試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(50 points) (所有題目必須有計算過程,否則不予計分)

1. **(10 pts):** The direction (in general) of a plane can be expressed by the wave vector $\vec{k}=\left(k_x,k_y,k_z\right)$ For example, a electric field of a plane wave (in free space) with the form

$$\vec{E}(\vec{r},t)=E_0\hat{y}\sin\left[4x-3z+\omega t\right]=\left(120\ \text{V/m}\right)\hat{y}\sin\left[k(\hat{k}\cdot\vec{r})-\omega t\right]$$

describes the wave traveling in the direction of $\hat{k}$ **with** wave number k (in unit of **m**$^{-1}$) and the position vector $\vec{r}=\left(x,y,z\right)$. Answer the following questions including **correct unit**. note: c = 3×10^8m/s, μ_0= $4\pi\times10^{-7}$ Tm/A, and $\varepsilon_0\mu_0=1/c^2$.

a) (2 pts) Find the wave number (k), wavelength (λ), and the angular frequency (ω) of this plane wave.

b) (2 pts) What is the direction of this plane wave propagating?

c) (4 pts) The magnetic field of this plane can be written as $\vec{B}(\vec{r},t)=\vec{B}_0\sin\left[k(\hat{k}\cdot\vec{r})-\omega t\right]$. Find the values of B_{0x} and B_{0z} in SI unit.

d) (2 pts) Find the Poynting vector $\vec{S}$ (magnitude and direction), and the intensity (I = <S>) of this plane wave.

2. (10 pts) Consider a copper ring of radius a and resistance R a magnetic field

$$\vec{B}(\vec{r},t)=\begin{cases}\hat{z}B_0\left(1-\dfrac{r}{r_0}\right)\left(1-\dfrac{t^2}{T^2}\right) & r<r_0, 0<t<T\\ 0 & r\geq r_0, T<t \text{ and } t<0\end{cases}$$

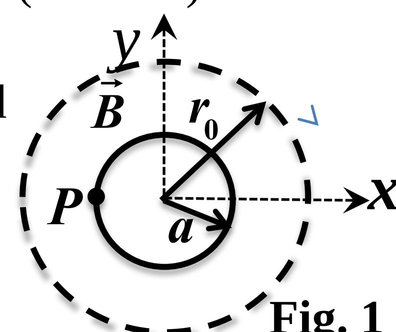


Fig. 1

Express your answer in terms of B_0, a, r_0, R, T, and μ_0 as needed.

a) (3 pts) What is the magnetic flux through the copper ring at some time t ($0<t<T$)?

b) (4 pts) What are the current and direction (c.w. or c.c.w.) of the current $I(t)$ in the ring?

c) (3 pts) what is the total charge Q has moved past a fixed point P in the ring during the time interval that the magnetic field is changing?

3. (15pts) (a) (7pts) As shown in Fig. 2(a), an AC circuit cinsists of a voltage source, and resister **R**, **R** = **10 Ohm**, and an inductor **L**, **L** = **10 mH**, If the current through the circuit is $I = I_0 \cos(\omega t)$, apply the phasor method to determine the ratio V_L/V_S as a function of ω, and the the limiting values for ***$\omega \to 0$, and $\omega \to \infty$.***

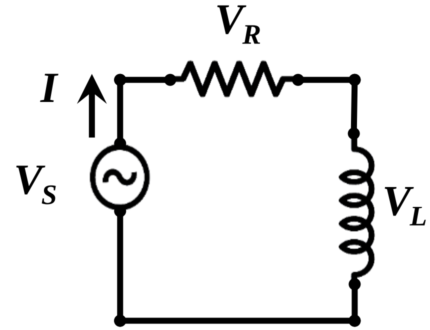


Fig. 2(a)

