Making informed predictions

Intro to Regression and Linear Prediction

Data $\rightarrow$ **Model** $\rightarrow$ Predicted Outcome

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We need to predict votes based on what we know (public opinion data)

Prediction

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$X \rightarrow \mathbf{f(X)} \rightarrow Y$

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$$X \rightarrow \mathbf{f(X)} \rightarrow Y$$

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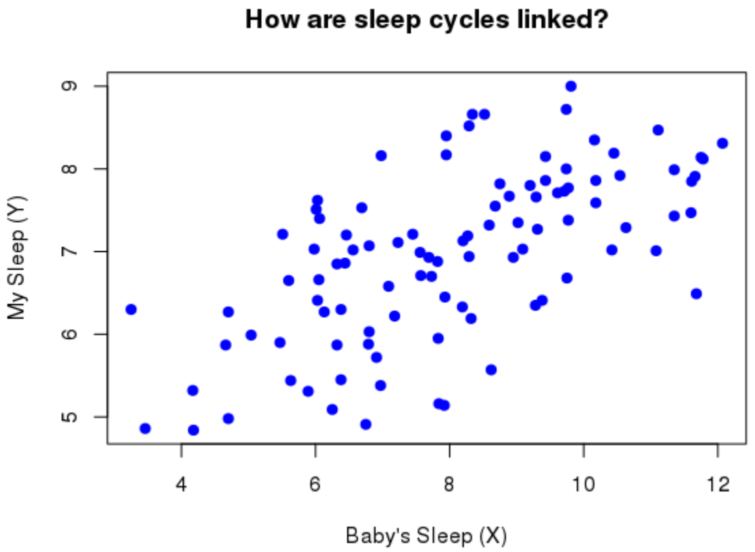
We can rewrite this relationship, by convention, as: $\mathbf{Y = f(X)}$

Prediction (using Jeremy's life choice)



Prediction

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$$? = f(7 \text{ hours})$$

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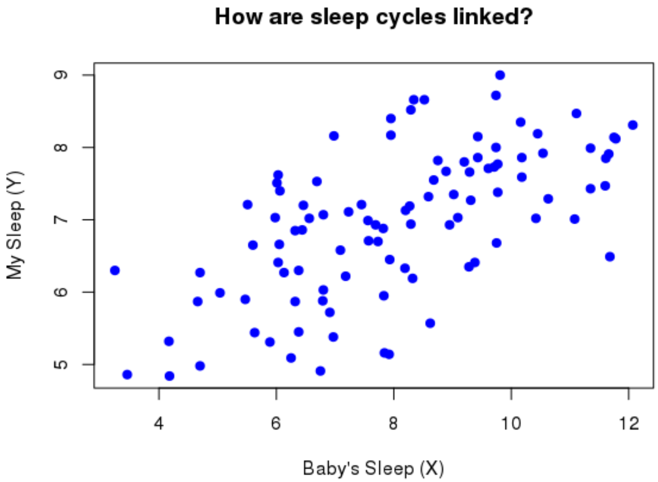
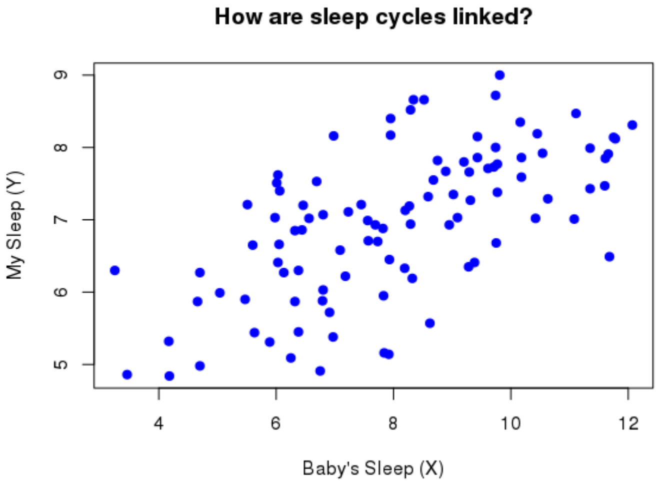
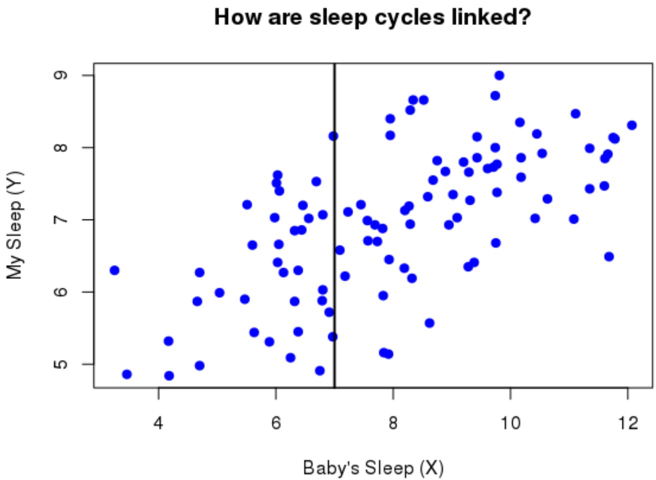
$? = \text{f}(8 \text{ hours})$


