

# Government 10: Quantitative Political Analysis

Sean Westwood

# Making informed predictions

# Intro to Regression and Linear Prediction

Data  $\rightarrow$  **Model**  $\rightarrow$  Predicted Outcome

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We need to predict votes based on what we know (public opinion data)

# Prediction

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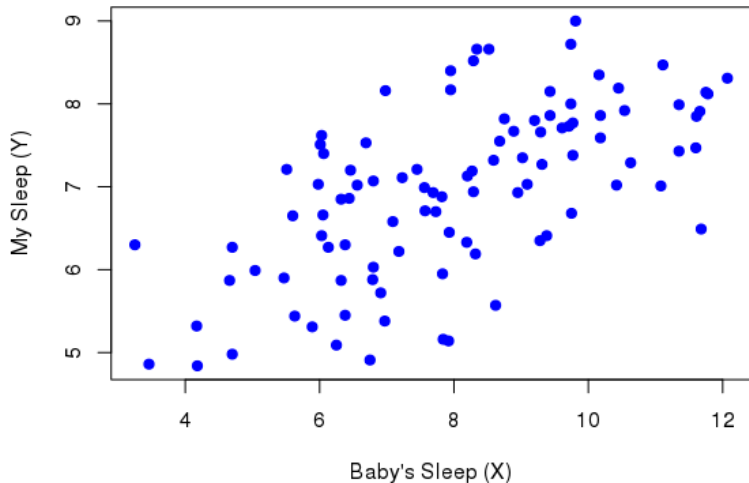
**X**  $\rightarrow$  **f(X)**  $\rightarrow$  **Y**

We assume that the dependent variable (Y) is a function of (or depends upon) the value of the independent variable (X)

We can rewrite this relationship, by convention, as: **Y = f(X)**

## Prediction (using Jeremy's life choice)

**How are sleep cycles linked?**



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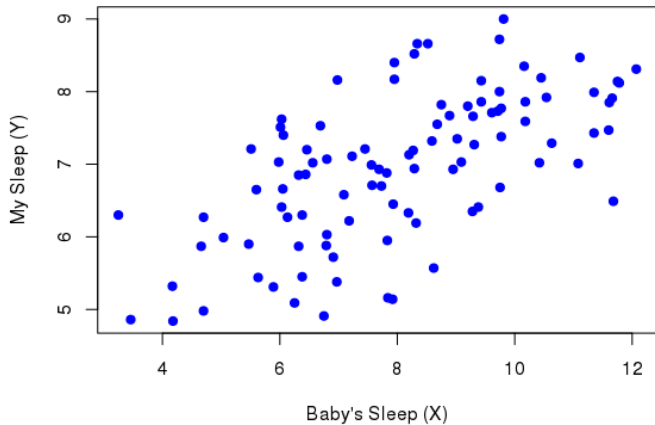
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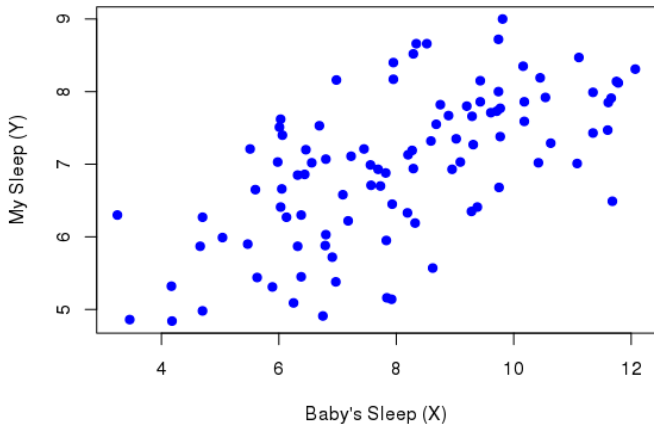
$$? = f(3.5 \text{ hours})$$

$$? = f(8 \text{ hours})$$

## How are sleep cycles linked?

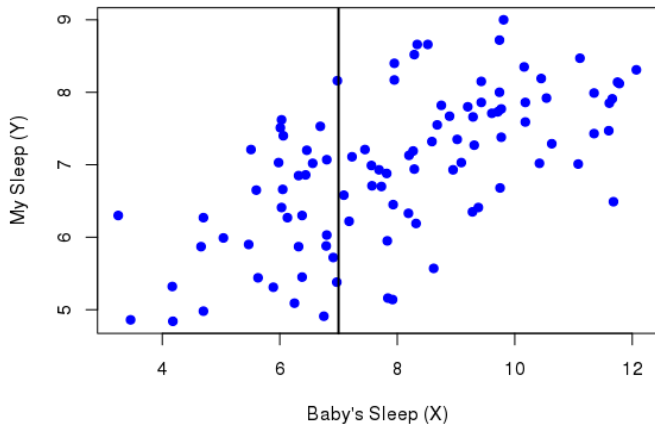


### How are sleep cycles linked?



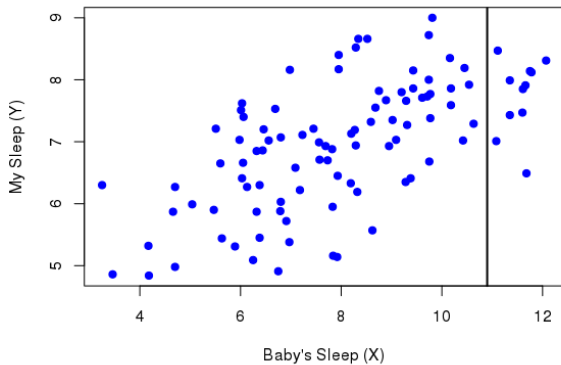
**What is my predicted level of sleep if Baby's Sleep = 7?**

