1.

a) $\frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$

b) $2WH_0 \operatorname{sinc}(2Wt)$

c) $M(f)=M^*(-f)$

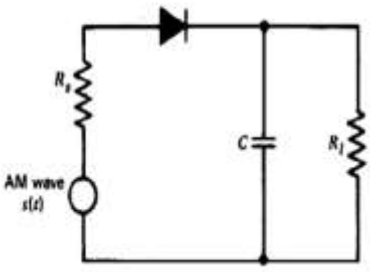
d) $|m_n(t)|\le 1$

e) $A_c[1+am_n(t)]\cos(2\pi f_c t)$ or $[1+am_n(t)]\cos(2\pi f_c t)$

f) $0\le a\le 1$

g) $A_c[1+am_n(t)]$ or $[1+am_n(t)]$

h)



i) $M(f+f_c)+M(f-f_c)$

j) $2W$

k) $2W$

l) $2W$

m) $\frac{1}{2}A_c m(t)\cos(2\pi f_c t)-\frac{1}{2}A_c\hat{m}(t)\sin(2\pi f_c t)$

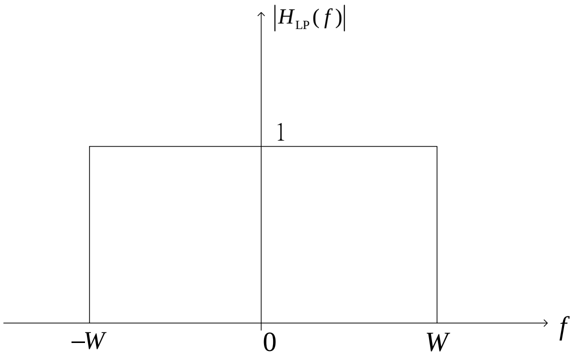
n) $H(f+f_c)+H(f-f_c)=\text{constant}\,,\ |f|\le W$

o) $2\cos\left[2\pi\left(f_c\pm f_{IF}\right)t\right]$

2.(a)

$$
\begin{aligned}
d(t) &= x_r(t)\cdot 2\cos(2\pi f_c t+\theta) \\
&= x_c(t)\cdot 2\cos(2\pi f_c t+\theta) \\
&= A_c\cdot m(t)\cdot\cos(2\pi f_c t)\cdot 2\cos(2\pi f_c t+\theta) \\
&= A_c\cdot m(t)\cdot 2\cdot\frac{1}{2}\left(\cos(4\pi f_c t+\theta)+\cos(\theta)\right) \\
&= A_c\cdot m(t)\cdot\cos(4\pi f_c t+\theta)+A_c\cdot m(t)\cdot\cos(\theta)
\end{aligned}
$$

2.(b)



2.(c)

$$
\begin{aligned}
y_D(t) &= \mathrm{Lp}\{d(t)\} \\
&= \mathrm{Lp}\{A_c\cdot m(t)\cdot\cos(4\pi f_c t+\theta)+A_c\cdot m(t)\cdot\cos(\theta)\} \\
&= A_c\cdot m(t)\cdot\cos(\theta)
\end{aligned}
$$

when $\theta=0,\ y_D(t)=A_c\cdot m(t)$

when $\theta=\dfrac{\pi}{4},\ y_D(t)=\dfrac{1}{\sqrt{2}}A_c\cdot m(t)$

when $\theta=\dfrac{\pi}{2},\ y_D(t)=0$

3.

a)

$$m_n(t) \triangleq \frac{m(t)}{\left|\max\left[m(t)\right]\right|} = \frac{9\cos(20\pi t)}{9} = \cos(20\pi t)$$

b)

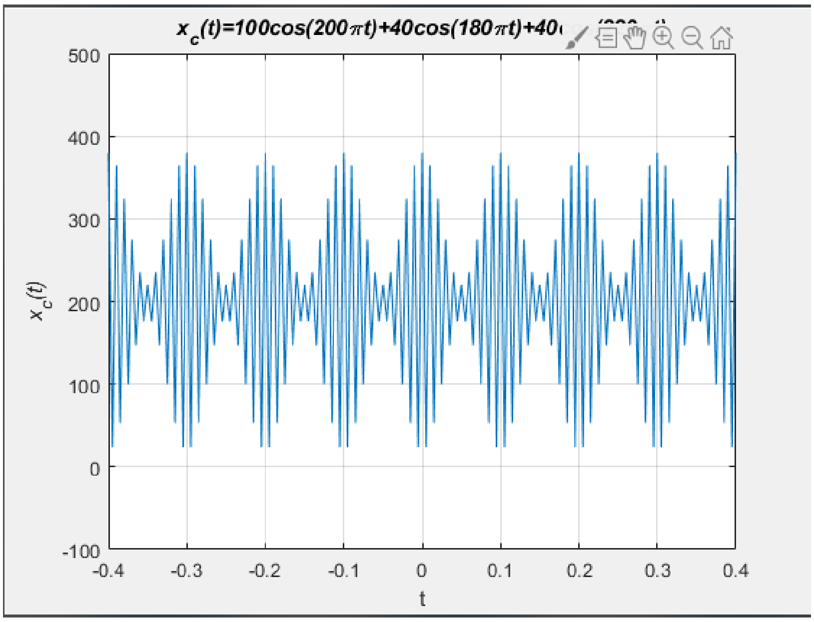
$$\left\langle m_n^2(t)\right\rangle = \left\langle \cos^2(20\pi t)\right\rangle = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\cos^2(20\pi t)dt = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\frac{1+\cos(40\pi t)}{2}dt = 0.5$$

