General Physics (II) **期中考 II** **May 14, 2021**

試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(55 points) (所有題目必須有計算過程,否則不予計分)

1. (20 pts) Fig. 1 shows a four-section conducting wire on x-y plane with current $\boldsymbol{I}$. From $\boldsymbol{A}$ to $\boldsymbol{B}$ is a horizontal segment on the x-axis. From $\boldsymbol{B}$ to $\boldsymbol{C}$ is a quarter-circle with radius $3\boldsymbol{R}$. From $\boldsymbol{C}$ to $\boldsymbol{D}$ is a vertical segment on the y-axis. From $\boldsymbol{D}$ to $\boldsymbol{A}$ is a quarter-circle with radius $\boldsymbol{R}$. Find the x-, y-, z-components of the magnetic field at point $\boldsymbol{P}$ on the z-axis due to
   (a) (7 pts) the current in the section from $\boldsymbol{A}$ to $\boldsymbol{B}$,
   (b) (3 pts) the current in the section from $\boldsymbol{C}$ to $\boldsymbol{D}$,
   (c) (7 pts) the current in the section from $\boldsymbol{B}$ to $\boldsymbol{C}$,
   (d) (3 pts) the current in the section from $\boldsymbol{D}$ to $\boldsymbol{A}$.

The coordinates of $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$ and $\boldsymbol{P}$ are $(-R, 0, 0)$, $(3R, 0, 0)$, $(0, 3R, 0)$, $(0, R, 0)$ and $(0, 0, z)$, respectively.

2. (15 pts.) A conducting bar of mass $\boldsymbol{m}$ is placed on two frictionless conducting rails which make an angle $\theta = 30^0$ relative to the horizontal surface. The rails are connected through a resistor $\boldsymbol{R}$ and are separated by a distance $\boldsymbol{L}$, as shown in Fig. 2. In addition, a uniform magnetic field $\vec{B}_0$ is applied vertically upward (as shown in Fig 2). Initially, the bar is at the bottom of the rails with velocity v_0 and starts to move upward.

a)(3pts) Find the magnitude and the direction (from "***a*** to ***b***" or "***b*** to ***a***") of the induced current in the conducting bar? Write the induced current $\boldsymbol{I(t)}$ in terms of B_0, L, R, g (gravitational acceleration), v (speed of the bar) and other necessary constants.

b)(4pts) Find the magnetic force (magnitude and direction) due to the induced current $\boldsymbol{I(t)}$. Draw the free body diagram of the conducting bar.

c)(5 points) Find the speed $(v(t))$ of the conducting bar as function of time. Write your answer in terms of m, B_0, L, R, g and/or other necessary constants.

d)(3pts) How long does it take for the conducting bar to reach the highest point (i.e., $v = 0$)

3.(a) (8 pts) Fig. 3(a) shows a cross-sectional view of an infinite long cylindrical conductor with radius $\boldsymbol{a}$ and current density $\vec{J}(r') = J_0[1-(r'/a)]\,\hat{z}$, where r' is the distance to the axis of this cylinder and the direction of current is out of page. Find the magnetic fields on the x-axis in the range $0 \le x \le 4\boldsymbol{a}$.

(b) (6 pts) Fig.3(b) shows the cross-sectional view of an infinite plate of thickness $4\boldsymbol{a}$ carries uniform current density of $-J_0\,\hat{z}$ (into the page), and besides, there is an infinitely long cylindrical hollow region with radius $\boldsymbol{a}$. Find the magnetic fields at $(-\boldsymbol{a}, 0, 0)$.

(c) (6 pts) In Fig.3(c), a cross section of an infinite long cylindrical conductor with radius $\boldsymbol{a}$ and current density $\vec{J}(r') = J_0[1-(r'/\boldsymbol{a})]\,\hat{z}$ in Fig.3(a) is completely put into hollow region in Fig.3(b). Find the magnetic field at $(-2\boldsymbol{a}, 4\boldsymbol{a}, 6\boldsymbol{a})$.

(Use Ampere's law, draw the loop path for the integral, and write down the answer in terms of $\boldsymbol{J_0}$, $\boldsymbol{a}$, $\boldsymbol{\mu_0}$, and $\boldsymbol{x}$.)

Fig. 1

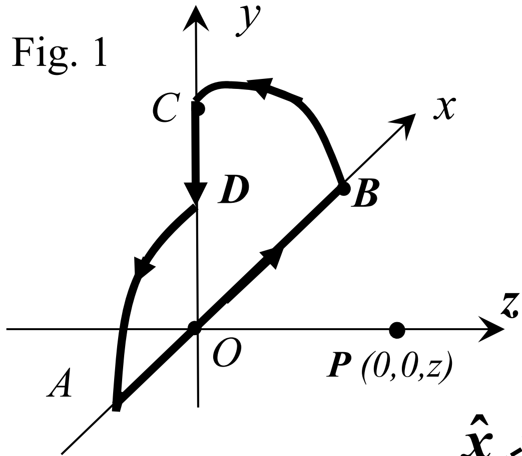


Fig. 3(a)

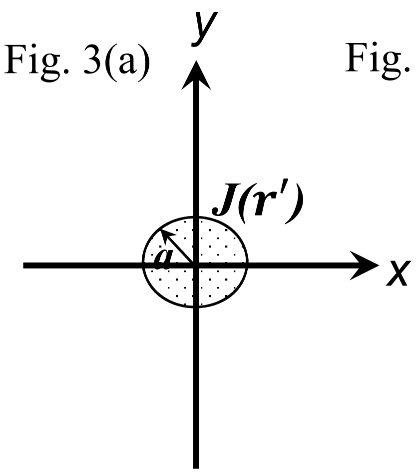


Fig. 3(b)

