General Physics (II) **期中考 I** **Apr. 9, 2021**

試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(50 points) (所有題目必須有計算過程,否則不予計分)

1. (A) (6pts)As shown in Fig. 1(a), a semi-circular line charge distribution is located on the x-y plane. The radius of the semi-circle is R and the charge density $\lambda_c(\phi) = \lambda_0 \cos^2 \phi$. Determine the magnitude and the direction of the ***E***-field on the ***z***-axis for ***z*** ≥ 0. (B) (9pts) Now consider a semi-infinitely-long line charge distribution(from y = -∞ to y = 0) is added onto the ***y***-axis as shown in Fig. 1(b). The charge density is $\lambda_Y(y) = \lambda_0 R^2/(y^2 + R^2)$. Determine the magnitude and the direction of the ***E***-field at point ***P*** at (0,0,***R***) resulted from the semi-circular line charge distribution and the semi-infinitely-long line charge distribution.

2. (a) (8 pts) As shown in Fig. 2(a), there is a uniform volume charge distribution with density ***ρ*** (>0) in the region 0≤x ≤2***a***, and infinite in the y- and z- directions. Determine the magnitude and the direction of the electric field on the x-axis, in the range 0≤x≤4***a***.
(b) (12 pts) As shown in Fig. 2(b), an infinitely long cylindrical (圓柱) section with its axis parallel to the ***z***-axis and with radius ***a*** in the charge distribution is removed from the above slab shown in Fig. 2(a). Besides, a sphere (球) of radius ***a*** locates at (3***a***, 0, 0) with volume charge density of ***ρr'***/***a***, where ***r'*** is the distance to the center of the sphere. Determine the magnitude and the direction of the electric field at point ***P*** (5***a***/2, 0, 3***a***/8). (Use Gauss's Law, draw the Gaussian surfaces, and write down the answer in terms of ***ρ***, ***a***, and ε_0.)

3. (A) (8 pts) As shown in Fig.3(a), there are two concentric cylindrical conducting shells of radius ***a*** and 9***a***. The length of the shells is ℓ. Determine the capacitance of this system. (B) (7 pts) As shown in Fig.3(b), between these two shells there is another concentric cylindrical conductor with inner and outer radii 3***a*** and 4***a***, respectively. Determine the capacitance of this new system. (You need to indicate the Gaussian surface in your solution.)

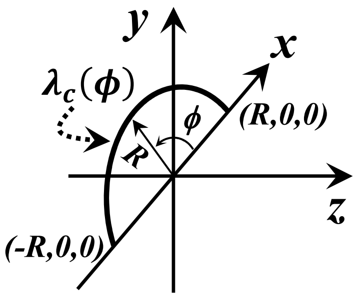


Fig. 1(a)

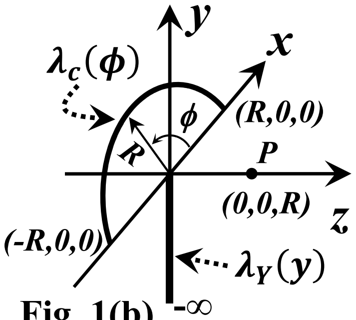


Fig. 1(b)

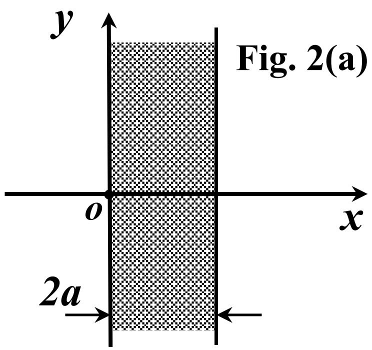


Fig. 2(a)

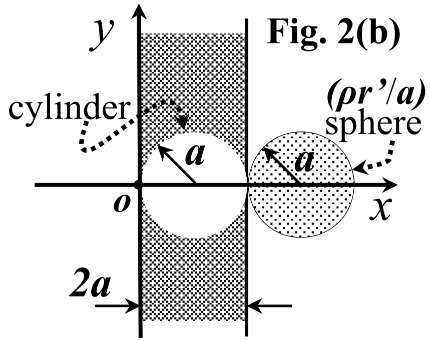


Fig. 2(b)

