General Physics (I) **期末考** **Jan. 12, 2018**

試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(50 points) (所有題目必須有計算過程,否則不予計分)

1. (10 pts) A particle moves where its potential energy is given by $\boldsymbol{U(r) = U_0[0.5/r^2 - 1/r\ ]}$.
   Suppose the particle has an energy $\boldsymbol{E_{total}=-0.495U_0}$.

   a) (4 pts) What is the classical turning points of the motion of this particle? (the range of the particle is moving? What is the force $\boldsymbol{F(r)}$ acting on this particle)

   b) (2 pts) What is the maximum kinetic energy of the particle (in term of $\boldsymbol{U_0}$), and for what value of $\boldsymbol{r}$ does it occur?

   c) (2 pts) Near the equilibrium point ($\boldsymbol{r = r_0}$), the system can be approximated as a simple harmonic motion (SHM). Let $\boldsymbol{x = r - r_0}$, write down the equation of motion as function of $\boldsymbol{x}$ by using the formula $(r_0 + x)^{-n} \approx r_0^{-n}(1 - nx/r_0 + ...)$, if $x << r_0$.

   d) (2 pts) Find the period of this particle in part (c).

2. (10 pts) A 30-g piece of iron at 900 K is thrown into 170-g insulated steel cup filled with 80-g water initially at 300 K, assuming no heat loss to the surroundings, find

(a) (2 pts) the final temperature T when equilibrium is reached;

(b) (6 pts) the change of the entropy for the water, the change of the entropy for the steel cup, and the change of the entropy for the iron,

(c) (2 pts) the change of the entropy for the surroundings, and the change of the entropy for the universe as whole.

Use 3 significant digits for your answers

(the specific heat and the latent heat of vaporization of water are 4200 J/kg^{o}C and 2.26 x 10^6 J/kg, respectively, and the specific heat of steel or iron is about 420 J/kg^{o}C).

3. (15 pts) As shown in the Fig. 1 , a bead is fired at the end of uniform rod and stick inside the rod. The rod (mass **3*m*** and length ***l***) is hung on the top and the bead has mass ***m*** and velocity ***v*** . After the collision, the rod with the bead begins to swing.

(a) (5 pts) What is the angular velocity of the pendulum right after the collision.

(b) (6 pts) Find the equation of motion and the period of this new physical pendulum. (Assume that the angle is small such that we can use $\sin\theta \approx \theta$.)

(c) (4 pts) What is the maximum angle θ_{max} of this simple harmonic oscillation?

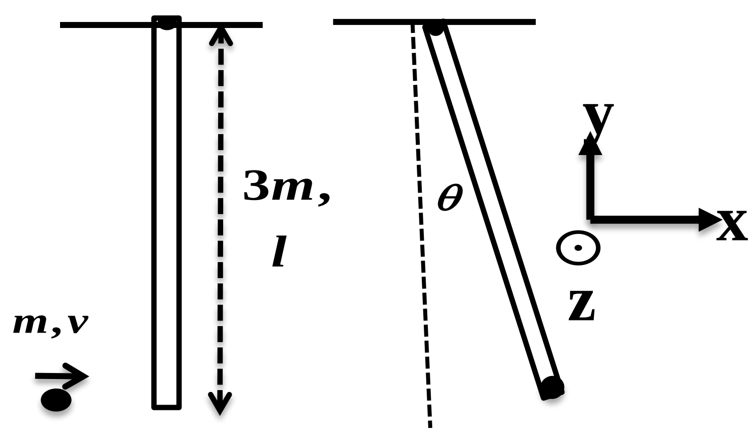


Fig. 1

4. (15pts) As shown in Fig. 2, an 1 mole of mono-atomic ideal gas system executes a cyclic process in the ***P-V*** diagram. It starts with ***a→b*** an isothermal process, ***b→c*** adiabatic, ***c→d*** isbaric, and ***d→a*** isovolumetric.

(a) (11 pts) Determine the efficiency of this cycle working as an heat engine. $\left(\ln(2) \approx 0.7\right)$

(b) (4 pts) Determine the entropy change of the ideal gas after each process, i.e. Δs_{ab}, Δs_{bc}, Δs_{cd}, ΔS_{da} , with Δs_{xy} defined as S_y-S_x .