(b) (8pts) As shown in Fig 2(b), a capacitor **C**, **C** = **10 mF** is inserted to the circuit, apply the phasor method again to determine the ratio V_L/V_S as a function of ω, and the the limiting values for ***$\omega \to 0$, and $\omega \to \infty$. (Draw your phasor diagram with the relevant physical quantities clearly labeled).***

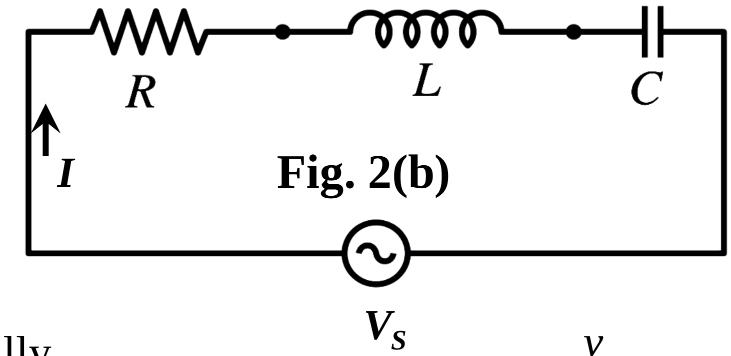


Fig. 2(b)

4. (15 pts) A plane wave of wavelength λ passes a set of six equally spaced slits and forms an interference pattern on a screen located a distance L away (Fig. 3). The spacing between neighboring slits is d, and $\lambda \ll d \ll L$. The electric field components at the point P on the screen caused by waves from first slit from the top is $E_1 = E_0 \sin(kr - \omega t)$, and the electric field from the ith slit from the top is $E_i = E_0 \sin(kr - \omega t + (i-1) * \phi)$, $i = 2, 3, 4, 5, 6$. Consider only the interference pattern.

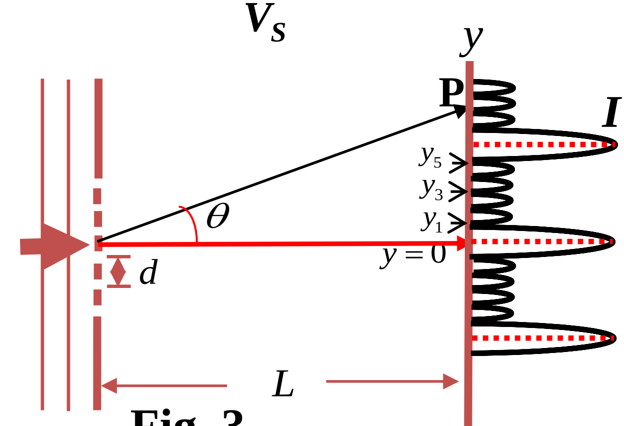


Fig. 3

(A) (3 pts) Find ϕ in terms of d, λ, θ and in terms of d, λ, y, L under condition $\sin\theta \approx \tan\theta$ as $\theta \ll 1$.

(B) (6 pts) The resultant electric field is $E_\theta = E_{\theta 0} \sin(\omega t + \delta)$ at point P. Find $E_{\theta 0}$ in terms of E_0 and ϕ.

(C) (6 pts) Find the conditions of δ's and the phasor diagrams for the locations at the first minimum y_1, the third minimum y_3 and fifth minimum y_5 on the screen.

II.選擇題(50 points)

1. (5 pts) Fig. 4 shows a L-C circuit. The top view of the electric and magnetic field inside the capacitor is shown in Fig.4(a) at time ***t*.** Wh of the following statement about the current in the circuit is correct?

(A) The current is counter-clock-wise (c.c.w.) and increasing at time t.
(B) The currentis clock-wise (c.w.) and increasing at this moment.
(C) The current is c.c.w. and decreasing at this moment.
(D) The current is c.w. and decreasing at this moment.
(E) (A) and (B) are correct ; **(F)** (C) and (D) are correct,
(G) both of (A) and (C) are possible; **(H)** (B) and (D) are correct,
(I) all (A) to (D) are correct.

