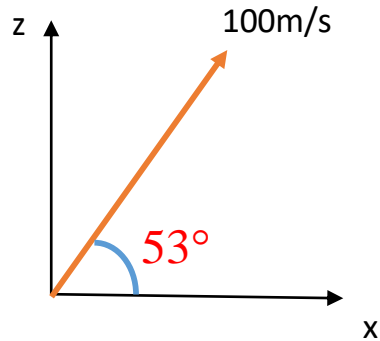


Problem 1

As shown in Fig. 1, a cannon located at the origin of the coordinate system launched a iron ball in the x-direction with a take off angle $\theta = 53^\circ$, and the initial speed of the ball is 100 m/s. Assume the gravitational acceleration $g = 10\text{m/s}^2$. ($\sin 53^\circ = 4/5$)

- (A) A cannonball is fired at $t = 0$ sec. In the meantime, a strong wind begins to blow in the +y-direction, and it accelerated the cannonball in the same direction with the acceleration $a_y = 0.06t$ (m/s^2). If the wind stops at $t = 2.0$ sec. Find the coordinate of the position where the ball lands.



$$v_{x(t=0)} = 60\text{m/s}$$

$$v_{z(t=0)} = 80\text{m/s}$$

x座標:

$$t = 2 \frac{v_{z(t=0)}}{g} = 16\text{s}$$

$$x = v_{x(t=0)}t = 960\text{m}$$

y座標:

當 $t \leq 2$

$$a_y = 0.06t(\text{m/s}^2)$$

$$v_y(t) = \int_0^t 0.06t dt + v_y(0) \\ = 0.03t^2(\text{m/s})$$

當 $t > 2$

$$a_y = 0(\text{m/s}^2)$$

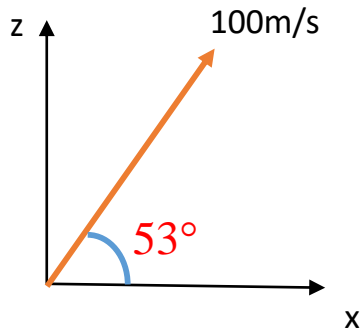
$$v_y(t) = v_y(2) = 0.12(\text{m/s})$$

$$y = \int_0^{16} v_y dt = \int_0^2 0.03t^2 dt + \int_2^{16} 0.12 dt + y(0) \\ = \frac{0.03}{3} t^3 \Big|_0^2 + 0.12t \Big|_2^{16} + 0 \\ = 1.76\text{m}$$

$$\rightarrow (x, y) = (960\text{m}, 1.76\text{m})$$

Problem 1

(B) A second cannon ball is fired again at $t = 0$ sec. Now the wind blows very strongly in the $-x$ direction such that it accelerates the cannon ball with $a_x = -30.0t$ (m/s^2). At $t = t_1$, the winds stops and the cannon ball also stops moving in the x -direction and then reach ground at $t = t_2$. Determine t_1 and t_2 and how far it lands from the origin?



$$v_{x(t=0)} = 60 \text{ m/s}$$

$$v_{z(t=0)} = 80 \text{ m/s}$$

$$t_2 = 2 \frac{v_{z(t=0)}}{g} = 16 \text{ s}$$

$$\text{當 } t \leq t_1$$

$$v_x(t) = \int_0^t a_x dt + v_x(0)$$

$$= \int_0^t -30t dt + 60$$

$$= -15t^2 + 60$$

$$\rightarrow v_x(t_1) = -15t_1^2 + 60 = 0$$

$$t_1 = 2 \text{ s}$$

$$\text{當 } t > t_1$$

$$a_x = 0 (\text{m/s}^2)$$

$$\rightarrow v_x(t) = v_x(t_1) = 0 (\text{m/s})$$

$$x(16) = \int_0^{16} v_x dt + x(0)$$

$$= \int_0^2 (-15t^2 + 60) dt + \int_2^{16} 0 dt + x(0)$$

$$= -5t^3 + 60t \Big|_0^2$$

$$= 80 \text{ m}$$

Problem 2

The water in the **300-m-wide** river has a speed of **6.00 km/h** due east relative to the Earth. A boat with a speed of **12.0 km/h** relative to the water is to travel from point A to point B, where B is 37° W of N relative to A, as shown in Fig. 2. (a) What should boat heading be? (b) How long will it take.

