General Physics (I) **期中考 I** **Nov. 3, 2017**

試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(50 points) (所有題目必須有計算過程,否則不予計分)

1. (10 pts) A small ball on an incline surface with angle $\boldsymbol{\theta}$ is thrown with velocity $\boldsymbol{v_o}$ at angle vertical to the incline surface, as shown in Fig.1. What is the distance $\boldsymbol{d}$ it can reach? And how long $\boldsymbol{t_f}$ has it been in the air?

2. (10 pts) A small block of mass $\boldsymbol{m}$ is placed insided an invertied core that is rotating about vertical axis such that the time for one revolution of the cone is $\boldsymbol{T}$ (Fig. 2). The static friction between the block and the inside surface of the cone is $\boldsymbol{\mu}$. Write your answers in terms of ***g, m, R,*** and ***h.***
   (a) (3 pts) Draw the free body diagram for the small block in this motion.
   (b) (7 pts) Find the periods of the block, $\boldsymbol{T = 2\pi R/v = 2\pi/\omega = 1/f}$, for (i) the friction force is zero, (ii)$\boldsymbol{T}$ is maxima and minima, to keep the block at distance $\boldsymbol{h}$ above the apex of the cone.

3. (15 pts). As shown in Fig.3, on the top of the inclined table which makes an angle $\boldsymbol{\phi}$ with horizontal, a block with mass $\boldsymbol{m_1}$ is placed on top of another block with mass $\boldsymbol{m_2}$, and they are connected with a massless rope through the pulleys. The coefficient of the kinetic friction between all surfaces is $\boldsymbol{\mu}$. A constant force $\boldsymbol{F}$ is pulling block $\boldsymbol{m_1}$ in the direction at an angle $\boldsymbol{\theta}$ relative to inclined plane.

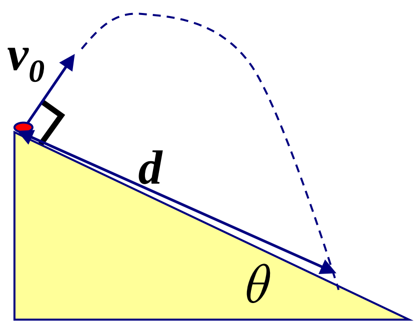


Fig. 1

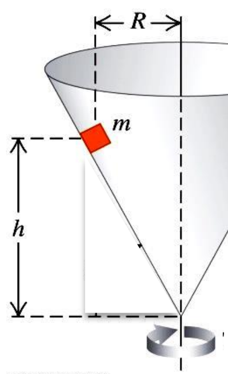


Fig. 2

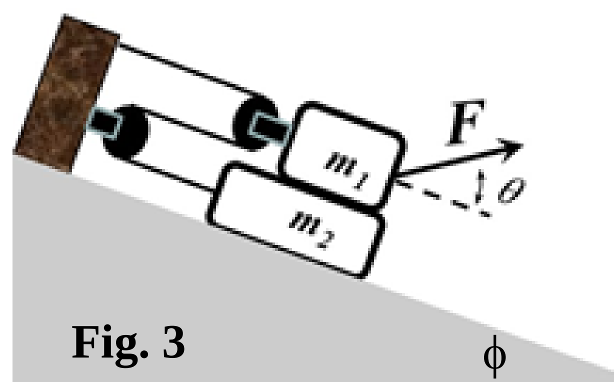


Fig. 3

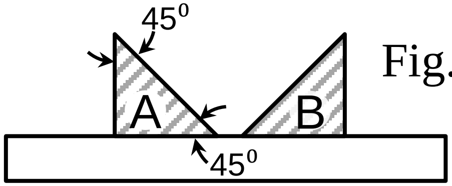


Fig. 4(a)

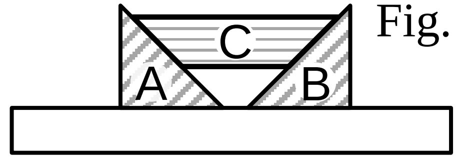


Fig. 4(b)

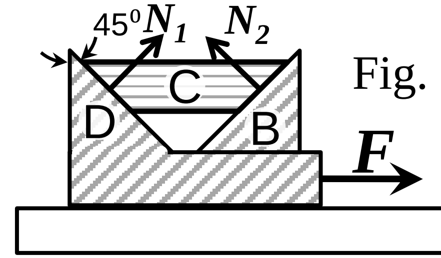


Fig. 4(c)

(a) (5 pts) List the all forces that act on block $\boldsymbol{m_1}$ and those act on block $\boldsymbol{m_2}$. Define a coordinate system for the system in Fig.3, and draw free body diagram of block $\boldsymbol{m_1}$ and block $\boldsymbol{m_2}$.

