試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(55 points) (所有題目必須有計算過程,否則不予計分)

1&2. Fig. 1 shows a cannon located at the origin of the coordinate system launches a ball in the x-direction with a take-off angle θ = 37^0, and the initial speed of the ball is 50 m/s. Assume the gravitational acceleration $\boldsymbol{g}$ = 10m/s^2 in $-\hat{z}$ direction.

(A) (5pts)A cannonball is fired at $t = 0$ sec, find the time the ball flies and the coordinate of the position where the ball lands.

(B) (15pts)A cannonball is fired at $t = 0$ sec, again. In the meantime, a strong wind begins to accelerate the cannonball with the acceleration $\vec{a}(t) = 6t\hat{x} + e^{-t}\hat{y}$ (m/s^2). If the wind stops at $t = 1$ sec, find the coordinate of the position where the ball lands.

3. (a) (10 pts) As shown in Fig. 2(a), the force $\boldsymbol{F}$ is applied to block $\boldsymbol{m_1}$ with block $\boldsymbol{m_2}$ on top of it. There is no friction between $\boldsymbol{m_1}$ and the flat surface. If $\boldsymbol{m_1}$.= 3kg, $\boldsymbol{m_2}$ = 5kg, $\boldsymbol{\theta}$=53° , and the static and kinetic friction coefficients $\boldsymbol{\mu_s}$ and $\boldsymbol{\mu_k}$ between $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ are 1/2 and 1/3, respectively. Determine the range of $\boldsymbol{F}$ that can keep $\boldsymbol{m_2}$ static on $\boldsymbol{m_1}$. (b) (10 pts)As shown in Fig. 2(b), if the force $\boldsymbol{F}$ is removed and both of the blocks are initially static, what would be the acceleration of $\boldsymbol{m_1}$? (g = 10 m/s^2, sin(53^o)=4/5)
(You need to draw free-body diagram in your solution.)

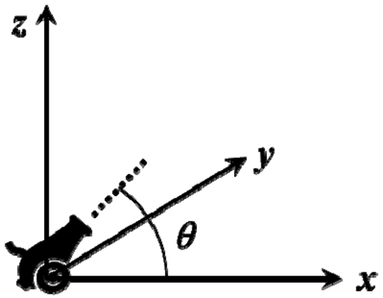


Fig. 1

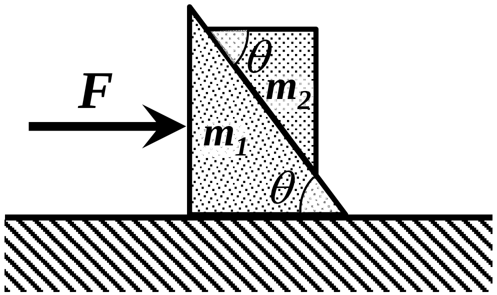


Fig. 2(a)

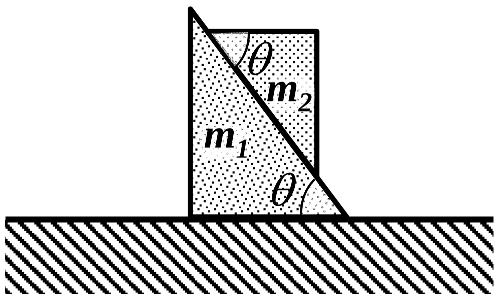


Fig. 2(b)

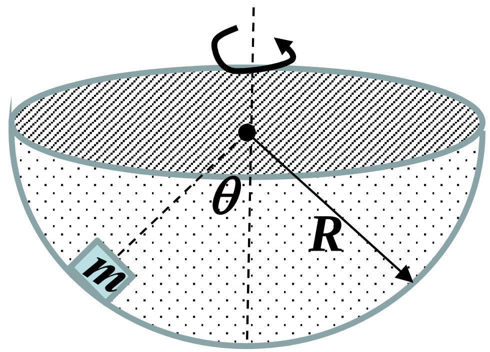


Fig. 3

4. (15 pts) A small block of mass ***m*** is at rest inside an rotating half sphere with the angle $\boldsymbol{\theta}$ relative to the axis of rotation (Fig. 3) . The period of time for one revolution of the half sphere is $\boldsymbol{T}$. The static friction coefficient between the block and the inside surface of the cone is $\boldsymbol{\mu}$. Write your answers in terms of ***g, m, R,*** and $\boldsymbol{\theta}$.

(a) (10 pts) Find the maximum period $\boldsymbol{T_{max}}$ and the minium period $\boldsymbol{T_{min}}$ of the block, such that the small block can remain the same angle $\boldsymbol{\theta}$.

