General Physics (II) **期中考 II** May 5, 2017

試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(60 points) (所有題目必須有計算過程,否則不予計分)

1.(10 pts) Consider the classical model of motion of an electron (charge ***-e*** , mass ***m***) in a hydrogen atom.

a) (3 pts) Suppose the electron follows a circular orbit of radius ***R*** around a proton. What is the angular frequency ω_0 of this orbital motion? Write your answer in terms of ***e, m, R, k*** **(=** $(4\pi\varepsilon_0)^{-1}$ and other necessary constants.

b) (7 pts) Now that a small magnetic field ***B*** perpendicular to the plane of the orbit is switched on. Assuming that the radius ***R*** of the orbit does not change, calculate the angular frequency ω of the orbital motion in terms of ***B, e, m, R,*** **and/or** ω_0. (Hint: consider the circular motion is clockwise or counter-clockwise relative to the direction of the magnetic field.)

2. (10 points) A R-C circuit ,shown in Fig. 1, consists of $R_1 = R$, $R_2 = 2R$, two batteries $\mathcal{E}$ and $3\mathcal{E}$, and a capacitor ***C***. The capacitor is initially uncharged. The switch *S* is closed at $t = 0$. Write your answers in terms of $\mathcal{E}$, ***C,*** and/or ***R.***

(a) (2 pts) Immediately after the switch is closed, what are the currents i_1, and i_2?

(b) (3 pts) After a long time, the circuit is steady, what are the currents i_1, and i_2? What is the charge ***Q*** on the capacitor ***C***?

(c) (5 points) Find the charge ***Q(t)*** on the capacitor as a function of time ***t*** and the time constant τ of the circuit during this charging process

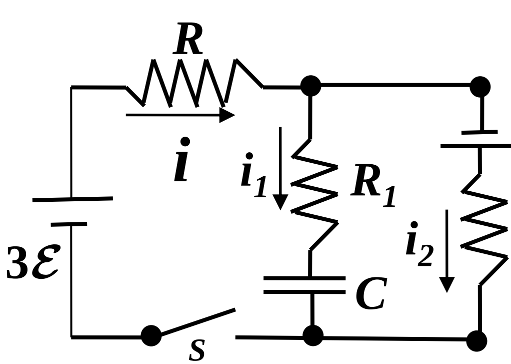


Fig. 1

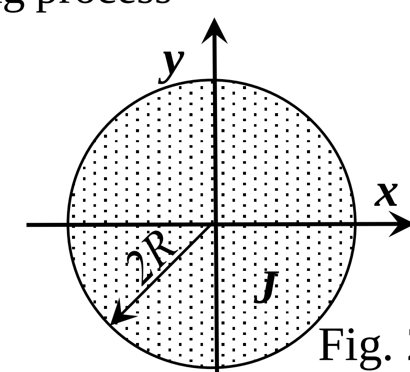


Fig. 2(a)

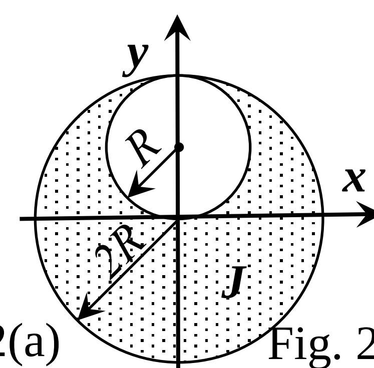
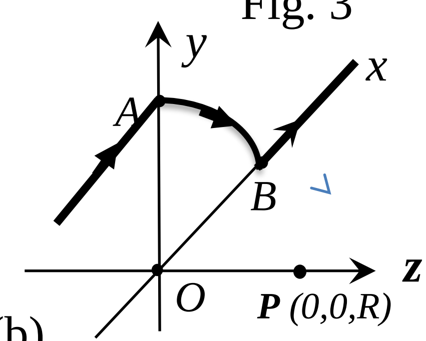


Fig. 2(b)

Fig. 3



3. (20pts) As shown in Fig. 2(a) an infinitely long cylindrical conductor with radius ***2R*** carries a uniform current with density ***J*** in the +z-axis direction (***J*** >0).

(a) (7 pts) Determine the magnitude and direction of the magnetic field on the y-axis for the range of $0 \le \boldsymbol{y} < \infty$.

(b) Now that a cylindrical portion of radius R is removed from the conductor, which is shown in Fig. 2(b), if the current density remains the same, determine the direction and the magnitude of the magnetic field distribution on the y-axis for the range of $0 \le \boldsymbol{y} < \infty$ (7 pts),

(c) and direction and the magnitude of the magnetic field distribution on the x-axis for the range of $0 \le \boldsymbol{x} < \infty$ (6 pts).

4. (20 pts) Fig. *3* shows a three-section conducting wire on x-y plane with current I. The first section is from -∞ to A and is parallel to the x-axis. The section is from A to B is a quarter of a circle with radius ***R***. The last section is from B to ∞ on the x-axis. Find the *x*-, *y*-, *z*-components of the magnetic field at point ***P*** on the *z*-axis due to

(a) (7pts) current in the section from B to ∞,

(b) (5pts) current in the section from -∞ to A, and

(c) (8pts) current in the section from A to B.

