試卷請註明、姓名、班級、學號，請遵守考場秩序

# I.計算題(50 points) (所有題目必須有計算過程,否則不予計分)

1. (10 pts). Two cylindrical shells consist of two shells have radii ***a***, ***4a***, and length L( Fig. 1 without the shaded area). The most inner shell ($r=a$) carries total charge Q and the outer shell (***r =4a***) carries total charge -Q. Ignoring the end effects and express your answers in terms of the total charge Q, a, L, and/or any other constant.)

(a) (7 pts) Draw the Guassian surfaces and calculate the electric fields (both the direction and magnitude) and electric potential in the region $a \le r < 4a$. (Assume $V(4a)=0$.)

(b) (3 pts) Find out the capacitance of this cylinder ?

2. (10 pts) 1.The capacitance of a parallel plate capacitor is $C_0$ with surface charge density $\sigma_0$. Now a slab of dielectric material with dielectric constant K ( K = 5 ) is inserted a distance xL into the space between the square parallel plates, as shown in Fig.2 .

(A) (4 pts) Now we want the capacitor to be $3C_0$, how much distance xL should the material inserted in?

(B) (6 pts) What is surface charge $\sigma_1$ , $\sigma_2$ and $\sigma_{ind}$ in terms of $\sigma_0$ .

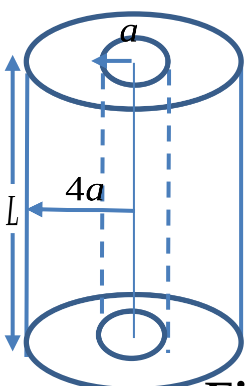


Fig. 1

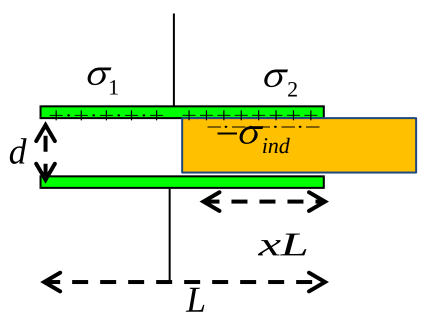


Fig. 2

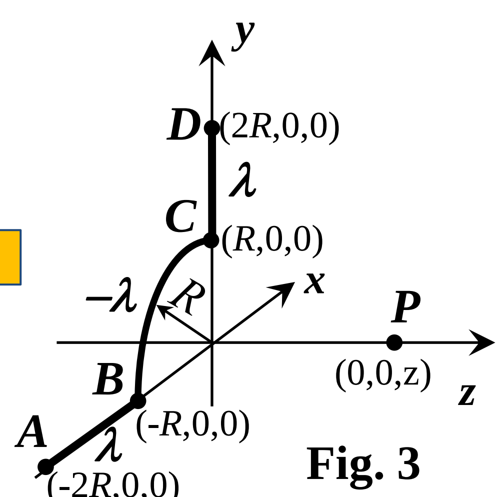


Fig. 3

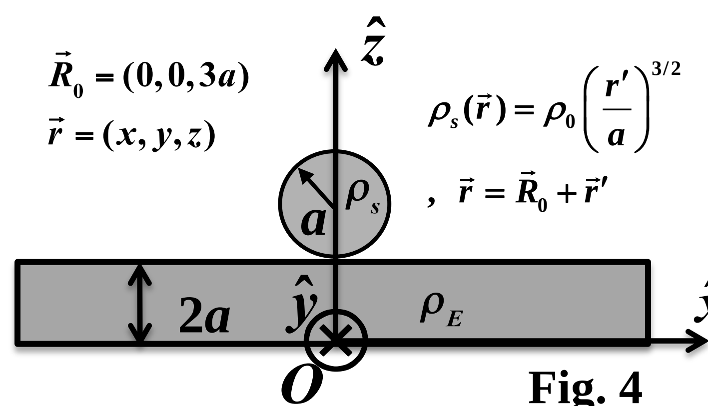


Fig. 4

3. (15 pts) As shown in Fig. 3, there are continuous charge distribution between line segment ***AB***, ***CD*** , with density $\lambda$, and between the arc ***BC*** with density $-\lambda$. Apply the Coulomb's law to determine the direction and the magnitude of the electric field at point ***P*** on the z-axis. (Details of the calculation need to presented in your answer)

4. (15 pts) A sphere of radius ***a*** locates at $\vec{R}_0 = (0,0,3a)$(Fig. 4) with charge density $\rho_s(\vec{r}') = \rho_0\left(\frac{r'}{a}\right)^{3/2}$ where $\rho_0$ is a constant and ***r'*** is the distance to the center of the sphere. An infinite wide slab of thickness 2***a*** with uniform volume charge density $\rho_E$ is placed between **z = 0** and **2*a*.**

(a) (3 pts) What is the total charge ***Q*** in the sphere? (write your answers in terms of ***a*** and $\rho_0$.)

