General Physics (I) **期中考 I** **Oct.16, 2020**

試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(50 points) (所有題目必須有計算過程,否則不予計分)

1. (15 pts) As shown in Fig. 1(a), a coast guard（海岸巡防隊）are firing a cannon (at $\boldsymbol{t} = 0$ sec) with an angle $\theta = 37^0$ above the horizontal surface at a escaping (逃逸) boat whose speed is $v_B = 16\ m/sec$. and *12 m* east away from the coast at $\boldsymbol{t} = 0$ sec ..
   (a) (7 pts) (i) Find the magnitude of the shell's initial velocity v_c ? and (ii) how much does time it take to hit the boat? (Let $g = 10\ m/s^2$)
   (b) (8 pts) As shown in Fig. 1b, if the boat is heading $+y$ with the same speed. The coast guard adjusts the angle θ, but keep the cannon at a horizontal angle $\alpha = 37^0$ relative to the x-axis to hit the boat. Find the angle θ and the magnitude of the shell's velocity v_c? ($\boldsymbol{g} = 10\ m/s^2$)

2. (a) (5 pts)As shown in Fig. 2(a) blocks of mass $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ are connected to a massless string and sitting on the block of mass $\boldsymbol{M}$. block $\boldsymbol{M}$ is fixed and all contacting surfaces are frictionless, let $\boldsymbol{m_1} = 2\ \boldsymbol{kg}$, and $\boldsymbol{m_2} = 1\ \boldsymbol{kg}$. Determine the acceleration of $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ (b) (12 pts) If the whole system is now placed on a frictionless surface (Fig. 2(b)), and a force $\boldsymbol{F}$ is applied to block $\boldsymbol{M}$ to make it accelerate in the horizontal direction with block $\boldsymbol{m_1}$ and block $\boldsymbol{m_2}$ resting on block $\boldsymbol{M}$, Let $\boldsymbol{M} = 7\ \boldsymbol{kg}$. Determine the magnitude of $\boldsymbol{F}$.(The free-body diagrams of $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$, <u>$\boldsymbol{M}$ and the pulley (for part (2b) only)</u> are required in your answer) ($\mathbf{g} = 10\mathrm{m/s^2}$;$sin37^\circ \cong 3/5$)

3. Fig. 3(a) shows that a car is driving on a track with two semi-circular racing paths of radius 3R and R respectively. The whole track is banked inward with angle θ, shown in Fig. 3(b). A car racer is driving a car, with mass *m* and constant speed $\boldsymbol{v}$, along the track. The static friction coefficient μ_s is for the whole track. At point A, the car reaches the minimum speed on this semi-circle without sliding. At the point B, the car reaches the maximum speed on this curve.
   (a) (10 pts) Draw the free-body diagram and write down the force equations for the car at points A and B, respectively.
   (b) (3 pts) Express the speed $\boldsymbol{v}$ of the car, in terms of m, μ_s, R, g, and θ.
   (c) (5 pts) Find the static friction coefficient μ_s in terms of m, R, g, and θ.

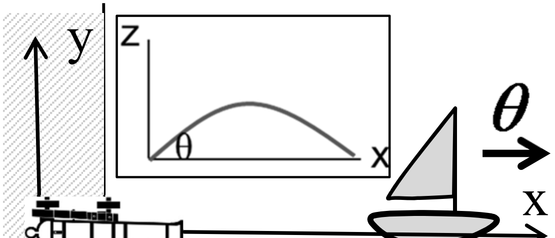


Fig. 1(a)

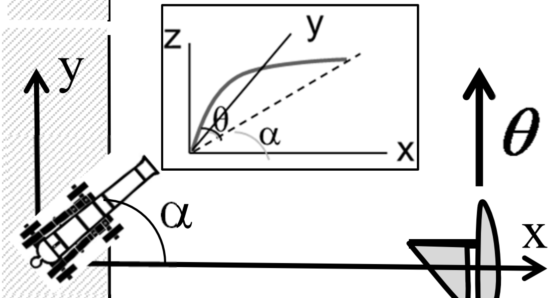


Fig. 1(b)

