HW3-1: There are three infinite lines with line charge density λ_0. O is the origin of the coordinate and line B crosses the ***x***-axis at **(-2,0,0).**

(a) What is the electric field at position $\vec{r} = (2,3,5)$ due to line B?

(b) What is the electric field at position $\vec{r} = (2,3,5)$ due to line C?

(c) What is the total electric field at position $\vec{r} = (2,3,5)$ due to line A, B and C?

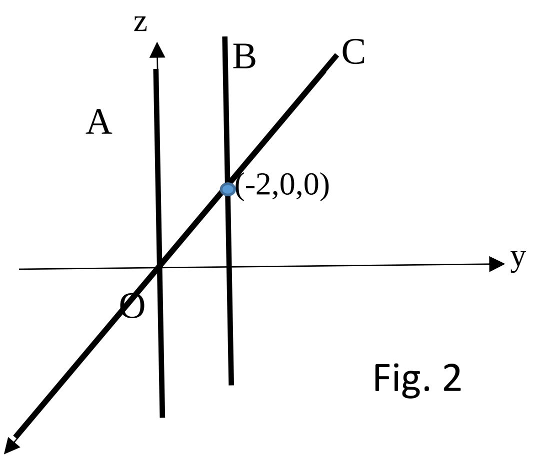


Fig. 2

HW3-2: Fig. 3 shows a cross sectional view of uniform charge distribution in an infinitely long cylindrical shell. The charge density is **ρ (ρ > 0),** the inner radius is **2*R***, and the outer radius **3*R***, the axis of the shell coincides with the ***z***-axis. A second uniform cylindrical charge distribution is added to the system, with the axis of symmetry parallel to the ***z***-axis but passing **(*R*,0,0).** The radius of the cylinder is ***R***, and the charge density is **–*r*,** Determine the magnitude and direction of the ***E***-field along the x-axis **(0 ≤ *x*< 4*R*).**

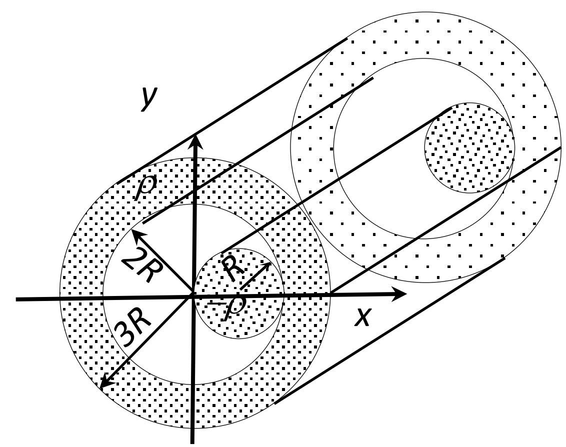


Fig. 3

HW 3-3: A very large thin plane has uniform surface charge density $\boldsymbol{\sigma}$. Touching it on the right (see the Fig. 1) is a long wide slab of thickness $\boldsymbol{d}$ with uniform volume charge density ρ_E.

Determine the electric field (a) to the left of the plane, (b) to the right of the slab, and (c) everywhere inside the slab.

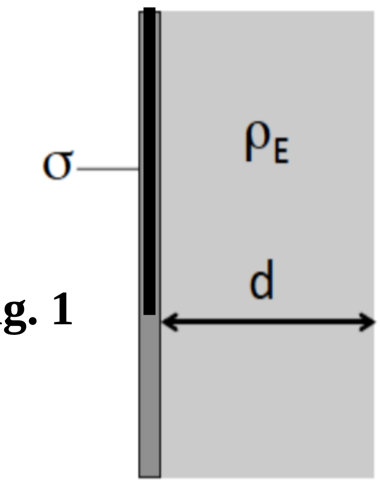
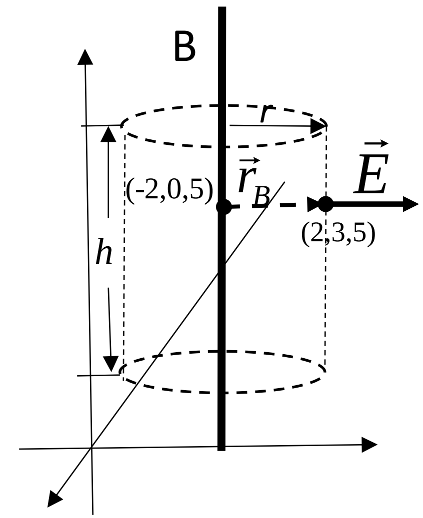


Fig. 1

HW3-1sol (a) For line B, we choose the cylindrical Gauss's surface with radius *r* and height *h*, as shown in figure on the right.



