試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(55 points) (所有題目必須有計算過程,否則不予計分)

1. Fig. 1 shows 3 line charge distributions in *x*-*z* plane. The charge densities are λ_0, $-\lambda_0$, and $\lambda_1\cos\theta$ for the charges on lines ***AO*** and ***DO*** and arc ***BC***, where θ is the angle relative to +z-axis, and λ_0 and λ_1 are positive constants, respectively. Find the *x*-, *y*-, *z*-components of the electric field at point ***P*** on the *y*-axis due to (a) (6 pts) line charge ***AO***, (b) (3pts) line charge ***DO***, and (c) (8 pts) line charge ***BC***, and (d) (3 pts) the electrical potential at *P* for this whole system. The coordinates of ***O, A, B, C, D,*** and ***P*** are (*0,0,0*), (*0,0,2R*), (*0,0,R*), (*R,0,0*), (*2R,0,0*), and (*0,y,0*), respectively.

2. (20pts) As shown in Fig. 2, a uniform spherical charge distribution of radius ***a*** (***a=R***) and charge density ***13ρ*** (*ρ*>0) is placed in side a concentric spherical conductor shell with inner and outer radius ***b*** (***b=3R***) and ***c*** (***c=5R***), respectively. Outside of the conductor shell, there is a second charge distribution with density of ***–ρ /13*** in a concentric spherical shell region with inner and outer radius ***d*** (***d=7R***) and ***e*** (***e=8R***). Determine (a) (10pts) the direction and magnitude of the ***E***-field for 0≤ ***r*** < ∞.(b) (10pts) the electric potential ***V*** for 0≤ ***r*** < ∞, assuming V(∞)=0.

