HW13-1: Problem 34-58

Lloyd's mirror provides one way of obtaining a double-slit interference pattern from a single source so the light is coherent. As shown in Fig. 34-31, the light that reflects from the plane mirror appears to come from the virtual image of the slit. Describe in detail the interference pattern on the screen.

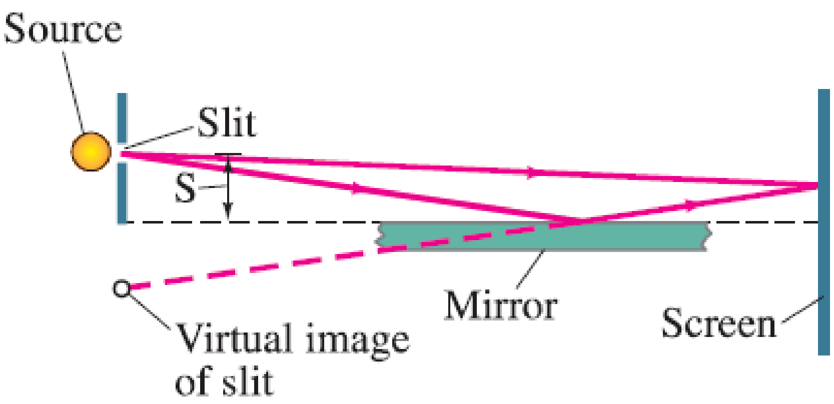


FIGURE 34–31 Problem 58.

HW13-1 sol (Problem 34-58):

Lloyd's mirror provides one way of obtaining a double-slit interference pattern from a single source so the light is coherent. As shown in Fig. 34-31, the light that reflects from the plane mirror appears to come from the virtual image of the slit. Describe in detail the interference pattern on the screen.

Sol:

The reflected wave appears to be coming from the virtual image, so this corresponds to a double slit.

∗The reflection light change phase by π

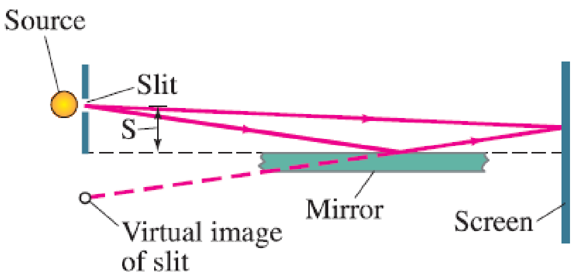
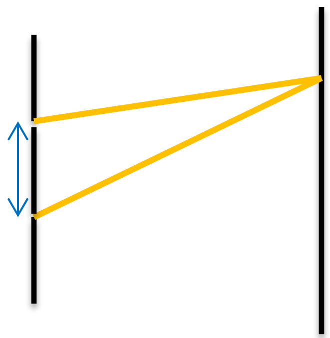


FIGURE 34–31 Problem 58.

$d = 2S$



$$d\sin\theta_{\min} = n\lambda \quad \Rightarrow \sin\theta_{\min} = n\frac{\lambda}{2S}, n = 0, 1, 2, \ldots.$$

$$d\sin\theta_{\max} = (n+\frac{1}{2})\lambda \quad \Rightarrow \sin\theta_{\max} = (n+\frac{1}{2})\frac{\lambda}{2S}, n = 0, 1, 2, \ldots.$$

HW13-2:

Apply the Phasor construction to obtain the sum of the following:

$$\text{(a) } E_0 \sin\left(\omega t\right) + E_0 \sin\left(\omega t + \varphi\right) + E_0 \sin\left(\omega t + 2\varphi\right) + E_0 \sin\left(\omega t + 3\varphi\right)$$

$$\text{(b) } E_0 \cos\left(\omega t - \frac{\pi}{2}\right) + 3E_0 \cos\left(\omega t\right) + 2E_0 \cos\left(\omega t + \frac{\pi}{2}\right)$$

