General Physics (I) The Final Examination Jan. 6, 20121

試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(50 points) (所有題目必須有計算過程，否則不予計分)

1. (10 pts) A uniform rod with length ***l*,** mass ***M*** hang from one end and the other end is attached with a horizontal massless spring with spring constant ***k*** (shown as Fig. 1). This spring is initially at the equilibrium position ($\theta = 0$). Now the rod is displaced by a small angle θ from the vertical position and is then released.
   Assume the angle θ is so small such that $\sin\theta \approx \theta$, $\cos\theta \approx 1$
   (A) (2 pts) Draw the time dependence of θ.
   (B) (2 pts) Draw the free body diagram of the rod.
   (C) (6 pts) Find the period ***T*** of the motion of the rod.

Note: the moment of inertia about the center of mass for rod $I_{cm} = \frac{1}{12}Ml^2$

2. If ***200*** cm^3 of water at ***95*** ^0C is poured into a ***150 g*** glass cup initially at ***25*** ^0C,
   (A) what will be the final temperature ***T*** of the tea and cup when equilibrium is reached, assuming no heat loss to the surroundings ?
   (B) Determine the change of the entropy of (a) (3 pts) the water, (b) (3 pts) the cup, (c) (1 pt) the surroundings, and (d) (1 pt) the universe as whole.

(the specific heat of water is ***4200*** J/kg^0C ,and the specific heat of glass is ***840*** J/Kg^0C)

$$\rho_{water} = 1.0\, g/cm^3$$

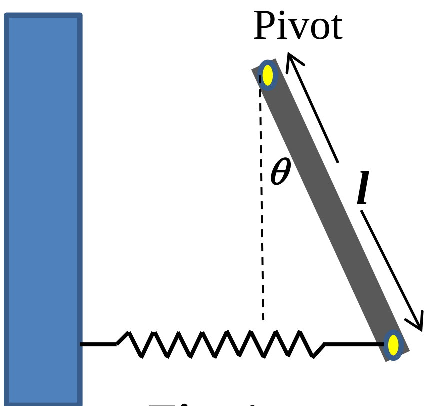


Fig. 1

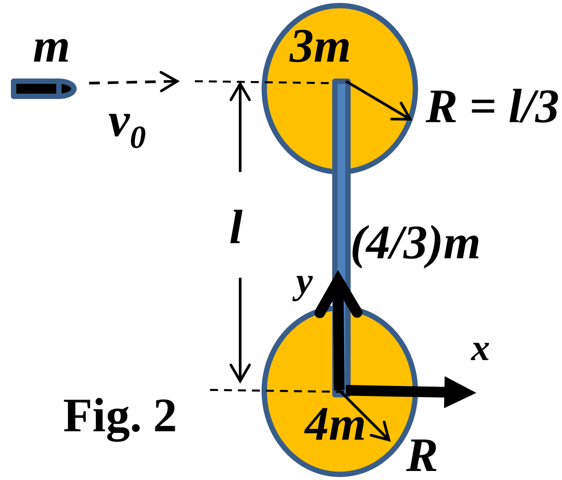


Fig. 2

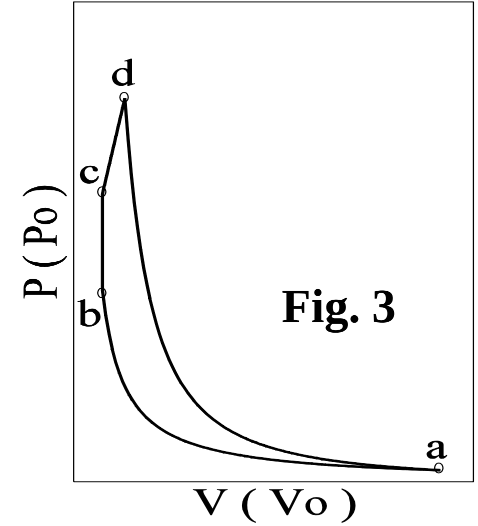


Fig. 3

3. (15 pts) Two solid disks with same radius ***R= l/3*** are connected by a thin rod with mass ***(4/3)m***, length ***l*** rest on a frictionless table as shown in Fig. 2. The top disk has mass ***3m*** and the bottom one has ***4m*** , The upper disk is struck by a bullet of mass ***m*** moving at speed v_0 along ***+x*** direction from the left. After the collision, the bullet sticks inside the center of the upper disk Assume the origin the coordinate is located at the center of the bottom disk.

