General Physics (I) **期末考** **Jan. 4, 2019**

試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(53points) (所有題目必須有計算過程,否則不予計分)

1&2. (16 pts) As shown in Fig. 1(A) , a plastic ball with mass $\boldsymbol{m}$ is fired with velocity $\boldsymbol{v_0}$ at the end of a uniform rod with the other end fixed on the wall, and bounces back with velocity $\boldsymbol{v}$. Right after the collision, the angular velocity of the rod (mass $\mathbf{4}\boldsymbol{m}$ and length $\boldsymbol{l}$) is $\boldsymbol{\omega_o = v_o/\ell}$ and begins to swing (Fig. 1(B)).

(a)(6 pts) What is the velocity $\boldsymbol{v}$ of plastic ball right after the collision?

(b) (6 pts) Draw the free-body diagram and derive the equation of motion for this swinging rod. The general form of the solution is $\boldsymbol{\theta(t) = A\sin(\omega t + \delta)}$, and note that the collision occurs at $\boldsymbol{t = 0}$. What is the period of this physical pendulum?

(c) (4 pts) What are the amplitude $\boldsymbol{A}$ and the phase $\boldsymbol{\delta}$?
(hint: apply the initial condition to the general solution of $\boldsymbol{\theta(t)}$)

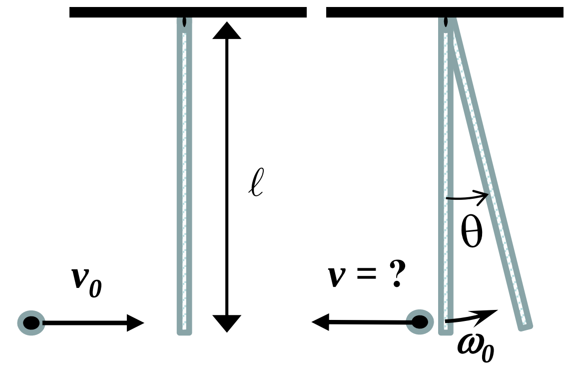


Fig. 1 (A) (B)

3. (17pts) A particle is confined to move in x-direction between $\boldsymbol{x}$=0 and $\boldsymbol{x}$= ∞, and it experiences an conservative force $\boldsymbol{F}(\boldsymbol{x})$ such that its potential energy $\boldsymbol{U}(\boldsymbol{x}) = -\boldsymbol{b}x^2 \cdot e^{-\boldsymbol{a}x}$, where $\boldsymbol{a},\boldsymbol{b}>\boldsymbol{0}$,

(a) (6pts) Determine this conservative force $\boldsymbol{F}(\boldsymbol{x})$ as a function of x,

(b) (4pts) At the equilibrium point $\boldsymbol{x} = \boldsymbol{S}$, $\boldsymbol{F}(\boldsymbol{S}) = 0$, determine the value of $\boldsymbol{S}$.

(c) (4pts) If the particle is moving around $\boldsymbol{S}$, and if we define $\boldsymbol{z = x\text{-}S}$, write down the equation of motion of the particle in terms of $\boldsymbol{z}$,

(d) (3pts) For the case if $\boldsymbol{z/S \ll 1}$, the particle executes a simple harmonic oscillation around S, determine the period of the oscillation of the particle near S.

Useful formula: $(1+z)^n \approx 1+nz, \quad e^{az} \approx 1+az, \quad for\ |z| \ll 1, |az| \ll 1$

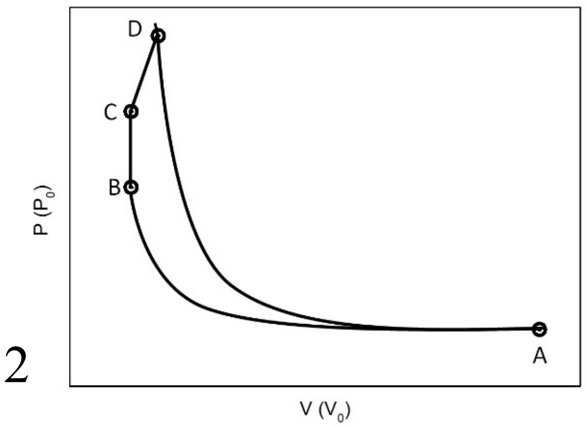


Fig. 2

4. (20 pts) A one mole monatomic ideal gas engine is operated by the cycle shown in Fig. 2, where process A→B is isothermal, B→C: isovolumetric, C→D: straight-line, and D→A: adiabatic. The volumes at A, B, C, and D are $128V_0$, $8V_0$, $8V_0$, and $16V_0$, respectively. The pressures at point A and C are P_0 and $24P_0$, respectively.