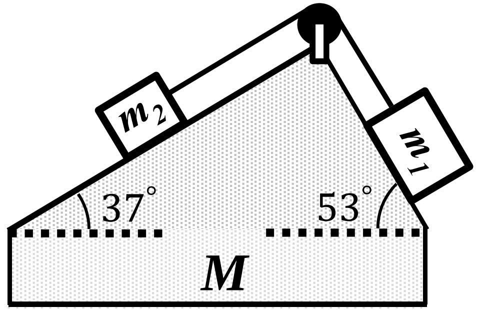


Fig. 2(a)

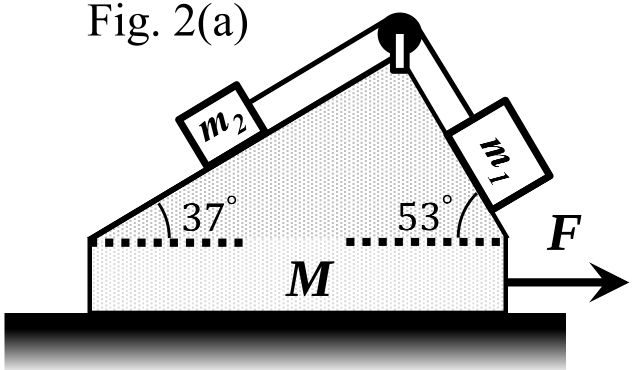


Fig. 2(b)

Fig. 3(a)

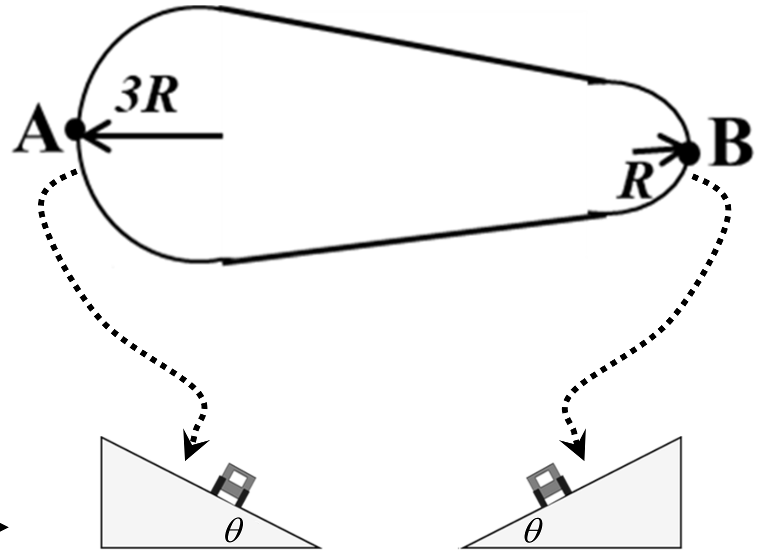


Fig. 3(b)

