

試卷請註明、姓名、班級、學號，請遵守考場秩序

I. 計算題(55 points) (所有題目必須有計算過程,否則不予計分)

1. (10 pts) Fig. 1 shows a cross sectional view of uniform charge distribution in an infinitely long cylindrical shell. The charge density is ρ ($\rho > 0$), the inner radius is $2R$, and the outer radius $3R$, the axis of the shell coincides with the z -axis. A second uniform cylindrical charge distribution is added to the system, with the axis of symmetry parallel to the z -axis but passing $(R, 0, 0)$. The radius of the cylinder is R , and the charge density is $-\rho$, Determine the magnitude and direction of the E -field along the x -axis ($0 \leq x < 3R$).
2. (a) (5 pts) As shown in Fig.2(a), two conducting coaxial cylinders with inner and outer radius a , $6a$ and length ℓ . Calculate the capacitance of this device. Ignore the end effects.
- (b)(5 pts) Now the region between the radii $3a$ and $4a$ is filled with a metal shell, as shown in Fig. 2(b). What is the capacitance of this new device?

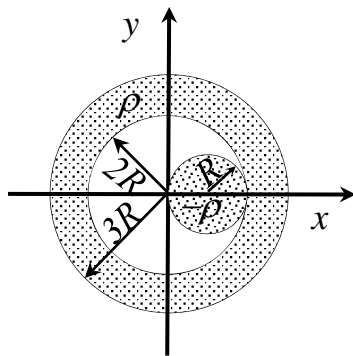


Fig. 1

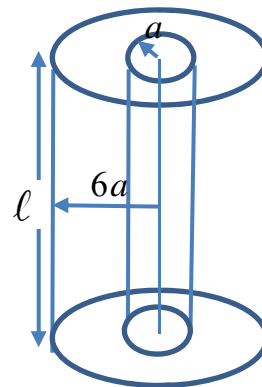


Fig. 2(a)

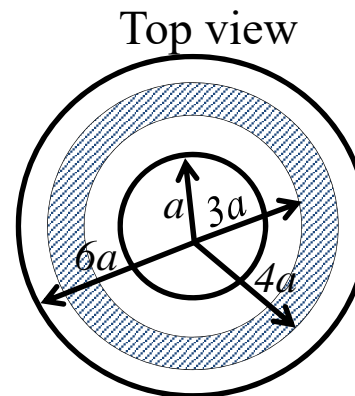


Fig. 2(b)

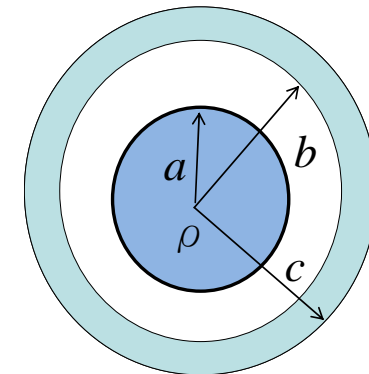


Fig. 3

3. (15 pts) As shown in Fig. 3, the volume charge density ρ within the sphere of radius a is distributed in spherically symmetric fashion with $\rho(r) = \rho_0[1 - r^2/a^2]$, and it is concentric with a spherical conducting shell of inner radius b and outer radius c . This conducting shell has no charge. Determine **the electric field \mathbf{E} (magnitude and direction), the electric potential V** , as a function of the radial distance r in the regions of (1) $r > c$; (2) $b < r < c$, (3) $a < r < b$, (4) $r < a$, and **the total charges** on the inner and outer surfaces of the conducting shell, respectively. Let electric potential $V=0$ at infinity.

4. (20 pts) Fig. 4 shows two line charge distributions in the x - y plane.

The charge density is $\lambda_1 = \lambda_0(1 - x/a)$ for the rod on x -axis ($-a < x < a$) and $\lambda_2 = \lambda_0 \sin \theta$ for the semicircle. Here a is the radius of the semi-circle, λ_0 is a positive constant and θ is the angle from $+x$ -axis.

- (a) (11 pts) Evaluate the electric field (x -, y -, and z -components) and the potential at point P on the z -axis due to the AB line segment .
- (b) (9 pts) Evaluate the electric field (x -, y -, and z -components) and the potential at point P on the z -axis due to the semicircle in Fig. 4.

