試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(45 points) (所有題目必須有計算過程,否則不予計分)

1&2. (15 pts)As shown in Fig. 1, a conducting bar of mass $\boldsymbol{m}$ slides down two frictionless conducting rails which make an angle $\boldsymbol{\theta}$ with the horizontal and one end with a resistor R, and the distance between two rails is ℓ. A uniform magnetic field $\boldsymbol{B}$ is applied horizontally. The bar is released from the top with velocity $\boldsymbol{0}$.

(A) (6 pts) Find the current, magnitude and direction, through the conducting bar.

(B) (6 pts) Draw the free body diagram of the conducting bar sliding down the rail, and write down the equation of motion.

(C) (3 pts) What is the terminal velocity $\boldsymbol{v_T}$ of the bar?

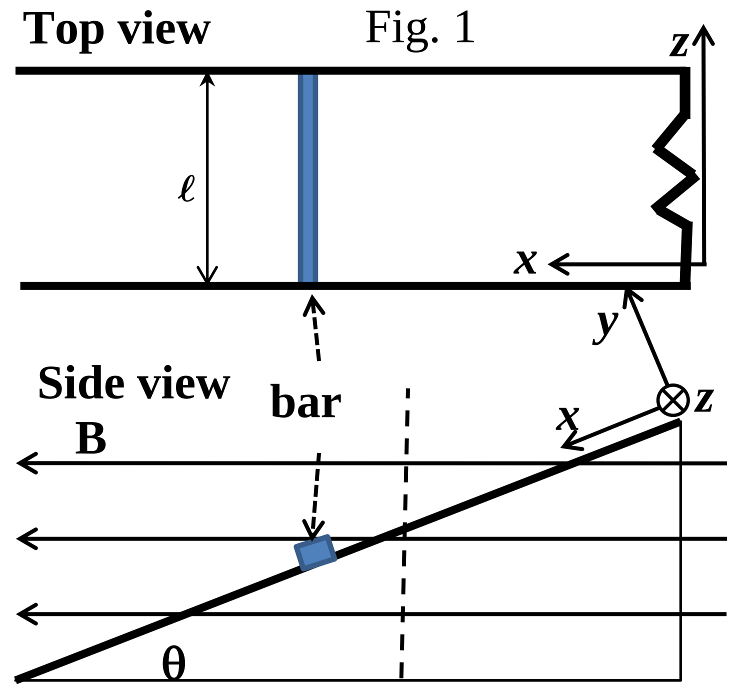


Fig. 1

3. (15pts) As shown in Fig. 2, the circuit consists of an inductor $\boldsymbol{L}$, a switch $\boldsymbol{S}$, and a capacitor, which consists of two conducting disks with radius $\boldsymbol{R}$. Before $\boldsymbol{t}$ = 0, the are charge $\boldsymbol{Q_0}$ ($\boldsymbol{Q_0}$ >0) stored in the capacitor and no current through the inductor. At $\boldsymbol{t}$ = 0, the switch **S** is closed. For t > 0,

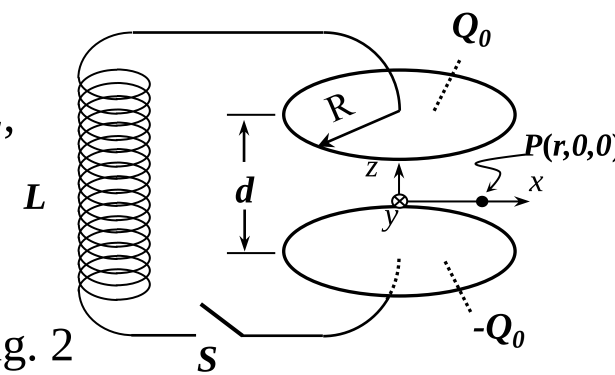


Fig. 2

(A) (3pts)The charge on the upper conductor of capacitor can be expressed as $\boldsymbol{Q(t)=Q_0\cos\omega t}$. Determine $\boldsymbol{\omega}$ in terms of $\boldsymbol{L}$,$\boldsymbol{R}$, $\boldsymbol{d}$, $\boldsymbol{\varepsilon_0}$,$\boldsymbol{\mu_0}$, and other necessary constants.

(B) (3pts) Determine the direction and the magnitude as a function of time of the E-field at point $\boldsymbol{P}(\boldsymbol{r},\boldsymbol{0},\boldsymbol{0})$ ($\boldsymbol{r} \leq \boldsymbol{R}$) indicated in Fig. 2, where the z-axis passes the centers of the disks, and the origin is equally distant from the disks.

(C) (4pts) Determine the direction and the magnitude of the B-field at point $\boldsymbol{P}$ as a function of time.

