General Physics (II) **期中考 II** **May. 15, 2020**

試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(50 points) (所有題目必須有計算過程,否則不予計分)

1&2. (20 pts) Fig. *1* shows a three-section conducting wire on x-y plane with current I. The first section is from -∞ to A on the x-axis. The second section is from A to B is a semi-circle with radius ***R***. The last section is from B to ∞ on the x-axis. Find the *x*-, *y*-, *z*-components of the magnetic field at point ***P*** on the *z*-axis due to

(a)(8 pts) current in the section from -∞ to A,

(b)(4 pts) current in the section from B to ∞,

(c)(8 pts) current in the section from A to B.

The coordinates of ***A***,***B***, and ***P*** are **(- *R, 0,0*)**, **(*R, 0,0*),** and **(*0,0,z*),** respectively.

3. (a) (5pts) As shown in Fig. 2(a), a uniform infinite cylindrical current distribution of radius R and density *J* has its axis coincides with the z-axis, and the current is in the +z-direction. Determine the B-filed (magnitude and direction) at point P(R/2,-R/2,0) (b) (10pts) As shown in Fig. 2(b), out side of this cylindrical current distribution is surrounded by current running in the (-z) direction, i.e. current density $-J$, in the region between $\boldsymbol{-R \le x \le R}$ ($-\infty < \boldsymbol{y} < \infty$, and $-\infty < \boldsymbol{z} < \infty$), Determine the direction and the magnitude of the B-field on the x-axis for $\boldsymbol{0 \le x \le 2R}$.

(If you applies Ampere's law, you need to draw the loop path for the integral.)

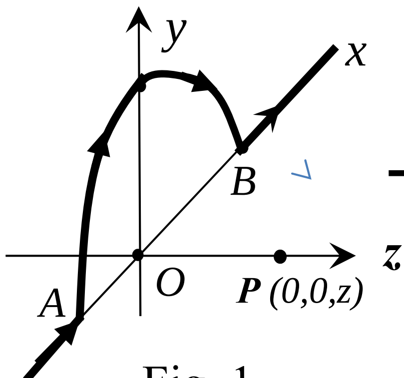


Fig. 1

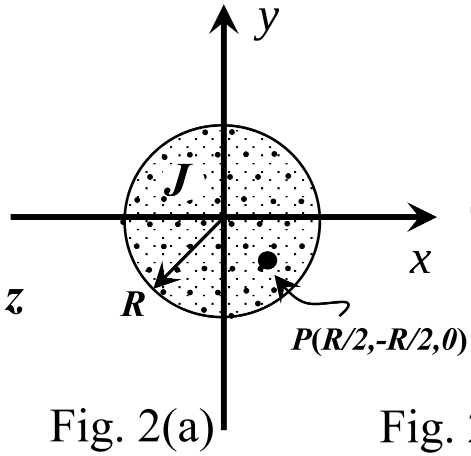


Fig. 2(a)

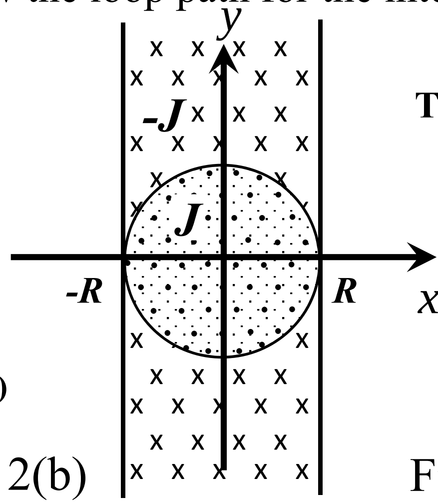


Fig. 2(b)

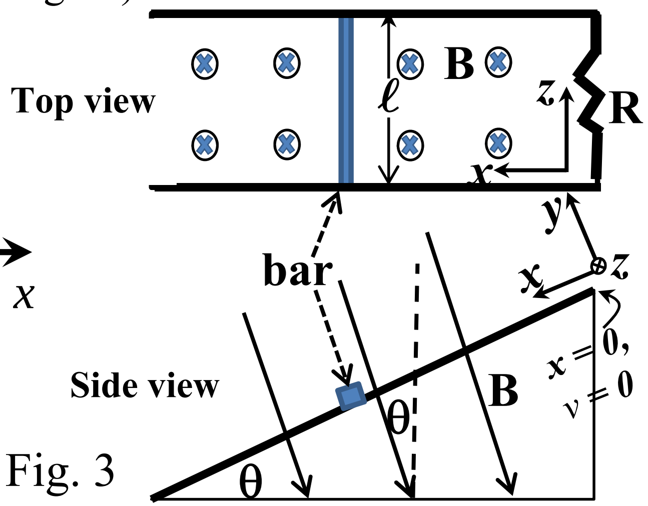


Fig. 3

4. (15 pts) As shown in Fig. 3, a conducting bar of mass $\boldsymbol{m}$ slides down two frictionless conducting rails which make an angle $\boldsymbol{\theta}$ with the horizontal and one end with a resistor $\boldsymbol{R}$, and the distance between two rails is $\boldsymbol{\ell}$. A uniform magnetic field $\boldsymbol{B}$ is applied with an angle $\boldsymbol{\theta}$ with respective to vertical. The bar is released from the top with the velocity of $\boldsymbol{0}$.

