Signals and Systems –Final 06/22/2021

1. (25%) An LTI system with impulse response $h_1[n] = (\frac{1}{4})^n u[n]$ is connected in parallel with another causal LTI system with impulse response $h_2[n]$. The resulting parallel interconnection has the frequency response:

$$H(e^{j\omega}) = \frac{1}{(1-\frac{1}{2}e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})^2}$$

   (a) Determine $h_2[n]$.
   (b) Find the difference equation describing the overall system.
   (c) Determine the impulse response of the system.

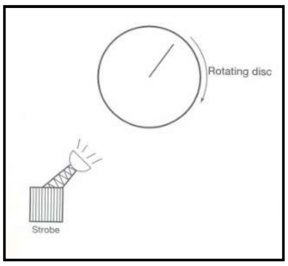
2. (20%) Consider a two-point differentiator by $y[n] = x[n] - x[n-1]$,
   (a) Determine the frequency response $Y\left(e^{j\omega}\right)$ for input $x[n] = 4 + 0.1cos(2\pi n)$.
   (b) Determine the output $y[n]$.

3. (15%) Determine the discrete-time Fourier transform of the sequence

$$x[n] = \begin{cases} \frac{7}{15}\frac{sin\left(\frac{7\pi n}{15}\right)}{sin\left(\frac{\pi n}{15}\right)}, & n \neq multiple\ of\ 15 \\ \frac{49}{15}, n = multiple\ of\ 15 \end{cases}$$

4. (10%) Consider a case in which we have a disc rotating clockwise at a constant rate ω_0 with a single radial line marked on the disc, as shown in the figure below. The flashing strobe acts as sampling system with frequency ω_s. What are the observed rotating frequency (in terms of ω_0) and the rotating direction of the disc for cases as below?

(a) $\omega_s = \frac{1}{4}\omega_0$, (b) $\omega_s = \frac{5}{4}\omega_0$



5. (35%) The figure below shows a system that process a continuous-time signals using a discrete-time filter $h[n]$ that is linear and causal with difference equation:

$$y[n] = \frac{3}{4}y[n-1] + x[n]$$

If $X_c(j\omega)$ is shown as Figure 3, and the sampling rate $1/T$=11kHz,

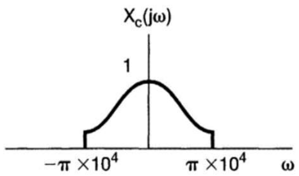
(a) Does aliasing occur during sampling process? What is aliasing and it will occur under which condition?

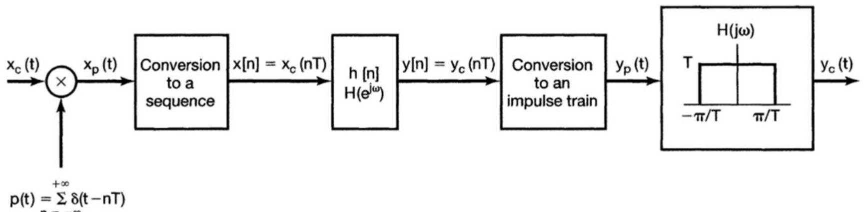
(b) Determine and plot the frequency response of the discrete-time system.

(c) Determine the impulse response of the discrete-time system.

(d) Sketch $X_p(j\omega)$ and $X(e^{j\omega})$.

(e) Determine the frequency response $H_c(j\omega)$ of the equivalent overall system with input $x_c(t)$ and output $y_c(t)$.





(a)

6. (15%) Consider a discrete-time sequence $x[n]$ from which we form two new sequences, $x_p[n]$ and $x_d[n]$, where $x_p[n]$ corresponds to sampling $x[n]$ with a sampling period of 3 and $x_d[n]$, corresponds to decimating $x[n]$ by a factor of 3, so that

$$x_p[n] = \begin{cases} x[n], & n = 0, \pm 3, \pm 6.. \\ 0, & n \neq 0, \pm 3, \pm 6.. \end{cases}, \quad x_d[n] = x[3n]$$

If $X(e^{j\omega})$ is as shown in the figure below, sketch $X_p(e^{j\omega})$ and $X_d(e^{j\omega})$.