Chapter 12 Problem Solutions

12.1 A 60-Hz turbogenerator is rated 500 MVA, 22 kV. It is Y-connected and solidly grounded and is operating at rated voltage at no load. It is disconnected from the rest of the system. Its reactances are $X_d'' = X_1 = X_2 = 0.15$ and $X_0 = 0.05$ per unit. Find the ratio of the subtransient line current for a single line-to-ground fault to the subtransient line current for a symmetrical three-phase fault.

Solution:

Single line-to-ground fault:

$$I_a^{(1)} = \frac{1}{j0.15 + j0.15 + j0.15} = -j2.857 \text{ per unit}$$

$$I_a = 3I_a^{(1)} = -j8.571 \text{ per unit}$$

Three-phase fault:

$$I_a = \frac{1}{j0.15} = -j6.667 \text{ per unit}$$

The ratio is $8.571/6.667 = 1.286/1$.

12.2 Find the ratio of the subtransient line current for a line-to-line fault to the subtransient current for a symmetrical three-phase fault on the generator of Prob. 12.1.

Solution:

Line-to-line fault:

$$I_a^{(1)} = \frac{1}{j0.15 + j0.15} = -j3.333 \text{ per unit}$$

$$I_a^{(2)} = -I_a^{(1)} = j3.333 \text{ per unit}$$

$$I_b^{(1)} = a^2 I_a^{(1)} = 3.333 \underline{/150°} \text{ per unit}$$

$$I_{b2} = a I_a^{(2)} = 3.333 \underline{/210°} \text{ per unit}$$

$$I_b = I_b^{(1)} + I_{b2} = -5.773 \text{ per unit}$$

Using the three-phase fault value for I_a from Prob. 12.1, the ratio is now

$$5.773/6.667 = 0.866/1$$

12.3 Determine the inductive reactance in ohms to be inserted in the neutral connection of the generator of Prob. 12.1 to limit the subtransient line current for a single line-to-ground fault to that for a three-phase fault.

Solution:

From Prob. 12.1, $I_a = -j6.667$ per unit for a three-phase fault. Let x be the inductive reactance in per unit to be inserted. Then, for a single line-to-ground fault,

$$I_a = 3I_a^{(1)} = \frac{3}{j(0.15 + 0.15 + 0.05 + 3x)}$$

For a three-phase fault, $I_a = 1/j0.15 = -j6.667$ per unit. Equating the values for I_a, we have

$$3 = -j^2(0.35 + 3x)(6.667)$$
$$x = 0.0333 \text{ per unit}$$
$$\text{Base } Z = \frac{(22)^2}{500} = 0.968 \ \Omega$$
$$x = 0.0333 \times 0.968 = 0.3226 \ \Omega$$

12.4 With the inductive reactance found in Prob. 12.3 inserted in the neutral of the generator of Prob. 12.1, find the ratios of the subtransient line currents for the following faults to the subtransient line current for a three-phase fault:
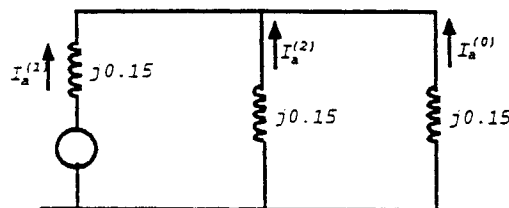(a) single line-to-ground fault, (b) line-to-line fault and (c) double line-to-ground fault.

Solution:

(a) The ratio equals one since reactance was added to achieve this ratio.

(b) The ratio is 0.866 (see Prob. 12.2; this is because the fault current in the line-to-line fault is not affected by the reactance on the neutral).

(c) For a double line-to-ground fault,

$$Z_0 = j(0.05 + 3 \times 0.0333) = j0.15$$

$$I_a^{(1)} = \frac{1.0}{j0.15 + \frac{j0.15 \times j0.15}{j0.15 + j0.15}} = \frac{1}{j0.225} = -j4.44 \text{ per unit}$$

$$I_a^{(2)} = I_a^{(0)} = j4.44\left(\frac{0.15}{0.30}\right) = j2.22 \text{ per unit}$$

$$I_f = 3I_a^{(0)} = j6.67 \text{ per unit}$$

$$I_b = 4.44\underline{/-90° + 240°} + 2.22\underline{/90° + 120°} + 2.22\underline{/90°}$$
$$= -3.85 + j2.22 - 1.923 - j1.11 + j2.22 = -5.773 + j3.33 = 6.67\underline{/150°} \text{ per unit}$$

For a three-phase fault,

$$I_a = \frac{1}{j0.15} = -j6.67 \text{ per unit} \qquad \text{Ratio} = \frac{6.67}{6.67} = 1.0$$

12.5 How many ohms of resistance in the neutral connection of the generator of Prob. 12.1 would limit the subtransient line current for a single line-to-ground fault to that for a three-phase fault?

Solution:

For a three-phase fault:

$$I_a = \frac{1}{j0.15} = -j6.667 \text{ per unit}$$

$$|-6.667| = 3\left|\frac{1}{j(0.15 + 0.15 + 0.05) + 3R}\right|$$

$$2.222 = \frac{1}{\sqrt{9R^2 + (0.35)^2}}$$

$$4.9383 = \frac{1}{9R^2 + 0.1225}$$

$$R = 0.0943 \text{ per unit}$$

$$\text{Base } Z = \frac{(22)^2}{500} = 0.968 \ \Omega$$

$$R = 0.0943 \times 0.968 = 0.0913 \ \Omega$$

12.6 A generator rated 100 MVA, 20 kV has $X_d'' = X_1 = X_2 = 20\%$ and $X_0 = 5\%$. Its neutral is grounded through a reactor of 0.32 Ω. The generator is operating at rated voltage without load and is disconnected from the system when a single line-to-ground fault occurs at its terminals. Find the subtransient current in the faulted phase.

Solution:

$$\text{Base } Z = \frac{(20)^2}{100} = 4.0 \ \Omega$$

$$X_n = \frac{0.32}{4} = 0.08 \text{ per unit}$$

$$Z_0 = j0.05 + 3 \times j0.08 = j0.29 \text{ per unit}$$

$$I_a^{(1)} = \frac{1}{j0.2 + j0.2 + j0.29} = -j1.449 \text{ per unit}$$

$$I_a = 3I_a^{(1)} = -j4.347 \text{ per unit}$$

$$\text{Base } I = \frac{100,000}{\sqrt{3} \times 20} = 2887 \text{ A}$$

$$|I_a''| = 4.347 \times 2887 = 12,550 \text{ A}$$

12.7 A 100-MVA 18-kV turbogenerator having $X_d'' = X_1 = X_2 = 20\%$ and $X_0 = 5\%$ is about to be connected to a power system. The generator has a current-limiting reactor of 0.162 Ω in the neutral. Before the generator is connected to the system, its voltage is adjusted to 16 kV when a double line-to-ground fault develops at terminals b and c. Find the initial symmetrical rms current in the ground and in line b.

Note to Instructor: Since $Z_1 = Z_2 = Z_0$ we see from Prob. 12.4 that $|I_f| = |I_b| = |I_c|$ and fewer calculations are needed. Of course, the problem may be worked in the usual manner, but here we will take advantage of Prob. 12.4. A good variation is to make $Z_n = 0.243 \ \Omega$.

