General Physics (I) **期中考 II** **Nov.30, 2018**

試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(51points) (所有題目必須有計算過程,否則不予計分)

Fig. 1

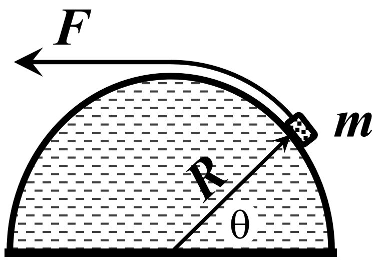
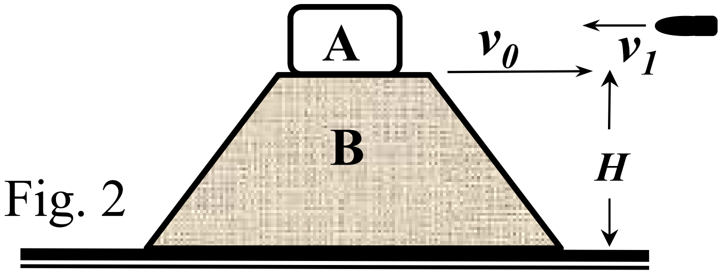
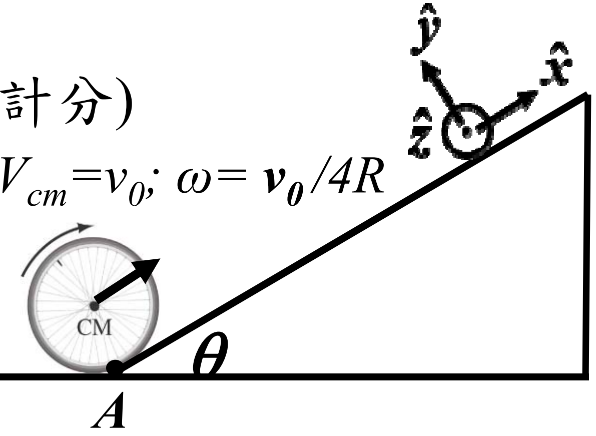


Fig. 2



$t = 0,\ V_{cm}=v_0;\ \omega= \boldsymbol{v_0}/4R$

Fig. 3



1&2. (15pts) As shown in Fig. 1, a block of mass $\boldsymbol{m}$ is placed on top of a semi-circular surface with radius $\boldsymbol{R}$. The force $\boldsymbol{F}$ is pulling the block upward through a string such that $\boldsymbol{F}$ is always tangential to the surface. If the kinetic friction coefficient between the block and the surface is $\boldsymbol{\mu_k = \mu_0 \cdot \cot\theta}$, where $\boldsymbol{\mu_0}$ is a constant. Calculate the work down by $\boldsymbol{F}$ if the block is pulled from the position of $\boldsymbol{\theta = 60^\circ}$ to $\boldsymbol{\theta = 90^\circ}$ under the condition: (a) (7pts) The block moves in a quasi-static manner. (b) (8pts) The block moves with a constant speed of $\boldsymbol{v}= (2\boldsymbol{gR})^{1/2}$.(You need to show the free-body diagram in your solution.)

3. (15pts) As shown in Fig.2, block A with mass $\boldsymbol{0.9\ m}$ is placed on block B with mass $3\boldsymbol{m}$. Both are initially moving with velocity v_0. A bullet with mass $\boldsymbol{0.1m}$ and unknown velocity $\boldsymbol{v_1}$ hits and is embedded (嵌入) in block A, and then the velocity of the block A becomes zero. Assume all the surface are frictionless. (a) (4 pts) What is the velocity $\boldsymbol{v_1}$ of the bullet before the collision? (b) (11 pts) Eventually, the block A slides down block B. Assume the height $\boldsymbol{H = 9v_0^2/g}$, what is the velocity $\boldsymbol{u_A}$ of block A and velocity $\boldsymbol{u_B}$ of block B after they become separated?

