6.1 An uncharged 100-μF capacitor is charged by a constant current of 1 mA. Find the voltage across the capacitor after 4 s.

SOLUTION:

$$V(t) = \frac{1}{C}\int_0^T i(t)\,dt$$

$$V(t) = \frac{1}{100\mu}\int_0^4 1\,m\,dt$$

$$V(t) = \frac{1}{100\mu}\left[1m(4)-0\right]$$

$$V(t) = 40V$$

or

$$V = 40V$$

6.2 A capacitor has an accumulated charge of 600 μC with 5 V across it. What is the value of capacitance?

SOLUTION:

$$C = \frac{Q}{V}$$

$$C = \frac{600\mu}{5}$$

$$C = 120\,\mu F$$

6.3 A 25-μF capacitor initially charged to –10 V is charged by a constant current of 2.5 μA. Find the voltage across the capacitor after $2\frac{1}{2}$ min.

SOLUTION:

$$V(t) = \frac{1}{C}\int_0^t i(t)\,dt + V(0)$$

$$V(t) = \frac{1}{25\mu}\int_0^{150} 2.5\mu\,dt - 10$$

$$V(t) = \frac{2.5\mu}{25\mu}\left[150\right] - 10$$

$$V(t) = 5\text{V}$$

6.4 The energy that is stored in a 25-mF capacitor is $w(t) = 12 \sin^2 377t$. Find the current in the capacitor.

SOLUTION:

$$w(t) = \frac{1}{2} C V^2(t)$$

$$w(t) = 12 \sin^2 377t \quad J$$

$$V^2(t) = \frac{(12 \sin^2 377t)(2)}{25 \times 10^{-6}}$$

$$V(t) = \pm\ 979.8 \sin 377t \ \ V$$

$$i(t) = C \frac{dv(t)}{dt}$$

$$i(t) = (25 \times 10^{-6}) \left[\pm\ 979.8\,(377) \cos 377t \right]$$

$$i(t) = \pm\ 9.23 \cos 377t \ \ A$$

6.5 A capacitor is charged by a constant current of 2 mA and results in a voltage increase of 12 V in a 10-s interval. What is the value of the capacitance?

SOLUTION:

$$v(t_2) - v(t_1) = \frac{1}{C}\int_{t_1}^{t_2} i(t)\, dt$$

$$v(t_2) - v(t_1) = \frac{1}{C}\left[I(t_2 - t_1)\right]$$

$$12 = \frac{I}{C}(10)$$

$$C = \frac{2 \times 10^{-3}(10)}{12}$$

$$C = 1.67 \text{ mF}$$

6.6 The current in a 100-µF capacitor is shown in Fig. P6.6. Determine the waveform for the voltage across the capacitor if it is initially uncharged.

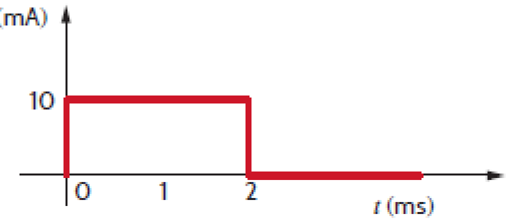


Figure P6.6

SOLUTION:

$t_1 = 2ms$

$$V = \frac{1}{C}\int i\,dt + V_0$$

$t < 0$, $i = 0$, and $V = 0$

$0 \leq t < t$, $i = 10mA$, and

$$V = \frac{1}{100\mu}\int_0^{2m} 10m\,dt$$

$$V = \frac{10m}{100\mu}\left[t\right]_0^{2m} = 100t \text{ V}$$

$t \geq t_1$

$$V = \frac{1}{C}\int 0\,dt + 100(2m)$$

$$V = 0.2 \text{ V}$$

$$V(t) = \begin{cases} 0\,\text{V}, & t < 0 \\ 100t\,\text{V}, & 0 \leq t < t_1 \\ 0.2\,\text{V}, & t \geq t_1 \end{cases}$$

6.7 Draw the waveform for the current in a 12-μF capacitor when the capacitor voltage is as described in Fig. P6.7.

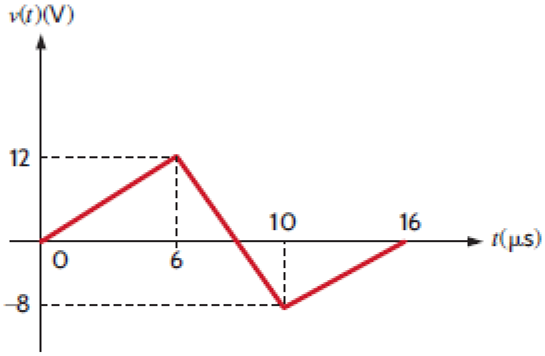
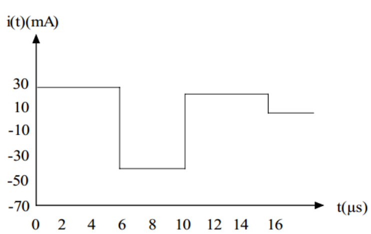


Figure P6.7

SOLUTION:

C =12μF $\quad i = C\frac{dv}{dt}$

Time (μs)	$\frac{dv}{dt}$ (V/μs)	i(t) (A)
$0 \le t \le 6$	2	24
$6 \le t \le 10$	−5	−60
$10 \le t \le 16$	1.33	16
$t > 16$	0	0



6.8 Derive the waveform for the current in a 60-μF capacitor in the voltage across the capacitor as shown in Fig. P6.8.

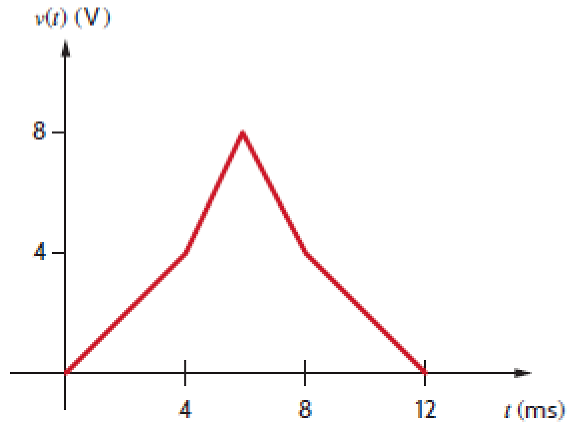
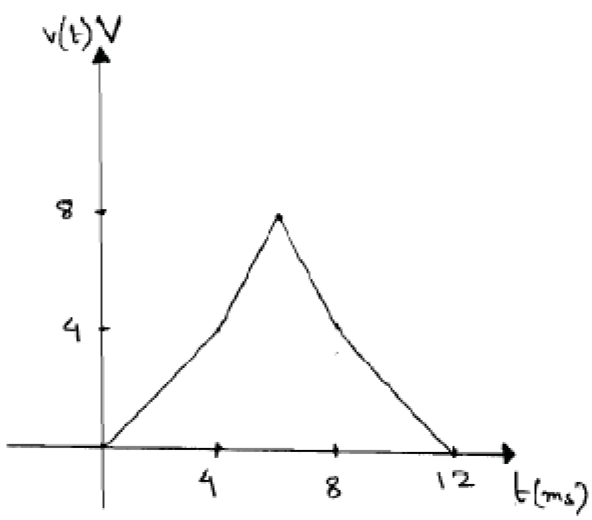


Figure P6.8

SOLUTION:



$$v(t) = \frac{4}{4\times10^{-3}}t \qquad 0 \le t \le 4\,ms$$

$$= \frac{4}{2\times10^{-3}}t \qquad 4 \le t \le 6\,ms$$

$$= \frac{-4}{2\times10^{-3}}t + a \qquad 6 \le t \le 8\,ms$$

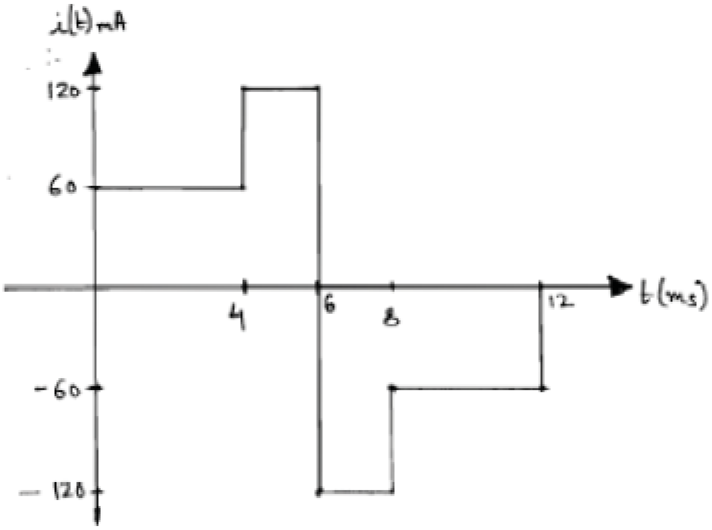
$$= \frac{-4}{4\times10^{-3}}t + c \qquad 8 \le t \le 12\,ms$$

$$i(t) = (60\times10^{-6})(10^{3}) = 60\,mA \qquad 0 \le t \le 4\,ms$$

$$= (60\times10^{-6})(2\times10^{3}) = 120\,mA \qquad 4 \le t \le 6\,ms$$

$$= (60\times10^{-6})(-2\times10^{3}) = -120\,mA \qquad 6 \le t \le 8\,ms$$

$$= (60\times10^{-6})(-10^{3}) = -60\,mA \qquad 8 \le t \le 12\,ms$$


i(t) mA
120
60
-60
-120
4
6
8
12
t (ms)

6.9 Draw the waveform for the current in a 3-µF capacitor when the voltage across the capacitor is as given in Fig. P6.9.

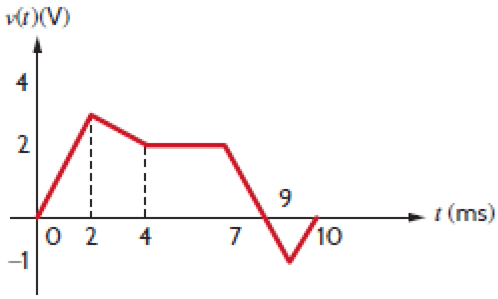
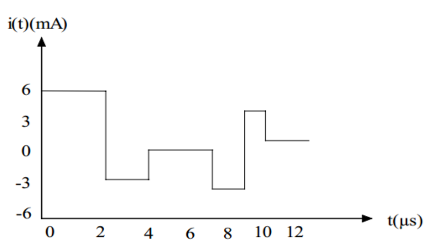


Figure P6.9

SOLUTION:

C =12µF $\quad i = C\frac{dv}{dt}$

Time(ms)	$\frac{dv}{dt}$ (V/µs)	i(t) (A)
0 ≤t ≤2	2	6
2 ≤t ≤4	−1	−3
4 ≤t ≤7	0	0
7 ≤t ≤9	$-\frac{3}{2}$	−4.5
9 ≤t ≤10	1	3
t >0	0	0



6.10 The current flowing through a 5-μF capacitor is shown in Fig. P6.10. Find the energy stored in the capacitor at $t = 1.4$ ms, $t = 3.3$ ms, $t = 4.3$ ms, $t = 6.7$ ms, and $t = 8.5$ ms.

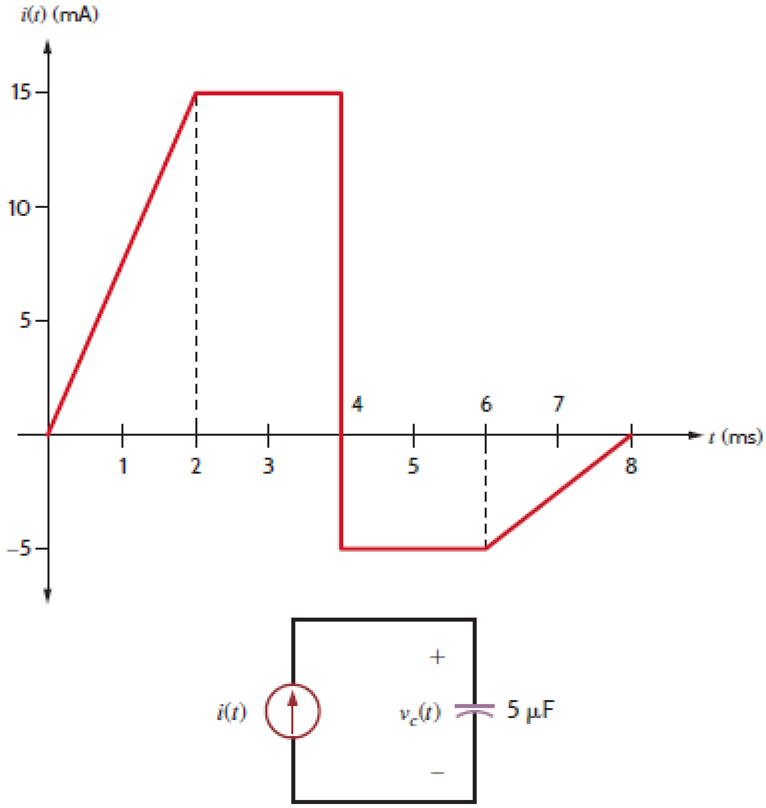


Figure P6.10

SOLUTION:

(See Next Page)

$v_c(t) = \frac{1}{C}\int_0^t i\,dt + v_c(0)$

$w_c = \frac{1}{2} C\, v_c^2$

Assume $v_c(0) = 0$

$\underline{0 \le t \le 2\,ms:}$

$i(t) = 7.5\,t \text{ mA}$

$v_c(t) = \frac{1}{5\times10^{-6}}\int_0^t 7.5\,t\,dt$

$v_c(t) = 7.5\times10^{5}\,t^2 \text{ V}$

$v_c(t = 2\times10^{-3}) \quad 7.5\times10^{5}(2\times10^{-3})^2 = 3\text{ V}$

<u>$2 \le t \le 4$ ms :</u>

$i(t) = 15\text{ mA}$

$$v_c(t) = \frac{1}{5\times10^{-6}}\int_{2\times10^{-3}}^{t} 15\times10^{-3}\,dt + 3$$

$$v_c(t) = 3000(t - 2\times10^{-3}) + 3 \text{ V}$$

$$v_c(t = 4\times10^{-3}) = 3000(4\times10^{-3} - 2\times10^{-3}) + 3$$
$$= 9\text{ V}$$

<u>$4 \le t \le 6$ ms :</u>

$i(t) = -5\text{ mA}$

$$v_c(t) = \frac{1}{5\times10^{-6}}\int_{4\times10^{-3}}^{t} -5\times10^{-3}\,dt + 9$$

$$v_c(t) = -1000(t - 4\times10^{-3}) + 9 \text{ V}$$

$$v_c(t = 6\times10^{-3}) = -1000(6\times10^{-3} - 4\times10^{-3}) + 9$$
$$= 7\text{ V}$$

<u>$6 \le t \le 8$ ms :</u>

$i(t) = 2.5t - 0.02\text{ A}$

$$v_c(t) = \frac{1}{5\times10^{-6}}\int_{6\times10^{-3}}^{t} (2.5t - 0.02)\,dt + 7$$

$v_c(t) = 2.5\times10^5 t^2 - 4\times10^3 t + 22\,V$

$v_c(t = 8\times10^{-3}) = 6V$

<u>$t > 8ms$:</u>

$i(t) = 0 \qquad v_c(t) = 6V$

<u>At $t = 1.4$ ms:</u>

$v_c(t = 1.4\times10^{-3}) = 7.5\times10^5(1.4\times10^{-3})^2 = 1.47\,V$

$w_c = \frac{1}{2}(5\times10^{-6})(1.47)^2 = 5.4\,\mu J$

<u>At $t = 3.3$ ms :</u>

$v_c(t = 3.3\times10^{-3}) = 3000(3.3\times10^{-3} - 2\times10^{-3}) + 3$

$= 6.9\,V$

$w_c = \frac{1}{2}(5\times10^{-6})(6.9)^2 = 119\,\mu J$

<u>At $t = 4.3$ ms</u>:

$v_c(t = 4.3\times10^{-3}) = -1000(4.3\times10^{-3} - 4\times10^{-3}) + 9$

$= 8.7\,V$

$w_c = \frac{1}{2}(5\times10^{-6})(8.7)^2 = 189.23\,\mu J$

<u>At $t = 6.7$ ms :</u>

$v_c(t = 6.7\times10^{-3}) = 2.5\times10^5(6.7\times10^{-3})^2$
$-4\times10^3(6.7\times10^{-3}) + 22$

$v_c(t) = 6.42\ \text{V}$

$w_c = \frac{1}{2}(5\times10^{-6})(6.42)^2 = 103\ \mu J$

<u>At $t = 8.5$ ms;</u>

$v_c(t = 8.5\ \text{ms}) = 6\ \text{V}$

$w_c = \frac{1}{2}(5\times10^{-6})(6)^2 = 90\ \mu J$

6.11 The voltage across a 25-μF capacitor is shown in Fig. P6.11. Determine the current waveform.

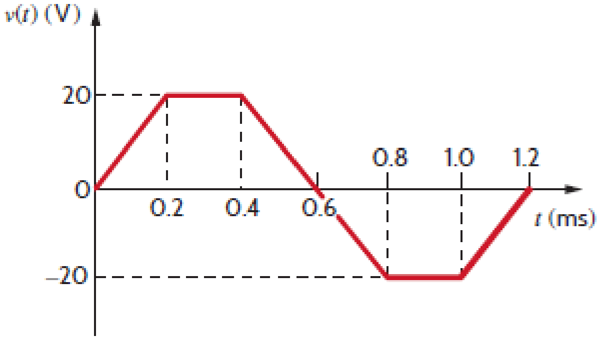


Figure P6.11

SOLUTION:

$t_1 = 0.2ms$

$t_2 = 0.4ms$

$t_3 = 0.8ms$

$t_4 = 1ms$

$t_5 = 1.2ms$

$$i = C\frac{dV}{dt}$$

$t < 0$, $V = 0$, and $i = 0$

$0 \leq t < t_1$, $V = 10^5 t$ V

$i = 25\mu[10^5] = 2.5A$

$t_1 \leq t < t_2$, $V = 20V$, $i = 0$

$t_2 \leq t < t_3$, $V = 60 - 10^5 t$ V

$i = 25\mu[-10^5] = -2.5\text{ A}$

$t_3 \le t < t_4$, $v=0$; $i=0$

$t_4 \le t < t_5$, $v = -120 \times 10^5 t$

$i = 25\mu[10^5] = 2.5\text{A}$

$$i(t) = \begin{cases} 0 & t < 0 \\ 2.5 & 0 \le t < 0.2\text{ms} \\ 0 & 0.2\text{ms} \le t < 0.4\text{ ms} \\ -2.5 & 0.4\text{ms} \le t < 0.8\text{ms} \\ 0 & 0.8\text{ms} \le t < 1\text{ms} \\ 2.5 & 1\text{ms} \le t < 1.2\text{ms} \\ 0 & t \geq 1.2\text{ms} \end{cases}$$

6.12 Find the energy stored in the electric field of the capacitor of E6.2 at $t = 6$ ms.

SOLUTION:

From the graph we know that

$$\begin{aligned} v(t) &= -2t + 24 \; 0 \le t \le 6 \\ i(t) &= 24 \\ p(t) &= v(t) \times i(t) \\ p(t) &= -48t + 576 \\ E(t) &= \int_0^t p(x)dx \\ E(t) &= -24t^2 + 576t \; mJ \\ &= 3.455 \; mJ \;\; at \; t = 6ms \end{aligned}$$

6.13 Draw the waveform for the current in a 24-μF capacitor when the capacitor voltage is as described in Fig. P6.13.

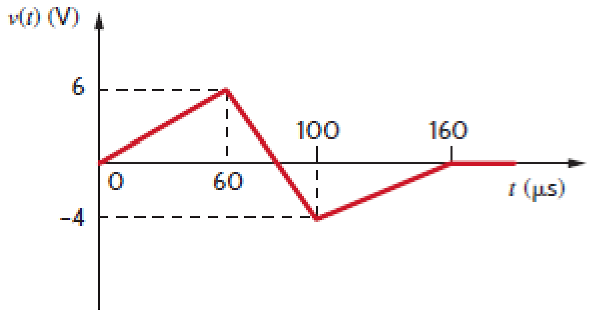


Figure P6.13

SOLUTION:

$t<0$, $V=0$, $i=0$

$0 \le t < 60\mu s$, $V = 10^5 t$ V

$i = C\frac{dV}{dt}$

$i = (24\mu)(10^5)$

$i = 2.4A$

$60\mu s \le t < 100\mu s$, $V = 21 - 2.5\times10^5 t$ V

$i = (24\mu)(-2.5\times10^5)$

$i = -6A$

$100\mu s \le t < 160\mu s$, $V = \frac{-32}{3} + \frac{10^6}{15} t$ V

$i = 24\mu\left[\frac{10^6}{15}\right]$

$i = 1.6A$

$t \geq 160\mu s$, $v = 0$, $i = 0$

$$i(t) = \begin{cases} 0 & t < 0 \\ 2.4A & 0 \leq t < 60\mu s \\ -2.4A & 60\mu s \leq t < 100\mu s \\ 1.6A & 100\mu s \leq t < 160\mu s \\ 0 & t \geq 160\mu s \end{cases}$$

6.14 The voltage across a 10-μF capacitor is given by the waveform in Fig. P6.14. Plot the waveform for the capacitor current.