Solution:

$$\text{Base } Z = \frac{(18)^2}{100} = 3.24 \ \Omega \qquad\qquad Z_0 = j0.05 + 3 \times j0.05 = j0.20 \text{ per unit}$$

$$X_n = \frac{0.162}{3.24} = 0.05 \text{ per unit} \qquad\qquad E_{g^{.}} = \frac{16}{18} = 0.8889 \text{ per unit}$$

At this point we may recognize that Prob. 12.4 has shown that when $Z_1 = Z_2 = Z_0$, both $|I_b|$ and $|I_f|$ for a double line-to-ground fault are equal to $|I_a|$ for a single line-to-ground fault, which is

$$\frac{0.8889}{0.20} = 4.444 \text{ per unit}$$

where

$$\text{Base } I = \frac{100,000}{\sqrt{3} \times 18} = 3207.5 \text{ A}$$

$$|I_b| = |I_f| = 4.444 \times 3207.5 = 14,254 \text{ A}$$

Note that the above calculation is valid only because $Z_1 = Z_2 = Z_0$. If this were not the case, I_b must be found by first determining $I_a^{(1)}$, $I_a^{(2)}$ and $I_a^{(0)}$.

12.8 The reactances of a generator rated 100 MVA, 20 kV, are $X_d'' = X_1 = X_2 = 20\%$ and $X_0 = 5\%$. the generator is connected to a Δ-Y transformer rated 100 MVA, 20Δ-230Y kV, with a reactance of 10%. The neutral of the transformer is solidly grounded. The terminal voltage of the generator is 20 kV when a single line-to-ground fault occurs on the open-circuited, high-voltage side of the transformer. Find the initial symmetrical rms current in all phases of the generator.

Solution:
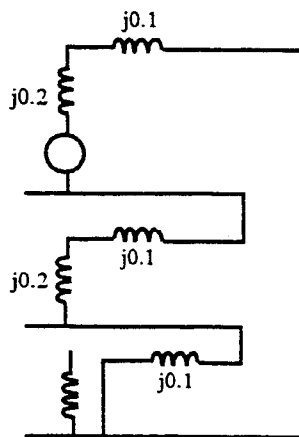
On the high-voltage side,

$$I_A^{(2)} = I_A^{(0)} = I_A^{(1)} = \frac{1}{j0.3 + j0.3 + j0.1} = -j1.429 = 1.429 \underline{/-90^\circ} \text{ per unit}$$

In the generator, $I_a^{(0)} = 0$ and

$$I_a^{(1)} = I_A^{(1)}\angle{-30°} = 1.429\angle{-120°} \text{ per unit}$$

$$I_a^{(2)} = I_A^{(2)}\angle{30°} = 1.429\angle{-60°} \text{ per unit}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 1.429\angle{-120°} \\ 1.429\angle{-60°} \end{bmatrix} = \begin{bmatrix} 2.475\angle{-90°} \\ 2.475\angle{90°} \\ 0 \end{bmatrix} \text{ per unit}$$

Calculating current values we have

$$\text{Base } I = \frac{100,000}{\sqrt{3}\times 20} = 2887 \text{ A}$$

$$|I_a| = |I_b| = 2.475\times 2887 = 7145 \text{ A}$$

$$|I_c| = 0 \text{ A}$$

12.9 A generator supplies a motor through a Y-Δ transformer. The generator is connected to the Y side of the transformer. A fault occurs between the motor terminals and the transformer. The symmetrical components of the subtransient current in the motor flowing toward the fault are
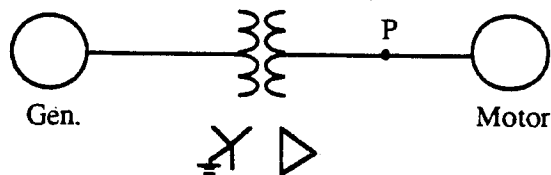
$$I_a^{(1)} = -0.8 - j2.6 \text{ per unit}$$

$$I_a^{(2)} = -j2.0 \text{ per unit}$$

$$I_a^{(0)} = -j3.0 \text{ per unit}$$

From the transformer toward the fault

$$I_a^{(1)} = 0.8 - j0.4 \text{ per unit}$$

$$I_a^{(2)} = -j1.0 \text{ per unit}$$

$$I_a^{(0)} = 0 \text{ per unit}$$

Assume $X_d'' = X_1 = X_2$ for both the motor and the generator. Describe the type of fault. Find (a) the prefault current, if any, in line a, (b) the subtransient fault current in per unit and (c) the subtransient current in each phase of the generator in per unit.

Solution:

Gen.   Motor

The fault occurs at point P in the above diagram. The sum of $I_a^{(1)}$ from the motor and from the transformer gives $I_a^{(1)}$ in the fault. So in the fault,
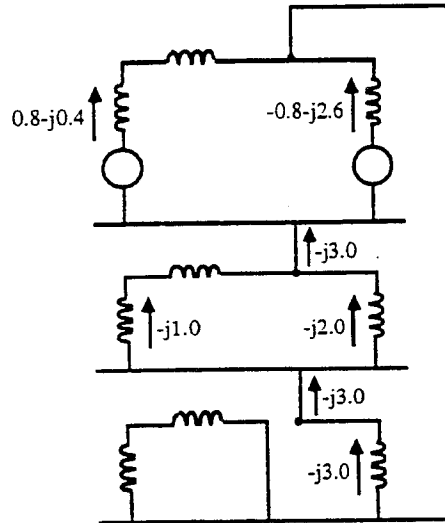
$$I_a^{(1)} = -0.8 - j2.6 + 0.8 - j0.4 = -j3.0 \text{ per unit}$$

Similarly,

$$I_a^{(2)} = 0 - j1.0 + 0 - j2.0 = -j3.0 \text{ per unit}$$
$$I_a^{(0)} = -j3.0 \text{ per unit}$$

which means a single line-to-ground fault. The connection of the sequence networks is shown as

(a) Currents are marked on the network diagram. The division of the current due to the fault between the branches of the positive-sequence network is the same as between the branches of the negative-sequence network since all X_1 values equal the X_2 values in corresponding branches. The prefault current in line a toward the motor is

$$0.8 - j0.4 - (-j1.0) = 0.8 + j0.6 \text{ per unit}$$
$$\text{or} \quad -[-0.8 - j2.6 - (-j2.0)] = 0.8 + j0.6 \text{ per unit}$$

(b) $I_f'' = 3(-j3.0) = -j9.0 \text{ per unit}$

(c) Assume that the generator is on the high-voltage side of the transformer such that

$$I_A^{(1)} = I_a^{(1)}e^{j30°} = (0.8 - j0.4)e^{j30°} = 0.894\underline{/3.43°} \text{ per unit}$$
$$I_A^{(2)} = I_a^{(2)}e^{-j30°} = (-j1.0)e^{-j30°} = 1.0\underline{/-120°} \text{ per unit}$$
$$I_A^{(0)} = 0$$

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.894\underline{/3.43°} \\ 1.0\underline{/-120°} \end{bmatrix} = \begin{bmatrix} 0.903\underline{/-64.3°} \\ 1.0\underline{/-53.1°} \\ 1.893\underline{/121.6°} \end{bmatrix} \text{ per unit}$$

12.10 Using Fig. 12.18, calculate the bus impedance matrices $Z_{bus}^{(1)}$, $Z_{bus}^{(2)}$ and $Z_{bus}^{(0)}$ for the network of Example 12.6.

