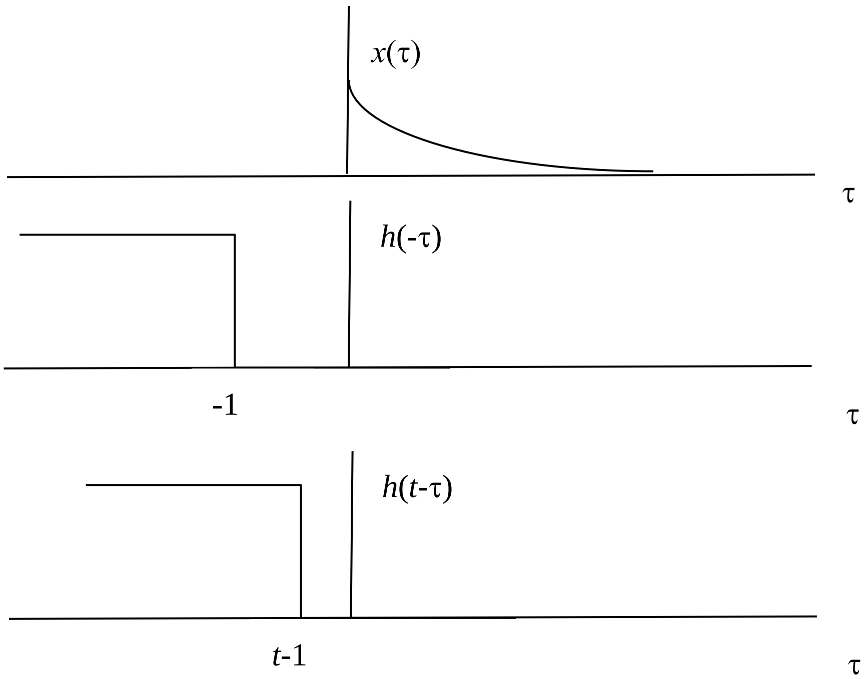
Signals and Systems – Quiz 2 03/29/2021

Name: Student ID:

1. (10%) Compute $y(t) = x(t) * h(t)$, where $x(t) = e^{-at}u(t), a > 0, h(t) = u(t-1)$



Ans:

$$y(t) = x(t) * h(t) = \int x(\tau)h(t-\tau)\,d\tau$$

For $t-1 < 0$, the product of $x(\tau)$ *and* $h(t-\tau)$ is zero.

For $t-1 > 0$, $x(\tau)h(t-\tau) = \begin{cases} e^{-a\tau}, 0 < \tau < t-1 \\ 0, otherwise \end{cases}$

$$y(t) = \int_0^{t-1} e^{-a\tau} d\tau = \frac{1}{a}(1 - e^{-a(t-1)})$$

$$y(t) = \frac{1}{a}(1 - e^{-a(t-1)})u(t-1)$$

2. (20%) Compute and plot $y[n] = x[n] * h[n]$, where

$$x[n] = \begin{cases} 1, & 4 \le n \le 7 \\ 0, & \text{otherwise} \end{cases}, \qquad h[n] = \begin{cases} 1, & 5 \le n \le 9 \\ 0, & \text{otherwise} \end{cases}$$

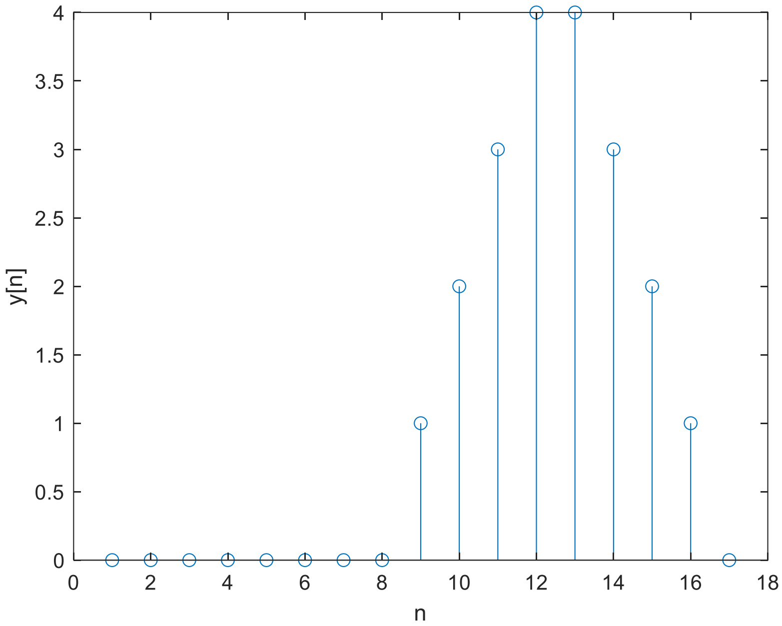
Ans:

$$x[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$= x[4]h[n-4] + x[5]h[n-5] + x[6]h[n-6] + x[7]h[n-7]$$

This give

$$y[n] = \begin{cases} n-8, & 9 \le n \le 11 \\ 4, & 12 \le n \le 13 \\ 17-n, & 14 \le n \le 16 \\ 0, & \text{otherwise} \end{cases}$$



3. (30%) The input-output relationship of an LTI system is described as

$$y[n] - \frac{1}{3}y[n-1] = x[n].$$

(a) What is the impulse response of this system?

