29. We have $y_c = c_1 e^{-x/5} + c_2$ and we assume $y_p = Ax^2 + Bx$. Substituting into the differential equation we find $A = -3$ and $B = 30$. Thus $y = c_1 e^{-x/5} + c_2 - 3x^2 + 30x$. From the initial conditions we obtain $c_1 = 200$ and $c_2 = -200$, so

$$y = 200e^{-x/5} - 200 - 3x^2 + 30x.$$

2, $4y''-y=xe^{\frac{x}{2}}$, $y(0)=1$, $y'(0)=0$

$4m^2-1=0$, $m=\pm\frac{1}{2}$, $y_c=c_1e^{\frac{1}{2}x}+c_2e^{-\frac{1}{2}x}$

$$W=\begin{vmatrix} e^{\frac{1}{2}x} & e^{-\frac{1}{2}x} \\ \frac{1}{2}e^{\frac{1}{2}x} & -\frac{1}{2}e^{-\frac{1}{2}x} \end{vmatrix}=-1$$

$u_1'=\frac{W_1}{W}=\frac{1}{4}x \Rightarrow u_1=\frac{1}{8}x^2$

$$W_1=\begin{vmatrix} 0 & e^{-\frac{1}{2}x} \\ \frac{1}{4}xe^{\frac{x}{2}} & -\frac{1}{2}e^{-\frac{1}{2}x} \end{vmatrix}=-\frac{1}{4}x$$

$u_2'=\frac{W_2}{W}=-\frac{1}{4}xe^{x}$, $u_2=-\frac{1}{4}(xe^{x}-e^{x})$

$y_p=u_1y_1+u_2y_2=\frac{1}{8}x^2e^{\frac{1}{2}x}-\frac{1}{4}xe^{\frac{1}{2}x}+\frac{1}{4}e^{\frac{1}{2}x}$

$$W_2=\begin{vmatrix} e^{\frac{1}{2}x} & 0 \\ \frac{1}{2}e^{\frac{1}{2}x} & \frac{1}{4}xe^{\frac{x}{2}} \end{vmatrix}=\frac{1}{4}xe^{x}$$

$y=y_c+y_p$

$=c_1e^{\frac{1}{2}x}+c_2e^{-\frac{1}{2}x}+\frac{1}{8}x^2e^{\frac{1}{2}x}-\frac{1}{4}xe^{\frac{1}{2}x}$

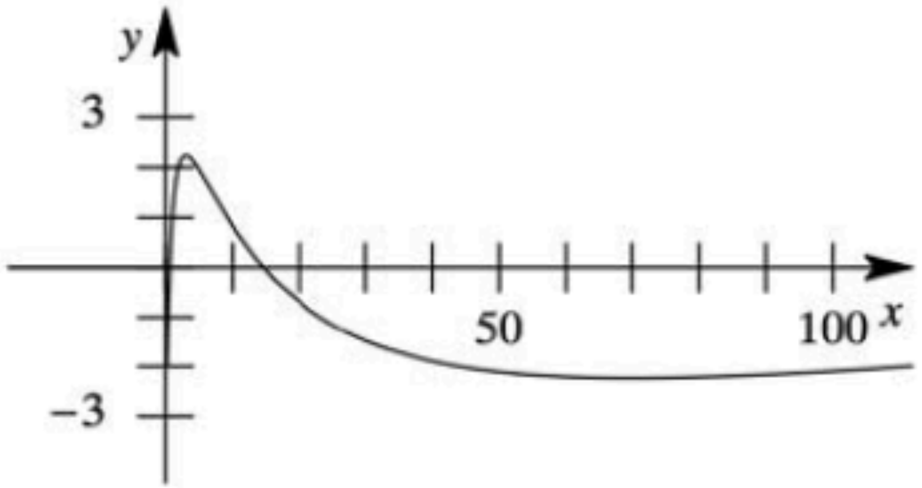
$y(0)=c_1+c_2=1$

$y'(0)=\frac{1}{2}c_1-\frac{1}{2}c_2-\frac{1}{4}=0$

$\Rightarrow c_1=\frac{3}{4}$, $c_2=\frac{1}{4}$

$y=\frac{3}{4}e^{\frac{1}{2}x}+\frac{1}{4}e^{-\frac{1}{2}x}+\frac{1}{8}x^2e^{\frac{1}{2}x}-\frac{1}{4}xe^{\frac{1}{2}x}$

27. The auxiliary equation is $m^2 + 1 = 0$, so that



$$y = c_1 \cos(\ln x) + c_2 \sin(\ln x)$$

and

$$y' = -c_1 \frac{1}{x} \sin(\ln x) + c_2 \frac{1}{x} \cos(\ln x).$$

The initial conditions imply $c_1 = 1$ and $c_2 = 2$. Thus $y = \cos(\ln x) + 2\sin(\ln x)$. The graph is given to the right.

4. $y'' - 10y' + 25y = e^{5x}$, $y(0) = -1$, $y'(0) = 1$

$m^2 - 10m + 25 = 0$, $(m-5)^2 = 0$, $m = 5, 5$

$y = C_1 e^{5x} + C_2 x e^{5x}$

$y(0) = C_1 = -1$

$y'(0) = 5C_1 + C_2 = 1$, $C_2 = 6$

$y_h = -e^{5x} + 6xe^{5x}$

$$W = \begin{vmatrix} e^{5x} & xe^{5x} \\ 5e^{5x} & e^{5x} + 5xe^{5x} \end{vmatrix} = e^{10x} + 5xe^{10x} - 5xe^{10x} - e^{10x}$$

$$G(x,t) = \frac{e^{5t} \cdot xe^{5x} - e^{5x} \cdot te^{5t}}{e^{10t}} = \frac{(x-t)e^{5(t+x)}}{e^{10t}}$$

$$y_p(x) = \int_0^x \frac{(x-t)e^{5(t+x)}}{e^{10t}} e^{5t}\,dt = \int_0^x (x-t)e^{5x}\,dt = e^{5x}\left(xt - \frac{1}{2}t^2 \Big|_0^x\right) = \frac{1}{2}x^2 e^{5x}$$

$$y = y_p(x) + y_h(x) = -e^{5x} + 6xe^{5x} + \frac{1}{2}x^2 e^{5x}$$