General Physics II: HW 6

Due on 4/23/2020

<u>HW6-1:</u> Problem 25-92 and 25-93 in Giancoli (pp. 782) (pp. 676)

<u>HW6-2:</u> Problem 26-36 in Giancoli (pp. 810) (pp. 702)

<u>HW6-3:</u> Problem 26-51 and 26-52 in Giancoli (pp. 812) (pp. 703)

HW6-1:

A. (prob. 25-92) For the wire in Fig. 25–39, whose diameter varies uniformly from ***a*** to ***b*** as shown, suppose a current ***I = 2.0A*** enters at ***a***. If ***a = 2.5mm*** and ***b = 4.0mm***, what is the current density (assume uniform) at each end?

B. (prob. 25-93) The cross section of a portion of wire increases uniformly as shown in Fig. 25–39, so it has the shape of a truncated cone. The diameter at one end is ***a*** and at the other it is ***b***, and the total length along the axis is ***l***. If the material has resistivity ***ρ***, determine the resistance ***R*** between the two ends in terms of ***a***, ***b***, ***l***, and ***ρ***. Assume that the current flows uniformly through each section, and that the taper is small, i.e., (***b-a***)<< ***l***.

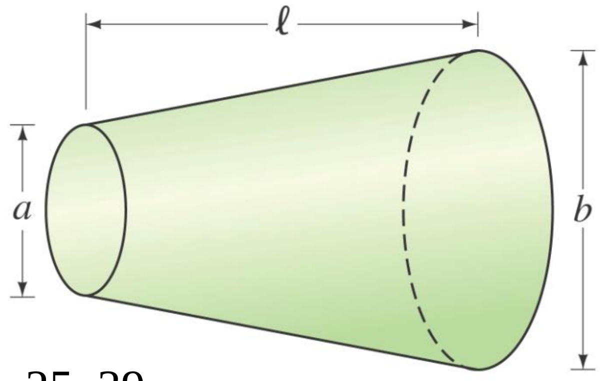
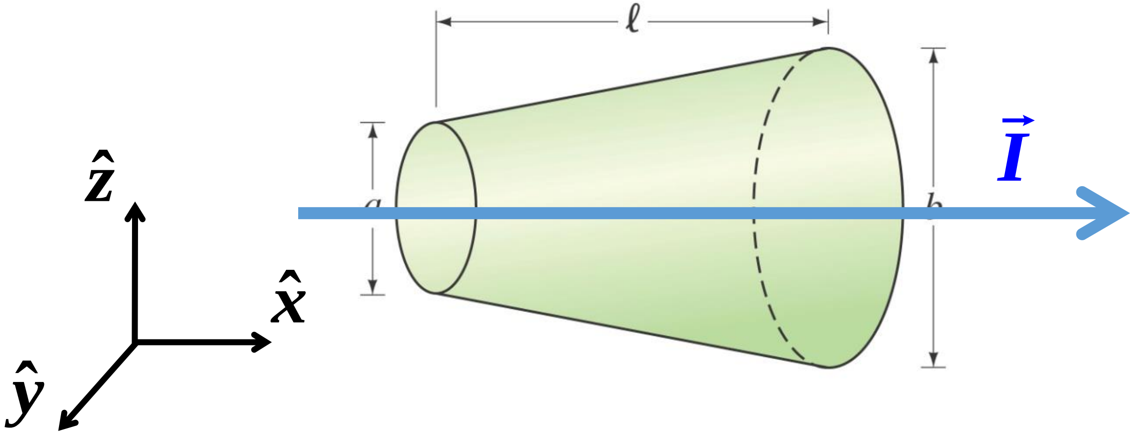
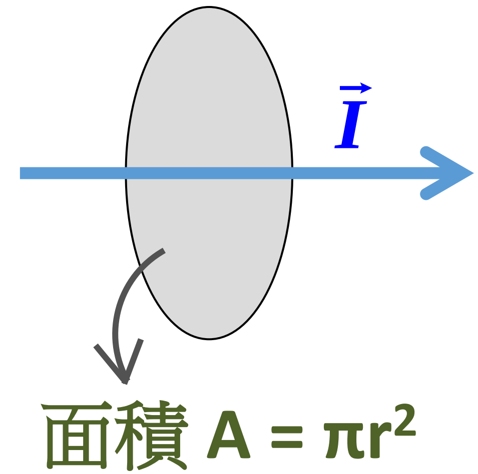


Fig. 25–39

(prob. 25-92)

