General Physics (II) **期末考** **Jun. 22, 2018**

試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(50 points) (所有題目必須有計算過程,否則不予計分)

1. (12pts) The direction (in general) of a electromagnetic plane wave can be expressed by the wave vector $\vec{k} = (k_x, k_y, k_z)$. The following equation describes the magnetic field of a plane wave traveling in free space with wave number $\boldsymbol{k} = |\vec{\boldsymbol{k}}|$(in unit of $\mathbf{m^{-1}}$)

$$\vec{B}(\vec{r},t) = B_0 \hat{z} \sin(\vec{k}\cdot\vec{r} - \omega t) = 2\times 10^{-7}(T)\hat{z}\sin(x - \sqrt{3}y - \omega t)$$

Answer the following questions in SI unit. (Note: $\vec{r} = (x, y, z)$, $c = 3\times 10^8$ m/s, $\mu_0 = 4\pi \times 10^{-7}$ $m/\boldsymbol{A}$, and $\varepsilon_0\mu_0 = 1/c^2$)

(a) Find the wave number (k), wavelength (λ), and the angular frequency (ω) of this plane wave.

(b) What is the direction of propagation of this plane wave?

(c) The electric field of this wave plane can be written as $\vec{E}(\vec{r},t) = \vec{E}_0 \sin(\vec{k}\cdot\vec{r} - \omega t)$, where $\vec{E}_0 = (E_{0x}, E_{0y}, E_{0z})$ Find the values of $\boldsymbol{E_{0x}}$ and $\boldsymbol{E_{0y}}$.

(d) Find the Poynting vector $\vec{S}$ (magnitude and direction), and the intensity (I = <S>) of this plane wave.

2. (8pts) As shown in Fig. 1., an AC circuit with a power supply of voltage V_{in}, is connected to a capacitor C and a resistor R. Assume $I_{in} = I_0 \cos(\omega t)$. Applying the phasor method to find

(a) $V_C(t)$, $V_R(t)$, $V_{in}(t)$

(b) the ratio V_{out0}/V_{in0} as a function of ω, where the V_{in0} and V_{out0} are the voltage amplitudes of the power supply and the resistor, respectively.

(c) the ratio V_{out0}/V_{in0} as $\omega \rightarrow 0$, and $\omega \rightarrow \infty$.

