GP(II).spring.2020
HW11.solution

HW11-1:

(a) For the AC circuit shown in Fig. 1(a), given that $\boldsymbol{I(t) = I_0\cos(\omega t)}$, find the voltage $\boldsymbol{V(t)}$ for the circuit in Fig. 1(a) and $|V_L/V|$ as a function of $\boldsymbol{\omega}$.

(b) For the AC circuit shown Fig. 1(b), given that $\boldsymbol{V(t) = V_0\cos(\omega t)}$, find the current $\boldsymbol{I(t)}$ and $|I_C/I|$ as a function of $\boldsymbol{w}$.

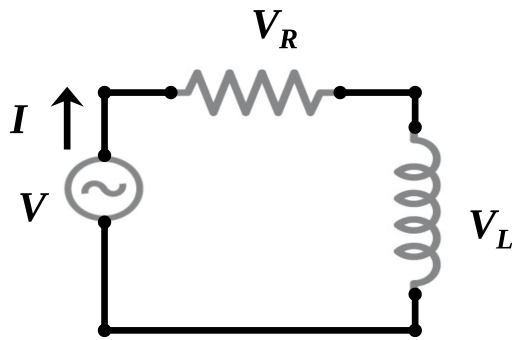


Fig. 1(a)

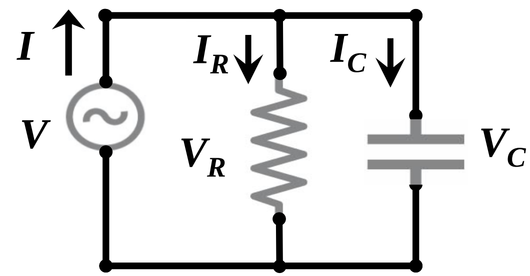


Fig. 1(b)

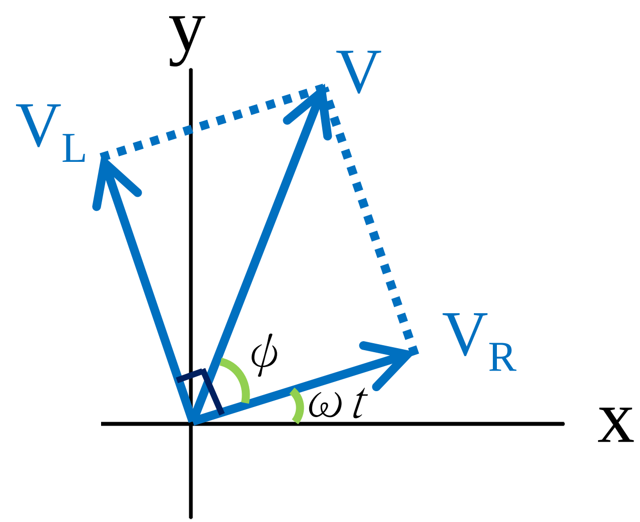
Solution of HW12-1:

(a) RL 串聯 → 電流 I(t) 一樣

$$V_R = IR = RI_0 \cos\omega t$$

$$V_L = L\frac{dI}{dt} = LI_0\left(-\omega \sin\omega t\right)$$

$$= LI_0\omega\cos\left(\omega t + \frac{\pi}{2}\right)$$



$$V = V_0\cos\left(\omega t + \phi\right)$$

$$\text{where}\begin{cases} V_0 = \sqrt{V_{R0}{}^2 + V_{L0}{}^2} = I_0\sqrt{R^2 + (\omega L)^2} \\ \phi = \tan^{-1}\left(\dfrac{V_{L0}}{V_{R0}}\right) = \tan^{-1}\left(\dfrac{\omega L}{R}\right) \end{cases}$$

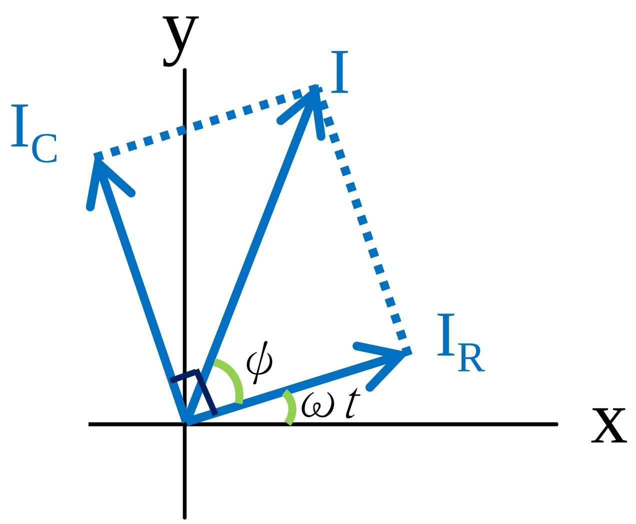
Solution of HW12-1:

(b) RC 並聯 → 電壓 V(t) 一樣

$$I_R = \frac{V}{R} = \frac{V_0 \cos \omega t}{R}$$

$$I_C = \frac{dQ}{dt} = C\frac{dV}{dt}$$

$$= CV_0 \omega \cos\left(\omega t + \frac{\pi}{2}\right)$$

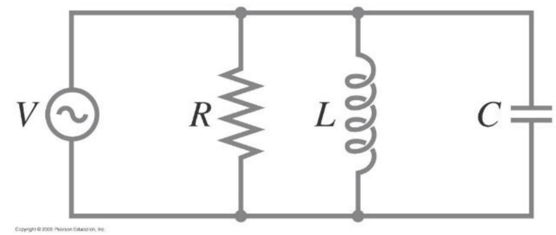


$$I = I_0 \cos(\omega t + \phi)$$

$$\textit{where} \begin{cases} I_0 = \sqrt{{I_{R0}}^2 + {I_{C0}}^2} = V_0 \sqrt{\dfrac{1}{R^2} + (\omega C)^2} \\ \phi = \tan^{-1}\left(\dfrac{I_{C0}}{I_{R0}}\right) = \tan^{-1}(R\omega C) \end{cases}$$

HW11-2:

A resistor *R*, capacitor *C*, and inductor *L* are connected in parallel across an ac generator as shown in the Fig. 30–34. The source emf is $\boldsymbol{V(t)}=\boldsymbol{V_0}\cos(\boldsymbol{\omega t})$. Determine the current as a function of time (including amplitude and phase): (a) in the resistor, (b) in the inductor, (c) in the capacitor. (d) What is the total current leaving the source ***V*** ?

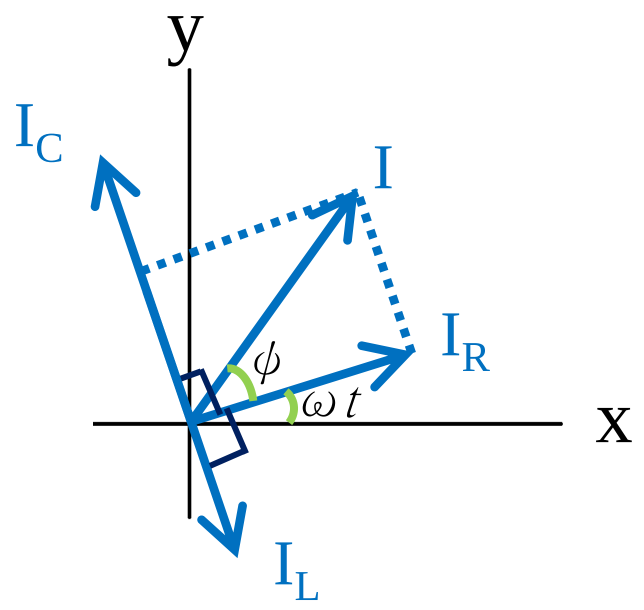


Solution of HW12-2:
並聯 → 電壓 V(t) 一樣

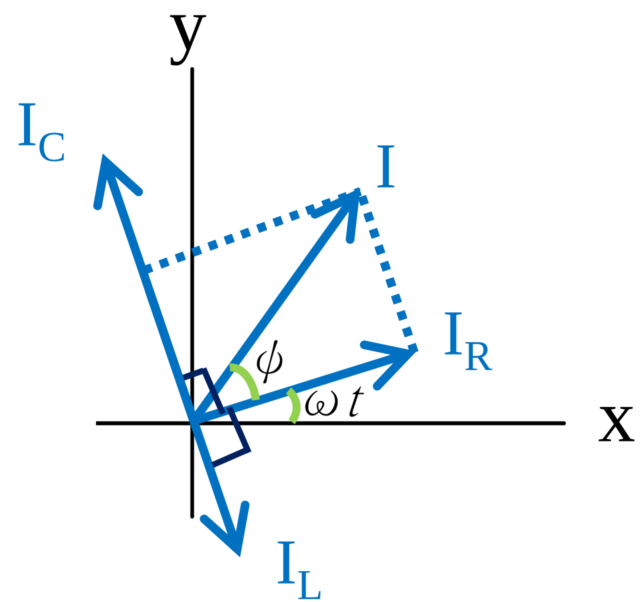
(a) $I_R = \dfrac{V_0 \cos \omega t}{R}$

