General Physics (II) **期末考** **Jun. 17, 2016**

試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(53 points) (所有題目必須有計算過程,否則不予計分)

1. **(10 pts):** The direction (in general) of a plane can be expressed by the wave vector $\vec{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$. For example, a magnetic field of a plane wave (in free space) with the form

$$\vec{B}(\vec{r},t) = B_0\hat{z}\sin\left(\vec{k}\cdot\vec{r} - \omega t\right) = B_0\hat{z}\sin\left[\left(2.0\ \mathrm{m}^{-1}\right)\frac{x+y}{\sqrt{2}} - \omega t\right], \quad \textbf{and } B_0 = 5.0\times10^{-7}\ \mathrm{T}.$$

describes the wave moving toward the direction $\hat{k} = \frac{\hat{x}+\hat{y}}{\sqrt{2}}$ with wave number k = 2.0 **m^{-1}**. Answer the following questions including **correct unit**. note: $c = 3\text{x}10^8$m/s, $\mu_0 = 4\pi\text{x}10^{-7}$ Tm/A, and $\varepsilon_0\mu_0 = 1/c^2$

a) (2 pts) Find the wave length (λ), and the angular frequency (ω) of this plane wave.

b) (1 pts) What is the direction of this plane wave propagating?

c) (4 pts) The Electric field of this plane can be written as $\vec{E}(\vec{r},t) = \left(E_{0x}\hat{x} + E_{0y}\hat{y}\right)\sin\left[2.0\frac{x+y}{\sqrt{2}} - \omega t\right]$. Find the values of E_{0x} and E_{0y} in SI unit.

d) (3 pts) Find the Poynting vector $\vec{S}$ (magnitude and direction), and the intensity (I = <S>) of this plane wave.

2. (8 pts) As shown in Fig. 1, an ***AC*** circuit with a power supply, voltage V_{in} ,is connected to a capacitor ***C*** and a resistor ***R***. Assume $I_{in} = I_0 \cdot \cos(\omega t)$.

(A) (6 pts) Find the ratio V_{out0}/V_{in0} as a function of ω, where the V_{in0} and V_{out0} are the **amplitude (or magnitude)** of the power supply and the resistor, respectively.

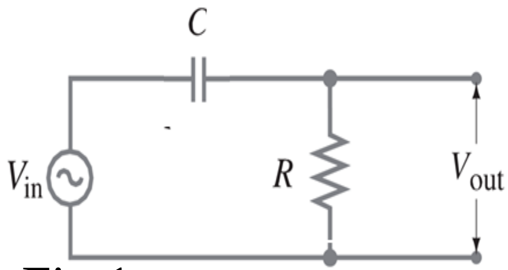


Fig.1

(B) (2 pts) Determine the ratio V_{out0}/V_{in0} as $\omega \rightarrow 0$, and $\omega \rightarrow \infty$.

3.(17 pts) A N-turn solenoid of radius a and length h is connected with a resistor R_1 and battery with voltage V as shown in the Fig.2. The switch have been at position A for a long time. At time $t = 0$, the switch changes from A to B. Assume the current $I(t) = I_0 e^{-t/\tau}$, and the direction of current I is shown in Fig. 2. Consider a point P (inside the solenoid) at radius a.

(A) (5 pts) Find the inductance L of the solenoid in terms of N, a, h and μ_0)

(B) (5 pts) Express I_0 and τ in terms of V, R_1, and L..

(C) (7 pts) Find the Poynting vector $S(t)$ (magnitude and direction) at P. And what is the total power flow into/out of the inductor? By using L and τ in previous parts, you can prove $P(t) = I^2(t)R$.

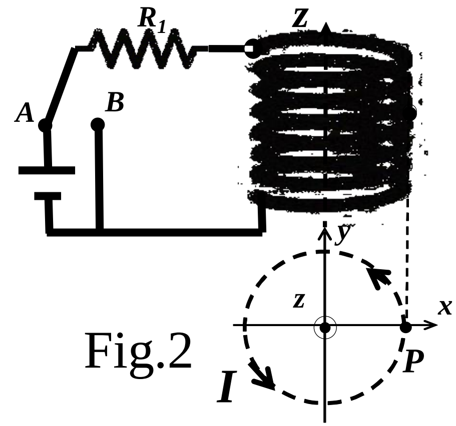


Fig.2

4. (18 pts) A plane wave, with wavelength λ = 500**nm**, travels through a diffraction grating with N-slit and the interference pattern on the screen is shown in Fig. 3. The distance between the N-slit and the screen is $5m$ and the spacing between nearest neighboring slits is $100\mu m$.

