General Physics (I) **期中考 II** **Nov. 29, 2019**

試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(47 points) (所有題目必須有計算過程,否則不予計分)

1&2. (15 pts) As shown in Fig. 1, a small disk of mass $\boldsymbol{m}$ is tied to a thin rod fixed to the z-axis with a massless string. The length of the string is $\boldsymbol{R}$ and the disk is initially placed on a $\boldsymbol{x}$-$\boldsymbol{y}$ plane surface at the position $(0,-\boldsymbol{R},0)$. A force $\boldsymbol{F}$ pointing toward $+x$-axis is pushing the disk to move along the circular path of radius $\boldsymbol{R}$, from $\boldsymbol{\theta} = \mathbf{0}$ to $\boldsymbol{\theta} = \boldsymbol{\pi}/\mathbf{2}$ in a quasi-static (近乎靜止) manner . The coefficient of the kinetic friction for the disk moving on the x-y plane is μ_k. Determine (a) (9 pts) the work done by the force $\boldsymbol{F}$ and (b)(6pts) the work done by the kinetic frictional force.

3. (17pts) (A) (6pts) As shown in Fig. 2, the sphere A, suspended by a light wire, swings from height $\boldsymbol{h}$ to the bottom, and then elastically collides with block B which is initially at rest. The mass of sphere A is $\mathbf{3}\boldsymbol{m}$ and the mass of block B is $\boldsymbol{m}$. Find the velocities of block A and block B after collision in terms of $\boldsymbol{m}$, $\boldsymbol{h}$, and $\boldsymbol{g}$ (gravitational acceleration).

(B) (4pts) The collided block B moves toward block C, which is free to move around. Find the minimum height $\boldsymbol{h}_{min}$ that block B can climb up to the top of block C. There is no friction between all surfaces. The mass of block C is $\mathbf{5}\boldsymbol{m}$.

(C) (7pts)The collided block B travels toward block C. Assume that $\boldsymbol{h}$ is so large that block B can climb up to the top of block C, then slide down on the other side of block C and continue to travel forward. Find the velocities of block B and block C in terms of $\boldsymbol{m}$, $\boldsymbol{h}$, and $\boldsymbol{g}$ after they separate from each other.

Fig. 1

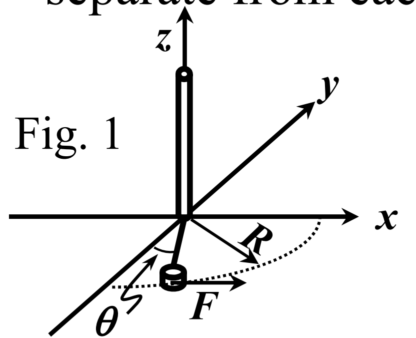


Fig. 2

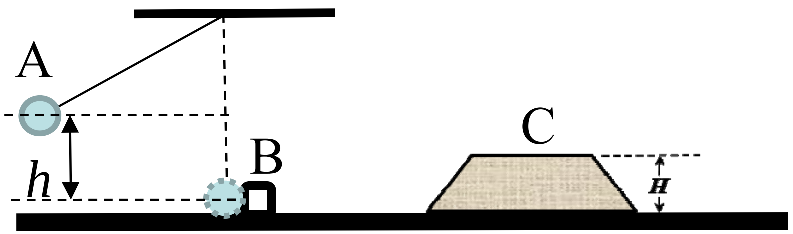
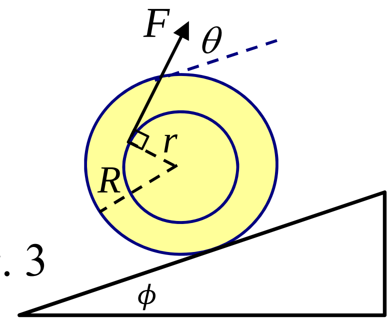


Fig. 3



4. (15 pts) A disk with mass m and radius R is pulling upward in a pure roll motion by a constant force F at the radius r with an angle θ relative to the inclined surface, as shown in Fig. 3. the surface of incline is tilted with an angle ϕ. Find the acceleration a and the frictional force f. ($I = MR^2/2$)

II.選擇題(57 points)

1. (5 pts) A solid disk (with mass M and radius R) and a ring (with mass $m = M/7$ and radius R), are cut half as arranged in Fig. 4. Let I_P be the moment of inertia of this object when it rotates around point P with the rotating axis out of the plane. What is the value a if we write $I_P = a\,MR^2$. ($I_{CM,\,disk} = ½\,MR^2$ and $I_{CM,\,ring} = mR^2$.)