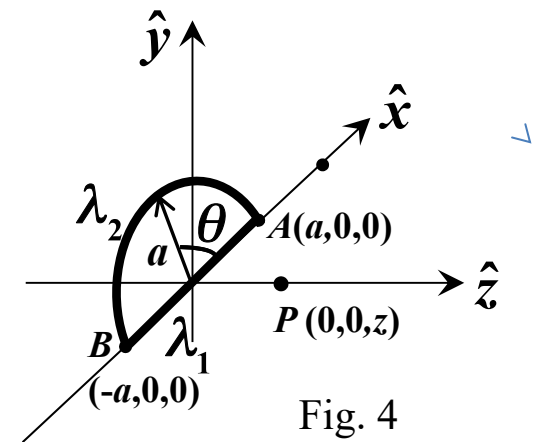


Fig. 4

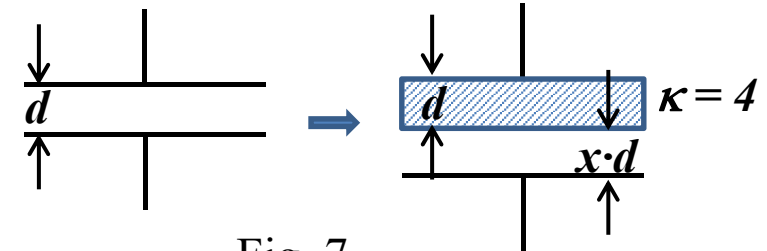
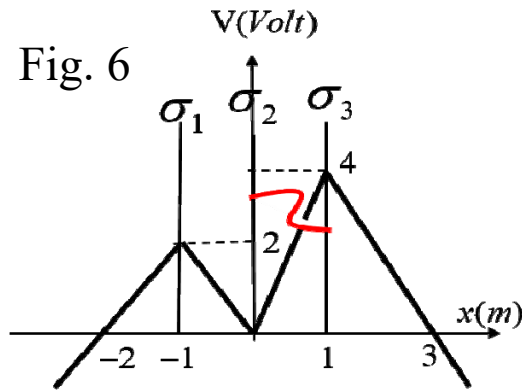
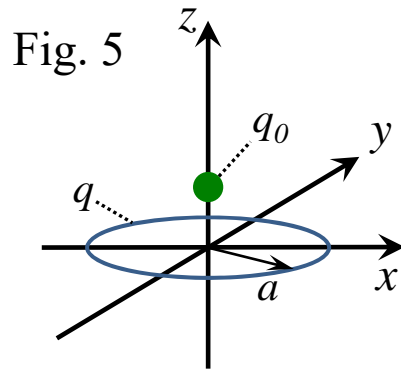
Integration Formula for reference

$$\int \frac{dx}{\sqrt{x^2 \pm b^2}} = \ln \left(x + \sqrt{x^2 \pm b^2} \right)$$

$$\int \frac{x^2 dx}{\sqrt{x^2 \pm b^2}} = \frac{x\sqrt{x^2 \pm b^2}}{2} - \frac{b^2}{2} \ln \left(x + \sqrt{x^2 \pm b^2} \right)$$

$$\int \frac{dx}{(x^2 \pm b^2)^{3/2}} = \frac{\pm x}{b^2 \sqrt{x^2 \pm b^2}}$$

$$\int \frac{x^2 dx}{(x^2 \pm b^2)^{3/2}} = \frac{-x}{\sqrt{x^2 \pm b^2}} + \ln \left(x + \sqrt{x^2 \pm b^2} \right)$$



II. 選擇題(45 points)

- (5 pts) Fig. 5 shows a uniformly charged ring in the x - y plane, centered at the origin, with radius $a=0.1\text{m}$ and total charge $q=10^{-7}\text{C}$. Imagine a small ball of mass 0.001kg and negative charge $q_0 = -10^{-8}\text{C}$. The ball is released from rest at the point $d=10^{-3}\text{m}$ and constrained to move along the z axis only, with no damping. The ball oscillates along the z axis between $z=d$ and $z=-d$ in a simple harmonic motion. What is the period T of this oscillation (in SI unit and ignore the gravitation)?
 (A) $T < 0.1$ (B) $0.1 \leq T < 0.3$ (C) $0.3 \leq T < 0.5$ (D) $0.5 \leq T < 0.8$ (E) $0.8 \leq T < 1.0$
 (F) $1.0 \leq T < 3.0$ (G) $3 \leq T < 5$ (H) $5 \leq T < 7$ (J) $7 \leq T < 9$ (K) $9 \leq T < 10$
 (L) $10 \leq T < 30$ (M) $30 \leq T < 50$ (N) $50 \leq T < 70$ (O) $70 \leq T < 90$ (P) $90 \leq T < 100$
- (5 pts) Three uniformly charged planes are located at $x = -1\text{ m}$, $x = 0$, and $x = 1\text{ m}$ with surface charge density σ_1 , σ_2 and σ_3 , respectively. The potential as a function of the x -coordinate is shown in Fig. 6. Now the surfaces with σ_2 and σ_3 are connected with a conducting wire (the curved line in Fig. 6) and wait till the charge is redistributed. The new charge densities σ'_2 and σ'_3 on the corresponding planes are
 (A) $(3\epsilon_0, -3\epsilon_0)$ (B) $(-3\epsilon_0, 3\epsilon_0)$ (C) $(2\epsilon_0, -2\epsilon_0)$ (D) $(-2\epsilon_0, 2\epsilon_0)$ (E) $(\epsilon_0, -\epsilon_0)$
 (F) $(-\epsilon_0, \epsilon_0)$ (G) $(0, 0)$ (H) $(3\epsilon_0, 3\epsilon_0)$

