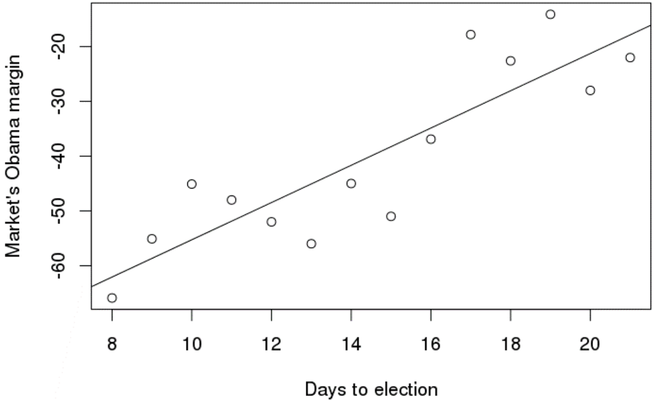
Prediction and Multivariate Regression

Within-Sample Prediction



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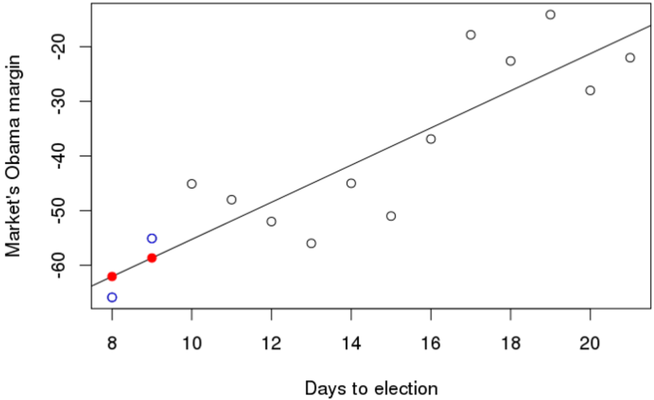

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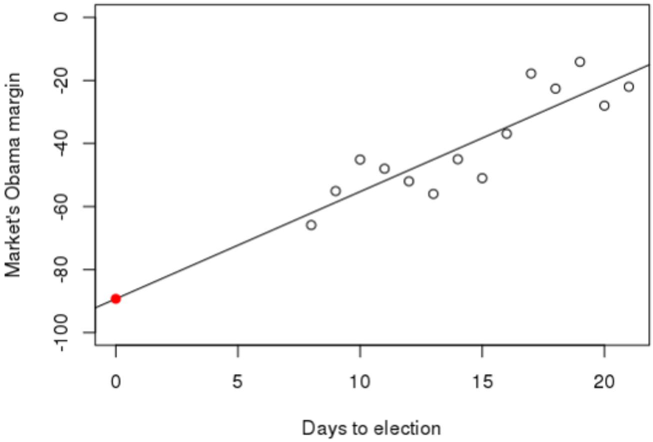
  - For instance, it's 2 weeks before the election, and I want to predict the margin if DaysLeft=0

Out-of-Sample Prediction



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- We can predict the results in 2016 using what we know about the 2012 relationship

The Process of Out-of-Sample Prediction

2012 Polling Data

1. Model, Obtain Coefficients
2. Compare to actual results
3. Revise the model

   Repeat...

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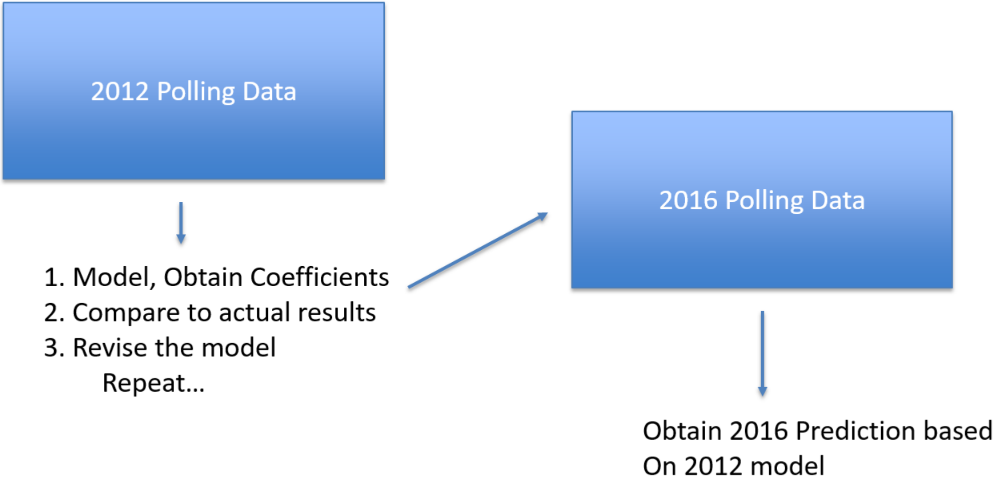

Out-of-Sample Prediction Assumptions

- Key Assumption: The relationship between the independent variable(s) and the dependent variable are the same in each dataset.

