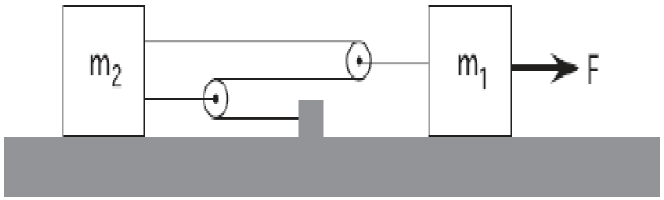
1. Two blocks 1 and 2 rest on a frictionless horizontal surface. They are connected by three massless strings and two frictionless, massless pulleys as shown below. .

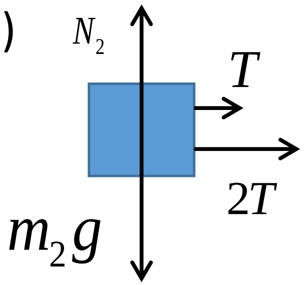
(a) Draw the free-body diagram for each block and each pulley.

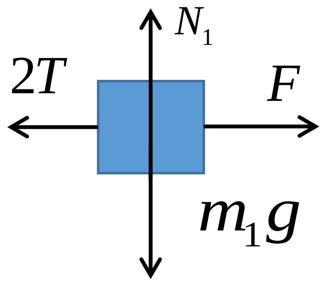
(b) Find the acceleration of the each block

Fig. 1



(a)





(b)

$$\begin{cases} 3T = m_2a_2 \cdots\cdots\cdots(1) \\ F - 2T = m_1a_1 \cdots(2) \\ 2a_1 = 3a_2 \cdots\cdots\cdots(3) \end{cases}$$

因為 m_1 受滑輪組之拉力是m_2受滑輪組之拉力的2/3倍
故m_1之位移是m_2之位移的3/2倍

(1),(3)→ $3T = \dfrac{2}{3}m_2a_1 \cdots\cdots(4)$

(2),(4)→ $$\begin{cases} a_1 = \dfrac{9F}{9m_1 + 4m_2} \\ a_2 = \dfrac{6F}{9m_1 + 4m_2} \end{cases}$$

2. A small block of mass *m* rests on the sloping side of a wedge of mass *M* which itself rests on a horizontal table as shown in Fig. 2. Assuming all surfaces are frictionless.

(a) Draw the free-body diagram for each block and determine the acceleration of each block.

Assume $M = 3m$, $\theta = 37^0$, $g = 10\ m/s^2$

Find the minimum value of the external force (F) such that the mass m leave the surface of the wedge .

(b) Find the acceleration of each block when (i) F = 2mg and (ii) F = 5mg.

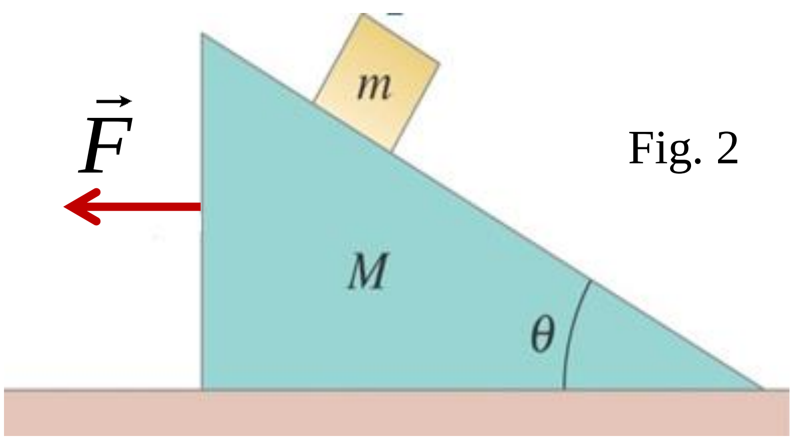
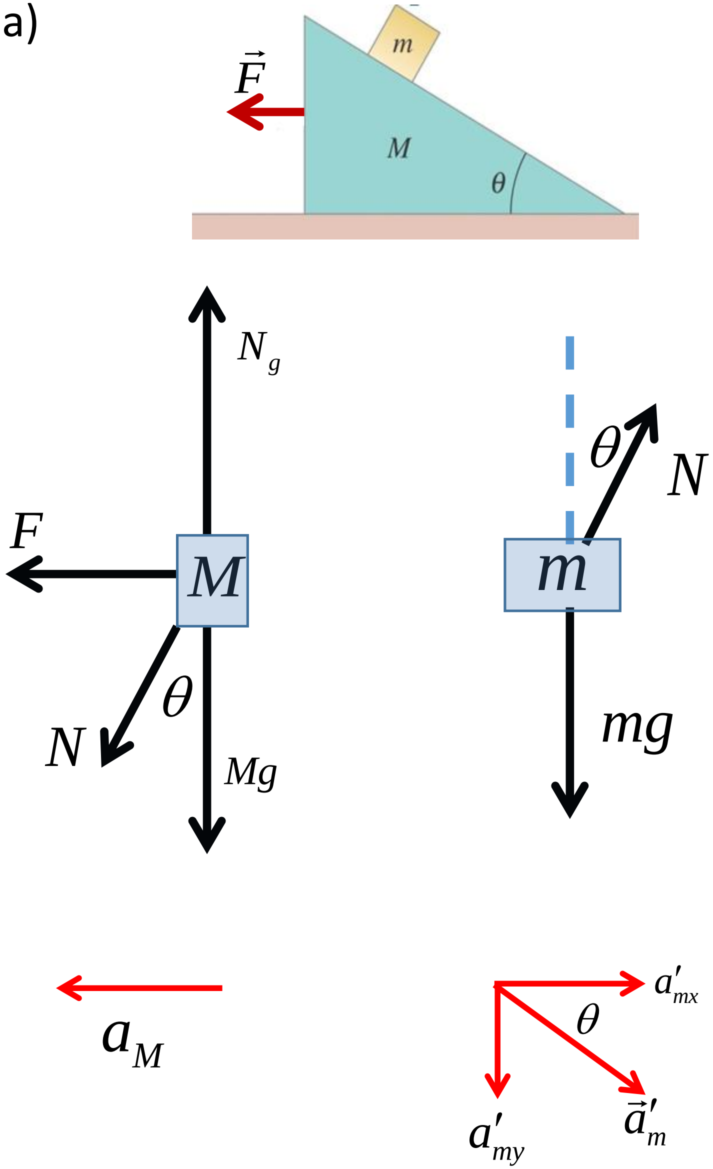


