試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(50 points) (所有題目必須有計算過程,否則不予計分)

1. (10 points) A R-C circuit is shown in Fig. 1. $R_1 = R_2 = R_3 = R$, a capacitor C, and a battery $\mathcal{E}$. The capacitors are initially uncharged. The switch S is open initially.

(a) (4 points) What are the currents $\boldsymbol{i_0}$, $\boldsymbol{i_2}$, and $\boldsymbol{i_3}$ immediately after the switch S is move to position 1 ? And $\boldsymbol{i_0}$, $\boldsymbol{i_2}$, and $\boldsymbol{i_3}$ after t → ∞ ?

(b) (6 points) When the currents are steady, then the switch S moves to position 2. (i) Find the charges on the capacitor and the currents $\boldsymbol{i_1}$, $\boldsymbol{i_2}$, and $\boldsymbol{i_3}$ immediately after moving to position 2? (iii)What is the time constant τ for the capacitor to discharge?

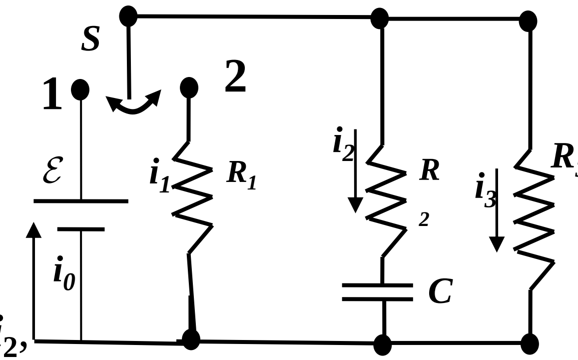


Fig.1.

2. (10 pts.) A conducting rod of mass $\boldsymbol{m}$ is rest at $\boldsymbol{x = 0}$. At $\boldsymbol{t = 0}$, it starts to fall down on two frictionless conducting rails (separated by a distance $\boldsymbol{L}$, as shown in Fig. 2). There is a uniform magnetic field (along –$\boldsymbol{z}$-axis) enclosed the conducting rails. In addition, a resistor $\boldsymbol{R}$ connects the two rails at $\boldsymbol{x = x_0}$. The rod will pass the resistor freely. **(a)** (2 pts.) Find the **magnitude** and **direction** of the induced current $\boldsymbol{I}$ in the rod in terms of $\boldsymbol{B, L, R, g}$ (gravitational acceleration), $\boldsymbol{v}$ (speed of the bar) and other necessary constants before it passes the resistor. **(b)** (5 pts.) Solve the velocity of the rod $\boldsymbol{v(t)}$ before it passes the resistor. (assume the rod is rest at $\boldsymbol{x = 0}$ at $\boldsymbol{t = 0}$) **(c) (3 pts.)** The rod passes the resistor with velocity v_0 at time $\boldsymbol{t_0}$ (you don't need to solve them). What is the velocity of the rod $\boldsymbol{v(t)}$ after it passes the resistor? Your answer can include $\boldsymbol{v_0}$ and/or $\boldsymbol{t_0}$ if it is necessary.

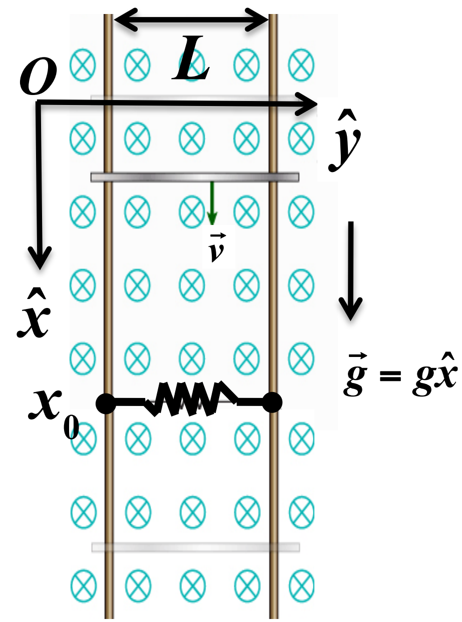


Fig. 2

3. (15 pts) Fig. *3* shows 3 lines with current I in *x*-*y* plane. Find the *x*-, *y*-, *z*-components of the ***B***-field at point ***P*** on the *z*-axis due to

(a) (6 pts) current line ***OB***,

(b) (b) (3pts) current line charges ***CO***, and

(c) (c) (6 pts) current line charges ***BC***.