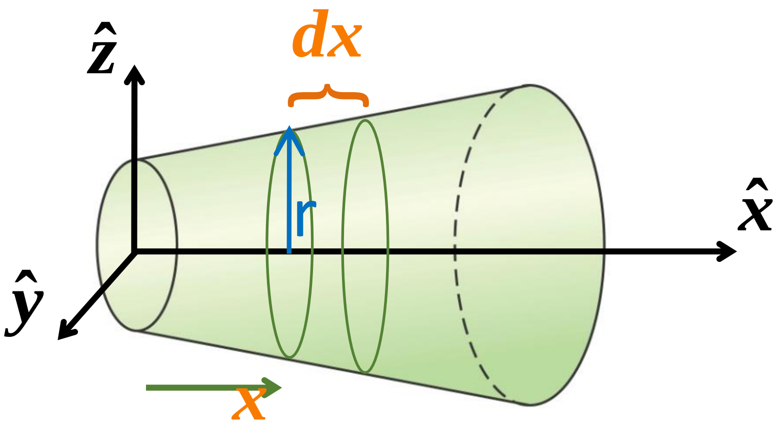




$$\vec{J}_a = \frac{\vec{I}}{A_a} = \frac{\vec{I}}{\pi\left(\frac{a}{2}\right)^2} = \frac{2.0A}{\pi\left(\frac{2.5\times10^{-3}}{2}m\right)^2}\hat{x} = 4.07\times10^5\,\hat{x}\ \ A/m^2$$

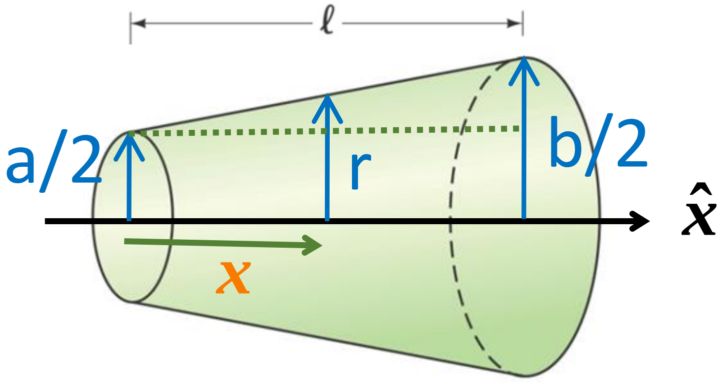
$$\vec{J}_b = \frac{\vec{I}}{A_b} = \frac{\vec{I}}{\pi\left(\frac{b}{2}\right)^2} = \frac{2.0A}{\pi\left(\frac{4.0\times10^{-3}}{2}m\right)^2}\hat{x} = 1.59\times10^5\,\hat{x}\ \ A/m^2$$

(prob. 25-93)



$$R = \rho \frac{L}{A}$$

$$\Rightarrow \quad dR = \rho \frac{dL}{A} = \rho \frac{dx}{\pi r^2}$$



$$r = \frac{a}{2} + \frac{x}{l}\left[\frac{(b-a)}{2}\right]$$

$$= \frac{al + bx - ax}{2l}$$

$$dR = \rho \frac{dx}{\pi r^2} = \rho \frac{dx}{\pi \left(\frac{al + bx - ax}{2l} \right)^2}$$

$$= \frac{4\rho l^2}{\pi} \left[al + (b-a)x \right]^{-2} dx$$

$$R = \int dR = \int_0^l \frac{4\rho l^2}{\pi} \left[al + (b-a)x \right]^{-2} dx$$

$$= \frac{4\rho l^2}{\pi} \left[-\frac{1}{b-a} \frac{1}{al + (b-a)x} \right] \Bigg|_0^l$$

$$= \frac{4\rho l^2}{\pi} \frac{-1}{b-a} \left[\frac{1}{bl} - \frac{1}{al} \right] = \frac{4\rho l}{\pi} \frac{1}{a-b} \frac{a-b}{ab} = \frac{4\rho l}{\pi ab}$$

Solution HW6-2:

(a)Determine the currents I_1, I_2 and I_3 in Fig.26-53. Assume the internal resistance of each battery is $r = 1.0\,\Omega$.

(b)What is the terminal voltage of the ***6.0-V*** battery?