(D) (5pts) Determine the direction and the magnitude of the Poynting vector at point $\boldsymbol{P}$ as a function of time.

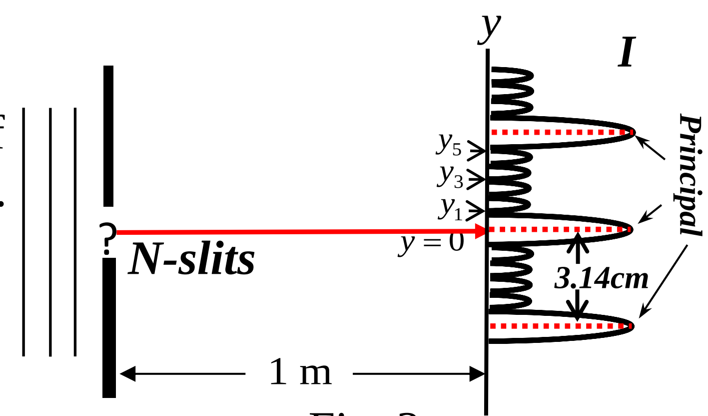


Fig. 3

4. (15pts) A plane electromagnetic wave travels in the direction normal to a screen with N-parallel slits, and the spacing between neighboring slits is $20\ \mu m$. As shown in Fig. 3, the wave emitted from each slit produces an interference pattern on a screen $1\ m$ away. The spacing between the principal maximum intensity peaks is $3.14\ cm$.

(A) (3pts) Determine the number of the slits and the wavelength of the electromagnetic wave.

(B) (4pts)Draw the phasor diagram for the E-fields emitted from each slit at the intensity minimum y_1 and y_3 on the screen.

(C) (5pts) Determine the intensity on the screen as a function of y on the screen, assuming that the initial intensity is I_0 at each slit and the unit of y is cm.

(D) (3pts) If the width of each slit is $4\ \mu m$, how many principal bright fringes will be observed in the central diffraction band?

II.選擇題(55 points)

1. (5 pts) Fig. 4 shows V_R (resistor), V_C (Capacitor) and $V_{\text{Power Supply}}$ (frequency ω) in a series ac-RLC circuit. Which of the following actions will make the circuit to reach resonance?
(1) increasing ***L***; (2) decreasing ***L***; (3) increasing ***C***;
(4) decreasing **C**; (5) increasing ω; (6) decreasing ω;

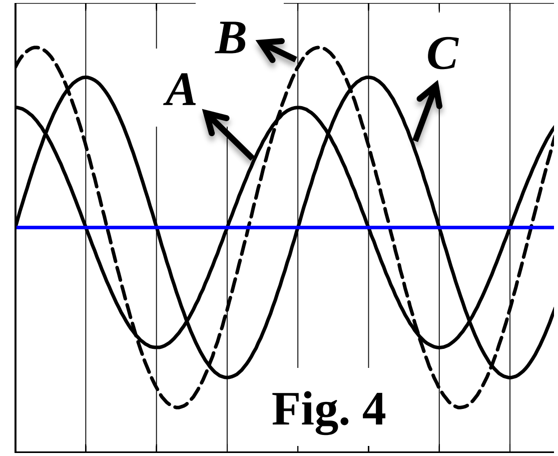


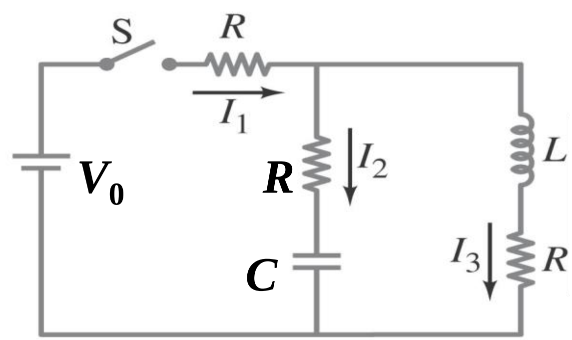
Fig. 4

(A) **1,3,5** (B) **1,3,6** (C) **1,4,5** (D) **1,4,6** (E) **2,3,5**
(F) **2,3,6** (G) **2,4,5** (H) **2,4,6** (J) None of above.

2. (5 pts.) At $t = 0$, the switch in the circuit shown in Fig. 5 is closed. After a sufficiently long time, what are the magnitude of current I_2 and I_3 **(in unit of *Amperes*)**? Let V_0**=12V,** C**=1.0 μF,** $R = 1\Omega$ and L**=0.035H.**

(A) **6, 0** (B) **0, 6** (C) **4,4** (D) **4,0**
(E) **0,4** (F) **2, 2** (G) **0,2** (H) **2, 0**
(J) None of above.