How are sleep cycles linked?
My Sleep (Y)
9
8
7
6
5
4
6
8
10
12
Baby's Sleep (X)



What is my predicted level of sleep if Baby's Sleep = 7?



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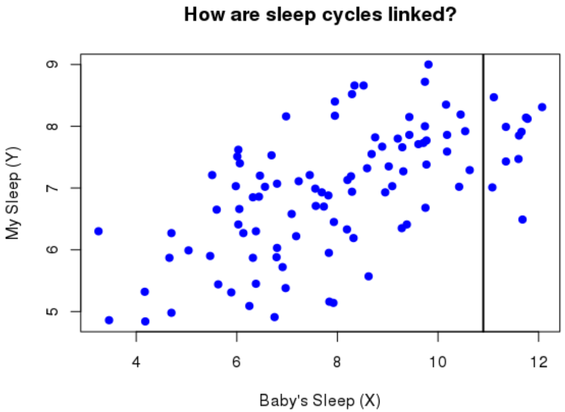
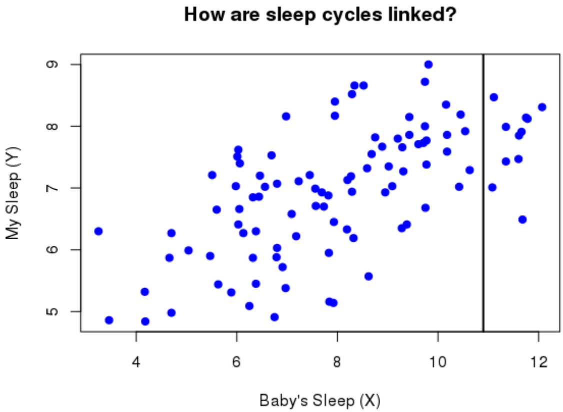


What is my predicted level of sleep if Baby's Sleep = 10.9?



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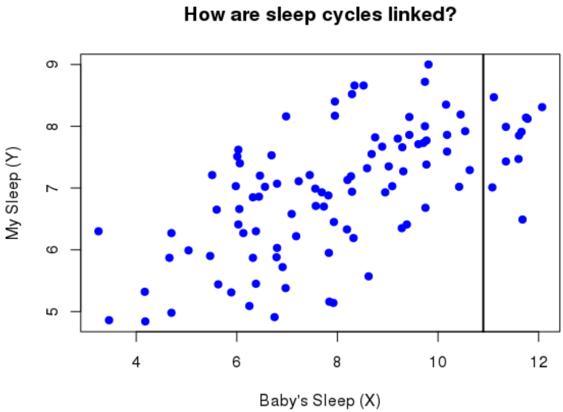
Option 1: We could divide the space into bins, and take the average in each bin



