

### HW14-1 Problem 20-41 and 20-50 in [Giancoli \(pp. 642\)](#) ; in Giancoli (pp. 554)

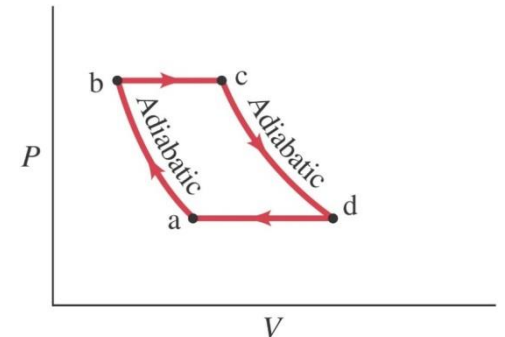
41.(a) An ice cube of mass  $m$  at  $0^\circ\text{C}$  is placed in a large  $20^\circ\text{C}$  room. Heat flows (from the room to the ice cube) such that the ice cube melts and the liquid water warms to  $20^\circ\text{C}$ . The room is so large that its temperature remains nearly  $20^\circ\text{C}$  at all times. Calculate the change in entropy for the (water + room) system due to this process. Will this process occur naturally? (b) A mass  $m$  of liquid water at  $20^\circ\text{C}$  is placed in a large  $20^\circ\text{C}$  room. Heat flows (from the water to the room) such that the liquid water cools to  $0^\circ\text{C}$  and then freezes into a  $0^\circ\text{C}$  ice cube. The room is so large that its temperature remains  $20^\circ\text{C}$  at all times. Calculate the change in entropy for the (water + room) system due to this process. Will this process occur naturally?

50. The specific heat per mole of potassium at low temperatures is given by  $C_v = aT + bT^3$ , where  $a = 2.08 \text{ mJ/mol} \cdot \text{K}^2$  and  $b = 2.57 \text{ mJ/mol} \cdot \text{K}^4$ . Determine (by integration) the entropy change of  $0.15 \text{ mol}$  of potassium when its temperature is lowered from  $3.0 \text{ K}$  to  $1.0 \text{ K}$ .

### HW14-2 Problem 20-78 in [Giancoli \(pp. 645\)](#) ; in Giancoli (pp. 557)

A gas turbine operates under the *Brayton cycle*, which is depicted in the PV diagram of Fig. 20-28. In process ab the air-fuel mixture undergoes an adiabatic compression. This is followed, in process bc, with an isobaric (constant pressure) heating, by combustion. Process cd is an adiabatic expansion with expulsion of the products to the atmosphere. The return step, da, takes place at constant pressure. If the working gas behaves like an ideal gas, show that the efficiency of the Brayton cycle is