(b) (12 pts) Use Gauss's Law (you need to draw the Gaussian surfaces in your calculation to get full credits) find the electric fields (magnitude and direction) in the following cases

(i) $\vec{r} = (0,0,z)$, for $z > 4a$, $2a < z < 4a$, and $0 < z < 2a$. (ii) $\vec{r} = (2a,0,3a)$.

## II.選擇題(51 points)

1. (5pts) As shown in Fig. 5, a point charge ***Q*** (***Q***>***0***) is located at the origin, and an electric dipole $\vec{p}$ is place on the **z**-axis, with is dipole moment pointing in the +**z**-direction. If the dipole $\vec{p}$ is released to be free to move, which of the following statement is correct regarding the motion of the dipole soon after being released?

(A) The dipole will only rotate

(B) The dipole will only drift in the +z-direction.

(C) The dipole will only drift in the –z direction.

(D) The dipole will rotate and direction in the +z-direction.

(E) The dipole will rotate and direction in the -z-direction.

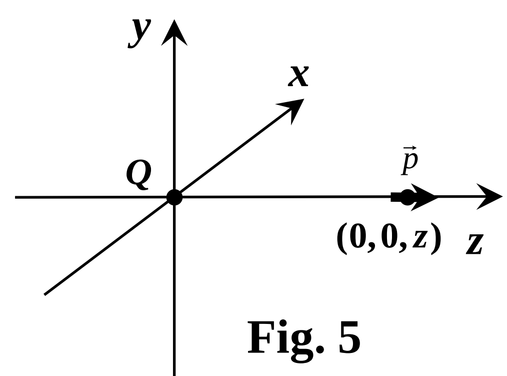


**Fig. 5**

2. (5pts) As shown in Fig, 6, two positive point charges $\boldsymbol{Q}$ are placed on y-z plane with coordinates as indicated. A negative change $-\boldsymbol{q}$ located at ($\boldsymbol{x}$,0,0) is allowed to travel on the x-axis only. For the case that $\boldsymbol{x} \ll \boldsymbol{a}$, the total force acting on $-\boldsymbol{q}$ can be expressed as $\boldsymbol{F_{total}} \approx \boldsymbol{s} \cdot \boldsymbol{kQq{\cdot}x/a^3}$ , which of the following is correct?

(A) $s \le -4$ (B) $-4 < s \le -2$ (C) $-2 < s \le 0$ (D) $0 < s \le 2$
(E) $2 < s \le 4$ (F) $4 < s$

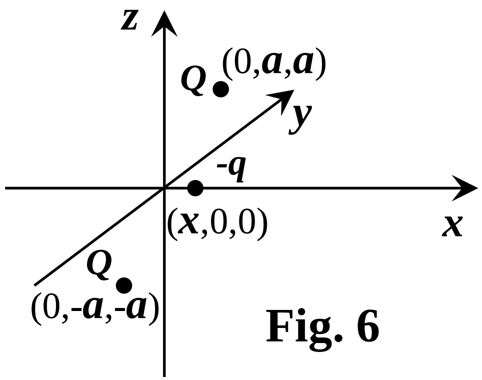

**Fig. 6**

3. (5 pts) There are three large plates with charge density $\boldsymbol{\sigma_1}$, $\boldsymbol{\sigma_2}$ and $\boldsymbol{\sigma_3}$ respectively. The potential is shown in Fig. 7. What is the ratio of the surface charge density $x = \sigma_3 / (\sigma_1 + \sigma_2)$ if an dielectric material ($\kappa = 4$) filled between $\boldsymbol{x}$**=0** and $\boldsymbol{x}$**=2**?

(A) $x \le -6$ (B) $-6 < x \le -5$ (C) $-5 < x \le -4$ (D) $-4 < x \le -3$
(E) $-3 < x \le -2$ (F) $-2 < x \le -1$ (G) $-1 < x \le +1$
(H) $+1 < x \le +2$ ( I ) $+2 < x < +3$ (J) $+3 < x < +4$
(K) $+4 < x < +5$ (L) $+5 < x < +6$ (M) $+6 < x$

4. (5 pts) Same as question **3**, now we connect plates **1** and **2** by a wire. After the equilibrium reaches, what is the ratio of the surface charge density $x = \sigma_{2,new} / \sigma_3$ ? (use the same data of the multiple choices in question **3**.)

