General Physics II: HW 8 **Due on 5/07/2020**

HW8-1: Problem 28.23 in Giancoli (pp. 868) (pp. 753)

A very long flat conducting strip of width ***d*** and negligible thickness lies in a horizontal plane and carries a uniform current ***I*** across its cross section.

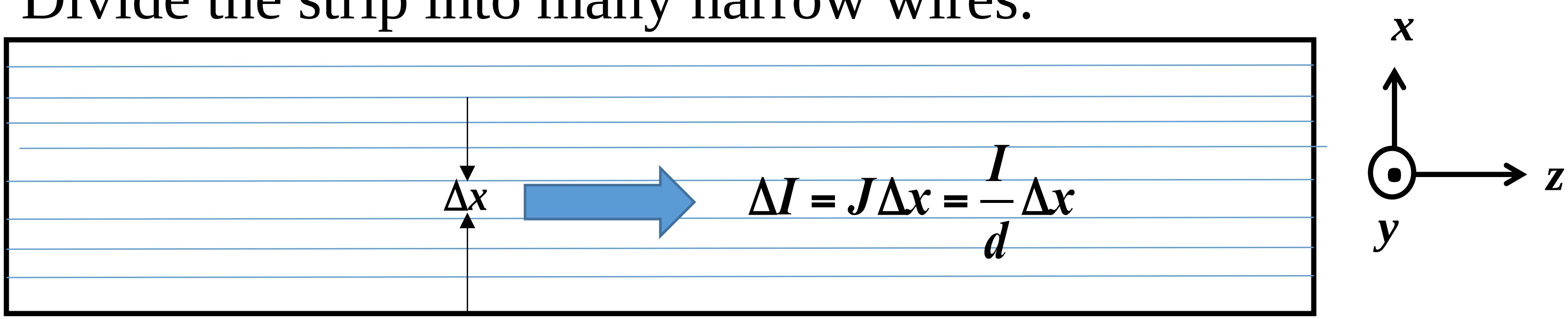
(a)Show that at points a distance ***y*** directly above its center, the field is given by

$$\boldsymbol{B = \frac{\mu_0 I}{\pi d} tan^{-1} \frac{d}{2y}}$$

Assuming the strip is infinitely long. [Hint: Divide the strip into many thin "wires," and sum (integrate) over these.]

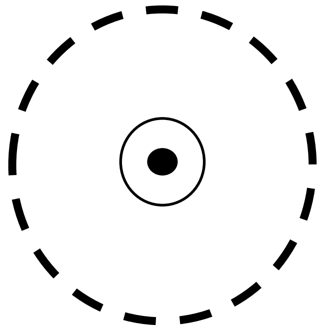
(b)What value does ***B*** approach for y>>d? Does this make sense? Explain.

(a) Divide the strip into many narrow wires.



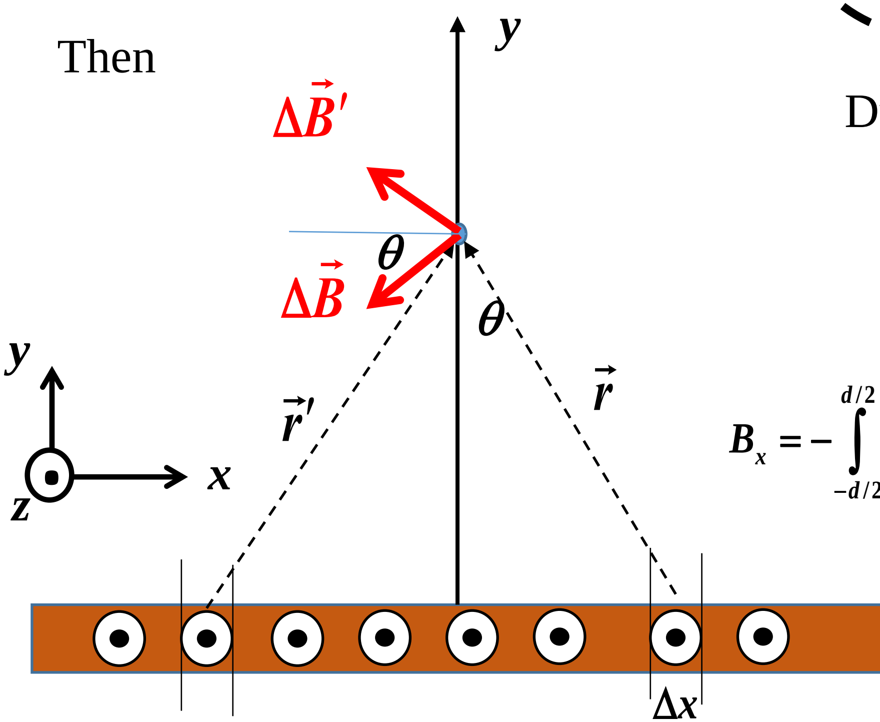
$$\Delta I = J\Delta x = \frac{I}{d}\Delta x$$

