試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(50 points) (所有題目必須有計算過程，否則不予計分)

1. (10 points) A particle has a velocity of $\vec{v} = (2.0e^{-2t}\,\hat{i} + 3.0t\,\hat{j})\,m/s$ .
   The particle starts at $\vec{r} = (1.0\,\hat{i} - 2.0\,\hat{j})m$ at $\boldsymbol{t = 0}$ .

Give the position and acceleration as a function of time .

2. (15 points) A cart ( mass $\boldsymbol{m_c}$ ) lying on a fixed frictionless incline. At $\boldsymbol{t = 0}$ , the cart starts from rest at point $\boldsymbol{A(\,x_c = 0\,,\,y_c = 0\,)}$ and moves in the incline. We choose the x axis along to the incline as shown in Fig. 1. At $\boldsymbol{t = 0}$ , a ball ( mass $\boldsymbol{m_b}$ ) is shot from the cart perpendicularly to the incline with a speed of $\boldsymbol{v}$ relative to the incline.

(A) Draw the free-body diagram for each object.

(B) Find the $\boldsymbol{x}$ and $\boldsymbol{y}$ components of the acceleration for each object.

(C) Determine $\boldsymbol{x_c(t)}$, $\boldsymbol{y_c(t)}$ , $\boldsymbol{x_b(t)}$ and $\boldsymbol{y_b(t)}$.

(D) Show that the ball will land before, in, or after the cart.

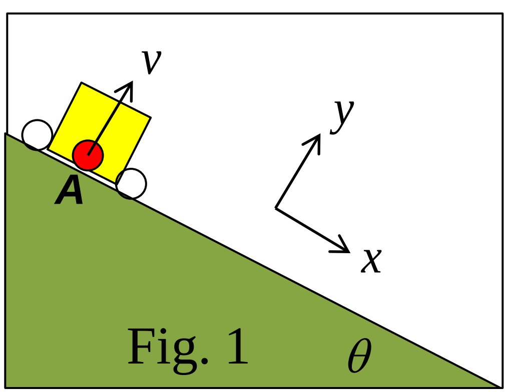


Fig. 1

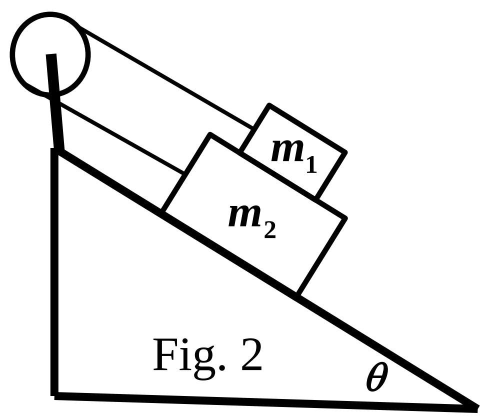


Fig. 2

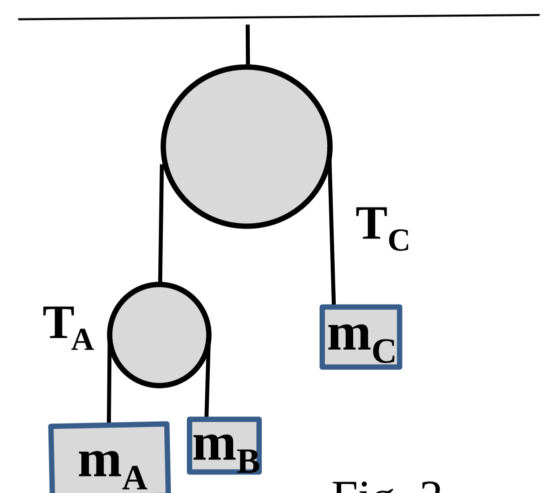


Fig. 3

3. Block 1 and block 2, with masses $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ such that $\boldsymbol{m_1} << \boldsymbol{m_2}$ ($\boldsymbol{m_1}$ is much less than $\boldsymbol{m_2}$), are connected by a massless inextensible string wrapped around a massless ideal pulley. The pulley is rigidly connected to the top of an inclined plane which makes an angle $\boldsymbol{\theta}$ with the horizontal, as shown in the Fig. 2 above. If $\boldsymbol{m_1}$ moves down, and $\boldsymbol{m_2}$ moves up. The coefficient of kinetic friction between the blocks is $\boldsymbol{\mu_k}$. The surface between the block 2 and the inclined plane is frictionless. Gravity is directed vertically downward with acceleration $\boldsymbol{g}$.