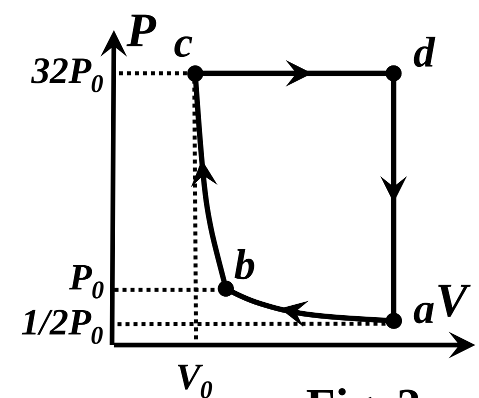
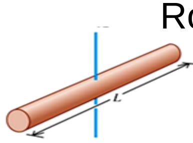
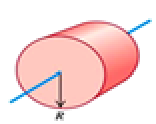


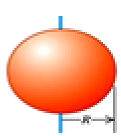
Fig. 2



$I = \frac{1}{12}ML^2$



$I = \frac{1}{2}MR^2$



$I = \frac{2}{5}MR^2$

II.選擇題(50 points)

1. (5 pts) A gyroscope has a flying wheel with mass $\boldsymbol{m_0}$ and a weight $\boldsymbol{m_0}$ at the other end of an axle, which is pivoted at point ***O*** as shown in Fig. 3a. The wheel spins about the axle in the direction shown by the arrow. At the moment shown in the fig. 3a, the axle is along x-axis and the gyroscope is stable. Let $\vec{L} = -0.5 m_0 r_0^2 \omega_0 \hat{x}$ be the angular momentum of the gyroscope at this moment. Now we put another weight with mass $\boldsymbol{m_0}$, at the other end of the axle at distance $\boldsymbol{r_1 = 2r_0}$ from ***O*** (Fig. 3b). Ignoring the mass of the axle. (1)What is the direction of the torque for the gyroscope at the moment shown in the figure 3b? (2) Viewing from the top, will the gyroscope rotate (precession) clockwise (***c.w.***) or counter-clockwise (***c.c.w***)?
   (A) **+*x*, *c.w.*** (B) **–*x*, *c.w.*** (C) **+*x*, *c.c.w*** (D) **–*x*, *c.c.w.*** (E) **+*y*, *c.w.*** (F) **–*y*, *c.w.*** (G) **+*y*, *c.c.w***
   (H) **-*y*, *c.c.w*** (J) **+*z*, *c.w.*** (K) **–*z*, *c.c.w.***

2. (5 pts) Same as above, Let ***a*** be the angular velocity of the precession of the gyroscope and $\boldsymbol{r_0 = 0.1}$**m**, $\boldsymbol{m_0 = 1.5}$**kg**, $\boldsymbol{\omega_0 = 100}$ $\mathbf{s^{-1}}$, and $\boldsymbol{g = 10}$ $\mathbf{m/s^2}$. What the value ***a*** is?
   (A) $0 < a \le 1$ (B) $1 < a \le 2$ (C) $2 < a \le 3$ (D) $3 < a \le 4$ (E) $4 < a \le 5$ (F) $5 < a \le 6$
   (G) $6 < a \le 7$ (H) $7 < a \le 8$ (J) $8 < a \le 9$ (K) $9 < a \le 10$ (L) $10 < a$

3. (5 pts) Let a simple harmonic oscillation described as $x(t) = A\cos(\omega t + \delta)$. Now we recorded a figure for velocity ***v(t)*** as shown in Fig. 4. What is the phase angle $\boldsymbol{\delta}$?
   (A) 0.1π (B) -0.1π (C) 0.2π (D) -0.2π (E) 0.9π (F) -0.9π (H) 0.8π (J) -0.8π

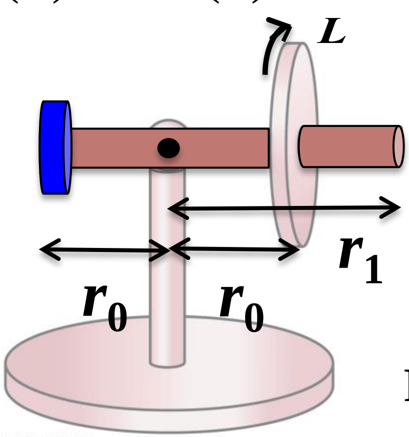
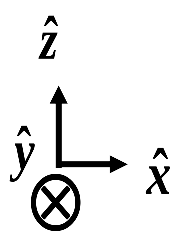