(A) (3 pts) What is the coordinate (x_{cm}, y_{cm}) of the new center of mass of the system right after the collision?

(B) (5 pts) What is the moment of inertia about its new center of mass right after the collision?

(C) (7 pts) What is the angular velocity ω of the system about its center of mass after collision?

Note: the moment of inertia about the center of mass for disk $I_{cm} = \frac{1}{2}MR^2$

4. (20 pts) An ideal gas engine consist of ***1*** mole monatomic molecules is operated by the cycle shown in Fig. 3 (***a→b*** **:** isothermal , ***b→c*** :isovolumetric, ***c→d*****:** straight line segment, and ***d→a*** **:** adiabatic). The volume at ***a*** **,** ***b, c,*** and ***d*** are $64V_0$, $4V_0$, $4V_0$, and $8V_0$, respectively. The pressure at point ***a*** is P_0. (write your answer in term of P_0, V_0 and ***R***)

a) (4 pts) Determine the thermal dynamic variables (***P, V,*** and ***T***) at points ***a, b, c,*** and ***d***.

b) (12 pts) Calculate the work ***W*** done (by the gas), heat transfer ***Q***, change of the internal energy ΔE_{int}, and the change of entropy ΔS for each process (*a* → *b*, *b* → *c*, *c* → *d*, and *d* → *a*).

c) (4 pts) Determine the efficiency of this ideal gas engine.

請將下列表格抄至答案紙上，否則不予批改。

	$P\ (P_0)$	$V\ (V_0)$	$T\ (P_0V_0/R)$
a	***1***	***64***	
b		***4***	
c	***24***	***4***	
d		***8***	

	$W\ (P_0V_0)$	$Q\ (P_0V_0)$	$\Delta E_{int}\ (P_0V_0)$	$\Delta S\ (nR)$
a→b				
b→c				
c→d				
d→a				

II.選擇題(50 points)

1. (5 pts) Two circular rings, ***X*** and ***Y,*** are hanging on nails on a wall. The mass of ***X*** is four-times that of ***Y***, and the diameter of ***X*** is also four times that of ***Y***. If the period of small oscillations of X is ***T***, the period of small oscillations of Y is

(A) ***T*** (B) ***T***/2 (C) 2***T*** (D) ***T***/4 (E) 4***T***

(F) ***T***/8 (G) 8***T*** (H) ***T***/16 (I) 16***T*** (J) None of above

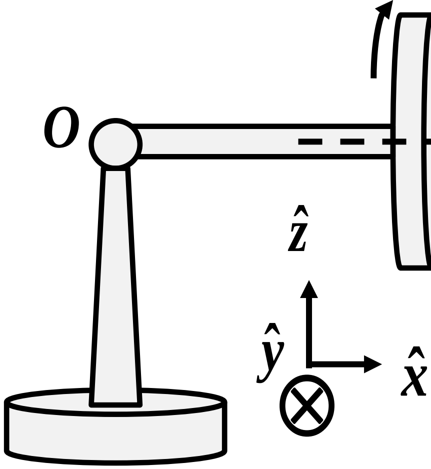


Fig. 4

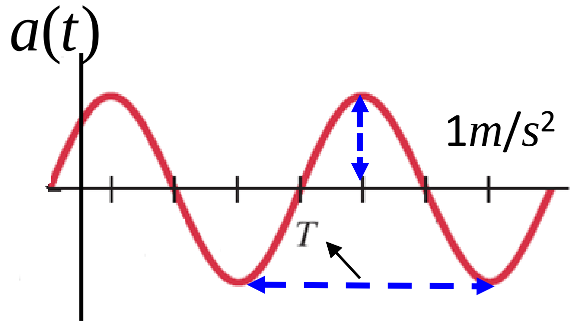


Fig. 5

2. (5 pts) A gyroscope has a wheel at one end of an axle, which is pivoted at point O as shown in Fig. 4. The wheel spins about the axle in the direction shown by the arrow. At the moment shown in the figure, the axle is horizontal and in the *x*-*y* plane. Let $\vec{L}$ be the angular momentum of the gyroscope about the center of mass of the gyroscope. Ignoring the mass of the axle and assume the spin angular velocity is much greater than the precessional angular velocity. (1)What is the direction of $d\vec{L}/dt$ of the gyroscope at the moment shown in the figure? (2) Viewing from the top, the gyroscope will rotate clockwise (***c.w.***) or counter-clockwise (***c.c.w***)?