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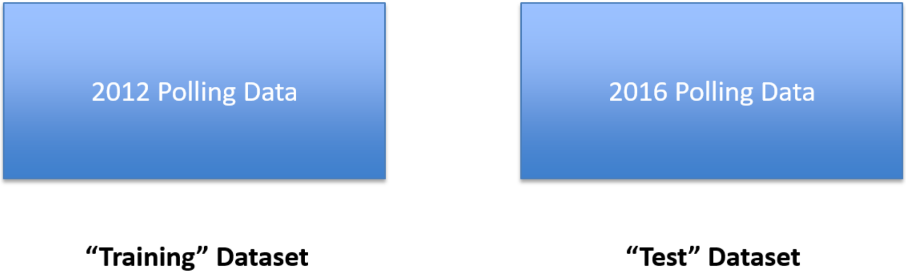

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- With a model!

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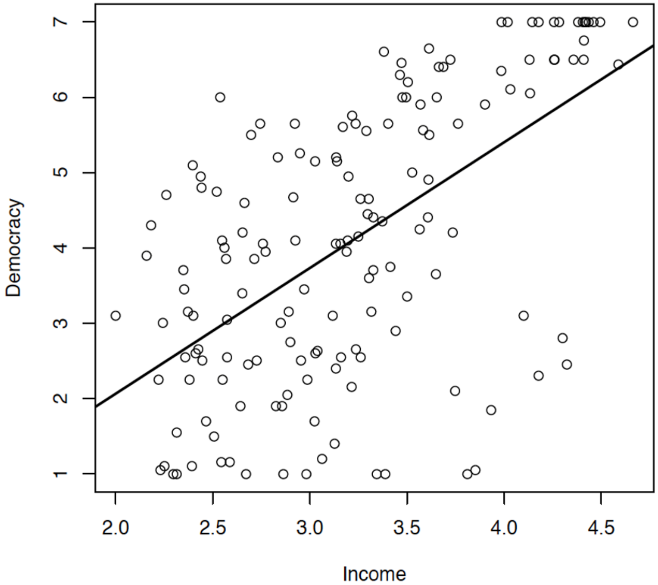
- We specify the following model:
$Democracy\,Index_{Country\,i} = Income\,Index_{Country\,i}$

Visualizing a Multivariate Regression



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= -1.26 + 1.6 (Income)

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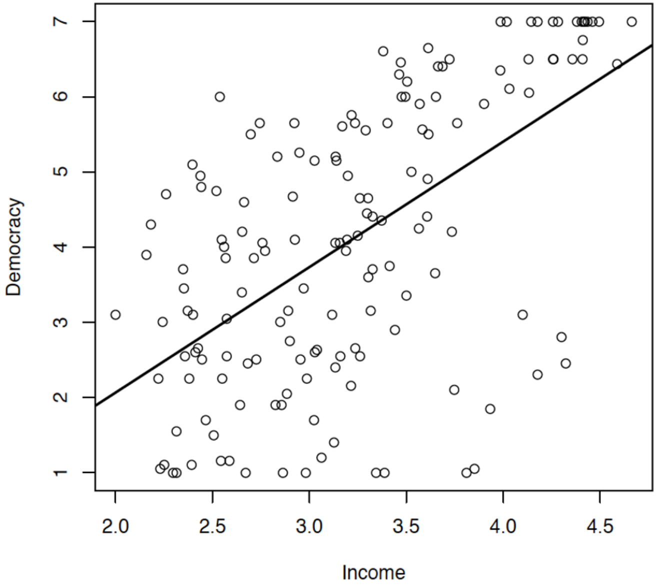