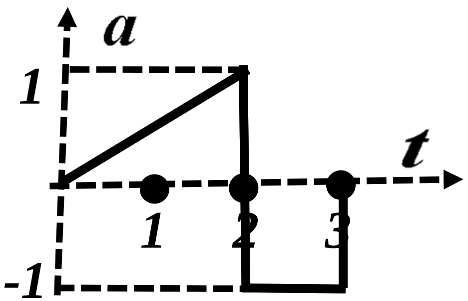


Fig 4

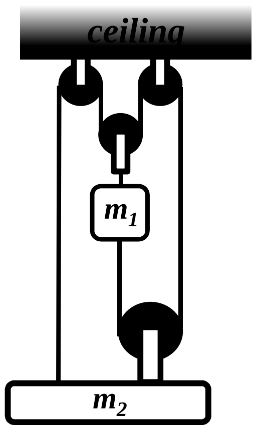


Fig. 5

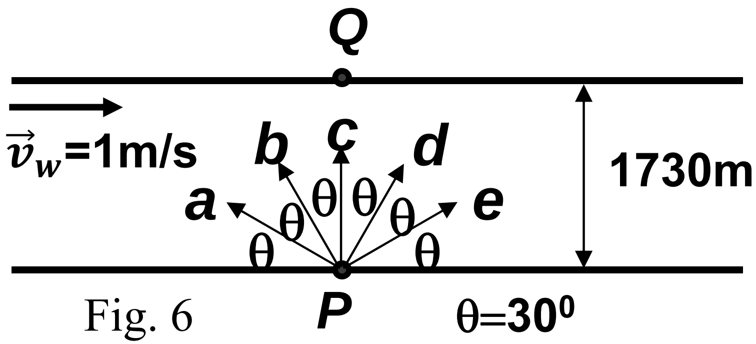


Fig. 6

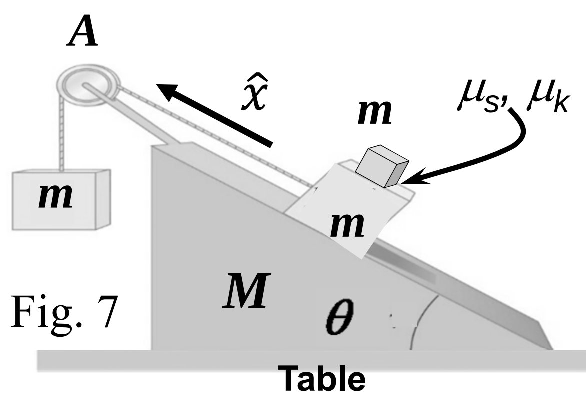


Fig. 7

II.選擇題(50 points)

1. (5pts) Consider a particle moving with the acceleration ***a*** (in unit of ***m/s***^{2}) vs. time ***t*** (in unit of ***s***) graph as shown in Fig. 4. Assume the particle is at rest and at ***x* = 0** at ***t* = 0 sec.** What is the position ***x*** of the particle at ***t* = 3 sec.**

(A) $x = 0$ (B) $0 < x \le 0.2$ (C) $0.2 < x \le 0.4$ (D) $0.4 < x \le 0.6$ (E) $0.6 < x \le 0.8$
(F) $0.8 < x \le 1.0$ (G) $1.0 < x \le 1.2$ (H) $1.2 < x \le 1.4$ (J) $1.4 < x$

2. (5 pts) As shown in Fig. 5, blocks m_1 and m_2 ($m_1 << m_2$) are connected with a string through a set of pulleys and are hung from the ceiling, and are released to motion at $t = 0$ sec. Let a_1 and a_2 be the acceleration of m_1 and m_2, respectively, and $x = |a_1/a_2|$, which of the following is correct? (A) $x < 0.1$ (B) $0.1 \le x < 0.25$ (C) $0.25 \le x < 0.5$ (D) $0.5 \le x < 1$ (E) $1 \le x < 2$ (F) $2 \le x < 4$ (G) $4 \le x < 10$ (H) $10 \le x$

3. (5 pts) In a hot pursuit, an FBI agent at ***P*** (Fig. 6) must get across a 1730 -m-wide river to ***Q*** in minimum time by rowing a boat (划船) across the river and then running along the shore. The river's current is **1.0 m/s**, he can row a boat at **2.0** m/s, and he can run at speed **3.00 m/s**. For the minimum-time path, which direction should the boat be heading?
(A) ***a*** (B) between ***a*** and ***b***, (C) ***b*** (D) between ***b*** and ***c***, (E) ***c***. (F) between ***c*** and ***d***, (G) ***d*** (H) between ***d*** and ***e***, (J) ***e*** (K) none of above.