For each infinite long wire



$$\Rightarrow \oint \vec{B}\bullet d\vec{l} = \mu_0 \Delta I \Rightarrow \Delta B = \frac{\mu_0 \Delta I}{2\pi r}$$

Then



Due to symmetry, only the x-component remains.

$$\Delta B_x = \frac{\mu_0 J \Delta x}{2\pi\sqrt{x^2+y^2}} \cdot \left(-\cos\theta\right) = -\frac{\mu_0 J y \Delta x}{2\pi\left(x^2+y^2\right)}$$

$$B_x = -\int_{-d/2}^{d/2} \frac{\mu_0 I y dx}{2\pi d\left(x^2+y^2\right)} = -2\frac{\mu_0 I y}{2\pi d}\int_0^{d/2} \frac{dx}{x^2+y^2} = -\frac{\mu_0 I y}{\pi d}\cdot\frac{1}{y}\arctan\left(\frac{x}{y}\right)\Bigg|_0^{d/2}$$

$$= -\frac{\mu_0 I}{\pi d}\cdot\arctan\left(\frac{d}{2y}\right)$$ (積分查表)

(b)

By **(a)** $y >> d$

$$\tan^{-1}\frac{d}{2y} = \frac{d}{2y} - \frac{1}{3}\left(\frac{d}{2y}\right)^3 + \frac{1}{5}\left(\frac{d}{2y}\right)^5 - \ldots \simeq \frac{d}{2y}$$

Taylor's expansion: $\left(|x| < 1\right)$

$$\tan^{-1} x = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \ldots \simeq x, \quad \text{for } x \ll 1$$

$$\Rightarrow B = \frac{\mu_0 I}{\pi d} tan^{-1}\frac{d}{2y} \approx \frac{\mu_0 I}{\pi d}\frac{d}{2y} = \frac{\mu_0 I}{2\pi y}$$

The result is the same with the infinite wire magnetic

HW8-2:

Part 1: A coaxial cable consists of a solid inner conductor of radius $\boldsymbol{R_1}$, surrounded by a concentric cylindrical tube of inner radius $\boldsymbol{R_2}$ and outer radius $\boldsymbol{R_3}$ (see figure). The conductors carry equal and opposite currents $\boldsymbol{I_0}$ distributed uniformly across their cross sections. Determine the magnetic field at a distance R from the axis for

(a) $R < R_1$;

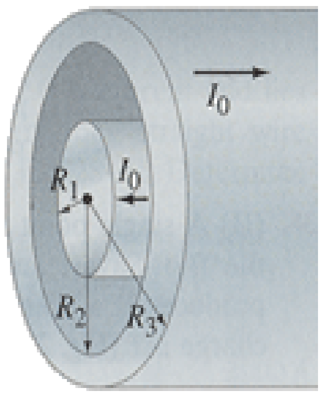
(b) $R_1 < R < R_2$;

(c) $R_2 < R < R_3$;

(d) $R > R_3$;

