Homework 9

1. A conducting rod of mass $\boldsymbol{m}$ is rest at $\boldsymbol{x = 0}$. At $\boldsymbol{t = 0}$, it starts to fall down on two frictionless conducting rails (separated by a distance $\boldsymbol{L}$, as shown in Fig. 1). There is a uniform magnetic field (along $-z$-axis) enclosed the conducting rails. In addition, a resistor $\boldsymbol{R}$ connects the two rails at $\boldsymbol{x = x_0}$. The rod will pass the resistor freely. **(a)** Find the **magnitude** and **direction** of the induced current $\boldsymbol{I}$ in the rod in terms of $\boldsymbol{B, L, R, g}$ (gravitational acceleration), $\boldsymbol{v}$ (speed of the bar) and other necessary constants before it passes the resistor. **(b)** Solve the velocity of the rod $\boldsymbol{v(t)}$ before it passes the resistor. (assume the rod is rest at $\boldsymbol{x = 0}$ at $\boldsymbol{t = 0}$) **(c)** The rod passes the resistor with velocity $\boldsymbol{v_0}$ at time $\boldsymbol{t_0}$ (you don't need to solve them). What is the velocity of the rod $\boldsymbol{v(t)}$ after it passes the resistor? Your answer can include $\boldsymbol{v_0}$ and/or $\boldsymbol{t_0}$ if it is necessary.

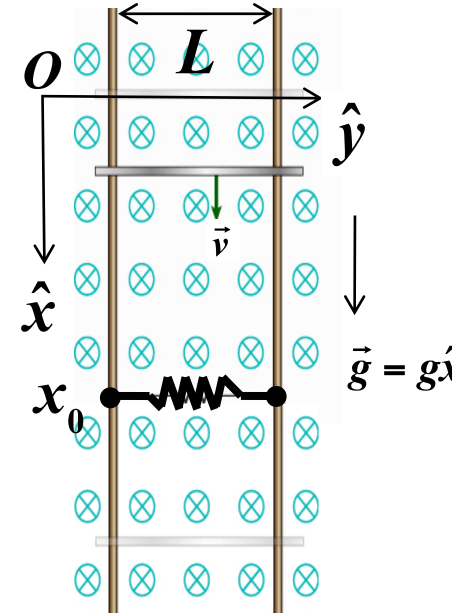

Fig. 1

2.A conducting bar of mass $\boldsymbol{m}$ slides down two frictionless conducting rails which make an angle θ with respect to the horizontal and one end with a resistor R, and the distance between two rails is ℓ, as shown in Fig. 2. A uniform magnetic field $\boldsymbol{B}$ is applied with an angle θ with respective to vertical. The bar is released from the top with zero velocity. Draw the free body diagram of the conducting bar as it slides down the rail. Find the velocity of the metal bar as a function of time. what is the terminal velocity $\boldsymbol{v_T}$ of the bar?

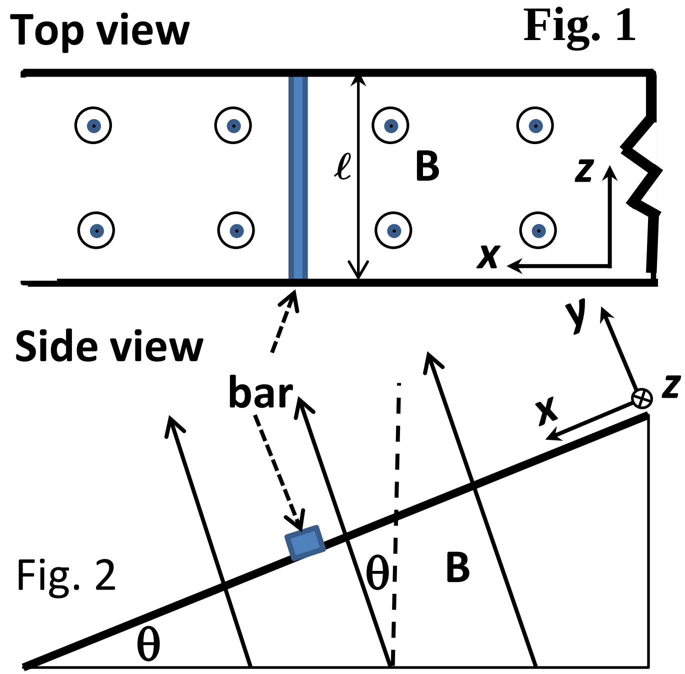

Fig. 2

3. Giancloi textbbok, page 902, problem 54
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1.(a)

$$\mathcal{E}_{ind} = -\frac{d\Phi_B}{dt},$$ Magnetic flux decreases→I : c.w. or *left* → *right* for $x < x_0$