(A) +***x, c.w.*** (B) –***x, c.w.*** (C) +***x, c.c.w*** (D) –***x, c.c.w.*** (E) +***y, c.w.*** (F) –***y, c.w.*** (G) +***y, c.c.w*** (H) ***-y, c.c.w*** (I) +***z, c.w.*** (J) –***z, c.c.w.***

3. (5 pts) Fig. 5 shows the acceleration ***a*(t)** for a small mass ***m*** at the end of a spring as a function of time. The period ***T*** = π (second). What is the position ***x(t)*** as a function of time?

(A) $\mathbf{0.25\sin(2t+\frac{\pi}{4})}$ (B) $\mathbf{0.25\sin(2t-\frac{\pi}{4})}$ (C) $\mathbf{-0.25\sin(2t+\frac{\pi}{4})}$ (D) $\mathbf{-0.25\sin(2t-\frac{\pi}{4})}$

(E) $\mathbf{0.25\cos(2t+\frac{\pi}{4})}$ (F) $\mathbf{0.25\cos(2t-\frac{\pi}{4})}$ (G) $\mathbf{-0.25\cos(2t+\frac{\pi}{4})}$ (I) $\mathbf{-0.25\cos(2t-\frac{\pi}{4})}$

(J) none of above

4. (5 pts) The speed of a rifle bullet is to be determined by using a ballistic pendulum consisting of a plank (厚木板)suspended from a nail, as shown in Fig. 6. If the friction is negligible, for any point of impact, here are three possible conserved quantities during the collision for the system of the bullet and the plank:

I. Linear momentum

II. Angular momentum about the nail

III. Angular momentum about the center of mass of the plank

which of the following is true during the collision?

(A) I only (B) II only (C) III only (D) I and II (E) I and III

(F) II and III (G) I, II, and III (H) None of above

5. An ideal gas engine is operated to follow the cycle shown in the ***P-V*** diagram in Fig. 7. The temperature of the gas $\boldsymbol{T}$ varies between $\boldsymbol{T_B}$ and $\boldsymbol{T_D}$, and its volume $\boldsymbol{V}$ varies between $\boldsymbol{V_A}$ and $\boldsymbol{V_C}$. Which of the following statements is correct?

(A) The efficiency of the engine is equal to $\boldsymbol{1-T_D/T_B}$

(B) $\boldsymbol{Q_{AB} > 0, Q_{BC} > 0}$

(C) $\boldsymbol{Q_{DA} < 0, Q_{CD} < 0}$

(D) $\boldsymbol{Q_{AB} > 0, Q_{CD} < 0}$

(E) $\boldsymbol{Q_{BC} > 0, Q_{DA} < 0}$

6. A system of ideal gas with $\boldsymbol{\gamma = 7/5}$ evolves from condition $\boldsymbol{O}$ in Fig.8 along four different paths in the ***P-V*** diagram shown in Fig. 8.i.e. 1: $\boldsymbol{O \rightarrow A}$ (Isobaric), 2: $\boldsymbol{O \rightarrow B}$ (Isothermal), 3: $\boldsymbol{O \rightarrow C}$ (Adiabatic), 4: $\boldsymbol{O \rightarrow D}$ (Isovolumetric). Which of the following item is correct?

(A) $\boldsymbol{Q_1 < Q_2 < Q_3 = 0 < Q_4}$

(B) $\boldsymbol{W_1 > W_2 > W_3 > W_4 = 0}$

(C) $\boldsymbol{\Delta E_{int,1} > \Delta E_{int,2} = 0 > \Delta E_{int,3} > \Delta E_{int,4}}$

(D) $\boldsymbol{\Delta S_1 > \Delta S_2 = 0 > \Delta S_3 > \Delta S_4}$

(E) Only two correct statements in (A)-(D).

(F) Only three correct statements in (A)-(D).

(G) (A)-(D) are all correct.