(e) Let $I_0 = 1.50A, R_1 = 1.00cm, R_2 = 2.00cm,$ and $R_3 = 2.50cm$

Graph B from $R = 0$ to $R = 3.00cm$

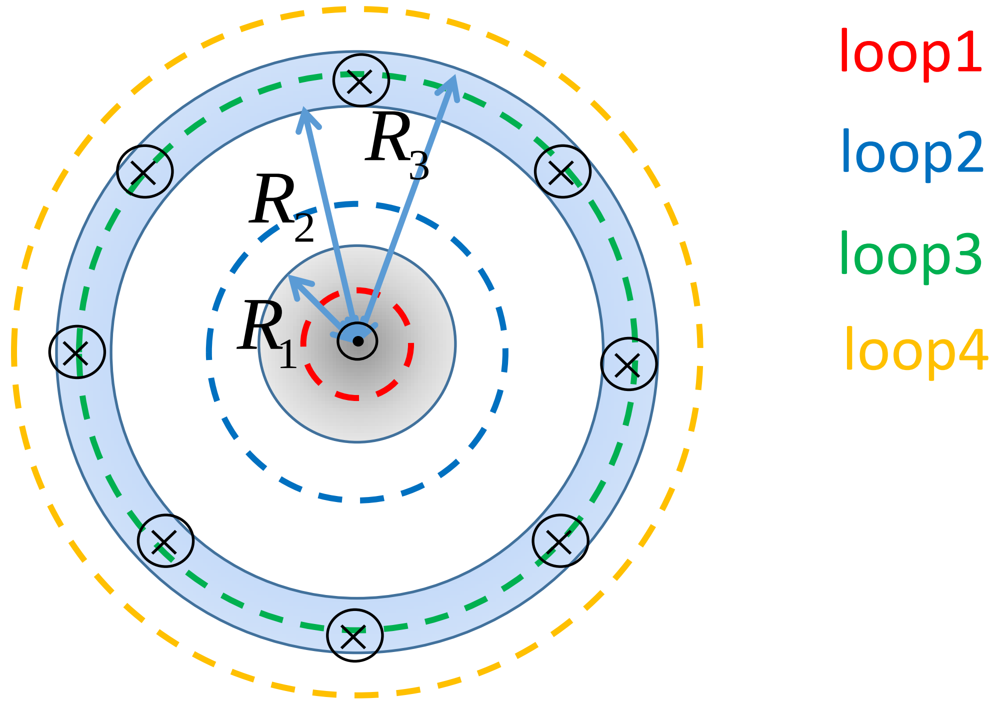


Part 2

Suppose the current in the coaxial cable of problem above , is not uniformly distributed, but instead the current density j varies linearly with distance from the center:
$\boldsymbol{j_1 = C_1 R}$ for the inner conductor and $\boldsymbol{j_2 = C_2 R}$ for the outer conductor. Each conductor still carries the same total current $\boldsymbol{I_0}$ in opposite directions. Determine the magnetic field in terms of $\boldsymbol{I_0}$ in the same four regions of space as in problem above.

Part1：

圓柱狀對稱：

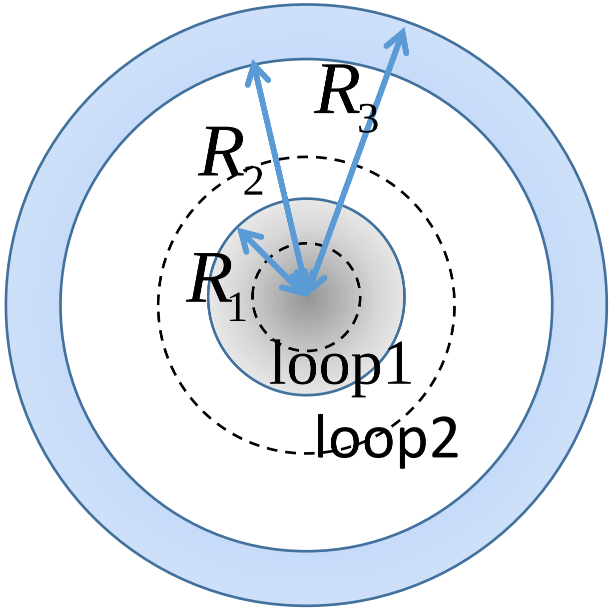


均勻電流密度：

$$\vec{j}_{inner}=\frac{I}{A}\hat{z}=\frac{I_0}{\pi R_1^{\ 2}}\hat{z}$$ (內圈電流密度出紙面)

$$\vec{j}_{outer}=\frac{I}{A}(-\hat{z})=-\frac{I_0}{\pi(R_3^{\ 2}-R_2^{\ 2})}\hat{z}$$ (外圈電流密度入紙面)

Ampere's law : $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$



(a): On loop1: $0 < R < R_1$ $\quad \vec{B} = B\hat{\varphi}, \quad d\vec{\ell} = \hat{\varphi} R d\varphi$

$$\oint_{loop\ 1} \vec{B}\cdot d\vec{\ell} = \int_0^{2\pi} BR\, d\varphi = 2\pi RB$$

$$\mu_0 I_{enc} = \mu_0 \int \vec{j}\cdot d\vec{A} = \mu_0 \int \left(\frac{I_0}{\pi R_1^{\,2}}\hat{z}\right)\cdot \hat{z} dA = \mu_0 (\pi R^2)\left(\frac{I_0}{\pi R_1^{\,2}}\right)$$

$$\Rightarrow \quad B\cdot 2\pi R = \mu_0 I_0 \cdot \frac{R^2}{R_1^{\,2}}, \quad \vec{B} = \frac{\mu_0 I_0}{2\pi}\frac{R}{R_1^{\,2}}\hat{\varphi}$$

