GP HW13

Solution

HW13-1:

A 3.65-mol sample of an ideal diatomic gas expands adiabatically from a volume of $0.1210\ \mathrm{m}^3$ to $0.750\ \mathrm{m}^3$.

Initially the pressure was 1.00 atm. Determine:

(a) the initial and final temperatures;

(*b*) the change in internal energy;

(*c*) the heat lost by the gas;

(*d*) the work done *on* the gas. (Assume no molecular vibration.)

Solution:

(a)

adiabatic(絕熱): $PV^{\gamma} = \textbf{constant for ideal gas}$

$\gamma = \frac{7}{5} = 1.4$ for diatomic atoms with rotation but without vibration

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

$$\Rightarrow P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 1atm\left(\frac{0.1210m^3}{0.750m^3}\right)^{1.4} \approx 7.777\times10^{-2}atm$$

Ideal gas law: $PV = nRT \Rightarrow T = \frac{PV}{nR}$

$$T_1 = \frac{P_1V_1}{nR} = \frac{(1.013\times10^5\,pa)(0.121\,\mathrm{m}^3)}{(3.65mol)(8.314\frac{J}{mol\cdot K})} \approx 404K$$

$$T_2 = \frac{P_2V_2}{nR} = \frac{(7.777\times10^{-2})(1.013\times10^5\,pa)(0.750\,\mathrm{m}^3)}{(3.65mol)(8.314\frac{J}{mol\cdot K})} \approx 195K$$

(b)

$$\Delta E_{\text{int}} = nC_V \Delta T$$

$$C_V = \frac{5}{2}R$$ for diatomic atoms with rotation but without vibration

$$\Delta E_{\text{int}} = (3.65 mo\ell)\frac{5}{2}(8.314\frac{J}{mo\ell \cdot K})(195K - 404K)$$

$$\approx -1.59 \times 10^4 J$$

(c)

adiabatic : no heat is transferred $\Rightarrow Q = 0$

(d)

$$W = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} \frac{P_1V_1^{\gamma}}{V^{\gamma}} dV = P_1V_1^{\gamma} \int_{V_1}^{V_2} V^{-\gamma} dV$$

$$= P_1V_1^{\gamma}\left(\frac{1}{-\gamma+1}\right)\left(V_2^{1-\gamma} - V_1^{1-\gamma}\right)$$

$$= 1(1.013\times10^5)(0.1210)^{1.4}\left(\frac{1}{-1.4+1}\right)\left(0.750^{-0.4} - 0.1210^{-0.4}\right)$$

$$\approx 1.59\times10^4\,J$$

Check :

$\Delta E_{\text{int}} = Q - W$ adiabatic : $Q = 0$

$\Rightarrow \Delta E_{\text{int}} = -W$

$W \equiv$ **work done by gas**

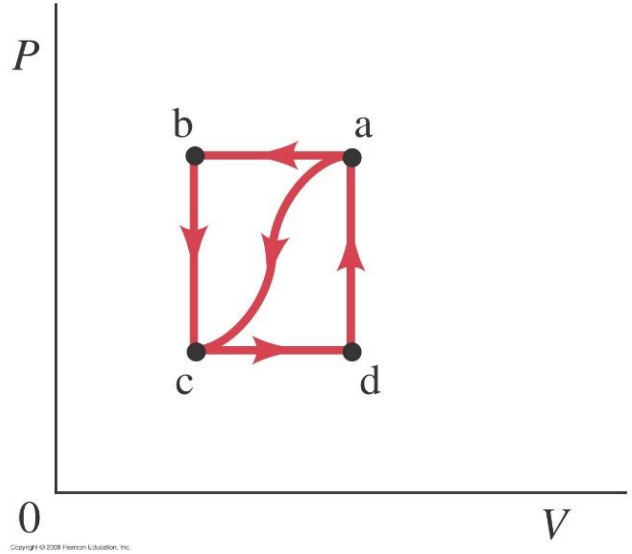
$\Rightarrow W' \equiv$ **work done on gas** $= -W = \Delta E_{\text{int}} \approx -1.59\times10^4\,J$

HW13-2 Problem 19-39 and 19-40 in Giancoli (pp. 524)

19-39. In the process of taking a gas from state a to state c along the curved path shown in Fig.19–32, 85 J of heat leaves the system and 55 J of work is done on the system. (*a*) Determine the change in internal energy, $E_{\text{int, a}} - E_{\text{int, c}}$. (*b*) When the gas is taken along the path cda, the work done by the gas is W = 38 J. How much heat Q is added to the gas in the process cda? (c) If $P_{\text{a}} = 2.2P_{\text{d}}$, how much work is done by the gas in the process abc? (d) What is Q for path abc? (e) If $E_{\text{int, a}} - E_{\text{int, b}} = 15\text{ J}$, what is Q for the process bc? Here is a summary of what is given:

$Q_{a\to c} = -85\ J$ $W_{\text{a}\to\text{c}} = -55\text{ J}$ $W_{\text{cda}} = 38\text{ J}$ $E_{\text{int, a}} - E_{\text{int, b}} = 15\text{ J}$ $P_{\text{a}} = 2.2P_{\text{d}}$

19-40. Suppose a gas is taken clockwise around the rectangular cycle shown in Fig.19–32, starting at b, then to a, to d, to c, and returning to b. Using the values given in Problem 39, (*a*) describe each leg of the process, and then calculate (*b*) the net work done during the cycle, (*c*) the total internal energy change during the cycle, and (*d*) the net heat flow during the cycle. (*e*) What percentage of the *intake* heat was turned into usable work: i.e., how efficient is this “rectangular” cycle (give as a percentage)?



Sol: for 39

(a) $E_{\text{int, a}} - E_{\text{int, c}} = \Delta E_{\text{int, ca}} = -\Delta E_{\text{int, ac}} = -\left(Q_{\text{ac}} - W_{\text{ac}}\right) = -\left(-85\text{J} + 55\text{J}\right) = 30\text{ J}$

(b) $\Delta E_{\text{int, cda}} = Q_{\text{cda}} - W_{\text{cda}}$

$\Rightarrow \quad Q_{\text{cda}} = \Delta E_{\text{int, cda}} + W_{\text{cda}} = \Delta E_{\text{int, ca}} + W_{\text{cda}} = 30\text{ J} + 38\text{ J} = 68\text{ J}$

(c) $W_{\text{abc}} = W_{\text{ab}} = P_{\text{a}}\Delta V_{\text{ab}} = P_{\text{a}}\left(V_{\text{b}} - V_{\text{a}}\right) = 2.2P_{\text{d}}\left(V_{\text{c}} - V_{\text{d}}\right) = -2.2W_{\text{cd}} = -2.2\left(38\text{ J}\right) = -84\text{ J}$

$(W_{\text{bc}} = 0 \Rightarrow W_{abc} = W_{ab} \quad ; \quad W_{\text{da}} = 0 \Rightarrow W_{\text{cda}} = W_{\text{cd}})$

(d) $\Delta E_{\text{int, abc}} = Q_{\text{abc}} - W_{\text{abc}}$

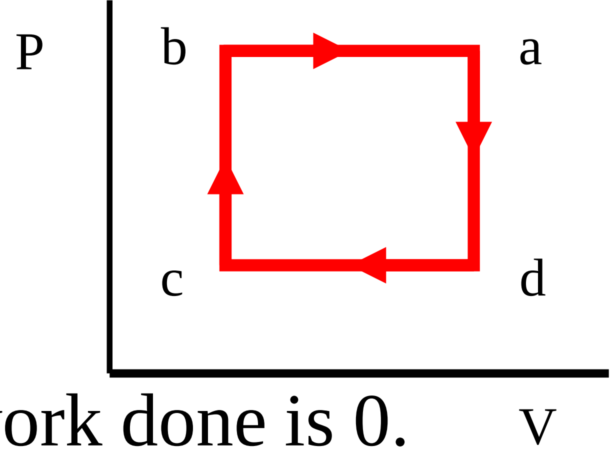
$\Rightarrow \quad Q_{\text{abc}} = \Delta E_{\text{int, abc}} + W_{\text{abc}} = \Delta E_{\text{int, ac}} + W_{\text{abc}} = -30\text{ J} - 84\text{J} = -114\text{ J}$

(e) $E_{\text{int,a}} - E_{\text{int,b}} = 15\text{ J} \quad \Rightarrow \quad E_{\text{int,b}} = E_{\text{int,a}} - 15\text{ J}$

$\Delta E_{\text{int, bc}} = E_{\text{int,c}} - E_{\text{int,b}} = E_{\text{int,c}} - \left(E_{\text{int,a}} - 15\text{ J}\right) = \Delta E_{\text{int, ac}} + 15\text{ J} = -30\text{ J} + 15\text{ J} = -15\text{ J}$

$\Delta E_{\text{int, bc}} = Q_{\text{bc}} - W_{\text{bc}} \quad \Rightarrow \quad Q_{\text{bc}} = \Delta E_{\text{int, bc}} + W_{\text{bc}} = -15\text{ J} + 0 = -15\text{ J}$

Sol: for 40



(a)

- ba: isobaric expansion, and the work done is positive.
- ad: isovolumetric/isochoric reduction in pressure, and the work done is 0.
- dc: isobaric compression, and the work done is negative.
- cb: isovolumetric/isochoric expansion in pressure, and the work done is 0.