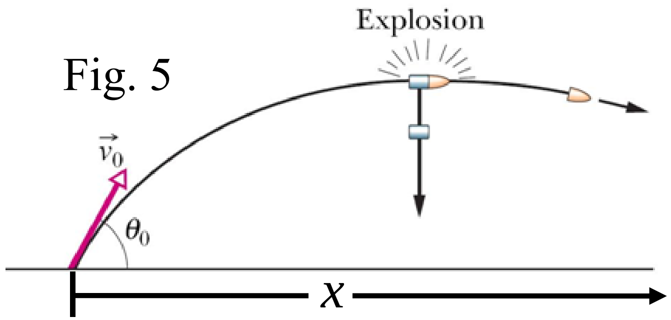
(A) $a \le 4$ (B) $4 < a \le 8$ (C) $8 < a \le 12$ (D) $12 < a \le 16$
(E) $16 < a \le 20$ (F) $20 < a \le 24$ (G) $24 < a \le 28$
(H) $28 < a \le 32$ (J) $32 < a \le 36$ (K) $36 < a \le 40$ (L) $40 < a$.

Fig. 4

2. (5 pts.) A shell is shot with an initial velocity of 14 m/s, at an angle of $\theta_0 = 45^o$ with respect to the horizon. At the top of the trajectory, the shell explodes into two fragments of equal mass, as shown in Fig. 5. One fragment, whose speed immediately after the explosion is zero, falls vertically. Assume that the terrain is level and that the air drag is negligible. The distance (x) between the position where the shell is shot and the location where the other fragment lands is

(A) $5 \le x < 10$ (B) $10 \le x < 15$ (C) $15 \le x < 20$ (D) $20 \le x < 25$ (E) $25 \le x < 30$ (F) $30 \le x < 35$
(G) $35 \le x < 40$ (H) $40 \le x < 45$ (J) $45 \le x < 50$ m.

Fig. 5



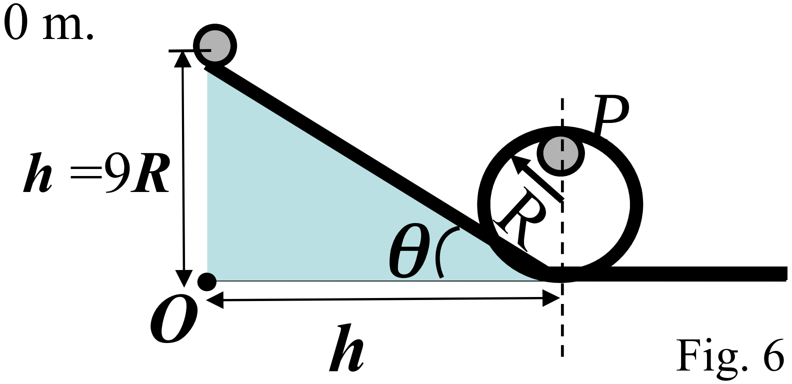


Fig. 6

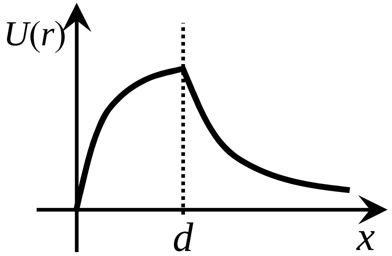
3. (5 pts) A solid sphere ($\boldsymbol{I = 2mr^2/5}$) with radius $\boldsymbol{r}$ and mass $\boldsymbol{m}$ roll downhill without slipping as shown in Fig. 6, at the end of the slide is a vertical circular track with radius $\boldsymbol{R}$ ($= 10\boldsymbol{r}$). If the sphere is at position $\boldsymbol{h=9R}$ initially, the velocity of the center of mass of the sphere at position $\boldsymbol{P}$ (top of the circlar track) is $\boldsymbol{v}$. Let $\boldsymbol{x = v^2/(gR)}$, what is the value of $\boldsymbol{x}$?