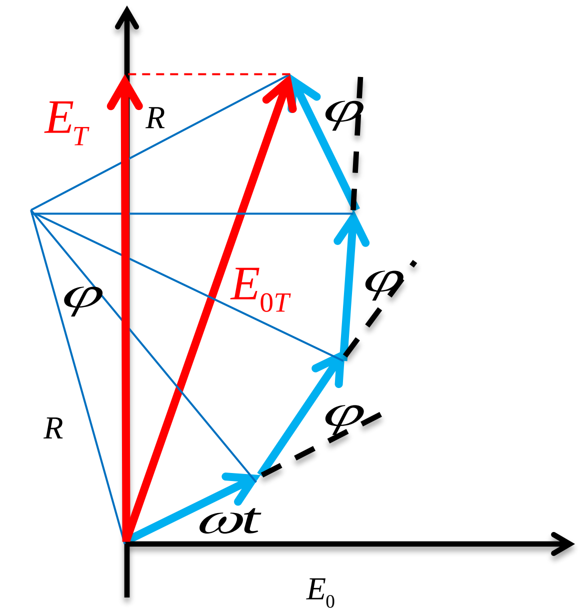
$$\text{(c) } E_0 \cos(\omega t) + E_0 \cos\left(\omega t + \frac{\pi}{6}\right) + 2E_0 \cos\left(\omega t + \frac{\pi}{2}\right)$$

HW13-2 sol

Apply the Phasor construction to obtain the sum of the following:

$(a)\ E_0\sin(\omega t)+E_0\sin(\omega t+\varphi)+E_0\sin(\omega t+2\varphi)+E_0\sin(\omega t+3\varphi)$

$(b)\ E_0\cos(\omega t-\frac{\pi}{2})+3E_0\cos(\omega t)+2E_0\cos(\omega t+\frac{\pi}{2})$

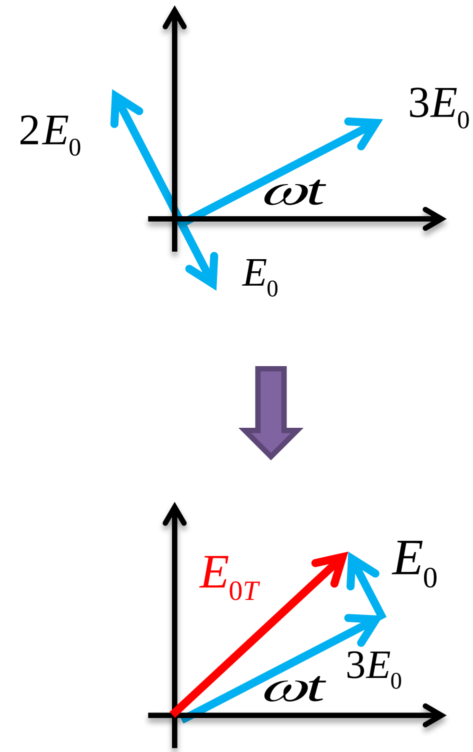


$$\frac{E_0/2}{R}=\sin\frac{\varphi}{2}$$

$$\frac{E_{0T}/2}{R}=\sin\frac{4\varphi}{2}$$

$$\Rightarrow \frac{E_{0T}}{E_0}=\frac{\sin 2\varphi}{\sin\frac{\varphi}{2}} \qquad \Rightarrow E_{0T}=\frac{\sin 2\varphi}{\sin\frac{\varphi}{2}}E_0$$

$$E_T=E_{0T}\sin(\omega t+\frac{3\varphi}{2})=\frac{\sin 2\varphi}{\sin\frac{\varphi}{2}}E_0\sin(\omega t+\frac{3\varphi}{2})$$



$$\Rightarrow E_{0T}=\sqrt{10}E_0$$

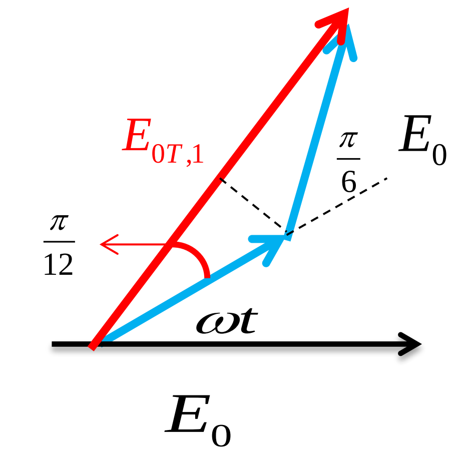
$$E_T=E_{0T}\cos(\omega t+\tan^{-1}\frac{1}{3})$$

