General Physics (I) **期末考** **Jan. 8, 2021**

試卷請註明、姓名、班級、學號，請遵守考場秩序

I.**計算題**(50points) (所有題目必須有計算過程,否則不予計分)

1. (a) (7 pts) As shown in Fig. 1(a) a ring of mass ***m*** and radius ***R*** is fixed to one end of a rod whose length is ***2R*** and the mass is ***6m***. The other end of the rod is attached to a pivot on the ceiling and the whole assembly is free to swing as a physical pendulum, Determine the period of the physical pendulum.

   (b) (8 pts) Now the ring is detached from the rod and placed on a circular track of radius *3R* as shown in Fig. 1(b), and released to roll. The ring executes pure roll motion and rolls back and forth on the track. Determine the period of the motion of the ring.

2. A heat engine takes one mole of ideal monatomic gas around the cycle shown in Fig.2 (adiabatic **a→b**, straight line **b→c**, isothermal **c→d**,, and isobaric **d→a**). The volume at points **a**, **b**, **c**, and d are given by $\mathbf{8V_0}$, $\mathbf{V_0}$, $\mathbf{8V_0}$, and $\mathbf{16V_0}$, respectively. The pressure at point **a** is $\mathbf{P_0}$. (Write your answer in term of $\mathbf{P_0}$, $\mathbf{V_0}$, **R**, **ln2**, **ln3**, **ln5**, and **ln7**)

a) (5pts) Determine the thermal dynamic variables (**P**, **V**, and **T**) at points **a**, **b**, **c**, and **d**.

b) (12pts) Calculate the work done (by the gas), heat, internal energy change, and entropy change for each process (**a→b**, **b→c**, **c→d**, and **d→a**).

c) (3pts) Determine the efficiency of the heat engine. (Summarize your answer as the table shown below.)

Fig. 1(a)

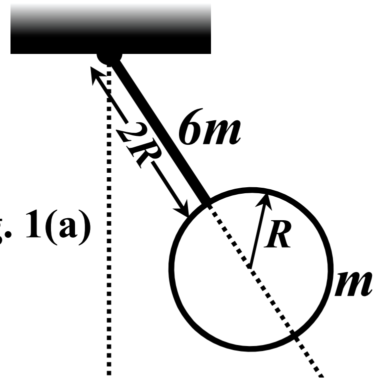
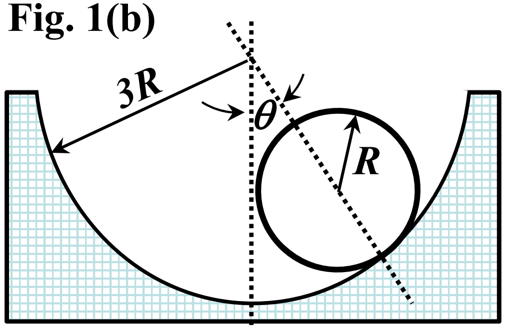


Fig. 1(b)



