General Physics II: HW 7

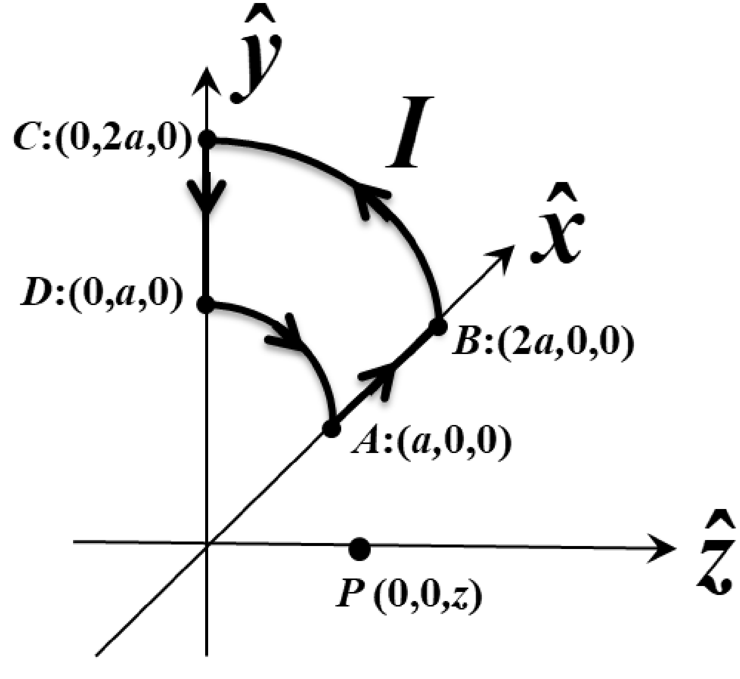
Due on 4/30/2020

HW7-1: Problem 27-12 in Giancoli (pp. 839) (pp. 727)

HW7-2: Problem 27-40 in Giancoli (pp. 840) (pp. 729)

HW7-3:

Figure shows a wire with current $\boldsymbol{I}$ from $\boldsymbol{A \rightarrow B \rightarrow C \rightarrow D \rightarrow A}$ in $\boldsymbol{x}$-$\boldsymbol{y}$ plane. The sections from $\boldsymbol{B \rightarrow C}$ and $\boldsymbol{D \rightarrow A}$ are two quarter circles with radius $\boldsymbol{2a}$ and $\boldsymbol{a}$. Evaluate the magnetic field ($\boldsymbol{x}$-, $\boldsymbol{y}$-, and $\boldsymbol{z}$-components) at point $\boldsymbol{P}$ on the $\boldsymbol{z}$-axis due to the current $\boldsymbol{I}$ from $\boldsymbol{A \rightarrow B}$, $\boldsymbol{C \rightarrow D}$, $\boldsymbol{B \rightarrow C}$, and $\boldsymbol{D \rightarrow A}$.



Solution HW7-1:

A circular loop of wire, of radius ***r***, carries current ***I***. It is placed in a magnetic field whose straight lines seem to diverge from a point a distance ***d*** below the loop on its axis. (This is, the field makes an angle ***θ*** with the loop at all points, Fig. 27-41, where **tan *θ* = *r*/*d***.) Determine the force on the loop.

$$d\vec{F} = I \cdot d\vec{l} \times \vec{B} = I(dl\hat{\phi}) \times B(\cos\theta\hat{z} + \sin\theta\hat{r})$$

$$= IB \cdot dl(\cos\theta\hat{r} + \sin\theta(-\hat{z}))$$

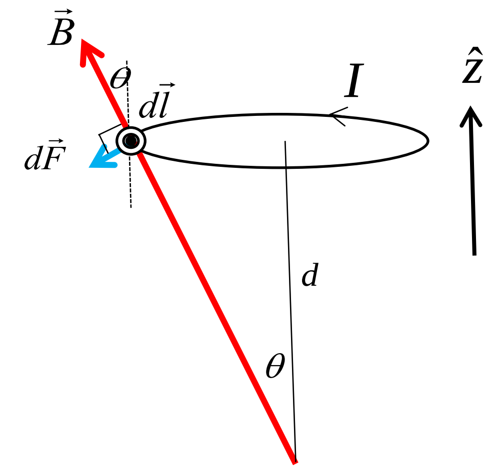
$$\Rightarrow \vec{F} = \int d\vec{F}$$

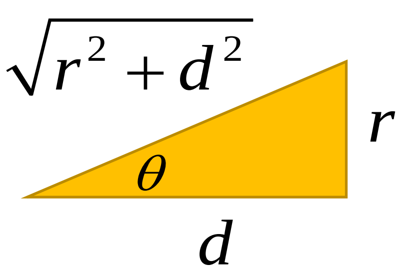
$$= \int IB \cdot dl(\cos\theta\hat{r} + \sin\theta(-\hat{z})) \quad ; dl = r\,d\phi$$

