General Physics (I) 期末考 Jan. 9, 2015

試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(55 points) (所有題目必須有計算過程,否則不予計分)

1. (10 pts) An unknown amount of -20 ^{o}C ice is added to a cup that contains 2.0 kg water at 20 ^{o}C. After reaching equilibrium, the cup contains 0 ^{o}C water. (Assume no heat lost to the cup.)

(a) (3pts) What is amount of ice is adding to the cup?(Use 2 significant digits for your answer(兩位有效數字))

(b) (2pts) What is the entropy change of the water? (3 significant digits accuracy.)

(c) (4pts) What is the entropy change of the ice? (3 significant digits accuracy.)

(d) (1pt) What is the entropy change of the universe (system + environment)? (3 significant digits accuracy.)

(C_{water}= 4200 J/kg, C_{ice}= 2100 J/kg, the latent heat of fusion of the water L_f = 3.330 x 10^5 J/kg)

Suggest: If you use integration method, please use the following technique to do the numerical calculation:

$$\ln(1+x) = x + O(x^2)$$

2. (10 pts) Consider a one-dimensional potential $U(r) = \frac{B}{r^9} - \frac{A}{r}$, where A and B are positive constants. There is a static equilibrium point at $r = r_0$ for this potential.

a) (2 pts) Find the equilibrium point r_0 in terms of A and B.

b) (3 pts) A particle with mass m and velocity $v = \frac{dr}{dt} = \dot{r}$ moves in this potential. Write down the total energy E_{tot} (= Kinetic energy + potential energy). Find the equation of motion for this system.

c) (3 pts) Near the equilibrium point r_0, the system can be approximated as a simple harmonic oscillator (SHO). Let $r = r_0 + x$, rewrite the equation of motion in part b) as function of x by using the formula $(r_0 + x)^{-n} \approx r_0^{-n}(1 - nx/r_0 + ...)$, if $x << r_0$.

d) (2 pts) Find the period of this particle in terms of r_0, A and/or B.

3. (10 pts) A wrench (扳鉗 or 扳手) of mass m is pivoted a distance L from its center of mass and allowed to swing as a physical pendulum as shown in Fig. 1. Express your answers below in terms of L, m, g and/or other necessary constants.

(2 pts) If the period for this small angle oscillation is T , what is the moment of inertia of the wrench about the axis through the pivot? (in terms of L, m, T, g)

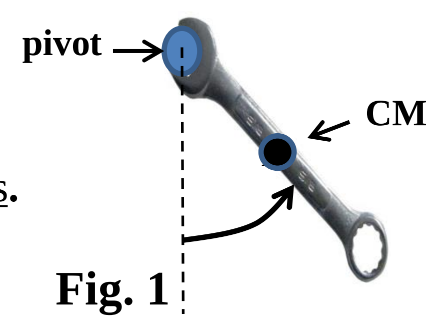


Fig. 1

b)(3 pts) If the wrench is initially displaced by a small angle θ_0 from its equilibrium position. what is the time dependence of the angular displacement $\theta(t)$? What is the angular speed of the wrench as it passes through the equilibrium?

c)(5 pts) Same as part (b) but now the angle of the initial displacement θ_0 is **NOT** small, what is the angular speed of wrench as it passes through the equilibrium position? Is it bigger or smaller then the result you expect for a small angle physical pendulum (as in part (b))?

4. (20 pts) As shown in Fig. 2, a ideal gas thermal engine contains one mole of mono-atomic gas, and it executes the cycle consisted of the following processes; a → b : isothermal, b → c : adiabatic, c → d : isothermal, d → a : isochoric.

(1) (8 pts) Determine the change of internal energy ΔE_{int}, heat exchanges $\boldsymbol{Q}$, and work done by the system $\boldsymbol{W}$, of each process in terms of P_0V_0. Express your answer according to the table in Fig. 2.

(2) (5 pts) Determine the efficiency of this thermal engine.

(3) (2 pts) If you are allowed to modify the processes used by this engine without changing the highest and lowest temperatures used in the existing cycle, what would be the highest efficiency achievable ?