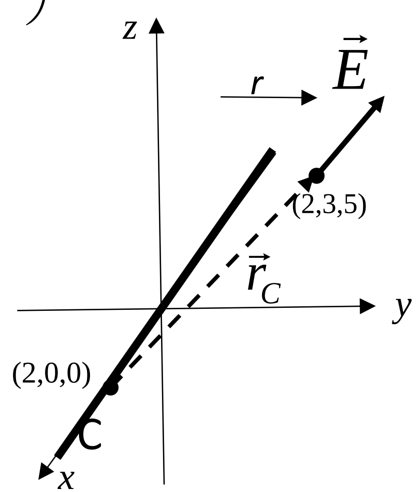
$$\oint \vec{E}\cdot d\vec{A} = E(r)\cdot 2\pi r\cdot h = \frac{q_{in}}{\varepsilon_0} = \frac{\lambda\cdot h}{\varepsilon_0}$$

$$\vec{E}_B = \frac{1}{2\pi\varepsilon_0}\cdot\frac{\lambda}{r_B}\hat{r}_B$$ in cylindrical coordinate (r, θ, z).

For point (2,3,5), the electric field is same as above with vector:

$$\vec{r}_B = (2,3,5)-(-2,0,5) = (4,3,0) \quad , \quad \hat{r}_B = \frac{(4,3,0)}{5} = \left(\frac{4}{5},\frac{3}{5},0\right)$$

$$\vec{E}_B = \frac{\lambda}{2\pi\varepsilon_0}\cdot\left(\frac{4}{25},\frac{3}{25},0\right)$$



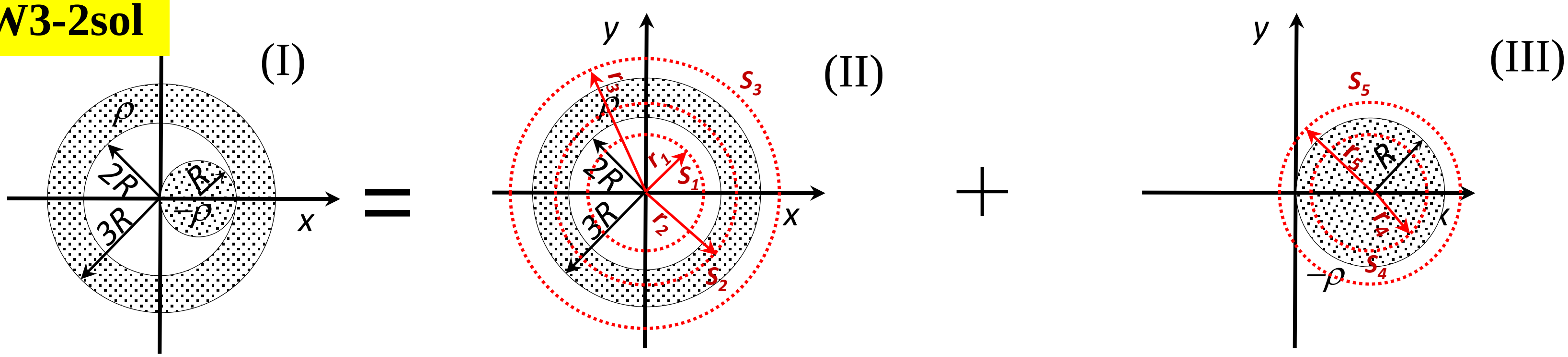
(b). Similar as above: $\vec{r}_C = (2,3,5)-(2,0,0) = (0,3,5)$

$$\vec{E}_C = \frac{\lambda}{2\pi\varepsilon_0}\cdot\left(0,\frac{3}{34},\frac{5}{34},\right)$$