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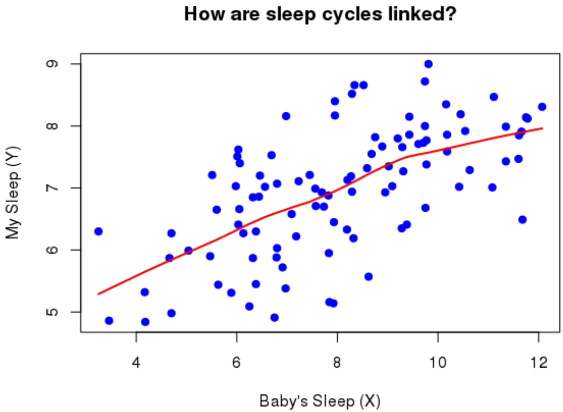
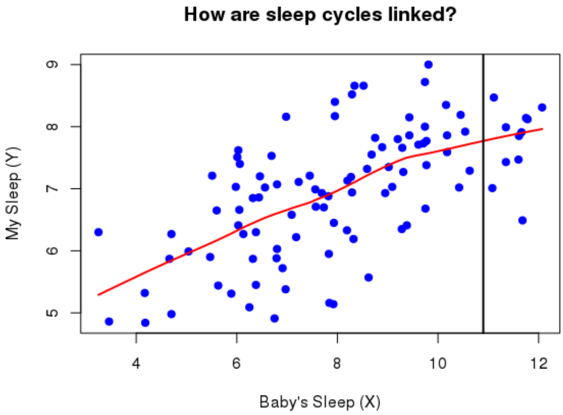


Option 2: Use a moving average

Where we smooth over time

Problem: windows are arbitrary

But with a moving average we can make predictions for any value in the range of our data



A better option

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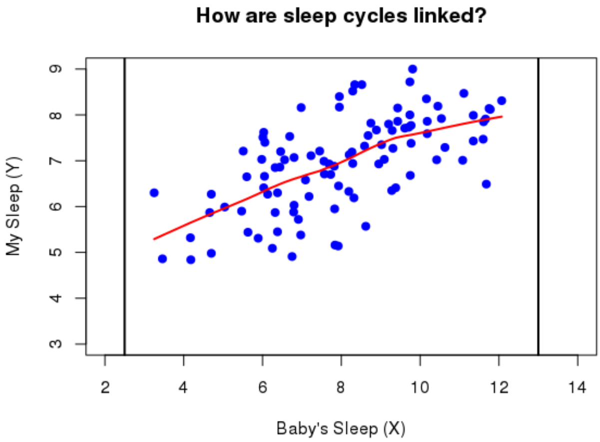
1. Interpretability
2. Out of sample prediction

Out of sample prediction



f(X) is undefined outside of our sample. To predict, we need a model that allows for any feasible value of X.

Regression

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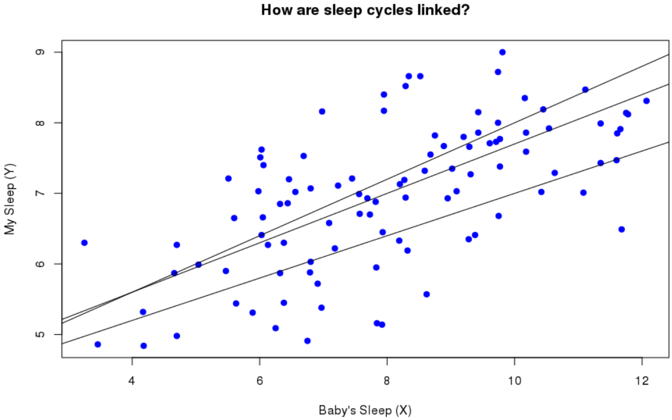
This means we can use some helpful algebra:

$$Y = mX + b \qquad (\text{slope} + \text{intercept})$$

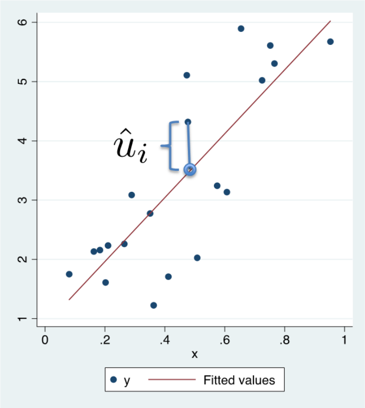
But how do we fit the slope and the intercept?