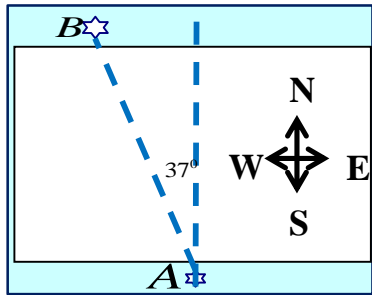
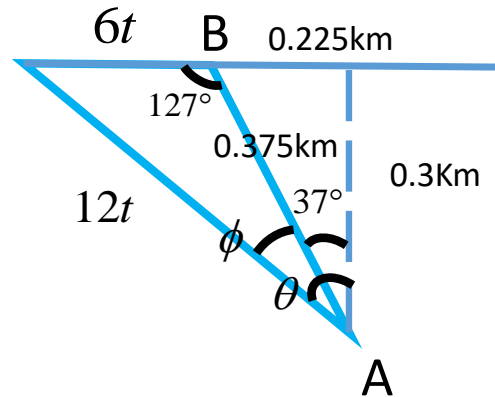


Fig. 2



Sol: 由正弦定理

$$\frac{6t}{\sin \phi} = \frac{12t}{\sin 127^\circ} = \frac{0.375}{\sin(\pi - \theta)}$$

$\pi/2$

$$\sin \phi = \frac{\sin 127^\circ}{2} = \frac{2}{5}$$

$$\cos \theta = \cos(\phi + 37^\circ) = \cos \phi \cos 37^\circ - \sin \phi \sin 37^\circ$$

$$= \frac{4\sqrt{21} - 6}{25} = \frac{0.3}{12t}$$

$$t = \frac{5}{8(4\sqrt{21} - 6)} \approx 0.0507\text{hr} \approx 3.04\text{min}$$

Problem 3

A person stands at the base of a hill that is a straight incline making an angle ϕ with the horizontal. (Fig3) For a given initial speed v_0 , at what angle θ (to the horizontal) should objects be thrown so that the distance d they land up the hill is as large as possible?

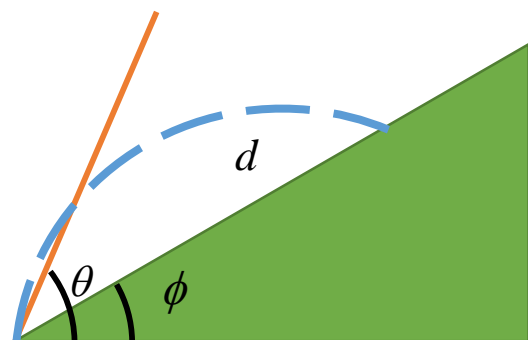


Fig3

$$\begin{aligned} x: (v_0 \cos \theta)t &= d \cos \phi \\ y: (v_0 \sin \theta)t - \frac{1}{2}gt^2 &= d \sin \phi \end{aligned} \quad \rightarrow \quad t = \frac{d \cos \phi}{v_0 \cos \theta}$$

$$(v_0 \sin \theta)\left(\frac{d \cos \phi}{v_0 \cos \theta}\right) - \frac{1}{2}g\left(\frac{d \cos \phi}{v_0 \cos \theta}\right)^2 = d \sin \phi$$

$$d = \frac{2v_0^2 \cos^2 \theta}{g \cos^2 \phi} \left(\frac{\sin \theta \cos \phi - \sin \phi \cos \theta}{\cos \theta} \right) \quad \rightarrow$$

$$d = \frac{2v_0^2 \cos \theta \sin(\theta - \phi)}{g \cos^2 \phi}$$

當d有最大時
d的一次微分為0

$$\frac{d}{d\theta}(d) = \frac{2v_0^2 (\cos \theta \cos(\theta - \phi) - \sin \theta \sin(\theta - \phi))}{g \cos^2 \phi}$$

$$\cos(2\theta - \phi) = 0$$

$$\rightarrow 2\theta - \phi = 90^\circ$$

$$\theta = 45^\circ + \frac{\phi}{2}$$