General Physics (II) **期末考** **Jun. 19, 2020**

試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(52 points) (所有題目必須有計算過程,否則不予計分)

1&2. (a) (10pts) A plane wave propagates in space with its B-field expressed as $\vec{B} = \vec{B}_0 \sin(kx - \omega t)$, where $\vec{B}_0 = (-\frac{3}{2}\hat{y} + 2\hat{z}) \times 10^{-5}(T)$ and $k = 5\pi \times 10^6$(1/m). Determine the E-field of this plane wave as a function of time and space, and its wavelength and frequencyω. (b) (7pts) Next, this wave is reflected from a planar object and changed the amplitudes of the B-field and the E-field, and its direction of propagation such that the B-field now becomes $\vec{B} = \vec{B}_1 \sin(k_x x + k_y y + k_z z - \omega t)$ with $\vec{B}_1 = 10^{-5}\hat{x}$ (T), and the E-field becomes $\vec{E} = E_1 \hat{E}_0 \sin(k_x x + k_y y + k_z z - \omega t)$ **with $E_1 > 0$ and the unit vector $\hat{E}_0$ remains the same as before**. Determine (k_x, k_y, k_z) and the Poynting vector of the reflected wave. ($c = 3 \times 10^8$ m/s, $\mu_0 = 4\pi \times 10^{-7}$m/A) (Your answers to all the questions above should include correct units.)

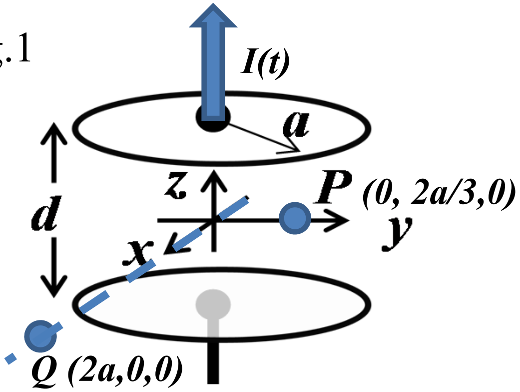
3. (17 pts) Fig. 1 shows a circular capacitor with spacing ***d*** and radius ***a*** which is connected to a circuit. The current ***I***(***t***) is zero for ***t < 0*** and ***t >T***, and ***I(t) =*** $I_0 (t/T)^3$ ($I_0 > 0$) for $0 \le t \le T$. At $t = 0$, there is no charge on the capacitor. Let point ***P*** locate inside the capacitor ($r < a$) and point ***Q*** locate outside the capacitor ($r > a$). The coordinates of ***P*** and ***Q*** are (*0,2a/3,0*), and (*2a,0,0*), respectively. Ignore the edge effect (i.e. ***E(r) = 0, for r > a***), for ***0 < t <T,***

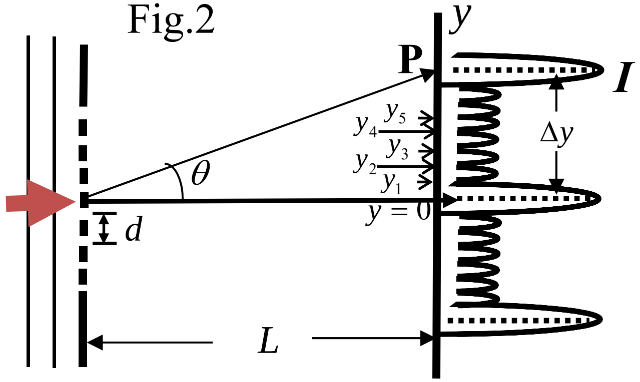
(A) (5 pts) find the direction and the magnitude of the electric field ***E*** at point ***P***;

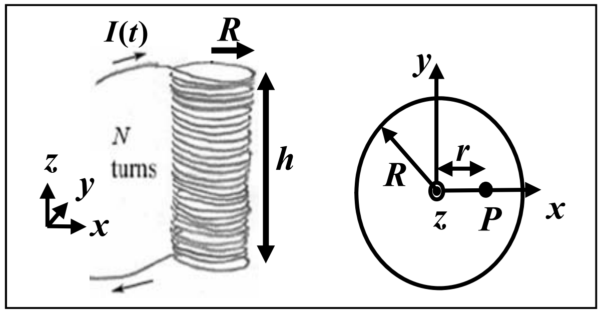
(B) (4 pts) find the the directions and the magnitudes of magnetic field B_1 at point ***P*** and