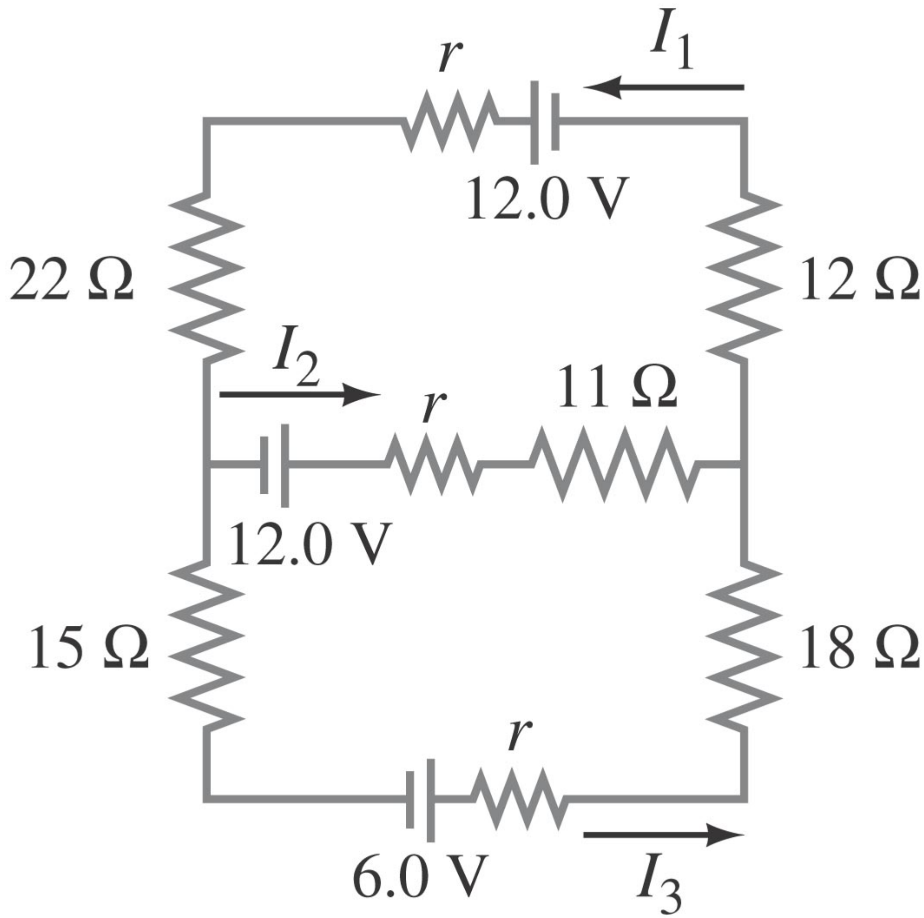
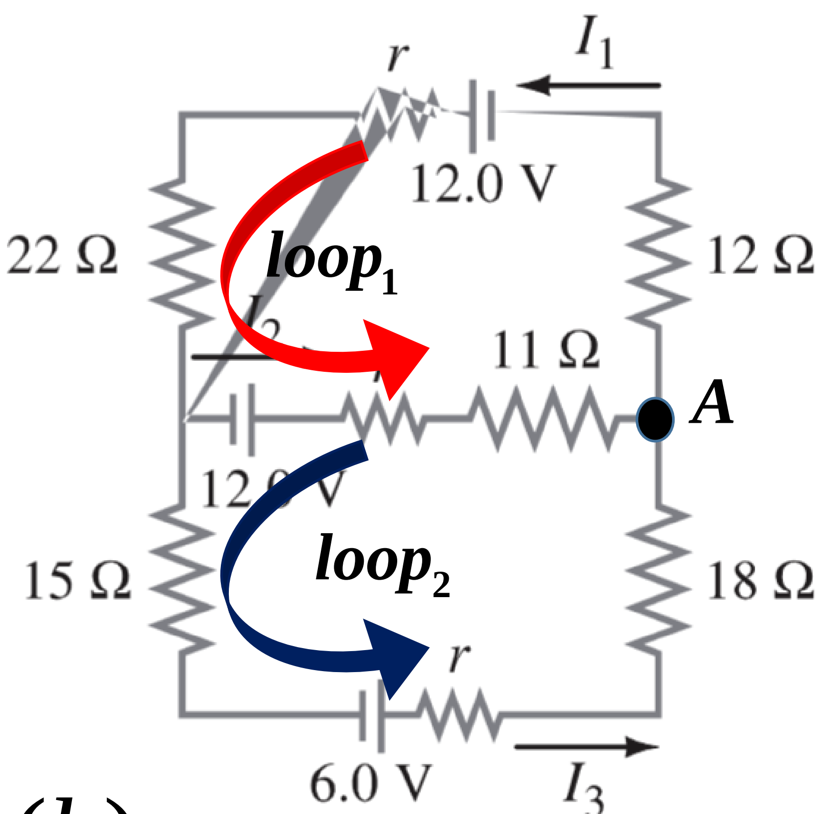