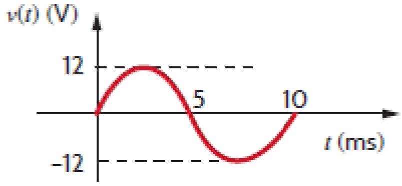


Figure P6.14

SOLUTION:

$$i = C\frac{dv}{dt}$$

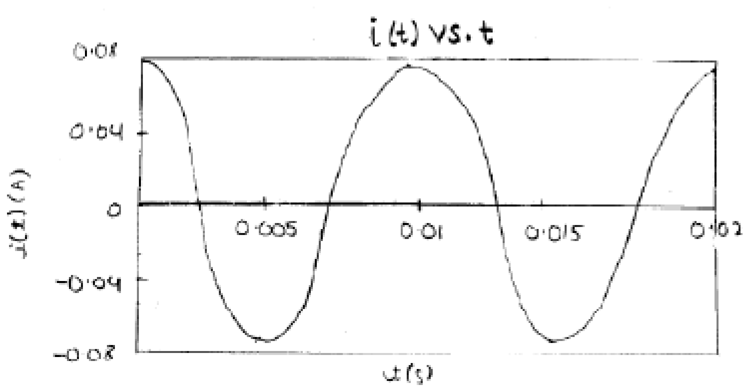
$$v(t) = 12\sin\omega t \quad , \quad T = 10ms$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{10m}$$

$$\omega = 200\pi \text{ rad/s}$$

$$i(t) = (10\mu)(12)(\omega)\cos\omega t$$

$$i(t) = 75.4\cos\omega t \text{ mA}$$



6.15 The waveform for the current in a 26-μF capacitor is shown in Fig. P6.15. Determine the waveform for the capacitor voltage.

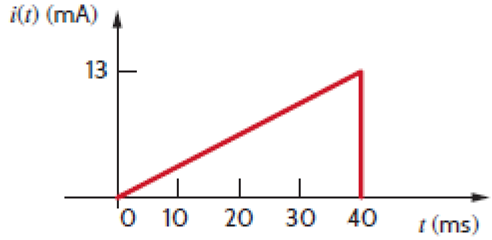


Figure P6.15

SOLUTION:

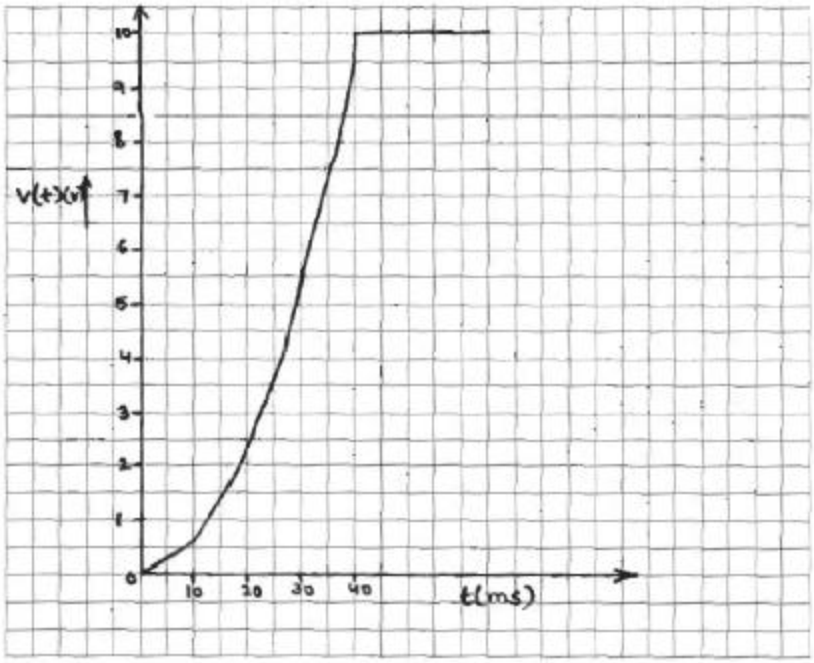
$t_1 = 40\text{ms}$ $\qquad V = \frac{1}{C}\int i\,dt + V_0$

$t < 0, \quad i(t) = 0, \quad V = 0$

$0 \leqslant t < t_1, \quad i(t) = \frac{13}{40}\,t = 0.325\,t, \quad V = 6250\,t^2\ \text{V}$

$t \geqslant t_1, \quad i(t) = 0, \quad V = 6250\,(40 \times 10^{-3})^2 + 0 = 10\ \text{V}$

$$\therefore\ v(t) = \begin{cases} 0\ \text{V} & t < 0\text{ms} \\ 6250\,t^2\ \text{V} & 0 \leqslant t < 40\ \text{ms} \\ 10\ \text{V} & t \geqslant 40\ \text{ms} \end{cases}$$


v(t)(v)
10
9
8
7
6
5
4
3
2
1
0
10
20
30
40
t(ms)

6.16 At $t = 0$, the voltage across a 50-mF capacitor is 10 V. Calculate the voltage across the capacitor for $t > 0$ when current $4t$ mA flows through it.

SOLUTION:

$$v = \frac{1}{C}\int idt + V(t0)$$

$$= \frac{1}{50 \times 10^{-3}}\int \; 4t \times 10^{-3}dt + 10$$

$$= \frac{2t^2}{50} + 10$$

6.17 The waveform for the voltage across 100-μF capacitor shown Fig. 6.17a is given in Fig. 6.17b. Determine the following quantities: (a) the energy stored in the capacitor at $t = 2.5$ ms, (b) the energy stored in the capacitor at $t = 5.5$ ms, (c) $i_c(t)$ at $t = 1.5$ ms, (d) $i_c(t)$ at $t = 4.75$ ms, and (e) $i_c(t)$ at $t = 7.5$ ms.

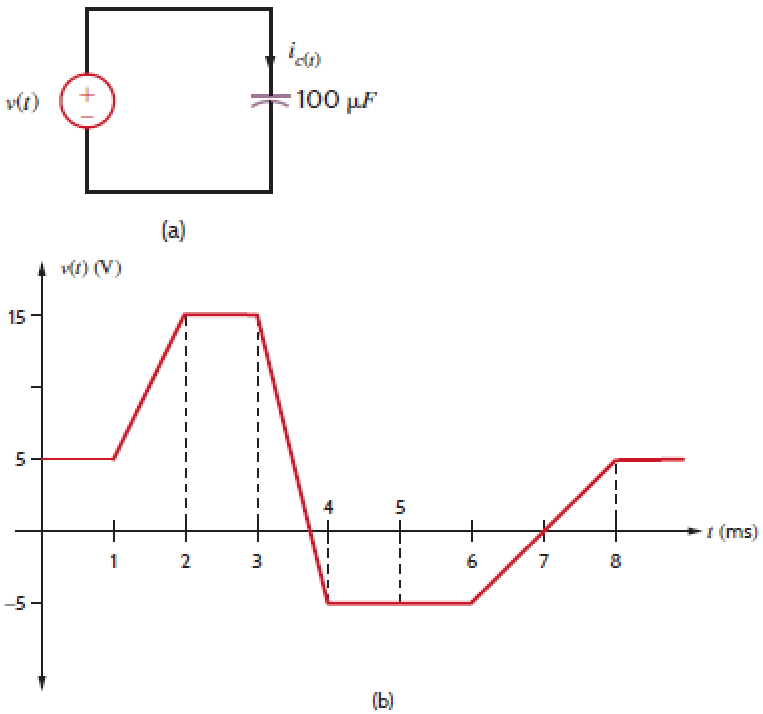


Figure P6.17

SOLUTION:

(See Next Page)

$C = 100\mu F$

(a) $w(t) = \frac{1}{2} C v^2(t)$

$w(2.5ms) = \frac{1}{2}(100\mu)(15)^2$

$w(2.5ms) = 11.25\ mJ$

(b) $w(t) = \frac{1}{2} C v^2(t)$

$w(5.5ms) = \frac{1}{2}(100\mu)(-5)^2$

$w(5.5ms) = 1.25\ mJ$

(c) $V(t) = mt + B$

$$m = \frac{15-5}{2\times10^{-3} - 1\times10^{-3}}$$

$m = 10{,}000$

$V(t) = 10000t + B$

$10 = 5000(1m) + B$

$B = -5$

$V(t) = 10000t - 5V$ for the interval of interest

$V(t) = 10{,}000 \times t - 5$ volts

$i_c(t) = C\frac{dV(t)}{dt}$

$i_c(t) = 100u\,[10000]$

$i_c(t) = 1A$

$i_c(1.5mS) = 1A$.

(d) $V(t) = -5V$ for the interval of interest

$i_c(t) = C\frac{dv(t)}{dt}$

$i_C(t) = 0A$

$i_C(4.75ms) = 0A$

(e) $m = \dfrac{5-0}{8\times10^{-3} - 7\times10^{-3}} = 5000$

$V(t) = 5000t + B$

$5 = 5000(8m) + B$

$B = -35$

$V(t) = 5000t - 35\,V$ for the interval of interest

$i_C(t) = C\dfrac{dv(t)}{dt}$

$i_C(t) = 100\mu[5000]$

$i_C(t) = 0.5A$

$i_C(7.5ms) = 0.5A$

6.18 The current in an 80-mH inductor increases from 0 to 60 mA. How much energy is stored in the inductor?

SOLUTION:

$$w = L\int_0^t Li^2 dt = \frac{1}{2}Lt^2(t) - \frac{1}{2}Li^2(-\infty)$$
$$= \frac{1}{2} \times 80 \times 10^{-3} \times (60 \times 10^{-3})^2 - 0$$
$$= 144\mu J$$

6.19 The current in a 100-mH inductor is $i(t) = 2 \sin 377t\ A$. Find (a) the voltage across the inductor and (b) the expression for the energy stored in the element.

SOLUTION:

(a)
$$V(t) = L\frac{di(t)}{dt}$$
$$V(t) = 0.1(2)(377)\cos 377t$$
$$V(t) = 75.4 \cos 377t \text{ V}$$

(b)
$$w(t) = \frac{1}{2} L i^2(t)$$
$$wt = \frac{1}{2}(0.1)\left[2 \sin 377t\right]^2$$
$$wt = 0.2 \sin^2 377t \text{ J}$$

6.20 A voltage of $6e^{-2000t}$ V appears across a parallel combination of a 100-mF capacitor and a 12 Ω resistor. Calculate the power absorbed by the parallel combination.

SOLUTION:

$I_R = \frac{V}{R} = \frac{6}{12}e^{-2000t} = 0.5e^{-2000t}$

$I_C = C\frac{dV}{dt} = 100 \times 10^{-3} \times 6(-2000)e^{-2000t}$

$= -120010^{-2000t}$

$I = I_R + I_C = -1199.5e^{-2000t}$

$p = vi = -7197e^{-4000t}W$

6.21 The current through a 0.5-F capacitor is $6(1-e^{-t})$A.
Determine the voltage and power at $t = 2$ s. Assume $v(0) = 0$.

SOLUTION:

$$v(t) = \frac{1}{\frac{1}{2}}\int_0^t 6(1-e^{-t})dt + 0$$
$$= 12(t+e^{-t}) - 12$$
$$v(2) = 12(2+e^{-t}) - 12 = 13.624V$$
$$p = iv = [12(t+e^{-t}) - 12]6(1-e^{-t})$$
$$p(2) = [12(2+e^{-2}) - 12]6(1-e^{-2}) = 70.66\,W$$

6.22 The voltage across a 4-H inductor is given by the waveform shown in Fig. P6.22. Find the waveform for the current in the inductor.

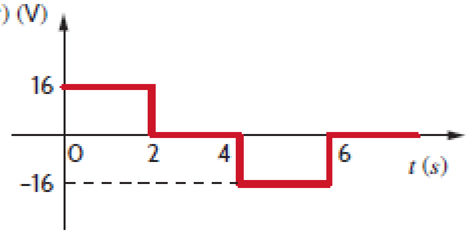


Figure P6.22

SOLUTION:

$$i(t) = \frac{1}{L}\int v(t)\,dt$$

$L = 4\ H$

For $t < 0$, $i(t) = 0$ since $v(t) = 0$

For $0 \leq t < 2s$

$$i(t) = \frac{1}{4}\int_0^t 16\,dt = 4t\ A$$

For $2 \leq t < 4s$

$$i(t) = i_0 + \frac{1}{4}\int_2^4 v(t)\,dt$$

$$= (4 \times 2)\ A + 0 \quad (\because V(t) = 0 \text{ in the interval})$$

$$= 8\ A$$

For $4 \leq t < 6s$

$$i(t) = 8 + \frac{1}{4}\int_4^t (-16)\,dt$$

$= 8 - 4t - (-4) \times 4$

$= 24 - 4t \quad A$

For $t > 6\,s$

$i(t) = 0\ A$

$$i(t) = \begin{cases} 0\ A & t < 0\,s \\ 4t\ A & 0 \le t < 2\,s \\ 8\ A & 2\,s \le t < 4\,s \\ 24 - 4t\ A & 4\,s \le t < 6\,s \\ 0\ A & t > 6\,s \end{cases}$$

6.23 The current in a 20-mH inductor is shown in Fig. P6.23. Derive the waveform for the inductor voltage.

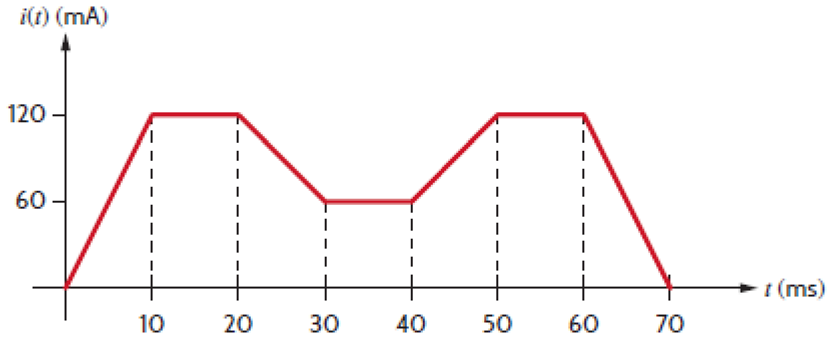
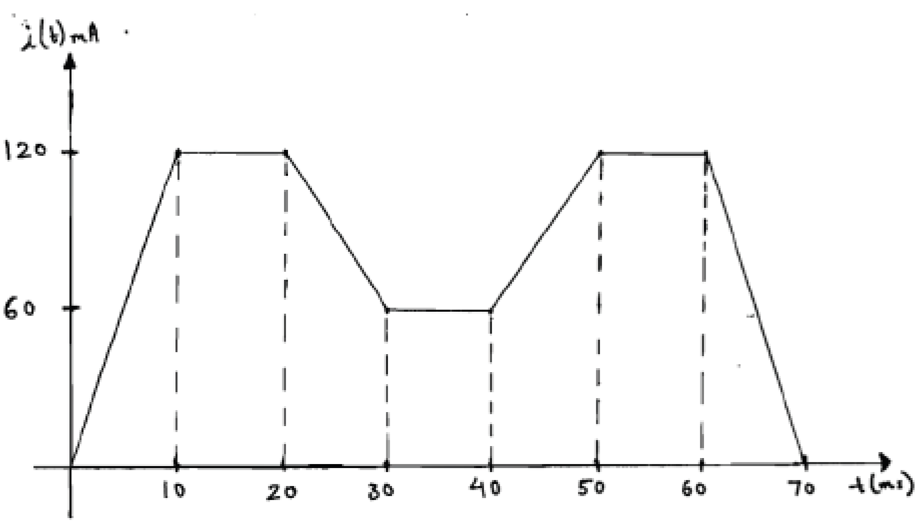


Figure P6.23

SOLUTION:



$$v(t) = L\frac{di}{dt} = (20 \times 10^{-3})\frac{di}{dt}$$

$$i(t) = \frac{120}{10}t \qquad 0 \le t \le 10\,\text{ms}$$

$$= 120 \times 10^{-3} \qquad 10 \le t \le 20\,\text{ms}$$

$$= -\frac{60}{10}t + b \qquad 20 \le t \le 30\,\text{ms}$$

$$= 60 \times 10^{-3} \qquad 30 \le t \le 40\,\text{ms}$$

$$= \frac{60}{10}t - a \qquad 40 \le t \le 50\,\text{ms}$$

$$= 120 \times 10^{-3} \qquad 50 \le t \le 60\,\text{ms}$$

$$= -\frac{120}{10}t \qquad 60 \le t \le 70\,\text{ms}$$

$$v(t) = (20 \times 10^{-3})(12) = 240\,\text{mV} \qquad 0 \le t \le 10\,\text{ms}$$

$$= 0 \qquad 10 \le t \le 20\,\text{ms}$$

$$= (20 \times 10^{-3})(-6) = -120\,\text{mV} \qquad 20 \le t \le 30\,\text{ms}$$