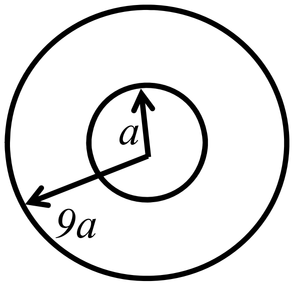


Fig. 3(a)

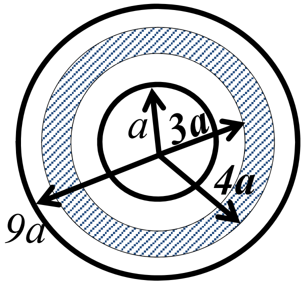


Fig. 3(b)

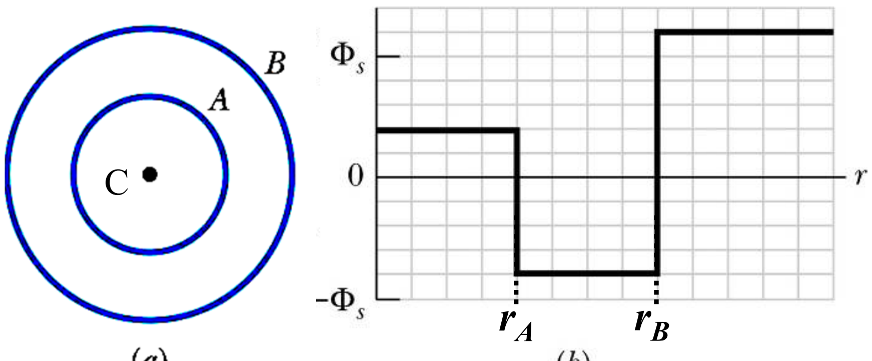


Fig. 4

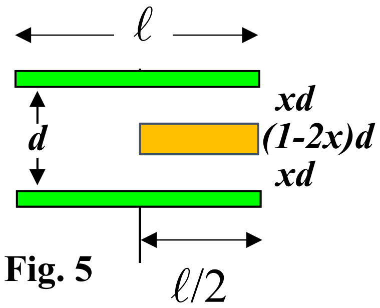


Fig. 5

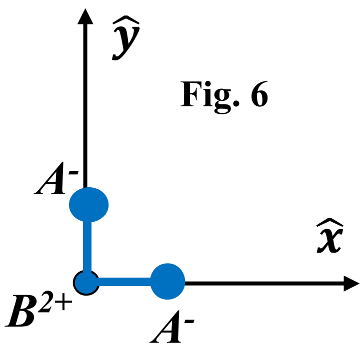


Fig. 6

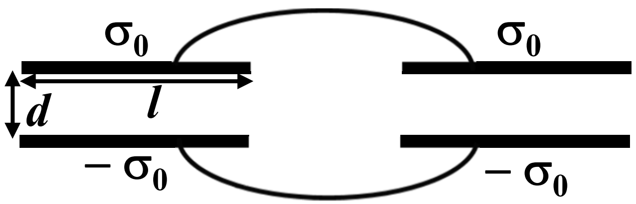


Fig. 7(a)

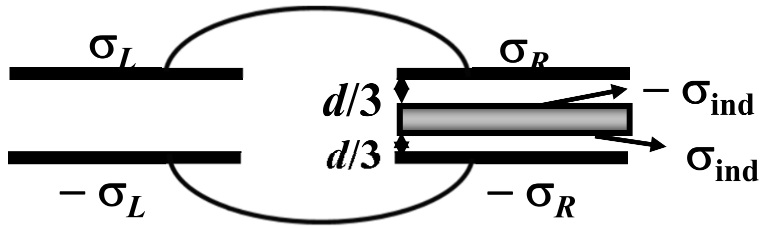


Fig. 7(b)

II.選擇題(50 points)

1. (5pts) A charge distribution is space results in a electric potential distribution $V(x,y,z) = \frac{2V_0}{3}x^2 - \frac{V_0}{2}y^2 - \frac{V_0}{6}z^2$, a dipole $\vec{p} = (0,-p,0)$ is located at $\vec{r} = (0,-\frac{a}{2},0)$. τ is the magnitude of the torque experienced by the dipole $\vec{p}$ due to the E-field associated to the potential $V(x,y,z)$.
   Let $x = \tau/(a \cdot p \cdot V_0)$, which of the following is correct?
   (A) $x<0.1$ (B) $0.1\leq x<1$ (C) $1\leq x<2$ (D) $2\leq x<5$ (E) $5\leq x<10$ (F) $10\leq x<100$ (G) $100\leq x$