(Note, $\ln 2 \sim 0.7$, $\ln 3 \sim 1.1$, and $\ln 5 \sim 1.6$)

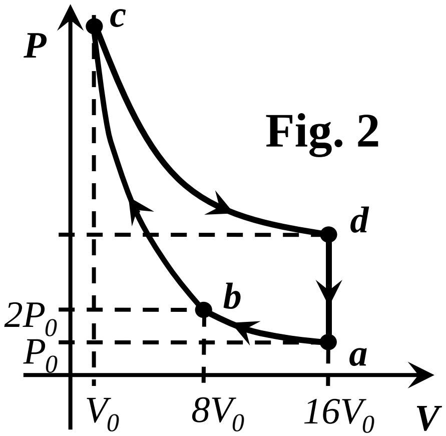


Fig. 2

	ΔE_{int}	Q	W
a → b			
b → c			
c → d			
d → c			

II.選擇題(52 points)

1. (4 pts)) The graph of velocity vs. time for a small block with mass m at the end of a spring is shown in Fig. 3. The equilibrium position of the block is at x = 0.The positive direction of the velocity is along +**x**. Which of the following statements are true about (i) the position (**x**), (ii) the direction of the velocity ($\pm\hat{x}$) and (iii) the direction of the force of spring ($\pm\hat{x}$) for the block at point ***P*** in Fig 3?

(A) (i) $x > 0$, (ii)$+\hat{x}$, and (iii)$+\hat{x}$

(B) (i) $x > 0$, (ii)$+\hat{x}$, and (iii) $-\hat{x}$

(C) (i) $x > 0$, (ii) $-\hat{x}$, and (iii)$+\hat{x}$

(D) (i) $x > 0$, (ii)$-\hat{x}$, and (iii) $-\hat{x}$

(E) (i) $x < 0$, (ii)$+\hat{x}$, and (iii)$+\hat{x}$

(F) (i) $x < 0$, (ii)$+\hat{x}$, and (iii) $-\hat{x}$

(G) (i) $x < 0$, (ii)$-\hat{x}$, and (iii)$+\hat{x}$

(H) (i) $x < 0$, (ii)$-\hat{x}$, and (iii)$-\hat{x}$

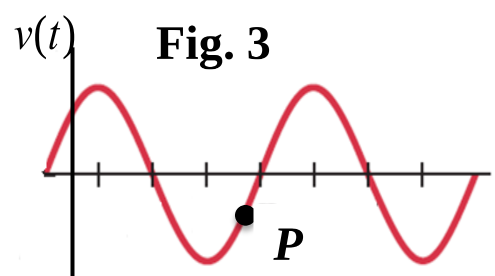


Fig. 3

2. (4 pts) A gyroscope has a wheel at one end of an axle, which is pivoted at point *O* as shown in Fig. 4. The wheel rotates about the axle (shown in the figure as dashed lines). At the moment shown in the figure, the axle is horizontal alone ***x***-axis and the gyroscope precesses **clock-wise** (looking from top). Assume the spin angular velocity is much greater than the precessional angular velocity. (1)What is the directions of the angular momentum of the gyroscope at the moment shown in the Fig. 4? (2) what is the direction of the torque acting on the wheel (i.e., the direction of $d\vec{L}/dt$) ?

(A) **+*x*, +*x*** ; (B) **+*x*, -*x***; (C) **+*x*, +*y*** ; (D) **+*x*, -*y***; (E) **+*x*, +z** ; (F) **+*x*, -z** ;
(G) **–*x*, +*x*** ; (H) **–*x*, -*x*** ; (I) ***-x*, +*y*** ; (J) ***-x*, -*y***; (K) **–*x*, +z** ; (L) **–*x*, -z** .

