General Physics (II) 期中考 I Apr. 8, 2016

試卷請註明、姓名、班級、學號，請遵守考場秩序

I.計算題(50 points) (所有題目必須有計算過程,否則不予計分)

1. (A)(5 pts) There are two concentric (同心圓) spherical conducting shells, of radius ***a*** and ***8a*** as shown in Fig. 1(a). Determine the capacitance of the shells.
(B)(5 pts) Now the space between radius ***2a*** and ***4a*** are filled with dielectrics with dielectric constant $\boldsymbol{\kappa = 3}$ as indicated by the dashed lines (虛線) in Fig. 1(b). What is the capacitance now?

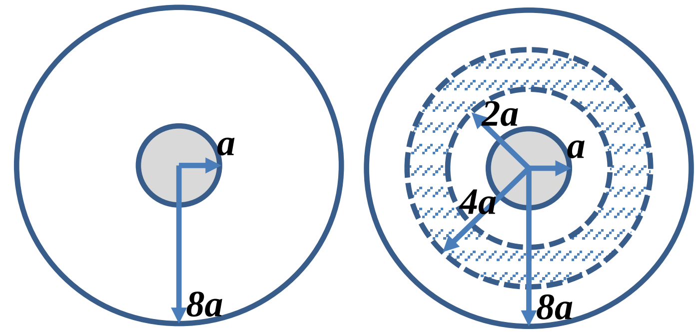


Fig. 1(a) Fig. 1(b)

2. There is charged sphere with charge density $\rho(r) = Ar^{5/2}$, and radius ***R***, as shown in Fig. 2.
(A)(2 pts) What is total charge of the sphere?
(B)(8 pts) Find the electric field, magnitude and direction, for $\boldsymbol{r > R}$ and $\boldsymbol{r < R}$.

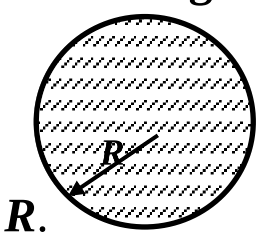


Fig. 2

3. (15 pts) Fig. *3* shows 3 line charge distributions in *x*-*y* plane. The charge densities are λ_1 (>0) for the charges on ***OB*** and ***OC*** and $\boldsymbol{\lambda_2 = \lambda_0 \cos\theta}$ ($\lambda_0 > 0$ and θ is the angle relative to +*x*-axis) for charges on the arc ***BC***. Find the *x*-, *y*-, *z*-components of the ***E***-field at point ***P*** on the *z*-axis due to (a) (6 pts) line charges ***OB***, (b) (3pts) line charges ***OC***, and (c) (6 pts) line charges ***BC***. The coordinates of ***B***,***C***, and ***P*** are (*R*,*0*,*0*), (*0*,*R*,*0*), and (*0*,*0*,*z*), respectively.

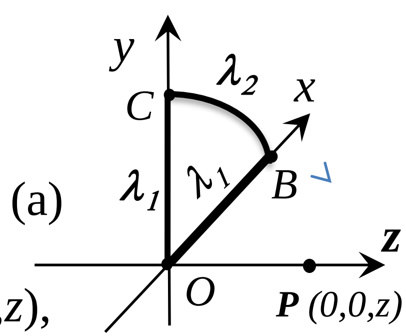


Fig. 3

4. (a) (8 pts) As shown in Fig. 4(a), there is a uniform charge distribution with density ρ (>0) in the region 0≤x ≤2R, and infinite in the y- and z- directions , Determine the magnitude and the direction of the electric field on the x-axis, in the range 0≤x ≤4R.

(b) (7 pts) As shown in Fig. 4(b), an infinitely long cylindrical (圓柱) section with the z-axis as its central axis and with radius R in the charge distribution was shifted to the position right next to the boundary of the original charge distribution. Determine the magnitude and the direction of the electric field on the x-axis in the range $2R \le x \le 4R$.