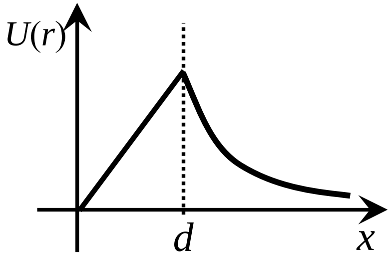
(A) $x \le 3$; (B) $3 < x \le 5$; (C) $5 < x \le 6$; (D) $6 < x \le 7$; (E) $7 < x \le 8$;
(F) $8 < x \le 9$; (G) $9 < x \le 10$; (H) $10 < x \le 12$; (J) $12 < x \le 15$; (K) $15 \le x$

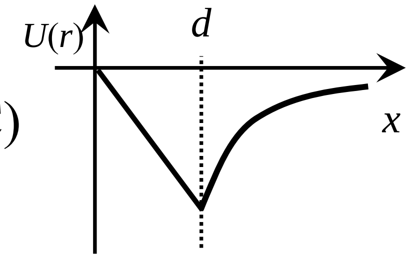
4. (5 pts) A conservative force $F(x)$ experienced by a particle with mass m and at distance x from the origin along the x-axis can be expressed with the following equation.

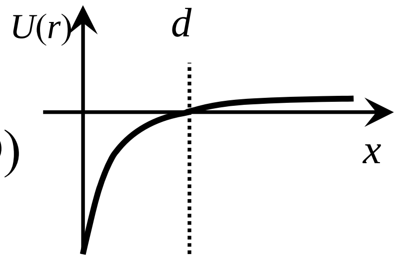
$$F(x) = \begin{cases} Ax, & 0 \le x < d \\ Ad, & d \le x \end{cases} \quad , A > 0$$

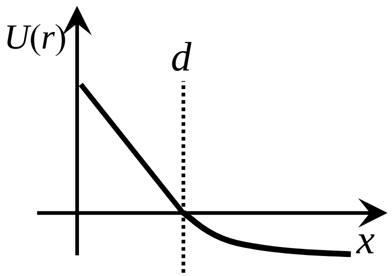
, where $\boldsymbol{A}$ and $\boldsymbol{d}$ are constants . Which of the following shows the correct potential $U(x)$ corresponding to $F(x)$?

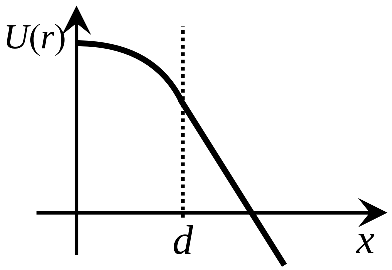
(A)


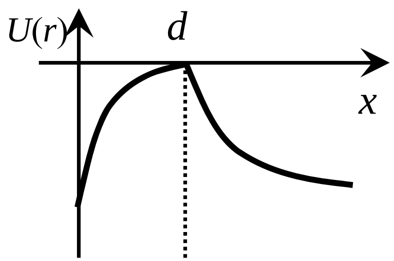
(B)


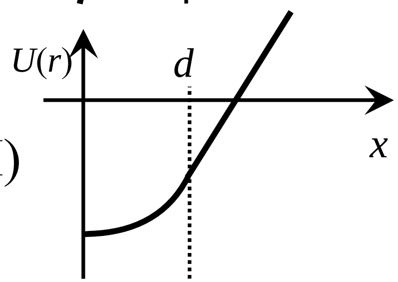
(C)


(D)


(E)


(F)


(G)


(H)