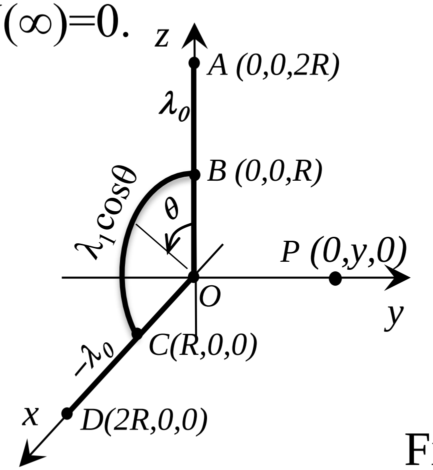


Fig. 1

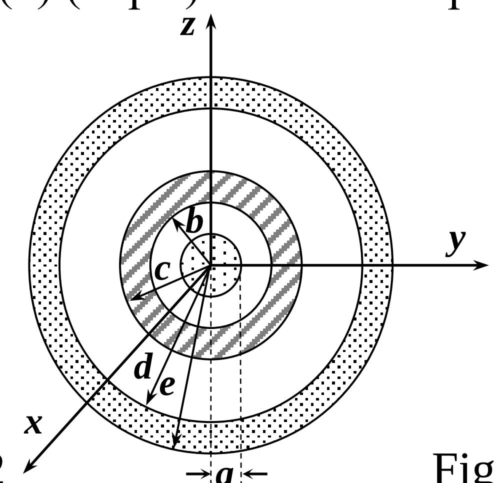


Fig. 2

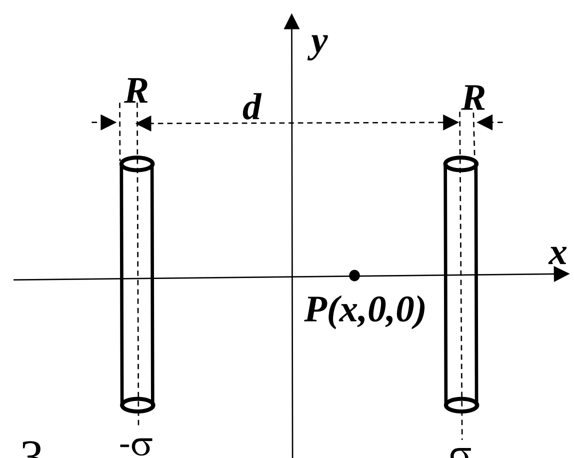


Fig. 3

3. (15 pts)As shown in the Fig. 3, Two infinitely long parallel conducting cylinders, with radius $\boldsymbol{R}$, carry uniform surface charge density $\boldsymbol{\sigma}$ and $-\boldsymbol{\sigma}$. The distance between the centers of the cylinders is $\boldsymbol{d}$. Assume $\boldsymbol{d}$ is large enough that the surface charges are uniformly distributed.
(a) (8 pts) By using the Gauss's Law, find the electric field, in vector form, at the point P (x,0.,0).
(b) (4 pts) Find the electric potential difference between these two conductors.
(c) (3 pts) What is the capacitance of these two cylinders per unit length?

II.選擇題(45 points)

1. (5pts) An electric dipole with dipole moment $\vec{p} = (3\hat{i} + 4\hat{j}) \times 10^{-30} (m \cdot C)$ is placed in an uniform electric field $\vec{E} = 5000\hat{j}(N/C)$.. The electric potential energy of the dipole and the magnitude of the torque acting on it in unit of 10^{-27} $\boldsymbol{m \cdot N}$ are (the potential energy =0, when $\vec{p} \perp \vec{E}$)
(A)15 and 15; (B) 15 and 20; (C) 20 and 15; (D) 20 and 20; (E) 25 and 15; (F) 25 and 20; (G) -15 and 15 (H) -15 and 20; (J) -20 and 15; (K) -20 and 20; (L) -25 and 15; (M) -25 and 20, respectively.

2. (5pts) As shown in the Fig. 4, the capacitor with separation $\boldsymbol{d}$ is connected with the battery. The charge on the plate is $\boldsymbol{Q}$. Now a slab of copper (conductor)of thickness $\boldsymbol{b}$ is thrust into the capacitor. The charge on the plate increases to $\boldsymbol{1.5\,Q}$. Assume $\boldsymbol{b = x \cdot d}$, what is $\boldsymbol{x}$?
(A) 1/5 (B) 1/4 (C) 1/3 (D) 1/2 (E) 2/3 (F) 3/4 (G) 3/2