(b) $I_C = V_0 \omega C \cos\left(\omega t + \dfrac{\pi}{2}\right)$

(c) $I_L = \dfrac{V_0}{\omega L} \cos\left(\omega t - \dfrac{\pi}{2}\right)$



Solution of HW12-2:



(d) $I = I_0 \cos(\omega t + \phi)$

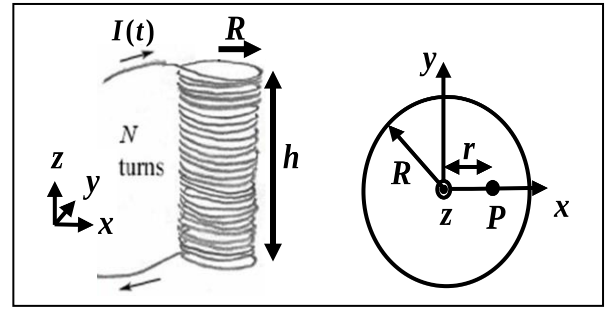
where
$$\begin{cases} I_0 = \sqrt{(I_{C0} - I_{L0})^2 + I_{R0}{}^2} = V_0 \sqrt{\left(\omega C - \dfrac{1}{\omega L}\right)^2 + \dfrac{1}{R^2}} \\ \phi = \tan^{-1}\left(\dfrac{I_{C0} - I_{L0}}{I_{R0}}\right) = \tan^{-1}\left(\dfrac{\omega C - \dfrac{1}{\omega L}}{R}\right) \end{cases}$$

HW11-3: A N-turn solenoid of radius R and length h has an alternating current $\boldsymbol{I(t) = I_0 \sin\omega t}$. Consider a point $\boldsymbol{P}$ (inside the solenoid) at radius $\boldsymbol{r}$

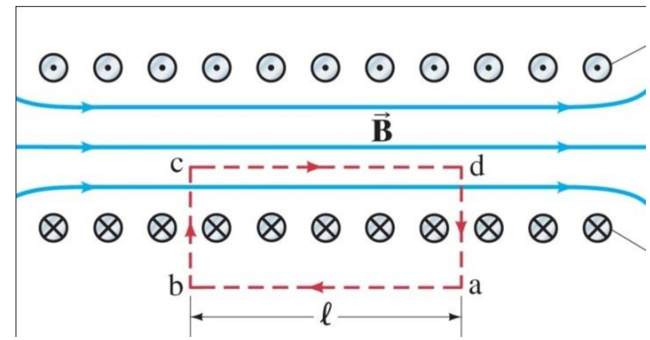
(a) Find the magnetic field $\boldsymbol{B(t)}$ at $\boldsymbol{P}$ as function of time $\boldsymbol{t}$

(b) Find the electric field $\boldsymbol{E(t)}$ at $\boldsymbol{P}$.

(c) Find the Poynting vector $\boldsymbol{S(t)}$ at $\boldsymbol{P}$



(a) $\oint \vec{B}\cdot d\vec{l} = Bl = \mu_0 I_{enc} = \mu_0 I \dfrac{N}{h} l$

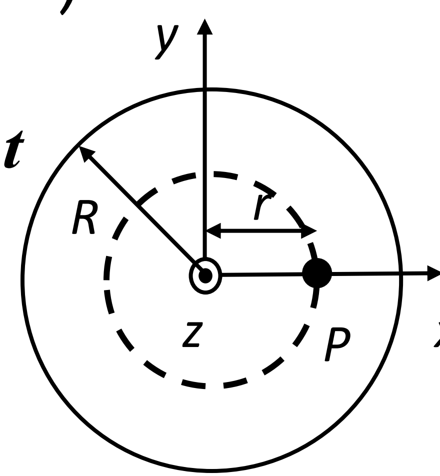


$\rightarrow \vec{B} = \mu_0 I_0 \dfrac{N}{h}\hat{z}\sin\omega t \quad \textbf{(or along +}z\textbf{ direction)}$

(b) $\oint \vec{E}\cdot d\vec{l} = -\dfrac{d\Phi_B}{dt} \Rightarrow E\cdot 2\pi r = -\dfrac{d}{dt}\iint \vec{B}\cdot d\vec{A} = -\dfrac{d}{dt}B\pi r^2$

$\Rightarrow \vec{E} = \left(-\hat{y}\right)\dfrac{\mu_0}{2}\dfrac{N}{h} r\omega I_0\cos\omega t \quad \left(\textbf{at point } P\right)$

(c) $S = \dfrac{1}{\mu_0}\left|\vec{E}\times\vec{B}\right| = \dfrac{\mu_0}{2}\left(\dfrac{NI_0}{h}\right)^2 r\omega\sin\omega t\cos\omega t$



along –*x* at point *P*