Solution:

From Fig. 12.18,

$$Y_{bus}^{(1)} = Y_{bus}^{(2)} = \begin{bmatrix} -j40/3 & j10 \\ j10 & -j20 \end{bmatrix} \text{ per unit}$$

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Hence,

$$\mathbf{Z}_{bus}^{(1)} = \mathbf{Z}_{bus}^{(2)} = \begin{bmatrix} -j40/3 & j10 \\ j10 & -j20 \end{bmatrix}^{-1} = \begin{bmatrix} j0.12 & j0.06 \\ j0.06 & j0.08 \end{bmatrix} \text{ per unit}$$

By inspection,

$$\mathbf{Z}_{bus}^{(0)} = \begin{bmatrix} j(0.09 + 0.06) & j0 \\ j0 & \frac{j(0.2 \times 0.1)}{(0.2 + 0.1)} \end{bmatrix} = \begin{bmatrix} j0.15 & j0 \\ j0 & j0.0667 \end{bmatrix} \text{ per unit}$$

12.11 Solve for the subtransient current in a single line-to-ground fault first on bus ① and then on bus ② of the network of Example 12.6. Use the bus impedance matrices of Prob. 12.10. Also find the voltages to neutral at bus ② with the fault at bus ①.

Solution:

From Eq. (12.12), under single line-to-ground fault conditions at bus ⓚ,

$$I_{fa}^{(0)} = I_{fa}^{(1)} = I_{fa}^{(2)} = \frac{V_f}{Z_{kk}^{(1)} + Z_{kk}^{(2)} + Z_{kk}^{(0)} + 3Z_f}$$

Fault at bus ①:

$$\begin{aligned} I_{fa}^{(0)} = I_{fa}^{(1)} = I_{fa}^{(2)} &= \frac{V_f}{Z_{11}^{(1)} + Z_{11}^{(2)} + Z_{11}^{(0)} + 3Z_f} \\ &= \frac{1.0 \angle 0°}{j(0.12 + 0.12 + 0.15 + 0)} = -j2.564 \text{ per unit} \end{aligned}$$

Fault current calculation yields

$$I_{fa} = I_{fa}^{(0)} + I_{fa}^{(1)} + I_{fa}^{(2)} = 3 \times -j2.564 = -j7.692 \text{ per unit}$$

Fault at bus ②:

$$\begin{aligned} I_{fa}^{(0)} = I_{fa}^{(1)} = I_{fa}^{(2)} &= \frac{V_f}{Z_{22}^{(1)} + Z_{22}^{(2)} + Z_{22}^{(0)} + 3Z_f} \\ &= \frac{1.0 \angle 0°}{j(0.08 + 0.08 + 0.0667 + 0)} = -j4.411 \text{ per unit} \end{aligned}$$

Fault current calculation yields

$$I_{fa} = I_{fa}^{(0)} + I_{fa}^{(1)} + I_{fa}^{(2)} = 3 \times -j4.411 = -j13.233 \text{ per unit}$$

Voltage at bus ② with fault at bus ①:

$$V_a^{(1)} = 1.0 \angle 0° - I_{fa}^{(1)} Z_{21}^{(1)} = 1.0 - (-j2.564)(j0.06) = 0.84616 \text{ per unit}$$
$$V_a^{(2)} = -I_{fa}^{(2)} Z_{21}^{(2)} = -(-j2.564)(j0.06) = -0.15384 \text{ per unit}$$
$$V_a^{(0)} = -I_{fa}^{(0)} Z_{21}^{(0)} = 0 \text{ per unit}$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_a^{(0)} \\ V_b^{(1)} \\ V_c^{(2)} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.84616 \\ -0.15384 \end{bmatrix} = \begin{bmatrix} 0.692 \underline{/0°} \\ 0.933 \underline{/-111.8°} \\ 0.933 \underline{/111.8°} \end{bmatrix} \text{ per unit}$$

12.12 Calculate the subtransient currents in all parts of the system of Example 12.6 with prefault current neglected if the fault on the low-voltage side of the transformer is a line-to-line fault. Use $\mathbf{Z}_{bus}^{(1)}$, $\mathbf{Z}_{bus}^{(2)}$ and $\mathbf{Z}_{bus}^{(0)}$ of Prob. 12.10.

Solution:

With a double line fault (between phases b and c) at bus ①,

$$I_{fa}^{(1)} = -I_{fa}^{(2)} = \frac{V_f}{Z_{11}^{(1)} + Z_{11}^{(2)}} = \frac{1.0}{j(0.12 + 0.12)} = -j4.167 \text{ per unit}$$

$$I_{fa}^{(0)} = 0$$

$$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j4.167 \\ j4.167 \end{bmatrix} = \begin{bmatrix} 0 \\ -7.217 \\ 7.217 \end{bmatrix} \text{ per unit}$$

During this type of fault, no zero-sequence current flows through the system.

(Positive Sequence)   (Negative Sequence)
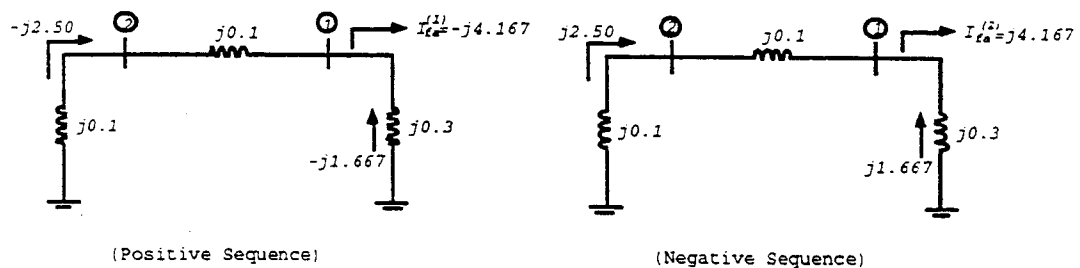
(i) No current flows in any of the grounding connections.

(ii) In the low-voltage side of the transformer:

$$I_a^{(0)} = 0 \qquad I_a^{(1)} = -j2.50 \text{ per unit} \qquad I_a^{(2)} = j2.50 \text{ per unit}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 0 \\ -j\sqrt{3} \\ j\sqrt{3} \end{bmatrix} (-j2.50) = \begin{bmatrix} 0 \\ -4.33 \\ 4.33 \end{bmatrix} \text{ per unit}$$

(iii) In the generator and the high-voltage side of the transformer:

The Y-Δ connection introduces a 30° phase lead to positive-sequence currents and a 30° phase lag to negative-sequence currents. Therefore,

$$I_a^{(1)} = 2.50 \underline{/-60°} \text{ per unit}$$
$$I_a^{(2)} = 2.50 \underline{/60°} \text{ per unit}$$

$$I_a^{(0)} = 0$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 2.50\underline{/-60°} \\ 2.50\underline{/60°} \end{bmatrix} = \begin{bmatrix} 2.50 \\ -5.0 \\ 2.50 \end{bmatrix} \text{ per unit}$$

(*iv*) In the motor section (towards the fault):

$$I_a^{(0)} = 0 \qquad I_a^{(1)} = -j1.667 \text{ per unit} \qquad I_a^{(2)} = j1.667 \text{ per unit}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 0 \\ -j\sqrt{3} \\ j\sqrt{3} \end{bmatrix}(-j1.667) = \begin{bmatrix} 0 \\ -2.88 \\ 2.88 \end{bmatrix} \text{ per unit}$$

(Each motor supplies 1/4 of these currents)

