試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(50 points) (所有題目必須有計算過程,否則不予計分)

1. (10 pts) A bullet of mass $\boldsymbol{m}$ is fired to embed into box A of mass $\mathbf{5}\boldsymbol{m}$ at rest, as shown in Fig. 1 Then, the combined body elastically collides with box B of mass $\mathbf{4}\boldsymbol{m}$ at rest, and box B moves along a circular vertical loop with a radius $\boldsymbol{R}$. What is the minimum velocity $\boldsymbol{v_0}$ of a bullet such that box B just passes the top of the loop? (Ignore the friction between all the contact surfaces).

2. (10 pts) An external force $\boldsymbol{F}$ is pushing horizontally a small box of mass $\boldsymbol{m}$ in the very slow motion from point **A**, through point **B** to the point **C** which is the top of the quarter circle of radius $\boldsymbol{R}$. The distance between **A** and **B** is $\boldsymbol{d}$, as shown in Fig. 2. The kinetic frictional coefficient of the surface is $\boldsymbol{\mu}$. Calculate the total work done by the external force $\boldsymbol{F}$ from position A to position C.

3. (15 pts) As shown in Fig. 3, on top of a frictionless surface a thin rod of length $4\boldsymbol{R}$ and mass $\boldsymbol{M}$ is rest along the $\boldsymbol{y}$-axis with its center at the origin. A ring with mass $\boldsymbol{M}$ and radius of $\boldsymbol{R}$ is spinning with angular speed $\boldsymbol{\omega_0}$ and traveling with speed of $\boldsymbol{v_0}$ in the $\boldsymbol{+x}$ direction. The ring collides with the rod. If the ring and the rod stick to each other right after the collision, determine the velocity (magnitude and direction) of the center of the mass of the assembly and its rotational velocity (magnitude and direction) around its center of mass in terms of $\boldsymbol{M}$, $\boldsymbol{R}$, $\boldsymbol{\omega_0}$, and $\boldsymbol{v_0}$ (ignore the thickness of the rod).

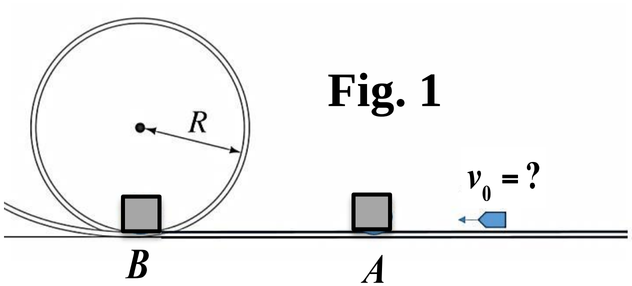
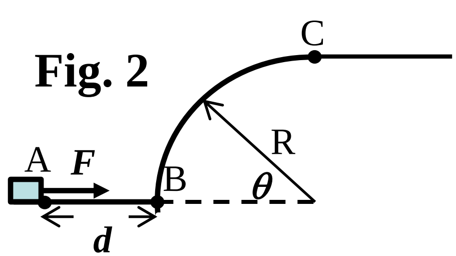


Fig. 1

Fig. 2



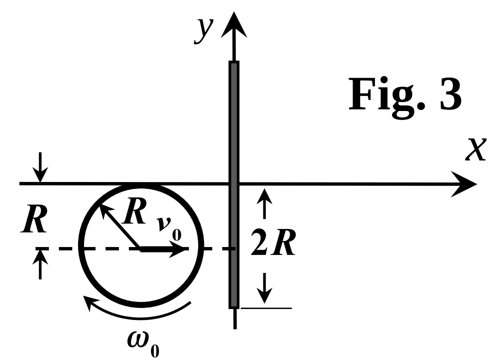


Fig. 3

4. (15 points) A dumb bell is dragged to roll without slipping upward along a slide as shown in the Fig. 4. The mass of the dumb bell is m and its moment of inertia for rotation around the axis passing the center of mass is ½ mR^2 ($R = 2r$). $\mu_K = 0.2$ and $\mu_s = 0.4$ are the kinetic and static frictional coefficients between the incline and the dumb bell. Write your answer in terms of F, R, m, $\mathbf{g}$ (gravitation acceleration), and θ in the following questions.

