General Physics (I) **期中考 I** **Oct.18, 2019**

試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(50 points) (所有題目必須有計算過程,否則不予計分)

1&2. (18pts) Fig. 1(a) shows a shell is fired with an angle $\theta = 53^0$ above the horizontal surface (x-y plane) toward a boat whose speed is $\boldsymbol{v_B} = 24$ m/s and 48 m east away from the coast. Assume the gravitational acceleration g = 10 m/s^2 in the $-z$ direction. At $\boldsymbol{t} = 0$ sec, a shell with initial velocity $\boldsymbol{v_s}$ is fired, and the boat with $\boldsymbol{v_B}$ starts to sail.

(a) (7 pts) (i) Write down the positions as a function of time ($\boldsymbol{t}$) for the shell and the boat in terms of $\boldsymbol{v_s}$ and $\boldsymbol{t}$. (ii) Find $\boldsymbol{v_s}$ and (iii) how much time it takes to hit the boat.

(b) (11 pts) Now assume the boat is heading the $+\boldsymbol{y}$ direction with the same speed and the sea water is flowing in the $\boldsymbol{-y}$ direction with a speed $\boldsymbol{v_w} = 18$ m/s. The shell is kept with angle $\boldsymbol{\theta} = 53^0$ above the horizontal surface, but with a horizontal angle α relative to the $+\boldsymbol{x}$-axis, as shown in Fig. 1(b). (i) Write down the positions as a function of time $\boldsymbol{t}$ for the shell and the boat in terms of $\boldsymbol{v_s}$, $\boldsymbol{t}$, and $\boldsymbol{\alpha}$. (ii) Find $\boldsymbol{v_s}$ and $\boldsymbol{\alpha}$ for the shell to hit the boat.

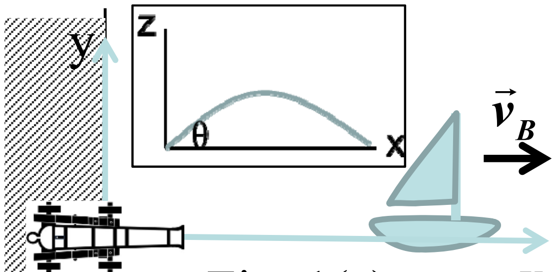


Fig. 1(a)

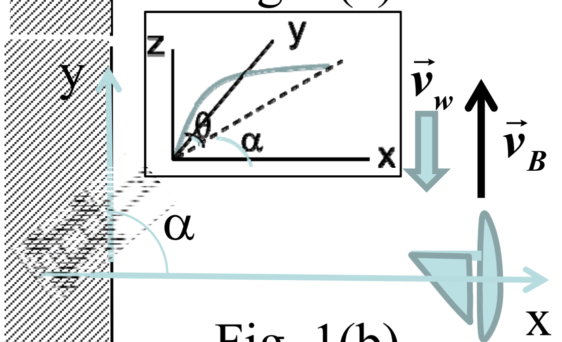


Fig. 1(b)