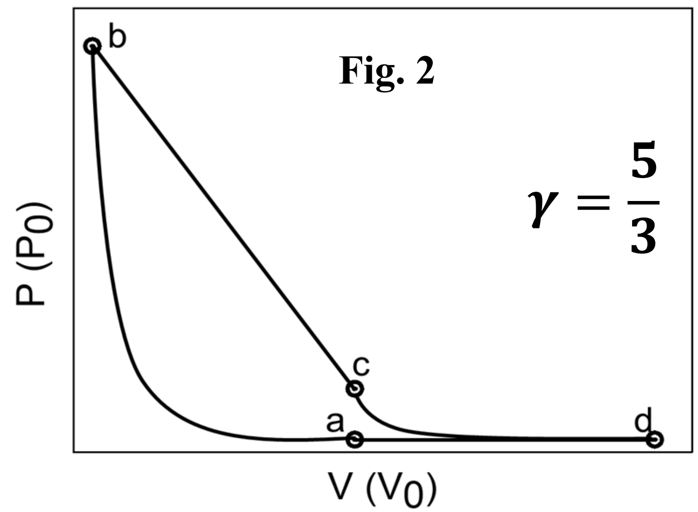


Fig. 2

	P (P_0)	V (V_0)	T (P_0V_0/R)
a	1	8	
b		1	
c		8	
d		16	

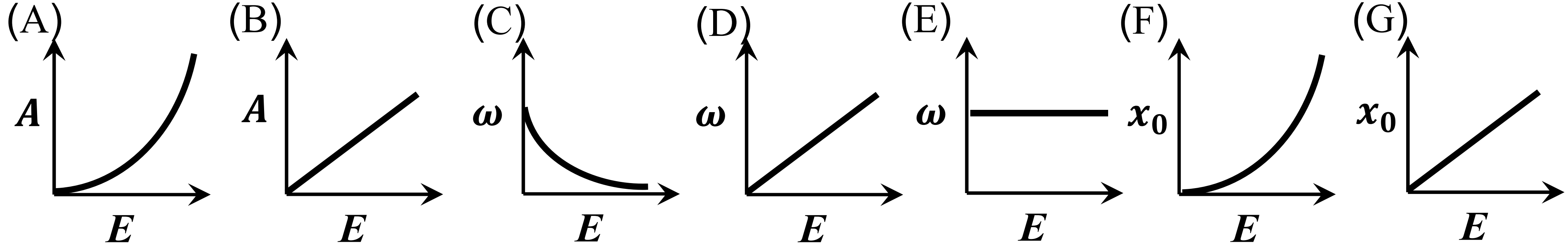
	W (P_0V_0)	Q (P_0V_0)	ΔE_{int} (P_0V_0)	ΔS (R)
a→b				
b→c				
c→d				
d→a				

3. (15 pts) A ***x***-kg iron at **900 K**, with specific heat **450 J/kg/K**, is in contact with **1 kg** ice at **0 ºC**. The final temperature is **27 ºC** (both the water/ice and iron). Assume no heat loss from the system. (You need to include correct units for all the answers to the questions below.)
   a) (3 pts) What is the mass of the iron?
   b) (7 pts) What is the entropy change of the ice when it becomes 27^0C water?
   c) (2 pts) What is the entropy change of the iron?
   d) (3 pts) What is the entropy change of the environment and universe ?
   (c = 4200 J/kg for water, the latent heat of fusion for water $\boldsymbol{L_f}$ = 3.34 x 10^5 J/kg, ln2 = 0.693, ln3 = 1.10, and $\ln(1 \pm x) \approx \pm x, for\ |x| \ll 1;$ 0^0C ≡ 273 K)

II 選擇題 (50points)

1. (5 pts) Which of the following is a true statement?
   (A) It is impossible to transfer heat from a cooler body to a hotter body.
   (B) It is not possible to convert work entirely into heat.
   (C) The free expansion of a gas is an example of an irreversible process.
   (D) All of these statements are false.

2. (5pts) A particle of mass m is confined to move on x-axis under the influence of a conservative force. The potential energy of the particle is $U(x) = \boldsymbol{a}(x - \boldsymbol{b})^2$, where $\boldsymbol{a}$ and $\boldsymbol{b}$ are positive constants. The total energy of the particle is $\boldsymbol{E}$ ($\boldsymbol{E} \geq \mathbf{0}$), and the position of the particle as a function of time is $\boldsymbol{x}(t) = \boldsymbol{A}\cos(\boldsymbol{\omega} t + \boldsymbol{\phi}) + \boldsymbol{x_0}$. Which of the following is correct?



3. (5pts) Fig. 3 shows a Carnot engine that works between $\boldsymbol{T_1}$=400 K and $\boldsymbol{T_2}$ =160 K drives a Carnot refrigerator that works between $\boldsymbol{T_3}$=300 K and $\boldsymbol{T_4}$ =210 K. What is the absolute value of $\boldsymbol{a}$, if $\mathbf{Q_3} = \boldsymbol{a}\cdot\mathbf{Q_1}$?
(A) $a<0.1$; (B) $0.1 \leqq a<0.5$; (C) $0.5 \leqq a<1.0$; (D) $1.0 \leqq a<1.5$; (E) $1.5 \leqq a< 2.0$; (F) $2.0 \leqq a< 2.5$; (G) $2.5 \leqq a< 3.0$; (H) $3.0 \leqq a< 3.5$; (J) $3.5 \leqq a< 4.0$; (K) $4.0 \leqq a$.