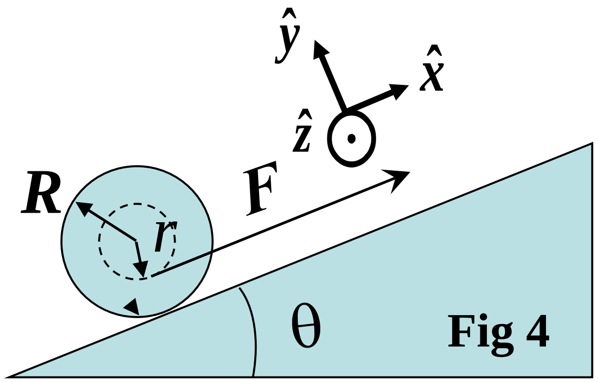


Fig 4

a) (5 pts) Draw the free body diagram for the dumb bell. Write down the equations of motion based on Newton's 2nd law for linear motion and rotation from your free body diagram.

b) (3 pts) If the dumb bell executes pure rolling along the incline, find the acceleration a and the friction force of the dumb bell in general case.

c) (4 pts) If the angle $\sin\theta = 0.25$, find the conditions of F/mg such that the dumb bell is rolling without slipping/skidding along $+x$ ($a > 0$)?

d) (3 pts) Is it possible the dumb bell is static (at rest) on the incline? If yes, find the applied force F (direction and magnitude) and the condition of the angle θ of the incline.

II.選擇題(53 points)

1. (5pts) A 0.25-kg skeet is fired at an angle of 37° to the horizon with a speed of **10 m/s**, shown in Fig. 5. When it reaches the maximum height h, it is hit from below by a **0.025kg** pellet traveling vertically upward with a speed of **484 m/s**. The pellet is embedded in the skeet. How much higher h (m) did the skeet go up due to the collision? (g = 10 m/s²)

(A) 1-20; (B) 21-40; (C) 41-60; (D) 61-80; (E) 81-100; (F) 101-120.

2. (5pts) Same as problem 1, how much the extra distance Δx (m) does the skeet travel due to the collision?

(A) 1-20; (B) 21-40; (C) 41-60; (D) 61-80;
(E) 81-100; (F) 101-120.

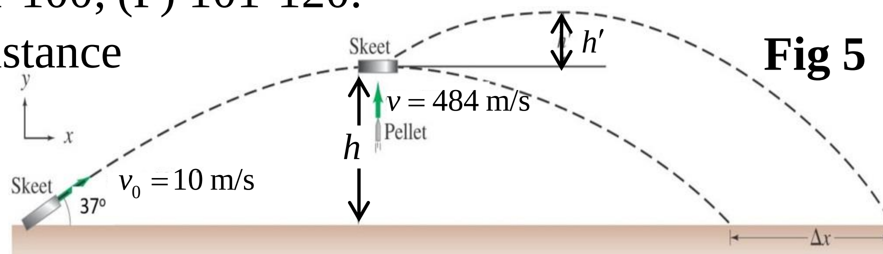


Fig 5

3. (5 pts.) Consider a uniform disk with mass $\boldsymbol{M}$ and radius $\boldsymbol{R}$ as shown in Fig. 6a. Let $\boldsymbol{I_P}$ be the moment of inertia of this object when it rotates around its center $\boldsymbol{O}$. It is then drilled out 6 identical circular holes of radius $\boldsymbol{r}$ **(= (1/3)$\boldsymbol{R}$)**. Each small hole is just touch the edge of the disk (Fig. 6b). $\boldsymbol{I_Q}$ be the moment of inertia of this object with 6 holes through $\boldsymbol{O}$. Let the ratio of the moment of inertia of these two disks is $\boldsymbol{a} = \boldsymbol{I_Q}/\boldsymbol{I_P}$. What is the value $\boldsymbol{a}$? ($\boldsymbol{I_{CM}}$ = ½ $\boldsymbol{MR^2}$ for a disk with mass $\boldsymbol{M}$ and radius $\boldsymbol{R}$.)

(A) $a \le 0.15$ (B) $0.15 < a \le 0.25$ (C) $0.25 < a \le 0.35$ (D) $0.35 < a \le 0.45$ (E) $0.45 < a \le 0.55$ (F) $0.55 < a \le 0.65$ (G) $0.65 < a \le 0.75$ (H) $0.75 < a \le 0.85$ (J) $0.85 < a \le 0.95$ (K) $0.95 < a$.

