General Physics (I) **期中考 II** **Nov.27, 2020**

試卷請註明、姓名、班級、學號，請遵守考場秩序

I.**計算題**(45points) (所有題目必須有計算過程,否則不予計分)

1. (A) (6pts) An external force F_{ext}, parallel to the displacement, is pushing a small particle of mass $\boldsymbol{m}$ in a quasi-static motion from the bottom to the top of the quarter circle of radius $\boldsymbol{R}$, shown in Fig. 1. The frictional coefficients of the circular surface is $\mu_k = \mu$. Draw the free-body diagram and calculate the work done by the external force from point A to point B.
   (B) (9pts) Same condition as above except that the friction coefficient $\mu_k = \mu_0 \sin\theta$ and the particle moves at constant speed $v = \sqrt{gR}$ along the path. Draw the free-body diagram and calculate the work done by the external force from point A to point B. Useful formula: $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$; $\sin 2\theta = 2\sin\theta\cos\theta$
2. (15 pts) (A) (8pts) As shown in Fig. 2, a bullet of mass $\boldsymbol{m}$ is fired to embed into box A of mass $\boldsymbol{3m}$ which is initially at rest. The combined system (the bullet and box A) moves toward block B, which is free to move around. The mass and height of block B are $\boldsymbol{16m}$ and $\boldsymbol{H}$, respectively. There is no friction between all surfaces. Find the minimum velocity, $\boldsymbol{v_{0,min}}$, of the bullet, such that the combined system can climb up to the top of block B. Write down $\boldsymbol{v_{0,min}}$ in terms of $\boldsymbol{m}$, $\boldsymbol{H}$, and $\boldsymbol{g}$ (gravitational acceleration).
   (B) (7pts) The bullet with the velocity of $\boldsymbol{v_0} >> \boldsymbol{v_{0,min}}$ can make the combined system climb up to the top of block B, then slide down on to the other side of block B and continue to travel forward. Find the velocities of the combined system and block B in terms of $\boldsymbol{m}$, $\boldsymbol{v_0}$, $\boldsymbol{H}$, and $\boldsymbol{g}$ after they separate from each other.

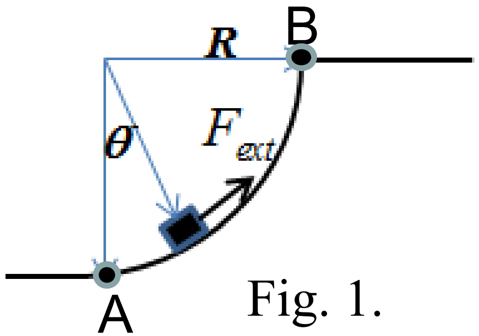


Fig. 1.

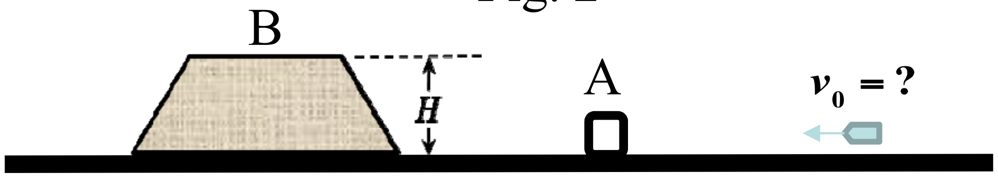


Fig. 2

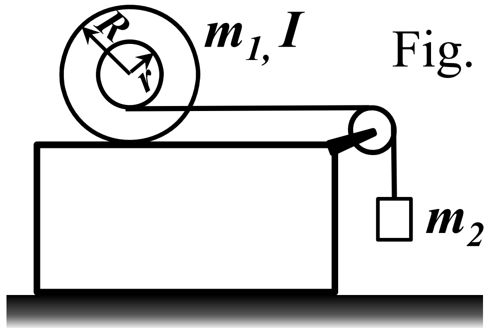


Fig. 3

3. (15pts) As shown in Fig. 3, a dumb bell on a table top is connected to a block of mass $\boldsymbol{m_2}$ with a massless string wrapping around a massless pulley, The mass of the dumbbell is $\boldsymbol{m_1}$, and its moment of inertia for rotation around the center of mass is $\boldsymbol{I} = 0.5\ \boldsymbol{m_1} \cdot \boldsymbol{R}^2$, and $\boldsymbol{R} = 2\boldsymbol{r}$. The coefficient of static friction is $\mu_s = 0.8$ for the dumb bell and the table surface. Let $\boldsymbol{m_1} = \boldsymbol{m}$, and $\boldsymbol{m_2} = 3\boldsymbol{m}$. (a) (10pts) Determine the acceleration of the dumb bell and the friction force between the dumb bell and the table. (b) (5pts) Determine the maximum $\boldsymbol{m_2}$ in terms of $\boldsymbol{m}$ for the dumb bell to execute pure roll motion without slipping on the table. (Free-body diagrams are required for the answers).