4. (5 pts) A gyroscope has a wheel (mass M) at position $2a/3$ from one end of the axle (length a), which is pivoted at point O as shown in Fig. 4a. The wheel rotates about its axle with spin angular velocity $\boldsymbol{\omega}$ and precessional angular velocity $\boldsymbol{\Omega_0}$. At the moment shown in the Fig. 4b, a weight with mass $\boldsymbol{m=0.2M}$ placed at one end of the axle such that the precessional velocity of the gyroscope changes to $\boldsymbol{x\Omega_0}$. Assume the spin angular velocity is much greater than the precessional angular velocity. What is the value of $\boldsymbol{x}$? (Ignore the mass of the axle)
(A) $x = 1$ (B) $0 < x \leq 0.25$ (C) $0.25 < x \leq 0.5$ (D) $0.5< x \leq 0.75$ (E) $0.75 < x < 1$
(F) $1< x \leq 1.2$ (G) $1.2 < x \leq 1.4$ (H) $1.4< x \leq 1.6$ (J) $1.6 < x \leq 1.8$ (K) $1.8 < x \leq 2.0$
(L) $2 < x$ (M) $x = 0$.

5. (5 pts)) In an insulated container there are 1.0 mole of nitrogen (N_2) gas (at pressure $\boldsymbol{P_0}$, and volume $\boldsymbol{V_0}$) and 1.0 mole of argon (Ar) gas (at pressure $\boldsymbol{P_0}$, and volume $2\boldsymbol{V_0}$) separated by an insulating wall initially .The temperature of $\mathbf{N_2}$ gas is $\boldsymbol{T_0}$. The insulating wall is then removed suddenly and the gases (assumed ideal) are allowed to mix. The final temperature becomes $\boldsymbol{xT_0}$. when the system reaches equilibrium. What is the value $\boldsymbol{x}$? Note: in this temperature range, **γ=5/3** for monatomic ideal gas and **γ=7/5** for diatomic ideal gas.

(A) $1 \le x \le 1.1$ (B) $1.1 < x \le 1.2$ (C) $1.2 < x \le 1.3$ (D) $1.3 < x \le 1.4$ (E) $1.4 < x \le 1.5$ (F) $1.5 < x \le 1.6$ (G) $1.6 < x \le 1.7$ (H) $1.7 < x \le 1.8$ (J) $1.8 < x \le 1.9$ (K) $1.9 < x$

6. (5 pts) Same as problem 5, if $\boldsymbol{T_0}$**= 480K,** the change of the entropy of the system (after the insulated wall removed) is $\Delta \boldsymbol{S/R} = \boldsymbol{a}$. What is the value $\boldsymbol{a}$? (Note: **ln2 ~ 0.7, ln3 ~1.1, ln5 ~ 1.6, ln7~1.9, ln11 ~2.4,** and **ln13~2.6**)

(A) $a \le 1$ (B) $1 < a \le 1.2$ (C) $1.2 < a \le 1.4$ (D) $1.4 < a \le 1.6$ (E) $1.6 < a \le 1.8$
(F) $1.8 < a \le 2$ (G) $2 < a \le 2.25$ (H) $2.25 < a \le 2.5$ (J) $2.5 < a \le 2.75$ (K) $2.75 < a \le 3$ (L) $3 < a$