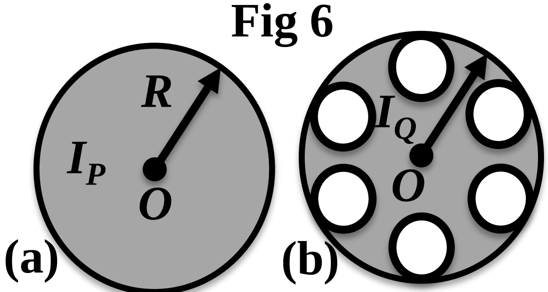
4. (5 pts)) The plot of Fig. 7 shows the potential energy of a particle, due to a conservative force exerted on it by another particle , as s function of distance. For which point(s) is the force exerted on the particle zero?

(A) point X (B) point Y (C) point Z (D) points X and Y (E) points Y and Z (F) points X and Z (G) all three points (H) none of above

5. (5 pts) A spherical shell (**$I = (2/3)Mr^2$**) with radius $\boldsymbol{r}$ and mass $\boldsymbol{M}$ rolls downhill without slipping as shown in Fig. 8, at the end of the slide is a vertical circular track with radius $\boldsymbol{R}$ =2 m (= 3$\boldsymbol{r}$). If the shell is at position $\boldsymbol{h}$**=10** m initially, what is the velocity of the center of mass ($\boldsymbol{v}$) of the shell at position $\boldsymbol{P}$, which is **2** m above ground? (let $\boldsymbol{g}$ = **10m/s²**, $\sqrt{2} \sim 1.4, \sqrt{3} \sim 1.7, \sqrt{5} \sim 2.2$)

(A) $v \le 5$; (B) $5 < v \le 6$; (C) $6 < v \le 7$; (D) $7 < v \le 8$; (E) $8 < v \le 9$; (F) $9 < v \le 10$ (G) $10 < v \le 11$; (H) $11 < v \le 12$; (J) $12 < v \le 13$; (K) $13 \le v$

Fig 6



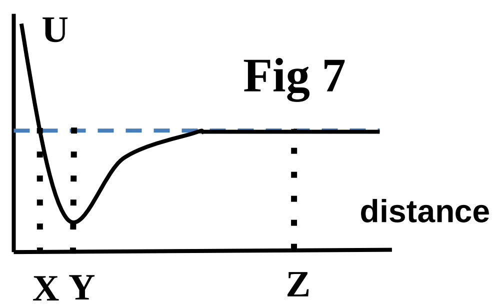


Fig 7

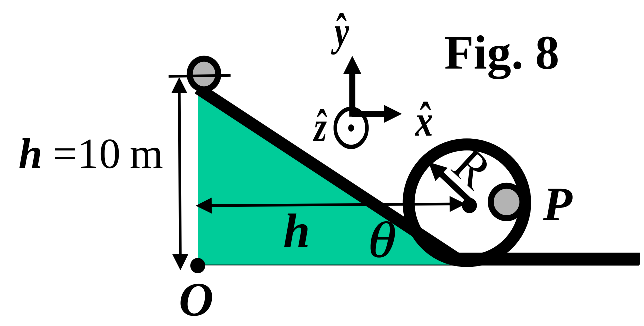


Fig. 8

6. (5 pts) As shown in Fig. 9a, a disk is executing pure roll with angular speed ω_0 on the floor toward a ring, which is static initially. Both the disk and the ring have the same mass and radius. Some time after the collision, the disk and ring execute pure roll with angular speed ω_1 and ω_2, respectively, as shown in Fig. 9(b). If $\omega_1 = \omega_0/2$, *and* $x = |\omega_2/\omega_1|$,which of the following statement is correct?

(A) $0 < x \le 1/4$ (B) $1/4 < x \le 1/2$ (C) $1/2 < x \le 1$ (D) $1 < x \le 5/4$ (E) $5/4 < x \le 3/2$ (F) $3/2 < x \le 2$ (G) $2 < x$

Fig. 9(a)

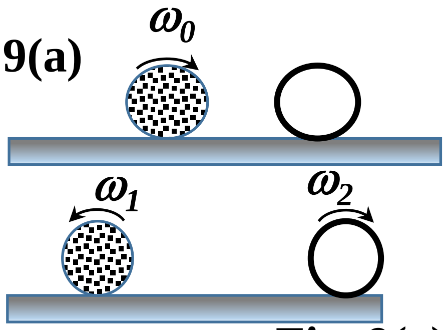


Fig. 9(a)

7. (5 pts) As shown in Fig. 10, a bob is connected to a string and executing a circular motion with constant speed. In the Fig. 10, point A is the center of the orbit, point B is the point where the string is fixed to the ceiling, and point C is midway between A and B, Let $\vec{L}_A$, $\vec{L}_B$, and $\vec{L}_C$ be the angular momenta measured from coordinate systems with origin set at point A, B, and C, respectively.(i.e. measured from point A, B, and C, respectively). Which of the following statement is correct?

