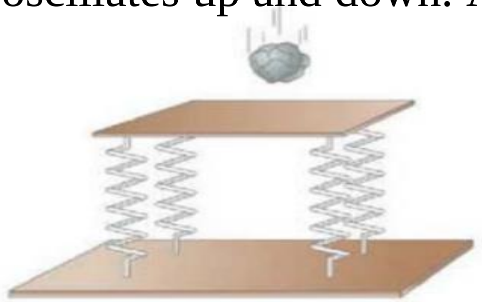
HW11-1: Problem 14-83 in Giancoli (pp. 454) ; Problem 14-83 in Giancoli (pp. 393)

A 1.6-kg table is supported on four springs. A 0.8-kg chunk of modeling clay is held above the table and dropped so that it hits the table with a speed of 1.65m/s (Fig. below). The clay makes an inelastic collision with table, and clay oscillates up and down. After a long time, the table comes to rest 6.0 cm below its original position.

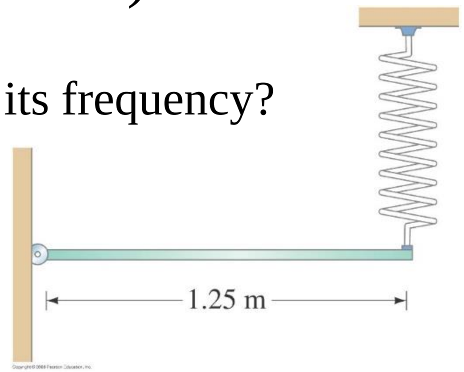


(a) What is the effective spring constant of all four springs taken together?

(b) With what maximum amplitude does the platform oscillate?

HW11-2: Problem 14-11 in Giancoli (pp. 448) ; Problem 14-11 in Giancoli (pp. 389)

A uniform meter stick of mass *M* is pivoted on a hinge at one end and held horizontal by a spring with spring constant *k* attached at the other end (Fig. 14–28). If the stick oscillates up and down slightly, what is its frequency? [*Hint*: Write a torque equation about the hinge.]



HW11-3:

A particle is confined to move in *x*-direction between $\boldsymbol{x}=0$ and $\boldsymbol{x}=\infty$, and it experiences an conservative force $\boldsymbol{F}(\boldsymbol{x})$ such that its potential energy $\boldsymbol{U}(\boldsymbol{x}) = -\boldsymbol{b}x^{2}\cdot e^{-\boldsymbol{a}x}$, where $\boldsymbol{a}, \boldsymbol{b}>0$,

(a) Determine this conservative force $\boldsymbol{F}(\boldsymbol{x})$ as a function of x,

(b) At the equilibrium point $\boldsymbol{x} = \boldsymbol{S}$, $\boldsymbol{F}(\boldsymbol{S}) = 0$, determine the value of $\boldsymbol{S}$.