(C) (4 pts) magnetic field B_2 at point ***Q;***

(D) (4 pts) find the Poynting vector ***S***(*t*) (direction and magnitude) at ***P*** and ***Q.***







4. (18 pts) Fig. 2 shows a plane wave, with wavelength $\lambda = 600nm$, traveling through a diffraction grating with ***8***-slit and forming the interference pattern on a screen. The distance between the slits and the screen is $\boldsymbol{L = 4m}$ and the spacing between nearest neighboring slits is $\boldsymbol{d = 60\mu m}$.

a) (6 pts) Determine $\boldsymbol{y_1}$, and $\boldsymbol{y_2}$ in Fig 2, and draw the phasor diagrams the of E-fields of the waves reaching positions $\boldsymbol{y_1}$, and $\boldsymbol{y_2}$, (You need to indicate the phase difference between the neighboring E-field phasors in your diagram.)

b) (3 pts) Find the spacing (Δy) between the principal maxima on the screen. (For $0<\theta<<1$, $\sin\theta \approx \tan\theta$).

c) (9 pts) Determine the intensity ***I*** on the screen as a function of ***y***. Consider only the effect of interference.

II.選擇題(48 points)

1. (5pts) Fig.3 shows that a N-turn solenoid of radius R and length h has an alternating current $\boldsymbol{I(t) = I_0 \sin(\pi t/6)}$ in SI units ($\boldsymbol{I_0 > 0}$). Consider a point ***P*** inside the solenoid at radius ***r***. The directions of the magnetic field ***B***, the electric field ***E***, and the Poynting vector ***S*** at P for t=1 sec are (A) +z, +y, +x; (B) +z, -y, +x; (C) +z, -y, -x; (D) +z, +y, -x; (E) -z, +y, +x; (F) -z, -y, +x; (G) -z, -y, -x; (H) -z, +y, -x; (J) +y, +x, +z; (K) +y, -x, +z; (L) +y, -x, -z; (M) +y, +x, -z; (N) -y, +x, +z; (O) -y, -x, +z; (P) -y, -x, -z; (Q) -y, +x, -z; , respectively.

2. (5pts) In outer space, a disk of mass m and area A is at rest. At t=0, a laser beam starts shining on the disk at normal incidence. The acceleration of the disk as a function of time is shown in Fig. 4, which of the following is the time dependence of the amplitude $\boldsymbol{E_0}$ of the E-field of the laser beam as a function of time.

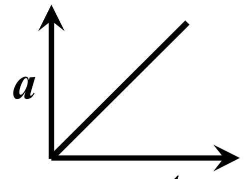
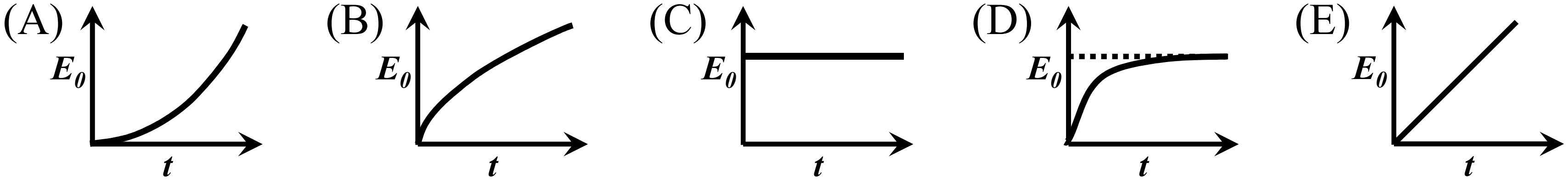


Fig.4



3. (5 pts) In Fig. 5(a), wave 1 and wave 2 are identical initially, with the same wavelength λ = 700 nm in air. Wave 1 goes through a material with length L and index of refraction n, These two waves are then reflected by mirrors to reach point $\boldsymbol{P}$ on a screen, ($\overline{ABP} = \overline{A'B'P}$). Suppose that we can vary n form n = 1.0 to n = 1.5, and the intensity $\boldsymbol{I}$ of the light at point $\boldsymbol{P}$ varies with n are given in Fig. 5(b). What can Length $\boldsymbol{L}$ be (in the unit of nm)?
(A)140 (B) 250 (C) 350 (D)500 (E) 700 (F) 875

4. (5 pts) Fig. 6(a) shows a ***LC*** circuit, viewing from the right, the electric and magnetic field inside the **inductor** is shown in Fig.6(b) at some time ***t.*** Which of the following statement is correct for the directions of the electric and magnetic fields inside the capacitor (top view)?