Fig. 5



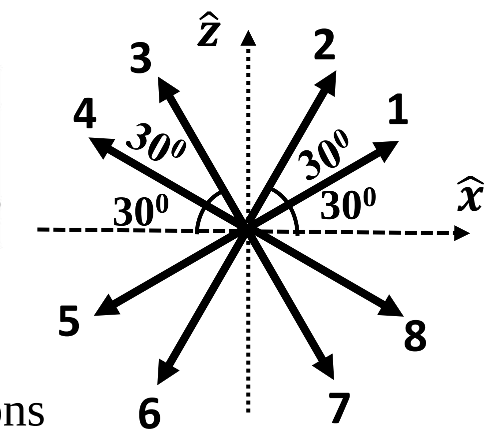


Fig. 6

3. (5 pts) A magnetic field of a plane wave (in free space) with the form $\vec{B}(\vec{r},t) = (-2\times 10^{-7}T)\hat{y}\sin(\sqrt{3}x - z + \omega t)$. What are the directions of the Poynting vector and the Electric field (indicated in Fig. 6)?

(A) **1, 3** (B) **1, 7** (C) **2,4** (D) **2,8** (E) **3,1** (F) **3,5** (G) **4,2** (H) **4,6** (J) **5,3** (K) **5,3**
(L) **5, 7** (M) **6,4** (N) **6,8** (O) **7,1** (P) **7,5** (Q) **8,2** (R) **8,6** (S) None of above.

4. (5pts) As shown in Fig. 7, in the region $x \geqq 0$, there exists a constant magnetic field **(*0,0,B*)**, and a square conductor loop is traveling at constant velocity $\boldsymbol{v}$. The total resistance of the conductor loop is $\boldsymbol{R}$**.** At time $\boldsymbol{t = 0}$, the tip of the loop reaches the position $x = 0$, If we define the **+** direction of current in this loop to be counter-clockwise**,** which of the following plot showing the current $\boldsymbol{i}$ in the loop as a function of time.

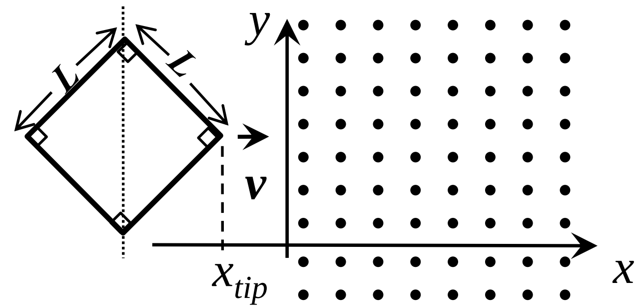
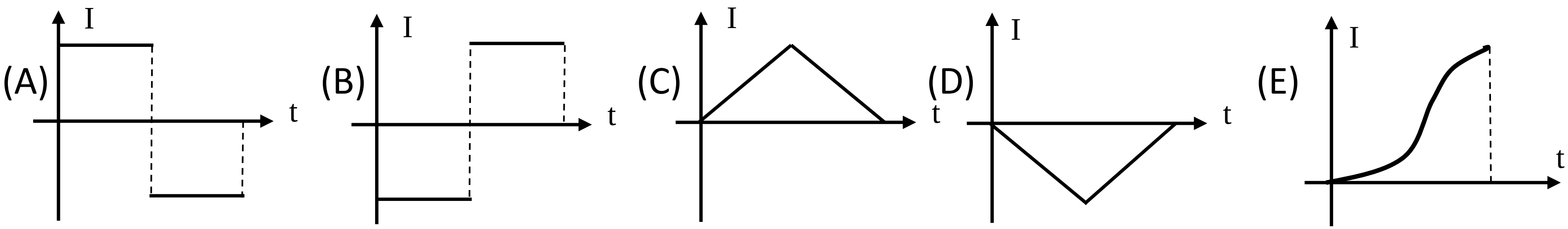


Fig. 7



5. (5pts) As shown in Fig. 8, the flash light shines a beam of light on a black disk, which is normal to the direction of the light beam. The wavelength $\boldsymbol{\lambda}$ of the light wave in the light beam varies from 400nm to 800nm, and suppose that the $\boldsymbol{E}$-field of the light wave of each wavelength $\boldsymbol{\lambda}$ can be expressed with the following equation: $\vec{E}(\lambda) = \vec{E}_0 \cos(2\pi(x - ct)/\lambda)$, where $\vec{E}_0$ is independent of $\boldsymbol{\lambda}$. Which of the following shows the correct trend of the $\boldsymbol{\lambda}$ dependence of the pressure $\boldsymbol{P}$ that the light beam exerted on the disk?