12.13 Repeat Prob. 12.12 for a double line-to-ground fault.

Solution:

Under double line-to-ground (phases b, c and ground) fault conditions,

$$I_{fa}^{(1)} = \frac{V_f}{Z_{11}^{(1)} + \frac{Z_{11}^{(2)} Z_{11}^{(0)}}{Z_{11}^{(2)} + Z_{11}^{(0)}}} = \frac{1.0\underline{/0°}}{j\left(0.12 + \frac{0.15 \times 0.15}{0.12 + 0.15}\right)} = -j5.357 \text{ per unit}$$
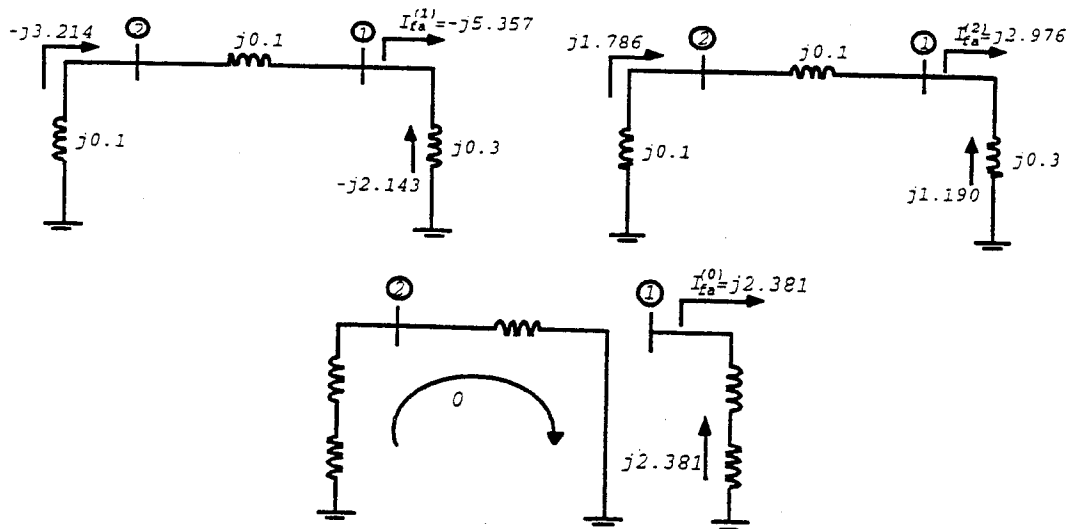
$$I_{fa}^{(2)} = -I_{fa}^{(1)} \frac{Z_{11}^{(0)}}{Z_{11}^{(0)} + Z_{11}^{(2)}} = j2.976 \text{ per unit}$$

$$I_{fa}^{(0)} = -I_{fa}^{(1)} - I_{fa}^{(2)} = j2.381 \text{ per unit}$$

Currents toward the fault from the three phases are

$$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} j2.381 \\ -j5.357 \\ j2.976 \end{bmatrix} = \begin{bmatrix} 0 \\ 8.052\underline{/153.7°} \\ 8.052\underline{/26.3°} \end{bmatrix} \text{ per unit}$$

Sequence current flows through various parts of the network as shown below:

(*i*) In the generator grounding connection, no current flows since zero-sequence current in that part of the network is zero.

(*ii*) Current in the motor grounding section (toward motors):

$$3I_a^{(0)} = 3 \times j2.381 = j7.143 \text{ per unit}$$

(Each motor handles 1/4 of this current)

(*iii*) In the low-voltage side of the transformer:

$$I_a^{(0)} = 0 \qquad I_a^{(1)} = -j3.214 \text{ per unit} \qquad I_a^{(2)} = j1.786 \text{ per unit}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j3.214 \\ j1.786 \end{bmatrix} = \begin{bmatrix} 1.429\angle-90° \\ 4.388\angle170.6° \\ 4.388\angle9.4° \end{bmatrix} \text{ per unit}$$

(*iv*) In the generator and high-voltage side of the transformer:
As in Prob. 12.12,

$$I_a^{(1)} = 3.214\angle-60° \text{ per unit}$$

$$I_a^{(2)} = 1.786\angle60° \text{ per unit}$$

$$I_a^{(0)} = 0$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 3.214\angle-60° \\ 1.786\angle60° \end{bmatrix} = \begin{bmatrix} 2.789\angle-26.3° \\ 5.0\angle180° \\ 2.789\angle26.3° \end{bmatrix} \text{ per unit}$$

(*v*) In the motor section:

$$I_a^{(0)} = j2.381 \text{ per unit} \qquad I_a^{(1)} = -j2.143 \text{ per unit} \qquad I_a^{(2)} = j1.190 \text{ per unit}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} j2.381 \\ -j2.143 \\ j1.190 \end{bmatrix} = \begin{bmatrix} 1.429\angle90° \\ 4.062\angle135.3° \\ 4.062\angle44.7° \end{bmatrix} \text{ per unit}$$

(Each motor carries 1/4 of these currents)

12.14 The machines connected to the two high-voltage buses shown in the single-line diagram of Fig. 12.32 are each rated 100 MVA, 20 kV with reactances of $X_d'' = X_1 = X_2 = 20\%$ and $X_0 = 4\%$. Each three-phase transformer is rated 100 MVA, 345Y/20Δ kV, with leakage reactance of 8%. On a base of 100 MVA, 345 kV the reactances of the transmission line are $X_1 = X_2 = 15\%$ and $X_0 = 50\%$. Find the 2 × 2 bus impedance matrix for each of the three sequence networks. If no prefault current is flowing in the network, find the subtransient current to ground for a double line-to-ground fault on lines B and C at bus ①. Repeat for a fault at bus ②. When the fault is at bus ②, determine the current in phase b of machine 2 if the lines are so named that $V_A^{(1)}$ leads $V_a^{(1)}$ by 30°. If the phases are named so that $I_a^{(1)}$ leads $I_A^{(1)}$ by 30°, what letter (a, b or c) would identify the phase of machine 2 which would carry the current found for phase b above?

Note to Instructor: Although this very simple two-node problem is solved just as easily by not resorting to the Z_{bus} method, the problem is useful to point out the great advantage of Z_{bus} where fault calculations are needed at many nodes beacuse the computer takes the labor out of finding Z_{bus}. With Z_{bus}, the problem is easily extended to find the voltage of unfaulted buses.

Solution:

The negative-sequence network shown below is identical to the positive-sequence network with emfs short-circuited.
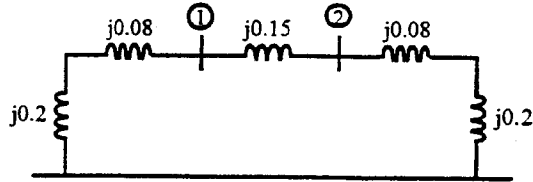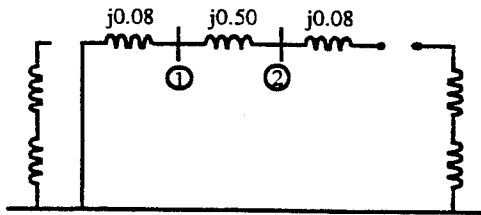

$$Y_{11}^{(2)} = Y_{11}^{(1)} = \frac{1}{j0.28} + \frac{1}{j0.15} = -j10.24 \text{ per unit}$$

$$Y_{22}^{(2)} = Y_{11}^{(2)} = Y_{11}^{(1)} = -j10.24 \text{ per unit}$$

$$Y_{12}^{(1)} = Y_{12}^{(2)} = \frac{-1}{j0.15} = j6.67 \text{ per unit}$$

The zero-sequence network is shown as

Since the impedance $j0.08$ connected to bus ② is not connected to any other bus, it is not included in $Y_{22}^{(0)}$.