(A)(5 pts) Find the current, magnitude and direction, through the conducting bar when the velocity of the bar is $\boldsymbol{v}$. (use the coordinate system in the figure for the current direction)

(B)(5 pts) Draw the free body diagram of the conducting bar sliding down the rail, and write down the equation of motion.

(C)(5 pts) Find the velocity as a function of time and the terminal velocity $\boldsymbol{v_T}$ for the bar.

II.選擇題(50 points)

1. (5 pts) A R-C circuit is shown in Fig. 4. $\boldsymbol{R = R_1 = 1}$ **kΩ**, $\boldsymbol{R_2 = 2}$ **kΩ**, $\boldsymbol{C = 5\ \mu F}$,and $\mathcal{E} = 3$V. The capacitor is initially uncharged. The switch S is closed at $\boldsymbol{t = 0}$. The time constant $\boldsymbol{t_{½}}$ is defined as the time when the capacitor is charging to half of its maxima value. Let $\boldsymbol{t_{½}} = \boldsymbol{b}\times10^{-3}$ s. What is the range of constant $\boldsymbol{b}$? (**ln2 ~ 0.7**)

(A) $b \le 1$ (B) $1 < b \le 2$ (C) $2 < b \le 3$ (D) $3 < b \le 4$ (E) $4 < b \le 5$ (F) $5 < \boldsymbol{b} \le 6$ (G) $6 < b \le 7$ (H) $7 < \boldsymbol{b} \le 8$ (J) $8 < \boldsymbol{b} \le 9$ (K) $9 < b \le 10$ (L) $10 < b$

2. (5 pts) Fig. 5 shows two infinite current sheets which are parallel to the $\boldsymbol{y}$-$\boldsymbol{z}$ plane. The direction of the current is along $+\boldsymbol{z}$ in the left plane current and $-\boldsymbol{z}$ for the other one. There is a counter-clock-wise (CCW) circular loop current $\boldsymbol{I}$ with area $\boldsymbol{A}$, placed in the $\boldsymbol{x}$-$\boldsymbol{y}$ plane between two current sheets. Which of the following statement is correct for (i) the net force and (ii) the direction of the torque $\boldsymbol{\tau}$ on the circular loop at this moment? (ignore the gravity)

(A)$\boldsymbol{F_{net}} \neq \boldsymbol{0}$; (+$x$)-axis. (B) $\boldsymbol{F_{net}} \neq \boldsymbol{0}$; (-$x$)-axis. (C) $\boldsymbol{F_{net}} \neq \boldsymbol{0}$; (+$y$)-axis.
(D) $\boldsymbol{F_{net}} \neq \boldsymbol{0}$; (-$y$)-axis. (E) $\boldsymbol{F_{net}} = \boldsymbol{0}$; (+$x$)-axis. (F) $\boldsymbol{F_{net}} = \boldsymbol{0}$; (-$x$)-axis.
(G) $\boldsymbol{F_{net}} = \boldsymbol{0}$; (+$y$)-axis. (H) $\boldsymbol{F_{net}} = \boldsymbol{0}$; (-$y$)-axis. (J) $\boldsymbol{F_{net}} \neq \boldsymbol{0}$; $\boldsymbol{\tau} = 0$. (K) $\boldsymbol{F_{net}} = \boldsymbol{0}$; $\boldsymbol{\tau} = 0$
(L) None of above