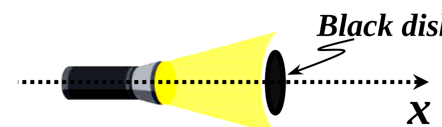
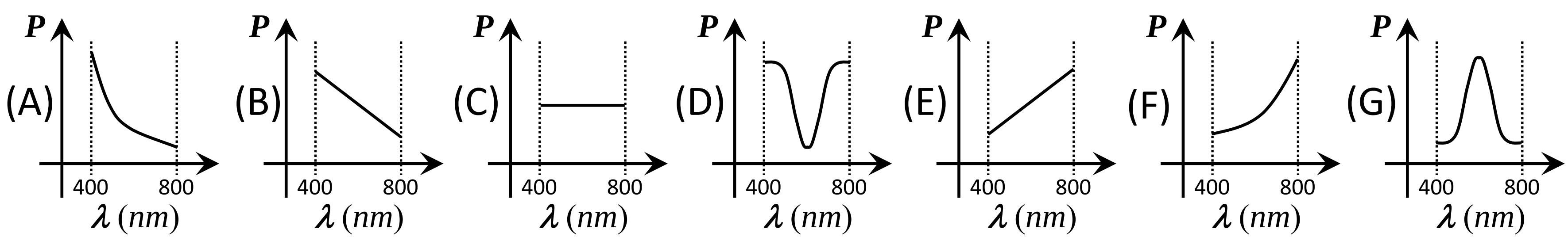


Fig. 8



(Problem 6 & 7 continue on the next page)

6. (5pts) A thin film of oil (n_o =1.50) with varying thickness floats on water (n_w = 1.33). When it is illuminated from above by white light, the reflected colors are shown in Fig. 9. In air, the wavelength of orange light is 600 nm (where constructive interference occurs). Find the oil's thickness **t** at **B** point.

(A) 133; (B) 150; (C) 200; (D) 266; (E) 300; (F) 400; (G) 450; (H) 532 nm.

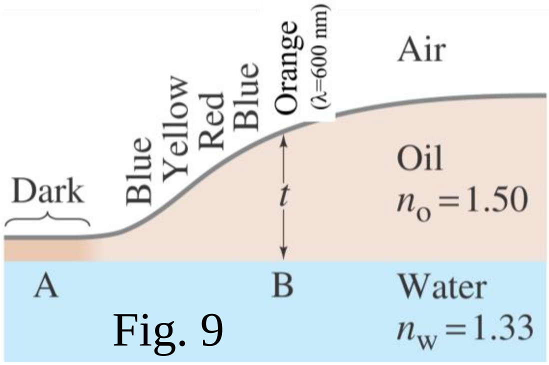


Fig. 9

7. (5 pts.) For a ***RLC*** circuit shown in Fig. 10. The AC voltage source is $\boldsymbol{V(t) = V_0 \sin \omega t}$. The current through the inductor, capacitor and resistor are $\boldsymbol{I_L}$, $\boldsymbol{I_C}$ and $\boldsymbol{I_R}$, respectively. Let $\boldsymbol{V_0}$**=0.4V, ω=10^4 rad./s, *C*=5.0 μF, *R* = 20 Ω** and ***L*=0.001H.** What is the magnitude of the current $\boldsymbol{I_{R,0}}$ **(in unit of *Amperes*)**?

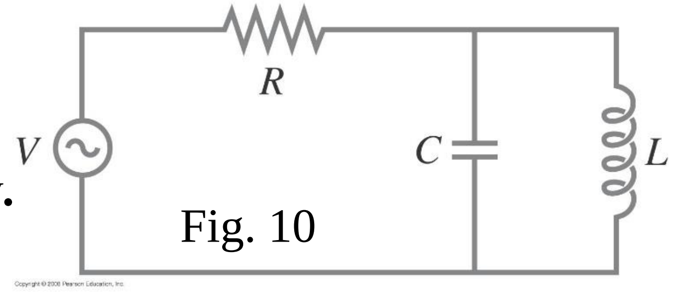
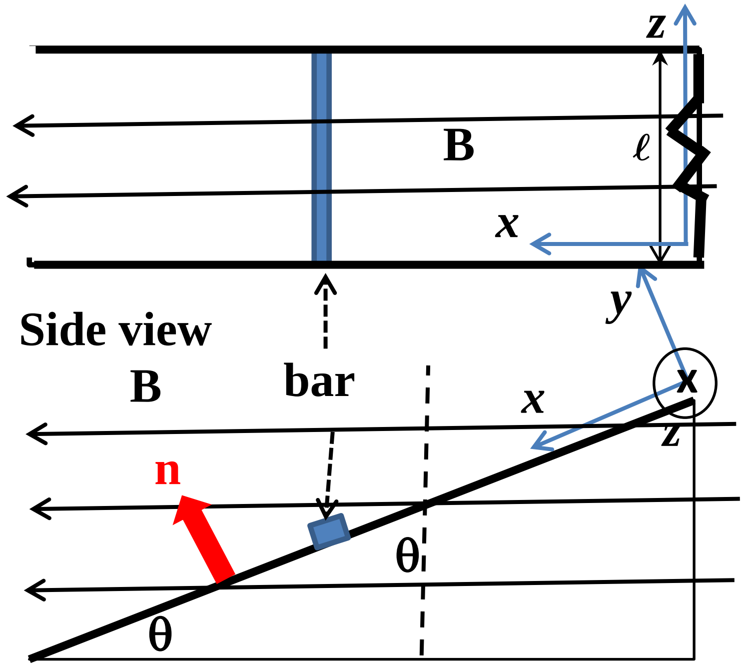