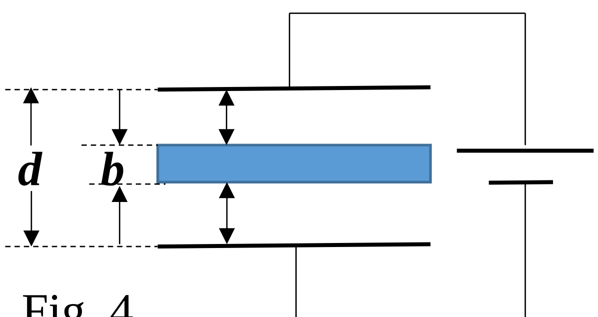


Fig. 4

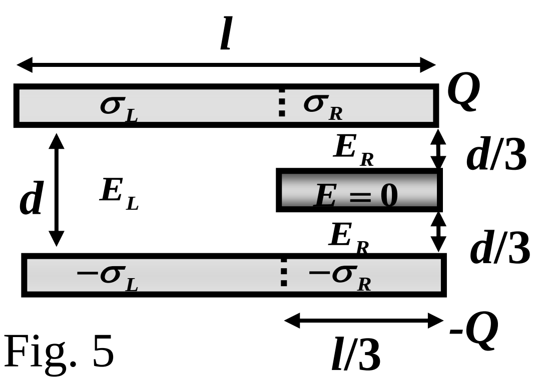


Fig. 5

3. (5 pts) As shown in Fig. 5, two flat, square metal plates have sides of length l , are arranged parallel to each other with a separation of d , where $d << l$. A charge Q is moved from the lower plate to the upper plate. Now a third uncharged conducting plate with thickness $d/3$ places between the other two plates to a depth $l/3$, maintaining the same spacing $d/3$ between its surface and the surfaces of the other two. You may neglect edge effects. Let the charge density $\sigma_0 = Q/l^2$ .What is the value $x = \sigma_R / \sigma_0$?
(A) $0 < x \le 0.25$ (B) $0.25 < x \le \frac{1}{3}$ (C) $\frac{1}{3} < x \le 0.5$ (D) $0.5 < x \le \frac{2}{3}$ (E) $\frac{2}{3} < x \le 0.75$
(F) $0.75 < x \le 1$ (G) $1 < x \le 1.2$ (H) $1.2 < x \le 1.4$ (J) $1.4 < x \le 1.6$
(K) $1.6 < x \le 2.0$ (L) $2.0 < x$

4. (5pts) Same structure as in problem 3, the capacitance of the system is $\alpha \cdot (\varepsilon_0 \ell^2)/d$. What is the value of α?
(A) $0 < \alpha \le 0.25$ (B) $0.25 < \alpha \le \frac{1}{3}$ (C) $\frac{1}{3} < \alpha \le 0.5$ (D) $0.5 < \alpha \le \frac{2}{3}$ (E) $\frac{2}{3} < \alpha \le 0.75$
(F) $0.75 < \alpha \le 1$ (G) $1 < \alpha \le 1.2$ (H) $1.2 < \alpha \le 1.4$ (J) $1.4 < \alpha \le 1.6$
(K) $1.6 < \alpha \le 2.0$ (L) $2.0 < \alpha$

5. (5pts) Free electrons in air can be accelerated due to high electric fields to ionize O_2 and N_2 molecules by collisions. The air then becomes conducting. This breakdown in dry air occurs for electric fields about 3×10^6 V/m. If you have a Van de Graff, which charges a conducting sphere of radius of $0.5m$ in the same air, what are the maximum charge q (in units of μC, i.e. $10^{-6}C$) that you can charge it?

(A) $0 < q \le 1$ (B) $1 < q \le 2$ (C) $2 < q \le 5$ (D) $5 < q \le 10$ (E) $10 < q \le 20$
(F) $20 < q \le 50$ (G) $50 < q \le 100$ (H) $100 < q \le 200$ (J) $200 < q \le 500$
(K) $500 < q \le 10^3$ (L) $10^3 < q$