(Write down your answer in term of P_0, V_0, R, ln2, ln3, ln5, and ln7)

(a) (4 pts) Find P, V, and T at points A, B, C, and D.

(b) (12 pts) Calculate the work W done (by the gas), the heat transfer Q, the change of the internal energy ΔE_{int}, and the change of entropy ΔS for each process.

(c) (4 pts) Find the efficiency of this ideal gas engine.

請將下列表格抄至答案紙上，否則不予批改。

	P (P_0)	V (V_0)	T (P_0V_0/R)
A			
B			
C			
D			

	W (P_0V_0)	Q (P_0V_0)	ΔE_{int} (P_0V_0)	ΔS (R)
A→B				
B→C				
C→D				
D→A				

II.選擇題(47 points)

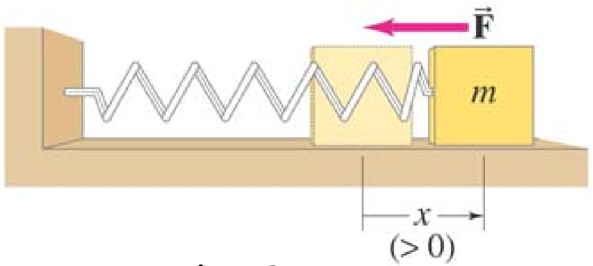


Fig. 3

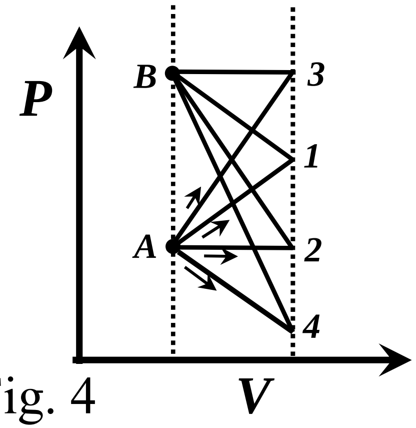
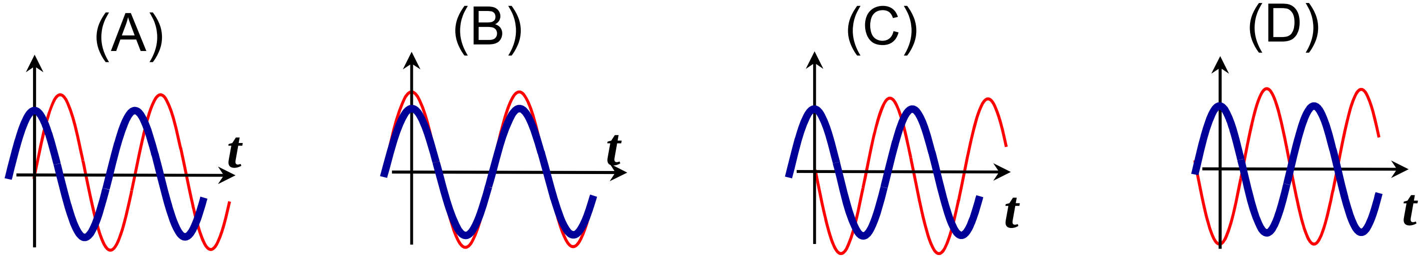


Fig. 4

1. (5pts) As shown in Fig. 3, the simple harmonic motion of the mass attached to the spring can be described as displacement $\boldsymbol{x}(t)$, the velocity $\boldsymbol{v}(t)$ and acceleration $\boldsymbol{a}(t)$. In the following diagrams, the velocity is represented by ——— , and acceleration represented by ——— . Which of the following diagram shows the correct $\boldsymbol{v}(t)$ and $\boldsymbol{a}(t)$, despite the amplitude?



2. (5pts) As shown in Fig. 4, an ideal gas system in the P-V diagram started from point A and goes through four different processes throught points labelled as 1,2,3,and 4 to point B, and exchange heat $\boldsymbol{Q_1}$, $\boldsymbol{Q_2}$, $\boldsymbol{Q_3}$, and $\boldsymbol{Q_4}$, with the environment, respectively. Which of the following statement is correction regarding the heat exchange between the system and the environment.