(c). $$\vec{E}_A = \frac{\lambda}{2\pi\varepsilon_0}\cdot\left(\frac{2}{13},\frac{3}{13},0\right)$$

$$\vec{E}_{total} = \vec{E}_A + \vec{E}_B + \vec{E}_C \frac{\lambda}{2\pi\varepsilon_0}\cdot\left(\frac{2}{13}+\frac{4}{25},\frac{2}{13}+\frac{3}{25}+\frac{3}{34},\frac{5}{34}\right)$$

HW3-2sol



The calculation for part (III):

For inner cylinder, and |x-R|<R,
Choose a cylindrical surface S_3 with radius r_3 length ℓ_3, and apply Gauss's law, i.e.

$$\Phi_E = \oiint_{S_3} \vec{E}\cdot d\vec{A} = E(r_3)2\pi r_3\ell = \frac{\rho\pi(r_3^2)\ell}{\varepsilon_0}$$

$$\Rightarrow E(r_3) = \frac{\rho r_3}{2\varepsilon_0} \qquad \Rightarrow E(x) = \frac{\rho|x-R|}{2\varepsilon_0}$$

For 0 ≤x<R , $|x-R| = -(x-R)$

$$\Rightarrow \vec{E}(x) = \frac{-\rho(x-R)}{2\varepsilon_0}\hat{x} \qquad \text{----- equ. (1)}$$

For R ≤x<2R , $|x-R| = (x-R)$

$$\Rightarrow \vec{E}(x) = \frac{-\rho(x-R)}{2\varepsilon_0}\hat{x} \qquad \text{----- equ. (2)}$$

For |x-R|≥R,
Choose a cylindrical surface S_4 with radius r_4 length ℓ_4, and apply Gauss's law, i.e.

$$\Phi_E = \oiint_{S_4} \vec{E}\cdot d\vec{A} = E(r_4)2\pi r_4\ell = \frac{\rho\pi(R^2)\ell}{\varepsilon_0}$$

$$\Rightarrow E(r_4) = \frac{\rho R^2}{2\varepsilon_0 r_4} \qquad \Rightarrow E(x) = \frac{\rho R^2}{2\varepsilon_0|x-R|}$$

For 2R ≤x<4R , $|x-R| = (x-R)$

$$\Rightarrow E(x) = \frac{-\rho R^2}{2\varepsilon_0(x-R)}\hat{x} \qquad \text{----- equ. (3)}$$

The calculation for part (II):

For x<2R,

Choose a cylindrical surface S_1 with length ℓ_1, and apply Gauss's law, i.e.

$$\Phi_E = \oiint_{S_1} \vec{E} \cdot d\vec{A} = E(r_1) 2\pi r_1 \ell = 0$$

$\Rightarrow E(r_1)=0 \quad \Rightarrow E(x)=0$, for $0 \leq x < 2R$ ----- equ. (4)

For 2R≤ x<2R,

Choose a cylindrical surface S_2 with length ℓ_2.

$$\Phi_E = \oiint_{S_2} \vec{E} \cdot d\vec{A} = E(r_2) 2\pi r_2 \ell = \frac{\rho\pi(r_2^2-4R^2)\ell}{\varepsilon_0}$$

$$\Rightarrow E(r_2) = \frac{\rho(r_2^2-4R^2)}{2\pi r_2 \varepsilon_0}$$

$$\Rightarrow \vec{E}(x) = \frac{\rho(x^2-4R^2)}{2x\varepsilon_0}\hat{x} \qquad \text{----- equ. (5)}$$

For 3R< x,

Choose a cylindrical surface S_3 with length ℓ_3.

$$\Phi_E = \oiint_{S_2} \vec{E} \cdot d\vec{A} = E(r_2) 2\pi r_2 \ell = \frac{\rho\pi(r_2^2-4R^2)\ell}{\varepsilon_0}$$

The final result for part (I)

= sum of results of part (II) and Part (III):

