Homework 10 (Chap11)

HW10-1 :

On a level billiards table a cue ball, initially at rest at point O on the table, is struck so that it leaves the cue stick with a center-of-mass speed v_0 and a “reverse” spin of angular speed ω_0 (see Fig). A kinetic friction force acts on the ball as it initially skids across the table.

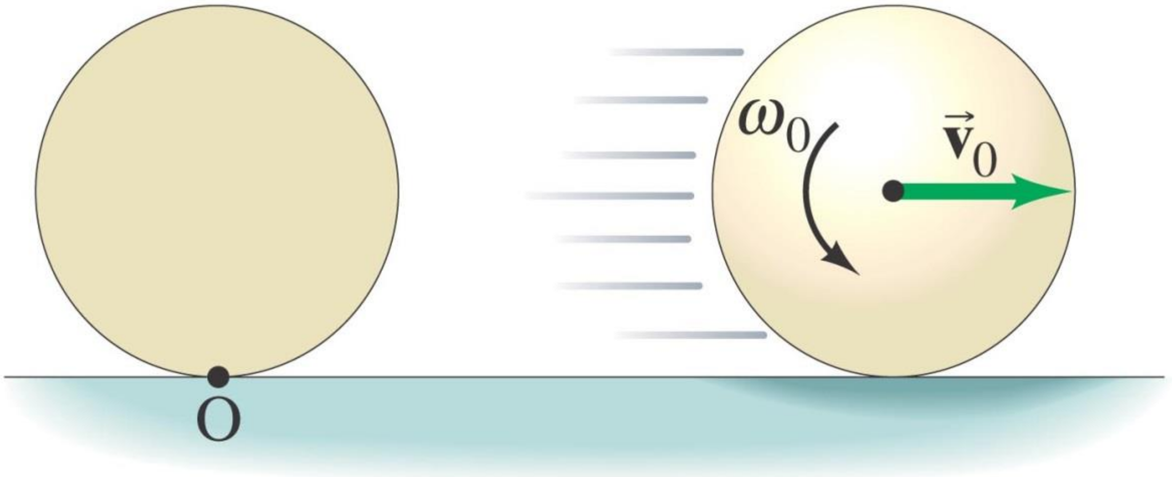
(*a*) Explain why the ball’s angular momentum is conserved about point O.

(*b*) Using conservation of angular momentum, find the critical angular speed ω_c such that, if $\omega_0 = \omega_c$ kinetic friction will bring the ball to a complete (as opposed to momentary) stop.

(*c*) If is 10% smaller than ω_c i.e., $\omega_0 = 0.90\omega_c$,determine the ball’s cm velocity when it starts to roll without slipping.

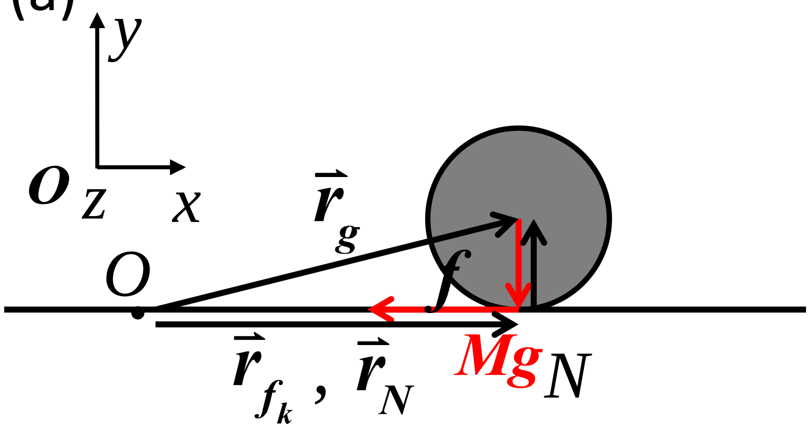
(*d*) If is 10% larger than ω_c i.e., $\omega_0 = 1.10\omega_c$,determine the ball’s cm velocity when it starts to roll without slipping.