4. (21 pts) As shown in Fig. 3, a wheel (mass $\boldsymbol{m}$, radius $\boldsymbol{R}$ and $\boldsymbol{I_{CM}=mR^2}$) with the velocity of the center of mass $\boldsymbol{v_0}$ and angular velocity ω= $\boldsymbol{v_0}$/4R is released to the bottom of an inclined surface. The kinetic and static friction coefficients are $\boldsymbol{\mu_k=0.25}$ and $\boldsymbol{\mu_s=0.40}$, respectively, between the wheel and the inclined surface with $\boldsymbol{\theta = 37^o}$.

(a) (3pts) Draw the free body diagram of the wheel before it reaches pure roll motion on the incline.

(b) (1pts) Is the friction static or kinetic before the wheel reaches pure rolling motion on the incline?

(c) (10pts) At what time t_r after its release, does the wheel execute pure rolling motion on the incline?

(d) (2pts) How far does the wheel travel from point A at this moment? (write your answer in terms of m, R, v_0, g)

(e) (5pts) What is the work W_T done by the frictional force on the translational motion and the work W_R on the rotational motion of the wheel when the wheel just reaches the pure rolling condition?
(write you answers in terms of m, R, v_0, g)

II.選擇題(49 points)

1. (5 pts) Fig. 4 shows a conservative force $F(x)$ experienced by a particle of mass m executing a one dimensional motion, which of the following is the potential energy $U(x)$ corresponding to $F(x)$?

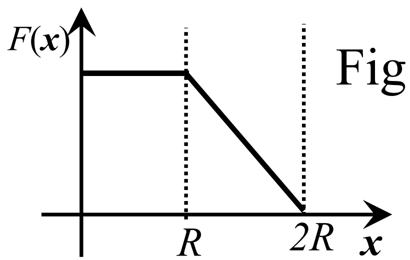
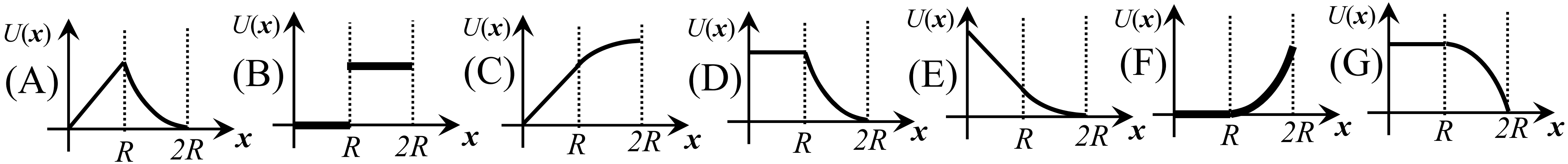


Fig. 4



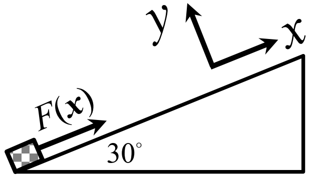


Fig. 5(a)

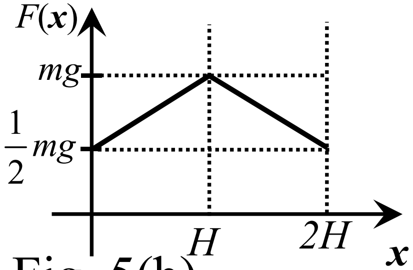


Fig. 5(b)

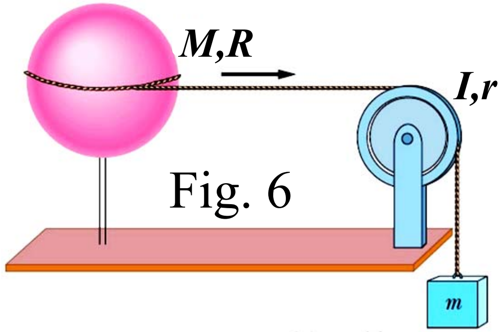


Fig. 6

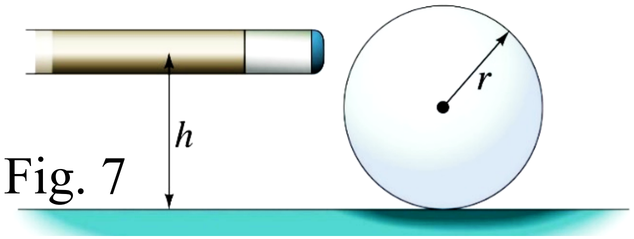


Fig. 7

2. (5pts) As shown in Fig. 5(a), a non-conservative force $F(x)$ is acting on a block of mass m, which initially is resting at the bottom of the inclined surface (i.e. $x = 0$). The magnitude of $F(x)$ as a function of the position along the frictionless inclined surface is shown in Fig. 5(b), If E_k is the kinetic energy of the block at $x = H$, and $z = E_k/mgH$, which of the following is correct?