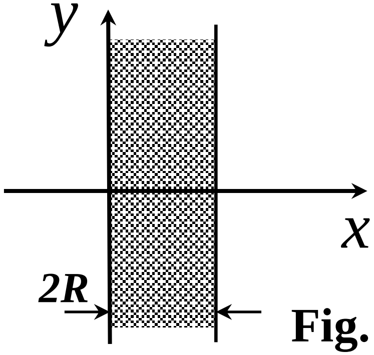


Fig. 4(a)

Useful formula:

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left(x + \sqrt{x^2 \pm a^2}\right) \quad ; \quad \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) = -\cos^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{dx}{\left(x^2 \pm a^2\right)^{\frac{3}{2}}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}} \quad ; \quad \int \frac{x\,dx}{\left(x^2 \pm a^2\right)^{\frac{3}{2}}} = \frac{-1}{\sqrt{x^2 \pm a^2}}$$

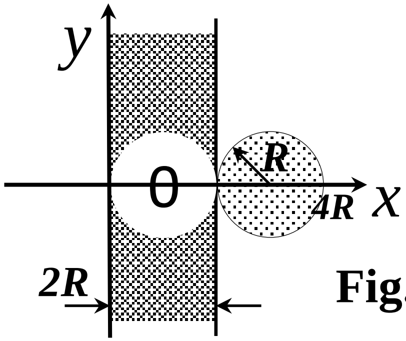


Fig. 4(b)

II.選擇題(52 points)

1. (4 pts) A parallel-plate capacitor, with C_0 = 3.0 μF, now is filled with two different dielectrics ,K_1 = 4 and K_2=3, each with half the space between the plates, as shown in Fig. 5. What is the new capacitance C?

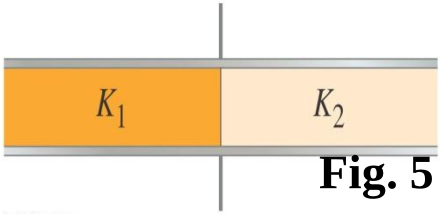


Fig. 5

(A) C ≤ 1.0 μF (B) 1.0 μF < C ≤ 5.0 μF (C) 5.0 μF < C ≤ 10.0 μF
(D) 15.0 μF < C ≤ 20.0 μF (E) 20.0 μF < C ≤ 25.0 μF (F) 25.0 μF < C (G) 10.0 μF < C ≤ 15.0 μF

2. (4 pts) A hollow metal sphere (內部中空金屬球) is placed between two parallel uniformly charged plates. There are induced charges densities on the outer and inner surfaces of the sphere in Fig. 6. Which of the following is correct?

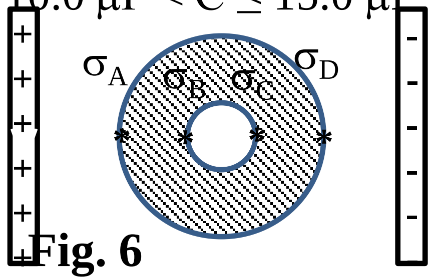


Fig. 6

(A) $\sigma_A = -\sigma_D > 0$, $\sigma_B = -\sigma_C > 0$ (B) $\sigma_A = -\sigma_D > 0$, $\sigma_B = -\sigma_C = 0$ (C) $\sigma_A = -\sigma_D > 0$, $\sigma_B = -\sigma_C < 0$
(D) $\sigma_A = -\sigma_D = 0$, $\sigma_B = -\sigma_C > 0$ (E) $\sigma_A = -\sigma_D = 0$, $\sigma_B = -\sigma_C = 0$ (F) $\sigma_A = -\sigma_D = 0$, $\sigma_B = -\sigma_C < 0$
(G) $\sigma_A = -\sigma_D < 0$, $\sigma_B = -\sigma_C > 0$ (H) $\sigma_A = -\sigma_D < 0$, $\sigma_B = -\sigma_C = 0$ (I) $\sigma_A = -\sigma_D < 0$, $\sigma_B = -\sigma_C < 0$

3. (4 pts) Fig. 7(a) shows a uninform cubic shell charge distribution with density σ_q (>0) and nearly zero thickness. The cross-sectional view shows the spherical surface S inside the shell, point a on surface S. Φ_S is the electric flux through S, and E_a the magnitude of the electric field at point a. Fig. 7(b) shows a uniform spherical shell charge distribution with density σ_s (>0) and nearly zero thickness. The cross-sectional view shows the cubic surface C inside the shell, and the point b on the surface C. Φ_C is the electric flux through C, and E_b the magnitude of the electric field at point b. Which of the following statements is correct? (In both figures, the centers of the surfaces coincide with origin of the coordinate system.)