4. (5 pts) Same as problem 4, the minimum time is ***x*** minutes for the agent to cross the river. What is the value of ***x***? ($\sqrt{2} \sim 1.41, \sqrt{3} \sim 1.73, \sqrt{5} \sim 2.24$, $\sin 15^0 \sim 0.26$)
(A) $x \le 10$ (B) $10 < x \le 15$ (C) $15 < x \le 20$ (D) $20 < x \le 25$ (E) $25 < x \le 30$ (F) $30 < x \le 35$ (G) $35 < x \le 40$ (H) $40 < x \le 45$ (J) $45 < x \le 50$ (K) $50 < x$.

5. (5 pts) Fig. 7 shows a system which contains three small blocks (each with mass ***m***) and a large block with mass ***M*** on a horizontal table. The large block is fixed on the table and no friction between block ***m*** and ***M*** but the kinetic and static friction coefficients are 0.4 and 0.5 between the two small blocks ***m***. If $\sin\theta = 1/5$, (the figure is not to scale), find the friction force ***f*** acting on the upper ***m*** block. Let $a = |f|/(mg)$, what is the value ***a*** and the direction of ***f*** ?

(A) $0 < a \le 0.15, +\hat{x}$ (B) $0.15 < a \le 0.25, +\hat{x}$ (C) $0.25 < a \le 0.35, +\hat{x}$ (D) $0.35 < a \le 0.45, +\hat{x}$ (E) $0.45 < a \le 0.55, +\hat{x}$ (F) $a \le 0.15, -\hat{x}$ (G) $0.15 < a \le 0.25, -\hat{x}$ (H) $0.25 < a \le 0.35, -\hat{x}$ (J) $0.35 < a \le 0.45, -\hat{x}$ (K) $0.45 < a \le 0.55, -\hat{x}$ (L) $a = 0$.

6. (5pts) An object with mass of 10 kg is constrained to move in a horizontal circular path with radius of 25 m. Its tangential acceleration as a function of time is given by $a_t = 3t^2$ in m/s^2. If v = 3 m/s at t = 0 s, the total force ***F*** on the object at t = 3 s is

(A)F<200 N; (B) 200 N≦***F***< 300 N; (C) 300 N≦***F***< 400 N; (D) 400 N≦***F***< 500 N; (E) 500 N≦***F***< 600 N; (F) 600 N≦***F***< 700 N; (G) 700 N≦***F***< 800 N; (H) 800 N≦***F***< 1000 N; (J) 1000 N≦***F***.

Multiple Choice Questions:

1	**2**	**3**	**4**	**5**	**6**				
G	**F**	**C**	**C**	**D**	**D**				
7	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	**B**	**D**	**C**	**B**	**A**	**C**	**G**	**C**	**F**

3.

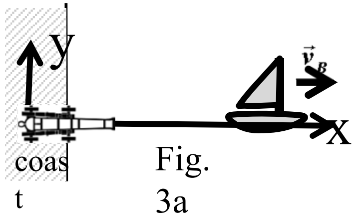


Fig. 3a

The position of cannon:

$$z_c(t) = v_c \sin\theta \cdot t - \tfrac{1}{2} g t^2$$

$$x_c(t) = v_c \cos\theta \cdot t = \frac{4}{5} v_c \cdot t$$

The position of the boat:

$$z_b(t) = 0$$

$$x_b(t) = v_b \cdot t + L = 16 \cdot t + 12$$

When the cannon hit the boat at time t_f:

$$z_c(t_f) = z_b(t_f) \Rightarrow v_c \sin\theta \cdot t_f - \tfrac{1}{2} g t_f^{\,2} = 0 \quad 2 \qquad \Rightarrow t_f = \frac{2v_c \sin\theta}{g} = \frac{3}{25} v_c \quad 1$$

$$x_c(t_f) = x_b(t_f) \Rightarrow v_c \cos\theta \cdot t_f = v_b \cdot t_f + L \quad 2 \qquad \Rightarrow \frac{2v_c^{\,2}}{g} \cos\theta \sin\theta - \frac{2v_c \cdot v_b}{g} \sin\theta - L = 0$$

$$\frac{12}{125} v_c^{\,2} - \frac{48}{25} \cdot v_c - 12 = 0 \qquad \Rightarrow v_c^{\,2} - 20 \cdot v_c - 125 = 0$$