(A) $\vec{L}_A$ is conserved. (B) $\vec{L}_B$ is conserved. (C) $\vec{L}_C$ is conserved.

(D) All above are correct. (E) Only (A) and (B) are correct.

(F) Only (A) and (C) are correct. (G) Only (B) and (C) are correct.

(H) None above is correct.

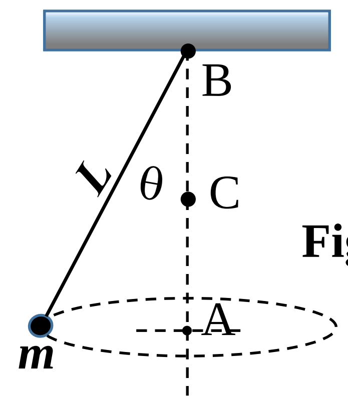
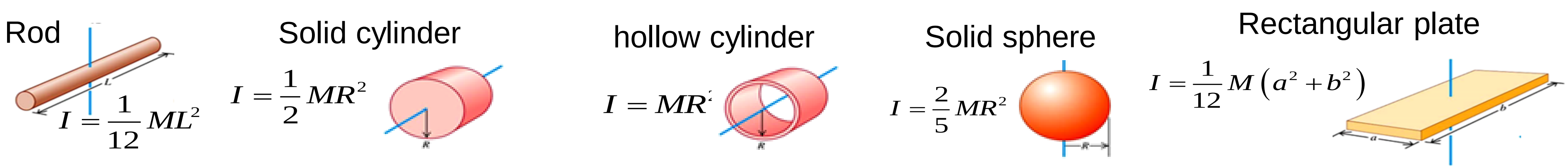


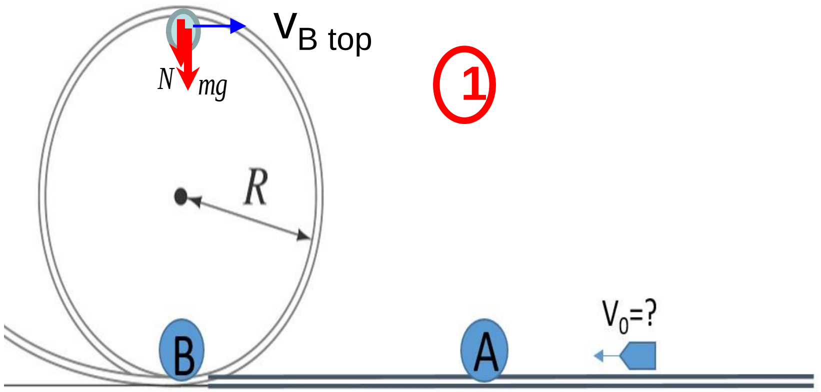
Fig. 10



Multiple Choice Questions:

1	**2**	**3**	**4**	**5**	**6**	**7**		
E	**D**	**C**	**E**	**D**	**G**	**A**		
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**
D	**D**	**F**	**D**	**C**	**B**	**B**	**E**	**C**

1. (10 pts) A bullet of mass *m* is fired to embed into box A of mass 5*m* at rest, as shown in Fig. Then, the combined body elastically collides with box B of mass 4*m* at rest, and box B moves along a circular vertical loop with a radius *R*. What is the minimum velocity v_0 of a bullet that box B can pass the top of the loop?