3. (5 pts) A particle with mass 0.14 g and a charge of $5.0 \times 10^{-6} \text{ C}$ is placed in a region of space where the potential is given by $V(x) = (2 \text{ V/m}^2) x^2 - (3 \text{ V/m}^3) x^3$. If the particle starts at $x = 2 \text{ m}$, the initial acceleration in x direction in unit m/s^2 of will be
 (A) -3 (B) -2 (C) -1 (D) 0 (E) 1 (F) 2 (G) 3
4. (5 pts) As shown in Fig. 7 ,A capacitor C_0 is consisted of two parallel plates with separation distance d . Now the space between two plates is filled with dielectric material with dielectric constant $\kappa = 4$. Then the separation distance increases by a distance of $x \cdot d$. The new capacitance of this new capacitor is still C_0 . What is the value of x ?
 (A) $1/4$ (B) $1/3$ (C) $1/2$ (D) $2/3$ (E) $3/4$ (F) 1 (G) $4/3$ (H) $5/4$
5. (5 pts) Fig. 8 shows a cylindrical Gaussian surface S , whose axis of symmetry coincides with the x -axis. The left end of S is fixed at $x = 0$, and the other end is adjustable. If there is a continuous charge distribution with charge density $\rho(\vec{r}) = Ax^2$, where A is a positive constant. Let Φ_E be the electric flux through S , Which of the following shows the correct relation between Φ_E and L ?

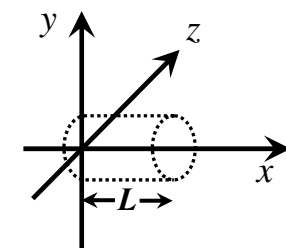
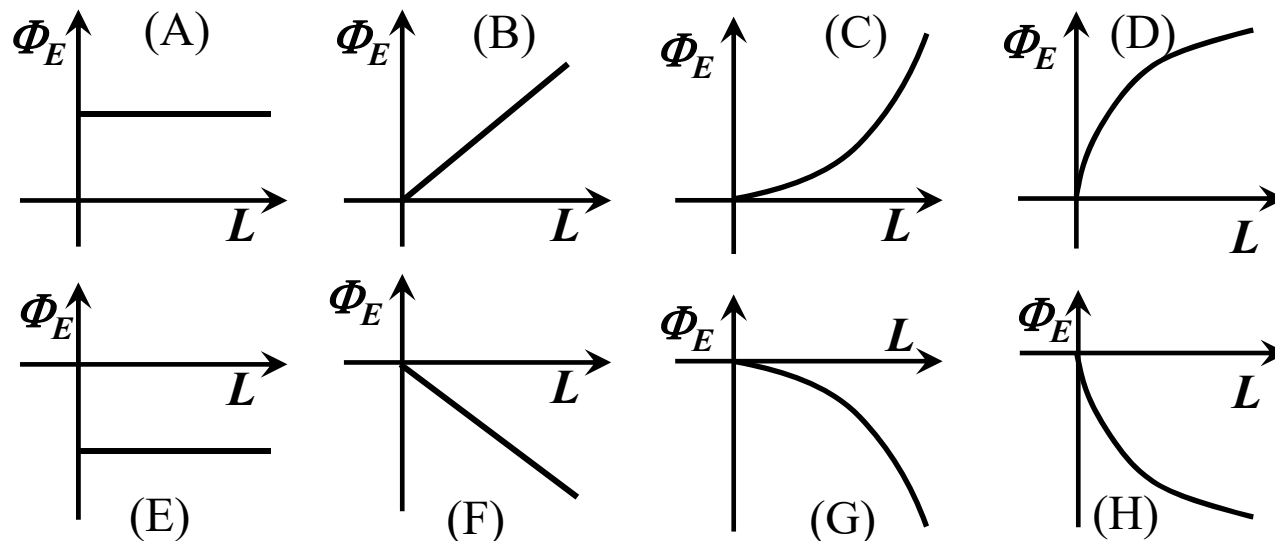
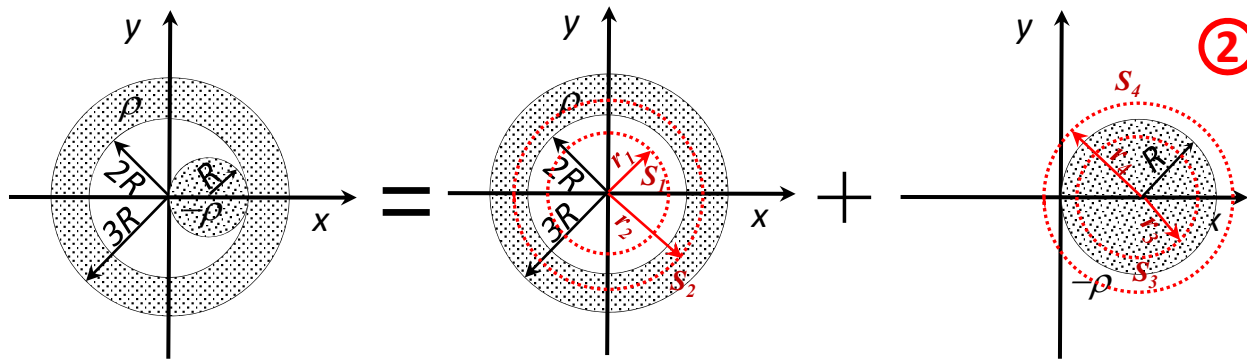


