HW12-1

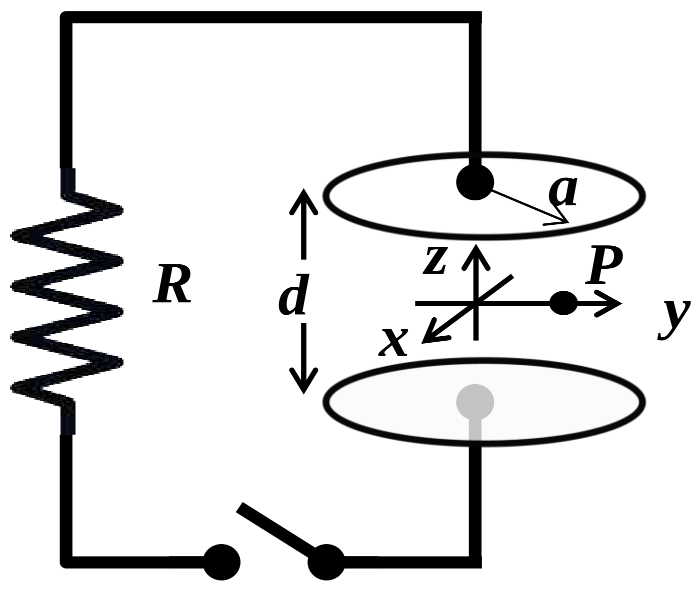
As shown in the figure below, for $t < 0$, the capacitor has charge Q_0 (>0). It consists of two conducting disks of radius ***a***, separated with a distance ***d***. At $t = 0$, the switch is closed. Consider at point ***P*** on the ***y***-axis, and its coordinate is **(0,*r*,0), *r* < *a*.** Find

(A) The capacitance ***C*** of the capacitor and the charge $Q(t)$ on the capacitor as a function of time.

(B) The electric field $\vec{E}(t)$ at ***P*** as a function of time.

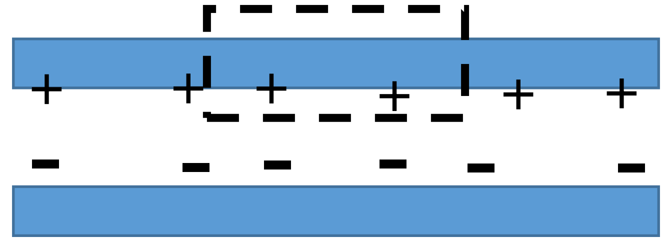
(C) The field $\vec{B}(t)$ at ***P*** as a function of time.

(D) The Poynting vector $\vec{S}(t)$ at ***P*** as a function of time



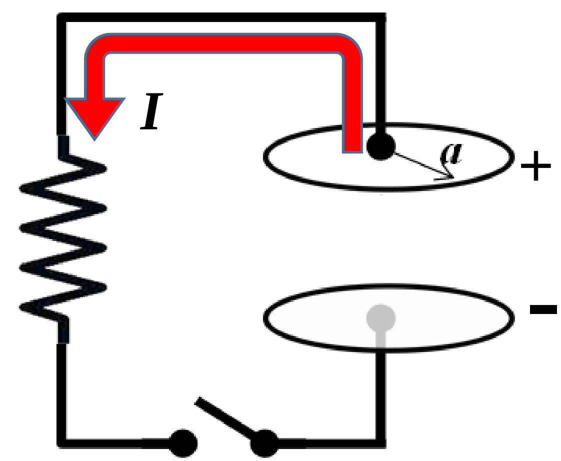
(A)

以無限大之平面電荷分布近似有限電荷於平面電極上之電場分布



By Gauss' Laws: $\oiint \vec{E}\bullet d\vec{A}=\frac{Q_{enc}}{\varepsilon_0}\Rightarrow EA=\frac{\sigma A}{\varepsilon_0}=\frac{Q}{a^2\pi\varepsilon_0}$

$$C=\frac{Q}{|\Delta V|}=\frac{Q}{|Ed|}=\frac{Q}{|\frac{Q}{\varepsilon_0 a^2\pi}d|}=\frac{\varepsilon_0 a^2\pi}{d}$$



電容的電場↓，導線內的電場：C.N. $I=-\frac{dQ}{dt}$

By Kirchhoff Circuit Laws:

$$\frac{Q}{C}-IR=0\Rightarrow -\frac{dQ}{dt}=\frac{Q}{RC}\Rightarrow \int_{Q_0}^{Q_{(t)}}\frac{dQ}{Q}=-\int_0^t\frac{dt}{RC}\Rightarrow \ln\left|\frac{Q(t)}{Q_0}\right|=\frac{-t}{RC}$$

$$\Rightarrow Q(t)=Q_0e^{\frac{-t}{RC}}=Q_0e^{\frac{-td}{R\varepsilon_0 a^2\pi}}$$