by symmetry; 0

$$= 0 + IrB\frac{r}{\sqrt{r^2 + d^2}}\int_0^{2\pi} d\phi(-\hat{z})$$

$$= \frac{2\pi I r^2 B}{\sqrt{r^2 + d^2}}(-\hat{z})$$





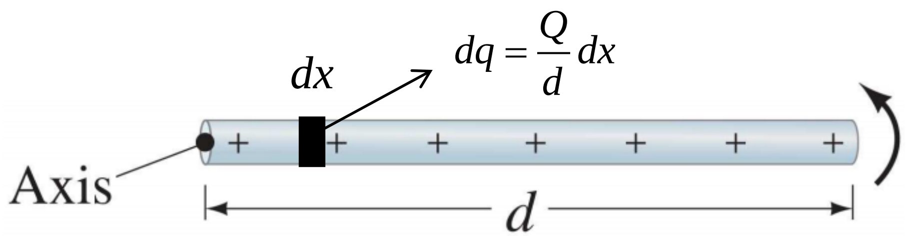
Solution HW7-2:

Suppose a nonconducting rod of length **d** carries a uniformly distributed charge **Q**. It is rotated with angular velocity **ω** about an axis perpendicular to the rod at one end, Fig. 27-48. Show that the magnetic dipole moment of this rod is **(Q ωd ²)/6**.

[Hint: Consider the motion of each infinitesimal length of the rod.]

Sol:

$$magnetic\ dipole\ moment\ \vec{\mu} = I\vec{A}$$



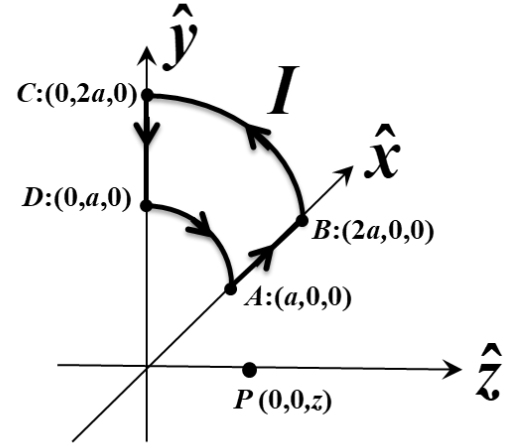
$$Divide\ the\ rod\ into\ an\ infinite\ number\ of\ segments, each\ of\ length\ is\ dx$$

$$a\ closed\ current\ caused\ by\ a\ short\ charge: dI = \frac{dq}{\boxed{t}} = \frac{\frac{Q}{d}dx}{\frac{2\pi}{\omega}} = \frac{Q\omega}{2\pi d}dx$$

the time required for enclosing a circle

$$\therefore \vec{\mu} = \int d\vec{\mu} = \int \vec{A}dI = \int (\pi x^2)\frac{Q\omega}{2\pi d}dx\odot = \frac{Q\omega}{2d}\int_0^d x^2 dx\odot = \frac{Q\omega d^2}{6}\odot$$

Solution HW7-3:



$$\vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I d\vec{r}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

For the B-field resulted from the current from ***A* → *B***

$$\vec{r} = (0,0,z), \quad \vec{r}' = (x',0,0), \quad d\vec{r}' = \hat{x}dx'$$

$$d\vec{r}' \times (\vec{r} - \vec{r}') = \hat{x}dx' \times (-x'\hat{x} + z\hat{z}) = \hat{y}(-zdx')$$

$$|\vec{r} - \vec{r}'| = |(-x',0,z)| = \sqrt{x'^2 + z^2}$$

$$\boxed{\int \frac{dx}{(x^2 \pm b^2)^{3/2}} = \frac{\pm x}{b^2\sqrt{x^2 \pm b^2}}}$$

$$\vec{B}_{A\to B} = \int d\vec{B} = \frac{\mu_0}{4\pi}\int_a^{2a} \frac{I d\vec{r}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} = \frac{\mu_0 I}{4\pi}\hat{y}\int_a^{2a} \frac{-zdx'}{(x'^2 + z^2)^{3/2}}$$

$$= \frac{\mu_0 I}{4\pi}\hat{y}\left(\frac{-zx'}{z^2\sqrt{x'^2 + z^2}}\right)_{x'=a}^{2a} = \frac{\mu_0 I}{4\pi z}\hat{y}\left[\frac{a}{\sqrt{z^2 + a^2}} - \frac{2a}{\sqrt{z^2 + 4a^2}}\right]$$