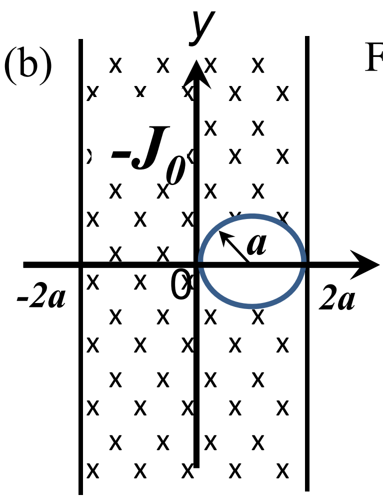
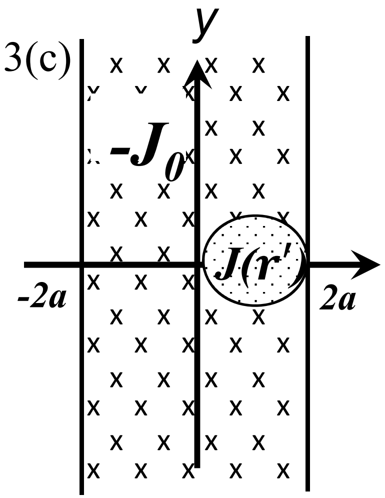


Fig. 3(c)



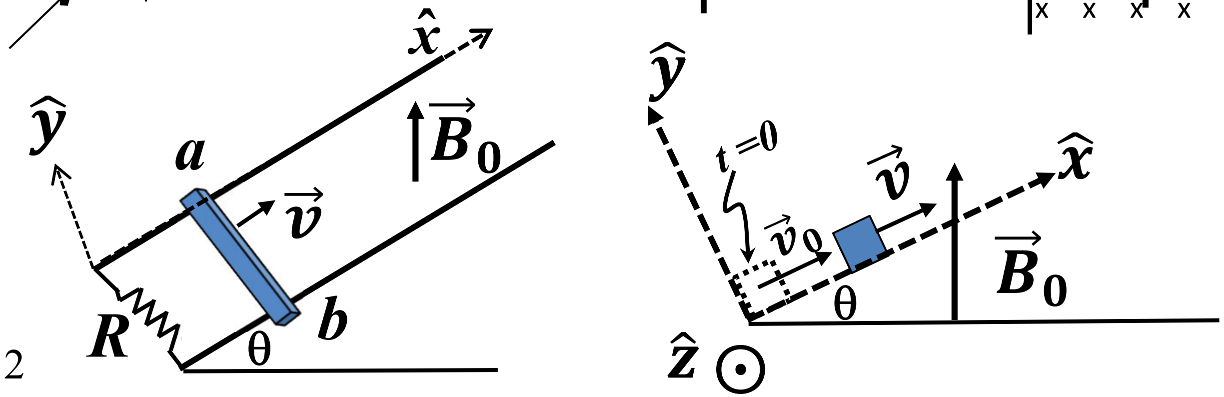
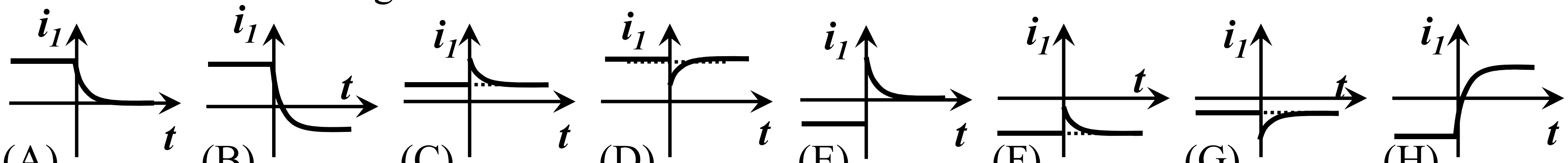


Fig. 2

II.選擇題(45 points)

1. (5pts)As shown in Fig. 4, the switch ***S*** in the circuit is initially open, and there is no charge stored in the capacitor. At $\boldsymbol{t}$ = 0 sec, the switch ***S*** is closed, at the same time (right after S being closed) which of the following is correct?
   (A) $i_3<-3/2$ (B) $-3/2\le i_3<-1$ (C) $-1\le i_3<-1/2$ (D) $-1/2\le i_3<0$ (E) $0\le i_3<1/2$
   (F) $1/2\le i_3<1$ (G) $1\le i_3<3/2$ (H) $3/2\le i_3$

2. (5pts)continue with problem 1, which of the following is the time dependence of $\boldsymbol{i_1}$ before and after the switch ***S*** being closed at $\boldsymbol{t}$ = 0 sec?



(A) (B) (C) (D) (E) (F) (G) (H)

3. (5 pts) Fig. 5(a) shows a wire of length ***L*** carrying a current ***I*** and is bent into a circular coil of one turn. In Fig. 5(b) the same length of wire that has been bent into a coil of two turns. Assume $\boldsymbol{B_a}$ and $\boldsymbol{B_b}$ are magnitudes of magnetic field at the center of the two coils, and $\boldsymbol{\mu}_a$ and $\boldsymbol{\mu_b}$ are the dipole moment magnitudes of the coil. What are the ratio ($\boldsymbol{B_b/B_a}$,$\boldsymbol{\mu_b/\mu_a}$)?
   (A)(1,1) (B)(2,1) (C)(2,1/2) (D)(4,1) (E)(4,2) (F)(4,1/2) (G)(8,1) (H)(8,2) (J)(8,1/2)

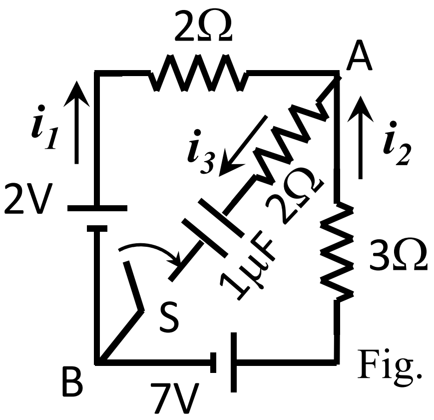


Fig. 4

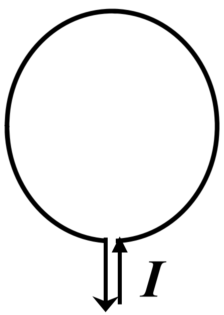


Fig. 5(a)

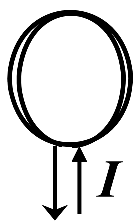
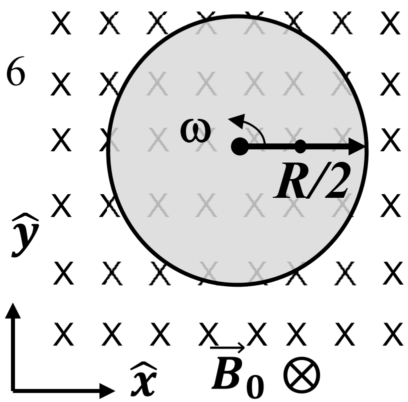