(A)*B*-field is counter-clock-wise (c.c.w.) looking from the right and ***E***-field is pointing to the right.

(B) ***B***-field is clock-wise (c.w.)looking from the right and ***E***-field is pointing to the right.

(C) ***B***-field is c.c.w. looking from the right and ***E***-field is pointing to the left.

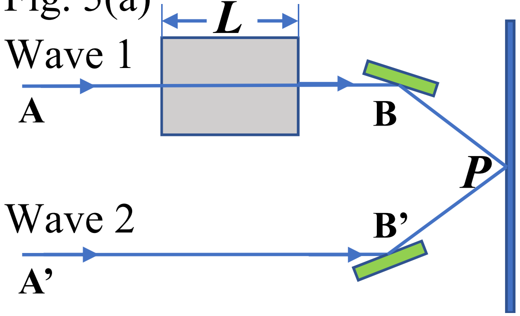
(D) ***B***-field is c.w. looking from the right and ***E***-field is pointing to the left.

(E) both of (A) and (B) are possible; **(F)** both of (C) and (D) are possible,

(G) both of (A) and (C) are possible; **(H)** both of (B) and (D) are possible,

(J) all (A) to (D) are possible

Fig. 5(a)



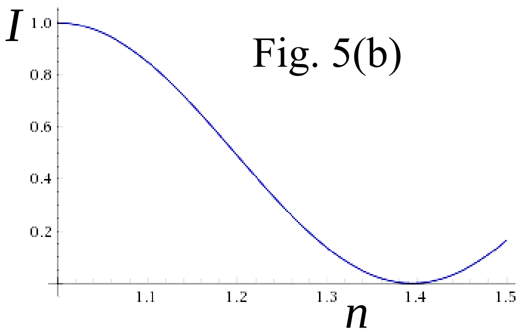


Fig. 5(b)

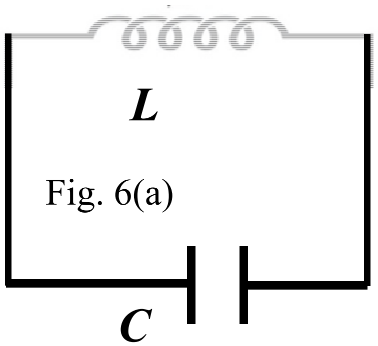


Fig. 6(a)

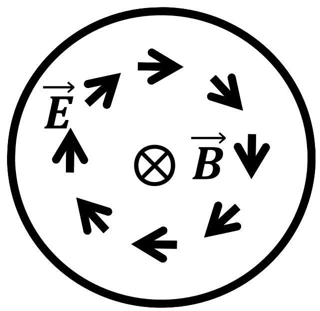


Fig. 6 (b)

5. (5 pts.) For a ***RLC*** circuit shown in Fig. 7. The AC voltage source is $\boldsymbol{V(t) = V_0 \sin \omega t}$ The current through the inductor, capacitor and resistor are $\boldsymbol{I_L}$, $\boldsymbol{I_C}$ and $\boldsymbol{I_R}$. Let **V_0=4V, ω=10^4 rad./s, *C*=5.0 μF, *R* = 40 Ω** and ***L*=1.0mH.** Which of the following action will result as the current through the resistor is zero?

(1) increasing ***L***; (2) **decreasing *L***; (3) increasing ***C***; (4) decreasing **C**; (5) increasing ω; (6) decreasing ω;

(A) **1,3,5** (B) **1,3,6** (C) **1,4,5** (D) **1,4,6** (E) **2,3,5** (F) **2,3,6** (G) **2,4,5** (H) **2,4,6** (J) None of above.