(b) (5 pts) Find the acceleration of block m_2.

(c) I (5 pts) f the angle $\boldsymbol{\theta}$ is adjustable, find the angle θ with which block $\boldsymbol{m_2}$ has maximum acceleration.

4. (A) (7pts) As shown in Fig. 4(a) two identical (相同的) blocks A and B rest on a table top. At $\boldsymbol{t}$ = 0 sec, a third block ***C*** is placed on top of these two blocks, as shown in Fig. 4(b). The masses of block ***A***, ***B*** and ***C*** are ***M***, ***M*** and ***2M***, respectively. If there is no friction between all contacting surfaces, determine the direction and the magnitude of the acceleration of block ***C***.

(B) (8pts) As shown in Fig. 4(c) block ***C*** and block ***B*** are now placed on top of block ***D***, and a constant force ***F*** drags the assembly toward the right to keep block ***B*** and block ***C*** resting on block ***D***. Let $\boldsymbol{N_1}$ be the magnitude of the normal force between block ***D*** and block ***C***, and $\boldsymbol{N_2}$ the normal force between block ***B*** and block ***C***. If again there is no friction between all surfaces in contact, determine the ratio $\boldsymbol{N_1/N_2}$.(Draw the force diagram for each object in the problem)

II.選擇題(50 points)

1. (4pts) The smallest meaningful measure of length is called the "Planck length," and is defined as $\lambda_p = G^l h^m c^n$, the speed of light $\boldsymbol{c}$ **= 3.00 x 10⁸ m/s**, the gravitation constants G **= 6.67 x 10⁻¹¹ m³/(kg·s²)**, the Planck's constant $\boldsymbol{h}$ **= 6.63 x 10⁻³⁴ kg·m²/s**. Then

(A) ***l = 1, m = 1/2, n= -2*** (B) ***l = 1/2, m = 1, n= -3/2*** (C) ***l = 1/2, m = -1/2 , n= -2***

(D) ***l = 1/2, m = 1/2 , n= -2*** (E) ***l = -1/2, m = 1, n= -3/2*** (F) ***l = 1/2, m = 1/2, n= -3/2***

2. (4pts) Consider a particle moving with the acceleration $\boldsymbol{a}$ (in unit of $\boldsymbol{m/s^2}$)vs. time $\boldsymbol{t}$ (in unit of $\boldsymbol{s}$) graph as shown in Fig. 5. Assume the particle is at rest and at $\boldsymbol{x} = \mathbf{1}$ at $\boldsymbol{t} = \mathbf{0}$ **sec.** What is the position $\boldsymbol{x}$ of the particle at $\boldsymbol{t} = \mathbf{2}$ **sec.**

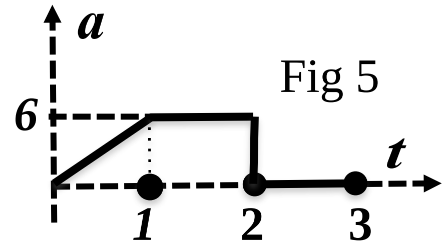


Fig 5

(A) $0 < \boldsymbol{x} \le 2$ (B) $2 < \boldsymbol{x} \le 4$ (C) $4 < \boldsymbol{x} \le 6$ (D) $6 < \boldsymbol{x} \le 8$ (E) $8 < \boldsymbol{x} \le 10$
(F) $10 < \boldsymbol{x} \le 12$ (G) $12 < \boldsymbol{x} \le 14$ (H) $14 < \boldsymbol{x} \le 16$ (J) $16 < \boldsymbol{x}$

3. (4pts) A boat must cross a **280-m**-wide river and arrive at a point **210 m** upstream from where it starts (Fig. 6). The speed of the river is **3 m/s**. To do so, the pilot must head the boat at a **45°** upstream angle. What is the speed of the boat $\boldsymbol{v}$ relative to the bank (in unit of **m/s**) ?