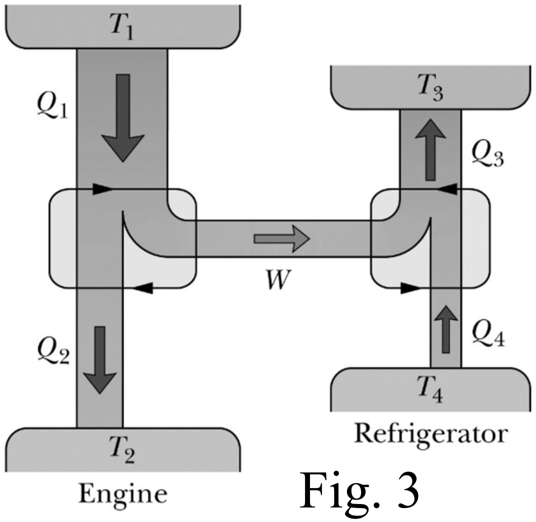


Fig. 3

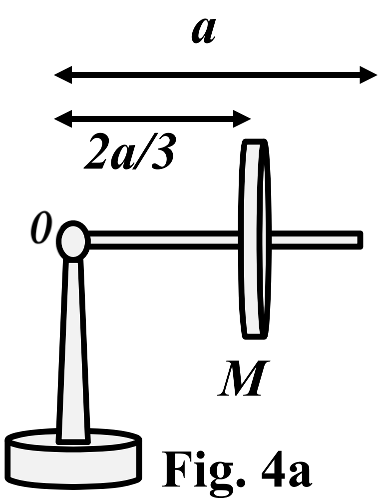


Fig. 4a

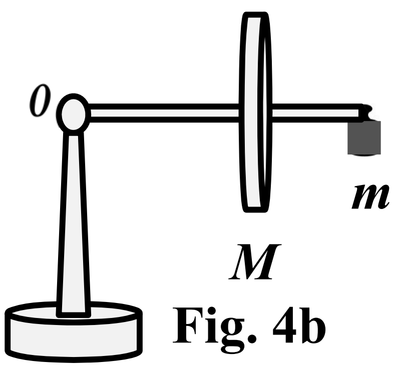
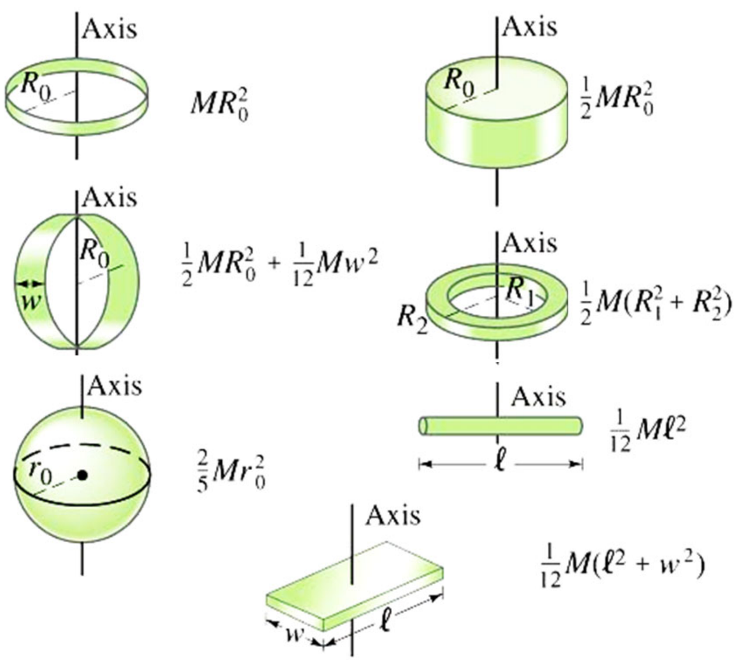


Fig. 4b

Reference for moment of inertia



Multiple Choice Questions:

1	**2**	**3**	**4**	**5**	**6**				
C	**E**	**F**	**G**	**D**	**E**				
7	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
B	**B**	**A**	**C**	**E**	**E**	**E**	**E**	**C**	**G**

1. (a) (7 pts) As shown in Fig. 1(a) a ring of mass $\boldsymbol{m}$ and radius $\boldsymbol{R}$ is fixed to one end of a rod whose length is $\boldsymbol{2R}$ and the mass is $\boldsymbol{6m}$. The other end of the rod is attached to a pivot on the ceiling and the rod-ring assembly is free to swing as a physical pendulum, Determine the period of the physical pendulum in terms of m, R, and $\boldsymbol{\pi}$. (g=10 m/s^2)
(b) (8 pts) Now the ring is detached from the rod and placed on a circular track of radius $3R$ as shown in Fig. 1(b), and released to roll. The ring executes pure roll motion and rolls back and forth on the track. Determine the period of the motion of the ring.

