Differential Equations **Name:** ____________

Exam II, Fall 2019 **ID:** ____________

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1. (10%) **Reduction of Order**
   Use the one solution given to determine the second linearly independent solution of the following differential equation by *the method of reduction of order*.

$$y'' - \frac{2x}{x-1}y' - 4y = 0, \quad y_1 = x$$

   (Note: No credits will be given if other methods are used.)

   Solution: $y_2 = \frac{1}{2}(x-2)e^{2x}$

2. (10%) **Method of Undetermined Coefficients**
   Use *the method of undetermined coefficients* to determine the general solution of the differential equation

$$y'' - 4y' + 4y = 5xe^{2x}$$

   (Note: No credits will be given if other methods are used.)

   Solution: $y = c_1e^{2x} + c_2xe^{2x} + \frac{5}{6}x^3e^{2x}$

3. (10%) **Variation of Parameters**
   Use *the method of variation of parameters* to determine the particular solution of the differential equation

$$y'' + y' = x^3 - 1$$

   (Note: No credits will be given if other methods are used.)

   Solution: $y_p = \frac{1}{4}x^4 - x^3 + 3x^2 - 7x + 7$

4. (10%) **Cauchy-Euler Equation**
   Determine the general solution of the differential equation

$$x^3y''' + x^2y'' + 4y = 0, \quad x > 0$$

Solution: $y = \frac{c_1}{x} + x^{3/2}[c_2 \cos(\frac{\sqrt{7}}{2} \ln x) + c_2 \sin(\frac{\sqrt{7}}{2} \ln x)]$

5. (10%) **Green's Function**
   Use *the form of Green's function* to find a solution of the initial-value problem

$$y'' - y' = 1, \quad y(0) = 10, y'(0) = 1$$

   (Note: No credits will be given if Laplace transform is used.)

Solution: $y = 8 + 2e^x - x$

6. (10%) **Power Series Solutions about an Ordinary Point**
   Use *the power series method* to solve the differential equation

$$(x^2 - 1)y'' - y = 0$$

   The answer should include at least three terms for *each* independent solution. (Note: No credits will be given if other methods are used.)

Solution: $y = c_1(1 - \frac{1}{2}x^2 - \frac{1}{24}x^4 - \cdots) + c_2(x - \frac{1}{6}x^3 - \frac{1}{24}x^5 - \cdots)$

7. (10%) **Method of Frobenius**
   Use *the method of Frobenius* to obtain two linearly independent series solutions about $x = 0$ for the differential equation

$$2xy'' + 5y' + xy = 0$$

   The answer should include at least three terms for *each* independent solution. (Note: No credits will be given if other methods are used.)

Solution: $y = c_1 x^{-3/2}(1 - \frac{1}{2}x^2 + \frac{1}{40}x^4 + \cdots) + c_2(1 - \frac{1}{14}x^2 + \frac{1}{616}x^4 + \cdots)$

8. (10%) **Convergence of Power Series**
   Determine the interval of convergence and the radius of convergence of the power series

$$\sum_{n=0}^{\infty} 3^{n+1} x^n$$

Solution: Interval of convergence: $[-\frac{1}{3}, \frac{1}{3})$, radius of convergence: $\frac{1}{3}$

9. (10%) **Laplace Transform**
   Derive the Laplace transform of the function

$$f(t) = \begin{cases} 0, & t < 2\pi \\ \sin t, & 2\pi \leq t \leq 3\pi \\ 0, & t > 3\pi \end{cases}$$

Solution: $F(s) = \frac{e^{-2\pi s} + e^{-3\pi s}}{s^2+1}$

10. (10%) **Laplace Transform and Initial-Value Problem**
    Use *Laplace transform* to solve the initial-value problem

$$y''' - 8y = e^{-2t}, \quad y(0) = y'(0) = y''(0) = 0$$

(Note: No credits will be given if other methods are used.)

Solution: $y = -\frac{1}{16}e^{-2t} + \frac{1}{48}e^{2t} + \frac{1}{24}e^{-t}\cos\sqrt{3}t - \frac{1}{8\sqrt{3}}e^{-t}\sin\sqrt{3}t$

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