(b) (5 pts) Find the periods of the block $\boldsymbol{T_0}$ such that the small block can remain the same angle $\boldsymbol{\theta}$ with zero friction force between the small block and the half sphere

(You need to draw free-body diagram each case.)

II.選擇題(52 points)

1. (4pts) Newton's 2nd law of motion shows $F = ma$ where the force F acting on a particle is related to its mass m and acceleration a. Newton's law of gravitation shows $F = Gm_1m_2/r^2$ where an attractive force F exists between particles with mass m_1 and m_2, and r is the distance between them. The dimension of G is $[L]^{\alpha}[M]^{\beta}[T]^{\gamma}$, in which (A) $\alpha = 1, \beta = 2, \gamma = 3$; (B) $\alpha = -1, \beta = 2, \gamma = -3$; (C) $\alpha = 3, \beta = 1, \gamma = 2$; (D) $\alpha = 3, \beta = -1, \gamma = 2$; (E) $\alpha = -3, \beta = -1, \gamma = -2$; (F) $\alpha = 3, \beta = -1, \gamma = -2$; (G) $\alpha = -3, \beta = 1, \gamma = 2$, respectively.

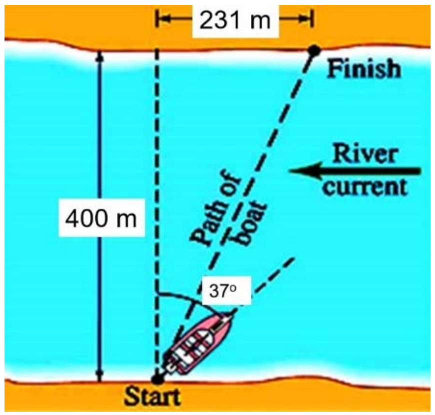
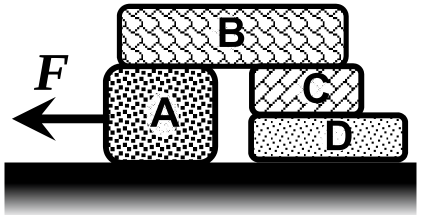


Fig. 4

Fig. 5



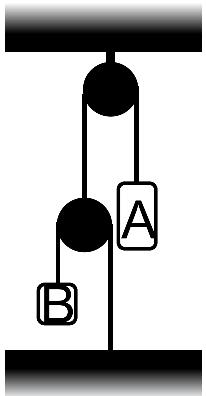


Fig. 6

Fig. 7(a)

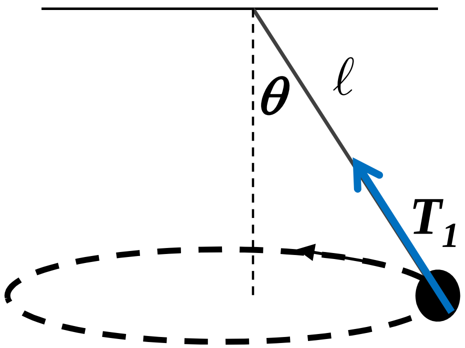
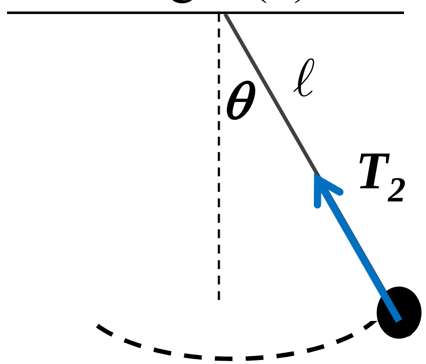


Fig. 7(b)



2. (4pts) The boat in Fig. 4 must cross a 400-m-wide river and arrive at a point 231($\sim 400/\sqrt{3}$) m upstream from where it starts as shown in figure. The speed of the river is 3 m/s. To do so, the captain must head the boat at a 37° upstream angle. The speeds of the boat relative to the bank and the boat relative to the river are about (A) 30 m/s, 32m/s; (B) 25 m/s, 26 m/s; (C) 20 m/s, 21m/s; (D) 15 m/s, 16 m/s; (E) 16 m/s, 15 m/s; (F) 21 m/s, 20 m/s; (G) 26 m/s, 25 m/s ; (H) 32 m/s, 30 m/s, respectively. ($\sqrt{2} = 1.41;\ \sqrt{3} = 1.73;\ \sin\theta \sim \theta,\ for\ \theta << 1$)