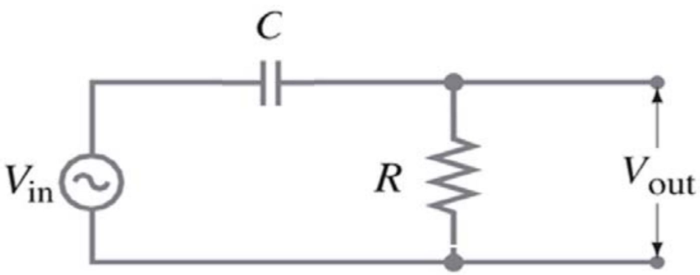


Fig 1

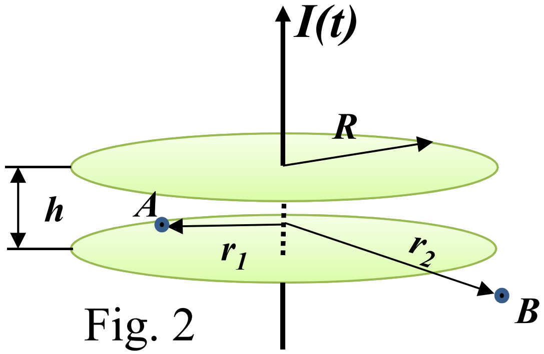


Fig. 2

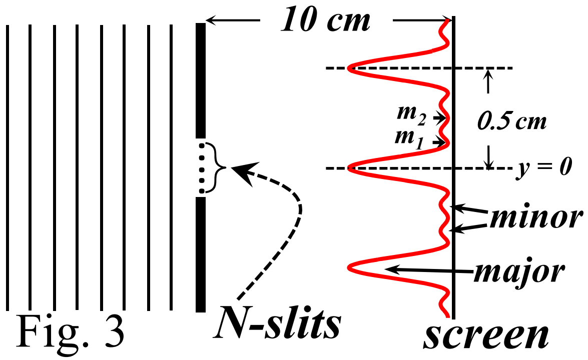


Fig. 3

3. (15 pts) A circular capacitor (as shown in Fig. 2) with spacing ***h*** and radius ***R*** is connected to a circuit. The current $I(t)$ is zero for $t < 0$ and $t > T$ and $I(t) = I_0\,(t/T)^2$ for $0 \le t \le T$. At $t = 0$, there is no charge on the capacitor. Let point ***A*** locates inside the capacitor ($r_1 < R$) and point ***B*** locates outside the capacitor ($r_2 > R$). Ignoring the edge effect (i.e. $E(r) = 0$, *for* $r > R$).

(A)(4 pts) Find the direction (up or down) and the magnitude of the electric field $E\,(r_1,t)$ at point ***A*** for $0 < t < T$.

(B) (4 pts) Find the the direction (clockwise, or counter clockwise viewed from the top) and the magnitude of magnetic fields $B_1(r_1,\, t)$ at point ***A*** and

(C) (4 pts) $B_2(r_2,\, t)$ at point ***B*** (direction and magnitude) due to the time-varying electric field. (You need to draw Ampere's loop for each case.)

(d) (3 pts) Plot the magnitude of the magnetic field $B(r,\, t{=}T)$ as a function of ***r***, specify the magnitude of $B(R)$.

4. (15 pts) As shown in Fig. 3, a plane electromagnetic wave travels in the direction normal to a screen with ***N***-parallel slits, and the spacing between neighboring slits is $10.64\mu m$. The wave emitted from each slit forms an interference pattern on a screen ***10 cm*** away. The spacing between the central maximum intensity peak and the neighboring peak is ***0.5 cm***.

(A) (5 pts) Determine the number of the slits and the wavelength of the electromagnetic wave.

(B) (5 pts) Draw the phasor diagram of the E-fields emitted from each slit and arriving at the intensity minimum m_1 and m_2 on the screen.

(C) (5 pts) If the width of each slit is $2.00\ \mu m$, how many major bright fringes (ignore the minor fringes) will appear in the central bright band due to the effect of the diffraction from individual slits?.

II.選擇題(54 points)

1. (5 pts) Fig. 4 shows the voltages of the resistor (V_R), the power supply (V_s), and the inductor (V_L) in the RLC circuit shown in Fig. 5. (i) which curve is V_R ? and (ii) is the frequency ω of the circuit above or below resonance frequency ($\omega_0 = 1/\sqrt{LC}$)?
   (A) Curve *A* is V_R ,and $\omega > \omega_0$; (B) Curve *B* , and $\omega > \omega_0$;
   (C) Curve *C* , and $\omega > \omega_0$; (D) Curve *A*, and $\omega < \omega_0$;
   (E) Curve *B*, and $\omega < \omega_0$; (F) Curve *C*, and $\omega < \omega_0$;
   (G) None of above is correct.

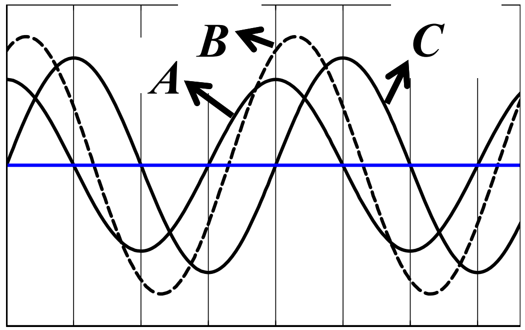


Fig. 4

2. (5 pts) Fig. 6 shows V_L (inductor) and $V_{Power\ Supply}$ in the RLC circuit shown in Fig. 5. Which of the following action will make the circuit to reach resonance?
   (A) Increasing *L*; (B) Decreasing *L*; (C) Increasing *C*;
   (D) Decreasing **C**; (E) *A* or *C*; (F) *A* or *D* ;
   (G) *B* or *C*; (H) *B* or *D*; (J) None of above.