Fig. 10

(A) 0 (B) $0 < I_{R,0} \leq 0.005$ (C) $0.005 < I_{R,0} \leq 0.01$ (D) $0.01 < I_{R,0} \leq 0.015$ (E) $0.015 < I_{R,0} \leq 0.02$ (F) $0.02 < I_{R,0} \leq 0.025$ (G) $0.025 < I_{R,0} \leq 0.03$ (H) $0.03 < I_{R,0} \leq 0.04$ (J) $0.04 < I_{R,0}$

Multiple Choice Questions:

1	**2**	**3**	**4**	**5**	**6**	**7**			
A	**B**	**G, H**	**D**	**C**	**E**	**D**			



(A)

The normal vector of the plane is indicated by the vector n in the figure above.

The movement of the bar ➔ Φ_B changes

$$\Phi_B = \int \vec{B}\cdot d\vec{A} = B\left(\ell \cdot x(\mathrm{t})\right)\cos\left(\frac{\pi}{2}-\theta\right)$$

$$= B\left(\ell \cdot x(\mathrm{t})\right)\sin\theta$$

2 pts

The change of Φ_B induces emf $\mathcal{E}$:

$$\left|\frac{d\Phi_B}{dt}\right| = \mathcal{E} = B\ell\frac{dx(\mathrm{t})}{dt}\sin\theta = B\ell v(t)\sin\theta$$

1 pt

$\mathcal{E}$ induces current I:

$$I = \frac{\mathcal{E}}{R} = \frac{B\ell v(t)\sin\theta}{R}$$

Top view: 順時針

1 pt

2 pt

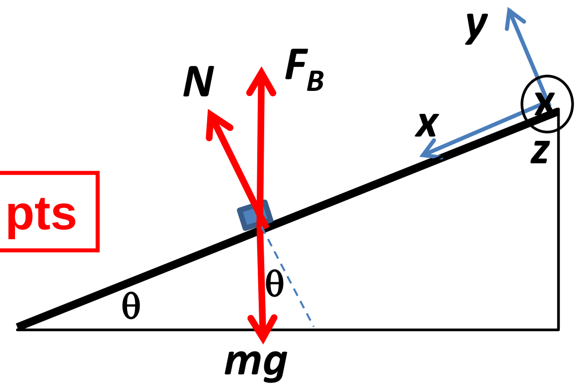
(B) the bar with I in B $\Rightarrow$ magnetic force.

$$\vec{F}_B = I\vec{\ell} \times \vec{B} = \frac{B^2\ell^2 v(t)\sin\theta}{R}$$

2pts

Side view Direction: $\uparrow$

2 pts



x : $mg\sin\theta - F_B\sin\theta = ma = m\frac{dv}{dt}$ **1 pt**

y : $N - mg\cos\theta + F_B\cos\theta = 0$ **1 pt**

(C) When the bar has reach "terminal velocity" $\boldsymbol{v_T}$, there is no acceleration.

1 pt

i.e $$mg\sin\theta - \frac{B^2\ell^2 v(t)\sin\theta}{R}\sin\theta = 0$$

$\Rightarrow$ $$v_T = \frac{R \cdot mg}{B^2\ell^2\sin\theta}$$ **2 pts**

3. (15pts) As shown in Fig. 2, the circuit consists of an inductor ***L***, a switch ***S***, and a capacitor, which consists of two conducting disks with radius ***R***. Before $t = 0$, the are charge Q_0 ($Q_0 > 0$) stored in the capacitor and no current through the inductor.

At $t = 0$, the switch **S** is closed. For $t > 0$,

(A) (3pts)The charge on the upper conductor of capacitor can be expressed as $Q(t)=Q_0\cos\omega t$. Determine ω in terms of $L, R, d, \varepsilon_0, \mu_0$, and other necessary constants.

(B) (3pts) Determine the direction and the magnitude as a function of time of the E-field at point $P(r,0,0)$ $(r \le R)$ indicated in Fig. 2, where the z-axis passes the centers of the disks, and the origin is equally distant from the disks.

