Homework 8 (Chap 10)

1. Gioncoli Textbook, problem 86. page 326
   Gioncoli Textbook, problem 86. page 281

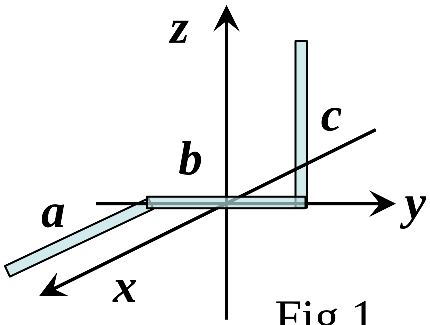


Fig 1

2. As shown in Fig. 1, three thin rods, ***a***, ***b***, and ***c*** are joined together such that rod ***a*** is parallel to the ***x***-axis, rod ***b*** lies on ***y***-axis with ***z***-axis passing its mid-point, and rod ***c*** is parallel to the ***z***-axis. Each rod has the same mass ***M*** and length ***L***, what would be the moment of inertia if the joined rod structure rotates around the ***z***-axis? (Assume the radius of the rod is nearly zero)

3. A marble of mass *m* and radius *r* rolls along the looped rough track of Fig. 2 below. What is the minimum value of the vertical height ***h*** that the marble must drop if it is to reach the highest point of the loop without leaving the track?

   (*a*) Assume $r \ll R$

   (*b*) do not make this assumption. Ignore frictional losses.

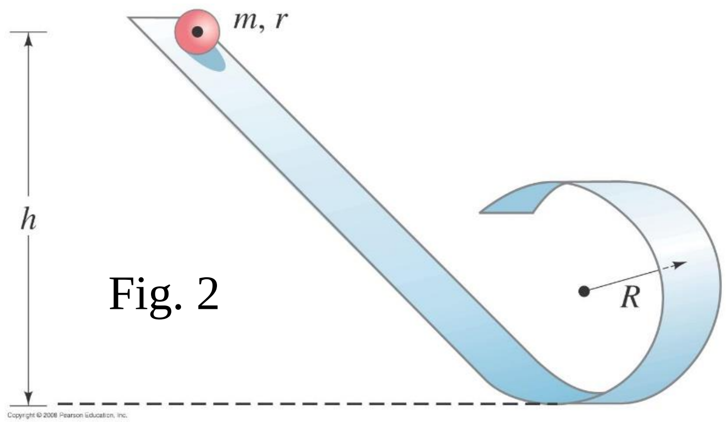
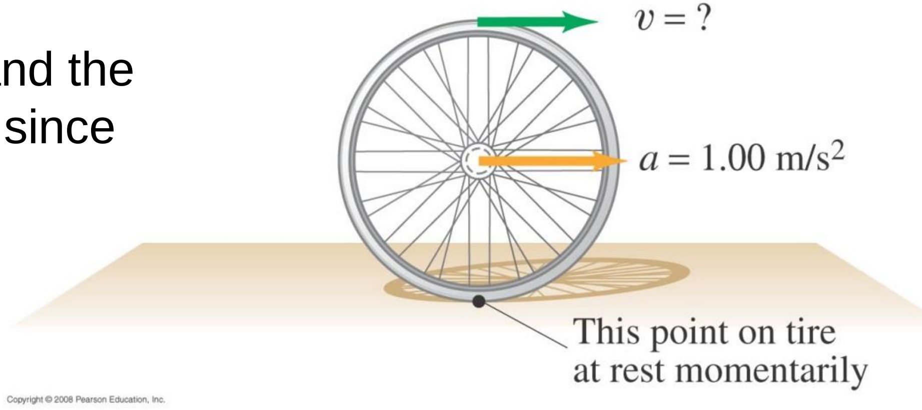


Fig. 2

86. A cyclist accelerates from rest at a rate of 1m/sec^2 How fast will a point at the top of the rim of the tire (diameter = 68cm) be moving after 2.5 s? [Hint: At any moment, the lowest point on the tire is in contact with the ground and is at rest — see Fig. 10–63.]