(A) $Q_4 < 0$ (B) $Q_4 < Q_3 < Q_2 < Q_1$ (C) $Q_1 = 0$ (D) $Q_4 = Q_1$
(E) $Q_4 > Q_3 > Q_2 > Q_1$ (F) $Q_1 > Q_3 = Q_2 > Q_4$ (G) $Q_1 < Q_3 = Q_2 < Q_4$

3. (5 pts)) 1.0 mole of Oxygen (O_2) gas at **240K** and 1.0 mole of argon (Ar) gas at **400K** are separated by a insulated wall with equal-sized and pressure in an insulated container. The insulated wall is removed suddenly and the gases (assumed ideal) allowed to mix. The change of the entropy of the system $\Delta \boldsymbol{S/R} = \boldsymbol{a}$. What is the value $\boldsymbol{a}$? Note: in this temperature range, $\boldsymbol{C_V = 3R/2, C_P = 5R/2}$ for monatomic ideal gas and $\boldsymbol{C_V = 5R/2, C_P = 7R/2}$ for diatomic ideal gas. (ln2 ~ 0.7, ln3 ~1.1, ln5 ~ 1.6)

(A) $a \leq -2$ (B) $-2 < a \leq -1.6$ (C) $-1.6 < a \leq -1.2$ (D) $-1.2 < a \leq -0.8$ (E) $-0.8 < a \leq -0.4$
(F) $-0.4 < a \leq 0$ (G) $0 < a \leq 0.4$ (H) $0.4 < a \leq 0.8$ (J) $0.8 < a \leq 1.2$ (K) $1.2 < a \leq 1.6$
(L) $1.6 < a \leq 2.0$ (M) $2.0 < a$.

4. (5 pts). The specific heat per mole of some metal at low temperatures is given by $C = \left(2T + 2.5T^3\right) \times 10^{-3}\ \mathrm{J/mol \cdot K}$. We drop 0.1 mole of this metal at ***6K*** into a cup full with ***4K*** liquid Helium. The entropy change of this metal is $\boldsymbol{a} \times \mathbf{10^{-3}\ J/K}$ when it reaches the thermal equilibrium with the liquid Helium. What is the value of ***a***? (assume the amount of liquid Helium is so large that its temperature rise is insignificant.)

(A) $a \leq -30$ (B) $-30 < a \leq -25$ (C) $-25 < a \leq -20$ (D) $-20 < a \leq -15$ (E) $-15 < a \leq -10$
(F) $-10 < a \leq -5$ (G) $-5 < a \leq 5$ (H) $5 < a \leq 10$ (J) $10 < a \leq 15$ (K) $15 < a \leq 20$
(L) $20 < a \leq 25$ (M) $25 < a \leq 30$ (N) $30 < a$.

5. (5 pts). Same as problem 4, the entropy change of the surrounding environment (the liquid Helium) is $\boldsymbol{b} \times \mathbf{10^{-3}\ J/K}$. What is the value of ***b***?

(A) $b \leq -30$ (B) $-30 < b \leq -25$ (C) $-25 < b \leq -20$ (D) $-20 < b \leq -15$ (E) $-15 < b \leq -10$
(F) $-10 < b \leq -5$ (G) $-5 < b \leq 5$ (H) $5 < b \leq 10$ (J) $10 < b \leq 15$ (K) $15 < b \leq 20$
(L) $20 < b \leq 25$ (M) $25 < b \leq 30$ (N) $30 < b$.

Multiple Choice Questions:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
A	**D**	**K**	**E**	**K**	**B**	**B**	**B**	
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**	
A	**E**	**E**	**C**	**C**	**A or E**	**E**	**G**	

*選擇題第十四題答案選A或E都計分.

1. (16 pts) As shown in the Fig. 1(A) , a plastic ball with mass ***m*** is fired with velocity $\boldsymbol{v_0}$ at the end of uniform rod with one end is fixed on the wall, and bounces back with velocity ***v***. Right after the collision, the angular velocity of the rod (mass **4*m*** and length ***l***) is $\omega_o = v_o/\ell$ and begin to swing (Fig. 1(B)).)