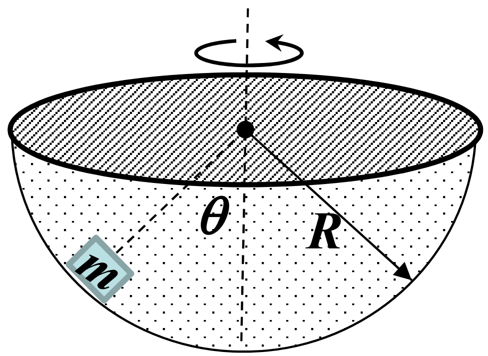


Fig. 2

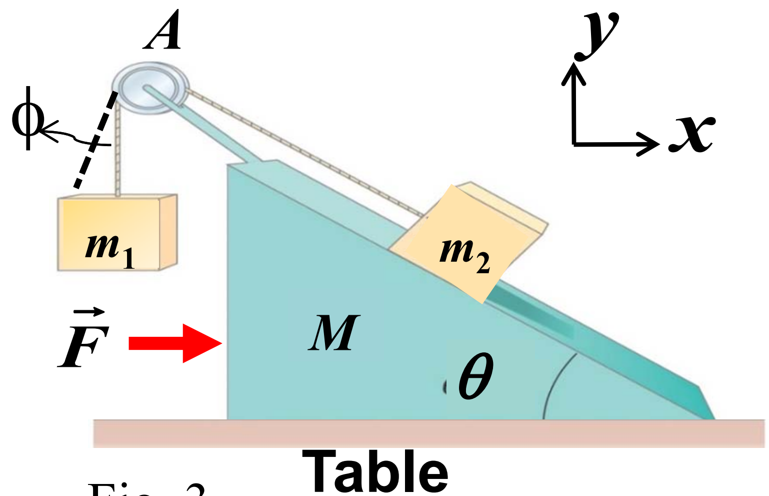


Fig. 3

3. (16 pts) A small block of mass $\boldsymbol{m}$ is at rest inside a rotating half sphere with the angle $\boldsymbol{\theta}$, relative to the axis of rotation (Fig. 2) . The period of the roation of the half sphere is $\boldsymbol{T}$. The static frictional coefficient between the block and the inner surface of the sphere is $\boldsymbol{\mu}$.
(Draw free-body diagram for all the cases below .)
(a) (6 pts) Find the period $\boldsymbol{T_0}$ such that the small block can remain rest at the angle $\boldsymbol{\theta}$ without frictional force.
(b) (10 pts) Now consider the effect of the friction, for a given angle $\boldsymbol{\theta}$, find out the maximum period $\boldsymbol{T_{max}}$ and the minimum period $\boldsymbol{T_{min}}$. that the small block remain rest at the same position in the half sphere.

4. (16 pts) Fig. 3 shows a system which contains two small blocks, masses $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$, and a large block with mass $\boldsymbol{M}$ on a horizontal table. The static and kinetic friction coefficients are $\boldsymbol{\mu_s}$ and μ_k between blocks $\boldsymbol{m_2}$ and $\boldsymbol{M}$ but no friction between $\boldsymbol{M}$ and the table. The system is moving by applying a horizontal constant force $\boldsymbol{F}$ on block $\boldsymbol{M}$ and both of the small blocks do not move relative to the large block $\boldsymbol{M}$. The string, connecting to block $\boldsymbol{m_1}$, is making a vertical angle $\boldsymbol{\phi}$ during this motion as indicated in Fig. 3. Ignore the masses of the pulley $\boldsymbol{A}$ and the string connecting the two small blocks.

a) (6 pts) Draw the free body diagram for all three blocks and the pulley separately, label each force you draw with proper symbol (e.g., $\boldsymbol{Mg}$, $\boldsymbol{F}$, **...** etc.)
b) (5 pts) Write down the equations of motion (in both the $\boldsymbol{x}$ and $\boldsymbol{y}$ directions indicated in Fig 3) for each object due to the free body diagrams you plotted in part a) by the Newton's 2nd law.
c) (5 pts) Let (only in this part), $\theta = \pi/4$, $\boldsymbol{m_1=m}$, $\boldsymbol{m_2=2m}$, and $\boldsymbol{M=5m}$, and ignore the friction between $\boldsymbol{m_2}$ and $\boldsymbol{M}$. If $\boldsymbol{a}$ is the acceleration of the system, find the ratio $\boldsymbol{a/g}$ and $\boldsymbol{F/mg}$, and the angle $\boldsymbol{\phi}$.

II.選擇題(50 points)

1. (5pts) A freshman student tries to extract an energy scale $\boldsymbol{E}$ where the gravitational force, the electromagtic force, and the nuclear force would play equally effective roles in a physical system. He tried the following relation $E \approx aG^x \cdot c^y \cdot h^z$, where $\boldsymbol{a}$ is a dimensionless constant, $G = 6.67 \times 10^{-11} m^3/kg \cdot s^2$, $c = 3 \times 10^8 m/s$, and $h = 6.63 \times 10^{-34} m^2 kg/s$,

If the energy $\boldsymbol{E}$ is measured in unit of *Joule* ($1\,Joule = 1\,m^2kg/s^2$). Let $w = x + y + z$, then which of the following is correct?

(A) $w < -7$ (B) $-7 \le w < -5$ (C) $-5 \le w < -3$ (D) $-3 \le w < -1$
(E) $-1 \le w < 1$ (F) $1 \le w < 3$ (G) $3 \le w < 5$ (H) $5 \le w < 7$ (J) $7 < w$

2. (5pts) A positively charged particle travels in a electric field along a line. Its position can be described with a coordinate $x(t) = 5\sin(\pi(t-1)/2)$ (μm), with $\boldsymbol{t}$ in units of second, for $\boldsymbol{t} = 0$ sec to $\boldsymbol{t} = 2$ sec. Which of the following statement about the motion of the particle is correct?
(A) The total displacement of the particle is 5 μm. (B) The particle is initially at rest.
(C) The DIRECTION of the velocity of the particle is constant.
(D) The DIRECTION of the acceleration of the particle is constant.
(E) The acceleration of the particle is never equal to zero. (F) All of the above are correct.
(G) Four of the above are correct. (H) Three of the above are correct.
(J) Two of the above are correct. (K) None of the above is correct.