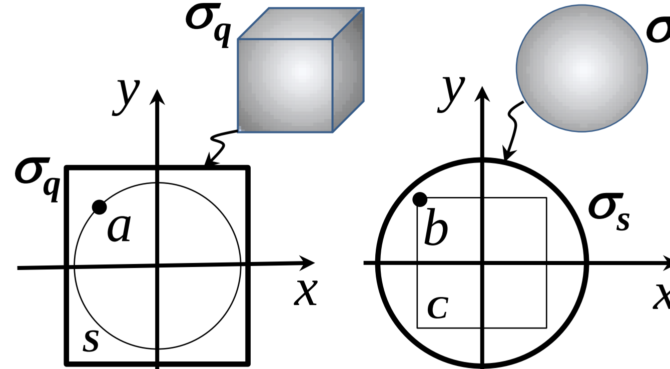


Fig. 7(a) **Fig. 7(b)**

(A) $\Phi_S = 0, \Phi_C = 0, E_a = 0, E_b = 0$ (B) $\Phi_S > 0, \Phi_C = 0, E_a = 0, E_b = 0$ (C) $\Phi_S = 0, \Phi_C > 0, E_a = 0, E_b = 0$

(D) $\Phi_S = 0, \Phi_C = 0, E_a \neq 0, E_b = 0$ (E) $\Phi_S > 0, \Phi_C = 0, E_a \neq 0, E_b = 0$ (F) $\Phi_S = 0, \Phi_C > 0, E_a \neq 0, E_b = 0$

(G) $\Phi_S = 0, \Phi_C = 0, E_a = 0, E_b \neq 0$ (H) $\Phi_S > 0, \Phi_C = 0, E_a = 0, E_b \neq 0$ (I) $\Phi_S = 0, \Phi_C > 0, E_a = 0, E_b \neq 0$

4. (4 pts) As shown in Fig. 8, a point charge Q(>0) is placed at the center of two infinitely thin concentric conducting shells with radii of a and b, and the origin of the coordinate system coincides with center of the spherical shells. Which of the following shows the correct E-field distribution as a function of r?

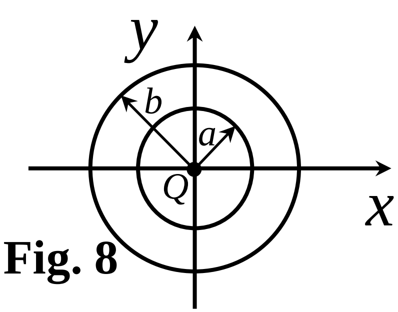
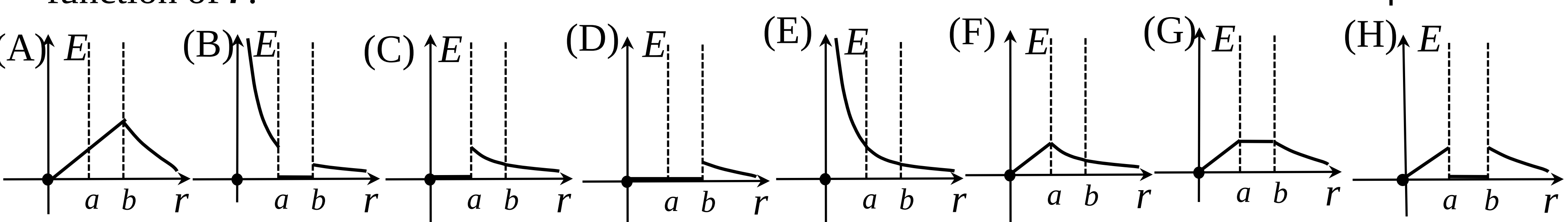
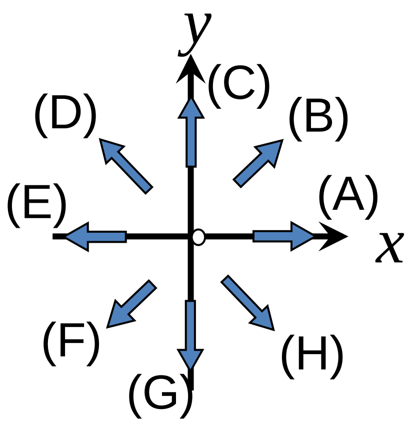


Fig. 8



5. (4 pts) An electron is shot from point $\boldsymbol{P}$ with velocity $\vec{v} = \left(-1.5\times10^{6}\,\mathrm{m/s}\right)\hat{y}$ through a region of constant electric field. The trajectory is show in Fig 9. After **1.0μs**, the electron leaves the region II at point $\boldsymbol{Q}$ with velocity $\vec{v} = \left(3.0\hat{x} + 2.5\hat{y}\right)\times10^{6}\,\mathrm{m/s}$. Which vector most closely shows the direction of the electric field?