Fig. 3a



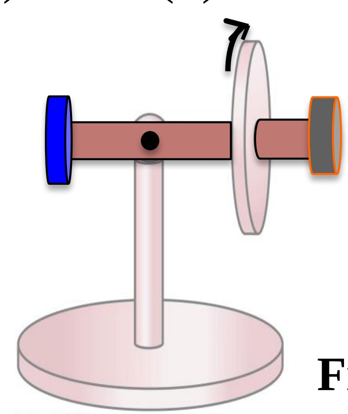


Fig. 3b

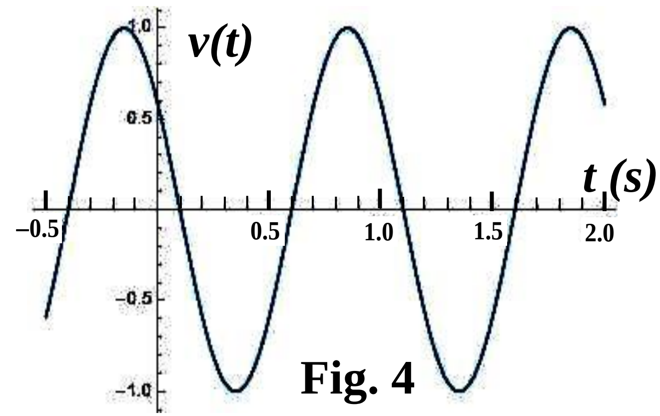


Fig. 4

4. (5 pts) A ideal gas system execute a quasi-equilibrium process from ***a*** to ***b*** in the P-V diagram shown in Fig. 5. Along this process, the pressure ***P*** can be expressed as a function of the volume ***V*** as $\boldsymbol{P(V) = 5P_0[(V/V_0)^{2/3} + (V_0/V)^{2/3}]}$, If $\boldsymbol{\gamma = 7/5}$, and ***W*** the work done by the gas system to the environment, let ***x***= $\boldsymbol{W/P_0V_0}$, which of the following is correct?

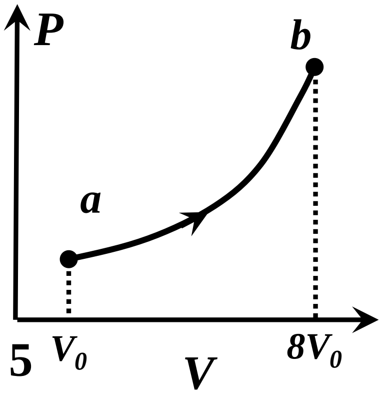


Fig. 5

(A) $0< x \leq 1$ (B) $1< x \leq 10$ (C) $10< x \leq 100$ (D) $100< x \leq 1000$
(E) $1000< x \leq 10000$ (F) $10000< x$

5. (5 pts) Imagine a Carnot engine that operates between the temperatures T_H = 800 ^oC and T_L = 200 ^oC. The engine performs 1200 J work each cycle, which take 0.1 second. The efficiency of this engine and the heat out of the engine are

(A)75% and 400 J; (B)65% and 640 J; (C)55% and 940 J; (D)75% and 1600 J;
(E)65% and 1840 J; (F)55% and 2140 J, respectively.

6. (5 pts) If a Brayton heat engine takes an ideal monatomic gas around the cycle shown in the fig. 6 where P_{max} = 16 atm, P_{min} = 0.5 atm, V_{max} = 800 m^3, and V_{min} = 100 m^3, the efficiency of this engine will be

(A)21-30%; (B)31-40%; (C)41-50%; (D)51-60%;
(E)61-70%; (F) 71-80%; (G)81-90%; (H)91-100%.