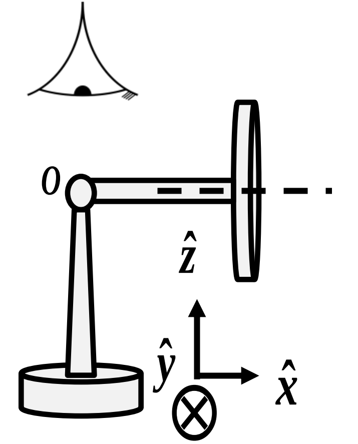
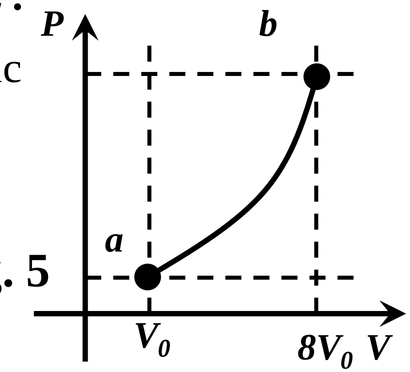


Fig. 4

3. (4 pts) As shown in Fig. 5, an ideal gas system consists of one mole of mono-atomic gas and undergoes a quasistatic process *a*→ *b*. Alone this path, the pressure can be express with the following equation: $P(V) = P_0 \cdot (V/V_0) + P_0 \cdot (V_0/V)$

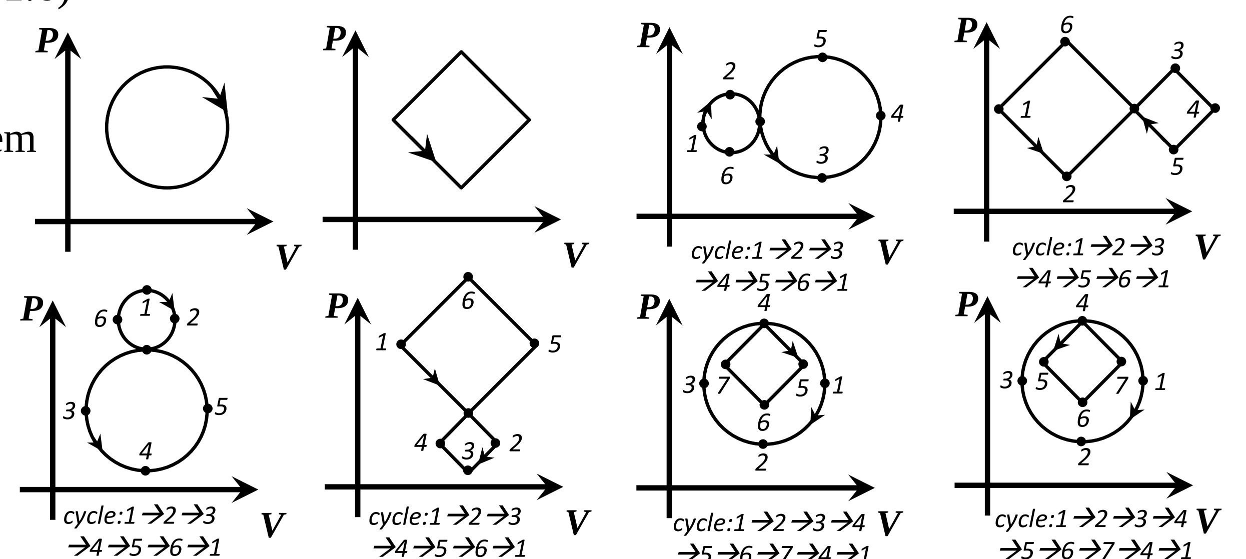
If the work done by the ideal gas system is ***W***, and ***x*** = ***W***/($\boldsymbol{P_0V_0}$), then

(A) $10 \geq x > 0$ (B) $20 \geq x > 10$ (C) $30 \geq x > 20$ (D) $40 \geq x > 30$
(E) $50 \geq x > 40$ (F) $60 \geq x > 50$
(Note, ***ln*** 2 ~ 0.7, ***ln*** 3~ 1.1, and ***ln*** 5 ~ 1.6)

Fig. 5


4. (4 pts) How many of the following cyclic processes of an ideal gas system would behave like a heat pump?

(A) 1 (B) 2 (C) 3
(D) 4 (E) 5 (F) 6
(G) 7 (H) 8



5. (4pts) An ideal gas with $C_p = 7R/2$ and $C_v=5R/2$ is carried through the cycle illustrated in Fig. 6. The expansion is adiabatic. What is the efficiency ***e*** of this engine?