Fig. 5(b)

Fig. 6



4. (5pts) A circular metal disk of radius $\boldsymbol{R}$=*0.5m* rotates counter-clock-wise with period **0.1s** about an axis through its center perpendicular to the disk (Fig. 6). The disk rotates in a uniform magnetic field $\vec{\boldsymbol{B}} = (\boldsymbol{-0.1\hat{z}})\boldsymbol{T}$. The magnitude of the electric field at $\vec{\boldsymbol{r}} = \frac{\boldsymbol{R}}{2}\hat{\boldsymbol{x}}$ is $\boldsymbol{E}$ (in SI unit). What is $\boldsymbol{E}$ and the direction of the electric field at this position?

(A) $\boldsymbol{E = 0}$ (B) $\boldsymbol{0 < E \leq 1, +\hat{x}}$ (C) $\boldsymbol{0 < E \leq 1, -\hat{x}}$, (D) $\boldsymbol{0 < E \leq 1, +\hat{y}}$, (E) $\boldsymbol{0 < E \leq 1, -\hat{y}}$,
(F) $1< \boldsymbol{E \leq 2, +\hat{x}}$ (G) $\boldsymbol{1 < E \leq 2, -\hat{x}}$, (H) $\boldsymbol{1 < E \leq 2, +\hat{y}}$, (J) $1< \boldsymbol{E \leq 2, -\hat{y}}$,
(K) $2< \boldsymbol{E \leq 3, +\hat{x}}$ (L) $\boldsymbol{2 < E \leq 3, -\hat{x}}$, (M) $\boldsymbol{2 < E \leq 3, +\hat{y}}$, (N) $2< \boldsymbol{E \leq 3, -\hat{y}}$,
(O) $3< \boldsymbol{E, +\hat{x}}$ (P) $\boldsymbol{3 < E, \ -\hat{x}}$, (Q) $\boldsymbol{3 < E, \ +\hat{y}}$, (R) $3< \boldsymbol{E, \ -\hat{y}}$,

5. (5pts) Fig. 7 shows that a long circular pipe with outer radius $\boldsymbol{R}$ carries a uniformly distributed current $+\boldsymbol{I_0}$ into the page ($\boldsymbol{I_0}$ >0) and a wire runs parallel to the pipe at a distance of 3R from center to center. The current in the wire is $\boldsymbol{I_{wire}} = \boldsymbol{a \cdot I_0}$ such that the net magnetic field at point $\boldsymbol{P}$ has the same magnitude as the net magnetic field at the center $\boldsymbol{O}$ of the pipe but is in the opposite direction. (A) a<-8; (B) -8≦a<-4; (C) -4≦a<-1; (D) -1≦a<-0.5; (E) -0.5≦ a <-0.25; (F) -0.25≦a<0.25; (G) 0.25≦a<0.5; (H) 0.5≦a <1; (J) 1≦ a <4; (K) 4≦ a <8; (L) 8≦ a.

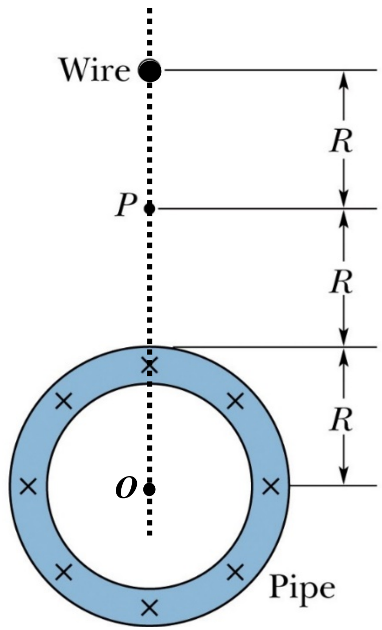


Fig. 7

Integration Formula for reference

$$\int \frac{dx}{\sqrt{x^2 \pm b^2}} = \ln\left(x + \sqrt{x^2 \pm b^2}\right) \qquad \int \frac{x^2 dx}{\sqrt{x^2 \pm b^2}} = \frac{x\sqrt{x^2 \pm b^2}}{2} - \frac{b^2}{2}\ln\left(x + \sqrt{x^2 \pm b^2}\right)$$

$$\int \frac{dx}{\left(x^2 \pm b^2\right)^{3/2}} = \frac{\pm x}{b^2\sqrt{x^2 \pm b^2}} \qquad \int \frac{x^2 dx}{\left(x^2 \pm b^2\right)^{3/2}} = \frac{-x}{\sqrt{x^2 \pm b^2}} + \ln\left(x + \sqrt{x^2 \pm b^2}\right)$$

Multiple Choice Questions:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
G	F	F	F	G	B	C	G	E	C
11	**12**	**13**	**14**	**15**					
A	D	B	B	F					

1. (20 pts) Fig. *1* shows a four-section conducting wire on x-y plane with current I. The first section is from A to B on the x-axis. The second section is from B to C is a semi-circle with radius 3***R***. The third section is from C to D. The last section is from D to A a quater-circle with radius ***R***. Find the *x*-, *y*-, *z*-components of the magnetic field at point ***P*** on the *z*-axis due to
(a)(8 pts) current in the section from A to B,
(b)(3 pts) current in the section from C to D,
(c)(7 pts) current in the section from B to C.
(d)(2 pts) current in the section from D to A
The coordinates of ***A***,***B***, C,Dand ***P*** are **(- *R*, *0*,*0*)**, **(3*R*, *0*,*0*)**, **(*0*, 3*R*,*0*)**, **(*0*, *R*,*0*)** and **(*0*,*0*,*z*)**, respectively.