3. (4pts) As shown in Fig. 5, block A,B,C, and D are placed on a frictionless surface. The force F drags block A toward the left, and the friction between the blocks make block B, C, and D to travel together with block A. If the number of forces applied on block A, B, C, and D are $\boldsymbol{n_A}$, $\boldsymbol{n_B}$, $\boldsymbol{n_C}$, and $\boldsymbol{n_D}$, respectively, and $\boldsymbol{N} = \boldsymbol{n_A} + \boldsymbol{n_B} + \boldsymbol{n_C} + \boldsymbol{n_D}$. Which of the following is correct?
(A) N $\leq$ 10 (B) 10<N$\leq$13 (C) 13<N$\leq$16 (D) 16<N$\leq$19 (E) 19<N$\leq$22 (F) 22<N$\leq$24 (F) 24<N

4. (4 pts) Fig. 7(a) shows a ball is held in circular motion horizontally. The force of the rope is $\boldsymbol{T_1}$. Fig. 7(b) shows that a ball is in pendulum motion and its highest point is at the angle $\boldsymbol{\theta} = 37^{\circ}$ and the tension force is $\boldsymbol{T_2}$. What is the ratio $\boldsymbol{r} = \boldsymbol{T_1} / \boldsymbol{T_2}$?.
(A) $\boldsymbol{r} < 0.4$ (B) $0.4 \leq \boldsymbol{r} < 0.6$ (C) $0.6 \leq \boldsymbol{r} < 0.8$ (D) $0.8 \leq \boldsymbol{r} < 1.0$ (E) $1.0 \leq \boldsymbol{r} < 1.2$
(F) $1.2 \leq \boldsymbol{r} < 1.4$ (G) $1.4 \leq \boldsymbol{r} < 1.6$ (H) $1.6 \leq \boldsymbol{r} < 1.8$ (J) $1.8 \leq \boldsymbol{r} < 2.0$ (K) $2.0 \leq \boldsymbol{r}$

5. (4pts) As shown in Fig. 6, block A and B are connected to the strings of the pulley system, let $\boldsymbol{a_A}$ and $\boldsymbol{a_B}$ be the acceleration of block A and B, respectively, and $\boldsymbol{x} = \boldsymbol{a_A} / \boldsymbol{a_B}$, which of the following is correct?

(A) $x \leq -3$ (B) $-3 < x \leq -1$ (C) $-1 < x \leq 0$ (D) $0 < x \leq 1$ (E) $1 < x \leq 3$ (F) $3 < x$

6 (4 pts) A merry-go-round is spinning with a fixed angular speed. As a person is walking radially from the center towards the edge, which of the following statement about the friction between the person and the mary-go-around is correct?

(A) The static frictional force increases. (B) The static frictional force decrease.
(C) The kinetic frictional force increases. (D) The kinetic frictional force decreas.
(E) The frictional force remains the same.

7. (4 pts) The elevator (Fig. 8) is moving upward at a ***decreasing*** speed.
The magnitude of the force of the bottom block on the top block is $\boldsymbol{F_1}$.
The magnitude of the force of the earth on the top block is $\boldsymbol{F_2}$.
The magnitude of the force of the top block on the bottom block is $\boldsymbol{F_3}$.

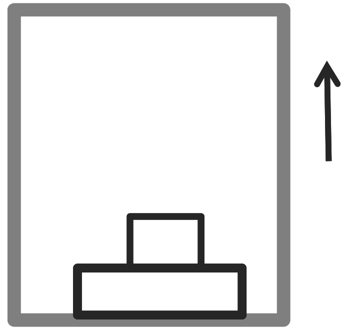


Fig. 8

(A) $F_1 = F_2 = F_3$ **(B)** $F_1 \neq F_2 \neq F_3$ **(C)** $F_1 > F_2 = F_3$ **(D)** $F_1 < F_2 = F_3$
(E) $F_2 > F_1 = F_3$ **(F)** $F_2 < F_1 = F_3$ **(G)** $F_3 > F_1 = F_2$ **(H)** $F_3 < F_1 = F_2$
(I) none of above

8. (4 pts) A small bead is rolling vertically inside a circular orbit with radius R, as shown in Fig 9. At some moment, the direction of the net force exerting on the bead is along $-\boldsymbol{x}$. What is the possible position(s) of the bead? (note the gravity: $\vec{g} = -9.8\hat{y}$)

