Signals and Systems – Quiz 2 04/20/2020

Name: Student ID:

1. (20%) Let $y(t)$ denote the convolution of the following two signals. Determine and plot $y(t)$

$$x(t) = e^{3t}u(-t)$$
$$h(t) = u(t-2)$$

Solution:

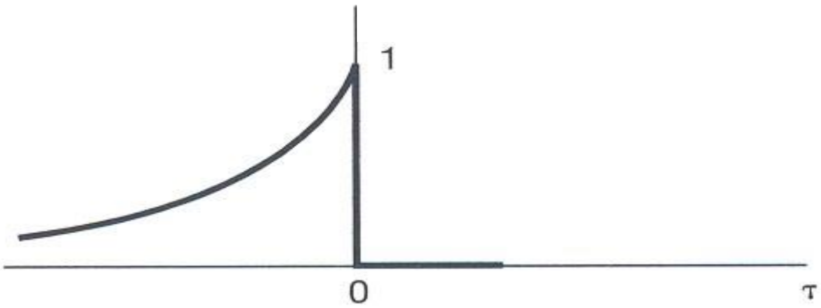
When $t-2 \leq 0$, the product of $x(\tau)$ and $h(t-\tau)$ is nonzero for $-\infty < \tau < t-2$, and the convolution integral becomes

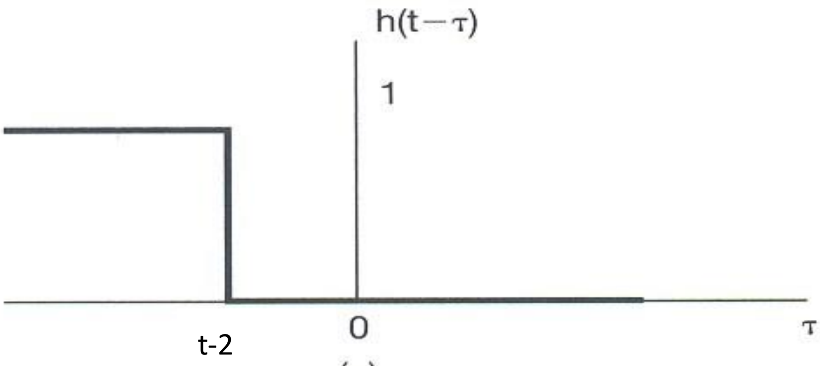
$$y(t) = \int_{-\infty}^{t-2} e^{3\tau} d\tau = \frac{1}{3} e^{3(t-2)}$$

When $t-2 \geq 0$, the product of $x(\tau)$ and $h(t-\tau)$ is nonzero for $-\infty < \tau < 0$, and the convolution integral becomes

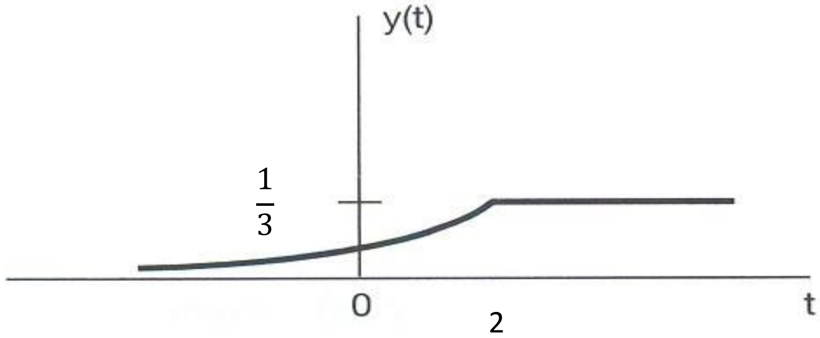
$$y(\mathrm{t}) = \int_{-\infty}^{0} e^{3\tau} d\tau = \frac{1}{3}$$

$$x(\tau) = e^{3\tau}u(-\tau)$$





(a)



2. (40%) The input-output relationship of an LTI system is described as

$$y[n] - \frac{1}{3}y[n-1] = x[n].$$

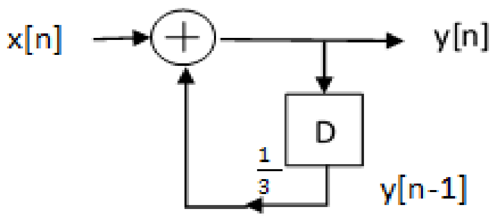
(a) Draw the block diagram representation for this system.

(b) What is the impulse response of this system?

(c) Suppose $x[n] = \left(\frac{1}{2}\right)^n u[n]$, find the particular and homogeneous solutions of this system.