Fig. 8

1	2	3	4	5	6	7	8	9	10
F	D	E	E	C	L	C	C	B	D
11	12	13	14	15					
H	F	F	B	A					

1. (10 pts) Fig. 1 shows a cross sectional view of uniform charge distribution in an infinitely long cylindrical shell. The charge density is ρ ($\rho > 0$), the inner radius is $2R$, and the outer radius $3R$, the axis of the shell coincides with the z -axis. A second uniform cylindrical charge distribution is added to the system, with the axis of symmetry parallel to the z -axis but passing $(R,0,0)$. The radius of the cylinder is R , and the charge density is $-\rho$, Determine the magnitude and direction of the E -field along the x -axis ($0 \leq x < 3R$).



For outer shell, and $x < 2R$,
Choose a cylindrical surface S_1 with length ℓ_1 ,
and apply Gauss's law, i.e.

$$\Phi_E = \oiint_{S_1} \vec{E} \cdot d\vec{A} = E(r_1)2\pi r_1 \ell = 0 \quad (1)$$

$$\Rightarrow E(r_1) = 0 \Rightarrow E(x) = 0, \text{ for } 0 \leq x < 2R$$

For $x \geq 2R$, Choose a cylindrical surface S_2 with length ℓ_2 .

$$\Phi_E = \oiint_{S_2} \vec{E} \cdot d\vec{A} = E(r_2)2\pi r_2 \ell = \frac{\rho\pi(r_2^2 - 4R^2)\ell}{\epsilon_0}$$

$$\Rightarrow E(r_2) = \frac{\rho(r_2^2 - 4R^2)}{2\pi r_2 \epsilon_0} \Rightarrow \vec{E}(x) = \frac{\rho(x^2 - 4R^2)}{2x\epsilon_0} \hat{x} \quad (1)$$

For inner cylinder, and $|x - R| < R$,

Choose a cylindrical surface S_3 with radius r_3 ,
length ℓ_3 , and apply Gauss's law, i.e.

$$\Phi_E = \oiint_{S_3} \vec{E} \cdot d\vec{A} = E(r_3)2\pi r_3 \ell = \frac{\rho\pi(r_3^2)\ell}{\epsilon_0}$$

$$\Rightarrow E(r_3) = \frac{\rho r_3}{2\epsilon_0} \Rightarrow E(x) = \frac{\rho|x - R|}{2\epsilon_0} \quad (1)$$