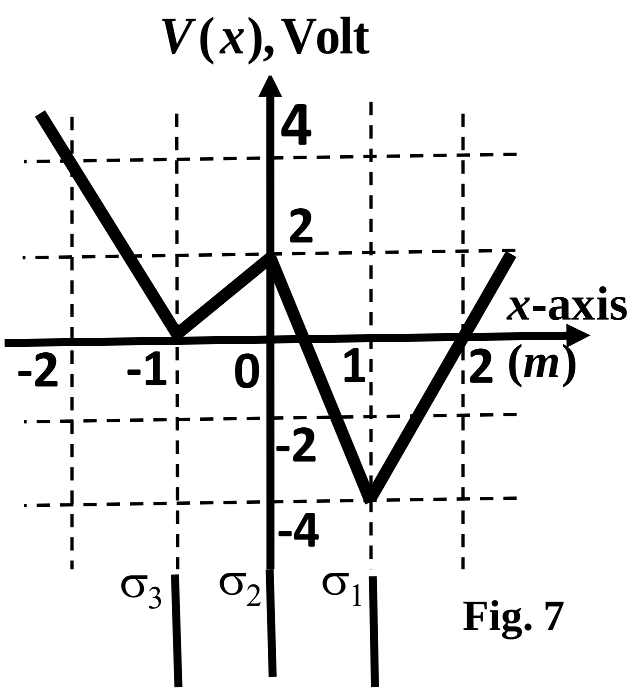

**Fig. 7**

5. (5 pts) A parallel plate capacitor is connected with a battery and allowed to charge up. Now a slab of dielectric material is placed between the plates of the capacitor while the capacitor is still connected to the battery. After this is done one would find out

(A) The energy stored in the capacitor decreases. (B) The voltage across the capacitor increases.
(C) The charge on the capacitor increases. (D) The charge on the capacitor does not change.

**6.** (5 pts) A closed cubic imaginary Gaussian surface is shown in Fig. 8. The surfaces of the cubic are labeled as $\boldsymbol{S_R}$ (right), $\boldsymbol{S_L}$ (left), $\boldsymbol{S_{1\text{-}4}}$ (top, front, bottom and back). A negative point charge, **-*Q***, is located on the axis through the center of the right Gaussian surface ($\boldsymbol{S_R}$) with a distance ***a***. The electric flux through surfaces $\boldsymbol{S_L}$ and $\boldsymbol{S_R}$ is $\Phi_A$. The electric flux through surfaces $\boldsymbol{S_1}+\boldsymbol{S_2}+\boldsymbol{S_3}+\boldsymbol{S_4}$ is $\Phi_B$. Which statement below is correct?

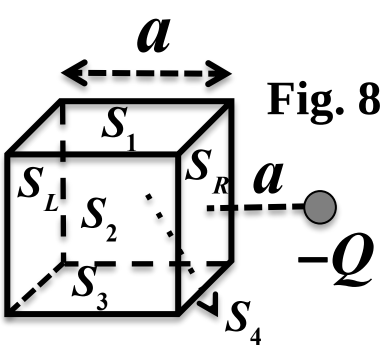


**Fig. 8**

(A) $\Phi_A>0, \Phi_B>0$ , $\Phi_A+\Phi_B=0$ (B) $\Phi_A>0$ , $\Phi_B<0$, $\Phi_A+\Phi_B=0$ (C) $\Phi_A<0$ , $\Phi_B>0$, $\Phi_A+\Phi_B=0$
(D) $\Phi_A<0, \Phi_B<0$ , $\Phi_A+\Phi_B=0$ (E) $\Phi_A>0, \Phi_B>0$, $\Phi_A+\Phi_B\neq 0$ (F) $\Phi_A>0$ , $\Phi_B<0$, $\Phi_A+\Phi_B\neq 0$
(J) $\Phi_A<0$ , $\Phi_B>0$ , $\Phi_A+\Phi_B\neq 0$ (K) $\Phi_A<0, \Phi_B<0$, $\Phi_A+\Phi_B\neq 0$

Multiple Choice Questions:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
| **C** | **C** | **I** | **G** | **C** | **B** | **L** | **C** | **B** | **D** |
| **11** | **12** | **13** | | | | | | | |
| **H** | **F** | **F** | | | | | | | |

1. (10 pts). Two cylindrical shells consist of two shells have radii ***a*, *4a*,** and length L( Fig. 1 without the shaded area). The most inner shell (*r* =*a*) carries total charge Q and the outer shell (***r* =*4a***) carries total charge -Q. Ignoring the end effects and express your answers in terms of the total charge Q, a, L, and/or any other constant.)

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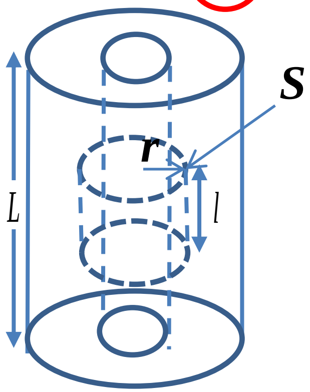
(b) (3 pts) Find out the capacitance of this cylinder ?