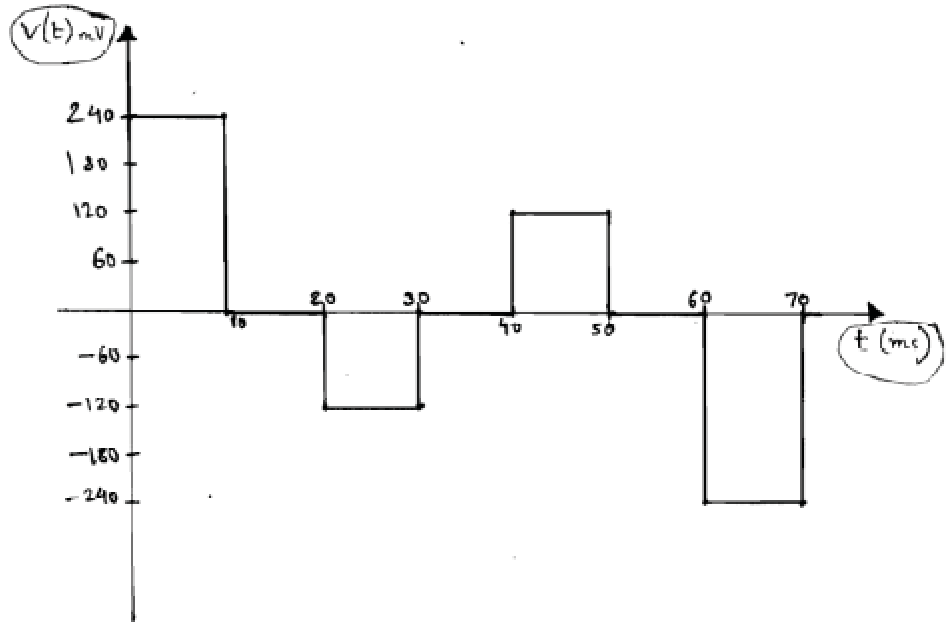
$$= 0 \qquad 30 \le t \le 40\,\text{ms}$$

$$= (20 \times 10^{-3})(6) = 120\,\text{mV} \qquad 40 \le t \le 50\,\text{ms}$$

$= 0 \qquad 50 \le t \le 60\,ms$

$= (20\times10^{-3})(-12) \qquad 60 \le t \le 70\,ms$

$= -240\,mV$



6.24 The current waveform in a 40-mH inductor is shown in Fig. P6.24. Derive the waveform for the inductor voltage.

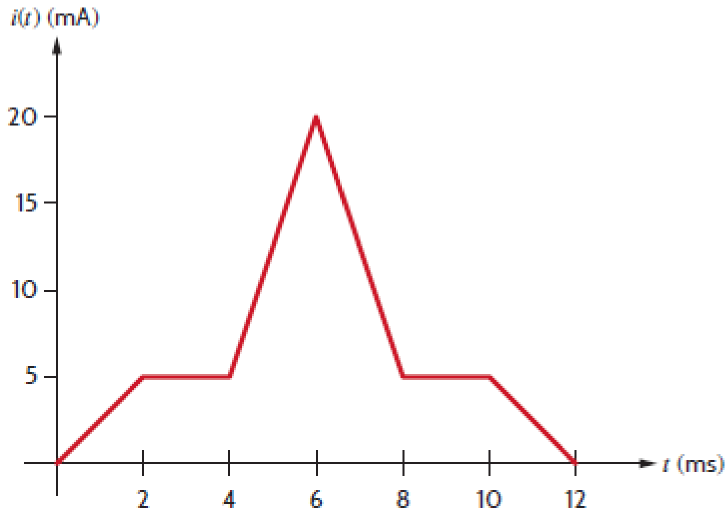


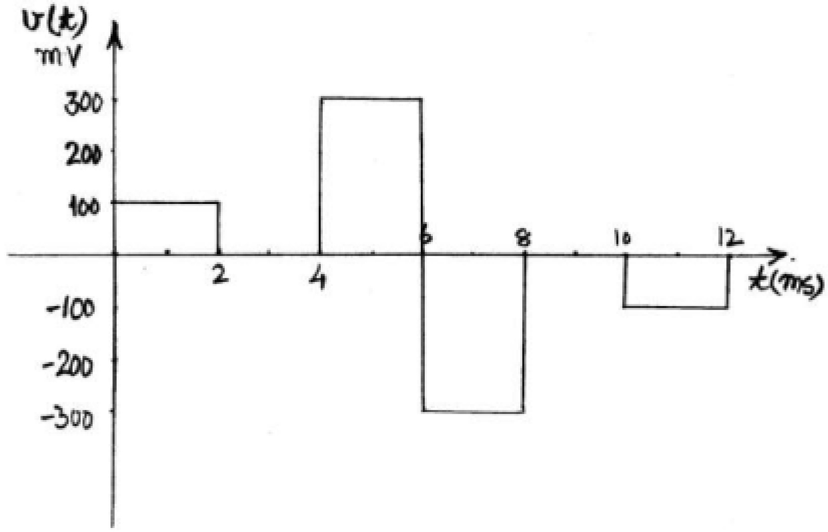
Figure P6.24

SOLUTION:

$$v(t) = L\frac{di}{dt} = (40\times 10^{-3})\frac{di}{dt}$$

$$\begin{aligned}
i(t) &= \frac{5}{2}t & 0 \le t \le 2\,ms \\
&= 5\times 10^{-3} & 2 \le t \le 4\,ms \\
&= \frac{15}{2}t - a & 4 \le t \le 6\,ms \\
&= \frac{-15}{2}t + b & 6 \le t \le 8\,ms \\
&= 5\times 10^{-3} & 8 \le t \le 10\,ms \\
&= \frac{-5}{2}t + c & 10 \le t \le 12\,ms
\end{aligned}$$

$$\begin{aligned}
v(t) &= (40\times 10^{-3})\left(\frac{5}{2}\right) = 100\,mV & 0 \le t \le 2\,ms \\
&= 0 & 2 \le t \le 4\,ms
\end{aligned}$$


v(t)
mV
300
200
100
-100
-200
-300
2
4
6
8
10
12
t(ms)

6.25 If the current in a 50-mH inductor is given by the waveform in Fig. P6.25, compute the waveform for the inductor voltage.

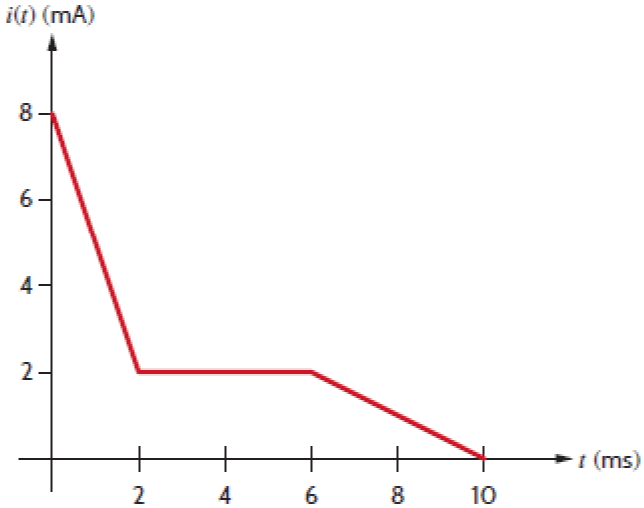
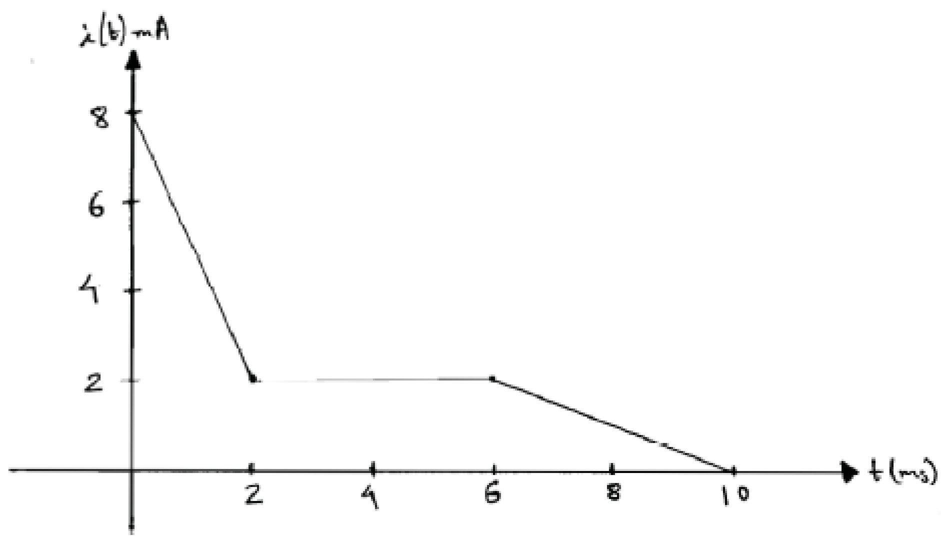


Figure P6.25

SOLUTION:



$$v(t) = L\frac{di}{dt} = (50 \times 10^{-3})\frac{di}{dt}$$

$$i(t) = -\frac{6}{2}t \qquad 0 \le t \le 2ms$$

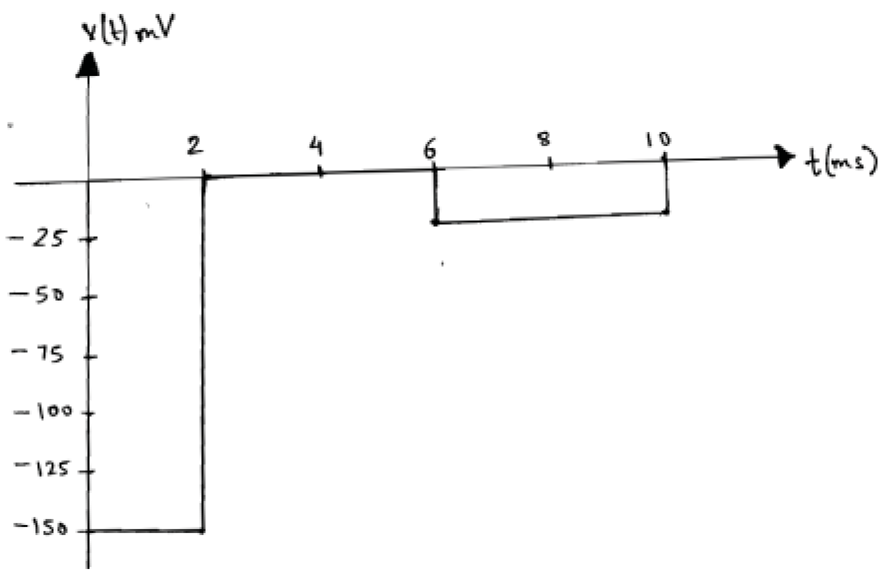
$$= 0 \qquad 2 \le t \le 6ms$$

$$= \frac{-2}{4}t + a \qquad 6 \le t \le 10ms$$

$$v(t) = (50 \times 10^{-3})(-3) = -150mV \qquad 0 \le t \le 2ms$$

$$= 0 \qquad 2 \le t \le 6ms$$

$$= (50 \times 10^{-3})\left(-\frac{1}{2}\right) = -25mV \qquad 6 \le t \le 10ms$$



6.26 The waveform for the current flowing through a 0.5-H inductor is shown in the plot in Fig. P6.26. Accurately sketch the inductor voltage versus time. Determine the following quantities: (a) the energy stored in the inductor at t = 1.7 ms, (b) the energy stored in the inductor at t = 4.2 ms, and (c) the power absorbed by the inductor at t = 1.2 ms, t = 2.8 ms, and t = 5.3 ms.

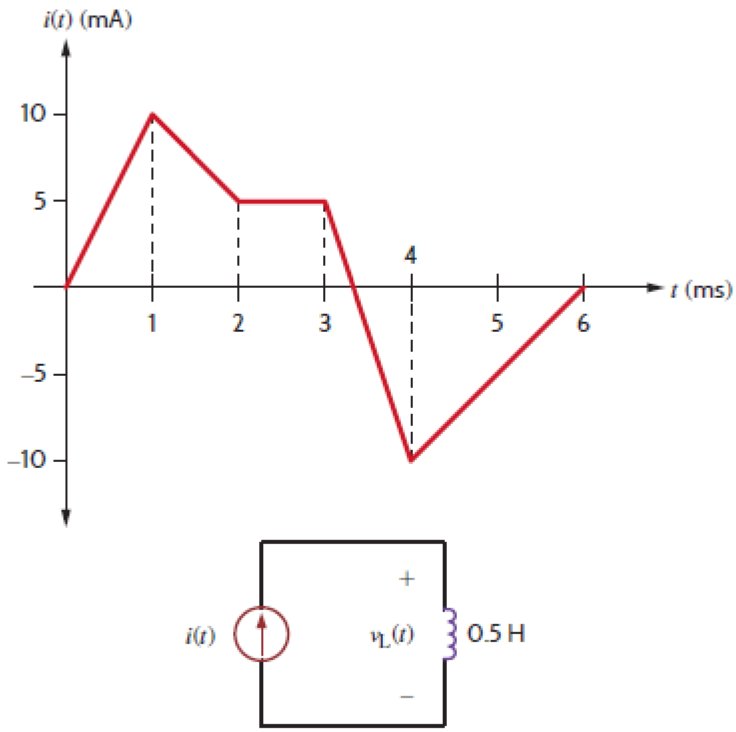


Figure P6.26

SOLUTION:

(See Next Page)

$v_L(t) = L \frac{di_L}{dt}$

$\underline{0 \le t \le 1\text{ms}:}$

$i(t) = 10t \text{ mA}$

$v_L(t) = 0.5 \frac{d}{dt}(10t) = 5 \text{ V}$

$\underline{1 \le t \le 2\text{ms}:}$

$i(t) = -5t + 15 \text{ mA}$

$v_L(t) = 0.5 \frac{d}{dt}(-5t + 15) = -2.5 \text{ V}$

$\underline{2 \le t \le 3\text{ms}:}$

$i(t) = 5 \text{ mA}$

$v_L(t) = 0$

$\underline{3 \le t \le 4\text{ms}:}$

$i(t) = -15t + 50 \text{ mA}$

$v_L(t) = 0.5 \frac{d}{dt}(-15t + 50) = -7.5 \text{ V}$

$\underline{4 \le t \le 6\text{ms}:}$

$i(t) = 5t - 30 \text{ mA}$

$v_L(t) = 0.5 \frac{d}{dt}(5t - 30) = 2.5 \text{ V}$

$\underline{t > 6\text{ ms};}$

$i(t) = 0 \quad v_L(t) = 0$

• $\omega_L = \frac{1}{2} L i^2$

$\underline{\text{At } t = 1.7\text{ms}:}$

$i(t = 1.7\text{ms}) = -5(1.7) + 15 = 6.5 \text{ mA}$

$\omega_L = \frac{1}{2}(0.5)(6.5)^2 = \underline{10.5625 \mu J}$

$\underline{\text{At } t = 4.2\text{ ms};}$

$i(t = 4.2\text{ms}) = 5(4.2) - 30 = -9 \text{ mA}$

$\omega_L = \frac{1}{2}(0.5)(-9)^2 = \underline{20.25\ \mu J}$

$P = vi$

<u>At $t = 1.2$ ms:</u>

$i(t = 1.2\,ms) = -5(1.2) + 15 = 9\ mA$

$P = (-2.5)(9m) = \underline{-22.5\ mW}$

<u>At $t = 2.8$ ms:</u>

$i(t = 2.8\,ms) = 5\ mA$

$\underline{P = (0)(5) = 0}$

<u>At $t = 5.3$ ms:</u>

$i(t = 5.3\,ms) = 5(5.3) - 30 = -3.5\ mA$

$P = (2.5)(-3.5m) = \underline{-8.75\ mW}$

6.27 The current in a 10-mH inductor is shown in Fig. P6.27. Determine the waveform for the voltage across the inductor.

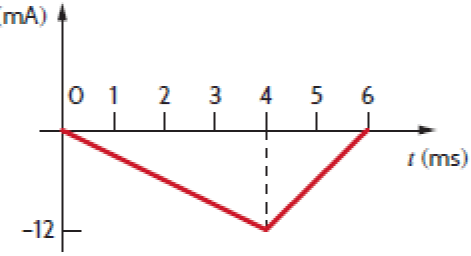


Figure P6.27

SOLUTION:

$$V(t) = L\frac{di(t)}{dt}$$

for $t < 0$, $V(t) = 0$

for $0 < t \le 4ms$

$$V(t) = 10m\left[\frac{-12m}{4m}\right] = -30mV$$

for $4ms < t \le 6ms$

$$V(t) = 10m\left[\frac{12m}{2m}\right] = 60mV$$

for $t > 6ms$

$V(t) = 0$

$$Vt = \begin{cases} 0 & t < 0 \\ -30mV & 0 < t \le 4ms \\ 60mV & 4ms < t \le 6ms \\ 0 & t > 6ms \end{cases}$$

6.28 The current in a 50-mH inductor is given in Fig. P6.28. Find the inductor voltage.

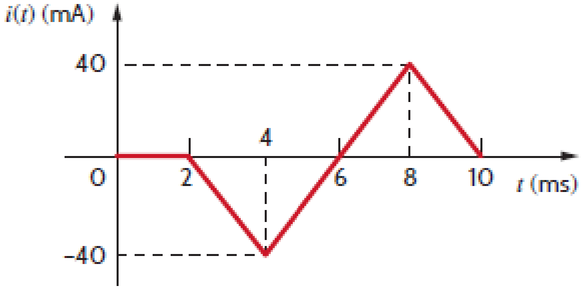


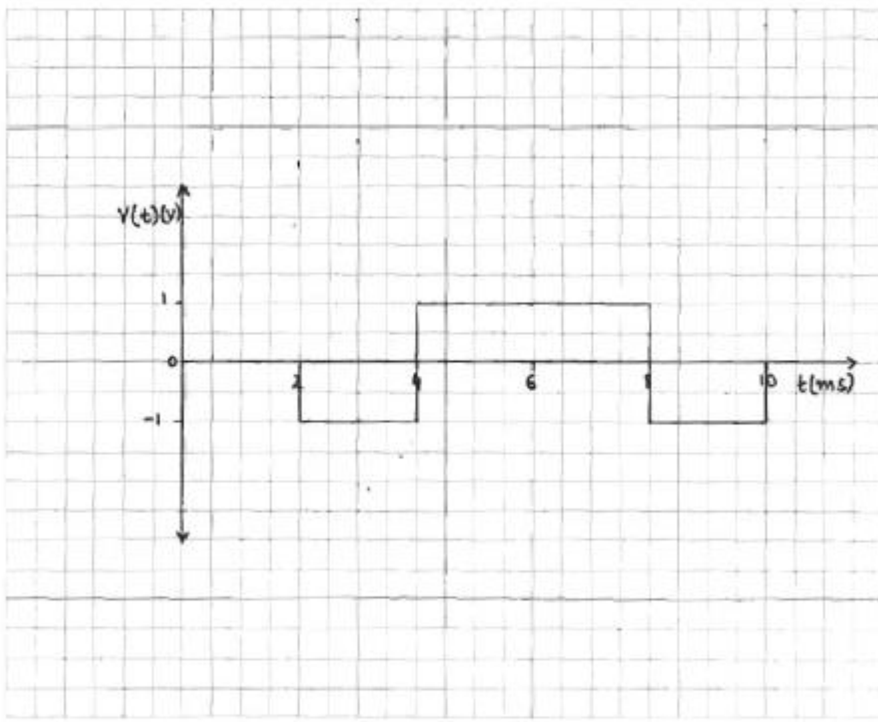
Figure P6.28

SOLUTION:

$$V = L\frac{di}{dt}$$

$$\frac{di}{dt} = \begin{cases} 0 & 0 \leq t < 2 \\ -20 \text{ A/s} & 2 \leq t < 4 \\ 20 \text{ A/s} & 4 \leq t < 8 \\ -20 \text{ A/s} & 8 \leq t < 10 \end{cases}$$

$$V = \begin{cases} 0 & 0 \leq t < 2 \\ -1\text{V} & 2 \leq t < 4 \\ 1\text{V} & 4 \leq t < 8 \\ -1\text{V} & 8 \leq t < 10 \end{cases}$$


V(t)(V)
1
0
-1
2
4
6
8
10
t(ms)

6.29 Draw the waveform for the voltage across a 24-mH inductor when the inductor current is given by the waveform shown in Fig. P6.29.