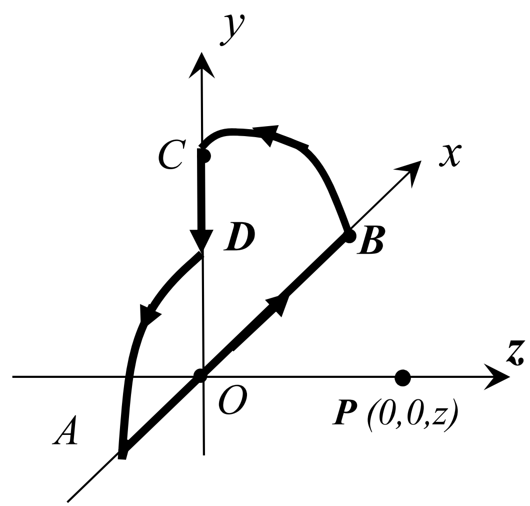
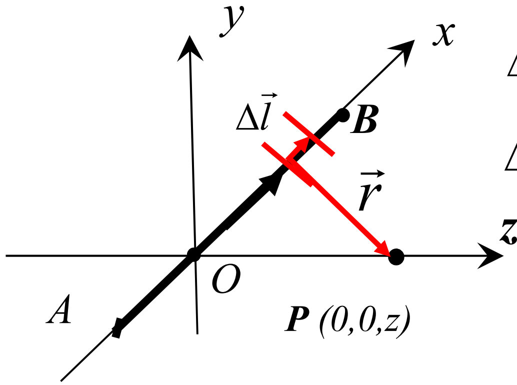


Fig. 1

4. (A) current in the section from A to B B:



$$\Delta\vec{B}=\frac{\mu_0}{4\pi}\cdot\frac{I\Delta\vec{l}\times\hat{r}}{r^2}=\frac{\mu_0}{4\pi}\cdot\frac{I\Delta\vec{l}\times\vec{r}}{r^3} \quad 1$$

$$\Delta\vec{l}=\Delta x\,(\hat{x}) \quad 1$$

$$\vec{r}=(0,0,z)-(x,0,0)=(-x,0,z)$$

$$\hat{r}=\frac{(-x,0,z)}{\sqrt{z^2+x^2}} \quad 1$$

$$\Delta\vec{l}\times\vec{r}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \Delta x & 0 & 0\\ -x & 0 & z\end{vmatrix}=z\Delta x\cdot\left(-\hat{j}\right) \quad 1$$

$$\therefore\ \Delta\vec{B}\left(=\frac{\mu_0 I}{4\pi}\frac{\Delta x\left(-\hat{j}\right)}{x^2+z^2}\sin\theta\right)=\frac{\mu_0 I}{4\pi}\frac{z\Delta x\left(-\hat{j}\right)}{\sqrt{x^2+z^2}^{\,3}}$$

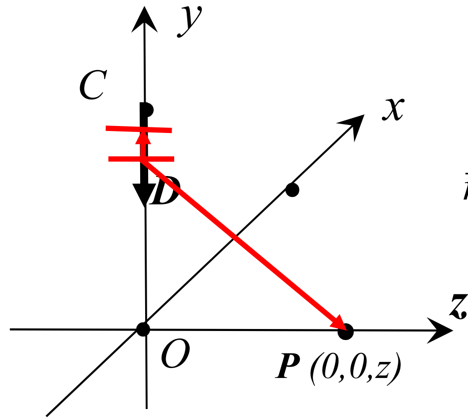
(i) 查積分表: $\displaystyle\int\frac{dx}{\left(x^2\pm a^2\right)^{\frac{3}{2}}}=\frac{\pm x}{a^2\sqrt{x^2\pm a^2}}$

$$\vec{B}_1=\frac{\mu_0 Iz}{4\pi}\left(-\hat{j}\right)\int_{-R}^{3R}\frac{dx}{\sqrt{x^2+z^2}^{\,3}}=\frac{\mu_0 Iz}{4\pi}\left(-\hat{j}\right)\left.\frac{x}{z^2\sqrt{x^2+z^2}}\right|_{-R}^{3R} \quad 1$$

$$=\frac{\mu_0 Iz}{4\pi}\left(-\hat{j}\right)\cdot\left(\left.\frac{x}{z^2\sqrt{x^2+z^2}}\right|_{-R}^{3R}\right)$$

$$=\frac{\mu_0 I}{4\pi z}\left(\frac{3R}{\sqrt{9R^2+z^2}}+\frac{R}{\sqrt{R^2+z^2}}\right)\left(-\hat{j}\right) \quad 2$$

(B) current in the section from C to D:



$$\Delta\vec{l}=\Delta y\,(\hat{y})$$

$$\vec{r}=(0,0,z)-(y,0,0)$$
$$=(-x,0,z)$$

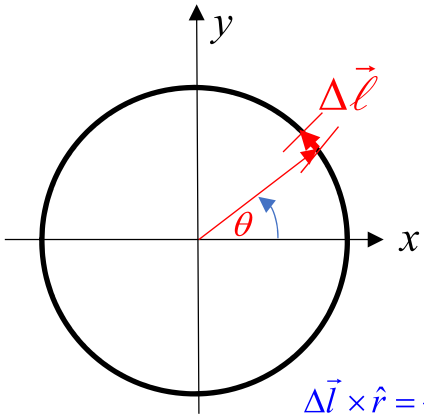
$$\Delta\vec{l}\times\vec{r}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & \Delta y & 0\\ 0 & -y & z\end{vmatrix}=z\Delta y\cdot\left(\hat{i}\right) \quad 1$$

$$\vec{B}_2=\frac{\mu_0 Iz}{4\pi}\left(\hat{i}\right)\int_{3R}^{R}\frac{dy}{\sqrt{y^2+z^2}^{\,3}}$$

$$=\frac{\mu_0\left(-I\right)z}{4\pi}\left(\hat{i}\right)\int_{R}^{3R}\frac{dy}{\sqrt{y^2+z^2}^{\,3}}$$

$$=\frac{\mu_0(-I)}{4\pi z}\left(\frac{3R}{\sqrt{9R^2+z^2}}-\frac{R}{\sqrt{R^2+z^2}}\right)(\hat{x}) \quad 2$$

current in the section from B to C:



$$\vec{r} = (0,0,z) - (3R\cos\theta, 3R\sin\theta, 0)$$
$$= (-3R\cos\theta, -3R\sin\theta, z) \quad 1$$