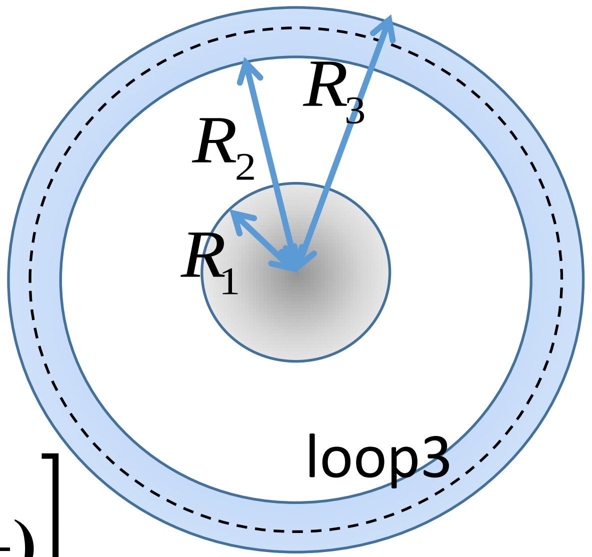
(b): On loop2: $R_1 < R < R_2$ $\quad \vec{B} = B\hat{\varphi}, \quad d\vec{\ell} = \hat{\varphi} R d\varphi$

$$\oint_{loop\ 2} \vec{B}\cdot d\vec{\ell} = \int_0^{2\pi} BR\, d\varphi = 2\pi RB$$

$$\mu_0 I_{enc} = \mu_0 I_{inner} = \mu_0 I_0$$

$$\Rightarrow B 2\pi R = \mu_0 I_0, \quad \vec{B} = \frac{\mu_0 I_0}{2\pi R}\hat{\varphi}$$

(c): On loop3: $R_2 < R < R_3$



$$\oint_{loop\ 3} \vec{B}\cdot d\vec{\ell} = \int_0^{2\pi} BR\,d\varphi = 2\pi RB$$

$$\mu_0 I_{enc} = \mu_0 (I_{inner} + \Delta\vec{A}_{out}\cdot\vec{j}_{outer})$$

$$= \mu_0 \left[I_0 + \pi(R^2 - R_2{}^2)(-\frac{I_0}{\pi R_3{}^2 - \pi R_2{}^2}) \right]$$

$$= \mu_0 I_0 (1 - \frac{R^2 - R_2{}^2}{R_3{}^2 - R_2{}^2})$$

$$= \mu_0 I_0 (\frac{R_3{}^2 - R^2}{R_3{}^2 - R_2{}^2})$$

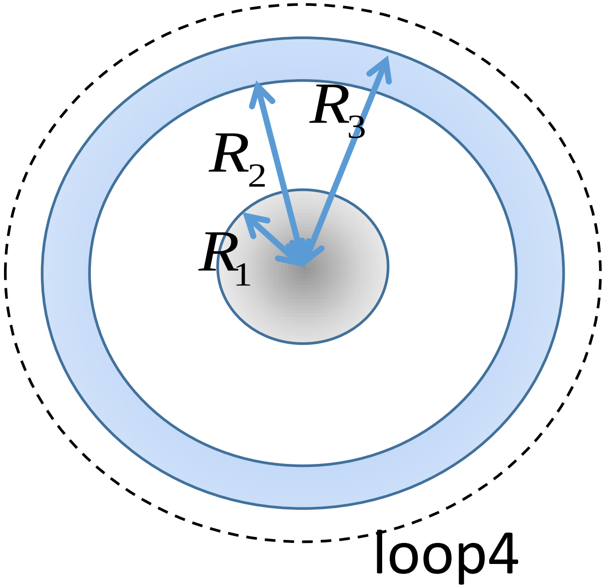
$$\Rightarrow B 2\pi R = \mu_0 I_0 (\frac{R_3{}^2 - R^2}{R_3{}^2 - R_2{}^2}), \qquad \vec{B} = \frac{\mu_0 I_0}{2\pi R}(\frac{R_3{}^2 - R^2}{R_3{}^2 - R_2{}^2})\hat{\varphi}$$

