11 $y = c_1 x^{-1} + c_2 x + c_3 x \ln x + 4x^2$

$y' = -c_1 x^{-2} + c_2 + c_3(1+\ln x) + 8x$

$y'' = 2c_1 x^{-3} + c_3 x^{-1} + 8$

$y''' = -6c_1 x^{-4} - c_3 x^{-2}$

$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 12x^2$

$x^3(-6c_1 x^{-4} - c_3 x^{-2}) + 2x^2(2c_1 x^{-3} + c_3 x^{-1} + 8)$

$- x[-c_1 x^{-2} + c_2 + c_3(1+\ln x) + 8x] + (c_1 x^{-1} + c_2 x + c_3 x \ln x + 4x^2)$

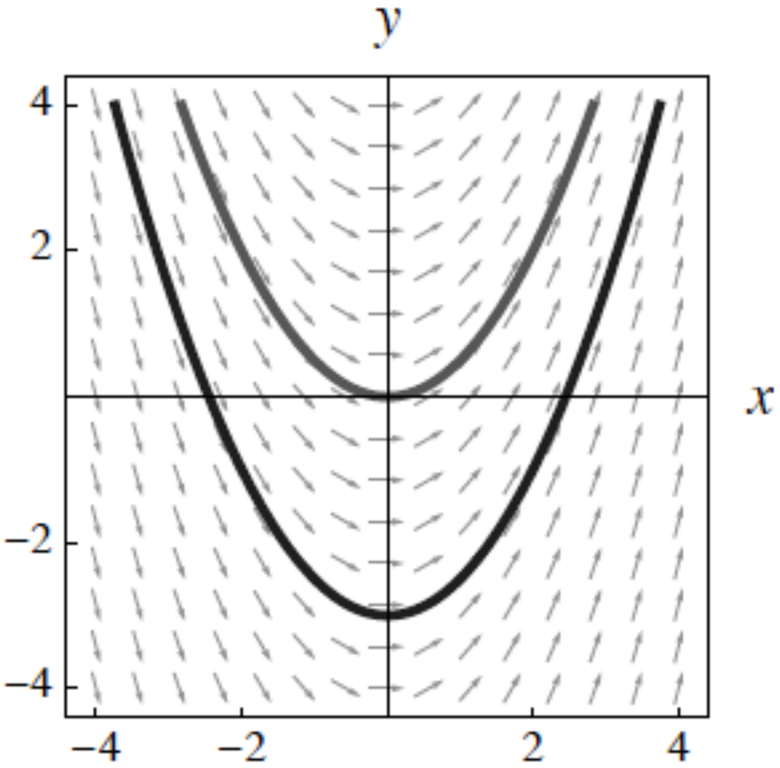
$= -6c_1 x^{-1} - c_3 x + 4c_1 x^{-1} + 2c_3 x + 16x^2 + c_1 x^{-1} - c_2 x - c_3 x - c_3 x \ln x - 8x^2$

$+ c_1 x^{-1} + c_2 x + c_3 x \ln x + 4x^2$

$= x^{-1}(-6c_1 + 4c_1 + c_1 + c_1) + x(-c_3 + 2c_3 - c_2 - c_3 + c_2)$

$+ x\ln x(-c_3 + c_3) + x^2(16 - 8 + 4) = 12x^2$

For $f(x, y) = \dfrac{y}{x}$ we have $\dfrac{\partial f}{\partial y} = \dfrac{1}{x}$. Thus, the differential equation will have a unique solution in any region where $x > 0$ or where $x < 0$.



y
x
4
2
−2
−4
−4
−2
2
4

$$\int (2y-2)\,dy = \int (3x^2+4x+2)\,dx$$

$$y^2-2y = x^3+2x^2+2x+c$$

$y(1)=-2$ 代入

$$4+4=1+2+2+c$$

$$c=3$$

$$y^2-2y = x^3+2x^2+2x+3$$

$$(y-1)^2 = (x+2)(x^2+2)$$

$$y = 1-\sqrt{(x+2)(x^2+2)}$$

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