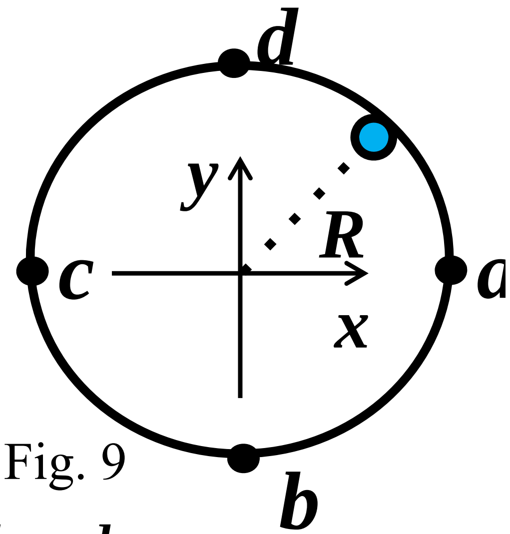


Fig. 9

(A) point ***a*** (B) point ***b*** (C) point ***c*** (D) point ***d*** (E) point ***a*** or ***c*** (F) point ***b*** or ***d***
(G) Some point between ***a*** and ***b*** (H) Some point between ***b*** and ***c***
(J) Some point between ***c*** and ***d*** (K) Some point between ***d*** and ***a***
(L) Both **(G)** and **(H)** are possible (M) Both **(J)** and **(K)** are possible (N) None of above

Multiple Choice Questions:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
F	C	D	G	C	A	E	G	F
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**
A	B	C	D	G	C	C	F	B

1. (20 pts) Figure shows a cannon located at the origin of the coordinate system launches a ball in the x-direction with a take-off angle $\theta = 37^0$, and the initial speed of the ball is 50 m/s. Assume the gravitational acceleration $\boldsymbol{g} = 10\text{m/s}^2$ in $-\hat{z}$ direction.

(A) A cannonball is fired at $\boldsymbol{t = 0}$ sec, find the time the ball flies and the coordinate of the position where the ball lands.

(B) A cannonball is fired at $\boldsymbol{t = 0}$ sec, again. In the meantime, a strong wind begins to accelerate the cannonball with the acceleration $\vec{a} = 6t\,\hat{x} + e^{-t}\,\hat{y}$ (m/s²). If the wind stops at $\boldsymbol{t = 1}$ sec, find the coordinate of the position where the ball lands.

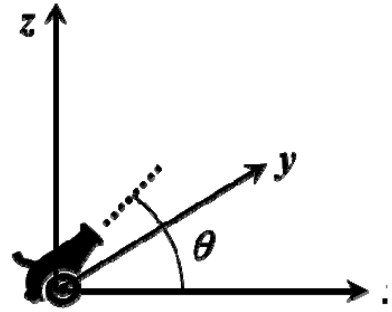


Fig.

(A) $\vec{v}(t=0) = 40\hat{x} + 30\hat{z}(m/x), \quad \vec{r}(t=0) = 0\hat{x} + 0\hat{z}(m)$

$$\vec{a} = -g\hat{z} = -10\hat{z}(m/s^2) \Rightarrow \begin{cases} v_z(t) = 30 - 10t \\ v_x(t) = 40 \end{cases} \Rightarrow \begin{cases} z(t) = 30t - 5t^2 \text{ ①} \\ x(t) = 40t \text{ ①} \end{cases}$$

$$z(t_1) = 30t_1 - 5t_1^2 = 0$$
$$\Rightarrow t_1 = 0, or\ \boxed{6\sec} \text{ ①}$$
$$\Rightarrow x(t_1) = 240m \text{ ①}$$

The coordinate of the position where the ball lands is (240 m, 0, 0). ①

(B) $0 \le t < 1, \quad \vec{a} = 6t\hat{x} - e^{-t}\hat{y} - 10\hat{z}(m/s^2) \qquad \vec{v}(t=0) = 40\hat{x} + 30\hat{z}(m/x),$

$1 \le t \qquad \vec{a} = 6t\hat{x} - e^{-t}\hat{y} - 10\hat{z}(m/s^2) \qquad \vec{r}(t=0) = 0\hat{x} + 0\hat{z}(m)$

In x-direction,

$0 \le t < 1, \quad v_x(t) = 3t^2 + 40$ ① $\Rightarrow v_x(t=1) = 43(m/s)$ ①

$x(t) = t^3 + 40t$ ① $\Rightarrow x(t=1) = 41(m)$ ①

$1 \le t, \quad v_x(t) = 43(m/s)$ ①

$\Rightarrow x(t=1) = 43(t-1) + 41 = 43t - 2(m)$ ①

In y-direction,

$0 \le t < 1, \quad v_y(t) = -e^{-t} + 1$ ① $\Rightarrow v_y(t=1) = 1 - e^{-1}(m/s)$ ①

$y(t) = e^{-t} + t - 1$ ① $\Rightarrow y(t=1) = e^{-1}(m)$ ①

$1 \le t, \quad v_y(t) = 1 - e^{-1}(m/s)$ ①

$y(t) = e^{-1} + (1 - e^{-1})(t-1) = (1 - e^{-1})t - 1 + 2e^{-1}(m)$ ①

z-component in (B) is the same with that in (A), and therefore it land at t = 6 *sec*.

