Signals and Systems – Quiz 5 06/10/2020

Name: Student ID:

1. Given the fact that

$$a^{|n|} \overset{F}{\leftrightarrow} \frac{1-a^2}{1-2a\cos\omega+a^2}, |a|<1$$

Use duality to determine the Fourier series coefficients of the following continuous time signal with period $T = 1$:

$$x(t)=\frac{1}{5-3\cos(2\pi t)}$$

Ans:

Knowing that

$$\left(\frac{1}{3}\right)^{|n|} \overset{F}{\leftrightarrow} \frac{1-\frac{1}{9}}{1-\frac{2}{3}\cos\omega+\frac{1}{9}}=\frac{4}{5-3\cos\omega}$$

We may use the Fourier transform analysis equation to write:

$$\frac{4}{5-3\cos\omega}=\sum_{n=-\infty}^{\infty}\left(\frac{1}{3}\right)^{|n|} e^{-j\omega n}$$

Putting $\omega=-2\pi t$ in this equation, and replacing the variable n by the variable k

$$\frac{1}{5-3\cos(2\pi t)}=\sum_{n=-\infty}^{\infty}\frac{1}{4}\left(\frac{1}{3}\right)^{|k|} e^{j2\pi kt}$$

Since Fourier series $x(t)=\sum_{n=-\infty}^{\infty} a_k e^{jk\omega_0 t}$

$$\Rightarrow\ a_k=\frac{1}{4}\left(\frac{1}{3}\right)^{|k|}$$

2. Consider a signal $x[n]$ which is the product of two signals; that is

$$x[n] = x_1[n]x_2[n], \text{ where } x_1[n] = \frac{sin(\pi n/4)}{\pi n} \text{ and } x_2[n] = \frac{sin(\pi n/2)}{\pi n}$$

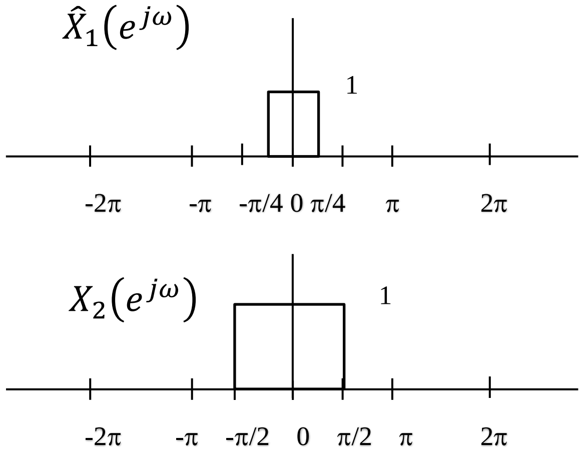
Sketch the Fourier transform of $x[n]$ from the multiplication property.

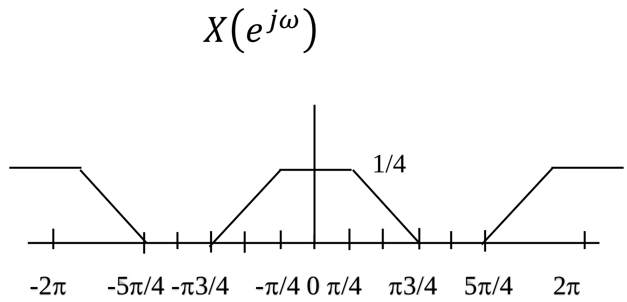
Ans:

From multiplication property $X(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(e^{j\theta})X_2(e^{j(\theta-\omega)})d\theta$, where $X_1(e^{j\omega})$ and $X_2(e^{j\omega})$ are the Fourier transform of $x_1[n]$ and $x_2[n]$.

Let $\hat{X}_1(e^{j\omega}) = \begin{cases} X_1(e^{j\omega}) & for - \pi < \omega \le \pi \\ 0 & otherwise \end{cases}$

$$X(e^{j\omega}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{X}_1(e^{j\theta})X_2(e^{j(\theta-\omega)})d\theta$$





3. An LTI system with impulse response $h_1[n] = \left(\frac{1}{4}\right)^n u[n]$ is connected in parallel with another causal LTI system with impulse response $h_2[n]$. The resulting parallel interconnection has the frequency response:

$$H(e^{j\omega}) = \frac{2}{(1-\frac{1}{2}e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})^2}$$

(a) Determine $h_2[n]$.
(b) Find the difference equation describing the overall system.
(c) Determine the impulse response of the system.

Ans:

$$h[n] = h_1[n] + h_2[n] \leftrightarrow H(e^{j\omega}) = H_1(e^{j\omega}) + H_2(e^{j\omega})$$

$$h_1[n] = \left(\frac{1}{4}\right)^n u[n] \leftrightarrow H_1(e^{j\omega}) = \frac{1}{1-\frac{1}{4}e^{-j\omega}}$$

$$H_2(e^{j\omega}) = \frac{2}{(1-\frac{1}{2}e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})^2} - \frac{1}{1-\frac{1}{4}e^{-j\omega}} = \frac{8}{1-\frac{1}{2}e^{-j\omega}} + \frac{-5}{1-\frac{1}{4}e^{-j\omega}} + \frac{-2}{\left(1-\frac{1}{4}e^{-j\omega}\right)^2}$$

$$\rightarrow h_2[n] = \left\{8\left(\frac{1}{2}\right)^n - 5\left(\frac{1}{4}\right)^n u[n] - 2(n+1)\left(\frac{1}{4}\right)^n\right\}u[n]$$

$$H(e^{j\omega}) = \frac{2}{\left(1-\frac{1}{2}e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right)^2} = \frac{2}{1-e^{-j\omega}+\frac{5}{16}e^{-2j\omega}-\frac{1}{32}e^{-3j\omega}}$$

$$= \frac{8}{1-\frac{1}{2}e^{-j\omega}} + \frac{-4}{1-\frac{1}{4}e^{-j\omega}} + \frac{-2}{\left(1-\frac{1}{4}e^{-j\omega}\right)^2}$$

$$\rightarrow h[n] = \left\{8\left(\frac{1}{2}\right)^n - 4\left(\frac{1}{4}\right)^n u[n] - 2(n+1)\left(\frac{1}{4}\right)^n\right\}u[n]$$

$$\rightarrow y[n] - y[n-1] + \frac{5}{16}y[n-2] - \frac{1}{32}y[n-3] = 2x[n]$$