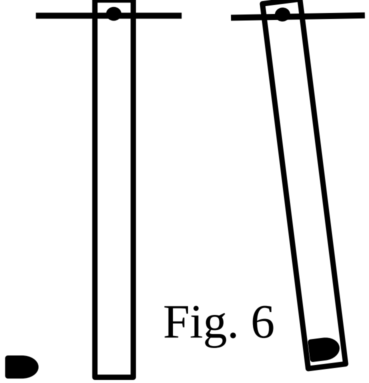


Fig. 6

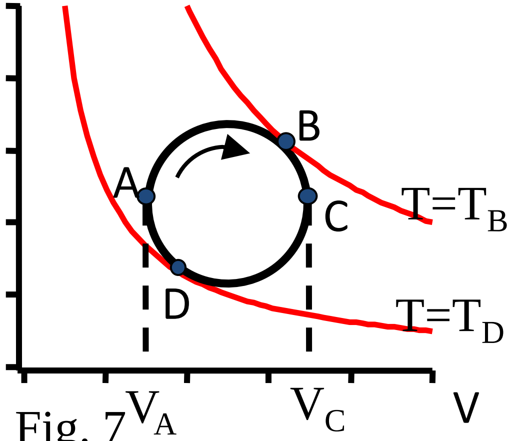


Fig. 7

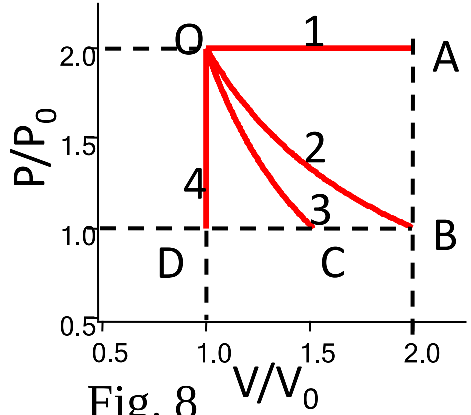


Fig. 8

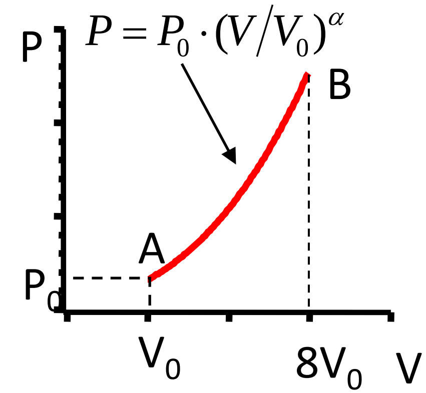


Fig. 9

7. An ideal gas engine is expanded to follow a path from ***A*** to ***B*** as shown in Fig. 9. If $\boldsymbol{\alpha = 2/3}$, the work ***W*** done by the system in terms of $\boldsymbol{P_0V_0}$ is

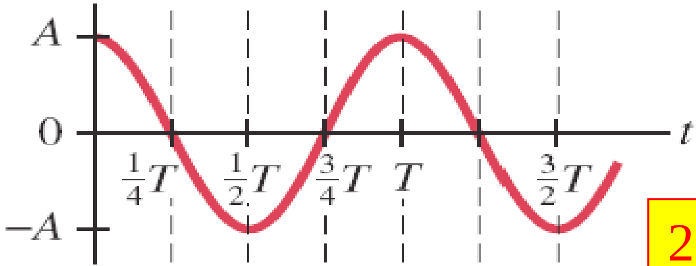
(A) **ln*8*** (B) ***21*** (C) ***24*** (D) ***93/5*** (E) ***93***

8. A piece of ***5 mole*** metal whose specific heat is ***0.1R/mole·°K*** (***R*** is the ideal gas constant) is put to be in contact with a tank of ***1 mole*** ideal gas with $\boldsymbol{\gamma = 7/5}$. At the moment of the contact, the temperature of the ideal gas is $\boldsymbol{T_0}$ and the temperature of metal the is $\boldsymbol{9T_0}$, and both parts are allowed to exchange heat with each other only. If the pressure of the ideal gas is fixed and its volume is allowed to change. What is the entropy change $\boldsymbol{\Delta S}$ of the ideal gas after the system has reached equilibrium?

(A) $\dfrac{R}{2}\ln 2$ (B) $\dfrac{3R}{2}\ln 2$ (C) $\dfrac{5R}{2}\ln 2$ (D) $\dfrac{7R}{2}\ln 2$ (E) $\dfrac{9R}{2}\ln 2$ (F) $\dfrac{11R}{2}\ln 2$