Solution:

(a)



(b) Let $x[n] = K\delta[n]$ and $y[n] = 0$ at $n < 0$

$$y[0] = \frac{1}{3}y[-1] + x[0] = K$$

$$y[1] = \frac{1}{3}y[0] + x[1] = \frac{1}{3}K$$

$$y[2] = \frac{1}{3}y[1] + x[2] = (\frac{1}{3})^2 K$$

$$\vdots$$

$$y[n] = \frac{1}{3}y[n-1] + x[n] = (\frac{1}{3})^n K$$

$$h[n] = (\frac{1}{3})^n u[n]$$

(a) Let guess the homogeneous solution $y_h[n] = A(1/3)^n u[n]$

It shows that $A\left(\frac{1}{3}\right)^n - \frac{1}{3}A\left(\frac{1}{3}\right)^{n-1} = 0$

Particular solution $y_p[n] = B(1/2)^n u[n]$

$$B\left(\frac{1}{2}\right)^n - \frac{1}{3}B\left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^n \quad \text{Therefore, } \mathrm{B} = 3.$$

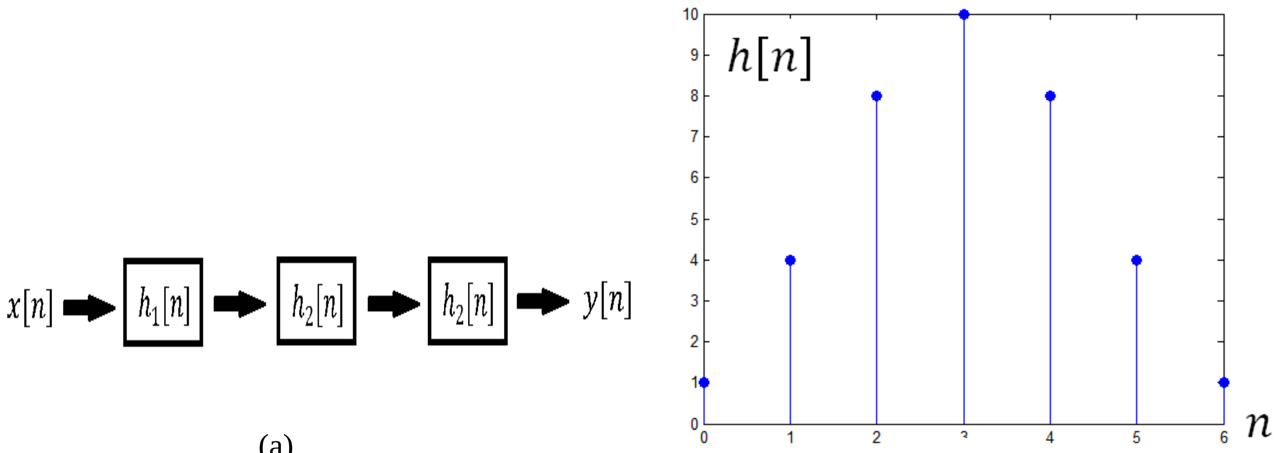
Initial rest, y[-1]=0, $y[0] = x[0] + (1/3)y[-1] = \mathrm{x}[0] = 1$. Now we also have

$$y[n] = y_p[n] + y_h[n] = A(1/3)^n u[n] + B(1/2)^n u[n]$$

$$y[0] = \mathrm{A} + \mathrm{B} = 1,\ \mathrm{A} = 1 - \mathrm{B} = -2$$

$$y[n] = y_p[n] + y_h[n] = \left(3\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{3}\right)^n\right)u[n]$$

3. (40%) Consider the cascade interconnection of three *causal* LTI systems, illustrated in Fig. 1(a). The impulse response $h_2[n]$ is : $h_2[n] = u[n] - u[n-3]$,
and the overall impulse response is as shown in Fig. 1(b).



(a) (b)

Fig. 1

(a) Find the impulse response $h_1[n]$.

(b) Find the response of the overall system to the input $x[n] = \delta[n] - \delta[n-1] - \delta[n-2]$.

Solution:

(a) Given that $h_2[n] = u[n] - u[n-3] = \delta[n] + \delta[n-1] + \delta[n-2]$.

$$h_2[n] * h_2[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 2\delta[n-3] + \delta[n-4].$$

Since $h[n] = h_1[n] * \left[h_2[n] * h_2[n]\right]$,

We get

$$h[n] = h_1[n] + 2h_1[n-1] + 3h_1[n-2] + 2h_1[n-3] + h_1[n-4].$$

Therefore

$$h[0] = h_1[0] \rightarrow h_1[0] = 1$$
$$h[1] = h_1[1]+2h_1[0] \rightarrow h_1[1] = 2$$
$$h[2] = h_1[2]+2h_1[1]+3h_1[0] \rightarrow h_1[2] = 1$$
$$h[3] = h_1[3]+2h_1[2]+3h_1[1]+2h_1[0] \rightarrow h_1[3] = 0$$
$$h[4] = h_1[4]+2h_1[3]+3h_1[2]+2h_1[1]+h_1[0] \rightarrow h_1[4] = 0$$
$$h[5] = h_1[5]+2h_1[4]+3h_1[3]+2h_1[2]+h_1[1] \rightarrow h_1[5] = 0$$
$$h[6] = h_1[6]+2h_1[5]+3h_1[4]+2h_1[3]+h_1[2] \rightarrow h_1[6] = 0$$

$$h_1[n] = 0 \text{ for } n < 0.$$

(b) In this case, $y[n] = x[n] * h[n] = h[n] - 2h[n-1] - h[n-2]$.