II.選擇題(57points)

1. (5pts) A ball is initially at rest on an ice surface with negligible friction. At time $t = 0$, a horizontal force begins to move the ball. The force is given by $F(t) = 12\text{-}3t^2$ in SI units, and it acts until its magnitude becomes zero. The momentum (p in SI unit) of the ball when $F = 0$ is (A) $p < 1$; (B) $1 \leqq p < 10$; (C) $10 \leqq p < 20$; (D) $20 \leqq p < 30$; (E) $30 \leqq p < 40$; (F) $40 \leqq p < 50$; (G) $50 \leqq p < 70$; (H) $70 \leqq p < 100$; (I) $100 \leqq p$.
2. (5pts) As shown in Fig. 4, a block of mass $\boldsymbol{m_1}$ is connected to the other block of mass $\boldsymbol{m_2}$ with a massless string which wrap around a pulley with radius R and moment of inertia $\boldsymbol{I}$ for rotation around its axis. After the blocks are released to move, the tensions of string on block $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ are $\boldsymbol{T_1}$ and, $\boldsymbol{T_2}$ respectively. Let $\boldsymbol{m_1} = \boldsymbol{m}$, $\boldsymbol{m_2} = 5\boldsymbol{m}$, $\boldsymbol{I} = \boldsymbol{x} \cdot \boldsymbol{m}\boldsymbol{R}^2$, and if $T_2/T_1 = 3$, which of the following is correct?
   (A) $0 < x \leq 0.5$ (B) $0.5 < x \leq 1$ (C) $1 < x \leq 2$ (D) $2 < x \leq 4$ (E) $4 < x \leq 8$
   (F) $8 < x \leq 16$ (G) $16 < x$
3. (5pts) A solid disk (with mass $\boldsymbol{M}$, radius $\boldsymbol{R}$ and moment of inertia $I_0 = \frac{1}{2} MR^2$) and a same disk but cutoff by a small disk (with radius $r = \frac{1}{2} R$) as shown in Fig. 5. Let $I_{CM,cut}$ be the moment of inertia of the cutoff disk when it rotates around its center of mass (CM) with the rotating axis out of the plane. What is the value $\boldsymbol{a}$ if we write $I_{CM,cut} = a\, I_0$?
   (A) $a \leq 0.5$ (B) $0.5 < a \leq 0.55$ (C) $0.55 < a \leq 0.6$ (D) $0.6 < a \leq 0.65$ (E) $0.65 < a \leq 0.7$
   (F) $0.7 < a \leq 0.75$ (G) $0.75 < a \leq 0.8$ (H) $0.8 < a \leq 0.85$ (J) $0.85 < a \leq 0.9$ (K) $0.9 < a$.

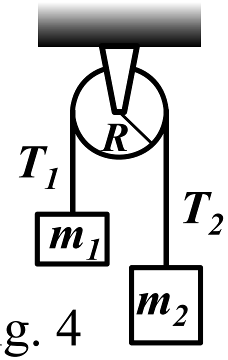


Fig. 4

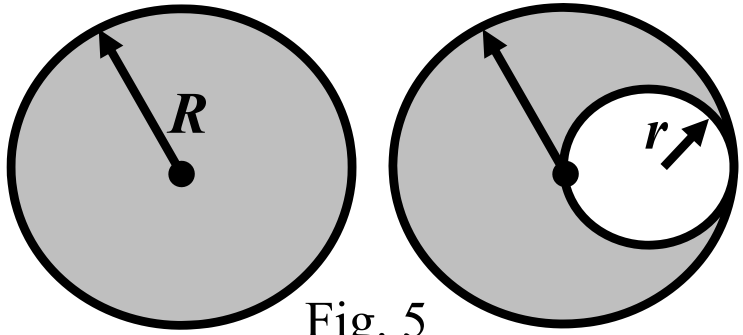
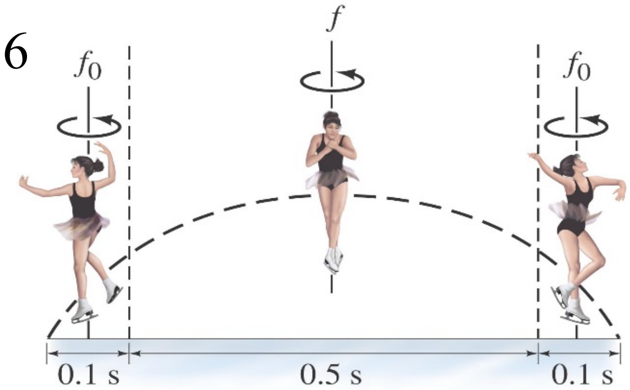
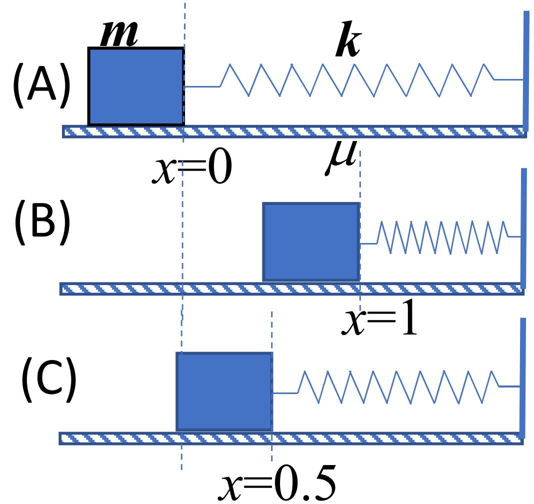