$$=\sqrt{10}E_0\cos(\omega t+\tan^{-1}\frac{1}{3})$$

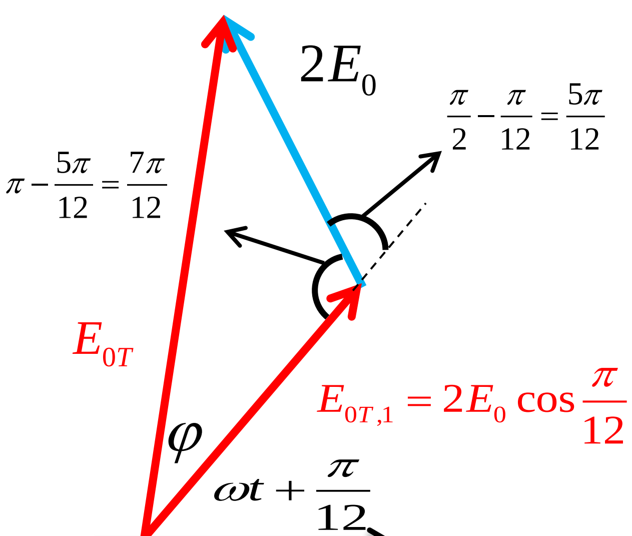
Problem 2

Apply the Phasor construction to obtain the sum of the following:

$$(c)\ E_0\cos(\omega t) + E_0\cos(\omega t + \frac{\pi}{6}) + 2E_0\cos(\omega t + \frac{\pi}{2})$$



$$\frac{\frac{E_{0T,1}}{2}}{E_0} = \cos\frac{\pi}{12} \qquad \Rightarrow E_{0T,1} = 2E_0\cos\frac{\pi}{12} = 1.9E_0$$



Use Law of cosines

$$E_{0T} = \sqrt{(2E_0)^2 + E_{0T,1}{}^2 - 2\cdot(2E_0)\cdot E_{0T,1}\cos\left(\frac{7\pi}{12}\right)}$$

$$= \sqrt{4+\sqrt{3}}E_0 = 2.4E_0$$

$$\varphi = \cos^{-1}\left(\frac{E_{0T}{}^2 + E_{0T,1}{}^2 - (2E_0)^2}{2E_{0T}E_{0T,1}}\right) = 0.94$$

$$\Rightarrow E_T = E_{0T}\cos(\omega t + \frac{\pi}{12} + \varphi) = 2.4E_0\cos(\omega t + 1.2)$$

HW13-3: Consider a plane wave with wave length λ passes a diffraction grating with 6 slits. The intensity on the screen is shown in Fig. 3(a). The grating located a distance L away from the screen and the spacing between neighboring slits is d. Assume $\lambda << d << L$ and only the effect of interference is considered in this problem.

(A) Fig. 3(b) shows the phase difference (δ) and the phasor configuration for the light intensity at position y_1. Find the phase difference and draw the phasor configuration for all the other four the minima points (y_2, … y_5).

(B) Evaluate the positions (y_1, … y_5) relative to the central maximum (y_0) in terms of L, λ, d, and other necessary constants (you may just set $y_0 = 0$).

(C) Express the light intensity on the screen as a function of phase difference (δ) (assume the intensity of the principal maxima is I_0).

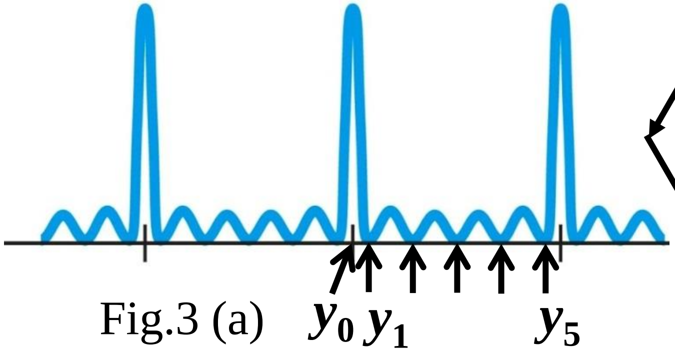


Fig.3 (a)

