General Physics (I) **期末考** **Jan. 3, 2020**

試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(50 points) (所有題目必須有計算過程,否則不予計分)

1& 2.(20 pts)A heat engine takes one mole of ideal monatomic gas around the cycle shown in Fig. 1 (isobaric a→b, isothermal b→c, straight line c→d, and adiabatic d→a). The volume at points ***a, b, c***, and ***d*** are given by **$8V_0$, $4V_0$, $2V_0$, and V_0**, respectively. The pressure at point ***a*** is P_0. (Copy the tables below to your answer sheet and write your answer in term of P_0, V_0, R, **ln2, ln3, ln5**, and **ln7**)

a) (5 pts) Determine the thermal dynamic variables (P, V, and T) at points ***a, b, c,*** and ***d***.

b) (12 pts) Calculate the work done (by the gas), heat, internal energy change, and entropy change for each process (***a→b, b→c, c→d,*** and ***d→a***).

c) (3 pts) Determine the efficiency of the heat engine.

	P/P_0	V/V_0	$T/(P_0V_0/R)$
a	1	8	
b	1	4	
c		2	
d		1	

	$W/(P_0V_0)$	$Q/(P_0V_0)$	$\Delta E_{int}/(P_0V_0)$	$\Delta S/R$
a→b				
b→c				
c→d				
d→a				

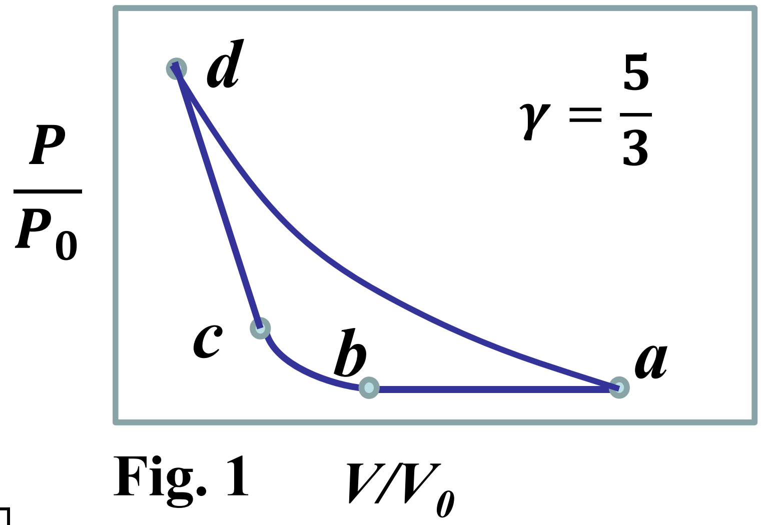


Fig. 1

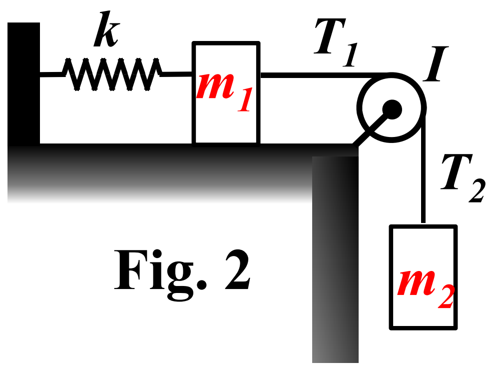
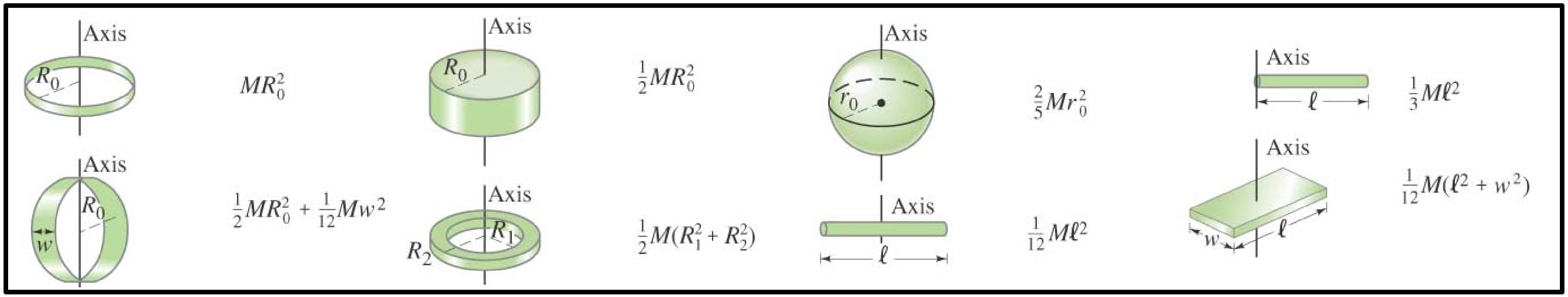


