will result in the maximum future reward. It is important that we don't look at immediate rewards only, since, in Pong we don't get the point until we have hit the ball back to the other side.

In order to look at future rewards, we use the discount factor γ . When γ is close to zero, we are “*myopic*” in our evaluation (we only look for short-term rewards). However, as γ gets closer towards 1, we are “*far-sighted*” in our evaluation.

Overall, we need to know a strategy that provides the best action to take in a given state which maximises the expected total reward. This is called a *policy*, denoted by π .

Definition 3.2. A *policy* π is a distribution over actions given states,

$$\pi(a \mid s) = \mathbb{P}[A_t = a \mid S_t = s]$$

The policy of an agent fully describes the behaviour of the agent (TODO: add ref) which only depends on the current state, not the history. In order to describe the optimal policy for an agent to follow, we first need some more definitions.

Definition 3.3. G_t is the total discounted reward for time-step t

$$G_t = R_{t+1} + \gamma R_{t+2} + \cdots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

Definition 3.4. The *action-value* function $q_{\pi}(s, a)$ is expected return starting from state s , taking action a , following policy π

$$q_{\pi}(s, a) = \mathbb{E}_{\pi}[G_t \mid S_t = s, A_t = a]$$

In addition to the action-value function $q_{\pi}(s, a)$ we also define the state-value function for a given state s .

Definition 3.5. The *state-value* function $v_{\pi}(s, a)$ is the expected value of a being in a given state s while under a policy π

$$v_{\pi}(s, a) = \mathbb{E}_{a \sim \pi(s)} [q_{\pi}(s, a)]$$

Definition 3.6. The *optimal action-value* function is denoted by $q_*(s, a)$ and is the maximum action-value function over all possible policies

$$q_*(s, a) = \max_{\pi} q_{\pi}(s, a)$$

Once we have found the optimal action-value function we consider the MDP “solved”. Additionally, we know that we can, given some state, take actions that will lead to the highest possible future reward.

3.1.1 Markov property

With Markov decision processes we assume that in each state, we have all the information we require in order to produce an optimal action. However, in games such as Pong and Breakout, we may not necessarily have all the information we require at a single time. For example, Figure 3.1 shows a screenshot of the Atari game Breakout. In this situation, we cannot discern the optimal action given only this frame as it is impossible to tell if the ball is moving towards or away from the paddle.

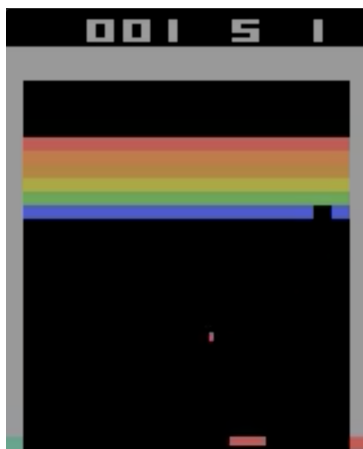


Figure 3.1: Screenshot of a single frame from the game Breakout

In order to solve this problem, we store a small rolling history of k states (where typically, $k = 4$) called the “*frame-stack*”. During forward passes through the network we pass the whole frame-stack to the network. This provides a history of frames to the network which then allows the network to be able to predict the path of the ball, allowing the network to make good actions leading to a higher score.

3.2 Reinforcement learning

Following on from the previous section on Markov decision processes, this section describes Reinforcement learning and how these two methods are tightly connected to each other.

In its basic form, RL can be modelled graphically as shown in Figure 3.2. The agent gets the state from the environment, using its policy, it chooses an action to take – updating the environment. The environment then produces some new state and a reward signal which the agent uses to pick the next action.

3.2.1 Exploration vs Exploitation

A key idea in RL is the problem of exploration vs exploitation. This means that if we have an environment, and a policy that dictates how we should navigate the environment, should we always follow the policy, or should we deviate and try to find a better path resulting in a higher reward.

In this project we follow a method called ϵ -greedy in which we explore forever, but with a linearly decreasing probability (denoted by ϵ) of random actions, this is decreased over a predefined number of timesteps. Below is the method for choosing the actions.

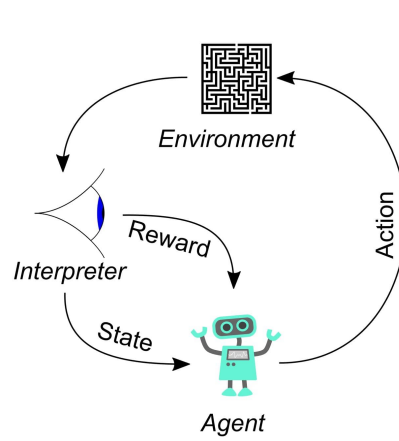


Figure 3.2: Diagram of reinforcement learning

- Choose random action with probability ϵ
- With probability $1 - \epsilon$ select action = $\arg \max_{a \in \mathcal{A}} \hat{Q}(a)$

Although ϵ -greedy is a simple method, it is surprisingly effective in exploring the environment and produces high scoring models. In fact, this method was used in the original DQN paper by V.Mnih et al. in which they scored better than human averages in the majority of Atari 2600 games.

In the area of reinforcement learning, there are some more advanced methods such as Upper Confidence Bounds (UCB) and Thompson Sampling. However, these Bayesian-based methods are not implemented/explored in this project and is left for future work.