5. (5 pts) A bullet of mass $\boldsymbol{m}$ moving with velocity $\boldsymbol{v_0}$**=3 m/s** strikes and becomes embedded at the end of a rod with mass 3$\boldsymbol{m}$ and length $\boldsymbol{L}$**=1m** (Fig. 7). The rod, initially at rest hangs on the ceiling at the other end, starts to rotate about point $\boldsymbol{A}$ after the collision. Find the highest point $\boldsymbol{h}$ the bullet (embedded in the rod) can reach.($\boldsymbol{h}$ in meters, $\boldsymbol{g}$ **= 10 m/s²**)

(A) $h \le 0.1$ (B) $0.1 < h \le 0.2$ (C) $0.2 < h \le 0.3$ (D) $0.3 < h \le 0.4$
(E) $0.4 < h \le 0.5$ (F) $0.5 < h \le 0.6$ (G) $0.6 < h \le 0.7$ (H) $0.7 < h \le 0.8$
(J) $0.8 < h$.

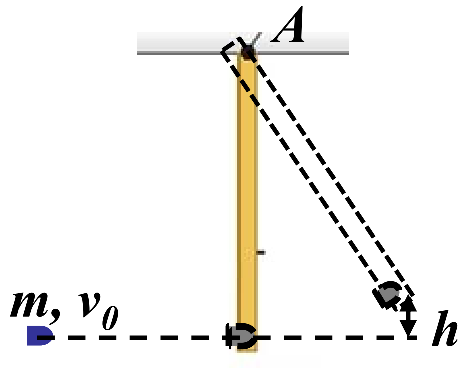


Fig. 7

6. (5 pts) As shown in Fig. 8, a spinning wheel of radius $\boldsymbol{R}$ and mass $\boldsymbol{m}$ with counter-clockwise angular speed ω_0 is released to the floor with center of mass velocity $\boldsymbol{v_0}$, and $\boldsymbol{v_0} = 3\boldsymbol{\omega_0} \boldsymbol{R}$. Let $\boldsymbol{v_{cm}} = x\, \boldsymbol{v_0}$ after the wheel begins to execute pure roll. What is x? ($\boldsymbol{I = mR^2}$)

(A) $x < -0.5$ (B) $-0.5 \le x < -0.25$ (C) $-0.25 \le x < 0$ (D) $0 \le x < 0.25$
(E) $0.25 \le x < 0.5$ (F) $0.5 \le x < 0.75$ (G) $0.75 \le x < 1.0$ (H) $1.0 \le x$

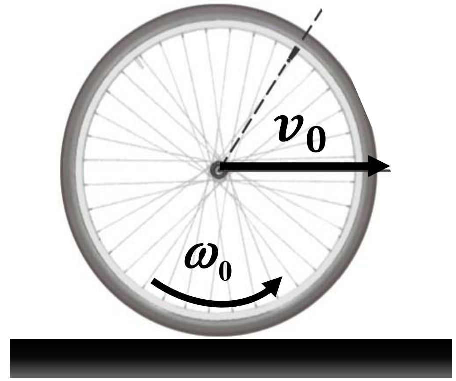
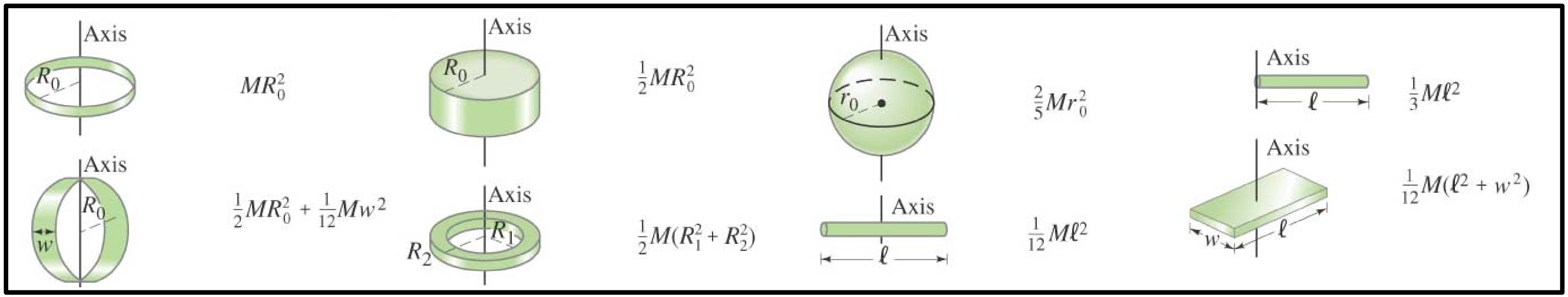