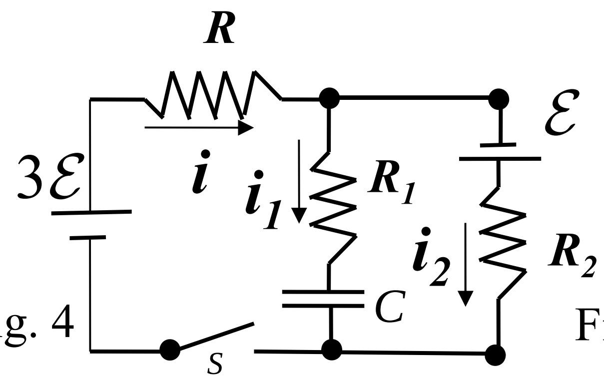


Fig. 4

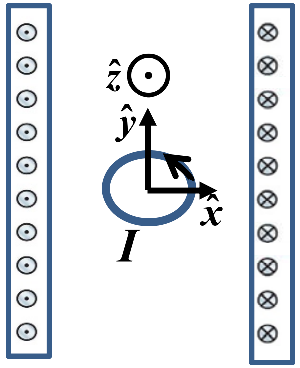


Fig. 5

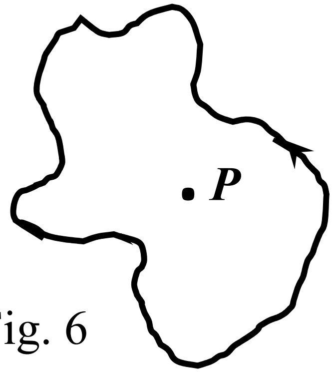


Fig. 6

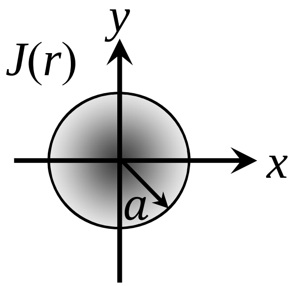
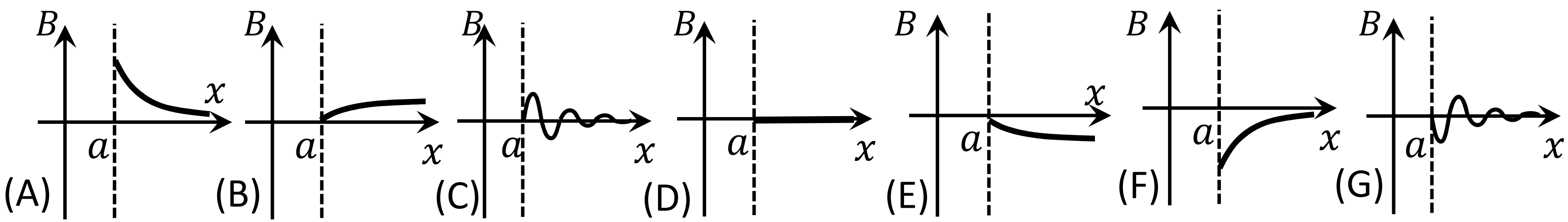


Fig. 7

3. (5 pts) In the fig. 6, an irregular loop of wire carrying a current lines in the plane of the paper. Suppose that now the loop is distorted into some other shape wile remaining in the same plane. Point ***P*** is still within the loop. Which of the following is a true statement concerning this situation?

(A) The direction of the magnetic field at point ***P*** will always lie in the plane of the paper.
(B) It is possible that the magnetic field at point ***P*** is zero.
(C) The magnetic filed at point ***P*** will never change in magnitude when the loop is distorted.
(D) The magnetic field at ***P*** will not change in direction when the loop is distorted.
(E) None of the is true.

4. (5 pts) Fig. 7 shows an infinitely long cylindrical current distribution with radius *a* and with its axis coincides with the z-axis. The current density $J(r) = J_0 \sin(2\pi r/a)/r$, where J_0 is a positive constant and $r = \sqrt{x^2 + y^2}$, and the direction of $J(r)$ is in the +z-direction. Which of the following is the *B*-field along the *x*-axis for $x \geq a$ resulted from $J(r)$?



5. (5 pts) A square wire loop with 2.00 m sides is perpendicular to a uniform magnetic field, with half the area of the loop in the field as shown in Fig 8. The loop contains an ideal battery with emf of 5 V. If the magnitude of the B-field varies with time (t) according to B = 0.1 + 0.8t, with B in Tesla and t in sec. The magnitude of the emf ε (in V) in the circuit: (A) $\varepsilon \leqq 0$ (B) $0 < \varepsilon \leqq 1$ (C) $1 < \varepsilon \leqq 2$ (D) $2 < \varepsilon \leqq 3$ (E) $3 < \varepsilon \leqq 4$ (F) $4 < \varepsilon \leqq 5$ (G) $5 < \varepsilon \leqq 6$ (H) $6 < \varepsilon \leqq 7$ (J) $7 < \varepsilon \leqq 8$ (K) $8 < \varepsilon \leqq 9$ (L) $9 < \varepsilon$.

6. (5 pts) Fig 9, shows on x-y plane an infinitely long wire carrying current ***I*=10 *A*.** A conducting rod sits on two parallel frictionless rails (the spacing of the rail is ***a***) with a distance ***a*** from the current. Assume the velocity of the rod is **10^3 m/s** moving toward right**.** What is the current ***b*** (in SI unit) through the conducting rod? (assume there is a resistor with ***R*=10^{-3} Ω** in the far left end of the rails. ***a*=1.0 m, ln2 ~ 0.7, μ_0=4π x 10^{-7}**)

(A) $b \le 0.5$ (B) $0.5 < b \le 1.0$ (C) $1.0 < b \le 1.5$ (D) $1.5 < b \le 2.0$ (E) $2 < b \le 2.5$ (F) $2.5 < b \le 3$ (G) $3 < b \le 4$ (H) $4 < b \le 8$ (J) $8 < b \le 10$ (K) $10 < b$

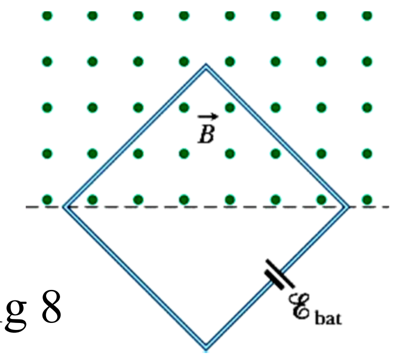


Fig 8

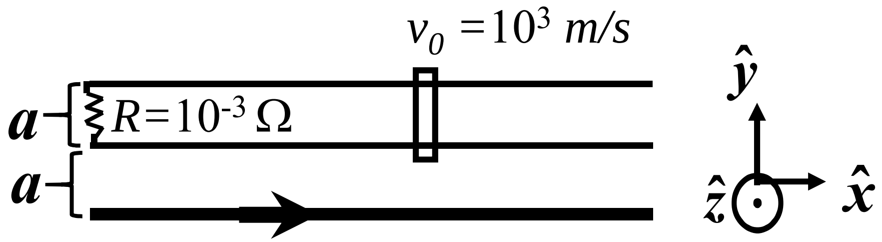


Fig. 9

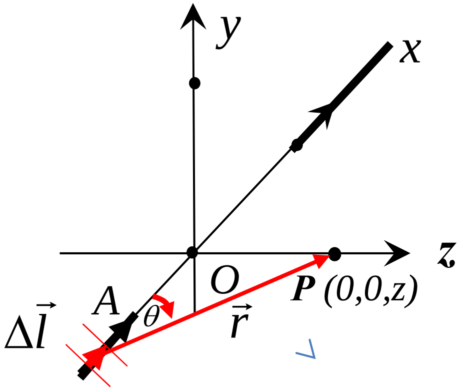
I=10 A

$$\int \frac{dx}{\sqrt{x^2 \pm b^2}} = \ln\left(x + \sqrt{x^2 \pm b^2}\right) \qquad \int \frac{x^2 dx}{\sqrt{x^2 \pm b^2}} = \frac{x\sqrt{x^2 \pm b^2}}{2} - \frac{b^2}{2}\ln\left(x + \sqrt{x^2 \pm b^2}\right)$$