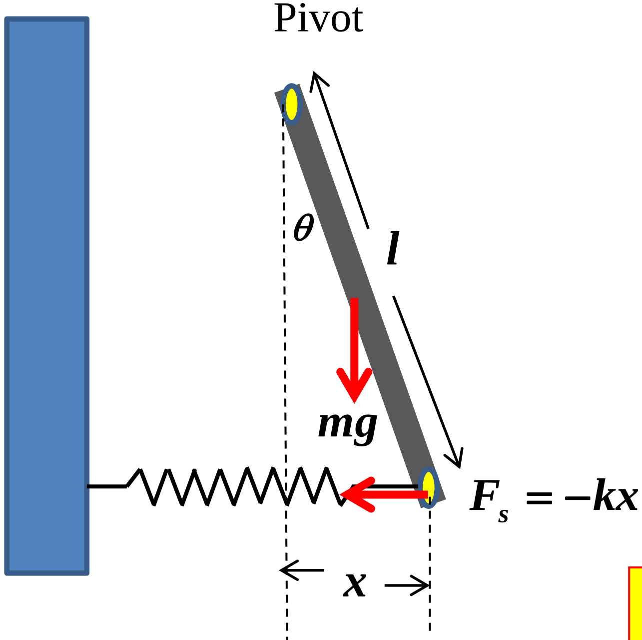
1	2	3	4	5	6	7	8	9	10
B	E	C,I	B	D	E	D	D	B	B
11	12	13	14	15	16	17	18		
E	B	A	B	C	A	A	D		

1(A) Draw the time dependence of $\boldsymbol{\theta}$



2 pts

(B) Free body diagram



2 pts

(C) Calculation of the time period $\boldsymbol{T}$

$$\sum \tau = I\alpha$$

$$-\frac{l}{2}\cdot mg\sin\theta - l\cdot kx\cos\theta = I\frac{d^2\theta}{dt^2}$$

2 pts

since $\sin\theta \approx \theta\,,\cos\theta \approx 1$ for the angle θ

and $x = l\sin\theta \approx l\theta$

$$\Rightarrow \frac{d^2\theta}{dt^2} + \frac{\left(\frac{l}{2}\cdot mg + l^2\cdot k\right)}{\frac{1}{3}ml^2}\theta = 0$$

2 pts

$$\Rightarrow \omega^2 = \left(\frac{3g}{2l} + \frac{3k}{m}\right) = \frac{3mg + 6kl}{2ml}$$

$$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2ml}{3mg + 6kl}}$$

2 pts

2. If ***200 cm³*** of water at ***95 ⁰C*** is poured into a ***150 g*** glass cup initially at ***25 ⁰C***, what will be the common final temperature ***T*** of the tea and cup when equilibrium is reached, assuming no heat flows to the surroundings ? Determine the change in entropy of (a) the water, (b) the cup, (c) the surroundings, and (d) the universe as a whole. (specific heat of water is ***4200 J/kgC⁰*** , specific heat of glass is ***840 J/KgC⁰***)

$$\rho_{water} = 1g/cm^3 \Rightarrow m_{water} = 200cm^3 \times 1g/cm^3 = 200g = 0.2Kg$$

$$m_{water}c_{water}(95\,^0C - T) = m_{cup}c_{cup}(T - 25\,^0C)$$

$$(0.2Kg)(4200J/Kg\,^0C)(95\,^0C - T) = (0.15Kg)(840J/Kg\,^0C)(T - 25\,^0C)$$

$$\Rightarrow T = 86\,^0C$$

2 points

Method 1

$$\Delta S_{water} = mc\ln\frac{T_f}{T_i} = (0.2Kg)(4200J/Kg\,^0C)\ln\left(\frac{86+273}{95+273}\right) = 840\ln\left(\frac{359}{368}\right)$$

3 points

$$\Delta S_{cup} = mc\ln\frac{T_f}{T_i} = (0.15Kg)(840J/Kg\,^0C)\ln\left(\frac{86+273}{25+273}\right) = 126\ln\left(\frac{359}{298}\right)$$

3 points

$$\Delta S_{surr} = 0$$

1 point

$$\Delta S_{univ} = \Delta S_{water} + \Delta S_{cup} + \Delta S_{surr} = 840\ln\left(\frac{359}{368}\right) + 126\ln\left(\frac{359}{298}\right)$$

1 point

$$= (-20.80 + 23.46)\,J/{}^0K = 2.66J/{}^0K$$

Method 2

$$\Delta S_{water} = (0.2Kg)\left(4200J/Kg\,{}^0C\right)\frac{359-368}{363.5} = -20.80J/{}^0K$$

3 points

$$\Delta S_{cup} = (0.15Kg)\left(840J/Kg\,{}^0C\right)\frac{359-298}{328.5} = 23.40J/{}^0K$$