(d): On loop4: $R_1 < R < R_2$

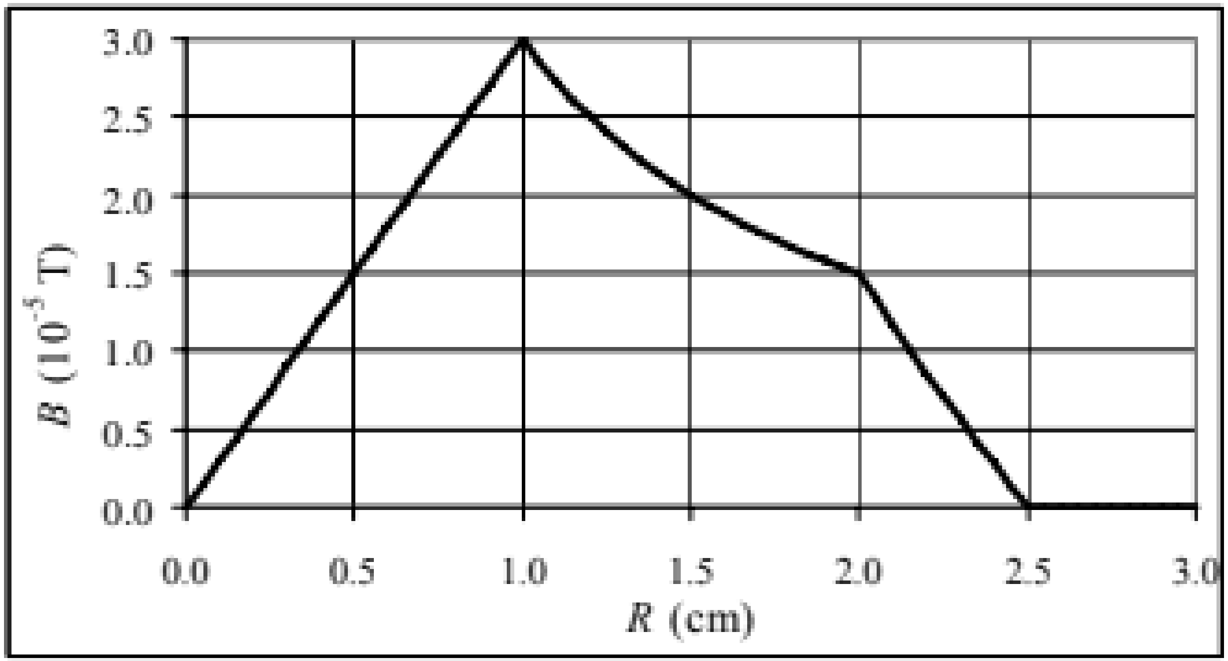
$$\oint_{loop\ 4} \vec{B}\cdot d\vec{\ell} = \int_0^{2\pi} BR\,d\varphi = 2\pi RB$$

$$\mu_0 I_{enc} = \mu_0 (I_{inner} + I_{outer}) = \mu_0 \left[I_0 + (-I_0) \right] = 0$$

$$\Rightarrow \quad B 2\pi R = 0, \quad \vec{B} = \hat{0}$$



(e):



Part2 :

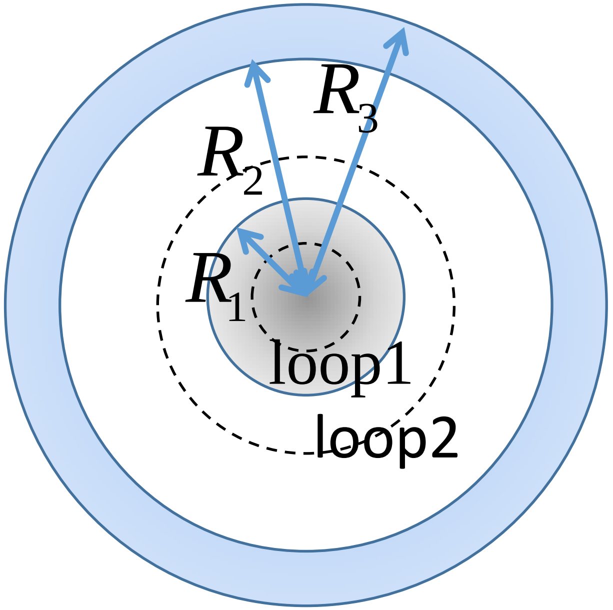
$$I_{inner} = \int\limits_{\substack{inner\\surface}} \vec{j}\cdot d\vec{A} = \int_0^{R_1} \left(C_1 R\hat{z}\right)\cdot\left(2\pi R dR\hat{z}\right) = 2\pi C_1 \int_0^{R_1} R^2\, dR = 2\pi C_1 \cdot \frac{R_1^{\ 3}}{3} = I_0$$

$$\Rightarrow C_1 = \frac{3I_0}{2\pi R_1^{\ 3}}$$

$$I_{outer} = \int\limits_{\substack{outer\\surface}} \vec{j}\cdot d\vec{A} = \int_{R_1}^{R_3} \left(-C_2 R\hat{z}\right)\cdot\left(2\pi R dR\hat{z}\right) = 2\pi C_2 \int_{R_2}^{R_3} R^2\, dR$$

$$= 2\pi C_2 \cdot \frac{R_3^{\ 3} - R_2^{\ 3}}{3} = -I_0 \quad \Rightarrow C_2 = \frac{3I_0}{2\pi(R_3^{\ 3} - R_2^{\ 3})}$$

