Homework 7 (Chap 9)

1. Block A with mass ***0.8 m*** is on block B with mass ***m***. Both are initially at rest. Now a bullet with mass ***0.2m*** and velocity $\boldsymbol{5\,v_0}$ hits and is embedded in block A. Assume all the surface are frictionless.

a) What is the velocity $\boldsymbol{v_A}$ of block A immediately after the collision?

b) Eventually, the block A slides down block B. Assume the height $\boldsymbol{H = 2v_0^2/g}$, what is the velocity $\boldsymbol{u_A}$ of block A and velocity $\boldsymbol{u_B}$ of block B after separation? .

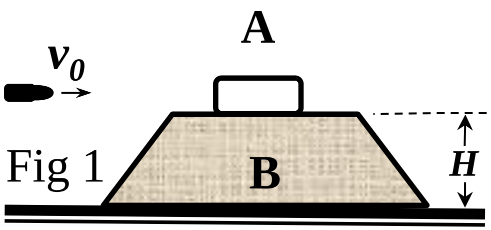

Fig 1

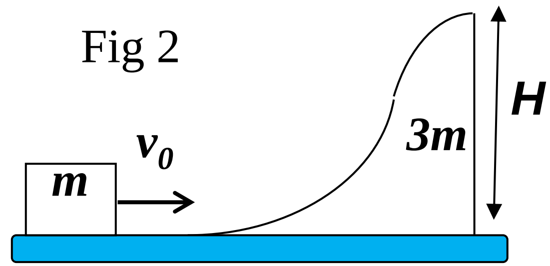

Fig 2

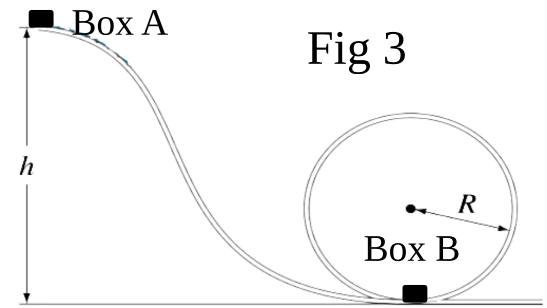

Fig 3

2.(15pts) As shown in Fig. 2, a block of mass ***3m*** with height ***H*** sits at rest on a frictionless table. A small cube of mass ***m*** with velocity $\boldsymbol{v_0}$ moves toward the block. Assume that all the surfaces between the inclined block, cube and the table are frictionless.

(A) (4pts) What are the velocities of the cube and the block respectively when the cube reach the highest position on the block but remains on the block?

(B) (6pts) What is the maximum velocity of $\boldsymbol{v_0}$, such that the cube reaches the height ***H*** but does not run over it? Write your answer in terms of ***m***, ***H***, and ***g***

(C) (5pts) Once the cube reach the height ***H***, it begins to slide down. What are the velocities of cub $\boldsymbol{v_1}$ and the block $\boldsymbol{v_2}$ when they separate. Write your answer in terms of ***m*** and $\boldsymbol{v_0}$.

3. Box A of mass ***m*** is released from rest at the top of height ***h***, shown in Fig. 3. It collides with box B of mass 2***m*** elastically, and the Box B move along a circular vertical loop. What is the minimum height ***h*** such that the box B can complete the circular motion?

1. Block A with mass ***0.8 m*** is on block B with mass ***m***. Both are initially at rest. Now a bullet with mass ***0.2m*** and velocity ***5 v_0*** hits and is embedded in block A. Assume all the surface are frictionless.