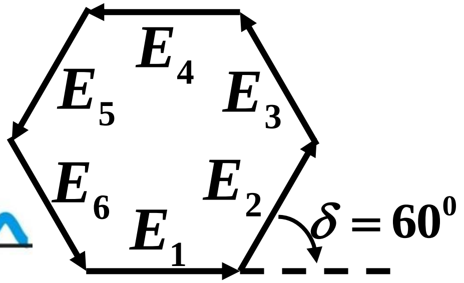
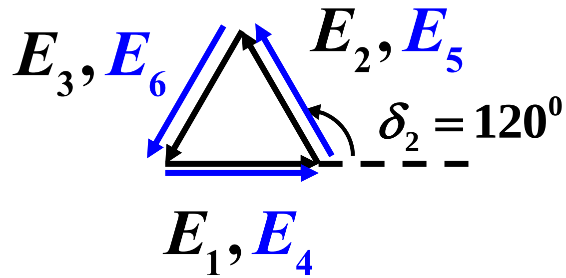


Fig.3 (b)

3.

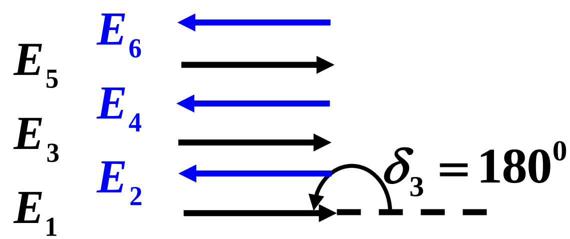
8 pts

(a) y_2 : $\delta_2 = \dfrac{2\pi}{3}$, or 120^0



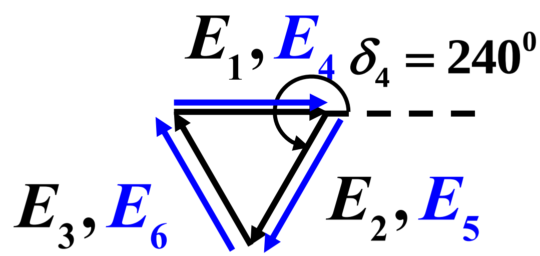
2 pts

y_3 : $\delta_3 = \pi$, or 180^0



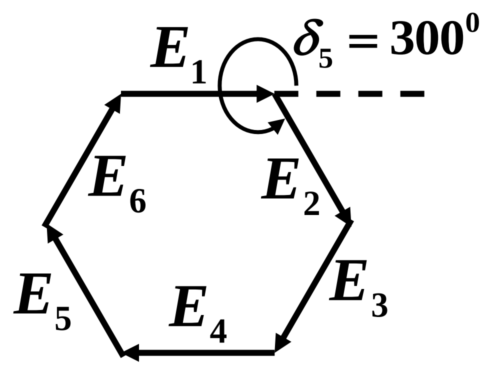
2 pts

y_4 : $\delta_4 = \dfrac{4\pi}{3}$, or 240^0



2 pts

y_5 : $\delta_5 = \dfrac{5\pi}{3}$, or 300^0



2 pts

4 pts (b)

2 pts

$$y_i = L\tan\theta_i$$

$$\delta_i = k\Delta L = kd\sin\theta_i$$

2 pts

$$\Rightarrow y_i \simeq L\frac{\delta_i}{kd} = \frac{\lambda L}{6d}i, \qquad i = 1, 2, \ldots, 5$$

3 pts (c)

$$\frac{E_{T,0}}{2} = R\sin\frac{6\delta}{2}$$

1 pts

$$\frac{E_0}{2} = R\sin\frac{\delta}{2}$$

$$\Rightarrow \frac{E_{T,0}}{E_0} = \frac{\sin\frac{6\delta}{2}}{\sin\frac{\delta}{2}} \quad \text{and } I = I_0\frac{\sin^2 3\delta}{\sin^2\frac{\delta}{2}}$$

1 pts

1 pts