# 109 學年度第一學期「通訊原理」Ans#1

Date:11/05/2020

1. 每題 2 分，共 30 分

a) Ans: $\cos(2\pi f_0 t)$

b) Ans: $X(f)=\int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$

c) Ans: $\int_{-\infty}^{\infty} x(\tau)y(t-\tau)d\tau$

d) Ans: $X(f)e^{-j2\pi ft_0}$

e) Ans: $H_{LP}(f)=H_0\Pi\left(\frac{f}{2B}\right)$ or $H_{LP}(f)=H_0\Pi\left(\frac{f}{2B}\right)e^{-j2\pi ft_0}$

f) Ans: $T_0\mathrm{sinc}\left(fT_0\right)$

g) Ans: $\sum_{k=-\infty}^{\infty} a_k\delta(f-kf_0)$

h) Ans: $\frac{1}{2}[X(f-f_0)+X(f+f_0)]$

i) Ans: $\frac{1}{\pi t}$

j) Ans: $R_x\left(\tau\right)=\lim_{T\to\infty}\frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} x(t)\,x^*(t-\tau)\,dt$ or $R_x\left(\tau\right)=\lim_{T\to\infty}\frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} x(t+\tau)\,x^*(t)\,dt$

k) Ans: $-\cos(2\pi f_0 t)$

l) Ans: $x(t)+j\widehat{x}(t)$

m) Ans: 1

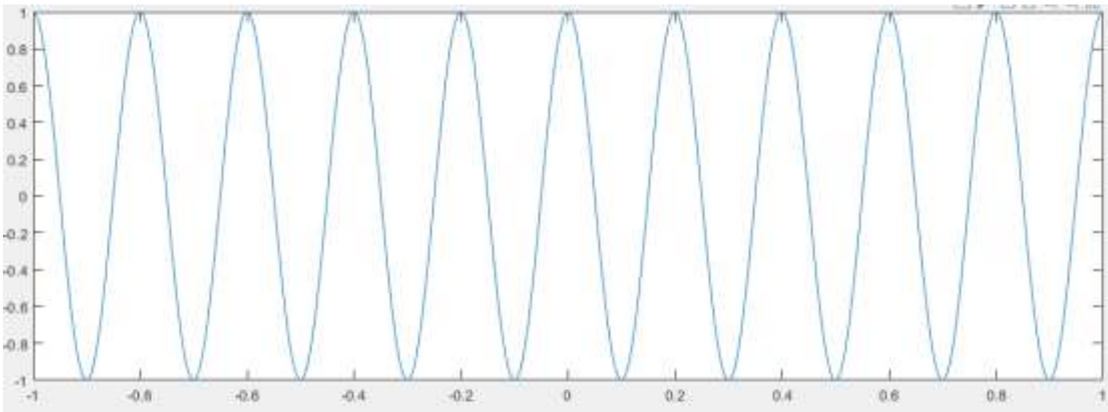
n) Ans: $x(t)=x_R(t)\cos\left(2\pi f_0 t\right)-x_I(t)\sin\left(2\pi f_0 t\right)$

o) Ans: $\cos(\alpha+\beta)=\cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)$

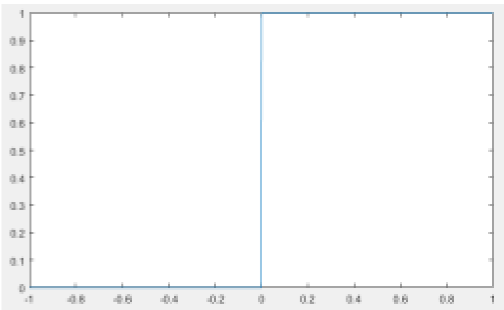
2. 每題 10 分，共 20 分

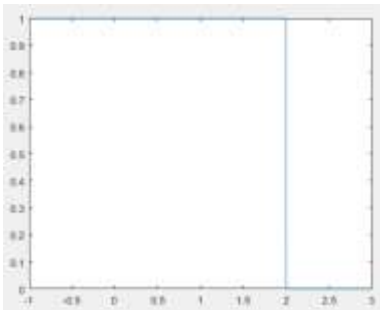
(a)

