

Probability

Midterm Exam II, May 4, 2017

1. (10%) Let X be a Poisson random variable with $P(X = 0) = e^{-2}$. Find $P(X > 2)$.
2. (15%) Let X be an exponential random variable with probability density function (pdf) $f_X(x) = e^{-x}$, $0 \leq x < \infty$. Let the new random variable Y be defined as $Y = 2 - X^3$. Find $F_Y(y)$ (the CDF of Y).
3. (10%) Let X be a random variable, where $E\{X\} = 1$ and $\text{Var}\{X\} = 4$. Let $Y = aX + b$, where a and b are two constants to be designed such that $E\{Y\} = 2$ and $\text{Var}\{Y\} = 9$. Find a and b .
4. (10%) Let X be a Gaussian random variable, where $E\{X\} = 1$ and $\text{Var}\{X\} = 4$. Let $Y = 2X - 1$. Use Q function we learned in class to express $P(Y > 10)$.
5. (25%) Let X be a random variable with the following probability density function (pdf):

$$f_X(x) = 0.3\delta(x + 1) + 0.7\delta(x - 1).$$

- (a) (5%) Find the Probability mass function (PMF) of X .
 - (b) (5%) Find the Cumulative distribution function (CDF) of X . (Hint: it may be convenient to use unit-step functions.)
 - (c) (5%) Find the mean of X .
 - (d) (5%) Find the variance of X .
 - (e) (5%) Let C be the event that $X > 0$. Find the conditional CDF $F_X(x|C)$.
6. (30%) The cumulative distribution function (CDF) of random variable X is given by
- $$F_X(x) = \begin{cases} 0 & x < -1 \\ (x + 3)/4 & -1 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$
- (a) (5%) What is the type (discrete, continuous, or mixed) of the random variable X ? Please explain your answer. (0 point if the explanation is wrong)
 - (b) (5%) Plot the probability density function (pdf) of X .
 - (c) (5%) Find $P[X = -1]$.
 - (d) (5%) Find $P[-2 < X \leq 0]$.
 - (e) (5%) Find $P[-1 \leq X \leq 0]$.
 - (f) (5%) Find $E\{X\}$.