$$\hat{r} = \frac{(-3R\cos\theta, -3R\sin\theta, z)}{\sqrt{z^2+9R^2}}$$

$$d\vec{\ell} = (3Rd\theta)\left(\cos\left(\frac{\pi}{2}+\theta\right), \sin\left(\frac{\pi}{2}+\theta\right), 0\right)$$
$$= (3Rd\theta)(-\sin\theta, \cos\theta, 0) \quad 1$$

$$\Delta\vec{l} \times \hat{r} = \frac{3R\Delta\theta}{\sqrt{9R^2+z^2}} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\sin\theta & \cos\theta & 0 \\ -3R\cos\theta & -3R\sin\theta & z \end{vmatrix}$$

$$= \frac{3R\Delta\theta}{\sqrt{9R^2+z^2}}\left(z\cos\theta\hat{i} + z\sin\theta\hat{j} + 3R\hat{k}\right) \quad 1$$

$$\Delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I3R\Delta\theta\left(z\cos\theta\hat{i} + z\sin\theta\hat{j} + 3R\hat{k}\right)}{\sqrt{9R^2+z^2}^3}$$

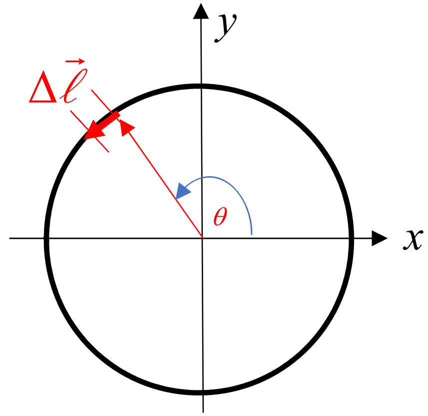
1

$$\vec{B} = \frac{\mu_0 I3R}{4\pi\sqrt{9R^2+z^2}^3}\left\{z\cdot\hat{i}\int_0^{\pi/2}\cos\theta d\theta + z\cdot\hat{j}\int_0^{\pi/2}\sin\theta d\theta + 3R\cdot\hat{k}\int_0^{\pi/2}d\theta\right\}$$

$$\boxed{= \frac{\mu_0 I3R}{4\pi\sqrt{9R^2+z^2}^3}\left\{z\hat{i} + z\cdot\hat{j} + \frac{3\pi R}{2}\cdot\hat{k}\right\}}$$

1 1 1

current in the section from D to A: I



同左邊做法: 3R→R

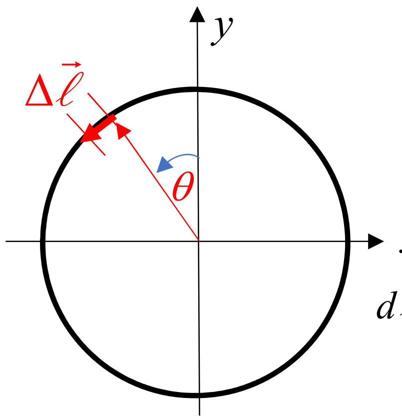
1

$$\vec{B} = \frac{\mu_0 IR}{4\pi\sqrt{R^2+z^2}^3}\left\{z\cdot\hat{i}\int_{\pi/2}^{\pi}\cos\theta d\theta + z\cdot\hat{j}\int_{\pi/2}^{\pi}\sin\theta d\theta + R\cdot\hat{k}\int_{\pi/2}^{\pi}d\theta\right\}$$

$$\boxed{= \frac{\mu_0 IR}{4\pi\sqrt{R^2+z^2}^3}\left\{-z\hat{i} + z\cdot\hat{j} + \frac{\pi R}{2}\cdot\hat{k}\right\}}$$

2

current in the section from D to A: II



$$\vec{r}=(0,0,z)-(-R\sin\theta,R\cos\theta,0)$$
$$=(R\sin\theta,-R\cos\theta,z)$$

$$\hat{r}=\frac{(R\sin\theta,-R\cos\theta,z)}{\sqrt{z^2+R^2}}$$

$$d\vec{\ell}=(Rd\theta)\left(-\sin\left(\frac{\pi}{2}+\theta\right),\cos\left(\frac{\pi}{2}+\theta\right),0\right)$$
$$=(Rd\theta)(-\cos\theta,-\sin\theta,0)$$

$$\Delta\vec{l}\times\hat{r}=\frac{R\Delta\theta}{\sqrt{R^2+z^2}}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -\cos\theta & -\sin\theta & 0\\ R\sin\theta & -R\cos\theta & z\end{vmatrix}$$

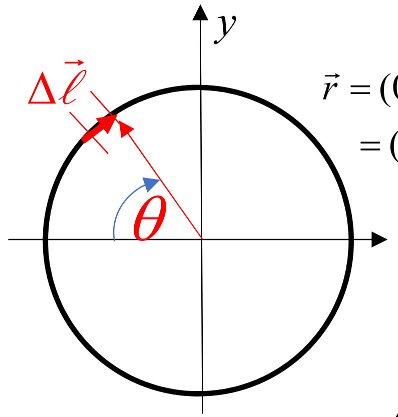
$$=\frac{R\Delta\theta}{\sqrt{R^2+z^2}}\left(-z\sin\theta\hat{i}+z\cos\theta\hat{j}+R\hat{k}\right)$$ ①

$$\Delta\vec{B}=\frac{\mu_0}{4\pi}\frac{IR\Delta\theta\left(-z\sin\theta\hat{i}+z\cos\theta\hat{j}+R\hat{k}\right)}{\sqrt{R^2+z^2}^3}$$

$$\vec{B}=\frac{\mu_0 IR}{4\pi\sqrt{R^2+z^2}^3}\left\{-z\cdot\hat{i}\int_0^{\pi/2}\sin\theta d\theta+z\cdot\hat{j}\int_0^{\pi/2}\cos\theta d\theta+R\cdot\hat{k}\int_0^{\pi/2}d\theta\right\}$$

$$\boxed{=\frac{\mu_0 IR}{4\pi\sqrt{R^2+z^2}^3}\left\{-z\hat{i}+z\cdot\hat{j}+\frac{\pi R}{2}\cdot\hat{k}\right\}}$$ ②

current in the section from D to A: III



$$\vec{r}=(0,0,z)-(-R\cos\theta,R\sin\theta,0)$$
$$=(R\cos\theta,-R\sin\theta,z)$$