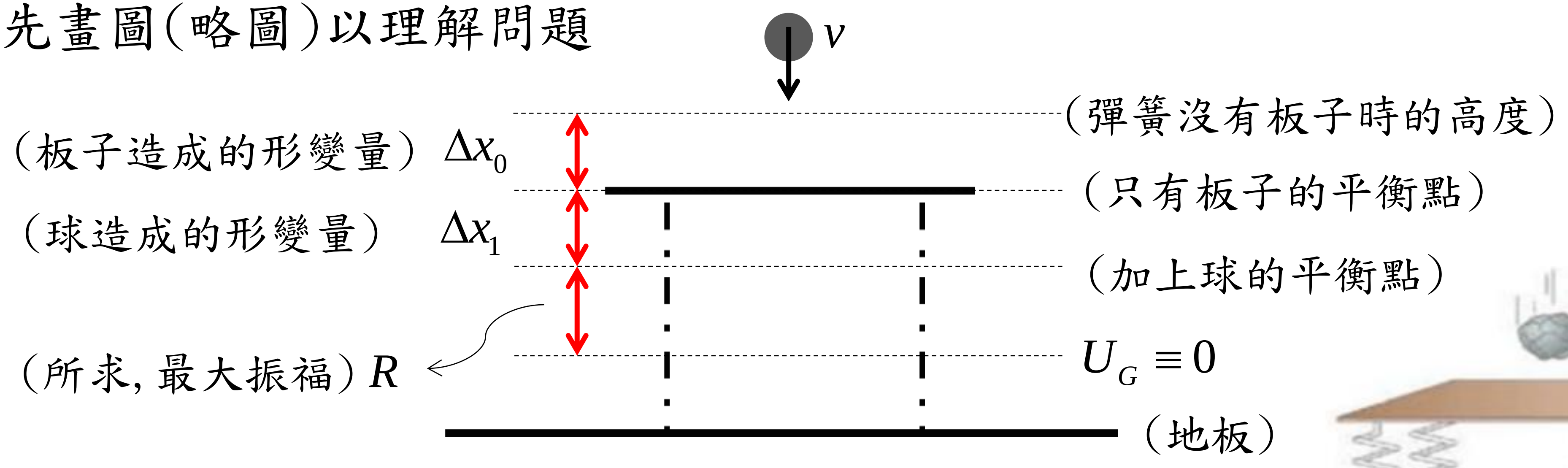
(c) If the particle is moving around $\boldsymbol{S}$, and if we define $\boldsymbol{z} = \boldsymbol{x}-\boldsymbol{S}$, write down the equation of motion of the particle in terms of $\boldsymbol{z}$,

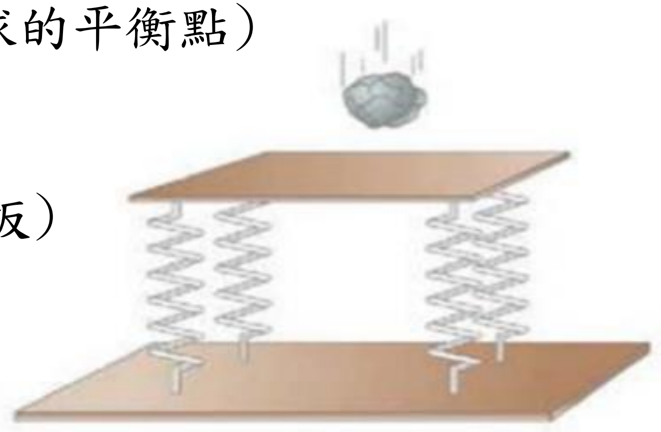
(d) For the case if $\boldsymbol{z}/\boldsymbol{S} \ll 1$, the particle executes a simple harmonic oscillation around S, determine the period of the oscillation of the particle near S.