[*Hint*: The ball possesses two types of angular momentum, the first due to the linear speed of its cm relative to point O, the second due to the spin at angular velocity about its own cm. The ball’s total *L* about O is the sum of these two angular momenta.]



Solution HW10-1 （sol 1, 角動量定義進紙面為正）

(a)



$$\vec{\tau}_{net,O} = \vec{r}_{f_k} \times \vec{f}_k + \vec{r}_N \times \vec{N} + \vec{r}_g \times m\vec{g}$$

$$= \vec{r}_{f_k} \times \vec{f}_k + r_N (N - mg)\hat{z}$$

$$= 0$$

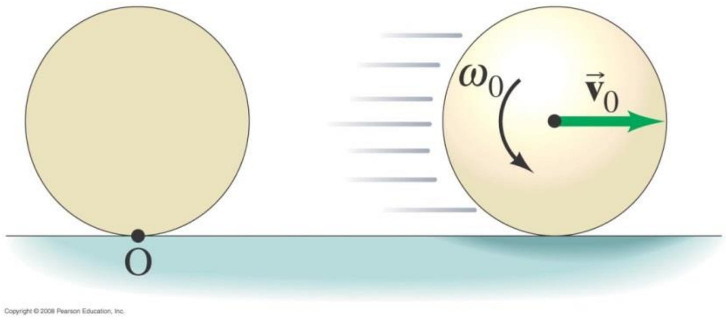
$$\frac{d\vec{L}}{dt} = \vec{\tau}_{net,O} = 0 \quad \Rightarrow \quad \vec{L} \text{ is conservation about point O}$$

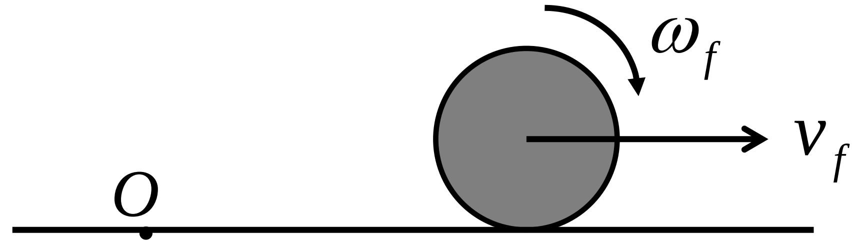
(b)

$$\vec{L}_{i,tot} = \vec{L}_{f,tot} , \vec{L}_{tot} = \vec{L}_{spin} + \vec{L}_{orb}$$

$$\vec{L}_{orb} = \vec{r} \times \vec{p} \propto \hat{z}$$

$$\vec{L}_{spin} = I_{CM}\vec{\omega} = I_{CM}\omega\hat{z}$$





$$\vec{L}_{i,tot} = I_{CM}\vec{\omega}_0 - mrv_0\hat{z}$$
$$= (I_{CM}\omega_0 - mrv_0)\hat{z}$$

$$\vec{L}_{f,tot} = I_{c.m.}\vec{\omega}_f - mrv_f\hat{z}$$
$$= (-I_{CM}\omega_f - mrv_f)\hat{z}$$

注意:

對於起始條件, 初角動量為逆時鐘出紙面的量(+z), 根據假設出紙面為+, 列式是毫無疑慮的.

要注意的是:

未知數的方向應依該未知數屬於的軸的正向假設,

在此即順時鐘, 進紙面

若依照假設”定義進紙面為-”, 代表你已指出純滾動的條件為

pure roll $\Rightarrow \quad v_f = r\omega_f$

最後若 ω_f, v_f 解出為負值則代表結果與假設方向相反, 即向左純滾動

pure roll $\Rightarrow v_f = r\omega_f \Rightarrow \vec{L}_{f,tot} = (-I_{CM} - mr^2)\omega_f \hat{z}$

$$\vec{L}_{i,tot} = \vec{L}_{f,tot} \Rightarrow (I_{CM}\omega_0 - mrv_0)\hat{z} = -(I_{CM} + mr^2)\omega_f \hat{z}$$