3.3 Q-Learning

Q-Learning is a model-free algorithm that is used to solve, iteratively, the Bellman equation for the MDP. By model-free we mean that the algorithm does not require a model of the environment, therefore it can easily handle stochastic rewards. The method was introduced in 1989 by Chris Watkins in his PhD thesis titled “*Learning from delayed rewards*”.

We can use Q-Learning in order to approximate $q_*(s, a)$, this produces the optimal policy π based on the Q-values. The Q-Learning algorithm stores the state-action values in a large table of values, as represented below in Table 3.1. For each possible state that the environment can produce, we store a single value (Q-value) for each action that can be performed in that state. These Q-values represents the ‘*quality*’ of being in a state s and taking action a .

Q-Table		Actions			
		NOOP	FIRE	LEFT	RIGHT
States	0	0	0	0	0
	1	-2.3452	-1.8375	-2.3634	-1.5463
	\vdots	\vdots	\vdots	\vdots	\vdots
	128	2.4456	1.2345	6.3462	3.8356

Table 3.1: Example of how state-action pairs are stored in a Q-Table (NOOP abbreviates ‘No action/No operation’)

Since the Q-Learning algorithm is iterative, at each timestep, we need to update the Q-value corresponding to the occupied state and the action we took. In Chapter 3.5 this idea will be expanded upon, by using neural networks as a function approximator, replacing the need to store the whole table of Q-values. Algorithm 1 shows how the algorithm is implemented in order to both, predict the next action to take, and updating the Q-table. In order to update the Q-values for each state we use the Q-value update equation which is shown below.

Definition 3.7.

$$Q_{t+1}(s_t, a_t) = Q_t(s_t, a_t) + \alpha(R_t + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t))$$

- $Q(s, a)$, q-value for state-action pair
- α , learning rate
- R , immediate reward from environment
- γ discount factor, $\gamma \in [0, 1]$
- $\max_a Q(s_{t+1}, a)$, estimate of optimal future reward

A major issue with Q-Learning is that since we perform a maximisation of all future rewards when updating the Q-values, this can lead to the algorithm over-estimating the

quality of actions and slowing down the learning. During training, we use the same Q-function in order to both produce the expected maximum action-value of future rewards and, select the best possible action in the current state.

Algorithm 1: Tabular Q-Learning

```

Initialize action-value function  $Q$  with random values
for  $timestep = 1$  do
  Initialise  $S$ 
  while  $S$  is not done state do
    With probability  $\varepsilon$  choose random action  $a_t$ 
    Else, pick  $a_t = \max_a Q(s_t, a)$ 
    Take action  $a_t$  and observe reward  $r_t$  and state  $s_{t+1}$ 
     $Q_{t+1}(s_t, a_t) = Q_t(s_t, a_t) + \alpha(R_t + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t))$ 
    Update  $s_t = s_{t+1}$ 
  end
end

```

As shown above, we can use Q-learning to find the optimal policy by storing all the $q_*(s, a)$ values. However, in practise we run into a issue with the size of the Q-table in that it quickly becomes too big to store in memory. This issue is especially prevalent for large MDPs such as those from Atari games due to the raw pixel input. In order to handle the high-dimensionsonal input from the games, we can instead use a function approximator to estimate the values of $q_*(s, a)$. This idea will be developed in Section 3.5 where RL is combined with Deep Learning.

3.4 Deep Learning

This section will describe the basics used in the project for Deep Learning and some of the techniques used in order to construct the Deep Q-Networks discussed in Section 3.5.

3.4.1 Multi-layer perceptron (MLP)

The main technique that underpins most of the artificial neural networks (ANN) that are used today is how we combine single neurons to for networks of neurons. Figure 3.3 shows how we model an artificial neuron with weights and activation functions, this is supposed to loosely model how neurons work inside the brain. The input connections are acting in place of synapses which connect together to form networks.

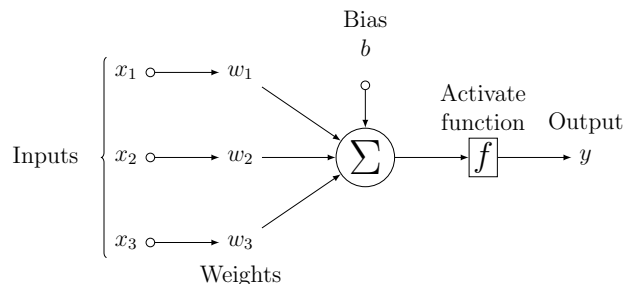


Figure 3.3: Schematic the modelling of a single neuron

The artificial neuron takes inputs $x_1 \dots x_n$ with corresponding weights $w_1 \dots w_n$. Usually, the bias b is stored in w_0 with $x_0 = 1$. During a forward pass of the network we process the input vector \mathbf{x} and producing an output y . In the center of the figure we have a single node that represents the function to calculate u which is the weighted sum of the input vector \mathbf{x} and vector of weights \mathbf{w} . The equation to calculate this quantity is $u = \sum_{i=0}^n w_i x_i$.

Finally, each neuron has an activation function associated, there are many possible options for these functions such as Logistic (Sigmoid), TanH and ReLU. Each of these functions defines a threshold for when the output u results in a 1/0 output in y .