The coordinates of ***B***,***C***, and ***P*** are (*R*,*0*,*0*), (*0*,*R*,*0*), and (*0*,*0*,*z*), respectively.

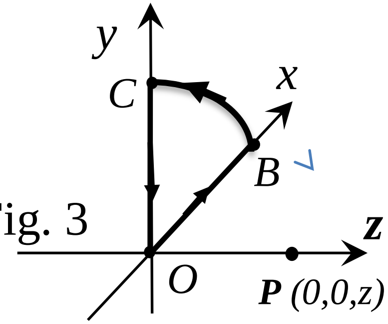


Fig. 3

4. (15 pts) In Fig. 4, an cross section of an infinite long cylindrical conductor with radius **2*R*.** At point ***B***, coordinate **(*R*,0)**, is cut by an infinite long cylindrical tube with radius ***R***. There is a uniform current ***I*** flowing in the conductor (direction is out of page). (1) (2 pts) Find the current density J_0 in terms of ***I*, *R*** and other necessary constants. (b) (13 pts) Find the ***x*-** and ***y*-** components of the magnetic field at positions (i) ***A*** : **(0, *R*)** (ii) ***B*: (*R*,0)** and ***C*: (2*R*, 2*R*).**

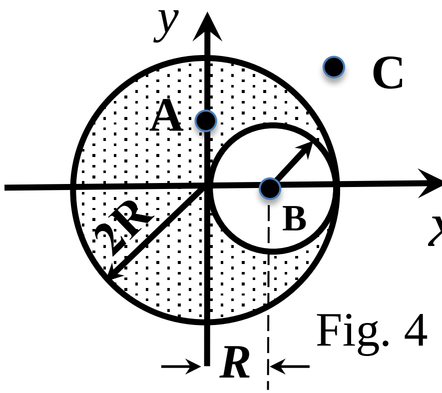


Fig. 4

Useful formula:

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left(x + \sqrt{x^2 \pm a^2}\right); \int \frac{dx}{\left(x^2 \pm a^2\right)^{\frac{3}{2}}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}}; \int \frac{xdx}{\left(x^2 \pm a^2\right)^{\frac{3}{2}}} = \frac{-1}{\sqrt{x^2 \pm a^2}}; \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) = -\cos^{-1}\left(\frac{x}{a}\right)$$

II.選擇題(52 points)

1. (5pts) Consider a solenoid of length ***L*** , ***N*** windings, and radius ***r (L*** >> ***r***). A current I is flowing through the wire. If the radius of the solenoid becomes ***2r***, and all other quantities remain the same, the magnetic field

(A) would be the same. (B) would be twice as strong. (C) would become one half as strong.

2. (5pts) As shown in Fig. 5, the switch S is closed at t=0, and at time $t = t_{1/2}$ the voltage across the capacitor reaches ½ of its maximum voltage. What would be the current i_1 at this moment ($t = t_{1/2}$)?

(A) -1.5 A (B) -1.25 A (C) -1.0 A (D) -2/3 A
(E) 0 A (F) 2/3 A (G) 1.0 A (H) 1.25A
(I) 1.5 A

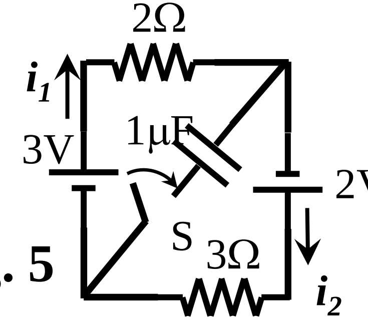


Fig. 5

3. (5pts)As shown in Fig. 6, a infinitely conducting plate with thickness d carries a uniform current density J in +z-direction, in the middle of the plate there is a infinitely long hollow cylindrical region with radius b ($2b<d$) its axis coincides with the z-axis. Which of the following could be the direction of the magnetic field at point P outside of the plate?