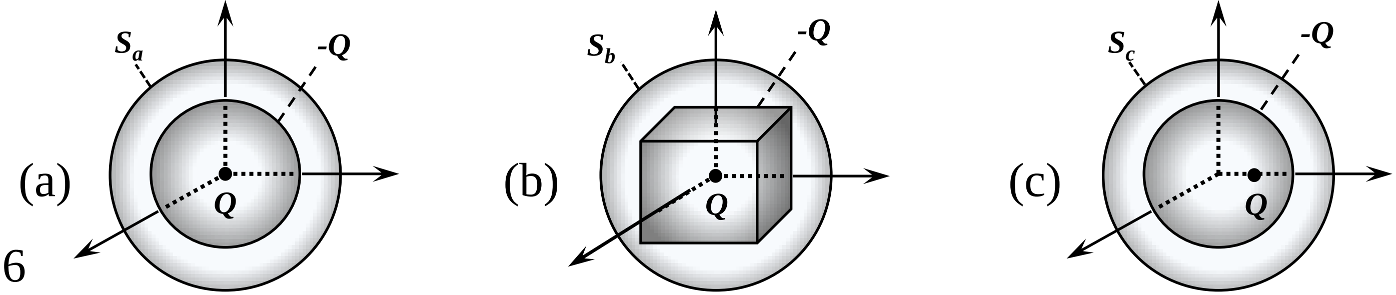


Fig. 6

6. As shown in Fig. 6(a),(b), and (c), A point charge $\boldsymbol{Q}$ is placed inside an uniform spherical shell charge distribution (Fig. 6(a) and Fig. 6(c)), and a cubic shell charge distribution (Fig. 6(b)). Spherical Gaussian surfaces labelled $\boldsymbol{S_a}$, $\boldsymbol{S_b}$, and $\boldsymbol{S_c}$ are defined in Fig 6(a), (b), and (c), respectively. The total charge of all shells is $-\boldsymbol{Q}$. Let $\boldsymbol{\Phi_{E,A}}$, $\boldsymbol{\Phi_{E,B}}$, and $\boldsymbol{\Phi_{E,C}}$ be the total electric flux through the surface $\boldsymbol{S_a}$, $\boldsymbol{S_b}$, and $\boldsymbol{S_c}$, respectively, and $\boldsymbol{E_{E,A}}$, $\boldsymbol{E_{E,B}}$, $\boldsymbol{E_{E,C}}$ be the electric field distribution on the surface $\boldsymbol{S_a}$, $\boldsymbol{S_b}$, and $\boldsymbol{S_c}$, respectively. Which of the following statement is correct?

(A) $\boldsymbol{\Phi_{E,A}}= 0$, $\boldsymbol{E_A}= 0$, $\boldsymbol{\Phi_{E,B}}= 0$, $\boldsymbol{E_B}= 0$, $\boldsymbol{\Phi_{E,C}}= 0$, and $\boldsymbol{E_C} = 0$
(B) $\boldsymbol{\Phi_{E,A}}= 0$, $\boldsymbol{E_A}= 0$, $\boldsymbol{\Phi_{E,B}}= 0$, $\boldsymbol{E_B}= 0$, $\boldsymbol{\Phi_{E,C}}= 0$, and $\boldsymbol{E_C}\neq 0$
(C) $\boldsymbol{\Phi_{E,A}}= 0$, $\boldsymbol{E_A}= 0$, $\boldsymbol{\Phi_{E,B}}= 0$, $\boldsymbol{E_B}\neq 0$, $\boldsymbol{\Phi_{E,C}}= 0$, and $\boldsymbol{E_C}\neq 0$
(D) $\boldsymbol{\Phi_{E,A}}= 0$, $\boldsymbol{E_A}\neq 0$, $\boldsymbol{\Phi_{E,B}}= 0$, $\boldsymbol{E_B}\neq 0$, $\boldsymbol{\Phi_{E,C}}= 0$, and $\boldsymbol{E_C}\neq 0$
(E) $\boldsymbol{\Phi_{E,A}}= 0$, $\boldsymbol{E_A}\neq 0$, $\boldsymbol{\Phi_{E,B}}\neq 0$, $\boldsymbol{E_B}\neq 0$, $\boldsymbol{\Phi_{E,C}} = 0$, and $\boldsymbol{E_C}\neq 0$
(F) $\boldsymbol{\Phi_{E,A}}= 0$, $\boldsymbol{E_A}= 0$, $\boldsymbol{\Phi_{E,B}}\neq 0$, $\boldsymbol{E_B}\neq 0$, $\boldsymbol{\Phi_{E,C}}\neq 0$, and $\boldsymbol{E_C}\neq 0$
(G) $\boldsymbol{\Phi_{E,A}}= 0$, $\boldsymbol{E_A}= 0$, $\boldsymbol{\Phi_{E,B}}\neq 0$, $\boldsymbol{E_B}\neq 0$, $\boldsymbol{\Phi_{E,C}} = 0$, and $\boldsymbol{E_C}\neq 0$