3. (5pts) Fig. 4 shows the acceleration vs. time graph for a particle, assume the particle is at rest and at origin at $t = 0$. The velocity and position of the particle at $t = 2$ s are
(A) 9 m/s and 13 m; (B) 9 m/s and 21 m; (C) 9 m/s and 31 m; (D) 15 m/s and 13 m;
(E) 15 m/s and 21 m; (F) 15 m/s and 31 m; (G) 21 m/s and 13 m; (H) 21 m/s and 21 m;
(J) 21 m/s and 31 m, respectively.

4. (5pts) As shown in Fig. 5, ***block 1***, ***2*** and ***3*** are placed on a frictionless surface. They have the same mass $\boldsymbol{M}$. A force $\boldsymbol{F}$ is pushing ***block 1*** toward the right, and the friction between the blocks keep all three blocks to move together and remain static relative to each other. The frictional force between ***block 1*** and ***3*** is $\boldsymbol{F_{13}}$, and the static friction coefficient is $\mu_1 = 0.1$. The frictional force between ***block 2*** and ***3*** is $\boldsymbol{F_{23}}$ and the static friction coefficient is $\mu_2 = 0.4$. The ratio $|\boldsymbol{F_{13}}|/|\boldsymbol{F_{23}}| =$

(A) 1/4 (B) 1/3 (C) 1/2 (D) 1 (E) 2 (F) 3 (G) 4

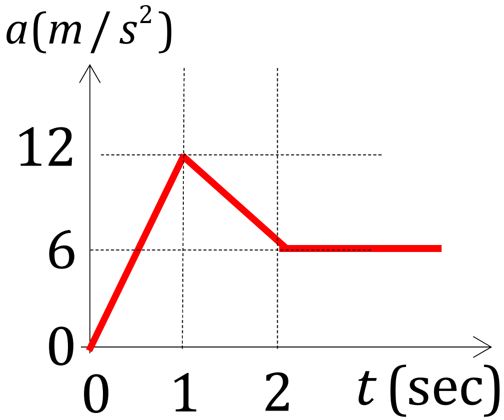


Fig. 4

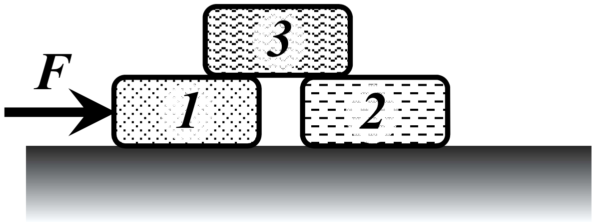


Fig. 5

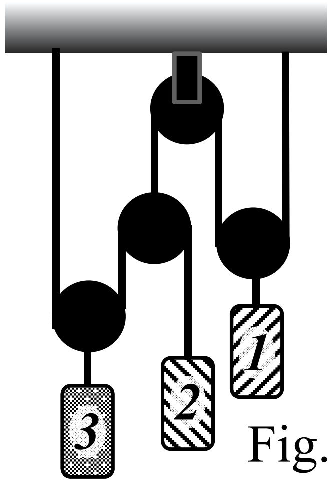


Fig. 6

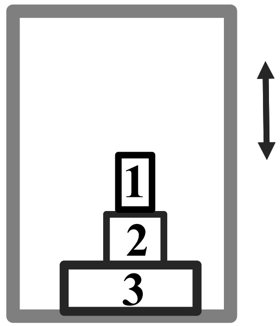


Fig. 7

5. (5pts) As shown in Fig. 6, block ***1***, ***2*** and ***3*** are hanging from the ceiling with ropes and pulleys and accelerate due to gravitation. Let $\boldsymbol{a}_1$, $\boldsymbol{a}_2$ and $\boldsymbol{a}_3$ are the acceleration of block ***1***, ***2*** and ***3***, respectively. Assuming the ropes and the pulleys are massless, there exists a relation between $\boldsymbol{a}_1$, $\boldsymbol{a}_2$ and $\boldsymbol{a}_3$, which can be expressed as $\boldsymbol{\alpha} \cdot \boldsymbol{a}_1 + \boldsymbol{a}_2 + \boldsymbol{\beta} \cdot \boldsymbol{a}_3 = \mathbf{0}$. Let $\boldsymbol{Z} = |\boldsymbol{\alpha}| + |\boldsymbol{\beta}|$, which of the following is correct?