Fig. 2

3. (15 pts) Assume that the universe is consisted of iron, water and surrounding in this problem. The **3-kg** piece of iron at **800K** is thrown into **X**-kg water initially at **280K**, assuming no heat loss to the surroundings. The final temperature is **300K**. Find (a) (4 pts) The mass **X** of the water; (b) (4 pts) the change of the entropy of the water, (c) (4 pts) the change of the entropy of the iron, (d) (3 pts) the change of the entropy of the surrounding and the universe as whole. (the specific heat of water is **4200 J/kg°C**, and the specific heat of iron is about **420 J/kg°C**.).

4. (15 pts) As shown in Fig. 2, a block of mass $\boldsymbol{m_1}$ on the frictionless surface is attached to a spring. The spring constant is $\boldsymbol{k}$. the block is connected to another block of the same mass $\boldsymbol{m_2}$ with a string. The string wrap around a pulley which is free to rotate around its axis. The radius and the moment of inertia of the pulley are $\boldsymbol{R}$, $\boldsymbol{I}$. The system is initially in equilibrium. At $\boldsymbol{t} = 0$ the block $\boldsymbol{m_2}$ is pulled downward and then released to start a simple harmonic motion with the rotation of the pulley due to the static friction between the pulley and the string. If $\boldsymbol{m_1 = m_2 = m}$, determine the period of the simple harmonic motion in terms of $\boldsymbol{m}$, $\boldsymbol{k}$, $\boldsymbol{R}$, and $\boldsymbol{I}$. (Note that the tension $\boldsymbol{T_1}$ and $\boldsymbol{T_2}$ of the string on either side of the pulley are not always the same due to the rotational motion of the pulley).

Reference for moment of inertia



II.選擇題(50 points)

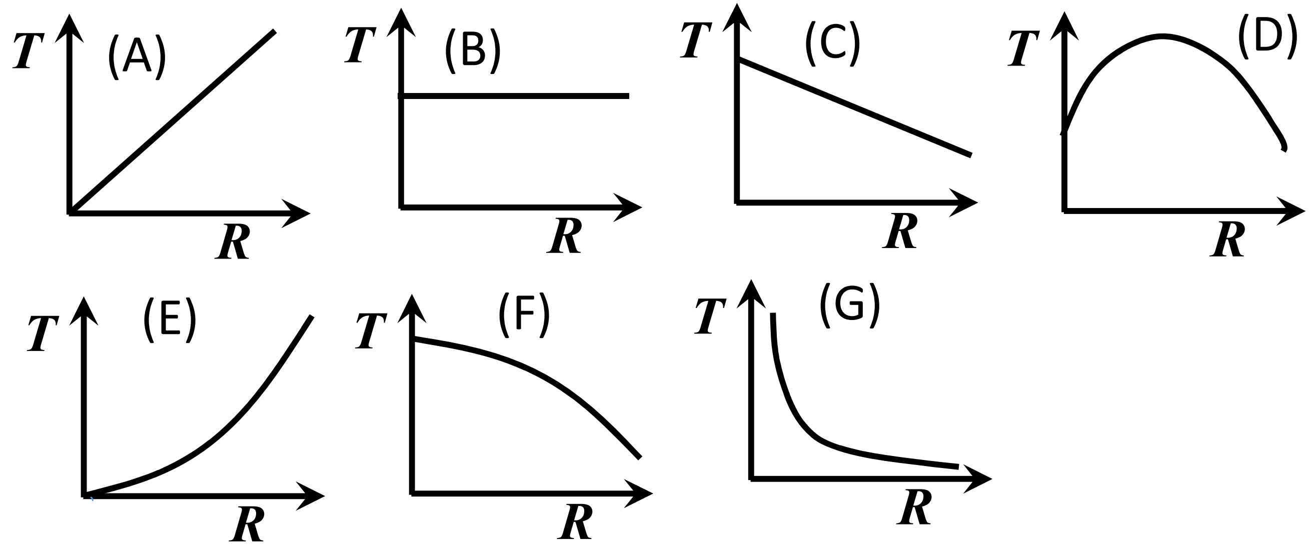
1. (5 pts) A Carnot engine (two isothermal and two adiabatic processes) operates at the temperature between **50 °C** and **200 °C,** and performs **10J** of work each cycle, which takes **0.2s.** The efficiency of the engine is ***a*%**, the average power is ***b*(Watt),** and the heat from the low-temperature reservoir every cycle$|Q_L|$ is ***c*J**, respectively. Let $N = a + b + c$. Which of the following is correct?
(A) $N < 80$ **(B)** $80 \le N < 90$ **(C)** $90 \le N < 100$ **(D)** $100 \le N < 110$ **(E)** $110 \le N < 120$ **(F)** $120 \le N < 130$
(G) $130 \le N < 140$ **(H)** $140 \le N < 150$ **(J)** $150 \le N < 160$ **(K)** $160 \le N$.