Fig.26-53

(a) We get three equations in Fig.26-53 by Kirchhoff Circuit Laws :

$$\begin{cases} \boldsymbol{A} \\ \boldsymbol{loop_1} \\ \boldsymbol{loop_2} \end{cases} \Rightarrow \begin{cases} I_1 = I_2 + I_3 \\ 24 - I_1(1+22+12) - I_2(1+11) = 0 \\ -6 - I_3(15+1+18) + I_2(11+1) = 0 \end{cases}$$



$$\Rightarrow \begin{cases} 35I_3 + 47I_2 = 24 \\ 34I_3 - 12I_2 = -6 \end{cases} \Rightarrow 2018I_2 = 1026$$

$$\Rightarrow \begin{cases} I_3 = \dfrac{3}{1009}(\text{A}) \\ I_2 = \dfrac{513}{1009}(\text{A}) \\ I_1 = \dfrac{516}{1009}(\text{A}) \end{cases}$$

(b) $V = 6 - 1I_3 \approx 5.997\,\text{V}$

Solution HW6-3:

A. (prob. 26-51) Two resistors and two uncharged capacitors are arranged as shown in Fig. 26–62. Then a potential difference of ***24V*** is applied across the combination as shown.

(a) What is the potential at point ***a*** with switch ***S*** open? (Let ***V=0*** at the negative terminal of the source.)

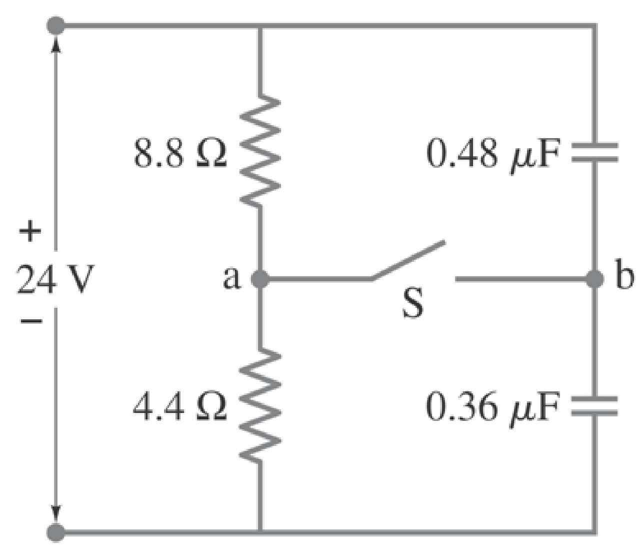
(b) What is the potential at point ***b*** with the switch open?

(c) When the switch is closed, what is the final potential of point ***b***?

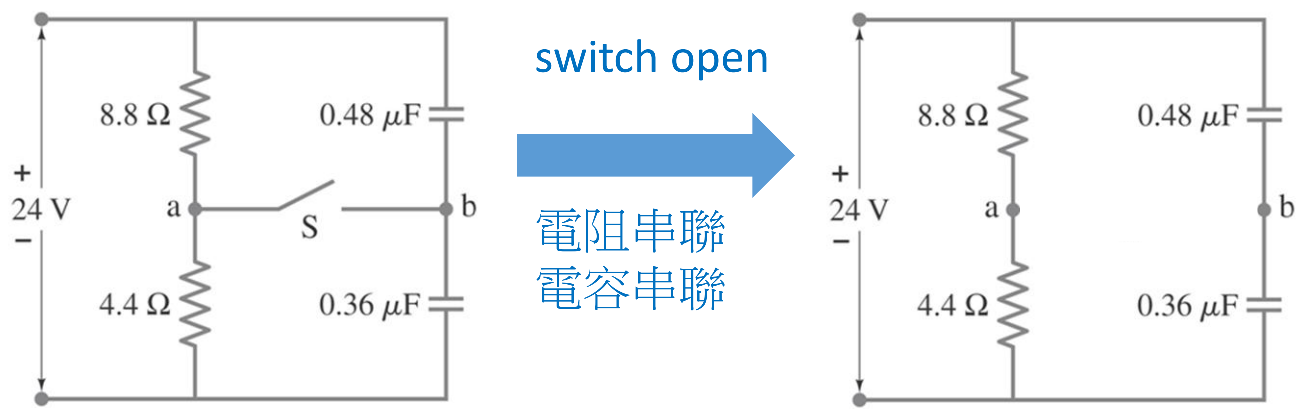
(d) How much charge flows through the switch ***S*** after it is closed ?