$$\hat{r}=\frac{(R\cos\theta,-R\sin\theta,z)}{\sqrt{z^2+R^2}}$$

$$d\vec{\ell}=(Rd\theta)\left(-\cos\left(\frac{\pi}{2}+\theta\right),\sin\left(\frac{\pi}{2}+\theta\right),0\right)$$
$$=(Rd\theta)(\sin\theta,\cos\theta,0)$$

$$\Delta\vec{l}\times\hat{r}=\frac{R\Delta\theta}{\sqrt{R^2+z^2}}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \sin\theta & \cos\theta & 0\\ R\cos\theta & -R\sin\theta & z\end{vmatrix}$$

$$=\frac{R\Delta\theta}{\sqrt{R^2+z^2}}\left(z\cos\theta\hat{i}-z\,c\sin\theta\hat{j}-R\hat{k}\right)$$ ①

$$\Delta\vec{B}=\frac{\mu_0}{4\pi}\frac{IR\Delta\theta\left(z\cos\theta\hat{i}-z\sin\theta\hat{j}+R\hat{k}\right)}{\sqrt{R^2+z^2}^3}$$

$$\vec{B}=\frac{\mu_0(-I)R}{4\pi\sqrt{R^2+z^2}^3}\left\{z\hat{i}-z\cdot\hat{j}-\frac{\pi R}{2}\cdot\hat{k}\right\}$$

$$\boxed{=\frac{\mu_0 IR}{4\pi\sqrt{R^2+z^2}^3}\left\{-z\hat{i}+z\cdot\hat{j}+\frac{\pi R}{2}\cdot\hat{k}\right\}}$$ ②

2. (15 pts.) A conducting bar of mass $\boldsymbol{m}$ is placed on two frictionless conducting rails which make an angle $\theta = 30^0$ relative to the horizontal surface. The rails are connected through a resistor $\boldsymbol{R}$ and are separated by a distance $\boldsymbol{L}$, as shown in Fig. 2. In addition, a uniform magnetic field $\vec{B}_0$ is applied vertically upward (as shown in Fig 2). Initially, the bar is at the bottom of the rails with velocity v_0 and starts to move upward.

a)(3pts) Find the magnitude and the direction (from "***a*** to ***b***" or "***b*** to ***a***") of the induced current in the conducting bar? Write the induced current $I(t)$ in terms of B_0, L, R, g (gravitational acceleration), v (speed of the bar) and other necessary constants.

b)(4pts) Find the magnetic force (magnitude and direction) due to the induced current $I(t)$. Draw the free body diagram of the conducting bar.

c)(5 points) Find the speed $(v(t))$ of the conducting bar as function of time. Write your answer in terms of m, B_0, L, R, g and/or other necessary constants.

d)(3pts) How long does it take for the conducting bar to reach the highest point (i.e., $v = 0$)

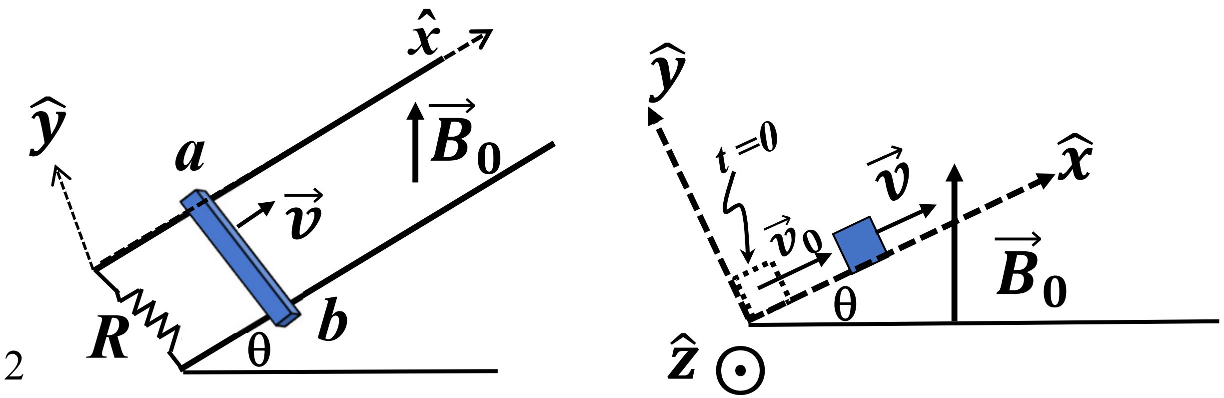
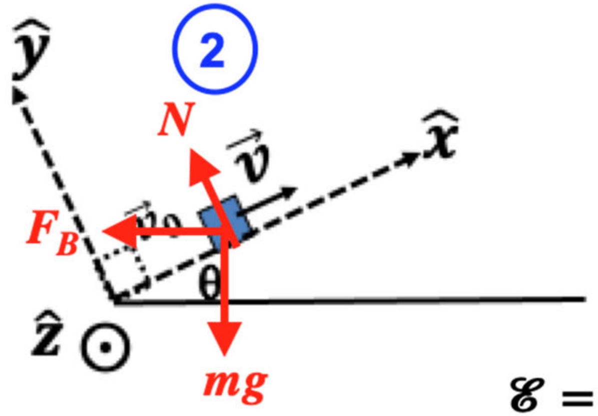


Fig. 2

2(a)



2

$$\vec{B} = B_0 \sin\theta \hat{x} + B_0 \cos\theta \hat{y} = B_0 \left(\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{y} \right)$$

I: $a \to b$ or $\vec{I} = I\hat{z}$ 1

$$\mathcal{E} = IR = \frac{d\Phi_B}{dt} = \frac{d}{dt}\vec{B} \cdot \vec{A} = \frac{d}{dt}B_0 L x \cos\theta = \frac{\sqrt{3}}{2}B_0 L v(t)$$

$$I(t) = \frac{B_0 L v(t) \cos\theta}{R} = \frac{\sqrt{3}}{2}\frac{B_0 L v(t)}{R}$$ 2

(b)

$$\vec{F}_B = IL\hat{z} \times \vec{B} = ILB_0 \sin\theta \hat{y} - ILB_0 \cos\theta \hat{x} = ILB_0 \left(-\frac{\sqrt{3}}{2}\hat{x} + \frac{1}{2}\hat{y} \right)$$

$$= \frac{\sqrt{3}L^2 B_0^2}{2R} v(t) \left(-\frac{\sqrt{3}}{2}\hat{x} + \frac{1}{2}\hat{y} \right)$$ 2