2. (5pts) Fig.4(a) shows a charged particle is suspended at the center of two concentric spherical shells that are very thin and made of nonconducting material. Fig.4(b) gives the net flux $\boldsymbol{\Phi}$ through a Gaussian sphere centered on the particle, as a function of the radius $\boldsymbol{r}$ of the sphere. The scale of the vertical axis is set by $\boldsymbol{\Phi_s}$=5 Nm²/C. The net charges on the central particle, shell $\boldsymbol{A}$, and shell $\boldsymbol{B}$ are $\boldsymbol{Q_C}$, $\boldsymbol{Q_A}$, and $\boldsymbol{Q_B}$, respectively, and let $\boldsymbol{a}=(\boldsymbol{Q_C}+\boldsymbol{Q_B})/\boldsymbol{Q_A}$.
   (A) $\boldsymbol{a}$<-9; (B) -9≦$\boldsymbol{a}$<-7; (C) -7≦$\boldsymbol{a}$<-5; (D) -5≦$\boldsymbol{a}$<-3; (E) -3≦$\boldsymbol{a}$< -1; (F) -1≦$\boldsymbol{a}$<1; (G) 1≦$\boldsymbol{a}$< 3;
   (H) 3≦$\boldsymbol{a}$< 5; (J) 5≦$\boldsymbol{a}$< 7; (K) 7≦$\boldsymbol{a}$< 9; (L) 9≦$\boldsymbol{a}$.

3. (5 pts) A capacitor with original capacitance $\boldsymbol{C_0}$ is inserted a conductor half way and equal spacing between the plates, as indicated in Fig. 5. Now the capacitance of this capacitor turns to be $\boldsymbol{3C_0/2}$. What is x?
   (A) 1/10 (B) 1/8 (C) 1/6 (D) 1/5 (E) 1/4 (F) 1/3 (G) 2/5

4. (5pts) Consider a polar molecular $\boldsymbol{A_2B}$. Each $\boldsymbol{A}$ atom gains one $\boldsymbol{e}$; becomes $\boldsymbol{A^-}$, and the $\boldsymbol{B}$ atom loses 2$\boldsymbol{e}$; becomes $\boldsymbol{B^{2+}}$ such that the molecule has non-zero electric dipole moment (Fig. 6). Now it is placed in a uniform electric field $\vec{\boldsymbol{E}} = \boldsymbol{E_0\hat{z}}$ $(\boldsymbol{E_0 > 0})$. What is the rotating axis right after the electric field is turn on?
   (A) $\hat{z}$ (B) $-\hat{z}$ (C) $\hat{x}$ (D) $-\hat{x}$ (E) $\hat{y}$ (F) $-\hat{y}$ (G) $\hat{x}+\hat{y}$ (H) $-\hat{x}+\hat{y}$ (J) $\hat{x}-\hat{y}$
   (K) $-\hat{x}-\hat{y}$ (L) none of above

5. (5 pts) Two identical capacitors (Fig. 7(a)), containing two flat square conducting plates with area $\boldsymbol{A}$ **(=** $\boldsymbol{l^2}$**)** and a separation $\boldsymbol{d}$ **(**$\boldsymbol{d << l}$**).** Both capacitors are charging in parallel to store charge $\boldsymbol{Q}$ **(=** $\boldsymbol{\sigma_0 A}$**)** on each capacitor with a battery, then the battery is removed (Fig. 7(a)) but these two capacitors are still connected in parallel. Now an uncharged conducting plate with thickness $\boldsymbol{d/3}$ is placed into one of the capacitor and maintaining the same spacing $\boldsymbol{d/3}$ between the two conducting plates of the capacitor (Fig. 7(b)). Let $\boldsymbol{x = \sigma_{ind}/\sigma_0}$ **,** What is true for the value $\boldsymbol{x}$? You may neglect edge effects.

(A) $x = 0$ (B) $0 < x \le 0.2$, (C) $0.2 < x \le 0.4$, (D) $0.4 < x \le 0.6$, (E) $0.6 < x \le 0.8$,
(F) $0.8 < x \le 1.0$, (G) $1.0 < x \le 1.2$, (H) $1.2 < x \le 1.4$, (J) $1.4 < x \le 1.6$,
(K) $1.6 < x \le 1.8$, (L) $1.8 < x \le 2.0$, (M) $2.0 < x$.