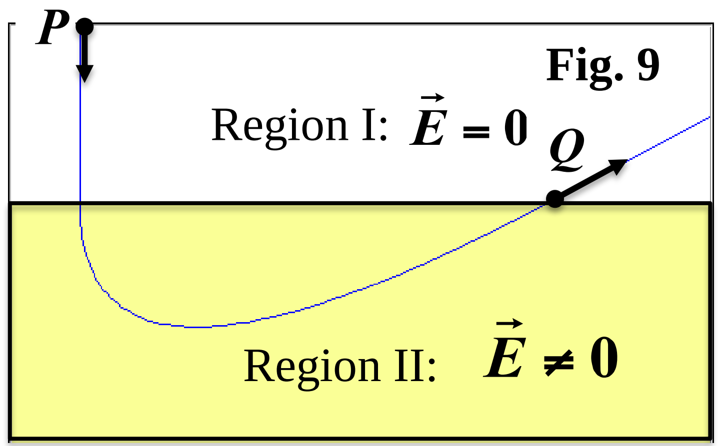


Fig. 9

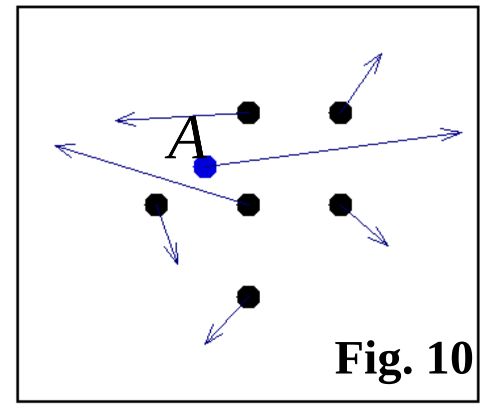


Fig. 10

6. (4 pts) Same as problem 1, the magnitude of the electric field in region II is $\boldsymbol{b}$ **N/C**. ($\boldsymbol{m_e \sim 10^{-30}}$ **kg**, $|e| \sim$ **1.6x10^{-19}C.**)

(A) $b \le 5$ (B) $5 < b \le 10$ (C) $10 < b \le 15$ (D) $15 < b \le 20$ (E) $20 < b \le 25$
(F) $25 < \boldsymbol{b} \le 30$ (G) $30 < b \le 35$ (H) $35 < \boldsymbol{b} \le 40$ (I) $40 < \boldsymbol{b} \le 45$ (J) $45 < b \le 50$
(K) $50 < b$

7. (4 pts) In figure 10, the electrostatic force on each charge is indicated by an arrow (the length of each vector represents the strength of the Coulomb's force on each charge). The magnitude of each charge is **1C**, and the charge $\boldsymbol{A}$ is negative. What is the net charge shown in this figure?

(A) 0 ; (B) 1 ; (C) 2; (D) 3; (E) 4; (F) 5; (G) 6; (H) 7; (I) none of above

Multiple Choice Questions:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
G	**H**	**D**	**E**	**F**	**G**	**D**	**L**	**C**	**B**
11	**12**	**13**	**14**	**15**					
D	**H**	**F**	**F**	**C**					

1. (A)(5 pts) There are two concentric (同心圓) spherical conducting shells, of radius ***a*** and ***8a*** as shown in Fig. 1(a). Determine the capacitance of the shells.
(B)(5 pts) Now the space between radius ***2a*** and ***4a*** are filled with dielectrics with dielectric constant ***κ = 3*** as indicated by the dashed lines (虛線) in Fig. 1(b). What is the capacitance now?

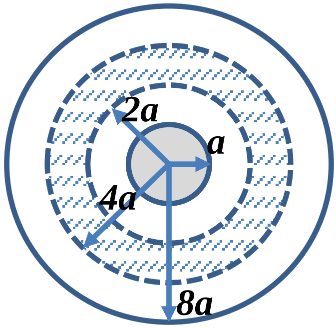
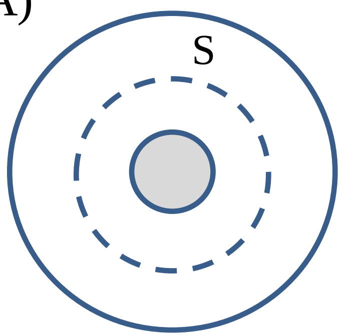


Fig. 1(a) **Fig. 1(b)**

Sol: (A)



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

Choose Gauss's surface S.

$$\mathrm{E}\cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\vec{\mathrm{E}} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$ 2