The coordinates of ***A***,***B***, and ***P*** are **(*0, R,0*)**, **(*R, 0,0*)**, and **(*0,0,R*)**, respectively.

Useful formula:

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left(x + \sqrt{x^2 \pm a^2}\right); \int \frac{dx}{\left(x^2 \pm a^2\right)^{\frac{3}{2}}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}};$$

$$\int \frac{x\,dx}{\left(x^2 \pm a^2\right)^{\frac{3}{2}}} = \frac{-1}{\sqrt{x^2 \pm a^2}}; \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) = -\cos^{-1}\left(\frac{x}{a}\right)$$

II.選擇題(44 points)

1. (5 pts) Then a battery with potential difference of $\mathcal{E}$ is connected to two resistors and two uncharged capacitors are arranged as shown in Fig. 4.. The charges on the capacitor is Q when the circuit is static. Now the switch S is closed at time t = 0. At $t \to \infty$, the charge on C_1 changes to Q_1 when the circuit is static again . What is the ratio $b = Q_1/Q$?

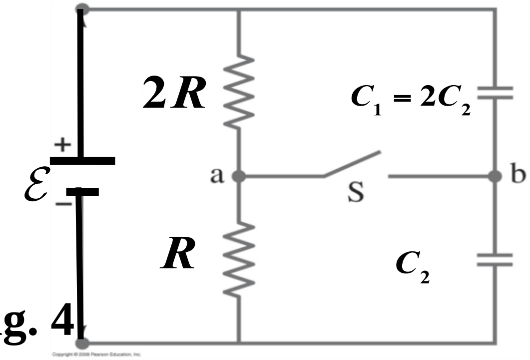


Fig. 4

(A) $0 < b \le 0.5$ (B) $0.5 < b \le 1$ (C) $1 < b \le 1.5$ (D) $1.5 < b \le 2$ (E) $2 < b \le 2.5$
(F) $2.5 < \boldsymbol{b} \le 3$ (G) $3 < b \le 3.5$ (H) $3.5 < \boldsymbol{b} \le 4$ (J) $4 < \boldsymbol{b} \le 4.5$ (K) $4.5 < b \le 5$ (L) $5 < b$

2. (5 pts)As shown in Fig. 5, a infinite conducting plate with thickness ***4R*** carries a uniform current density ***J*** in **+z-**direction, and in the plate there is a infinitely long hollow cylindrical region with radius ***R*** . Which of the following could be the direction of the magnetic field at point P?

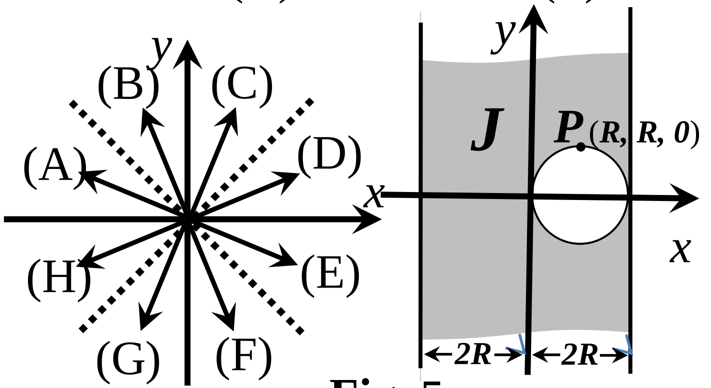


Fig. 5

3. (5 pts) A circular loop has radius ***R*** and carries current $\boldsymbol{I_2}$ in a clockwise direction. The center of the loop is a distance ***2R*** above a long, straight wire carrying current $\boldsymbol{I_1}$. What are ratio $\boldsymbol{r} = \boldsymbol{I_1}/\boldsymbol{I_2}$ and direction of the current $\boldsymbol{I_1}$ in the wire if the magnetic field at the center of the loop is zero?

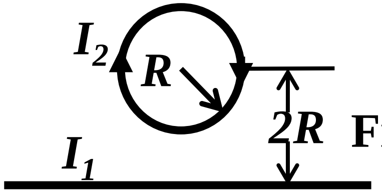


Fig. 6

(A) $0 < \boldsymbol{r} \le 2$, $\rightarrow$. (B) $0 \le \boldsymbol{r} \le 2$, $\leftarrow$. (C) $2 < \boldsymbol{r} \le 4$, $\rightarrow$. (D) $2 \le \boldsymbol{r} \le 4$, $\leftarrow$.
(E) $4 < \boldsymbol{r} \le 6$, $\rightarrow$. (F) $4 \le \boldsymbol{r} \le 6$, $\leftarrow$. (G) $6 < \boldsymbol{r} \le 8$, $\rightarrow$. (H) $6 \le \boldsymbol{r} \le 8$, $\leftarrow$.
(J) $8 < \boldsymbol{r} \le 10$, $\rightarrow$. (K) $8 \le \boldsymbol{r} \le 10$, $\leftarrow$. (L) $10 < \boldsymbol{r}$, $\rightarrow$. (M) $10 \le \boldsymbol{r}$, $\leftarrow$.