(A) Draw free-body diagrams for the two blocks. Note that you should clearly define your choice of coordinate system.

(B) Solve for the acceleration of each block in terms of $\boldsymbol{m_1}$, $\boldsymbol{m_2}$, $\boldsymbol{\mu_k}$, $\boldsymbol{\theta}$, *and* $\boldsymbol{g}$.

4. . The double Atwood machine in Fig. 3 has frictionless, massless pulleys and cords. Assume $\boldsymbol{m_A} = 2m$, $\boldsymbol{m_B} = m$, $\boldsymbol{m_C} = m$. Determine the acceleration of masses $\boldsymbol{m_A}$, $\boldsymbol{m_B}$, and $\boldsymbol{m_C}$, and the tensions $\boldsymbol{T_A}$ and $\boldsymbol{T_C}$ in the cords.

# II.選擇題(50 points)

1. (5 pts) The dimension of the gravitational constant $\boldsymbol{G}$ is $[\boldsymbol{G}]=[L]^x[M]^y[T]^z$. Which of the following is true?

(A). $x+y-z = 0$ (B) $2x+y = 0$ (C) $2x-3y = 0$ (D) $y+2z = 0$ (E) $3x+y = 0$ (F) $2y-z = 0$

2. (5 pts) The acceleration of a car as function of time is shown in Fig. 4. Assume at $t = 0$ sec, the car starts from rest. What would be the displacement of the car at $t = 1$ sec.

(A) 1 m (B) ½ m (C) 1/3 m (D) 1/6 m (E) 0 m (F) -1/6 m (G) -1/3m (H) – ½ m (J) -1 m

3. (5 pts) An electron travels in space under the influence of an static electric field and a static magnetic field, and its trajectory can be described as $\vec{r}(t) = (5\cos\pi t, 5\sin(-\pi t), 5t^2)$ Which of the following statement about the acceleration $\vec{a} = (a_x, a_y, a_z)$ of the electron at $t$ = *1.5* sec is true ?

(A) $\boldsymbol{a}_x = 5\pi$ (B) $\boldsymbol{a}_y = -5\pi$ (C) $\boldsymbol{a}_z = 15$ (D) $\boldsymbol{a}_x = 0$ (E) $\boldsymbol{a}_x = 5\pi^2$ (F) $\boldsymbol{a}_y = 5\pi^2$

4. (5 pts) A boat whose speed in still water is 4.8m/s must cross a 300 meter wide river from point ***A*** to point ***B*** as shown in Fig. 5. The speed of the river current is 3 ***m/s*** and its direction is indicated by the arrow. If the pilot set the boat to head in a direction as indicated by the arrrow at point ***A*** such that the boat travel along a straight line from ***A*** to ***B***. What is the angle β?

($\sin 15^\circ = 0.26$, $\sin 37^\circ = 0.6$, $\sin 53^\circ = 0.8$, $\sin 75^\circ = 0.97$)

(A) 15° (B) 30 ° (C) 37 ° (D) 53 ° (E) 75 °

5. (5 pts) A skier of mass *M* slides down a ramp shaped as a circle of radius *R*, as shown in the Fig. 6. At the end point of the ramp just before the skier is in the air, the magnitude of the normal force exerted by the ramp on the skier is *N*. The acceleration constant is *g*. Then:

(A) $N > Mg$ (B) $N = Mg$ (C) $N < Mg$

(D) All of (A), (B), and (C) are possible depending on the speed of the skier.