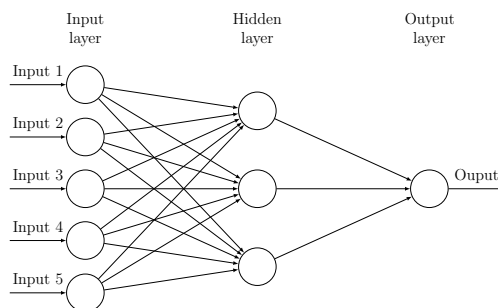


Figure 3.4: Diagram representation of MLP

Figure 3.4 shows the structure of an MLP (also called artificial neural network), we construct this network by combining multiple neurons together. The MLP consists of the n input neurons (each having a single input value), with then at least one hidden layer. The last hidden layer is connected to the output node/nodes.

In general, for each node in the ANN, it will be connected to every other node in the following layer, whether that is another hidden layer or the output layer. This network structure is referred to as “*fully-connected*” as every node in a layer is connected to every other node in the next layer. One downside to this ‘fullyconnected-ness’ is that it tends to make them prone to overfitting to the input. Therefore, many techniques have been developed in order to reduce overfitting. Some examples are using regularization, dropout layers¹, early stopping and batch normalisation. This is not the only type of network consisting of artificial neurons; by connecting the neurons in different ways the networks exhibit different behaviours such as recurrent neural networks (RNNs) that have a small ‘memory’ capacity due to the network structure.

3.4.2 Convolutional Neural Network (CNN)

Neural networks are excellent approximators for complex functions that need to be learnt, however, when we have high-dimensional inputs the size of the network can become too difficult to manage. An important note is that any CNN can in theory be implemented as a neural network individual neurons. However, if we take as an example a single frame from Atari which is 210 x 160 pixels each with RGB values. This results in having over

¹Dropout layer. A layer in a ANN that randomly ignores the input from the previous layer with a given probability. It aids in preventing neurons from becoming dependent on other neurons in previous layers, thereby reducing the effect of overfitting.

100,000 weights on just the input layers and as such, too many weights to process in an efficient manner.

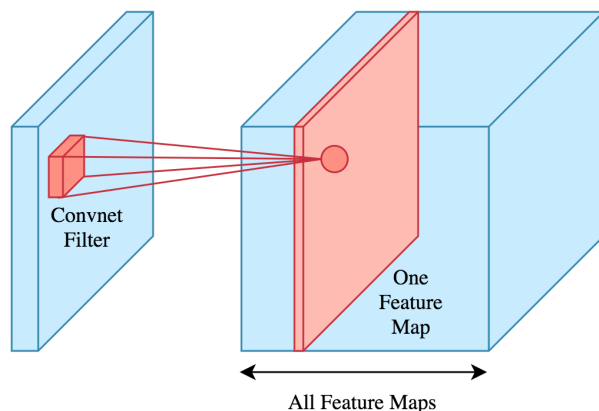


Figure 3.5: Single convolutional layer using filters

In addition, CNNs can exploit the spatial structure of the data by using different building blocks in order to construct the network. Some of the layers help to avoid problems such as overfitting which is a key problem when training these networks. Others such as ReLU layers help reduce the training time since it simplifies the activation functions with having too much impact on the overall network accuracy. Below is a list of the most common layers used in CNNs with a short explanation the role of each.

- Convolution. Layer with filters that have learnable weights. Computing output is dot product of each filter with the input.
- Pooling. Layer that acts as non-linear downsampling of the input. Helps control overfitting by reducing size of network and number of learnable parameters.
- ReLU. Layer which applies an activation function to the input of the layer that removes negative values from the input by setting to those values to zero. [9] Improves speed of training the network.
- Fully-connected. At the end of the network, a fully-connected layer can be added in order to perform high-level decisions such as predicting MNIST digits. This is usually an MLP (described in Section 3.4.1).

By carefully combining these different layers, we form a convolutional neural network. The main applications are in image recognition and visual processing such as images and video; in this project we a CNN is used to process frames from Atari. As previously touched upon in Section 2.3, we can view what information the network has chosen to learnt by looking at the activation in each layer, and each filter. It can provide insight into how well the network is learning and if it requires fine-tuning.

Figure 3.6 provides an example network that is used to predict the value of a handwritten digit from the MNIST² dataset. It uses a series of convolution (CONV) and pooling (POOL) layers before connecting to a fully-connected network (with 10 outputs) which is used to perform the final prediction of the number.

²MNIST. Large database of 60,000 training image of 28x28 images showing handwritten digits from 0-9

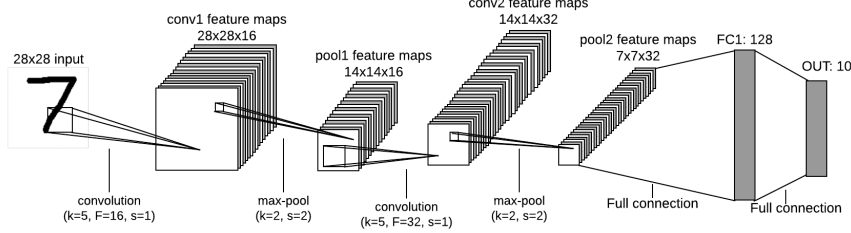


Figure 3.6: Architecture of a network to predict MNIST digits

3.5 Deep Q-Learning

The following sections will outline how the previous sections have laid the foundation for introducing Deep Q-Learning. It is the method by which the computer can learn to play the Atari games to human-level. It will use the information from the previous sections and also introduce some new concepts such as Experience replay which is key to the learning process.