(C) (4pts) Determine the direction and the magnitude of the B-field at point ***P*** as a function of time.

(D) (5pts) Determine the direction and the magnitude of the Poynting vector at point ***P*** as a function of time.

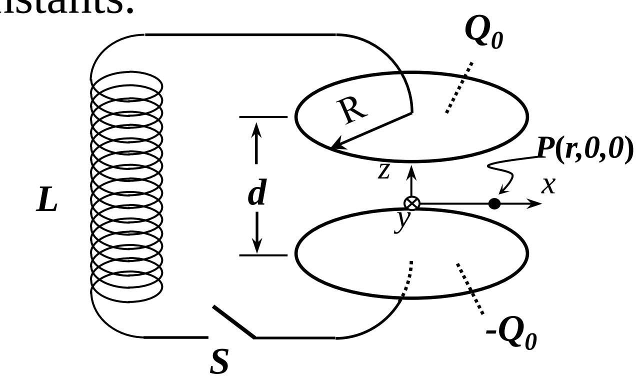


Fig. 2

(A) For t>0, the charge on the capacitor executes LC ossccilation, i.e.

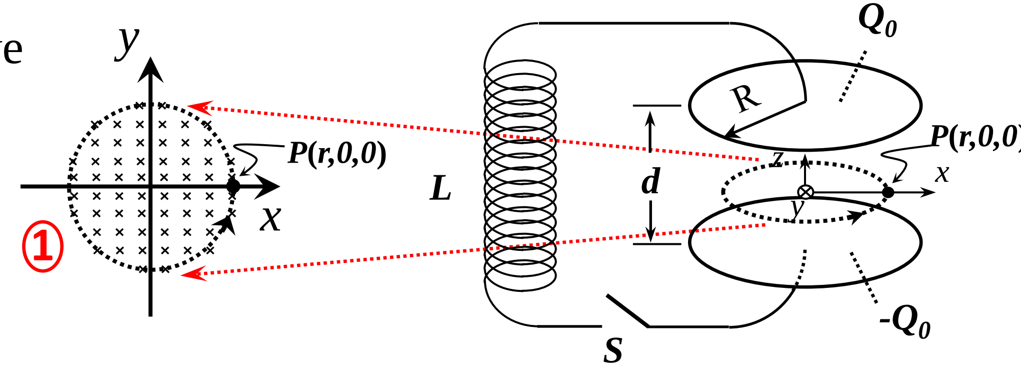
$$\omega = \frac{1}{\sqrt{LC}}, \quad and\ C = \varepsilon_0\frac{\pi R^2}{d}, \text{①} \quad Therefore\ , \omega = \frac{1}{\sqrt{L\varepsilon_0\frac{\pi R^2}{d}}} = \frac{1}{R}\sqrt{\frac{d}{\pi L\varepsilon_0}} \quad \text{②}$$

(B) From Gauss's Law, the magnitude of the E-field in between a two infinite uniform plane charge distributions with charge density σ and –σ is

$$E = \frac{\sigma}{\varepsilon_0}, \text{ For the case of the capacitor } \sigma = \frac{Q(t)}{\pi R^2} = \frac{Q_0}{\pi R^2}cos(\omega t) = \frac{Q_0}{\pi R^2}cos\left(\frac{1}{R}\sqrt{\frac{d}{\pi L\varepsilon_0}}t\right), \quad \text{①}$$

$$\vec{E} = (o, o, -\frac{\sigma}{\varepsilon_0}) = (o, o, \frac{-Q_0}{\pi\varepsilon_0 R^2}cos\left(\frac{1}{R}\sqrt{\frac{d}{\pi L\varepsilon_0}}t\right)) \quad \text{②}$$

(C) To calculate the B-field at point P, first we choose a loop at x-y plane shown in the figure on the right. And according to the Maxwell-Ampere's law



$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0\varepsilon_0\frac{d\Phi_E}{dt},$$

With this loop,

$$\Phi_E = \vec{E}\cdot\vec{B} = -\pi r^2 E(t) = -\pi r^2\frac{Q_0}{\pi\varepsilon_0 R^2}cos\left(\frac{1}{R}\sqrt{\frac{d}{\pi L\varepsilon_0}}t\right) = -\frac{Q_0 r^2}{\varepsilon_0 R^2}cos\left(\frac{1}{R}\sqrt{\frac{d}{\pi L\varepsilon_0}}t\right),$$ ①

$$\Rightarrow \oint \vec{B}\cdot d\vec{\ell} = 2\pi r B(t) = \mu_0\varepsilon_0\frac{d}{dt}\left(-\frac{Q_0 r^2}{\varepsilon_0 R^2}cos\left(\frac{1}{R}\sqrt{\frac{d}{\pi L\varepsilon_0}}t\right)\right),$$

$$\Rightarrow B(t) = \mu_0\frac{Q_0 r}{2\pi R^3}\sqrt{\frac{d}{\pi L\varepsilon_0}}sin\left(\frac{1}{R}\sqrt{\frac{d}{\pi L\varepsilon_0}}t\right), \quad or\ B(t) = \mu_0\frac{Q_0 r\omega}{2\pi R^2}sin(\omega t),$$