6. (5pts) Same structure as in **problem 5,** but the battery is **connected to the capacitors all the time.** What is true for the value $\boldsymbol{x}$ **(**$\boldsymbol{x = \sigma_{ind}/\sigma_0}$**)**?

(A) $x = 0$ (B) $0 < x \le 0.2$, (C) $0.2 < x \le 0.4$, (D) $0.4 < x \le 0.6$, (E) $0.6 < x \le 0.8$,
(F) $0.8 < x \le 1.0$, (G) $1.0 < x \le 1.2$, (H) $1.2 < x \le 1.4$, (J) $1.4 < x \le 1.6$,
(K) $1.6 < x \le 1.8$, (L) $1.8 < x \le 2.0$, (M) $2.0 < x$.

Integration Formula for reference

$$\int \frac{dx}{\sqrt{x^2 \pm b^2}} = \ln\left(x + \sqrt{x^2 \pm b^2}\right) \qquad \int \frac{x^2 dx}{\sqrt{x^2 \pm b^2}} = \frac{x\sqrt{x^2 \pm b^2}}{2} - \frac{b^2}{2}\ln\left(x + \sqrt{x^2 \pm b^2}\right)$$

$$\int \frac{dx}{\left(x^2 \pm b^2\right)^{3/2}} = \frac{\pm x}{b^2\sqrt{x^2 \pm b^2}} \qquad \int \frac{x^2 dx}{\left(x^2 \pm b^2\right)^{3/2}} = \frac{-x}{\sqrt{x^2 \pm b^2}} + \ln\left(x + \sqrt{x^2 \pm b^2}\right)$$

Multiple Choice Questions:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A	E	E	H	G	J	M	B	C	F
11	**12**	**13**	**14**	**15**	**16**				
F	B	B	D	H	A				

1. (A) (6pts)As shown in Fig. 1(a), a semi-circular line charges distribution is located on the x-y plane. The radius of the semi-circle is R and the charge density $\lambda_c(\phi) = \lambda_0 \cos^2\phi$. Determine the magnitude and the direction of the ***E***-field on the ***z***-axis for ***z*** ≥ 0. (B) (9pts) Now consider a semi-infinite line charge distribution (from $y = -\infty$ to $y = 0$) is added onto the ***y***-axis as shown in Fig. 1(b). The charge density is $\lambda_Y(y) = \lambda_0 R^2/(y^2 + R^2)$. Determine the magnitude and the direction of the ***E***-field at point ***P*** at (0,0,***R***).

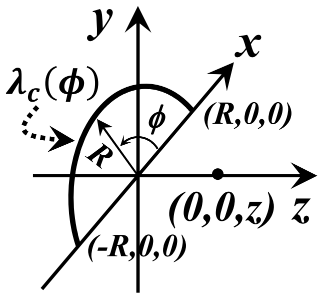


Fig. 1(a)

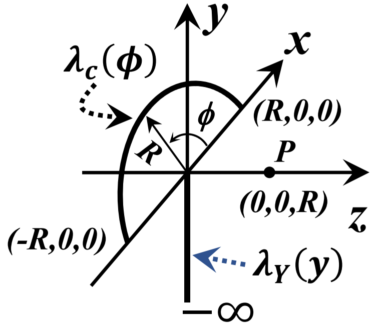


Fig. 1(b)

(A) For an arbitrary point (0,0,z) on z-axis,

$$\vec{r} = (0,0,z) - (R\cos\phi, R\sin\phi, 0)$$

$$= (-R\cos\phi, -R\sin\phi, z) \quad 1$$

$$dq = \lambda_0 \cos^2\phi \cdot R d\phi \quad 1$$

$$d\vec{E} = \frac{k\lambda_0 \cos^2\phi \cdot R d\phi(-R\cos\phi, -R\sin\phi, z)}{\sqrt{z^2+R^2}^3} \quad 1$$

$$\vec{E} = \int_0^\pi \frac{k\lambda_0(-R^2\cos^3\phi, -R^2\cos^2\phi\sin\phi, zR\cos^2\phi)}{\sqrt{z^2+R^2}^3} d\phi$$