$$\Rightarrow \omega_f = \frac{mrv_0 - I_{CM}\omega_0}{I_{CM} + mr^2} \quad , \quad I_{CM} = \frac{2}{5}mr^2 , \quad v_f = r\omega_f$$

$$\Rightarrow v_f = \frac{2}{7}\left(\frac{5}{2}v_0 - r\omega_0\right)\cdots(\text{eq}^*)$$

$$v_f = \frac{2}{7}\left(\frac{5}{2}v_0 - r\omega_c\right) = 0 \quad \Rightarrow \omega_c = \frac{5}{2}\frac{v_0}{r}$$

(c),(d): bring $\omega_0 = 0.90\omega_c$, $\omega_0 = 1.10\omega_c$ in ep*

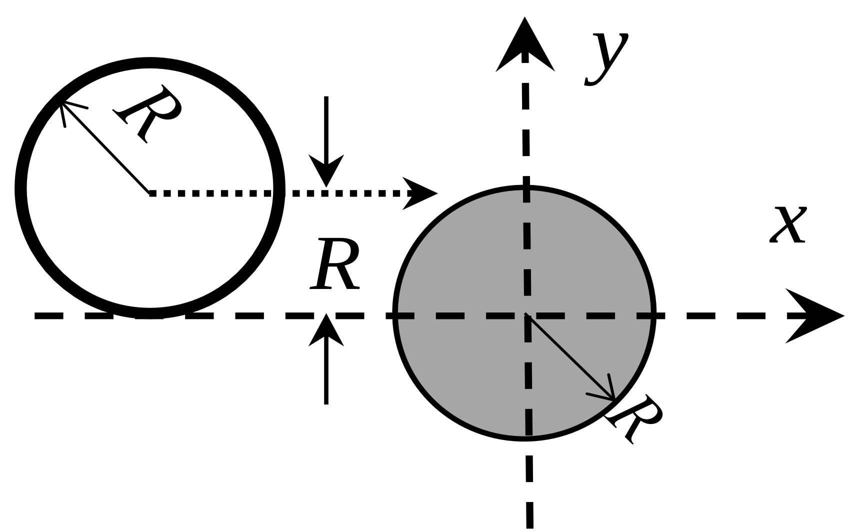
$$\Rightarrow v_f = \frac{1}{14}v_0 \ , \ v_f = -\frac{1}{14}v_0$$

(c),(d): bring $\omega_0 = 0.90\omega_c$, $\omega_0 = 1.10\omega_c$ in ep*

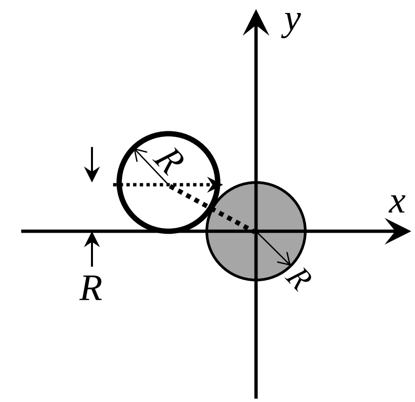
$$\Rightarrow v_f = \frac{1}{14} v_0 \ , \ v_f = -\frac{1}{14} v_0$$

HW10-2 :

As shown in the figure below, an uniform disk with radius ***R*** and mass ***M*** is rest on a horizontal frictionless surface. A ring of the same mass and radius is traveling at speed ***v*** with its center along a line parallel to x-axis, which is displaced from the ***x***-axis by a distance of ***R***. The ring collides the disk and both of them stick together at the contact point. Determine the final velocity (magnitude and direction) and the angular velocity (magnitude and direction) of the center of mass of the ring-disk assembly ?



2. As shown in Fig. z, an uniform disk with radius ***R*** and mass ***M*** is rest on a horizontal frictionless surface. A ring of the same mass and radius is traveling at speed ***v*** with its center along a line parallel to x-axis which is displaced from the ***x***-axis by a distance of ***R***. The ring collides the disk and both of them stick together at the contact point. Determine the final velocity (magnitude and direction) and the angular velocity (magnitude and direction) of the ring-disk assembly ?