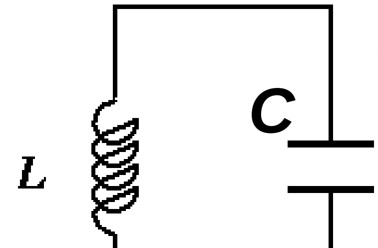


Fig. 4

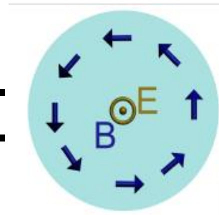


Fig. 4(a)

2. (5 pts.) A conducting bar of mass $\boldsymbol{m}$**=0.01kg** and resistance $\boldsymbol{R}$**=1Ω** slides down two frictionless conducting rails which make an angle $\boldsymbol{\theta}$**=30⁰** with the horizontal, and are separated by a distance $\boldsymbol{\ell}$**=0.2m**, as shown in Fig. 5. In addition, a uniform magnetic field $\boldsymbol{B}$**=0.5T** is applied vertically upward. The bar is released from rest and slides down. What is the terminal speed $\boldsymbol{v_T}$ of the conducting bar can reached (**m/s**)? (assuming the rails are infinite long and $\boldsymbol{g}$ **=10 m/s²**)

(A) $v_T \le 1$ (B) $1 < v_T \le 5$ (C) $5 < v_T \le 10$ (D) $10 < v_T \le 15$ (E) $15 < v_T \le 20$
(F) $20 < v_T \le 25$ (G) $25 < v_T \le 30$ (H) $30 < v_T \le 35$ (J) $35 < v_T \le 40$ (K) $40 < v_T$

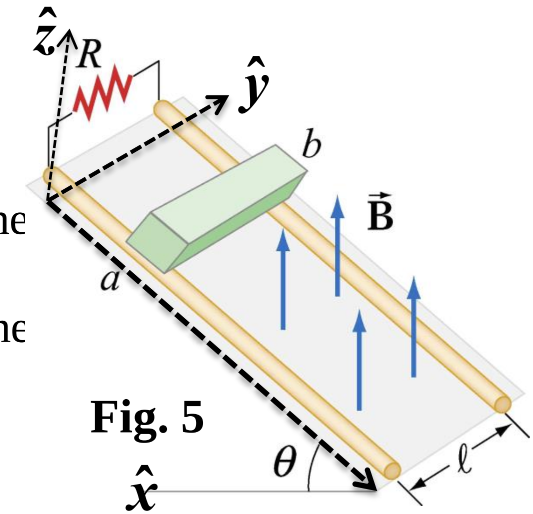


Fig. 5

3. (5 pts) A toroidal inductor has a square cross section as show in Fig. 6(a). There number of windings is $\boldsymbol{N}$ and its inductance is $\boldsymbol{L_T}$. And a solenoidal inductor has the same cross section and length $\boldsymbol{2\pi R}$, as shown in Fig. 6(b), with the same number of windings and its inductance is $\boldsymbol{L_S}$. Let $\boldsymbol{x=L_T/L_S}$. Which of the following is correct? ($\boldsymbol{ln}$ 2 = 0.7, $\boldsymbol{ln}$ 3 = 1.1, $\boldsymbol{ln}$ 10 =2.3)

(A) $0< x <0.2$ (B) $0.2\le x <0.4$ (C) $0.4\le x <0.6$ (D) $0.6\le x <0.8$
(E) $0.8\le x <01.0$ (F) $1.0\le x <1.5$ (G) $1.5\le x <2.0$ (H) $2.0 \le x$

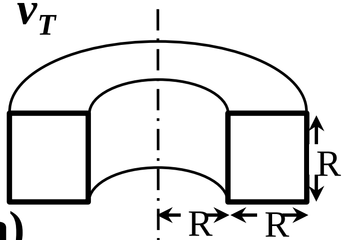


Fig. 6 (a)

Fig. 6 (b)