$$E_x = \int_0^\pi \frac{-k\lambda_0 R^2 \cos^3\phi}{\sqrt{z^2+R^2}^3} d\phi$$

$$E_x = \int_0^\pi \frac{-k\lambda_0 R^2(1-\sin^2\phi)}{\sqrt{z^2+R^2}^3} d(\sin\phi)$$

$$= \frac{-k\lambda_0 R^2}{\sqrt{z^2+R^2}^3}\left(\sin\phi - \frac{\sin^3\phi}{3}\right)\Bigg|_0^\pi = 0 \quad 1$$

$$E_y = \int_0^\pi \frac{-k\lambda_0 R^2\cos^2\phi\sin\phi}{\sqrt{z^2+R^2}^3} d\phi$$

$$= \int_0^\pi \frac{k\lambda_0 R^2\cos^2\phi}{\sqrt{z^2+R^2}^3} d(\cos\phi)$$

$$= \frac{k\lambda_0 R^2}{\sqrt{z^2+R^2}^3}\left(\frac{\cos^3\phi}{3}\Bigg|_0^\pi\right) = -\frac{2}{3}\frac{k\lambda_0 R^2}{\sqrt{z^2+R^2}^3} \quad 1$$

$$E_z = \int_0^\pi \frac{k\lambda_0 zR\cos^2\phi}{\sqrt{z^2+R^2}^3} d\phi$$

$$= \frac{k\lambda_0 zR}{\sqrt{z^2+R^2}^3}\int_0^\pi \left(\frac{1+\cos 2\phi}{2}\right) d\phi$$

$$= \frac{\pi}{2}\frac{k\lambda_0 zR}{\sqrt{z^2+R^2}^3} \quad 1$$

For $z>0$, the $\boldsymbol{E}$-field on the z-axis is

$$\vec{E}=(0,-\frac{2}{3}\frac{k\lambda_0 R^2}{\sqrt{z^2+R^2}^3},\frac{\pi}{2}\frac{k\lambda_0 zR}{\sqrt{z^2+R^2}^3})$$

(B) The E-field at point (0,0,R) from the semi-infinite line charge distribution,

$$\vec{r}=(0,0,R)-(0,y,0)=(0,-y,R) \quad 1$$

$$dq=\frac{\lambda_0 R^2}{y^2+R^2}dy \quad 1$$

$$d\vec{E}=\frac{k(0,-y,R)}{\sqrt{y^2+R^2}^3}\frac{\lambda_0 R^2}{y^2+R^2}dy \quad 1$$

$$\vec{E}=\int_{-\infty}^{0}\frac{k(0,-y,R)}{\sqrt{y^2+R^2}^3}\frac{\lambda_0 R^2}{y^2+R^2}dy$$

$$E_x=0$$

$$E_y=\int_{-\infty}^{0}\frac{-k\lambda_0 R^2 y}{\sqrt{y^2+R^2}^5}dy, \quad u\equiv y^2+R^2$$

$$E_y=\frac{-k\lambda_0 R^2}{2}\int_{\infty}^{R^2}\frac{du}{u^{5/2}}$$

$$=\frac{-k\lambda_0 R^2}{2}\left(-\frac{2}{3u^{3/2}}\Big|_0^{R^2}\right)=\frac{k\lambda_0}{3R} \quad 2$$

$$E_z=\int_{-\infty}^{0}\frac{k\lambda_0 R^3}{\sqrt{y^2+R^2}^5}dy, \quad y\equiv R\tan\theta$$

$$=k\lambda_0 R^3\int_{-\frac{\pi}{2}}^{0}\frac{R\sec^2\theta\, d\theta}{\sqrt{R^2\tan^2\theta+R^2}^5}$$

$$=\frac{k\lambda_0}{R}\int_{-\frac{\pi}{2}}^{0}\frac{\sec^2\theta\, d\theta}{\sec^5\theta}=\frac{k\lambda_0}{R}\int_{-\frac{\pi}{2}}^{0}\cos^3\theta\, d\theta$$

$$=\frac{k\lambda_0}{R}\int_{-\frac{\pi}{2}}^{0}(1-\sin^2\theta)d(\sin\theta)$$

$$=\frac{k\lambda_0}{R}\left(\sin\theta-\frac{\sin^3\theta}{3}\right)\Big|_{-\frac{\pi}{2}}^{0}=\frac{2k\lambda_0}{3R} \quad 2$$