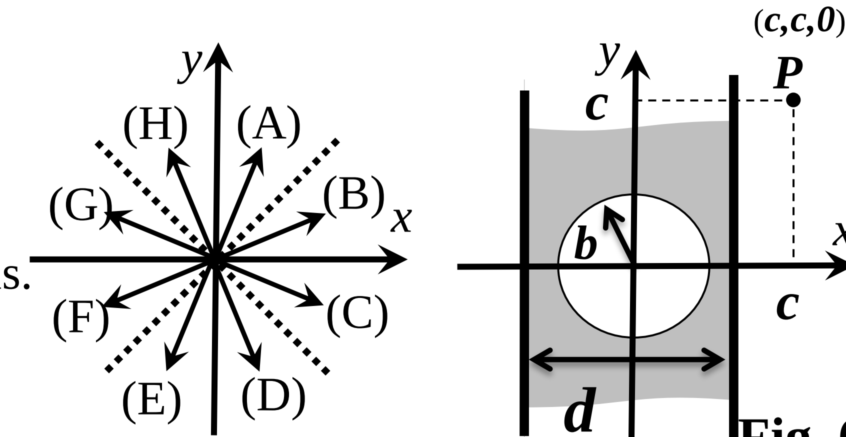


Fig. 6

4. (5pts)As shown in Fig. 7, a conducting ring carries a current I flowing clockwise. The center of the ring is at the origin with radius R. Which of the following is B_z along x-axis ?

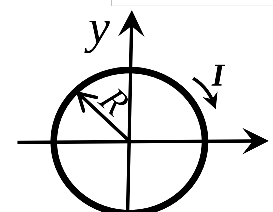
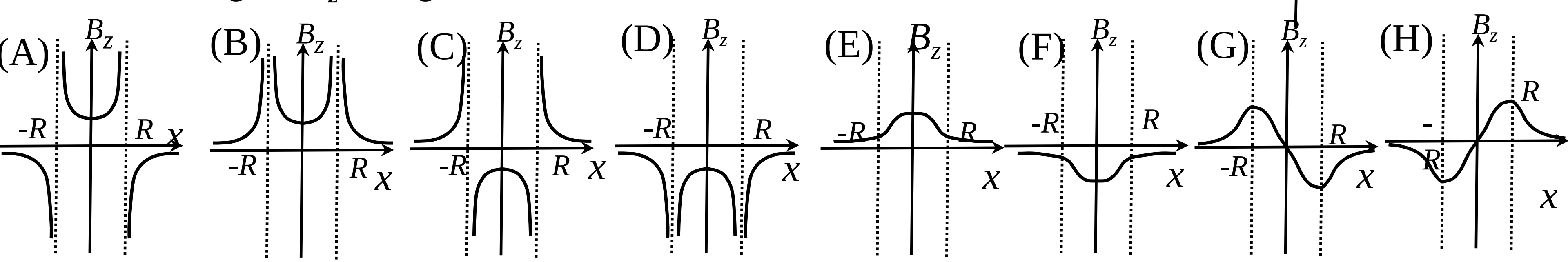


Fig. 7



5. (5pts) Fig.8 shows the cross section of an infinitely long hollow (中空) conducting rod along the z-axis (out of page), which carries a uniform current density $\boldsymbol{J}$ ($\boldsymbol{J} > 0$) in the +z-direction. Consider two circular closed loops (dotted lines in the figure) labelled as ***1*** and ***2*** on the x-y plane with their centers at the origin. Let $\boldsymbol{B_1}(x,y,z)$ and $\boldsymbol{B_2}(x,y,z)$ be the B-field generated by $\boldsymbol{J}$ at each point along loop ***1*** and loop ***2***, respectively, and

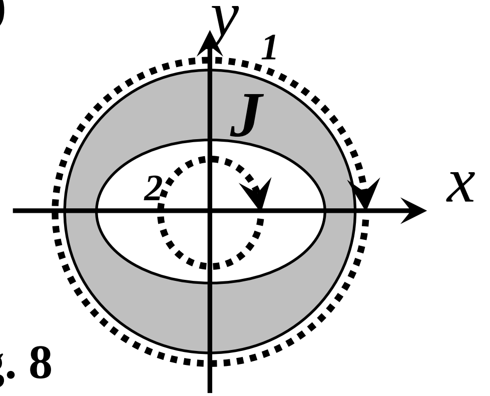


Fig. 8

$$K_1 = \oint_{Loop1} \vec{B}_1 \cdot d\vec{\ell}, \; and \quad K_2 = \oint_{Loop2} \vec{B}_2 \cdot d\vec{\ell}$$

Which of the following statement is correct?