Fig. 8

Reference for moment of inertia



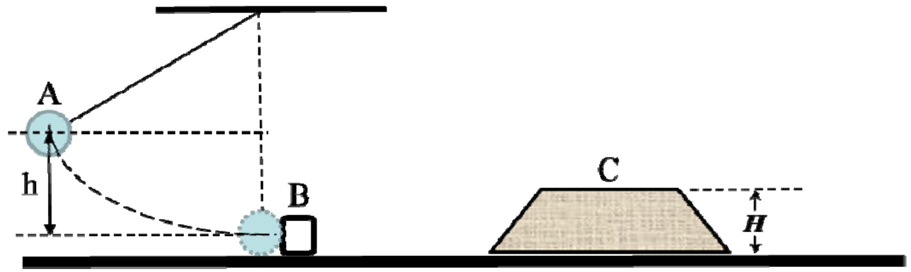
Multiple Choice Questions:

1	**2**	**3**	**4**	**5**	**6**				
F	**F**	**G**	**F**	**D**	**E**				
7	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	**E**	**C**	**D**	**D**	**F**	**D**	**C**	**B**	

1. (A) As shown in Fig., the block A, suspended by a light wire, swings from height **h** to bottom, and then elastically collides with block B which is initially at rest. The mass of block A is **3m** and the mass of block B is **m**. Find the velocities of block A and block B after collision in terms of m, h, and g.

(B) The collided block B moves toward block C, which is free to move around. Find the minimum height $\mathbf{h_{min}}$ that block B can climb up to the top of block C? There is no friction between all surfaces. The mass of block C is **5m**.

(C) The collided block B travels toward block C. Assume that **h** is so large that block B can climb up to the top of block C, then slide down on the other side of block C and continue to travel forward. Find the velocities of block B and block C after they separate from each other.



(A) Before collision, $3mgh = \frac{1}{2}3mv_A{}^2 \Rightarrow v_A = \sqrt{2gh}$ 2

When block A is colliding with block B, the conservation of momentum gives

$$3m\sqrt{2gh} + m\cdot 0 = 3mv_A{}' + mv_B{}' \quad (1)$$ 1

, and conservation of energy gives 1

$$\frac{1}{2}3m\cdot 2gh + \frac{1}{2}m\cdot 0 = \frac{1}{2}3mv_A{}'^2 + \frac{1}{2}mv_B{}'^2 \quad (2)$$

1 $$\Rightarrow v_A{}' = \frac{\sqrt{2gh}}{2} \quad and \quad v_B{}' = \frac{3\sqrt{2gh}}{2}$$ 1

(B) $$mv_B{}' + 5m\cdot 0 = (m+5m)v_{cm} \Rightarrow v_{cm} = \frac{1}{4}\sqrt{2gh}$$ 1 1

$$\frac{1}{2}mv_B{}'^2 = \frac{1}{2}(m+5m)v_{cm}{}^2 + mgH \Rightarrow h_{min} = \frac{8}{15}H$$ 1 1

$$\begin{cases} mv_B{}' = mv_{B,f} + 5mv_{C,f} \\ \frac{1}{2}mv_B{}'^2 = \frac{1}{2}mv_{B,f}^2 + \frac{1}{2}\cdot 5mv_{C,f}^2 \end{cases}$$ 1 1

1st solution is as follows

$$v_{C,f} = \frac{\sqrt{2gh}}{2}, \qquad v_{B,f} = -\sqrt{2gh}$$ 2

2nd solution is as follows

$$v_{C,f} = 0, v_{B,f} = v_B{}' = \frac{3\sqrt{2gh}}{2}$$ 2

1st solution is not consistent with the problem.