For the B-field resulted from the current from ***C* → *D***,

$$\vec{r} = (0,0,z), \quad \vec{r}' = (0,y',0), \qquad d\vec{r}' = \hat{y}dy'$$

$$d\vec{r}' \times \left(\vec{r} - \vec{r}'\right) = \hat{y}dy' \times \left(-y'\hat{y} + z\hat{z}\right) = \hat{x}\left(zdy'\right)$$

$$\left|\vec{r} - \vec{r}'\right| = \left|(0,y',z)\right| = \sqrt{y'^2 + z^2}$$

$$\boxed{\int \frac{dx}{\left(x^2 \pm b^2\right)^{3/2}} = \frac{\pm x}{b^2\sqrt{x^2 \pm b^2}}}$$

$$\vec{B}_{C\to D} = \int d\vec{B} = \frac{\mu_0}{4\pi}\int_{2a}^{a} \frac{Id\vec{r}' \times \left(\vec{r} - \vec{r}'\right)}{\left|\vec{r} - \vec{r}'\right|^3} = \frac{\mu_0 I}{4\pi}\hat{x}\int_{2a}^{a} \frac{zdy'}{\left(y'^2 + z^2\right)^{3/2}}$$

$$= \frac{\mu_0 I}{4\pi}\hat{x}\left(\frac{zy'}{z^2\sqrt{y'^2 + z^2}}\right)_{y'=2a}^{a} = \frac{\mu_0 I}{4\pi z}\hat{x}\left[\frac{a}{\sqrt{z^2 + a^2}} - \frac{2a}{\sqrt{z^2 + 4a^2}}\right]$$

For the B-field resulted from the current from ***B*** → ***C***, (θ: angular from +***x***-axis)

$$\vec{r} = (0,0,z), \quad \vec{r}' = (x',y',0), \qquad d\vec{r}' = \hat{x}dx' + \hat{y}dy'$$

$$x' = 2a\cos\theta \qquad dx' = -2a\sin\theta d\theta$$

$$y' = 2a\sin\theta \qquad dy' = 2a\cos\theta d\theta \qquad \left|\vec{r} - \vec{r}'\right| = \left|(-x',-y',z)\right| = \sqrt{4a^2 + z^2}$$

$$d\vec{r}' \times \left(\vec{r} - \vec{r}'\right) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ dx' & dy' & 0 \\ -x' & -y' & z \end{vmatrix} = \hat{x}\left(zdy'\right) + \hat{y}\left(-zdx'\right) + \hat{z}\left(x'dy' - y'dx'\right)$$

$$= \hat{x}\left(2az\cos\theta d\theta\right) + \hat{y}\left(2az\sin\theta d\theta\right) + \hat{z}\left(4a^2 d\theta\right)$$

$$\vec{B}_{B\to C} = \frac{\mu_0 I}{4\pi}\frac{2a}{\left(4a^2 + z^2\right)^{3/2}}\left[\hat{x}\int_0^{\pi/2} z\cos\theta\, d\theta + \hat{y}\int_0^{\pi/2} z\sin\theta\, d\theta + \hat{z}\int_0^{\pi/2} 2a\, d\theta\right]$$

$$= \frac{\mu_0 I}{4\pi}\frac{2a}{\left(4a^2 + z^2\right)^{3/2}}\left[\hat{x}z + \hat{y}z + \hat{z}\left(\pi a\right)\right]$$

For the B-field resulted from the current from ***D*** → ***A***,

$$\vec{r} = (0,0,z), \quad \vec{r}' = (x',y',0), \qquad d\vec{r}' = \hat{x}dx' + \hat{y}dy'$$

$$x' = a\cos\theta \quad dx' = -a\sin\theta d\theta$$

$$y' = a\sin\theta \quad dy' = a\cos\theta d\theta$$

$$\left|\vec{r} - \vec{r}'\right| = \left|(-x',-y',z)\right| = \sqrt{a^2 + z^2}$$

$$d\vec{r}' \times \left(\vec{r} - \vec{r}'\right) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ dx' & dy' & 0 \\ -x' & -y' & z \end{vmatrix} = \hat{x}\left(zdy'\right) + \hat{y}\left(-zdx'\right) + \hat{z}\left(x'dy' - y'dx'\right)$$

$$= \hat{x}\left(az\cos\theta d\theta\right) + \hat{y}\left(az\sin\theta d\theta\right) + \hat{z}\left(a^2 d\theta\right)$$

$$\vec{B}_{D\to A} = \frac{\mu_0 I}{4\pi}\frac{a}{\left(a^2+z^2\right)^{3/2}}\left[\hat{x}\int_{\pi/2}^{0} z\cos\theta\, d\theta + \hat{y}\int_{\pi/2}^{0} z\sin\theta\, d\theta + \hat{z}\int_{\pi/2}^{0} a\, d\theta\right]$$

$$= \frac{\mu_0 I}{4\pi}\frac{-a}{\left(a^2+z^2\right)^{3/2}}\left[\hat{x}z + \hat{y}z + \hat{z}\left(\frac{\pi a}{2}\right)\right]$$