The E-field at point (0,0,R) from the semi circular line charge is,

$$\vec{E}=\left(0,-\frac{2}{3}\frac{k\lambda_0 R^2}{\sqrt{R^2+R^2}^3},\frac{\pi}{2}\frac{k\lambda_0 RR}{\sqrt{R^2+R^2}^3}\right)$$

$$=\left(0,-\frac{1}{3\sqrt{2}}\frac{k\lambda_0}{R},\frac{\pi}{4\sqrt{2}}\frac{k\lambda_0}{R}\right) \quad 2$$

The total E-field at point (0,0,R) is,

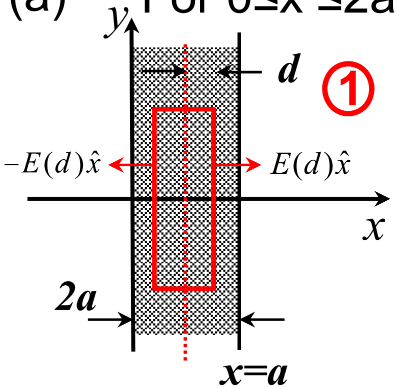
$$\vec{E}=\left(0,-\frac{1}{3\sqrt{2}}\frac{k\lambda_0}{R},\frac{\pi}{4\sqrt{2}}\frac{k\lambda_0}{R}\right)+\left(0,\frac{k\lambda_0}{3R},\frac{2k\lambda_0}{3R}\right)$$

$$=\left(0,\frac{1}{3}-\frac{1}{3\sqrt{2}},\frac{2}{3}+\frac{\pi}{4\sqrt{2}}\right)\frac{k\lambda_0}{R}$$

1.(a) (8 pts) As shown in Fig. 2(a), there is a uniform volume charge distribution with density ρ (>0) in the region 0≤x ≤2a, and infinite in the y- and z- directions. Determine the magnitude and the direction of the electric field on the x-axis, in the range 0≤x≤4a.

(b) (12 pts) As shown in Fig. 2(b), an infinitely long cylindrical (圓柱) section with the z-axis as its central axis and with radius a in the charge distribution is removed. Besides, a sphere of radius *a* locates at (3a, 0, 0) with volume charge density of ρr'/a, where r' is the distance to the center of the sphere. Determine the magnitude and the direction of the electric field at point P (5a/2, 0, 3a/8). (Use Gauss's Law, draw the Gaussian surfaces, and write down the answer in terms of ρ, a, and ε_0.)

(a) For 0≤x ≤2a



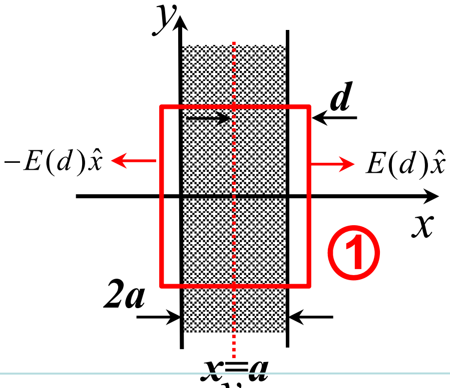
$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

$$2E(d)A = \frac{\rho\cdot 2dA}{\varepsilon_0},$$ ①

$$\Rightarrow E(d) = \frac{\rho}{\varepsilon_0}d$$ ①

$$\vec{E}(x) = \frac{\rho}{\varepsilon_0}(x-a)\hat{x}$$ ①

For 2a<x ≤4a

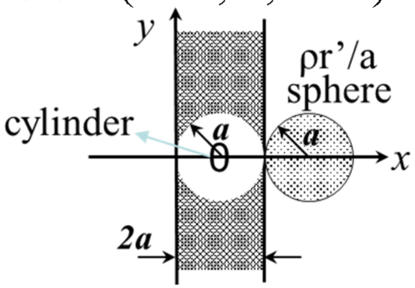


$$2E(d)A = \frac{\rho\cdot 2aA}{\varepsilon_0},$$ ①

$$\Rightarrow E(d) = \frac{\rho a}{\varepsilon_0}$$ ①

$$\vec{E}(x) = \frac{\rho a}{\varepsilon_0}\hat{x}$$ ①

(b) (5a/2, 0, 3a/8). ②



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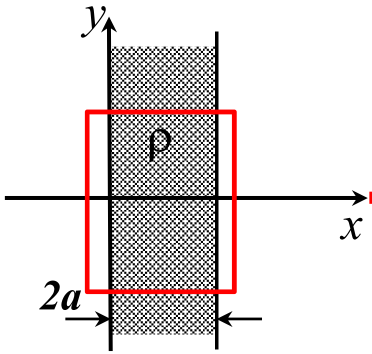