(c)

$$m\frac{dv(t)}{dt} = \sum F_x = -mg\sin\theta + F_{B,x} = -\frac{mg}{2} - \frac{3}{4}\frac{B_0^2 L^2}{R}v(t)$$ 2

$$\frac{dv(t)}{dt} = -\frac{g}{2} - \frac{3}{4}\frac{B_0^2 L^2}{mR}v(t) = -\frac{g}{2} - \frac{1}{\tau}v, \qquad \tau^{-1} = \frac{3}{4}\frac{B_0^2 L^2}{mR}$$

$$\int_{v_0}^{v(t)} \frac{dv}{v + \frac{g\tau}{2}} = \frac{-1}{\tau}\int_0^t dt = -\frac{t}{\tau} \qquad \to v(t) = \frac{-g\tau}{2} + \left(v_0 + \frac{g\tau}{2}\right) e^{-t/\tau}$$

1 2

$$\tau^{-1} = \frac{3}{4}\frac{B_0^2 L^2}{mR}$$

(d) $v(t_f) = 0 \to \frac{g\tau}{2} = \left(v_0 + \frac{g\tau}{2}\right) e^{-t_f/\tau}$ or $t_f = \tau \ln\left(\frac{2v_0}{g\tau} + 1\right)$

1

$$t_f = \frac{4mR}{3B_0^2L^2} \ln\left(\frac{3v_0B_0^2L^2}{2mgR} + 1\right)$$

3. (a) (8 pts) Fig. 2(a) shows a cross section of an infinite long cylindrical conductor with radius a and current density $\vec{J}(r') = J_0[1-(r'/a)]\,\hat{z}$, where r' is where r' is the distance to the axis of this cylinder and the direction of current is out of page. Find the magnetic fields on the x-axis in the range $0 \le x \le 4a$.

(b) (6 pts) Fig.2(b) shows the cross sectional view of an infinite plate of thickness 4a carries uniform current density of $-J_0\,\hat{z}$ (into page), and besides, there is an infinitely long cylindrical hollow region with radius a. Find the magnetic fields at (-a, 0, 0).

(c) (6 pts) In Fig.2(c), a cross section of an infinite long cylindrical conductor with radius a and current density $\vec{J}(r') = J_0[1-(r'/a)]\,\hat{z}$ in Fig.2(a) is completely put into hollow region in Fig.2(b). Find the magnetic field at (-2a, 4a, 6a).

(Use Ampere's law, draw the loop path for the integral, and write down the answer in terms of J_0, a, μ_0, and x.)

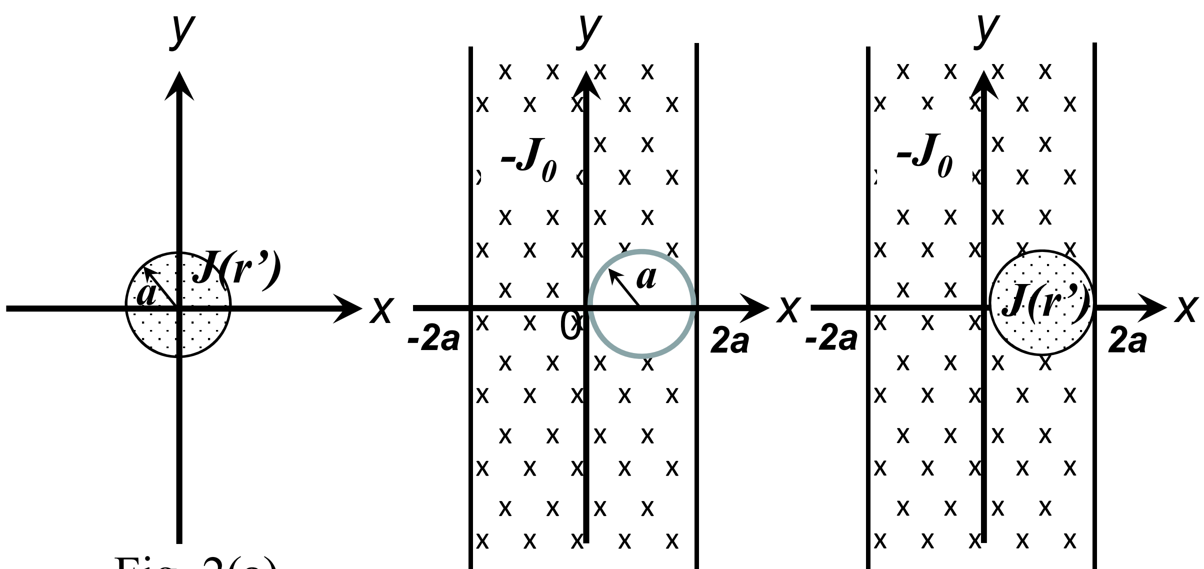
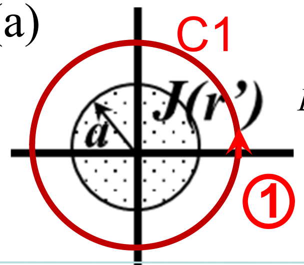


Fig. 2(a) Fig. 2(b) Fig. 2(c)

3. (a) (8 pts) Fig. 2(a) shows a cross section of an infinite long cylindrical conductor with radius a and current density $\vec{J}(r') = J_0[1-(r'/a)]\,\hat{z}$ (direction of current is out of page). Find the magnetic fields on the x-axis in the range $0 \le x \le 4a$. (b) (6 pts) Fig.2(b) shows the cross sectional view of an infinite plate of thickness 4a carries uniform current density of $-J_0\,\hat{z}$ (into page), and besides, there is an infinitely long cylindrical hollow region with radius a. Find the magnetic fields at (-a, 0, 0).(c) (6 pts) In Fig.2(c), a cross section of an infinite long cylindrical conductor with radius a and current density $\vec{J}(r') = J_0[1-(r'/a)]\,\hat{z}$ in Fig.2(a) is completely put into hollow region in Fig.2(b). Find the magnetic field at (-2a, 4a, 6a). (Use Ampere's law, draw the loop path for the integral, and write down the answer in terms of J_0, a, μ_0 and x.)