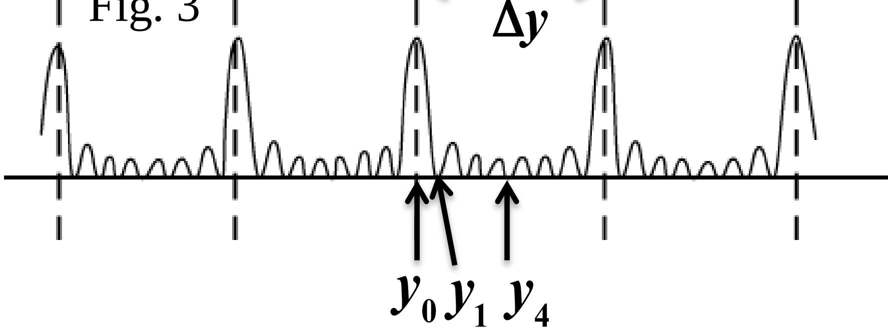
a) (3 pts) What is the $N=$? Find the spacing (Δy) between the principal maxima on the screen.

b) (6 pts) Draw the phasor diagrams for positions y_1 and y_4 indicated in Fig 3. What are the phase differences corresponding to these two points? Find these positions y_1 and y_4. Assume the position y_0 is at the central peak.

c) (4 pts) If the width of each slit of this diffraction gating is $20\mu m$, find the position (y_d) of the first minimum of the **diffraction pattern** (not shown in the figure) on the screen. How many interference fringes in the central diffraction peak?

d) (5 pts) Determine the intensity on the screen as a function of y (including both the interference and diffraction). Assuming that the initial intensity is I_0 for each slit.

Fig. 3



II.選擇題(50 points)

1. (5 pts) As shown in Fig. 4(a) , an capacitor is connected to an AC power supply with the current $\boldsymbol{I(t)} = \boldsymbol{I_0\cos(\omega t)}$ flowing through. Assume that at $\boldsymbol{t=0}$, the charge on the capacitor $\boldsymbol{Q=0}$, then which of the following shows the correct time dependence of the electric field $\boldsymbol{E}=(0,0,E_z)$ and the magnetic field $\boldsymbol{B}=(0,B_y,0)$ at point $\boldsymbol{P}$ inside the capacitor. In the following diagrams $E_z=$ ——— , $B_y=$ ———

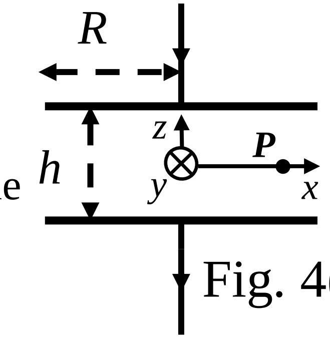


Fig. 4(a)

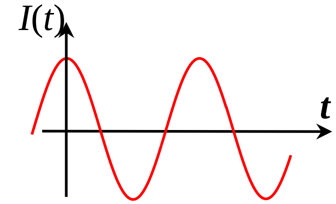
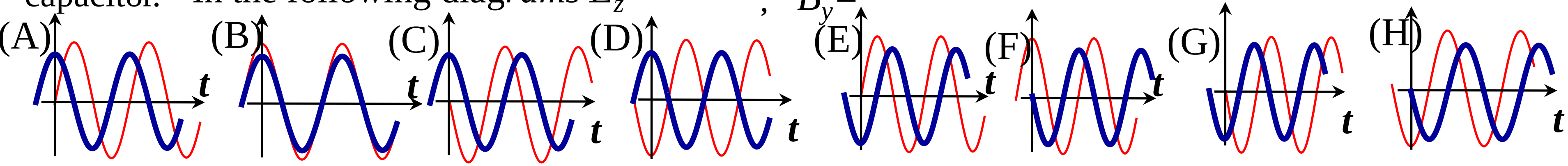


Fig. 4(b)