4. (5 pts) A thin ring of radius **1.0*cm*** and mass **10*g*** carrying a uniform charge **0.01*C* (*coulombs*)** rotates about its axis with constant angular speed **ω=100 rad./s**. Let **α** be the ratio of the magnitudes of its magnetic dipole moment to its angular momentum. What is the value of **α** in SI unit? ($\boldsymbol{I_{CM,ring}= MR^2}$)

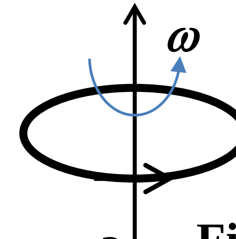


Fig. 7

(A) $0< \alpha \leq 0.1$ (B) $0.1< \alpha \leq 0.5$ (C) $0.5 < \alpha \leq 1$ (D) $1 < \alpha \leq 1.5$ (E) $1.5 < \alpha \leq 2$
(F) $2 < \alpha \leq 2.5$ (G) $2.5 < \alpha \leq 3$ (H) $3 < \alpha \leq 3.5$ (J) $3.5 < \alpha \leq 4$ (K) $4 < \alpha$

5. (5 pts) A long, straight wire has a constant current flowing to the right. A square ring is situated above the wire , and also has a constant current flowing through it (as shown in Fig. 8). What is the net magnetic force $\boldsymbol{F_B}$ (with direction ↑ or ↓)and the net torque $\boldsymbol{\tau}$ acting on the ring?

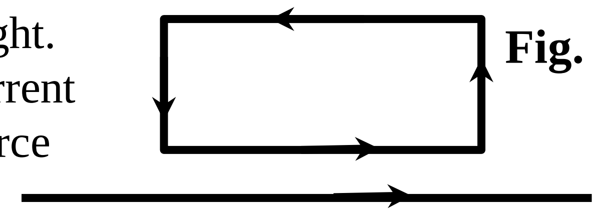


Fig. 8

(A) $\boldsymbol{F_B} = 0$, $\boldsymbol{\tau} = 0$ (B) $\boldsymbol{F_B} = 0$, $\boldsymbol{\tau} \neq 0$ (C) $\boldsymbol{F_B} \neq 0$ and ↑, $\boldsymbol{\tau} = 0$ (D) $\boldsymbol{F_B} \neq 0$ and ↑, $\boldsymbol{\tau} \neq 0$
(E) $\boldsymbol{F_B} \neq 0$ and ↓, $\boldsymbol{\tau} = 0$ (F) $\boldsymbol{F_B} \neq 0$ and ↓, $\boldsymbol{\tau} \neq 0$

6. (5 pts)Fig. 9 shows the cross section of an infinitely long hollow (中空) conducting rod along the z-axis (out of page), which carries an uniform current density ***J*** (***J*** > 0) in the +z-direction. Consider two closed loops labelled as ***1*** and ***2*** on the x-y plane with their centers at the origin. Let $\boldsymbol{B_1}(x,y,z)$ and $\boldsymbol{B_2}(x,y,z)$ be the B-field generated by ***J*** at each point along loop ***1*** and loop ***2***, respectively, and

$$K_1 = \oint_{Loop1} \vec{B}_1 \cdot d\vec{\ell}, \ and \quad K_2 = \oint_{Loop2} \vec{B}_2 \cdot d\vec{\ell}$$

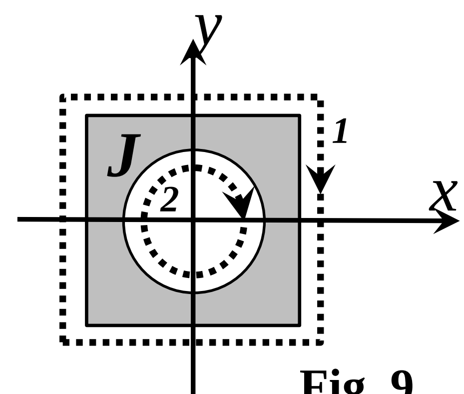


Fig. 9

Which of the following statement is correct?

(A) $|\boldsymbol{B_1}| = \boldsymbol{constant}$, $\boldsymbol{K_1} = 0$, $|\boldsymbol{B_2}| = \boldsymbol{0}$, $\boldsymbol{K_2} = 0$ (B) $|\boldsymbol{B_1}| \neq \boldsymbol{constant}$, $\boldsymbol{K_1} \neq 0$, $|\boldsymbol{B_2}| \neq \boldsymbol{0}$, $\boldsymbol{K_2} = 0$
(C) $|\boldsymbol{B_1}| = \boldsymbol{constant}$, $\boldsymbol{K_1} \neq 0$, $|\boldsymbol{B_2}| \neq \boldsymbol{0}$, $\boldsymbol{K_2} \neq 0$ (D) $|\boldsymbol{B_1}| = \boldsymbol{constant}$, $\boldsymbol{K_1} \neq 0$, $|\boldsymbol{B_2}| = \boldsymbol{0}$, $\boldsymbol{K_2} = 0$
(E) $|\boldsymbol{B_1}| = \boldsymbol{constant}$, $\boldsymbol{K_1} \neq 0$, $|\boldsymbol{B_2}| \neq \boldsymbol{0}$, $\boldsymbol{K_2} = 0$ (F) $|\boldsymbol{B_1}| \neq \boldsymbol{constant}$, $\boldsymbol{K_1} \neq 0$, $|\boldsymbol{B_2}| = \boldsymbol{0}$, $\boldsymbol{K_2} = 0$
(G) $|\boldsymbol{B_1}| \neq \boldsymbol{constant}$, $\boldsymbol{K_1} \neq 0$, $|\boldsymbol{B_2}| \neq \boldsymbol{0}$, $\boldsymbol{K_2} \neq 0$

