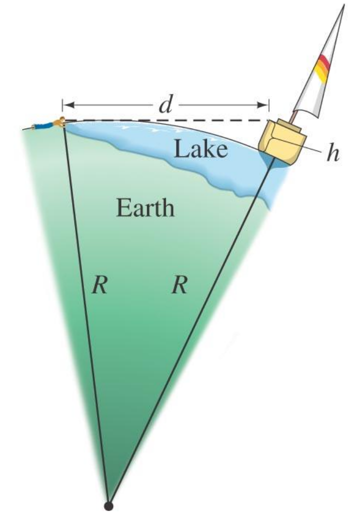
Homework 1

Due on 9/19/2018 for class meeting on Mon & Wed
Due on 9/20/2018 for class meeting on Tue & Thu

<u>1. Problem 1.33</u> Many sailboats are moored at a marina 4.4 km away on the opposite side of a lake. You stare at one of the sailboats because, when you are lying flat at the water's edge, you can just see its deck but none of the side of the sailboat. You then go to that sailboat on the other side of the lake and measure that the deck is 1.5 m above the level of the water. Using Fig. below, where ***h*** =1.5m, estimate the radius *R* of the Earth.



2. The smallest meaningful measure of length is called the "Planck length," and is defined as $\lambda_p = G^l h^m c^n$, the speed of light ***c* = 3.00 x 10^{8} m/s**, the gravitation constants G **= 6.67 x 10^{-11} m^3/(kg·s^2)**, the Planck's constant ***h* = 6.63 x 10^{-34} kg·m^2/s**. What are ***l***, ***m***, and ***n***?

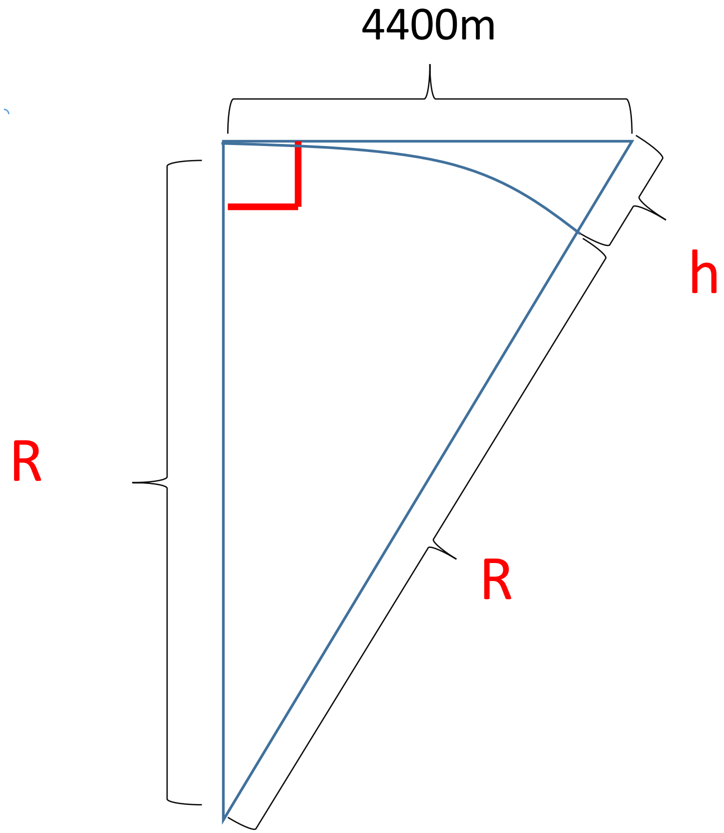
3. Find the answer for the following.

$(a)\dfrac{d}{dx}\left(x^2+2\right)^5 \qquad (b)\dfrac{d}{dx}((\sqrt{2x+1})\cdot\sin x) \qquad (c)\dfrac{d}{dx}\left[\cot\sqrt{2x}\right]$

(d) (i) Draw the diagram of the function $f(x)=(1-e^{-x})$ for 0< x<10.
(ii) Differentiate the function f(x), and obtain $f'(x)$,

Homework 1 solution:

1.



From the figure on the right, one can obtain the equation:

$$R^2 + 4400^2 = (R+h)^2$$

Do the calculation,

$$3R = 19359997.75$$

$$R \cong 6.45 \times 10^6 \, m$$

2. The smallest meaningful measure of length is called the "Planck length," and is defined as $\lambda_p = G^l h^m c^n$, the speed of light ***c* = 3.00 x 10^8 m/s**, the gravitation constants G **= 6.67 x 10^{-11} m^3/(kg·s^2)**, the Planck's constant ***h* = 6.63 x 10^{-34} kg·m^2/s**. What are ***l***, ***m***, and ***n***?