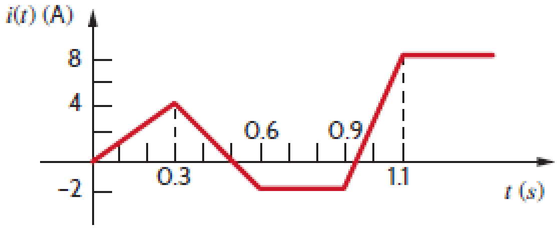


Figure P6.29

SOLUTION:

$$V(t) = L\frac{di(t)}{dt}$$

for $t \leq 0$, $V(t) = 0$

for $0 \leq t \leq 0.3s$

$$V(t) = 24m\left[\frac{40}{3}\right]$$

$$V(t) = 320mV$$

for $0.3s < t \leq 0.6s$

$$V(t) = 24m[-20]$$

$$V(t) = -480mV$$

for $0.6s < t \leq 0.9s$

$V(t) = 0$

for $0.9s < t \leq 1.1s$

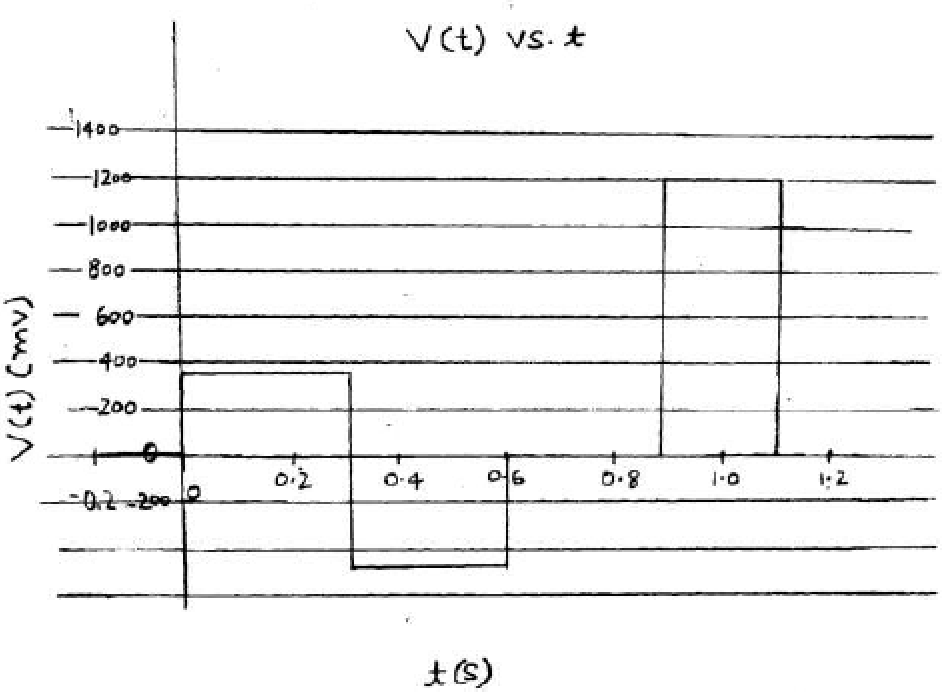
$$V(t) = 24m[50]$$

$$V(t) = 1200mV$$

for $t > 1.1s$

$$V(t) = 0$$

$$V(t) = \begin{cases} 0 & t \leq 0 \\ 320mV & 0 < t \leq 0.3s \\ -480mV & 0.3s < t \leq 0.6s \\ 0 & 0.6s < t \leq 0.9s \\ 1200mV & 0.9s < t \leq 1.1s \\ 0 & t > 1.1s \end{cases}$$



6.30 The current in a 4-mH inductor is given by the waveform in Fig. P6.30. Plot the voltage across the inductor.

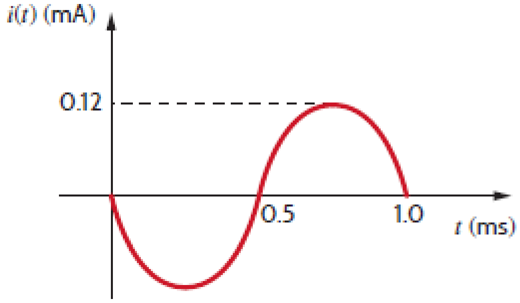


Figure P6.30

SOLUTION:

$i(t) = -120 \sin \omega t \ \mu A$

$\omega = \frac{2\pi}{T}$

$T = 1 ms$

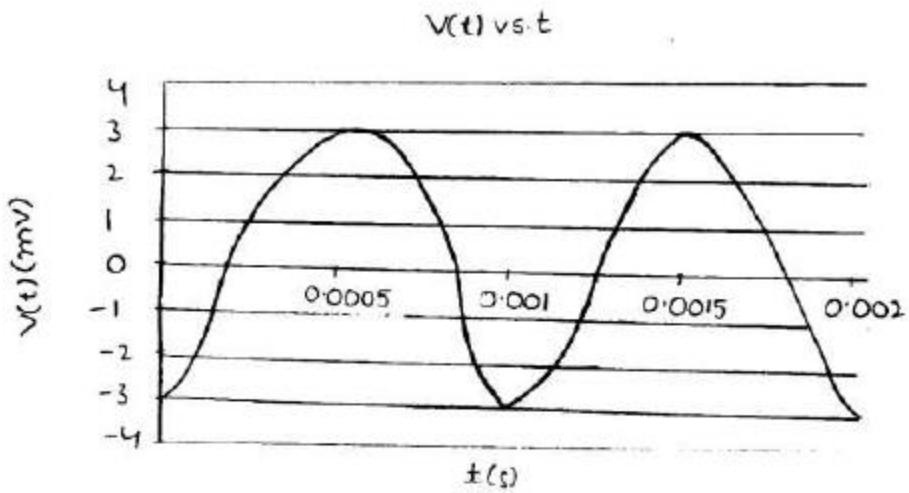
$\omega = 2000\pi \ rad/s$

$i(t) = -120 \sin 2000\pi t \ \mu A$

$v(t) = L \frac{di(t)}{dt}$

$v(t) = 4m \left[-120\mu (2000\pi) \cos 2000\pi t \right]$

$v(t) = -3.02 \cos 2000\pi t \ mV$


V(t) vs. t
4
3
2
1
0
-1
-2
-3
-4
V(t) (mV)
0.0005
0.001
0.0015
0.002
t (s)

6.31 Find the possible capacitance range of the following capacitors.

(a) 0.068 μF with a tolerance of 10%.

(b) 120 pF with a tolerance of 20%.

(c) 39 μF with a tolerance of 20%.

SOLUTION:

(a) $C = 0.0068\ \mu F$ with 10% tolerance

Range:

$$61.2\,nF \le C \le 74.8\,nF$$

$$6.12\,n \le C \le 7.48\,n$$

(b) $C = 120\,pF$ with 20% tolerance

Range:

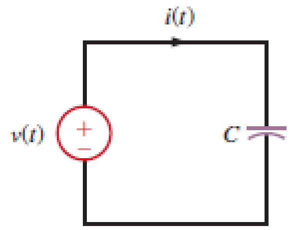
$$96\,pF \le C \le 144\,pF$$

(c) $C = 39\,\mu F$ with 20% tolerance

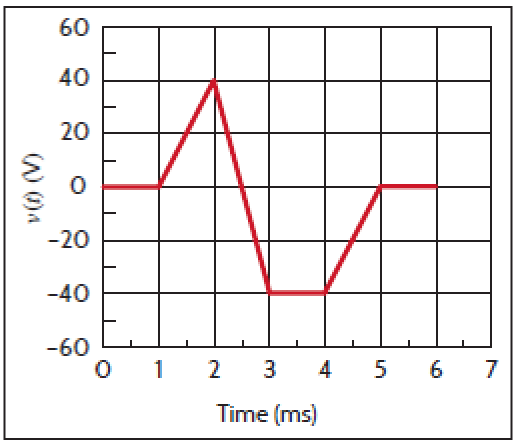
Range:

$$31.2\,\mu F \le C \le 46.8\,\mu F$$

6.32 The capacitor in Fig. P6.32 (a) is 53 nF with a tolerance of 10%. Given the voltage waveform in Fig. 6.32 (b), find the current $i(t)$ for the minimum and maximum capacitor values.



(a)



(b)

Figure P6.32

SOLUTION:

$$i(t) = C\frac{dv}{dt}$$

$$\frac{dv}{dt} = \begin{cases} 0 \text{ V/s} & 0s \le t < 1s \\ 4\times10^{4} \text{ V/s} & 1s \le t < 2s \\ -8\times10^{4} \text{ V/s} & 2s \le t < 3s \\ 0 \text{ V/s} & 3s \le t < 4s \end{cases}$$

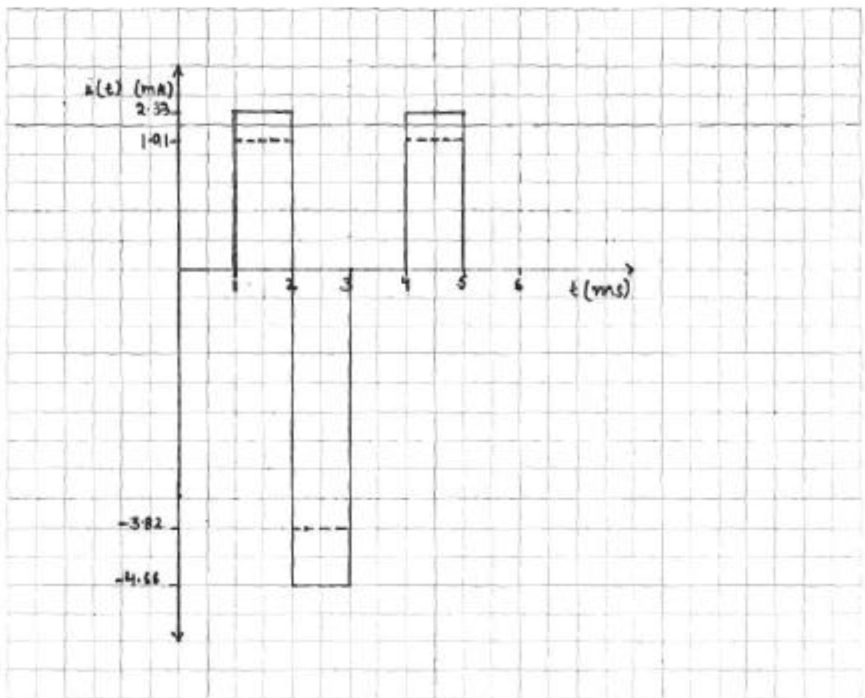
$$4\times10^4 \text{ V/s} \qquad 4s \le t < 5s$$
$$0 \text{ V/s} \qquad 5s \le t < 6s$$

$$C_{max} = 53 + 5.3 = 58.3 \text{ nF}$$

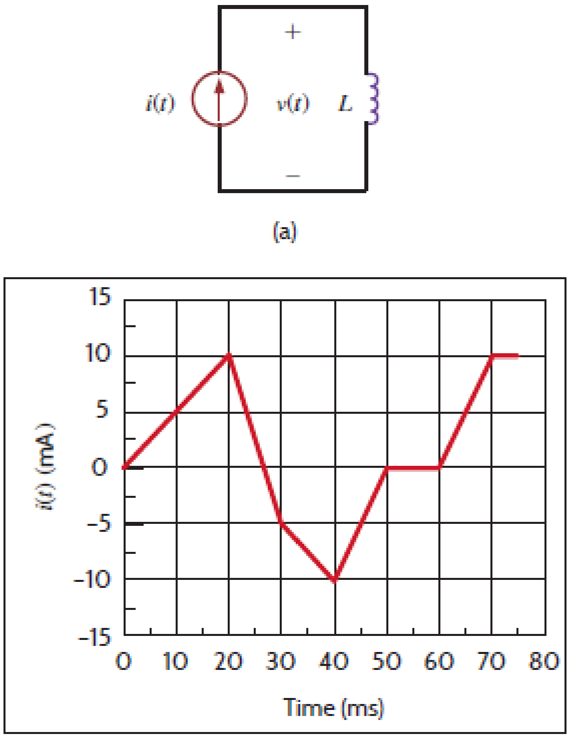
$$C_{min} = 53 - 5.3 = 47.7 \text{ nF}$$

$$i(t)\,[max] = \begin{cases} 0 \text{ mA} & 0 \le t < 1s \\ 2.33 \text{ mA} & 1s \le t < 2s \\ -4.66 \text{ mA} & 2s \le t < 3s \\ 0 \text{ mA} & 3s \le t < 4s \\ 2.33 \text{ mA} & 4s \le t < 5s \\ 0 \text{ mA} & 5s \le t < 6s \end{cases}$$

$$i(t)\,[min] = \begin{cases} 0 \text{ mA} & 0 \le t < 1s \\ 1.91 \text{ mA} & 1s \le t < 2s \\ -3.82 \text{ mA} & 2s \le t < 3s \\ 0 \text{ mA} & 3s \le t < 4s \\ 1.91 \text{ mA} & 4s \le t < 5s \\ 0 \text{ mA} & 5s \le t < 6s \end{cases}$$


i(t) (mA)
2.33
1.91
-3.82
-4.66
1
2
3
4
5
6
t (ms)

6.33 The inductor in Fig. P6.33a is 4.7 μH with a tolerance of 20%. Given the current waveform in Fig. 6.33b, graph the voltage $v(t)$ for the minimum and maximum inductor values.



(a)

(b)

Figure P6.33

SOLUTION:

$$V(t) = L\frac{di(t)}{dt}$$

for $0 \le t < 20ms$

$V_{max}(t) = (1.2)(4.7\mu)(\frac{1}{2}) = 2.82\mu V$

$V_{min}(t) = (0.8)(4.7\mu)(\frac{1}{2}) = 1.88\mu V$

for $20ms \le t \le 30ms$

$V_{max}(t) = (1.2)(4.7\mu)(-1.5) = -8.46\mu V$

$V_{min}(t) = (0.8)(4.7\mu)(-1.5) = -5.46\mu V.$

for $30ms \le t \le 40ms$

$V_{max}(t) = (1.2)(4.7\mu)(-0.5) = -2.82\mu V$

$V_{min}(t) = (0.8)(4.7\mu)(-0.5) = -1.88\mu V$

for $40ms \le t \le 50ms$

$V_{max}(t) = (1.2)(4.7\mu)(1) = 5.64\mu V$

$V_{min}(t) = (0.8)(4.7\mu)(1) = 3.76\mu V$

for $50ms \le t < 60ms$

$V(t) = 0V$

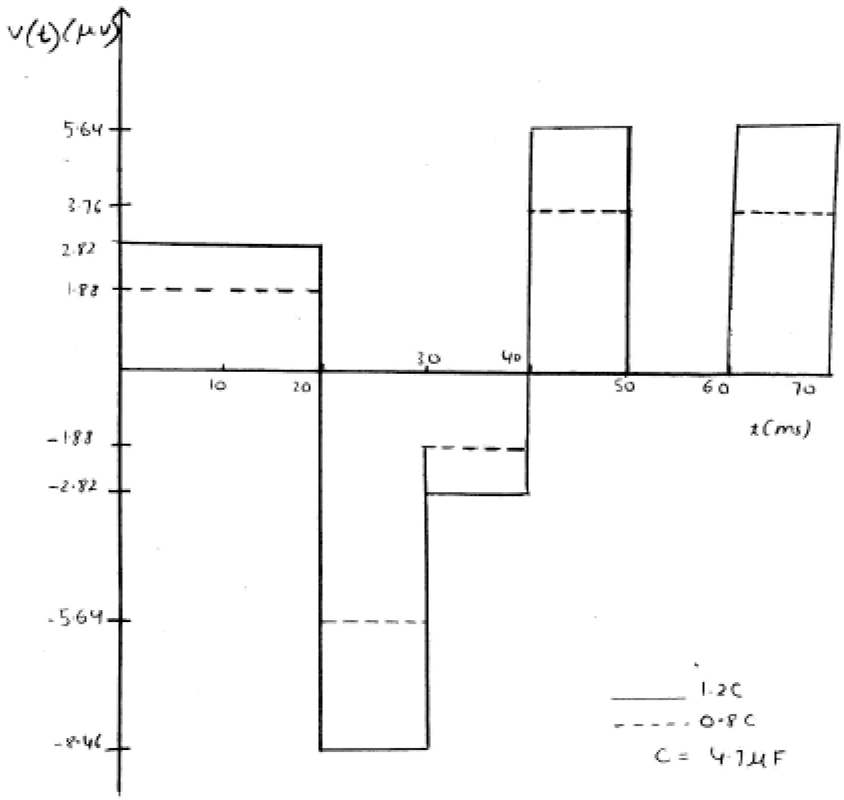
for $60ms \le t \le 70ms$

$V_{max}(t) = (1.2)(4.7\mu)(1) = 5.64\mu V$

$V_{min}(t) = (0.8)(4.7\mu)(1) = 3.76\mu V$

for $t > 70ms$

$V(t) = 0V$


v(t)(μv)
5.64
3.76
2.82
1.88
-1.88
-2.82
-5.64
-8.46
10
20
30
40
50
60
70
t(ms)
1.2C
0.8C
C = 4.7μF

6.34 Determine the voltage across each capacitor and energy across each capacitor in the given circuit.

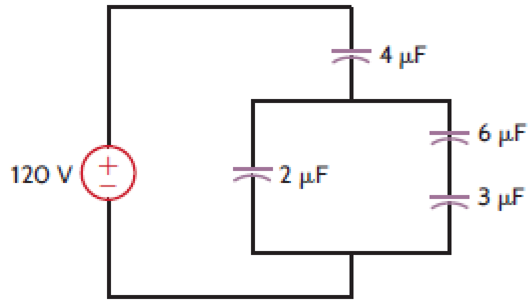


Figure P6.34

SOLUTION:

(a) 3μF is in series with 6μF 3x6/(9) = 2μF

$V_{4\mu F}$= 1/2 x 120 = 60V
$V_{2\mu F}$ = 60V
$V_{6\mu F}= \frac{3}{6+3}(60) = 20V$
$V_{3\mu F} = 60 - 20 = 40\ V$

(b) Hence $w = \frac{1}{2}Cv^2$
$W_{4\mu F} = \frac{1}{2} \times 4 \times 10^{-6} \times 3600 = 7.2mJ$
$W_{2\mu F}= \frac{1}{2} \times 2 \times 10^{-6} \times 3600 = 3.6mJ$
$W_{6\mu F}= \frac{1}{2} \times 2 \times 10^{-6} \times 400 = 1.2mJ$
$W_{3\mu}= \frac{1}{2} \times 2 \times 10^{-6} \times 1600 = 2.4mJ$

6.35 Find the value of C if the energy stored in the capacitor in Fig. P6.35 equals the energy stored in the inductor.

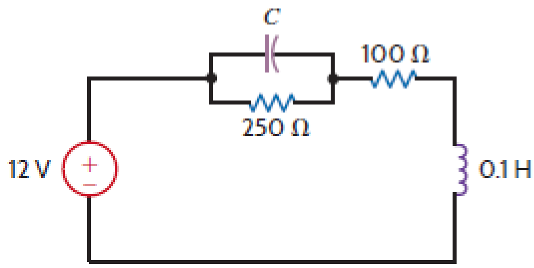
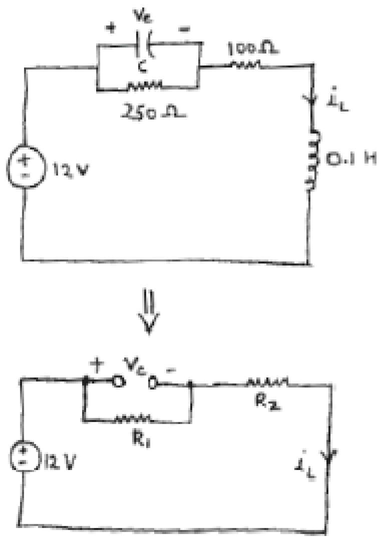


Figure P6.35

SOLUTION:



Since voltage source is constant, v_C and i_L are constant

$i_C = C\dfrac{dv_C}{dt} = 0$ and

$V_L = L\dfrac{di_L}{dt} = 0$

$i_L = \dfrac{12}{R_1+R_2} = 34.29$ mA

$V_C = \dfrac{12R_1}{R_1+R_2} = 8.57$ V

$$W_C = \frac{1}{2}CV_C^2 = W_L = \frac{1}{2}Li_L^2 \Rightarrow C = L\left(\frac{i_L}{V_C}\right)^2$$

$$\boxed{C = 1.60\,\mu F}$$

6.36 Given the network in Fig. P6.36, find (a) the power dissipated in the 3-Ω resistor and (b) the energy stored in the capacitor.