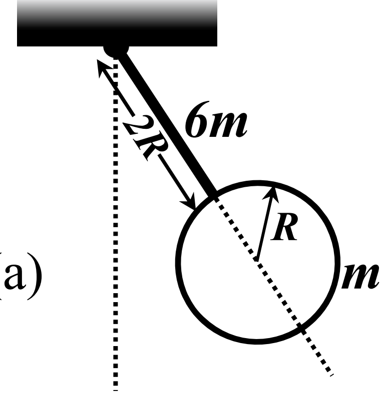


Fig. 1(a)

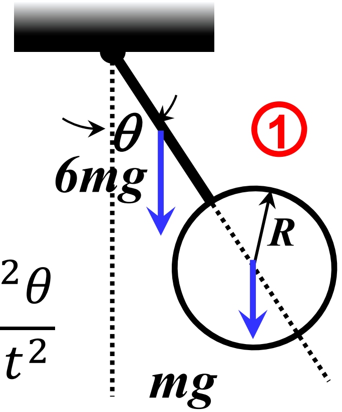
(a) **Solution 1:** $T = 2\pi\sqrt{\dfrac{I}{mgh}}$

$$I = I_{rod} + I_{ring}$$
$$= \frac{1}{3}\cdot 6m\cdot(2R)^2 + mR^2 + m(3R)^2$$
$$= 18mR^2 \quad ②$$

$$h = \frac{6m\cdot R + m\cdot 3R}{6m+m} = \frac{9}{7}R \quad ②$$

$$T = 2\pi\sqrt{\frac{18mR^2}{7mg\frac{9}{7}R}} = 2\pi\sqrt{\frac{2R}{g}} = 2\pi\sqrt{\frac{R}{5}} \quad ③$$

Solution 2:



$$\sum \vec{\tau} = I\vec{\alpha}$$

$$-6mgR\sin\theta - mg\cdot 3R\sin\theta = I\frac{d^2\theta}{dt^2} \quad ①$$

$$\sin\theta \sim \theta, for\ \theta \ll 1$$

$$I = I_{rod} + I_{ring} = 18mR^2 \quad ②$$

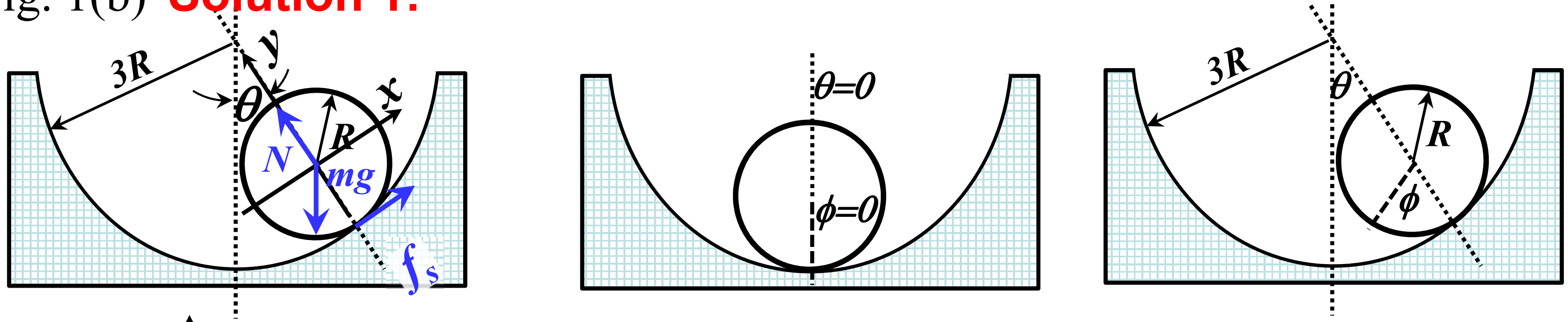
$$\Rightarrow -6mgR\theta - 3mgR\theta = 18mR^2\frac{d^2\theta}{dt^2}$$