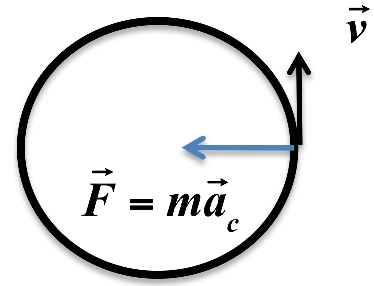
Multiple Choice Questions:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
D	**C**	**G**	**B**	**E**	**B**	**B**	**C**	**A**	**B**
11	**12**	**13**							
C	**G**	**B**							

Problem 1

(a) 3 pts

$$F = k\frac{e^2}{r^2} = ma_c = m\frac{v^2}{r} = mr\omega_0^2 \quad \text{with } v = r\omega_0,$$

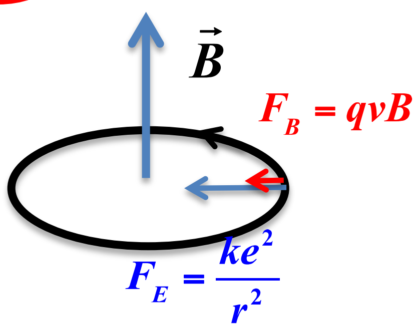


$$\boxed{\omega_0 = \left(\frac{ke^2}{mr^3}\right)^{\frac{1}{2}} = \left(\frac{e^2}{4\pi\varepsilon_0 mr^3}\right)^{\frac{1}{2}}}$$

3 pts

(b) 7 pts

Add a B-field normal to the plane of circular motion



$$F = F_E - F_B = k\frac{e^2}{r^2} + q\left|\vec{v}\times\vec{B}\right| = k\frac{e^2}{r^2} + eBr\omega = mr\omega^2 \ ,$$

2 pts

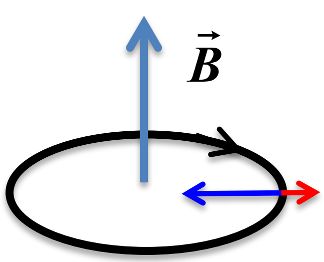
$$\omega^2 - \frac{eB}{m}\omega - \frac{ke^2}{mr^3} = \omega^2 - \frac{eB}{m}\omega - \omega_0^2 = 0$$

$$\boxed{\omega = \frac{1}{2}\left[\frac{eB}{m} + \sqrt{\left(\frac{eB}{m}\right)^2 + 4\omega_0^2}\right]},$$

1 pts

2 pts

only + solution is valid



$$F = k\frac{e^2}{r^2} - q\left|\vec{v}\times\vec{B}\right| = k\frac{e^2}{r^2} - eBr\omega = mr\omega^2 \ ,$$

1 pts

$$\boxed{\omega = \frac{1}{2}\left[-\frac{eB}{m} + \sqrt{\left(\frac{eB}{m}\right)^2 + 4\omega_0^2}\right]},$$

1 pts

only + solution is valid

The shift of the frequency (ω-ω_0) due to the external B-field is known as the Zeeman effect (classical picture)

2 pts

Problem 2

(a) Right after the switch is closed, the charges on the capacitors are zero V_C=0, then

$$3\varepsilon - iR - i_1R_1 = 3\varepsilon - iR - i_1R = 0$$

$$3\varepsilon - iR + \varepsilon - i_2R_2 = 4\varepsilon - iR - 2i_2R = 0$$

$$i = i_1 + i_2$$

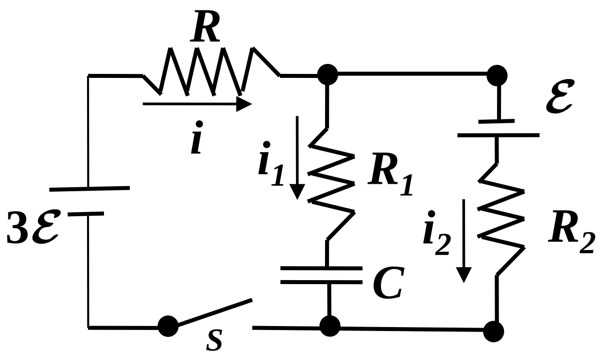


Fig. XX

$$3\varepsilon - i_2R - 2i_1R = 0$$

$$4\varepsilon - i_1R - 3i_2R = 0 \quad \rightarrow i_1 = \frac{\varepsilon}{R},\ \ i_2 = \frac{\varepsilon}{R},$$

2 pts

3 pts

(b) The current is steady → i_1 =0, $i = i_2$ 1 pts

$$3\varepsilon - iR + \varepsilon - 2iR = 0 \rightarrow i = \frac{4\varepsilon}{3R}$$