(b) Suppose $x[n] = \left(\frac{1}{2}\right)^n u[n]$, find the particular and homogeneous solutions of this system.

Ans:

(a) Let $x[n] = \delta[n]$ and $y[n] = 0$ at $n < 0$

$$y[0] = \frac{1}{3}y[-1] + x[0] = 1$$

$$y[1] = \frac{1}{3}y[0] + x[1] = \frac{1}{3}$$

$$y[2] = \frac{1}{3}y[1] + x[2] = (\frac{1}{3})^2$$

$$\vdots$$

$$y[n] = \frac{1}{3}y[n-1] + x[n] = (\frac{1}{3})^n$$

$$h[n] = (\frac{1}{3})^n u[n]$$

(a) Let guess the homogeneous solution $y_h[n] = A(1/3)^n u[n]$

It shows that $A\left(\frac{1}{3}\right)^n - \frac{1}{3}A\left(\frac{1}{3}\right)^{n-1} = 0$

Particular solution $y_p[n] = B(1/2)^n u[n]$

$$B\left(\frac{1}{2}\right)^n - \frac{1}{3}B\left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^n \quad \text{Therefore, } B = 3.$$

Initial rest, y[-1]=0, $y[0] = x[0] + (1/3)y[-1] = x[0] = 1$. Now we also have

$$y[n] = y_p[n] + y_h[n] = A(1/3)^n u[n] + B(1/2)^n u[n]$$

$$y[0] = A + B = 1, \qquad A = 1 - B = -2$$

$$y[n] = y_p[n] + y_h[n] = \left(3\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{3}\right)^n\right)u[n]$$

4. (30%) Consider a system *S* with input $x[n]$ and output $y[n]$. This system is obtained through a series interconnection of a system S_1 followed by a system S_2. The input-output relationships for S_1 and S_2 are

$$S_1\colon\ y_1[n] = 2x_1[n] + 3x_1[n-2]$$

$$S_2\colon\ y_2[n] = x_2[n-1] + \frac{1}{2}x_2[n-3]$$

where $x_1[n]$ and $x_2[n]$ denote input signals.

(a) Determine the input-output relationship for system S.

(b) Draw a block diagram representation of S.

Ans:

(a) The signal $x_2[n]$, which is the input to S_2, is $y_1[n]$.

$$y_2[n] = x_2[n-1] + \frac{1}{2}x_2[n-3]$$

$=y_1[n-1] + \frac{1}{2}y_1[n-3]$

$=2x_1[n-1] + 3x_1[n-3]$

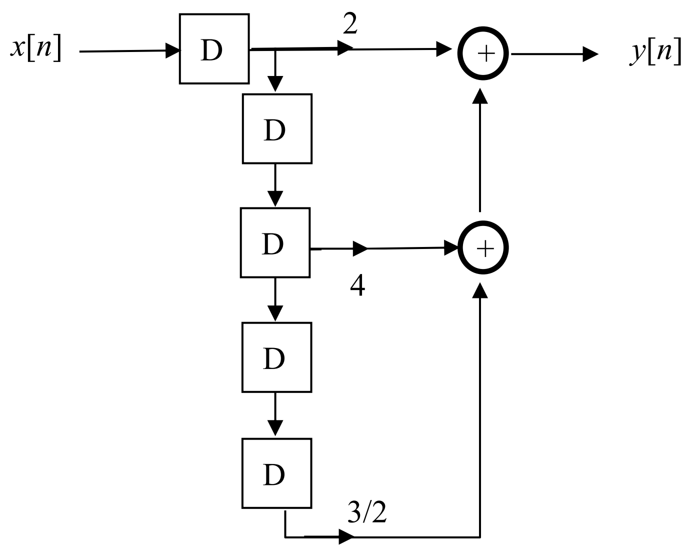
$+\frac{1}{2}(2x_1[n-3] + 3x_1[n-5])$

$=2x_1[n-1] + 4x_1[n-3]+\frac{3}{2}x_1[n-5]$

The input-output relationship for *S* is

$y[n]=2x[n-1] + 4x[n-3]+\frac{3}{2}x[n-5]$

(b)



5. (20%) Consider the LTI system consisting a pure time shift

$$y(t) = x(t - t_0)$$

   (a) Determine the impulse response of this system.
   (b) Determine the impulse response of its inverse system.

Ans:

   (a) $\delta(t - t_0)$
   (b) $\delta(t + t_0)$

6. (10%) For a discrete-time causal LTI system, the impulse response should satisfy one condition. Show this condition.
   Ans: $h[n] = 0, n < 0$