(1) For a bullet embedded into box A, the momentum is conserved.

$$mv_0 + 5m(0) = (m + 5m)v_C \quad 1$$

$$v_C = \frac{1}{6}v_0 \quad 1$$

(2) For the combined body elastically collided with box B,

$$6m\frac{1}{6}v_0 + 4m(0) = 6mvA' + 4mvB' \quad 1$$

$$\frac{1}{2}6m\left(\frac{1}{6}v_0\right)^2 + \frac{1}{2}4m(0)^2 = \frac{1}{2}6m(v_A')^2 + \frac{1}{2}4m(vB')^2 \quad 1$$

Then you can get $\quad v_B' = \frac{1}{5}v_0 \quad 1$

(3) Box B moves along the circular vertical loop and must go through the top

as N = 0 $\quad mg + N = m\frac{v_{B\ top}^2}{R} \quad 1$

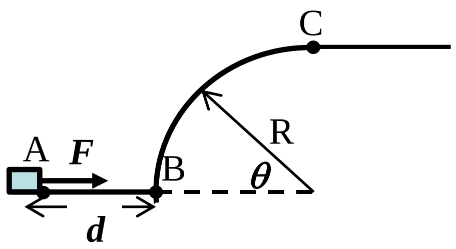
$$(v_{B\ top})_{min} = \sqrt{gR} \quad 1$$

Finally, energy is conserved for Box B moving along loop.

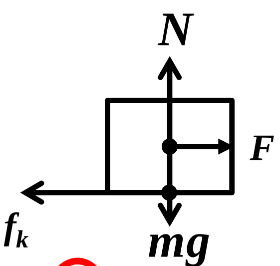
$$\frac{1}{2}4m\left(\frac{1}{5}v_0\right)^2 = 4mg(2R) + \frac{1}{2}4m(\sqrt{gR})^2 \quad 1$$

$$v_0 = 5\sqrt{5gR} \quad 1$$

2.



1. Work done from point A to point B:



1

$$x:\quad F - f_k = 0$$
$$y:\quad N - mg = 0$$
$$f_k = \mu N = \mu mg$$

1

$$W_{A\to B} = \vec{F}\cdot\vec{d} = \mu mg\cdot d$$

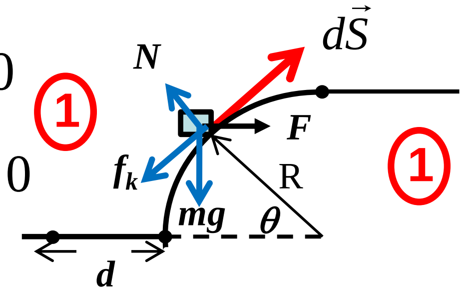
1

2. Work done from point B to point C:

$$x:\quad F - N\cos\theta - f\sin\theta = 0$$

1

$$y:\quad N\sin\theta - mg - f\cos\theta = 0$$



1

$$f_k = \mu N = \frac{\mu mg}{\sin\theta - \mu\cos\theta}$$

1

$$F = \frac{mg(\cos\theta + \mu\sin\theta)}{\sin\theta - \mu\cos\theta}$$

1

以下送分

$d\vec{S}$的分析: $\boxed{dS = Rd\theta}$ 1

$$W_{B\to C} = \int_1^2 \vec{F}\cdot d\vec{S}$$

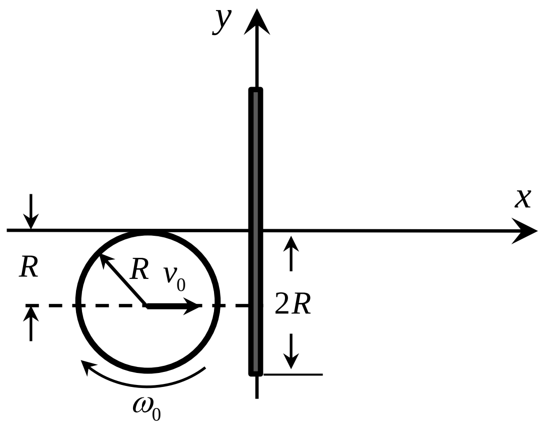
1

$$= \int_0^{\pi/2} \frac{mg(\cos\theta + \mu\sin\theta)}{\sin\theta - \mu\cos\theta}\cdot\sin\theta\cdot Rd\theta$$

1

很難解.

1. As shown in Fig. x, on top of a frictionless surface a thin rod of length 4***R*** and mass ***M*** is rest along the ***y***-axis with its center at the origin. A ring with mass ***M*** and radius of ***R*** is spinning with angular speed $\boldsymbol{\omega_0}$ and traveling with speed of $\boldsymbol{v_0}$ in the **+*x*** direction. The ring collides with the rod. If the ring and the rod stick to each other right after the collision, determine the velocity (magnitude and direction) of the center of the mass of the assembly and its rotational velocity (magnitude and direction) around its center of mass in terms of ***M, R,*** $\boldsymbol{\omega_0}$, and $\boldsymbol{v_0}$ (ignore the thickness of the rod).