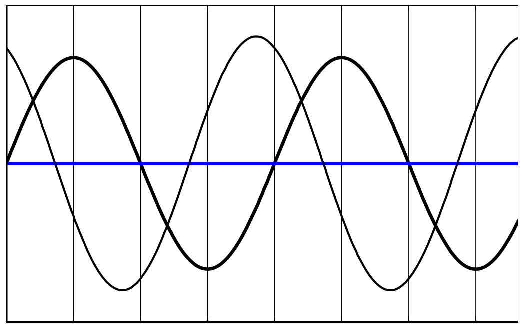


Fig. 6

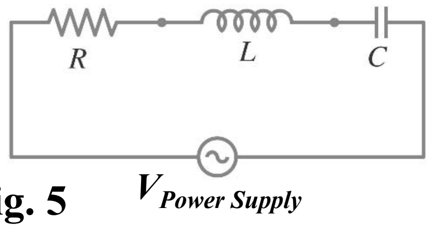


Fig. 5

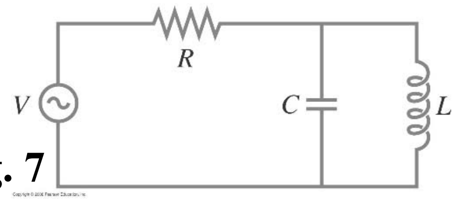


Fig. 7

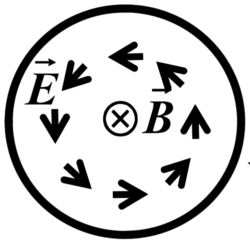


Fig. 8(b)

Fig. 8(a)

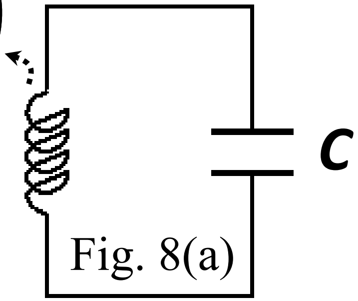


Fig. 9(a)

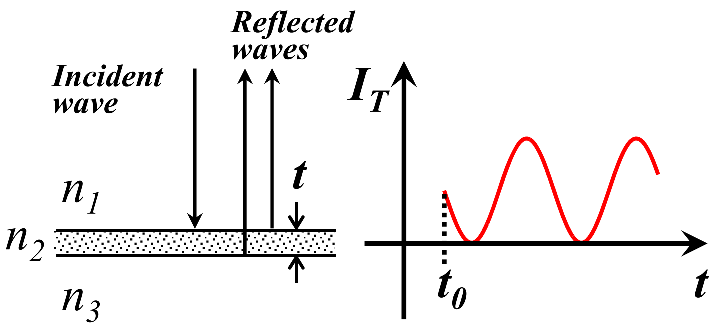


Fig. 9(b)

3. (5 pts.) For a ***RLC*** circuit shown in Fig. 7. The AC voltage source is $V(t) = V_0 \sin\omega t$. The current through the inductor, capacitor and resistor are I_L, I_C and I_R, respectively. Let V_0=1V, $\omega=10^4\ s^{-1}$, C=1.0 μF, and L=0.01H. What is the magnitude of the current $I_{R,0}$ **(in SI unit, *A*)**?

(A) 0 (B) $0 < I_{R,0} \leq 0.005$ (C) $0.005 < I_{R,0} \leq 0.01$ (D) $0.01 < I_{R,0} \leq 0.015$ (E) $0.015 < I_{R,0} \leq 0.02$ (F) $0.02 < I_{R,0} \leq 0.025$ (G) $0.025 < I_{R,0} \leq 0.03$ (H) $0.03 < I_{R,0} \leq 0.04$ (J) $0.04 < I_{R,0}$

4. (5 pts) Fig.8(a) shows a ***LC*** circuit, the top view of the electric and magnetic field inside the **inductor** is shown in Fig.8(b) at some time ***t.*** Which of the following statement is correct for the directions of the electric and magnetic fields inside the capacitor (top view)?