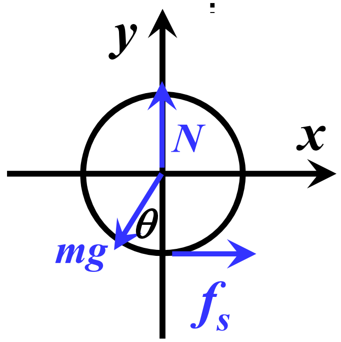
$$\Rightarrow \frac{d^2\theta}{dt^2} + \frac{9mgR}{18mR^2}\theta = 0$$

$$\Rightarrow \frac{d^2\theta}{dt^2} + \frac{g}{2R}\theta = 0 \quad \Rightarrow \omega = \sqrt{\frac{5}{R}} = \sqrt{\frac{5}{R}} \quad ①$$

$$\Rightarrow T = 2\pi\sqrt{\frac{R}{5}} \quad ③$$

Fig. 1(b) **Solution 1:**





$$\sum \vec{F} = m\vec{a} \quad \Rightarrow x: -mg\sin\theta + f_s = ma \qquad (1)$$

$$\sum \vec{\tau} = I\vec{\alpha} \quad \Rightarrow z: f_s R = mR^2\alpha = mR^2\frac{d^2\phi}{dt^2} \qquad (2)$$

For pure roll on the track, $\Rightarrow \phi = -\dfrac{3R-R}{R}\theta = -2\theta$ (3)

and $\quad a = 2R\dfrac{d^2\theta}{dt^2}, \Rightarrow \dfrac{d^2\phi}{dt^2} = -2\dfrac{d^2\theta}{dt^2}$ (4)

From (2),(3),(4): $\Rightarrow f_s R = -2mR^2\dfrac{d^2\theta}{dt^2}$

$$\Rightarrow f_s = -2mR\frac{d^2\theta}{dt^2} \qquad (5)$$

From (1),(4),(5): $-mg\sin\theta - 2mR\dfrac{d^2\theta}{dt^2} = 2mR\dfrac{d^2\theta}{dt^2}$

For θ<<1,

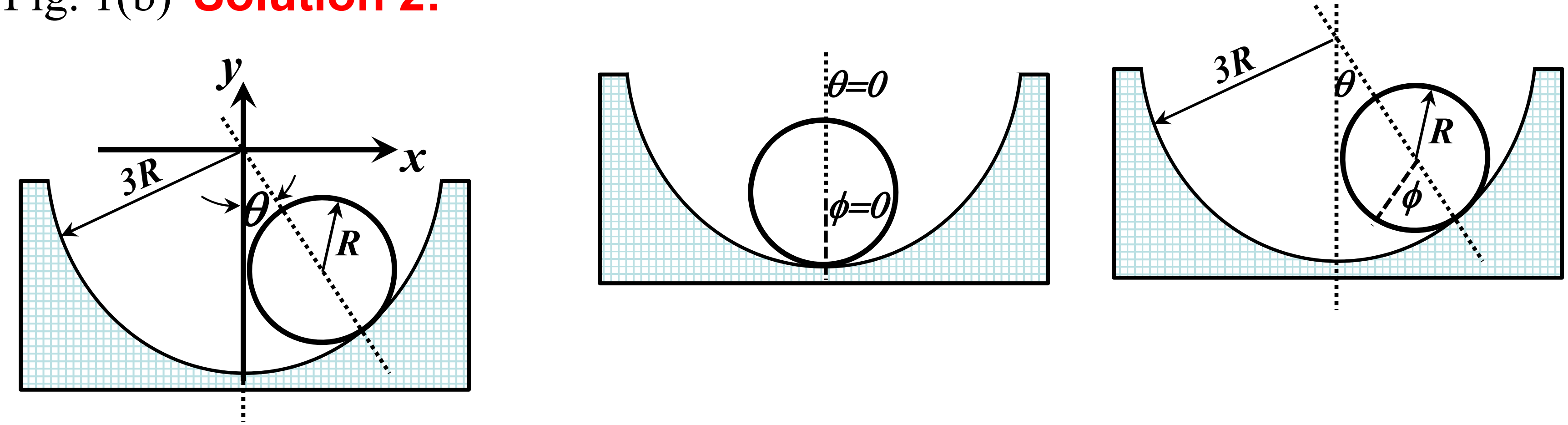
$$mg\theta + 4mR\frac{d^2\theta}{dt^2} = 0$$

$$\Rightarrow \frac{g}{4R}\theta + \frac{d^2\theta}{dt^2} = 0$$

$$\Rightarrow \omega = \sqrt{\frac{g}{4R}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{4R}{g}} = 2\pi\sqrt{\frac{2R}{5}}$$