$$e = 1 - \left( \frac{P_b}{P_a} \right)^{\frac{1-\gamma}{\gamma}}.$$

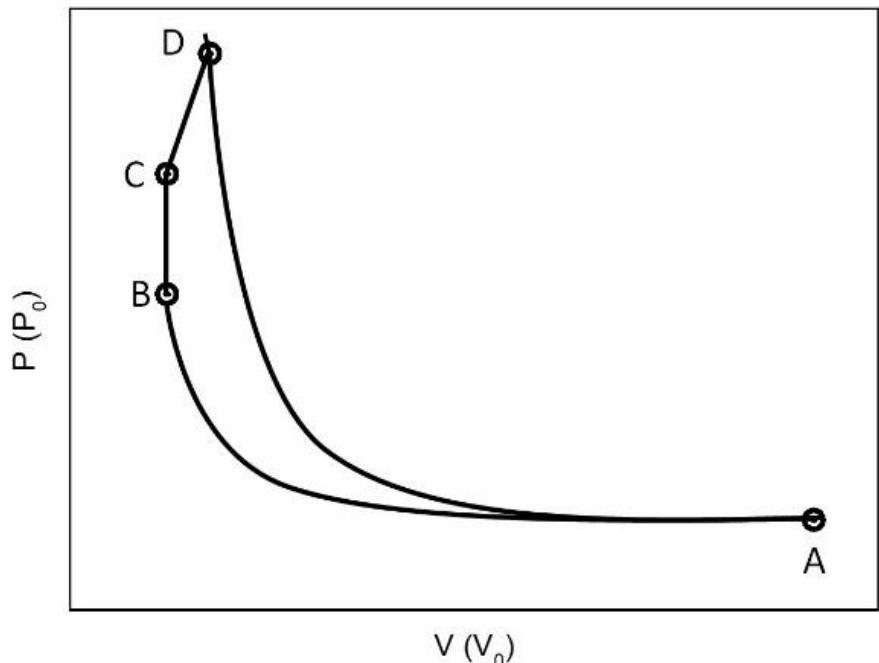


HW14-3

A one mole monatomic ideal gas engine is operated by the cycle shown in Fig., where process  $A \rightarrow B$  is isothermal,  $B \rightarrow C$ : isovolumetric,  $C \rightarrow D$ : straight-line, and  $D \rightarrow A$ : adiabatic. The volumes at A, B, C, and D are  $128V_0$ ,  $8V_0$ ,  $8V_0$ , and  $16V_0$ , respectively. The pressures at point A and C are  $P_0$  and  $24P_0$ , respectively.

(Write down your answer in term of  $P_0$ ,  $V_0$ ,  $R$ ,  $\ln 2$ ,  $\ln 3$ ,  $\ln 5$ , and  $\ln 7$ )

- (a) Find  $P$ ,  $V$ , and  $T$  at points A, B, C, and D.
- (b) Calculate the work  $W$  done (by the gas), the heat transfer  $Q$ , the change of the internal energy  $\Delta E_{\text{int}}$ , and the change of entropy  $\Delta S$  for each process.
- (c) Find the efficiency of this ideal gas engine.



	P ( $P_0$ )	V ( $V_0$ )	T ( $P_0 V_0 / R$ )
A			
B			
C			
D			

	W ( $P_0 V_0$ )	Q ( $P_0 V_0$ )	$\Delta E_{\text{int}}$ ( $P_0 V_0$ )	$\Delta S$ (R)
A $\rightarrow$ B				
B $\rightarrow$ C				
C $\rightarrow$ D				
D $\rightarrow$ A				

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**Sol:**

(a) **This process will occur naturally.**

Assume the mass  $m$  is in kg

$$\Delta S_{\text{melt}} = \frac{Q_{\text{melt}}}{T_{\text{melt}}} = \frac{mL_{\text{fusion}}}{T_{\text{melt}}} = m \frac{3.33 \times 10^5 \text{ J/kg}}{273 \text{ K}} = (1219.78m) \text{ J/K}$$

$$\Delta S_{\text{warming}} = \int \frac{dQ_{\text{warming}}}{T} = \int_{T_{\text{melt}}}^{T_{\text{room}}} \frac{mcdT}{T} = mc \ln \frac{T_{\text{room}}}{T_{\text{melt}}} = m(4186 \text{ J/kg} \cdot \text{K}) \ln \frac{293 \text{ K}}{273 \text{ K}} = (295.95m) \text{ J/K}$$

$$\Delta S_{\text{room}} = \frac{-mL - mc(T_{\text{room}} - T_{\text{melt}})}{T_{\text{room}}} = -m \left[ \frac{3.33 \times 10^5 \text{ J/kg} + (4186 \text{ J/kg} \cdot \text{K})(20 \text{ K})}{293 \text{ K}} \right] = (-1422.25m) \text{ J/K}$$

$$\Delta S_{\text{total}} = \Delta S_{\text{melt}} + \Delta S_{\text{warming}} + \Delta S_{\text{room}} = (1219.78m) \text{ J/K} + (295.95m) \text{ J/K} - (1422.25m) \text{ J/K} = \mathbf{93.48 \text{ m J/K}}$$

(b) This process will NOT occur naturally.