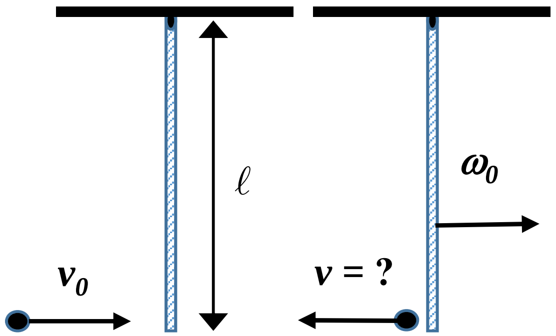


Fig. 1(A) Fig. 1(B)

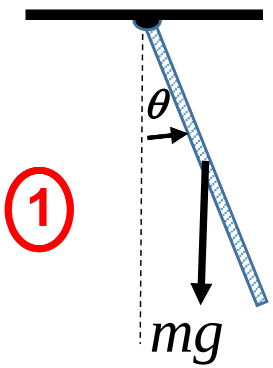
(a) Angular momentum conservation:

$$l \cdot mv_0 (\hat{z}) = I\omega_0 (\hat{z}) \pm l \cdot mv(\mp\hat{z})$$ 2

1 $$l \cdot mv_0 = \frac{1}{3}(4m)l^2 \cdot \omega_0 \pm l \cdot mv$$ 1

$$\Rightarrow v = \mp\frac{1}{3}v_0$$ 2 Direction: due left

(b) Derive the equation of motion:



1

$$-4mg\frac{l}{2}\sin\theta = I\alpha = \frac{4}{3}ml^2\frac{d^2\theta}{dx^2}$$ 2

$$\sin\theta \approx \theta \quad if \ |\theta| \ll 1$$

$$\frac{d^2\theta}{dx^2} + \frac{3g}{2l}\theta \approx 0$$ 1

$$\omega^2 = \frac{3g}{2l} \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2l}{3g}}$$ 2

(c) Amplitude and phase:

Initial condition:

$$\theta(t=0) = A\sin\delta = 0$$ 1

$$\dot{\theta}(t=0) = \omega A\cos\delta = \omega_0 = v_0/l$$ 1

$$\delta = 0$$ 1

$$A = \frac{\omega_0}{\omega} = \frac{v_0}{\omega l} = v_0\sqrt{\frac{2}{3gl}}$$ 1

3. (17pts) A particle is confined to move in x-direction between $\boldsymbol{x}$=0 and $\boldsymbol{x}$= ∞, and it experiences an conservative force $\boldsymbol{F}(\boldsymbol{x})$ such that its potential energy $\boldsymbol{U}(\boldsymbol{x}) = -\boldsymbol{b}x^{2}\cdot e^{-\boldsymbol{ax}}$, where $\boldsymbol{a},\boldsymbol{b}>\boldsymbol{0}$,

(a) (6pts) Determine this conservative force $\boldsymbol{F}(\boldsymbol{x})$ as a function of x,

(b) (4pts) At the equilibrium point $\boldsymbol{x} = \boldsymbol{S}$, $\boldsymbol{F}(\boldsymbol{S}) = 0$, determine the value of $\boldsymbol{S}$.

(c) (4pts) If the particle is moving around $\boldsymbol{S}$, and if we define $\boldsymbol{z} = \boldsymbol{x}-\boldsymbol{S}$, write down the equation of motion of the particle in terms of $\boldsymbol{z}$,

(d) (3pts) For the case if $\boldsymbol{z}/\boldsymbol{S} \ll \boldsymbol{1}$, the particle executes a simple harmonic oscillation around S, determine the period of the oscillation of the particle near S.

Useful formula: $(1+z)^n \approx 1+nz, \quad e^{az} \approx 1+az, \quad for\ |z| \ll 1, |az| \ll 1$

(a) $$U(x) = -bx^2 e^{-ax}$$

$$F(x) = -\frac{dU(x)}{dx} \overset{2}{=} -\frac{d(-bx^2 e^{-ax})}{dx}$$

$$= 2bxe^{-ax} - abx^2 e^{-ax} = -(ax-2)bxe^{-ax}$$ 4

(b) $$for\ F(S) = 0, \Rightarrow 2bSe^{-aS} - abS^2 e^{-aS} = 0$$ 1

$$\Rightarrow S = \frac{2}{a}$$ 3

(c) $$\sum \vec{F} = m\vec{a}, \quad F(x) = -(ax-2)bxe^{-ax} = m\frac{d^2x}{dt^2}$$