For the total E-field with 0 ≤ x < R:

= equ. (1) + equ. (4)

$$\vec{E}(x) = \frac{-\rho(x-R)}{2\varepsilon_0}\hat{x}$$

For the total E-field with R ≤ x < 2R,

= equ. (2) + equ. (4)

$$\vec{E}(x) = \frac{-\rho(x-R)}{2\varepsilon_0}\hat{x}$$

For the total E-field with 2R ≤ x < 3R,

= equ. (3) + equ. (5)

$$E(x) = \frac{-\rho R^2}{2\varepsilon_0(x-R)}\hat{x} + \frac{\rho(x^2-4R^2)}{2x\varepsilon_0}\hat{x}$$

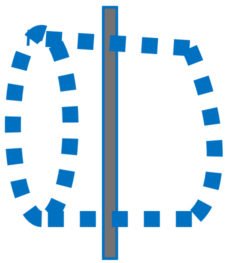
For the total E-field with 3R < x,

= equ. (3) + equ. (6)

$$E(x) = \frac{-\rho R^2}{2\varepsilon_0(x-R)}\hat{x} + \frac{\rho(x^2-4R^2)}{2x\varepsilon_0}\hat{x}$$

Solution HW3-3 :

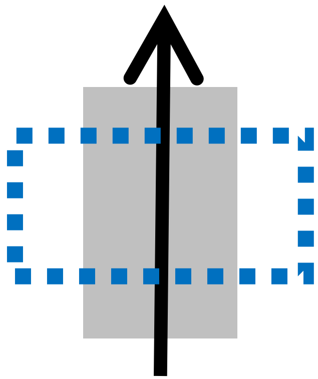
把兩個東西拆開來分析，令*slab*中心為*x*軸



$$\Phi_E = \oiint \vec{E}\cdot d\vec{A} = 2EA$$

$$= \frac{Q_{in}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

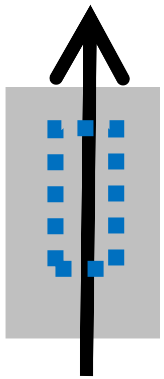
$$\vec{E} = \frac{\sigma}{2\varepsilon_0} n$$



$$\Phi_E = \oiint \vec{E}\cdot d\vec{A} = 2EA$$

$$= \frac{Q_{in}}{\varepsilon_0} = \frac{\rho_E (Ad)}{\varepsilon_0}$$

$$\vec{E} = \frac{\rho_E d}{2\varepsilon_0} n$$



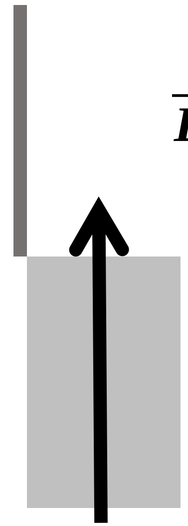
$$\Phi_E = \oiint \vec{E}\cdot d\vec{A} = 2EA$$

$$= \frac{Q_{in}}{\varepsilon_0} = \frac{\rho_E (A|x|)}{\varepsilon_0}$$

$$\vec{E} = \frac{\rho_E x}{2\varepsilon_0} x$$

$$\vec{E} = -\frac{\sigma}{2\varepsilon_0}\,x$$

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\,x$$



$$\vec{E} = -\frac{\rho_E d}{2\varepsilon_0}\,x$$

$$\vec{E} = \frac{\rho_E d}{2\varepsilon_0}\,x$$

$$\vec{E} = \frac{\rho_E x}{2\varepsilon_0}\,x$$

$$\vec{E} = \frac{-\sigma}{2\varepsilon_0}\,x + \frac{-\rho_E d}{2\varepsilon_0}\,x = -\frac{\sigma + \rho_E d}{2\varepsilon_0}\,x$$

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\,x + \frac{\rho_E x}{2\varepsilon_0}\,x = \frac{\sigma + \rho_E x}{2\varepsilon_0}\,x$$

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\,x + \frac{\rho_E d}{2\varepsilon_0}\,x = \frac{\sigma + \rho_E d}{2\varepsilon_0}\,x$$