In this project we use neural networks as the function approximator for $Q(s, a)$, we represent this by using θ to represent the parameterisation of the Q-function, $Q(s, a; \theta)$. Referring back to Section 3.1, the aim of Q-learning is to find a function $Q(s, a; \theta)$ that closely approximated the optimal action-value function $Q_*(s, a)$. The *value iteration* algorithm will converge to the optimal action-value function, $Q_i \rightarrow Q_*$ as $i \rightarrow \infty$ [12]. In practise however, we need to use a function approximator as it can generalise the solution, given by $Q(s, a; \theta) \approx Q_*(s, a)$.

In order to train the network, we need to define the ‘loss’ function for the Q-function. This will allow us to update the weights of the network in the direction of the gradient, bringing the Q-function closer to $Q_*(s, a)$ with every timestep.

Definition 3.8. The loss function of the Q-network is defined by

$$L_i(\theta_i) = \mathbb{E} [(y_i - Q(s, a; \theta_i))^2]$$

where $y_i = \mathbb{E} [r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) | s, a]$ is the target value for iteration i and the function $L_i(\theta_i)$ is updated at every timestep. It is also important to note that we hold the parameters of the network fixed from the previous iteration θ_{i-1} in order to stabilize learning.

By differentiating the loss function defined above, with respect to the weights of the network (denoted by θ), we get the following gradient.

Definition 3.9.

$$\nabla_{\theta_i} L_i(\theta_i) = \mathbb{E} \left[\left(r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) - Q(s, a; \theta_i) \right) \nabla_{\theta_i} Q(s, a; \theta_i) \right]$$

In practise, when implementing the Q-learning algorithm, it is more computationally efficient to not compute the full expectation of the gradient, rather we optimise the loss using stochastic gradient descent. This leads us onto the Deep Q-Learning algorithm

which is presented below.

Algorithm 2: Deep Q-Learning with Experience replay

```

Initialize replay memory  $\mathcal{D}$  with maximum capacity  $N$ 
Initialize action-value function  $Q$  with random weights  $\theta$ 
Initialize target action-value function  $\hat{Q}$  with weights  $\theta^- = \theta$ 
for  $timestep = 1, T$  do
    Initialise  $S$ 
    while  $S$  is not done state do
        With probability  $\varepsilon$  choose random action  $a_t$ 
        otherwise, pick  $a_t = \max_a Q(s_t, a; \theta)$ 
        Take action  $a_t$  and observe reward  $r_t$  and state  $s_{t+1}$ 
         $Q(s, a) = Q(s_t, a_t) + \alpha(R + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t))$ 
        Store tuple  $(s_t, a_t, r_t, s_{t+1})$  in  $\mathcal{D}$ 
        Randomly sample a minibatch of tuples  $(s_t, a_t, r_t, s_{t+1})$  from  $\mathcal{D}$ 
        Every  $C$  steps copy weights  $\theta^- = \theta$ 
    end
end

```

3.5.1 Experience Replay

One of the problems with the Q-learning algorithm is that if we try and learn from consecutive frames, the algorithm will take a long time to converge. This is due to the large correlation between consecutive frame of the Atari games. By splitting up the training samples it breaks this correlation and reduces the variance of the gradient updates.

This approach is known as “*experience replay*” [6]. Every timestep, we store the tuple $e_t = (s_t, a_t, r_t, s_{t+1})$ of experience in the replay memory which is denoted by \mathcal{D} in Algorithm 2. Therefore, over time we accumulate a collection of experiences such that $D = e_1, e_2, \dots, e_N$.

In practise, we keep a rolling collection of experiences for a memory with capacity N ; in this project the experience is stored in a Double-Ended Queue³. It means that at timestep $N + 1$, we remove the tuple e_1 and replace it with tuple e_{N+1} .

During the Q-learning updates, we take a mini-batch of samples, $e \sim \mathcal{D}$, randomly from the replay memory in order to perform the stochastic gradient descent. This method comes with several benefits that are outlined below.

- We randomly sample the replay memory at each step, we can reuse experience samples multiple times, providing much greater data efficiency.
- As mentioned previously, there is a strong correlation between consecutive frames, experience replay provides a method to remove this correlation.

Despite the benefits that experience replay brings, this approach has limitations. The main being that each experience sample is treated with the same importance i.e. each tuple has an equal probability of being chosen in a sample e . Therefore, in 2015 Schaul

³Double-ended queue is an abstract data type that can generalise a queue. It typically provides operations to add/remove elements from the head and tail.

et al. [10] introduced a new method called “Prioritized Experience Replay” which aimed to ensure that more important experiences are replayed with a greater frequency.

3.6 Q-Learning improvements

As discussed in Section 3.5, due to how Q-learning works, it can lead to an overestimation of the quality of some actions. Therefore, this section describes two different approaches in order to improve upon the algorithm and reduce the maximisation bias and thereby improving the stability of learning.