$$\int \frac{dx}{\left(x^2 \pm b^2\right)^{3/2}} = \frac{\pm x}{b^2\sqrt{x^2 \pm b^2}} \qquad \int \frac{x^2 dx}{\left(x^2 \pm b^2\right)^{3/2}} = \frac{-x}{\sqrt{x^2 \pm b^2}} + \ln\left(x + \sqrt{x^2 \pm b^2}\right)$$

1	2	3	4	5	6	7	8	9	10
F	F	D	D	E	C	B	C	G	B
11	12	13	14	15	16				
F	E	C	A	D	B				

4. (a) Line segment on x-axis:



$$\Delta\vec{B}=\frac{\mu_0}{4\pi}\cdot\frac{I\Delta\vec{l}\times\hat{r}}{r^2}=\frac{\mu_0}{4\pi}\cdot\frac{I\Delta\vec{l}\times\vec{r}}{r^3}$$ 1

$$\Delta\vec{l}=\Delta x\,\hat{x}$$ 1

$$\vec{r}=(0,0,z)-(x,0,0)=(-x,0,z)$$ 1

$$\hat{r}=\frac{(-x,0,z)}{\sqrt{z^2+x^2}}$$

另一種做法:

$$\Delta\vec{l}\times\vec{r}=\Delta x\cdot|\vec{r}|\cdot\sin\theta(-\hat{y})$$

$$\sin\theta=\frac{z}{\sqrt{z^2+x^2}}$$

$$\Delta\vec{l}\times\vec{r}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \Delta x & 0 & 0\\ -x & 0 & z\end{vmatrix}=z\Delta x\cdot(-\hat{j})$$

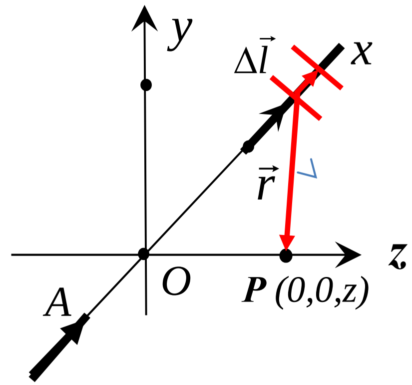
$$\therefore\Delta\vec{B}\left(=\frac{\mu_0 I}{4\pi}\frac{\Delta x(-\hat{j})}{x^2+z^2}\sin\theta\right)=\frac{\mu_0 I}{4\pi}\frac{z\Delta x(-\hat{j})}{\sqrt{x^2+z^2}^3}$$ 2

(i) 查積分表: 1

$$\int\frac{dx}{(x^2\pm a^2)^{\frac{3}{2}}}=\frac{\pm x}{a^2\sqrt{x^2\pm a^2}}$$

$$\vec{B}_1=\frac{\mu_0 Iz}{4\pi}(-\hat{j})\int_{-\infty}^{-R}\frac{dx}{\sqrt{x^2+z^2}^3}=\frac{\mu_0 Iz}{4\pi}(-\hat{j})\frac{x}{z^2\sqrt{x^2+z^2}}\Big|_{-\infty}^{-R}$$

$$=\frac{\mu_0 Iz}{4\pi}(-\hat{j})\cdot\left(\frac{x}{z^2\sqrt{x^2+z^2}}\Bigg|_{-\infty}^{-R}\right)=\frac{\mu_0 I}{4\pi z}\left(1-\frac{R}{\sqrt{R^2+z^2}}\right)(-\hat{j})$$ 2



$$\Delta\vec{B}=\frac{\mu_0}{4\pi}\cdot\frac{I\Delta\vec{l}\times\vec{r}}{r^3},\quad\Delta\vec{l}=\Delta x\,\hat{x}$$

$$\vec{r}=(-x,0,z),\quad\hat{r}=\frac{(-x,0,z)}{\sqrt{z^2+x^2}}$$

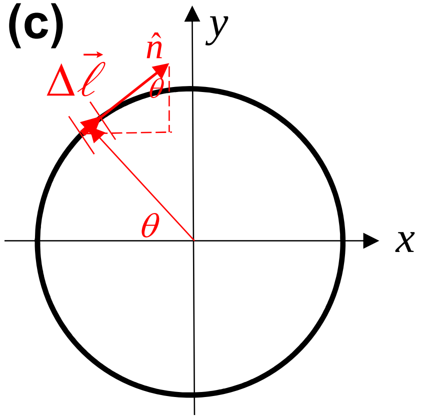
$$\Delta\vec{l}\times\vec{r}=z\Delta x\cdot\hat{j}$$

$$\therefore\Delta\vec{B}=\frac{\mu_0 I}{4\pi}\frac{z\Delta x(-\hat{j})}{\sqrt{x^2+z^2}^3}$$ 1

$$\vec{B}_1=\frac{\mu_0 Iz}{4\pi}(-\hat{j})\int_{R}^{\infty}\frac{dx}{\sqrt{x^2+z^2}^3}$$ 1

$$=\frac{\mu_0 I}{4\pi z}\left(1-\frac{R}{\sqrt{R^2+z^2}}\right)(-\hat{j})$$ 2

(c)