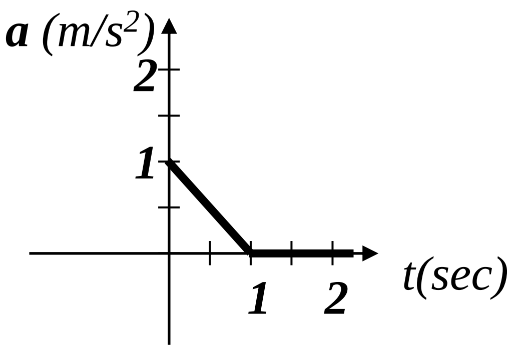


Fig. 4

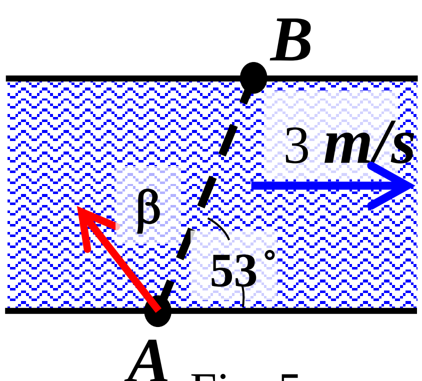


Fig. 5

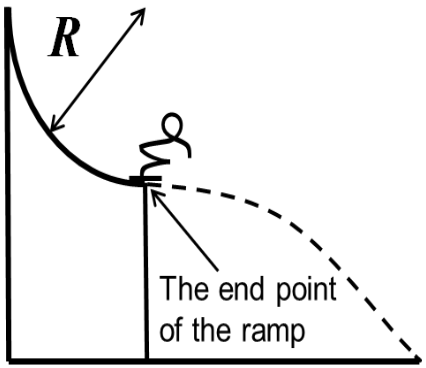


Fig. 6

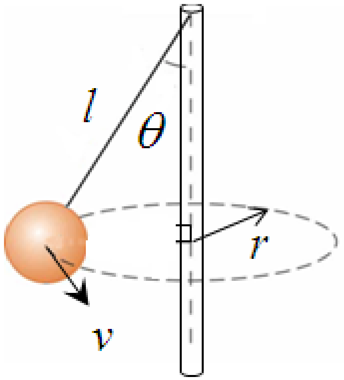


Fig. 7

6. (5 pts) The fig. 7 represents a point mass $\boldsymbol{m}$ attached a cord of fixed length $\boldsymbol{l}$. If the mass moves in a horizontal circle of radius $\boldsymbol{r}$ with uniform velocity $\boldsymbol{v}$, the tension in the cord is

(A) $\boldsymbol{mg\cos\theta}$ (B) $\boldsymbol{\dfrac{mv}{\sin\theta}}$ (C) $\boldsymbol{mg\dfrac{r}{l}}$ (D) $\boldsymbol{m\sqrt{v^2+g^2}}$ (E) $\boldsymbol{m\sqrt{\dfrac{v^4}{r^2}+g^2}}$

(F) None above

7. (5 pts) A rock is thrown vertically upward with initial speed $\boldsymbol{v_0}$. Assume a drag force proportional to $-\vec{v}$ , where $\vec{v}$ is the velocity of the rock. Which of the following is correct?

(A) The acceleration of the rock is always equal to **g**

(B) The acceleration of the rock is equal to **g** only at the top of the flight.

(C) The acceleration of the rock is always less than **g**

(D) The speed of the rock upon return to its starting point is $\boldsymbol{v_0}$

(E) The rock can attain a terminal speed greater than $\boldsymbol{v_0}$ before it returns to its starting point.

8. (5 pts) You place two bricks (Fig. 8) on a level board and then slowly increase $\boldsymbol{\theta}$. One of the brick ($\boldsymbol{m_2}$) has twice the mass and twice the surface contact area as the other brick ($\boldsymbol{m_1}$). The coefficients of static friction and kinetic friction are the same for each of the two bricks. Which of the following statement(s) is (are) true?

1. The brick ($\boldsymbol{m_1}$) slides before the brick ($\boldsymbol{m_2}$) .
2. The brick ($\boldsymbol{m_1}$) slides after the double brick.
3. Both bricks ($\boldsymbol{m_1}$ and $\boldsymbol{m_2}$) begin to slide at the same time.
4. The brick ($\boldsymbol{m_1}$) has a greater acceleration than the brick.
5. The brick ($\boldsymbol{m_1}$) has a smaller acceleration than the brick.
6. Both bricks ($\boldsymbol{m_1}$ and $\boldsymbol{m_2}$) have the same acceleration.