(a)

1




Choose surface S as the Gaussian surface.

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$ 1

$$\Rightarrow E\,2\pi r l = \frac{\lambda l}{\varepsilon_0}$$

$$\text{where } \lambda = \frac{Q}{L}$$

$$\Rightarrow \vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$$ 2

$$V(r) - V(4a) = -\int_{4a}^{r} \vec{E}\cdot d\vec{l}$$ 1

$$V(r) = -\int_{4a}^{r} \frac{\lambda}{2\pi\varepsilon_0 r}dr + V(4a)$$

$$= -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r}{4a}\right)$$ 2

(b)

$$C = \frac{Q}{|\Delta V|}$$ 1

$$C = \frac{Q}{|V(4a) - V(a)|} = \frac{Q}{\frac{Q/L}{2\pi\varepsilon_0}\ln 4}$$

$$= \frac{2\pi\varepsilon L}{\ln 4} = \frac{\pi\varepsilon L}{\ln 2}$$ 2

2. (10 pts) 1.The capacitance of a parallel plate capacitor is $C_0$ with surface charge density $\boldsymbol{\sigma_0}$. Now a slab of dielectric material with dielectric constant K ( K = 5 ) is inserted a distance xL into the space between the square parallel plates, as shown in Fig.2 .

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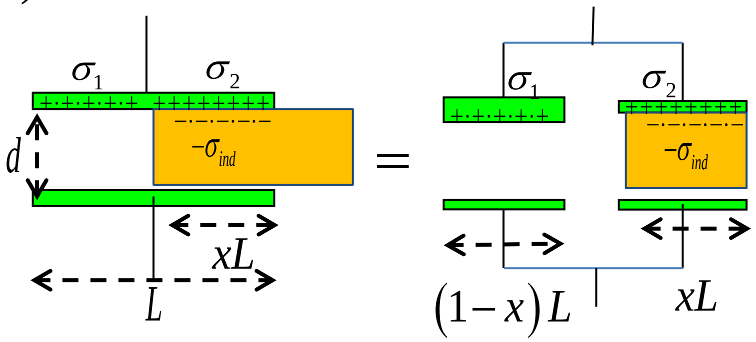
(a)




$$C = \frac{(1-x)L}{L}C_0 + K\frac{xL}{L}C_0 \qquad 2$$

$$3C_0 = (1-x)C_0 + 5 \cdot xC_0$$

$$x = \frac{1}{2} \qquad 2$$

(b)

The total electric field on the right:

$$1 \quad E_2 = E_{20} - E_{ind} \quad \Rightarrow E_2 = \frac{\sigma_2}{\varepsilon_0} - \frac{\sigma_{ind}}{\varepsilon_0}$$

The property of dielectric material:

$$1 \quad E_2 = \frac{E_{20}}{K} \quad \Rightarrow \frac{\sigma_2}{k} = \sigma_2 - \sigma_{ind}$$

$$\Rightarrow \sigma_{ind} = \sigma_2\left(1 - \frac{1}{K}\right) = \frac{4}{5}\sigma_2$$

The voltage on the right and the left are the same.

$$1 \quad E_1 = E_2 = \frac{E_{20}}{K} \quad \Rightarrow \sigma_2 = K\sigma_1$$

The total charge is unchanged in this case.

$$\sigma_0 \cdot A = \sigma_1 \cdot (1-x) \cdot A + \sigma_2 \cdot x \cdot A \qquad 1$$

$$\sigma_0 = (1-x)\sigma_1 + xK\sigma_1 = 3\sigma_1$$

$$\Rightarrow \sigma_1 = \frac{\sigma_0}{3}, \quad \sigma_2 = \frac{5\sigma_0}{3}, \quad \sigma_{ind} = \frac{4\sigma_0}{3} \qquad 2$$

$$\text{or} \quad \sigma_1 = \frac{\sigma_0}{1+4x}, \quad \sigma_2 = \frac{5\sigma_0}{1+4x}, \quad \sigma_{ind} = \frac{4\sigma_0}{1+4x}$$

3.