$$\Rightarrow (v_c - 25)(v_c + 5) = 0$$

$$v_c = 25\ m/\sec \quad 2 \quad (v_c = -5\ m/\sec \text{不合理})$$

$$t_f = \frac{2v_c \sin\theta}{g} = 3 \sec. \quad 2$$

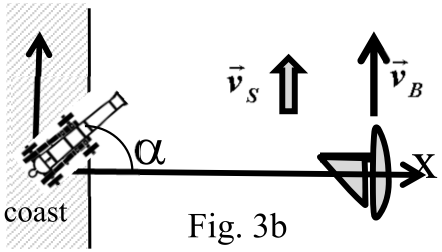


Fig. 3b

$$z_c(t) = v_c \sin\theta \cdot t - \tfrac{1}{2} g t^2 = z_b(t) = 0 \quad \cdots(1)$$ 1

$$x_c(t) = v_c \cos\theta \cos\alpha \cdot t = x_b(t) = L = 12 \quad \cdots(2)$$ 1

$$y_c(t) = v_c \cos\theta \sin\alpha \cdot t = y_b(t) = v_b \cdot t = 16t \quad \cdots(3)$$ 1

$$eq(3) \Rightarrow v_c \cos\theta \cdot \frac{3}{5} = 16 \quad \Rightarrow v_c \cos\theta = \frac{80}{3}\, m/s \quad \cdots(4)$$

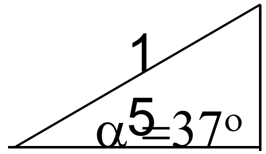
$$eq(2) \Rightarrow \frac{80}{3} \cdot \frac{4}{5} \cdot t_f = 12 \quad \Rightarrow t_f = \frac{9}{16} \quad \cdots(5)$$ 1

put the result of eq (5) into $eq(1) \Rightarrow v_c \sin\theta = \tfrac{1}{2} g t_f = \frac{45}{16} \quad \cdots(6)$

$$\Rightarrow \frac{eq.(6)}{eq.(4)} = \tan\theta = \frac{27}{16 \cdot 16} = \frac{27}{256}$$ 1

$$eq.(6)^\wedge 2 + eq.(4)^\wedge 2 \Rightarrow v_c^2 = \sqrt{\left(\frac{80}{3}\right)^2 + \left(\frac{45}{16}\right)^2} \approx 26.8\, m/s$$ 1

另法:



$$\Rightarrow t_f = \frac{9}{16}$$ $\boxed{2}$

$$v_c \cos\theta = \frac{15}{9/16} = \frac{80}{3}\, m/s \quad \cdots(4)$$

$$v_c \sin\theta \cdot t_f - \tfrac{1}{2} g t_f^{\,2} = 0$$

$$\Rightarrow v_c \sin\theta = \tfrac{1}{2} g t_f = \frac{45}{16}$$

$$\Rightarrow \tan\theta = \frac{27}{256}$$ $\boxed{2}$

$$\Rightarrow v_c^2 = \sqrt{\left(\frac{80}{3}\right)^2 + \left(\frac{45}{16}\right)^2} \approx 26.8\, m/s$$ $\boxed{2}$

2. (a) (5 pts)As shown in Fig. 2(a) blocks of mass $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ are connected to a massless string and sitting on the block of mass ***M***. block ***M*** is fixed and all contacting surfaces are frictionless, let $\boldsymbol{m_1}$ = 2 ***kg***, and $\boldsymbol{m_2}$ =1 ***kg*.** Determine the acceleration of $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ (b) (12 pts) If the whole system is now placed on a frictionless surface (Fig. 2(b)), and a force ***F*** is applied to block ***M*** to make it accelerate in the horizontal direction with block $\boldsymbol{m_1}$ and block $\boldsymbol{m_2}$ resting on block ***M***, Let ***M*** = 7 ***kg***. Determine the magnitude of ***F***.(The free-body diagrams of $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$, <u>***M*** and the pulley (for part (2) only)</u> are required in your answer) (**g** = 10m/s^2 ;$sin37^\circ \cong 3/5$)