$\Rightarrow x(t=6) = 43 \cdot 6 - 2 = 256m,$ ① $\quad y(t=6) = (1 - e^{-1})6 - 1 + 2e^{-1} = 5 - 4e^{-1}(m)$ ①

The coordinate of the position where the ball lands is (256*m*, (5 - 4e⁻¹)*m* ,0) ①

3. (a) (10pts) As shown in Fig. 2(a), the force $\boldsymbol{F}$ is applied to block $\boldsymbol{m_1}$ with block $\boldsymbol{m_2}$ on top of it. There is no friction between $\boldsymbol{m_1}$ and the flat surface. If $\boldsymbol{m_1}$.= 3kg, $\boldsymbol{m_2}$ = 5kg, $\boldsymbol{\theta}$=53°, and the static and kinetic friction coefficients $\boldsymbol{\mu_s}$ and $\boldsymbol{\mu_k}$ between $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ are 1/2 and 1/3, respectively. Determine the range of $\boldsymbol{F}$ that can keep $\boldsymbol{m_2}$ static on $\boldsymbol{m_1}$. (b) (10 points) As shown in Fig. 2(b) if the force $\boldsymbol{F}$ is removed, what would be the acceleration of $\boldsymbol{m_1}$? (g = 10 m/s^2)

(a)

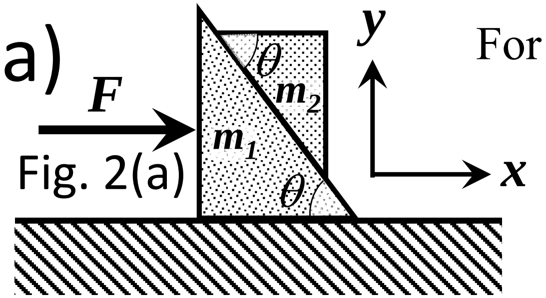
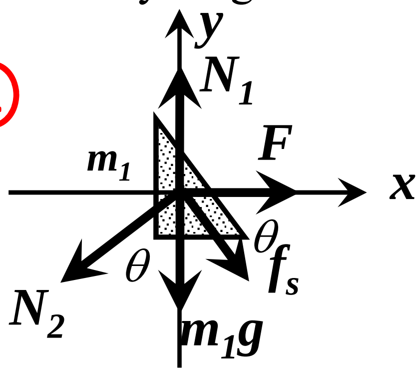


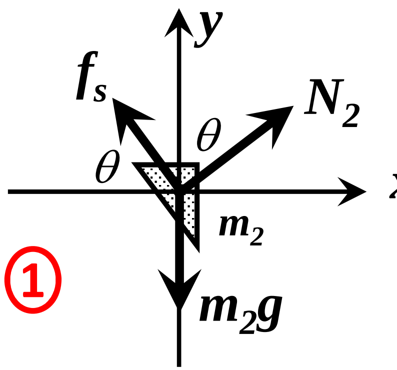
Fig. 2(a)

Free-body diagram for m_1:

①



Free-body diagram for m_2:



①

For $\boldsymbol{m_1}$: $\sum \vec{F} = \vec{F} + \vec{N}_1 + \vec{N}_2 + \vec{f}_s + m_1\vec{g} = m_1\vec{a}_1$

$x: F + f_s\cos\theta - N_2\sin\theta = m_1 a_1$ ①

$\Rightarrow F + \frac{3}{5}f_s - \frac{4}{5}N_2 = 3a_1$ (1)

$y: N_1 - N_2\cos\theta - f_s\sin\theta - m_1 g = 0$

$\Rightarrow N_1 - \frac{3}{5}N_2 - \frac{4}{5}f_s - 30 = 0$ (2) ①

For $\boldsymbol{m_2}$: $\sum \vec{F} = \vec{N}_2 + \vec{f}_s + m_2\vec{g} = m_2\vec{a}_2$

$x: N_2\sin\theta - f_s\cos\theta = m_2 a_2$

$\Rightarrow \frac{4}{5}N_2 - \frac{3}{5}f_s = 5a_2$ (3) ①

$y: N_2\cos\theta + f_s\sin\theta - m_2 g = 0$

$\Rightarrow \frac{3}{5}N_2 + \frac{4}{5}f_s - 50 = 0$ (4) ①

The physical relation between $\boldsymbol{a_1}$ and $\boldsymbol{a_2}$ ***is***

$a_1 = a_2 \equiv a$ (5) ①

From (1)+(3), and (5), we get

$F = 8a \Rightarrow a = \frac{F}{8}$ (6)

