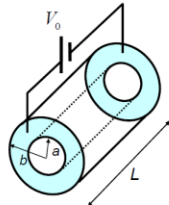


期末考題

1.(20%)

Ex. Calculate the end-to-end DC resistance of a cylindrical conductor shell

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Sol :  $\sigma = \text{const.}$

$$dR_s = \frac{\ell}{\sigma S} = \frac{dz}{\sigma 2\pi r dr}$$

$$dG_s = \int_{r=a}^b \frac{\sigma 2\pi r}{dz} dr = \sigma \pi (b^2 - a^2) \frac{1}{dz}$$

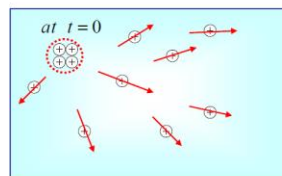
$$dR_s = \frac{1}{dG_s} = \frac{1}{\sigma \pi (b^2 - a^2)} dz$$

$$R = \int_{z=0}^{z=L} dR_s = \int_{z=0}^L \frac{1}{\sigma \pi (b^2 - a^2)} dz = \frac{1}{\sigma \pi (b^2 - a^2)} L$$

2.(20%)題目改  $i(t)=0.15e^{-25t}$

EX.

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$$i(t) = 0.125e^{-25t}$$

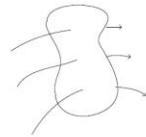
(a)  $\tau = ?$

(b) Charge transported through the surface in time  $t = 5\tau$

(c)  $Q_0 = ?$  (Initial charge)

Sol:

$$(a) \tau = \frac{1}{25} = 40 \text{ ms}$$

(b) Charge passing through the surface in time  $t$  interval

$$i(t) \quad Q(t) = \int_0^t i(t) dt = 0.125 \int_0^t e^{-25t} dt = 5(1 - e^{-25t}) \text{ mC}$$

$$t = 5\tau = 0.2 \text{ s} \quad Q(t)|_{t=5\tau} = 4.97 \text{ mC}$$

$$(c) Q_0 = Q(t \rightarrow \infty) = 5 \text{ mC}$$

### 3.應用最後一行的公式

$$\left. \frac{-dQ}{dt} \right|_{\text{in } V} = I$$

$$\Rightarrow \frac{-d}{dt} \iiint_V \rho_v dv = \oint_S \vec{J} \cdot d\vec{s} = \iiint_V \nabla \cdot \vec{J} dv$$

$$\Rightarrow \iiint_V \left( -\frac{\partial \rho}{\partial t} \right) dv = \iiint_V \nabla \cdot \vec{J} dv$$

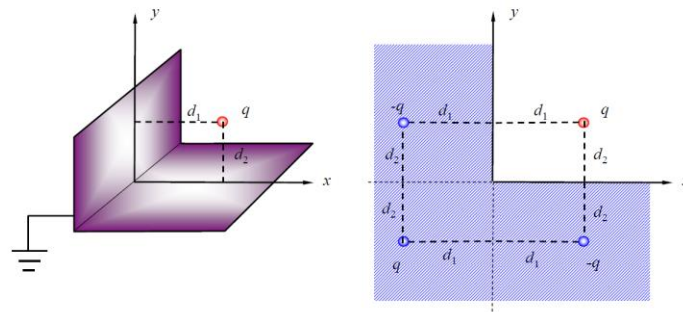
$$\Rightarrow \iiint_V \left( \nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} \right) dv = 0$$

$$\Rightarrow \boxed{\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0} \quad \text{Equation of Continuity}$$

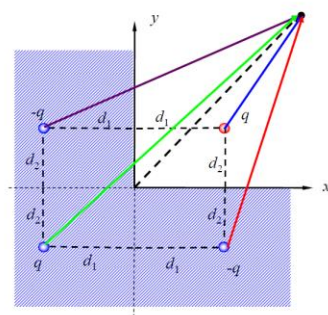
### 4.(20%)

Ex:

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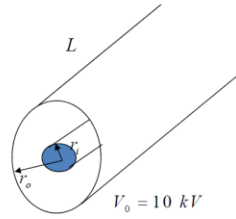
$$V = \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{\sqrt{(x-d_1)^2 + (y-d_2)^2 + z^2}} - \frac{1}{\sqrt{(x+d_1)^2 + (y-d_2)^2 + z^2}} + \frac{1}{\sqrt{(x-d_1)^2 + (y+d_2)^2 + z^2}} - \frac{1}{\sqrt{(x+d_1)^2 + (y+d_2)^2 + z^2}} \right]$$

5.(20%)

題目有改數字

EX.

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$$\begin{cases} \epsilon_r = 2.6 \\ E_{ds} = 20 \times 10^6 \end{cases}$$

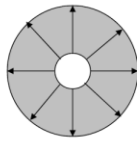
$$E_{\max} \leq \frac{1}{4} E_{ds}$$

$$r_i = 2 \text{ mm}$$

$$r_o = ?$$

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Sol:



$$\oint \vec{D} \cdot d\vec{s} = \int \rho_i d\ell$$

$$\oint \epsilon \vec{E} \cdot d\vec{s} = \int \rho_i d\ell$$

$$E 2\pi r L = \frac{\rho_i L}{\epsilon_0 \epsilon_r}$$

$$\vec{E} = \frac{\rho_i}{2\pi r \epsilon_0 \epsilon_r} \hat{r}$$



$$V_0 = V(r = r_i) - V(r = r_o) = - \int_{r_o}^{r_i} \vec{E} \cdot d\vec{\ell}$$

$$= - \int_{r_o}^{r_i} \frac{\rho_i}{2\pi \epsilon_0 \epsilon_r} \frac{1}{r} dr$$

$$= \frac{\rho_i}{2\pi \epsilon_0 \epsilon_r} \ln \frac{r_o}{r_i}$$



$$\begin{cases} \ln \frac{r_o}{r_i} = \frac{2\pi\epsilon_0\epsilon_r V_0}{\rho_\ell} \\ E_{r, \max} = \frac{\rho_\ell}{2\pi\epsilon_0\epsilon_r r_i} = \frac{1}{4} E_{ds} \Rightarrow \frac{2\pi\epsilon_0\epsilon_r}{\rho_\ell} = \frac{4}{r_i} \frac{1}{E_{ds}} \end{cases}$$

$$\therefore \ln \frac{r_o}{r_i} = \frac{4}{r_i} \frac{V_0}{E_{ds}}$$

$$r_o = r_i e^{\frac{4 V_0}{r_i E_{ds}}}$$

$$= 2 \times 10^{-3} e^{\frac{4}{2 \times 10^{-3}} \frac{10 \times 10^3}{20 \times 10^6}}$$

$$= 2 \times 10^{-3} e^1 = 5.4 \text{ mm}$$