$\cos(10\pi t)$

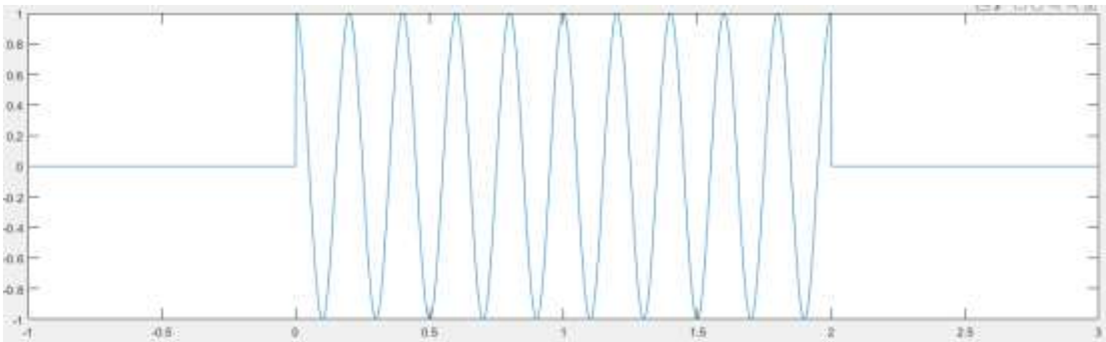



$u(t)$ $u(\ 2t$-



$x_1(t) = \cos(10\pi t)u(t)u(2-t)$




This is a cosine burst nonzero between 0 and 2 seconds.

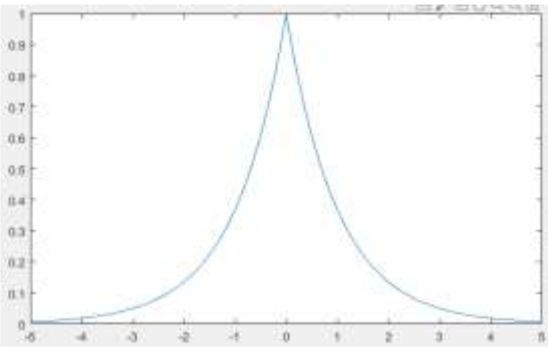
Its power is $P \triangleq \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \left| x_1(t) \right|^2 dt = 0.$

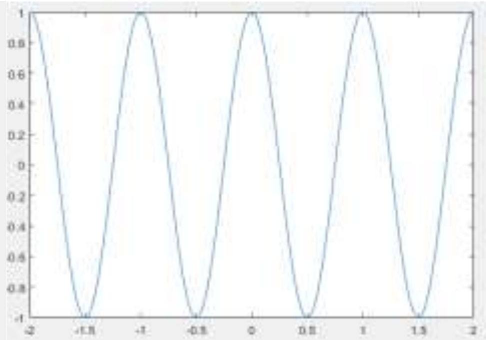
Its energy is $E_1 \triangleq \lim_{T\to\infty}\int_{-T}^{T} |x_1(t)|^2\, dt = \int_0^2 \cos^2(10\pi t)\, dt = \int_0^2 \frac{1+\cos(20\pi t)}{2}\, dt = 1$ J . It is an energy signal.

2.

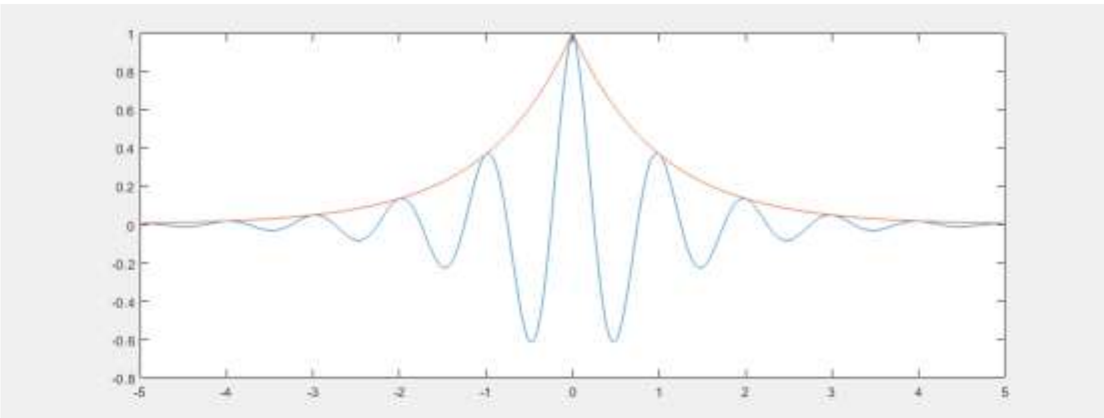
(b)