2. (5 pts)A 1.0mm^2 square thin black paper mass of 0.5 mg is found resting in air with a laser beam shinning from below. Assume all the area of the paper was illuminated with uniform light intensity from the laser and all the incident light was absorbed by the paper, what would be the amplitude of the E-field of the laser beam in units of V/m? ($\varepsilon_0 = 8.85\times10^{-12}$ C^2/N·m, $\mu_0 = 4\pi \times 10^{-7}$ T·m/A)

(A) E~10^{-6} V/m (B) E~10^{-4} V/m (C) E~10^{-2} V/m (D) E~1 V/m (E) E~10^{2}V/m (F) E~10^{4}V/m (G) E~10^{6} V/m

3. (5 pts) An LRC in-series circuit is driven by a AC voltage source $\boldsymbol{V(t) = V_0\sin(\omega t)}$. The rate of energy dissipated by the resistor

(A) does not depends on the value of R. (B) increase as the angular frequency ω is increased.
(C) increase as the angular frequency ω is decreased. (D) None of above.

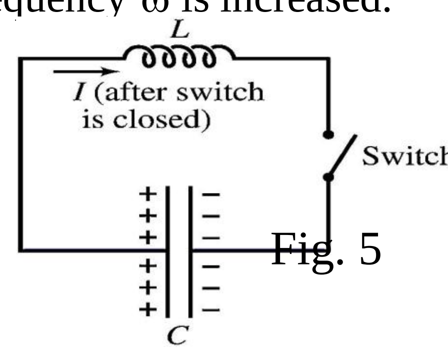


Fig. 5

4. (5 pts) Fig. 5 shows a L-C circuit with total charge $\boldsymbol{Q_0}$ on the capacitor. At $\boldsymbol{t}=0$ the switch is closed, and the current $\boldsymbol{I}$ starts to flow and oscillates with period $\boldsymbol{T}$. If the capacitor consists of two parallel conducting disks with radius $\boldsymbol{r_C}$ and separation $\boldsymbol{d}$. The inductor is a $\boldsymbol{N}$-turn solenoid coil with radius $\boldsymbol{r_L}$. Which of the following modification of the devices would decrease the period $\boldsymbol{T}$ by a factor of 2?

(A) $\boldsymbol{r_C \rightarrow r_C/2}$ (B) $\boldsymbol{d \rightarrow d/2}$ (C) $\boldsymbol{N \rightarrow N/2}$ (D) $\boldsymbol{r_L \rightarrow r_L/2}$ (E) $\boldsymbol{r_C \rightarrow r_C/4}$ (F) $\boldsymbol{d \rightarrow d/4}$
(G) $\boldsymbol{N \rightarrow N/4}$ (H) $\boldsymbol{r_L \rightarrow r_L/4}$ (**I**) 2 above (J) 3 above (K) 4 above

5. (5 pts) Fig.6 shows the voltages across a capacitor $\boldsymbol{C}$, and the voltage across a inductor $\boldsymbol{L}$ as functions of time. Which of the following AC circuits ($V_s = V_0\cos(\omega t)$) will match $V_L(t)$ and $V_C(T)$ in fig. 6?

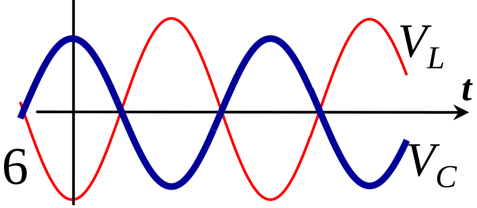
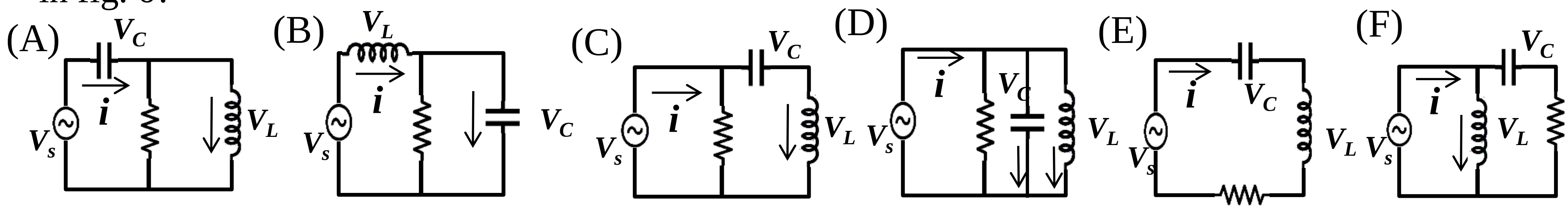


