# Homework 1

**1.** The combination of the three fundamental constants of nature ( that is G, c, and h) forms a quantity with dimensions of time

$$t_p = G^x h^y c^z$$

where G = 6.67 $\cdot 10^{-11}$ $m^3/(kg \cdot s^2)$, h = 6.63 $\cdot 10^{-34}$ kg $\cdot m^2/s$ and c = 3.00 * $10^8$ m/s. What are the values of x, y, and z ? This quantity , $t_p$, is called the Planck time.

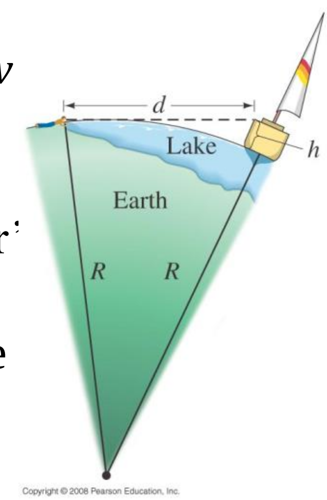
2. Find the answer for the following.

$$(a)\frac{d}{dx}\left[\cot\sqrt{3x}\right] \qquad (b)\frac{d}{dx}\left(\sqrt{2x^2-1}\cdot\cos\sqrt{2x}\right) \qquad (c)\frac{d}{dx}\left(3x^2+x-7\right)^{3/2}$$

(d) (i) Use Excel to sketch the graph of the function ( ***v(t)*** )

$$v(t) = \frac{mg}{b}\left(1-e^{-\frac{b}{m}t}\right) \quad where \quad \frac{mg}{b} = 2.0(m/s) \;;\; \frac{b}{m} = 2.0\left(s^{-1}\right)$$

(ii) Find the derivative of the above function (***dv/dt)*** and show that $\frac{dv}{dt} = g - \frac{b}{m}v$




**3.** Many sailboats are moored at a marina 4.4 km away on the opposite side of a lake. You stare at one of the sailboats because, when you are lying flat at the water's edge, you can just see its deck but none of the side of the sailboat. You then go to that sailboat on the other side of the lake and measure that the deck is 1.5 m above the level of the water. Using Fig. , where ***h*** =1.5m, estimate the radius *R* of the Earth.

**Problem 1**

The combination of the three fundamental constants of nature ( that is G, c, and h) forms a quantity with dimensions of time

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**Sol:**

dimensional analysis

G $\rightarrow \left[\frac{L^3}{MT^2}\right]$

h $\rightarrow \left[\frac{ML^2}{T}\right]$

c $\rightarrow \left[\frac{L}{T}\right]$

$$[T] = [\frac{L^3}{MT^2}]^x \bullet [\frac{ML^2}{T}]^y \bullet [\frac{L}{T}]^z$$

$$\rightarrow [L]: 3x + 2y + z = 0 \quad \rightarrow$$

$$[M]: -x + y = 0$$

$$[T]: -2x - y - z = 1$$

$$x = \frac{1}{2}$$

$$y = \frac{1}{2}$$

$$z = -\frac{5}{2}$$

**Problem 2**

Find the answer for the following.

$$(a)\frac{d}{dx}\left[\cot\sqrt{3x}\right]$$

[Key:利用chain rule ]

Sol:

$$\frac{d}{dx}[\cot\sqrt{3x}] \qquad \text{let } u=\sqrt{3x}$$

$$=\frac{d}{du}[\cot u]\bullet\frac{du}{dx}$$

$$=-\csc^2 u\bullet\frac{d}{dx}\sqrt{3x}$$

$$=-\csc^2\sqrt{3x}\bullet\frac{d\sqrt{3x}}{d3x}\bullet\frac{d3x}{dx}$$

$$=-\csc^2\sqrt{3x}\bullet\frac{1}{2}\bullet(3x)^{-\frac{1}{2}}\bullet 3$$

$$=-\frac{3}{2\sqrt{3x}}\csc^2\sqrt{3x}$$

$$(b)\frac{d}{dx}(\sqrt{2x^2-1}\cdot\cos\sqrt{2x})$$

Sol:

$$\frac{d}{dx}(\sqrt{2x^2-1}\bullet\cos\sqrt{2x})$$

$$=\frac{d}{dx}(\sqrt{2x^2-1})\bullet\cos\sqrt{2x}+\frac{d}{dx}(\cos\sqrt{2x})\bullet\sqrt{2x^2-1}$$

$$=\cos\sqrt{2x}\bullet\frac{d}{d(2x^2-1)}(\sqrt{2x^2-1})\bullet\frac{d}{dx}(2x^2-1)+\sqrt{2x^2-1}\bullet\frac{d}{d\sqrt{2x}}\cos\sqrt{2x}\bullet\frac{d\sqrt{2x}}{dx}$$

$$=\cos\sqrt{2x}\bullet\frac{1}{2\sqrt{2x^2-1}}\bullet 4x+\sqrt{2x^2-1}\bullet(-\sin\sqrt{2x})\bullet\frac{\sqrt{2}}{2\sqrt{x}}$$