(A) $|\boldsymbol{B_1}| = constant,\ \ K_1 = 0,\ |\boldsymbol{B_2}| = 0,\ K_2 = 0$

(B) $|\boldsymbol{B_1}| = constant,\ \ K_1 \neq 0,\ |\boldsymbol{B_2}| \neq 0,\ K_2 = 0$

(C) $|\boldsymbol{B_1}| = constant,\ \ K_1 \neq 0,\ |\boldsymbol{B_2}| \neq 0,\ K_2 \neq 0$

(D) $|\boldsymbol{B_1}| = constant,\ \ K_1 \neq 0,\ |\boldsymbol{B_2}| = 0,\ K_2 = 0$

(E) $|\boldsymbol{B_1}| \neq constant,\ \ K_1 \neq 0,\ |\boldsymbol{B_2}| \neq 0,\ K_2 = 0$

(F) $|\boldsymbol{B_1}| \neq constant,\ \ K_1 \neq 0,\ |\boldsymbol{B_2}| = 0,\ K_2 = 0$

(G) $|\boldsymbol{B_1}| \neq constant,\ \ K_1 \neq 0,\ |\boldsymbol{B_2}| \neq 0,\ K_2 \neq 0$

6. (5 pts) In Fig. 9, a uniform and constant magnetic field $\boldsymbol{B}$ is directed perpendicularly into the page in upper half plane but out of the page in the lower half plane everywhere within a rectangular region. A semicircle wire circuit is rotated counterclockwise in the plane of the page about an axis A at constant frequency. The axis A is perpendicular to the page at the edge of the fields and directed through the center of the straight-line portion of the circuit. Which of the following graphs best approximates the emf $\mathcal{E}$ induced in the circuit as a function of time $\boldsymbol{t}$?

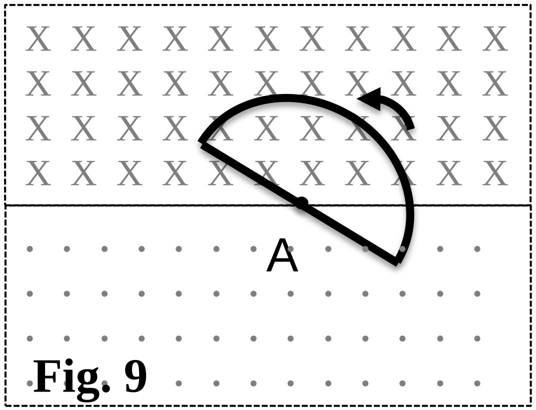
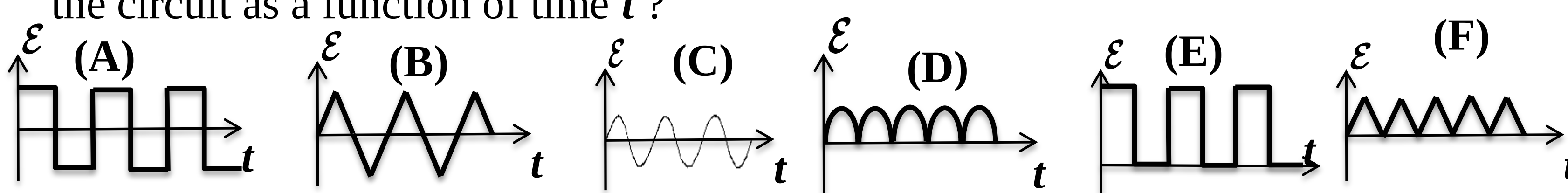


Fig. 9

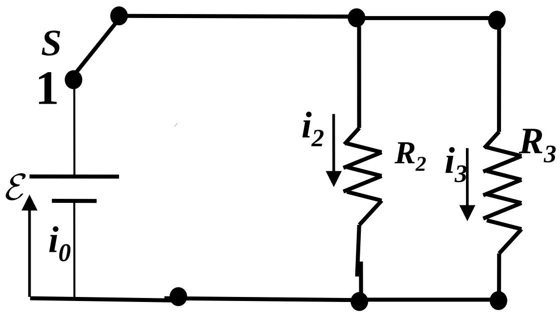


(G) None of above

Multiple Choice Questions:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A	**H**	**A**	**C**	**E**	**A**	**A**	**E**	**B**	**C**
11	**12**	**13**							
A	**A**	**B**							

(a) At t =0, C acts like a short circuit.