Fig. 5

Fig. 6


4. (5pts) Competitive ice skaters commonly perform single, double, and triple axle jumps in which they rotate 1½, 2½, and 3½ revolutions, respectively, about a vertical axis while airborne. A typical skater remains airborne for about **0.7s**. Suppose a skater leaves the ground in an "open" position (e.g., arms outstretched) with moment of inertia $\boldsymbol{I_0}$ and rotational frequency $\boldsymbol{f_0}$ **=*1.25* rev/s** maintaining this position for **0.1s**. The skater then assumes a "closed" position (arms brought closer, Fig. 6) with moment of inertia $\boldsymbol{I}$, acquiring a rotational frequency $\boldsymbol{f}$ which is maintained for **0.50s**. Finally, the skater immediately returns to the "open" position for **0.1s** until landing. Let I_s (I_T) is the moment of inertia in midflight (closed arms) in order to complete a **single (triple)** axle jumps. What is the ratio $\boldsymbol{x = I_S/I_T}$?
(A) $x \le 1$; (B) $1 < x \le 1.5$; (C) $1.5 < x \le 2$; (D) $2 < x \le 2.5$; (E) $2.5 < x \le 3$;
(F) $3 < x \le 3.5$; (G) $3.5 < x \le 4$; (H) $4 < x \le 4.5$; (J) $4.5 < x \le 5$; (K) $5 \le x$

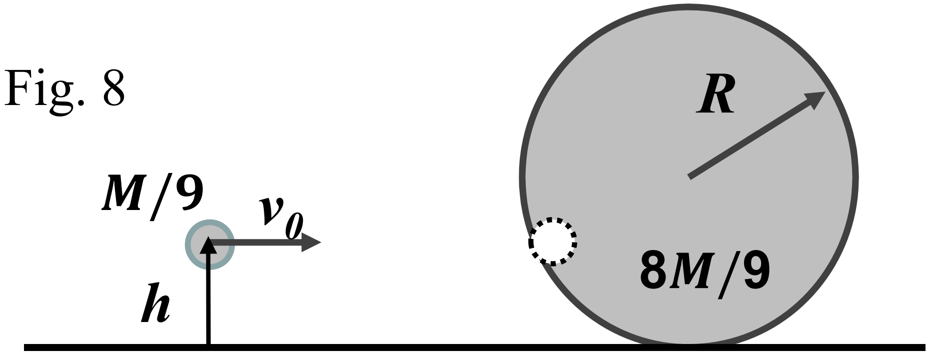
5. (5pts) As shown in Fig. 7(A), a box of mass $\boldsymbol{m}$ = 2 kg is moving with velocity $\boldsymbol{v}$ = 0.5 m/s, and hit a spring. The spring constant is $\boldsymbol{k}$ and the frictional constant of the surface is $\boldsymbol{\mu}$. After the box is hits the spring, it moves forward 1 meter (Fig. 7(B)), then travels back 0.5 m and stop there (Fig. 7(C)). What is the ratio $\boldsymbol{y = k/\mu}$ (in SI units) ?
(A) $0 < y \le 3$ (B) $3 < y \le 6$ (C) $6 < y \le 9$
(D) $9 < y \le 12$ (E) $12 < y \le 15$ (F) $15 < y$