$e^{-|t|}$ $\cos(2\pi t)$





$x_2(t) = e^{-|t|}\cos(2\pi t)$



Its power is $P \triangleq \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\left|x_1(t)\right|^2 dt = 0.$

Using evenness of the integrand, its energy is

$$E_2 \triangleq \lim_{T\to\infty}\int_{-T}^{T}\left|x_2(t)\right|^2 dt = 2\int_0^\infty e^{-2t}\cos^2(2\pi t)\,dt = \int_0^\infty e^{-2t}\,dt + \int_0^\infty e^{-2t}\cos(4\pi t)\,dt \qquad (*)$$

Next, we compute the two integration terms on the right of (*). First,

$$\int_0^\infty e^{-2t}\,dt = -\frac{1}{2}e^{-2t}\Big|_{t=0}^{\infty} = \frac{1}{2} \qquad (**)$$

Then,

$$\int_0^{\infty} e^{-2t}\cos(4\pi t)\,dt, \text{ integration by parts}$$

$$= \underbrace{e^{-2t}\,\frac{1}{4\pi}\sin(4\pi t)\Big|_{t=0}^{\infty}}_{0-0=0} - \int_0^{\infty}\frac{-2}{4\pi}\sin(4\pi t)e^{-2t}\,dt, \text{ integration by parts again}$$

$$= -\left(\frac{1}{8\pi^2}\underbrace{e^{-2t}\cos(4\pi t)\Big|_{t=0}^{\infty}}_{0-1=-1} - \int_0^{\infty}\frac{-1}{4\pi^2}\cos(4\pi t)e^{-2t}dt\right).$$

Now, we have

$$\frac{4\pi^2+1}{4\pi^2}\int_0^{\infty} e^{-2t}\cos(4\pi t)\,dt = \frac{1}{8\pi^2},$$

which implies

$$\int_0^{\infty} e^{-2t}\cos(4\pi t)\,dt = \frac{1}{8\pi^2+2}. \qquad (***)$$

Finally, by substituting (***) and (**) into (*), we have

$$E_2 \square \int_0^{\infty} e^{-2t}\,dt + \int_0^{\infty} e^{-2t}\cos(4\pi t)\,dt = \frac{1}{2} + \frac{1}{2+8\pi^2}.$$

3. 共 10 分

$$x(t) = \sum_{m=-\infty}^{\infty}\delta(t-mT_s) = \sum_{n=-\infty}^{\infty} X_n e^{jn2\pi f_s t} \quad , \quad f_s = \frac{1}{T_s}$$

$$X_n = \frac{1}{T_s}\int_{\frac{-T}{2}}^{\frac{T}{2}} \delta(t)\,e^{-jn2\pi f_s t}\,dt = f_s$$

$$\Rightarrow x(t) = f_s\sum_{n=-\infty}^{\infty} e^{j2\pi f_s t}$$

4. 共 10 分

$$x(t) = \Pi\left(\frac{t}{\tau}\right)\cos(2\pi f_0 t)$$

$$\cos(2\pi f_0 t) \overset{\text{CTFT}}{\longleftrightarrow} \frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$$

$$\Pi\left(\frac{t}{\tau}\right) \overset{\text{CTFT}}{\longleftrightarrow} \tau\mathrm{sinc}(f\tau)$$

$$X(f) = \mathcal{F}\left\{\Pi\left(\frac{t}{\tau}\right)\cos(2\pi f_0 t)\right\} = [\tau\mathrm{sinc}(f\tau)] * \left[\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)\right]$$

$$= \frac{1}{2}\tau\left\{\sin c\left[(f - f_0)\tau\right] + \sin c\left[(f + f_0)\tau\right]\right\}$$

5. 每題 5 分，共 20 分

The autocorrelation function must be (1) even, (2) have an absolute maximum at $\tau = 0$ and (3) have a Fourier transform that is real and nonnegative.