(b)

$$W_{abc} = -84\text{ J} = W_{ab} + W_{bc} = W_{ab} + 0 = W_{ab}\ \text{(from prblem 39)} \Rightarrow W_{ba} = -W_{ab} = 84\text{ J}$$

$$W_{ad} = 0$$

$$W_{cda} = 38\text{ J} = W_{cd} + W_{da} = W_{cd} + 0 = W_{cd}\text{(from prblem 39)} \Rightarrow W_{dc} = -W_{cd} = -38\text{ J}$$

$$W_{cb} = 0$$

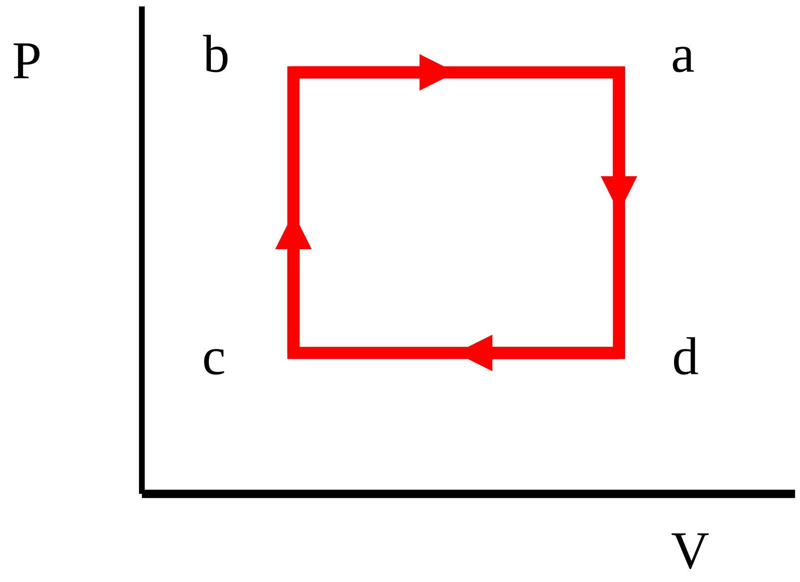
$$\Rightarrow W_{badc} = W_{ba} + W_{ad} + W_{dc} + W_{cb} = 46\text{ J}$$

(c) For a cycle $\Rightarrow \Delta E_{\text{int}} = 0$

(d) $\Delta E_{\text{int, tot}} = Q_{\text{net}} - W_{\text{net}} \Rightarrow Q_{\text{net}} = \Delta E_{\text{int}} + W_{\text{net}} = 0 + 46\,\text{J} = 46\ \text{J}$

(e) $Q_{\text{net}} = Q_{\text{adc}} + Q_{\text{cba}} = -68\,\text{J} + Q_{\text{cba}} = 46\,\text{J} \Rightarrow Q_{\text{cba}} = 114\,\text{J} = Q_{\text{input}}$

$$\text{efficiency} = \frac{W_{\text{net}}}{Q_{\text{input}}} \times 100\% = \frac{46\,\text{J}}{114\,\text{J}} \times 100\% \cong 40\%$$



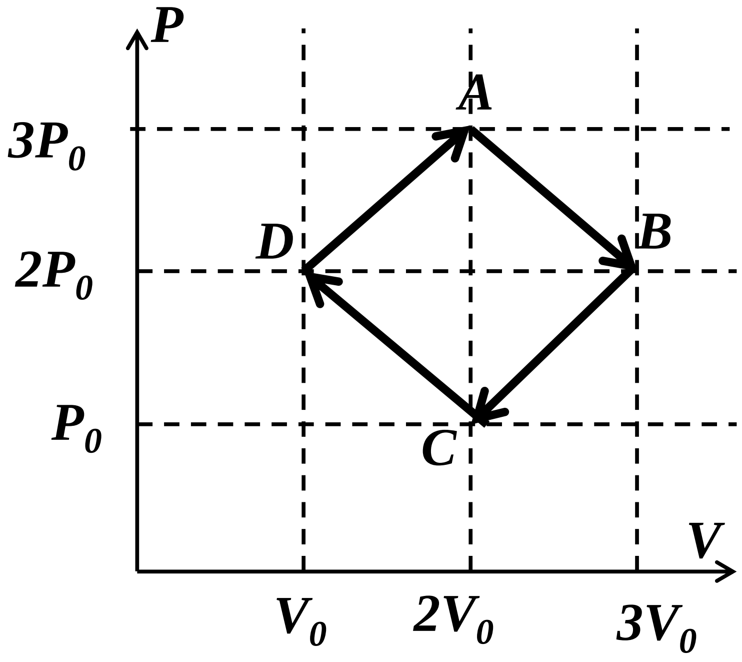
HW13-3

A thermal cycle runs clockwise from A to B, then C and finally D with straight lines and the pressures and the volumes for each point are shown in the figure on the right.

a) Calculate the work done by each process.
b) Calculate the change of internal energy in each process. (assume diatomic gas.)
c) Rank the intake heat $Q_{A\rightarrow B}$, $Q_{B\rightarrow C}$, $Q_{C\rightarrow D}$ **and** $Q_{D\rightarrow A}$, as it moves reversibly from points A, B, C, and D.