Alternative solution

and at $\boldsymbol{P}$, $\vec{B}(t) = \left(0, \mu_0\frac{Q_0 r}{2\pi R^3}\sqrt{\frac{d}{\pi L\varepsilon_0}}sin\left(\frac{1}{R}\sqrt{\frac{d}{\pi L\varepsilon_0}}t\right), 0\right)$ $\boxed{\text{or, } \vec{B}(t) = \left(0, \mu_0\frac{Q_0 r\omega}{2\pi R^2}sin(\omega t), 0\right)}$ ②

(D) The Poynting vector at point $\boldsymbol{P}$, $\vec{P} = \vec{E}\times\vec{B}/\mu_0$, $i.e.$

$$\vec{P} = \frac{1}{\mu_0}\left(o, o, \frac{-Q_0}{\pi\varepsilon_0 R^2}cos\left(\frac{1}{R}\sqrt{\frac{d}{\pi L\varepsilon_0}}t\right)\right)\times\left(0, \mu_0\frac{Q_0 r}{2\pi R^3}\sqrt{\frac{d}{\pi L\varepsilon_0}}sin\left(\frac{1}{R}\sqrt{\frac{d}{\pi L\varepsilon_0}}t\right), 0\right)$$ ①

$$\Rightarrow \vec{P} = (0,0,-1)\times(0,1,0)\frac{1}{\mu_0}\frac{Q_0}{\pi\varepsilon_0 R^2}cos\left(\frac{1}{R}\sqrt{\frac{d}{\pi L\varepsilon_0}}t\right)\cdot\mu_0\frac{Q_0 r}{2\pi R^3}\sqrt{\frac{d}{\pi L\varepsilon_0}}sin\left(\frac{1}{R}\sqrt{\frac{d}{\pi L\varepsilon_0}}t\right)$$

$$= (1,0,0)\frac{1}{\mu_0}\frac{Q_0}{\pi\varepsilon_0 R^2}\mu_0\frac{Q_0 r}{2\pi R^3}\sqrt{\frac{d}{\pi L\varepsilon_0}}sin\left(\frac{2}{R}\sqrt{\frac{d}{\pi L\varepsilon_0}}t\right) = \left(\frac{Q_0^2 r}{2\pi^2\varepsilon_0 R^5}\sqrt{\frac{d}{\pi L\varepsilon_0}}sin\left(\frac{2}{R}\sqrt{\frac{d}{\pi L\varepsilon_0}}t\right), 0, 0\right)$$ ④

(continue on next page)

(C) $$\boxed{\vec{P} = \frac{1}{\mu_0}\left(o, o, \frac{-Q_0}{\pi\varepsilon_0 R^2} cos(\omega t)\right) \times \left(0, \mu_0 \frac{Q_0 r\omega}{2\pi R^2} sin(\omega t), 0\right)}$$ ①

$$\Rightarrow \vec{P} = (0,0,-1) \times (0,1,0) \frac{1}{\mu_0} \frac{Q_0}{\pi\varepsilon_0 R^2} cos(\omega t) \cdot \mu_0 \frac{Q_0 r\omega}{2\pi R^2} sin(\omega t)$$

Alternative solution

$$\boxed{= (1,0,0) \frac{Q_0^2 r\omega}{2\pi^2 \varepsilon_0 R^4} cos(\omega t) sin(2\omega t) = \left(\frac{Q_0^2 r\omega}{4\pi^2 \varepsilon_0 R^4} sin(2\omega t), 0,0\right)}$$ ④

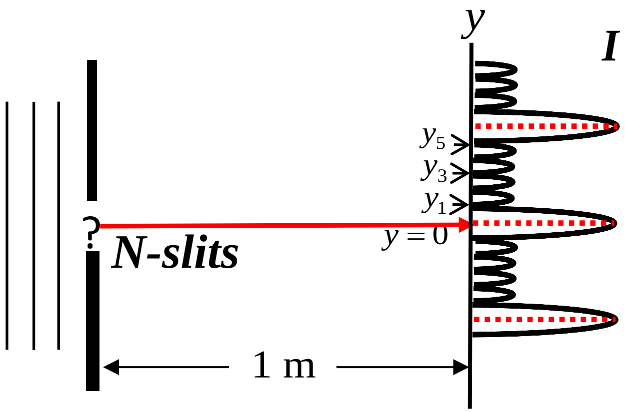
1. A plane electromagnetic wave travels in the direction normal to a screen with N-parallel slits, and the spacing between neighboring slits is $20\ \mu m$. As shown in Fig., the wave emitted from each slit produces an interference pattern on a screen $1\ m$ away. The spacing between the central maximum intensity peak and the neighboring peak is $3.14\ cm$.