(A) $0 \leq z < 1/16$ (B) $1/16 \leq z < 3/16$ (C) $3/16 \leq z < 5/16$ (D) $5/16 \leq z < 9/16$ (E) $9/16 \leq z < 1$ (F) $1 \leq z < 2$ (G) $2 \leq z$

3. (5pts) A uniform solid sphere of mass $\boldsymbol{M}$=15 kg and radius $\boldsymbol{R}$=0.15 m can rotate about a vertical axis on frictionless bearings shown in Fig. 6. A massless cord passes around the equator of the shell, over a pulley of the rotational inertia $\boldsymbol{I}$=0.02 kg m^2 and radius $\boldsymbol{r}$ =0.1 m, and is attached to a small object of mass $\boldsymbol{m}$=2 kg. There is no friction on the pulley's axis; the cord does not slip on the pulley. What is the speed $\boldsymbol{v}$ (m/s) of the object when it fall by 1.0 m?
(A) $v \leq 1$; (B) $1 < v \leq 1.5$; (C) $1.5 < v \leq 2$; (D) $2 < v \leq 2.5$; (E) $2.5 < v \leq 3$; (F) $3 < v \leq 3.5$; (G) $3.5 < v \leq 4$; (H) $4 < v \leq 4.5$; (I) $4.5 < v \leq 5$; (J) $5 < v$.

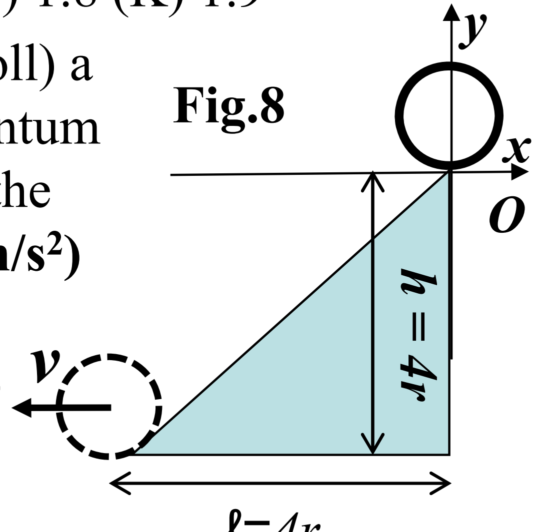
4. (5pts) In Fig. 7, if a billiard ball is hit in just the right way by a cue stick, the ball will roll without slipping immediately after losing contact with the stick. Consider a billiard ball (radius r, mass M) at rest on a horizontal pool table. A cue stick exerts a constant horizontal force $F=10Mg$ on the ball for a time $\boldsymbol{t}$ at a point that is a height $\boldsymbol{h}$ above the table's surface. Assume that the coefficients of kinetic friction and the static friction between the ball and table are μ_k=0.2 and μ_s=0.4, respectively. The ball will roll without slipping immediately after losing contact with the stick, if $\boldsymbol{h/r}$ is

(A) 0.9 (B) 1.0 (C) 1.1 (D) 1.2 (E) 1.3 (F) 1.4 (G) 1.5 (H) 1.6 (I) 1.7 (J) 1.8 (K) 1.9

5. (5 pts) A ring of radius $\boldsymbol{r} = \mathbf{0.1}\boldsymbol{m}$ and mass $\boldsymbol{m}=\mathbf{2.0kg}$ rolls down (pure roll) a incline from rest at point $\boldsymbol{O}$ as shown in Fig. 8. The total angular momentum of the ring relative to point $\boldsymbol{O}$ is $\boldsymbol{a}$ ($\mathbf{kg{\cdot}m^2/s}$) when the ring just reaches the bottom of the incline. Which is the following answer correct? ($\boldsymbol{g} = \mathbf{10\ m/s^2}$)

(A) $a \le -1$ (B) $-1 < a \le -0.8$ (C) $-0.8 < a \le -0.6$ (D) $-0.6 < a \le -0.4$
(E) $-0.4 < a \le -0.2$ (F) $-0.2 < a \le 0$ (G) $0 < a \le 0.2$ (H) $0.2 < a \le 0.4$
(J) $0.4 < a \le 0.6$ (K) $0.6 < a \le 0.8$ (L) $0.8 < a \le 1.0$ (M) $1.0 < a$.

