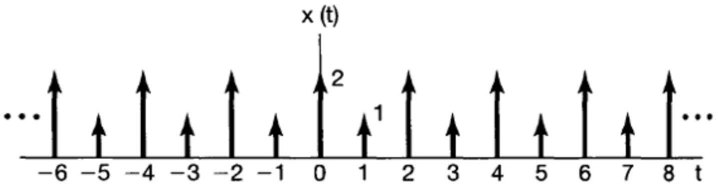
Signals and Systems – Quiz 4 05/20/2020

Name: Student ID:

1. (20%) Determine the Fourier transform of $x(t)$ as shown below:



Solution:

If

$$x_1(t) = \sum_{k=-\infty}^{\infty} \delta(t-2k),$$

Then

$$x(t) = 2x_1(t) + x_1(t-1).$$

Therefore,

$$X_1(j\omega) = \pi \sum_{k=-\infty}^{\infty} \delta(\omega - k\pi)$$

$$X(j\omega) = X_1(j\omega)\left[2 + e^{-j\omega}\right] = \pi \sum_{k=-\infty}^{\infty} \delta(\omega - k\pi)[2 + (-1)^k].$$

2. (30%) Consider the Fourier transform pair $\delta(t) \leftrightarrow 1$. Use duality to find the Fourier transform $G(j\omega)$ of the signal $g(t) = 1$.

Solution:

$$x(t) = \delta(t) \leftrightarrow X(j\omega) = 1$$

$$\delta(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} 1 \cdot e^{j\omega t} d\omega$$

Replace t wit $-t$

$$\delta(-t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} 1 \cdot e^{-j\omega t} d\omega$$

Exchange t and ω

$$\delta(-\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} 1 \cdot e^{-j\omega t} dt$$

$$\Rightarrow \; G(j\omega) = 2\pi\, \delta(-\omega) = 2\pi\, \delta(\omega)$$

3. (50%) Consider an LTI system whose response to the input and output are as follows:

$$x(t) = [2e^{-t} + 2e^{-3t}]u(t)$$
$$y(t) = [4e^{-t} - 4e^{-4t}]u(t)$$

(a) Find the frequency response of this system.

(b) Determine the system's impulse response.

(c) Find the differential equation relating the input and the output of this system.

Solution:

$$X(j\omega) = \frac{2}{j\omega + 1} + \frac{2}{j\omega + 3}$$
$$Y(j\omega) = \frac{4}{j\omega + 1} - \frac{4}{j\omega + 4}$$
$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{3(3 + j\omega)}{(4 + j\omega)(2 + j\omega)}$$

(a) Using the partial fraction expansion of the result of (a) and taking its inverse Fourier transform, it can obtain

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{3(3 + j\omega)}{(4 + j\omega)(2 + j\omega)} = \frac{3/2}{j\omega + 4} + \frac{3/2}{j\omega + 2}$$
$$h(t) = \frac{3}{2}[e^{-4t} + e^{-2t}]u(t)$$

(c)

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{9 + 3j\omega}{8 + 6j\omega - \omega^2}$$

Therefore, the differential equation is

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 3\frac{dx(t)}{dt} + 9x(t)$$