1. (25%) **Reduction of Order**
   Given that the function $y_1 = \cos 4x$ is a solution of the differential equation

$$y'' + 16y = 0$$

   use *reduction of order* to find another solution.

Solution: $y_2 = \sin 4x$

2. (20%) **Cauchy-Euler Equation**
   Solve

$$4x^2y'' + 17y = 0, \quad y(1) = -1, y'(1) = -\frac{1}{2}$$

Solution: $y = -x^{1/2}\cos(2\ln x)$

3. (40%) **Method of Undetermined Coefficients**
   Solve the differential equation by undetermined coefficients

$$y'' - 2y' + 2y = e^{2x}(\cos x - 3\sin x)$$

Solution: $y = e^x(c_1\cos x + c_2\sin x) + \frac{7}{5}e^{2x}\cos x - \frac{1}{5}e^{2x}\sin x$

4. (20%) **Variation of Parameters**
   Solve

$$y'' - 4y' + 4y = (12x^2 - 6x)e^{2x}$$

   by *variation of parameters*, subject to the initial condition $y(0) = 1, y'(0) = 0$.

Solution: $y = e^{2x}(x^4 - x^3 - 2x + 1)$