(A) $70\% \geq e > 60\%$ (B) $60\% \geq e > 50\%$ (C) $50\% \geq e > 40\%$
(D) $40\% \geq e > 30\%$ (E) $30\% \geq e > 20\%$

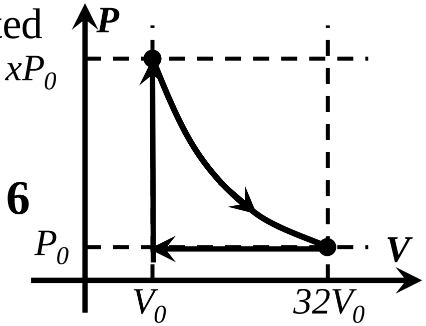


Fig. 6

6. (4 pts) Which statements are true?

a. It is possible to completely convert work into heat.
b. It is impossible to transfer heat from a cooler to a hotter body.
c. For a free-expansion process and isothermal process with the same beginning and end points, the entropy changes of the system are different since one is reversible process and the other one is not. .
d. The spontaneous (自發性) flow of heat from cold body to a hot boy will result a negative entropy change of the universe. So it is prohibited(禁止) process.

(A) a (B) b (C) c (D) d (E) a, b (F) a, c (G) a, d (H) b, c (I) b, d (J) c, d
(K) a, b, c (L) a, b, d (M) a, c, d (N) b, c, d (O) all above are false.

7. (4 pts) As shown in Fig. 7, an ideal gas system consists of one mole of mono-atomic gas and undergoes three process, an adiabatic process $a \rightarrow b$, an isobaric process $a \rightarrow c$ with entropy change ΔS_1 , and isochoric process $a \rightarrow d$ with entropy change ΔS_2 , Assume $V_b = V_c$, and $P_b = P_d$, then what is the ratio $\Delta S_1/\Delta S_2$

(A) $-\infty$ (B) $-\gamma$ (C) -1 (D) $-1/\gamma$ (E) 0
(F) $1/\gamma$ (G) 1 (H) γ (I) ∞

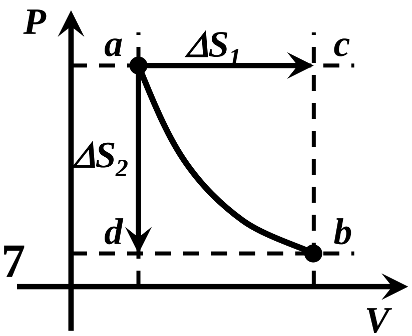


Fig. 7

8. (4 pts) Shown in Fig. 8, one contains 1 mole of nitrogen and the other contains 1 mole of oxygen, and both are at the same temperature ***T***. The volume of each container is ***V***. Those two vessels are connected with one narrow tube with a valve (閥). Now we open the valve and let the nitrogen and oxygen expand freely. What is the change of entropy after the opening?

(A) ***0*** (unchanged) (B) $R \cdot \ln 2$ (C) $2R \cdot \ln 2$ (D) $3R \cdot \ln 2$
(E)) $4R \cdot \ln 2$ (F) $5R \cdot \ln 2$ (G) insufficient information