Both of the ring and the disk are on a frictionless surface, and only the mutual interaction between them occur during the collision. Therefore, the linear momentum and the angular momentum of the system are conserved.

$\vec{P}_{i,total} = \vec{P}_{f,total} \quad \Rightarrow Mv = 2Mv_f,$ in x- direction. $\Rightarrow v_f = v/2, \quad or\ \vec{v}_f = v/2 \cdot \hat{x}.$

$$\vec{L}_{i,total} = \vec{L}_{f,total}$$

$$\Rightarrow \vec{r}_{ring,\mathrm{CM}} \times \vec{p}_{ring,\mathrm{CM}} = \vec{r}_{ring+disk,\mathrm{CM}} \times \vec{p}_{ring+disk,\mathrm{CM}} + I_{ring+disk,\mathrm{CM}} \cdot \vec{\omega}_f$$

$$\vec{r}_{ring,\mathrm{CM}} \times \vec{p}_{ring,\mathrm{CM}} = -MvR\hat{z}$$

After the collision, the y-coordinate of the center of mass is y_{cm} = M*R/2M = R/2, and the new momentum of inertia around the center of mass is

$$I_{ring+disk,\mathrm{CM}} = MR^2 + MR^2 + \frac{1}{2}MR^2 + MR^2 = \frac{7}{2}MR^2$$

$$\Rightarrow \vec{r}_{ring+disk,\mathrm{CM}} \times \vec{p}_{ring+disk,\mathrm{CM}} + I_{ring+disk,\mathrm{CM}} \cdot \vec{\omega} = -2M \cdot v_f \cdot \frac{R}{2}\hat{z} + \frac{7}{2}MR^2\omega_f\hat{z} = (-\frac{MvR}{2} + \frac{7}{2}MR^2\omega_f)\hat{z}$$

$$\Rightarrow -MvR\hat{z} = (-\frac{MvR}{2} + \frac{7}{2}MR^2\omega_f)\hat{z}$$

$$\Rightarrow -\frac{MvR}{2} = \frac{7}{2}MR^2\omega_f$$

$$\Rightarrow \omega_f = -\frac{v}{7R}, \quad or\ \omega_f = \frac{v}{7R}, \ clockwise.$$

HW10-3 :

As shown in the figure, a long pole, mass $\boldsymbol{m}$ ($\boldsymbol{I_{CM}=ml^2/12}$) and length $\boldsymbol{l}$, is hold horizontally and one of its end is fixed. It starts to fall and hit a uniform hollow sphere ($\boldsymbol{I_{CM} = 2/3\ MR^2}$). The collision is inelastic such that the pole is just bounced back to its original position. The hollow sphere is moving forward with $\boldsymbol{V_{CM}}$ and no angular velocity right after the collision. The kinetic (static) friction is μ_k (μ_s) between the contact surfaces.

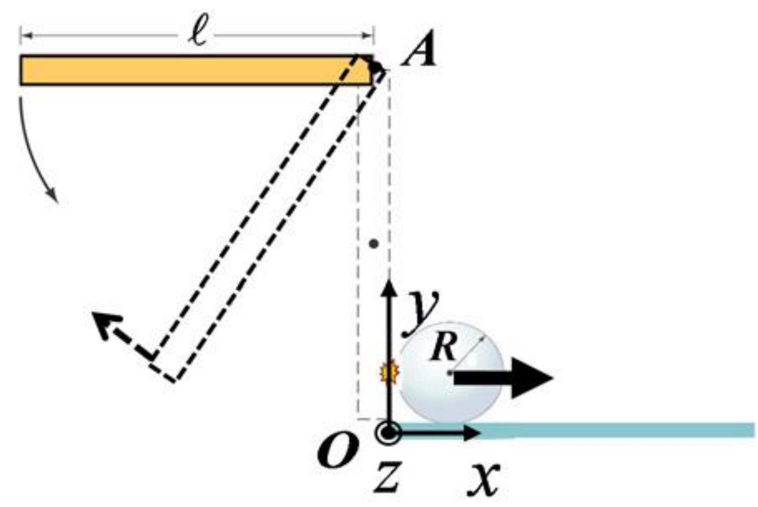
Let $\boldsymbol{l = 5R,\ m = 3M}$ and write your answers in terms of $\boldsymbol{M, R, g, \mu_s}$ and μ_k.