From (3), and (6), we get

$\frac{4}{5}N_2 - \frac{3}{5}f_s = \frac{5}{8}F$ (7)

①

From (7), and (4), we get

$N_2 = \frac{1}{2}F + 30;\quad f_s = 40 - \frac{3}{8}F$

$$\Rightarrow -\frac{1}{2}(\frac{F}{2}+30) \le (40 - \frac{3}{8}F) \le \frac{1}{2}(\frac{F}{2}+30)$$

$$-\frac{1}{2}(\frac{F}{2}+30) \le (40 - \frac{3}{8}F) \Rightarrow F \le 440$$

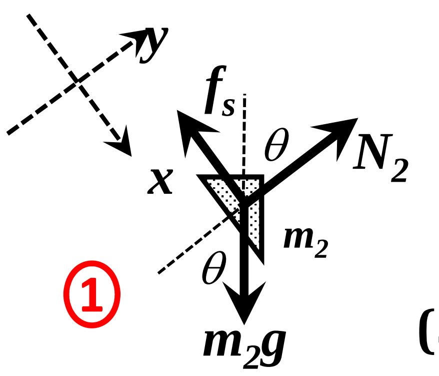
$$(40 - \frac{3}{8}F) \le \frac{1}{2}(\frac{F}{2}+30) \Rightarrow 40 \le F$$

②

$$\Rightarrow 40N \le F \le 440N$$

(a)

Free-body diagram for m_2:



For $\boldsymbol{m_2}$:

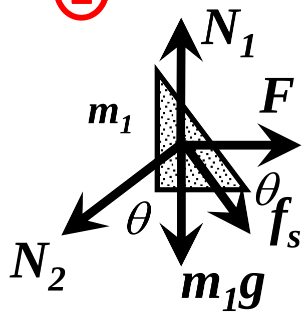
$$x: m_2 g\sin\theta - f_s = m_2 a_2\cos\theta \quad (3) \quad 1$$

$$y: N_2 - m_2 g\cos\theta = m_2 a_2\sin\theta \quad (4) \quad 1$$

$$(3)^2 + (4)^2 \rightarrow m_2 a = N_2\sin\theta - f_s\cos\theta$$

$$\Rightarrow \frac{4}{5}N_2 - \frac{3}{5}f_s = 5a_2 \quad (4) \quad 1$$

Free-body diagram for m_1: 1



For $\boldsymbol{m_1 + m_2}$:

$$F = (m_1 + m_2)a = 8a \quad (1)$$

$$\Rightarrow a = \frac{F}{8} \quad 1$$

The physical relation between $\boldsymbol{a_1}$ and $\boldsymbol{a_2}$ ***is***

$$a_1 = a_2 \equiv a \quad (5) \quad 1$$

From (4), and (1), we get

$$\frac{4}{5}N_2 - \frac{3}{5}f_s = \frac{5}{8}F \quad (7)$$

(1), (4) and (7), we get

$$N_2 = m_2 g\cos\theta + m_2 a\sin\theta \qquad f_s = 40 - \frac{3}{8}F$$

$$= 30 + 4a = 30 + \frac{1}{2}F$$

1

$$-f_{s,\max} \le f_s \le f_{s,\max} \quad \text{or} \quad -\frac{1}{2}N_2 \le 40 - \frac{3}{8}F \le \frac{1}{2}N_2$$

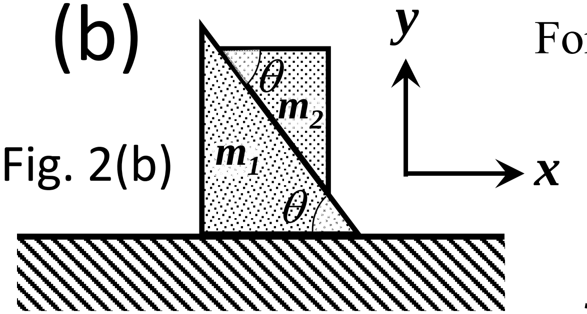
$$\Rightarrow -\frac{1}{2}\left(\frac{F}{2}+30\right) \le \left(40 - \frac{3}{8}F\right) \le \frac{1}{2}\left(\frac{F}{2}+30\right)$$

$$-\frac{1}{2}\left(\frac{F}{2}+30\right) \le \left(40 - \frac{3}{8}F\right) \Rightarrow F \le 440$$

$$\left(40 - \frac{3}{8}F\right) \le \frac{1}{2}\left(\frac{F}{2}+30\right) \Rightarrow 40 \le F$$

2

$$\Rightarrow 40\,N \le F \le 440\,N$$



For $\boldsymbol{m_1}$: $\sum \vec{F} = \vec{N}_1 + \vec{N}_2 + \vec{f}_k + m_1\vec{g} = m_1\vec{a}_1$