$$e^{az} \approx 1+az,\ (1+z)^{n} \approx 1+nz,\ \ for\ |z| << 1, |az| << 1$$

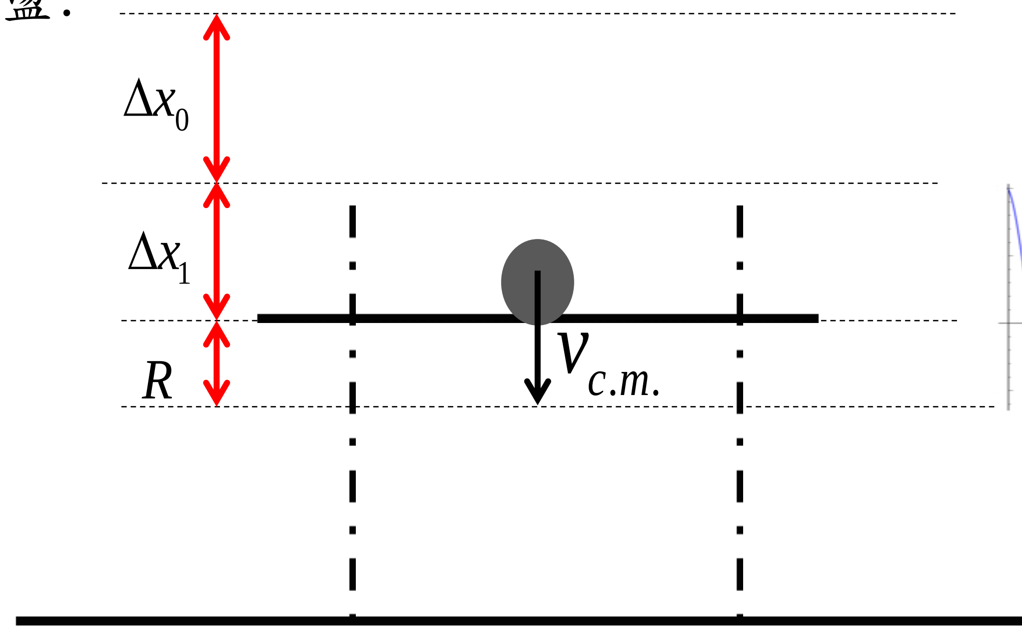
HW11-1: Problem 14-83 in Giancoli (pp. 454) ； Problem 14-83 in Giancoli (pp. 393)

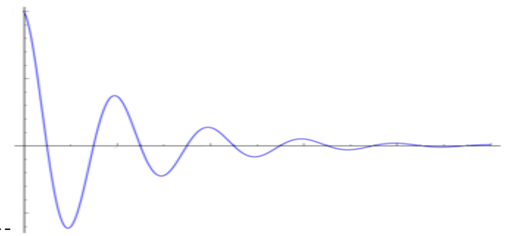
先畫圖(略圖)以理解問題





黏在一起後的震盪:





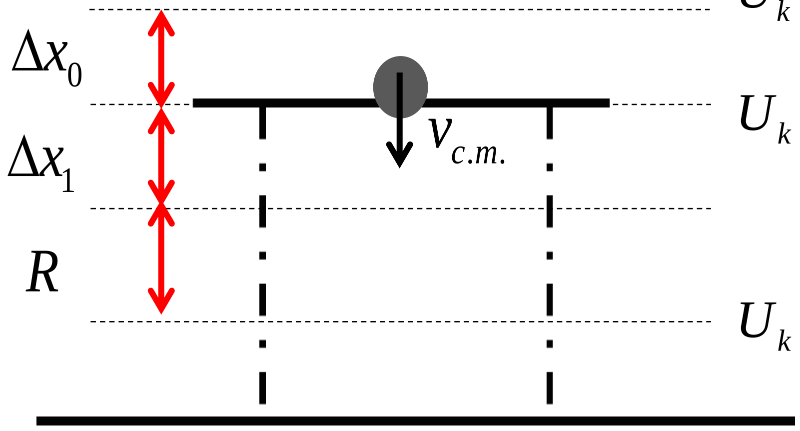
(a):

$$F = -k\Delta x \Rightarrow \begin{cases} -Mg = -k_{eff}\Delta x_0 \\ -(M+m)g = -k_{eff}(\Delta x_0 + \Delta x_1) \end{cases}$$

$$\Rightarrow mg = k_{eff}\Delta x_1 \,,\ k_{eff} = \frac{mg}{\Delta x_1} \approx 130.67(\mathrm{kg/s^2})$$

(b):