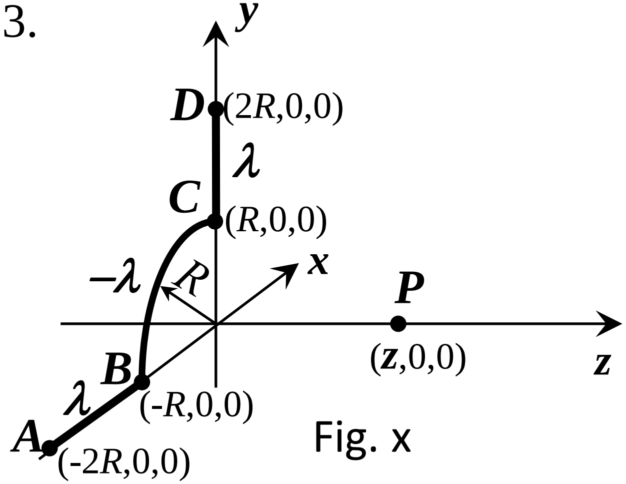



Fig. x

For the field from segment AB,

$$dq = \lambda dx$$

$$d\vec{E} = \frac{k\lambda dx(-x,0,z)}{\sqrt{x^2+z^2}^3}$$ 2

$$\vec{E}_{AB} = \int_{-2R}^{-R} \frac{k\lambda dx(-x,0,z)}{\sqrt{x^2+z^2}^3},$$

define $u \equiv x^2 + z^2$

$$E_{AB,x} = \int_{-2R}^{-R} \frac{-k\lambda x dx}{\sqrt{x^2+z^2}^3} = -\frac{k\lambda}{2}\int_{4R^2+z^2}^{R^2+z^2} \frac{du}{\sqrt{u}^3}$$

$$= -\frac{k\lambda}{2}(-2)\left(\frac{1}{\sqrt{u}}\Big|_{4R^2+z^2}^{R^2+z^2}\right)$$

$$= \frac{k\lambda}{\sqrt{R^2+z^2}} - \frac{k\lambda}{\sqrt{4R^2+z^2}}$$ 1

$$E_{AB,z} = \int_{-2R}^{-R} \frac{k\lambda z dx}{\sqrt{x^2+z^2}^3}$$

$$= k\lambda z \int_{-2R}^{-R} \frac{dx}{\sqrt{x^2+z^2}^3}$$

define $x \equiv z\tan\theta$

$$E_{AB,z} = k\lambda z \int_{\tan^{-1}(-2R/z)}^{\tan^{-1}(-R/z)} \frac{z\sec^2\theta d\theta}{z^3\sec^3\theta}$$

$$= \frac{k\lambda}{z}\int_{\tan^{-1}(-2R/z)}^{\tan^{-1}(-R/z)} \cos\theta d\theta$$

$$= \frac{k\lambda}{z}\left(\sin\theta\Big|_{\tan^{-1}(-2R/z)}^{\tan^{-1}(-R/z)}\right)$$

$$= \frac{k\lambda}{z}\left(\frac{-R}{\sqrt{R^2+z^2}} + \frac{2R}{\sqrt{4R^2+z^2}}\right)$$

$$= \frac{k\lambda}{z}\left(\frac{2R}{\sqrt{4R^2+z^2}} - \frac{R}{\sqrt{R^2+z^2}}\right)$$ 1

For the field from segment CD,

$$dq = \lambda dy$$

$$d\vec{E} = \frac{k\lambda dy(0,-y,z)}{\sqrt{x^2+z^2}^3}$$ 1

$$\vec{E}_{CD} = \int_{R}^{2R} \frac{k\lambda dx(0,-y,z)}{\sqrt{y^2+z^2}^3},$$

$$\vec{E}_{CD,y} = \int_{R}^{2R} \frac{-k\lambda y dx}{\sqrt{y^2+z^2}^3},$$

define $u \equiv y^2 + z^2$

$$E_{CD,y} = -\frac{k\lambda}{2}\int_{R^2+z^2}^{2R^2+z^2} \frac{du}{\sqrt{u}^3}$$

$$= -\frac{k\lambda}{2}(-2)\left(\frac{1}{\sqrt{u}}\Big|_{R^2+z^2}^{4R^2+z^2}\right)$$

$$= \frac{k\lambda}{\sqrt{4R^2+z^2}} - \frac{k\lambda}{\sqrt{R^2+z^2}}$$

$$E_{CD,z} = \int_{R}^{2R} \frac{k\lambda z dx}{\sqrt{x^2+z^2}^3}$$ 1

define $x \equiv z\tan\theta$

$$E_{CD,z} = k\lambda z \int_{\tan^{-1}(R/z)}^{\tan^{-1}(2R/z)} \frac{z\sec^2\theta d\theta}{z^3\sec^3\theta}$$

$$= \frac{k\lambda}{z}\int_{\tan^{-1}(R/z)}^{\tan^{-1}(2R/z)} \cos\theta d\theta$$

$$= \frac{k\lambda}{z}\left(\sin\theta\Big|_{\tan^{-1}(R/z)}^{\tan^{-1}(2R/z)}\right)$$

$$= \frac{k\lambda}{z}\left(\frac{2R}{\sqrt{4R^2+z^2}} - \frac{R}{\sqrt{R^2+z^2}}\right)$$ 1