Fig. 6



(G) 2 above (H) 3 above (I) 4 above

6. (5 pts) A laser beam produces light that impinges on two long narrow apertures (slits) separated by a distance $\boldsymbol{d}$. Each aperture has width $\boldsymbol{a}$, with $\boldsymbol{a} \ll \boldsymbol{d}$. The resulting pattern on a screen 10 meters away from the slits is shown in Fig. 7. If the separation $\boldsymbol{d}$ between the slits is 0.04 mm, then the wavelength λ of the laser light must be (1 nm = 10^{-9} meter)

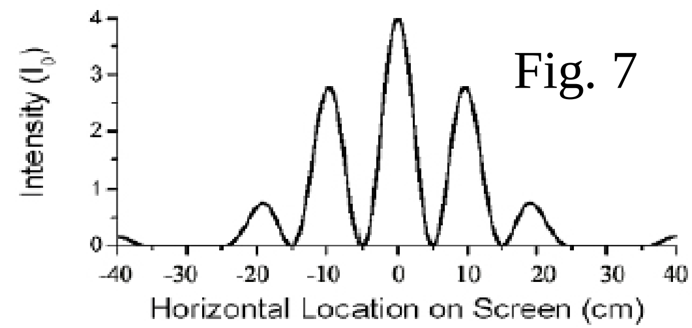


Fig. 7

(A) 20nm (B) **40nm** (C) **60nm** (D) **80nm** (E) **200nm** (F) **400nm** (G) **600nm** (H) **800nm** (I) **1200nm**

7. (5 pts) In Fig. 8, drawings A and B show two different interference fringe patterns from light on a screen passing through two slits of width $\boldsymbol{a}$, separated by distance $\boldsymbol{d}$. The wavelength of the light is the same in both cases, as is the distance to the screen, but $\boldsymbol{a}$ and $\boldsymbol{d}$ could be different (or the same). Which of the following is true when compared ***case A*** to ***case B***?

(A) ***a*** is unchanged but ***d*** is greater,

(B) ***a*** is unchanged but ***d*** is smaller

(C) ***d*** is unchanged but ***a*** is greater,

(D) ***d*** is unchanged but ***a*** is smaller

(E) (A) or (C) is correct; **(F)** (A) or (D) is correct,

(G) (B) or (C) is correct; **(H)** (B) or (D) is correct

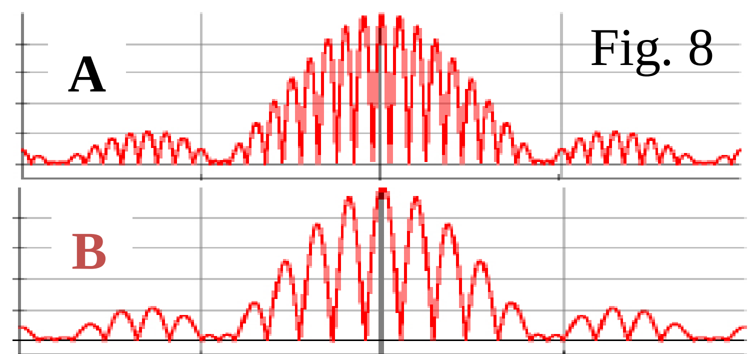


Fig. 8

Multiple Choice Questions:

1	**2**	**3**	**4**	**5**	**6**	**7**			
G	**G**	**D**	**J**	**G**	**F**	**A**			
8	**9**	**10**	**11**	**12**					
H	**B**	**D**	**H**	A					

$$\vec{B}(\vec{r},t) = B_0\hat{z}\sin\left[2.0\frac{x+y}{\sqrt{2}} - \omega t\right] = \left(5.0\times 10^{-7}T\right)\hat{z}\sin\left[2.0\frac{x+y}{\sqrt{2}} - \omega t\right]$$

$$\hat{k} = \left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0\right), \quad \left|\vec{k}\right| = 2.0\text{m}^{-1}$$

a) 2 pts

$$\omega = kc = 6\times 10^8 s^{-1}, \quad \lambda = \frac{2\pi}{k} = 3.14 \text{ m},$$

b) 1 pts

Propagating along the direction of wave vector $\hat{k} = \left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0\right)$

c) 4 pts

$$\vec{E} = E_0\hat{E}_0\sin\left(2.0\frac{x+y}{\sqrt{2}} - \omega t\right), \quad E_0 = cB_0 = 150 \text{ V/m}$$

$$\hat{E}_0\times\hat{B}_0 = \hat{E}_0\times\hat{z} = \hat{k} = \left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0\right) \rightarrow \hat{E}_0 = \frac{-\hat{x}+\hat{y}}{\sqrt{2}}$$

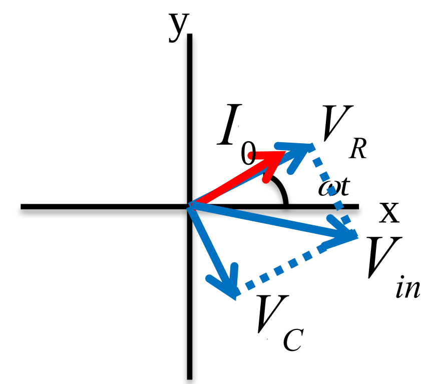
$$E_{0x} = \frac{-150}{\sqrt{2}} \text{ V/m}, \quad E_{0y} = \frac{150}{\sqrt{2}} \text{ V/m},$$

d) 3 pts

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{E_0B_0}{\mu_0}\left(\hat{k}\right)\sin^2\left(2.0\frac{x+y}{\sqrt{2}} - \omega t\right) = \frac{750}{4\pi}\frac{\hat{x}+\hat{y}}{\sqrt{2}}\sin^2\left(2.0\frac{x+y}{\sqrt{2}} - \omega t\right), \text{ W/m}^2$$

$$\langle I\rangle = \langle S\rangle = \frac{E_0B_0}{2\mu_0} = 30 \text{ W/m}^2$$

(a) resistor R and V_{out} are connected in parallel



2

$$I_{in} = I_0 \cdot \cos(\omega t)$$

$$\Rightarrow \quad V_R(t) = V_{R0}\cos(\omega t) \quad ; \quad V_C(t) = V_{C0}\cos(\omega t + \pi/2)$$

Assume $\quad V_{in}(t) = V_{in0}\cos(\omega t + \delta) = V_R(t) + V_C(t)$

$$V_{in0} = \sqrt{V_R^{\ 2} + V_C^{\ 2}}$$

$$= \sqrt{(I_0 R)^2 + (I_0 X_C)^2} = I_0\sqrt{R^2 + X_C^{\ 2}}$$

$$= I_0 Z \qquad \Rightarrow Z = \sqrt{R^2 + X_C^{\ 2}}$$

$\left(X_C = \dfrac{1}{wC} = \dfrac{1}{2p\,fC} \right)$ 2

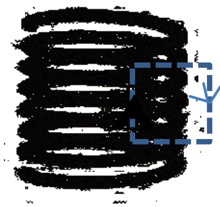
$$V_{out0} = V_{R0} = I_0 R$$ 1

$$A = V_{out0}/V_{in0} = \frac{R}{\sqrt{R^2 + \dfrac{1}{4\pi^2 f^2 C^2}}} = \frac{2\pi fCR}{\sqrt{4\pi^2 f^2 C^2 R^2 + 1}} = \frac{1}{\sqrt{1 + \dfrac{1}{4\pi^2 f^2 C^2 R^2}}}$$ 1

(b) $f \to 0:$ $\quad A \to 0$ 1

$f \to \infty:$ $\quad A \to 1$ 1

(A)



