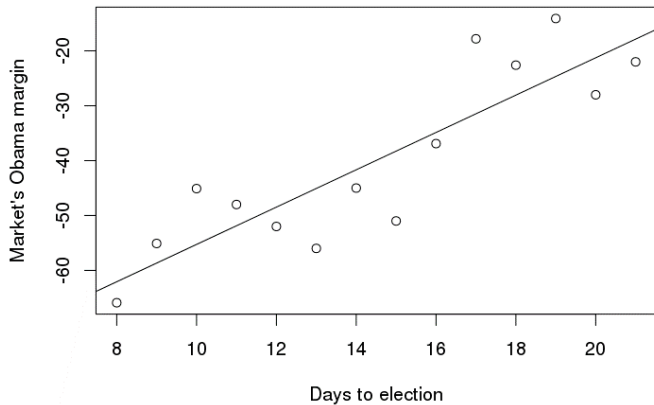


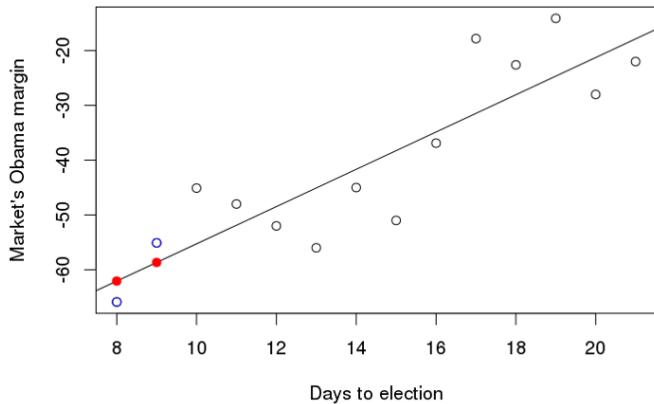
Government 10: Quantitative Political Analysis

Sean Westwood

Within-Sample Prediction



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A Refresher: Out-of-Sample Prediction

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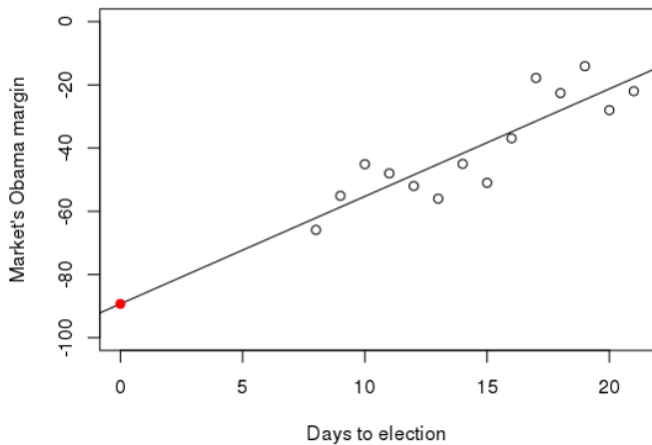
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- ▶ We can predict the results in 2016 using what we know about the 2012 relationship