$$IR = \left|\mathcal{E}_{ind}\right| = \frac{d}{dt} BLx \rightarrow \boxed{I = \frac{BL}{R} v}$$

(b)

$$\vec{F}_{tot} = \vec{F}_{mg} + \vec{F}_B = mg\hat{x} + I\vec{L} \times \vec{B} = mg\hat{x} + I\left(L\hat{y}\right) \times \left[-B\hat{z}\right] = \left(mg - ILB\right)\hat{x} = m\frac{dv}{dt}\hat{x}$$

$$\frac{dv}{dt} = g - \frac{B^2L^2}{mR} v = g - \alpha v, \quad \alpha \equiv \frac{B^2L^2}{mR}$$

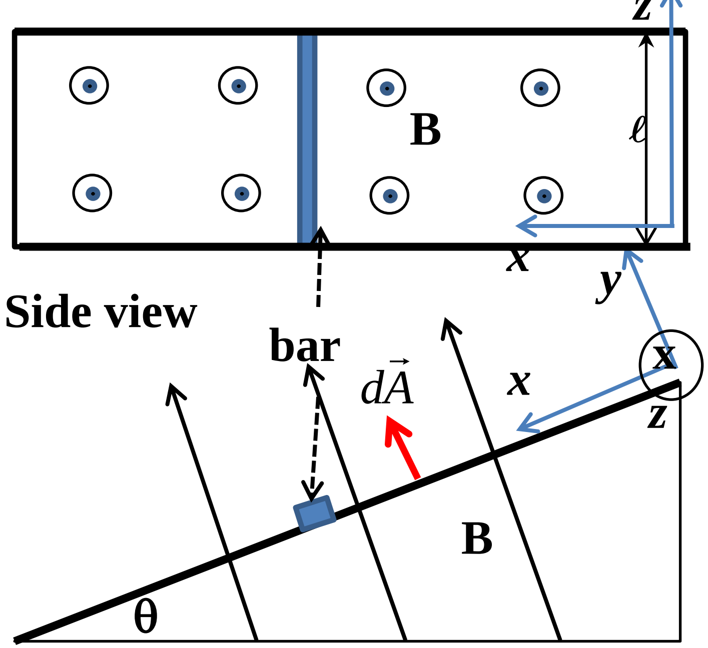
$$\int_0^{v(t)} \frac{dv}{g - \alpha v} = \int_0^t dt \quad \text{or } v(t) = \boxed{\frac{mgR}{B^2L^2}\left[1 - e^{-\alpha t}\right]}$$

(c)

Magnetic flux changes sign (start to increases after it pass the resistor)
→I : c.c.w. or still flowing from ***left* → *right*** for $x > x_0$ the equation of motion is the same as in part (b). But the initial condition becomes $v(t_0) = v_0$

$$\int_{v_0}^{v(t)} \frac{dv}{g - \alpha v} = \int_{t_0}^t dt \quad \text{or } v(t) = \frac{mgR}{B^2L^2}\left[1 - e^{-\alpha(t-t_0)}\right] + v_0 e^{-\alpha(t-t_0)}$$

2.