$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$$ 1

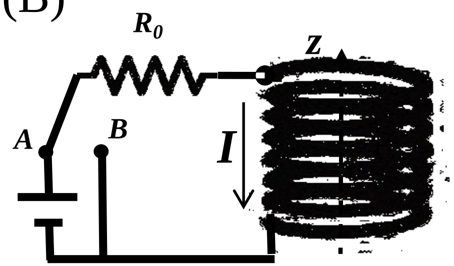
$$B(t) = \mu_0 \frac{N'}{l} I$$

$$\vec{B}(t) = \mu_0 \frac{N}{h} I_0 e^{-t/\tau}\hat{z}$$ 1

$$\mathcal{E} = -\frac{N d\Phi_B}{dt} = -L\frac{dI}{dt}$$ 1

$$L = \frac{N\Phi_B}{I} = \frac{N\mu_0 \frac{N}{h} I\pi a^2}{I} = \mu_0\left(\frac{N}{h}\right)^2\left(h\pi a^2\right)$$ 2

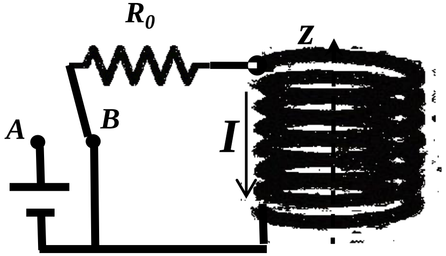
(B)



For a long time, L acts like a short circuit.

$$V - R_1 I_0 = 0$$ 2

$$\Rightarrow I_0 = \frac{V}{R_1}$$



Kirchhoff's law:

$$-L\frac{dI}{dt} - R_1 I = 0$$ 1

$$\frac{dI}{dt} + \frac{R_1}{L} I = 0$$

$$\Rightarrow I = I_0 e^{-t/\tau} \quad \text{and} \quad \tau = \frac{L}{R_1}$$ 2

(A)

$$\vec{B}(t) = \mu_0 \frac{N}{h} I_0 e^{-t/\tau}\hat{z}$$

1

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$ 1

$$\Rightarrow 2\pi a E = -\frac{dB}{dt}\pi a^2$$

$$\vec{E} = \frac{\mu_0 N I_0 a}{2\tau h} e^{-t/\tau}\left(\hat{y}\right)$$ 1

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0} = \frac{\mu_0 N^2 I_0{}^2 a}{2\tau h^2} e^{-2t/\tau}\left(\hat{x}\right)$$ 2

流出線圈

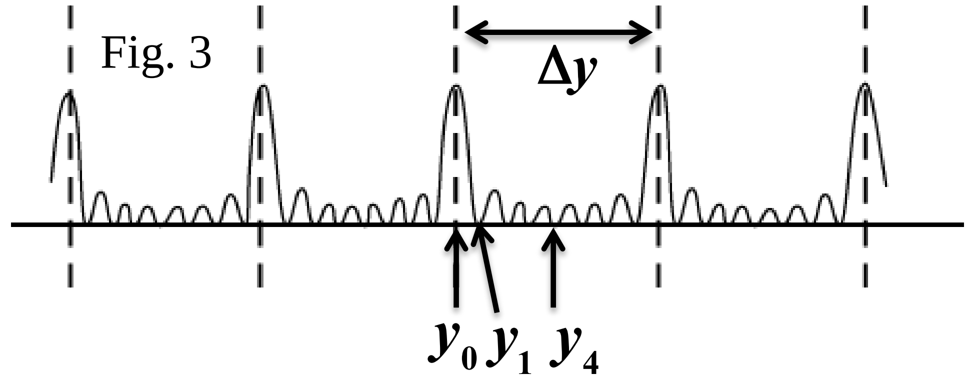
$$P = \int \vec{S}\cdot d\vec{A} = \frac{\mu_0 N^2 I_0{}^2 \pi a^2}{\tau h} e^{-2t/\tau}$$ 1

$$L = \mu_0\left(\frac{N}{h}\right)^2\left(h\pi a^2\right)$$

$$\Rightarrow P = \frac{\mu_0\left(\frac{N}{h}\right)^2\left(h\pi a^2\right) I_0{}^2 e^{-2t/\tau}}{L/R_1} = R_1 I^2(t)$$ 1

(a) 3 pts

$N = 8$ (7 dark lines between principal maxima) 1 pts



$$\frac{\Delta y}{L} = \tan\theta \simeq \sin\theta = \frac{\lambda}{d}\frac{\delta}{2\pi} = \frac{\lambda}{d}$$ 2 pts