Assume that the velocity of the bike is $\boldsymbol{v_b}$, and the velocity at the top of the rim of the tire is $\boldsymbol{v}$, since the wheel execute pure rotation, therefore

$$v = v_b + \omega R, \qquad \omega = v_b / R$$

$$\Rightarrow v = 2v_b$$

,where $\boldsymbol{\omega}$ is the angular speed of the wheel rotation, and R=0.68 m the radius of the wheel. Since the acceleration a of the bike is 1m/sec^2, we have

$$v_b = 0 + 1 \cdot 2.5 = 2.5(m/\sec)$$

$$\Rightarrow v = 2v_b = 5(m/\sec)$$

2. As shown in Fig. 1, three thin rods, ***a***, ***b***, and ***c*** are joined together such that rod ***a*** is parallel to the ***x***-axis, rod ***b*** lies on ***y***-axis with ***z***-axis passing its mid-point, and rod ***c*** is parallel to the ***z***-axis. Each rod has the same mass ***M*** and length ***L***, what would be the moment of inertia if the joined rod structure rotates around the ***z***-axis? (Assume the radius of the rod is nearly zero)

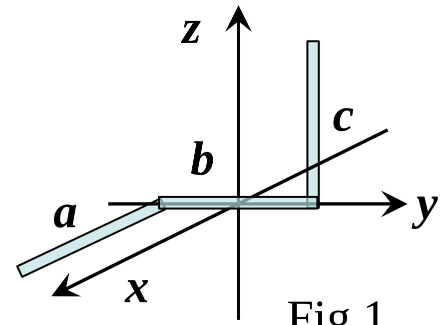


Fig 1

$$I_{tot} = I_a + I_b + I_c$$

$$I_a = M_a D^2 + I_{C,a}$$

$$= M\left(\left(\frac{L}{2}\right)^2 + \left(-\frac{L}{2}\right)^2\right) + \frac{ML^2}{12}$$

$$= \frac{1}{2}ML^2 + \frac{ML^2}{12} = \frac{7}{12}ML^2$$

$$I_b = I_{C,\mathrm{b}} = \frac{ML^2}{12}$$

$$I_a = M_c D^2 + I_{C,\mathrm{c}}$$

$$= M\left(\frac{L}{2}\right)^2 + ML^2$$

$$= \frac{1}{4}ML^2 + ML^2 = \frac{5}{4}ML^2$$

$$I_{tot} = I_a + I_b + I_c = \frac{23}{12}ML^2$$

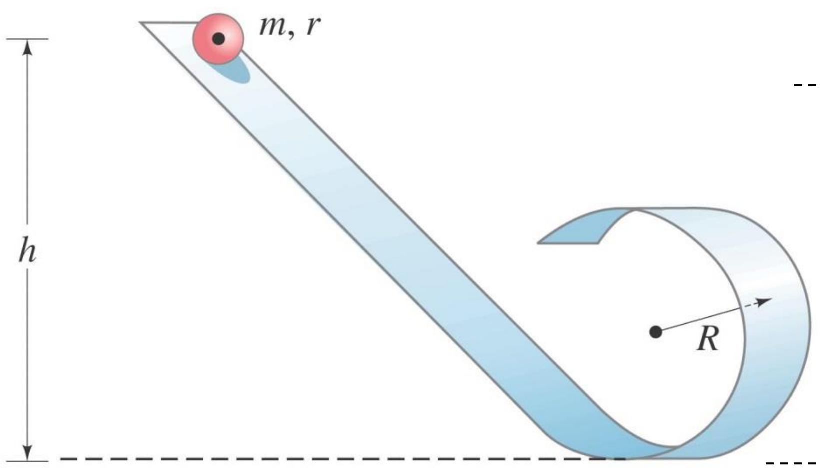
HW8-3 :

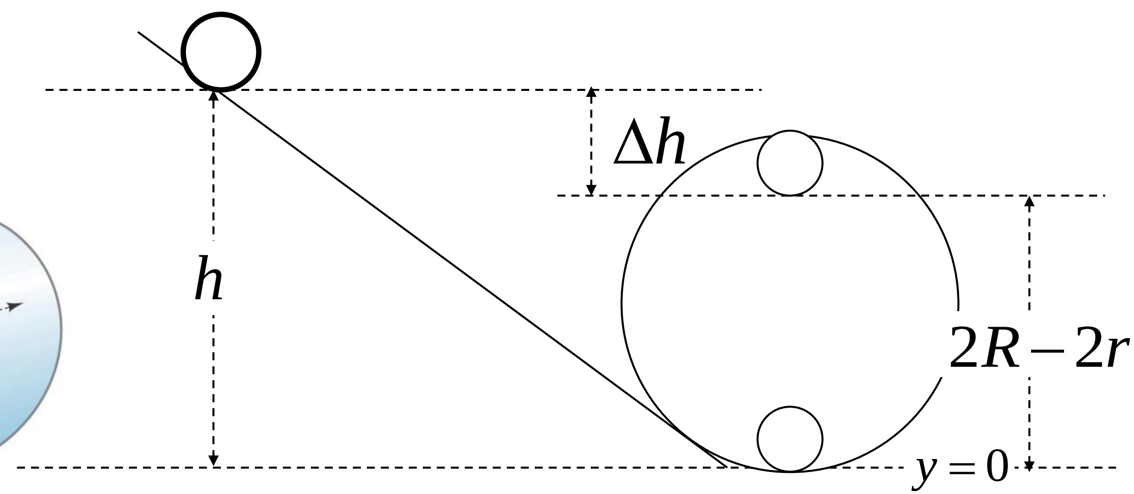
A marble of mass *m* and radius *r* rolls along the looped rough track of Fig.