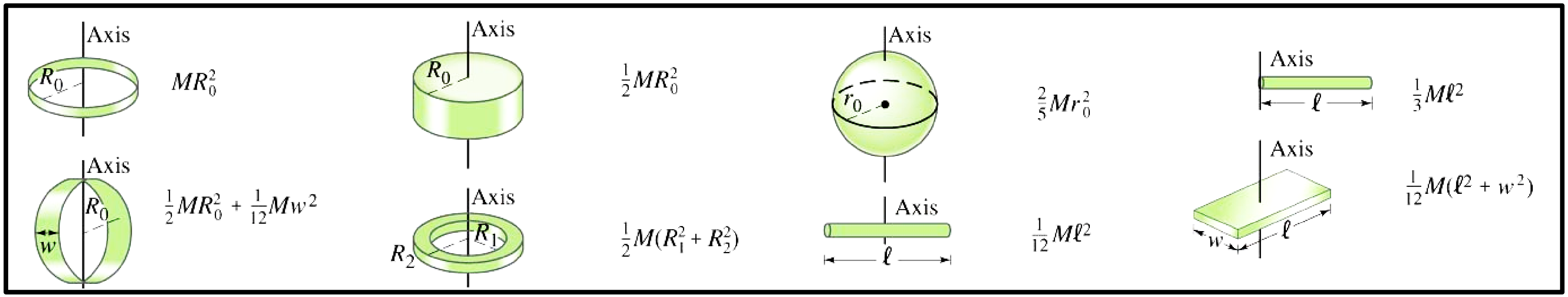
6. (5 pts.) In Fig. 8, a spherical clay (with radius $\boldsymbol{R}$ and mass $\boldsymbol{M}$), cut by a small piece (with mass $\boldsymbol{M/9}$) is at rest on the table initially. Now, the small piece with velocity $\boldsymbol{v_0}$ hits the clay at height $\boldsymbol{h} = \frac{1}{2}\boldsymbol{R}$ above the table and embeds into the clay. Right after the collision, the clay starts to move with the velocity $\boldsymbol{v}$ (relative to its center of mass) and the angular velocity $\boldsymbol{\omega}$. The static and kinetic friction coefficients are 0.3 (μ_s) and 0.1 (μ_k) between the clay and table. Let $\boldsymbol{v_f}$ ($\boldsymbol{\omega_f}$) be the velocity (angular velocity) of the clay in pure rolling. What is the value of $\boldsymbol{x}$ if we write the ratio $\boldsymbol{x} = \boldsymbol{\omega_f}/\boldsymbol{\omega}$? (assume the collision is completely inelastic and the small piece plus the spherical clay can be treated as an isolated system, *i.e.*, the friction between the clay and table can be ignored during the collision)

(A) $\boldsymbol{x} = -1$ (B) $-1 < \boldsymbol{x} \le -0.8$ (C) $-0.8 < \boldsymbol{x} \le -0.6$ (D) $-0.6 < \boldsymbol{x} \le -0.4$ (E) $-0.4 < \boldsymbol{x} \le -0.3$
(F) $-0.3 < \boldsymbol{x} \le -0.2$ (G) $-0.2 < \boldsymbol{x} \le -0.1$ (H) $-0.1 < \boldsymbol{x} \le 0$ (J) $0 < \boldsymbol{x} \le 0.1$ (K) $0.1 < \boldsymbol{x} \le 0.2$
(L) $0.2 < \boldsymbol{x} \le 0.3$ (M) $0.3 < \boldsymbol{x} \le 0.4$ (N) $0.4 < \boldsymbol{x} \le 0.6$ (O) $0.6 < \boldsymbol{x} \le 0.8$ (P) $0.8 < \boldsymbol{x} \le 1.0$
(Q) $\boldsymbol{x} = \mathbf{1}$.

Fig. 8



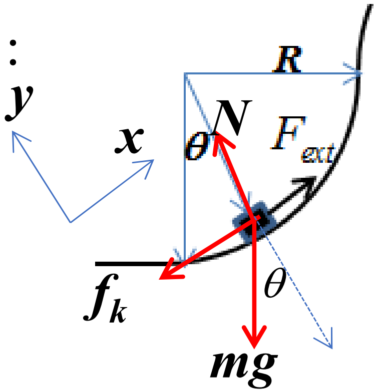
Reference for moment of inertia



Multiple Choice Questions:

1	**2**	**3**	**4**	**5**	**6**				
C	**F**	**G**	**E**	**F**	**F**				
7	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	
D	**D**	**F**	**D**	**C**	**B**	**B**	**E**	**C**	