Fig.8



6. (5 pts) A bullet of mass $\boldsymbol{m}$ moving with velocity $\boldsymbol{v_0}$ strikes and becomes embedded at the edge of a disk of mass $\boldsymbol{M}$ and radius $\boldsymbol{R_0}$ (Fig. 9). The disk, initially at rest on the table and fixed at point $\boldsymbol{A}$, starts to rotate about point $\boldsymbol{A}$. Which of the following quantities is (are) conserved right after the collision?

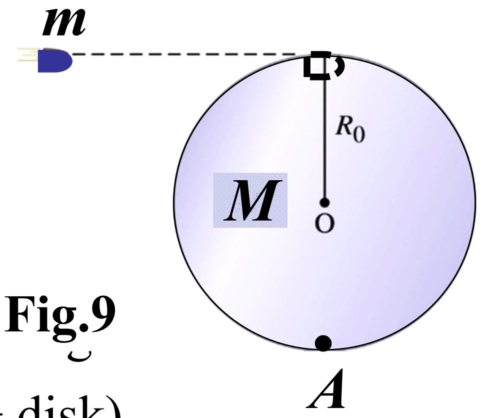
I. Linear momentum

II. Angular momentum about point $\boldsymbol{A}$

III. Angular momentum about the center of mass of the system (bullet + disk)

(A) I only; (B) II only; (C) III only; (D) I and II; (E) I and III; (F) II and III; (G) I, II, and III; (H) None of above

Fig.9



7. (5 pts) Same question as problem 6, but the disk is free to move on the table. The disk, initially at rest on the table, begins to move and rotates about its center of mass (bullet + disk). Assuming no friction between the disk and the table. Which of the quantities (in problem 7) is (are) conserved right after the collision?

(A) I only; (B) II only; (C) III only; (D) I and II; (E) I and III; (F) II and III; (G) I, II, and III; (H) None of above

1. (15pts) As shown in Fig. z, a block of mass $\boldsymbol{m}$ is placed on top of a semi-circular surface with radius $\boldsymbol{R}$. The force $\boldsymbol{F}$ is pulling the block upward through a string such that $\boldsymbol{F}$ is always tangential to the surface. If the kinetic friction coefficient between the block and the surface is $\mu_k = \mu_0 \cdot \cot\theta$, where μ_0 is a constant. Calculate the work down by $\boldsymbol{F}$ if the block is pulled from the position of $\theta = 60^\circ$ to $\theta = 90^\circ$ under the condition: (a) (7pts) The block moves in a quasi-static manner. (b) (8pts) The block moves with a constant speed of $v = (2gR)^{1/2}$.

(a)

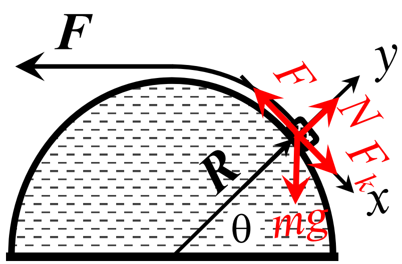
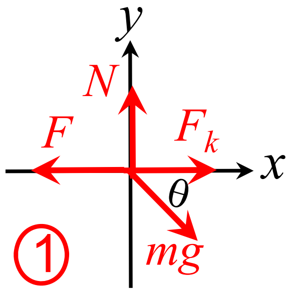


Fig. z



1

$$\sum \vec{F} = m\vec{a}$$

1

$x: F - F_k - mg\cos\theta = ma = 0$ (quasi-static) (1)