(A) ***B***-field is counter-clock-wise (c.c.w.) and ***E***-field is pointing out of page (up in Fig.8(a)).
(B) ***B***-field is clock-wise (c.w.) and ***E***-field is pointing out of page.
(C) ***B***-field is c.c.w. and ***E***-field is pointing into page (down in Fig.8(a)).
(D) ***B***-field is c.w. and ***E***-field is pointing into page.
(E) both of (A) and (B) are possible; **(F)** both of (C) and (D) are possible,
(G) both of (A) and (C) are possible; **(H)** both of (B) and (D) are possible,
(I) all (A) to (D) are possible

5. (5pts) As shown in Fig. 9(a) , a plane electromagnetic wave is traveling in a direction normal to a thin film with reflective index n_2 and initial thickness $t_0 = \lambda/(8n_2)$, where λ is the wavelength of this wave in air. The reflective index is n_1 for the medium on the top of the film, and n_3 the medium on the other side. If Fig. 9(b) shows the total wave intensity I_T resulted from the interference of the wave reflected from the n_1-n_2 interface with that reflected from the n_2-n_3 interface as the thickness of the film increases, which of the following statement is correct?

(A) $n_1 > n_2 > n_3$ (B) $n_2 > n_1 > n_3$ (C) $n_3 > n_2 > n_1$ (D) $n_3 > n_1 > n_2$
(E) **(A) and (C)** (F) **(B) and (D)** (G) **(A) and (B)**
(H) **(C) and (D)** (I) **(B),(C), and (D)** (J) **(A),(B), and (D)**

6. (5pts) The current amplitude $\boldsymbol{I}$ versus driving angular frequency ω_d for a driven RLC series circuit (Fig. 5) is given in Fig.10, where the vertical axis scale is set by I_s = 4.00 A. The inductance is 200 μH, and the emf amplitude is 8.0 V. C and R are (A) 2 μF and 2 Ω; (B) 4 μF and 2 Ω; (C) 6 μF and 2 Ω; (D) 8 μF and 2 Ω; (E) 10 μF and 2 Ω; (F) 10 μF and 4 Ω; (G) 10 μF and 6 Ω; (H) 10 μF and 8 Ω; (J) 10 μF and 10 Ω, respectively.

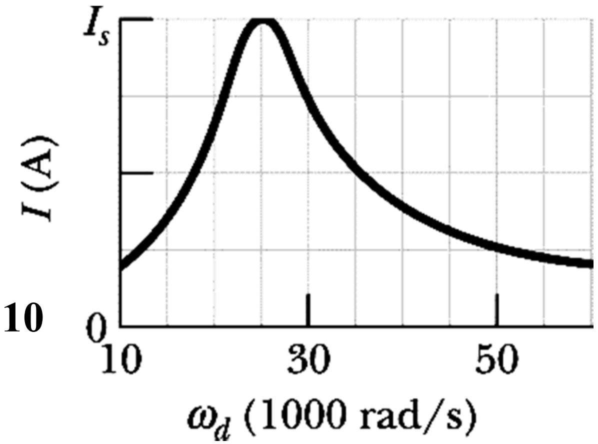


Fig. 10

Multiple Choice Questions:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
C	E	A	C	E	D	H	C	B	E
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
D	H	H	A						

1. (12pts) The direction (in general) of a plane can be expressed by the wave vector $\vec{k} = (k_x, k_y, k_z)$. The magnetic field of a plane wave (in free space) with the form of

$$\vec{B}(\vec{r}, \mathrm{t}) = B_0\hat{z}\ \sin\left[x - \sqrt{3}y - \omega t\right] = (2 \times 10^{-7} T)\ \hat{z}\ \sin\left[k(\hat{k} \cdot \vec{r}) - \omega t\right]$$

describes the wave traveling in the direction of $\hat{\boldsymbol{k}}$ with wave number $\boldsymbol{k}$ (in unit of $\mathbf{m^{-1}}$) and the position vector $\vec{r} = (x.y.z)$. Answer the following questions including correct unit. Note that $c = 3 \times 10^8$ m/s, $\mu_0 = 4\pi \times 10^{-7}$ Tm/A, and $\varepsilon_0\mu_0 = 1/c^2$.

(a) Find the wave number (k), wavelength (λ), and the angular frequency (ω) of this plane wave.

(b) What is the direction of this plane wave propagating?

(c) The electric field of this plane can be written as $\vec{\boldsymbol{E}}(\vec{\boldsymbol{r}}, \boldsymbol{t}) = \vec{\boldsymbol{E}}_{\mathbf{0}}\boldsymbol{sin}[\boldsymbol{k}(\hat{\boldsymbol{k}} \cdot \vec{\boldsymbol{r}}) - \boldsymbol{\omega t}]$

Find the values of $\boldsymbol{E_{0x}}$ and $\boldsymbol{E_{0y}}$ in SI unit.