2. (5pts) At low temperature the specific heat of diamond is $\mathbf{c} = 1.70 \times 10^{-7}\ T^3$ ($\mathrm{J \cdot mol^{-1} \cdot K^{-1}}$). Determine the entropy change ΔS of **1.00** mole of diamond when it is heated from **2 K** to **20 K**? (ΔS in unit of $\mathrm{J \cdot K^{-1}}$)
(A) $\Delta S < 10^{-4}$ **(B)** $10^{-4} \le \Delta S < 3 \times 10^{-4}$ **(C)** $3 \times 10^{-4} \le \Delta S < 5 \times 10^{-4}$ **(D)** $5 \times 10^{-4} \le \Delta S < 7 \times 10^{-4}$
(E) $7 \times 10^{-4} \le \Delta S < 9 \times 10^{-4}$ **(F)** $9 \times 10^{-4} \le \Delta S$

3. (5 pts) An ideal gas system consists of one mole of mono-atomic gas at state ***a*** with pressure P_0, volume V_0, and temperature T_0. W_1 ($a \rightarrow b \rightarrow d$) is the sign of the total work done by this system for an isothermal process from the state ***a*** to a middle state ***b*** ($P=2P_0$) then an isobaric process from ***b*** to the final state ***d*** ($T = 2T_0$). W_2 ($a \rightarrow d$) is the sign of the work done by the system for an isovolumetric process from the state ***a*** to the final state ***d***. W_3 ($a \rightarrow c \rightarrow d$) is the sign of the total work done by this system for an isobaric process from the state ***a*** to a middle state ***c*** ($T=2T_0$) then an isothermal process from ***c*** to the final state ***d***. Which one of the following is correct for (W_1, W_2, W_3)?
(A) (+,+,+); **(B)** (+,+, -); **(C)** (-,+,+); **(D)** (-,+,-); **(E)** (+,0,+); **(F)** (+,0, -); **(G)** (-,0,+);
(H) (-,0,-); **(J)** (+,-,+); **(K)** (+,-, -); **(L)** (-,-,+); **(M)** (-,-,-) **(N) None of above**

4. (5 pts) Same as in problem 3, the entropy change for system from ***a*** to ***d*** is $\Delta S = xR$. What is the value x? (R is the ideal gas constant, and **ln2 ~ 0.7**, **ln3~ 1.1**, and **ln5 ~ 1.6**)
(A) $x < -2.5$ **(B)** $-2.5 \le x < -2.0$ **(C)** $-2.0 \le x < -1.5$ **(D)** $-1.5 \le x < -1.0$ **(E)** $-1.0 \le x < -0.5$
(F) $-0.5 \le x < 0$ **(G)** $0 \le x < 0.5$ **(H)** $0.5 \le x < 1.0$ **(J)** $1.0 \le x < 1.5$ **(K)** $1.5 \le x < 2.0$
(L) $2.0 \le x < 2.5$ **(M)** $2.5 \le x$