(a)

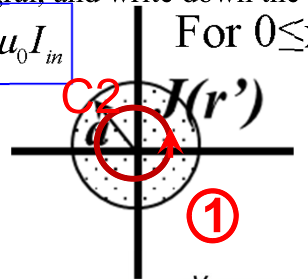


For a≤x≤4a

$$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{in}}$$

$$B2\pi x = \mu_0\int_0^a J_0[1-\frac{r'}{a}]2\pi r'\,dr' = \frac{\mu_0 J_0 \pi a^2}{3}$$ ① ①

$$\vec{B} = \frac{\mu_0 J_0 a^2}{6x}\hat{j}$$ ①

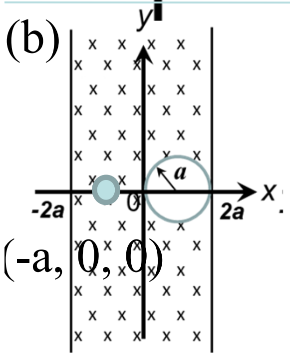


For 0≤x≤a

$$B2\pi x = \mu_0\int_0^x J_0[1-\frac{r'}{a}]2\pi r'\,dr' = \mu_0 J_0 \pi[x^2 - \frac{2x^3}{3a}]$$ ① ①

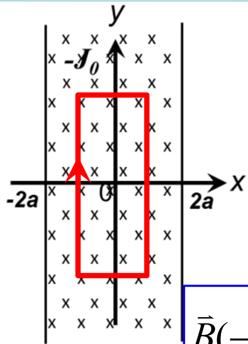
$$\vec{B} = \frac{\mu_0 J_0}{2x}[x^2 - \frac{2x^3}{3a}]\hat{j} = \frac{\mu_0 J_0}{2}[x - \frac{2x^2}{3a}]\hat{j}$$ ①

(b)



(-a, 0, 0)

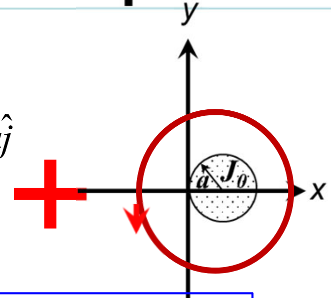
② =



$$2B_p L = \mu_0 J_0 2aL$$

$$\vec{B}_p(-a,0,0) = \mu_0 J_0 a\hat{j}$$ ①

+



$$B_c 2\pi r = \mu_0 J_0 \pi a^2$$

$$\vec{B}_c(r) = \frac{\mu_0 J_0 a^2}{2r}\hat{\phi}$$ ①

$$\vec{B}_c(-a,0,0) = -\frac{\mu_0 J_0 a^2}{2(2a)}\hat{j} = -\frac{\mu_0 J_0 a}{4}\hat{j}$$ ①

$$\boxed{\vec{B}(-a,0,0) = \mu_0 J_0 a\hat{j} - \frac{\mu_0 J_0 a}{4}\hat{j} = \frac{3\mu_0 J_0 a}{4}\hat{j}}$$ ①

(c)(-2a, 4a, 6a)

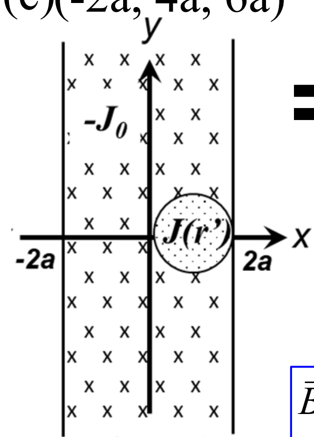


Fig. 2(c)

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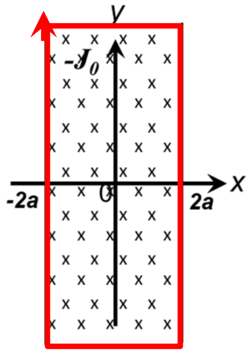
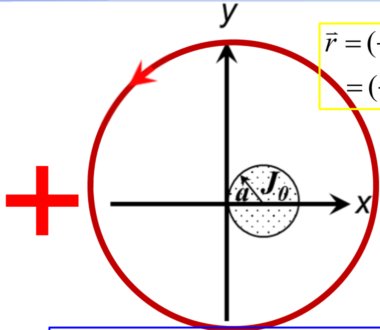


$$\boxed{\vec{B}_p(-2a,4a,6a) = 2\mu_0 J_0 a\hat{j}}$$ ①

+

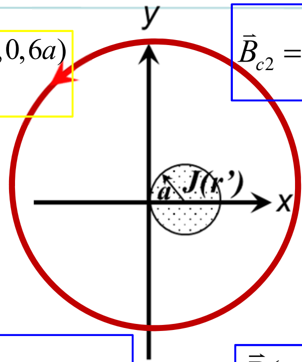


$$\boxed{\begin{aligned}\vec{r} &= (-2a,4a,6a)-(a,0,6a)\\ &= (-3a,4a,0)\end{aligned}}$$

$$\vec{B}_{c1}(-2a,4a,6a) = \frac{\mu_0 J_0 a^2}{2(5a)}\hat{l}\times\hat{r}$$ ①

$$= \frac{\mu_0 J_0 a^2}{2(5a)}[(0,0,1)\times(\frac{-3}{5},\frac{4}{5},0)] = \frac{\mu_0 J_0 a}{10}[-\frac{4}{5}\hat{i} - \frac{3}{5}\hat{j}]$$ ①

+



$$\boxed{\vec{B}_{c2} = \frac{\mu_0 J_0 a^2}{6(5a)}\hat{l}\times\hat{r} = \frac{\mu_0 J_0 a}{30}[-\frac{4}{5}\hat{i} - \frac{3}{5}\hat{j}]}$$ ① ①

$$\boxed{\begin{aligned}\vec{B}(-2a,4a,6a) &= \vec{B}_P + \vec{B}_{c1} + \vec{B}_{c2}\\ &= \frac{-8\mu_0 J_0 a}{75}\hat{i} + \frac{48\mu_0 J_0 a}{25}\hat{j}\end{aligned}}$$ ①