(a)

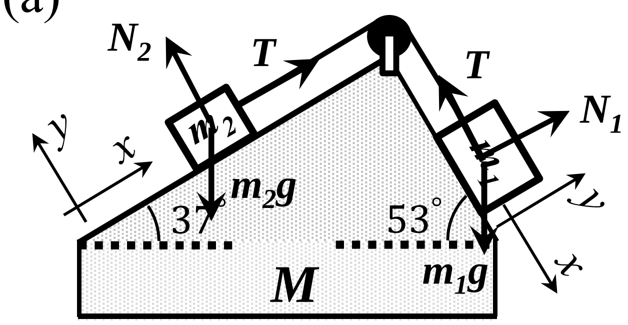


Fig. 2(a)

(b)

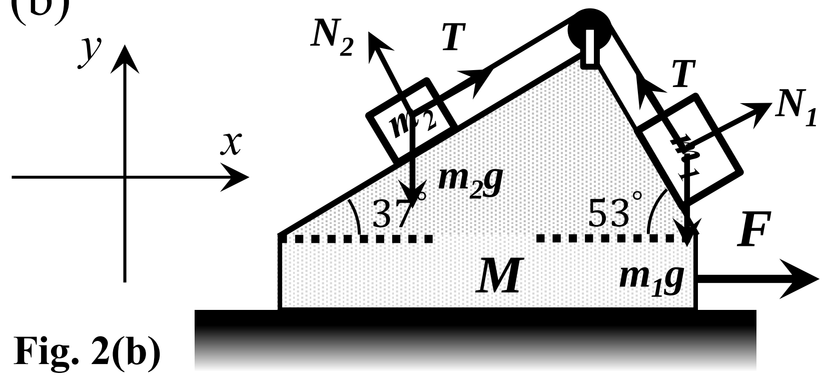
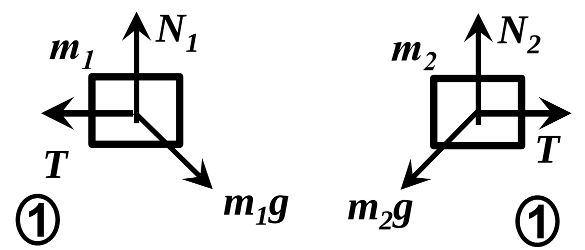


Fig. 2(b)



For m_2, $\sum \vec{F} = m_2\vec{a}_2$

$x : T - m_2 g sin37^\circ = m_2 a_2$

$y : N_2 - m_2 g cos37^\circ = 0$ ①

$\Rightarrow x: T - 6 = a_2$ **(2)**

$y : N_2 - 8 = 0$

$a_1 = a_2 \equiv a$ **(constraint)** **(3)**

For m_1, $\sum \vec{F} = m_1\vec{a}_1$

$x : m_1 g \cdot sin53^\circ - T = m_1 a_1$ **(1),(2),&(3)**$\Rightarrow 16 - T = 2a$

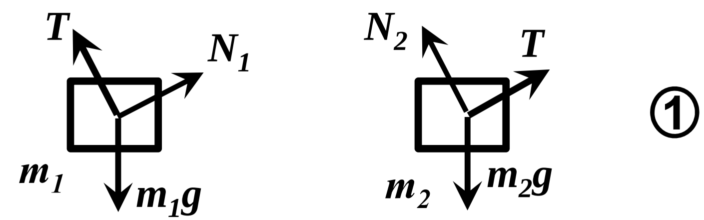
$y : N_1 - m_1 g \cdot cos53^\circ = 0$ ①