(B) 從(A)高斯定律得到的電場結果可知:

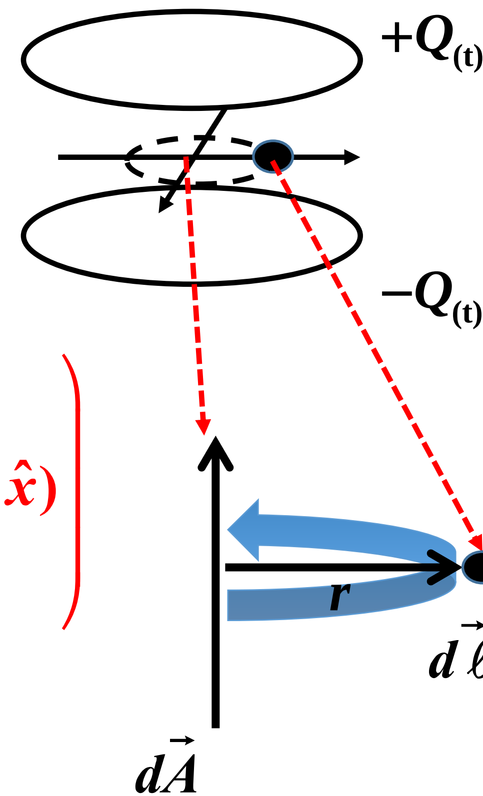
$$E=\frac{\sigma}{\varepsilon_0}=\frac{Q(t)}{a^2\pi\varepsilon_0}\Rightarrow \vec{E}=\frac{Q(t)}{a^2\pi\varepsilon_0}(-\hat{z})=\frac{Q_0e^{\frac{-td}{R\varepsilon_0a^2\pi}}}{a^2\pi\varepsilon_0}(-\hat{z})$$

(C)

Apply Ampere-Maxwell's equation to the circular path show:

$$\oint\vec{B}\bullet d\vec{\ell}=2\pi rB=\mu_0\varepsilon_0\frac{d\Phi_E}{dt}=\mu_0\varepsilon_0\frac{d}{dt}\frac{Q_0e^{\frac{-td}{R\varepsilon_0a^2\pi}}\left(-\hat{z}\right)\cdot\left(\pi r^2\hat{z}\right)}{a^2\pi\varepsilon_0}$$

$$=\frac{\mu_0Q_0rde^{\frac{-td}{R\varepsilon_0a^2\pi}}}{2\varepsilon_0R\left(a^2\pi\right)^2},\quad \text{c.c.w.}\Rightarrow \vec{B}_P=\frac{-\mu_0rdQ_0e^{\frac{-td}{R\varepsilon_0a^2\pi}}}{2\varepsilon_0R(a^2\pi)^2}(\hat{x})$$



(D)

$$\vec{S}=\frac{1}{\mu_0}\vec{E}\times\vec{B}=\frac{1}{\mu_0}\left(\frac{Q_0e^{\frac{-td}{R\varepsilon_0a^2\pi}}}{a^2\pi\varepsilon_0}(-\hat{z})\right)\times\left(\frac{\mu_0dQ_0re^{\frac{-td}{R\varepsilon_0a^2\pi}}}{2\varepsilon_0R(a^2\pi)^2}(-\hat{x})\right)$$

$$=\frac{Q_0{}^2dre^{\frac{-2td}{R\varepsilon_0a^2\pi}}}{2\pi^3\varepsilon_0{}^2Ra^6}(\hat{y})=\frac{Q_0{}^2dre^{\frac{-2td}{R\varepsilon_0a^2\pi}}}{2a^6\pi^4\varepsilon_0{}^2R}(\hat{y})$$

HW12-2: The direction (in general) of a plane can be expressed by the wave vector $\vec{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$. For example, a magnetic field of a plane wave (in free space) with the form

$$\vec{B}(\vec{r},t) = B_0\hat{z}\sin\left(\vec{k}\cdot\vec{r} - \omega t\right) = B_0\hat{z}\sin\left[\left(2.0\ \mathrm{m}^{-1}\right)\frac{x+y}{\sqrt{2}} - \omega t\right], \quad \textbf{and } B_0 = 5.0\times 10^{-7}\,\mathrm{T}.$$

describes the wave moving toward the direction $\hat{k} = \frac{\hat{x}+\hat{y}}{\sqrt{2}}$ with wave number $\boldsymbol{k} = 2.0\ \mathbf{m}^{-1}$. Answer the following questions including **correct unit**. note: $c = 3\times10^8$ m/s, $\mu_0 = 4\pi\times10^{-7}$ Tm/A, and $\varepsilon_0\mu_0 = 1/c^2$

a) Find the wave length (λ), and the angular frequency (ω) of this plane wave.
b) What is the direction of this plane wave propagating?
c) The Electric field of this plane can be written as $\vec{E}(\vec{r},t) = \left(E_{0x}\hat{x} + E_{0y}\hat{y}\right)\sin\left[2.0\frac{x+y}{\sqrt{2}} - \omega t\right]$.
   Find the values of $\boldsymbol{E_{0x}}$ and $\boldsymbol{E_{0y}}$ in SI unit.
d) Find the Poynting vector $\vec{S}$ (magnitude and direction), and the intensity ($I = \langle S \rangle$) of this plane wave.