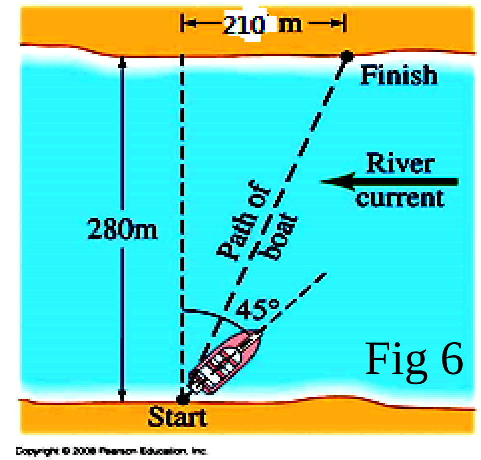


Fig 6

(A) $0 < \boldsymbol{v} \le 3$ (B) $3 < \boldsymbol{v} \le 6$ (C) $6 < \boldsymbol{v} \le 9$ (D) $9 < \boldsymbol{v} \le 12$
(E) $12 < \boldsymbol{v} \le 15$ (F) $15 < \boldsymbol{v} \le 18$ (G) $18 < \boldsymbol{v} \le 21$ (H) $21 < \boldsymbol{v}$

Hint: $\sin(\theta \pm \varphi) = \sin\theta\cos\varphi \mp \cos\theta\sin\varphi$

4. (4pts) As shown in Fig. 7, block A, B, and C are connected with a thin rope and pulleys. The mass of block A, B, and C, are $\boldsymbol{m}$, $3\boldsymbol{m}$, $5\boldsymbol{m}$, respectively. If there is no friction between all contacting surfaces, and the rope and the pulleys are massless, which of the following statement is correct regarding the $\boldsymbol{a_A}$ the acceleration of block $\boldsymbol{A}$, and $\boldsymbol{a_B}$ the acceleration of block **B**?

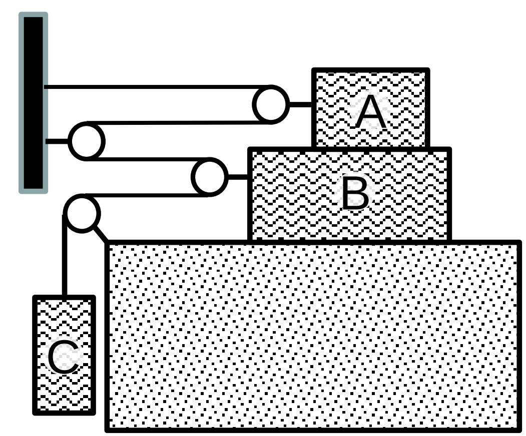


Fig 7

(A) $0 < |\boldsymbol{a_A}/\boldsymbol{a_B}| \le 1/8$ (B) $1/8 < |\boldsymbol{a_A}/\boldsymbol{a_B}| \le 1/4$ (C) $1/4 < |\boldsymbol{a_A}/\boldsymbol{a_B}| \le 1/2$
(D) $1/2 < |\boldsymbol{a_A}/\boldsymbol{a_B}| \le 1$ (E) $1 < |\boldsymbol{a_A}/\boldsymbol{a_B}| \le 2$ (F) $2 < |\boldsymbol{a_A}/\boldsymbol{a_B}| \le 4$
(G) $4 < |\boldsymbol{a_A}/\boldsymbol{a_B}| \le 8$ (H) $8 < |\boldsymbol{a_A}/\boldsymbol{a_B}|$

5. (4 pts) A racing car is constrained to move in a circular track of radius **100*m*** (in Fig. 8). At **t=0,** the car is at rest **(v=0)** and at position **x=100*m*** and **y=0** (the origin of the reference system is at the center of the circle)**.** The velocity of this car is increasing due to a time dependent tangential acceleration (切線加速度), which is given by $\boldsymbol{a_{tan}=1+t^2/3}$ **(m/s²)**. What is the magnitude of the acceleration $\boldsymbol{a}$ $(=(a^2_{tan}+a^2_{normal})^{1/2}$ at time $\boldsymbol{t}$ **= 3sec**?