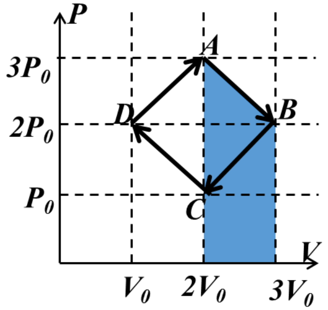
All the answer should be in terms of $\boldsymbol{P_0 V_0}$.

	$\triangle E$	Q	W
A→B			
B→C			
C→D			
D→A			



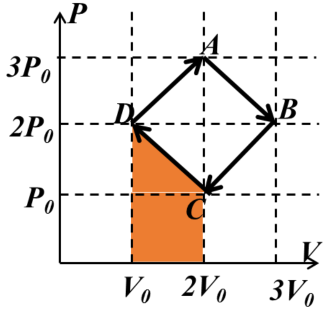
Sol: (a)

A ➔ B



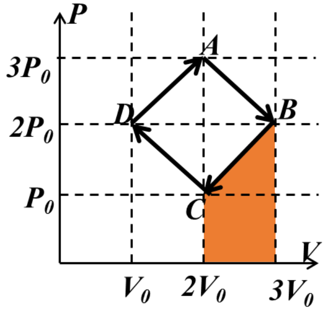
$$W_{A\to B}=\int_{V_A}^{V_B}PdV=\int_{2V_0}^{3V_0}(-\frac{P_0}{V_0}(V-3V_0)+2P_0)dV=\frac{5}{2}P_0V_0$$

C ➔ D



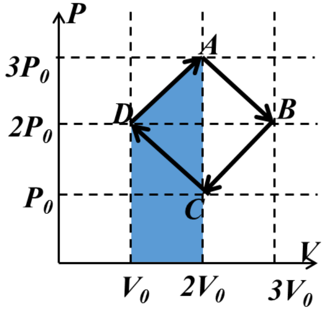
$$W_{C\to D}=\int_{V_C}^{V_D}PdV=\int_{2V_0}^{V_0}(-\frac{P_0}{V_0}\left(V-2V_0\right)+P_0)dV=-\frac{3}{2}P_0V_0$$

B ➔ C



$$W_{B\to C}=\int_{V_B}^{V_C}PdV=\int_{3V_0}^{2V_0}(\frac{P_0}{V_0}\left(V-2V_0\right)+P_0)dV=-\frac{3}{2}P_0V_0$$

D ➔ A



$$W_{D\to A}=\int_{V_D}^{V_A}PdV=\int_{V_0}^{2V_0}(\frac{P_0}{V_0}\left(V-V_0\right)+2P_0)dV=\frac{5}{2}P_0V_0$$

(b)

diatomic gas➔ $E=\frac{5}{2}NkT$ （只和 T 有關 $T\propto PV$ ）

A➔B $$\Delta E=\frac{5}{2}Nk(T_B-T_A)=\frac{5}{2}Nk(\frac{6P_0V_0}{Nk}-\frac{6P_0V_0}{Nk})=0$$

B➔C $$\Delta E=\frac{5}{2}Nk(T_C-T_B)=\frac{5}{2}Nk(\frac{2P_0V_0}{Nk}-\frac{6P_0V_0}{Nk})=-10P_0V_0$$

C➔D $$\Delta E=\frac{5}{2}Nk(T_D-T_C)=\frac{5}{2}Nk(\frac{2P_0V_0}{Nk}-\frac{2P_0V_0}{Nk})=0$$

D➔A $$\Delta E=\frac{5}{2}Nk(T_A-T_D)=\frac{5}{2}Nk(\frac{6P_0V_0}{Nk}-\frac{2P_0V_0}{Nk})=10P_0V_0$$

(c) 已知 $\boldsymbol{E}$ 和 $\boldsymbol{W}$ $\boxed{E = Q - W}$ ➔ $Q = E + W$

	W	Q	ΔE
A➔B	$\frac{5}{2}P_0V_0$	$\frac{5}{2}P_0V_0$	0
B➔C	$-\frac{3}{2}P_0V_0$	$-\frac{23}{2}P_0V_0$	$-10P_0V_0$
C➔D	$-\frac{3}{2}P_0V_0$	$\frac{-3}{2}P_0V_0$	0
D➔A	$\frac{5}{2}P_0V_0$	$\frac{25}{2}P_0V_0$	$10P_0V_0$

$$Q_{D\to A} > Q_{A\to B} > 0 > Q_{C\to D} > Q_{B\to C}$$