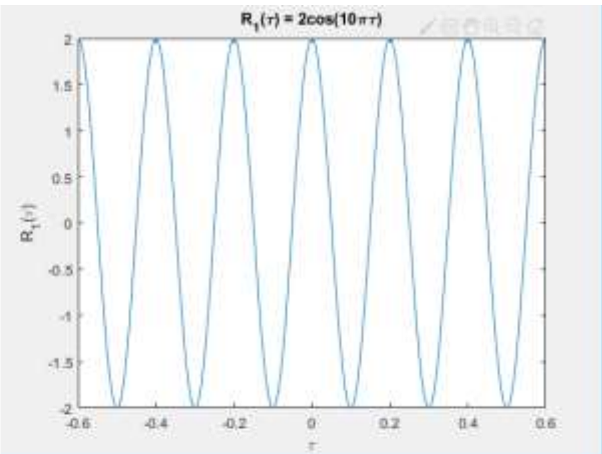
a. Legitimate. All properties satisfied.

(1) $R_1(\tau) = R_1(-\tau) = 2\cos(10\pi\tau)$

(2) $\tau = 0, R_1(\tau)$ have an absolute maximum.

(3) $S_1(f) = \int_{-\infty}^{\infty} 2\cos(10\pi\tau)e^{-j2\pi f\tau}d\tau = \int_{-\infty}^{\infty} 2\left(\frac{e^{j2\pi 5\tau} + e^{-j2\pi 5\tau}}{2}\right)e^{-j2\pi f\tau}d\tau = \delta(f\text{ -}5) + \delta(f + 5)$

$S_1(f)$ is real and nonnegative.



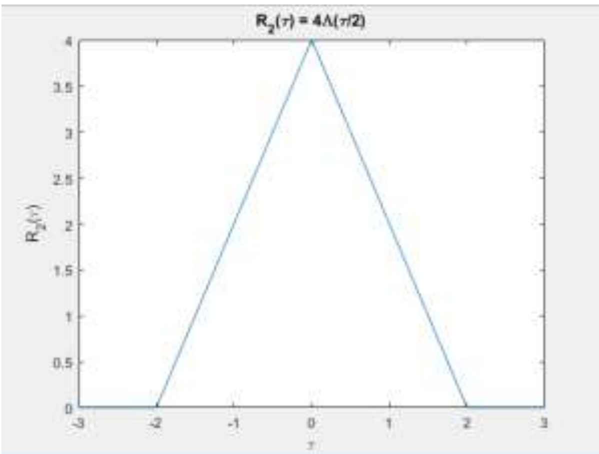

b. Legitimate. All properties satisfied.

(1) $R_2(\tau) = R_2(-\tau) = 4\Lambda(\tau/2)$

(2) $\tau = 0$, $R_2(\tau)$ have an absolute maximum.

(3) $S_2(f) = \int_{-\infty}^{\infty} 4\Lambda(\tau/2) e^{-j2\pi f\tau} d\tau = 8\,\mathrm{sinc}^2(2f)$

$S_2(f)$ is real and nonnegative.



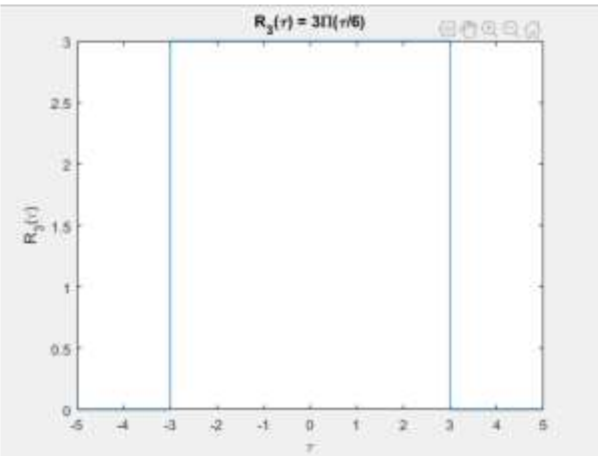
c. Illegitimate. Property (3) not satisfied

(1) $R_3(\tau) = R_3(-\tau) = 3\Pi(\tau/6)$

(2) $\tau = 0$, $R_3(\tau)$ have an absolute maximum.

(3) $S_3(f) = \int_{-\infty}^{\infty} 3\Pi(\tau/6)e^{-j2\pi f\tau}d\tau = 18\,\mathrm{sinc}(6f)$

$S_3(f)$ is not real and nonnegative.