Sol.

$$[\mathrm{L}]=[\frac{L^3}{MT^2}]^x \bullet [\frac{ML^2}{T}]^y \bullet [\frac{L}{T}]^z$$

$$\rightarrow \quad \begin{aligned} [L]&: 3x+2y+z=1 \\ [M]&: -x+y=0 \\ [T]&: -2x-y-z=0 \end{aligned} \quad \rightarrow \quad \begin{aligned} x&=\frac{1}{2} \\ y&=\frac{1}{2} \\ z&=-\frac{3}{2} \end{aligned}$$

Problem 3

$$(a)\quad \frac{d}{dx}\left(x^2+2\right)^5$$

$$=\frac{dy^5}{dy}\cdot\frac{d\left(x^2+2\right)}{dy}$$

$$=5\left(x^2+2\right)^4\cdot(2x)$$

$$\boxed{=10x\cdot\left(x^2+2\right)^4}$$

Key: set $y=\left(x^2+2\right)^5$

$$(b)\ \frac{d}{dx}((\sqrt{2x+1})\cdot\sin x)$$

$$=\frac{d(\sqrt{2x+1})}{dx}\cdot\sin x+(\sqrt{2x+1})\cdot\frac{d\sin x}{dx}$$

$$=\frac{d(\sqrt{y})}{dy}\cdot\frac{d(2x+1)}{dx}\cdot\sin x+\sqrt{2x+1}\cos x$$

$$=\frac{1}{2}\frac{1}{\sqrt{2x+1}}\cdot 2\cdot\sin x+\sqrt{2x+1}\cos x$$

$$\boxed{=\frac{\sin x}{\sqrt{2x+1}}+\sqrt{2x+1}\cos x}$$

$$(c)\ \frac{d}{dx}\left[\cot\sqrt{2x}\right]$$

$$=\frac{d\cot y}{dy}\cdot\frac{d\sqrt{2x}}{dx}$$

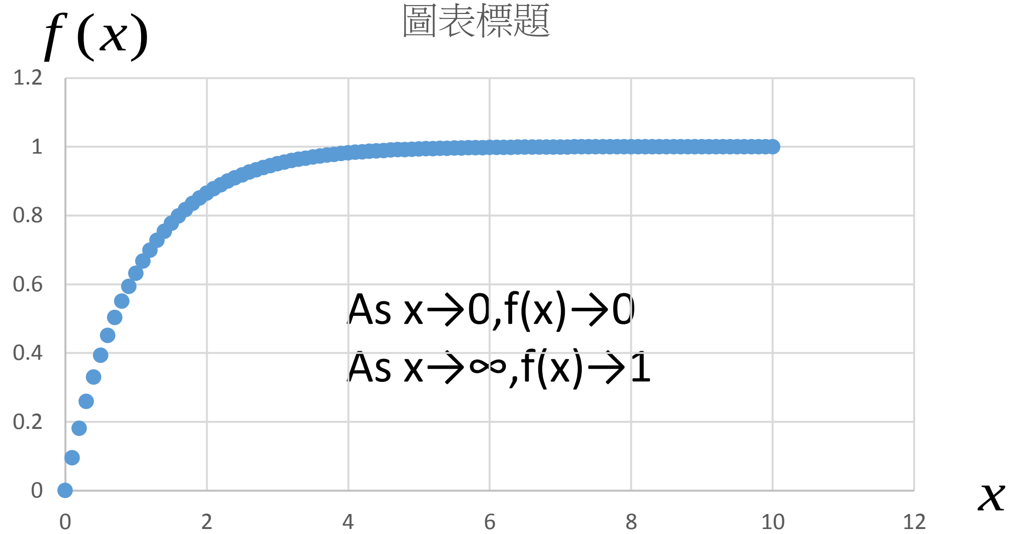
$$=\left(-\csc^2\sqrt{2x}\right)\cdot\sqrt{2}\left(\frac{1}{2}\right)\frac{1}{\sqrt{x}}$$

$$\boxed{=-\frac{1}{\sqrt{2x}}\cdot\csc^2\sqrt{2x}}$$

Key: set $y=\sqrt{2x}$

- **<u>Problem 3</u>**
- (d) (i) Draw the diagram of the function $f(x) = (1 - e^{-x})$

sol:



(ii) Differentiate the function f(x) and obtain $f'(x)$,

$$\frac{df(x)}{dx} = -\frac{de^{-x}}{dx} = e^{-x}$$

$$= 1 - f(x)$$