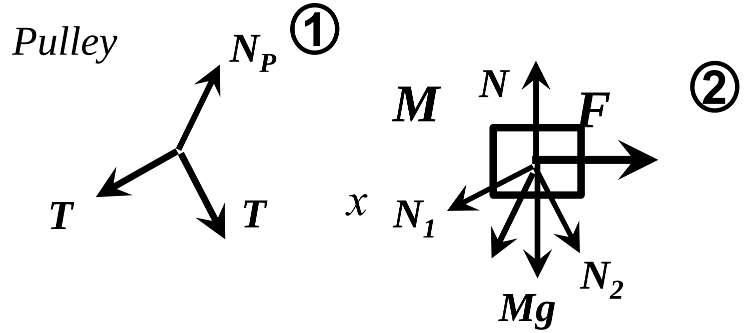
$T - 6 = a$

$\Rightarrow x: 16 - T = 2a_1$ **(1)**

$y : N_1 - 12 = 0$

$\Rightarrow a = \frac{10}{3}(m/s^2)$ ①





For m_1, $\sum \vec{F} = m_1 \vec{a}_1$

$x : N_1 sin53^\circ - Tcos53^\circ = m_1 a_1$

$y : N_1 cos53^\circ + Tsin53^\circ - m_1 g = 0$

$\Rightarrow x: \frac{4}{5} N_1 - \frac{3}{5} T = 2a_1$ (4)

$y: \frac{3}{5} N_1 + \frac{4}{5} T = 20$ (5) ①

For m_2, $\sum \vec{F} = m_2 \vec{a}_2$

$x : -N_2 sin37^\circ + Tcos37^\circ = m_2 a_2$

$y : N_2 cos37^\circ + Tsin37^\circ - m_2 g = 0$

$\Rightarrow x: -\frac{3}{5} N_2 + \frac{4}{5} T = a_2$ (6) ①

$y: \frac{4}{5} N_2 + \frac{3}{5} T = 10$ (7)

For *pulley*, $\sum \vec{F} = 0$

$x : N_{p,x} - Tsin53^\circ + Tsin37^\circ = 0$

$y : N_{py} - Tcos53^\circ - Tcos37^\circ = 0$

$\Rightarrow x: N_{p,x} - \frac{4}{5} T + \frac{3}{5} T = 0$ (8) ①

$y: N_{p,x} - \frac{3}{5} T - \frac{4}{5} T = 0$ (9)

For M, $\sum \vec{F} = M \vec{a}_M$

$x : F - N_1 sin53^\circ + N_2 sin37^\circ - N_{p,x} = M a_M$

$y : N - N_1 cos53^\circ - N_2 cos37^\circ - N_{py} - Mg = 0$

$\Rightarrow x: F - \frac{4}{5} N_1 + \frac{3}{5} N_2 - N_{p,x} = 7a_M$ (10) ①

$y: N - \frac{3}{5} N_1 - \frac{4}{5} N_2 - N_{py} - 70 = 0$ (11) ①

$a_1 = a_2 = a_M \equiv a$ **(constraint)** (12)

(4)-(10),&(12) $\Rightarrow \frac{4}{5} N_1 - \frac{3}{5} T = 2a$ (13)

$\frac{3}{5} N_1 + \frac{4}{5} T = 20$ (14)

$-\frac{3}{5} N_2 + \frac{4}{5} T = a$ (15)

$\frac{4}{5} N_2 + \frac{3}{5} T = 10$ (16)

$F - \frac{4}{5} N_1 + \frac{3}{5} N_2 + \frac{3}{5} T - \frac{4}{5} T = 7a$ (17)

4/5*(14)-3/5*(13) $\Rightarrow T = 16 - \frac{6}{5} a$ (18)

3/5*(16)+4/5*(15) $\Rightarrow T = 6 + \frac{4}{5} a$ (19) ①

(18)-(19) $\Rightarrow 0 = -10 + 2a \Rightarrow a = 5$ (20)

(13)+(15)+(17),&(20) $F = 10a \Rightarrow F = 50\ (N)$ ②

1. Figure 1(a) shows that a car is driving on a track with two semi-circular racing paths of radius 3R and R respectively. The whole track is banked inward with angle θ, shown in Fig. 1(b). A car racer is driving a car, with mass m and constant speed v, along the track. The static friction coefficient μ_s is constant for the whole track. At point A, the car reaches the minimum speed on this semi-circle without sliding. At the point B, the car reaches the maximum speed on this curve.
   (a) Draw the free-body diagram and write down the force equations for the car at points A and B, respectively.
   (b) Find the speed v of the car, in terms of m, μ_s, R, g, and θ.
   Find the static coefficient μ_s in terms of m, R, g, and θ.