3 points

$$\Delta S_{surr} = 0$$

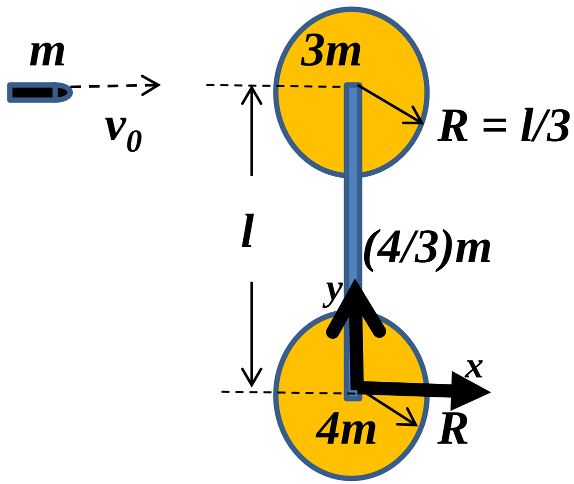
1 point

$$\Delta S_{univ} = \Delta S_{water} + \Delta S_{cup} + \Delta S_{surr}$$

$$= (-20.80 + 23.40)\,J/{}^0K = 2.60J/{}^0K$$

1 point

3.



(A) The coordinate of the center of mass

$$(x_{cm}, y_{cm}) = \frac{m(l,0)+3m(l,0)+\left(4/3\right)m(l/2,0)+4m(0,0)}{\left(28/3\right)m}$$

$$= (l/2, 0)$$

3 pts

(B) The moment of inertia relative to new CM.

$$I_{cm} = \underbrace{\left(m\left(\frac{l}{2}\right)^2\right)}_{\text{bullet}} + \underbrace{\left(\frac{1}{2}(3m)\left(\frac{l}{3}\right)^2 + 3m\left(\frac{l}{2}\right)^2\right)}_{\text{upper disk}}$$

$$+ \underbrace{\left(\frac{1}{2}(4m)\left(\frac{l}{3}\right)^2 + 4m\left(\frac{l}{2}\right)^2\right)}_{\text{lower disk}} + \underbrace{\left(\frac{1}{12}\left(\frac{4}{3}m\right)(l)^2\right)}_{\text{rod}}$$

$$= \frac{5}{2}ml^2$$

5 pts

(C) Angular velocity w after the collision

兩種算法:

(i)以底部為原點的算法:

$$m\vec{v}_0 = \left(1+3+4/3+4\right)m\vec{v}_{cm}$$

$$\vec{v}_{cm} = \frac{3}{28}\vec{v}_0 = \left(\frac{3}{28}v_0, 0\right)$$

1 pt

$$\vec{L}_i = \vec{L}_f$$

1 pt

$$lmv_0(-\hat{z}) = \frac{l}{2}\left(\frac{28}{3}m\right)\left(\frac{3}{28}v_0\right)(-\hat{z}) + I_{cm}\vec{\omega}_f$$

1 pts

2 pts

$$\Rightarrow \omega_f = \frac{v_0}{5l}$$

2 pts

(ii)以新質心為原點的算法:

$$\vec{L}_i = \vec{L}_f$$

1 pt

1 pts

$$\frac{l}{2}mv_0(-\hat{z}) = I\vec{\omega}_f = \frac{5}{2}ml^2\vec{\omega}_f$$

3 pts

$$\Rightarrow \omega_f = \frac{v_0}{5l}$$

2 pts

1. A heat engine takes one mole of ideal monatomic gas around the cycle shown in Fig. 1 (isothermal a→b, isovolumetric b→c , straight line from *c*→*d*, and adiabatic from *d*→*a*). The volume at points a , b, c, and d are given by $64V_0$, $4V_0$, $4V_0$, and $8V_0$, respectively. The pressure at point a is P_0. (write your answer in term of P_0, V_0 and R)

a) Determine the thermal dynamic variables (***P, V,*** and ***T***) at points a, b, c, and d.

b) Calculate the work done (by the gas), heat, internal energy, and entropy change for each process (a ➔ b, b ➔ c, c ➔ d, and d ➔ a).

c) Determine the efficiency of the heat engine.