Fig. 1(b) **Solution 2:**



$$E = \frac{1}{2} I_C \omega^2 + \frac{1}{2} m v^2 + (-mg2R\cos\theta) \qquad (1)$$

$$I_C = mR^2, \qquad v = 2R\frac{d\theta}{dt}, \quad \omega = \frac{d\phi}{dt}$$

For pure roll on the track, $\Rightarrow \phi = -\frac{3R - R}{R}\theta = -2\theta$ (2)

$$\Rightarrow \omega = \frac{d\phi}{dt} = -2\frac{d\theta}{dt}, \quad (4)$$

$$\Rightarrow E = \frac{1}{2} mR^2 \left(\frac{d\phi}{dt}\right)^2 + \frac{1}{2} m\left(2R\frac{d\theta}{dt}\right)^2 - 2mgR\cos\theta$$

$$\Rightarrow E = 2mR^2 \left(\frac{d\theta}{dt}\right)^2 + 2mR^2 \left(\frac{d\theta}{dt}\right)^2 - 2mgR\cos\theta$$

$$\Rightarrow E = 4mR^2 \left(\frac{d\theta}{dt}\right)^2 - 2mgR\cos\theta$$

$$\frac{dE}{dt} = 0 \Rightarrow \left(8mR^2\frac{d^2\theta}{dt^2} + 2mgR\sin\theta\right)\frac{d\theta}{dt} = 0$$

For θ<<1,

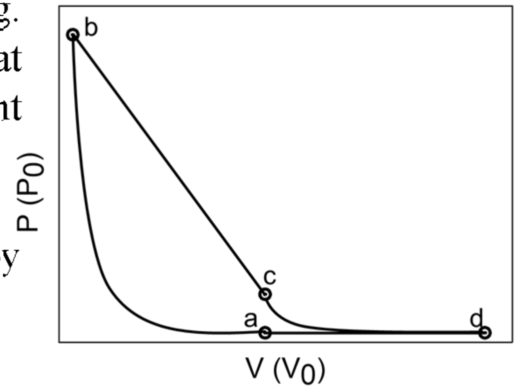
$$mg\theta + 4mR\frac{d^2\theta}{dt^2} = 0$$

$$\Rightarrow \frac{g}{4R}\theta + \frac{d^2\theta}{dt^2} = 0$$

$$\Rightarrow \omega = \sqrt{\frac{g}{4R}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{4R}{g}} = 2\pi\sqrt{\frac{2R}{5}}$$

1. A heat engine takes one mole of ideal monatomic gas around the cycle shown in Fig. (adiabatic a→b, straight line b→c, isothermal c→d,, and isobaric d→a). The volume at points a, b, c, and d are given by $8V_0$, V_0, $8V_0$, and $16V_0$, respectively. The pressure at point a is P_0. (Write your answer in term of P_0, V_0, R, ln2, ln3, ln5, and ln7)



a) (5pts) Determine the thermal dynamic variables (P, V, and T) at points a, b, c, and d.

b) (12pts) Calculate the work done (by the gas), heat, internal energy change, and entropy change for each process (a→b, b→c, c→d, and d→a).

c) (3pts) Determine the efficiency of the heat engine.

(a) $p_b v_b^\gamma = p_a v_a^\gamma$ (adiabatic) $\Rightarrow p_b\left(v_0\right)^{5/3} = p_0\left(8v_0\right)^{5/3} \Rightarrow p_b = 32p_0$

$T_d = T_c$ (isothermal) $\Rightarrow p_d v_d = p_c v_c \Rightarrow p_c = 2p_0$

$T_a = \frac{8p_0v_0}{R}; T_b = \frac{32p_0v_0}{R};\quad T_c = \frac{16p_0v_0}{R};\quad T_d = \frac{16p_0v_0}{R}$