4. (5 pts) As shown in Fig. 7, a plane electromagnetic wave is traveling in a direction normal to a thin film with reflective index $\boldsymbol{n_2}$ and thickness of $\boldsymbol{\lambda/4}$, where λ is the wavelength of this wave in medium $\boldsymbol{n_2}$. The reflective index is $\boldsymbol{n_1}$ for the medium on the left side of the film, and $\boldsymbol{n_3}$ on the other side. If the wave reflected from the $\boldsymbol{n_1}$-$\boldsymbol{n_2}$ interface interferes destructively with that reflected from the $\boldsymbol{n_2}$-$\boldsymbol{n_3}$ interface, which of the following statement is correct?

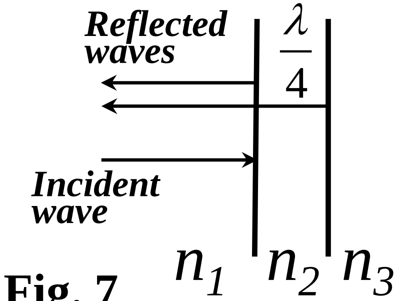


Fig. 7

(A) $n_1 > n_2 > n_3$ **(B)** $n_2 > n_1 > n_3$ **(C)** $n_3 > n_2 > n_1$ **(D)** $n_3 > n_1 > n_2$ **(E)** (A) and (C)
(F) (B) and (D) **(G)** (A) and (B) **(H)** (C) and (D) **(I)** (B),(C), and (D) **(J)** (A),(B), and (D)

5. (5pts) Fig. 8 shows a DC circuit with the switch S being closed for $t<0$, and at $t=0$, the switch is opened, which of the following shows the time dependence of the current i_L through the inductor L?

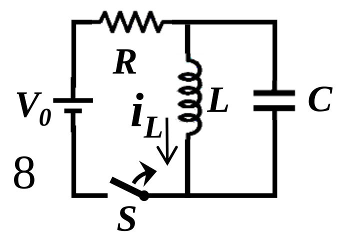
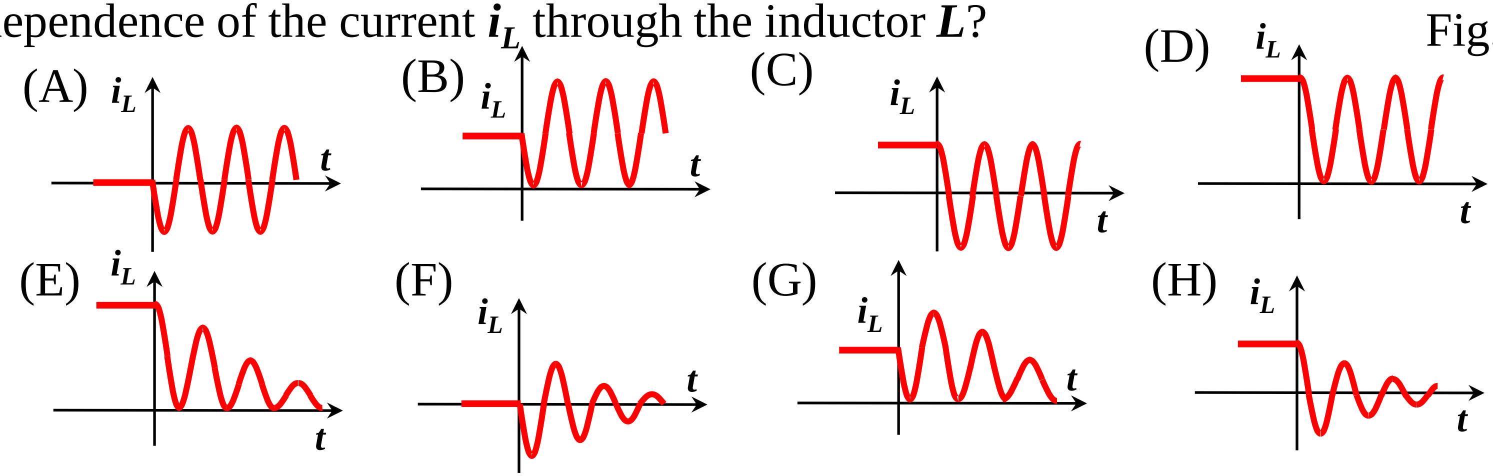