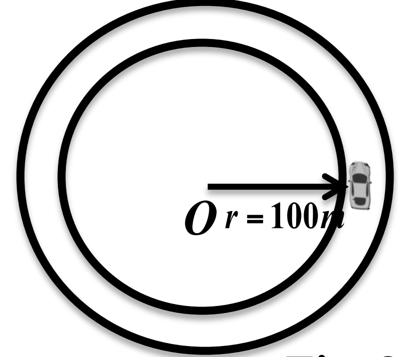
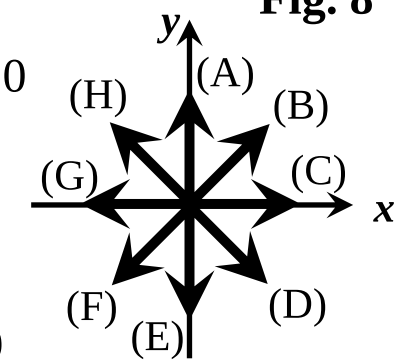


Fig. 8

(A) $0 < a \le 2$ (B) $2 < a \le 4$ (C) $4 < a \le 6$ (D) $6 < a \le 8$ (E) $8 < a \le 10$
(F) $10 < a \le 12$ (G) $12 < a \le 14$ (H) $14 < a \le 16$ (J) $16 < a$



6. (4 pts) Same as question 5, which answer in Fig 9, is the most close to describe the direction of the velocity of the car at this moment?

Fig. 9

7. (4 pts) A block of mass **1kg** slides along a horizontal (水平) surface lubricated with a thick oil which provides a drag force $F_D = -4\sqrt{v}$ (N). Let $\boldsymbol{v_0 = 4}$ ***m/s*** at **t=0.** How long (**t_f**) does it take for the block to stop?

(A) $0 < t_f \le 0.2$ (B) $0.2 < t_f \le 0.4$ (C) $0.4 < t_f \le 0.6$ (D) $0.6 < t_f \le 0.8$ (E) $0.8 < t_f \le 1.0$
(F) $1.0 < t_f \le 1.2$ (G) $1.2 < t_f \le 1.5$ (H) $1.5 < t_f \le 2.0$ (J) $2.0 < t_f$

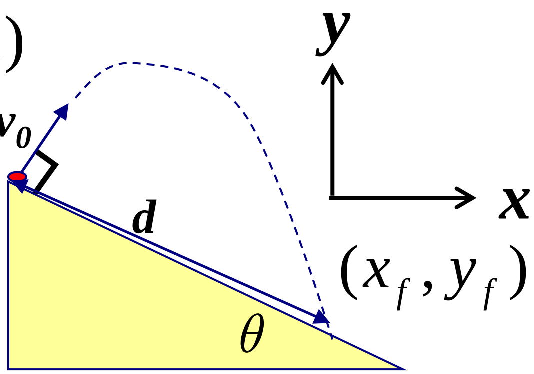
8. (4 pts) Continue with question 7, what is the distance (***S***) that the block travels (in meter, ***m***) before it stops?

(A) $0 < S \le 0.5$ (B) $0.5 < S \le 1.0$ (C) $1.0 < S \le 1.5$ (D) $1.5 < S \le 2.0$ (E) $2.0 < S \le 2.5$
(F) $2.5 < S \le 3.0$ (G) $3.0 < S \le 3.5$ (H) $3.5 < S \le 4.0$ (J) $4.0 < S$

Multiple Choice Questions:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
F	**D**	**E**	**F**	**C**	**H**	**E**	**C**	
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**
B	**C**	**D**	**G**	**C**	**C**	**F**	**B**	**A**

(1)



$$x_f = d\cos\theta$$

$$y_f = -d\sin\theta$$

$$x(t_f) = v_0 \sin\theta \cdot t_f = d\cos\theta \quad \text{--- (1)}$$

1 1 1

$$y(t_f) = v_0\cos\theta \cdot t_f - \frac{1}{2} g \cdot t_f^{\,2} = -d\sin\theta \quad \text{--- (2)}$$

2

There are two equations and two variables, t_f and d. **Solvable**.

