$\cancel{(+1)}$ $(s^2+4)Y(s)\cdot +1$

1. $s^2\mathcal{L}[y] - s\cdot y(0) - y'(0) + 4\cdot\mathcal{L}[y] = \dfrac{1}{s} - \dfrac{e^{-s}}{s}$ (+2)

$$\mathcal{L}[y] = \frac{1-s-e^{-s}}{s(s^2+4)} = \frac{1}{4}\cdot\frac{1}{s} - \frac{1}{4}\cdot\frac{s}{s^2+4} - \frac{1}{2}\cdot\frac{2}{s^2+4} - e^{-s}\left[\frac{1}{4}\cdot\frac{1}{s} - \frac{1}{4}\cdot\frac{s}{s^2+4}\right]$$

$\cancel{(+5)}$ (+3)

$$y = \frac{1}{4} - \frac{1}{4}\cdot\cos 2t - \frac{1}{2}\cdot\sin 2t - \left[\frac{1}{4} - \frac{1}{4}\cos 2(t-1)\right]u(t-1) \quad ※$$

(+5)

2. $\mathcal{L}[f*g] = \mathcal{L}[f]\,\mathcal{L}[g] = \dfrac{1}{s+1}\cdot\dfrac{1}{s^2+1}$ (+3)

$F(s)\cdot G(s)$

$$\frac{1}{s+1}\cdot\frac{1}{s^2+1} = \frac{1}{2}\cdot\frac{1}{s+1} - \frac{s}{2}\cdot\frac{1}{s^2+1} + \frac{1}{2}\cdot\frac{1}{s^2+1} \quad (+5)$$

$$= \frac{1}{2}\cdot\left(e^{-t} + \sin t - \cos t\right) \quad ※$$

(+2)

3. $\mathcal{L}[t^2\cdot\sin kt] = (-1)^2\cdot\dfrac{d^2}{ds^2}\cdot\dfrac{k}{s^2+k^2}$ (+3)

$$= \frac{d}{ds}\left[\frac{-2ks}{(s^2+k^2)^2}\right]$$

$$= \frac{-2k}{(s^2+k^2)^2} + (-2ks)\cdot(-2)\cdot\frac{1}{(s^2+k^2)^3}\cdot 2s \quad (+5)$$

$$= \frac{-2k(s^2+k^2)}{(s^2+k^2)^3} + \frac{8ks^2}{(s^2+k^2)^3}$$

$$= \frac{6ks^2 - 2k^3}{(s^2+k^2)^3} \quad ※ \quad +(2)$$

$x^{-n} = \dfrac{1}{x^n}$

$-n\,x^{-(n+1)}$

$= -n\cdot\dfrac{1}{x^{n+1}}$

4. $y'' + 6y' + 10y = 3\delta(t-\pi)$, $y(0)=0$, $y'(0)=0$

$\underline{(s^2+6s+10)Y(s) = 3e^{-\pi s}}$ (+5分)

$y(t) = \underline{3e^{-3(t-\pi)}\sin(t-\pi)\,u(t-\pi)}$ (+5分)

5. $y_1' = 4y_1 + 6y_2$

$y_2' = -3y_1 - 5y_2$

$\underline{k_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix} e^{t}}$ $\underline{k_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-2t}}$. $y = c_1k_1 + c_2k_2$

(+5分) (+5分)

6. $\underline{\Phi^{-1}(t) = \frac{1}{-e^{-t}} \begin{bmatrix} e^{-2t} & e^{-2t} \\ -e^{t} & -2e^{t} \end{bmatrix}}$ (+5分)

$y_p = \begin{pmatrix} 2 \\ -1 \end{pmatrix} xe^{x} + \frac{2}{3}\begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{t} + \frac{1}{2}\begin{pmatrix} -5 \\ 3 \end{pmatrix}$

7. $$e^{At} = \mathcal{L}^{-1}\left((SI-A)^{-1}\right)$$ +5

$$(SI-A) = \begin{pmatrix} s-3 & -4 & -5 \\ 0 & s-5 & -4 \\ 0 & 0 & s-3 \end{pmatrix}$$

$$(SI-A)^{-1} = \begin{pmatrix} \frac{1}{s-3} & \frac{4}{(s-5)(s-3)} & \frac{5s-9}{(s-3)^2(s-5)} \\ 0 & \frac{1}{s-5} & \frac{4}{(s-3)(s-5)} \\ 0 & 0 & \frac{1}{s-3} \end{pmatrix}$$ +3

$$e^{At} = \begin{pmatrix} e^{3t} & -2e^{3t}+2e^{5t} & -4e^{3t}+4e^{5t}-3te^{3t} \\ 0 & e^{5t} & -2e^{3t}+2e^{5t} \\ 0 & 0 & e^{3t} \end{pmatrix}$$ +2

8. $y' = xy$

$$\frac{dy}{dx} = xy$$

so $y = e^{\frac{x^2}{2}}$ +5

$$y''(0.1) = e^{\frac{(0.1)^2}{2}}\left((0.1)^2+1\right)\cdot\frac{(0.1)^2}{2}$$

+5

9. $$\int_{-\pi/2}^{\pi/2} x \cos 2x \, dx$$

10%

$$= \frac{1}{2}\left(\frac{1}{2}\cos 2x + x\sin 2x\right)\Bigg|_{-\pi/2}^{\pi/2} = 0$$

10.

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi}(x+\pi)\,dx = 2\pi$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi}(x+\pi)\cos nx\,dx = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{2}{n}(-1)^{n+1}$$

$$f(x) = \pi + \sum_{n=1}^{\infty}\frac{2}{n}(-1)^{n+1}\sin nx$$

f in continous on the interval

$a_0 \Rightarrow 2.5\%$

$a_n \Rightarrow 2.5\%$

$b_n \Rightarrow 2.5\%$

$f(x) \Rightarrow 2.5\%$