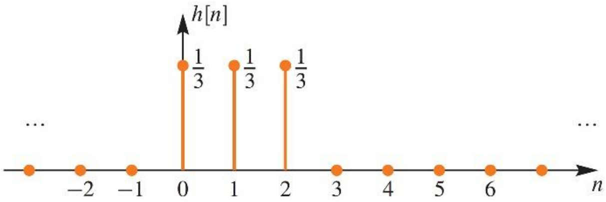
***Introduction to Digital Signal Processing Final Examination, June 22, 2021***

1. The impulse response $h[n]$ of an FIR filter is shown below. (10%)
   (a) Draw the implementation of this system as a block diagram in direct form. (5%)
   (b) If an input x[$n$] = {2, 1, -1, 1, 2} is applied to it, obtain the output. (5%)




2. Suppose that three systems are connected in cascade. In other words, the output of $S_1$ is the input of $S_2$, and the output of $S_2$ is the input of $S_3$. The three systems are specified by the following difference equations: (10%)

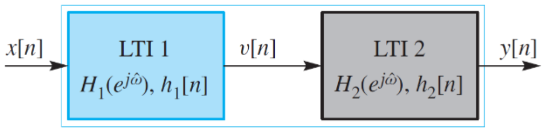
$$S_1: \quad y_1[n] = x_1[n] - x_1[n-1]$$

$$S_2: \quad y_2[n] = x_2[n] + x_2[n-2]$$

$$S_3: \quad y_3[n] = x_3[n-1] + x_3[n-2]$$

   (a) Determine the impulse response $h[n]$ of the overall system. (5%)
   (b) If a new system is defined as $(S_1 + S_2) * S_3$ determine its $h[n]$. Thus $x_1[n] = x_2[n] = x[n]$, $x_3[n] = y_2[n] + y_1[n]$, and $y[n] = y_3[n]$. (5%)

3. For the following cascade configuration, the first system is a 4-point moving average and the second system is a first difference. (15%)




   (a) Obtain a single difference equation that relates $y[n]$ to $x[n]$ for the overall cascade system. (5%)
   (b) If the input is $x[n] = 10u[n]$, determine the output $y[n]$. (5%)
   (c) Determine the frequency response function of the overall cascade system. (5%)

4. (a) Determine the DTFT of each of the following sequence: (5%)

$$x_1[n] = 2u[n-2] - 2u[n-7] = \begin{cases} 2 & 2 \le n \le 6 \\ 0 & otherwise \end{cases}$$

   (b) Determine the inverse DTFT of the following transform: (5%)

$$V_1(e^{j\hat{\omega}}) = \begin{cases} 1 & |\hat{\omega}| \le 0.3\pi \\ 0 & 0.3\pi < |\hat{\omega}| \le \pi \end{cases}$$

(c) Evaluate the following operation using the denoted properties: (5%)

$$x_3[n] = \frac{\sin(0.25\pi n)}{3\pi n} * \frac{\sin(0.45\pi n)}{5\pi n} = ? \text{ (Convolution property of the DTFT)}$$

5. Suppose that the following continuous-time signal (10%)

   $x(t) = 4\cos(35\pi t) + 6\cos(15\pi t - 0.5\pi)$

   is sampled with rate $f_s = 50$ Hz to obtain the discrete-time signal $x[n]$ which is periodic with period $N$, and we want to determine the DFS representation of $x[n]$.

   (a) Determine the values and indices $k$ of the nonzero Fourier Series coefficients $\{a_k\}$ for the DFS summation. Recall that the range of the DFS summation is from $-M$ to $M$, where $M \le N/2$. Express each nonzero $a_k$ value in polar form.

   (b) If $a_k = \sin(\frac{\pi k}{10}) - \cos(\frac{\pi k}{5})$, determine the new $x[n]$.

6. An LTI system is described by the difference equation (10%)

   $$y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2])$$

   (a) Determine the system function $H(z)$ for this system.

   (b) What is the output if the input is

   $x[n] = 4 + \cos[0.25\pi(n-1)] - 3\cos[(2\pi/3)n]$

7. Determine the inverse z-transform of the following: (10%)

   (a) $H_a(z) = 2 - 0.5z^{-1} - 3z^{-3} + 4z^{-4}$

   (b) $H_b(z) = \frac{1+z^{-2}}{1+0.8z^{-1}+0.64z^{-2}}$

8. Given an IIR filter defined by the difference equation (10%)

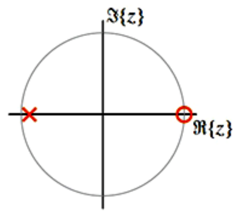
   $$y(n) = \frac{1}{4}y[n-1] + x[n]$$

   (a) When the input to the system is unit-step sequence, $u[n]$, determine the functional form for the output signal $y[n]$. Use the inverse z-transform method. Assume that the output signal $y[n]$ is zero for $n < 0$.

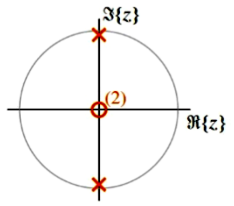
   (b) Find the output when $x[n]$ is a complex exponential that start at $n = 0$:

   $x(n) = e^{j(\pi/6)n}u[n]$

9. In the following figures, match each pole-zero plot (PZ 1-2) with the correct one of five possible frequency responses (A-E) and one of five possible impulse responses (J-N). Explain your reason for each of the answers. (10%)




Pole-Zero Plot #1




Pole-Zero Plot #2