Accordingly,

$$v_{C,f} = 0, v_{B,f} = v_B{}' = \frac{3\sqrt{2gh}}{2}$$ 1

1&2. (15 pts) As shown in Fig. 1, a small disk of mass $\boldsymbol{m}$ is tied to a thin rod fixed to the $\boldsymbol{z}$-axis with a massless string. The length of the string is $\boldsymbol{R}$ and the disk is initially placed on a $\boldsymbol{x}$-$\boldsymbol{y}$ plane surface at the position (0,-$\boldsymbol{R}$,0). A force $\boldsymbol{F}$ pointing toward +$\boldsymbol{x}$-axis is pushing the disk to move along the circular path of radius $\boldsymbol{R}$, from $\boldsymbol{\theta} = \mathbf{0}$ to $\boldsymbol{\theta} = \boldsymbol{\pi}/\mathbf{2}$ in a quasi-static (近乎靜止) manner . The coefficient friction for the disk moving on the x-y plane is μ_k. Determine (a) (9 pts) the work done by the force F and (b)(6pts) the work done by the kinetic friction force.

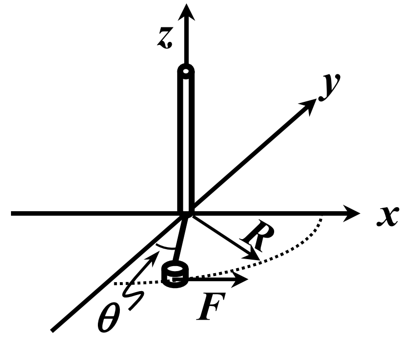
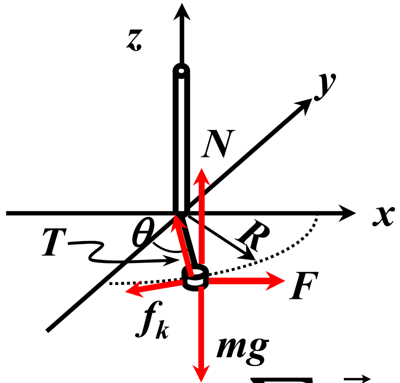
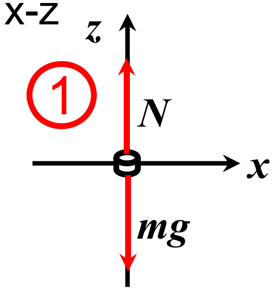


