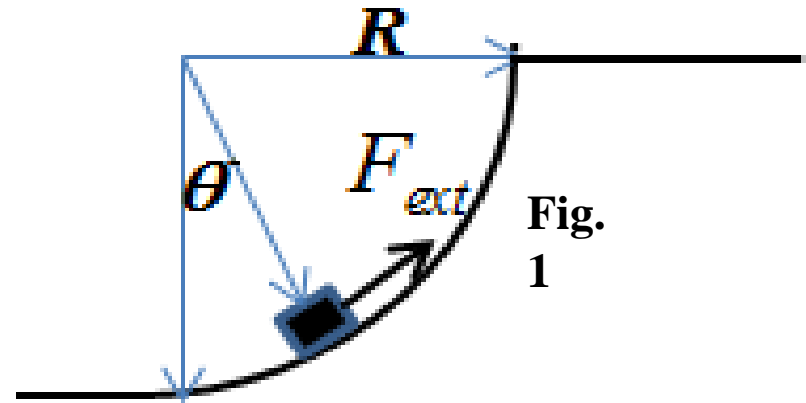


Homework 6 (Chap 7-8)

1. An external force, parallel to the displacement, is pushing a small particle of mass m in the very slow motion from the bottom to the top of the quarter circle of radius R , shown in Fig. 1. The frictional coefficients of the circle surface is $\mu_k = \mu_0 \cos \theta$. Calculate the work done by the external force. Useful information: $\cos 2\theta = 2\cos^2 \theta - 1 = 1 - 2\sin^2 \theta$



2. A particle is confined to move along the x-axis with the following potential energy,

$$U(x) = 3x^3 - 9x + 4$$

where x is the coordinate of the particle in unit of μm , and $U(x)$ are in units of electron volts (eV).

- Determine the force F experienced by the particle as a function of x .
- At what values of x is $F(x)$ equal to zero?
- Indicate the positions of the stable and the unstable equilibrium of the particle at the potential energy at these positions.
- If the total energy of the particle at the stable equilibrium position is 4eV, determine the range of the motion of this particle.

3. Problem 8-85 in Giancoli (pp. 247)

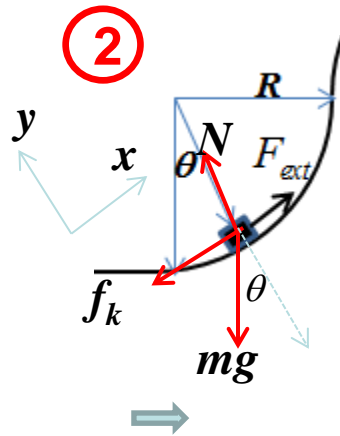
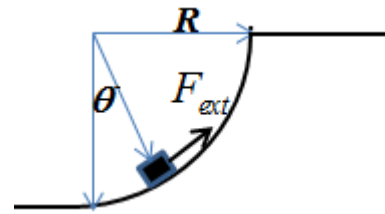
Problem 8-85 in Giancoli (pp. 211)

A ball is attached to a horizontal cord of length ℓ whose other end is fixed, Fig. 8–42.

- (a) If the ball is released, what will be its speed at the lowest point of its path?
- (b) A peg is located a distance h directly below the point of attachment of the cord. If $h = 0.8\ell$ what will be the speed of the ball when it reaches the top of its circular path about the peg?

1. (10 pts) An external force is pushing a small particle of mass m in the very slow motion from the bottom of the quarter circle of radius R , to the top. Due to snow yesterday, the frictional coefficient of the circle surface is $\mu_k = \mu_0 \cos \theta$. Calculate the work done by the external force.

$$2\cos^2 \theta - 1 = 1 - 2\sin^2 \theta = \cos 2\theta$$



$$x: F_{ext} - f_k - mg \sin \theta = 0$$

$$y: N - mg \cos \theta = 0$$

$$f_k = \mu_k N$$

$$f_k = mg \cdot \mu_0 \cos^2 \theta$$

$$F_{ext} = mg (\sin \theta + \mu_0 \cos^2 \theta) \quad (1)$$

(3)

$$\begin{aligned} W_F &= \int_0^{\pi/2} \vec{F}_{ext} \cdot d\vec{s} = mg \int_0^{\pi/2} (\sin \theta + \mu_0 \cos^2 \theta) \cdot (R d\theta) = mgR \int_0^{\pi/2} \sin \theta d\theta + mgR \int_0^{\pi/2} \mu_0 \frac{\cos 2\theta + 1}{2} d\theta \\ &= mgR \left(-\cos \theta \Big|_0^{\pi/2} + \mu_0 \frac{-(\sin 2\theta / 2) + \theta}{2} \Big|_0^{\pi/2} \right) \\ &= mgR + mgR \mu_0 \cdot \frac{\pi}{4} \end{aligned}$$