$\therefore$  The heat exchange is exactly opposite as in part (a)

$$\therefore \Delta S_{\text{total}} = -93.48m \text{ J/K}$$

50. The specific heat per mole of potassium at low temperatures is given by  $C_V = aT + bT^3$ , where  $a = 2.08 \text{ mJ/mol} \cdot \text{K}^2$  and  $b = 2.57 \text{ mJ/mol} \cdot \text{K}^4$ . Determine (by integration) the entropy change of 0.15 mol of potassium when its temperature is lowered from 3.0 K to 1.0 K.

Sol:

Assume the process is reversible  $\Rightarrow dQ = nC_V dT$

$$\begin{aligned} S &= \int_{T_1}^{T_2} \frac{dQ}{T} = \int_{T_1}^{T_2} \frac{nC_V dT}{T} = \int_{T_1}^{T_2} \frac{n(aT + bT^3) dT}{T} = \int_{T_1}^{T_2} n(a + bT^2) dT = n \left( aT + \frac{1}{3}bT^3 \right)_{T_1}^{T_2} \\ &= (0.15 \text{ mol}) \left[ (2.08 \text{ mJ/mol} \cdot \text{K}^2)(1.0 \text{ K} - 3.0 \text{ K}) + \frac{1}{3}(2.57 \text{ mJ/mol} \cdot \text{K}^4) \left[ (1.0 \text{ K})^3 - (3.0 \text{ K})^3 \right] \right] \\ &= -4.0 \text{ mJ/K} \end{aligned}$$

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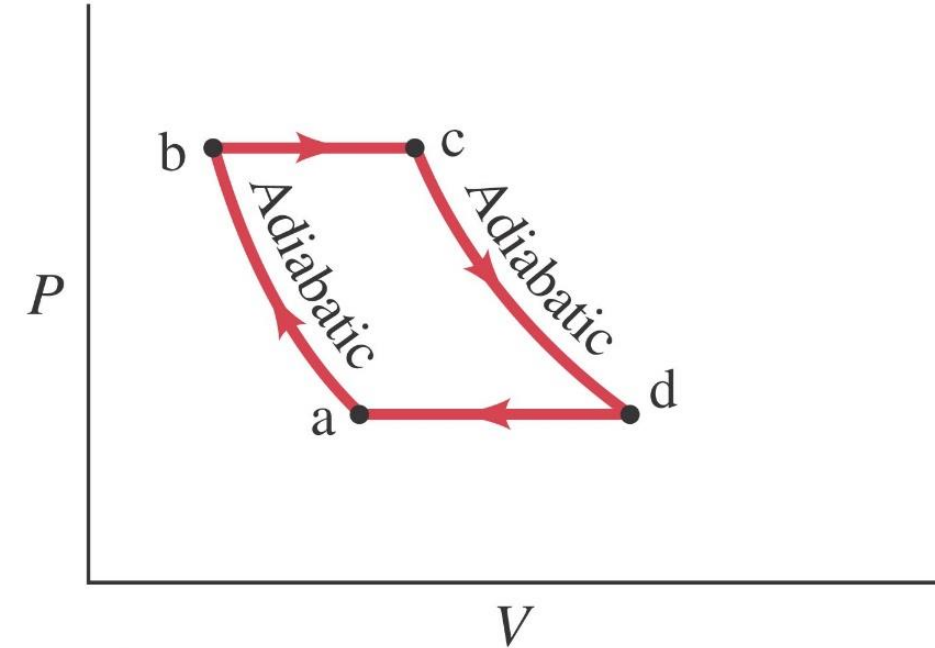
**Sol:**

$$Q_H = Q_{bc} = nC_P(T_c - T_b) \text{ and } Q_L = Q_{da} = nC_P(T_d - T_a)$$

$$\Rightarrow e = 1 - \frac{Q_L}{Q_H} = 1 - \frac{nC_P(T_d - T_a)}{nC_P(T_c - T_b)} = 1 - \frac{(T_d - T_a)}{(T_c - T_b)}$$

substitute the equation by  $PV = nRT$

$$\Rightarrow e = 1 - \frac{(T_d - T_a)}{(T_c - T_b)} = 1 - \frac{\left( \frac{P_d V_d}{nR} - \frac{P_a V_a}{nR} \right)}{\left( \frac{P_c V_c}{nR} - \frac{P_b V_b}{nR} \right)} = 1 - \frac{(P_d V_d - P_a V_a)}{(P_c V_c - P_b V_b)} = 1 - \frac{P_a (V_d - V_a)}{P_b (V_c - V_b)}$$