Fig. 8



6. (5 pts) In the double slit interference phenomenon, which of the following adjustment would increase the separation between the bright fringes (亮紋) on the screen?
(A) Increase the wavelength of the light used. (B) Increase the separation between the slits.
(C) Immerse (沉浸) the apparatus in water. (D) more than one of these. (E) None of above.

Multiple Choice Questions:

1	**2**	**3**	**4**	**5**	**6**				
A	**C**	**D**	**E**	**C**	**A**				
7	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	**E**	**D**	**H**	**A**	**B**	**A**	**D**	**H**	**A**

(1) $\vec{E}(\vec{r},t) = E_0\hat{y}\sin\left[4x - 3z + \omega t\right] \rightarrow \left(120 \text{ V/m}\right)\hat{y}\sin\left[k(\hat{k}\cdot\vec{r}) - \omega t\right]$

$$\vec{k} = 5(\frac{-4}{5}, 0, \frac{+3}{5}), \quad k = \left|\vec{k}\right| = 5.0\text{m}^{-1}$$

a) 2 pts $k = 5\text{m}^{-1}, \ \lambda = \frac{2\pi}{k} = \frac{2\pi}{5} = 1.26 \text{ m} \ , \omega = kc = 15\times 10^8 s^{-1},$

b) 2 pts Propagating along the direction of wave vector $\hat{k} = (\frac{-4}{5}, 0, \frac{3}{5})$

c) 4 pts

$$\vec{B} = B_0\hat{B}_0\sin\left(-4x + 3z - \omega t\right), \ \text{1 pts} \quad B_0 = E_0 / c = 4.0\times 10^{-7}\text{T} \ \text{1 pts}$$

$$\hat{E}_0 \times \hat{B}_0 = \hat{y}\times\hat{B}_0 = \hat{k} = (\frac{-4}{5}, 0, \frac{3}{5}) \rightarrow \hat{B}_0 = \frac{-3\hat{x} - 4\hat{z}}{5} \ \text{2 pts}$$

$$B_{0x} = -2.4\times 10^{-7} \text{ T}, \quad B_{0z} = -3.2\times 10^{-7} \text{ T},$$

a) 2 pts

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{E_0B_0}{\mu_0}\left(\hat{k}\right)\sin^2\left(-4x + 3z - \omega t\right) = \frac{120}{\pi}\frac{-4\hat{x} + 3\hat{z}}{5}\sin^2\left(-4x + 3z - \omega t\right), \text{ W/m}^2 \ \text{1 pts}$$

$$\left\langle I\right\rangle = \left\langle S\right\rangle = \frac{E_0B_0}{2\mu_0} = \frac{60}{\pi} \text{ W/m}^2 \text{ or } 19.1 \text{ W/m}^2 \ \text{1 pts}$$

(2) (a) 3 pts

$$\vec{B}(\vec{r},t)=\begin{cases}\hat{z}B_0\left(1-r/r_0\right)\left(1-t^2/T^2\right) & r<r_0, 0<t<T\\ 0 & r\geq r_0, T<t \text{ and } t<0\end{cases}$$

$$\Phi_B=\int\vec{B}\cdot d\vec{A}=\int_0^a\left(\hat{z}B_0\left(1-\frac{r}{r_0}\right)\left[1-\left(\frac{t}{T}\right)^2\right]\right)\cdot\hat{z}2\pi r dr$$ 2 pts

$$=2\pi B_0\left(\frac{a^2}{2}-\frac{a^3}{3r_0}\right)\left[1-\left(\frac{t}{T}\right)^2\right]$$ 1 pts