5. (5 pts) As shown in Fig. 3, a torsional pendulum (扭擺) consists of a string and a disk of radius $\boldsymbol{R}$ and mass $\boldsymbol{M}$, the torque $\boldsymbol{\tau}$ of the string resulted from the twist angle of the string $\boldsymbol{\theta}$, is $\boldsymbol{\tau = -\kappa \cdot \theta}$, where $\boldsymbol{\kappa}$ is a positive constant. If the mass of the disk $\boldsymbol{M}$ is fixed, which of the following shows the correct relation between the period $\boldsymbol{T}$ and the radius $\boldsymbol{R}$ of the pendulum?



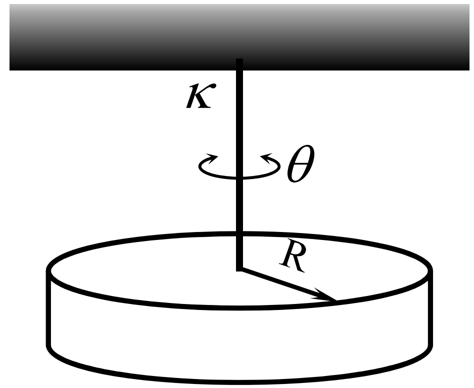


Fig. 3

6. (5 pts) A uniform rod with length $\boldsymbol{L}$, mass $\boldsymbol{m}$ hang from one end and the other end of the rod is attached with a horizontal massless spring with spring constant $\boldsymbol{k}$ (Fig. 4). This spring is initially at the equilibrium position ($\boldsymbol{\theta = 0}$). Assume the angle $\boldsymbol{\theta}$ is so small such that $\boldsymbol{\sin\theta \sim \theta, \cos\theta \sim 1}$. At $\boldsymbol{t = 0}$, a bullet of mass $\boldsymbol{m/3}$ and speed $\boldsymbol{v}$ strikes and becomes embedded inside the rod at position $\boldsymbol{L/2}$ from the pivot. The rod is then oscillating with angular displacement as: $\boldsymbol{\theta(t) = \theta_0 \cos(\omega t + \phi)}$. Which of the following is correct for the constants θ_0 and ϕ? (assume $\boldsymbol{\theta}$ is positive for counter-clockwise rotation)

(A) $\theta_0 > 0$, $\phi = 0$ (B) $\theta_0 < 0$, $\phi = 0$ (C) $\theta_0 > 0$, $\phi = \pi/2$ (D) $\theta_0 < 0$, $\phi = \pi/2$ (E) None of above

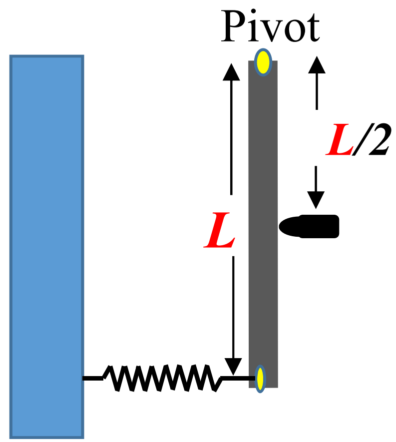


Fig. 4

Multiple Choice Questions:

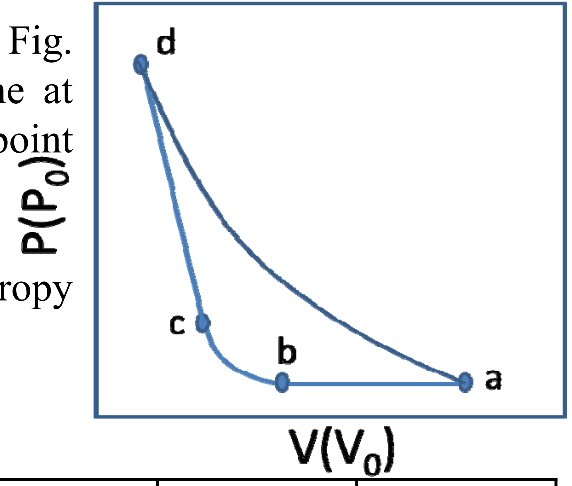
1	**2**	**3**	**4**	**5**	**6**				
D	**C**	**F**	**J**	**A**	**C**				
7	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	**B**	**E**	**G**	**A**	**E**	**C**	**E**	**C**	**E**