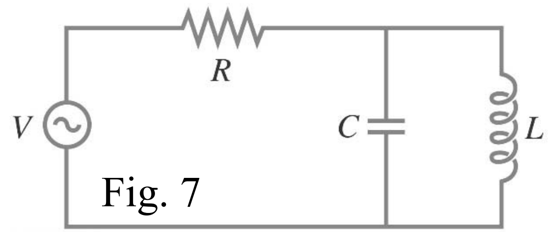


Fig. 7

6.(5 pts) Fig. 8 shows the current of resistor ($\boldsymbol{I_R}$), inductor ($\boldsymbol{I_L}$) and the total current ($\boldsymbol{I_{tot}}$) in a parallel **ac-*RLC*** circuit in Fig. 9 . Let ω_0 be the frequency such that $\boldsymbol{I_{tot}}$ is minima. (i) Which curve is $\boldsymbol{I_R}$, and (ii) is the circuit above or below ω_0 ?

(A) Curve ***A*** and it is above ω_0; (B) Curve ***B*** and it is above ω_0;
(C) Curve ***C*** and it is above ω_0; (D) Curve ***A*** and it is below ω_0;
(E) Curve ***B*** and it is below ω_0; (F) Curve ***C*** and it is below ω_0;
(G) None of above.

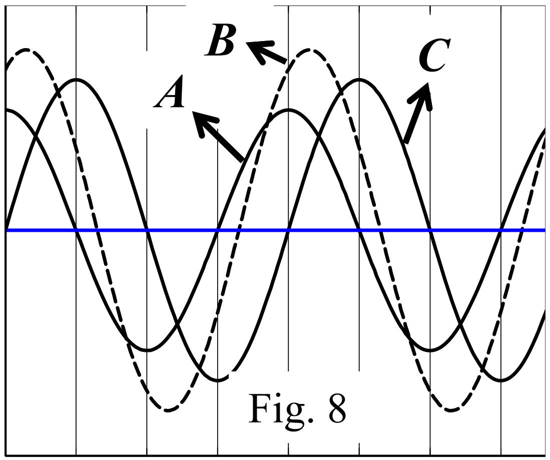


Fig. 8

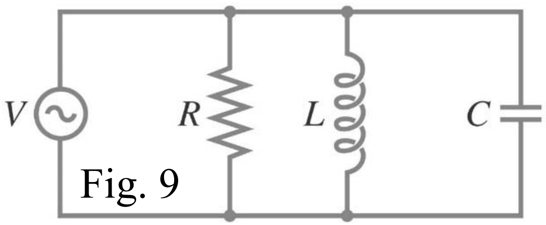


Fig. 9

1	2	3	4	5	6	7	8	9	10
C	B	F	D	A	D	H	C	A	B
11	12	13	14	15	16				
A	E	D	H	H					

1&2. (a) (10pts) A plane wave propagates in space with its B-field expressed as $\vec{B} = \vec{B}_0 \sin(kx - \omega t)$, where $\vec{B}_0 = (-\frac{3}{2}\hat{y} + 2\hat{z}) \times 10^{-5}(T)$ and $k = 5\pi \times 10^6$(1/m). Determine the E-field of this plane wave as a function of time and space, and its wavelength and frequencyω. (b) (7pts) Next, this wave is reflected from a planar object and changed the amplitudes of the B-field and the E-field, and its direction of propagation such that the B-field now becomes $\vec{B} = \vec{B}_1 \sin(k_x x + k_y y + k_z z - \omega t)$ with $\vec{B}_1 = 10^{-5}\hat{x}$ (T), and the E-field becomes $\vec{E} = E_1 \hat{E}_0 \sin\left(k_x x + k_y y + k_z z - \omega t\right)$ **with $E_1 > 0$ and the unit vector $\hat{E}_0$ remains the same**. Determine (k_x, k_y, k_z) and the Poynting vector of the reflected wave. ($c = 3 \times 10^8$ m/s, $\mu_0 = 4\pi \times 10^{-7}$m/A) (Your answers to all the questions above should include correct units.)