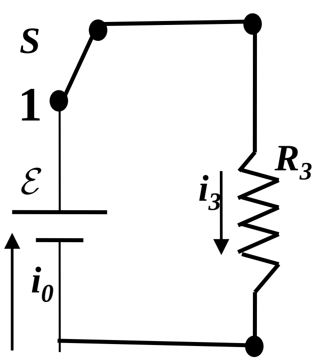


$$i_0 = i_2 + i_3 = 2i_2$$

並聯： $\dfrac{1}{R_{eff}} = \dfrac{1}{R_2} + \dfrac{1}{R_3} \Rightarrow R_{eff} = \dfrac{R}{2}$ 2

$\mathcal{E} - i_0 R_{eff} = 0 \Rightarrow \boxed{i_0 = \dfrac{2\mathcal{E}}{R}\ ;\ i_2 = i_3 = \dfrac{\mathcal{E}}{R}}$

At t =∞, C is fully charged and acts like a open circuit.



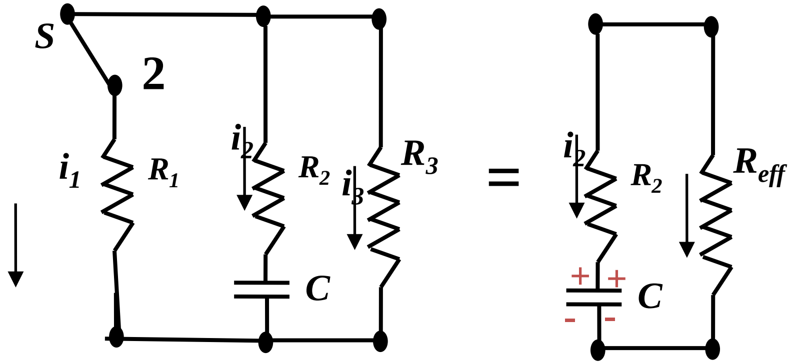
$i_2 = 0$ and $i_0 = i_3$

$\mathcal{E} - i_3 R = 0$

$\Rightarrow \boxed{i_2 = 0\ ;\ i_0 = i_3 = \dfrac{\mathcal{E}}{R}}$

2

(b) (i)



From (a) part, t =∞: $\mathcal{E} = i_3 R_3 = \dfrac{Q_0}{C}$ 2

$\Rightarrow \boxed{Q_0 = C\mathcal{E}}$

Since R_1 and $_{R3}$ are in parallel,

$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_3} \Rightarrow R_{eff} = \frac{R}{2}$$

and $-i_2 = i_1 + i_3 = 2i_1 = i_{eff}$ 1 1

$\mathcal{E} + i_2 R_2 - i_{eff} R_{eff} = 0 \Rightarrow \boxed{i_2 = -\dfrac{2\mathcal{E}}{3R}\ ;\ i_1 = i_3 = \dfrac{\mathcal{E}}{3R}}$

(b) (ii) $\dfrac{Q}{C} + i_2 R_2 - i_{eff} R_{eff} = 0$ and $\dfrac{dQ}{dt} = i_2$

$\Rightarrow \boxed{\dfrac{dQ}{dt} + \dfrac{2Q}{3RC} = \dfrac{dQ}{dt} + \dfrac{Q}{\tau} = 0}$ 1 $\boxed{\therefore \tau = \dfrac{3RC}{2}}$ 1

Solution is : $Q(t) = Q_0 \cdot e^{-t/\tau}$

2.(a) 2 pts

1 pts

$$\mathcal{E}_{ind} = -\frac{d\Phi_B}{dt},$$ Magnetic flux decreases → I : c.w. or *left* → *right* for $x < x_0$

(b) 5 pts

$$IR = \left|\mathcal{E}_{ind}\right| = \frac{d}{dt}BLx \rightarrow \boxed{I = \frac{BL}{R}v}$$ 1 pts

$$\vec{F}_{tot} = \vec{F}_{mg} + \vec{F}_B = mg\hat{x} + I\vec{L}\times\vec{B} = mg\hat{x} + I\left(L\hat{y}\right)\times\left[-B\hat{z}\right] = \left(mg - ILB\right)\hat{x} = m\frac{dv}{dt}\hat{x}$$

2 pts

$$\frac{dv}{dt} = g - \frac{B^2L^2}{mR}v = g - \alpha v, \quad \alpha \equiv \frac{B^2L^2}{mR}$$

2 pts

$$\int_0^{v(t)} \frac{dv}{g-\alpha v} = \int_0^t dt \quad \text{or } v(t) = \boxed{\frac{mgR}{B^2L^2}\left[1-e^{-\alpha t}\right]}$$ 1 pts