(1)（A） 作法一：



②

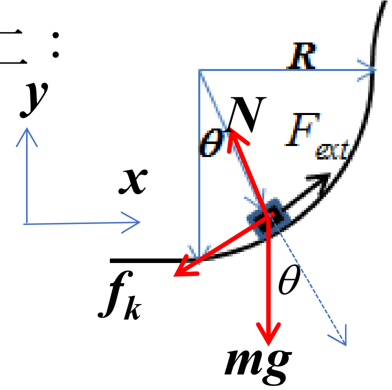
$$x:\ F_{ext} - f_k - mg\sin\theta = 0$$ ①

$$y:\ N - mg\cos\theta = 0$$ ①

$$f_k = \mu_k N$$ ①

$$\Rightarrow \quad F_{ext} = mg\left(\sin\theta + \mu\cos\theta\right)$$

(1)（A） 作法二：



②

$$x:\ F_{ext}\cos\theta - f_k\cos\theta - N\sin\theta = 0$$ ①

$$y:\ N\cos\theta - mg + F_{ext}\sin\theta - f_k\sin\theta = 0$$ ①

$$f_k = \mu_k N$$ ①

$$N = \frac{mg - F_{ext}\sin\theta}{\cos\theta - \mu_k\sin\theta}$$

$$\Rightarrow \quad F_{ext} = mg\left(\sin\theta + \mu_0\sin\theta\cos\theta\right)$$

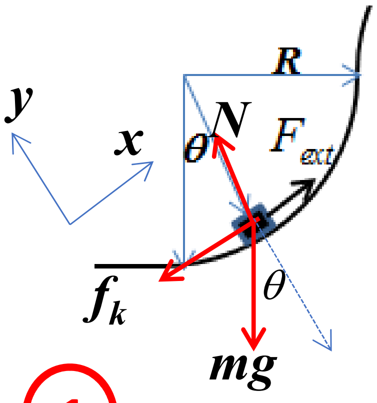
$$W_F = \int_0^{\pi/2} \vec{F}_{ext}\cdot d\vec{s} = mg\int_0^{\pi/2}\left(\sin\theta + \mu\cos\theta\right)\cdot\left(Rd\theta\right)$$ ①

$$= mgR\left\{\int_0^{\pi/2}\sin\theta d\theta + \mu_0\int_0^{\pi/2}\cos\theta d\theta\right\}$$

① ①

$$= mgR\left(-\cos\theta\Big|_0^{\pi/2} + \mu_0\left(\sin\theta\right)\Big|_0^{\pi/2}\right) = mgR\left(1+\mu_0\right)$$

(1)（B）：



1

$$x: \; F_{ext} - f_k - mg\sin\theta = 0$$

$$y: \; N - mg\cos\theta = m\frac{v^2}{R} = mg \qquad 2$$

$$f_k = \mu_k N = \mu_0 \sin\theta \cdot mg\left(1+\cos\theta\right) \qquad 1$$

$$\Rightarrow \quad F_{ext} = mg\left(\sin\theta + \mu_0 \sin\theta\left(1+\cos\theta\right)\right)$$

$$W_F = \int_0^{\pi/2} \vec{F}_{ext}\cdot d\vec{s} = mg\int_0^{\pi/2}\left(\sin\theta + \mu_0\sin\theta\left(1+\cos\theta\right)\right)\cdot\left(Rd\theta\right)$$

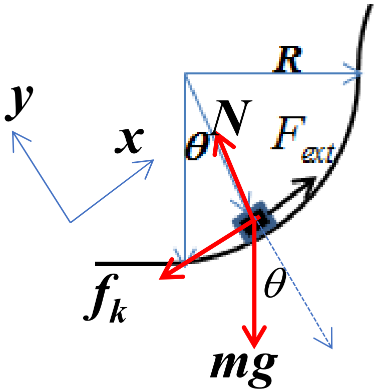
$$= mgR\left\{\left(1+\mu_0\right)\int_0^{\pi/2}\sin\theta d\theta + \frac{\mu_0}{2}\int_0^{\pi/2}\sin 2\theta d\theta\right\}$$

$$= mgR\left(\left(1+\mu_0\right)\left(-\cos\theta\right|\right)_0^{\pi/2} + \frac{\mu_0}{4}\left(-\cos 2\theta\right)\Big|_0^{\pi/2}\right)$$

$$= \left(1+\mu_0\right)\cdot mgR + mgR\cdot\frac{\mu_0}{2} = mgR\left(1+\frac{3\mu_0}{2}\right)$$