(b) 4 pts

$$\varepsilon_{mf}=IR=-\frac{d}{dt}\int\vec{B}\cdot d\vec{A}=-\frac{d}{dt}2\pi B_0\left(\frac{a^2}{2}-\frac{a^3}{3r_0}\right)\left[1-\left(\frac{t}{T}\right)^2\right]$$ 2 pts

$$\rightarrow IR=-\left(\frac{-2t}{T^2}\right)2\pi B_0\left(\frac{a^2}{2}-\frac{a^3}{3r_0}\right) \text{ or } I=\frac{2\pi B_0 a^2 t}{T^2 R}\left(1-\frac{2a}{3r_0}\right)$$ 1 pts

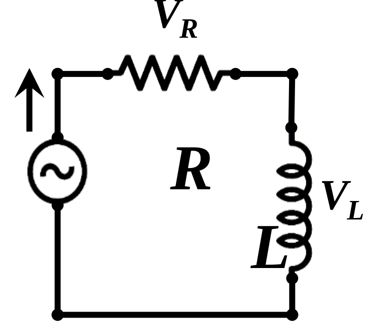
The current is c.c.w because the magnetic flux is decreasing as time (along +z direction) 1 pts

(c) 3 pts

$$I=\frac{dQ}{dt}\rightarrow Q=\int_0^T I(t)dt=\frac{2\pi B_0 a^2 T^2/2}{T^2 R}\left(1-\frac{2a}{3r_0}\right)=\frac{\pi B_0 a^2}{R}\left(1-\frac{2a}{3r_0}\right)$$

1 pts 1 pts 1 pts

(3a)

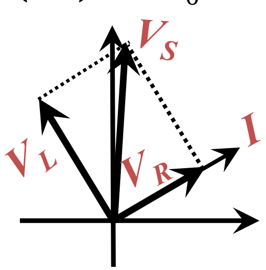


$$I = I_0 \cos(\omega t)$$

$$V_L = LI_0\omega \cos(\omega t + \frac{\pi}{2})$$ 1

$$V_R = RI_0 \cos(\omega t)$$ 1

$$V_S = V_R + V_L$$
$$= RI_0 \cos(\omega t) + LI_0\omega\cos(\omega t + \frac{\pi}{2})$$



1

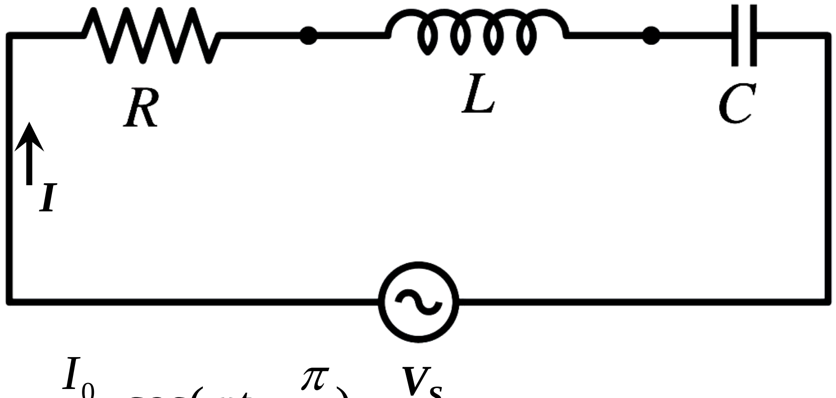
1

$$V_S = I_0\sqrt{R^2 + L^2\omega^2}\cos(\omega t + \tan^{-1}\frac{L\omega}{R})$$

$$\frac{V_L}{V_S} = \frac{I_0 L\omega}{I_0\sqrt{R^2+L^2\omega^2}} = \frac{L\omega}{\sqrt{R^2+L^2\omega^2}}$$ 1