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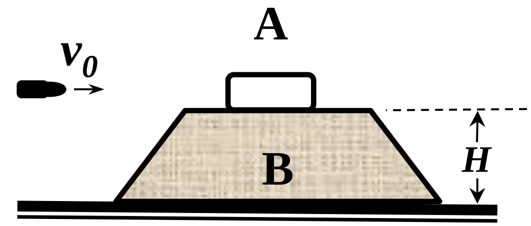


(a) Embedded process:

Momentum conservation: $P_i = P_f$

$$0.2m \cdot 5v_0 = (0.2m + 0.8m)v_A$$

$$\Rightarrow \quad v_A = v_0$$

(b) After separation:

Momentum conservation: $P_i = P_f$

$$mv_0 = mu_A + mu_B \qquad \text{--- (1)}$$

Mechanical Energy conservation: $E_i = E_f$

$$\frac{1}{2}mv_0^2 + mgH = \frac{1}{2}mu_A^2 + \frac{1}{2}mu_B^2 \qquad \text{--- (2)}$$

$$\frac{1}{2}mv_0^2 + 2mv_0^2 = \frac{1}{2}mu_A^2 + \frac{1}{2}mu_B^2$$

(1) ➔ $v_0 = u_A + u_B$ ➔ $u_B = v_0 - u_A$ --- (3

(2) ➔ $v_0^2 + 4v_0^2 = u_A^2 + u_B^2$

$\boldsymbol{u_B}$ is Replaced by (3): $5v_0^2 = u_A^2 + (v_0 - u_A$

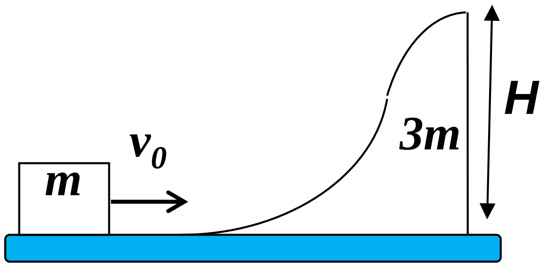
➔ $2u_A^2 - 2u_Av_0 - 4v_0^2 = 0$

$$(u_A + v_0)(u_A - 2v_0) = 0$$

$u_A = -v_0$ Make no sense, since block A is moving to the right.

$u_A = 2v_0$ and $u_B = -v_0$

2.(A)



Momentum conservation:

$$P_i = P_f$$ 1 pt

$$mv_0 = (m+3m)v_{cm}$$

$$\Rightarrow \quad v_{cm} = \tfrac{1}{4}v_0$$ 3 pts

When the cube reach the highest position on the block

$$v_{cube} = v_{block} = v_{cm} = \tfrac{1}{4}v_0$$

2.(B)

Energy conservation:

$$K_i + U_i = K_f + U_f$$

1 pt

$$\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}(4m)\left(\frac{v_0}{4}\right)^2 + mgH$$

2pts

$$\tfrac{3}{8}mv_0^2 = mgH$$

$$(v_0)_{\max} = \sqrt{\frac{8gH}{3}}$$

3pts

2.(C)

Momentum conservation:

$$mv_0 = mv_1 + 3mv_2$$ 1 pt

$$v_2 = \tfrac{1}{3}(v_0 - v_1)$$

Energy conservation:

$$\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}(3m)v_2^2$$ 1 pt

$$v_0^2 = v_1^2 + \tfrac{1}{3}(v_0 - v_1)^2$$

$$4v_1^2 - 2v_1v_0 - 2v_0^2 = 0$$

$$(2v_1 + v_0)(v_1 - v_0) = 0$$

$$\begin{cases} v_1 = -\tfrac{1}{2}v_0 \\ v_2 = \tfrac{1}{2}v_0 \end{cases}$$ 3 pts

$$\left(\begin{cases} v_1 = v_0 \\ v_2 = \end{cases} \text{不合題意,忽略}\right)$$

3. Box A of mass $\boldsymbol{m}$ starts release from rest at the top of height $\boldsymbol{h}$, as shown in Fig. 1. It collides with box B of mass $2\boldsymbol{m}$ elastically, and the Box B move along a circular vertical loop. What is the minimum height $\boldsymbol{h}$ such that the box B can pass the top of the loop?