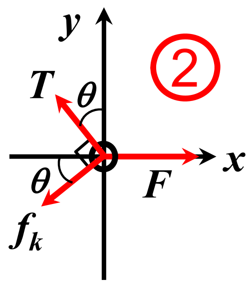
Fig. 1



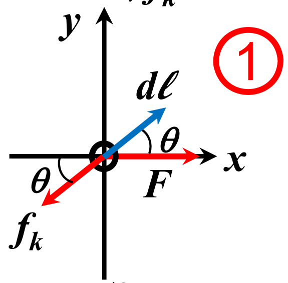
Free-body diagram in x-z



Free-body diagram in x-y



Mutual orientation betwneen $\boldsymbol{F}$, $\boldsymbol{f_k}$ and dl



(a) $\sum \vec{F} = m\vec{a}$

$\Rightarrow \vec{F} + \vec{T} + \vec{f}_k + m\vec{g} + \vec{N} = 0$

In z direction, $N - mg = 0$

$\Rightarrow N = mg$, and

$f_k = \mu_k N = \mu_k mg$ (1) 1

In x direction,

$F - T\sin\theta - f_k\cos\theta = 0$ (2) 1

In y direction,

$T\cos\theta - f_k\sin\theta = 0$ (3) 1

From (1),(3), we obatin

$T = \mu_k mg\tan\theta$ (4) 1

From (4),(2),(1) we obatin

$F = \mu_k mg(\tan\theta\sin\theta + \cos\theta)$

$= \mu_k mg/\cos\theta$ 2

The work done by $\boldsymbol{F}$ is $\boldsymbol{W_F}$

$$W_F = \int_0^{\pi/2} \vec{F}\cdot d\vec{\ell}$$

$$= \int_0^{\pi/2} F\cos\theta\cdot d\ell$$

$$= \int_0^{\pi/2} (\mu_k mg/\cos\theta)\cdot\cos\theta\cdot Rd\theta$$

$$= \int_0^{\pi/2} \mu_k mgRd\theta = \mu_k mg\pi R/2$$ 2

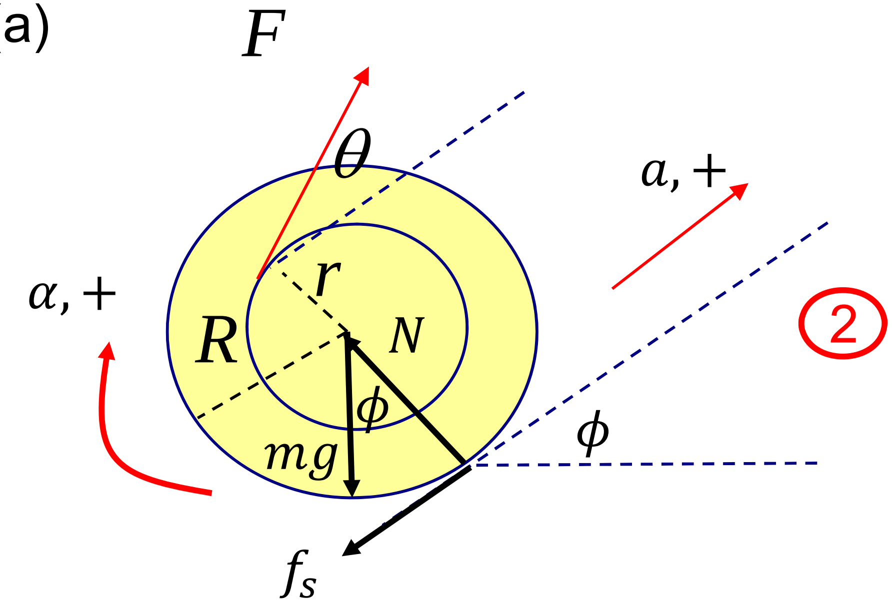
The work done by $\boldsymbol{f_k}$ is $\boldsymbol{W_{f_k}}$

$$W_{f_k} = \int_0^{\pi/2} \vec{f}_k\cdot d\vec{\ell} = \int_0^{\pi/2} f_k\cdot d\ell$$

$$= \int_0^{\pi/2} \mu_k mgR\cdot d\theta$$ 1

$$= \mu_k mg\pi R/2$$ 2

(a)



2

$$x: \ F cos\theta - f_s - mg \ sin\phi = ma \quad ---(1)$$

$$(y: \ \ N + F sin\theta - mg cos\phi = 0)$$ 1

$$\text{rotation}: \ \ f_s R + Fr = I\alpha \quad ---(2)$$ 2

Rolling condition: $a = R\alpha \quad ---(3)$ 2

(3個變數$\boldsymbol{f_s}$, $\boldsymbol{a}$, $\boldsymbol{\alpha}$, 3個方程式可解.)

(3) 帶入 (2),兩邊同時除以R：

$$f_s + F\frac{r}{R} = I\frac{\alpha}{R} = \frac{1}{2}ma \text{ ----(2')}$$

(2') + (1) :

$$F\left(cos + \frac{r}{R}\right) - mgsin\phi = \frac{3}{2}ma$$

$$\Rightarrow \quad a = \frac{2F}{3m}\left(cos + \frac{r}{R}\right) - \frac{2}{3}gsin\phi \quad --(4)$$

4

(4) 帶入 (1): $\ f_s = F cos\theta - mg \ sin\phi - ma$

$$f_s = \frac{F}{3m}\left(cos - \frac{2r}{R}\right) - \frac{1}{3}gsin\phi$$ 2

(b)