(A) 1,4 (B) 1,5 (C) 1,6 (D) 2,4 (E) 2,5 (F) 2,6

(G) 3,4 (H) 3,5 (I) 3,6 (J) None of above

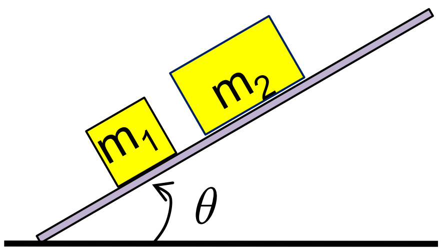



Fig.8

# SOLUTION

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| F | C | D | B | A | E | B | I |

| 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|
| A | A | B | D | E | C | E | B | B | A |

1. (10 points) A particle has a velocity of $\vec{v} = (2.0e^{-2t}\hat{i} + 3.0t\hat{j})m/s$ .

The particle starts at $\vec{r} = (1.0\hat{i} - 2.0\hat{j})m$ at ***t* = *0*** .

Give the position and acceleration as a function of time .

$$\int_1^x dx = \int_0^t 2.0e^{-2t}dt$$

$$x - 1 = -\int_0^t e^{-2t}d(-2t) = -\int_0^{-2t} e^u du = -e^u\Big|_0^{-2t} = 1 - e^{-2t}$$

$$x = 2 - e^{-2t}\ (m)$$ 3 points

$$\int_{-2}^y dy = \int_0^t 3tdt = \frac{3t^2}{2}\Bigg|_0^t = \frac{3t^2}{2} \quad y = \frac{3t^2}{2} - 2 \quad (m)$$ 3 points

$$a_x = \frac{dv_x}{dx} = \frac{d}{dx}(2.0e^{-2t}) = -4.0e^{-2t} \quad (m/s^2)$$ 2 points

$$a_y = \frac{dv_y}{dx} = \frac{d}{dx}(3.0t) = 3 \quad (m/s^2)$$ 2 points

2. (15 pts) A cart ( mass $\boldsymbol{m_c}$ ) lying on a fixed frictionless incline. At $\boldsymbol{t = 0}$ , the cart starts from rest at $\boldsymbol{( x_c = 0 , y_c = 0 )}$ and moves in the incline. We choose the x axis along to the incline as shown in Fig. 1. At the same time, a ball ( mass $\boldsymbol{m_b}$ ) is thrown from the cart perpendicularly to the incline with a speed of $\boldsymbol{v}$ relative to the incline.

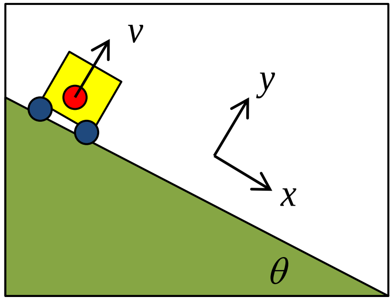


Fig. 1

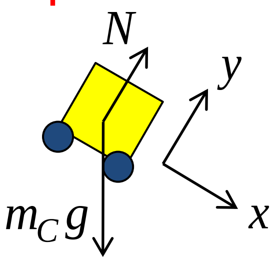
a. Draw the free-body diagram for each object ( the cart and the ball ) after the ball is thrown .

b. Find the ***x*** and ***y*** components of the acceleration for each object.

c. Determine $\boldsymbol{x_c(t)}$, $\boldsymbol{y_c(t)}$ , $\boldsymbol{x_b(t)}$ and $\boldsymbol{y_b(t)}$.

d. Show that the ball will land in the cart.

1 point




$$\sum F_y = N - m_c g\cos\theta = m_c a_{cy} = 0$$

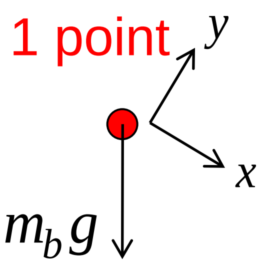
$$\Rightarrow \quad N = m_c g\cos\theta; \qquad a_{cy} = 0$$

1 point

$$\sum F_x = m_c g\sin\theta = m_c a_{cx} \quad \Rightarrow \quad a_{cx} = g\sin\theta$$

1 point

1 point




$$\sum F_y = -m_b g\cos\theta = m_b a_{by} \quad \Rightarrow \quad a_{by} = -g\cos\theta$$