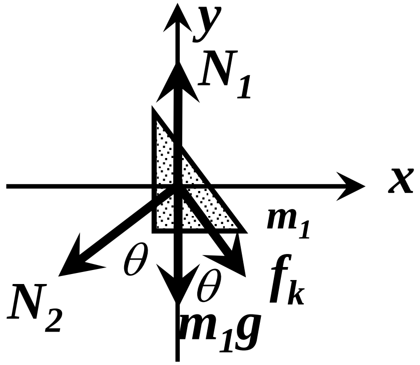
$x: f_k \cos\theta - N_2 \sin\theta = m_1 a_1$

$\Rightarrow \frac{3}{5} f_k - \frac{4}{5} N_2 = 3a_1$ (1) 1

$y: N_1 - N_2 \cos\theta - f_k \sin\theta - m_1 g = 0$

$\Rightarrow N_1 - \frac{3}{5} N_2 - \frac{4}{5} f_k - 30 = 0$ (2) 1

Free-body diagram for m_1:



For $\boldsymbol{m_2}$: $\sum \vec{F} = \vec{N}_2 + \vec{f}_k + m_2\vec{g} = m_2\vec{a}_2$

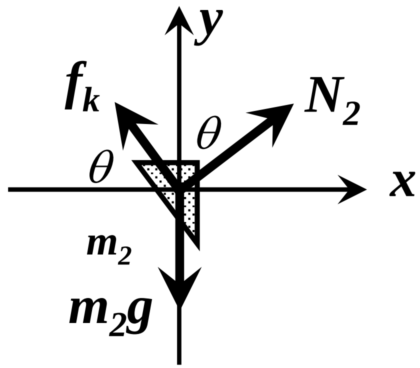
$x: N_2 \sin\theta - f_k \cos\theta = m_2 a_{2x}$

$\Rightarrow \frac{4}{5} N_2 - \frac{3}{5} f_k = 5a_{2x}$ (3) 1

$y: N_2 \cos\theta + f_k \sin\theta - m_2 g = a_{2y}$

$\Rightarrow \frac{3}{5} N_2 + \frac{4}{5} f_k - 50 = 5a_{2y}$ (4) 1

Free-body diagram for m_2:



The physical relation between $\boldsymbol{a_1}$ and $\boldsymbol{a_2}$ ***is***

$\frac{a_{2y}}{a_{2x} - a_1} = -\tan\theta \Rightarrow a_{2y} = -\frac{4}{3}(a_{2x} - a_1)$ (5) 1

$$\frac{y_2}{x_1 - x_2} = \tan\theta$$

$$\rightarrow \frac{\ddot{y}_2}{\ddot{x}_1 - \ddot{x}_2} = \tan\theta$$

And, $f_k = \mu_k N_2 = \frac{1}{3} N_2$ (6) 1

From (1),(3),(4) and (6), we get

$\frac{1}{5} N_2 - \frac{4}{5} N_2 = 3a_1 \Rightarrow a_1 = -\frac{1}{5} N_2$ (7)

$\frac{4}{5} N_2 - \frac{3}{5}\frac{1}{3} N_2 = 5a_{2x} \Rightarrow a_{2x} = \frac{3}{25} N_2$ (8)

$\frac{3}{5} N_2 + \frac{4}{5}\frac{1}{3} N_2 - 50 = 5a_{2y} \Rightarrow a_{2y} = \frac{13}{75} N_2 - 10$ (9)

From (7),(8),(9) and (5), we get

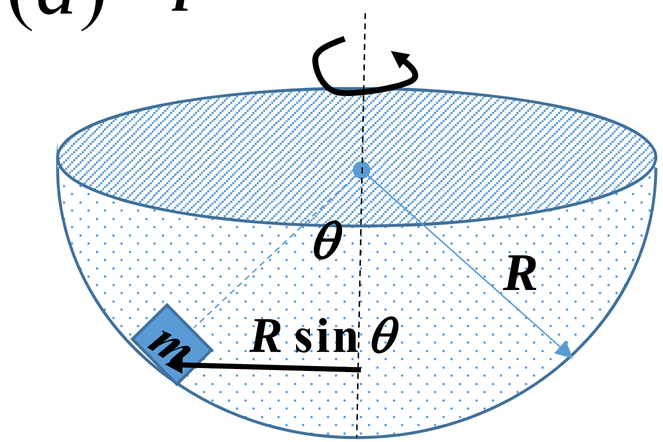
$\Rightarrow \frac{13}{75} N_2 - 10 = -\frac{4}{3}\left(\frac{3}{25} N_2 + \frac{1}{5} N_2\right)$