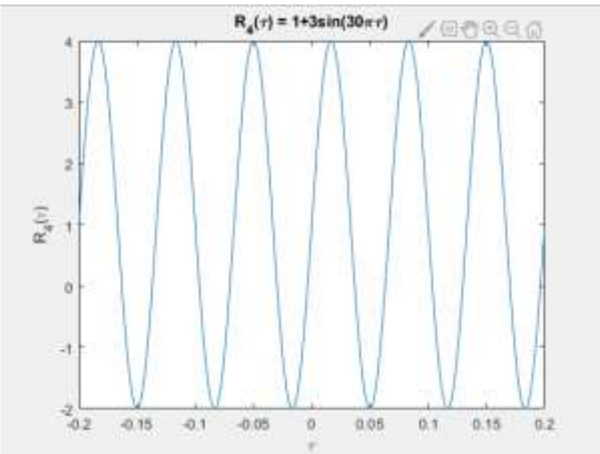

d. Illegitimate. None of properties satisfied.

(1) $R_4(\tau) = 1 + 3\sin(30\pi\tau) \neq R_4(-\tau) = 1 - 3\sin(30\pi\tau)$

(2) $\tau = 0$, $R_4(\tau)$ isn't the maximum. $\tau = \frac{1}{60}$, $R_4(\tau)$ have an absolute maximum.

(3) $S_4(f) = \int_{-\infty}^{\infty}(1 + 3\sin(30\pi\tau))e^{-j2\pi f\tau}d\tau = \int_{-\infty}^{\infty}\left(1 + 3\left(\frac{e^{j2\pi 15\tau} - e^{-j2\pi 15\tau}}{2j}\right)\right)e^{-j2\pi f\tau}d\tau$

$= \delta(f) + \frac{3(\delta(f\text{-}15) - \delta(f+15))}{2j}$, $S_4(f)$ isn't real and nonnegative.



6. 共 10 分

$$x(t)=\cos(2\pi f_0 t)$$

$$x(t)\overset{\text{CTFT}}{\longleftrightarrow} X(f)=\frac{1}{2}\delta(f-f_0)+\frac{1}{2}\delta(f+f_0)$$

$$\begin{aligned}\hat{X}(f)&=-j\,\mathrm{sgn}(f)X(f)\\&=-j\,\mathrm{sgn}(f)\left[\frac{1}{2}\delta(f-f_0)+\frac{1}{2}\delta(f+f_0)\right]\\&=-\frac{j}{2}\delta(f-f_0)\mathrm{sgn}(f)+\frac{j}{2}\delta(f+f_0)\mathrm{sgn}(f),\ \text{assume that } f_0>0\\&=-\frac{j}{2}\delta(f-f_0)+\frac{j}{2}\delta(f+f_0)\end{aligned}$$

$$\begin{aligned}\hat{x}(t)&=\mathcal{F}^{-1}\left\{\hat{X}(f)\right\}=\mathcal{F}^{-1}\left\{-\frac{j}{2}\delta(f-f_0)+\frac{j}{2}\delta(f+f_0)\right\}\\&=-\frac{j}{2}e^{j2\pi f_0 t}+\frac{j}{2}e^{-j2\pi f_0 t}\\&=-\frac{j}{2}(e^{j2\pi f_0 t}-e^{-j2\pi f_0 t})=\sin(2\pi f_0 t),\ \text{Euler's relation}\end{aligned}$$

Time-average power: $R_x(0)$, $R_{\hat{x}}(0)$

$$\begin{aligned}R_x(0)&=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\cos(2\pi f_0 t)\cos^*(2\pi f_0 t)dt\\&=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\left|\cos(2\pi f_0 t)\right|^2\,dt=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\frac{1+\cos(4\pi f_0 t)}{2}dt\\&=\frac{1}{T_0}\int_0^{T_0}\frac{1+\cos(4\pi f_0 t)}{2}dt\\&=\frac{1}{2}\end{aligned}$$

$$\begin{aligned}R_{\hat{x}}(0)&=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\sin(2\pi f_0 t)\sin^*(2\pi f_0 t)dt\\&=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\left|\sin(2\pi f_0 t)\right|^2\,dt=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\frac{1-\cos(4\pi f_0 t)}{2}dt\\&=\frac{1}{T_0}\int_0^{T_0}\frac{1-\cos(4\pi f_0 t)}{2}dt\\&=\frac{1}{2}\end{aligned}$$