1,)

a、 second order , linear

b、 third order , non-linear

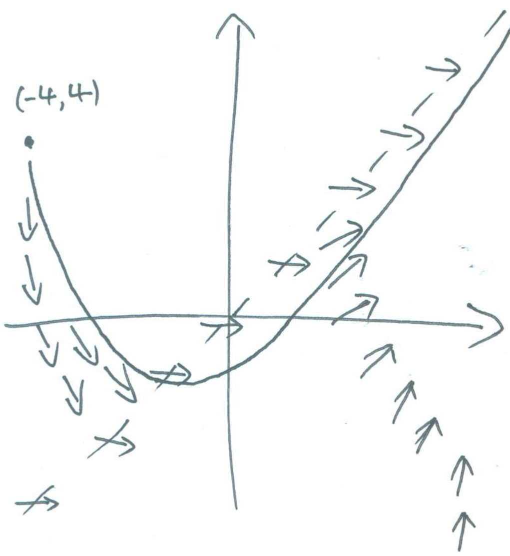
c、 second order , linear , homogenus

d、 second order , non-linear

e、 third order , non-linear

共,11个答案，一个一分

2、 配分, 斜率 4分, Feild 4分, curve 2分.



3、

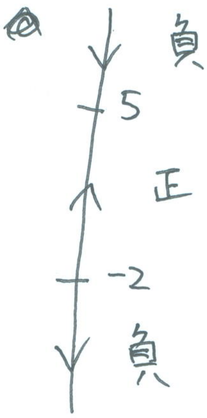
$10+3c-c^2=0$

$-(c-5)(c+2)=0$

so critical point , 5, -2

5分

phase portrait



5分

4. $y' = 3x^2 y$, $y(2) = 1$

$$\frac{dy}{dx} = 3x^2 y$$

$$\int \frac{dy}{y} = \int 3x^2 dx$$

$$\ln|y| = x^3 + C \quad (5分)$$

$$C + 8 = 0$$

$$C = -8 \quad (5分)$$

$$y = e^{x^3 - 8} \ \#$$

5. $\dfrac{dy}{dx} = (2x+2y+3)^2 + 4x + 4y + 6$, $y(0) = -\dfrac{3}{4}$

令 $u = 2x+2y+3 \Rightarrow y = \dfrac{u-2x-3}{2}$, $\dfrac{dy}{dx} = -1 + \dfrac{1}{2}\dfrac{du}{dx}$

$$-1 + \frac{1}{2}\frac{du}{dx} = u^2 + 2u \Rightarrow \frac{1}{2}\frac{du}{dx} = u^2 + 2u + 1$$

$$\frac{1}{u^2+2u+1} du = 2dx \Rightarrow \int \frac{1}{u^2+2u+1} du = \int 2 dx$$

$$\int \frac{1}{(u+1)^2} du = 2x + C = -\frac{1}{u+1} = -\frac{1}{2x+2y+4} \quad (5分)$$

$$y(0) = -\frac{3}{4} \Rightarrow C = \frac{-2}{5} \quad (5分)$$

$$\left(2x - \frac{2}{5}\right)(2x+2y+4) + 1 = 0 \ \#$$

6. $(x+1)y' + y = 5x^2(x+1)$, $y(2) = 3$

$\Rightarrow \frac{dy}{dx} + \frac{1}{x+1} y = 5x^2$

令 $p(x) = \frac{1}{x+1}$ $f(x) = 5x^2$ $e^{\int \frac{1}{x+1} dx} = e^{\ln|x+1|} = x+1$

一定要有

$\Rightarrow \frac{d}{dx}[(x+1)\cdot y] = (x+1)x^2 \cdot 5$

$\Rightarrow \int \frac{d}{dx}[(x+1)\cdot y]\, dx = \int 5(x^3 + x^2)\, dx$

$\Rightarrow (x+1)\cdot y = 5\cdot\left(\frac{x^4}{4} + \frac{x^3}{3} + C_1\right)$