1 pts

$$Q = CV_C = C\left|\varepsilon - iR_2\right| = \frac{5}{3}\varepsilon C$$

1 pts

5 pts

(c)

$$i = i_1 + i_2$$

$$3\varepsilon - iR - i_1R_1 - \frac{Q}{C} = 3\varepsilon - i_2R - 2i_1R - \frac{Q}{C} = 0$$

1 pts

$$3\varepsilon - iR + \varepsilon - i_2R_2 = 4\varepsilon - i_1R - 3i_2R = 0$$

$$i_2 = \frac{4\varepsilon - i_1R}{3R}, \quad i_1 = \frac{dQ}{dt}$$

1 pts

$$\rightarrow \frac{dQ}{dt} + \frac{Q}{\frac{5}{3}RC} = \frac{\varepsilon}{R} \quad \text{or} \quad \frac{dQ}{\frac{\varepsilon}{R} - \frac{3Q}{5RC}} = dt$$

1 pts

$$\frac{-5RC}{3}\ln\left(\frac{\varepsilon}{R} - \frac{3Q}{5RC}\right) = t + \text{Constant}$$

$$\rightarrow Q(t) = \frac{5\varepsilon C}{3}\left(1 - e^{-\frac{3t}{5RC}}\right), \quad \text{by } Q(t=0) = 0$$

1 pts

$$\text{time constant } \tau = \frac{5}{3}RC$$

1 pts

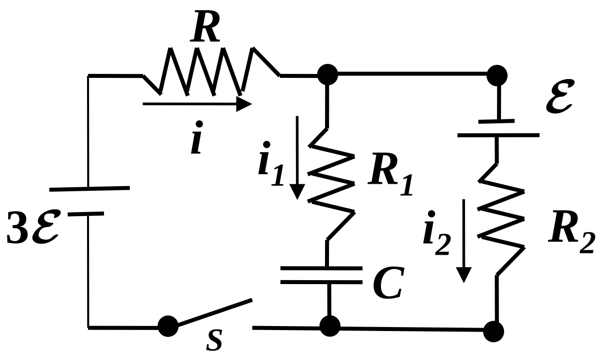
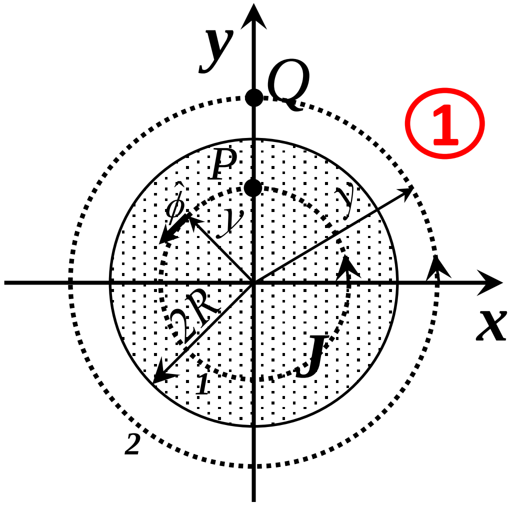
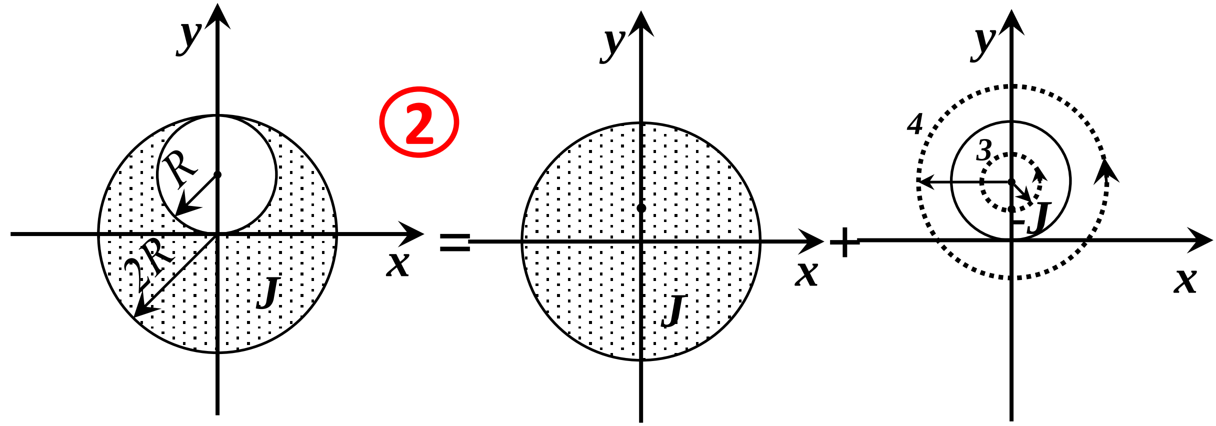