(a) Determine the number of the slits and the wavelength of the electromagnetic wave.

(b) Draw the phasor diagram for the E-fields emitted from each slit at the intensity minimum y_1 and y_3 on the screen.

(c) Determine the intensity on the screen as a function of y on the screen, assuming that the initial intensity is I_0 at each slit and the unit of y is cm.

(d) If the width of each slit is 4 μ m, how many interference bright fringes will be observed in the central diffraction peak?

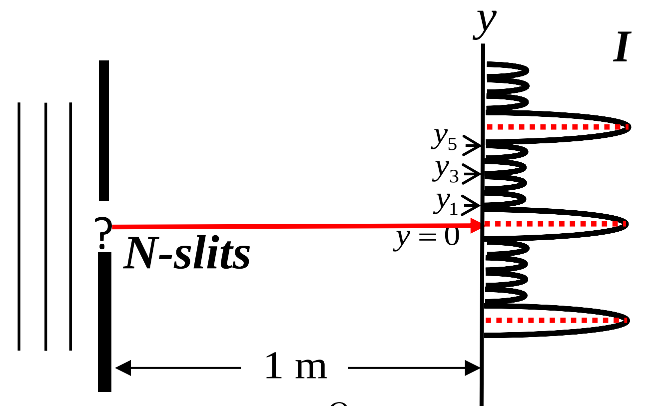


(a) N = 6 1

$$\delta = \frac{2\pi}{\lambda} d \sin\theta \sim \frac{2\pi}{\lambda} d \tan\theta = \frac{2\pi}{\lambda} d \frac{y}{L}$$ 1

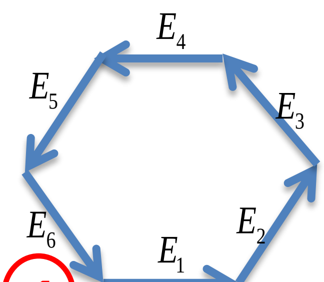
$$2\pi = \frac{2\pi}{\lambda}(20\mu m)\frac{3.14cm}{1m}$$

$$\lambda = 628nm = 0.628\mu m$$ 1



(b) The phasor of E at y_1,

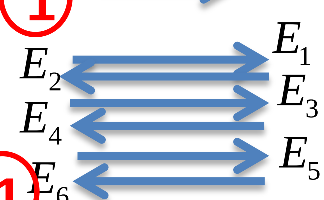
$$\delta = \frac{2\pi}{6} = \frac{\pi}{3}$$ 1



1

The phasor of E at y_3,

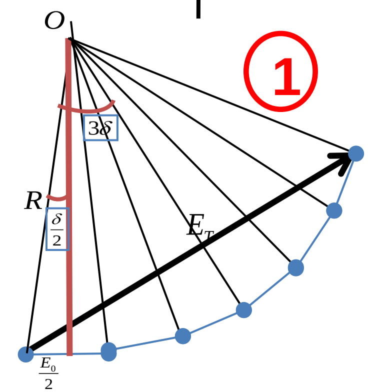
$$\delta = \frac{2\pi}{6} 3 = \pi$$ 1



1

(c)

$$\sin\frac{\delta}{2} = \frac{E_0/2}{R}; \quad \sin\frac{6\delta}{2} = \frac{E_{\theta 0}/2}{R}$$ 1



1

$$\frac{E_{\theta 0}}{E_0} = \frac{\sin 3\delta}{\sin \delta/2}$$ 1

$$\delta = \frac{2\pi}{\lambda} d \sin\theta = 2y$$ for y in cm 1

$$I(y) = I_0\left(\frac{\sin 3\delta}{\sin \delta/2}\right)^2 = I_0\left(\frac{\sin 6y}{\sin y}\right)^2$$ 1

(d) For the first destructive interference of diffraction from individual slits,

a sinθ = λ 1

For constructive interference from the slits,

d sinθ = mλ

The interference (principal maxima) occurs at $\delta = 2\pi m = \frac{2\pi}{\lambda} d \sin\theta < \frac{2\pi}{\lambda} d \frac{\lambda}{a} \rightarrow m < \frac{d}{a} = \frac{20}{4} = 5$ 1

$(m = 0, \pm 1, \pm 2, \pm 3, \pm 4)$ There are 9 bright fringes. 1