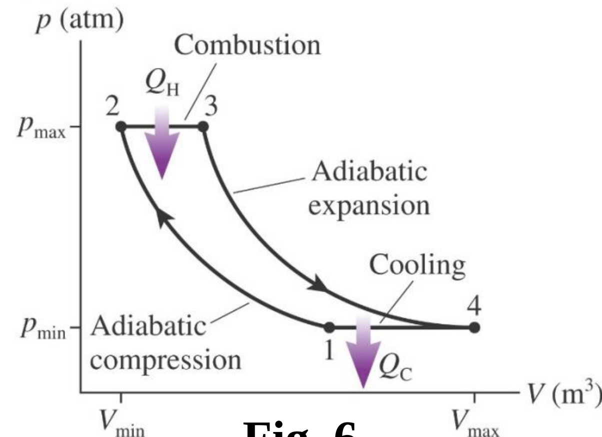


Fig. 6

Multiple Choice Questions:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
E	**D**	**D**	**D**	送分	**F**	**C**	**C**
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
E	**E**	**G**	**B**	**B**	**A**	**E**	**E**

Problem 1:

(a) Classical turning points occur at $E_{tot} = U(r)$ (kinetic energy $= 0$)

$$-0.495U_0 = U_0\left(\frac{0.5}{r^2} - \frac{1}{r}\right) \to \frac{1}{2}\left(\frac{1}{r}\right)^2 - \left(\frac{1}{r}\right) + 0.495 = 0$$ **1 pts**

$$\frac{1}{r} = 1 \pm \sqrt{1 - 2\cdot 0.495} = 1 \pm 0.1 = 1.1,\quad 0.9 \quad \text{or } r = \frac{1}{1.1},\ \frac{1}{0.9} \approx 0.91,\ 1.11$$ **2 pts**

(b)

$$F(r) = -\frac{dU(r)}{dx} = U_0\left(\frac{1}{r^3} - \frac{1}{r^2}\right)$$ **1 pts**

Maximum kinetic energy occurs at $F = 0$, (or U_{min}, equilibrium point), i.e. $dU(r)/dr = 0$

$$F(r_0) = 0 = U_0\left(\frac{1}{r^3} - \frac{1}{r^2}\right) \to r_0 = 1$$ **1 pts**

$(c)\,\&\,(d)$

$$-0.495U_0 = KE_{\max} + U_0\left(\frac{0.5}{r_0^2} - \frac{1}{r_0}\right) \to KE_{\max} = 0.005U_0$$ **1 pts**

$$F(r) = U_0\left(\frac{1}{r^3} - \frac{1}{r^2}\right) = U_0\left(\frac{1}{(1+x)^3} - \frac{1}{(1+x)^2}\right) \simeq U_0(1 - 3x - 1 + 2x) = -U_0 x$$

$$m\ddot{r} = m\ddot{x} = F = -U_0 x \to \ddot{x} + \frac{U_0}{m}x = 0$$ **2 pts**

$$\omega^2 = \frac{U_0}{m} \to T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{U_0}}$$ **2 pts**

1. 1 $m_{Fe}c_{Fe}[T-900]+m_sc_s[T-300]+m_wc_w[T-300]=0$

1 $T = 318$ K

2. 2

$$\Delta S_w = \int_{300}^{318} \frac{m_w c_w}{T} dT$$

$$= m_w c_w \ln\frac{318}{300}$$

$$= 80 \times 10^{-3} \times 4200 \times \left(\frac{18}{300}\right)$$

$= 20.2$ J/K

2

$$\Delta S_s = m_s c_s \ln\frac{318}{300}$$

$$= 170 \times 10^{-3} \times 420 \times \left(\frac{18}{300}\right)$$

$= 4.28$ J/K

2

$$\Delta S_{Fe} = m_{Fe} c_{Fe} \ln\frac{318}{900}$$

$$= 30 \times 10^{-3} \times 420 \times (-1.04)$$

$= -13.1$ J/K

3.

1 $$\Delta S_{en} = 0$$

1 $\Delta S_{total} = 11.4$ J/K

Another solution:

$$\Delta S_w = \frac{80\times10^{-3}\times4200\times18}{309} = 19.6 \text{ J/K}$$

$$\Delta S_s = \frac{170\times10^{-3}\times420\times18}{309} = 4.16 \text{ J/K}$$

$$\Delta S_{Fe} = \frac{30\times10^{-3}\times420\times(-582)}{609} = -12.0 \text{ J/K}$$

$$\Delta S_{en} = 0$$

$$\Delta S_{total} = \Delta S_w + \Delta S_s + \Delta S_{Fe} + \Delta S_{en} = 19.6+4.16-12.0+0 = 11.8 \text{ J/K}$$

4. (15pts) As shown in Fig. x, an one mole of mono-atomic ideal gas system executes a cyclic process in the ***P-V*** diagram. It starts with ***a→b*** an isothermal process, ***b→c*** adiabatic, ***c→d*** isbaric, and ***d→a*** isovolumetric.(a) (11pts) Determine the efficiency of this cycle working as an heat engine. (b) (4pts) Determine the entropy change of the ideal gas after each process, i.e. ΔS_{ab}, ΔS_{bc}, ΔS_{cd}, ΔS_{da} , with ΔS_{xy} defined as S_y-S_x .