(d) Find the Poynting vector $\vec{\boldsymbol{S}}$ (magnitude and direction), and the intensity (I = <S>) of this plane wave.

$$\vec{B}(\vec{r},t) = B_0\hat{z}\sin(\vec{k}\cdot\vec{r} - \omega t) = 2\times10^{-7}(T)\hat{z}\sin(x - \sqrt{3}y - \omega t)$$

$$k\hat{k} = 2(\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0)$$

(a) $k = 2$ (1/m); $\lambda = \pi$ (m); ω=kc=2c=6 $\times 10^8$ (rad/s)

1 1 1

$E_{0y} = 30$ (V/m)

$E_{0x} = 30\sqrt{3}$ (V/m)

(b) $\hat{k} = (\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0)$ 1

(c) $E_0 = B_0$c= (2x10^{-7})(3x10^8)=60 V/m; 1

$\hat{E}\times\hat{B} = \hat{k} \longrightarrow \hat{E} = (\frac{\sqrt{3}}{2}, \frac{1}{2}, 0)$ 1 $\Rightarrow$ $E_{0x} = 30\sqrt{3}(V/m)$ 1

$E_{0y} = 30(V/m)$ 1

(d)

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0} = \frac{60\times2\times10^{-7}}{4\pi\times10^{-7}}(\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0)\sin^2(x - \sqrt{3}y - \omega t) = \frac{30}{\pi}(\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0)\sin^2(x - \sqrt{3}y - \omega t)(\mathrm{W/m^2})$$

2

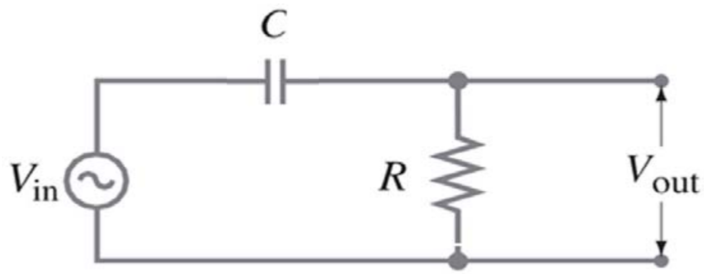
the intensity $I = \langle S \rangle = \frac{30}{\pi}\frac{1}{2} = \frac{15}{\pi}(\mathrm{W/m^2})$ 2

2. (8pts) As shown in Fig., an *AC* circuit with a power supply of voltage V_{in}, is connected to a capacitor C and a resistor R. Assume $I_{in} = I_0 \cos(\omega t)$. Applying the phasor method to find

(a) $V_C(t)$, $V_R(t)$, $V_{in}(t)$

(b) the ratio $V_{out\,0}/V_{in\,0}$ as a function of ω, where the $V_{in\,0}$ and $V_{out\,0}$ are the voltage amplitudes of the power supply and the resistor, respectively.

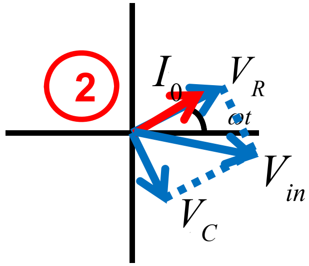
(c) the ratio $V_{out\,0}/V_{in\,0}$ as $\omega \to 0$, and $\omega \to \infty$.



1 1

(a) $V_C(t) = \dfrac{I_0}{\omega C}\cos(\omega t - \dfrac{\pi}{2});\quad V_R(t) = I_0 R\cos(\omega t)$

2



$$V_{in}(t) = I_0\sqrt{R^2 + \frac{1}{(\omega C)^2}}\cos(\omega t - \tan^{-1}\frac{1}{\omega RC})$$

