

# Signals and Systems – Quiz 1

03/11/2021

Name:

Student ID:

1. (15%) Express each of the following complex numbers in polar form:

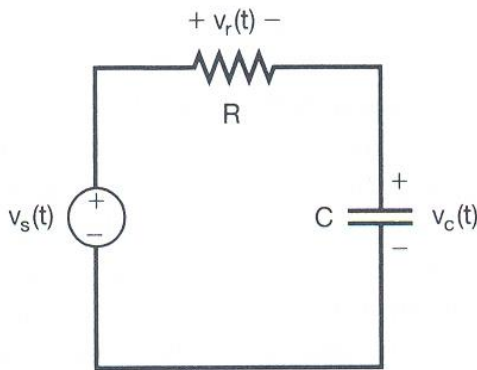
(a)  $\frac{\sqrt{3}}{2} - j\frac{1}{2}$  (b)  $(1+j)(1-j)$  (c)  $(1-j)^2$

**Solution:** (a)  $\frac{\sqrt{3}}{2} - j\frac{1}{2} = e^{-j\frac{\pi}{6}}$  (b)  $(1+j)(1-j) = \sqrt{2}e^{j\frac{\pi}{4}} \cdot \sqrt{2}e^{j\frac{-\pi}{4}} = 2e^{j0}$

(c)  $(1-j)^2 = 2e^{-j\frac{\pi}{2}}$

2. (30%) Consider the RC circuit depicted in Fig.1. If we regard  $v_s(t)$  as the input and  $v_c(t)$  as the output signal, derive the differential equation describing the relationship between the input  $v_s(t)$  and the output  $v_c(t)$ .

**Solution:**



$$\begin{aligned} i(t) &= \frac{v_s(t) - v_c(t)}{R} & i(t) &= C \frac{dv_c(t)}{dt} \\ \Rightarrow \frac{v_s(t) - v_c(t)}{R} &= C \frac{dv_c(t)}{dt} \\ \Rightarrow \frac{dv_c(t)}{dt} + \frac{1}{RC} v_c(t) &= \frac{1}{RC} v_s(t) \\ \Rightarrow \frac{dy(t)}{dt} + ay(t) &= bx(t) & a=b &= \frac{1}{RC} \end{aligned}$$

Fig. 1

3. (30%) Determine whether or not the discrete-time system below is linear

$$y[n] = nx[n]$$

**Solution:**

$$y_1[n] = nx_1[n], \quad y_2[n] = nx_2[n]$$

$$\text{Let } x_3[n] = ax_1[n] + bx_2[n]$$

$$y_3[n] = nx_3[n] = anx_1[n] + bnx_2[n] = ay_1[n] + by_2[n],$$

$\Rightarrow$  this system is linear

4. (25%) Consider an LTI system whose response to the signal  $x_1(t)$  in Fig. 2(a) is the signal  $y_1(t)$  illustrated in Fig. 2(b). Determine and sketch carefully the response of the system to the input  $x_2(t)$  depicted in Fig. 2(c).

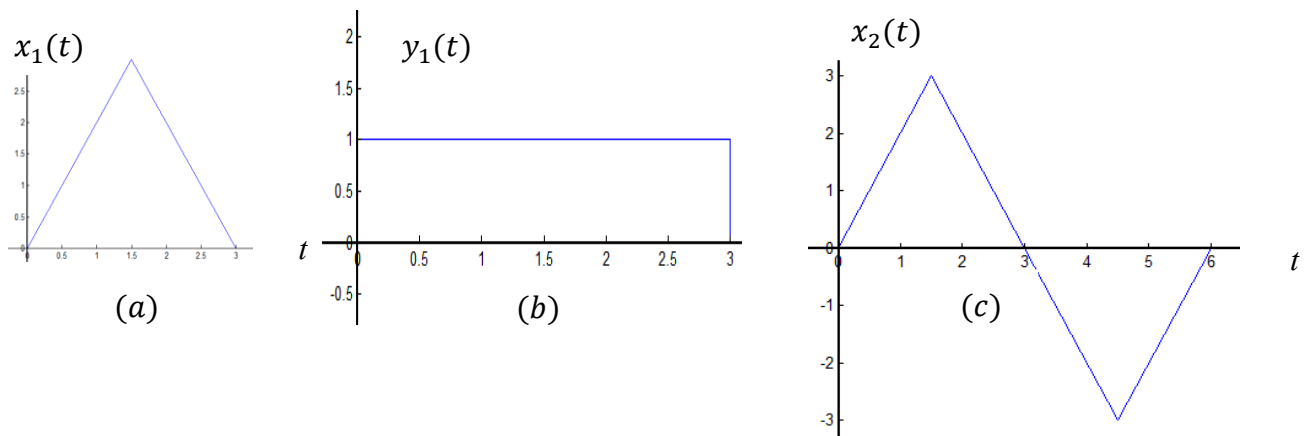


Fig. 2

**Solution:**

$x_2(t) = x_1(t) - x_1(t - 3)$ . Therefore, using linearity we get  $y_2(t) = y_1(t) - y_1(t - 3)$

