1. $\frac{dy}{dx} - 2y = e^{x}$

$e^{-2x} y = e^{-2x} \cdot e^{x}$

$\int e^{-2x} y \frac{d}{dx} \cdot dx = \int e^{-x} dx$

$e^{-2x} y = - e^{-x} + C$

$y = -e^{x} + Ce^{2x}$

$y(1) = 2 \Rightarrow 2 = -e + Ce^{2},$

$C = (2+e)e^{-2}$

$y = -e^{x} + (2+e)e^{-2} e^{2x}$

$= -e^{x} + 2e^{2x-2} + e^{2x-1}$

2. $\frac{dy}{dx} = x^{2} y^{3}$

$\frac{dy}{y^{3}} = x^{2} dx$

$\int y^{-3} dy = \int x^{2} dx$

$-\frac{1}{2} y^{-2} = \frac{1}{3} x^{3} + C$

$y(-2) = 1 \Rightarrow C = \frac{13}{6}$

$-\frac{1}{2} y^{-2} = \frac{1}{3} x^{3} + \frac{13}{6}$

$y = \frac{3}{\sqrt{-6x^{3}-39}}$ (負不合)

3. $(2x+3y-1)dx + (3x-2y+3)dy = 0$

$\frac{\partial M}{\partial y} = 3 = \frac{\partial N}{\partial x}$

$f(x,y) = x^{2} + 3xy - x + g(y)$

$= 3xy - y^{2} + 3y + g(x)$

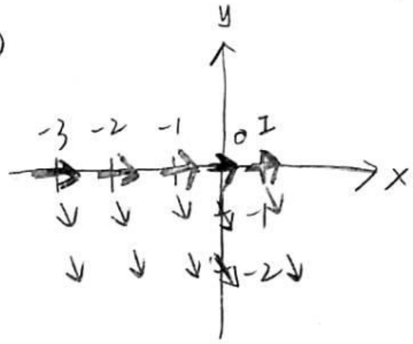
$\Rightarrow x^{2} - x + 3xy - y^{2} + 3y + C = f(x,y)$

$y(0) = 0 \Rightarrow C = 0$

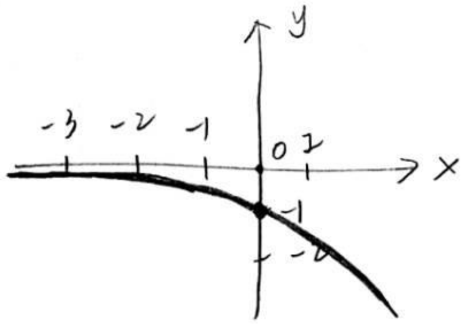
$f(x,y) = x^{2} - x + 3xy - y^{2} + 3y$

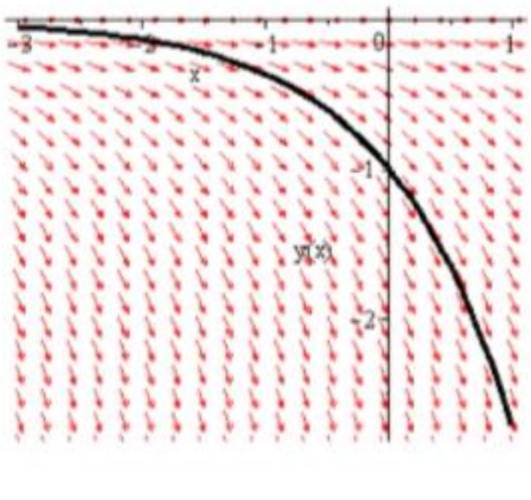
$= x^{2} - y^{2} + 3y(x+1) - x = 0$

4. (a)



(b)





4.

5, $z = y^{-2}$

$$\frac{dz}{dy} = -2y^{-3}$$

$$\frac{dz}{dx} \cdot \frac{dx}{dy} = -2y^{-3}$$

$$\Rightarrow \frac{dz}{dx} = -2y^{-3} \cdot \frac{dy}{dx}$$

$$-\frac{1}{2}\frac{dz}{dx} = y^{-3}\frac{dy}{dx}$$

$$-\frac{1}{2}z' + 2z = -4$$

$$\Rightarrow z' - 4z = 8$$

$$\frac{d}{dx}(e^{-4x} \cdot z) = 8 \cdot e^{-4x}$$

$$e^{-4x} \cdot z = 8\int e^{-4x} dx$$

$$= -2e^{-4x} + C$$

$$z = \frac{1}{y^2} = -2 + Ce^{4x}$$

$$\Rightarrow y^2 = \frac{1}{-2 + Ce^{4x}}$$

$$\Rightarrow y = \frac{1}{\sqrt{-2 + Ce^{4x}}}$$

$$y(0) = 1 \Rightarrow 1 = \frac{1}{\sqrt{-2+C}},\ C = 3$$

$$y = \frac{1}{\sqrt{-2+3e^{4x}}}$$

6,

$$\frac{d^2y}{dx^2} + \frac{3}{x}\frac{dy}{dx} + \frac{1}{x^2}y = 0,\ y_1 = \frac{1}{x}$$

$$y_2(x) = y_1(x)\int \frac{e^{-\int p(x)dx}}{[y_1(x)]^2} dx$$

$$= \frac{1}{x}\int \frac{e^{-\int \frac{3}{x}dx}}{(\frac{1}{x})^2} dx$$

$$= \frac{1}{x}\int\left(x^2 \cdot e^{-\int \frac{3}{x}dx}\right)dx$$

$$= \frac{1}{x}\int(x^2 \cdot e^{\ln x^{-3}})dx$$

$$= \frac{1}{x}\int(x^2 \cdot x^{-3})\,dx$$

$$= \frac{1}{x}\int x^{-1}dx = \frac{1}{x}\ln x$$

$$y_2 = \frac{1}{x}\ln x$$ #

7.

(a) $\begin{vmatrix} 2x & e^{2x} & e^{-2x} \\ 2 & 2e^{2x} & -2e^{-2x} \\ 0 & 4e^{2x} & 4e^{-2x} \end{vmatrix} = 16x+8+8x-8=24 \neq 0$

$\Rightarrow$ independent

(b) $\begin{vmatrix} x^3 & x^3-2 & 1 \\ 3x^2 & 3x^2 & 0 \\ 6x & 6x & 0 \end{vmatrix} : 18x^3-18x^3=0$

$\Rightarrow$ depentdent

8. $y''+3y'-4y=0$

$m^2+3m-4=0, \quad m=1,-4$

$y=c_1e^x+c_2e^{-4x}$

$y'=c_1e^x-4c_2e^{-4x}$

$\begin{cases} 2=c_1e+c_2e^{-4} \\ -3=c_1e-4c_2e^{-4} \end{cases}$

$5c_2e^{-4}=5, \ c_2e^{-4}=1, \ c_2=e^4, \ c_1=e^{-1}$

$y=e^{(x-1)}+e^{(-4x+4)}$ #

9. $y_{n+1}=y_n+0.2y'=y_n+0.2(x^2y^3)$

(a) $y(0.2)=3+0.2\cdot 0\cdot 3^3=3$ #

(b) $y(0.4)=3+0.2\cdot(0.2)^2\cdot 3^3=3.216$ #

Solution: (a) 0, 2, 4, (b) See below for the phase portrait.

Solving $y(2-y)(4-y)=0$ we obtain the critical points 0, 2, and 4. From the phase portrait we see that 2 is asymptotically stable (attractor) and 0 and 4 are unstable (repellers).