$y: N - mg\sin\theta = 0$ (2) 1

$F_k = \mu_k N$ (3) 1

From (1),(2), and (3), we get

$$F = F_k + mg\cos\theta = \mu_k mg\sin\theta + mg\cos\theta$$
$$= \mu_0 \cot\theta \cdot mg\sin\theta + mg\cos\theta = (\mu_0 + 1)mg\cos\theta$$

1

$$dW = \vec{F}\cdot d\vec{\ell} = Fd\ell = FRd\theta$$

$$W = \int_{\theta=\pi/3}^{\theta=\pi/2} dW = \int_{\pi/3}^{\pi/2} FRd\theta = \int_{\pi/3}^{\pi/2} (\mu_0 + 1)mg\cos\theta Rd\theta$$ 1

$$= (\mu_0 + 1)mgR(\sin\theta\big|_{\pi/3}^{\pi/2}) = (\mu_0 + 1)mgR(1 - \sqrt{3}/2)$$ 1

(b) The block is moving with a constant speed $v = (2gR)^{1/2}$. The free-body diagram of theblock remains the same with Eq. (2) changed as the following.

$$y: N - mg\sin\theta = \frac{mv^2}{R} = -2mg \Rightarrow N = mg\sin\theta - 2mg \quad (4)$$

2

and eq.(3) becomes

$F_k = \mu_k(mg\sin\theta - 2mg)$ (5) 1

From (1),(4), and (5), we get

$$F = F_k + mg\cos\theta = \mu_k(mg\sin\theta - 2mg) + mg\cos\theta$$
$$= (\mu_0 + 1)mg\cos\theta - 2\mu_0 mg\cot\theta$$ 1

$$W = \int_{\pi/3}^{\pi/2} [(\mu_0 + 1)mg\cos\theta - 2\mu_0 mg\cot\theta]Rd\theta$$ 1

$$= \int_{\pi/3}^{\pi/2} (\mu_0 + 1)mg\cos\theta Rd\theta - \int_{\pi/3}^{\pi/2} 2\mu_0 mg\cot\theta Rd\theta$$

$$\int_{\pi/3}^{\pi/2} \frac{\cos\theta}{\sin\theta} d\theta = \int_{\pi/3}^{\pi/2} \frac{d(\sin\theta)}{\sin\theta} = \ln(\sin\theta)\big|_{\pi/3}^{\pi/2}$$
$$= \ln(1) - \ln(\sqrt{3}/2) = -\ln(\sqrt{3}/2)$$

$$\Rightarrow W = (\mu_0 + 1)mgR(1 - \sqrt{3}/2) + \mu_0 2mgR\ln(\sqrt{3}/2)$$

1 2

(a) Embedded process:

Momentum conservation: $P_i = P_f$

$$0.1m * v_1 + 0.9m * v_0 = 0$$ **2 pts**

$$\Rightarrow \quad v_1 = -9v_0$$ **2 pt**

(b) After separation:

Momentum conservation: $P_i = P_f$

$$m_B * v_0 = mA * uA + mB * u_B$$ **2 pts**

$$3v_0 = uA + 3u_B \quad \text{--- (1)}$$

$$u_A = 3v_0 - 3u_B$$

Mechanical Energy conservation: $E_i = E_f$ **2 pt**

$$\frac{1}{2}m_B * v_0^2 + mgH = \frac{1}{2}m_A * uA^2 + \frac{1}{2}m_B * u_B^2 \quad \text{--- (2)}$$

$$3v_0^2 + 18v_0^2 = (3v_0 - 3u_B)^2 + 3u_B^2$$

$$\rightarrow \quad 12u_B^2 - 18v_0 \cdot u_B - 12v_0^2 = 0$$

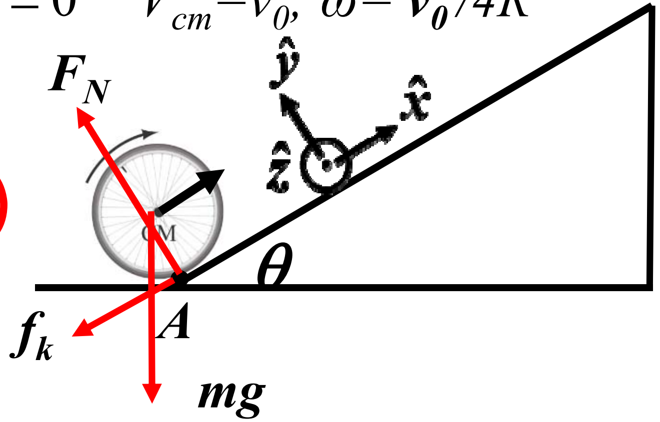
$$2u_B^2 - 3v_0 \cdot u_B - 2v_0^2 = 0$$