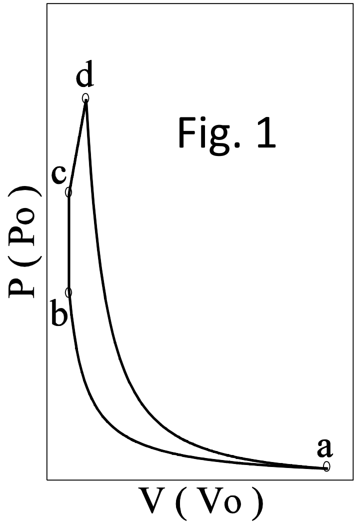


Fig. 1

(a)

3 points

	$P\,(p_0)$	$V\,(v_0)$	$T\,(T_a)$
a	1	64	64
b	16	4	64
c	24	4	96
d	32	8	256

14 points

(b)

	$W\,(p_0v_0)$	$Q\,(p_0v_0)$	$\Delta E_{int}\,(p_0v_0)$	$\Delta S\,(R)$
a→b	- 256ln2	- 256ln2	0	- 4ln2
b→c	0	48	48	3/2 ln(3/2)
c→d	112	352	240	11/2 ln2 – 3/2 ln3
d ➔ a	288	0	-288	0

(c)

$$e = \frac{W_{tot}}{Q_{in}} = \frac{400 - 256\ln 2}{400} = 1 - \frac{16\ln 2}{25}$$

3 points

(a)

$$T_a = T_b \quad \text{(isothermal)} \Rightarrow p_a v_a = p_b v_b \Rightarrow p_b = 16 p_0$$

$$p_d v_d^\gamma = p_a v_a^\gamma \quad \text{(adiabatic)} \Rightarrow p_d \left(8v_0\right)^{5/3} = p_0 \left(64v_0\right)^{5/3} \Rightarrow p_d = 32 p_0$$

$$p_a v_a = nRT_a \Rightarrow T_a = \frac{64 p_0 v_0}{R}$$

$$T_b = \frac{64 p_0 v_0}{R}; \quad T_c = \frac{96 p_0 v_0}{R}; \quad T_d = \frac{256 p_0 v_0}{R}$$

(b)

a→b (isothermal)

$$W_{a\to b} = \int_a^b pdV = \int_a^b nRT\frac{dV}{V} = nRT_a \ln\frac{V_b}{V_a} = \left(-256\ln 2\right) p_0 v_0$$

$$Q_{a\to b} = W_{a\to b}, \Delta E_{\text{int},a\to b} = 0$$

$$\Delta S_{a\to b} = nC_v \ln\frac{T_b}{T_a} + nR\ln\frac{V_b}{V_a} = \frac{3}{2}R\left(\ln\frac{4}{64}\right) = -4R\ln 2$$

b→c (constant V)

$$W_{b\to c} = 0$$

$$Q_{b\to c} = \Delta E_{\text{int},b\to c} = \frac{3}{2}nR\Delta T = \frac{3}{2}R\left(\frac{32p_0v_0}{R}\right) = 48p_0v_0$$

$$\Delta S_{b\to c} = \frac{3}{2}nR\ln\frac{T_c}{T_b} = \frac{3}{2}R\ln\left(\frac{3}{2}\right)$$

c→d (straight line)

$$W_{c\to d} = \frac{\left(24+32\right)4p_0v_0}{2} = 112p_0v_0$$

$$\Delta E_{\text{int},c\to d} = \frac{3}{2}nR\Delta T = \frac{3}{2}R\left(\frac{160p_0v_0}{R}\right) = 240p_0v_0$$

$$Q_{c\to d} = \Delta E_{\text{int},c\to d} + W_{c\to d} = 352p_0v_0$$

$$\Delta S_{c\to d} = \frac{3}{2}nR\ln\frac{T_d}{T_c} + R\ln\frac{V_d}{V_c} = \frac{3}{2}R\ln\left(\frac{256}{96}\right) + R\ln\left(\frac{8}{4}\right)$$

$$= \frac{3}{2}R\ln\left(\frac{8}{3}\right) + R\ln\left(2\right) = \frac{11}{2}R\ln 2 - \frac{3}{2}\ln 3$$

d→a (adiabatic)

$$Q_{d\to a} = 0; \qquad \Delta S_{d\to a} = 0$$

$$\Delta E_{\text{int},d\to a} = \frac{3}{2}nR\Delta T = \frac{3}{2}nR(64-256)p_0v_0 = -288p_0v_0$$

$$W = -\Delta E_{\text{int},d\to a} = 288p_0v_0$$