1 point

$$\sum F_x = m_b g\sin\theta = m_b a_{bx} \quad \Rightarrow \quad a_{bx} = g\sin\theta$$

1 point

$$a_{cy} = 0 \Rightarrow y_c = 0$$ 1 point

$$a_{cx} = g\sin\theta \Rightarrow x_c = \frac{1}{2} g\sin\theta t^2$$ 1 point

$$a_{by} = -g\cos\theta \Rightarrow y_b = vt - \frac{1}{2} g\cos\theta t^2$$ 1 point

$$a_{bx} = g\sin\theta \Rightarrow x_b = \frac{1}{2} g\sin\theta t^2$$ 1 point

Find the time ( $t_1$ ) when the ball hit the incline

$$y_b = 0 \Rightarrow t_1 = \frac{2v}{g\cos\theta}$$

$$x_c = \frac{1}{2} g\sin\theta \left( \frac{2v}{g\cos\theta} \right)^2 = \frac{2v^2 \sin\theta}{\cos^2\theta}$$

$$x_b = \frac{1}{2} g\sin\theta \left( \frac{2v}{g\cos\theta} \right)^2 = \frac{2v^2 \sin\theta}{\cos^2\theta}$$

At $t_1$ , $x_c = x_b$ and $y_c = y_b$ , therefore the ball will land in the cart. 5 points

3. (10 pts) Block 1 and block 2, with masses $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ such that $\boldsymbol{m_1} << \boldsymbol{m_2}$ ($\boldsymbol{m_1}$ is much less than $\boldsymbol{m_2}$ ), are connected by a massless inextensible string wrapped around a massless ideal pulley. The pulley is rigidly connected to the top of an inclined plane which makes an angle $\boldsymbol{q}$ with the horizontal, as shown in the Fig. 2 above. If $\boldsymbol{m_1}$ moves down, and $\boldsymbol{m_2}$ moves up. The coefficient of kinetic friction between the blocks is $\boldsymbol{\mu_k}$ . The surface between the lower block and the inclined plane is frictionless. Gravity is directed vertically downward with acceleration $\boldsymbol{g}$ .

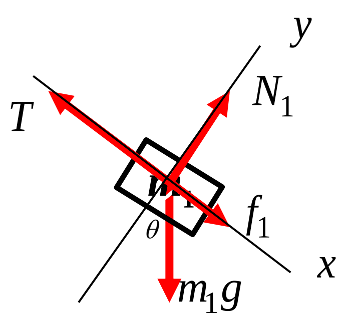
(A) Draw free-body diagrams for the two blocks. Note: You should clearly define your choice of coordinate system.

(B) Solve for the acceleration of each block in terms of $\boldsymbol{m_1}$ , $\boldsymbol{m_2}$ , $\boldsymbol{\mu_k}$ , $\boldsymbol{q}$ , *and* $\boldsymbol{g}$ .

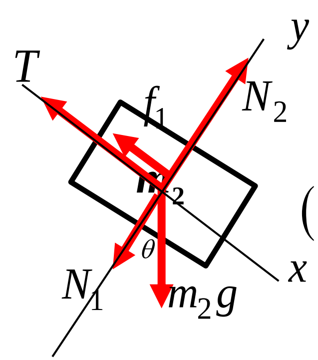
Solution:

(A)

2 pts




2 pts




$f$方向相反:4 pts→ 1pt.

(B)

$$m_1: \quad T - f_1 - m_1 g \sin\theta = m_1 a \quad (1)$$

$$N_1 - m_1 g \cos\theta = 0$$

$$f_1 = \mu_k N_1 = \mu_k m_1 g \cos\theta$$

2 pts

$$m_2: \quad m_2 g \sin\theta - T - f_1 = m_2 a \quad (2)$$

$$N_2 - N_1 - m_2 g \cos\theta = 0$$

1 pt

Equ. (1)+Equ .(2): (eliminate ***T*** )