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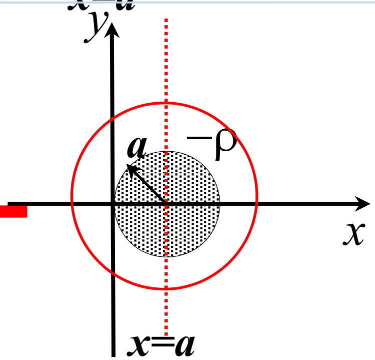

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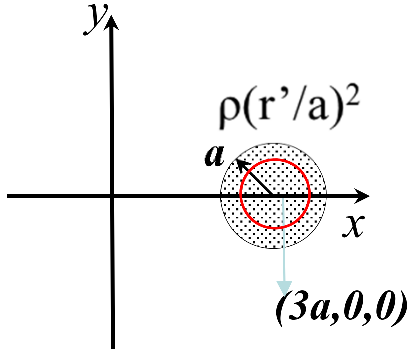


$$\vec{E}_s(P) = \frac{\rho a}{\varepsilon_0}\hat{x}$$ ①

$$2\pi r\ell E_c(r) = \frac{(-\rho)\cdot\pi a^2\ell}{\varepsilon_0}$$ ①

$$\vec{E}_c(r) = -\frac{\rho\cdot a^2}{2r\varepsilon_0}\hat{r}$$ ①

$\vec{r} = (5a/2, 0, 3a/8) - (a, 0, z)$, z=3a/8 ①

$$\vec{E}_c(P) = -\frac{\rho a}{3\varepsilon_0}\hat{x}$$ ①

$$4\pi r^2 E_{sp}(r) = \frac{1}{\varepsilon_0}\int_0^r \rho\left(\frac{r'}{a}\right)^2 4\pi r'^2\, dr'$$ ①

$$\vec{E}_{sp}(r) = \frac{\rho\cdot r^2}{4\varepsilon_0 a}\hat{r}$$ ①

① $\vec{r} = (5a/2, 0, 3a/8) - (3a, 0, 0) = \frac{a}{8}(-4, 0, 3)$

$$\vec{E}_{sp}(P) = \frac{5\rho a}{256\varepsilon_0}[-4\hat{x}+3\hat{z}]$$ ①

$$\boxed{\begin{aligned}\vec{E}_{total}(P) &= \vec{E}_s(P) + \vec{E}_c(P) + \vec{E}_{sp}(P)\\ &= \frac{2\rho a}{3\varepsilon_0}\hat{x} + \frac{5\rho a}{256\varepsilon_0}[-4\hat{x}+3\hat{z}] = \frac{113\rho a}{192\varepsilon_0}\hat{x} + \frac{15\rho a}{256\varepsilon_0}\hat{z}\end{aligned}}$$ ①

(a) Calculate the capacitance of this system:

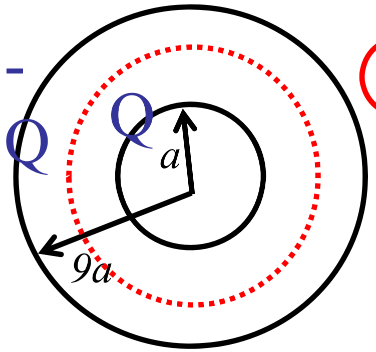


Fig. 2(a)

apply Gauss's Law with radius a<r<9a 1

$$\Phi_{E.S1} = \oiint_{S1} \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

$$E_I \cdot 2\pi r \cdot d = \frac{1}{\varepsilon_0}\frac{Q_a}{l}\cdot d$$

$$E_I = \frac{Q_a/l}{2\pi\varepsilon_0}\frac{1}{r}$$ 2

$$\Delta V = V(9a)-V(a) = -\int_a^{9a} \vec{E}\cdot d\vec{r}$$ 1

$$= -\int_a^{9a} \frac{Q_a/l}{2\pi\varepsilon_0}\frac{1}{r}\cdot dr = -\frac{Q_a/l}{2\pi\varepsilon_0}\ln 9$$ 2