$$Y_{11}^{(0)} = \frac{1}{j0.08} + \frac{1}{j0.50} = -j14.50 \text{ per unit}$$

$$Y_{12}^{(0)} = Y_{22}^{(0)} = \frac{-1}{j0.5} = j2.0 \text{ per unit}$$

$$Z_{bus}^{(1)} = Z_{bus}^{(2)} = \begin{bmatrix} -j10.24 & j6.67 \\ j6.67 & -j10.24 \end{bmatrix}^{-1} = \begin{bmatrix} j0.170 & j0.110 \\ j0.110 & j0.170 \end{bmatrix} \text{ per unit}$$

$$Z_{bus}^{(0)} = \begin{bmatrix} -j14.50 & j2.00 \\ j2.00 & -j2.00 \end{bmatrix}^{-1} = \begin{bmatrix} j0.08 & j0.08 \\ j0.08 & j0.58 \end{bmatrix} \text{ per unit}$$

Z_{bus} equivalent networks are shown below (mutual impedances not indicated) for a double line-to-ground fault on bus ①:
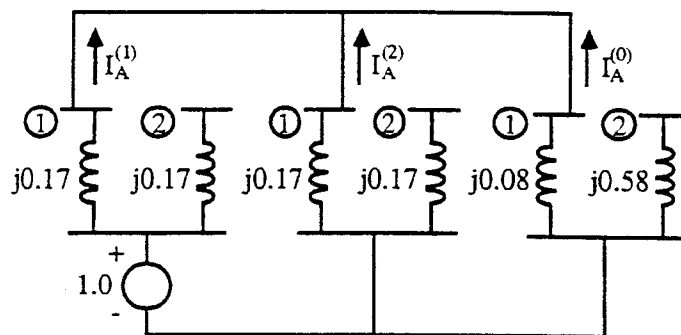
For the fault on bus ① (as shown):

$$I_A^{(1)} = \frac{1}{j\left(0.170 + \frac{0.170\times0.08}{0.170+0.08}\right)} = -j4.456 \text{ per unit}$$

$$V_A^{(1)} = 1.0 - j0.170(-j4.456) = 0.2425 \text{ per unit}$$

$$I_A^{(2)} = \frac{-0.2425}{j0.170} = j1.426 \text{ per unit}$$

$$I_A^{(0)} = \frac{-0.2425}{j0.08} = j3.031 \text{ per unit}$$

$$I_n = 3I_A^{(0)} = 3(j3.031) = j9.093 \text{ per unit}$$

Calculations for magnitude of the fault current yields

$$\text{Base } I = \frac{100,000}{\sqrt{3}\times345} = 167.3 \text{ A}$$

$$|I_n| = 167.3 \times 9.093 = 1521 \text{ A}$$

For the fault on bus ②:

$$I_A^{(1)} = \frac{1}{j\left(0.170 + \frac{0.170\times0.58}{0.170+0.58}\right)} = -j3.317 \text{ per unit}$$

$$V_A^{(1)} = 1.0 - j0.170(-j3.317) = 0.4361 \text{ per unit}$$

$$I_A^{(2)} = \frac{-0.4361}{j0.170} = j2.565 \text{ per unit}$$

$$I_A^{(0)} = \frac{-0.4361}{j0.58} = j0.7519 \text{ per unit}$$

$$I_n = 3I_A^{(0)} = j2.256 \text{ per unit}$$

The magnitude of the fault current is

$$|I_n| = 167.3 \times 2.256 = 377 \text{ A}$$

From the transformer at bus ② to the fault at bus ②:

$$I_A^{(1)} = -j3.317 \times \frac{j0.28 + j0.15}{j0.28 + j0.28 + j0.15} = -j2.009 \text{ per unit}$$

$$I_A^{(2)} = j2.565 \times \frac{j0.28 + j0.15}{j0.28 + j0.28 + j0.15} = j1.553 \text{ per unit}$$

In generator 2:

$$I_a^{(0)} = 0$$

$$I_a^{(1)} = I_A^{(1)}e^{-j30°} = 2.009\underline{/-120°} \text{ per unit}$$

$$I_a^{(2)} = I_A^{(2)}e^{j30°} = 1.553\underline{/120°} \text{ per unit}$$

$$I_b = \begin{bmatrix} 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0 \\ 2.009\underline{/-120°} \\ 1.553\underline{/120°} \end{bmatrix} = 1.824\underline{/167.5°} \text{ per unit}$$

Calculating for the fault current value yields
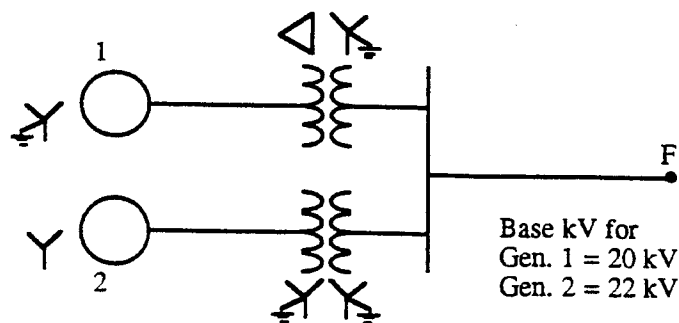
$$\text{Base } I = \frac{100,000}{\sqrt{3} \times 20} = 2887 \text{ A}$$

$$|I_b| = 2887 \times 1.824 = 5266 \text{ A}$$

12.15 Two generators G_1 and G_2 are connected respectively through transformers T_1 and T_2 to a high-voltage bus which supplies a transmission line. The line is open at the far end at which point F a fault occurs. The prefault voltage at point F is 515 kV. Apparatus ratings and reactances are

G_1	1000 MVA, 20 kV, $X_s = 100\%$	$X_d'' = X_1 = X_2 = 10\%$	$X_0 = 5\%$
G_2	800 MVA, 22 kV, $X_s = 120\%$	$X_d'' = X_1 = X_2 = 15\%$	$X_0 = 8\%$
T_1	1000 MVA, 500Y/20Δ kV, $X = 17.5\%$		
T_2	800 MVA, 500Y/22Y kV, $X = 16.0\%$		
Line	$X_1 = 15\%$, $X_0 = 40\%$ on a base of 1500 MVA, 500 kV		

The neutral of G_1 is grounded through a reactance of 0.04 Ω. The neutral of G_2 is not grounded. Neutrals of all transformers are solidly grounded. Work on a base of 1000 MVA, 500 kV in the transmission line. Neglect prefault current and find subtransient current (a) in phase c of G_1 for a three-phase fault at F, (b) in phase B at F for a line-to-line fault on lines B and C, (c) in phase A at F for a line-to-ground fault on line A and (d) in phase c of G_2 for a line-to-ground fault on line A. Assume $V_A^{(1)}$ leads $V_a^{(1)}$ by 30° in T_1.

