Signals and Systems – Quiz 1 03/23/2020

Name: Student ID:

1. (15%) Express each of the following complex numbers in polar form: (a) $-2j$ (b) $\frac{1}{2}j(1-j)$ (c) $(1-j)^2$

Ans: (a) $-2j = 2e^{-j\frac{\pi}{2}}$ (b) $\frac{1}{2}j(1-j) = \frac{\sqrt{2}}{2}e^{j\frac{\pi}{4}}$

(C) $(1-j)^2 = 2e^{-j\frac{\pi}{2}}$

2. (30%) Consider the RC circuit depicted in Fig.1. If we regard $v_s(t)$ as the input and $v_r(t)$ as the output signal, derive the differential equation describing the relationship between the input $v_s(t)$ and the output $v_r(t)$.

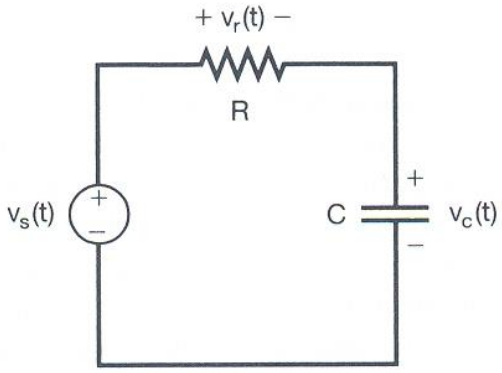

Fig. 1.

$$v_r(t) + v_c(t) = v_s(t)$$

$$\boxed{RC\frac{dv_c(t)}{dt} + v_c(t) = v_s(t)}$$

$$v_c(t) = v_s(t) - v_r(t)$$

$$RC\left(\frac{dv_s(t)}{dt} - \frac{dv_r(t)}{dt}\right) + v_s(t) - v_r(t) = v_s(t)$$

$$\Rightarrow RC\frac{dv_r(t)}{dt} + v_r(t) = RC\frac{dv_s(t)}{dt}$$

3. (30%) Determine whether or not the discrete-time system below is linear

$$y[n] = \Re e\{x[n]\}$$

Ans: let $x_1[n] = r[n] + js[n]$ *be the inpout* $\Rightarrow$ *the output* $y_1[n] = r[n]$

Consider another input $x_2[n]$ by scaling $x_1[n]$ with a complex number *a*, *a=j*

$$x_2[n] = jx_1[n] = j(r[n] + js[n]) = -s[n] + jr[n]$$

The output corresponding to $x_2[n]$ is $y_2[n] = \Re e\{x_2[n]\} = -s[n]$, which is not equal to the scaled version of $y_1[n]$,

$$ay_1[n] = jr[n]$$

Hence this system is not linear.

4. (25%) Consider an LTI system whose response to the signal $x_1(t)$ in Fig. 2(a) is the signal $y_1(t)$ illustrated in Fig. 2(b). Determine and sketch carefully the response of the system to the input $x_2(t)$ depicted in Fig. 2(c).

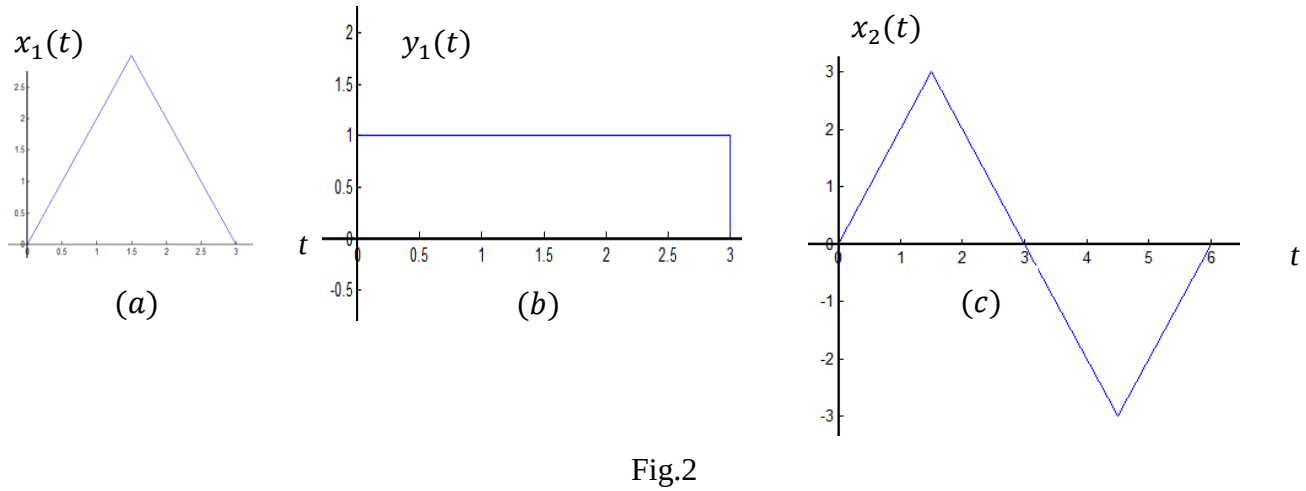

Fig.2

$x_2(t) = x_1(t) - x_1(t-3)$. Therefore, using linearity we get $y_2(t) = y_1(t) - y_1(t-3)$