The Process of Out-of-Sample Prediction

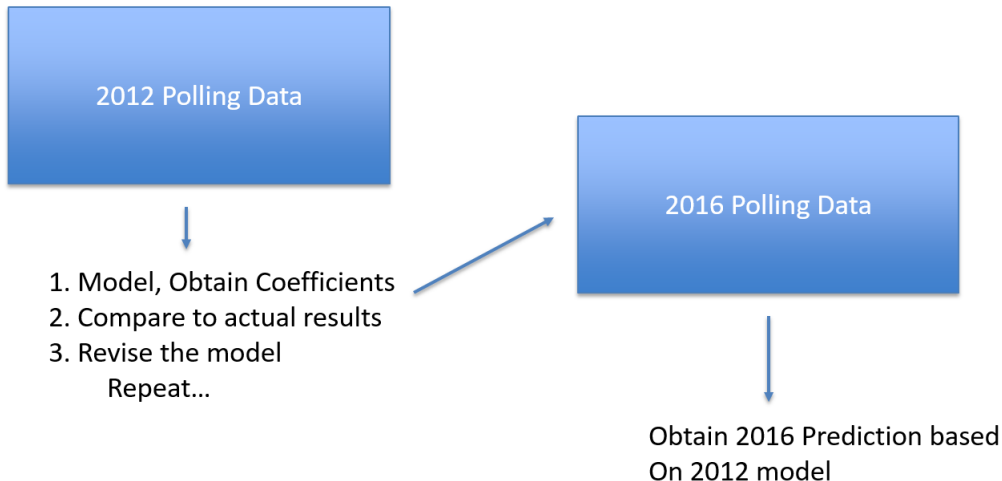


2012 Polling Data



1. Model, Obtain Coefficients
 2. Compare to actual results
 3. Revise the model
- Repeat...

The Process of Out-of-Sample Prediction



Out-of-Sample Prediction Assumptions

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2012 Polling Data

“Training” Dataset



2016 Polling Data

“Test” Dataset

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- ▶ With a model!

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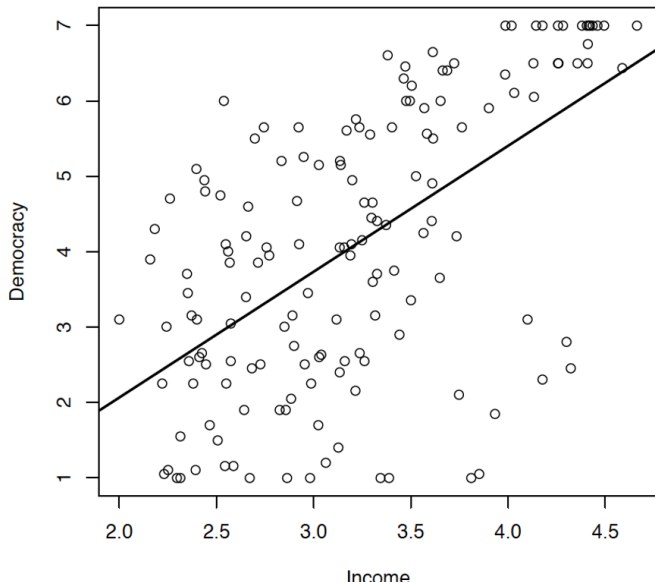
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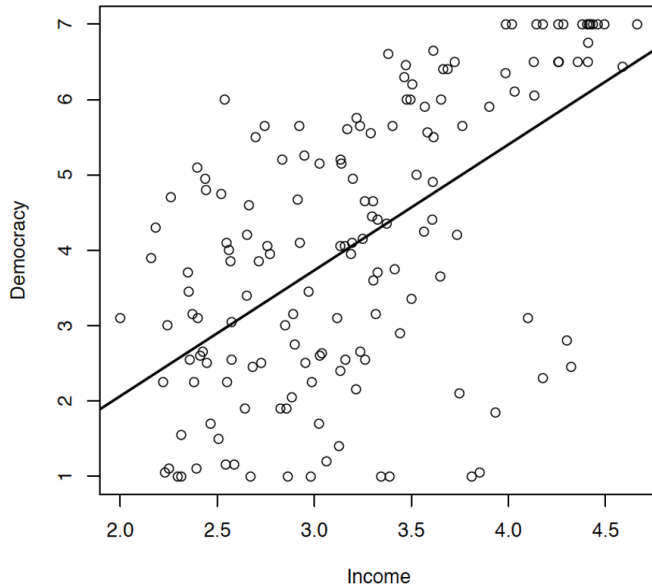
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$$\text{Democracy} = -1.26 + 1.6 (\text{Income})$$