(a) Let $\vec{E} = E_0 \hat{E} \sin(kx - \omega t)$

$$\frac{E_0}{B_0} = c$$

$$B_0 = \sqrt{(\frac{3}{2})^2 + 2^2} \times 10^{-5} = 2.5 \times 10^{-5}(T) \quad 1$$

$$E_0 = 2.5 \times 10^{-5} \times 3 \times 10^8 = 7500(\mathrm{V/m}) \quad 2$$

$$\hat{E} \times \hat{B} = \hat{k} \Rightarrow \hat{B} \times \hat{k} = \hat{E}$$

$$\hat{B} = (0, -\frac{3}{5}, \frac{4}{5}), \quad \hat{k} = (1,0,0) \quad 1$$

$$\hat{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & -3/5 & 4/5 \\ 1 & 0 & 0 \end{vmatrix} = (0, 4/5, 3/5) \quad 2$$

$$\Rightarrow \vec{E} = 7500(\frac{4}{5}\hat{y} + \frac{3}{5}\hat{z}) \sin(kx - \omega t)(V/m)$$

$$k = \frac{2\pi}{\lambda} = 5\pi \times 10^6 (1/m)$$

$$\Rightarrow \lambda = 4 \times 10^{-7}(m) = 0.4(\mu\mathrm{m}) \quad 2$$

$$c = \frac{\omega}{k} \Rightarrow \omega = ck$$

$$\Rightarrow \omega = 3 \times 10^8 \times 5\pi \times 10^6 = 1.5\pi \times 10^{15}(1/s)$$

$$(or) \quad = 4.7 \times 10^{15}(1/s) \quad 2$$

(b) $\hat{E}\times\hat{B}=\hat{k}$

$$\hat{B}=(1,0,0),\ \ \hat{E}=(0,4/5,3/5)$$

$$\Rightarrow \hat{k}=\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ 0 & 4/5 & 3/5\\ 1 & 0 & 0\end{vmatrix}=(0,3/5,-4/5)$$

$$\Rightarrow \vec{k}=k\hat{k}=5\pi\times10^{6}\times(0,3/5,-4/5)$$ 3

$$\vec{E}=E_1(\frac{4}{5}\hat{y}+\frac{3}{5}\hat{z})\sin(5\pi\times10^{6}\cdot(\frac{3}{5}y-\frac{4}{5}z)-\omega t)$$

$$\frac{E}{B}=c\ \Rightarrow \frac{E_1}{10^{-5}}=c$$

$$\Rightarrow E_1=3\times10^{8}\times10^{-5}=3000(V/m)$$ 1

$$\vec{S}=\frac{\vec{E}\times\vec{B}}{\mu_0}$$

$$\Rightarrow\vec{S}=\frac{1}{4\pi\times10^{-7}}\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ 0 & \frac{4}{5} & \frac{3}{5}\\ 1 & 0 & 0\end{vmatrix}3000\times10^{-5}\times\sin^{2}(5\pi\times10^{6}\cdot(\frac{3}{5}y-\frac{4}{5}z)-\omega t)$$

$$\Rightarrow\vec{S}=(\frac{3}{5}\hat{y}-\frac{4}{5}\hat{z})\frac{3}{4\pi}\times10^{5}\times\sin^{2}((3\pi y-4\pi z)\times10^{6}-\omega t)$$

$$\Rightarrow\vec{S}=2.38\times10^{4}\times(\frac{3}{5}\hat{y}-\frac{4}{5}\hat{z})\times\sin^{2}((3\pi y-4\pi z)\times10^{6}-\omega t)\,\boldsymbol{W/m^2}$$ 3

1. (17 pts) Fig. 1 shows a circular capacitor with spacing $\boldsymbol{d}$ and radius $\boldsymbol{a}$ which is connected to a circuit. The current $\boldsymbol{I}(\boldsymbol{t})$ is zero for $\boldsymbol{t < 0}$ and $\boldsymbol{t > T}$ and $\boldsymbol{I(t) = I_0\,(t/T)^3}$ for $\boldsymbol{0 \le t \le T}$. At $\boldsymbol{t = 0}$, there is no charge on the capacitor. Let point $\boldsymbol{P}$ locate inside the capacitor $(\boldsymbol{r < a})$ and point $\boldsymbol{Q}$ locate outside the capacitor $(\boldsymbol{r > a})$. The coordinates of $\boldsymbol{P}$ and $\boldsymbol{Q}$ are $(0,2a/3,0)$, and $(2a,0,0)$, respectively. Ignoring the edge effect (i.e. $\boldsymbol{E(r) = 0}$, ***for*** $\boldsymbol{r > a}$). For $\boldsymbol{0 < t < T}$,