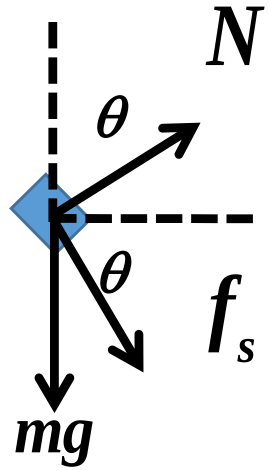
$\Rightarrow N_2 = \frac{50}{3}$ (10)

From (10)and (7), we get

$\Rightarrow a_1 = -\frac{10}{3}$ (m/sec²) 4

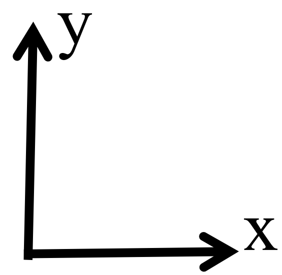
(a) ***Speed maximum:***

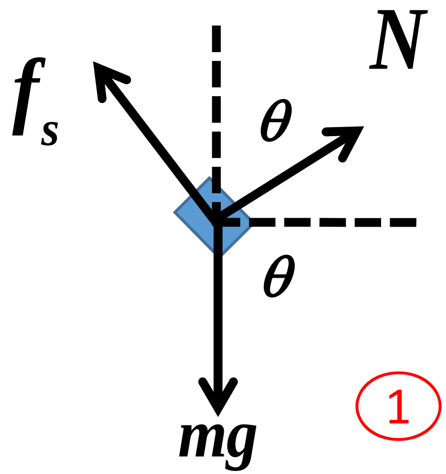




1

or ***Speed minimum:***





1

$$\hat{x}: \quad N\sin\theta + f_s\cos\theta = ma_c = m\frac{v^2}{R\sin\theta} \qquad (1)$$ 1

$$\hat{y}: \quad N\cos\theta - f_s\sin\theta - mg = 0 \qquad (2)$$ 1

(i) **maximum:** $f_s = \mu N$

(ii) **minimum:** $f_s = -\mu N$ 2

(i) speed maximum, $f_s = \mu N$

$$(2) \rightarrow N = \frac{mg}{\cos\theta - \mu\sin\theta}$$

$$(1) \rightarrow v_{max}^{\ 2} = gR\sin\theta\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}$$ 1

$$T_{min} = \frac{2\pi R\sin\theta}{v_{max}} = 2\pi\sqrt{\frac{R\sin\theta}{g}\frac{\cos\theta - \mu\sin\theta}{\sin\theta + \mu\cos\theta}}$$

1

(ii) speed minimum: $f_s = -\mu N$

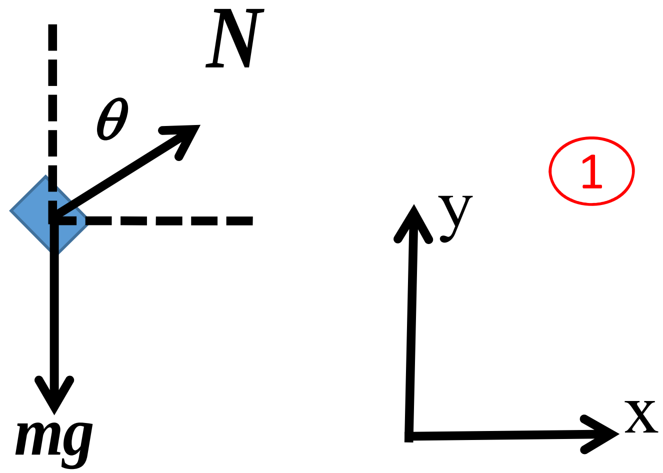
For T_{max}, $\mu \rightarrow -\mu$ 1

$$v_{min}^{\ 2} = gR\sin\theta\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}$$

$$T_{max} = \frac{2\pi R}{v_{max}} = 2\pi\sqrt{\frac{R\sin\theta}{g}\frac{\cos\theta + \mu\sin\theta}{\sin\theta - \mu\cos\theta}}$$

1

(b) *No friction force:*



1

$$\hat{x}: \quad N\sin\theta = ma_c = m\frac{v^2}{R\sin\theta} \qquad (1)$$ 1

$$\hat{y}: \quad N\cos\theta - mg = 0 \qquad (2)$$ 1

$$(2) \rightarrow N = \frac{mg}{\cos\theta}$$

$$(1) \rightarrow v^2 = gR\frac{\sin^2\theta}{\cos\theta}$$ 1

$$T_0 = \frac{2\pi R\sin\theta}{v} = 2\pi\sqrt{\frac{R\cos\theta}{g}}$$ 1