1.

a.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1 & 5 & 8 \end{array}\right] \Rightarrow P_{B' \to B} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 3 \\ 1 & 5 & 8 \end{bmatrix}$$

b.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 0 & 3 & 0 & 1 & 0 \\ 1 & 5 & 8 & 0 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 4 & -3 & -2 & 1 & 0 \\ 0 & 3 & 5 & -1 & 0 & 1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 3/4 & 1/2 & -1/4 & 0 \\ 0 & 0 & 11/4 & -5/2 & 3/4 & 1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 41/11 & -9/11 & -12/11 \\ 0 & 1 & 0 & 13/11 & -5/11 & -3/11 \\ 0 & 0 & 11/4 & -5/2 & 3/4 & 1 \end{array}\right] \Rightarrow P_{B \to B'} = \begin{bmatrix} \frac{15}{11} & \frac{1}{11} & \frac{-6}{11} \\ \frac{13}{11} & \frac{-5}{11} & \frac{-3}{11} \\ \frac{-10}{11} & \frac{3}{11} & \frac{4}{11} \end{bmatrix}$$

c.

$$[P]_B \Rightarrow a \times 1 + b \times x^2 + c \times x^2 = 1 + x + x^2 \qquad \Rightarrow [P]_B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow a = 1 \quad b = 1 \quad c = 1$$

2.

a.

$$\begin{bmatrix} -2 & 1 & 2 & 5 \\ 2 & 0 & 4 & 5 \\ 3 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 & 5 & 2 & 5 \\ 2 & -4 & 4 & 5 \\ 3 & -5 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \Rightarrow (-1) \begin{vmatrix} 5 & 2 & 5 \\ -4 & 4 & 5 \\ -5 & 0 & 0 \end{vmatrix} \Rightarrow (-1)(-5) \begin{vmatrix} 2 & 5 \\ 4 & 5 \end{vmatrix}$$

$$\Rightarrow (-1)(-5)(-10) = -50.$$

b.

$$\det(M)\det(M^{-1}) = 1 \Rightarrow \det(M^{-1}) = \frac{-1}{50}$$

c.

$$\left[\begin{array}{cccc|cccc} -2 & 1 & 2 & 5 & 1 & 0 & 0 & 0 \\ 2 & 0 & 4 & 5 & 0 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 2/5 & -1/5 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1/5 & 3/5 \\ 0 & 0 & 1 & 0 & -1/2 & 1/2 & -7/10 & 7/10 \\ 0 & 0 & 0 & 1 & 2/5 & -1/5 & 14/25 & -12/25 \end{array}\right]$$

d)

$$2^4 \det(M) = -15 \times 16 = -800$$

(e)

$$2^4 \det(M^{-1}) = \frac{-16}{50}$$

3.

$\overrightarrow{P_1P_2}:(0,-1,2)$ $\overrightarrow{P_1P_3}:(-1,1,3)$

(a) $\frac{1}{2}\|\overrightarrow{P_1P_2}\times\overrightarrow{P_1P_3}\| = \frac{1}{2}\|(-5,-2,-1)\| = \frac{1}{2}\sqrt{25+4+1} = \frac{1}{2}\sqrt{30}$ #

(b)

法向量: $(-5,-2,-1)$.

$-5x-2y-z=d$, $(1,1,0)$ 代入 $\Rightarrow -5x-2y-z=-7$ #

(c) $5x=7-2y-z$

$x=\frac{1}{5}(7-2y-z)$

$y=t_1$

$z=t_2$

$(x,y,z)=\left(\frac{7}{5}-\frac{2}{5}t_1-\frac{t_2}{5}, t_1, t_2\right)$. #

4.

(a) Let R be reduced row-echelon form of A; $A, B \in n\times n$ matrix

$R=E_1E_2\cdots E_r A$

$\det(R)=\det(E_1)\det(E_2)\cdots\det(E_r)\det(A)$ (by thm ?) Lemma 2.3.2

$\det(E_i)\neq 0$

So $\det(R)$ and $\det(A)$ are both zero or both nonzero.

if $\det(A)=0 \Rightarrow \det(R)=0 \Rightarrow R$ can't be an I_n 2.2.1

→ The bottom row(s) in R is (are) all zeros. (by thm ?)

(b) if $Ax=0$ has nontrivial solution

$\Rightarrow A^{-1}$ not exit

$\Rightarrow A^{-1}=\frac{1}{\det(A)}\operatorname{adj}(A)$

$\Rightarrow \det(A)=0$.

$A^{-1}(Ax)=A^{-1}0$

$Ix=0$

$x=0$

Assume $\det(A)\neq 0$.

A is invertible. (by thm 2.3.3)

$A^{-1}\cdot A(x)=A^{-1}\cdot 0$

$x=0$ exactly one solution

$\rightarrow\leftarrow$ ($Ax=0$ has nontrivial solution)

$\therefore \det(A)=0$.