(A) (5 pts) find the direction and the magnitude of the electric field $\boldsymbol{E}$ at point $\boldsymbol{P}$;

(B) (4 pts) find the the directions and the magnitudes of magnetic field $\boldsymbol{B_1}$ at point $\boldsymbol{P}$ and

(C) (4 pts) magnetic field $\boldsymbol{B_2}$ at point $\boldsymbol{Q}$;

(D) (4 pts) find the Poynting vector $\boldsymbol{S}(t)$ (direction and magnitude) at $\boldsymbol{P}$ and $\boldsymbol{Q}$.

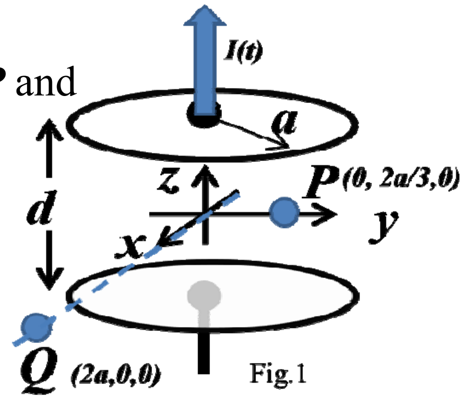
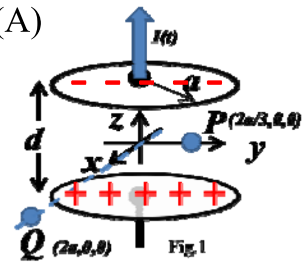


Fig.1

(A)

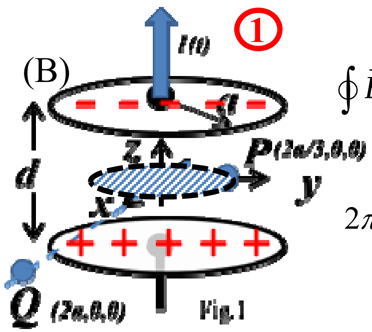


$$Q(t) = \int I(t)dt = I_0 \cdot \frac{t^4}{4T^3} \quad \text{for } 0 \le t \le T \quad 2$$

$$\vec{E}(P) = \frac{\sigma}{\varepsilon_0}\hat{k} = \frac{I_0\left(t^4/T^3\right)}{4\varepsilon_0 \pi a^2}\hat{k} \quad 1$$

2

(B)



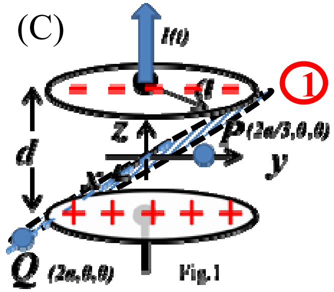
1

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\left(I_{inc.} + \varepsilon_0 \frac{d\Phi_E}{dt}\right) = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$$

$$2\pi r B = \mu_0\varepsilon_0 \frac{dE(t)}{dt}\pi r^2 = \mu_0 I_0 \cdot \frac{t^3}{T^3}\cdot\frac{r^2}{a^2} \quad 2$$

$$\vec{B}(P) = \frac{\mu_0 I_0}{3\pi a}\frac{t^3}{T^3}(-\hat{i}) \quad 1$$

(C)



1

$$2\pi r B = \mu_0\varepsilon_0 \frac{dE(t)}{dt}\pi a^2 = \mu_0 I_0 \cdot \frac{t^3}{T^3}\cdot\frac{a^2}{a^2}$$