3.6.1 Fixed Q-Network

One of the problems with the original tabular Q-learning algorithm is that if we use the same neural network to pick the best action and perform our estimation of the future reward, we are chasing a constantly moving target. This would lead to unstable learning and takes a very long time for the algorithm to converge (which it may not do in this case). However, a solution to this problem is to keep a copy of the Q-network, one that “lags” behind the other. We call the copied network the “fixed” or “frozen” Q-network.

Therefore, we maintain two networks, the “online” and “fixed” networks. Although this doubles the memory requirements to store the weights, we aren’t performing back-propagation on the fixed network, we only perform forward passes, thereby we don’t have the same increase in computational requirements. This approach also introduces another hyperparameter that must be chosen, that is, how often do we update the “fixed” network. Experimental results have shown that the algorithm is quite resistant to changes, leading to similar performance for a range of values. For Atari, a good choice is to update the fixed network every 10000 timesteps.

3.6.2 Double Q-Learning

In the original Q-learning algorithm, experimental results have shown that under certain conditions the algorithm tends to overestimate the quality of some actions. Therefore, in order to reduce the bias, H. V. Hasselt introduced the double Q estimator function in his 2010 paper [5]. It presents an off-policy reinforcement learning algorithm where we use two Q-functions, one for the value evaluation and the other for selecting the next action.

Definition 3.10. Double Q-Learning update equation where

$$Q_{t+1}^A(s_t, a_t) = Q_t^A(s_t, a_t) + \alpha_t \left(R_t + \gamma Q_t^B \left(s_{t+1}, \arg \max_a Q_t^A(s_{t+1}, a) \right) - Q_t^A(s_t, a_t) \right),$$

and

$$Q_{t+1}^B(s_t, a_t) = Q_t^B(s_t, a_t) + \alpha_t \left(R_t + \gamma Q_t^A \left(s_{t+1}, \arg \max_a Q_t^B(s_{t+1}, a) \right) - Q_t^B(s_t, a_t) \right).$$

This definition means that the estimated value of future rewards is calculated using a different policy, solving the original overestimation issue with Q-learning. An additional note is that although this prevents the overestimation, it can lead to underestimation of the Q-values in some situations.

For this project, we use double Q-learning in combination with deep learning. As proposed by H. V. Hasselt et al. (2015) [15], the original Double Q-Learning algorithm can be simplified to only need minor changes to the DQN algorithm in order to achieve a similar effect in reducing the overestimation.

This means that rather than maintaining two different networks and performing back-propagation on both, use the target (online) Q-network and fixed network in order to determine the best action to take given a state. The fixed (or frozen) network is the network described in Section 3.6.1 which uses a frozen copy of the network that is only updated periodically.

3.6.3 Duelling Q-Learning

This section will describe another improvement to the original Q-learning algorithm that makes changes to the underlying network architecture in order to improve approximation of $q_*(s, a)$. The architecture was first proposed by Z. Wang et al, 2015 [16].

By using Definition 3.4 and Definition 3.5 we can define a quantity called the Advantage, denoted by $A_\pi(\cdot, \cdot)$.

Definition 3.11. The *advantage function* relates the Q and value functions.

$$A_\pi(s, a) = Q_\pi(s, a) - V_\pi(s)$$

In order to know what quantity the advantage function produces, recall that the Q-function describes the quality of taking an action a in state s , and that the value function describes how good it is to be in a state s . Therefore, since the advantage function subtracts the value of a function from the Q-function, it obtains a relative measure of the importance of each action a in state s .

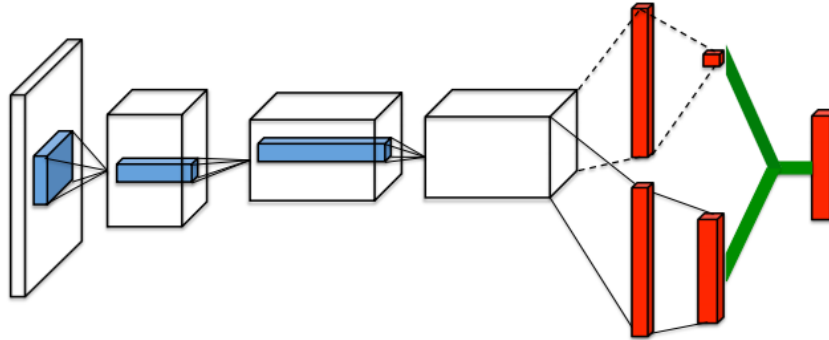


Figure 3.7: Duelling Q-Network architecture

As shown above, rather than having a single fully-connected layer that predicts a Q-value for each output (like in Section 3.5), the Duelling Q-Network maintains two discrete paths. One path attempts to predict the function $V_\pi(s)$, the other predicts $A_\pi(s, a)$. Finally, these two paths are combined together to calculate $Q_\pi(s, a)$. It is important to note, this is not an extra step in the training process, rather, it is built directly into the network and calculated during a forward pass. This is represented by the green lines combining the two paths to the output layer in Figure 3.7.

Definition 3.12. Q-function combining advantage and value functions

$$Q(s, a; \theta, \alpha, \beta) = V(s, a; \theta, \beta) + \left(A(s, a; \theta, \alpha) - \frac{1}{A} \sum_{a' \in A} A(s, a'; \theta, \alpha) \right)$$

As the above definition shows we calculate estimates of the Q-values for each action, and choosing the best action requires finding $a_* = \arg \max_{a' \in A} Q(s, a'; \theta)$. α and β are the parameters for the value and advantage functions respectively.