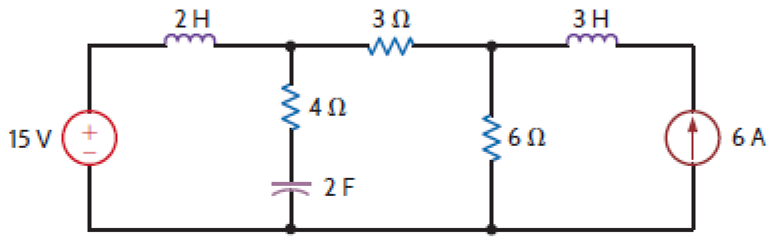
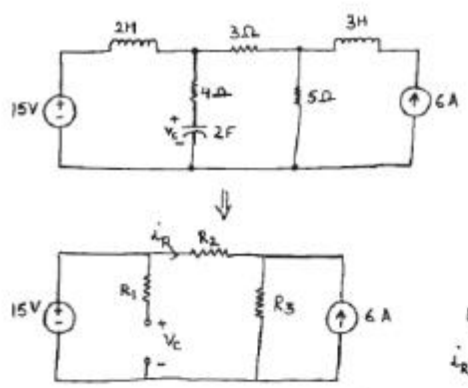


Figure P6.36

SOLUTION:



Since all sources are constant, all voltages and current are constant

$$V_L = L\frac{di_L}{dt} = 0 \quad \text{and}$$

$$i_C = C\frac{dV_C}{dt} = 0$$

$$P_{R_2} = R_2 i_R^2, \quad W_C = \frac{1}{2} C V_C^2$$

By superposition

$$i_R = 15\left[\frac{1}{R_2+R_3}\right] - 6\left[\frac{R_3}{R_2+R_3}\right]$$

$$= -1.875 \text{ A}$$

$$\boxed{P_{R_2} = 10.54 \text{ W}}$$

$$\boxed{W_C = 225 \text{ J}}$$

6.37 Calculate the energy stored in the inductor in the circuit shown in Fig. P6.37.

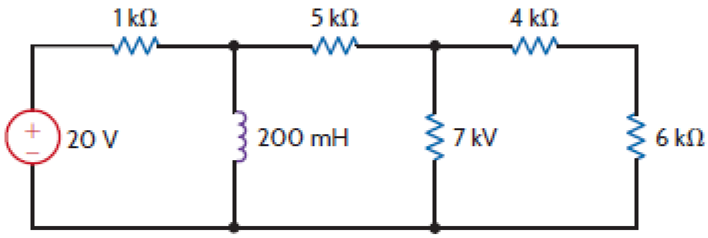
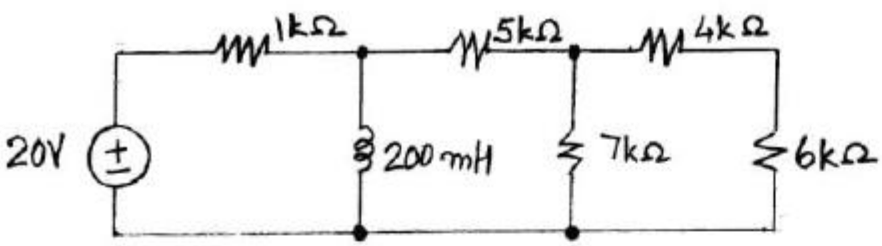
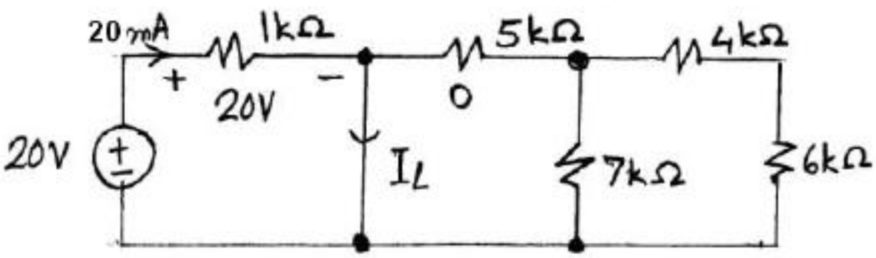


Figure P6.37

SOLUTION:





$$I_L = 20\ mA$$

$$W_L = \frac{1}{2} L I_L^2 = \frac{1}{2}(200\times10^{-3})(20\times10^{-3})^2$$

$$= 40\ \mu J$$

6.38 Calculate the energy stored in both the inductor and the capacitor shown in Fig. P6.38.

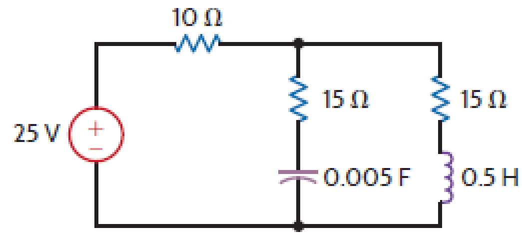
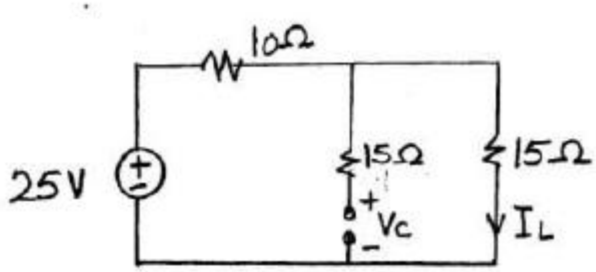


Figure P6.38

SOLUTION:



$$I_L = \frac{25}{10+15} = 1\,A$$

$$V_c = 15\,I_L = 15\,V$$

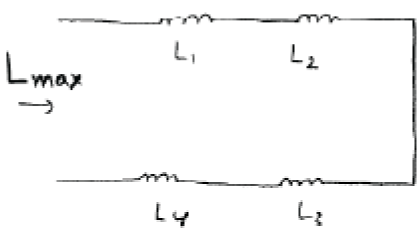
$$\omega_L = \frac{1}{2}LI_L^2 = \frac{1}{2}(0.5)(1)^2 = 0.25\,J$$

$$\omega_c = \frac{1}{2}CV_c^2 = \frac{1}{2}(0.005)(15)^2 = 0.5625\,J$$

6.39 Given four 4-mH inductors, determine the maximum and minimum values of inductance that can be obtained by interconnecting the inductors in series/parallel combinations.

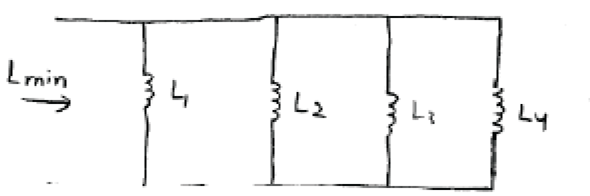
SOLUTION:

$$L_1 = L_2 = L_3 = L_4 = 4mH$$



$$L_{max} = L_1 + L_2 + L_3 + L_4$$

$$L_{max} = 16mH$$



$$L_{min} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \frac{1}{L_4}}$$

$$L_{min} = 1mH$$

6.40 Find the equivalent capacitance in Fig. P6.40.

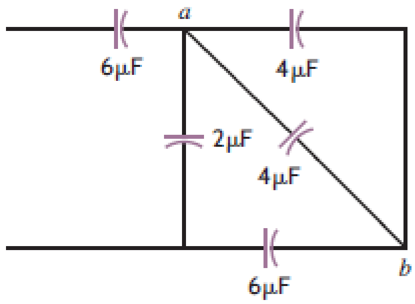
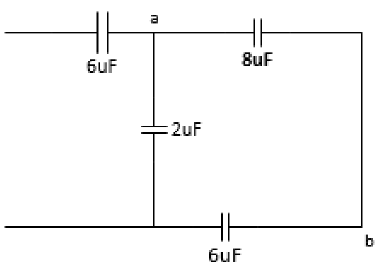
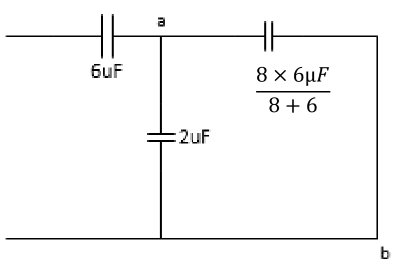


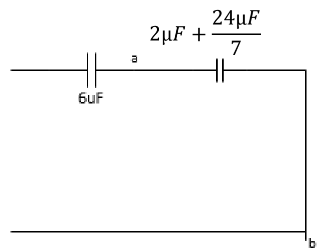
Figure P6.40

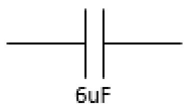
SOLUTION:

The equivalent capacitor can be evaluated as following









So the $C_{equivalent}$= 2.85μF

6.41 Find C_T in the network shown in Fig. P6.41.

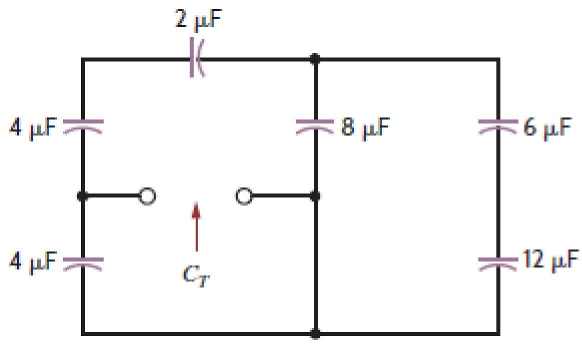
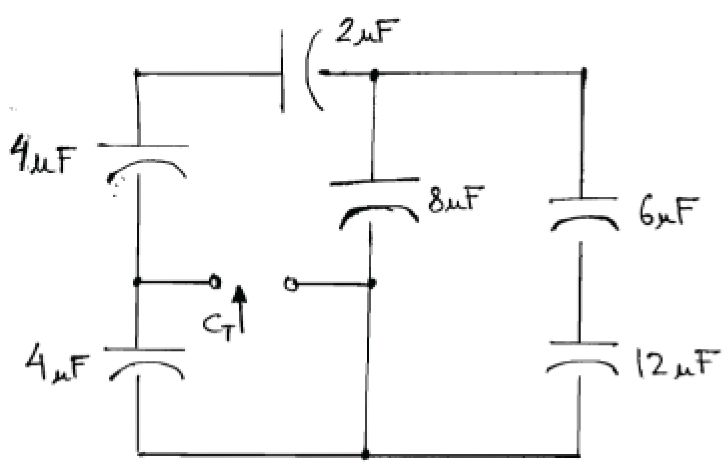
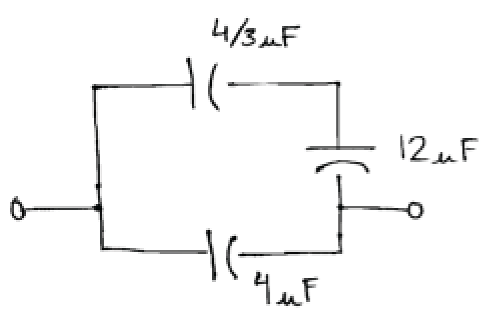


Figure P6.41

SOLUTION:





$$\frac{1}{C_s} = \frac{1}{4/3} + \frac{1}{12} = \frac{10}{12}$$

$$\Rightarrow \frac{1}{C_s} = \frac{10}{12}$$

$$\Rightarrow C_s = \frac{12}{10} = \frac{6}{5}\,\mu F$$

$$\therefore \quad C_T = \frac{6}{5} + 4 = \frac{6+20}{5} = \frac{26}{5}\,\mu F$$

6.42 Find the equivalent capacitance in the circuit in Fig. P6.42.

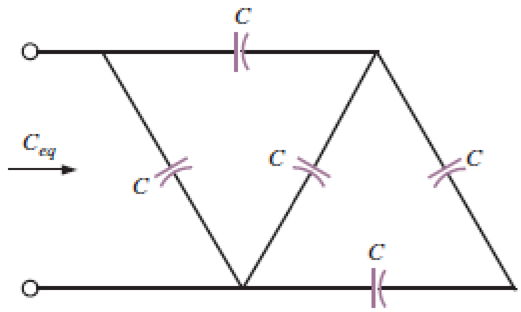


Figure P6.42

SOLUTION:

C in series with C = C/(2)

C/2 in parallel with C = 3C/2

$\frac{3}{2}$in series with C =$\frac{C\times\frac{3C}{2}}{5\frac{C}{2}}$

$\frac{3C}{2}$In parallel with C = C+$\frac{3C}{5}$

$$= 1.6\ C$$

6.43 Determine the value of C_T in the circuit in Fig. P6.43.

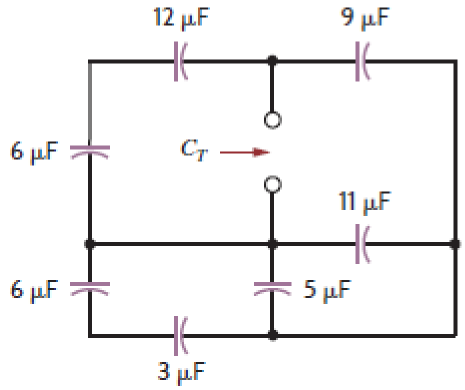
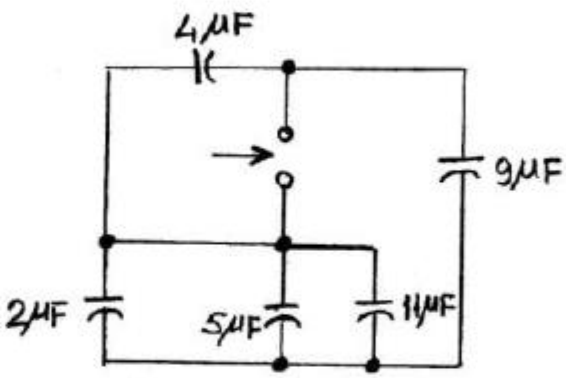
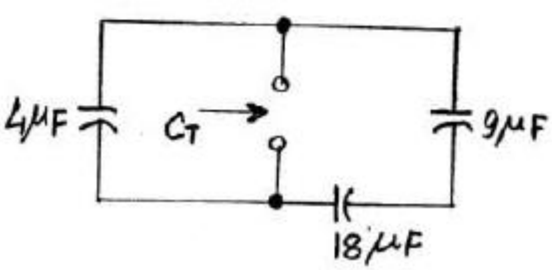


Figure P6.43

SOLUTION:





$C_T = 4\mu F + 6\mu F = 10\mu F$

6.44 Find the C_{eq} between the terminals A and B. Values of all capacitance are in μF.

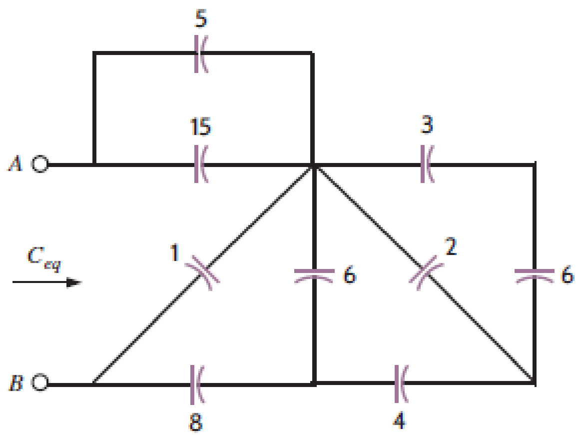
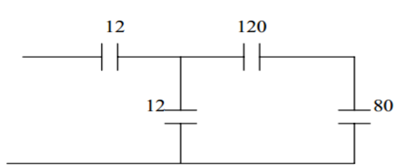


Figure P6.44

SOLUTION:

We combine 10, 20, and 30μF capacitors in parallel to have 60μF. Then 60μF capacitor in series with another 60μF capacitor gives 30μF. 30 + 50 = 80μF, 80 + 40 = 120μF the circuit is reduced to that shown below.



120-μF capacitor in series with 80μF gives (80x120)/200 = 48μF
48 + 12 = 60μF
60μF capacitor in series with 12μF gives
(60x12)/72 = **10μF**

6.45 Find C_T in the circuit in Fig. P6.45 if all capacitors are 6 μF.

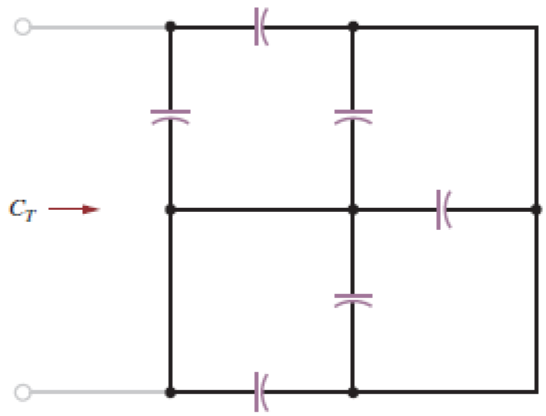
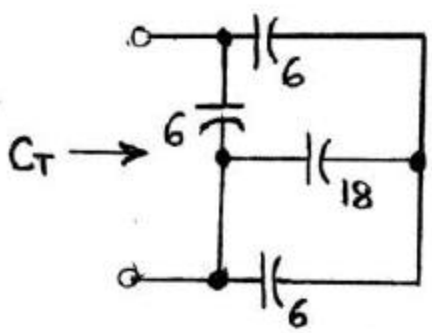
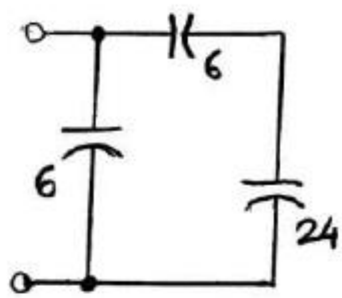
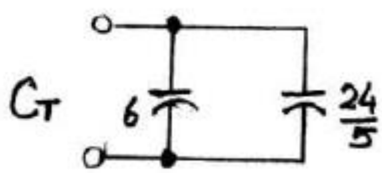


Figure P6.45

SOLUTION:







$$\frac{24}{5} + 6 = \frac{24}{5} + \frac{30}{5} = \frac{54}{5}\,\mu F = C_T$$

6.46 Determine the value of C_T in the circuit in Fig. P6.46 if all capacitors are 12 µF.

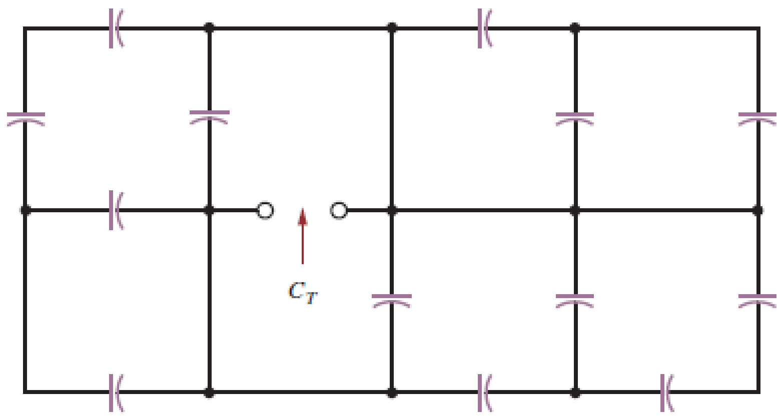
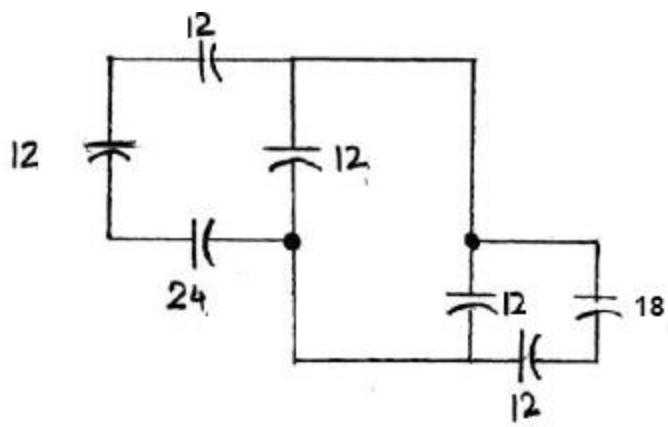
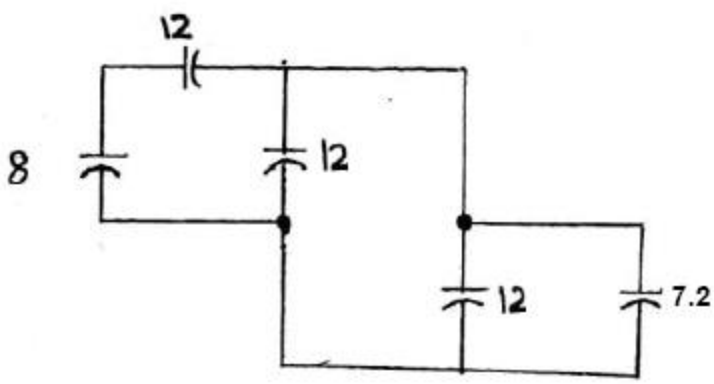
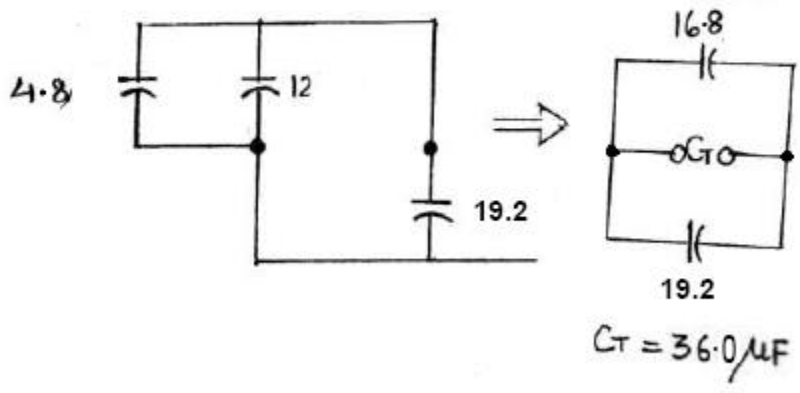