c)

$$\begin{aligned} x_c(t) &= 100\left[1+0.8\cos(20\pi t)\right]\cos(200\pi t) \\ &= 100\cos(200\pi t) + 40\cos(180\pi t) + 40\cos(220\pi t) \end{aligned}$$

$$\begin{aligned} \left\langle x_c^2(t)\right\rangle &= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}x_c^2(t)dt \\ &= \left\langle 100^2\left[1+0.8\cos(20\pi t)\right]^2\cos^2(200\pi t)\right\rangle \\ &= \frac{1}{2}\left\langle 100^2\left[1+0.8\cos(20\pi t)\right]^2\left(1+\cos\left(400\pi t\right)\right)\right\rangle \\ &= \frac{1}{2}\left\langle 100^2\left[1+0.8\cos(20\pi t)\right]^2\right\rangle + \frac{1}{2}\underbrace{\left\langle 100^2\left[1+0.8\cos(20\pi t)\right]^2\cos\left(400\pi t\right)\right\rangle}_{\approx 0,\text{ when } f_c \gg W} \\ &= \frac{1}{2}\left\langle 100^2\left[1+2\times 0.8\cos(20\pi t)+0.64\cos^2(20\pi t)\right]\right\rangle \\ &= \frac{1}{2}\left\langle 100^2\right\rangle + \frac{1}{2}\times 100^2\times 2\times 0.8\underbrace{\left\langle\cos(20\pi t)\right\rangle}_{\approx 0} + \frac{1}{2}\left\langle 100^2\times 0.64\cos^2(20\pi t)\right\rangle \\ &= \underbrace{5000}_{\text{Carrier Power}} + \underbrace{1600}_{\text{Message Power}} = 6600 \end{aligned}$$

d)



x_c(t)=100cos(200πt)+40cos(180πt)+40
500
400
300
200
100
0
-100
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
t
x_c(t)

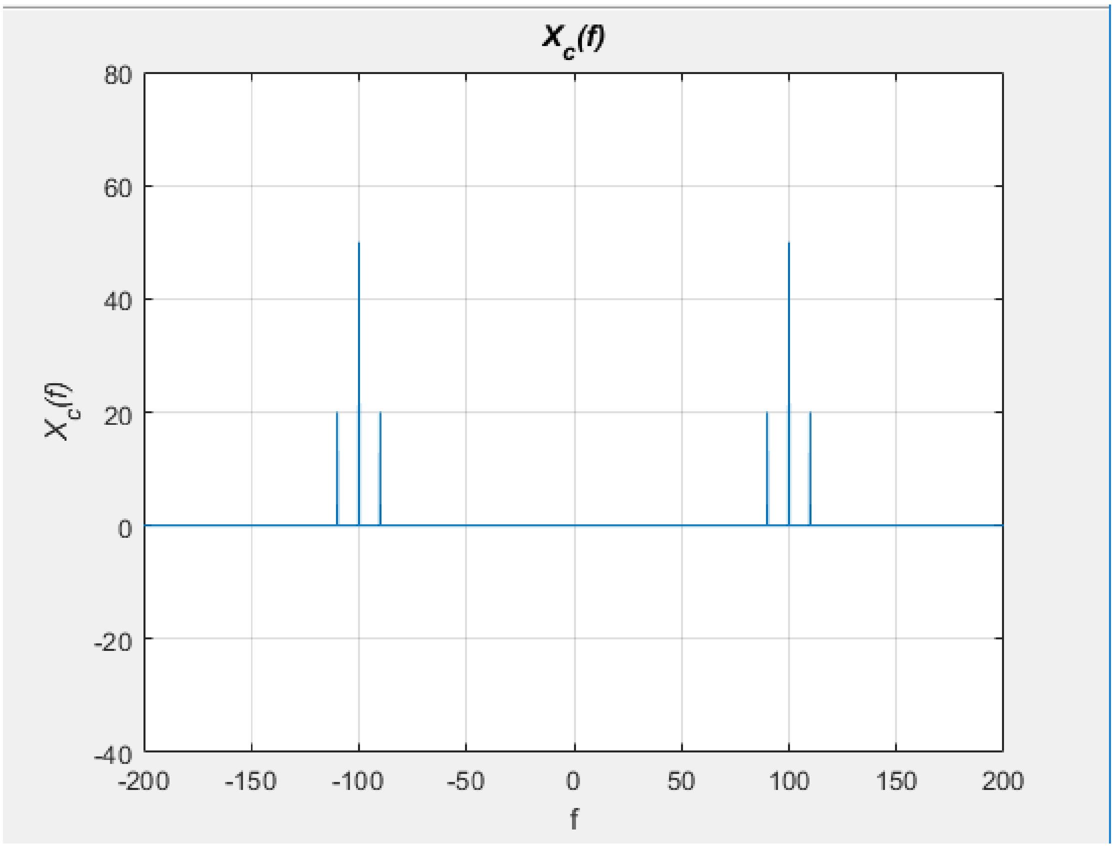
e)