1 $$C = \frac{Q}{|\Delta V|} = \frac{2\pi\varepsilon_0 l}{\ln 9}$$ 1

(b) Calculate the capacitance of the new system:

$$\Delta V = V(9a)-V(a) = -\int_a^{9a} \vec{E}\cdot d\vec{r}$$

$$= -\int_a^{3a} \vec{E}_I\cdot d\vec{r} - \int_{3a}^{4a} \vec{E}_{II}\cdot d\vec{r} - \int_{4a}^{9a} \vec{E}_{III}\cdot d\vec{r}$$

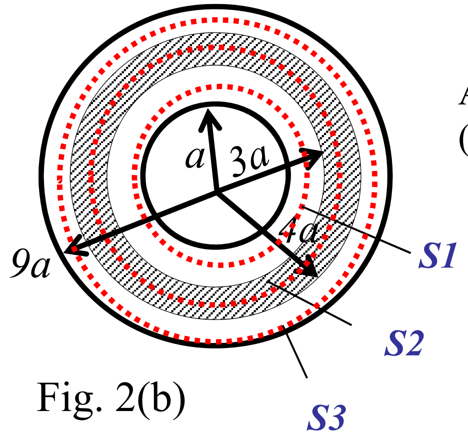


Fig. 2(b)

Apply similar method in problem (A) to the region a<r<3a

apply Gauss's Law to sphere S1,

$$\Rightarrow E_I = \frac{Q_a/l}{2\pi\varepsilon_0}\frac{1}{r}$$ 1

$$V(3a)-V(a) = -\int_a^{3a} \vec{E}_I\cdot d\vec{r} = -\frac{Q_a/l}{2\pi\varepsilon_0}\ln 3$$ 1

Apply similar method in problem (A) to the region 3a<r<4a

There is no electric field inside the metal. $E_{II} = 0$ 1

$$\Rightarrow V(4a)-V(3a) = -\int_{3a}^{4a} \vec{E}_{II}\cdot d\vec{r} = 0$$ 1

Apply similar method in problem (A) to the region 4a<r<9a

apply Gauss's Law to sphere S3, $\Rightarrow E_{III} = \frac{Q_a/l}{2\pi\varepsilon_0}\frac{1}{r}$ 1

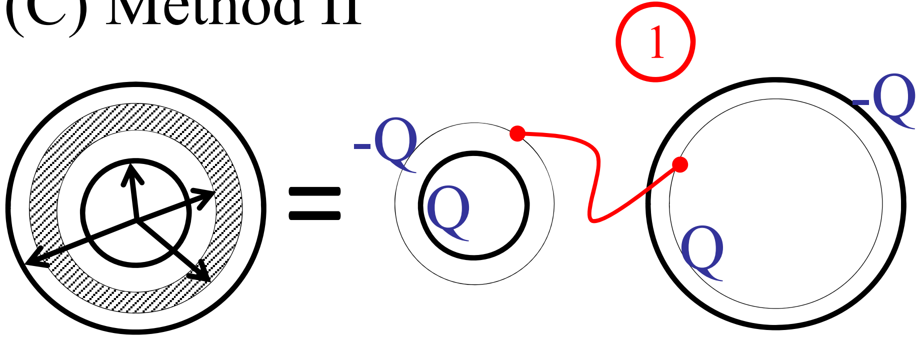
$$V(9a)-V(4a) = -\int_{4a}^{9a} \frac{Q_a/l}{2\pi\varepsilon_0}\frac{1}{r}\cdot dr = -\frac{Q_a/l}{2\pi\varepsilon_0}\ln\frac{9}{4}$$ 1

$$\Rightarrow C = \frac{Q}{|\Delta V|} = \frac{2\pi\varepsilon_0 l}{\ln(9/4)+\ln 3} = \frac{2\pi\varepsilon_0 l}{\ln(27/4)}$$ 1

a $3a$ $6a$ $4a$

a

(C) Method II



in series (串聯)

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} \quad 1$$

$$C_1 = \frac{2\pi\varepsilon_0 l}{\ln(3)} \quad 2 \quad \text{and} \quad C_2 = \frac{2\pi\varepsilon_0 l}{\ln(9/4)} \quad 2$$

$$C_{total} = \frac{2\pi\varepsilon_0 l}{\ln(9/4)+\ln 3} = \frac{2\pi\varepsilon_0 l}{\ln(27/4)} \quad 1$$