2

$$\vec{B}(Q) = \frac{\mu_0 I_0}{4\pi a}\frac{t^3}{T^3}(\hat{j}) \quad 1$$

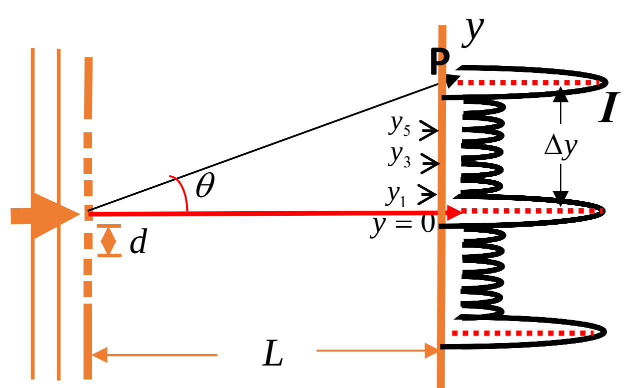
(D)

$$\vec{S}(P) = \frac{\vec{E}(P)\times\vec{B}(P)}{\mu_0} = \frac{1}{\mu_0}\frac{I_0\left(t^4/T^3\right)}{4\varepsilon_0\pi a^2}\hat{k}\times\frac{\mu_0 I_0}{3\pi a}\frac{t^3}{T^3}(-\hat{i})$$

$$= \frac{I_0^2}{12\varepsilon_0}\frac{1}{\pi^2 a^3}\frac{t^7}{T^6}(-\hat{j}) \quad 2$$

$$\vec{S}(Q) = \frac{\vec{E}(P)\times\vec{B}(P)}{\mu_0} = \frac{1}{\mu_0} 0 \times \frac{\mu_0 I_0}{4\pi a}\frac{t^3}{T^3}(\hat{j}) = 0 \quad 2$$

4. (15 pts) A plane wave, with wavelength $\lambda = 600\mathbf{nm}$, travels through a diffraction grating with ***8***-slit and the interference pattern on the screen is shown in Fig. 3. The distance between the ***8***-slit and the screen is ***L = 4m*** and the spacing between nearest neighboring slits is ***d* = 60*μm***.



a) (3 pts) Find the spacing (Δy) between the principal maxima ***y=0*** on the screen. Assume $0<\theta<<1$, then $\sin\theta \approx \tan\theta$ can be use.?

b) (6 pts) What are the phase differences and position of y_1, and y_2? Draw the phasor diagrams for y_1, and y_2.

c) (9 pts) Determine the intensity ration $I(\theta)/I(\theta=0)$ on the screen as a function of $\phi = k\Delta x$ where Δx is the path difference. You need to draw the appropriate phasor diagram.

(a)

$$\delta = k\Delta x = \frac{2\pi}{\lambda}\cdot d\sin\theta$$

$$\approx \frac{2\pi}{\lambda}\cdot d\tan\theta = \frac{2\pi}{\lambda}\cdot d\cdot\frac{y}{L} \quad 2$$

$$\delta = 2\pi \rightarrow \Delta y = \frac{L\lambda}{d}$$

$$= \frac{4\cdot 600\cdot 10^{-9}}{60\cdot 10^{-6}} = 40\text{ mm} \quad 1$$

$$\frac{\delta}{2\pi} = \frac{\Delta x}{\lambda} = \frac{d\sin\theta}{\lambda} \approx \frac{d}{\lambda}\cdot\frac{y}{L} \quad 2$$

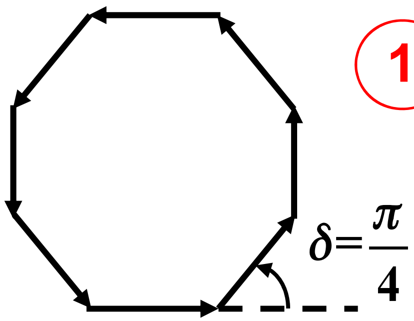
$$\delta = 2\pi \rightarrow \Delta y = \frac{\lambda}{d}L$$

$$\Delta y = \frac{4\cdot 600\cdot 10^{-9}}{60\cdot 10^{-6}} = 40\text{ mm} \quad 1$$