1. A heat engine takes one mole of ideal monatomic gas around the cycle shown in Fig. (isobaric a→b, isothermal b→c, straight line c→d, and adiabatic d→a). The volume at points a, b, c, and d are given by $8V_0$, $4V_0$, $2V_0$, and V_0, respectively. The pressure at point a is P_0. (Write your answer in term of P_0, V_0, R, ln2, ln3, ln5, and ln7)

a) (5pts) Determine the thermal dynamic variables (P, V, and T) at points a, b, c, and d.

b) (12pts) Calculate the work done (by the gas), heat, internal energy change, and entropy change for each process (a→b, b→c, c→d, and d→a).

c) (3pts) Determine the efficiency of the heat engine.



(a) $T_b = T_c$ (isothermal) $\Rightarrow p_b v_b = p_c v_c \Rightarrow p_c = 2p_0$

$p_d v_d^\gamma = p_a v_a^\gamma$ (adiabatic) $\Rightarrow p_d (v_0)^{5/3} = p_0 (8v_0)^{5/3} \Rightarrow p_d = 32 p_0$

$$T_a = \frac{8p_0v_0}{R};\ T_b = \frac{4p_0v_0}{R};\quad T_c = \frac{4p_0v_0}{R};\quad T_d = \frac{32p_0v_0}{R}$$

5	P (P_0)	V (V_0)	T (P_0V_0/R)
a	1	8	8
b	1	4	4
c	2	2	4
d	32	1	32

(b) a→b (isobaric)

$$\Delta E_{\text{int},a\to b} = \frac{3}{2}R(-4\frac{p_0v_0}{R}) = -6p_0v_0$$

$$W_{a\to b} = \int_a^b pdV = p\Delta V = -4p_0v_0$$

$$Q_{a\to b} = \Delta E_{a\to b} + W_{a\to b} = -10p_0v_0$$

$$\Delta S_{a\to b} = nC_v \ln\frac{T_b}{T_a} + nR\ln\frac{V_b}{V_a} = \frac{-5}{2}R\ln 2$$

c→d (line)

$$\Delta E_{\text{int},c\to d} = \frac{3}{2}R(28\frac{p_0v_0}{R}) = 42p_0v_0$$

$$W_{c\to d} = -\frac{(32+2)p_0}{2}v_0 = -17p_0v_0$$

$$Q_{c\to d} = \Delta E_{c\to d} + W_{c\to d} = 25p_0v_0$$

$$\Delta S_{c\to d} = \frac{3}{2}R\ln\frac{32}{4} + R\ln\frac{1}{2} = \frac{7}{2}R\ln 2$$

b→c (isothermal)

$$\Delta E_{\text{int},b\to c} = 0$$

$$W_{b\to c} = \int_b^c \frac{nRT}{V}dV = R\frac{4p_0v_0}{R}\ln\frac{2}{4}$$

$$= (-4\ln 2)p_0v_0$$

$$Q_{b\to c} = W_{b\to c} = (-4\ln 2)p_0v_0$$

$$\Delta S_{b\to c} = nR\ln\frac{V_c}{V_b} = -R\ln 2$$

d→a (adiabatic)

$$\Delta E_{\text{int},d\to a} = \frac{3}{2}R(-\frac{24p_0v_0}{R}) = -36p_0v_0$$

$$W_{d\to a} = -\Delta E_{d\to a} = 36p_0v_0$$

$$Q_{d\to a} = 0$$

$$\Delta S_{d\to a} = 0$$

12	W (P_0V_0)	Q (P_0V_0)	ΔE_{int} (P_0V_0)	ΔS (R)
a→b	-4	-10	-6	-5ln2/2
b→c	-4ln2	-4ln2	0	-ln2
c→d	-17	25	42	7ln2/2
d→a	36	0	-36	0