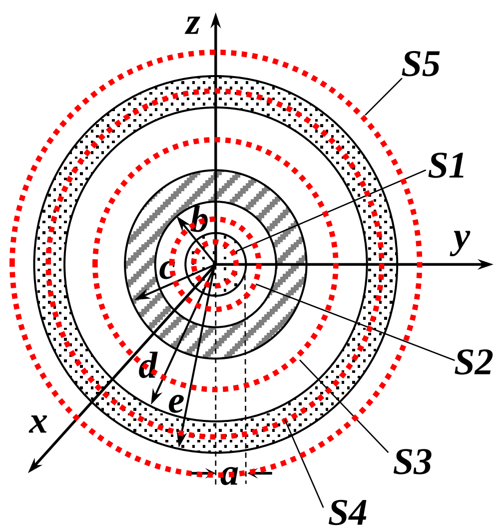
Integration Formula for reference

$$\int \frac{dx}{\sqrt{x^2 \pm b^2}} = \ln\left(x + \sqrt{x^2 \pm b^2}\right) \qquad \int \frac{x^2 dx}{\sqrt{x^2 \pm b^2}} = \frac{x\sqrt{x^2 \pm b^2}}{2} - \frac{b^2}{2}\ln\left(x + \sqrt{x^2 \pm b^2}\right)$$

$$\int \frac{dx}{\left(x^2 \pm b^2\right)^{3/2}} = \frac{\pm x}{b^2\sqrt{x^2 \pm b^2}} \qquad \int \frac{x^2 dx}{\left(x^2 \pm b^2\right)^{3/2}} = \frac{-x}{\sqrt{x^2 \pm b^2}} + \ln\left(x + \sqrt{x^2 \pm b^2}\right)$$

1	2	3	4	5	6		
J	C	H	G	G	C		

2. (20pts) As shown in Fig. 2, a uniform spherical charge distribution of radius ***a*** (***a=R***) and charge density ***13ρ*** (*ρ*>0) is placed in side a concentric spherical conductor shell with inner and outer radius ***b*** (***b=3R***) and ***c*** (***c=5R***), respectively. Outside of the conductor shell, there is a second charge distribution with density of ***–ρ /13*** in a concentric spherical shell region with inner and outer radius ***d*** (***d=7R***) and ***e*** (***e=8R***). Determine (a) (10pts) the direction and magnitude of the ***E***-field for $0 \le r < \infty$.(b) (10pts) the electric potential ***V*** for $b \le r < \infty$, assuming $V(\infty)=0$.



3

(a) For r < a, apply Gauss's Law to sphere S1,

$$\Phi_{E,S1} = \oiint_{S1} \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} = \frac{4\pi r^3}{3\varepsilon_0} \cdot 13\rho \quad 1$$

$$\Rightarrow 4\pi r^2 E(r) = \frac{4\pi r^3}{3\varepsilon_0} \cdot 13\rho \Rightarrow \vec{E}(r) = \frac{13 r\rho}{3\varepsilon_0}\hat{r} \quad 1$$

For a≤ r <b, apply Gauss's Law to sphere S2,

$$\Phi_{E,S2} = \oiint_{S2} \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} = \frac{4\pi R^3}{3\varepsilon_0} \cdot 13\rho \quad 1$$

$$\Rightarrow 4\pi r^2 E(r) = \frac{4\pi R^3}{3\varepsilon_0} \cdot 13\rho \Rightarrow \vec{E}(r) = \frac{13R^3\rho}{3\varepsilon_0 r^2}\hat{r} \quad 1$$

For b≤ r <c, due to the presence of the conductor

$$\vec{E}(r) = 0 \quad 1$$

For c≤ r <d, apply Gauss's Law to sphere S3,

$$\Phi_{E,S3} = \oiint_{S3} \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} = \frac{4\pi R^3}{3\varepsilon_0} \cdot 13\rho$$

$$\Rightarrow 4\pi r^2 E(r) = \frac{4\pi R^3}{3\varepsilon_0} \cdot 13\rho \Rightarrow \vec{E}(r) = \frac{13R^3\rho}{3\varepsilon_0 r^2}\hat{r} \quad 1$$