## Solution 1

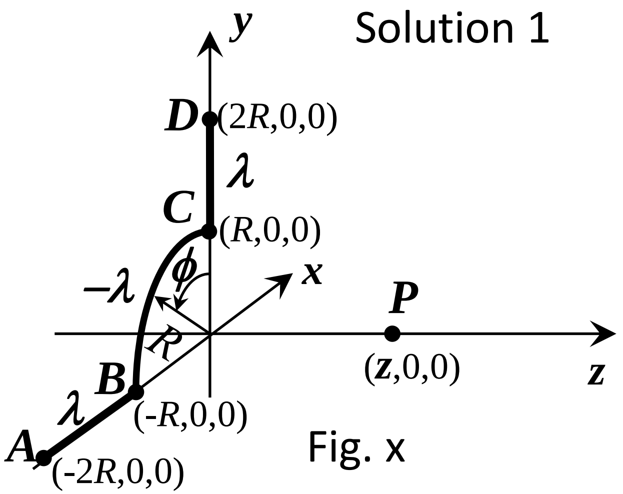



Fig. x

For the field from segment CB,

According to the angle $\phi$ defined in the figure,

$$dq = -\lambda R d\phi$$

$$d\vec{E} = \frac{-k\lambda R d\phi (R\sin\phi, -R\cos\phi, z)}{\sqrt{z^2+R^2}^3}$$ 2

$$\vec{E}_{CB} = \int_0^{\pi/2} \frac{-k\lambda R d\phi (R\sin\phi, -R\cos\phi, z)}{\sqrt{z^2+R^2}^3}$$

$$E_{CB,x} = \int_0^{\pi/2} \frac{-k\lambda R^2 \sin\phi d\phi}{\sqrt{z^2+R^2}^3}$$

$$= \frac{-k\lambda R^2}{\sqrt{z^2+R^2}^3} \int_0^{\pi/2} \sin\phi d\phi$$

$$= \frac{-k\lambda R^2}{\sqrt{z^2+R^2}^3} (-\cos\phi|_0^{\pi/2}) = \frac{-k\lambda R^2}{\sqrt{z^2+R^2}^3}$$ 1

$$E_{CB,y} = \int_0^{\pi/2} \frac{k\lambda R^2 \cos\phi d\phi}{\sqrt{z^2+R^2}^3}$$

$$= \frac{k\lambda R^2}{\sqrt{z^2+R^2}^3} \int_0^{\pi/2} \cos\phi d\phi$$

$$= \frac{k\lambda R^2}{\sqrt{z^2+R^2}^3} \sin\phi|_0^{\pi/2}$$

$$= \frac{k\lambda R^2}{\sqrt{z^2+R^2}^3}$$ 1

$$E_{CB,z} = \int_0^{\pi/2} \frac{-k\lambda R z d\phi}{\sqrt{z^2+R^2}^3}$$

$$= \frac{-k\lambda R z}{\sqrt{z^2+R^2}^3} \int_0^{\pi/2} d\phi$$

$$= \frac{-k\lambda \pi R z}{2\sqrt{z^2+R^2}^3}$$ 1

$$\vec{E} = \vec{E}_{AB} + \vec{E}_{CD} + \vec{E}_{CB}$$

$$E_x = \frac{k\lambda}{\sqrt{R^2+z^2}} - \frac{k\lambda}{\sqrt{4R^2+z^2}} - \frac{k\lambda R^2}{\sqrt{z^2+R^2}^3}$$ 1

$$E_y = \frac{k\lambda}{\sqrt{4R^2+z^2}} - \frac{k\lambda}{\sqrt{R^2+z^2}} + \frac{k\lambda R^2}{\sqrt{z^2+R^2}^3}$$ 1

$$E_z = \frac{2k\lambda}{z}\left(\frac{2R}{\sqrt{4R^2+z^2}} - \frac{R}{\sqrt{R^2+z^2}}\right) - \frac{k\lambda \pi R z}{2\sqrt{z^2+R^2}^3}$$ 1