(A) $\mathbf{Z} < 2$ (B) $2 \le \mathbf{Z} < 4$ (C) $4 \le \mathbf{Z} < 7$ (D) $7 \le \mathbf{Z} < 9$ (E) $9 \le \mathbf{Z} < 15$ (F) $15 \le \mathbf{Z}$

6. (5 pts) In Fig. 7, there are three blocks placed in the bottom of an elevator. Let $\boldsymbol{N}_1$ be the normal force between block 1 and 2 (similarly, $\boldsymbol{N}_2$ between block 2 and 3 and $\boldsymbol{N}_3$ between block 3 and the floor).Under the condition of $\boldsymbol{N}_1 \neq 0$, $\boldsymbol{N}_2 \neq 0$,and $\boldsymbol{N}_3 \neq 0$, consider the following three cases: (1) when the elevator is moving at constant speed, $\boldsymbol{a} = \boldsymbol{N}_3 / \boldsymbol{N}_1$, (2) when the elevator is accelerating upward, $\boldsymbol{b} = \boldsymbol{N}_3 / \boldsymbol{N}_1$, and (3) when the elevator is accelerating downward, $\boldsymbol{c} = \boldsymbol{N}_3 / \boldsymbol{N}_1$. Then which of the following is true?

(A) *b>a* and *c>a* (B) *b>a* and *c=a* (C) *b>a* and *c<a* (D) *b=a* and *c>a* (E) *b=a* and *c=a*
(F) *b=a* and *c<a* (G) *b<a* and *c>a* (H) *b<a* and *c=a* (J) *b<a* and *c<a* (K) none of above

Multiple Choice Questions:

1	**2**	**3**	**4**	**5**	**6**				
F	**J**	**D**	**E**	**C**	**E**				
7	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	**B**	**C**	**D**	**G**	**C**	**C**	**F**	**B**	**A**

Solutions:

1&2. (18pts) Fig. 1(a) shows a shell is fired with an angle $\theta = 53^0$ above the horizontal surface (x-y plane) toward a boat whose speed is $\boldsymbol{v_B}$ =24 m/s and 48 m east away from the coast. Assume the gravitational acceleration g = 10 m/s^2 in the $-z$ direction. At $\boldsymbol{t}$ = 0 sec, a shell with initial velocity $\boldsymbol{v_s}$ is fired, and the boat with $\boldsymbol{v_B}$ starts to sail.

(a) (7 pts) (i) Write down the positions as a function of time ($\boldsymbol{t}$) for the shell and the boat in terms of $\boldsymbol{v_s}$ and $\boldsymbol{t}$. (ii) Find $\boldsymbol{v_s}$ and (iii) how much time it takes to hit the boat.

(b) (11 pts) Now assume the boat is heading the +$\boldsymbol{y}$ direction with the same speed and the sea water is flowing in the -$\boldsymbol{y}$ direction with a speed $\boldsymbol{v_w}$= 18 m/s. The shell is kept with angle $\boldsymbol{\theta} = 53^0$ above the horizontal surface, but with a horizontal angle α relative to the +$\boldsymbol{x}$-axis, as shown in Fig. 1(b). (i) Write down the positions as a function of time $\boldsymbol{t}$ for the shell and the boat in terms of $\boldsymbol{v_s}$, $\boldsymbol{t}$, and $\boldsymbol{\alpha}$. (ii) Find $\boldsymbol{v_s}$ and $\boldsymbol{\alpha}$ for the shell to hit the boat.

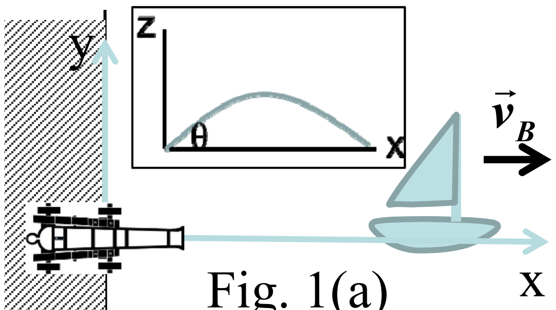


Fig. 1(a)

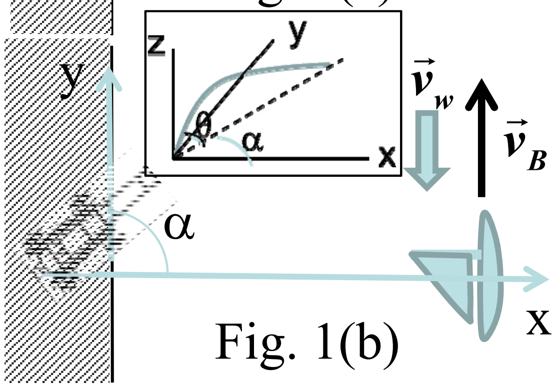


Fig. 1(b)