Figure P6.46

SOLUTION:






4.8
12
19.2
16.8
C_T
19.2
$C_T = 36.0 \mu F$

6.47 If the total capacitance of the network in Fig. P6.47 is 15 µF, find the value of C.

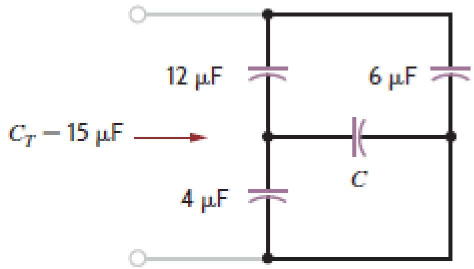
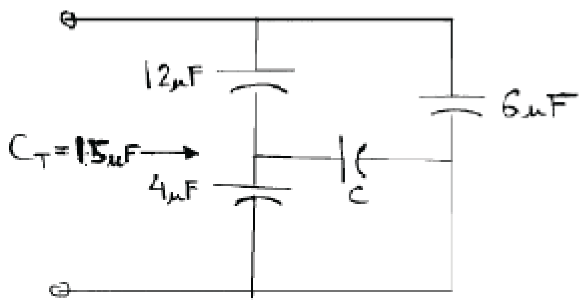


Figure P6.47

SOLUTION:



$$\frac{12(C+4)}{12+C+4} + 6 = 15$$

$$\Rightarrow \frac{12C+48}{C+16} = 9$$

$$\Rightarrow 12C+48 = 9C+144$$

$$\Rightarrow 3C = 96$$

$$\Rightarrow C = 32\ \mu F$$

$\therefore$ $C = 32\ \mu F$

6.48 Find the value of C in Fig. 6.48.

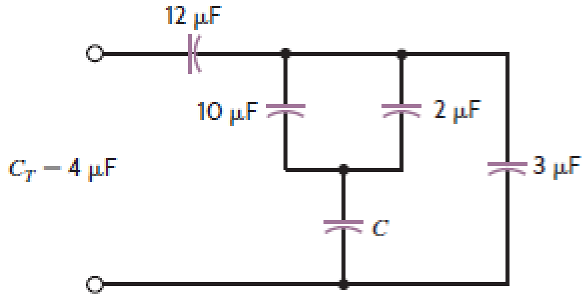


Figure P6.48

SOLUTION:

$$4 = \frac{12\left(\frac{12C}{12+C} + 3\right)}{12 + \frac{12C}{12+C} + 3}$$

$$4 = \frac{12(12C + 36 + 3C)}{12(12+C) + 12C + 36 + 3C}$$

$$1 = \frac{45C + 108}{180 + 27C}$$

$$180 + 27C = 45C + 108$$

$$72 = 18C$$

$$C = 4\,\mu F$$

6.49 Find the total capacitance C_T shown in the network in Fig. P6.49.

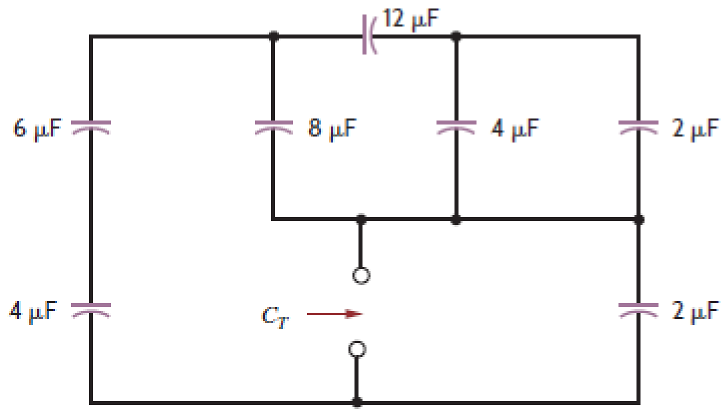
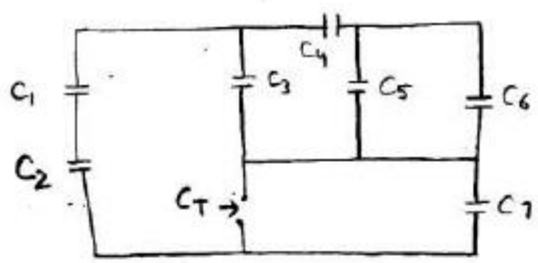


Figure P6.49

SOLUTION:



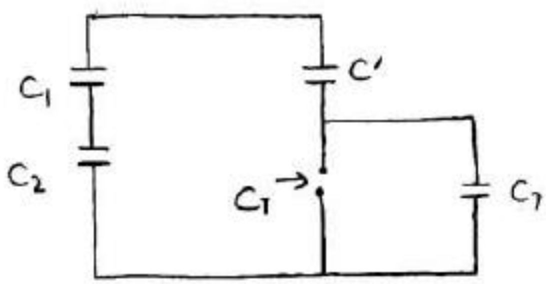
$C_1 = 6\mu F$, $C_2 = 4\mu F$, $C_3 = 8\mu F$,
$C_4 = 12\mu F$, $C_5 = 4\mu F$, $C_6 = 2\mu F$
and $C_7 = 2\mu F$

$$C' = \frac{(C_5 + C_6)(C_4)}{C_4 + C_5 + C_6} + C_3$$

$$C' = \frac{(4\mu + 2\mu)(12\mu)}{12\mu + 4\mu + 2\mu} + 8\mu$$

$$C' = 12\mu F$$

Redraw:



$$\frac{1}{C''} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C'}$$

$$\frac{1}{C''} = \frac{1}{6\mu} + \frac{1}{4\mu} + \frac{1}{12\mu}$$

$$\frac{1}{C''} = 500{,}000$$

$$C'' = 2\,\mu F$$

$$C_T = C'' + C_7 = 2\mu + 2\mu$$

$$C_T = 4\,\mu F$$

6.50 In the network in Fig. P6.50 below, find the capacitance C_T if

(a) the switch is open and

(b) the switch is closed.

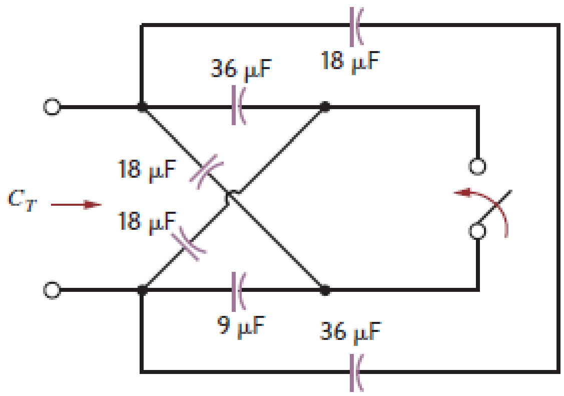
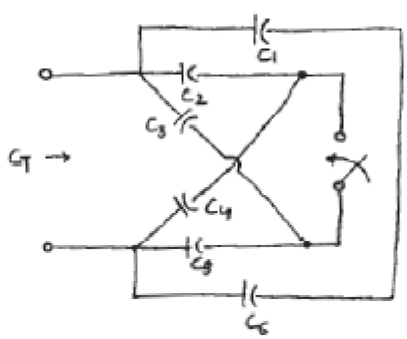
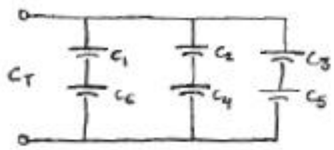


Figure P6.50

SOLUTION:



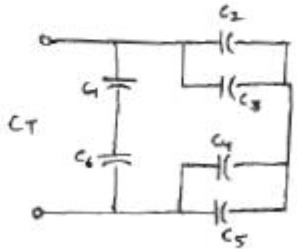
(a)



$$C_T = \frac{C_1C_6}{C_1+C_6} + \frac{C_2C_4}{C_2+C_4} + \frac{C_3C_5}{C_3+C_5}$$

$$\boxed{C_T = 30\,\mu F}$$

(b)



$$C_x = \frac{(C_2+C_3)(C_4+C_5)}{C_2+C_3+C_4+C_5}$$

$$= 18\,\mu F$$

$$C_T = C_x + \frac{C_1C_6}{C_1+C_6}$$

$$\boxed{C_T = 30\,\mu F}$$

6.51 Select the value of C to produce the desired total capacitance of $C_T = 10\ \mu F$ in the circuit in Fig. P6.51.

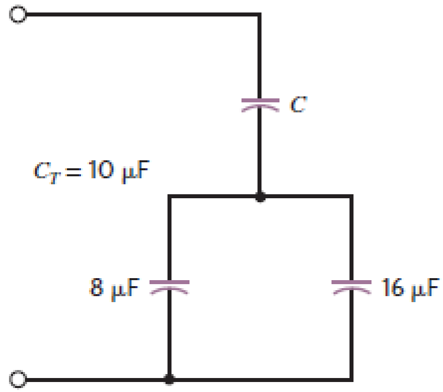
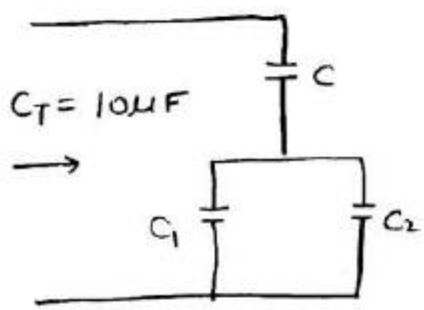


Figure P6.51

SOLUTION:



$C_1 = 8\mu F$ and $C_2 = 16\mu F$

$$C_T = \frac{(C_1 + C_2)(C)}{C_1 + C_2 + C}$$

$$C_T(C_1 + C_2) + CC_T = (C_1 + C_2)\,C$$

$$C[C_1 + C_2 - C_T] = C_T(C_1 + C_2)$$

$$C = \frac{C_T(C_1+C_2)}{C_1+C_2-C_T}$$

$$C = \frac{10\mu(8\mu+16\mu)}{8\mu+16\mu-10\mu}$$

$$C = 17.14\mu F$$

6.52 Select the value of C to produce the desired total capacitance of $C_T = 1\ \mu F$ in the circuit in Fig. P6.52.

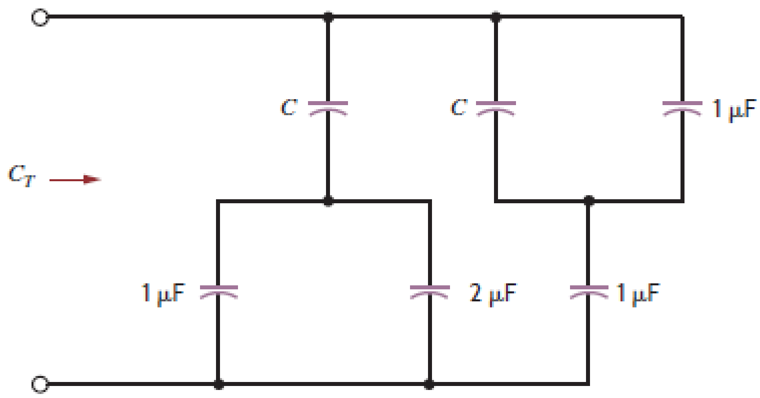
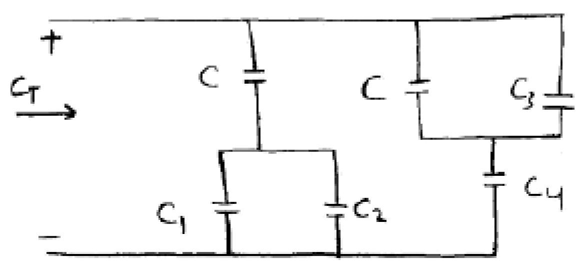


Figure P6.52

SOLUTION:



$C_1 = 1\mu F$, $C_2 = 2\mu F$, $C_3 = 1\mu F$, $C_4 = 1\mu F$

$$C_x = \frac{(C+C_3)(C_4)}{C+C_3+C_4}$$

$$C_y = \frac{(C_1+C_2)(C)}{C+C_1+C_2}$$

$$C_T = C_x + C_y$$

$$C_T = \frac{(C+C_3)(C_4)}{C+C_3+C_4} + \frac{(C_1+C_2)(C)}{C+C_1+C_2}$$

$$1\mu = \frac{(C+1\mu)(1\mu)}{C+1\mu+1\mu} + \frac{(1\mu+2\mu)(C)}{C+1\mu+2\mu}$$

$$1\mu = \frac{(C+1\mu)(1\mu)}{C+2\mu} + \frac{(3\mu)(C)}{C+3\mu}$$

$$1 = \frac{C+1}{C+2} + \frac{3C}{C+3}$$

Note: C is in μF

$$C+2 = C+1 + \frac{3C(C+2)}{C+3}$$

$$(C+2)(C+3) = (C+1)(C+3) + 3C(C+2)$$

$$C^2 + 5C + 6 = C^2 + 4C + 3 + 3C^2 + 6C$$

$$3C^2 + 5C - 3 = 0$$

$$C = \frac{-5 \pm \sqrt{25-4(3)(-3)}}{2(3)}$$

$$C = \frac{-5 \pm 7.81}{6}$$

$$C = 468 \text{ nF}$$

6.53 The two capacitors are connected as shown in Fig. P6.53. Now a 2 μF capacitor is connected across C_1 and the voltage now measured across C_2 is 3 V. Find the value of C_1 and C_2.

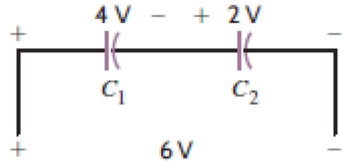
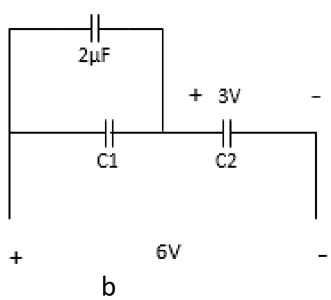


Figure P6.53

SOLUTION:

The has been modified as following



From the figure in P6.53,

$4C_1=2C_2$

From figure above

$3(C_1+C_2) = 3C_2$

Solving for C_1 and C_2.yields

$C_1= 2\mu F$

$C_2 = 4\mu F$

6.54 The three capacitors shown in Fig. P6.54 have been connected for some time and have reached their present values. Find (a) V_1 and (b) V_2.

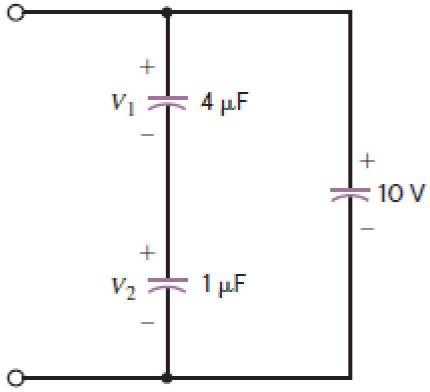
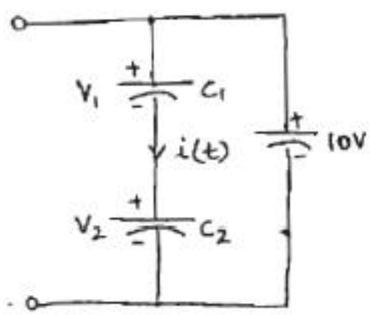


Figure P6.54

SOLUTION:



$C_1 = 4\mu F$, $C_2 = 1\mu F$

$V_1 = \frac{1}{C_1}\int i\,dt \quad V_2 = \frac{1}{C_2}\int i\,dt$

$\frac{V_1}{V_2} = \frac{C_2}{C_1} = \frac{1}{4}$

$V_1 + V_2 = 10V$

$\boxed{V_1 = 2V}$

$\boxed{V_2 = 8V}$

6.55 Determine the inductance at terminals A-B in the network in Fig. P6.55.

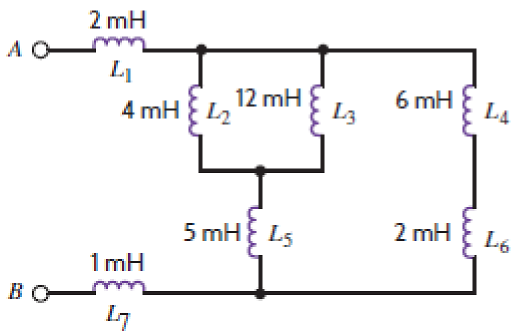
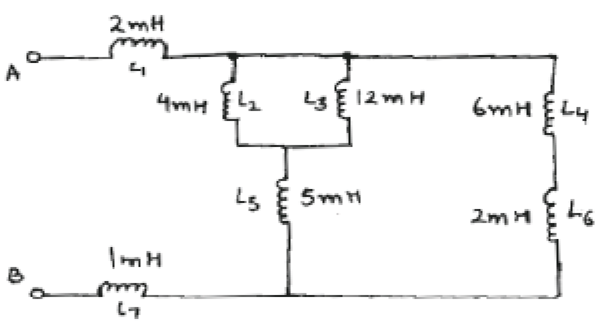


Figure P6.55

SOLUTION:



$$L_x = L_5 + \frac{L_2 L_3}{L_2 + L_3}$$

$$= 8\ mH$$

$$L_y = L_4 + L_6$$

$$= 8\ mH$$

$$L_{AB} = L_1 + L_7 + \frac{L_x L_y}{L_x + L_y}$$

$$\boxed{L_{AB} = 9\ mH}$$

6.56 Find the total inductance at the terminals of the network in Fig. P6.56.

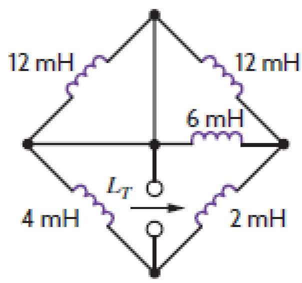
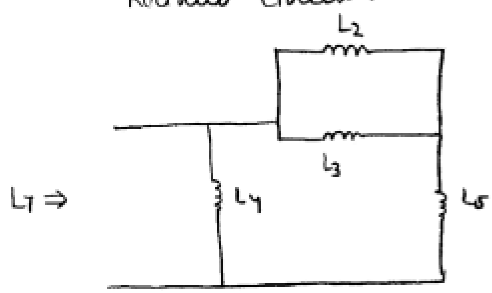


Figure P6.56

SOLUTION:

Redraw circuit :



$L_2 = 12\,mH$, $L_3 = 6\,mH$, $L_4 = 4\,mH$, and $L_5 = 2\,mH$

note: $L_1 = 12\,mH$ was shorted.