Solution:

Base kV for
Gen. 1 = 20 kV
Gen. 2 = 22 kV

The base currents are calculated as

$$\text{Line:} \quad \frac{1,000,000}{\sqrt{3} \times 500} = 1155 \text{ A}$$

$$\text{Gen. 1:} \quad \frac{1,000,000}{\sqrt{3} \times 20} = 28,868 \text{ A}$$

$$\text{Gen. 2:} \quad \frac{1,000,000}{\sqrt{3} \times 22} = 26,243 \text{ A}$$

Impedances in per unit are

$$\text{Gen. 1:} \quad X_d'' = X_2 = 0.10 \qquad X_0 = 0.05$$

$$\text{Gen. 2:} \quad X_d'' = X_2 = 0.15 \times \frac{1000}{800} = 0.1875$$

$$T_1: \quad X = 0.175 \qquad T_2: \quad X = 0.16 \times \frac{1000}{800} = 0.20$$

$$\text{Line:} \quad X_1 = X_2 = 0.15 \times \frac{1000}{1500} = 0.10$$

$$X_0 = 0.40 \times \frac{1000}{1500} = 0.267$$

$$\text{Operating voltage} = \frac{515}{500} = 1.03 \text{ per unit}$$

Positive Sequence Network
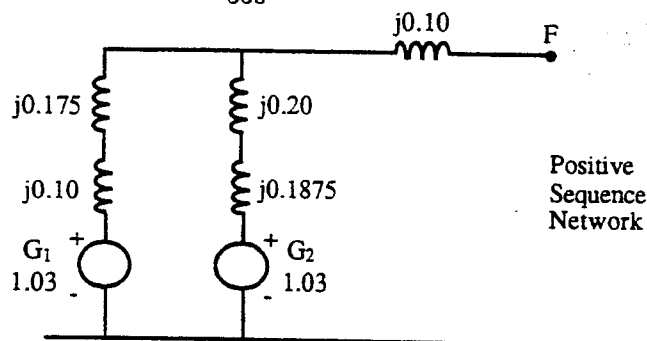
The above network reduces to

Zero-sequence Network
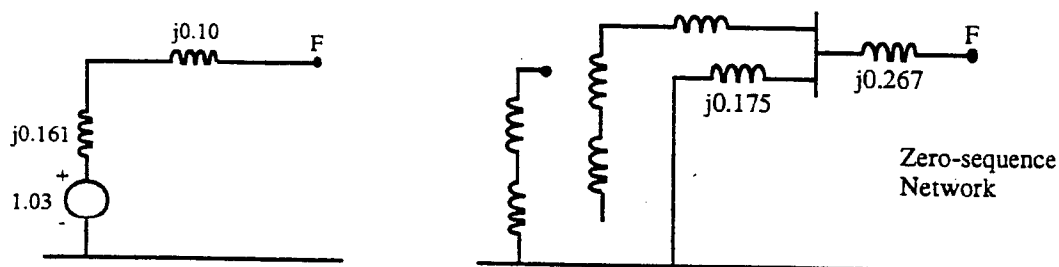
(a) Three-phase fault at F

$$I_A^{(1)} = \frac{1.03}{j0.261} = -j3.946 \text{ per unit}$$

$$\text{In Gen. 1:} \quad I_a^{(1)} = \left(-j3.946 \frac{j0.3875}{j0.275 + j0.3875} \right) e^{-j30°} = 2.308 \underline{/-120°} \text{ per unit}$$

$$I_c = 2.308 \underline{/ 0°} \text{ per unit}$$

In all three phases $|I''_f| = 2.308 \times 28.868 = 66,630$ A.

(b) Line-to-line fault at F ($Z_1 = Z_2$)

$$I_A^{(1)} = -I_A^{(2)} = \frac{1.03}{2(j0.261)} = -j1.973 \text{ per unit}$$

$$I_B = (a^2 - a) I_A^{(1)} = -j\sqrt{3}(-j1.973) = 3.417\angle 180° \text{ per unit}$$
$$|I''_B| = 3.417 \times 1155 = 3947 \text{ A}$$

(c) Single line-to-ground fault at F

$$I_A^{(1)} = \frac{1.03}{j0.261 + j0.261 + j0.175 + j0.267} = -j1.068 \text{ per unit}$$
$$|I''_A| = 3(1.068) \times 1155 = 3700 \text{ A}$$

(d) Single line-to-ground fault at F

In Gen. 2: $I_a^{(1)} = I_a^{(2)}$ $I_a^{(0)} = 0$

$$I_a^{(1)} = -j1.068 \frac{j0.275}{j0.275 + j0.3875} = -j0.443 \text{ per unit}$$

$$I_c = 0.443\angle 120° - 90° + 0.443\angle 240° - 90°$$
$$= 0.384 + j0.222 - 0.384 + j0.222 = j0.444 \text{ per unit}$$
$$|I''_c| = 0.444 \times 26,243 = 11,652 \text{ A}$$

12.16 In the network shown in Fig. 10.17, Y-Y connected transformers, each with grounded neutrals, are at the ends of each transmission line not terminating at bus ③. The transformers connecting the lines to bus ③ are Y-Δ with the neutral of the Y solidly grounded and the Δ sides connected to bus ③. All line reactances shown in Fig. 10.17 between buses include the reactances of the transformers. Zero-sequence values for these lines including transformers are 2.0 times those shown in Fig. 10.17.

Both generators are Y-connected. Zero-sequence reactances of the generators connected to buses ① and ③ are 0.04 and 0.08 per unit, respectively. The neutral of the generator at bus ① is connected to ground through a reactor of 0.02 per unit; the generator at bus ③ has a solidly grounded neutral.

Find the bus impedance matrices $\mathbf{Z}_{\text{bus}}^{(1)}$, $\mathbf{Z}_{\text{bus}}^{(2)}$ and $\mathbf{Z}_{\text{bus}}^{(0)}$ for the given network and then compute the subtransient current in per unit (a) in a single line-to-ground fault on bus ② and (b) in the faulted phase of line ①-②. Assume no prefault current is flowing and all prefault voltages at all the buses is $1.0\angle 0°$ per unit.
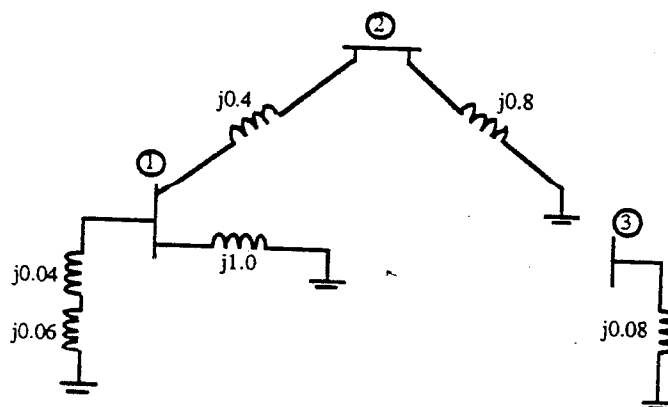
Solution:

The schematic diagram is shown as
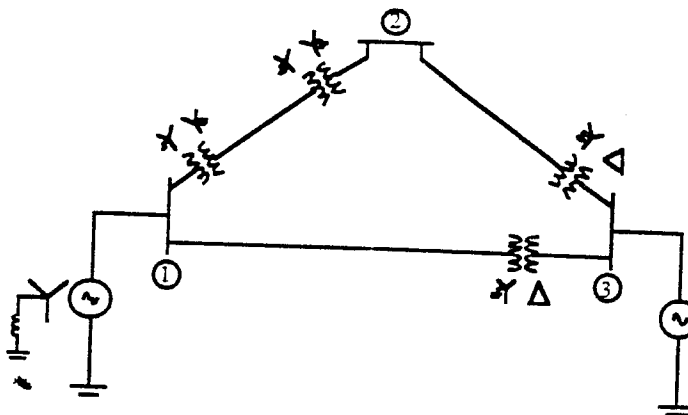
The positive- and negative-sequence network is shown as
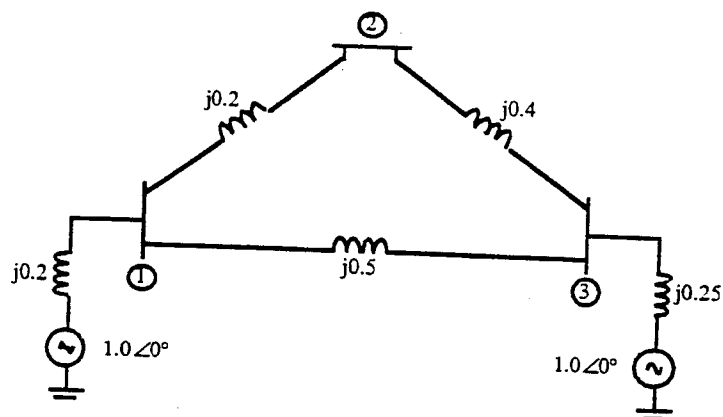
The zero-sequence network is shown as

From Prob. 10.9,

$$\mathbf{Z}_{bus}^{(1)} = \mathbf{Z}_{bus}^{(2)} = \begin{bmatrix} j0.1447 & j0.1195 & j0.0692 \\ j0.1195 & j0.2465 & j0.1006 \\ j0.0692 & j0.1006 & j0.1635 \end{bmatrix} \text{ per unit}$$

By examining the zero-sequence network,

$$\mathbf{Y}_{\text{bus}}^{(0)} = \begin{bmatrix} -j13.5 & j2.5 & j0 \\ j2.5 & -j3.75 & j0 \\ j0 & j0 & -j12.5 \end{bmatrix} \text{ per unit}$$

$$\mathbf{Z}_{\text{bus}}^{(0)} = \left[\mathbf{Y}_{\text{bus}}^{(0)}\right]^{-1} = \begin{bmatrix} j0.0845 & j0.0563 & j0 \\ j0.0563 & j0.3042 & j0 \\ j0 & j0 & j0.08 \end{bmatrix} \text{ per unit}$$

For a fault at bus ②,

$$I_{fa}^{(0)} = I_{fa}^{(1)} = I_{fa}^{(2)} = \frac{V_f}{Z_{22}^{(0)} + Z_{22}^{(1)} + Z_{22}^{(2)}}$$

$$= \frac{1.0\underline{/0°}}{j\,(0.2465 + 0.2465 + 0.3042)} = -j1.254 \text{ per unit}$$

$$I_{fa}'' = I_{fa}^{(0)} + I_{fa}^{(1)} + I_{fa}^{(2)} = -j3.762 \text{ per unit}$$

Currents through line ①-②:

$$I_{12,a}^{(1)} = \frac{\left(Z_{12}^{(1)} - Z_{22}^{(1)}\right)\left(-I_{fa}^{(1)}\right)}{z_{12}^{(1)}} = \frac{j(0.1195 - 0.2465)(j1.254)}{j0.2} = -j0.796 \text{ per unit}$$

$$I_{12,a}^{(2)} = I_{12,a}^{(1)} = -j0.796 \text{ per unit}$$

$$I_{12,a}^{(0)} = \frac{\left(Z_{12}^{(0)} - Z_{22}^{(0)}\right)\left(-I_{fa}^{(0)}\right)}{z_{12}^{(0)}} = \frac{j(0.0563 - 0.3042)(j1.254)}{j0.4} = -j0.777 \text{ per unit}$$

$$I_{12,a} = I_{12,a}^{(0)} + I_{12,a}^{(1)} + I_{12,a}^{(2)} = -j2.369 \text{ per unit}$$

12.17 The network of Fig. 9.2 has the line data specified in Table 9.2. The two generators connected to buses ① and ④ have $X_d'' = X_1 = X_2 = 0.25$ per unit each. Making the usual simplifying assumptions of Sec. 10.6, determine the sequence matrices $\mathbf{Z}_{\text{bus}}^{(1)} = \mathbf{Z}_{\text{bus}}^{(2)}$ and use them to calculate

(a) the subtransient current in per unit in a line-to-line fault on bus ② of the network

(b) the fault current contributions from line ①-② and line ④-②.

Assume that lines ①-② and ④-② are connected to bus ② directly (not through transformers) and that all positive- and negative-sequence reactances are identical.

<u>Note to Instructor</u>: The text requests for current contributions from line ③-② while it does not exist in Fig. 9.2. The solution below uses line ④-②.

Solution:

$\mathbf{Z}_{bus}$ is shown in the solution for Prob. 10.13 and

$$\mathbf{Z}_{bus}^{(1)} = \mathbf{Z}_{bus}^{(2)} = \mathbf{Z}_{bus}$$

(*a*) For a fault at bus ②:

$$I_{fa}^{(1)} = -I_{fa}^{(2)} = \frac{V_f}{Z_{22}^{(1)} + Z_{22}^{(2)}} = \frac{1.0\angle 0°}{j0.1466 + j0.1466} = -j3.411 \text{ per unit}$$

$$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ I_{fa}^{(1)} \\ -I_{fa}^{(1)} \end{bmatrix} = \begin{bmatrix} 0 \\ -j\sqrt{3}\,I_{fa}^{(1)} \\ j\sqrt{3}\,I_{fa}^{(1)} \end{bmatrix} = \begin{bmatrix} 0 \\ -5.908 \\ 5.908 \end{bmatrix} \text{ per unit}$$

(*b*) Currents through line ①–②:

$$I_{12,a}^{(1)} = \frac{\left(Z_{12}^{(1)} - Z_{22}^{(1)}\right)\left(-I_{fa}^{(1)}\right)}{z_{12}^{(1)}} = \frac{j(0.1234 - 0.1466)(j3.411)}{j0.0504} = -j1.570 \text{ per unit}$$

$$\begin{bmatrix} I_{12,a} \\ I_{12,b} \\ I_{12,c} \end{bmatrix} = \begin{bmatrix} 0 \\ -j\sqrt{3}\,I_{12,a}^{(1)} \\ j\sqrt{3}\,I_{12,a}^{(1)} \end{bmatrix} = \begin{bmatrix} 0 \\ -2.720 \\ 2.720 \end{bmatrix} \text{ per unit}$$

Currents through line ④–②:

$$I_{42,a}^{(1)} = \frac{\left(Z_{42}^{(1)} - Z_{22}^{(1)}\right)\left(-I_{fa}^{(1)}\right)}{z_{42}^{(1)}} = \frac{j(0.1266 - 0.1466)(j3.411)}{j0.0372} = -j1.834 \text{ per unit}$$

$$\begin{bmatrix} I_{42,a} \\ I_{42,b} \\ I_{42,c} \end{bmatrix} = \begin{bmatrix} 0 \\ -j\sqrt{3}\,I_{42,a}^{(1)} \\ j\sqrt{3}\,I_{42,a}^{(1)} \end{bmatrix} = \begin{bmatrix} 0 \\ -3.176 \\ 3.176 \end{bmatrix} \text{ per unit}$$

12.18 In the system of Fig. 12.9*a* consider that Machine 2 is a motor drawing a load equivalent to 80 MVA at 0.85 power factor lagging and nominal system voltage of 345 kV at bus ③. Determine the change in voltage at bus ③ when the transmission line undergoes (*a*) a one-open-conductor fault and (*b*) a two-open-conductor fault along its span between buses ② and ③. Choose a base of 100 MVA, 345 kV in the transmission line. Consult Examples 12.1 and 12.2 for $\mathbf{Z}_{bus}^{(0)}$, $\mathbf{Z}_{bus}^{(1)}$ and $\mathbf{Z}_{bus}^{(2)}$.