Ampere's law : $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$



(a): On loop1: $0 < R < R_1$ $\vec{B} = B\hat{\varphi}$, $d\vec{\ell} = \hat{\varphi} R d\varphi$

$$\oint_{loop\ 1} \vec{B}\cdot d\vec{\ell} = \int_0^{2\pi} BR\,d\varphi = 2\pi RB$$

$$\mu_0 I_{enc} = \mu_0 \int_{\substack{inner\\ surface}} \vec{j}\cdot d\vec{A} = 2\pi\mu_0 C_1 \int_0^R R^2\,dR$$

$$= 2\pi\mu_0 \cdot \frac{3I_0}{2\pi R_1{}^3}\frac{R^3}{3} = \mu_0 I_0 \frac{R^3}{R_1{}^3}$$

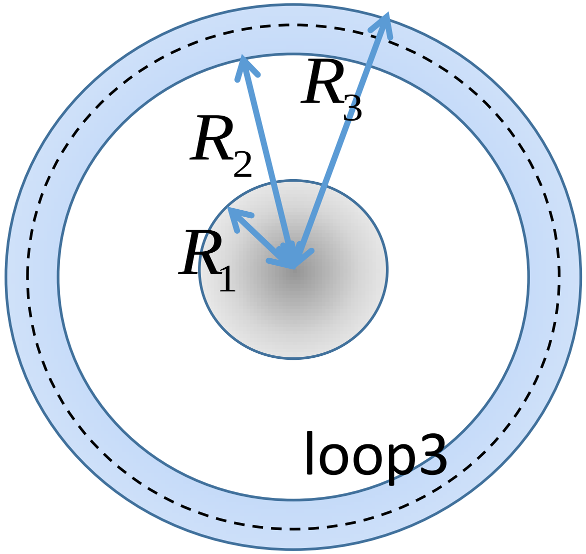
$$\Rightarrow \vec{B} = \frac{\mu_0 I_0}{2\pi}\frac{R^2}{R_1{}^3}\hat{\varphi}$$

(b): On loop2: $R_1 < R < R_2$ $\vec{B} = B\hat{\varphi}$, $d\vec{\ell} = \hat{\varphi} R d\varphi$

$$\oint_{loop\ 2} \vec{B}\cdot d\vec{\ell} = \int_0^{2\pi} BR\,d\varphi = 2\pi RB$$

$$\mu_0 I_{enc} = \mu_0 I_{inn} = \mu_0 I_0 \quad \Rightarrow B2\pi R = \mu_0 I_0, \quad \vec{B} = \frac{\mu_0 I_0}{2\pi R}\hat{\varphi}$$

(c): On loop3: $R_2 < R < R_3$



$$\oint_{loop\ 3} \vec{B}\cdot d\vec{\ell} = \int_0^{2\pi} BR\,d\varphi = 2\pi RB$$

$$\mu_0 I_{enc} = \mu_0 (I_{inner} + \int_{\substack{outer\\surface}} C_2 R(-\hat{z})\cdot\hat{z}\,dA)$$

$$= \mu_0 (I_{inner} - \int_{R_2}^{R} C_2 R 2\pi R dR)$$

$$= \mu_0 I_0 (1 - 2\pi C_2 \frac{R^3 - R_2^{\ 3}}{3}) = \mu_0 I_0 (1 - \frac{R^3 - R_2^{\ 3}}{R_3^{\ 3} - R_2^{\ 3}})$$

$$= \mu_0 I_0 \frac{R_3^{\ 3} - R^3}{R_3^{\ 3} - R_2^{\ 3}}$$

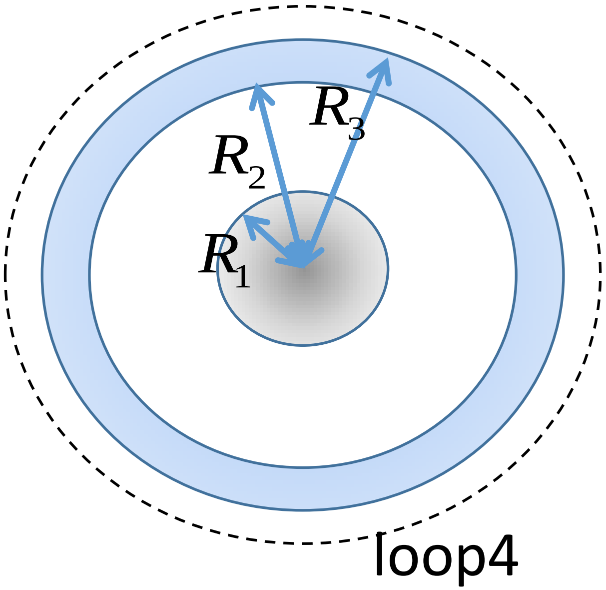
$$\Rightarrow B 2\pi R = \mu_0 I_0 \cdot \frac{R_3^{\ 3} - R^3}{R_3^{\ 3} - R_2^{\ 3}}, \quad \vec{B} = \frac{\mu_0 I_0}{2\pi R}\frac{R_3^{\ 3} - R^3}{R_3^{\ 3} - R_2^{\ 3}}\hat{\varphi}$$