In order to calculate the Q-function, it would be intuitive to perform the arithmetic sum of the value and advantage function based on Definition 3.11, however, using that method leads to two problems [16].

- It can be problematic to assume the functions give good estimates of the true values, especially using parameterised versions of the functions
- Naive sum of two values is ‘unidentifiable’. Given a Q-value, we cannot recover $V_\pi(\cdot)$ and $A_\pi(\cdot, \cdot)$ uniquely

This architecture has shown to achieve best in-class performance over the different methods that are described in the sections above. In addition, these methods don’t have to be used in isolation, some of the methods can be combined to further improve the performance of the Deep Q-Networks. For example, this project uses a Duelling Double Q-network architecture which outperforms the basic DQN since it is much more stable and prevents the overestimation problem.

Chapter 4

Implementation

In the previous chapter we discussed some of the mathematical and technical details of the project. Based on the information provided in those sections on reinforcement learning and deep learning, this section will discuss the implementation details. Although this project was primarily a research project into the details of reinforcement learning, it does contain a significant software engineering aspect as such this section will also include some code listing and system architecture diagrams.

4.1 Environment

4.1.1 OpenAI Gym

Referring back to Section 3.2, reinforcement learning is based upon a simple data flow between the agent (the thing that takes actions) and the environment (the system that produces a state and reward) which is shown diagrammatically in Figure 3.2.

OpenAI Gym provides a well-developed API that interacts with the Arcade Learning Environment (ALE) which is used for emulating Atari 2600 games. The ALE was developed to support reinforcement researchers to develop agents for Atari[2][7]. Additionally, OpenAI Gym provides emulation for robotics simulations and text-based environments, however, these were not used in this project. In order to emulate the Atari 2600 console, Gym provides a Python API for ALE, which then uses the Stella¹ emulator which actually executes the Atari games on the machine [4].

The code listing 4.1 shows the basic API used in order to step through the environment. The agent is initialised first which contains the code for optimising the neural network for predicting the actions based on raw observations. Gym offers the function `step(action)` which will update the environment, taking the action provided, and returns a new observation, reward, done signal, and any additional information.

During the experiments performed for this project, the number of timesteps was fixed, with each episode lasting a variable number of timesteps depending on the game. For example, Pong would last an average of 1000 timesteps, whereas Breakout would be around 300 timesteps. This difference was since we considered an episode finished when the agent lost a single life, not when all five lives are lost (as is the standard in the games tested in this project).

¹Stella is a Atari 2600 emulator released under a GNU General Public License for multiple platforms

During the initial prototyping phase of the project, the environments would be ran for a short time, around 100,000 timesteps, this is in comparison to the final evaluation in which we ran the environment for 10 million timesteps. This was primarily since after 100k steps, one could identify is the agent was beginning to improve.

Source Code 4.1: Training an agent with OpenAI Gym

```

1 import gym
2 env = gym.make("Pong-v0")
3 observation = env.reset()
4
5 agent = Agent()
6
7 for _ in range(1000):
8     # Draw the environment in the UI
9     env.render()
10
11     # Predict the next action, based on the observation
12     action = agent.step(observation)
13
14     # Update environment, taking the action
15     # Get a new observation and reward
16     observation, reward, done, info = env.step(action)
17
18     if done:
19         observation = env.reset()
20 env.close()

```

4.2 Agent

This section will provide implementation details for the agent, how it is trained and motivate the choice of the hyperparameters used during training and evaluation. The section will be split into sections, converging the different aspects of the agent, convolutional layers for handling the raw pixel input, and the fully-connected network for predicting the Q-values of each action.

4.2.1 CNN

The convolutional network will be described in this section, it is also shown diagrammatically in Figure 4.1. First, from the emulator, we receive frames with a size of 210x160 pixels. Since the project uses a GPU implementation for performing convolution, we require a square input to the network. As such, each frame is downsampled to 84x84 pixels using a function $\phi(\cdot)$ performing a bilinear interpolation.

Next, we construct a processed frame history $(\phi(s_{t-3}), \phi(s_{t-2}), \phi(s_{t-1}), \phi(s_t))$, the past four frames are processed using the function $\phi(\cdot)$. We briefly mentioned this in Section 3.1.1 in which we referred to this concept as a “frame-stack” the past k frames.

The first convolutional layer performs a convolution of 32 20x20 filters, each with a stride of 4, to the input layer. The second layer then convolves 64 9x9 filters of stride 2. Next, the third layer convolves 64 7x7 filters of stride 1, producing the final output before the fully-connected network. The output from the last layer is flattened to a vector of 3136 nodes which is fully-connected to 256 ReLU units. Finally, we have between 4 and 6 outputs (depending on the game) which contain the predicted Q-values for each action in the action space for the environment. This architecture was kept the same for all the Atari games during the evaluation of this project [8].