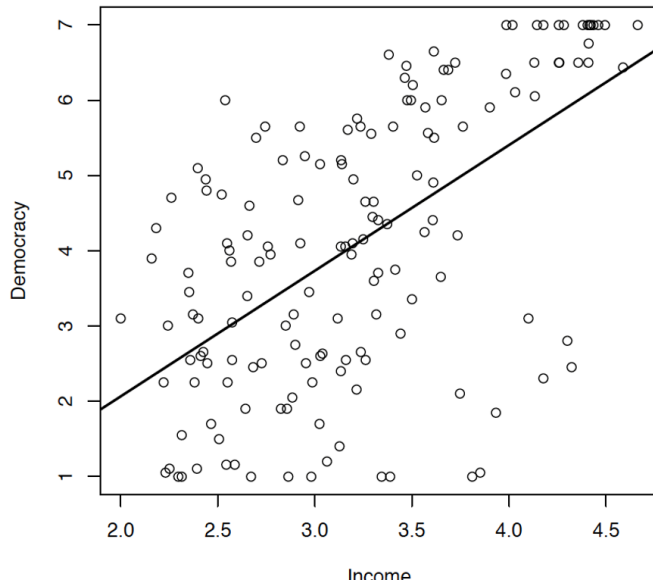
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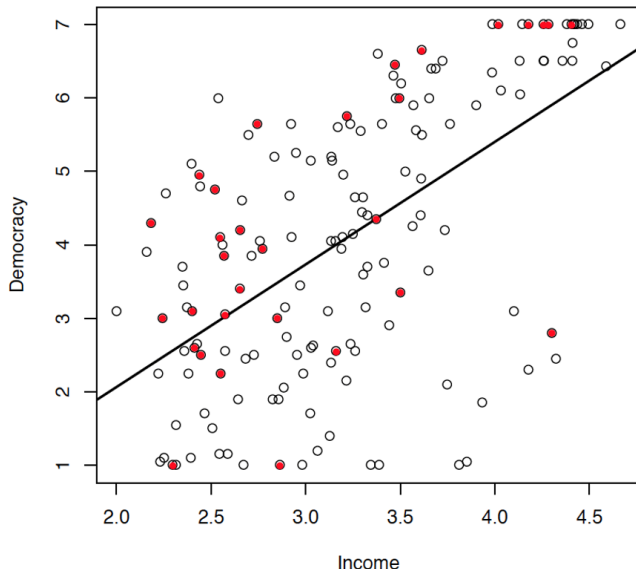


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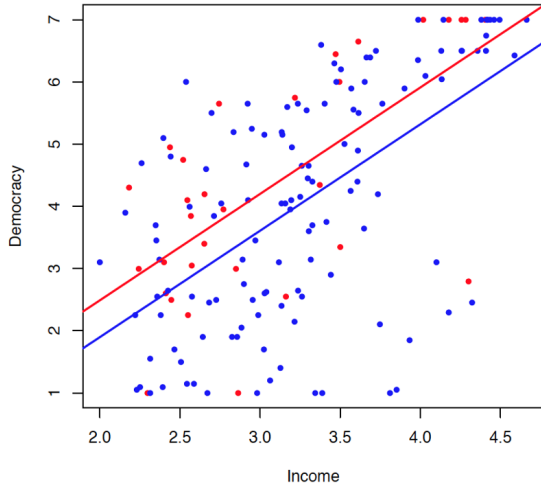
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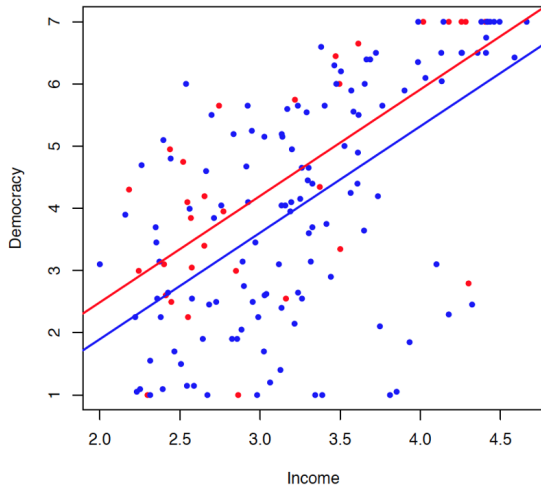
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We are fitting 2 lines with the same slope but different intercepts.

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- ▶ Continues for any number of additional independent variables.

Examples of How to Interpret Multivariate Regression

Suicide Interpretation Example

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$$\hat{Y} = 30.88 + -0.005 * (28) + 0.01 * (4000) + -0.0019 * (140) + 0.725 * (7.2)$$

$$\hat{Y} = 75.69$$

Republican Vote Share Interpretation Example

We are predicting `republican_vote_share` (a proportion) with four predictors:

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	-2.01	

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Consider someone in the control condition:

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