1

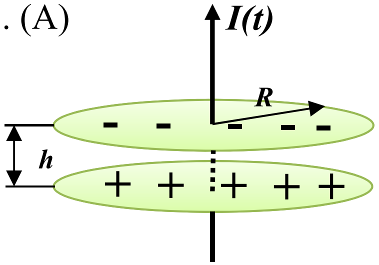
(b) $\dfrac{V_{out}}{V_{in}} = \dfrac{R}{\sqrt{R^2 + \dfrac{1}{(\omega C)^2}}}$ 1 $,or\quad \dfrac{V_{out}}{V_{in}} = \dfrac{1}{\sqrt{1 + \dfrac{1}{(\omega RC)^2}}}$

(c)

$\dfrac{V_{out}}{V_{in}} \to 0, as\ \omega \to 0$ 1

$\dfrac{V_{out}}{V_{in}} \to 1, as\ \omega \to \infty$ 1

3. (A)

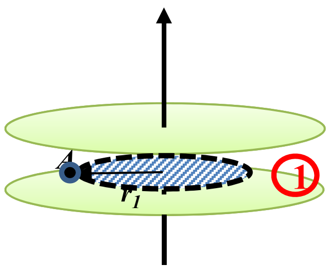


$$Q(t)=\int I(t)dt = I_0\cdot\frac{t^3}{3T^2} \quad \text{for } 0\le t\le T$$ 1

$$E(\vec{r}_1,t)=\frac{\sigma}{\varepsilon_0}=\frac{Q(t)/A}{\varepsilon_0}=\frac{I_0\left(t^3/T^2\right)}{3\varepsilon_0\pi R^2}$$ 2

Direction: up 1

(B) $0\le r_1\le R$ case:



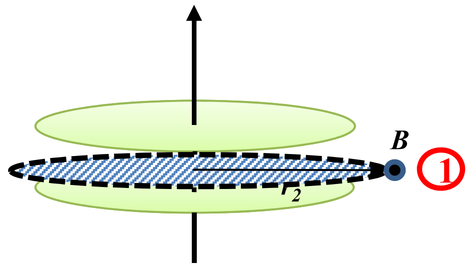
$$\oint \vec{B}\cdot d\vec{l} = \mu_0\left(I_{inc.}+\varepsilon_0\frac{d\Phi_E}{dt}\right)=\mu_0\varepsilon_0\frac{d\Phi_E}{dt}$$

$$2\pi rB=\mu_0\varepsilon_0\frac{dE(t)}{dt}\pi r^2=\mu_0 I_0\cdot\frac{t^2}{T^2}\cdot\frac{r^2}{R^2}$$

$$B=\frac{\mu_0 I_0}{2\pi}\cdot\frac{r}{R^2}\cdot\frac{t^2}{T^2}$$ 2

Direction: c.c.w. from top view 1

(C) $R\le r_2$ case:



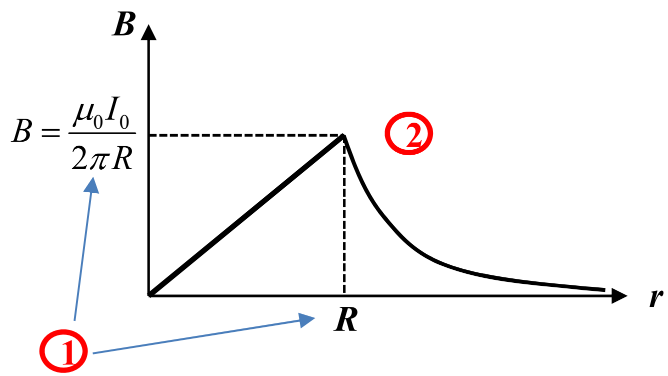
$$\oint \vec{B}\cdot d\vec{l} = \mu_0\left(I_{inc.}+\varepsilon_0\frac{d\Phi_E}{dt}\right)$$

$$2\pi rB=\mu_0\varepsilon_0\frac{dE(t)}{dt}\pi R^2=\mu_0 I_0\cdot\frac{t^2}{T^2}$$

$$B=\frac{\mu_0 I_0}{2\pi r}\cdot\frac{t^2}{T^2}\cdot$$ 2

1 Direction: c.c.w. from top view

(D)



4. (15 pts) As shown in Fig. 3, a plane electromagnetic wave travels in the direction normal to a screen with ***N***-parallel slits, and the spacing between neighboring slits is ***10.64μm***. The wave emitted from each slit forms an interference pattern on a screen ***10 cm*** away. The spacing between the central maximum intensity peak and the neighboring peak is ***0.5 cm***.