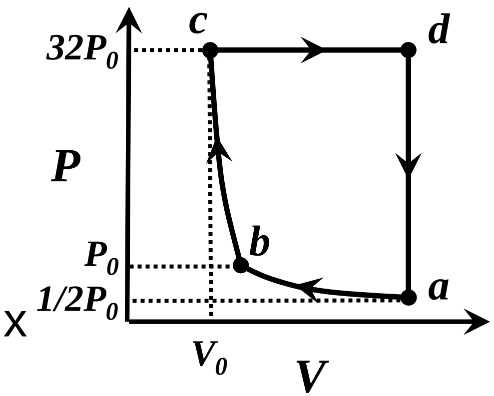


Fig. x

(a) Process b→c is an adiabatic process,

① $$P_bV_b^\gamma = P_cV_c^\gamma \Rightarrow P_0V_b^{5/3} = 32P_0V_0^\gamma \quad \Rightarrow V_b = 8V_0$$ ①

Process a→b is an isothermal process,

$$P_aV_a = P_bV_b \Rightarrow \frac{P_0}{2}V_a = P_0 \cdot 8V_0 \Rightarrow V_a = 16V_0$$ ①

ΔE $\boldsymbol{\Delta E_{int} = Q - W}$

For process *a→b*, $\Delta E_{\text{int}} = 0$, $\quad W = \int_{V_a}^{V_b} PdV$

$$P_aV_a = P_bV_b = nRT_a$$

$$\Rightarrow W = \int_{V_a}^{V_b} \frac{nRT_a}{V} dV = nRT_a \ln\frac{V_b}{V_a}$$

$$= -nRT_a \ln 2 = (-8\ln 2)P_0V_0$$ ①

$$\Rightarrow Q_{a\to b} = (8\ln 2)P_0V_0$$

For process *b→c*, $Q_{b\to c} = 0$ ①

For process *c→d*,

$$\Delta E_{\text{int}} = \frac{3}{2}nRT_d - \frac{3}{2}nRT_c = \frac{3}{2}(P_dV_d - P_cV_c)$$

$$= \frac{3}{2}(32P_0 \cdot 16V_0 - 32P_0V_0) = 720P_0V_0$$ ①

$$W = \int_{V_c}^{V_d} PdV = 32P_0 \cdot 15V_0 = 480P_0V_0$$ ①

$$\Rightarrow Q_{c\to d} = 720P_0V_0 + 480P_0V_0 = 1200P_0V_0$$ ①

For process *d→a*,

$$\Delta E_{\text{int}} = \frac{3}{2}nRT_a - \frac{3}{2}nRT_d = \frac{3}{2}(P_aV_a - P_dV_d)$$ ①

$$= \frac{3}{2}\left(\frac{1}{2}P_0 \cdot 16V_0 - 32P_0 \cdot 16V_0\right) = -756P_0V_0$$

$$W = 0 \quad \Rightarrow Q_{d\to a} = -756P_0V_0$$ ①

$$\Rightarrow e = 1 - \frac{(8\ln 2)P_0V_0 + 756P_0V_0}{1200P_0V_0} = 1 - \frac{(8\ln 2) + 756}{1200}$$ ①

$$= 1 - \frac{5.6 + 756}{1200} \approx 0.37$$

(b) For ideal gas system,

$$\Delta S=\frac{3}{2}nR\ln\frac{T_f}{T_i}+nR\ln\frac{V_f}{V_i}$$

$$=\frac{3}{2}nR\ln\frac{P_fV_f}{P_iV_i}+nR\ln\frac{V_f}{V_i}$$

$$=\frac{3}{2}nR\ln\frac{P_f}{P_i}+\frac{5}{2}nR\ln\frac{V_f}{V_i}$$

For process *a*→*b*,

$$\Delta S=nR\ln\frac{8V_0}{16V_0}=-R\ln 2$$ ①

For process *b*→*c*, (adiabatic)