B. (prob. 26-52) Suppose the switch ***S*** in Fig. 26–62 is closed. What is the time constant (or time constants) for charging the capacitors after the ***24V*** is applied?

Fig. 26–62



(prob. 26-51)



(a) $V_a = 24V - IR_1 = IR_2 = 24V \times \frac{R_2}{R_1 + R_2} = 8V$

(b) 串聯：$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C = \frac{C_1 C_2}{C_1 + C_2} = 0.2057\mu F$

$Q = VC = (24.0V)(0.2057\mu F) = 4.937\mu C = Q_1 = Q_2$

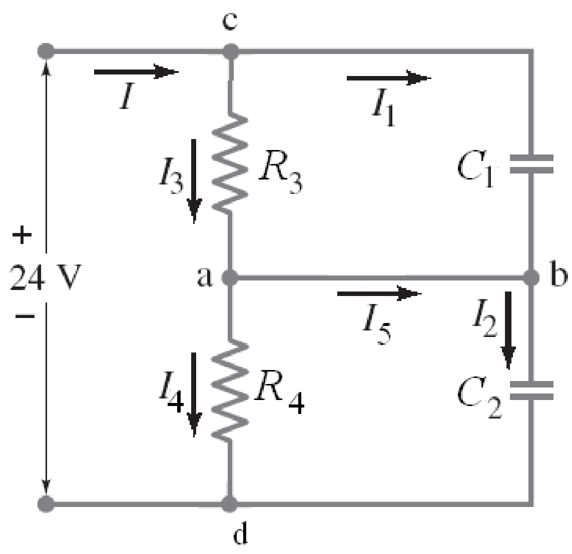
$V_b = 24V - \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{4.937\mu C}{0.2057\mu F} = 13.7V$

(c) 經過長時間，電容已充飽，電子便過不去 → 斷路
只剩剩下電阻那邊可走，回到(a)， $V_b = V_a = 8V$

(d) $Q = VC \Rightarrow \begin{cases} Q_1 = (16V)(0.48\mu F) = 7.68\mu C \\ Q_2 = (8V)(0.36\mu F) = 2.88\mu C \end{cases}$ 電荷由b至a通過開關S

$Q = -7.68\mu C + 2.88\mu C = -4.8\mu C$

(prob. 26-52)



$$I = I_1 + I_3 \quad \cdots\cdots\cdots\cdots\cdots (1)$$

$$I = I_2 + I_4 \quad \cdots\cdots\cdots\cdots\cdots (2)$$

$$\varepsilon - \frac{Q_1}{C_1} - \frac{Q_2}{C_2} = 0 \quad \cdots\cdots\cdots\cdots (3)$$

$$\frac{Q_1}{C_1} - I_3 R_3 = 0 \quad \cdots\cdots\cdots\cdots (4)$$

$$\frac{Q_2}{C_2} - I_4 R_4 = 0 \quad \cdots\cdots\cdots\cdots (5)$$

微分(3)： $\frac{d}{dt}\varepsilon - \frac{d}{dt}\frac{Q_1}{C_1} - \frac{d}{dt}\frac{Q_2}{C_2} = 0$ ， $I = \frac{dQ}{dt}$

$$\Rightarrow \quad 0 - \frac{I_1}{C_1} - \frac{I_2}{C_2} = 0 \quad \Rightarrow \quad I_2 = -I_1 \frac{C_2}{C_1}$$

代換(4)(5)： $I_3 = \frac{Q_1}{R_1 C_1}$ ， $I_4 = \frac{Q_2}{R_4 C_2}$

$$I_1 + I_3 = I_2 + I_4$$

$$I_1 + \frac{Q_1}{R_1 C_1} = -I_1 \frac{C_2}{C_1} + \frac{Q_2}{R_4 C_2} = -I_1 \frac{C_2}{C_1} + \frac{1}{R_4}\left(\varepsilon - \frac{Q_1}{C_1}\right)$$

$$\Rightarrow \quad \varepsilon = I_1 R + \frac{Q_1}{C} = I_1 R_4 \left(\frac{C_1 + C_2}{C_1}\right) + Q_1 \left(\frac{R_4 + R_3}{R_3 C_1}\right)$$

$$\boxed{\begin{aligned} \tau &= RC \\ &= R_4 \left(\frac{C_1 + C_2}{C_1}\right)\left(\frac{R_3 C_1}{R_4 + R_3}\right) \\ &= 2.5\mu s \end{aligned}}$$