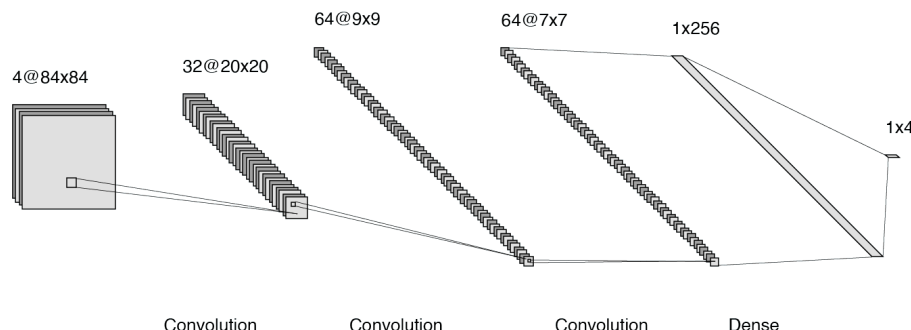


Figure 4.1: CNN Architecture

The above architecture was decided upon after reading different papers which implemented the agent. In the original DQN paper by Mnih et al. (2013) [8] they proposed a slightly smaller network with only two convolutional layers. In papers released later, the network architecture was deepened, as deeper networks have shown to perform better in control tasks such as Atari. An additional factor to take into account was that the input image was already quite small at 84x84 pixels and as such, by using a very deep network could lead to having lots of dead filters due to the learning rate.

In this project, there was much room to explore network architectures. One area which was not explored but would be promising is using pooling layers. These have been shown to increase performance with rotational/position feature extraction which would be important for some of the more complex Atari games that require fine control such as Pitfall.

Overall, OpenAI Gym was an excellent choice for this project, it provided an excellent abstraction of the environment, exposing an API that could be used as necessary. Being able to just focus on the implementation of the agent provided more research time initially, and more time for performing a variety of tests for hyperparameter choice and network architecture. The following section will focus on the details of the DQN algorithm.

4.2.2 DQN

In this section, the details on how the DQN agent was implemented, along with the hyperparameters used during training. It was the area that the most difficult to implement in this project, primarily since Q-Networks are very sensitive to hyperparameter choice and implementation. During development of the project, many iterations of the algorithm were tried until a setup that evaluated well with all the games in the test setup.

In the 2013 paper authored by DeepMind [8], they described the error function as the following. “We fixed all positive rewards to be 1 and all negative rewards to be -1,

leaving 0 rewards unchanged”. It was initially difficult to interpret, nevertheless, after reading some implementations of the algorithm and Medium articles, it was clear that one such use the Huber loss before computing the gradients for updating the network. The Huber loss 4.1 is used in most good implementations of the DQN algorithm and has shown to reach a similar level of performance.

Definition 4.1. Huber loss function.

$$L(a) = \begin{cases} \frac{1}{2}a^2, & \text{for } |a| \leq 1, \\ (|a| - \frac{1}{2}), & \text{otherwise.} \end{cases} \quad (4.1)$$

Another aspect of the DQN algorithm that up to this point, we have not discussed in the report. That is of an idea of a “*frame-skip*”. We have already talked about the frame-stack which is a rolling history of the past 4 frames which is used for forward passes through the network. While a frame-skip is when we take an action in the environment, then for a series of k frames we perform the last action taken. This is very important and the frame-skip hyperparameter has been shown to be strong determining factor in the performance of algorithms when playing Atari [3].

A small note with the frameskip hyperparameter is that it was not kept the same for all the games that was tested. In the game Space Invaders it the frameskip was changed from $k = 4$ to $k = 3$. The reason for this was that the bullets (fired by both the aliens and player) flash every fourth frame; by using a frameskip of 4 it means that the bullets would never be visible during training. This was the only changed hyperparameter, all others were kept the same.

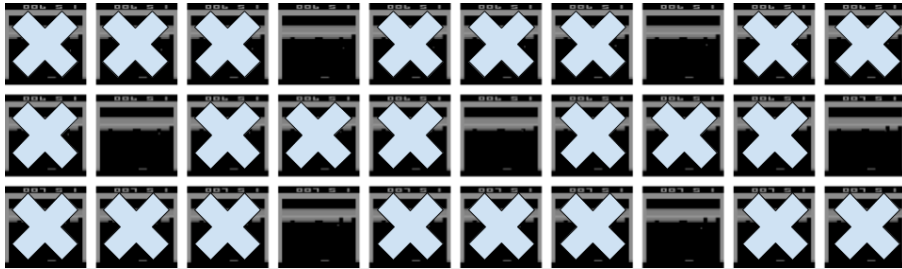


Figure 4.2: Frameskip on Atari Breakout

Figure 4.2 shows how frames are sampled using frameskips. Using a frameskip, $k = 4$, as shown in the figure, we use every fourth frame from the game. If we denote every ‘seen’ frame (those frames without the cross) as x_1, x_2, \dots, x_7 then our first input, s_1 , to the network will be (x_1, x_2, x_3, x_4) and the the following input, s_2 , would be (x_2, x_3, x_4, x_5) . An important observation of this method is that the states are overlapping in consecutive forward passes. It means that we provide the information to the network in order to predict the movement of the ball, such that we can move the paddle under the ball in time.

Another important consideration in the DQN algorithm is that of “cold-starts”. This is when the environment is initialised and always starts from the same state. The worry with the cold starts is that the agent may not learn to look at a state and choose the optimal action, rather, it learns to cheat and learn a sequence of actions. This would lead to poor generalisation in the environment.