黏在一起後的瞬間:



$$U_k = \frac{1}{2}k\Delta x^2 = 0$$

$$U_k = \frac{1}{2}k\Delta x_0^{\ 2},\ U_G = (m+M)g(\Delta x_1 + R),\ K_{c.m.}$$

$$U_k = \frac{1}{2}k(\Delta x_0 + \Delta x_1 + R)^2,\ U_G \equiv 0$$

最大震幅,

即考慮動能、重力位能轉換成彈力位能 $\Rightarrow$ 最低點動能為零

先考慮碰撞(完全非彈性碰撞)時間甚短, 所以動量變化量可忽略, 即動量守恆:

$$\vec{p} = const \;\Rightarrow\; mv = (m+M)v_{c.m}\,,\quad v_{c.m} = \frac{mv}{m+M}$$

$$\Rightarrow\; (K_{c.m})_i = \frac{1}{2}M_{c.m}v^2{}_{c.m} = \frac{1}{2}\frac{m^2}{m+M}v^2$$

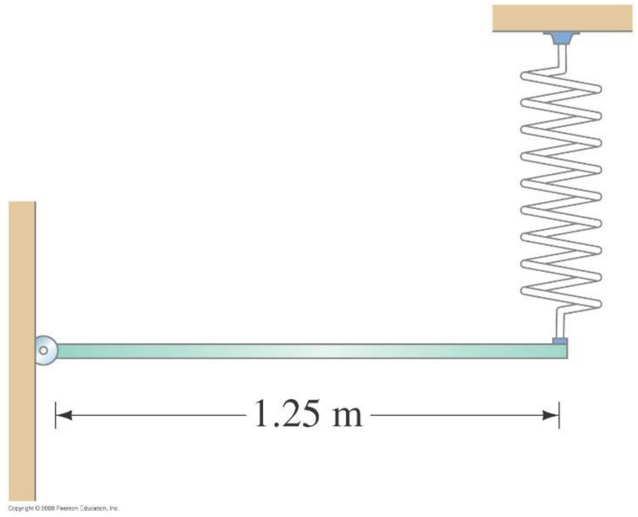
力學能守恆: $E_{tot} = K + U_k + U_G = const$

$$(K_{c.m})_i + \frac{1}{2}k\Delta x_0{}^2 + M_{c.m.}g(\Delta x_1 + R) = 0 + \frac{1}{2}k(\Delta x_0 + \Delta x_1 + R)^2 + 0$$

所求為R ,將等式整理,並用一元二次公式解, 得 $R \approx 9.56(cm)$

HW11-2: Problem 14-11 in Giancoli (pp. 448)； Problem 14-11 in Giancoli (pp. 389)

A uniform meter stick of mass M is pivoted on a hinge at one end and held horizontal by a spring with spring constant k attached at the other end (Fig. 14–28). If the stick oscillates up and down slightly, what is its frequency?
[*Hint*: Write a torque equation about the hinge.]



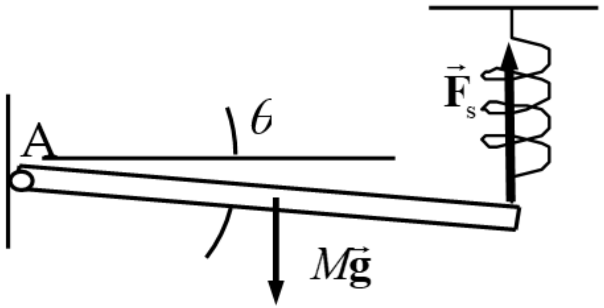
達平衡時伸長量為y_0，淨力矩：$\sum\tau = Mg\left(\tfrac{1}{2}\ell\right) - F_s\ell = \tfrac{1}{2}Mg\ell - ky_0\ell = 0$

伸長y時力矩：$\sum\tau = Mg\left(\tfrac{1}{2}\ell\right) - F_s\ell = \tfrac{1}{2}Mg\ell - k\left(y + y_0\right)\ell = I\alpha = \tfrac{1}{3}M\ell^2\dfrac{d^2\theta}{dt^2}$