Equ. (1) ➔ $t_f = \dfrac{l\cos\theta}{v_0\sin\theta}$ --- (3) 1

Equ. (3) 帶入 equ. (2)➔

$$v_0\cos\theta \cdot \frac{l\cos\theta}{v_0\sin\theta} - \frac{1}{2} g \cdot \left(\frac{l\cos\theta}{v_0\sin\theta}\right)^2 = -l\sin\theta$$

$$\frac{1}{2} g \cdot l \left(\frac{\cos\theta}{v_0\sin\theta}\right)^2 = \frac{\cos^2\theta}{\sin\theta} + \sin\theta = \frac{1}{\sin\theta}$$

$$\boxed{l = \frac{v_0^2}{2g} \cdot \frac{\sin\theta}{\cos^2\theta}}$$

3

$$\boxed{t_f = \frac{v_0}{2g} \cdot \frac{1}{\cos\theta}}$$

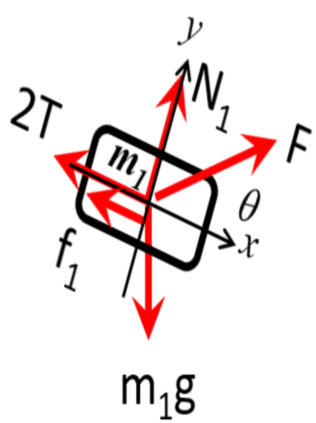
1

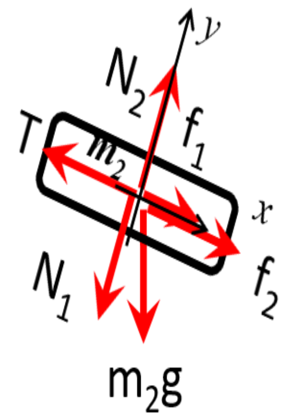
(a)

The forces which act on m_1 include
(1) Gravitational force $m_1 g$,
(2) Normal force N_1 from m_2,
(3) External Force F,
(4) Tension of the rope 2T,
(5) Friction force f_1 from surface between m_1 and m_2.

The forces which act on m_2 include
(1) Gravitational force $m_2 g$,
(2) Normal force N_1 from m_1,
(3) Normal force N_2 from table top,
(4) Tension of the rope T,
(5) Friction force f_1 from surface between m_1 and m_2,
(6) Friction force f_2 from surface between m_2 and table.

2pts,





2pts,

(b)

2pts,

x: $F\cos\theta + m_1 g\sin\emptyset - 2T - f_1 = m_1 a_1$

y: $F\sin\theta + N_1 - m_1 g\cos\emptyset = 0$

$f_1 = \mu N_1$

2pts,

x: $m_2 g\sin\emptyset - T + f_2 + f_1 = m_2 a_2$

y: $N_2 - N_1 - m_2 g\cos\emptyset = 0$

$f_2 = \mu N_2$

1pt,

$\boxed{constraint\ equation: a_2 = -2a_1}$

$$N_1 = m_1 g cos\emptyset - \mathrm{Fsin}\theta$$

$$N_2 = N_1 + m_2 g cos\emptyset = m_1 g cos\emptyset - \mathrm{Fsin}\theta + m_2 g cos\emptyset$$

$$a_2 = \frac{2[F(-cos\theta - 5\mu sin\theta) + g sin\emptyset(2m_2 - m_1) + \mu g cos\emptyset(5m_1 + 2m_2)]}{m_1 + 4m_2}$$

2pts,

(c) $\frac{da_2}{d\theta} = \frac{2F(sin\theta - 5\mu cos\theta)}{m_1 + 4m_2} = 0$ 2pts,