A solution to the problem of “cold starts” is to take between 0 and k NOOP (No operation/No action) actions, sampled uniformly at random when the environment is initialised, putting the environment in a slightly different state every time. This means that it would be more difficult for the agent to cheat in the environment by learning a set of actions from a starting state. The original DQN paper [8] used $k = 30$ which was the value used in this project.

On the otherhand, recent reviews have cast doubt over the effectiveness of the NOOP actions helping the problem of cold starts. One example of this was a paper by M. C. Machado et al. (2017) [7] which argued that the NOOP sequence was not as effective as previously thought. Instead, it is proposed that it would be better to instead enforce random actions to be taken while the agent is being trained. By doing this, it adds a factor of stochasticity to the environment which also prevents the cold start problem.

4.3 Visualisation

Another important part of this project is the CNN visualisation. It was described in section 2.3 however, in this section we will go into more detail about how it was implemented. It will first describe the motivation behind the choice of following a client-server architecture and then show the architecture diagrammatically. Additionally, it covers some of the problems that were encountered during development and how they were overcome.

The main consideration for this part of the project was how the model was executed, for which there was two options.

- Executing the model in browser (using Tensorflow.js)
- Server-client architecture, the trained model executes on the server, sending updates to client.

Firstly, since this project used Tensorflow, programmed in Python, we could have loaded the trained model in Tensorflow.js and executed the model directly in the browser. However, for this approach to be viable, we would need to have also loaded the emulator in the browser in order to generate the frames of the game, this would require more engineering work. Additionally, not all client machines would be able to handle processing forward passes of the model.

Therefore, a client-server architecture was implemented instead using WebSockets to perform communication. The client would initiate the connection with the server and request a new environment. The server would take requests from the client for performing actions such as, running the model, performing a single step in the model; then sending the states back to the client over the socket. The main benefit of using Websockets is that it provides full-duplex communication channels over a single TCP connection, it reduces the overhead for sending messages between client and server (unlike using standard HTTP requests) and means that we can run the model at high frame rates (usually at least 30 FPS).

The visualisation of the CNN also includes showing the raw filters and their activations on the site. In order to make this happen, a forward pass of the network is performed using the current state. We store the history of the pass which includes the activations of each layer of the network (and of each filter), and importantly the resulting Q-values

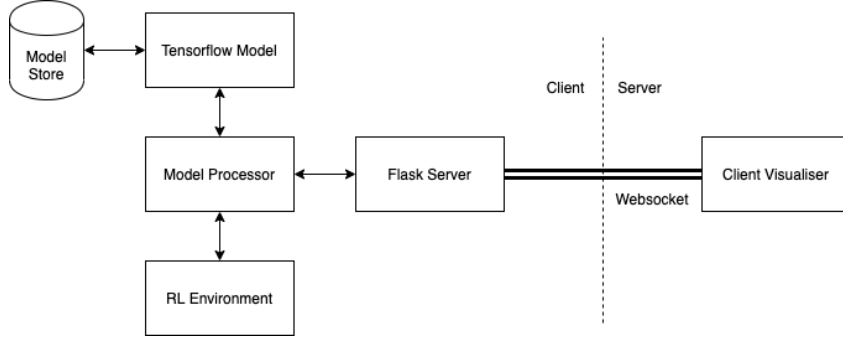


Figure 4.3: Visualisation system architecture which shows how data flows between the major components. The top of the diagram show the server. Websocket commands are processed in the Flask server and delegated to either the tensorflow model or the environment by the Model Processor. The client can choose the model to visualise based on those available in the Model Store. The websocket established between client and server performs all the necessary transfer of information and the client is purley drawing the generated visualisation on the client screen.

which are used to determine the best action to take given the current state. All this information is packaged into a predefined JSON format and sent over the socket to the client. The client parses this information and draws the pre-processed frame, the filters of the selected convolutional layer and the Q-values for each action. We also plot the Q-value of the chosen action over the past 50 frames, this allows the user to see if the agent is predicting a future reward. Figure 4.4 shows how these Q-values are plotted, we see a slow rise, showing the agent is confident in the series of actions and then a sharp drop when the agent recieved the reward.

In Figure 4.4, the first screenshot show the ball moving upwards towards the row of bricks. At this point the network is confident in the series of actions and it is expecting a reward (show by point A in the plot). The second screenshot shows the ball has just broken a brick, at this point the agent recieves the reward and the Q value has a sudden and sharp drop (shown by point B). Finally, point C shows that the Q-value has dropped down to its original value (corresponding to the third screenshot as the ball is travelling down to the paddle). This shows that the agent can learn to predict a complex series of events.



Figure 4.4: The Q-value function output is shown in the left-most figure (the max Q-value from the network predictions), shown over a 45 frame history. The next three screenshots correspond to the frames labelled by A, B and C in the left-most figure respectively.

Finally, the web server which handles the emulation of the environment and running

the model uses a selection of commands (which are sent over the socket) in order for a client to control the environment. The server runs Flask which is a micro-framework for creating web applications. The framework was chosen since it is lightweight, extensible and quick to setup. Additionally, it had a module which supported WebSockets, making the implementation of the visualisation easier as it provided an abstraction from the details of maintaining the Socket.

Chapter 5

Experiments

5.1 Pong

hi [11]

5.2 Breakout

5.3 Space Invaders

Chapter 6

Conclusion

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