(a) What is the angular velocity ω_0 of the pole just before hit the hollow sphere?

(b) Find $\boldsymbol{V_{CM}}$ (the velocity of the center mass of the hollow sphere) right after the collision?

(c) After the collision, the hollow sphere starts moving and reaches in pure roll after a while. What is the velocity of its center mass $\boldsymbol{V_f}$ when the sphere is in pure roll? Evaluate the total angular momentum $\boldsymbol{L}$ (magnitude and direction) of the sphere relative to the point $\boldsymbol{O}$ at this moment.

(d) How far does the hollow sphere travel when it just reaches in pure roll?



Problem 10-3

(a) Mechanical energy conservation before the collision:

$$mg\ell = mg\frac{\ell}{2} + \frac{1}{2}I_A\omega_0^2 \Rightarrow \omega_0^2 = \frac{mg\frac{\ell}{2}}{\frac{1}{2}\frac{1}{3}m\ell^2} = \frac{3g}{\ell} = \frac{g}{2R}$$

$$I_A = \frac{1}{3}m\ell^2$$

$$\Rightarrow \omega_0 = \sqrt{\frac{3g}{\ell}} = \sqrt{\frac{g}{2R}}$$

(b) Angular momentum conservation relative to point A during the collision:

Just before collision: $\vec{L}_i = I_A\omega_0\left(\hat{z}\right), \quad \odot$

Right after collision: $\vec{L}_f = -I_A\omega_0\hat{z} + M(\ell - R)V_{CM}\left(\hat{z}\right),$

$\otimes \qquad \odot$

$$L_i = L_f \Rightarrow V_{CM} = \frac{2I_A\omega_0}{(\ell - R)M} = \frac{2\frac{1}{3}(3M)(5R)^2\omega_0}{4RM} = \frac{25}{2}\sqrt{\frac{gR}{2}}$$

(c) Hollow sphere: angular momentum conservation relative to point O after the collision:

Angular momentum right collision: $\vec{L}_i = MRV_{CM}\left(-\hat{z}\right), \quad \otimes$

Pure roll: $\vec{L}_f = I_{hollow}\omega_f\left(-\hat{z}\right) + MRV_f\left(-\hat{z}\right), \quad V_f = R\omega_f$

$$= \frac{5}{3}MRV_f\left(-\hat{z}\right)$$

$$L_i = L_f \Rightarrow V_f = \frac{3}{5}V_{CM} = \frac{15}{2}\sqrt{\frac{gR}{2}}$$

(d) Before the hollow sphere reaches pure roll, there is kinetic friction between the contact surface: $f_k = -\mu_k N = -\mu_k Mg$

$$V_f(t_f) = V_{CM}(t=0) - at_f = V_{CM} - \left(\mu_k g\right)t_f$$

$$\rightarrow t_f = \frac{1}{\mu_k g}\left(V_{CM} - V_f\right) = \frac{5}{\mu_k g}\sqrt{\frac{gR}{2}}$$

$$\Delta x = V_{CM}t_f - \frac{1}{2}at_f^2 = 25\frac{R}{\mu_k}$$

$$\text{or} \quad \Delta x = \frac{1}{2a}\left(V_{CM}^2 - V_f^2\right) = \frac{1}{2\mu_k g}\left[\left(\frac{25}{2}\right)^2 - \left(\frac{15}{2}\right)^2\right]\frac{gR}{2} = 25\frac{R}{\mu_k}$$