$$X_c(f) = F\ \{100(1+0.8\cos(20\pi t))\cos(200\pi t)\} = F\ \{100\times 0.8\cos(20\pi t)\cos(200\pi t)\} + F\ \{100\cos(200\pi t)\}$$

$$= F\ [0.8\cos(20\pi t)] * F\ [100\cos(200\pi t)] + F\ \{100\cos(200\pi t)\}$$

$$= \frac{0.8}{2}[\delta(f-10)+\delta(f+10)] * \frac{100}{2}[\delta(f-100)+\delta(f+100)] + \frac{100}{2}[\delta(f-100)+\delta(f+100)]$$

$$= \frac{100}{2}\left[\underbrace{0.4\delta(f-110)+0.4\delta(f-90)+0.4\delta(f+90)+0.4\delta(f+110)}_{aM_n(f)} + \delta(f-100)+\delta(f+100)\right]$$



4.(a)

$$\text{Hilbert Transform : } \hat{m}(t) = 4\sin(2\pi f_m t) + \sin(4\pi f_m t)$$

$$x_c(t)$$

$$= \frac{1}{2} A_c m(t)\cos(2\pi f_c t) + \frac{1}{2} A_c \hat{m}(t)\sin(2\pi f_c t)$$

$$= \frac{1}{2} A_c (4\cos(2\pi f_m t) + \cos(4\pi f_m t))\cos(2\pi f_c t) + \frac{1}{2} A_c (4\sin(2\pi f_m t) + \sin(4\pi f_m t))\sin(2\pi f_c t)$$

$$= A_c \cos[2\pi(f_c + f_m)t] + A_c \cos[2\pi(f_c - f_m)t] + \frac{1}{4} A_c \cos[2\pi(f_c + 2f_m)t] + \frac{1}{4} A_c \cos[2\pi(f_c - 2f_m)t]$$

$$+ \{-A_c \cos[2\pi(f + f_m)t] + A_c \cos[2\pi(f_c - f_m)t]\} + \{-\frac{1}{4} A_c \cos[2\pi(f_c + 2f_m)t] + \frac{1}{4} A_c \cos[2\pi(f_c - 2f_m)t]\}$$

$$= 10\cos[2\pi(f_c + f_m)t] + 10\cos[2\pi(f_c - f_m)t] + \frac{5}{2}\cos[2\pi(f_c + 2f_m)t] + \frac{5}{2}\cos[2\pi(f_c - 2f_m)t]$$

$$+ \{-10\cos[2\pi(f_c + f_m)t] + 10\cos[2\pi(f_c - f_m)t]\} + \{-\frac{5}{2}\cos[2\pi(f_c + 2f_m)t] + \frac{5}{2}\cos[2\pi(f_c - 2f_m)t]\}$$

$$= 20\cos[2\pi(f_c - f_m)t] + 5\cos[2\pi(f_c - 2f_m)t]$$

b)

$$\begin{aligned}
X_c(f) &= \mathrm{F}\left\{20\cos\left[2\pi(f_c - f_m)t\right] + 5\cos\left[2\pi(f_c - 2f_m)t\right]\right\} \\
&= \mathrm{F}\left\{20\cos\left[2\pi(f_c - f_m)t\right]\right\} + \mathrm{F}\left\{5\cos\left[2\pi(f_c - 2f_m)t\right]\right\} \\
&= \frac{20}{2}\left[\delta(f - f_c + f_m) + \delta(f + f_c - f_m)\right] + \frac{5}{2}\left[\delta(f - f_c + 2f_m) + \delta(f + f_c - 2f_m)\right] \\
&= 10\delta(f - f_c + f_m) + 10\delta(f + f_c - f_m) + 2.5\delta(f - f_c + 2f_m) + 2.5\delta(f + f_c - 2f_m)
\end{aligned}$$

c)