(1)

(2)

2. A particle is confined to move along the x-axis with the following potential energy,

$$U(x) = 3x^3 - 9x + 4$$

where x is the coordinate of the particle in unit of μm , and $U(x)$ are in units of electron volts (eV).

- Determine the force \mathbf{F} experienced by the particle as a function of \mathbf{x} .
- At what values of x is $\mathbf{F(x)}$ equal to zero?
- Indicate the positions of the stable and the unstable equilibrium of the particle at the potential energy at these positions.
- If the kinetic energy of the particle at the stable equilibrium position is 6eV, determine the range of the motion of this particle.

$$\begin{aligned} \text{(a)} F &= -\frac{dU(x)}{dx} = -\frac{d}{dx}(3x^3 - 9x + 4) \\ &= -9x^2 + 9 \quad (\text{eV} / \mu\text{m}) \end{aligned}$$

$$\text{(b)} F(x) = -9x^2 + 9 = 0 \Rightarrow x = \pm 1 (\mu\text{m})$$

$$\text{(c)} \frac{dU(x)}{dx^2} = 18x \Rightarrow x = 1 (\mu\text{m}), \frac{d^2U}{dx^2} = 18 > 0$$

Therefore, $U(x)$ has a stable equilibrium at $x = 1 \mu\text{m}$.

$$\text{for } x = -1 (\mu\text{m}), \frac{d^2U}{dx^2} = -18 < 0$$

Therefore, $U(x)$ has a unstable equilibrium at $x = -1 \mu\text{m}$.

$$\text{for } x = 1 (\mu\text{m}), U = -2\text{eV}$$

$$\text{Total Energy } E = -2\text{eV} + E_k = 4\text{eV}$$

$$\Rightarrow E = 4\text{eV} = 3x^3 - 9x + 4 + E_k$$

$$\Rightarrow E_k = -3x^3 + 9x$$

$$\text{For } E_k = \frac{1}{2}mv^2 > 0,$$

$$\Rightarrow E_k = -3x^3 + 9x > 0$$

$$\Rightarrow x^3 - 3x < 0$$

$$\Rightarrow (x - \sqrt{3})(x + \sqrt{3})x < 0$$

$$\Rightarrow 0 < x < \sqrt{3}$$

3. Problem 8-85 in Giancoli (pp. 247)

Problem 8-85 in Giancoli (pp. 211)

A ball is attached to a horizontal cord of length ℓ whose other end is fixed, Fig. 8–42.

- (a) If the ball is released, what will be its speed at the lowest point of its path?
- (b) A peg is located a distance h directly below the point of attachment of the cord. If $h = 0.8\ell$ what will be the speed of the ball when it reaches the top of its circular path about the peg?

(a) $E_1 = E_2, \quad mgh_1 + \frac{1}{2}mv_1^2 = mgh_2 + \frac{1}{2}mv_2^2$

$$mg(h_1 - h_2) + \frac{1}{2}m \cdot 0^2 = \frac{1}{2}mv_2^2$$

$$\frac{1}{2}mv_2^2 = mg\ell \Rightarrow v_2 = \sqrt{2g\ell}$$

(b) $E_2 = E_3, \quad mgh_2 + \frac{1}{2}mv_2^2 = mgh_3 + \frac{1}{2}mv_3^2$

$$mg(h_2 - h_3) + \frac{1}{2}mv_2^2 = \frac{1}{2}mv_3^2$$

$$mg(-0.4\ell) + \frac{1}{2}m2g\ell = \frac{1}{2}mv_3^2$$

$$v_3^2 = 1.2g\ell$$

$$\Rightarrow v_3 = \sqrt{\frac{6g\ell}{5}}$$