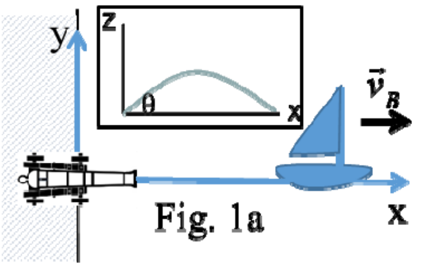


Fig. 1a

The position of shell:

$$x_s(t)=\frac{3}{5}v_s\cdot t$$

$$y_s(t)=0$$

$$z_s(t)=\frac{4}{5}v_st-5t^2$$

The position of the boat:

$$x_b(t)=24t+48$$

$$y_s(t)=0$$

$$z_b(t)=0$$

2pts

When the cannon hit the boat at time t_1:

$$z_s(t_1)=z_b(t_1)\Rightarrow\frac{4}{5}v_st_1-5t_1^{\ 2}=0\quad\Rightarrow t_1=\frac{4}{25}v_s$$ **1pt**

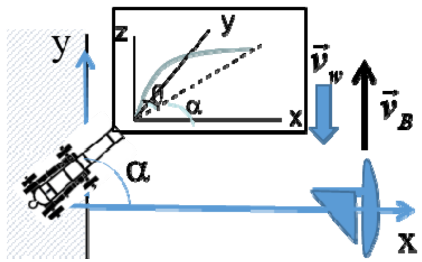
$$x_s(t_1)=x_b(t_1)\Rightarrow\frac{3}{5}v_st_1=24t_1+48$$ **1pt**

$$v_s^{\ 2}-40v_s-500=0$$

$$(v_s-50)(v_s+10)=0$$ **2pts**

$$v_s=50\ m/s\qquad(v_s=-10\ m/s\text{不合理})$$

$$t_1=\frac{4v_s}{25}=8\,\text{s}.$$ **1pt**



The position of shell:

$$x_s(t)=\frac{3\cos\alpha}{5}v_st$$

$$y_s(t)=\frac{3\sin\alpha}{5}v_st$$

$$z_s(t)=\frac{4}{5}v_st-5t^2$$

The position of the boat:

$$x_b(t)=48$$

$$y_b(t)=(v_b+v_S)t=(24-18)t=6t$$

$$z_b(t)=0$$

3pts

When the cannon hit the boat at time t_2:

$$z_s(t_2)=z_b(t_2)\Rightarrow\frac{4}{5}v_st_2-5t_2^{\ 2}=0\Rightarrow t_2=\frac{4}{25}v_s$$ **1pt**

$$y_s(t_2)=y_b(t_2)\Rightarrow\frac{3\sin\alpha}{5}v_st_2=6t_2\ \Rightarrow v_s\sin\alpha=10\quad\text{--(1)}$$ **1pt**

$$x_s(t_2)=x_b(t_2)\Rightarrow v_s^{\ 2}\cos\alpha=500\qquad\text{---(2)}$$ **1pt**

$((1)v_s)^2+(2)^2$: $v_s^{\ 4}-(10v_s)^2-500^2=0$

$$v_s^{\ 2}=\frac{100\pm\sqrt{10000+1000000}}{2}=50\pm50\sqrt{101}$$ **1pt**

$$\therefore v_s^{\ 2}=50+50\sqrt{101}$$

$$v_s=\sqrt{50+50\sqrt{101}}\ m/s\approx 23.5\ m/s$$ **2pts**

$$v_s\sin\alpha=10$$

$$\Rightarrow\alpha=\sin^{-1}\frac{10}{v_s}$$

$$=\sin^{-1}\frac{10}{\sqrt{50+50\sqrt{101}}}$$ **2pts**

$$\sim\sin^{-1}\frac{10}{23.5}$$

$$\sim\sin^{-1}0.43$$

$$\sim 25.5^o$$

3. (16 pts) A small block of mass $\boldsymbol{m}$ is at rest inside a rotating half sphere with the angle $\boldsymbol{\theta}$, relative to the axis of rotation (Fig. 2) . The period of the roation of the half sphere is $\boldsymbol{T}$. The static frictional coefficient between the block and the inner surface of the sphere is $\boldsymbol{\mu}$.
(Draw free-body diagram for all the cases below .)
(a) (6 pts) Find the period $\boldsymbol{T_0}$ such that the small block can remain rest at the angle $\boldsymbol{\theta}$ without frictional force.
(b) (10 pts) Now consider the effect of the friction, for a given angle $\boldsymbol{\theta}$, find out the maximum period $\boldsymbol{T_{max}}$ and the minimum period $\boldsymbol{T_{min}}$. that the small block remain rest at the same position in the half sphere.