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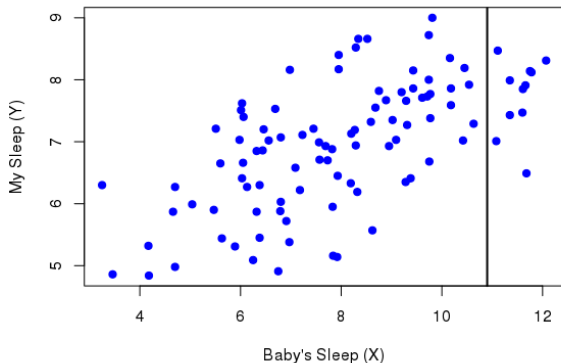
**What is my predicted level of sleep if Baby's Sleep = 7?**

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**What is my predicted level of sleep if Baby's Sleep = 10.9?**

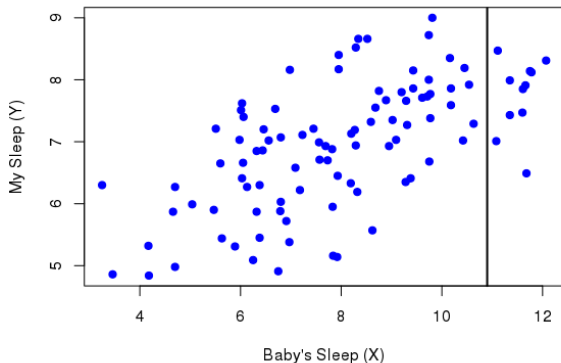
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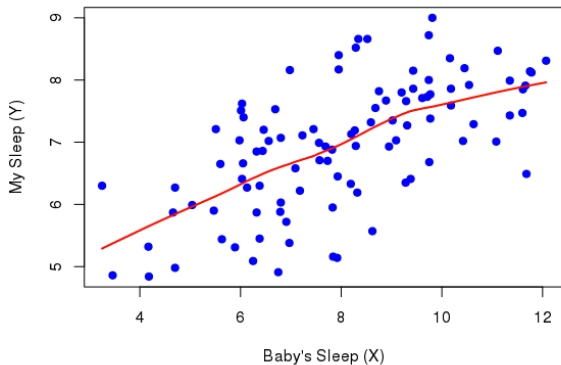


**What is my predicted level of sleep if Baby's Sleep = 10.9?**

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**Problem: Bins are arbitrary**

### How are sleep cycles linked?



Option 2: Use a moving average

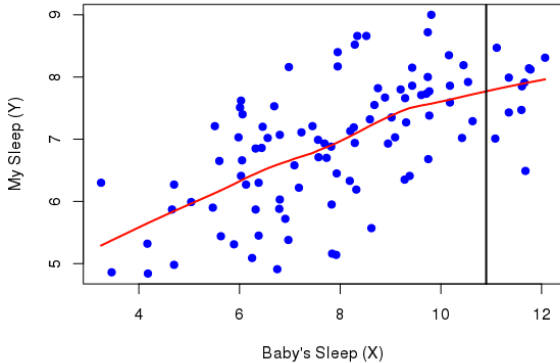
Where we smooth over time

**Problem: windows are arbitrary**



But with a moving average we can make predictions for any value in the range of our data

### How are sleep cycles linked?



A better option

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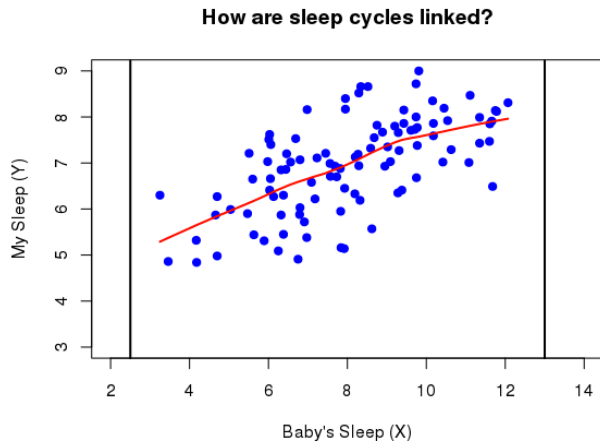
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1. Interpretability
2. Out of sample prediction

## Out of sample prediction



$f(X)$  is undefined outside of our sample. To predict, we need a model that allows for any feasible value of  $X$ .