(d): On loop4: $R_1 < R < R_2$

$$\oint_{loop\ 4} \vec{B} \cdot d\vec{\ell} = \int_0^{2\pi} BR\,d\varphi = 2\pi RB$$

$$\mu_0 I_{enc} = \mu_0 (I_{inn} + I_{out}) = \mu_0 \left[I_0 + (-I_0) \right] = 0$$

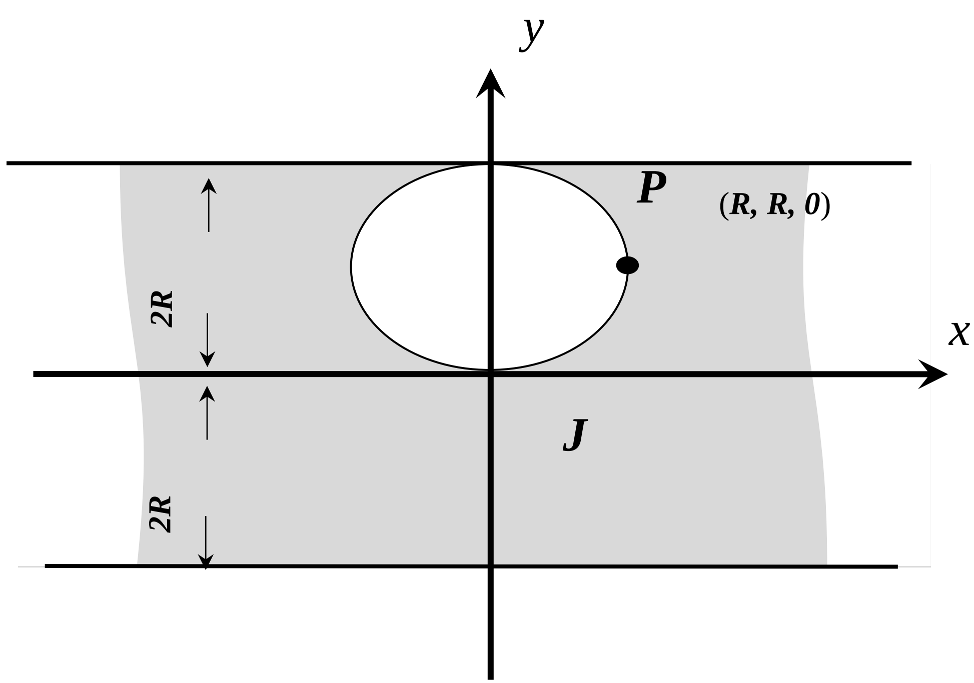
$$\Rightarrow \quad B 2\pi R = 0, \ \vec{B} = \hat{0}$$