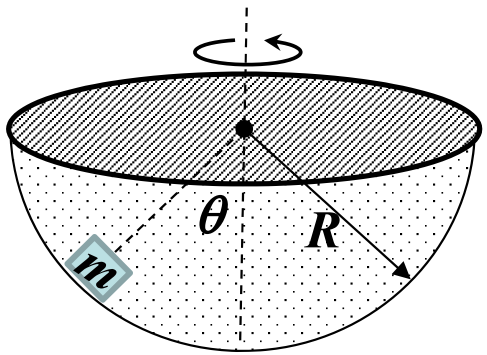
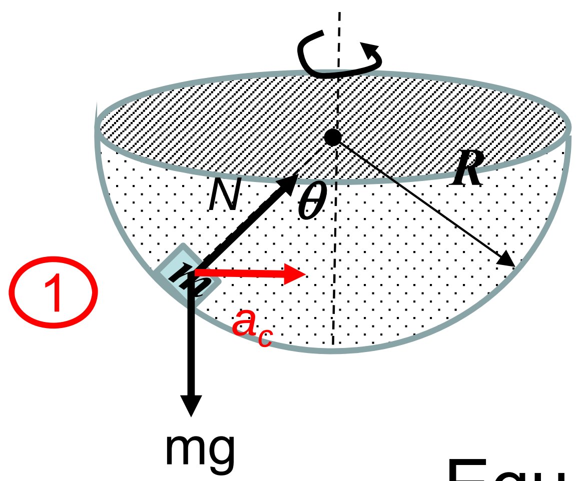


Fig. 2

(a)



$$x: N\sin\theta = ma_c \quad (1)$$
$$y: N\cos\theta - mg = 0 \quad (2)$$

1

Equ.(2): $N = \dfrac{mg}{\cos\theta} = mg$

$$v = \frac{2\pi r}{T} = \frac{2\pi R\sin\theta}{T}$$

1

Equ.(1): $mg\dfrac{\sin\theta}{\cos\theta} = m\dfrac{v^2}{r} = m\dfrac{\frac{4\pi^2R^2\sin^2\theta}{T^2}}{R\sin\theta} = m\dfrac{4\pi^2R\sin\theta}{T^2}$

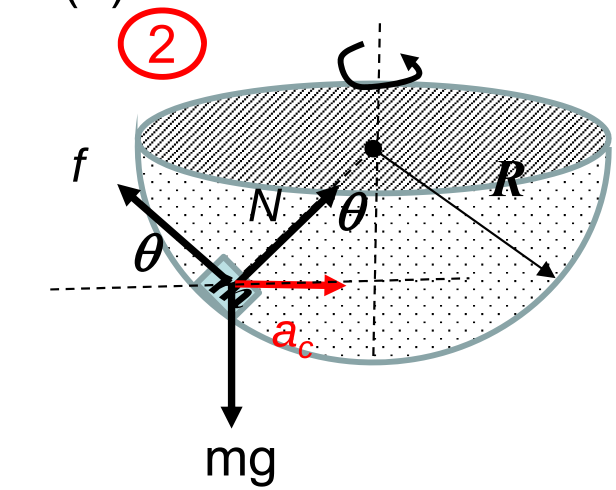
$$\Rightarrow \quad T^2 = \frac{4\pi^2R\cos\theta}{g}$$

2

or

$$T = 2\pi\sqrt{\frac{R\cos\theta}{g}}$$

(b)



for lowest speed case:

$$x: N\sin\theta - f\cos\theta = ma_c \quad (3)$$
$$y: N\cos\theta + f\sin\theta - mg = 0 \quad (4)$$

1

$$Set \quad f = f_{min} = \mu N \quad (5)$$

1

$$equ.(4)\ : N(cos\theta + \mu sin\theta) = mg$$

$$N = \frac{mg}{cos\theta + \mu sin\theta} \qquad (4')$$

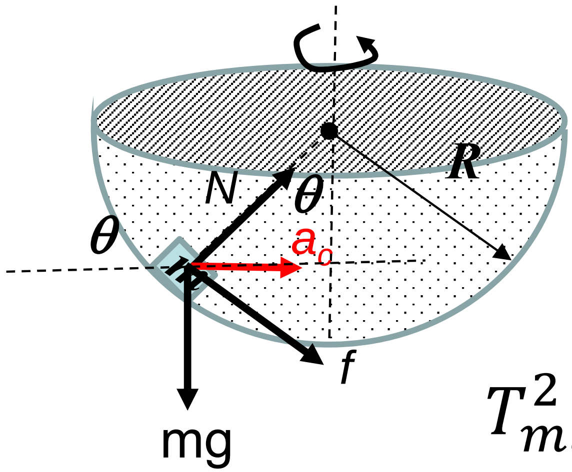
將equ.. $(4')$結果代入$equ.(3)$:

$$N(sin\theta - \mu cos\theta) = ma_c = m\frac{v^2}{r}$$

$$g\frac{sin\theta - \mu cos\theta}{cos\theta + \mu sin\theta} = \frac{4\pi^2 R sin\theta}{T_{max}^2} \quad (4')$$

$$T_{max}^2 = \frac{4\pi^2 R sin\theta}{g}\frac{cos\theta+\mu sin\theta}{sin\theta-\mu cos\theta} = \frac{4\pi^2 R}{g}\frac{cos\theta+\mu sin\theta}{1-\mu cot\theta} \qquad 3$$

Note: when μ=0 , the result T_{max} is exactly the one in part (a).