(c) 3 e = (15-4ln2)/25

Solution for Problem 3

(a)

$$|m_{Fe}C_{Fe}\Delta T_{Fe}| = |m_{water}C_{water}\Delta T_{water}|$$ 2

$$3 \cdot 420 \cdot (800 - 300) = x \cdot 4200 \cdot (300 - 280)$$

$$x = 7.5kg$$ 2

(b)

$$\Delta S_w \cong \frac{m_w C_w \Delta T_w}{\frac{T_f + T_i}{2}} = \frac{m_{Fe} C_{Fe} \Delta T_{Fe}}{\frac{T_f + T_i}{2}}$$ 2

$$\Delta S_w \cong \frac{30 \cdot 420 \cdot (800 - 300)}{\frac{300 + 280}{2}} = 2.17 \times 10^5\, JK^{-1}$$ 2

(C)

$$\Delta S_{Fe} = \int_{800}^{300} \frac{m_{Fe} C_{Fe} dT}{T} = -30 \cdot 420 \cdot (\ln(8) - \ln(3))$$ 2

$$\Delta S_{Fe} == -1.26 \cdot \times 10^4 (\ln(8) - \ln(3)) JK^{-1} = 1.24 \times 10^4\, JK^{-1}$$ 2

(D)

$$\Delta S_{surrounding} = 0$$ 1

$$\Delta S_{total} = \Delta S_w + \Delta S_{Fe} + \Delta S_{surrounding} = 2.06 \times 10^5\, JK^{-1}$$ 2

4. (15 pts) As shown in Fig. 2, a block of mass $\boldsymbol{m_1}$ on the frictionless surface is attached to a spring. The spring constant is $\boldsymbol{k}$. the block is connected to another block of the same mass $\boldsymbol{m_2}$ with a string. The string wrap around a pulley which is free to rotate around its axis. The radius and the moment of inertia of the pulley are $\boldsymbol{R}$, $\boldsymbol{I}$. The system is initially in equilibrium. At $\boldsymbol{t} = 0$ the block $\boldsymbol{m_2}$ is pulled downward and then released to start a simple harmonic motion with the rotation of the pulley due to the static friction between the pulley and the string. If $\boldsymbol{m_1 = m_2 = m}$, determine the period of the simple harmonic motion in terms of $\boldsymbol{m}$, $\boldsymbol{k}$, $\boldsymbol{R}$, and $\boldsymbol{I}$. (Note that the tension $\boldsymbol{T_1}$ and $\boldsymbol{T_2}$ of the string on either side of the pulley are not always the same due to the rotational motion of the pulley).

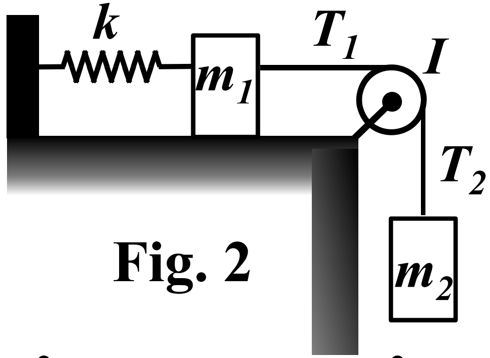
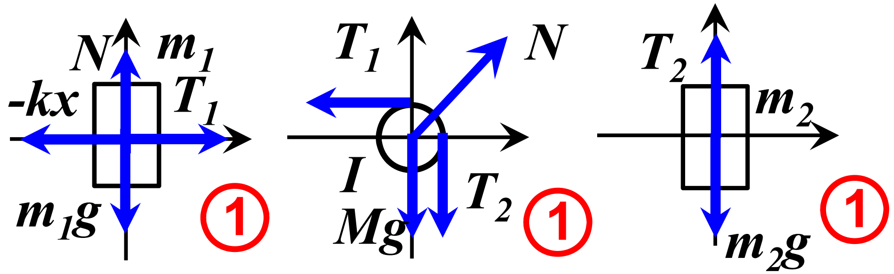