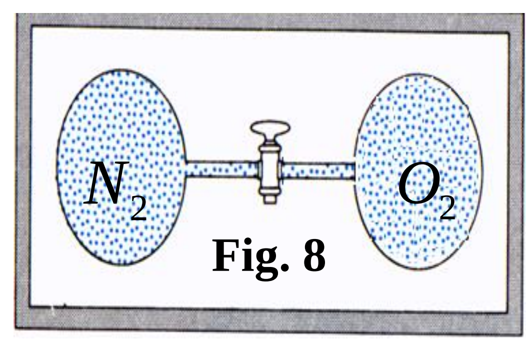
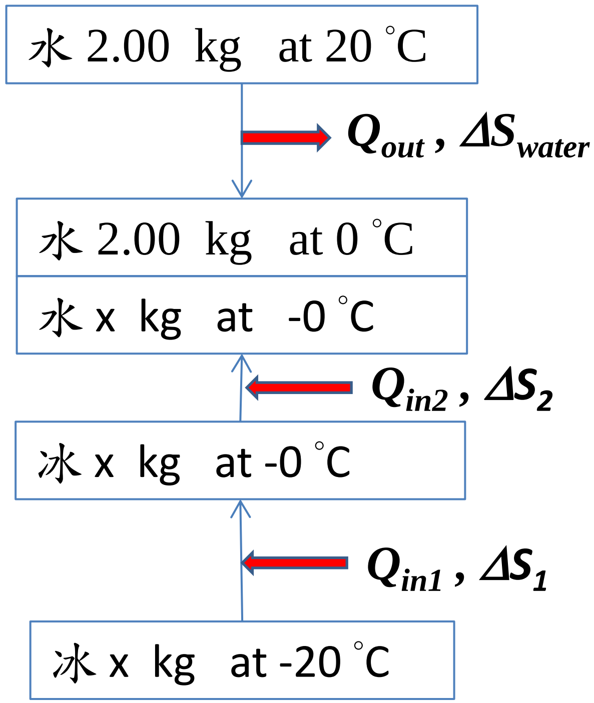


Fig. 8

Multiple Choice Questions:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
G	I	D	E	A	G	C	C	B	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**		
B	E	C	E	E	E	C	C		



(a) $|Q_{out}| = |Q_{in1}| + |Q_{in2}|$ **1pt**

$$2.0 \cdot 4200 \cdot 20 = x \cdot 2100 \cdot 20 + x \cdot 333000$$

$$x = \frac{168000}{337200} = 0.498 \approx 0.50\,kg$$

2pts

The amount of ice is ~ 0.5kg.

(b) **1pt**

$$\Delta S_{water} = \int \frac{dQ}{T} = \int_{293}^{273} \frac{mC_{water}\,dT}{T} \approx \frac{mC_{water}(273-293)}{\bar{T}} = \frac{2.0 \cdot 4200 \cdot (-20)}{283} \approx -594\,(J/K)$$

2pts

$$= mC_{ice} \ln\left(\frac{273}{293}\right) \approx 2 \cdot 4200 \frac{-20}{293} \approx 573(J/K)$$

(c) $\Delta S_{ice} = \Delta S_1 + \Delta S_2$

$$\Delta S_1 = \int \frac{dQ}{T} = \int_{253}^{273} \frac{mC_{ice}\,dT}{T} \approx \frac{mC_{ice}(273-253)}{\bar{T}} \approx \frac{0.5 \cdot 2100 \cdot (20)}{263} \approx 79.8(J/K)$$

1pt

$$= mC_{ice} \ln\left(\frac{273}{253}\right) \approx 0.5 \cdot 2100 \frac{20}{253} \approx 83.0(J/K$$

$$\Delta S_2 = \frac{\Delta Q}{T} = \frac{mL_f}{T}$$

1pt

$$\approx \frac{0.5 \cdot 333000}{273} \approx 610\,(J/K)$$

$$\approx \frac{0.498 \cdot 333000}{273} \approx 607\,(J/K)$$

2pts

$$\Delta S_{ice} \approx 690\,(J/K) \text{ or } 687\,(J/K)$$

(d)

1pt

$$\Delta S_{universe} = \Delta S_{water} + \Delta S_1 + \Delta S_2 = 96\,(J/K) \text{ or } 93\,(J/K)$$

$$\Delta S_{universe} \approx 120\,J/K$$

2 pts

Problem 2

(a)

$$I\ddot{\theta} + mgL\theta = 0 \quad \rightarrow \quad \omega^2 = \frac{mgL}{I} = \left(\frac{2\pi}{T}\right)^2 \text{ or } I = mgL\frac{T^2}{4\pi^2}$$

$\boxed{\text{2 pts}}$

(b)

$$\theta(t) = \theta_0 \cos(\omega t) = \theta_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

3 pts

$\boxed{\text{1 pts}}$

$$\dot{\theta}(t) = -\omega\theta_0 \sin(\omega t) \text{ or } \omega\theta_0 \cos\left(\omega t + \frac{\pi}{2}\right)$$