1 2

(1)（A）作法一：



2

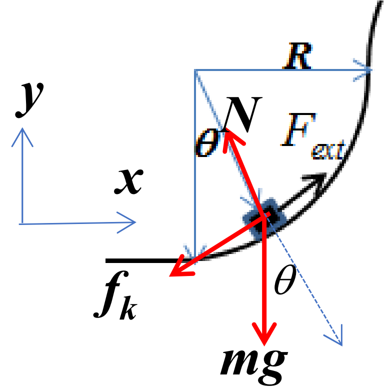
$$x:\ F_{ext}-f_k-mg\sin\theta=0 \quad 1$$

$$y:\ N-mg\cos\theta=0 \quad 1$$

$$f_k=\mu_k N \quad 1$$

$$\Rightarrow \quad F_{ext}=mg\left(\sin\theta+\mu\cos\theta\right)$$

(1)（A）作法二：



2

$$x:\ F_{ext}\cos\theta-f_k\cos\theta-N\sin\theta=0 \quad 1$$

$$y:\ N\cos\theta-mg+F_{ext}\sin\theta-f_k\sin\theta=0 \quad 1$$

$$f_k=\mu_k N \quad 1$$

$$N=\frac{mg-F_{ext}\sin\theta}{\cos\theta-\mu_k\sin\theta}$$

$$\Rightarrow \quad F_{ext}=mg\left(\sin\theta+\mu_0\sin\theta\cos\theta\right)$$

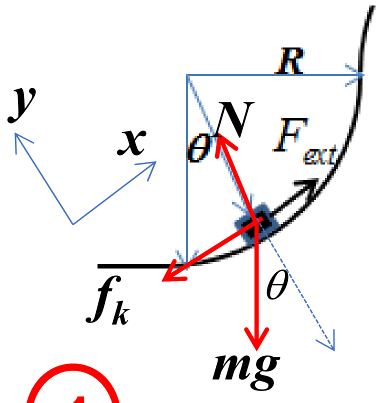
$$W_F=\int_0^{\pi/2}\vec{F}_{ext}\cdot d\vec{s}=mg\int_0^{\pi/2}\left(\sin\theta+\mu\cos\theta\right)\cdot\left(Rd\theta\right) \quad 1$$

$$=mgR\left\{\int_0^{\pi/2}\sin\theta d\theta+\mu_0\int_0^{\pi/2}\cos\theta d\theta\right\}$$

1 1

$$=mgR\left(-\cos\theta\Big|_0^{\pi/2}+\mu_0\left(\sin\theta\right)\Big|_0^{\pi/2}\right)=mgR\left(1+\mu_0\right)$$

(1)（B）：



1

$$x:\ F_{ext}-f_k-mg\sin\theta=0$$

$$y:\ N-mg\cos\theta=m\frac{v^2}{R}=mg \qquad 2$$

$$f_k=\mu_k N=\mu_0\sin\theta\cdot mg\left(1+\cos\theta\right) \qquad 1$$

$$\Rightarrow\quad F_{ext}=mg\left(\sin\theta+\mu_0\sin\theta\left(1+\cos\theta\right)\right)$$

$$W_F=\int_0^{\pi/2}\vec{F}_{ext}\cdot d\vec{s}=mg\int_0^{\pi/2}\left(\sin\theta+\mu_0\sin\theta\left(1+\cos\theta\right)\right)\cdot\left(Rd\theta\right)$$

$$=mgR\left\{\left(1+\mu_0\right)\int_0^{\pi/2}\sin\theta d\theta+\frac{\mu_0}{2}\int_0^{\pi/2}\sin 2\theta d\theta\right\}$$

$$=mgR\left(\left(1+\mu_0\right)\left(-\cos\theta\big|\right)_0^{\pi/2}+\frac{\mu_0}{4}\left(-\cos 2\theta\right)\Big|_0^{\pi/2}\right)$$

$$=\left(1+\mu_0\right)\cdot mgR+mgR\cdot\frac{\mu_0}{2}=mgR\left(1+\frac{3\mu_0}{2}\right)$$

1 2

2. (15 pts) (A) (8 pts) As shown in Fig. 2, a bullet of mass ***m*** is fired to embed into box A of mass **3*m*** which is initially at rest. The combined system (a bullet and box A) moves toward block B, which is free to move around. The mass and height of block B are **16*m*** and ***H***, respectively. There is no friction between all surfaces. Find the minimum velocity of a bullet, v_{0min}, that the combined system can climb up to the top of block B. Write v_{0min} in terms of ***m***, ***H***, and ***g*** (gravitational acceleration).

(B)(7 pts) The bullet with the velocity of $v_0 >> v_{0min}$ can make the combined system climb up to the top of block B, then slide down on the other side of block B and continue to travel forward. Find the velocities of the combined system and block B in terms of ***m***, v_0, ***H***, and ***g*** after they separate from each other.