For $0 \leq x < R$, $|x - R| = -(x - R)$

$$\Rightarrow \vec{E}(x) = \frac{-\rho(x - R)}{2\epsilon_0} \hat{x} \quad (1)$$

For $R \leq x < 2R$, $|x - R| = (x - R)$

$$\Rightarrow \vec{E}(x) = \frac{-\rho(x - R)}{2\epsilon_0} \hat{x} \quad (1)$$

For inner cylinder, and $|x-R| \geq R$,

Choose a cylindrical surface S_4 with radius r_4 length ℓ_4 , and apply Gauss's law, i.e.

$$\Phi_E = \oiint_{S_4} \vec{E} \cdot d\vec{A} = E(r_4)2\pi r_4 \ell = \frac{\rho\pi(R^2)\ell}{\epsilon_0}$$

$$\Rightarrow E(r_4) = \frac{\rho R^2}{2\epsilon_0 r_4} \quad \textcircled{1} \Rightarrow E(x) = \frac{\rho R^2}{2\epsilon_0 |x-R|}$$

For $2R \leq x < 3R$, $|x-R| = (x-R)$

$$\Rightarrow E(x) = \frac{-\rho R^2}{2\epsilon_0 (x-R)} \hat{x} \quad \textcircled{1}$$

For the total E-field with $0 \leq x < R$,

$$\vec{E}(x) = \frac{-\rho(x-R)}{2\epsilon_0} \hat{x}$$

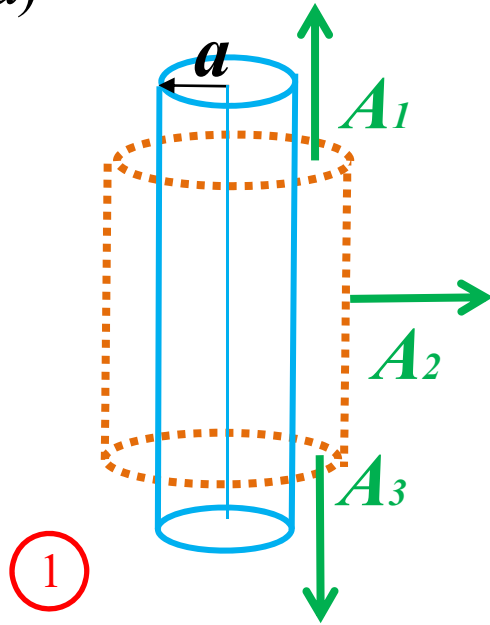
For the total E-field with $R \leq x < 2R$,

$$\vec{E}(x) = \frac{-\rho(x-R)}{2\epsilon_0} \hat{x}$$

For the total E-field with $2R \leq x < 3R$,

$$E(x) = \frac{-\rho R^2}{2\epsilon_0 (x-R)} \hat{x} + \frac{\rho(x^2 - 4R^2)}{2x\epsilon_0} \hat{x}$$

(a) Problem 2



$$\Phi_E = \oiint \vec{E} \cdot d\vec{A} = \int_{A_1} \vec{E} \cdot d\vec{A} + \int_{A_2} \vec{E} \cdot d\vec{A} + \int_{A_3} \vec{E} \cdot d\vec{A}$$

$$= 0 + EA_2 + 0 = E(2\pi rL)$$

$$= \frac{Q_{in}}{\epsilon_0} = \frac{Q(\frac{L}{l})}{\epsilon_0}$$

$$\vec{E} = \frac{Q}{\epsilon_0(2\pi rl)} \hat{r} \quad (1)$$

$$\begin{aligned} \Delta V &= - \int_a^{6a} \vec{E} \cdot d\vec{l} = - \int_a^{6a} \frac{Q}{\epsilon_0(2\pi rl)} \hat{r} \cdot d\hat{r} \\ &= - \frac{Q}{\epsilon_0(2\pi l)} \int_a^{6a} \frac{1}{r} dr = - \frac{Q}{\epsilon_0(2\pi l)} \ln(6) \quad (1) \end{aligned}$$

$$C = \frac{Q}{|\Delta V|} = \frac{2\pi\epsilon_0 l}{\ln(6)} \quad (1)$$

(b) Method I

(I) in $a < r < 3a$

$$\vec{E} = \frac{Q}{\epsilon_0(2\pi rl)} \hat{r}$$

$$V(r) - V(a) = - \int_a^r \vec{E} \cdot d\vec{l} = - \frac{Q}{\epsilon_0(2\pi l)} \ln\left(\frac{r}{a}\right)$$

$$V(3a) - V(a) = - \frac{Q}{\epsilon_0(2\pi l)} \ln(3) \quad (1)$$

(II) in $3a < r < 4a$

$$\vec{E} = 0$$

$$V(4a) - V(3a) = V(r) - V(3a) = 0 \quad (1)$$

(III) in $4a < r < 6a$

$$\vec{E} = \frac{Q}{\epsilon_0(2\pi rl)} \hat{r}$$

$$V(r) - V(4a) = - \int_{4a}^r \vec{E} \cdot d\vec{l} = - \frac{Q}{\epsilon_0(2\pi l)} \ln\left(\frac{r}{4a}\right)$$