1

$$\hat{r} = \frac{(R\cos\theta, -R\sin\theta, z)}{\sqrt{z^2+R^2}}$$

1

$\hat{n}$: the unit vector in the direction of current.

$$\Delta\vec{l}\times\hat{r} = \frac{R\Delta\theta}{\sqrt{R^2+z^2}}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \sin\theta & \cos\theta & 0 \\ R\cos\theta & -R\sin\theta & z \end{vmatrix}$$

$$= \frac{R\Delta\theta}{\sqrt{R^2+z^2}}\left(z\cos\theta\hat{i} - z\sin\theta\hat{j} - R\hat{k}\right)$$

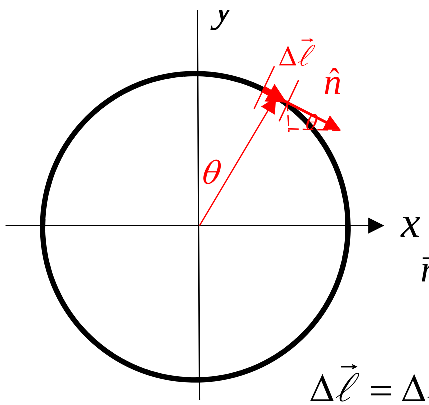
$$\Delta\vec{B} = \frac{\mu_0}{4\pi}\frac{IR\Delta\theta\left(z\cos\theta\hat{i} - z\sin\theta\hat{j} - R\hat{k}\right)}{\sqrt{R^2+z^2}^{3}}$$ 3

$$\vec{B} = \frac{\mu_0 IR}{4\pi\sqrt{R^2+z^2}^{3}}\left\{z\cdot\hat{i}\int_0^{\pi}\cos\theta d\theta - z\cdot\hat{j}\int_0^{\pi}\sin\theta d\theta - R\cdot\hat{k}\int_0^{\pi}d\theta\right\}$$ 1

$$\boxed{= \frac{\mu_0 IR}{4\pi\sqrt{R^2+z^2}^{3}}\left\{-2z\cdot\hat{j} - R\cdot\pi\cdot\hat{k}\right\}}$$

1 1

θ取法不同:



$$\vec{r} = (0,0,z) - (R\sin\theta, R\cos\theta, 0)$$
$$= (-R\sin\theta, -R\cos\theta, z)$$
$$\Delta\vec{\ell} = \Delta\ell\cdot\hat{n} = \left(R\Delta\theta\right)(\cos\theta, -\sin\theta, 0)$$

$$\Delta\vec{\ell}\times\hat{r} = \frac{R\Delta\theta}{\sqrt{R^2+z^2}}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\theta & -\sin\theta & 0 \\ -R\sin\theta & -R\cos\theta & z \end{vmatrix}$$

$$= \frac{R\Delta\theta}{\sqrt{R^2+z^2}}\left(-z\sin\theta\hat{i} - z\cos\theta\hat{j} - R\hat{k}\right)$$

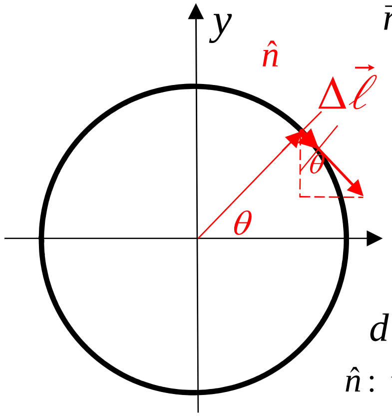
$$\Delta\vec{B} = \frac{\mu_0}{4\pi}\frac{IR\Delta\theta\left(-z\sin\theta\hat{i} - z\cos\theta\hat{j} - R\hat{k}\right)}{\sqrt{R^2+z^2}^{3}}$$ 3

$$\vec{B} = \frac{\mu_0 IR}{4\pi\sqrt{R^2+z^2}^{3}}\left\{z\cdot\hat{i}\int_{-\pi/2}^{\pi/2}\sin\theta d\theta - z\cdot\hat{j}\int_{-\pi/2}^{\pi/2}\cos\theta d\theta - R\cdot\hat{k}\int_{-\pi/2}^{\pi/2}d\theta\right\}$$

$$\boxed{= \frac{\mu_0 IR}{4\pi\sqrt{R^2+z^2}^{3}}\left\{-2z\cdot\hat{j} - R\cdot\pi\cdot\hat{k}\right\}}$$

1 1

$\boldsymbol{\theta}$取法不同:



$$\vec{r} = (0,0,z) - (R\cos\theta, R\sin\theta, 0)$$
$$= (-R\cos\theta, -R\sin\theta, z) \quad 1$$

$$\hat{r} = \frac{(-R\cos\theta, -R\sin\theta, z)}{\sqrt{z^2+R^2}} \quad 1$$

$$d\vec{\ell} = \Delta\ell \cdot \hat{n} = \left(Rd\theta\right)(\sin\theta, -\cos\theta, 0)$$

$\hat{n}$: the unit vector in the direction of current.