(c) 3 pts

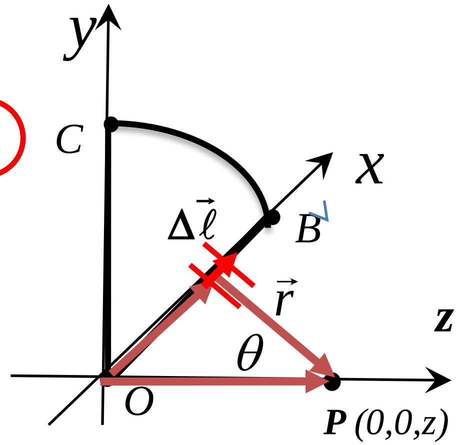
Magnetic flux changes sign (start to increases after it pass the resistor) → I : c.c.w. or still flowing from ***left*** → ***right*** for $x > x_0$ the equation of motion is the same as in part (b). But the initial condition becomes $v(t_0) = v_0$

2 pts $$\int_{v_0}^{v(t)} \frac{dv}{g-\alpha v} = \int_{t_0}^t dt \quad \text{or } v(t) = \frac{mgR}{B^2L^2}\left[1-e^{-\alpha(t-t_0)}\right] + v_0 e^{-\alpha(t-t_0)}$$ 1 pts

4. (a) Line segment in x-direction:

$$\Delta\vec{l} = \Delta x\,\hat{x} \quad , \quad \vec{r} = (0,0,z)-(x,0,0) = (-x,0,z) \quad , \quad \hat{r} = \frac{(-x,0,z)}{\sqrt{x^2+z^2}} \quad ①$$

$$\therefore \Delta\vec{B} = \frac{\mu_0 I}{4\pi}\frac{z\Delta x(-\hat{y})}{\sqrt{x^2+z^2}^{\,3}} \quad ②$$



(i) 查積分表:

$$\int \frac{dx}{\left(x^2 \pm a^2\right)^{\frac{3}{2}}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}}$$

$$① \quad \vec{B}_1 = \frac{\mu_0 I z}{4\pi}(-\hat{y})\left(\int_0^R \frac{dx}{\sqrt{x^2+z^2}^{\,3}}\right) = \frac{\mu_0 I z}{4\pi}(-\hat{y})\left(\left.\frac{x}{z^2\sqrt{x^2+z^2}}\right|_0^R\right) = \frac{\mu_0 I}{4\pi z}\frac{R}{\sqrt{R^2+z^2}}(-\hat{y}) \quad ②$$

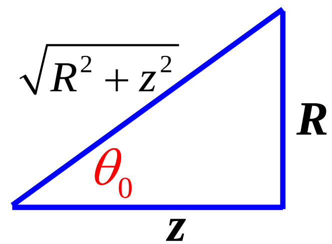
(ii) 變數變換:

$$\tan\theta = \frac{x}{z} \quad ①$$

$$dx = z\sec^2\theta\, d\theta$$

$$\Longrightarrow \vec{B}_1 = \frac{\mu_0 I z}{4\pi}(-\hat{y})\left\{\int_0^{\theta_0} \frac{z\sec^2\theta\, d\theta}{z^3\sec^3\theta}\right\} = \frac{\mu_0 I}{4\pi z}(-\hat{y})\left\{\sin\theta\Big|_0^{\theta_0}\right\}$$

$$= \frac{\mu_0 I}{4\pi z}\frac{R}{\sqrt{R^2+z^2}}(-\hat{y}) \quad ②$$

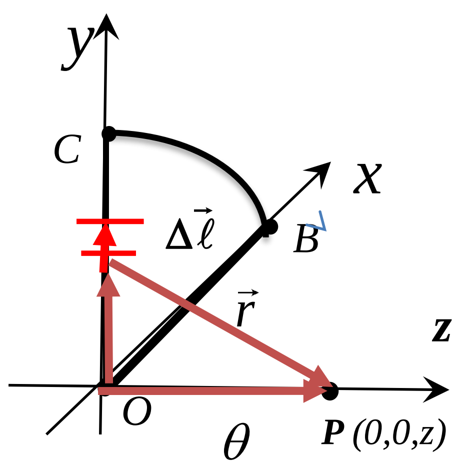


(b) Line segment in y-direction:

$$\Delta\vec{l} = \Delta y\,\hat{y} \quad , \quad \vec{r} = (0,0,z)-(0,y,0) = (0,-y,z) \quad , \quad \hat{r} = \frac{(0,-y,z)}{\sqrt{y^2+z^2}}$$

$$\therefore \Delta\vec{B} = \frac{\mu_0 I}{4\pi}\frac{z\Delta y(\hat{x})}{\sqrt{y^2+z^2}^{\,3}} \quad ②$$

$$① \quad \vec{B}_2 = \frac{\mu_0 I z}{4\pi}(\hat{x})\left(\int_R^0 \frac{dy}{\sqrt{y^2+z^2}^{\,3}}\right) = -\frac{\mu_0 I}{4\pi z}\frac{R}{\sqrt{R^2+z^2}}(\hat{x})$$