(A) When a bullet is combined with box A, the conservation of momentum gives

$$mv_0 = (m+3m)v_C \qquad 2$$

$\Rightarrow$ the velocity of the combined box $v_C = \dfrac{v_0}{4}$

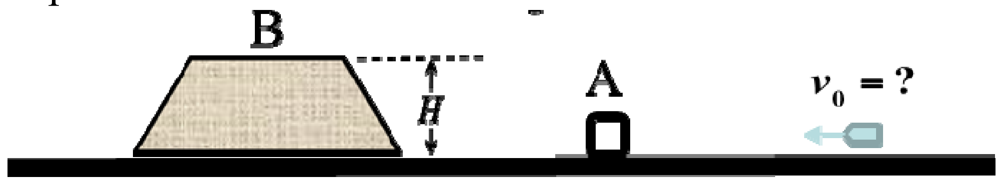
When the combined box elastically climb up to the top of block B, combined with box A, the conservation of momentum gives

$$4mv_C + 16m \cdot 0 = (4m+16\mathrm{m})v_{cm} \Rightarrow v_{cm} = \frac{v_0}{20} \qquad 2$$

, and conservation of energy gives

$$\frac{1}{2}4mv_C{}^2 = \frac{1}{2}(4m+16m)v_{cm}{}^2 + 4mgH \qquad 2$$

$$\Rightarrow v_{0\min} = \sqrt{40gH} = 2\sqrt{10gH} \qquad 2$$



(B) $$4mv_C = 16mv_{B,f} + 4mv_{C,f} \qquad 1$$

$$\frac{1}{2}4mv_C{}^2 = \frac{1}{2}16mv_{B,f}^2 + \frac{1}{2}\cdot 4mv_{C,f}^2 \qquad 1$$

1st solution is as follows

$$v_{B,f} = \frac{v_0}{10}, \qquad v_{C,f} = -\frac{3v_0}{20} \qquad 2$$

2nd solution is as follows

$$v_{B,f} = 0, \quad v_{C,f} = v_C = \frac{v_0}{4} \qquad 2$$

1st solution is not consistent with the problem.

Accordingly,

$$v_{B,f} = 0, \quad v_{C,f} = v_C = \frac{v_0}{4} \qquad 1$$

3. As shown in Fig. y, a dumb bell on a table top is connected to a block of mass m1 with a massless string wrapping around a massless pulley, The mass of the pulley is $\boldsymbol{m_1}$, its moment of inertia for rotation around the center of mass is $\boldsymbol{I} = 0.5\, \boldsymbol{m_1} \cdot \boldsymbol{R}^2$, and $\boldsymbol{R} = 2\boldsymbol{r}$. The coefficient of static friction $\mu_s = \boldsymbol{0.8}$ for the pulley and the table surface. Let $\boldsymbol{m_1} = \boldsymbol{m}$, and $\boldsymbol{m_2} = \boldsymbol{3m}$. (a) Determine the acceleration of the dumb bell and the friction force between the dumb bell and the table. (b) The maximum $\boldsymbol{m_2}$ in terms of $\boldsymbol{m}$ for the dumb bell to execute pure roll motion on the table. (Free-body diagrams are required for the answers).

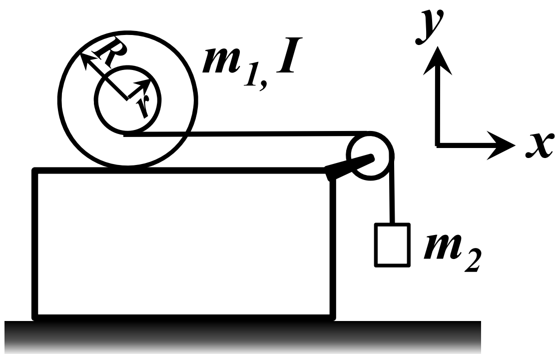
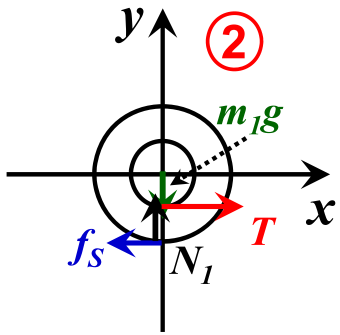
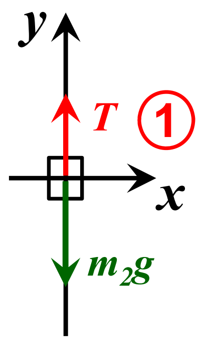