$$\Delta\vec{l} \times \hat{r} = \frac{R\Delta\theta}{\sqrt{R^2+z^2}} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \sin\theta & -\cos\theta & 0 \\ -R\cos\theta & -R\sin\theta & z \end{vmatrix}$$

$$= \frac{R\Delta\theta}{\sqrt{R^2+z^2}} \left(-z\cos\theta\hat{i} - z\sin\theta\hat{j} - R\hat{k}\right)$$

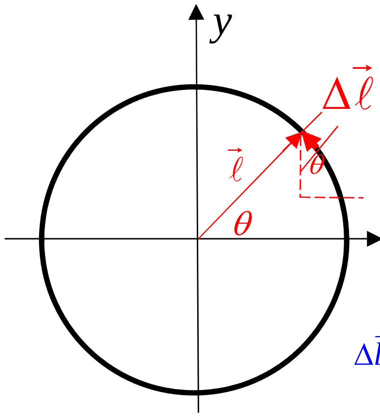
$$\Delta\vec{B} = \frac{\mu_0}{4\pi} \frac{IR\Delta\theta\left(-z\cos\theta\hat{i} - z\sin\theta\hat{j} - R\hat{k}\right)}{\sqrt{R^2+z^2}^3} \quad 3$$

$$\vec{B} = \frac{\mu_0 IR}{4\pi\sqrt{R^2+z^2}^3} \left\{ -z\cdot\hat{i}\int_0^\pi \cos\theta d\theta - z\cdot\hat{j}\int_0^\pi \sin\theta d\theta - R\cdot\hat{k}\int_0^\pi d\theta \right\} \quad 1$$

$$\boxed{= \frac{\mu_0 IR}{4\pi\sqrt{R^2+z^2}^3}\left\{-2z\cdot\hat{j} - R\cdot\pi\cdot\hat{k}\right\}}$$

1 1

另法:



$$\vec{\ell} = (R\cos\theta, R\sin\theta, 0)$$

$$\vec{r} = (0,0,z) - (R\cos\theta, R\sin\theta, 0)$$
$$= (-R\cos\theta, -R\sin\theta, z)$$

$$d\vec{\ell} = (-R\sin\theta, R\cos\theta, 0)d\theta$$

請注意, 此處 $d\vec{\ell}$ 的方向與電流方向相反.

$$\Delta\vec{l} \times \hat{r} = \frac{R\Delta\theta}{\sqrt{R^2+z^2}} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\sin\theta & \cos\theta & 0 \\ -R\cos\theta & -R\sin\theta & z \end{vmatrix}$$

$$= \frac{R\Delta\theta}{\sqrt{R^2+z^2}} \left(z\cos\theta\hat{i} + z\sin\theta\hat{j} + R\hat{k}\right)$$

$$\Delta\vec{B} = \frac{\mu_0}{4\pi} \frac{IR\Delta\theta\left(z\cos\theta\hat{i} + z\sin\theta\hat{j} + R\hat{k}\right)}{\sqrt{R^2+z^2}^3}$$

$$\vec{B} = \frac{\mu_0 IR}{4\pi\sqrt{R^2+z^2}^3} \left\{ z\cdot\hat{i}\int_\pi^0 \cos\theta d\theta + z\cdot\hat{j}\int_\pi^0 \sin\theta d\theta + R\cdot\hat{k}\int_\pi^0 d\theta \right\} \quad 1$$

請注意, 此處積分是 π 到0. 用此來決定電流方向.

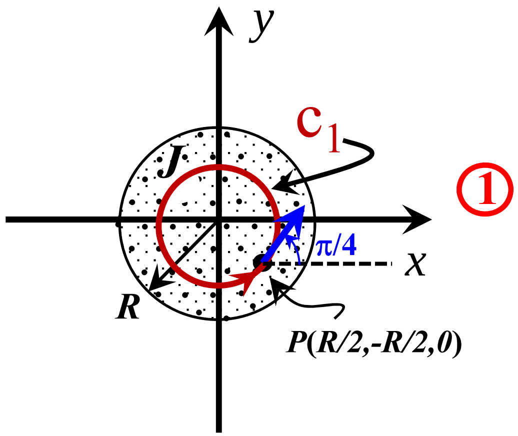
$$\boxed{= \frac{\mu_0 IR}{4\pi\sqrt{R^2+z^2}^3}\left\{-2z\cdot\hat{j} - R\cdot\pi\cdot\hat{k}\right\}}$$

1 1

3. (a) (5pts) As shown in Fig. 2(a), a uniform infinite cylindrical current distribution of radius R and density J has its axis coincides with the z-axis, and the current is in the +z-direction. Determine the B-filed (magnitude and direction) at point P(R/2,-R/2,0) (b) (10pts) As shown in Fig. 2(b), out side of this cylindrical current distribution is surrounded by current running in the (-z) direction, i.e. current density $-J$, in the region between $-\boldsymbol{R} \leq \boldsymbol{x} \leq \boldsymbol{R}$ ($-\infty < \boldsymbol{y} < \infty$, and $-\infty < \boldsymbol{z} < \infty$), Determine the direction and the magnitude of the B-field on the x-axis for $\boldsymbol{0} \leq \boldsymbol{x} \leq 2\boldsymbol{R}$.

(If you applies Ampere's law, you need to draw the loop path for the integral.)