During the collision, the only forces that occurs is the action and reaction force between the two objects. Therefore, the total force and the total torque with respect to the origin of the coordinate system is 0. The total momentum and the Total angular momentum respect to the origin is conserved. ①

$$\vec{P}_{i,total} = \vec{P}_{f,total} \quad \Rightarrow M\vec{v}_0 = (M + M)\vec{v}_f \quad \Rightarrow \vec{v}_f = \frac{\vec{v}_0}{2} = \frac{v_0}{2}\hat{x}$$ ① ①

$$\vec{L}_{i,total} = \vec{L}_{f,total} \quad \Rightarrow \vec{r}_{CM,ring} \times \vec{P}_{CM,ring} + I_{CM,ring}\vec{\omega}_{CM,ring} = \vec{r}_{CM,f} \times \vec{P}_{CM,f} + I_{CM,f}\vec{\omega}_{CM,f}$$ ①

$$\Rightarrow RMv_0(\hat{z}) + MR^2\omega_0(-\hat{z}) = \vec{r}_{CM,f} \times \vec{P}_{CM,f} + I_{CM,f}\vec{\omega}_{CM,f}$$

① ①

At the moment of collision, $\vec{r}_{CM,f} = \dfrac{M(-R,-R) + M(0,0)}{2M} = (-R/2, -R/2)$ ①

$$I_{CM,f} = I_{C,ring} + M\frac{R^2}{2} + I_{C,rod} + M\frac{R^2}{2} = MR^2 + M\frac{R^2}{2} + \frac{M(4R)^2}{12} + M\frac{R^2}{2} = \frac{10}{3}MR^2$$ ③

Right after the collision, $\vec{P}_{CM,f} = 2M\dfrac{\vec{v}_0}{2} = M\vec{v}_0$

$$\Rightarrow RMv_0(\hat{z}) + MR^2\omega_0(-\hat{z}) = \frac{R}{2}Mv_0(\hat{z}) + \frac{10}{3}MR^2\vec{\omega}_{CM,f}$$

① ①

$$\Rightarrow (\frac{R}{2}Mv_0 - MR^2\omega_0)\hat{z} = \frac{10}{3}MR^2\vec{\omega}_{CM,f}$$

$$\Rightarrow \vec{\omega}_{CM,f} = \frac{3}{10}(\frac{v_0}{2R} - \omega_0)\hat{z}$$

① ②

Problem 4

5 pts

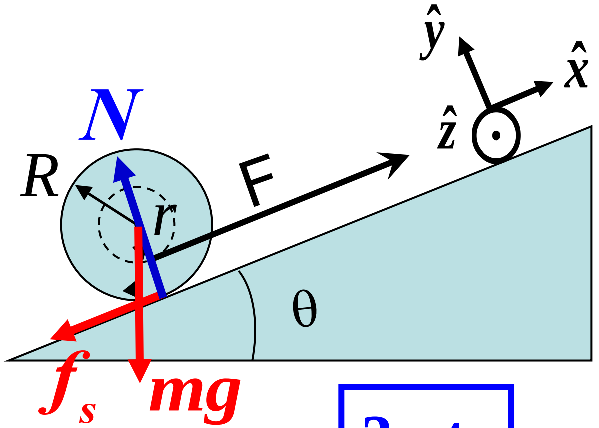
(a)

$\boxed{\text{2 pts}}$

$$\hat{x}: \quad F - mg\sin\theta - f_s = ma_{CM}$$

$$\hat{y}: \quad N - mg\cos\theta = 0$$

$$\sum \tau = Rf_s - rF = I_{CM}\alpha = \frac{1}{2}mR^2\alpha,$$



$\boxed{\text{3 pts}}$

(b)

pure roll: $a_{CM} = R\alpha \qquad F - mg\sin\theta - f_s = ma_{CM}$

$\boxed{\text{1 pts}}$

$$Rf_s - rF = I_{CM}\frac{a_{CM}}{R} = \frac{1}{2}mR^2\frac{a_{CM}}{R}$$

3 pts

$$a_{CM} = \frac{\left(1 - \frac{r}{R}\right)\frac{F}{m} - g\sin\theta}{1 + \frac{I_{CM}}{mR^2}} = \frac{1}{3m}\left(F - 2mg\sin\theta\right)$$