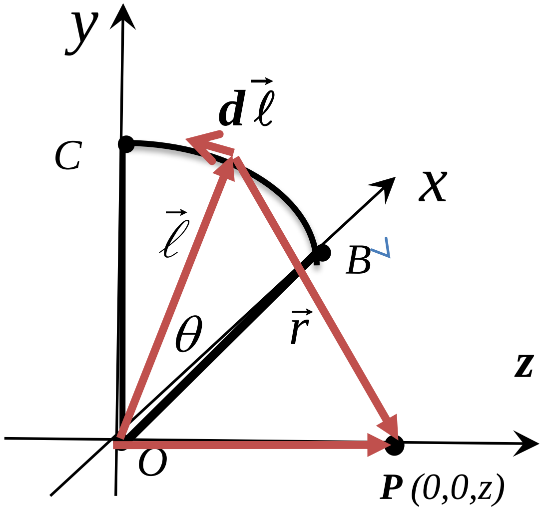
(c)

$$\vec{\ell} = (R\cos\theta, R\sin\theta, 0)$$ ①

$$d\vec{\ell} = R(-\sin\theta d\theta, \cos\theta d\theta, 0)$$ (R 不變情形下)

$$\vec{r} = (0,0,z) - (R\cos\theta, R\sin\theta, 0) = (-R\cos\theta, -R\sin\theta, z)$$

$$\hat{r} = \frac{(-R\cos\theta, -R\sin\theta, z)}{\sqrt{R^2 + z^2}}$$ ①



$$d\vec{l} \times \hat{r} = \frac{Rd\theta}{\sqrt{x^2 + R^2}} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -\sin\theta & \cos\theta & 0 \\ -R\cos\theta & -R\sin\theta & z \end{vmatrix} = \frac{Rd\theta}{\sqrt{x^2 + R^2}} \left(z\cos\theta\hat{x} + z\sin\theta\hat{y} + R\hat{z} \right)$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{\ell} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \frac{IRd\theta\left(z\cos\theta\hat{x} + z\sin\theta\hat{y} + R\hat{z} \right)}{\sqrt{x^2 + R^2}^3}$$ ②

$$\vec{B} = \frac{\mu_0 IR}{4\pi\sqrt{x^2 + R^2}^3} \left\{ z\hat{x} \int_0^{\pi/2} \cos\theta d\theta + z\hat{y} \int_0^{\pi/2} \sin\theta d\theta + R\hat{z} \int_0^{\pi/2} d\theta \right\}$$ ①

$$\boxed{\vec{B} = \frac{\mu_0 IR}{4\pi\sqrt{x^2 + R^2}^3} \left\{ z\hat{x} + z\hat{y} + \frac{\pi}{2} R\hat{z} \right\}}$$ ①

4.(a) 2 pts

2 pts

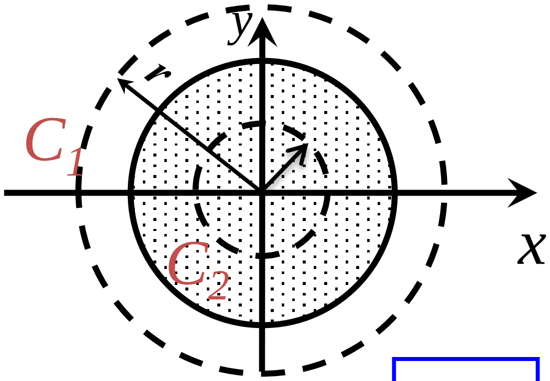
$$I = \int \vec{J} \cdot d\vec{A} = J_0\left[(2R)^2 - R^2\right] = 3J_0R^2 \rightarrow \boxed{J_0 = \frac{I}{3R^2}}$$

(b) 13 pts

$$r > R : \oint_{C_1} \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc} \rightarrow \vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\varphi}$$