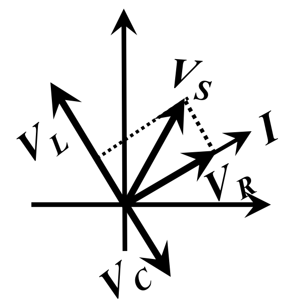
$$\omega \to 0,\quad \frac{V_L}{V_S} \to 0,\qquad \omega \to \infty,\quad \frac{V_L}{V_S} \to 1$$ 1 1

(3b)



$$V_C = \frac{I_0}{C\omega}\cos(\omega t - \frac{\pi}{2})$$ 1

$$V_S = V_R + V_L + V_C = RI_0\cos(\omega t) + LI_0\omega\cos(\omega t + \frac{\pi}{2}) + \frac{I_0}{C\omega}\cos(\omega t - \frac{\pi}{2})$$



2

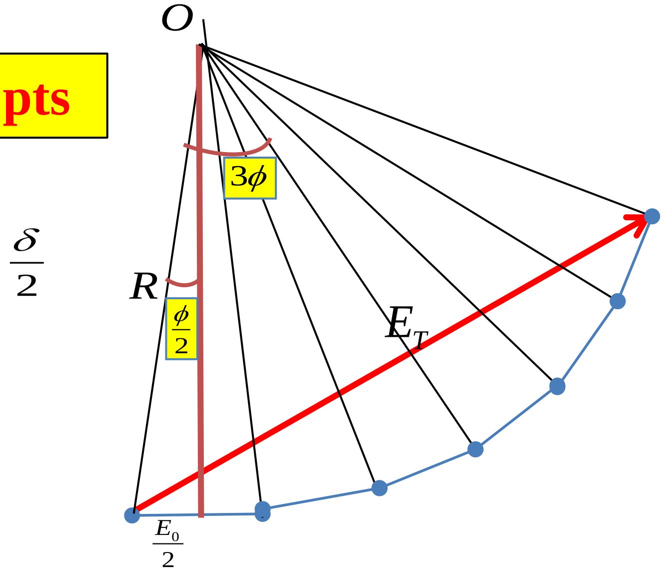
$$V_S = I_0\sqrt{R^2 + (L\omega - \frac{1}{C\omega})^2}\cos(\omega t + \tan^{-1}\frac{L\omega - \frac{1}{C\omega}}{R})$$ 1

$$\frac{V_L}{V_S} = \frac{I_0 L\omega}{I_0\sqrt{R^2 + (L\omega - \frac{1}{C\omega})^2}} = \frac{L\omega}{\sqrt{R^2 + (L\omega - \frac{1}{C\omega})^2}}$$ 2

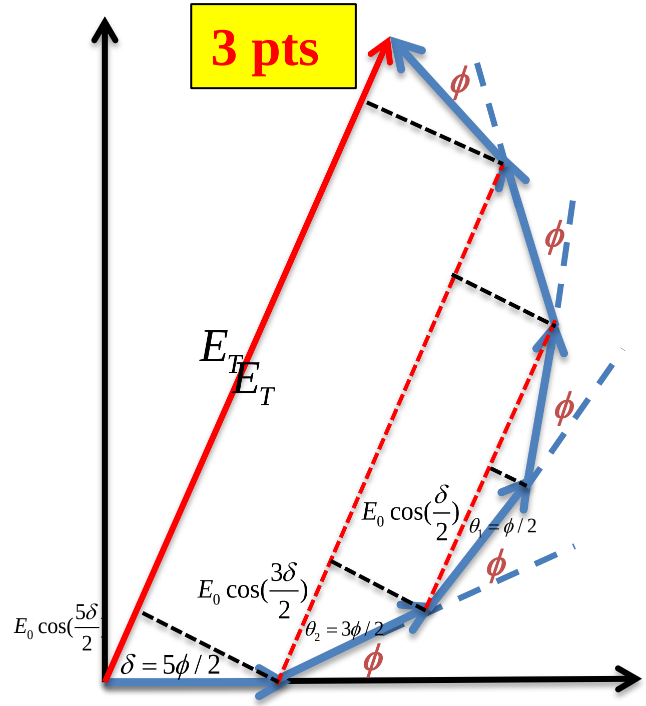
$$\omega \to 0,\quad \frac{V_L}{V_S} \to 0,\qquad \omega \to \infty,\quad \frac{V_L}{V_S} \to 1$$ 1 1