$$(m_2 - m_1) g \sin\theta - 2\mu_k m_1 g \cos\theta = (m_2 + m_1) a$$

$$a = g \cdot \frac{(m_2 - m_1)\sin\theta - 2\mu_k m_1 \cos\theta}{m_1 + m_2}$$

3 pts

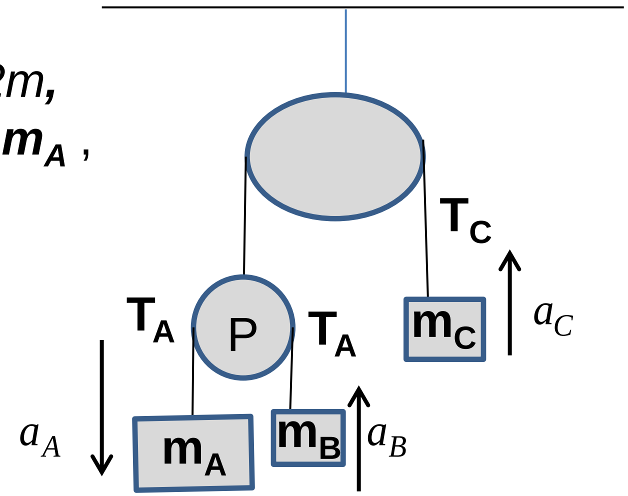
$f$方向相反時:

$$m_1: \quad m_1 g \sin\theta - T - f_1 = m_1 a$$

$$m_2: \quad T - f_1 - m_2 g \sin\theta = m_2 a$$

$$a = g \cdot \frac{(m_1 - m_2)\sin\theta - 2\mu_k m_1 \cos\theta}{m_1 + m_2}$$

4. . (15 pts) The double Atwood machine in Fig. 3 has frictionless, massless pulleys and cords. Assume $\boldsymbol{m_A} = 2m$, $\boldsymbol{m_B} = m$, $\boldsymbol{m_C} = m$. Determine the acceleration of masses $\boldsymbol{m_A}$, $\boldsymbol{m_B}$, and $\boldsymbol{m_C}$ and the tensions $\boldsymbol{T_A}$ and $\boldsymbol{T_C}$ in the cords.




Solution:

$$\left.\begin{aligned} m_A &: \quad 2mg - T_A = 2ma_A \downarrow \\ m_B &: \quad T_A - mg = ma_B \uparrow \\ m_C &: \quad T_C - mg = ma_C \uparrow \end{aligned}\right\} \text{5 pts}$$

Pully P: $T_C - 2T_A = 0$ 2 pts

Assume ***a'*** ↓ is the acceleration of $\boldsymbol{m_A}$ relative to the pulley *P* .

$$\Rightarrow a_A = a_{AG} = a_{AP} + a_{PG} = a' + a_C$$

$$a_B = a' - a_C$$

or $a_A - a_B = 2a_C$ 2 pts

$$m_A: \quad 2mg - T_A = 2m(a' + a_C) \quad \downarrow (1)$$

$$m_B: \quad T_A - mg = m(a' - a_C) \quad \uparrow (2)$$

$$m_C: \quad 2T_A - mg = ma_C \quad \uparrow (3)$$

(~9 pts)

Equ. (1)+Equ .(2): (eliminate $\boldsymbol{T_A}$ )

$$mg = 3ma' + ma_C$$

$$\Rightarrow a' = \frac{g - a_C}{3} \quad (4)$$

Insert Equ. (4) into Equ. (1):

$$2mg - T_A = \frac{2mg + 4ma_C}{3}$$

$$\Rightarrow \quad 4mg - 3T_A = 4ma_C \quad (1')$$

2*Equ. (1')+3*Equ .(3): (eliminating $\boldsymbol{T_A}$)

$$5mg = 11ma_C \quad \Rightarrow \quad a_C = \tfrac{5}{11}g$$

Equ. (4) ➔ $a' = \tfrac{2}{11}g \ \Rightarrow \ a_A = \tfrac{7}{11}g \ ; a_B = -\tfrac{3}{11}g$

or $a_A = \tfrac{7}{11}g \downarrow; \ a_B = \tfrac{3}{11}g \downarrow; a_C = \tfrac{5}{11}g \uparrow$ 4 pts

Equ. (3) $\Rightarrow \quad 2T_A = T_C = m\left(g + a_C\right)$

$$\Rightarrow \quad T_A = \tfrac{8}{11}mg \ ; T_C = \tfrac{16}{11}mg$$

2 pt