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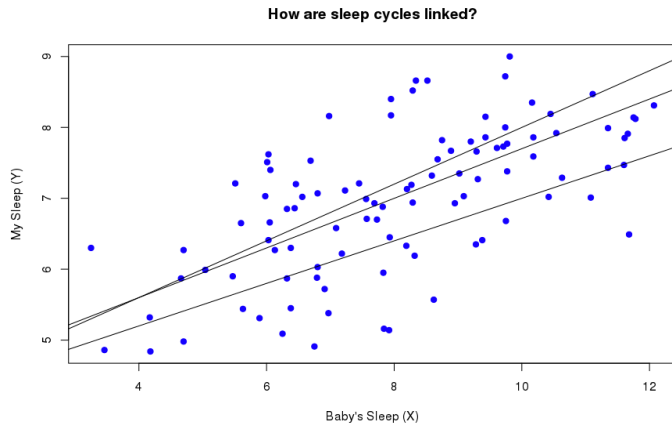
$$Y = f(X)$$

This means we can use some helpful algebra:

$$Y = mX + b \quad (\text{slope} + \text{intercept})$$

# But how do we fit the slope and the intercept?

We have to optimize something



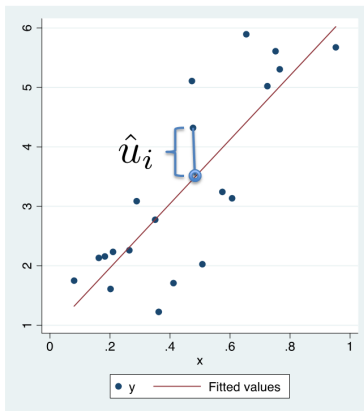


# Regression: Ordinary Least Squares (OLS)

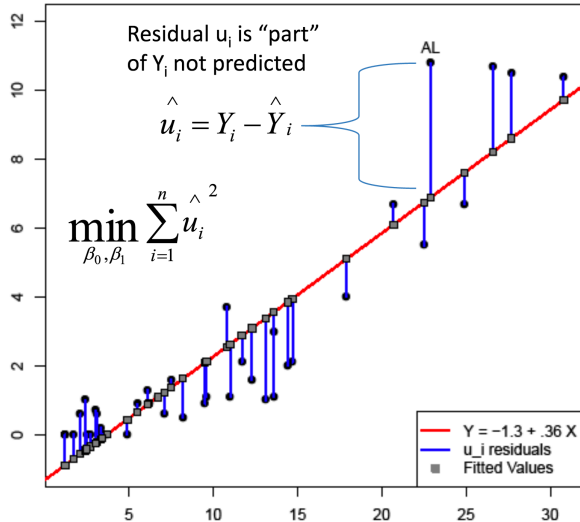
Mathematically, the least biased method selects the line that minimizes the squared differences between observed values of the dependent variable and its predicted values (the regression line)

$\hat{u}_i$  = the difference between the observed and predicted values

The subscript indicates each individual unit of observation (i.e., each country  $i$ , each person  $i$ , etc.)



# Regression: Ordinary Least Square (OLS)



Note: Similar to RMSE

## Regression

Bivariate Regression model components

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Bivariate Regression model components

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- ▶  $\hat{\alpha}$  : an estimated intercept coefficient
- ▶  $\hat{\beta}$  : an estimated slope coefficient
- ▶  $\hat{u}_i$  : an estimated error term, called a “residual”

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- ▶ More formally, it shows how many units y changes as x increases by one unit
  - ▶ Partial derivative of regression equation with respect to X

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The intercept (“alpha”):

$$Y_i = \hat{\alpha} + \hat{\beta}X_i + \hat{u}_i$$

- ▶ Measures the expected value of Y when the independent variable is 0
- ▶ Varies depending on the beta coefficient.

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- ▶ “What’s left over” / “Error”
- ▶ The difference between the observed value of Y (  $Y_i$  ) and the predicted value of Y (  $\hat{Y}_i$  ), based on the intercept and slope.

$$\hat{u}_i = Y_i - \hat{Y}_i$$



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For example:

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Intercept: Expected level of my sleep if baby slept 0 hours

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$$MySleep_i = 4.48 + 0.31 * 0 = 4.48$$

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Slope: Expected increase in my sleep (in hours)

Every additional hour of baby sleep is associated with an increase in my sleep of 0.31

## Practice: Interpreting Regression Coefficients

**Y = Student Exhaustion Scale:**

▶ 0 (Not Exhausted)  $\rightarrow$  100 (Exhausted)



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$$Y = -20 + 5(X)$$

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**Y = 2012 Vote Share for Obama (%)**

▶ 0 to 1 interval; i.e., .15 = 15%

**X = % of Registered Voters who are Republican:**

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$$Y = 0.98 - 0.96(X)$$

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The interpretation of a "1 unit change" depends on how the independent variable is measured

## Practice: Interpreting Regression Coefficients

**Y = 2008 Vote Share for Ralph Nader (%)**

▶ 0 to 1 interval; i.e., .15 = 15 percent

**X = % of Registered Voters who are Republican:**

▶ 0 to 100 interval; ie, 15 = 15 percent

$$Y = 0.13 - 0.002(X)$$

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$$My\ Sleep = 8.9$$