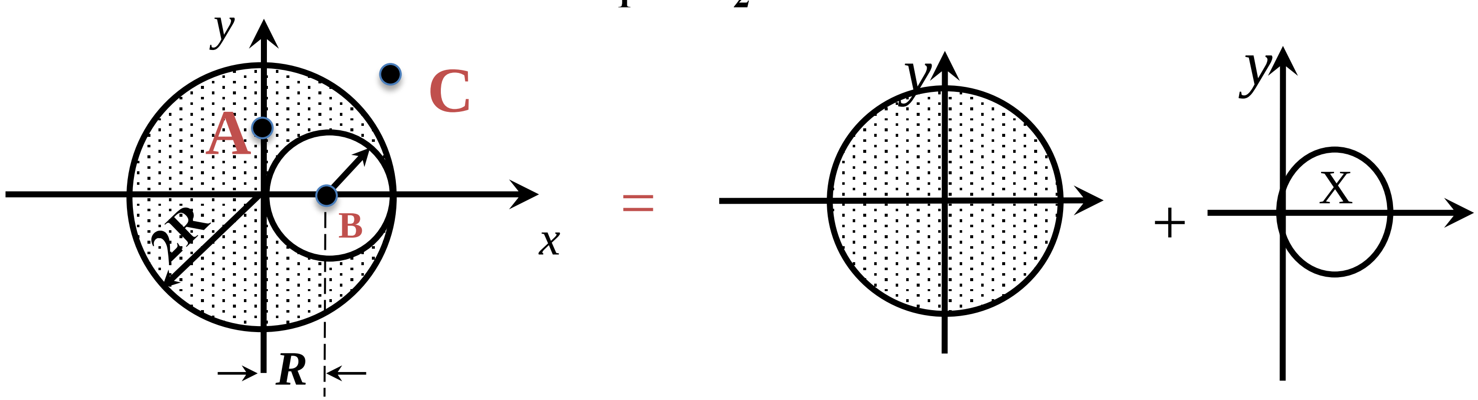
1 pts

$$r < R : \oint_{C_2} \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc} \rightarrow \vec{B} = \mu_0 J_0 \frac{r}{2}\hat{\varphi}$$



1 pts

$$\vec{B} = \vec{B}_1 + \vec{B}_2$$



2 pts

Point $A:(0,R)$ $\vec{B}_1: r=R<2R \rightarrow \vec{B}_1=\mu_0 J_0 \frac{r}{2}\hat{\varphi}=\mu_0 J_0 \frac{R}{2}(-\hat{x})$

3 pts

$$\vec{B}_2: r'=\sqrt{2}R>R \rightarrow \vec{B}_2=\mu_0 J_0 \frac{R^2}{2r'}\hat{\varphi}'=\mu_0 J_0 \frac{R}{2\sqrt{2}}\left(\frac{1}{\sqrt{2}}\hat{x}+\frac{1}{\sqrt{2}}\hat{y}\right)$$

$$\vec{B}=\vec{B}_1+\vec{B}_2=\boxed{\mu_0 J_0 R\left(-\frac{1}{4}\hat{x}+\frac{1}{4}\hat{y}\right)}$$

Point $B:(R,0)$

3 pts

$$\vec{B}_1: r=R<2R \rightarrow \vec{B}_1=\mu_0 J_0 \frac{r}{2}\hat{\varphi}=\mu_0 J_0 \frac{R}{2}(+\hat{y})$$

$$\vec{B}_2: r'=0<R \rightarrow \vec{B}_2=0 \qquad \vec{B}=\vec{B}_1+\vec{B}_2=\boxed{\mu_0 J_0 R\left(\frac{1}{2}\hat{y}\right)}$$

Point $C:(2R,2R)$

3 pts

$$\vec{B}_1: r=2\sqrt{2}R>2R \rightarrow \vec{B}_1=\mu_0 J_0 \frac{4R^2}{2r}\hat{\varphi}=\mu_0 J_0 \frac{R}{\sqrt{2}}\left(\frac{-1}{\sqrt{2}}\hat{x}+\frac{1}{\sqrt{2}}\hat{y}\right)$$

$$\vec{B}_2: r'=\sqrt{5}R>R \rightarrow \vec{B}_2=\mu_0 J_0 \frac{R^2}{2r'}\hat{\varphi}'=\mu_0 J_0 \frac{R}{2\sqrt{5}}\left(\frac{2}{\sqrt{5}}\hat{x}+\frac{-1}{\sqrt{5}}\hat{y}\right)$$

$$\vec{B}=\vec{B}_1+\vec{B}_2=\boxed{\mu_0 J_0 R\left(-\frac{3}{10}\hat{x}+\frac{2}{5}\hat{y}\right)}$$