$\boxed{\text{1 pts}}$

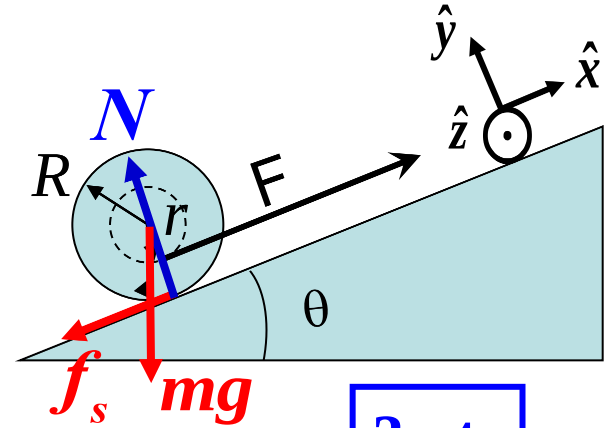
$$f_s = \frac{F\frac{r}{R} + \frac{I_{CM}}{mR^2}\left(F - mg\sin\theta\right)}{1 + \frac{I_{CM}}{mR^2}} = \frac{2}{3}\left(F - \frac{mg\sin\theta}{2}\right)$$

$\boxed{\text{1 pts}}$

Problem 4 (another way)

5 pts (a)

2 pts



$$\hat{x}: \quad F - mg\sin\theta - f_s = ma_{CM}$$

$$\hat{y}: \quad N - mg\cos\theta = 0$$

$$\sum \vec{\tau} = \left(-Rf_s + rF\right)\hat{z} = I_{CM}\vec{\alpha} = \frac{1}{2}mR^2\alpha\hat{z},$$

3 pts

(b)

pure roll: $a_{CM} = -R\alpha$

$$F - mg\sin\theta - f_s = ma_{CM}$$

1 pts

$$-Rf_s + rF = I_{CM}\frac{-a_{CM}}{R} = -\frac{1}{2}mR^2\frac{a_{CM}}{R}$$

3 pts

$$a_{CM} = \frac{\left(1 - \frac{r}{R}\right)\frac{F}{m} - g\sin\theta}{1 + \frac{I_{CM}}{mR^2}} = \frac{1}{3m}\left(F - 2mg\sin\theta\right)$$

1 pts

$$f_s = \frac{F\frac{r}{R} + \frac{I_{CM}}{mR^2}\left(F - mg\sin\theta\right)}{1 + \frac{I_{CM}}{mR^2}} = \frac{2}{3}\left(F - \frac{mg\sin\theta}{2}\right)$$

1 pts

Problem 4

4 pts

(c) **the condition is**

(i) $|f_s| < f_{s,\max} = \mu_s N = 0.4mg\cos\theta$ $\boxed{\textbf{1 pts}}$

(ii) $a_{CM} > 0$ **or** $\frac{F}{mg} > 2\sin\theta = 0.5$ $\boxed{\textbf{1 pts}}$

$$(i) \rightarrow 0.4\cos\theta < \frac{2}{3}\left(\frac{F}{mg} - \frac{\sin\theta}{2}\right) < 0.4\cos\theta$$

$$\rightarrow -0.6\cos\theta + 0.5\sin\theta < \frac{F}{mg} < 0.6\cos\theta + 0.5\sin\theta$$

$$\rightarrow -0.15\sqrt{15} + 0.125 < \frac{F}{mg} < 0.15\sqrt{15} + 0.125$$

3 pts

combine (i) and (ii), $0.125+0.15\sqrt{15} > \frac{F}{mg} > 0.5$ $\boxed{\textbf{2 pts}}$

(d) **Yes, if F =2 mg sinθ, along +x** $\boxed{\textbf{1 pts}}$

$a_{CM} = 0 \rightarrow \quad F = 2mg\sin\theta$, **and**

$$f_s = \frac{2}{3}\left(F - \frac{mg\sin\theta}{2}\right) = mg\sin\theta < f_{s,\max} = \mu_s mg\cos\theta \quad \tan\theta < \mu_s = 0.4$$ $\boxed{\textbf{2 pts}}$