Fig. 3





For $\boldsymbol{m_1}$, $\sum \vec{F} = m_1 \vec{a}_1$, Let $a_1 = a$

$x: T - f_S = m_1 a$ (1) 1

$y: N_1 - m_1 g = 0$ (2) 1

$\sum \vec{\tau} = I\vec{\alpha}$,

$\Rightarrow \vec{r} \times \vec{T} + \vec{R} \times \vec{f}_S = I\vec{\alpha}$,

$z: rT - Rf_S = I\alpha$ (3) 1

$a = -R\alpha$ (4) 1

For $\boldsymbol{m_2}$, $\sum \vec{F} = m_2 \vec{a}_2$, $a_2 = -a$

$y: T - m_2 g = -m_2 a$ (5) 1

From (1),(3),(4),(5), we get

$T - f_S = ma$ (6)

$\frac{R}{2} T - Rf_S = -\frac{Rma}{2}$ (7)

$T - 3mg = -3ma$ (8)

$(6) \cdot R - (7) - (R/2) \cdot (8)$, we get

$$\frac{3mgR}{2} = Rma + \frac{Rma}{2} + 3Rma/2$$

$a = g/2$ (9) 1

$(6) \cdot R/2 - (7)$, we get

$$\frac{R}{2} f_S = \frac{Rma}{2}$$

$f_S = mg$ 1

3. As shown in Fig. y, a dumb bell on a table top is connected to a block of mass m1 with a massless string wrapping around a massless pulley, The mass of the pulley is $\boldsymbol{m_1}$, its moment of inertia for rotation around the center of mass is $\boldsymbol{I} = 0.5\, \boldsymbol{m_1} \cdot \boldsymbol{R}^2$, and $\boldsymbol{R} = 2\boldsymbol{r}$. The coefficient of static friction $\boldsymbol{\mu_s} = \boldsymbol{0.8}$ for the pulley and the table surface. Let $\boldsymbol{m_1} = \boldsymbol{m}$, and $\boldsymbol{m_2} = \boldsymbol{3m}$. (a) Determine the acceleration of the dumb bell and the friction force between the dumb bell and the table. (b) The maximum $\boldsymbol{m_2}$ in terms of $\boldsymbol{m}$ for the dumb bell to execute pure roll motion on the table. (Free-body diagrams are required for the answers).

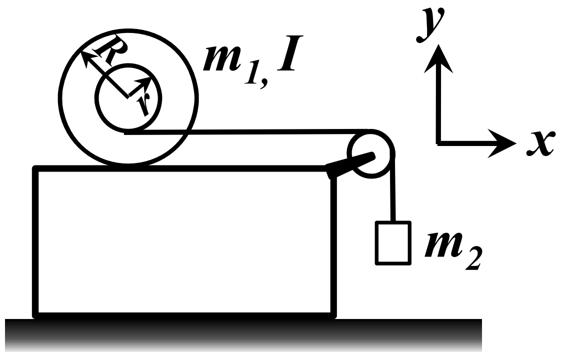
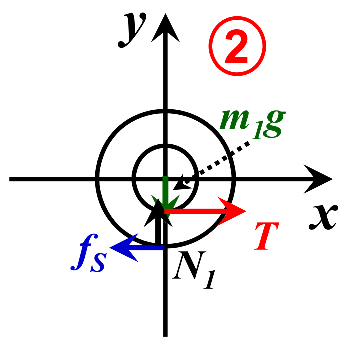
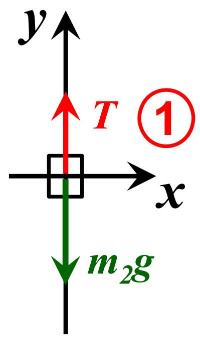


Fig. 3





For $\boldsymbol{m_1}$, $\sum \vec{F} = m_1 \vec{a}_1$, Let $a_1 = a$

x: $T - f_S = m_1 a$ (1) ①

y: $N_1 - m_1 g = 0$ (2) ①

$\sum \vec{\tau} = I\vec{\alpha}$,

$\Rightarrow \vec{r} \times \vec{T} + \vec{R} \times \vec{f_S} = I\vec{\alpha}$,

z: $-rT + Rf_S = I\alpha$ (3) ①

$a = R\alpha$ (4) ①

For $\boldsymbol{m_2}$, $\sum \vec{F} = m_2 \vec{a}_2$, $a_2 = -a$

y: $T - m_2 g = -m_2 a$ (5) ①

From (1),(3),(4),(5), we get

$T - f_S = ma$ (6)

$-\frac{R}{2}T + Rf_S = \frac{Rma}{2}$ (7)

$T - 3mg = -3ma$ (8)

$(6)\cdot R + (7) - (R/2)\cdot(8)$, we get

$$\frac{3mgR}{2} = Rma + \frac{Rma}{2} + 3Rma/2$$

$a = g/2$ (9) ①

$(6)\cdot R/2 - (7)$, we get

$$\frac{R}{2} f_S = \frac{Rma}{2}$$

$f_S = mg$ ①