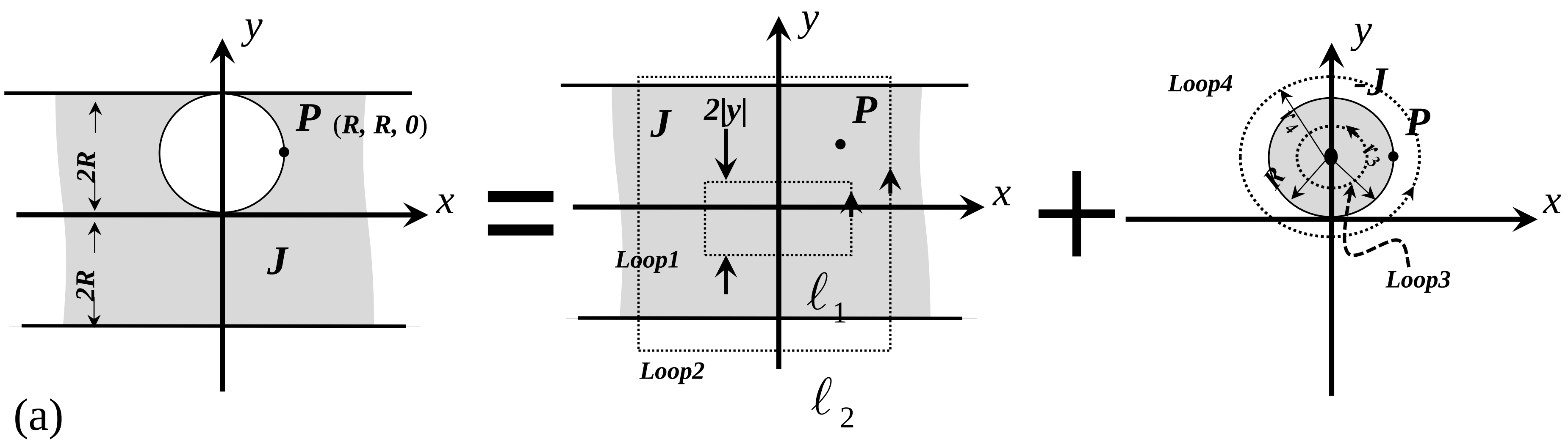


HW8-3:

As shown in Fig, an infinite conducting plate with thickness 4R carries a uniform current density J in +z-direction, and in the plate there is a infinitely long hollow cylindrical region with radius R and its cylindrical axis passes the y-axis at (0,R,0). Determine (a) The magnitude and the direction of the B-field on the y-axis for ($0 \leq y < \infty$).

(b)The direction and magnetic of the B-field at point P.





For the infinite plane block current and for 0≤y<2R, choose loop 1 (running counter clockwise) to apply Ampere's Law,

$$\oint_{Loop1} \vec{B}\cdot d\vec{\ell} = \mu_o I_{in} \Rightarrow \oint_{Loop1} \vec{B}\cdot d\vec{\ell} = 2B\ell_1 = \mu_o J\cdot 2|y|\ell_1$$

$$\Rightarrow B = \mu_o J\cdot|y| = \mu_o Jy \quad \Rightarrow \vec{B}(y) = \mu_o Jy(-\hat{x})$$

For 2R ≤ y, choose loop 2,

$$\oint_{Loop2} \vec{B}\cdot d\vec{\ell} = \mu_o I_{in} \Rightarrow 2B\ell_2 = \mu_o J(4R)\ell_2 \Rightarrow B(y) = \mu_o JR$$

$$\Rightarrow \vec{B}(y) = 2\mu_o JR(-\hat{x})$$

For the infinite cylindrical current and for (0,y) with 0≤|y-R|<R, choose loop 3,

$$\oint_{Loop3} \vec{B}\cdot d\vec{\ell} = \mu_o I_{in} \quad \Rightarrow B\cdot 2\pi \cdot r_3 = -\mu_o J\pi \cdot r_3^2$$

$$\Rightarrow B = -\frac{\mu_o J r_3}{2} = -\frac{\mu_o J\left|y-R\right|}{2}$$

for 0≤y<R, $\quad \vec{B}(\mathrm{y}) = \frac{\mu_o J\left|R-y\right|}{2}(-\hat{x}) = \frac{\mu_o J(y-R)}{2}\hat{x}$

for R≤y<2R, $\quad \vec{B}(y) = \frac{\mu_o J\left|R-y\right|}{2}\hat{x} = \frac{\mu_o J(y-R)}{2}\hat{x}$

For (0,y) with 2R≤|y-R|, choose loop 4,

$$\oint_{Loop4} \vec{B}\cdot d\vec{\ell} = \mu_o I_{in} \quad \Rightarrow B\cdot 2\pi \cdot r_4 = -\mu_o J\pi \cdot R^2$$

$$\Rightarrow B = -\frac{\mu_o J\cdot R^2}{2r_4} = -\frac{\mu_o J\cdot R^2}{2\left|y-R\right|}$$

For 2R≤y, $\quad \vec{B}(y) = \frac{\mu_o J\cdot R^2}{2\left|y-R\right|}\hat{x} = \frac{\mu_o J\cdot R^2}{2(y-R)}\hat{x}$

For the total B-field,

for 0≤y<2R, $$\vec{B}_{tot}(\mathrm{y}) = \frac{\mu_o J(y-R)}{2}\hat{x} + \mu_o Jy(-\hat{x}) = -\frac{\mu_o J(y+R)}{2}\hat{x}$$

for 2R≤y, $$\vec{B}_{tot}(y) = \frac{\mu_o J \cdot R^2}{2(y-R)}\hat{x} + 2\mu_o JR(-\hat{x}) = \mu_o J\,\frac{5R^2 - 4yR}{2(y-R)}\hat{x}$$

(b) For the B-field at P (R,R,0),

$$\vec{B}_P = \frac{\mu_o J r_3}{2}(-\hat{y}) + \mu_o JR(-\hat{x})$$

$$= \frac{\mu_o JR}{2}(-\hat{y}) + \mu_o JR(-\hat{x})$$

$$= \left(-\mu_o JR, -\frac{\mu_o JR}{2}, 0\right)$$