At the bottom of the pendulum $\left|\dot{\theta}_{\max}\right| = \omega\theta_0 = \frac{2\pi}{T}\theta_0$ $\boxed{\text{2 pts}}$

5 pts

(c)

$$E_{tot} = \frac{1}{2}I\dot{\theta}^2 + mgL(1-\cos\theta) = mgL(1-\cos\theta_0) = \frac{1}{2}I\dot{\theta}_{\max}^2$$

$\boxed{\text{2 pts}}$

$$\rightarrow \dot{\theta}_{\max} = \left(\frac{2mgL}{I}(1-\cos\theta_0)\right)^{1/2} = \frac{2\pi\sqrt{2}}{T}(1-\cos\theta_0)^{1/2}$$

$\boxed{\text{2 pts}}$

$$< \frac{2\pi}{T}\theta_0$$

$\boxed{\text{1 pts}}$

Problem 3

(a) $$U(r) = \frac{B}{r^9} - \frac{A}{r}$$

2 pts

$$\frac{dU}{dr} = \frac{A}{r^2} - \frac{9B}{r^{10}} = 0 \Rightarrow r = r_0 = \left(\frac{9B}{A}\right)^{1/8}$$ **2 pts**

(b) $$E_{tot} = KE + PE = \frac{1}{2}m\dot{r}^2 + U(r) = \frac{1}{2}m\dot{r}^2 - \frac{A}{r} + \frac{B}{r^8},$$ **1 pts**

3 pts

$$\frac{dE_{tot}}{dt} = 0 \Rightarrow 0 = m\dot{r}\frac{d\dot{r}}{dt} + \frac{A}{r^2}\dot{r} - \frac{9B}{r^{10}}\dot{r} = \dot{r}\left(m\ddot{r} + \frac{A}{r^2} - \frac{9B}{r^{10}}\right)$$ **1 pts**

Equation of motion: $$m\ddot{r} + \frac{A}{r^2} - \frac{9B}{r^{10}} = 0$$ **1 pts**

Or from $F = ma \rightarrow$ $$F = m\ddot{r} = -\frac{dU}{dr} = -\frac{A}{r^2} + \frac{9B}{r^{10}}$$

(c) Equation of motion: $m\ddot{r}+\frac{A}{r^2}-\frac{9B}{r^{10}}=0$

3 pts

$$x \equiv r - r_0 \rightarrow \begin{array}{c} r = x + r_0 \\ \dot{x} = \dot{r} \end{array}$$ 1 pts

$$\frac{1}{r^2}=\frac{1}{(x+r_0)^2}\simeq\frac{1}{r_0^2}\left(1-2\frac{x}{r_0}\right) \qquad \frac{1}{r^{10}}=\frac{1}{(x+r_0)^{10}}\simeq\frac{1}{r_0^{10}}\left(1-10\frac{x}{r_0}\right)$$ 1 pts

Equation of motion becomes:

$$m\ddot{x}+\frac{A}{r_0^2}\left(1-\frac{2x}{r_0}\right)-\frac{9B}{r_0^{10}}\left(1-\frac{10x}{r_0}\right)$$

$$=m\ddot{x}+\left(\frac{A}{r_0^2}-\frac{9B}{r_0^{10}}\right)+\left(\frac{90B}{r_0^{11}}-\frac{2A}{r_0^3}\right)x$$