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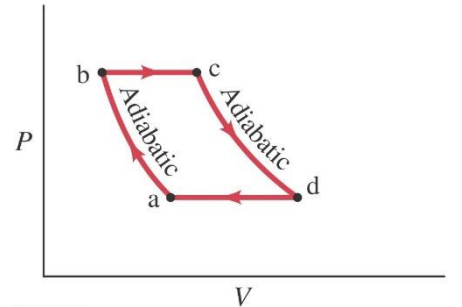
for adiabatic process ab and cd  $\Rightarrow P_a V_a^\gamma = P_b V_b^\gamma \Rightarrow V_a = V_b \left( \frac{P_b}{P_a} \right)^{1/\gamma}$

$$\Rightarrow P_b V_c^\gamma = P_a V_d^\gamma \Rightarrow V_d = V_c \left( \frac{P_b}{P_a} \right)^{1/\gamma}$$

substitute these equations into the efficiency expression

$$e = 1 - \frac{P_a (V_d - V_a)}{P_b (V_c - V_b)} = 1 - \frac{P_a \left( V_c \left( \frac{P_b}{P_a} \right)^{1/\gamma} - V_b \left( \frac{P_b}{P_a} \right)^{1/\gamma} \right)}{P_b (V_c - V_b)} = 1 - \frac{P_a \left( \frac{P_b}{P_a} \right)^{1/\gamma} (V_c - V_b)}{P_b (V_c - V_b)}$$

$$= 1 - \left( \frac{P_b}{P_a} \right)^{\frac{1}{\gamma} - 1} = 1 - \left( \frac{P_b}{P_a} \right)^{\frac{1-\gamma}{\gamma}}$$

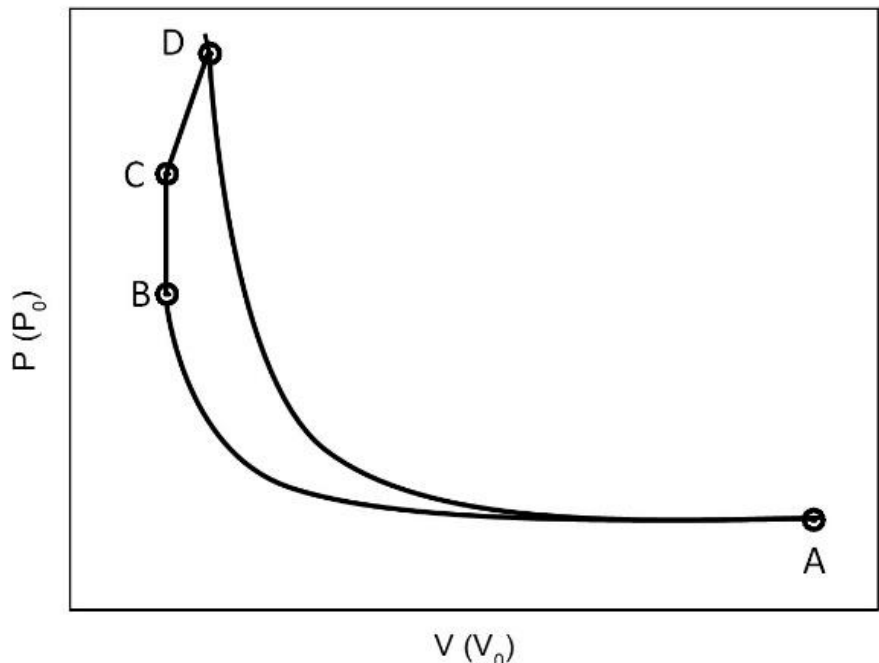


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(Write down your answer in term of  $P_0$ ,  $V_0$ ,  $R$ ,  $\ln 2$ ,  $\ln 3$ ,  $\ln 5$ , and  $\ln 7$ )

- (a) Find  $P$ ,  $V$ , and  $T$  at points A, B, C, and D.
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- (c) Find the efficiency of this ideal gas engine.