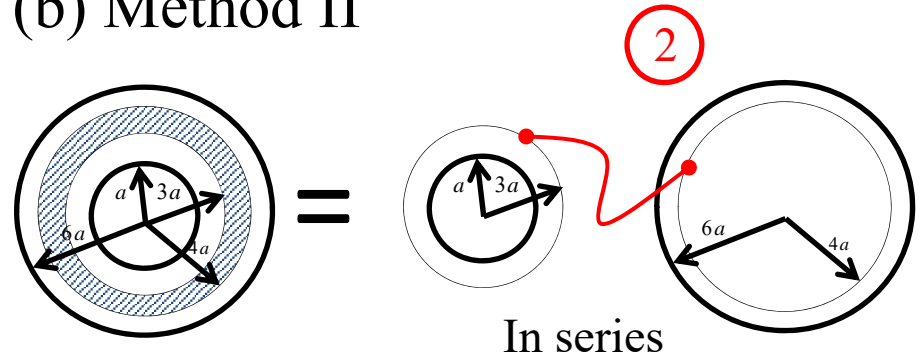
$$V(6a) - V(4a) = - \frac{Q}{\epsilon_0(2\pi l)} \ln\left(\frac{3}{2}\right) \quad (1)$$



$$\Delta V = V(6a) - V(a) = - \frac{Q}{\epsilon_0(2\pi l)} \ln\left(\frac{3}{2}\right) - 0 - \frac{Q}{\epsilon_0(2\pi l)} \ln(3) \quad (1)$$

$$C = \frac{Q}{|\Delta V|} = \frac{2\pi\epsilon_0 l}{2\ln(3) - \ln 2} \quad (1)$$

(b) Method II



$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} \quad (1)$$

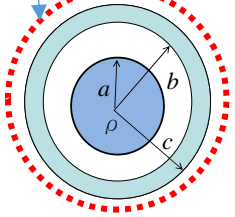
$$C_1 = \frac{2\pi\epsilon_0 l}{\ln(3)} \quad \text{and} \quad C_2 = \frac{2\pi\epsilon_0 l}{\ln\left(\frac{3}{2}\right)} \quad (1)$$

$$\Rightarrow C = \frac{Q}{|\Delta V|} = \frac{2\pi\epsilon_0 l}{2\ln(3) - \ln 2} \quad (1)$$

Problem 3

Guass's surface

(1) $r > c$



$$Q = \int_0^a \rho_0 \left[1 - \frac{r^2}{a^2}\right] 4\pi r^2 dr = \frac{8}{15} \pi \rho_0 a^3$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$E 4\pi r^2 = \frac{Q}{\epsilon_0} = \frac{1}{\epsilon_0} \frac{8}{15} \pi \rho_0 a^3$$

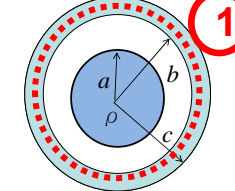
$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{8\pi\rho_0 a^3}{15r^2} \hat{r} = \frac{1}{\epsilon_0} \frac{2\rho_0 a^3}{15r^2} \hat{r}$$

$$V(r) - V(\infty) = - \int_{\infty}^r \vec{E} \cdot d\vec{r}$$

$$V(r) = - \int_{\infty}^r \frac{1}{4\pi\epsilon_0} \frac{8\pi\rho_0 a^3}{15r^2} dr = \frac{1}{4\pi\epsilon_0} \frac{8\pi\rho_0 a^3}{15r}$$

$$V(c) = \frac{1}{4\pi\epsilon_0} \frac{8\pi\rho_0 a^3}{15c}$$

(2) $b < r < c$



$$\vec{E} = 0 \text{ within metal}$$

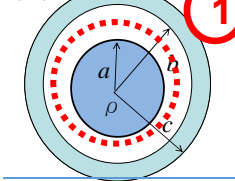
$$Q_b = -Q = -\frac{8}{15} \pi \rho_0 a^3$$