5	P (P_0)	V (V_0)	T (P_0V_0/R)
a	1	8	8
b	32	1	32
c	2	8	16
d	1	16	16

(b) a→b (adiabatic)

$$\Delta E_{\text{int},a\to b} = \frac{3}{2}R\left(\frac{24p_0v_0}{R}\right) = 36p_0v_0$$

$$W_{a\to b} = -\Delta E_{a\to b} = -36p_0v_0$$

$$Q_{d\to a} = 0$$

$$\Delta S_{d\to a} = 0$$

b→c (line)

$$\Delta E_{\text{int},b\to c} = \frac{3}{2}R\left(-16\frac{p_0v_0}{R}\right) = -24p_0v_0$$

$$W_{b\to c} = -\frac{(32+2)p_0}{2}7v_0 = 119p_0v_0$$

$$Q_{b\to c} = \Delta E_{b\to c} + W_{b\to c} = 95p_0v_0$$

$$\Delta S_{b\to c} = \frac{3}{2}R\ln\frac{16}{32} + R\ln\frac{8}{1} = \frac{3}{2}R\ln 2$$

c→d (isothermal)

$$\Delta E_{\text{int},c\to d} = 0$$

$$W_{c\to d} = \int_{v_c}^{v_d}\frac{nRT}{V}dV = R\frac{16p_0v_0}{R}\ln\frac{16}{8}$$

$$= 16\ln 2\, p_0v_0$$

$$Q_{c\to d} = W_{c\to d} = 16\ln 2\, p_0v_0$$

$$\Delta S_{c\to d} = nR\ln\frac{V_d}{V_c} = R\ln 2$$

d→a (isobaric)

$$\Delta E_{\text{int},d\to a} = \frac{3}{2}R\left(-8\frac{p_0v_0}{R}\right) = -12p_0v_0$$

$$W_{d\to a} = \int_{v_d}^{v_d} pdV = p\Delta V = -8p_0v_0$$

$$Q_{d\to a} = \Delta E_{d\to a} + W_{d\to a} = -20p_0v_0$$

$$\Delta S_{d\to a} = nC_v\ln\frac{T_a}{T_d} + nR\ln\frac{V_a}{V_d} = \frac{-5}{2}R\ln 2$$

12	W (P_0V_0)	Q (P_0V_0)	ΔE_{int} (P_0V_0)	ΔS (R)
a→b	-36	0	36	0
b→c	119	95	-24	3ln2/2
c→d	16ln2	16ln2	0	ln2
d→a	-8	-20	-12	-5ln2/2

3

(c) $$e = \frac{75 + 16\ln 2}{95 + 16\ln 2}$$

(a)

$$m_{water} \cdot L_f + m_{water} \cdot C_{water} \cdot (300-273) = x \cdot 450 \cdot (900-300)$$

2

$$3.34 \times 10^5 + 4200 \cdot 27 = x \cdot 450 \cdot 600$$

$$x = 1.66\ kg$$

1

(b)

$$\Delta S_{ice\ at\ 0^0 C \to water\ at\ 0^0 C} = \frac{m_{water} \cdot L_f}{273} = 1223\ J/K$$

2 1

$$\Delta S_{water\ at\ 0^0 C \to water\ at\ 27^0 C} = \int_{273}^{300} \frac{m_{water} \cdot C_{water} \cdot dT}{T} \approx \frac{1 \cdot 4200 \cdot 27}{286.5} \approx 396\ J/K$$

2 1

or

$$\Delta S_{water\ at\ 0^0 C \to water\ at\ 27^0 C} = 4200 \ln\left(\frac{300}{273}\right) \approx 4200 \cdot 0.099 \approx 416\ J/K$$

$$\Delta S_{ice\ at\ 0^0 C \to water\ at\ 27^0 C} = \begin{cases} 1223 + 396 \approx 1619\ J/K \\ 1223 + 416 \approx 1639\ J/K \end{cases}$$

1

(c)

$$\Delta S_{iron\ at\ 900\ K \to iron\ at\ 300\ K} = \int_{900}^{300} \frac{m_{iron} \cdot C_{iron} \cdot dT}{T} \approx 1.6 \cdot 450 \cdot \ln\left(\frac{1}{3}\right) \approx -822\ J/K$$

2

(d)

$$\Delta S_{environment} = 0$$

1

$$\Delta S_{universe} = \begin{cases} 1619 - 822 + 0 = 797\ J/K \\ 1639 - 822 + 0 = 817\ J/K \end{cases}$$

2