	P (P <sub>0</sub> )	V (V <sub>0</sub> )	T (P <sub>0</sub> V <sub>0</sub> /R)
A			
B			
C			
D			

	W (P <sub>0</sub> V <sub>0</sub> )	Q (P <sub>0</sub> V <sub>0</sub> )	ΔE <sub>int</sub> (P <sub>0</sub> V <sub>0</sub> )	ΔS (R)
A→B				
B→C				
C→D				
D→A				

Sol: (a)

	P (P <sub>0</sub> )	V (V <sub>0</sub> )	T (P <sub>0</sub> V <sub>0</sub> /R)
A	1	128	128
B	16	8	128
C	24	8	192
D	32	16	512

$T_A = T_B = 128P_0V_0/R$   
 $\Rightarrow P_A V_A = P_B V_B \Rightarrow P_B = 16P_0$

$P_D V_D^\gamma = P_A V_A^\gamma$   
 $\Rightarrow p_D (16V_0)^{5/3} = P_A (128V_0)^{5/3}$   
 $\Rightarrow P_D = 32P_0$

$T_C = 192P_0V_0/R, T_D = 512P_0V_0/R$

(c)

$$e = 1 - \frac{16\ln 2}{25} = \frac{25 - 16\ln 2}{25}$$

(b)

	W (P <sub>0</sub> V <sub>0</sub> )	Q (P <sub>0</sub> V <sub>0</sub> )	ΔE <sub>int</sub> (P <sub>0</sub> V <sub>0</sub> )	ΔS (R)
A→B	-512ln2	-512ln2	0	-4ln2
B→C	0	96	96	3(ln3-ln2)/2
C→D	224	704	480	(11ln2-3ln3)/2
D→A	576	0	-576	0

A→B (isotheraml)

$\Delta E_{\text{int}}=0$

$Q = W = \int_A^B P dV = \int_{V_A}^{V_B} nRT \frac{dV}{V} = nRT_A \ln \frac{V_B}{V_A} = (-512\ln 2)P_0V_0$

$\Delta S = nC_v \ln \frac{T_B}{T_A} + nR \ln \frac{V_B}{V_A} = \frac{3}{2}R \left( \ln \frac{4}{64} \right) = -4R \ln 2$

C→D (straight-line)

$W = \frac{(24 + 32)8P_0V_0}{2} = 224P_0V_0$

$\Delta E_{\text{int}} = \frac{3}{2}R(T_D - T_C) = \frac{3}{2}R \left( \frac{320P_0V_0}{R} \right) = 480P_0V_0$

$Q = \Delta E_{\text{int}} + W = 704P_0V_0$

$\Delta S = \frac{3}{2}nR \ln \frac{T_D}{T_C} + R \ln \frac{V_D}{V_C} = \frac{3}{2}R \ln \left( \frac{512}{128} \right) + R \ln \left( \frac{16}{8} \right)$   
 $= \frac{3}{2}R \ln \left( \frac{8}{3} \right) + R \ln(2) = \left( \frac{11}{2} \ln 2 - \frac{3}{2} \ln 3 \right) R$

B→C (isovolumetric)

$W=0$

$Q = \Delta E_{\text{int}} = 1(3R/2)[T_C - T_B] = 96P_0V_0$

$\Delta S = nC_v \ln \frac{T_C}{T_B} = \frac{3}{2}R \left( \ln \frac{192}{128} \right) = 3R(\ln 3 - \ln 2)/2$

D→A (adiabatic)

$Q=0$

$W = -\Delta E_{\text{int}} = -1(3R/2)[T_A - T_D] = 576P_0V_0$

$\Delta S = 0$