$$L_T = \left[(L_2 \,||\, L_3) + L_5 \right] || L_4$$

$$L_T = \left[(12m \,||\, 6m) + 2m \right] || 4m$$

$$L_T = \left[\frac{12m(6m)}{12m + 6m} + 2m \right] || 4m$$

$$L_T = 6m \,||\, 4m$$

$$L_T = 2.4\,mH$$

6.57 Find L_T in the circuit in Fig. P6.57.

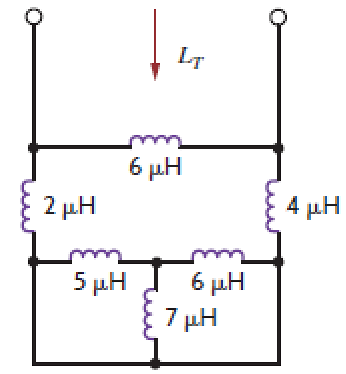
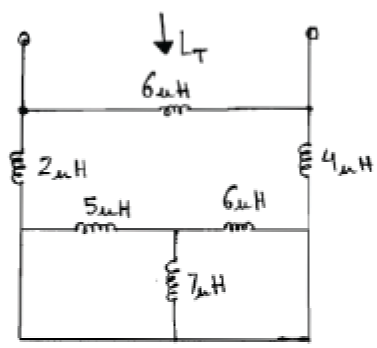
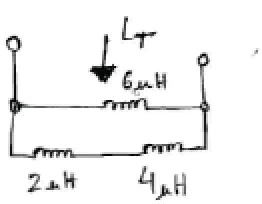


Figure P6.57

SOLUTION:





$L_T = 3\mu H$

6.58 Find L_T in the circuit in Fig. P6.58.

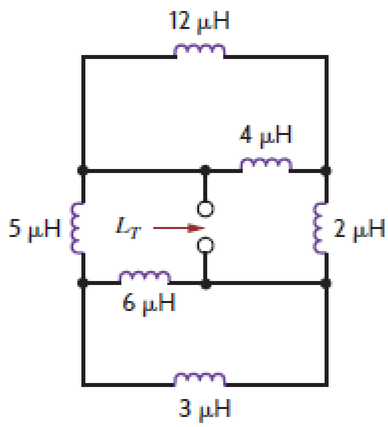
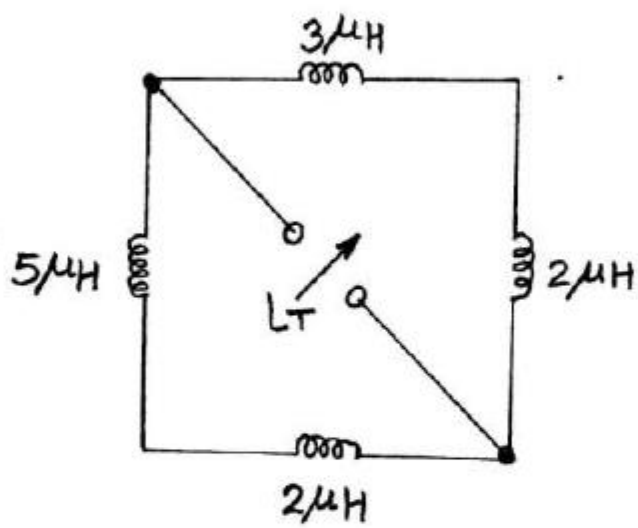


Figure P6.58

SOLUTION:



$$L_T = 7\mu H // 5\mu H$$
$$= \frac{35}{12}\ \mu H$$

6.59 Find L_T in the circuit in Fig. P6.59. All inductors are 12 μH.

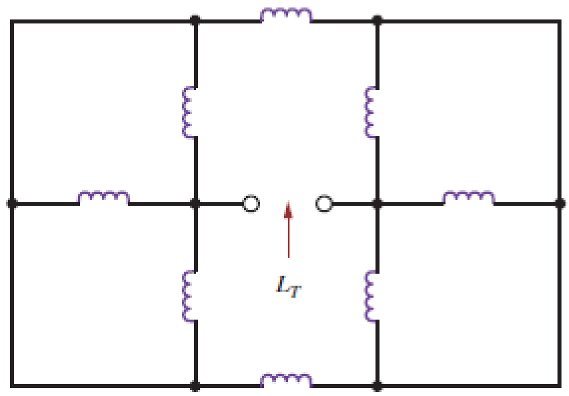
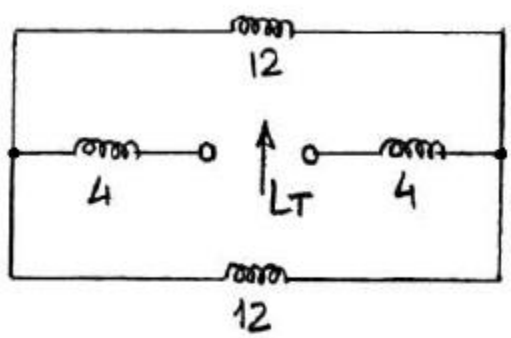


Figure P6.59

SOLUTION:



$L_T = 14\,\mu H$

6.60 Find the inductance between the terminals A and B.

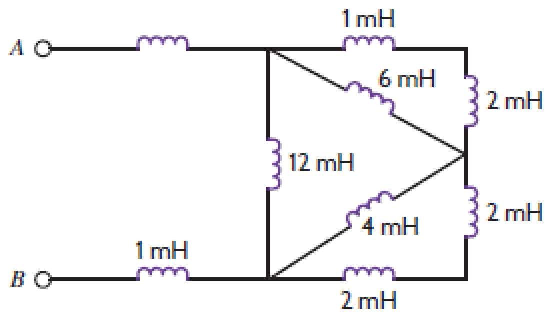
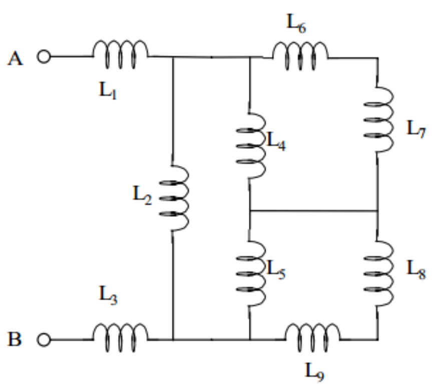


Figure P6.60

SOLUTION:

We can simplify the above figure by as following



L_1=L_3=L_6= 1 mH

L_2= 12 mH, L_4 = 6mH

L_5=4 mH, L_7= L_8= L_9=2 mH

L_{eq1} = L_6 + L_7 = 3 Mh

L_{eq2} = $L_4L_{wq1}/(L_4 + L_{eq1})$ = 2Mh

L_{eq3} = L_8 + L_9 + 4 Mh

L_{eq4} = $L_{eq3}L_5/(L_{eq3}+L_{eq5})$= 2Mh

$L_{eq5} = \frac{L2(Leq2+Leq4)}{L2+Leq2+Leq4} = 5mH$

$L_{AB} = 5$ mH

6.61 Find L_T in the circuit in Fig. P6.61.

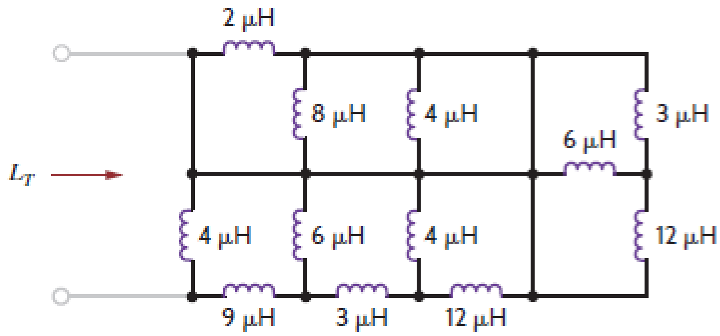
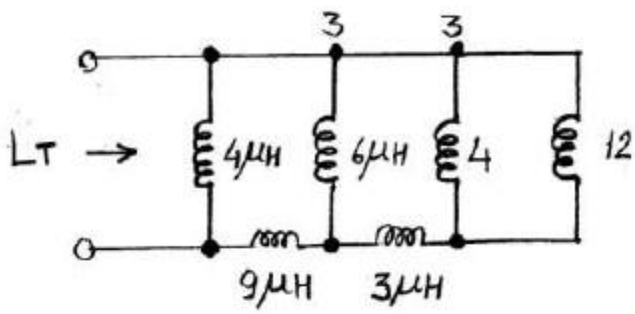


Figure P6.61

SOLUTION:



$$L_T = \frac{4 \times 12}{16} = 3\mu H$$

6.62 Find the value of inductance in the network in Fig. P6.62 so that the value of L_{eq} will be 2 mH.

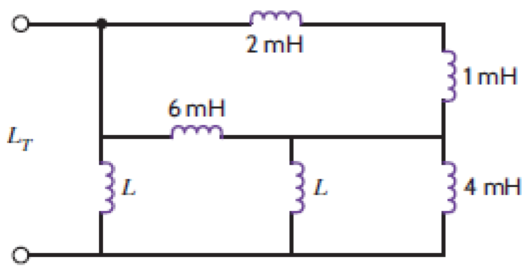
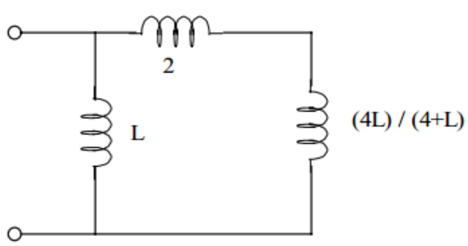


Figure P6.62

SOLUTION:

The figure in the question can be drawn as follows



$$\frac{\left(\frac{4L}{4+L}+2\right)L}{\frac{4L}{4+L}+2+L}=2$$

$$6L^2+8L=2L^2+20L+16$$

$$(l+4)(L+1)=0$$

$$L=4mH$$

6.63 If the total inductance, L_T, of the network in Fig. P6.63 is 6 μH, find the value of L.

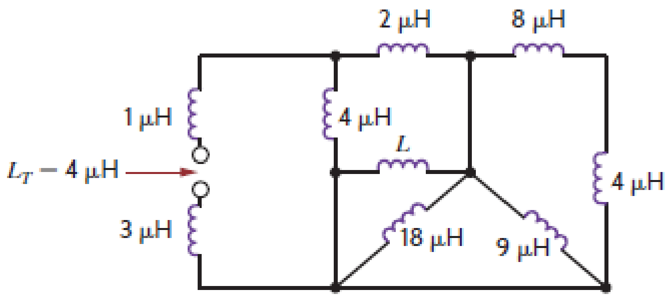
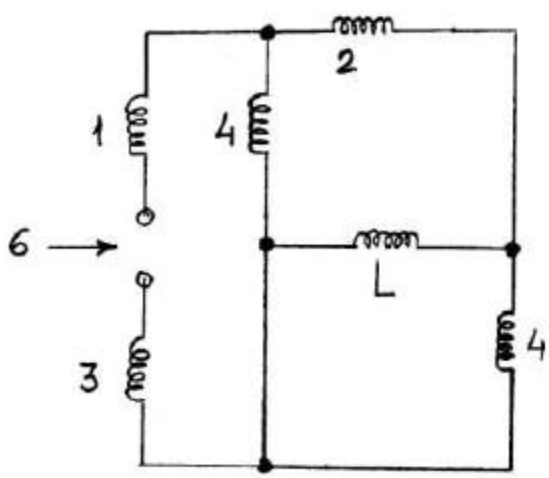


Figure P6.63

SOLUTION:



$$6 = 4 + \frac{\left[\left(\frac{4L}{4+L}\right)+2\right]4}{\frac{4L}{4+L}+2+4}$$

$$L = 4\,\mu H$$

6.64 Find L_T in the network in Fig. P6.64 (a) with the switch open and (b) with the switch closed. All inductors are 12 mH.

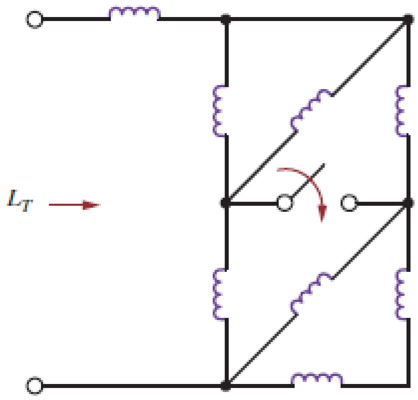
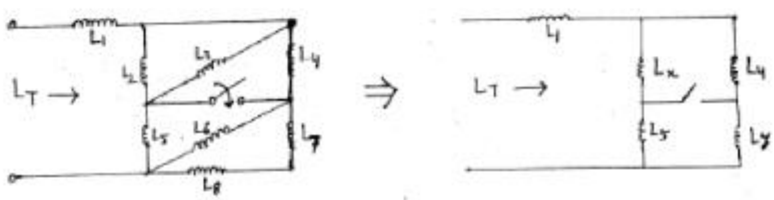


Figure P6.64

SOLUTION:



$$L_x = \frac{L_2 L_3}{L_2 + L_3} = 6\text{ mH} \qquad L_y = \frac{L_6 (L_7 + L_8)}{L_6 + L_7 + L_8} = 8\text{ mH}$$

(a) $$L_T = L_1 + \frac{(L_x + L_5)(L_4 + L_y)}{L_x + L_5 + L_4 + L_y} = 21.47\text{ mH}$$

$\boxed{L_T = 21.5\text{ mH}}$

(b) $$L_T = L_1 + \frac{L_x L_4}{L_x + L_4} + \frac{L_5 L_y}{L_5 + L_y} = 20.8\text{ mH}$$

$$\boxed{L_T = 20.8\text{ mH}}$$

6.65 Given the network shown in Fig. P6.65, find (a) the equivalent inductance at terminals *A*-*B* with terminals *C*-*D* short circuited, and (b) the equivalent inductance at terminals *C*-*D* with terminals A-B open circuited.

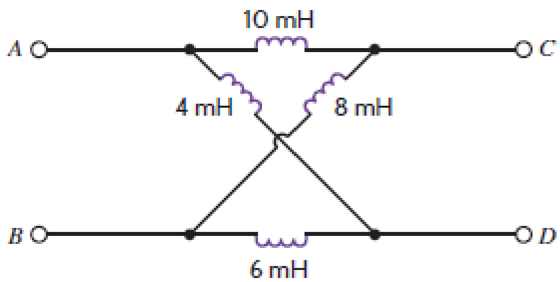
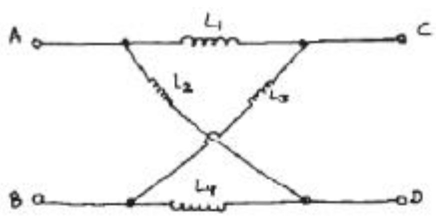
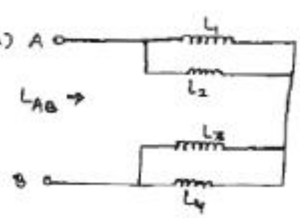


Figure P6.65

SOLUTION:

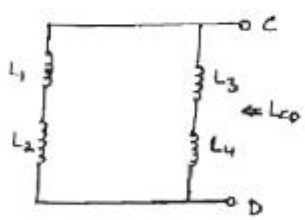


(a)



$$L_{AB} = \frac{L_1 L_2}{L_1+L_2} + \frac{L_3 L_4}{L_3+L_4}$$

$$\boxed{L_{AB} = 6.29 \text{ mH}}$$



$$L_{CD} = \frac{(L_1+L_2)(L_3+L_4)}{L_1+L_2+L_3+L_4}$$

$$\boxed{L_{CD} = 7 \text{ mH}}$$

6.66 A inductor of 100 mH is connected in parallel with a 2-kΩ resistor. The current through the inductor is $i(t) = 50e^{-400t}$ mA. (a) Find the voltage across the inductor. (b) Find the voltage across the resistor. (c) Calculate the energy in the inductor at $t = 0$.

SOLUTION:

(a) $V_L = L\frac{di}{dt} = 100 \times 10^{-3}(-400) \times 50 \times 10^{-3}e^{-400t}$
$= -2e^{-400t}\ V)$

(b) R and L are parallel to each other so $V_R = V_L$

(C) $w = \frac{1}{2}Li^2 = 0.5 \times 100 \times 10^{-3}(0.05)^2$
$= 125\mu J$

6.67 If the capacitors shown in Fig. P6.67 have been connected for some time and have reached their present values, determine (a) the voltage V_0 and (b) the total energy stored in the capacitors.

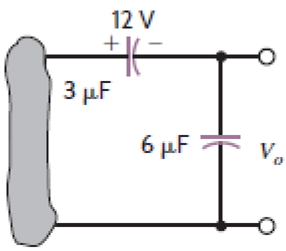


Figure P6.67

SOLUTION:

(a) $q = CV = (3\times10^{-6})(12) = (6\times10^{-6})V_o$

$$V_o = 6\,V$$

(b) $W(t) = \frac{1}{2}\left[3\times10^{-6}(12)^2 + 6\times10^{-6}(6)^2\right]$

$$= \frac{1}{2}\left[3(144) + 6(36)\right]10^{-6}$$

$$= \frac{1}{2}\left[432 + 216\right]10^{-6}$$

$$= 324\,\mu J$$

6.68 If the capacitors in the circuit in Fig. P6.68 have been connected for some time and have reached their present values, calculate (a) the voltage V_1 and (b) the total energy stored in the capacitors.

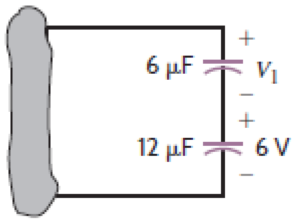


Figure P6.68

SOLUTION:

(a) $q = CV = (12\times10^{-6})(6) = (6\times10^{-6})V_1$

$V_1 = 12V$

(b) $W(t) = \frac{1}{2}\left[6\times10^{-6}(12)^2 + 12\times10^{-6}(6)^2\right]$

$= \frac{10^{-6}}{2}\left[864 + 432\right]$

$= 648\,\mu J$

6.69 If the capacitors shown in Fig. P6.69 have been connected for some time and the voltage has reached its present value, find (a) the voltages V_1 and V_2 and (b) the total energy stored in the capacitors.