$tan\theta = 5\mu \quad \theta = tan^{-1} 5\,\mu$ 2pts,

1. (A) (7pts) As shown in Fig. *X*(a) two identical (相同的) blocks A and B rest on a table top. At $\boldsymbol{t}$ = 0 sec, a third block ***C*** is placed on top of these two blocks, as shown in Fig. X(b). The mass of block ***A***, ***B*** and ***C*** are ***M***, ***M*** and ***2M***, respectively. If there is no friction between all contacting surfaces, determine the direction and the magnitude of the acceleration of block ***C***. (B) (8pts) As shown in Fig. X(c) block ***C*** and block ***B*** are now placed on top of block ***D***, and a constant force ***F*** drags the assembly toward the right to keep block ***B*** and block ***C*** resting on block ***D***. Let $\boldsymbol{N_1}$ be the magnitude of the normal force between block ***D*** and block ***C***, and $\boldsymbol{N_2}$ the normal force between block ***B*** and block ***C***. If again there is now friction between all surfaces in contact, determine the ratio $\boldsymbol{N_1/N_2}$.(Draw the force diagram for each object in the problem)

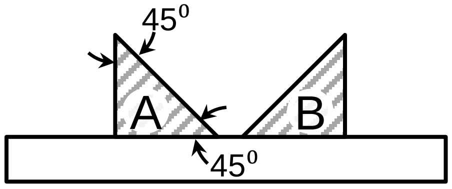


Fig. *X*(a)

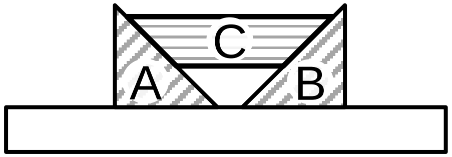


Fig. *X*(b)

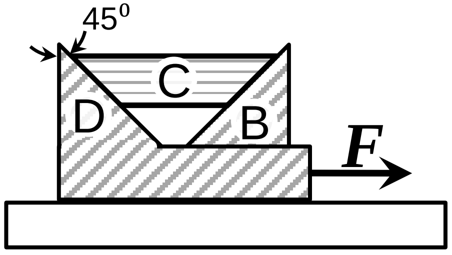
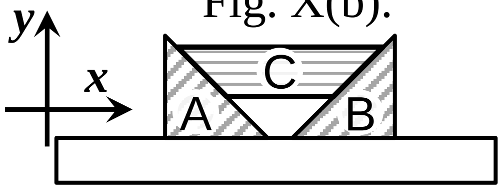


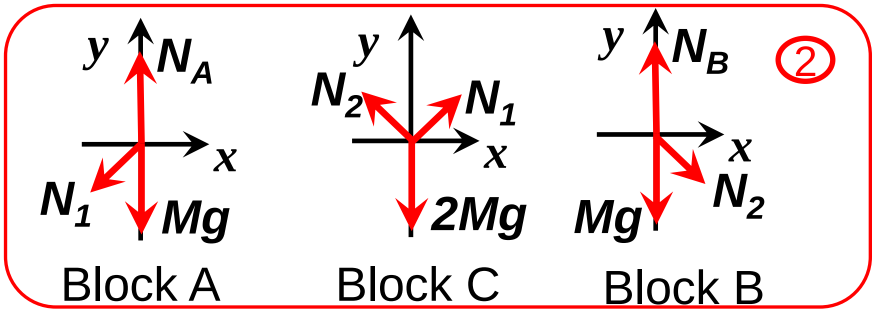
Fig. *X*(c)

1. (A) (7pts) As shown in Fig. *X*(a) two identical (相同的) blocks A and B rest on a table top. At $\boldsymbol{t} = 0$ sec, a third block ***C*** is placed on top of these two blocks, as shown in Fig. X(b). The mass of block ***A***, ***B*** and ***C*** are ***M***, ***M*** and ***2M***, respectively. If there is no friction between all contacting surfaces, determine the direction and the magnitude of the acceleration of block ***C***. (B) (8pts) As shown in Fig. X(c) block ***C*** and block ***B*** are now placed on top of block ***D***, and a constant force ***F*** drags the assembly toward the right to keep block ***B*** and block ***C*** resting on block ***D***. Let $\boldsymbol{N_1}$ be the magnitude of the normal force between block ***D*** and block ***C***, and $\boldsymbol{N_2}$ the normal force between block ***B*** and block ***C***. If again there is now friction between all surfaces in contact, determine the ratio $\boldsymbol{N_1/N_2}$.(Draw the force diagram for each object in the problem)

(A)

Fig. X(b).