$$Q_c = Q = \frac{8}{15} \pi \rho_0 a^3$$

$$V(r) - V(c) = 0$$

$$V(r) = V(c) = \frac{1}{4\pi\epsilon_0} \frac{8\pi\rho_0 a^3}{15c}$$

(3) $a < r < b$

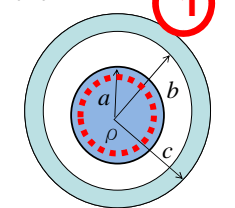


$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{8\pi\rho_0 a^3}{15r^2} \hat{r} = \frac{1}{\epsilon_0} \frac{2\rho_0 a^3}{15r^2} \hat{r}$$

$$V(r) - V(\infty) = - \int_{\infty}^r \vec{E} \cdot d\vec{r} = - \left\{ \int_{\infty}^c \vec{E} \cdot d\vec{r} + \int_c^b \vec{E} \cdot d\vec{r} + \int_b^r \vec{E} \cdot d\vec{r} \right\}$$

$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{8\pi\rho_0 a^3}{15c} - 0 + \frac{1}{4\pi\epsilon_0} \frac{8\pi\rho_0 a^3}{15r} - \frac{1}{4\pi\epsilon_0} \frac{8\pi\rho_0 a^3}{15b} = \frac{1}{4\pi\epsilon_0} \frac{8\pi\rho_0 a^3}{15} \left(\frac{1}{r} - \frac{1}{b} + \frac{1}{c} \right)$$

(4) $r < a$



$$E 4\pi r^2 = \frac{1}{\epsilon_0} \int_0^r \rho_0 \left[1 - \frac{r^2}{a^2}\right] 4\pi r^2 dr$$

$$\vec{E} = \frac{\rho_0}{\epsilon_0} \left[\frac{r}{3} - \frac{r^3}{5a^2} \right] \hat{r}$$

$$V(r) - V(\infty) = - \int_{\infty}^r \vec{E} \cdot d\vec{r} = - \left\{ \int_{\infty}^c \vec{E} \cdot d\vec{r} + \int_c^b \vec{E} \cdot d\vec{r} + \int_b^a \vec{E} \cdot d\vec{r} + \int_a^r \vec{E} \cdot d\vec{r} \right\}$$

$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{8\pi\rho_0 a^3}{15} \left(\frac{1}{a} - \frac{1}{b} + \frac{1}{c} \right) - \int_a^r \frac{\rho_0}{\epsilon_0} \left[\frac{r}{3} - \frac{r^3}{5a^2} \right] dr$$

$$= \frac{1}{4\pi\epsilon_0} \frac{8\pi\rho_0 a^3}{15} \left(\frac{1}{a} - \frac{1}{b} + \frac{1}{c} \right) + \frac{\rho_0}{\epsilon_0} \left[-\frac{r^2}{6} + \frac{r^4}{20a^2} + \frac{7a^2}{60} \right] = \frac{\rho_0}{\epsilon_0} \left[\frac{r^4}{20a^2} - \frac{r^2}{6} + \frac{a^2}{4} - \frac{2a^3}{15b} + \frac{2a^3}{15c} \right]$$

Problem 4

$$d\vec{E} = \frac{k dq}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}') \quad dV = \frac{k dq}{|\vec{r} - \vec{r}'|}$$

11 pts

1. For the E-field results from the charge on AB line segment,

$$\vec{r} = (0, 0, z), \quad \vec{r}' = (x', 0, 0) \quad dq = \lambda_1 dx' = \lambda_0 \left(1 - \frac{x'}{a}\right) dx' \quad 1 \text{ pts}$$

$$|\vec{r} - \vec{r}'| = |(-x', 0, z)| = \sqrt{x'^2 + z^2} \quad 1 \text{ pts}$$

0, odd function

$$V_P^{(1)} = \int_{-a}^a \frac{k dq}{|\vec{r} - \vec{r}'|} = k \int_{-a}^a \frac{\lambda_0 \left(1 - \frac{x'}{a}\right) dx'}{(x'^2 + z^2)^{1/2}} = k \int_{-a}^a \frac{\lambda_0 dx'}{(x'^2 + z^2)^{1/2}} - \frac{k}{a} \int_{-a}^a \frac{\lambda_0 x' dx'}{(x'^2 + z^2)^{1/2}} \quad 1 \text{ pts}$$

$$= 2k\lambda_0 \int_0^a \frac{dx'}{(x'^2 + z^2)^{1/2}} = 2k\lambda_0 \ln \frac{a + \sqrt{z^2 + a^2}}{z}$$