Fig. XX





(a) Consider the figure shown above, for y ≤ 2R, construct a close circular loop ***1*** passing ***P***, whose with coordinate is (0,y,0) and apply Ampere's law

$$\oint_1 \vec{B}\cdot d\vec{\ell} = \mu_0 I,$$

$$\vec{B}(\vec{r}) = B(r)\hat{\phi};\ d\vec{\ell} = d\ell\hat{\phi} \quad \text{on loop } \boldsymbol{1}.$$

$$\Rightarrow \oint_1 B\cdot d\ell = 2\pi y B(y) = \mu_0 J\pi y^2 \Rightarrow B(y) = \frac{\mu_0 J}{2}y$$ 2

$$\Rightarrow \vec{B}(y) = \left(-\frac{\mu_0 J}{2}y,0,0\right)$$ 1

For 2R <y, similarly use loop***2*** passing ***Q*** at (0,y,0),

$$\oint_2 \vec{B}\cdot d\vec{\ell} = \mu_0 I,$$

$$\vec{B}(\vec{r}) = B(r)\hat{\phi};\ d\vec{\ell} = d\ell\hat{\phi} \quad \text{on loop } \boldsymbol{2}.$$

$$\Rightarrow \oint_2 B\cdot d\ell = 2\pi y B(y) = \mu_0 J\pi 4R^2 \Rightarrow B(y) = \frac{2\mu_0 R^2 J}{y}$$ 2

$$\Rightarrow B(y) = \left(-\frac{2\mu_0 R^2 J}{y},0,0\right)$$ 1

(b) The current distribution is equivalent to the linear combination of the two current distributions shown above. According to part (a), the magnetic field generated by current density ***J*** is

$$B_J(y) = \begin{cases} \left(-\dfrac{\mu_0 J}{2}y,0,0\right) & 0 \le y \le 2R \\ \left(-\dfrac{2\mu_0 R^2 J}{y},0,0\right) & 2R < y \end{cases}$$

For the magnetic field generated by the current density ***–J***, for |y-R| ≤ R, we apply the Ampere's law along Loop ***3***.

$$\oint_3 \vec{B}\cdot d\vec{\ell} = \mu_0 I \Rightarrow \oint_3 \vec{B}\cdot d\vec{\ell} = 2\pi|R-y|B(y) = -\mu_0 J\pi(R-y)^2$$

$$\Rightarrow B(y) = -\frac{\mu_0 J|R-y|}{2}$$ 1

$$\Rightarrow \vec{B}_{-J}(y) = \begin{cases} \left(-\dfrac{\mu_0 J(R-y)}{2},0,0\right) & 0 \le y \le R \\ -\left(\dfrac{\mu_0 J(R-y)}{2},0,0\right) & R \le y \le 2R \end{cases}$$ 1

For the magnetic field generated by the current density **–*J***, for R ≤ |y-R|, we apply the Ampere's law along Loop ***4***.

$$\oint_4 \vec{B}\cdot d\vec{\ell} = \mu_0 I \Rightarrow \oint_4 \vec{B}\cdot d\vec{\ell} = 2\pi|y-R|B(y) = -J\mu_0\pi R^2$$

$$\Rightarrow B(y) = \frac{-\mu_0 J R^2}{2|R-y|}$$ 1

$$\Rightarrow \vec{B}_{-J}(y) = \begin{cases} (-\dfrac{\mu_0 J R^2}{2(R-y)},0,0) & 2R \le y \\ (-\dfrac{\mu_0 J R^2}{2(R-y)},0,0) & y \le 0 \end{cases}$$ 1

The total magnetic field is

$$\vec{B}_{Total}(y) = \vec{B}_J(y) + \vec{B}_{-J}(y)$$

$$= \begin{cases} (-\dfrac{\mu_0 J(R-y)}{2} - \dfrac{\mu_0 J}{2}y,0,0) & 0 \le y < R \\ (-\dfrac{\mu_0 J(R-y)}{2} - \dfrac{\mu_0 J}{2}y,0,0) & R \le y < 2R \\ (-\dfrac{\mu_0 J R^2}{2(R-y)} - \dfrac{2\mu_0 R^2 J}{y},0,0) & 2R < y \end{cases}$$

$$= \begin{cases} (-\dfrac{\mu_0 J R}{2},0,0) & 0 \le y < R \\ (-\dfrac{\mu_0 J R}{2},0,0) & R \le y < 2R \\ (-\dfrac{\mu_0 J R^2}{2(R-y)} - \dfrac{2\mu_0 R^2 J}{y},0,0) & 2R < y \end{cases}$$ 1

(c) According to part (a), the magnetic field generated by current density ***J*** is

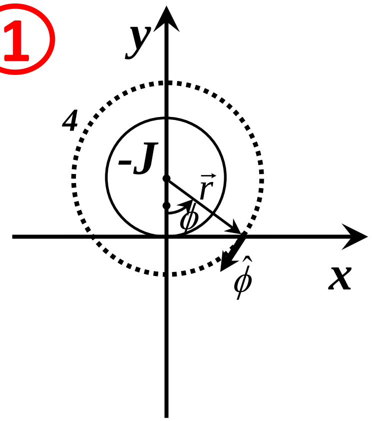
$$\Rightarrow B_J(x) = \begin{cases} (0,\dfrac{\mu_0 J}{2}x,0) & 0 \le x \le 2R \\ (0,\dfrac{2\mu_0 R^2 J}{x},0) & 2R < x \end{cases}$$ 2