Fig. 2



For $\boldsymbol{m_1}$, $\sum \vec{F} = m_1\vec{a}_1$

$$T_1 - kx = m_1 a_1 = m a_1 \quad (1)$$

For $\boldsymbol{m_2}$, $\sum \vec{F} = m_2\vec{a}_2$

$$T_2 - (m_2 g) = m_2 a_2 = m a_2 \quad (2)$$

For ***the pulley***, $\sum \vec{\tau} = I\vec{\alpha}$

$$\vec{R}\times\vec{T}_1 + \vec{R}\times\vec{T}_2 = I\vec{\alpha}$$

$$RT_1 - RT_2 = I\alpha \quad (3)$$

The relation between a_1, a_2, and α,

$$a_1 = -a_2 = a = -R\alpha \quad (4) \quad 1$$

$$T_1 - kx = ma \quad (5) \quad 1$$

$$T_2 - mg = -ma \quad (6) \quad 1$$

$$RT_1 - RT_2 = -I\frac{a}{R} \quad (7) \quad 1$$

(7)-R(5)+R(6):

$$Rkx - Rmg = -2Rma - I\frac{a}{R}$$

$$\Rightarrow k(x - \frac{mg}{k}) = -(2m + \frac{I}{R^2})a \quad 5$$

Define $z = x\text{-}mg/k$

$$\Rightarrow \frac{d^2z}{dt^2} = \frac{d^2}{dt^2}(x - \frac{mg}{k}) = \frac{d^2x}{dt^2} = a$$

$$\Rightarrow kz = -(2m + \frac{I}{R^2})\frac{d^2z}{dt^2}$$

$$\Rightarrow (2m + \frac{I}{R^2})\frac{d^2z}{dt^2} + kz = 0$$

$$\Rightarrow T = 2\pi\sqrt{\frac{2m + \frac{I}{R^2}}{k}} \quad 3$$

Solution that applies fact of the conservation of Mechanical Energy

$$E=\frac{1}{2}m_1v_1^2+\frac{1}{2}kx_1^2+\frac{1}{2}I\omega^2+\frac{1}{2}m_2v_2^2+m_2gy_2 \quad 2$$

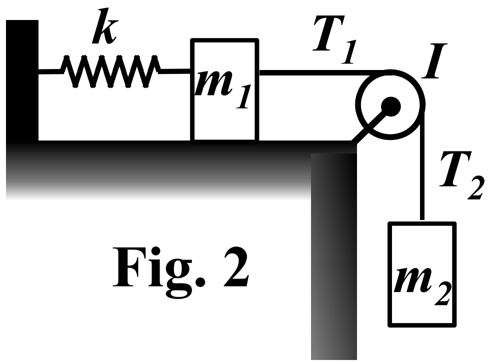


Fig. 2

The relation between v_1, v_2, and ω,

$$v_1=\frac{dx_1}{dt},\ v_2=\frac{dy_2}{dt},\ v\equiv\frac{dx_1}{dt}=v_1=-v_2=-R\omega \quad 1$$

$$\Rightarrow E=\frac{1}{2}mv^2+\frac{1}{2}kx_1^2+\frac{1}{2}I\omega^2+\frac{1}{2}m\ v^2+mgy_2 \quad 1$$

$$\frac{dE}{dt}=0=\frac{d}{dt}(mv^2+\frac{1}{2}kx_1^2+\frac{1}{2}I\omega^2+mgy_2)$$

$$\Rightarrow(2mva+kx_1v-I\frac{v}{R}\alpha-mgv)=0$$

$$\Rightarrow 2ma+kx_1-\frac{I}{R}\alpha-mg=0 \quad 1$$

The relation between a and ω, $a=-R\alpha$ 1

$$\Rightarrow 2ma+kx_1+\frac{I}{R^2}a-mg=0 \quad 1$$

$$\Rightarrow k(x-\frac{mg}{k})=-(2m+\frac{I}{R^2})a \quad 5$$

Define $z=x\text{-}mg/k$

$$\Rightarrow\frac{d^2z}{dt^2}=\frac{d^2}{dt^2}(x-\frac{mg}{k})=\frac{d^2x}{dt^2}=a$$

$$\Rightarrow kz=-(2m+\frac{I}{R^2})\frac{d^2z}{dt^2}$$

$$\Rightarrow(2m+\frac{I}{R^2})\frac{d^2z}{dt^2}+kz=0$$

$$\Rightarrow T=2\pi\sqrt{\frac{2m+\frac{I}{R^2}}{k}} \quad 3$$