2 pts

$$=m\ddot{x}+\frac{72B}{r_0^{11}}x=m\ddot{x}+\frac{8A}{r_0^3}x=0$$ 1 pts

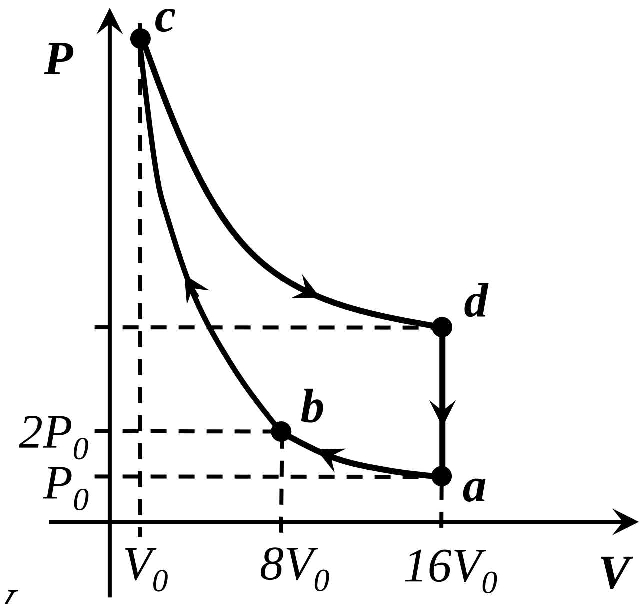
(d)

$$\omega^2=\frac{72B}{mr_0^{11}}=\frac{8A}{mr_0^3}\Rightarrow T=\frac{2\pi}{\omega}=2\pi\left(\frac{72B}{mr_0^{11}}\right)^{-1/2}$$

1 pts

$$=2\pi\left(\frac{8A}{mr_0^3}\right)^{-1/2}$$ 1 pts

	ΔE_{int}	Q	W
$a \rightarrow b$	0	-16ln2 1	-16ln2 1
$b \rightarrow c$	72 1	0	-72 1
$c \rightarrow d$	0	256ln2 1	256ln2 1
$d \rightarrow a$	-72 1	-72 1	0



(1) ***a → b, isothermal*** $\Rightarrow \Delta E_{\text{int}} = 0$

$$W = \int_{16V_0}^{8V_0} PdV = \int_{16V_0}^{8V_0} \frac{RT_a}{V} dV = 16P_0V_0 \int_{16V_0}^{8V_0} \frac{1}{V} dV = -16\ln 2 P_0V_0$$

$$\Rightarrow Q = -16\ln 2 \cdot P_0V_0$$

b → c, adiabatic $\Rightarrow Q = 0,\quad P_cV_c^{\gamma} = P_bV_b^{\gamma} \quad \Rightarrow P_cV_0^{\frac{5}{3}} = 2P_0(8V_0)^{\frac{5}{3}} \quad \Rightarrow P_c = 64P_0$ 2

$$W = \int_{8V_0}^{V_0} PdV = \int_{8V_0}^{V_0} \frac{64P_0V_0^{5/3}}{V^{5/3}} dV = 64P_0V_0^{5/3}\left(-\frac{3}{2}V^{-2/3}\Big|_{8V_0}^{V_0}\right) = 64P_0V_0^{5/3}\left(-\frac{3}{2}V_0^{-2/3} + \frac{3}{8}V_0^{-2/3}\right)$$

$$= -72P_0V_0$$

$$\Rightarrow \Delta E_{\text{int}} = 72 \cdot P_0V_0$$

c → d, isothermal $\Rightarrow \Delta E_{\text{int}} = 0$

$$W = \int_{V_0}^{16V_0} PdV = \int_{V_0}^{16V_0} \frac{RT_d}{V} dV = 64P_0V_0 \int_{V_0}^{16V_0} \frac{1}{V} dV = 256\ln 2 P_0V_0$$

$$\Rightarrow Q = 256\ln 2 \cdot P_0V_0$$

$d \rightarrow a$, ***isochoric*** $\Rightarrow W = 0$

$$\Delta E_{\text{int}} = \frac{3}{2}RT_a - \frac{3}{2}RT_d = \frac{3}{2}(16P_0V_0 - 64P_0V_0) = -72P_0V_0$$

$$\Rightarrow Q = -72P_0V_0$$

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(2) $$e = 1 - \frac{|Q_L|}{Q_H} = 1 - \frac{72 + 16\ln 2}{256\ln 2} = 1 - \frac{9 + 2\ln 2}{32\ln 2} \approx 1 - \frac{10.2}{19.2} = \frac{9}{19.2} \sim 0.49$$

(3) The highest efficiency would be of the Carnort cycle working between the same heighest and the lowest temperatures :

$$e = 1 - \frac{T_a}{T_c} = 1 - \frac{16P_0V_0}{64P_0V_0} = 0.75$$ 2

2