Fig. 2

(a)



$\vec{a}'_m$ 為the mass m 相對於wedge之加速度

$$\begin{cases} F + N\sin\theta = Ma_M \\ mg - N\cos\theta = ma_{my} \\ N\sin\theta = ma_{mx} \end{cases}$$

$$a_{my} = a'_{my}$$

$$a_{mx} = a'_{mx} - a_M$$

$$a'_{mx}\tan\theta = a'_{my}$$

$$\Rightarrow \begin{cases} F + N\sin\theta = Ma_M & \text{(1)} \\ mg - N\cos\theta = ma'_{mx}\tan\theta & \text{(2)} \\ N\sin\theta = m(a'_{mx} - a_M) & \text{(3)} \end{cases}$$

(1),(3) $\quad F = Ma_M - m(a'_{mx} - a_M)$

(2),(3) $\quad mg = ma'_{mx}\tan\theta + m(a'_{mx} - a_M)\cot\theta$

$$M = 3m \qquad \theta = 37^\circ \quad \Rightarrow \begin{cases} F = 4ma_M - ma'_{mx} \\ mg = \dfrac{25}{12}ma'_{mx} - \dfrac{4}{3}ma_M \end{cases}$$

$$\Rightarrow \begin{cases} a'_{mx} = \dfrac{4F + 12mg}{21m} \\ a_M = \dfrac{25F + 12mg}{84m} \end{cases} \quad \text{代回得} \quad N = \frac{5(12mg - 3F)}{84}$$

When $F \geq 4mg$ the mass m will leave the surface of whe wedge.

(b)

(i)

$$When \quad F \le 4mg, \quad N = \frac{5(12mg - 3F)}{84}$$

$$\rightarrow F = 2mg, \quad N = \frac{5}{14}mg$$

$$\begin{cases} F + N\sin\theta = Ma_M \\ mg - N\cos\theta = ma_{my} \\ N\sin\theta = ma_{mx} \end{cases}$$

$$\rightarrow \begin{cases} a_M = \frac{31}{42}g \\ a_{my} = \frac{5}{7}g \\ a_{mx} = \frac{3}{14}g \end{cases}$$

(ii)

$$When \quad F \ge 4mg, \quad N = 0$$

$$\rightarrow F = 5mg, \quad N = 0$$

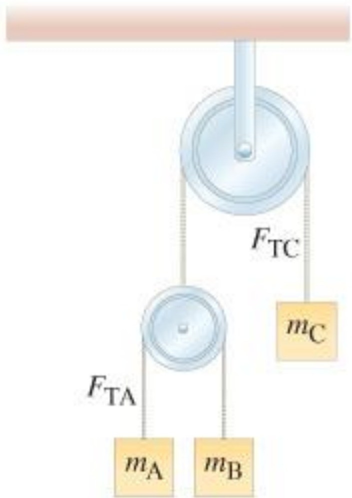
$$\begin{cases} F + N\sin\theta = Ma_M \\ mg - N\cos\theta = ma_{my} \\ N\sin\theta = ma_{mx} \end{cases}$$