For d≤ r <e, apply Gauss's Law to sphere S4, 1

$$\Phi_{E,S4} = \oiint_{S4} \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} = \frac{4\pi R^3}{3\varepsilon_0} \cdot 13\rho - \frac{4\pi(r^3 - d^3)}{3\varepsilon_0} \cdot \frac{\rho}{13}$$

$$\Rightarrow 4\pi r^2 E(r) = \frac{4\pi R^3}{3\varepsilon_0} \cdot 13\rho - \frac{4\pi(r^3 - 343R^3)}{3\varepsilon_0} \cdot \frac{\rho}{13}$$

$$\Rightarrow 4\pi r^2 E(r) = \frac{4\pi\rho}{39\varepsilon_0}(512R^3 - r^3)$$

$$\Rightarrow \vec{E}(r) = \frac{\rho}{39\varepsilon_0}\frac{(512R^3 - r^3)}{r^2}\hat{r} \quad 1$$

For e≤ r, apply Gauss's Law to sphere S5,

$$\Phi_{E,S5}=\oint\!\!\!\oint_{S5}\vec{E}\cdot d\vec{A}=\frac{Q_{in}}{\varepsilon_0}=\frac{4\pi R^3}{3\varepsilon_0}\cdot 13\rho-\frac{4\pi(e^3-d^3)}{3\varepsilon_0}\cdot\frac{\rho}{13}$$

$$\Rightarrow 4\pi r^2E(r)=\frac{4\pi R^3}{3\varepsilon_0}\cdot 13\rho-\frac{4\pi(512R^3-343R^3)}{3\varepsilon_0}\cdot\frac{\rho}{13}$$

$$\Rightarrow 4\pi r^2E(r)=\frac{4\pi R^3}{3\varepsilon_0}\cdot 13\rho-\frac{4\pi(169R^3)}{3\varepsilon_0}\cdot\frac{\rho}{13}=0$$

$$\Rightarrow E(r)=0 \qquad 1$$

(b) For e ≤ r , (8R ≤ r) ,

$$V(\infty)-V(r)=-\int_r^{\infty}\vec{E}\cdot d\vec{\ell}=0$$

$$\Rightarrow V(r)=V(\infty)=0 \qquad 1$$

For d ≤ r < e , (7R ≤ r < 8R) ,

$$V(e)-V(r)=-\int_r^{e}\vec{E}\cdot d\vec{\ell}=-\int_r^{8R}\frac{\rho}{39\varepsilon_0}\frac{(512R^3-r^3)}{r^2}\cdot dr$$

$$V(e)=0 \qquad 1$$

$$\Rightarrow 0-V(r)=-\frac{\rho}{39\varepsilon_0}\left(-\frac{512R^3}{r}-\frac{r^2}{2}\right)\Bigg|_r^{8R}$$

$$\Rightarrow V(r)=\frac{\rho}{39\varepsilon_0}\left(\frac{512R^3}{r}+\frac{r^2}{2}-96R^2\right) \qquad 2$$

For c ≤ r < d, (5R ≤ r < 7R) ,

$$V(d)-V(r)=-\int_r^{d}\vec{E}\cdot d\vec{\ell}=-\int_r^{d}\frac{13R^3\rho}{3\varepsilon_0 r^2}\cdot dr$$

$$V(d)=\frac{\rho}{39\varepsilon_0}\left(\frac{512R^3}{7R}+\frac{49R^2}{2}-96R^2\right) \qquad 1$$

$$=\frac{23}{14}\cdot\frac{\rho R^2}{39\varepsilon_0}$$

$$\Rightarrow \frac{23}{14}\frac{\rho R^2}{39\varepsilon_0}-V(r)=\frac{13R^3\rho}{3\varepsilon_0\cdot 7R}-\frac{13R^3\rho}{3\varepsilon_0 r}$$

$$\Rightarrow V(r)=\frac{13R^3\rho}{3\varepsilon_0 r}-\frac{15R^2\rho}{26\varepsilon_0} \qquad 2$$