$$-(ax-2)bxe^{-ax} = m\frac{d^2x}{dt^2}$$ 2

$$z \equiv x - S \Rightarrow x = z + S = z + 2/a$$

$$\Rightarrow -(a(z+2/a)-2)(z+2/a)be^{-a(z+2/a)} = m\frac{d^2(z+2/a)}{dt^2}$$

$$\Rightarrow -az(z+2/a)be^{-a(z+2b/a)} = m\frac{d^2z}{dt^2}$$ 2

$$\Rightarrow m\frac{d^2z}{dt^2} + az(z+S)be^{-az-2} = 0$$

(d) $$for\ |z| \ll 1, |az| \ll 1$$

$$az(z+S)be^{-a(z+S)} = azS(1+\frac{z}{S})be^{-az} \cdot e^{-aS}$$

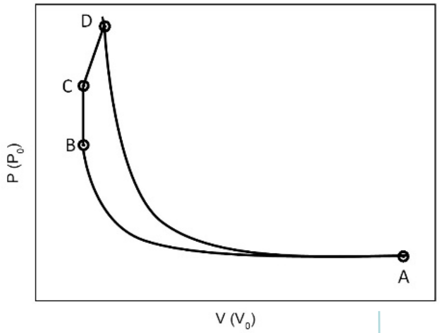
$$\approx azS(1+\frac{z}{S})(1-az) \cdot be^{-aS}$$

$$= aS(z + (\frac{1}{S} - a)z^2 - \frac{A}{S}z^3) \cdot be^{-aS} \approx aSz \cdot be^{-aS} = 2be^{-2} \cdot z$$

$$\Rightarrow m\frac{d^2z}{dt^2} + 2be^{-2} \cdot z = 0$$ 2

$$\Rightarrow \omega = \sqrt{\frac{2be^{-2}}{m}} = \sqrt{\frac{2b}{m}}e^{-1}$$

$$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi e\sqrt{\frac{m}{2b}}$$ 1



4

	P (P_0)	V (V_0)	T (P_0V_0/R)
A	1	128	128
B	16	8	128
C	24	8	192
D	32	16	512

$$T_A = T_B = 128P_0V_0/R$$

$$\Rightarrow P_AV_A = P_BV_B \Rightarrow P_B = 16P_0$$

$$P_DV_D^\gamma = P_AV_A^\gamma$$

$$\Rightarrow p_D\left(16V_0\right)^{5/3} = P_A\left(128V_0\right)^{5/3}$$

$$\Rightarrow P_D = 32P_0$$

$$T_C = 192P_0V_0/R,\ T_D = 512P_0V_0/R$$

4

$$e = 1 - \frac{16\ln 2}{25} = \frac{25 - 16\ln 2}{25}$$

12

b	W (P_0V_0)	Q (P_0V_0)	ΔE_{int} (P_0V_0)	ΔS (R)
A→B	-512ln2	-512ln2	0	-4ln2
B→C	0	96	96	3(ln3-ln2)/2
C→D	224	704	480	(11ln2-3ln3)/2
D→A	576	0	-576	0

A→B (isotheraml)

ΔE_{int}=0

$$Q = W = \int_A^B PdV = \int_{V_A}^{V_B} nRT\frac{dV}{V} = nRT_A\ln\frac{V_B}{V_A} = \left(-512\ln 2\right)P_0V_0$$

$$\Delta S = nC_v\ln\frac{T_B}{T_A} + nR\ln\frac{V_B}{V_A} = \frac{3}{2}R\left(\ln\frac{4}{64}\right) = -4R\ln 2$$

B→C (isovolumetric)

W=0

$$Q = \Delta E_{int} = 1(3R/2)[T_C - T_B] = 96P_0V_0$$

$$\Delta S = nC_v\ln\frac{T_C}{T_B} = \frac{3}{2}R\left(\ln\frac{192}{128}\right) = 3R(\ln 3 - \ln 2)/2$$

C→D (straight-line)

$$W = \frac{(24+32)8P_0V_0}{2} = 224P_0V_0$$

$$\Delta E_{int} = \frac{3}{2}R\left(T_D - T_C\right) = \frac{3}{2}R\left(\frac{320P_0V_0}{R}\right) = 480P_0V_0$$

$$Q = \Delta E_{int} + W = 704P_0V_0$$

$$\Delta S = \frac{3}{2}nR\ln\frac{T_D}{T_C} + R\ln\frac{V_D}{V_C} = \frac{3}{2}R\ln\left(\frac{512}{128}\right) + R\ln\left(\frac{16}{8}\right)$$

$$= \frac{3}{2}R\ln\left(\frac{8}{3}\right) + R\ln\left(2\right) = \left(\frac{11}{2}\ln 2 - \frac{3}{2}\ln 3\right)R$$

D→A (adiabatic)

Q=0

$$W = -\Delta E_{int} = -1(3R/2)[T_A - T_D] = 576P_0V_0$$

$$\Delta S = 0$$