**Problem 2**

$$(c)\ \frac{d}{dx}\left(3x^2+x-7\right)^{3/2}$$

Sol:

$$\frac{d}{dx}(3x^2+x-7)^{\frac{3}{2}}$$

$$=\frac{d}{d(3x^2+x-7)}(3x^2+x-7)^{\frac{3}{2}}\bullet\frac{d}{dx}(3x^2+x-7)$$

$$=\frac{3}{2}\bullet(3x^2+x-7)^{\frac{1}{2}}\bullet(6x+1)$$

**Problem 2**

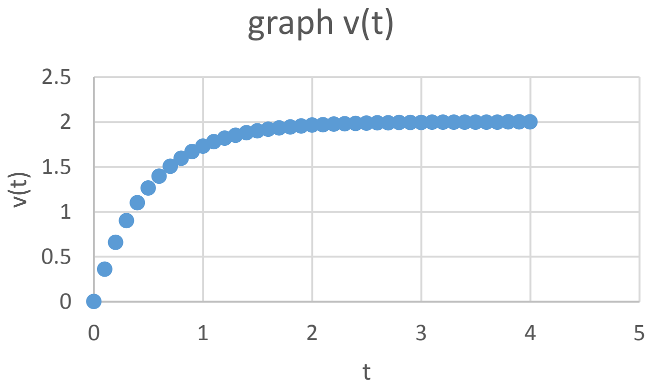
(d) (i) Use Excel to sketch the graph of the function ( ***v(t)*** )

$$v(t) = \frac{mg}{b}\left(1 - e^{-\frac{b}{m}t}\right) \quad where \quad \frac{mg}{b} = 2.0(m/s) \ ; \frac{b}{m} = 2.0\left(s^{-1}\right)$$

(ii) Find the derivative of the above function (***dv/dt***) and show that $\frac{dv}{dt} = g - \frac{b}{m}v$

(i)

Sol:




t➔ ∞

v➔2

(ii)

Sol:

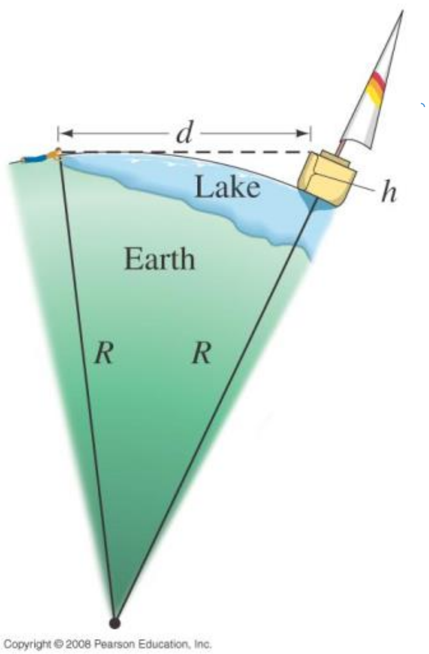
$$\frac{dv}{dt} = \frac{d}{dt}\frac{mg}{b}(1 - e^{-\frac{b}{m}t}) \qquad \text{Let } x = -\frac{b}{m}t$$

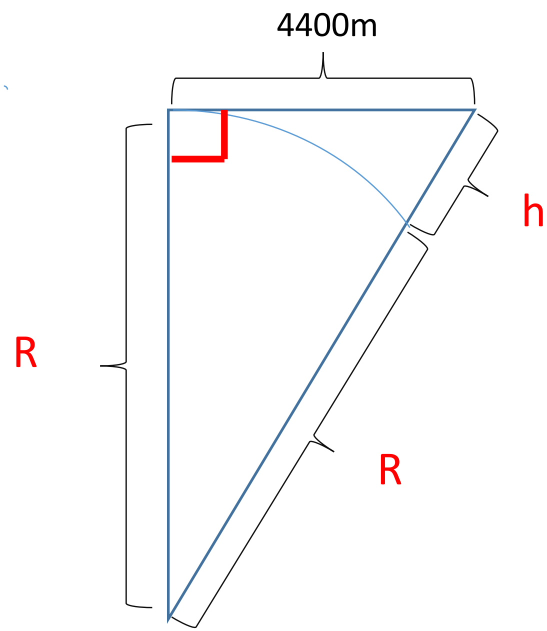
$$= \frac{d}{dt}\frac{mg}{b}(1 - e^{x}) = \frac{d}{dx}\frac{mg}{b}(1 - e^{x}) \bullet \frac{dx}{dt}$$

$$= \frac{mg}{b} \bullet (-e^{x}) \bullet (-\frac{b}{m}) = g \bullet e^{-\frac{b}{m}t} = g - \frac{b}{m}v$$

**Problem 3**

Many sailboats are moored at a marina 4.4 km away on the opposite side of a lake. You stare at one of the sailboats because, when you are lying flat at the water's edge, you can just see its deck but none of the side of the sailboat. You then go to that sailboat on the other side of the lake and measure that the deck is 1.5 m above the level of the water. Using Fig. below, where ***h*** =1.5m, estimate the radius *R* of the Earth.




$$R^2 + 4400^2 = (R + 1.5)^2$$

$$3R = 19359997.75$$

$$R \cong 6.45 \times 10^6\, m$$