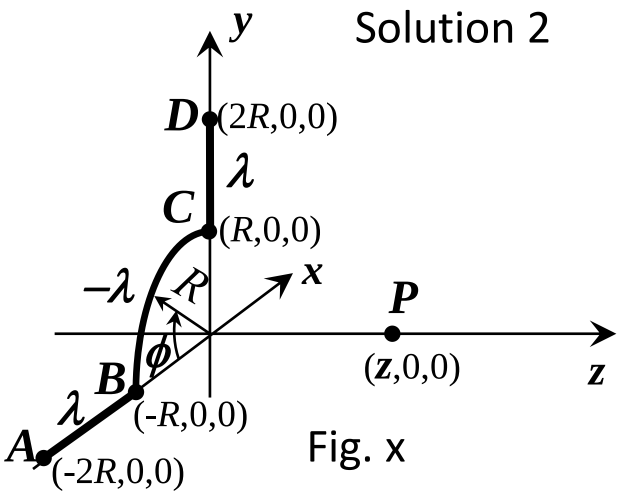


Fig. x

Solution 2

For the field from segment CB,

According to the angle $\phi$ defined in the figure,

$$dq = -\lambda R d\phi$$

$$d\vec{E} = \frac{-k\lambda R d\phi (R\cos\phi, -R\sin\phi, z)}{\sqrt{z^2+R^2}^3}$$

$$\vec{E}_{CB} = \int_0^{\pi/2} \frac{-k\lambda R d\phi (R\cos\phi, -R\sin\phi, z)}{\sqrt{z^2+R^2}^3}$$

$$E_{CB,x} = \int_0^{\pi/2} \frac{-k\lambda R^2 \cos\phi d\phi}{\sqrt{z^2+R^2}^3}$$

$$= \frac{-k\lambda R^2}{\sqrt{z^2+R^2}^3} \int_0^{\pi/2} \cos\phi d\phi$$

$$= \frac{-k\lambda R^2}{\sqrt{z^2+R^2}^3} (\sin\phi|_0^{\pi/2}) \quad = \frac{-k\lambda R^2}{\sqrt{z^2+R^2}^3}$$

$$E_{CB,y} = \int_0^{\pi/2} \frac{k\lambda R^2 \sin\phi d\phi}{\sqrt{z^2+R^2}^3}$$

$$= \frac{k\lambda R^2}{\sqrt{z^2+R^2}^3} \int_0^{\pi/2} \sin\phi d\phi$$

$$= \frac{k\lambda R^2}{\sqrt{z^2+R^2}^3} (-\cos\phi|_0^{\pi/2})$$

$$= \frac{k\lambda R^2}{\sqrt{z^2+R^2}^3}$$

$$E_{CB,z} = \int_0^{\pi/2} \frac{-k\lambda R z d\phi}{\sqrt{z^2+R^2}^3}$$

$$= \frac{-k\lambda R z}{\sqrt{z^2+R^2}^3} \int_0^{\pi/2} d\phi$$

$$= \frac{-k\lambda \pi R z}{2\sqrt{z^2+R^2}^3}$$

3 pts

# Problem 4

(a)

1 pts

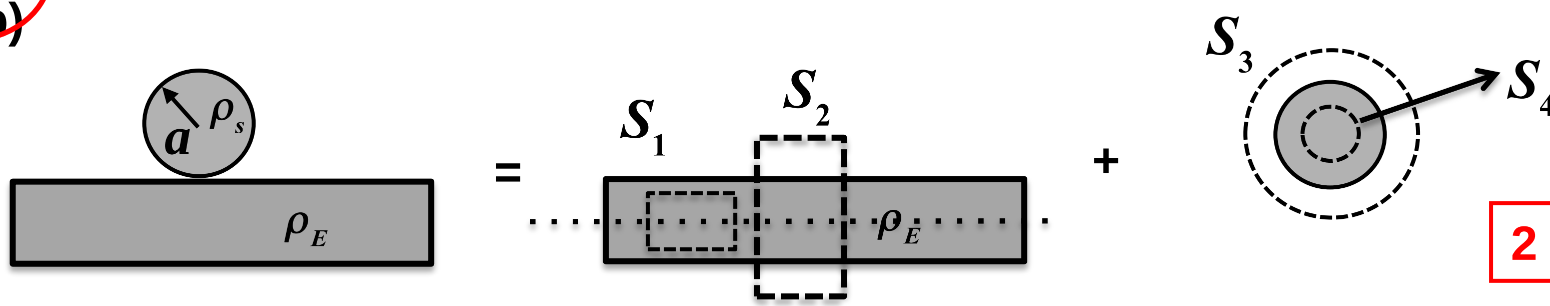
1 pts

$$Q = \int \rho(\vec{r}')d^3r' = \int \rho_0 \frac{r'^{3/2}}{a^{3/2}} 4\pi r'^2\, dr' = \frac{4\pi\rho_0}{a^{3/2}} \int_0^a r'^{7/2}\, dr' = \boxed{\frac{8\pi}{9}\rho_0 a^3}$$

1 pts

12 pts

(b)