What is the minimum value of the vertical height *h* that the marble must drop if it is to reach the highest point of the loop without leaving the track?

(*a*) Assume $r \ll R$

(*b*) do not make this assumption. Ignore frictional losses.





$$E_{i,tot} = E_{f,tot} \qquad mgh = mg(2\mathrm{R}-2\mathrm{r}) + \frac{1}{2}mv^2 + \frac{1}{2}I_C\omega^2$$

For pure roll on flat surface (R>>r), $v = -r\omega, \quad or \ |v| = r|\omega|,$

$$mgh = mg(2\mathrm{R}-2\mathrm{r}) + \frac{1}{2}mv^2 + \frac{1}{2}\frac{2mr^2}{5}\left(\frac{v}{r}\right)^2$$

$$\Rightarrow mgh = mg(2\mathrm{R}-2\mathrm{r}) + \frac{7}{10}mv^2$$

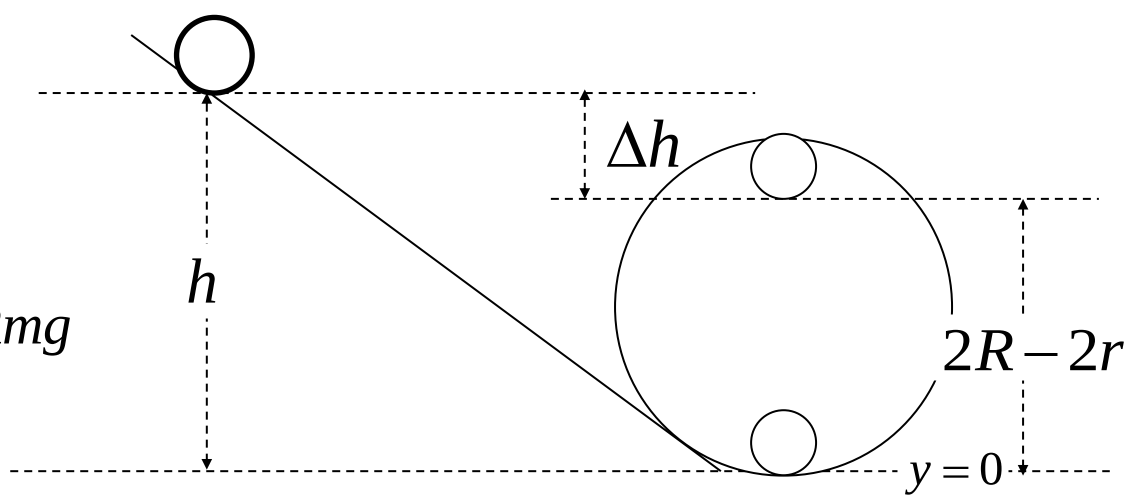
For the ball to pass the top, the gravitational force only should provide the acceleration for the boll, i.e.