Block A, $\sum \vec{F} = m\vec{a}_A$

$x: -N_1/\sqrt{2} = Ma_A$ (1)

$y: N_A - N_1/\sqrt{2} - Mg = 0$ (2)

Block C, $\sum \vec{F} = m\vec{a}_A$

$x: N_1/\sqrt{2} - N_2/\sqrt{2} = 2Ma_{C,x}$ (5)

$y: N_1/\sqrt{2} + N_2/\sqrt{2} - 2Mg = 2Ma_{C,y}$ (6)

Block B, $\sum \vec{F} = m\vec{a}_B$

$x: N_2/\sqrt{2} = Ma_B$ (3)

$y: N_B - N_2/\sqrt{2} - Mg = 0$ (4)

2

Condition of motion:

$a_A = -a_B = a_{C,y} = -a$ (7)

$a_{C,x} = 0$ (8)

1

(5),(8)→ $N_1/\sqrt{2} - N_2/\sqrt{2} = 0 \Rightarrow N_1 = N_2 \equiv N$ (9)

1

(1),(8),(9)→ $N/\sqrt{2} = Ma$ (10)

(6),(9),(10)→$N/\sqrt{2} + N/\sqrt{2} - 2Mg = -2Ma$

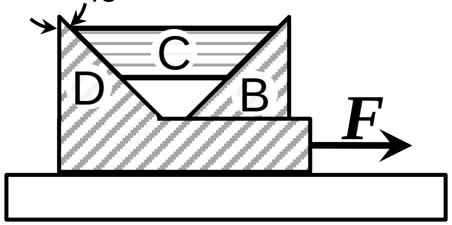
$\Rightarrow \sqrt{2}N - 2Mg = -2Ma$ (11)

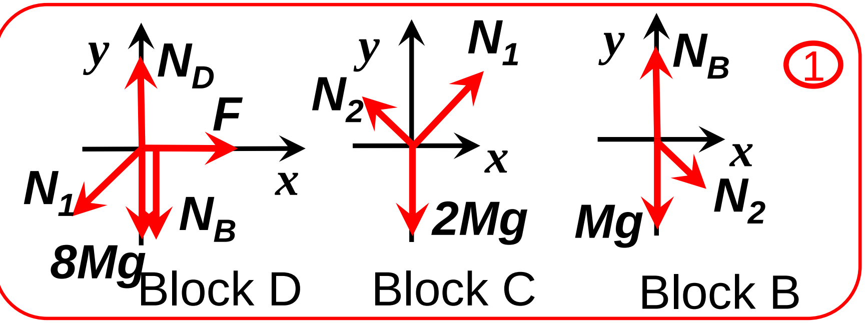
(10)x2-(11)→$2Mg = 4Ma$

$\Rightarrow a = \dfrac{g}{2}$ 1

The acceleration of block C is g/4, moving downward.

Fig. X(c)





Block D, $\sum \vec{F} = m\vec{a}_D$ 2

$x: F - N_1/\sqrt{2} = 8Ma_D$ (1)

$y: N_D - N_1/\sqrt{2} - N_B - 8Mg = 0$ (2)

Block C, $\sum \vec{F} = m\vec{a}_A$ 1

$x: N_1/\sqrt{2} - N_2/\sqrt{2} = 2Ma_{C,x}$ (5)

$y: N_1/\sqrt{2} + N_2/\sqrt{2} - 2Mg = 2Ma_{C,y}$ (6)

Block B, $\sum \vec{F} = m\vec{a}_B$ 1

$x: N_2/\sqrt{2} = Ma_B$ (3)

$y: N_B - N_2/\sqrt{2} - Mg = 0$ (4)

Conditions of motion: 1

$a_D = a_B = a_{C,x} = a$ (7)

$a_{C,y} = 0$ (8)

(7),(3)→ $N_2 = \sqrt{2}Ma$ (9)

(7),(5)→ $N_1 - N_2 = 2\sqrt{2}Ma$ (10)

(9),(10)→ $N_1 = 3\sqrt{2}Ma$

$\Rightarrow N_1/N_2 = 3$ 2

2. (10 pts) A small block of mass ***m*** is in a horizontal circle of radius ***R*** on the inside of a cone with a vertical axis as shown in Fig. XX. The plane of the circular motion is at a distance ***h*** above the apex of the cone. The static friction between the block and the inside surface of the cone is μ. Write your answers in terms of ***g, m, R,*** and ***h.***

(a) (3 pts) Draw the free body diagram for the small block in this motion.