For b ≤ r < c, (3R ≤ r < 5R) ,

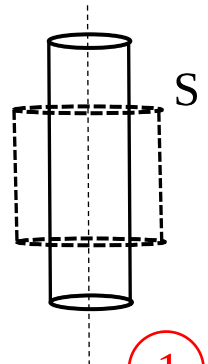
$$V(c)-V(r)=-\int_r^{c}\vec{E}\cdot d\vec{\ell}=0$$

$$\Rightarrow V(r)=V(c)=\frac{13R^3\rho}{3\varepsilon_0\cdot 5R}-\frac{15R^2\rho}{26\varepsilon_0}=\frac{113R^2\rho}{390\varepsilon_0}$$

2

(a) Total charge Q: $Q = \sigma 2\pi R h$ 2

(b) Choose S as the Gaussian surface:



$$E_{-}\cdot 2\pi r l = \frac{(-\sigma)\cdot 2\pi R l}{\varepsilon_0}$$

$$\therefore \vec{E}_{-} = \frac{(-Q/h)}{2\pi\varepsilon_0 \cdot r}(\hat{r})$$ 2

1

$$\vec{E}_{-} = -\frac{(Q/h)}{2\pi\varepsilon_0\cdot(x+d/2)}(\hat{i})$$ at P point.

In the similar method:

$$\vec{E}_{+} = \frac{(Q/h)}{2\pi\varepsilon_0\cdot(d/2-x)}(-\hat{i})$$ at P point.

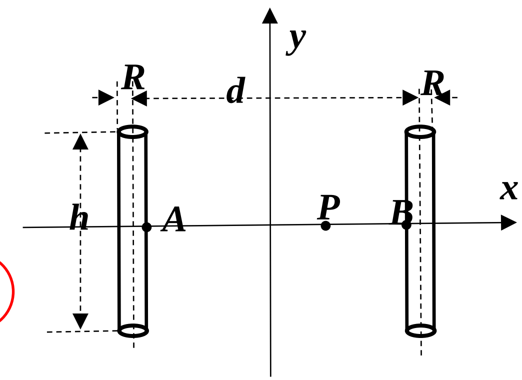
$$\vec{E}_{total} = \vec{E}_{+} + \vec{E}_{-}$$

$$= -\frac{(Q/h)}{2\pi\varepsilon_0\cdot(d/2-x)}(\hat{i})$$ 1

1

$$-\frac{(Q/h)}{2\pi\varepsilon_0\cdot(d/2+x)}(\hat{i})$$ 1

(c) Calculate the electric potential difference between the two cylindrical metal tubes:



$$\Delta V = V_B - V_A$$

$$= -\int_A^B \vec{E}_{total}\cdot d\vec{l}$$ 1

$$= -\int_{-\frac{d}{2}+R}^{\frac{d}{2}-R}\left(\frac{(Q/h)}{2\pi\varepsilon_0\cdot(d/2-x)}+\frac{(Q/h)}{2\pi\varepsilon_0\cdot(d/2+x)}\right)dx$$ 1

$$= -\frac{Q}{2\pi\varepsilon_0 h}\int_{-\frac{d}{2}+R}^{\frac{d}{2}-R}\left(\frac{-1}{(x-d/2)}+\frac{1}{(d/2+x)}\right)dx$$

1 1

$$= -\frac{Q}{2\pi\varepsilon_0 h}\left\{-\ln\left(\frac{|-R|}{d-R}\right)+\ln\left(\frac{d-R}{R}\right)\right\}$$

$$= -\frac{Q}{\pi\varepsilon_0 h}\ln\left(\frac{d-R}{R}\right)$$

(d) The capacitance of this system:

$$C = \frac{Q}{|\Delta V|} = \pi\varepsilon_0 h/\ln\left(\frac{d-R}{R}\right)$$

1 2