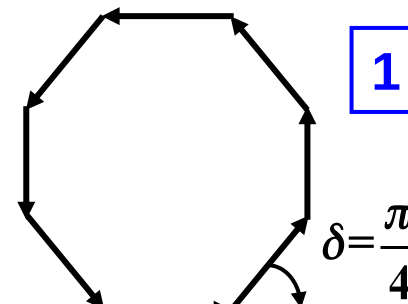
$$\rightarrow \Delta y = \frac{L\lambda}{d} = \frac{5 \cdot 500 \cdot 10^{-9}}{100 \cdot 10^{-6}} = 0.025 \text{ m (or 2.5 cm)}$$

6 pts

(b) y_1 : $\delta = \frac{2\pi}{8} = \frac{\pi}{4}$ (the phase difference) 1 pts

$$\frac{y_1}{L} = \tan\theta \simeq \sin\theta = \frac{\lambda}{d}\frac{\delta}{2\pi} = \frac{\lambda}{8d}$$ 1 pts

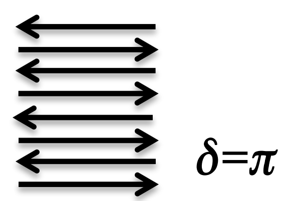
$$\rightarrow y_1 = \frac{L\lambda}{8d} = \frac{5 \cdot 500 \cdot 10^{-9}}{800 \cdot 10^{-6}} = 0.0031 \text{ m (or 0.31 cm)}$$ 1 pts



y_4 : $\delta = 4\frac{2\pi}{8} = \pi$ (the phase difference) 1 pts

$$\frac{y_4}{L} = \tan\theta \simeq \sin\theta = \frac{\lambda}{d}\frac{\delta}{2\pi} = \frac{\lambda}{2d}$$ 1 pts

$$\rightarrow y_4 = \frac{L\lambda}{2d} = \frac{5 \cdot 500 \cdot 10^{-9}}{200 \cdot 10^{-6}} = 0.0125 \text{ m (or 1.25 cm)}$$ 1 pts



(c) 4 pts

The first diffraction minimum occurs at $a\sin\theta = \frac{\lambda}{a}$ $\boxed{\text{1 pts}}$

$$\rightarrow y_d = \frac{L\lambda}{a} = \frac{5\cdot 500\cdot 10^{-9}}{20\cdot 10^{-6}} = 0.125\text{ m (or 12.5 cm)}$$ $\boxed{\text{2 pts}}$

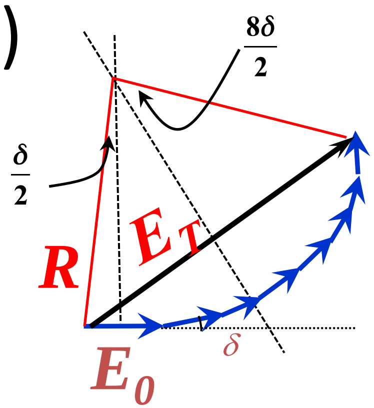
The interference (principal maxima) occurs at

$$\delta = 2n\pi = \frac{2\pi}{\lambda}d\sin\theta < \frac{2\pi}{\lambda}d\frac{\lambda}{a} \rightarrow n < \frac{d}{a} = 5$$ $\boxed{\text{1 pts}}$

There are **9 peaks** $\left(n = 0, \pm 1, \pm, 2, \pm 3, \pm 4\right)$

5 pts

(d)



$$\frac{E_T/2}{R} = \sin\frac{8\delta}{2}, \quad \frac{E_0/2}{R} = \sin\frac{\delta}{2} \quad \Rightarrow E_T = E_0\frac{\sin 4\delta}{\sin\delta/2}$$ $\boxed{\text{1 pts}}$

with the diffraction: $E_0 \rightarrow E_0\frac{\sin\beta/2}{\beta/2}$ $\boxed{\text{1 pts}}$

$$\Rightarrow I(y) = I_0\frac{\sin^2\beta/2}{(\beta/2)^2}\frac{\sin^2 4\delta}{\sin^2\delta/2}$$ $\boxed{\text{1 pts}}$

with

$$\delta = \frac{2\pi d}{\lambda}\frac{y}{L} \simeq 2.5\times 10^2 y$$

$$\beta = \frac{2\pi a}{\lambda}\frac{y}{L} \simeq 50 y$$ $\boxed{\text{2 pts}}$