5分

代入 $x = 2$ $y = 3$: $9 = 5\left(4 + \frac{8}{3} + C_1\right)$

$C_1 = -\frac{73}{15}$

5分

$\Rightarrow y = \frac{5x^3(3x+4)}{12(x+1)} - \frac{73}{3(x+1)}$

7. $x\frac{dy}{dx} + 6y = 3xy^{\frac{4}{3}}$, $y(1) = 8$

$\Rightarrow \frac{dy}{dx} + \frac{6y}{x} = 3y^{\frac{4}{3}}$. 令 $u = y^{-\frac{1}{3}}$. $\frac{du}{dy} = -\frac{1}{3}y^{-\frac{4}{3}}$. $\frac{dy}{du} = -3y^{\frac{4}{3}}$

一定要有

$\Rightarrow \frac{dy}{du}\frac{du}{dx} + \frac{6y}{x} = 3y^{\frac{4}{3}}$

$\Rightarrow -3y^{\frac{4}{3}}\frac{du}{dx} + \frac{6y}{x} = 3y^{\frac{4}{3}}$

$\Rightarrow \frac{du}{dx} - \frac{2}{x}u = -1$

令 $p(x) = -\frac{2}{x}$ $f(x) = -1$ $e^{\int -\frac{2}{x} dx} = e^{-2\ln x} = x^{-2}$

$\int \frac{d}{dx}(x^{-2}\cdot u)\, dx = \int -x^{-2}\, dx$

$\Rightarrow x^{-2}u = x^{-1} + C$

$\Rightarrow u = x + Cx^2$

$\Rightarrow y^{-\frac{1}{3}} = x + Cx^2$

5分

代入 $x = 1$ $y = 8$

$\frac{1}{2} = 1 + C$. $C = -\frac{1}{2}$

5分

$\Rightarrow y^{-\frac{1}{3}} = x - \frac{1}{2}x^2$

8. $(\underbrace{3x^2y + e^x\sin y}_{M}) + (\underbrace{x^3 + e^x\cos y}_{N})\,y' = 0$, $y(0) = \pi/2$

$\frac{\partial M}{\partial y} = 3x^2 + e^x\cos y$ $\quad \frac{\partial N}{\partial x} = 3x^2 + e^x\cdot\cos y$, then $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\frac{\partial F(x.y)}{\partial x} = 3x^2y + e^x\cdot\sin y$, $\overset{\int f(x.y)\,dx}{\Longrightarrow}$ $F(x.y) = x^3y + e^x\cdot\sin y + g(y)$

$\frac{\partial F(x.y)}{\partial y} = x^3 + e^x\cdot\cos y + g'(y) = x^3 + e^x\cdot\cos y$, $g'(y) = 0$

$g(y) = C$

$\therefore F(x.y) = x^3y + e^x\cdot\sin y + C$ +5 , $y(0) = \frac{\pi}{2}$, $F(0, \frac{\pi}{2}) = 0$, $\therefore$

$C = -1$

+5

9. $y' = x + \frac{1}{5}y$, $y(0) = -3$, $h = 1$

$y_{n+1} = y_n + (x_n + \frac{1}{5}y_n)\cdot 1 = x_n + \frac{6}{5}y_n$

$y(0) = -3$

$y(1) = 0 + \frac{6}{5}(-3) = -\frac{18}{5}$

$y(2) = 1 + \frac{6}{5}\cdot(\frac{-18}{5}) = -\frac{83}{25}$

$y(3) = 2 + \frac{6}{5}\cdot(\frac{-83}{25}) = -\frac{248}{125}$

} 少一個 " -3 "

10. Proof by Wronskian of the pairs function $\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

(a) $y_1 = \sin x$, $y_2 = \cos x$

$W = \begin{vmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{vmatrix} = -(\sin^2 x + \cos^2 x) = -1 \neq 0$ L-I (Independence)

+5

(b) $y_1 = x^3$, $y_2 = -2x^3$

$W = \begin{vmatrix} x^3 & -2x^3 \\ 3x^2 & -6x^2 \end{vmatrix} = (-6x^5 + 6x^5) = 0$ L-D (Dependent)

+5