We have to optimize something



Regression: Ordinary Least Squares (OLS)

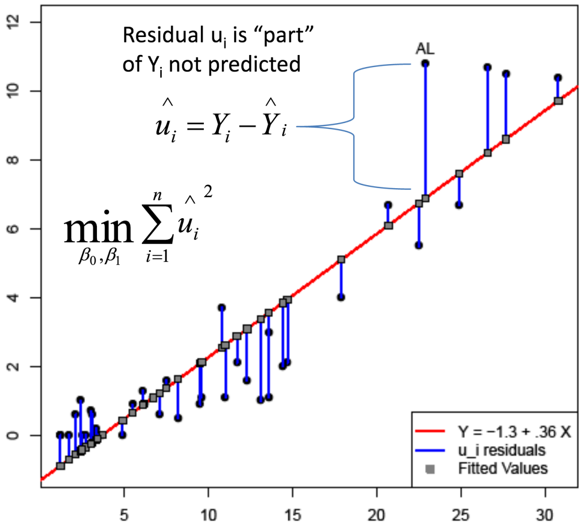
Mathematically, the least biased method selects the line that minimizes the squared differences between observed values of the dependent variable and its predicted values (the regression line)



$\hat{u}_i$ = the difference between the observed and predicted values

The subscript indicates each individual unit of observation (i.e., each country *i*, each person *i*, etc.)

Regression: Ordinary Least Square (OLS)



Note: Similar to RMSE

Regression

Bivariate Regression model components

$$Y_i = \hat{\alpha} + \hat{\beta} X_i + \hat{u}_i$$

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Bivariate Regression model components

$$Y_i = \hat{\alpha} + \hat{\beta} X_i + \hat{u}_i$$

- $\hat{\alpha}$: an estimated intercept coefficient
- $\hat{\beta}$: an estimated slope coefficient
- $\hat{u}_i$: an estimated error term, called a "residual"

Interpreting Regression Coefficients

The slope ("beta"):

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- More formally, it shows how many units y changes as x increases by one unit
  - Partial derivative of regression equation with respect to X

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The intercept ("alpha"):

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- Measures the expected value of Y when the independent variable is 0
- Varies depending on the beta coefficient.

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$$Y_i = \hat{\alpha} + \hat{\beta} X_i + \hat{u_i}$$

- "What's left over" / "Error"
- The difference between the observed value of Y (Y_i) and the predicted value of Y ($\hat{Y}_i$), based on the intercept and slope.

$$\hat{u_i} = Y_i - \hat{Y}_i$$

Performing regression

1. Obtain dataset with independent and dependent variables.

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For example:

$$MySleep_i = 4.48 + 0.31\ (Baby's\ Sleep)_i$$

Interpreting Regression Coefficients

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$$MySleep_i = 4.48 + 0.31 * 0 = 4.48$$

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Slope: Expected increase in my sleep (in hours)

<u>Every additional hour of baby sleep is associated with an increase in my sleep of 0.31</u>

Practice: Interpreting Regression Coefficients

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- 0 (Not Exhausted) $\rightarrow$ 100 (Exhausted)

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$$Y = 52 + 3.5(X)$$

Practice: Interpreting Regression Coefficients

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X = Hours Spent on PSET 4

$$Y = 50 - 2.2(X)$$

$$Y = 52 + 3.5(X)$$

$$Y = -20 + 5(X)$$

Practice: Interpreting Regression Coefficients

Y = 2012 Vote Share for Obama (%)

- 0 to 1 interval; i.e., .15 = 15%

X = % of Registered Voters who are Republican:

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$$Y = 0.98 - 0.96(X)$$

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X = % of Registered Voters who are Republican:

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$$Y = 0.98 - 0.96(X)$$

The interpretation of a "1 unit change" depends on how the independent variable is measured

Practice: Interpreting Regression Coefficients

Y = 2008 Vote Share for Ralph Nader (%)

- 0 to 1 interval; i.e., .15 = 15 percent

X = % of Registered Voters who are Republican:

- 0 to 100 interval; ie, 15 = 15 percent

$$Y = 0.13 - 0.002(X)$$

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Baby's Sleep $= 2$

My Sleep $= 4.48 + 0.31 * 2$

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Baby's Sleep $= 14.4$

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Baby's Sleep $= 14.4$

My Sleep $= 4.48 + 0.31$ * 14.4 My Sleep $= 8.9$