Solution:

For line ②–③:

$$Z_1 = j0.15 \text{ per unit} \qquad Z_2 = j0.15 \text{ per unit} \qquad Z_0 = j0.50 \text{ per unit}$$

$$I_{23} = \left|\frac{S}{V}\right|\angle -\theta = \frac{80/100}{1.0}\angle -\cos^{-1}0.85 = 0.8\angle -31.79° \text{ per unit}$$

From Eqs. (12.28) and (12.32),

$$Z_{pp'}^{(1)} = Z_{pp'}^{(2)} = \frac{-Z_1^2}{Z_{22}^{(1)} + Z_{33}^{(1)} - 2Z_{23}^{(1)} - Z_1} = \frac{-(j0.15)^2}{j(0.1696 + 0.1696 - 2 \times 0.1104 - 0.15)}$$
$$= j0.7120 \text{ per unit}$$

$$Z_{pp'}^{(0)} = \frac{-Z_0^2}{Z_{22}^{(0)} + Z_{33}^{(0)} - 2Z_{23}^{(0)} - Z_0} = \frac{-(j0.50)^2}{j(0.1999 + 0.1999 - 2 \times 0.0701 - 0.50)}$$
$$= j1.0399 \text{ per unit}$$

(a) One open conductor fault:

From Eq. (12.38),

$$V_a^{(0)} = V_a^{(1)} = V_a^{(2)} = \frac{I_{23}}{\frac{1}{Z_{pp'}^{(0)}} + \frac{1}{Z_{pp'}^{(1)}} + \frac{1}{Z_{pp'}^{(2)}}}$$

$$= \frac{0.8 \angle -31.79°}{\frac{1}{j0.7120} + \frac{1}{j0.7120} + \frac{1}{j1.0399}} = 0.2122 \angle 58.21° \text{ per unit}$$

From Eq. (12.27),

$$\Delta V_3^{(1)} = \Delta V_3^{(2)} = \frac{Z_{32}^{(1)} - Z_{33}^{(1)}}{Z_1} V_a^{(1)}$$

$$= \frac{j0.1104 - j0.1696}{j0.15} \times 0.2122 \angle 58.21° = 0.0837 \angle -121.79° \text{ per unit}$$

$$\Delta V_3^{(0)} = \frac{Z_{32}^{(0)} - Z_{33}^{(0)}}{Z_0} V_a^{(0)}$$

$$= \frac{j0.0701 - j0.1999}{j0.50} \times 0.2122 \angle 58.21° = 0.0551 \angle -121.79° \text{ per unit}$$

Change in a-phase voltage at bus ③ is

$$\Delta V_3 = \Delta V_3^{(1)} + \Delta V_3^{(2)} + \Delta V_3^{(0)} = (2 \times 0.0837 + 0.0551) \angle -121.79° = 0.2225 \angle -121.79° \text{ per unit}$$

(b) Two open conductor fault:

From Eq. (12.43),

$$V_a^{(1)} = I_{23} \frac{Z_{pp'}^{(1)} \left(Z_{pp'}^{(2)} + Z_{pp'}^{(0)} \right)}{Z_{pp'}^{(1)} + Z_{pp'}^{(2)} + Z_{pp'}^{(0)}}$$

$$= 0.8 \angle -31.79° \times \frac{j0.7120 \times (j0.7120 + j1.0399)}{j0.7120 + j0.7120 + j1.0399} = 0.4050 \angle 58.21° \text{ per unit}$$

$$V_a^{(2)} = I_{23} \frac{-Z_{pp'}^{(1)} Z_{pp'}^{(2)}}{Z_{pp'}^{(1)} + Z_{pp'}^{(2)} + Z_{pp'}^{(0)}}$$

$$= 0.8 \angle -31.79° \times \frac{-j0.7120 \times j0.7120}{j0.7120 + j0.7120 + j1.0399} = 0.1646 \angle -121.79° \text{ per unit}$$

$$V_a^{(0)} = I_{23} \frac{-Z_{pp'}^{(1)} Z_{pp'}^{(0)}}{Z_{pp'}^{(1)} + Z_{pp'}^{(2)} + Z_{pp'}^{(0)}}$$

$$= 0.8 \angle -31.79° \times \frac{-j0.7120 \times j1.0399}{j0.7120 + j0.7120 + j1.0399} = 0.2404 \angle -121.79° \text{ per unit}$$

From Eq. (12.27):

$$\Delta V_3^{(1)} = \frac{j0.1104 - j0.1696}{j0.15} \times 0.4050 \underline{/58.21°} = 0.1598 \underline{/-121.79°} \text{ per unit}$$

$$\Delta V_3^{(2)} = \frac{j0.1104 - j0.1696}{j0.15} \times 0.1646 \underline{/-121.79°} = 0.0650 \underline{/58.21°} \text{ per unit}$$

$$\Delta V_3^{(0)} = \frac{j0.0701 - j0.1999}{j0.50} \times 0.2404 \underline{/-121.79°} = 0.0624 \underline{/58.21°} \text{ per unit}$$

Change in a-phase voltage at bus ③ is

$$\Delta V_3 = \Delta V_3^{(1)} + \Delta V_3^{(2)} + \Delta V_3^{(0)} = (0.1598 - 0.0650 - 0.8624) \underline{/-121.79°} = 0.0324 \underline{/-121.79°} \text{ per unit}$$

Chapter 13 Problem Solutions

13.1 For a generating unit the fuel input in millions of Btu/h is expressed as a function of output P_g in megawatts by $0.032P_g^2 + 5.8P_g + 120$. Determine

   (a)   the equation for incremental fuel cost in dollars per megawatthour as a function of P_g in megawatts based on a fuel cost of \$2 per million Btu.

   (b)   the average cost of fuel per megawatthour when $P_g = 200$ MW.

   (c)   the approximate additional fuel cost per hour to raise the output of the unit from 200 MW to 201 MW. Also find this additional cost accurately and compare it with the approximate value.

Solution:

   (a)   The input-output curve in dollars per MWh is

$$
\begin{aligned}
f &= (0.032P_g^2 + 5.8P_g + 120) \times 2 \\
&= 0.064P_g^2 + 11.6P_g + 240 \text{ \$/MWh}
\end{aligned}
$$

The incremental fuel cost is

$$\frac{df}{dP_g} = 0.128P_g + 11.6 \text{ \$/MWh}$$

   (b)   The average cost of fuel when $P_g = 200$ MW is

$$\left.\frac{f}{P_g}\right|_{P_g=200} = \frac{0.064(202)^2 + 11.6(200) + 240}{200} = 25.6 \text{ \$/MWh}$$

   (c)   The approximate incremental cost for an additional 1 MW generation when $P_g = 200$ MW is

$$\left.\frac{df}{dP_g}\right|_{P_g=200} = 0.128(200) + 11.6 = 37.2 \text{ \$/h}$$