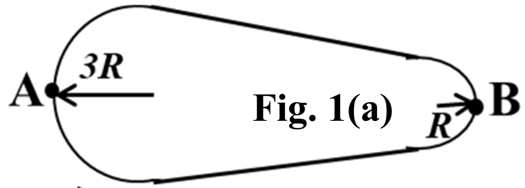


Fig. 1(a)

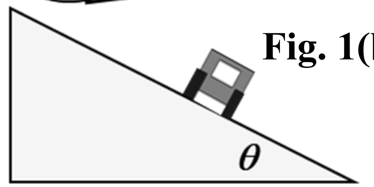
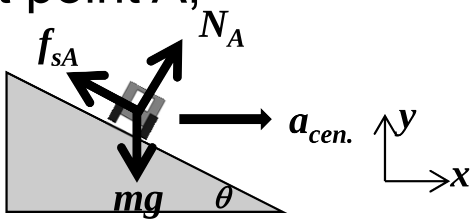


Fig. 1(b)

(a) At point A,

2



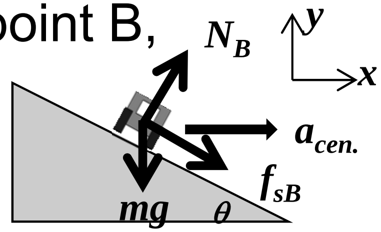
$$x:\quad N_A\sin\theta - f_{sA}\cos\theta = m\frac{v^2}{3R}$$

$$y:\quad N_A\cos\theta + f_{sA}\sin\theta - mg = 0$$ 3

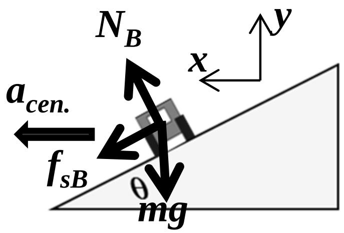
$$f_{sA} = \mu_s N_A$$

At point B,

2



or



$$x:\quad N_B\sin\theta + f_{sB}\cos\theta = m\frac{v^2}{R}$$ 3

$$y:\quad N_B\cos\theta - f_{sB}\sin\theta - mg = 0$$

$$f_{sB} = \mu_s N_B$$

(b) 1

$$\boxed{\frac{(\sin\theta - \mu_s\cos\theta)}{(\cos\theta + \mu_s\sin\theta)} = \frac{v^2}{3Rg}}$$

$$\boxed{\frac{(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)} = \frac{v^2}{Rg}}$$

$$v = \sqrt{Rg\frac{(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)}}$$ 2

or

$$v = \sqrt{3Rg\frac{(\sin\theta - \mu_s\cos\theta)}{(\cos\theta + \mu_s\sin\theta)}}$$

(c)

$$3\frac{(\sin\theta - \mu_s\cos\theta)}{(\cos\theta + \mu_s\sin\theta)} = \frac{(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)}$$ 1

$$2\sin\theta\cos\theta - 4\mu_s + 2\mu_s^2\sin\theta\cos\theta = 0$$

$$\mu_s^2\sin 2\theta - 4\mu_s + \sin 2\theta = 0$$

$$\mu_s = \frac{4 \pm \sqrt{16 - 4\sin^2 2\theta}}{2\sin 2\theta} = \frac{2 \pm \sqrt{4 - \sin^2 2\theta}}{\sin 2\theta}$$ 2

+ disagrees because v^2 will be negative in the case of point A.

$$\mu_s = \frac{2 - \sqrt{4 - \sin^2 2\theta}}{\sin 2\theta}$$ 2