1 pts

1 pts

$$\int \frac{dx}{\sqrt{x^2 \pm b^2}} = \ln(x + \sqrt{x^2 \pm b^2})$$

$$\vec{E}^{(1)} = \int d\vec{E} = \int_{-a}^a \frac{k dq}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}') = k \int_{-a}^a \frac{\lambda_0 \left(1 - \frac{x'}{a}\right) dx'}{(x'^2 + z^2)^{3/2}} (-x', 0, z) \quad \boxed{1 \text{ pts}}$$

0, odd function

$$E_x^{(1)} = k \int_{-a}^a \frac{-\lambda_0 x' dx'}{(x'^2 + z^2)^{3/2}} + \frac{k \lambda_0}{a} \int_{-a}^a \frac{x'^2 dx'}{(x'^2 + z^2)^{3/2}} = \frac{2k \lambda_0}{a} \int_0^a \frac{x'^2 dx'}{(x'^2 + z^2)^{3/2}} \quad \boxed{1 \text{ pts}}$$

$$= \frac{2k \lambda_0}{a} \left[\frac{-a}{\sqrt{a^2 + z^2}} + \ln \left(\frac{a + \sqrt{a^2 + z^2}}{z} \right) \right] \quad \int \frac{x^2 dx}{(x^2 \pm a^2)^{3/2}} = \frac{-x}{\sqrt{x^2 \pm a^2}} + \ln(x + \sqrt{x^2 \pm a^2}) \quad \boxed{1 \text{ pts}}$$

$$E_y^{(1)} = 0 \quad \boxed{1 \text{ pts}}$$

$$\int \frac{dx}{(x^2 \pm a^2)^{3/2}} = \frac{\pm x}{a^2 \sqrt{x^2 \pm a^2}}$$

$$E_z^{(1)} = k \int_{-a}^a \frac{\lambda_0 z dx'}{(x'^2 + z^2)^{3/2}} - \frac{k \lambda_0 z}{a} \int_{-a}^a \frac{x' dx'}{(x'^2 + z^2)^{3/2}} = 2k \lambda_0 z \int_0^a \frac{dx'}{(x'^2 + z^2)^{3/2}} \quad \boxed{1 \text{ pts}}$$

$$= 2k \lambda_0 \frac{a}{z \sqrt{a^2 + z^2}} \quad \boxed{1 \text{ pts}} \quad \text{0, odd function}$$

9 pts

2. For the E-field results from the charge on AB semi-circle,

$$\vec{r} = (0, 0, z), \quad \vec{r}' = (x', y', 0) = (a \cos \theta, a \sin \theta, 0) \quad dq = \lambda_2 d\ell = \lambda_0 \sin \theta (a d\theta) \quad 1 \text{ pts}$$

$$|\vec{r} - \vec{r}'| = |(-x', -y', z)| = \sqrt{a^2 + z^2} \quad 1 \text{ pts}$$

$$V_P^{(2)} = \int_0^\pi \frac{k dq}{|\vec{r} - \vec{r}'|} = k \int_0^\pi \frac{\lambda_0 a \sin \theta d\theta}{(a^2 + z^2)^{1/2}} = \frac{2k\lambda_0 a}{(a^2 + z^2)^{1/2}} \quad 2 \text{ pts}$$

$$\vec{E}^{(2)} = \int_0^\pi \frac{k dq}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}') = k \int_0^\pi \frac{\lambda_0 a \sin \theta d\theta}{(a^2 + z^2)^{3/2}} (-a \cos \theta, -a \sin \theta, z)$$

$$E_x^{(2)} = \frac{-k\lambda_0 a^2}{(a^2 + z^2)^{3/2}} \int_0^\pi \cos \theta \sin \theta d\theta = 0 \quad 1 \text{ pts} \quad E_z^{(2)} = \frac{k\lambda_0 a z}{(a^2 + z^2)^{3/2}} \int_0^\pi \sin \theta d\theta = \frac{2k\lambda_0 a z}{(a^2 + z^2)^{3/2}} \quad 1 \text{ pts}$$

$$E_y^{(2)} = \frac{-k\lambda_0 a^2}{(a^2 + z^2)^{3/2}} \int_0^\pi \sin^2 \theta d\theta = \frac{-\pi k\lambda_0 a^2}{2(a^2 + z^2)^{3/2}} \quad 1 \text{ pts}$$