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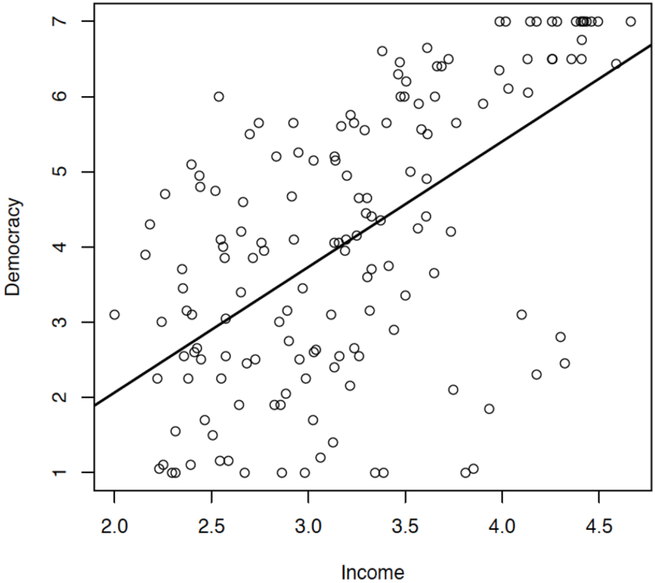

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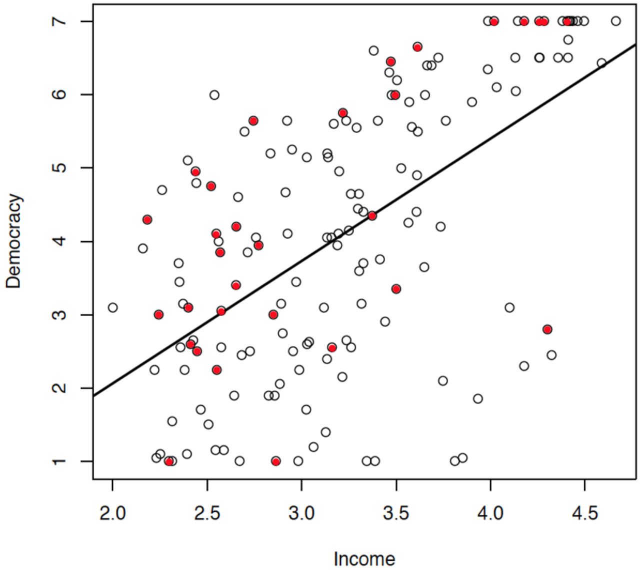

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We want to "control" for this

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  - In the multivariate model, it is the estimated value of $\hat{Y}_i$ when X *and* Z are zero.

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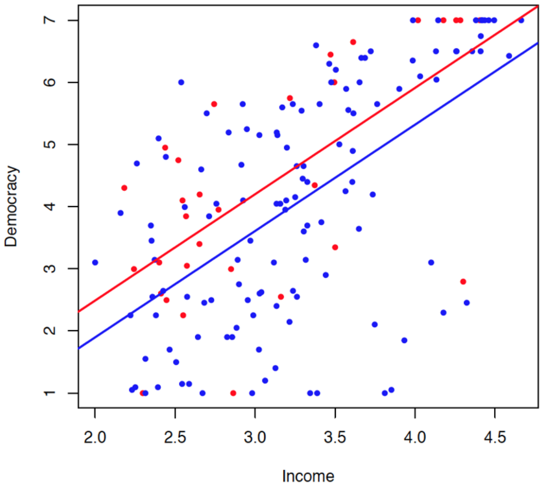

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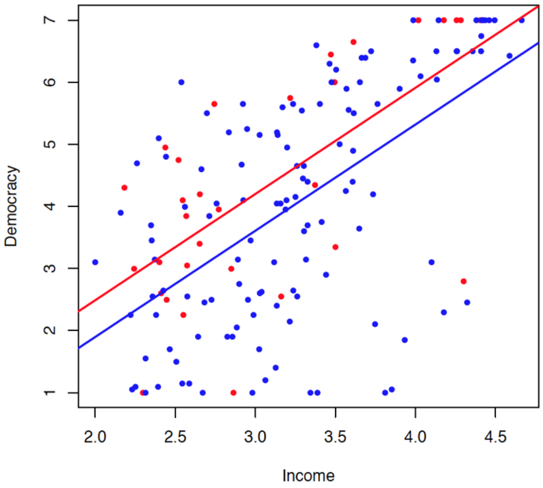

We are fitting 2 lines with the same slope but different intercepts.

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- Recall, for a continuous variable β will always be multiplied by the value of X_i

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- β_2 tells us how many units Y is expected to increase if Z increases by one unit and X does not change.
- Continues for any number of additional independent variables.

Examples of How to Interpret Multivariate Regression

Suicide Interpretation Example

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$$\hat{Y} = 276741.9$$

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$$\hat{Y} = 75.69$$

Republican Vote Share Interpretation Example

We are predicting `republican_vote_share` (a proportion) with four predictors: `voter_turnout_percentage` (0-100), `public_approval_rate` (0-100), `incumbant` (0 or 1), and `campaign_spending_millions` (0-??)

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Consider someone in the cash condition:

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Consider someone in the control condition:

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