$$\Delta S=0$$ ①

For process *c*→*d*,

$$\Delta S=\frac{5}{2}nR\ln\frac{16V_0}{V_0}=10R\ln 2$$ ①

For process *d*→*a*,

$$\Delta S=\frac{3}{2}nR\ln\frac{1/2P_0}{32P_0}=-9R\ln 2$$ ①

4.(a) conservation of angular momentum (relative to the point A)

$$\therefore \vec{L}_i = \vec{L}_f \quad ①$$

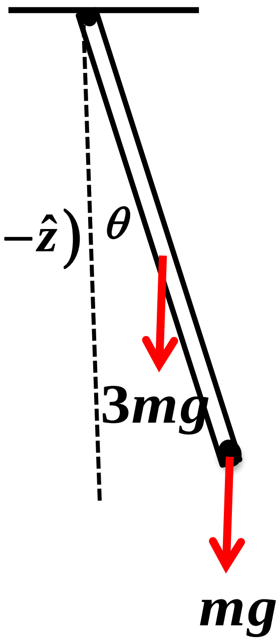
$$\vec{L}_i = lmv_0(\hat{z}) \quad ①$$

$$\vec{L}_f = I\omega(\hat{z}) \quad ①$$

and $\because I = \frac{1}{3}(3m)l^2 + ml^2 = 2ml^2 \quad ①$

$$\Rightarrow \omega = \frac{v_0}{2l} \quad ①$$

(b)



$$\tau = I\alpha$$

$$mgl\sin\theta(-\hat{z}) + 3mg\frac{l}{2}\sin\theta(-\hat{z}) = I\alpha(\hat{z}) \quad ②$$

$$\frac{d^2\theta}{dt^2} + \frac{\frac{5}{2}mgl}{2ml^2}\theta = 0$$

$$\Rightarrow \frac{d^2\theta}{dt^2} + \frac{5g}{4l}\theta = 0 \quad ①$$

$$\theta(t) = A\cos(\omega' t + \delta)$$

$$T = \frac{2\pi}{\omega'} = 2\pi\sqrt{\frac{4l}{5g}} \quad ②$$

(c) conservation of mechanical energy

$$\therefore E_i = E_f$$

$$E_i = \frac{1}{2} I\omega^2 \quad ①$$

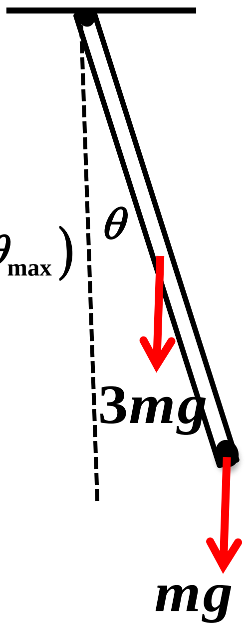
$$E_f = 3mg\frac{l}{2}\left(1-\cos\theta_{\max}\right)+mgl\left(1-\cos\theta_{\max}\right)$$

$$= \frac{5}{2}mgl\left(1-\cos\theta_{\max}\right) \quad ②$$

$$1-\cos\theta_{\max} = \frac{v^2}{10gl} \quad ①$$

or $\quad \theta_{\max} = \cos^{-1}\left(1-\frac{v^2}{10gl}\right)$

In small angle limit: $\quad \theta_{\max} = \sqrt{\frac{v^2}{5gl}}$



(c) Another method: boundary condition

$$\theta(t) = A\cos\left(\omega' t + \delta\right)$$

$$\theta(t=0) = 0 = A\cos\left(\delta\right) \Rightarrow \delta = \frac{\pi}{2} \text{ or } -\frac{\pi}{2} \quad ①$$

$$\frac{d\theta}{dt}(t=0) = \omega = -\omega' A\cos\left(\delta\right) \quad ①$$

$$\Rightarrow \quad \delta = -\frac{\pi}{2} \quad \text{and}$$

$$A = \theta_{\max} = \frac{\omega}{\omega'} = \frac{v_0/(2l)}{\sqrt{5g/(4l)}} = \frac{v_0}{\sqrt{5gl}} \quad ②$$