For highest speed case: Change the free-body diagram by

1 $$f \rightarrow -f \quad i.e. \quad \mu N \rightarrow -\mu N$$

$$T_{min}^2 = \frac{4\pi^2 R sin\theta}{g} * \frac{cos\theta+\mu sin\theta}{sin\theta-\mu cos\theta} = \frac{4\pi^2 R}{g}\frac{cos\theta-\mu sin\theta}{1+\mu cot\theta} \qquad 2$$

4. (16 pts) Fig. 3 shows a system which contains two small blocks, masses m_1 and m_2, and a large block with mass M on a horizontal table. The static and kinetic friction coefficients are μ_s and μ_k between blocks m_2 and M but no friction between M and the table. The system is moving by applying a horizontal constant force F on block M and both of the small blocks do not move relative to the large block M. The string, connecting to block m_1, is making a vertical angle ϕ during this motion as indicated in Fig. 3. Ignore the masses of the pulley A and the string connecting the two small blocks.

a) (6 pts) Draw the free body diagram for all three blocks and the pulley separately, label each force you draw with proper symbol (e.g., Mg, F, ... etc.)

b) (5 pts) Write down the equations of motion (in both the x and y directions indicated in Fig 3) for each object due to the free body diagrams you plotted in part a) by the Newton's 2nd law.

c) (5 pts) Let (only in this part), $\theta = \pi/4$, $m_1 = m$, $m_2 = 2m$, and $M = 5m$, and ignore the friction between m_2 and M. If a is the acceleration of the system, find the ratio a/g and F/mg, and the angle ϕ.

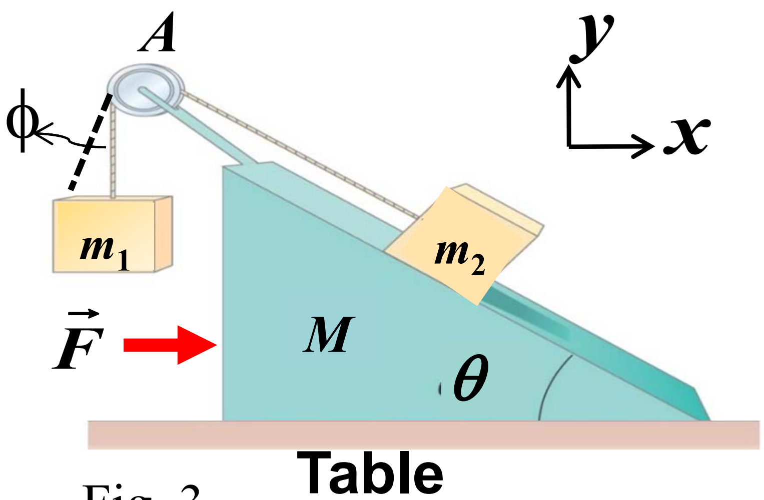
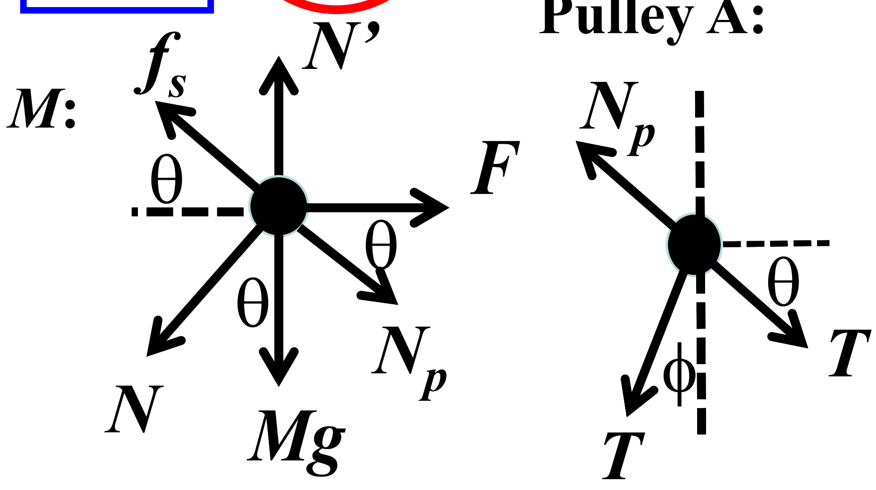
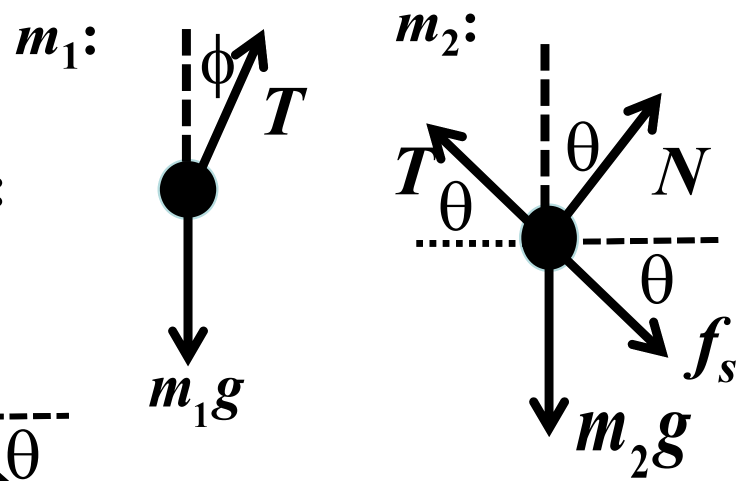