(b) (7 pts) Find the periods of the block, $\boldsymbol{T = 2\pi R/v = 2\pi/\omega = 1/f}$, for (i) the friction force is zero, (ii)***T*** is maxima and minima, to keep the block at distance ***h*** above the apex of the cone.

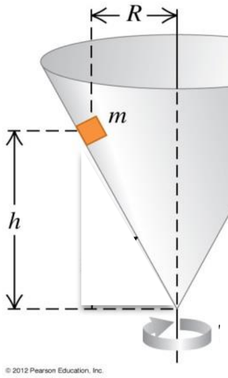
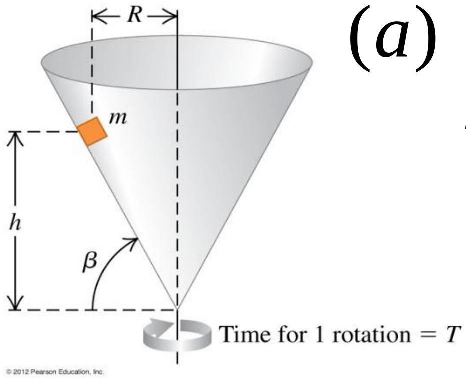
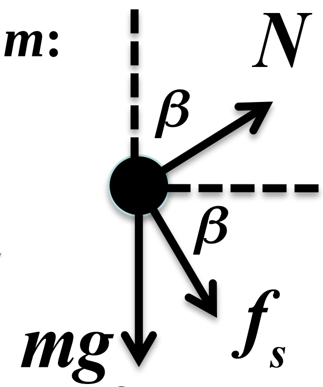


Fig. XX



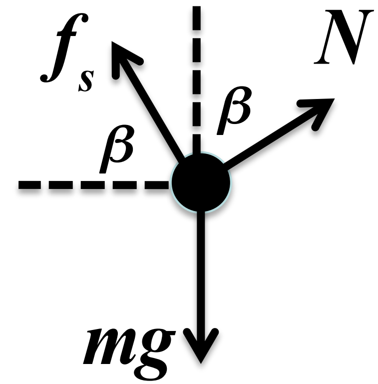
(a)

m:



3 pts

or



(b)

$$\hat{x}: \quad N\sin\beta + f_s\cos\beta = ma_c = m\frac{v^2}{R}$$

$$\hat{y}: \quad N\cos\beta - f_s\sin\beta - mg = 0$$

$$f_s \le f_{s,\max} = \mu N$$

$$\tan\beta = \frac{h}{R}$$

2 pts

$$\hat{x}: \quad N\sin\beta - f_s\cos\beta = ma_c = m\frac{v^2}{R}$$

$$\hat{y}: \quad N\cos\beta + f_s\sin\beta - mg = 0$$

$$f_s \le f_{s,\max} = \mu N, \quad \tan\beta = \frac{h}{R}$$

(i), $f_s = 0$

$$N\sin\beta = ma_c = m\frac{v^2}{R}$$

$$N\cos\beta - mg = 0$$

$$\rightarrow v^2 = gR\tan\beta = gh$$

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{gh}}, \quad \omega = \frac{\sqrt{gh}}{R}$$

2 pts

(ii), $f_s = f_{s,\max} = \mu N$

$$N\sin\beta + f_s\cos\beta = m\frac{v^2}{R}$$

$$N\cos\beta - f_s\sin\beta - mg = 0$$

$$f_s = f_{s,\max} = \mu N$$

2 pts

$$\rightarrow v^2 = gR\frac{\sin\beta + \mu\cos\beta}{\cos\beta - \mu\sin\beta} = gR\frac{\mu + h/R}{1 - \mu h/R}$$

$$T_{\min} = \frac{2\pi R}{v} = 2\pi\sqrt{\frac{R}{g}\frac{1 - \mu h/R}{h/R + \mu}}$$

For $T_{\max}$, $\mu \rightarrow -\mu$

$$T_{\min} = \frac{2\pi R}{v} = 2\pi\sqrt{\frac{R}{g}\frac{1 + \mu h/R}{h/R - \mu}}$$

1 pts