4 (b) Phasor : ΔOAB and ΔOAC

3 pts



3 pts



3 pts

$$\frac{E_0}{2R} = \sin\frac{\phi}{2}; \qquad \frac{E_{\theta 0}}{2R} = \sin\frac{6\phi}{2}$$

$$\Rightarrow \frac{E_{\theta 0}}{E_0} = \frac{\sin 6\phi/2}{\sin \phi/2}$$

3 pts

$$E_T = E_1 + E_2 + E_3 + E_4 + E_5 + E_6$$
$$= 2E_0\left[\cos\left(\frac{5\phi}{2}\right) + \cos\left(\frac{3\phi}{2}\right) + \cos\left(\frac{\phi}{2}\right)\right]\sin\left(kr - \omega t + \frac{5\phi}{2}\right)$$

(a) The phase difference is:

$$\begin{cases} \phi = k\Delta x = \dfrac{2\pi}{\lambda} d\sin\theta \approx \dfrac{2\pi}{\lambda} d\tan\theta \\ \tan\theta = \dfrac{y}{L} \end{cases}$$

2 pts

$$\Rightarrow \boxed{\phi = \frac{2\pi d}{\lambda L} y}$$

1 pt

(c)

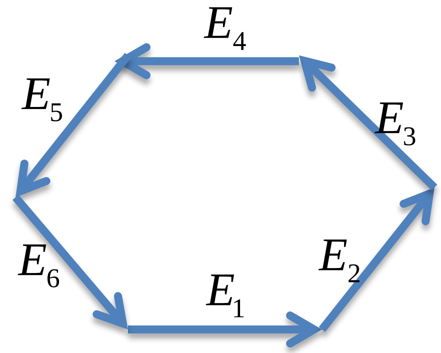
The first diffraction minimum occurs at

$$\phi = \frac{2\pi}{6} = \frac{\pi}{3} \Rightarrow \delta = \frac{5}{2}\phi = \frac{5\pi}{6}$$

1 pt

The phasor diagram is

1 pt

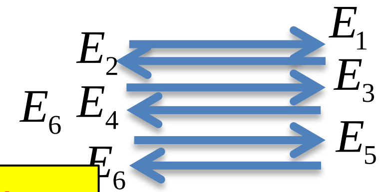


The third diffraction minimum occurs at

$$\phi = \frac{2\pi}{6} \bullet 3 = \pi \Rightarrow \delta = \frac{5}{2}\phi = \frac{5\pi}{2}$$

1 pt

The phasor diagram is



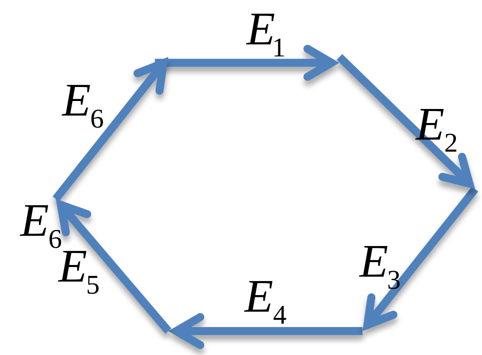
1 pt

The fifth diffraction minimum occurs at

$$\phi = \frac{2\pi}{6} \bullet 5 = \frac{10\pi}{3} \Rightarrow \delta = \frac{5}{2}\phi = \frac{25\pi}{2}$$

1 pt

The phasor diagram is



1 pt