According to part (b), the magnetic field generated by current density **-*J*** on the +x-axix is

$$\vec{B}_{-J}(x) = \frac{\mu_0 J R^2}{2r}\hat{\phi}, \quad 0 \le x, \textit{ and}$$ 1

$$\vec{r} = (x,0,0)-(0,R,0) = (x,-R,0)$$

$$r = \sqrt{x^2+R^2}$$



Consider the figure on the right: $\vec{r} = (r\sin\phi, -r\cos\phi, 0)$

$$\Rightarrow \hat{\phi} = (\sin(\phi-\frac{\pi}{2}), -\cos(\phi-\frac{\pi}{2}),0)$$

$$= (-\cos\phi, -\sin\phi, 0) = (-\frac{R}{r}, -\frac{x}{r}, 0)$$

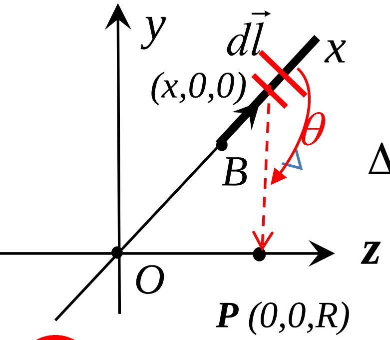
$$\Rightarrow \vec{B}_{-J}(x) = \frac{-\mu_0 J R^2}{2r}(\frac{R}{r},\frac{x}{r},0), \quad 0 \le x$$

$$\Rightarrow \vec{B}_{-J}(x) = \frac{-\mu_0 J R^2}{2(R^2+x^2)}(R,x,0), \quad 0 \le x$$ 2

$$\vec{B}_{Total}(x) = \vec{B}_J(x) + \vec{B}_{-J}(x)$$ 1

$$= \begin{cases} (-\dfrac{\mu_0 J R^3}{2(R^2+x^2)}, \dfrac{\mu_0 J}{2}x - \dfrac{\mu_0 J R^2}{2(R^2+x^2)}x, 0) & 0 \le x \le 2R \\ (-\dfrac{\mu_0 J R^3}{2(R^2+x^2)}, \dfrac{2\mu_0 R^2 J}{x} - \dfrac{\mu_0 J R^2 x}{2(R^2+x^2)}, 0) & 2R < x \end{cases}$$

4. (a) Line segment on x-axis:



$$\Delta\vec{B} = \frac{\mu_0}{4\pi}\cdot\frac{Id\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\cdot\frac{Id\vec{l}\times\vec{r}}{r^3}$$

1

$\Delta\vec{l} = \Delta x\,\hat{x}$ 1

1

$$\vec{r} = (0,0,R)-(x,0,0) = (-x,0,R)$$

$$\hat{r} = \frac{(-x,0,R)}{\sqrt{R^2+x^2}}$$

2

$$\therefore \Delta\vec{B} = \frac{\mu_0 I}{4\pi}\frac{R\Delta x\left(-\hat{j}\right)}{\sqrt{x^2+R^2}^3}\left(=\frac{\mu_0 I}{4\pi}\frac{\Delta x\left(-\hat{j}\right)}{x^2+R^2}\sin\theta\right)$$

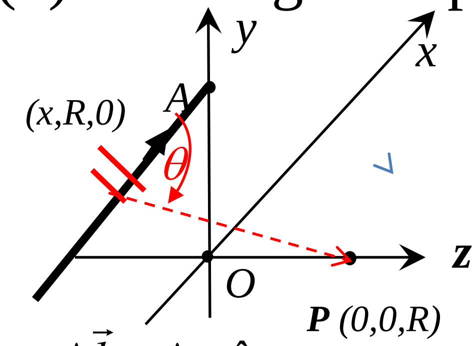
(i) 查積分表: $\displaystyle\int\frac{dx}{\left(x^2\pm a^2\right)^{\frac{3}{2}}} = \frac{\pm x}{a^2\sqrt{x^2\pm a^2}}$

$$\vec{B}_1 = \frac{\mu_0 IR}{4\pi}\left(-\hat{j}\right)\left(\int_R^\infty\frac{dx}{\sqrt{x^2+R^2}^3}\right) = \frac{\mu_0 IR}{4\pi}\left(-\hat{j}\right)\left(\left.\frac{x}{R^2\sqrt{x^2+R^2}}\right|_R^\infty\right)$$

1

$$= \frac{\mu_0 I}{4\pi R}\left(1-\frac{1}{\sqrt{2}}\right)\left(-\hat{j}\right)$$ 1

(b) Line segment parallel to x-axis:



$\Delta\vec{l} = \Delta x\,\hat{x}$

$$\vec{r} = (0,0,R)-(x,R,0) = (-x,-R,R)$$ 1

$$\hat{r} = \frac{(-x,-R,R)}{\sqrt{2R^2+x^2}}$$

$$\therefore \Delta\vec{B} = \frac{\mu_0 I}{4\pi}\frac{R\Delta x\left(-\hat{j}-\hat{k}\right)}{\sqrt{2R^2+x^2}^3}$$ 2

$$\vec{B}_1 = \frac{\mu_0 IR}{4\pi}\left(-\hat{j}-\hat{k}\right)\left(\int_{-\infty}^0\frac{dx}{\sqrt{2R^2+x^2}^3}\right)$$

$$= \frac{\mu_0 IR}{4\pi}\left(-\hat{j}-\hat{k}\right)\left(\left.\frac{x}{2R^2\sqrt{2R^2+x^2}}\right|_{-\infty}^0\right)$$ 1

$$= \frac{\mu_0 I}{8\pi R}\left(-\hat{j}-\hat{k}\right)$$ 1

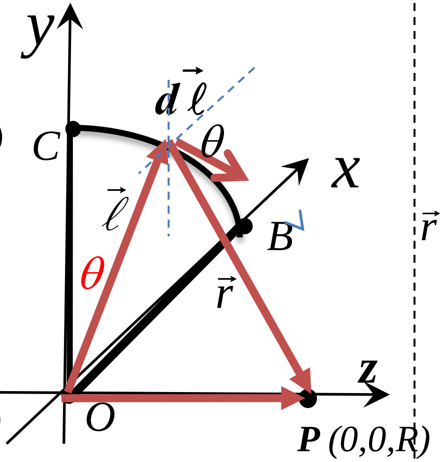
(c)

1 1

$$\vec r = (0,0,R) - (R\sin\theta, R\cos\theta, 0)$$

$$= (-R\sin\theta, -R\cos\theta, R)$$

$$\hat r = \frac{(-R\sin\theta, -R\cos\theta, R)}{\sqrt{2}R}$$



$$d\vec\ell = \left(Rd\theta\right)(\cos\theta, -\sin\theta, 0)$$

1 1

$$d\vec l \times \hat r = \frac{Rd\theta}{\sqrt{2}R}\begin{vmatrix} \hat i & \hat j & \hat k \\ \cos\theta & -\sin\theta & 0 \\ -R\sin\theta & -R\cos\theta & R \end{vmatrix}$$

$$= \frac{d\theta}{\sqrt{2}}\left(-R\sin\theta\hat i - R\cos\theta\hat j - R\hat k\right)$$ 2

$$d\vec B = \frac{\mu_0}{4\pi}\frac{Id\theta\left(-R\sin\theta\hat i - R\cos\theta\hat j - R\hat k\right)}{2\sqrt{2}R^2}$$

$$\vec B = \frac{\mu_0 I}{8\sqrt{2}\pi R}\left\{-\hat i\int_0^{\pi/2}\sin\theta\, d\theta - \hat j\int_0^{\pi/2}\cos\theta\, d\theta - \hat k\int_0^{\pi/2} d\theta\right\}$$ 1

$$\boxed{= \frac{\mu_0 I}{8\sqrt{2}\pi R}\left\{-\hat i - \hat j - \frac{\pi}{2}\hat k\right\}}$$ 1

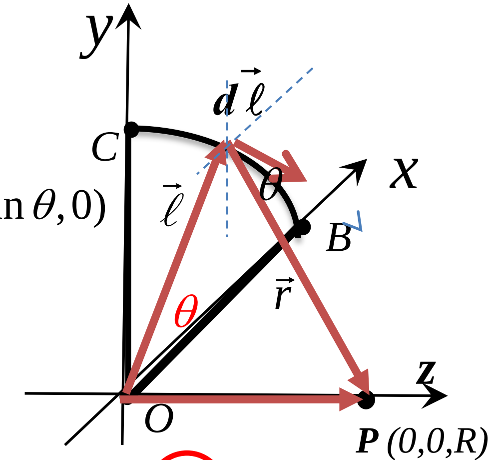
θ 取法不同:

1 1

$$\vec r = (0,0,R) - (Rs\cos\theta, R\sin\theta, 0)$$

$$= (-R\cos\theta, -R\sin\theta, R)$$

$$\hat r = \frac{(-R\cos\theta, -R\sin\theta, R)}{\sqrt{2}R}$$



$$d\vec\ell = \left(Rd\theta\right)(-\sin\theta, +\cos\theta, 0)$$ 1

1

$$d\vec l \times \hat r = \frac{Rd\theta}{\sqrt{2}R}\begin{vmatrix} \hat i & \hat j & \hat k \\ -\sin\theta & +\cos\theta & 0 \\ -R\cos\theta & -R\sin\theta & R \end{vmatrix}$$

$$= \frac{d\theta}{\sqrt{2}}\left(R\cos\theta\hat i + R\sin\theta\hat j + R\hat k\right)$$ 2

$$d\vec B = \frac{\mu_0}{4\pi}\frac{Id\theta\left(R\cos\theta\hat i + R\sin\theta\hat j + R\hat k\right)}{2\sqrt{2}R^2}$$

$$\vec B = \frac{\mu_0 I}{8\sqrt{2}\pi R}\left\{\hat i\int_{\pi/2}^{0}\cos\theta\, d\theta + \hat j\int_{\pi/2}^{0}\sin\theta\, d\theta + \hat k\int_{\pi/2}^{0} d\theta\right\}$$ 1

$$\boxed{= \frac{\mu_0 I}{8\sqrt{2}\pi R}\left\{-\hat i - \hat j - \frac{\pi}{2}\hat k\right\}}$$ 1