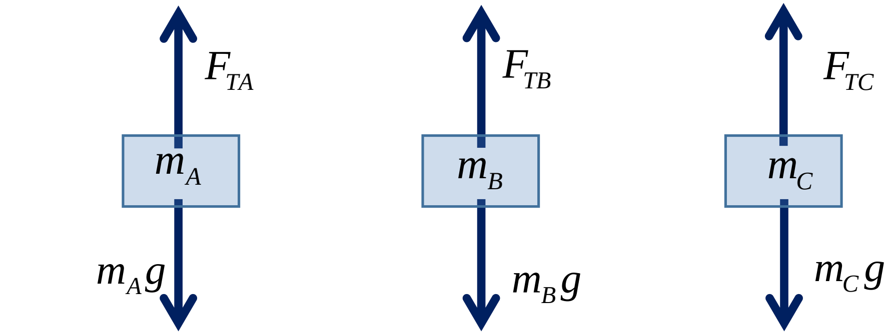
$$\rightarrow \begin{cases} a_M = \frac{5}{3}g \\ a_{my} = g \\ a_{mx} = 0 \end{cases}$$

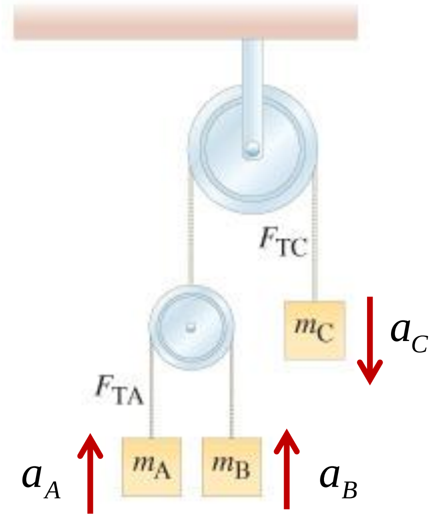
3.The double Atwood machine shown in Fig.3 has frictionless, massless pulleys and cords. Determine (a) the acceleration of mass m_A, m_B, and m_C, and (b) the tensions F_{TA} and F_{TC} in the cords.

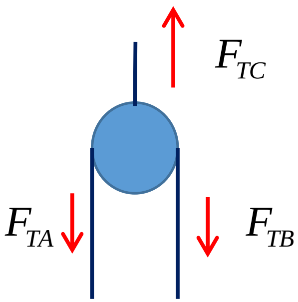
$m_A = 3Kg$, $m_B = 2Kg$ and $m_C = 5Kg$

Fig. 3



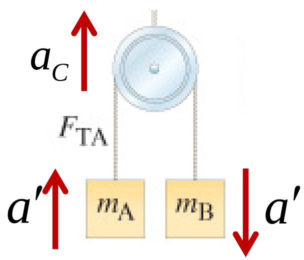






$$F_{TA} = F_{TB}$$

$$F_{TC} = 2F_{TA}$$ --> 因pulley無質量



$$a_A = a' + a_C$$

$$a_B = -a' + a_C$$

(a)

$$\begin{cases} F_{TA} - m_A g = m_A a_A \\ F_{TB} - m_B g = m_B a_B \\ m_C g - F_{TC} = m_C a_C \end{cases}$$

$$\downarrow$$

$$\begin{cases} F_{TA} - 3g = 3(a' + a_C) \cdots\cdots (1) \\ F_{TA} - 2g = 2(a_C - a') \cdots\cdots (2) \\ 5g - 2F_{TA} = 5a_C \cdots\cdots\cdots\cdots (3) \end{cases}$$

(1),(2) $g = -5a' - a_C$

(2),(3) $g = -4a' + 9a_C$

$$\to a' = -10a_C$$

$$\to \begin{cases} a_C = \frac{1}{49} g \\ a' = -\frac{10}{49} g \end{cases}$$

$$\to \begin{cases} a_A = -\frac{9}{49} g \\ a_B = \frac{11}{49} g \\ a_C = \frac{1}{49} g \end{cases}$$

$a_A < 0$ 代表m_A的加速度的方向與假設的方向相反

m_A的加速度為 $\frac{9}{49} g$ 向下

(b)

$$F_{TC} = 2F_{TA} = 5(g - a_C)$$

$$F_{TA} = \frac{120}{49} g \quad , \quad F_{TC} = \frac{240}{49} g$$