(a)



Select a CCW. circular path c_1 of radius r centered at the origin and passing $\boldsymbol{P}$.

$$\oint_{c_1} \vec{B} \cdot d\vec{\ell} = \mu_0 I_{in} \quad \Rightarrow \oint_{c_1} B \cdot d\ell = \mu_0 J \pi r^2 \text{ ①}$$

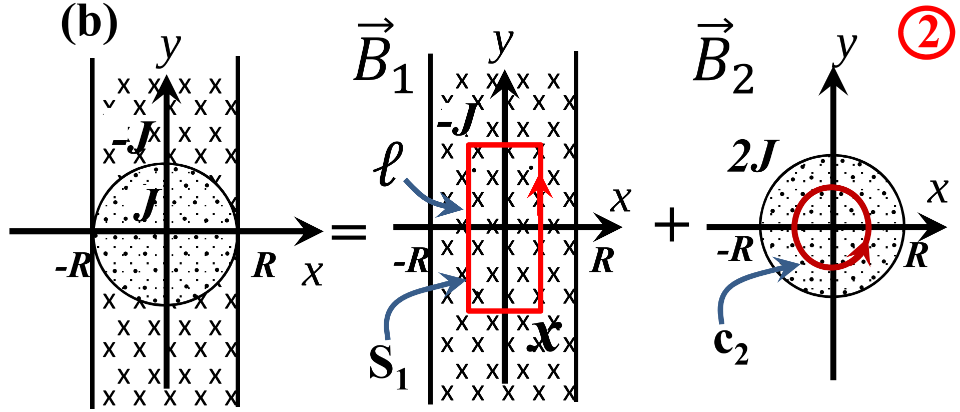
$$\Rightarrow B \cdot 2\pi r = \mu_0 J \pi r^2 \quad \Rightarrow B(r) = \frac{\mu_0 J r}{2} \text{ ①}$$

At point $\boldsymbol{P}$, $r = \sqrt{(R/2)^2 + (-R/2)^2} = \frac{\sqrt{2}}{2} R$

$$\Rightarrow B(r) = \frac{\mu_0 J \sqrt{2}}{4} R \text{ ①} \Rightarrow \vec{B}(r) = \frac{\mu_0 J \sqrt{2}}{4} R \cdot \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \text{ ①}$$

$$\Rightarrow \vec{B}(r) = \frac{\sqrt{2}\mu_0 J R}{4} \cdot \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), or \ \frac{\mu_0 J R}{4} \cdot (1,1)$$

(b)



For $0 \leq \boldsymbol{x} \leq$ R, for $\vec{B}_1$, select a rectangular loop path S_1, CCW., with width of $2x$ (x>0), centered at y-axis.

$$\oint_{S_1} \vec{B}_1 \cdot d\vec{\ell} = \mu_0 I_{in} \Rightarrow \oint_0^{2\pi} B_1 \cdot d\ell = -2\mu_0 J x \ell \text{ ①}$$

$$\Rightarrow 2B_1 \ell = -2\mu_0 J x \ell \Rightarrow B_1(x) = -\mu_0 J x \text{ ①}$$

$$\Rightarrow \vec{B}_1(x) = -\mu_0 J x \hat{y} \text{ ①}$$

For $0 \le \boldsymbol{x} \le R$, for $\vec{B}_1$, select a circular loop path c_2, CCW., with radius of x (x>0), centered at the origin.

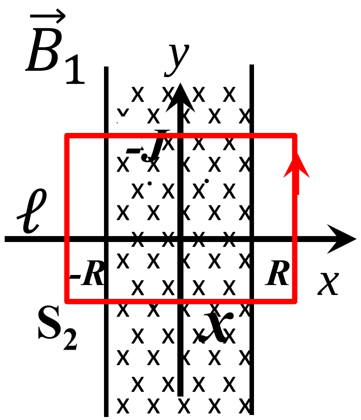
$$\oint_{c_2} \vec{B}_2 \cdot d\vec{\ell} = \mu_0 I_{in} \Rightarrow \oint_{c_2} B_2 \cdot d\ell = \mu_0 2J\pi x^2$$

$$\Rightarrow B_2 \cdot 2\pi x = 2\mu_0 J\pi x^2 \Rightarrow B_2(r) = \mu_0 J x$$

$$\Rightarrow \vec{B}_2(x) = \mu_0 J x \cdot \hat{y} \quad 1$$

$$\vec{B}(x) = \vec{B}_1(x) + \vec{B}_1(x) = -\mu_0 J x\hat{y} + \mu_0 J \mathrm{x} \cdot \hat{y}$$
$$= 0 \quad 1$$

For $R \le \boldsymbol{x} \le 2R$, for $\vec{B}_1$, select a rectangular loop path S_2, CCW., with width of $2x$ (x>0), centered at y-axis.



$$\oint_{S_1} \vec{B}_1 \cdot d\vec{\ell} = \mu_0 I_{in}$$

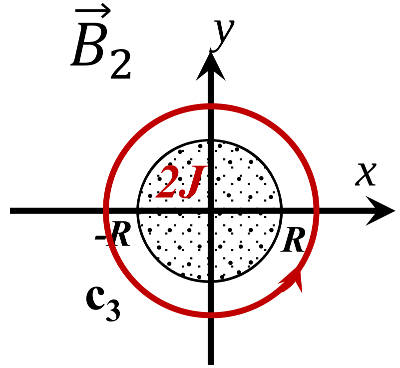
$$\Rightarrow \oint_0^{2\pi} B_1 \cdot d\ell = -2\mu_0 J R\ell$$

$$\Rightarrow 2B_1\ell = -2\mu_0 J R\ell$$

$$\Rightarrow B_1(x) = -\mu_0 J R$$

$$\Rightarrow \vec{B}_1(x) = -\mu_0 J R\hat{y} \quad 1$$

For $R \le \boldsymbol{x} \le 2R$, for $\vec{B}_2$, select a rectangular loop path S_2, CCW., with width of $2x$ (x>0), centered at y-axis.