(A) (5 pts) Determine the number of the slits and the wavelength of the electromagnetic wave.

(B) (5 pts) Draw the phasor diagram of the E-fields emitted from each slit and arriving at the intensity minimum m_1 and m_2 on the screen.

(C) (5 pts) If the width of each slit is ***2.00 μm***, how many major bright fringes (ignore the minor fringes) will appear in the central bright band due to the effect of the diffraction from individual slits?.

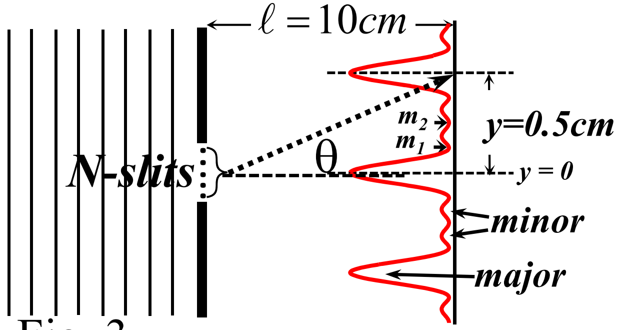


Fig. 3

(A) $N = 4$ 2

For constructive interference,

$d \sin\theta = m\lambda,$ and $\tan\theta = y/\ell$ 1

For θ<<1 ,sin θ ~ θ ~ tan θ,

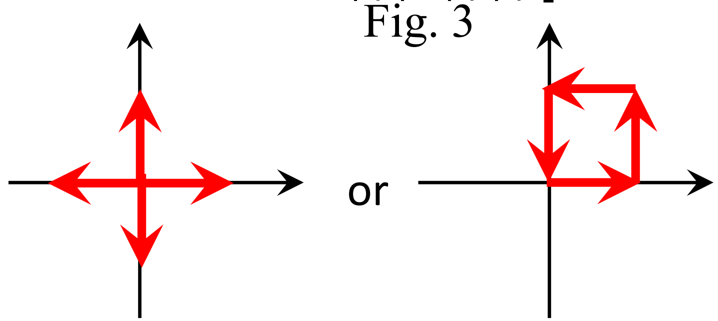
$$\Rightarrow \sin\theta = \frac{\lambda}{d} \approx \frac{y_{1st\ fringe}}{\ell} = \tan\theta$$ 1

$$\Rightarrow \frac{\lambda}{10.64\mu m} = \frac{0.5cm}{10cm}$$

$$\Rightarrow \lambda = \frac{1}{20} \times 10.64\mu m = 0.532\mu m$$ 1

(B) $N = 4$

The minima m_1 occurs at $\phi=\pi/2$,



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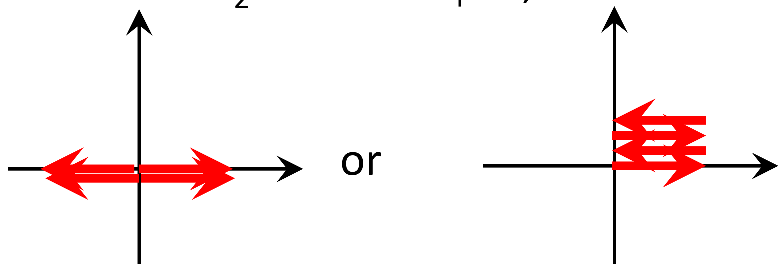
The minima m_2 occurs at $\phi=\pi$,



2

(C) For the first destructive interference of diffraction from individual slits,

$$D\sin\theta = \lambda,$$

For constructive interference from the slits,

$$d\sin\theta = m\lambda,$$

For mth constructive interference of the slits to overlap with the first destructive diffraction fringe,

$$\frac{d\sin\theta}{D\sin\theta} = \frac{m\lambda}{\lambda}, \quad 2 \quad \Rightarrow \frac{d}{D} = m, \quad \Rightarrow m = \frac{10.64\,\mu m}{2.0\,\mu m} = 5.32 \Rightarrow m = 5, \quad 1$$

There are 2*m+1 = 11 major bright fringes.

2