Fig. 3

Problem 4 16 pts

(a) **6 pts**





Grading policy: You need to plot all 4 free body diagrams with total 15 forces. Any one is missing deducts 1 point, up to 6 points total in this part.

(b) **5 pts**

M: $\boxed{F - f_s\cos\theta - N\sin\theta + N_p\cos\theta = Ma}$

$$N' + f_s\sin\theta - N\cos\theta - N_p\sin\theta - Mg = 0$$

m_1:

$$\boxed{\begin{aligned} &T\sin\varphi = m_1 a \\ &T\cos\varphi - m_1 g = 0 \end{aligned}}$$

Pulley A: $\boxed{T\cos\theta - N_p\cos\theta - T\sin\varphi = 0}$

$$N_p\sin\theta - T\sin\theta - T\cos\varphi = 0$$

m_2:

$$\boxed{\begin{aligned} &f_s\cos\theta - T\cos\theta + N\sin\theta = m_2 a \\ &T\sin\theta - f_s\sin\theta + N\cos\theta - m_2 g = 0 \end{aligned}}$$

Grading policy: All six equations (highlight as blue) are required. Any one is missing deducts 1 point, up to 5 points total in this part.

Problem 4 **5 pts**

(c) $M = 5m$, $m_1 = m$, $m_2 = 2m$, $f_s = 0$, **and** $\theta = 45^0$ (or $\pi/4$)

$$F - N\sin\theta + N_p\cos\theta = 5ma\,, \quad (1)$$

$$T\sin\varphi = ma\,, (2)$$

$$T\cos\varphi = mg, (3)$$

$$-T\cos\theta + N\sin\theta = 2ma\,, (4)$$

$$T\sin\theta + N\cos\theta = 2mg\,, (5)$$

$$T\cos\theta - N_p\cos\theta - T\sin\varphi = 0, (6)$$

$$N_p\sin\theta - T\sin\theta - T\cos\varphi = 0, (7)$$

2 pts

Grading policy: 1 point for wrote down any one of equations (1)-(6), (8) and (9). But you need wrote down all (1) to (6) to get 2nd point in this part.

(1)+(2)+(4)+(6) → $F = 8ma, (8)$

$\frac{(2)}{(3)} \to \tan\varphi = \frac{a}{g},\ (9)$

-(4) x cosθ + (5) x sinθ → $T = 2m(g\sin\theta - a\cos\theta) = \sqrt{2}m(g-a), (8)$

$(2)^2 + (3)^2$ → $T^2 = m^2(g^2 + a^2) = 2m^2(g-a)^2$

$\to \left(\frac{a}{g}\right)^2 - 4\left(\frac{a}{g}\right) + 1 = 0 \to \frac{a}{g} = 2 - \sqrt{3}$, the other solution $2 + \sqrt{3}$ is not valid

$\to \frac{F}{mg} = 8(2-\sqrt{3}) \sim 2.1,$ **1 pts**

$\left(\frac{T}{mg}\right)^2 = 8 - 4\sqrt{3},$ or $\frac{T}{mg} \sim 1.04$ **1 pts**

$\tan\varphi = 2 - \sqrt{3}$

or $\varphi = 15^0$ **1 pts**

If you got previous 2 points, each answer corrected get 1 more point. (keep square root in your answers is OK)