偏移角度極小時，由泰勒展開式可得： $y = \ell\sin\theta \approx \ell\theta$

將y代回可得： $\tfrac{1}{2}Mg\ell - ky\ell - ky_0\ell = \tfrac{1}{3}M\ell^2\dfrac{d^2\theta}{dt^2} \rightarrow \tfrac{1}{2}Mg\ell - ky\ell - \tfrac{1}{2}Mg\ell = \tfrac{1}{3}M\ell^2\dfrac{d^2\theta}{dt^2}$

$$\rightarrow -k\ell^2\theta = \tfrac{1}{3}M\ell^2\frac{d^2\theta}{dt^2} \rightarrow \frac{d^2\theta}{dt^2} + \frac{3k}{M}\theta = 0$$



又 $\dfrac{d^2\theta}{dt^2} + \omega^2\theta = 0$ 故可得知 $\Rightarrow \omega^2 = \dfrac{3k}{M} = 4\pi^2 f^2 \rightarrow f = \dfrac{1}{2\pi}\sqrt{\dfrac{3k}{M}}$

HW11-3:

A particle is confined to move in *x*-direction between **x**=0 and **x**= ∞, and it experiences an conservative force ***F***(***x***) such that its potential energy $\boldsymbol{U}(\boldsymbol{x}) = -bx^2 \cdot e^{-ax}$, where ***a***,***b>0***,

(a) (6pts) Determine this conservative force ***F***(***x***) as a function of *x*,

(b) (4pts) At the equilibrium point **x = *S***, ***F***(***S***) = 0, determine the value of ***S***.

(c) (4pts) If the particle is moving around ***S*,** and if we define ***z* =*x*-*S***, write down the equation of motion of the particle in terms of ***z***,

(d) (3pts) For the case if ***z/S <<1***, the particle executes a simple harmonic oscillation around S, determine the period of the oscillation of the particle near S.

Useful formula: $(1+z)^n \approx 1+nz, \quad e^{az} \approx 1+az, \qquad for\ |z|<<1, |az|<<1$

(a) $U(x) = -bx^2 e^{-ax}$

$$F(x) = -\frac{dU(x)}{dx} = -\frac{d(-bx^2 e^{-ax})}{dx}$$

$$= 2bxe^{-ax} - abx^2 e^{-ax} = -(ax-2)bxe^{-ax}$$

(b) $for\ F(S) = 0, \Rightarrow 2bSe^{-aS} - abS^2 e^{-aS} = 0$

$$\Rightarrow S = \frac{2}{a}$$

(c) $\sum \vec{F} = m\vec{a}, \quad F(x) = -(ax-2)bxe^{-ax} = m\frac{d^2x}{dt^2}$

$$-(ax-2)bxe^{-ax} = m\frac{d^2x}{dt^2}$$

$$z \equiv x - S \Rightarrow x = z + S = z + 2/a$$

$$\Rightarrow -(a(z+2/a)-2)(z+2/a)\mathrm{b}e^{-a(z+2/a)} = m\frac{d^2(z+2/a)}{dt^2}$$

$$\Rightarrow -az(z+2/a)be^{-a(z+2/a)} = m\frac{d^2z}{dt^2}$$

$$\Rightarrow m\frac{d^2z}{dt^2} + az(z+S)be^{-az-2} = 0$$

(d) $for\ |z/S|<<1, |az|<<1$

$$az(z+S)be^{-a(z+S)} = azS(1+\frac{z}{S})be^{-az} \cdot e^{-aS}$$

$$\approx azS(1+\frac{z}{S})(1-az) \cdot be^{-aS}$$

$$= aS(z+(\frac{1}{S}-a)z^2 - \frac{a}{S}z^3) \cdot be^{-aS} \approx aSz \cdot be^{-aS} = 2be^{-2} \cdot z$$

$$\Rightarrow m\frac{d^2z}{dt^2} + 2be^{-2} \cdot z = 0$$

$$\Rightarrow \omega = \sqrt{\frac{2be^{-2}}{m}} = \sqrt{\frac{2b}{m}}e^{-1}$$

$$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi e\sqrt{\frac{m}{2b}}$$