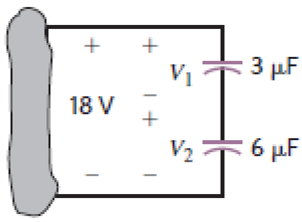


Figure P6.69

SOLUTION:

(a)

$$q = CV$$

$$q = (2\times10^{-6})(18)$$

$$= 36\times10^{-6}\ C$$

$$V_1 = \frac{36\times10^{-6}}{3\times10^{-6}} = 12V$$

$$V_2 = \frac{36\times10^{-6}}{6\times10^{-6}} = 6V$$

(b)

$$W(t) = \frac{1}{2}\left[3\times10^{-6}(12)^2 + 6\times10^{-6}(6)^2\right]$$

$$= \frac{10^{-6}}{2}\left[432 + 216\right]$$

$$= 324\,\mu J$$

6.70 For the network in Figure P6.70 choose C such that.
$v = -10 \int v_s dt$.

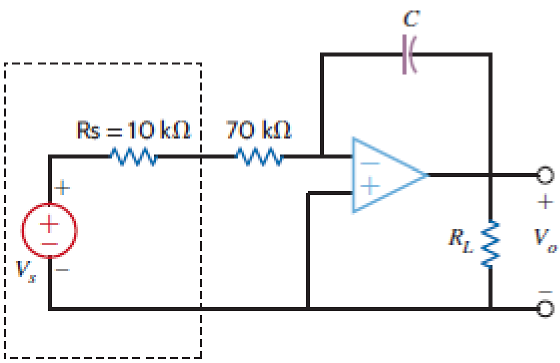


Figure P6.70

SOLUTION:

$R_s = 10K\Omega$

$$v = -10 \int v_s dt$$

$$R_{eq} = R_s + 70K = 80k$$

Considering $i_1 = i_2$ (ideal op-amp)

$$\frac{v_s - 0}{R_{eq}} = -C\frac{dv}{dt}$$

$$v_o = -\frac{1}{R_{eq}C}\int v_s dt$$

$$R_{eq}C = \frac{1}{10}$$

$$C = 1.25\mu F$$

6.71 For the network in Fig. P6.71 below, $v_{s_1}(t) = 80 \cos 324t$ V and $v_{s_2}(t) = 40 \cos 324t$ V Find $v_0(t)$.

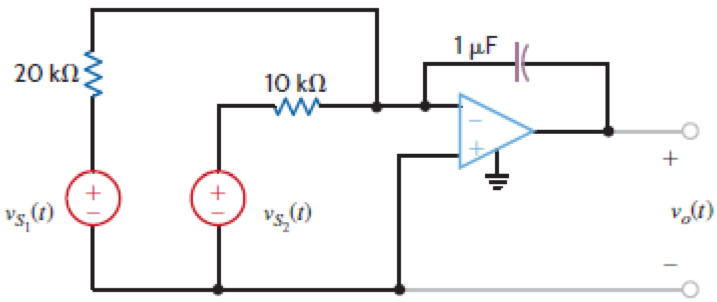
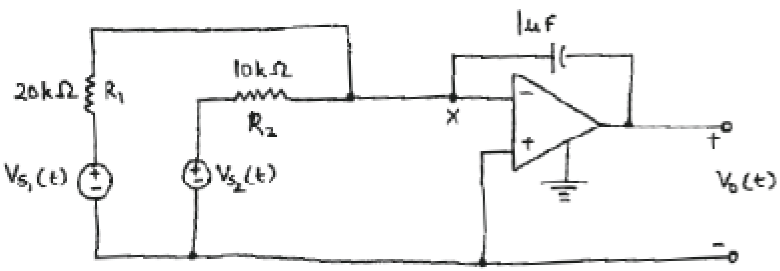


Figure P6.71

SOLUTION:



$$\frac{V_{s_1}}{R_1} + \frac{V_{s_2}}{R_2} = -C\frac{dV_o}{dt}$$

$$\therefore\ V_o(t) = -\frac{1}{C}\int\left(\frac{V_{s_1}}{R_1} + \frac{V_{s_2}}{R_2}\right)dt$$

$$\boxed{V_o(t) = -24.7 \sin 324t \text{ V}}$$

6.72 If the input to the network shown in Fig. P6.72a is given by the waveform in Fig. P6.72b, determine the output waveform $v_0(t)$ if $v_0(0) = 0$.

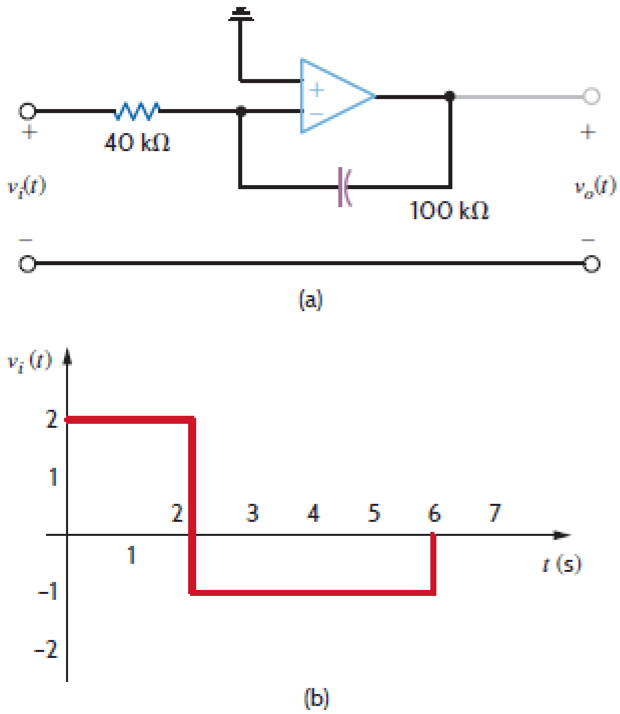


Figure P6.72

SOLUTION:

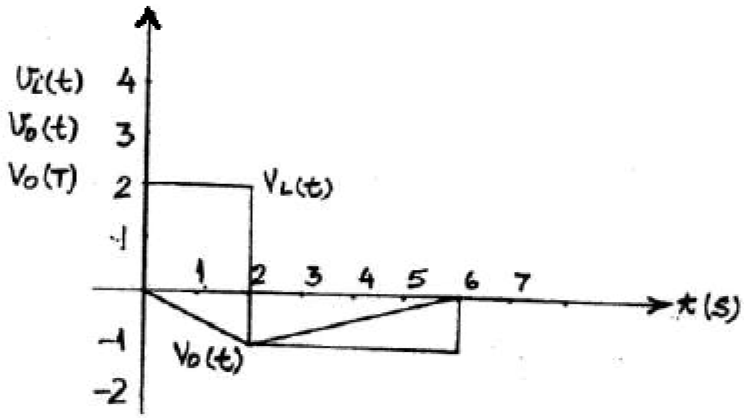
$$V_o(t) = \frac{-1}{4}\int 2\,dt = -\frac{1}{2}t \quad 0 < t < 2$$

$$t(2) = -1V$$

$$V_o(t) = -\frac{1}{4}\left[-1(t-2)\right] - 1$$

$$= \frac{1}{4}(t-2) - 1$$

$$V_o(t) = 0$$


$v_L(t)$
$v_o(t)$
$V_o(T)$
4
3
2
1
-1
-2
$V_L(t)$
$V_o(t)$
1 2 3 4 5 6 7
t (s)

6.73 The input to the network shown in Fig. P6.73a is shown in Fig. P6.73b. Derive the waveform for the output voltage $v_0(t)$ if $v_0(0) = 0$.

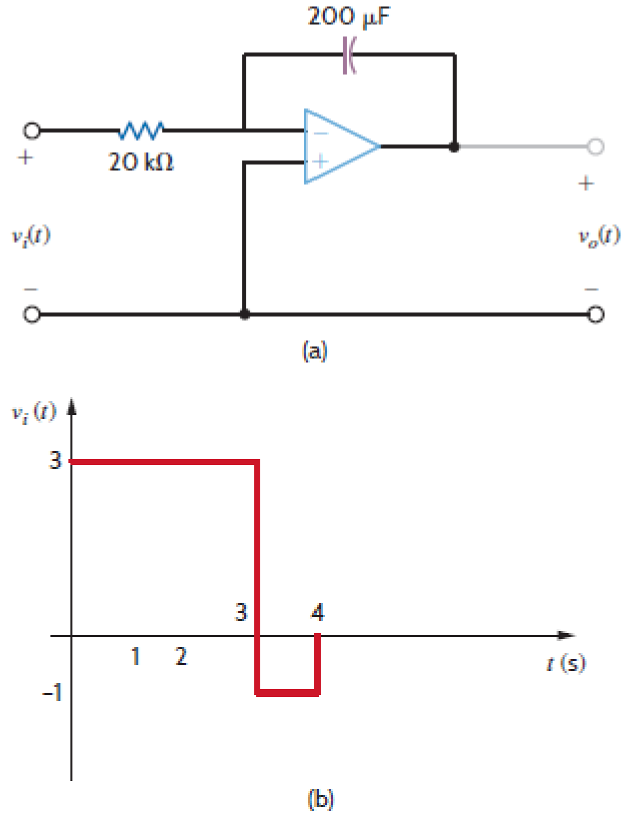


Figure P6.73

SOLUTION:

$$v_o(t) = \frac{-1}{RC}\int_0^t v_i(t)\,dt + v_o(t)$$

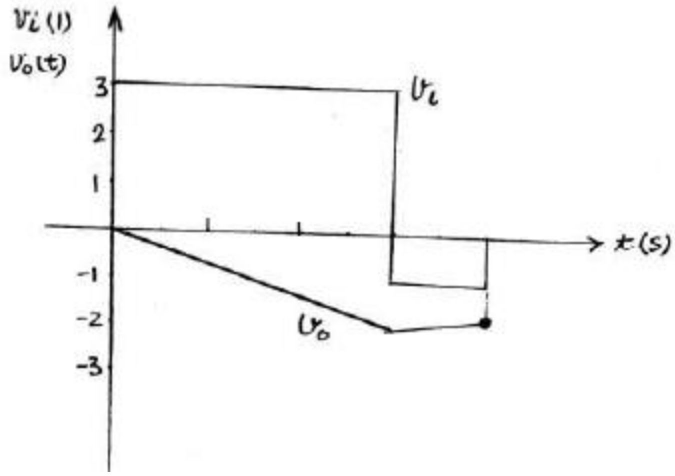
$$v_o = \frac{-1}{4}\int_0^3 3\,dt = \frac{-3}{4}t \qquad 0 < t < 3 \text{ seconds}$$

<u>At $t = 3$ seconds</u> $\quad v_o(3) = -\frac{9}{4}$ V

$$v_o(t) = -\frac{1}{4}\left[-1(t-3)\right] - \frac{9}{4} \qquad 3 < t < 4$$

$$= \frac{1}{4}(t-3) - \frac{9}{4} \qquad 3 < t < 4$$

$$v_o(4) = \frac{1}{4}(1) - \frac{9}{4} = -2\text{ V}$$


$v_i(t)$
$v_o(t)$
3
2
1
-1
-2
-3
v_i
v_o
$t(s)$

6.74 The inductors in Fig. P6.74 are initially charged and are connected to the black box at t = 0. If $i_1(0) = 4$ A, $i_2(0) = -2$ A, and $v(t)=50e^{-200t}$ mV, $t \geq 0$ find:

(a) the energy initially stored in each inductor,

(b) the total energy delivered to the black box from $t = 0$ to $t = \infty$,

(c) $i_1(t)$ and $i_2(t)$, $t \geq 0$,

(d) $i(t)$, $t \geq 0$

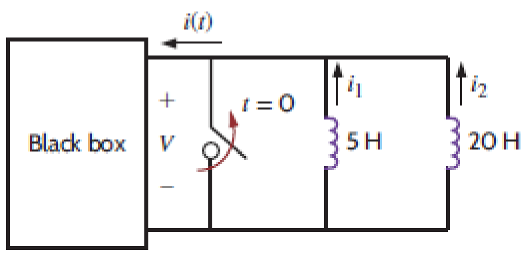


Figure P6.74

SOLUTION:

(a) $W_5 = \frac{1}{2}Li^2 = \frac{1}{2} \times 5 \times (4)^2 = 40\,J$

$W_{20} = \frac{1}{2}(20)(-2)^2 = 40J$

(b) w= $w_5 + w_{20}$ = 80J

(c) $i_1 = \frac{1}{L2}\int_0^t -50e^{-200t}\,dt + i1(0)$

$$= 5 \times 10^{-5}(e^{-200t} - 1) + 4\,A$$

$I_2 == \frac{1}{L2}\int_0^t -50e^{-200t}dt + i2(0)$

$$= 1.25 \times 10^{-5}(e^{-200t} - 1) - 2\,A$$

(d) I = $i_1 + i_2 = 6.25 \times 10^{-5}(e^{-200t} - 1) + 2\,A$

6.75 A voltage waveform similar to the given waveform in the figure is produced by a square wave generator. What kind of a circuit component is needed to convert the voltage waveform to the triangular current waveform shown in Fig. P6.75. Calculate the value of the component, assuming that it is initially uncharged.

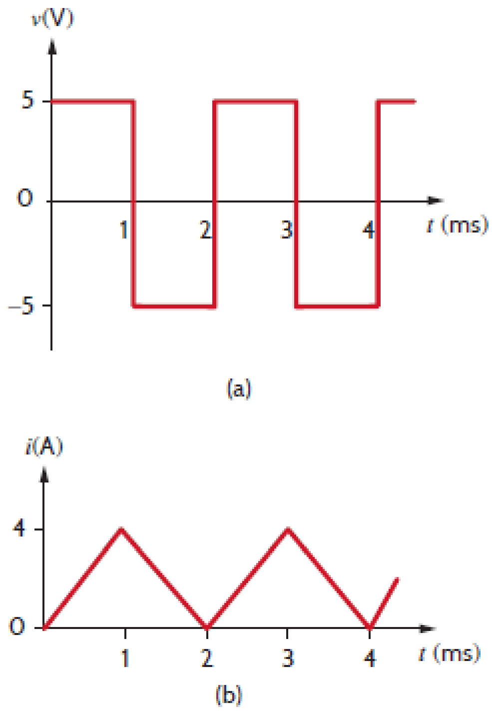


Figure P6.75

SOLUTION:

If we differentiate the wave form of I it will give the wave form similar to the voltage wave form So

$$v = L\frac{di}{dt}$$

$$i = \begin{cases} -4t, & 0 < t < 1\,ms \\ 8 - 4t, & 1 < t < 2\,ms \end{cases}$$

$$v = \begin{cases} 4000L, 0 < t < 1\,ms \\ -4000L, 1 < t < 2\,ms \end{cases}$$

From the graph

4000L = 5

L = 1.25 mH

6.76 Given the network in Fig. P6.76,

(a) Determine the equation for the closed-loop gain $|G| = \left|\frac{v_0}{v_i}\right|$.

(b) Sketch the magnitude of the closed-loop gain as a function of frequency if $R_1 = 1\ \text{k}\Omega$, $R_2 = 10\ \text{k}\Omega$, and $C = 2\ \mu\text{F}$.

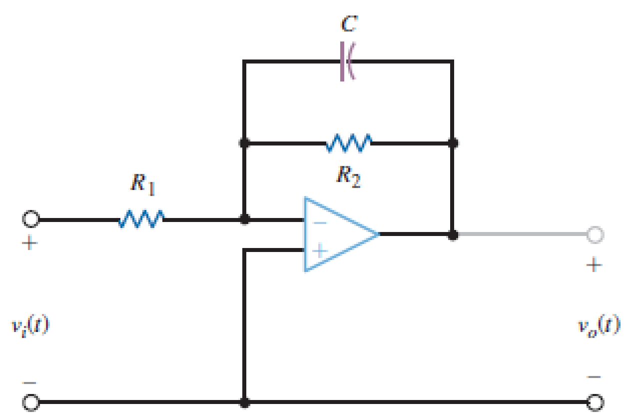


Figure P6.76

SOLUTION:

(a) $$|G| = -\frac{Z_2}{Z_1} = \frac{R_2\left(\frac{1}{j\omega C}\right)}{R_2 + \frac{1}{j\omega C}} \Big/ R_1$$

$$= \frac{-\frac{R_2}{R_1}}{1 + j\omega C R_2}$$

$$= \frac{\frac{R_2}{R_1}}{\sqrt{1 + \omega^2 C^2 R_2^2}}$$

(b) $$\frac{R_2}{R_1} = \frac{10K}{1K} = 10$$

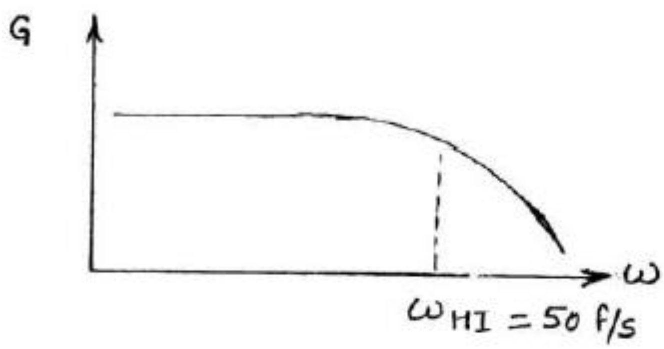
This is a low pass filter
The half power point is

$$\omega_{HI} = \frac{1}{R_2 C}$$

$$= \frac{1}{10\times10^3 \times 2\times10^{-6}}$$

$$= \frac{1}{0.02}$$

$$= 50$$



6.77 The output voltage v_o of the op-amp circuit of Fig. P6.77a is shown in Fig. P6.77b. Let $R_i = R_f = 1\ \text{M}\Omega$ and $C = 1\ \mu\text{F}$. Determine the input voltage waveform and sketch it.

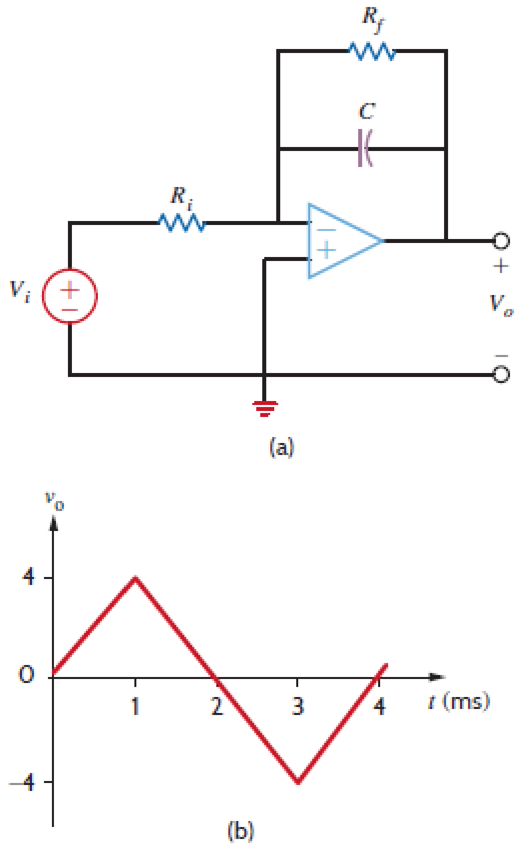


Figure P6.77

SOLUTION:

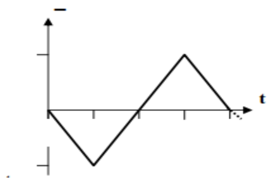
i = i_R+ i_C

$$\frac{v_i - 0}{R} = \frac{0 - v_0}{R_F} + C\frac{d(0 - v_0)}{dt}$$

$$R_F = 10^6 \times 10^{-6} = 1$$

$$v_i = -(v_0 + \frac{dv_0}{dt})$$

The graph of v_0 is shown below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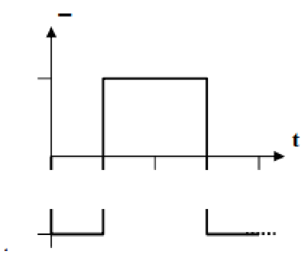
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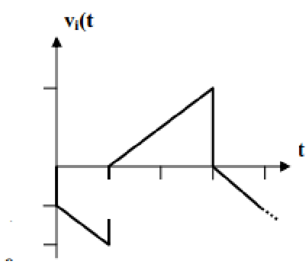
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6.78 Design an op-amp circuit such that:

$$v_0 = 10v_s + 2\int v_s dt$$

where v and v_0 are the input voltage and output voltage respectively.

SOLUTION:

The given equation can be modeled with a op amp summer, integrator and an inverter.

Summer equation

$$v_0 = -(\frac{R_F}{R_1}v_1 + \frac{R_{F2}}{R_2}v_2)$$

Integrator equation

$$v_0 = -\frac{1}{RC}\int v_s dt$$

Inverter equation

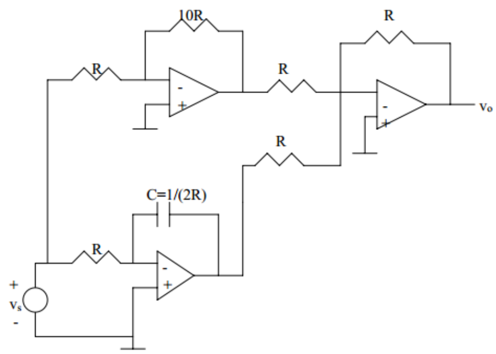
$$v_0 = -\frac{R_F}{R}v_s$$

Comparing the above equation with the given equation we get

$$R_F = 10R$$
$$C = \frac{1}{2R}$$

and for inverter $R_F = R$

So the circuit of the given equation is following


10R
R
R
R
R
C=1/(2R)
R
+
v_s
-
v_o