2 pts

$(i)$ $z > 4a$, use Gaussian surfaces $S_2$ (for slab) and $S_3$ (for sphere)

$$\oiint_{S_3} \vec{E}\cdot d\vec{A} = \oiint \left(E(r')\hat{r}'\right)\cdot \hat{r}'\, dA' = 4\pi r'^2 E = \frac{Q_{in}}{\varepsilon_0} = \frac{8\pi}{9\varepsilon_0}\rho_0 a^3$$

2 pts

$$\rightarrow \vec{E}_{sphere}(0,0,z) = \frac{Q_{in}}{4\pi\varepsilon_0}\frac{1}{r'^2}\hat{r}' = \frac{2\rho_0 a^3}{9\varepsilon_0}\frac{\vec{r}-\vec{R}_0}{\left|\vec{r}-\vec{R}_0\right|^3} = \boxed{\frac{2\rho_0 a^3}{9\varepsilon_0}\frac{\hat{z}}{(z-3a)^2}}$$

$$\oiint_{S_2} \vec{E}\cdot d\vec{A} = \oiint \left(E(z)\hat{z}\right)\cdot d\vec{A} = 2EA = \frac{\rho_E 2aA}{\varepsilon_0} \rightarrow \vec{E}_{slab}(0,0,z) = \boxed{\frac{\rho_E a}{\varepsilon_0}\hat{z}}$$

1 pts

$$\rightarrow \vec{E}(0,0,z) = \vec{E}_{slab} + \vec{E}_{sphere} = \left[\frac{\rho_E a}{\varepsilon_0} + \frac{2\rho_0 a^3}{9\varepsilon_0}\frac{1}{(z-3a)^2}\right]\hat{z}$$

(ii) $2a < z < 4a$, use Gaussian surfaces $S_2$ (for slab) and $S_4$ (for sphere)

$$\oiint_{S_4} \vec{E}\cdot d\vec{A} = 4\pi r'^2 E = \frac{Q_{in}}{\varepsilon_0} = \frac{\rho_0}{\varepsilon_0}\int_0^{r'} \frac{r^{3/2}}{a^{3/2}} 4\pi r^2\, dr = \frac{8\pi\rho_0}{9\varepsilon_0}\frac{r'^{9/2}}{a^{3/2}}$$

$$\rightarrow \vec{E}_{sphere}(0,0,z) = \frac{2\rho_0}{9\varepsilon_0}\frac{r'^{5/2}}{a^{3/2}}\hat{r}' = \frac{2\rho_0}{9\varepsilon_0}\frac{\left|\vec{r}-\vec{R}_0\right|^{3/2}}{a^{3/2}}\left(\vec{r}-\vec{R}_0\right) = \boxed{\frac{2\rho_0}{9\varepsilon_0}\frac{\left|z-3a\right|^{3/2}}{(a)^{3/2}}(z-3a)\hat{z}}$$

**2 pts**

$$\rightarrow \vec{E}(0,0,z) = \vec{E}_{slab} + \vec{E}_{sphere} \boxed{= \left[\frac{\rho_E a}{\varepsilon_0} + \frac{2\rho_0}{9\varepsilon_0}\frac{\left|z-3a\right|^{3/2}}{(a)^{3/2}}(z-3a)\right]\hat{z}}$$

**1 pts**

(iii) $0 < z < 2a$, use Gaussian surfaces $S_1$ (for slab) and $S_3$ (for sphere)

$$\oiint_{S_1} \vec{E}\cdot d\vec{A} = \oiint \left(E(z)\hat{z}\right)\cdot d\vec{A} = 2EA = \frac{\rho_E 2(z-a)A}{\varepsilon_0} \rightarrow \vec{E}_{slab}(0,0,z) = \frac{\rho_E (z-a)}{\varepsilon_0}\hat{z}$$

**2 pts**

$$\rightarrow \vec{E}(0,0,z) = \vec{E}_{slab} + \vec{E}_{sphere} \boxed{= \left[\frac{\rho_E (z-a)}{\varepsilon_0} - \frac{2\rho_0 a^3}{9\varepsilon_0}\frac{1}{(z-3a)^2}\right]\hat{z}}$$

(iii) $\vec{r} = (2a, 0, 3a)$, use Gaussian surfaces $S_2$ (for slab) and $S_3$ (for sphere)

$$\vec{E}(2a,0,3a) = \vec{E}_{slab} + \vec{E}_{sphere} = \frac{\rho_E a}{\varepsilon_0}\hat{z} + \frac{2\rho_0 a^3}{9\varepsilon_0}\frac{\vec{r}-\vec{R}_0}{\left|\vec{r}-\vec{R}_0\right|^3} = \frac{\rho_E a}{\varepsilon_0}\hat{z} + \frac{2\rho_0 a^3}{9\varepsilon_0}\frac{\hat{x}}{4a^2}$$

**2 pts**