$$mg \le \frac{mv^2}{R}$$

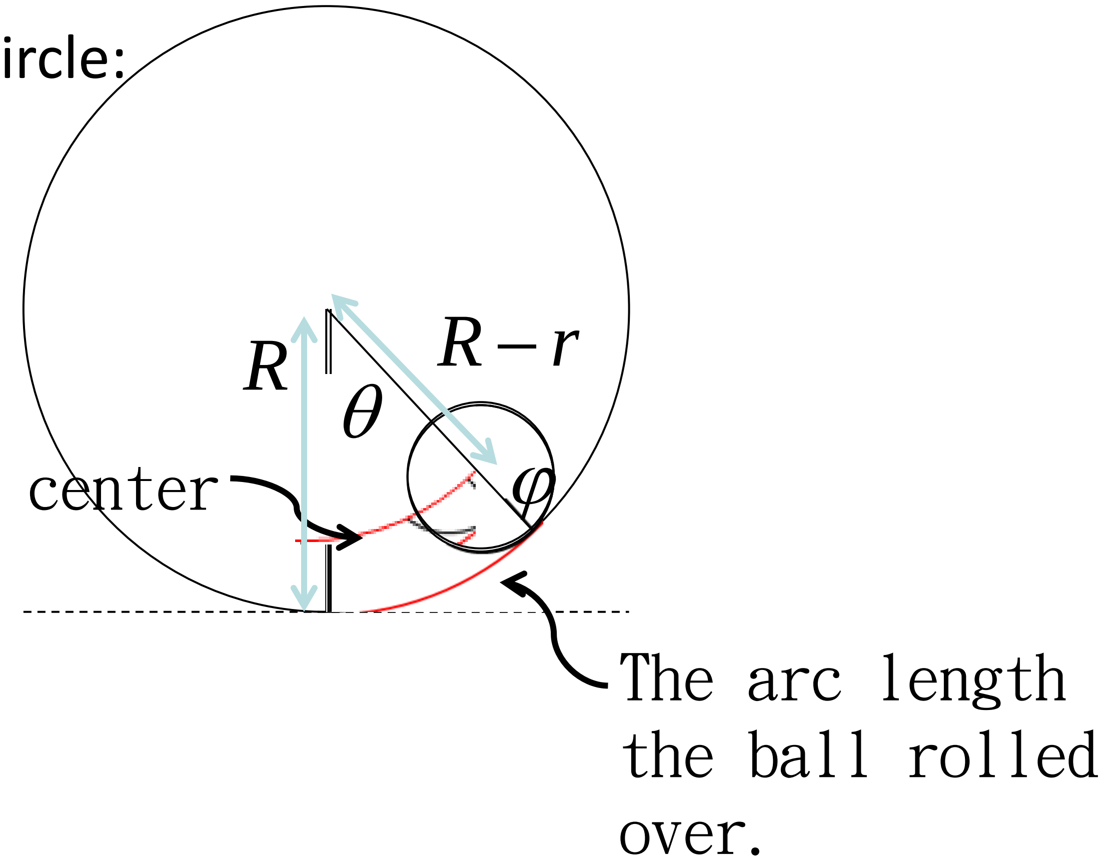
$$mgh = mg(2\mathrm{R}-2\mathrm{r}) + \frac{7}{10}mv^2$$

$$\Rightarrow mgh > mg(2\mathrm{R}-2\mathrm{r}) + \frac{7}{10}Rmg$$

$$\Rightarrow h > \frac{27}{10}R - 2\mathrm{r} \approx \frac{27}{10}R$$



Now for pure roll on a circle:



$$\begin{cases} \Delta X_{c.m.} = (R-r)\Delta\theta \\ R\Delta\theta = r\Delta\varphi \end{cases}$$

$$\begin{cases} v_{c.m.} = (R-r)\dfrac{d\theta}{dt} \\ R\dfrac{d\theta}{dt} = r\omega \Rightarrow \dfrac{d\theta}{dt} = \dfrac{r\omega}{R} \end{cases}$$

$$v_{c.m.} = (R-r)\frac{r\omega}{R} = (r - \frac{r^2}{R})\omega$$

$$E_{i,tot} = E_{f,tot} \qquad mgh = mg(2\mathrm{R}-2\mathrm{r}) + \frac{1}{2}mv^2 + \frac{1}{2}I_C\omega^2$$

For pure roll on a circle, $\left|v_{c.m.}\right| = (r - \frac{r^2}{R})\left|\omega\right|$

$$mgh = mg(2\mathrm{R}-2\mathrm{r}) + \frac{1}{2}mv^2 + \frac{1}{2}\frac{2mr^2}{5}\left(\frac{v}{r-\frac{r^2}{R}}\right)^2$$

$$mgh = mg(2\mathrm{R}-2\mathrm{r}) + \frac{1}{2}mv^2 + \frac{1}{2}\frac{2mv^2}{5}\left(\frac{R}{R-r}\right)^2$$

$$mgh = mg(2\mathrm{R}-2\mathrm{r}) + \frac{1}{2}mv^2\left(1+\frac{2}{5}\left(\frac{R}{R-r}\right)^2\right) \qquad mg \le \frac{mv^2}{(R-r)}$$

$$mgh \ge mg(2\mathrm{R}-2\mathrm{r})$$

$$+\frac{1}{2}mg(R-r)\left(1+\frac{2}{5}\left(\frac{R}{R-r}\right)^2\right)$$

$$h \ge (R-r)\left(\frac{5}{2}+\frac{1}{5}\left(\frac{R}{R-r}\right)^2\right)$$