(b)

$$y_1: \quad \delta = \frac{2\pi}{8} = \frac{\pi}{4} \text{ (the phase difference)} \quad 1$$

$$\rightarrow y_1 = \frac{\Delta y}{8} = 5\text{ mm} \quad 1$$

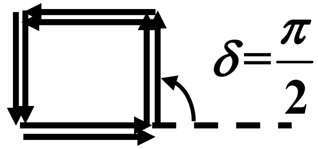


1

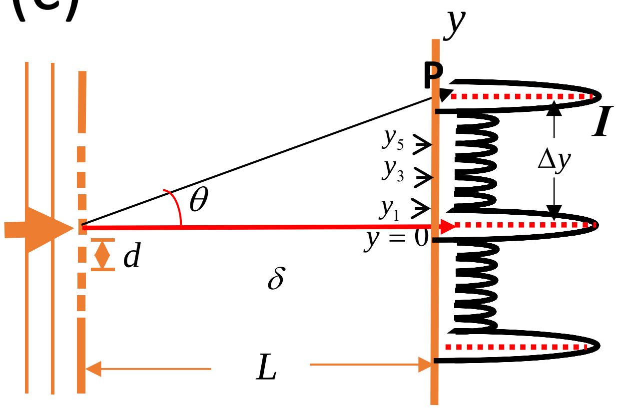
$$y_4: \quad \delta = 2\frac{2\pi}{8} = \frac{\pi}{2} \text{ (the phase difference)} \quad 1$$

1

$$\rightarrow y_1 = \frac{2\Delta y}{8} = 10\text{ mm} \quad 1$$



(c)



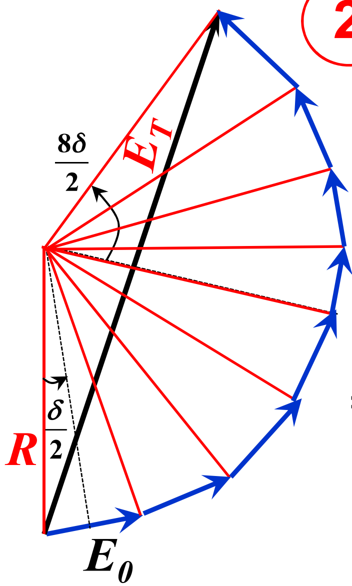
$\boldsymbol{E_i}$ is the electric field is from the ith slit, reaching at the position P.

$$E_i = E_0 \sin(\phi_1 + (i-1)\cdot\delta)\,,\ \delta = k\Delta x\,,\ \phi_1 = kx_1 - \omega t_1$$

The total electric field $\boldsymbol{E_T}$ at the position P is:

$$E(\theta) = \sum_{i=1}^{8} E_0 \sin(\phi_1 + (i-1)\cdot\delta)\,,\ \delta = k\Delta x$$

$$= E_0 \sin(\phi_1 + \varphi)$$



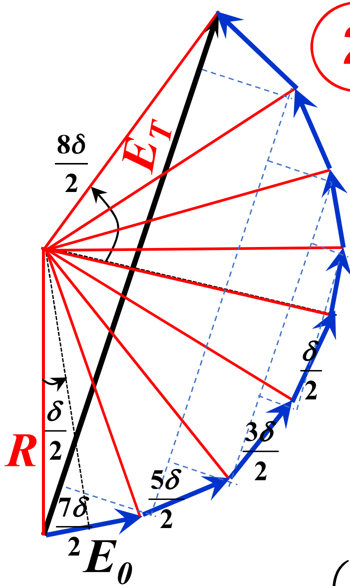
2

$$\frac{E_0/2}{R} = \sin\frac{\delta}{2}$$ 2

$$\frac{E_T/2}{R} = \sin\frac{8\delta}{2}$$ 2

$$\Rightarrow E_T = E_0 \frac{\sin 4\delta}{\sin \delta/2}$$ 1

$$\Rightarrow \frac{I(\theta)}{I(\theta=0)} = \frac{\sin^2 4\delta}{\sin^2 \delta/2}$$ 2



2

$$E_T = 2E_0\cos\frac{\delta}{2} + 2E_0\cos\frac{3\delta}{2} + 2E_0\cos\frac{5\delta}{2} + 2E_0\cos\frac{7\delta}{2}$$ 5

$$\Rightarrow \frac{I(\theta)}{I(\theta=0)} = \frac{\left(\cos\frac{\delta}{2} + \cos\frac{3\delta}{2} + \cos\frac{5\delta}{2} + \cos\frac{7\delta}{2}\right)^2}{16}$$ 2