The movement of the bar ➔ Φ_B changes

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = B\left(\ell \cdot x(\mathrm{t})\right)$$

The change of Φ_B induces emf $\mathcal{E}$:

$$\left|\frac{d\Phi_B}{dt}\right| = \mathcal{E} = B\ell\frac{dx(\mathrm{t})}{dt} = B\ell v(t)$$

$\mathcal{E}$ induces current I:

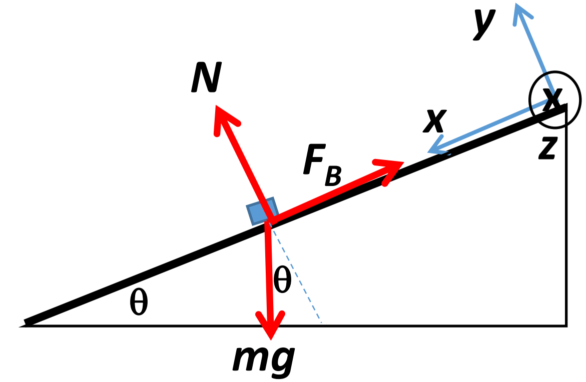
$$I = \frac{\mathcal{E}}{R} = \frac{B\ell v(t)}{R}$$

Top view: 順時針

the bar with I in B ➔ magnetic force.

$$\vec{F}_B = I\vec{\ell} \times \vec{B} = \frac{B^2\ell^2 v(t)}{R}$$

Side view Direction:



x : $$mg\sin\theta - F_B = ma = m\frac{dv}{dt}$$

y : $$N - mg\cos\theta = 0$$

When the bar has reach "terminal velocity" v_T, there is no acceleration.

i.e $$mg\sin\theta - \frac{B^2\ell^2}{R}v_T = 0 \Rightarrow v_T = \frac{R \cdot mg\sin\theta}{B^2\ell^2}$$

Solve the equation of motion:

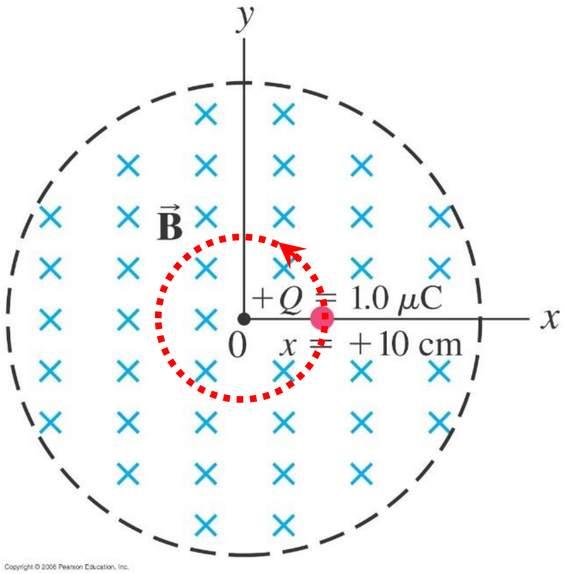
$$\frac{dv}{dt} = g\sin\theta - \frac{B^2\ell^2}{mR}v(t) = \frac{v(t) - v_0}{\tau}$$

where $$\tau = \frac{mR}{B^2\ell^2}, \quad v_0 = \frac{mgR\sin\theta}{B^2\ell^2} = v_T$$

$$\int_0^v \frac{dv}{v - v_T} = -\int_0^t \frac{dt}{\tau} \Rightarrow \ln\left|\frac{v - v_T}{-v_T}\right| = -\frac{t}{\tau}$$

$$\Rightarrow v(t) = v_T\left(1 - e^{-t/\tau}\right)$$

3.



Select a circular loop that is passing the charge Q and is concentric with the circular region of the B-field:

The magnetic flux $\boldsymbol{\Phi_B}$ through this loop is

$$\Phi_B = \vec{B}\cdot\vec{A} = B(-\hat{z})\cdot\pi x^2(\hat{z})$$

$$= -\pi \mathrm{B} x^2$$

According to Faraday's law the electromotive force along this loops is

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d(-\pi B x^2)}{dt} = \frac{dB}{dt}$$

$$= -0.1\cdot\pi x^2(V)$$

And on the other hand, the E-field along the loop can be obtained below,

$$\varepsilon = 2\pi x E = -0.1\cdot\pi x^2(V)$$

$$\Rightarrow E = 5\times10^{-3}(V/m)$$ (Counter clockwise)

The electric force on the charge Q can be obtained below,

$$F = QE = 10^{-6}C\cdot(5\times10^{-3})(V/m) = 5\times10^{-9}N$$