Signals and Systems Quiz 4

1. (30%) Determine the inverse Fourier transform of $X(j\omega)$, where $X(j\omega) = X_1(j\omega) + X_2(j\omega)$,

$$X_1(j\omega) = 4\delta(\omega) - \pi\delta(\omega - \pi) \; and \; X_2(j\omega) = \begin{cases} -5, & 0 \le \omega \le 1 \\ 3\pi, & -1 \le \omega < 0, \\ 0, & |\omega| > 1 \end{cases}$$

2. (40%) Consider an LTI system whose response to the input

$$x(t) = e^{-t}u(t) + 2te^{-t}u(t)$$

is

$$y(t) = \frac{1}{4}e^{-t}u(t) - \frac{1}{4}e^{-5t}u(t)$$

(a) Determine the frequency response of this system

(b) Determine the impulse response of this system

(c) Find the differential equation relating the input and the output of this system

3. (30%) Determine the Fourier transfer of $x(t) = \frac{12}{\pi(9+t^2)}$ using duality