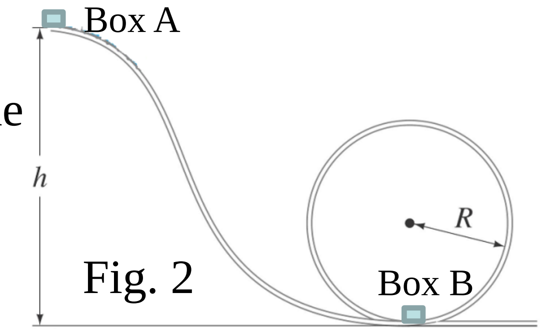


Fig. 2

The problem can be separated into three stages.

Stages I: the box A moves down.
The process is energy conservation.

$$m_A g h = \frac{1}{2} m_A v_{A,i}{}^2$$

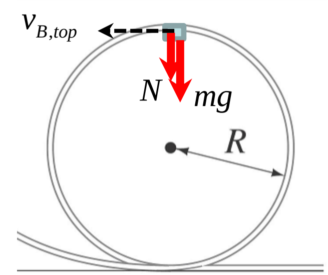
$$\Longrightarrow \quad v_{A,i} = \sqrt{2gh}$$

Stages II: Box A collides with box B.
The process is elastic.

$$v_{A,f} = \frac{m_A - m_B}{m_A + m_B} v_{A,i} + \frac{2m_B}{m_A + m_B} v_{B,i} = -\frac{1}{3} v_{A,i}$$

$$v_{B,f} = \frac{2m_A}{m_A + m_B} v_{A,i} + \frac{m_B - m_A}{m_A + m_B} v_{B,i} = \frac{2}{3} v_{A,i}$$

Stages III: the box B moves along the loop.
The criteria that the box B move to the top.



$$m_B g + N = m_B a_c = m_B \frac{v_{B,top}{}^2}{R}$$

$$\Longrightarrow \quad \left(v_{B,top}\right)_{\min} = \sqrt{gR}$$

when $\quad N = 0$

Along the loop, energy is conserved

$$\frac{1}{2} m_B v_{B,i}{}^2 = m_B g\left(2R\right) + \frac{1}{2} m_B \left(v_{B,top}\right)_{\min}^2$$

$$\frac{1}{2} m_B v_{B,i}{}^2 = m_B g\left(2R\right) + \frac{1}{2} m_B g R = \frac{5}{2} m_B g R$$

,and $\quad \frac{1}{2} m_B v_{B,i}{}^2 = \frac{1}{2} m_B \left(\left(\frac{2}{3}\right) v_{A,i}\right)^2 = \frac{1}{2} m_B \cdot \left(\frac{2}{3}\right)^2 \left(2gh\right)$

$$= \frac{4}{9} m_B g h$$

$$\Longrightarrow \quad \frac{4}{9} m_B g h = \frac{5}{2} m_B g R \quad \Longrightarrow \quad h = \frac{45}{8} R$$

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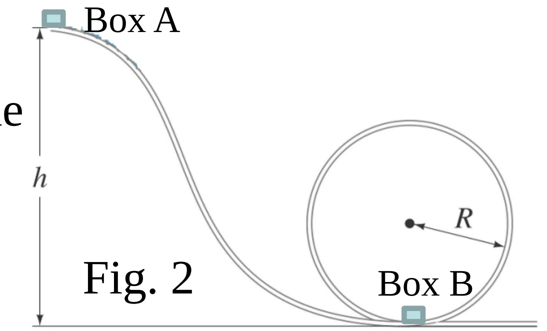

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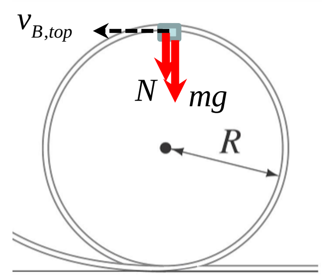

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