

Please show all work. Answers without justification may not receive full credits.

Do all problems. Each problem counts 18 points. Points total 144.

1. Determine the following limits.

(a) $\lim_{(x,y) \rightarrow (0,0)} (x^2 + y^2) \ln \sqrt{x^2 + y^2}$

(b) $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xyz}{x^6 + y^6 + z^2}$

2. Let $f(x, y) = (x^4 + y^4)^{1/4}$.

(a) Find $f_y(x, y)$.

(b) At what points, if any, does $f_y(x, y)$ fails to be continuous?

3. Compute the following partial derivatives.

(a) w_{rst} if $w = \ln(e^r + e^s + e^t)$;

(b) $\partial z / \partial x$, where $xe^y + 3 \sin 5z = 0$.

4. Find the point on the circle $x^2 + y^2 + z^2 = 4$ that is closest to the plane $2x + y - 2z = 9$.

5. Evaluate $\int_0^8 \int_{y^{1/3}}^2 e^{x^4} dx dy$.

6. Find the volume of the solid in the first octant bounded by the cylinders $x^2 + z^2 = 4$ and $x^2 + y^2 = 4$.

7. Let σ be the portion of the sphere $x^2 + y^2 + z^2 = 9$ on or above the plane $z = 1$. Give a parametric representation of σ and find its area.

8. Find the volume of the solid bounded above by the sphere $\rho = 5$ and below by the cone $\phi = \pi/3$.