$$\oint_{c_3} \vec{B}_2 \cdot d\vec{\ell} = \mu_0 I_{in}$$

$$\Rightarrow \oint_{c_2} B_2 \cdot d\ell = \mu_0 2J\pi R^2$$

$$\Rightarrow B_2 \cdot 2\pi x = 2\mu_0 J\pi R^2$$

$$\Rightarrow B_2(r) = \frac{\mu_0 J R^2}{x} \quad \Rightarrow \vec{B}_2(x) = \frac{\mu_0 J R^2}{x} \cdot \hat{y} \quad 1$$

$$\vec{B}(x) = \vec{B}_1(x) + \vec{B}_1(x) = -\mu_0 J R\hat{y} + \frac{\mu_0 J R^2}{x}\hat{y} \quad 1$$

$$= \mu_0 J R(-1 + \frac{R}{x})\hat{y}$$

4. (15 pts) As shown in Fig. x, a conducting bar of mass $\boldsymbol{m}$ slides down two frictionless conducting rails which make an angle $\boldsymbol{\theta}$ with the horizontal and one end with a resistor $\boldsymbol{R}$, and the distance between two rails is $\boldsymbol{\ell}$. A uniform magnetic field $\boldsymbol{B}$ is applied with an angle $\boldsymbol{\theta}$ with respective to vertical. The bar is released from the top with the velocity of $\boldsymbol{0}$.

(A) (5 pts) Find the current, magnitude and direction, through the conducting bar when the velocity of the bar is $\boldsymbol{v}$.

(B) (5 pts) Draw the free body diagram of the conducting bar sliding down the rail, and write down the equation of motion.

(C) (5 pts) Find the velocity as a function of time and the terminal velocity $\boldsymbol{v_T}$ for the bar.

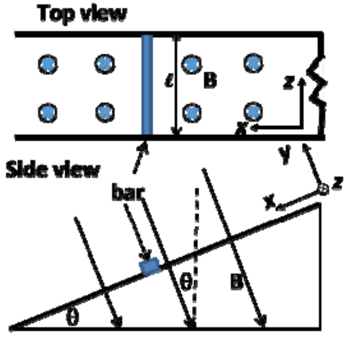
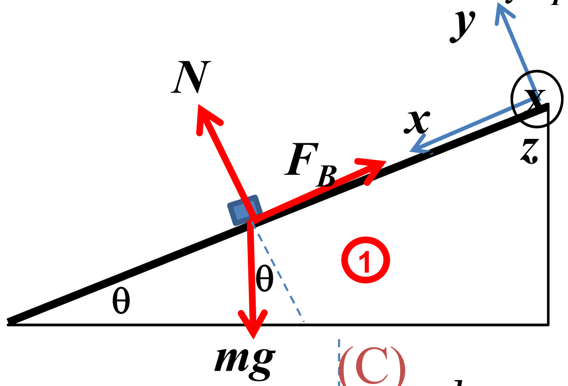


Fig. x



$$\text{x}:\quad mg\sin\theta - F_B = ma = m\frac{dv}{dt} \quad ②$$

$$\text{y}:\quad N - mg\cos\theta = 0$$

(A) $\boldsymbol{\Phi_B}$ is increased due to the movement of the bar

$$\Phi_B = \int \vec{B}\cdot d\vec{A} = B\left(\ell\cdot x(\text{t})\right) \quad ①$$

The change of $\boldsymbol{\Phi_B}$ induces emf $\mathcal{E}$:

$$\mathcal{E} = \left|\frac{d\Phi_B}{dt}\right| = B\ell\frac{dx(\text{t})}{dt} = B\ell v(t) \quad ②$$

$\mathcal{E}$ induces current $\boldsymbol{I}$:

$$I = \frac{\mathcal{E}}{R} = \frac{B\ell v(t)}{R} \quad ①$$

-z direction or

Top view: 逆時針 ①

(B)The bar with I in B ➔ magnetic force.

$$\vec{F}_B = I\vec{\ell}\times\vec{B} = \frac{B^2\ell^2 v(t)}{R}(\text{-}\hat{\text{x}}) \quad ②$$

Side view Direction:

(C)

$$\frac{dv}{dt} = g\sin\theta - \frac{B^2\ell^2}{mR}v(t) = -\frac{v(t)-v_0}{\tau} \quad ①$$

where $\tau = \dfrac{mR}{B^2\ell^2}$; $v_0 = \dfrac{mgR\sin\theta}{B^2\ell^2} = v_T$

$$\int_0^v \frac{dv}{v-v_T} = -\int_0^t \frac{dt}{\tau} \Rightarrow \ln\left|\frac{v-v_T}{-v_T}\right| = -\frac{t}{\tau} \quad ①$$

$$\Rightarrow v(t) = v_T\left(1-e^{-t/\tau}\right) \quad ①$$

When the bar has reach “terminal velocity” $\boldsymbol{v_T}$, there is no acceleration.

i.e $\quad mg\sin\theta - \dfrac{B^2\ell^2}{R}v_T = 0 \Rightarrow v_T = \dfrac{R\cdot mg\sin\theta}{B^2\ell^2} \quad ②$