$$\vec{B}(\vec{r},t) = B_0\hat{z}\sin\left[2.0\frac{x+y}{\sqrt{2}} - \omega t\right] = \left(5.0\times10^{-7}T\right)\hat{z}\sin\left[2.0\frac{x+y}{\sqrt{2}} - \omega t\right]$$

$$\hat{k} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0), \quad \left|\vec{k}\right| = 2.0\text{m}^{-1}$$

a) $$\omega = kc = 6\times10^8 s^{-1}, \quad \lambda = \frac{2\pi}{k} = 3.14 \text{ m},$$

a) the direction of the wave vector is propagating along $\hat{k} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)$

b) $$\vec{E} = E_0\hat{E}_0\sin\left(2.0\frac{x+y}{\sqrt{2}} - \omega t\right), \quad E_0 = cB_0 = 150 \text{ V/m}$$

$$\hat{E}_0\times\hat{B}_0 = \hat{E}_0\times\hat{z} = \hat{k} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0) \rightarrow \hat{E}_0 = \frac{-\hat{x}+\hat{y}}{\sqrt{2}}$$

$$E_{0x} = \frac{-150}{\sqrt{2}} \text{ V/m}, \quad E_{0y} = \frac{150}{\sqrt{2}} \text{ V/m},$$

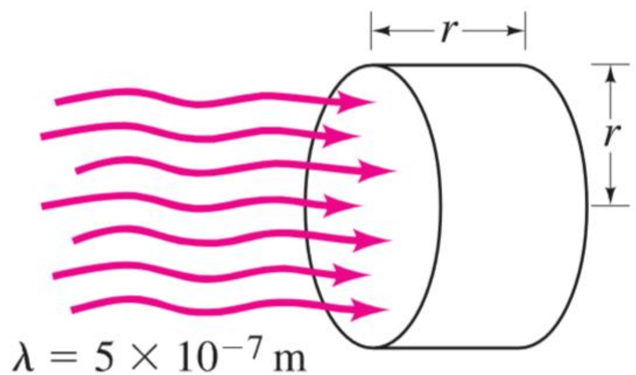
d) $$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{E_0B_0}{\mu_0}\left(\hat{k}\right)\sin^2\left(2.0\frac{x+y}{\sqrt{2}} - \omega t\right) = \frac{750}{4\pi}\frac{\hat{x}+\hat{y}}{\sqrt{2}}\sin^2\left(2.0\frac{x+y}{\sqrt{2}} - \omega t\right), \text{ W/m}^2$$

$$\langle I\rangle = \langle S\rangle = \frac{E_0B_0}{2\mu_0} = 30 \text{ W/m}^2$$

HW12-3

Laser light can be focused (at best) to a spot with a radius ***r*** equal to its wavelength λ . Suppose that a ***1.0-W*** bean of green laser light ($\lambda = 5\times10^{-7}m$) is used to form such a spot and that a cylindrical particle of that size (let the radius and height equal ***r***) is illuminated by the laser as shown in Fig.31-23. Estimate the acceleration of the particle, if its density equals that of water and it absorbs the radiation. [This order-of-magnitude calculation convinced researchers of the feasibility of "optical tweezers," p.829.]

FIGURE 31–23



光壓： $P = I / c = \langle S \rangle / c$ $\qquad P = F / A$

Intensity: $I = \langle S \rangle \Rightarrow I\pi r^2 = \langle S \rangle \pi r^2 = 1(\mathrm{W})$

（因為光全部聚焦在那個物體上，所以那個物體每秒獲得的能量剛好是光的功率）

$$\Rightarrow F = PA = P(\pi r^2) = \langle S \rangle \pi r^2 / c = \frac{1(\mathrm{W})}{3\times 10^8 (\mathrm{m/s})}$$

$$a = \frac{F}{m} = \frac{1(J/s)}{3\times 10^8 (\mathrm{m/s})} \frac{1}{(5\times 10^{-7})^3 \pi (\mathrm{m}^3) \times 1000(kg/m^3)}$$

$$= \frac{8\times 10^7}{3\pi} (\mathrm{J/kg\bullet m}) \approx 28.49\times 10^6 (\mathrm{J/kg\bullet m})$$