$$(2u_B + v_0) \cdot (u_B - 2v_0) = 0$$ **4 pts**

$$\begin{cases} u_B = -\frac{1}{2} v_0 \ ; u_A = \frac{9}{2} v_0 & \text{Make no sense, since block B is moving to the right.} \\ u_B = 2 v_0 \ ; u_A = -3 v_0 & \text{合理} \end{cases}$$ **3 pt**

(a) $t = 0 \quad V_{cm}=v_0;\ \omega = \boldsymbol{v_0}/4R$

3



(b) **Kinetic friction** 1

(c) $$\sum F_x = -f_k - \text{mg Sin}\theta = ma \quad 1$$

$$\sum F_y = F_N - \text{mg Cos}\theta = 0 \quad 1$$

$$\sum \tau = \text{R}f_k = \text{I}\,\alpha \quad 1$$

$$\boxed{\sum \tau = -\text{R}f_k = \text{I}\,\alpha}$$

$$\text{f}_\text{k} = \frac{mg}{5} \quad 1 \qquad \alpha = \frac{g}{5R} \quad 1 \qquad a = -\frac{4g}{5} \quad 1$$

$$\text{v(t)} = \text{v}_0 - \frac{4g}{5}\text{t} \quad 1$$

$$\omega(t) = \frac{\text{v}_0}{4R} + \frac{g}{5R}\text{t} \quad 1$$

$$\boxed{\omega(t) = -\frac{\text{v}_0}{4R} - \frac{g}{5R}\text{t}}$$

At t=t_r, a pure rolling just occurs.

$$\text{v}(t_r) = \omega(t_r)R \quad 1 \qquad \boxed{\text{v}(t_r) = -\omega(t_r)R}$$

$$\text{t}_r = \frac{3\text{v}_0}{4g} \quad 1$$

(d)

$$\Delta x(t_r) = \text{v}_0 t_r + \frac{1}{2}a{t_r}^2 = \text{v}_0\frac{3\text{v}_0}{4g} - \frac{1}{2}\frac{4g}{5}\left(\frac{3\text{v}_0}{4g}\right)^2 = \frac{21{\text{v}_0}^2}{40g} \quad 2$$

(e) $$W_T = -\text{f}_\text{k}\Delta x(t_r) = -\frac{mg}{5}\frac{21{\text{v}_0}^2}{40g} = -\frac{21}{200}\text{m}{\text{v}_0}^2 \quad 2$$

$$\Delta\theta(t_r) = \frac{\text{v}_0}{4R}\frac{3\text{v}_0}{4g} + \frac{1}{2}\frac{g}{5R}\left(\frac{3\text{v}_0}{4g}\right)^2 = \frac{39\,{\text{v}_0}^2}{160gR} \quad 1$$

$$\boxed{\Delta\theta(t_r) = -\frac{\text{v}_0}{4R}\frac{3\text{v}_0}{4g} - \frac{1}{2}\frac{g}{5R}\left(\frac{3\text{v}_0}{4g}\right)^2 = -\frac{39\,{\text{v}_0}^2}{160gR}}$$

$$W_R = \tau_\text{k}\Delta\theta(t_r) = R\frac{mg}{5}\frac{39{\text{v}_0}^2}{160gR} = \frac{39}{800}\text{m}{\text{v}_0}^2 \quad 2$$

$$\boxed{W_R = \tau_\text{k}\Delta\theta(t_r) = \left(-R\frac{mg}{5}\right)\left(-\frac{39{\text{v}_0}^2}{160gR}\right) = \frac{39}{800}\text{m}{\text{v}_0}^2}$$

Multiple Choice Questions:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
E	**C**	**C**	**F**	**B**	**B**	**G**		
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**	
D	**D**	**F**	**D**	**C**	**E**	**C**		