1

$$|\Delta V| = \int \vec{\mathrm{E}}\cdot d\hat{\ell} = \int_{8a}^{a} \frac{Q}{4\pi\varepsilon_0 r^2}\cdot dr$$

$$= -\frac{Q}{4\pi\varepsilon_0 r}\bigg|_{8a}^{a} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{8a}\right)$$ 1

$$= \frac{Q}{4\pi\varepsilon_0}\frac{7}{8a}$$

$$C = \frac{Q}{|\Delta V|} = 4\pi\varepsilon_0\frac{8a}{7}$$ 1

(B)

$$|\Delta V| = \int_{8a}^{a} \vec{\mathrm{E}}\cdot d\hat{\ell}$$

Since: $\vec{E}' = \frac{\vec{E}}{\kappa}$ in the dielectric region

$$= \int_{8a}^{4a}\frac{Q}{4\pi\varepsilon_0 r^2}\cdot dr + \frac{1}{\kappa}\int_{4a}^{2a}\frac{Q}{4\pi\varepsilon_0 r^2}\cdot dr + \int_{2a}^{a}\frac{Q}{4\pi\varepsilon_0 r^2}\cdot dr$$ 2 1

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{8a}+\frac{1}{3}\frac{1}{4a}+\frac{1}{2a}\right) = \frac{Q}{4\pi\varepsilon_0}\frac{17}{24a}$$ 1

$$C = \frac{Q}{|\Delta V|} = 4\pi\varepsilon_0\frac{24a}{17}$$ 1

另法： Because the capacitors are in series,

$$\frac{1}{C} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$ 1

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{8a}+\frac{1}{3}\frac{1}{4a}+\frac{1}{2a}\right) = \frac{Q}{4\pi\varepsilon_0}\frac{17}{24a}$$ 2 1

$$C = \frac{Q}{|\Delta V|} = 4\pi\varepsilon_0\frac{24a}{17}$$ 1

2. There is charged sphere with charge density $\rho(r) = Ar^{5/2}$, and radius $\boldsymbol{R}$, as shown in Fig. 2.

(A)(2 pts) What is total charge of the sphere?

(B)(8 pts) Find the electric field, magnitude and direction, for $\boldsymbol{r} > \boldsymbol{R}$ and $\boldsymbol{r} < \boldsymbol{R}$.

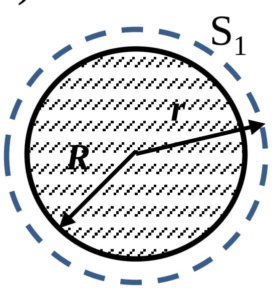
Sol: (A)

$$Q = \int_0^R \rho \cdot 4\pi r^2 dr$$ 1

$$= \int_0^R Ar^{5/2} \cdot 4\pi r^2 dr = 4\pi A \int_0^R r^{9/2} dr$$

$$= 4\pi A \frac{2R^{11/2}}{11} = \frac{8\pi A R^{11/2}}{11}$$ 1

(B) $\boldsymbol{r} > \boldsymbol{R}$:



1

Choose Gauss's surface S_1.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

Due to spherical symmetry,

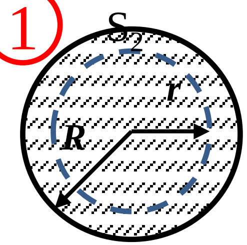
$$\vec{E} = E(r)\hat{r}$$

1 $$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$ 1

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$ 1

$\boldsymbol{r} < \boldsymbol{R}$:

1



Choose Gauss's surface S_2.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

Due to spherical symmetry,

$$\vec{E} = E(r)\hat{r}$$

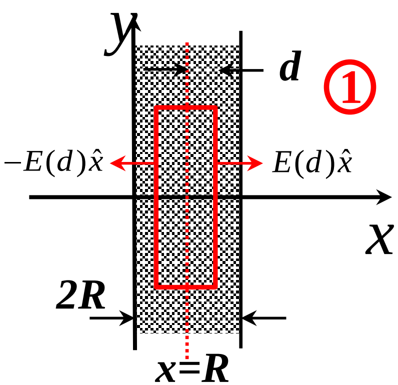
$$E \cdot 4\pi r^2 = \frac{1}{\varepsilon_0} \int_0^r \rho \cdot 4\pi r^2 dr$$ 1

$$= \frac{8\pi A r^{11/2}}{11\varepsilon_0} = \frac{Q}{\varepsilon_0} \frac{r^{11/2}}{R^{11/2}}$$ 1

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0} \frac{r^{7/2}}{R^{11/2}} \hat{r} = \frac{2Ar^{7/2}}{11\varepsilon_0} \hat{r}$$ 1

2. (a) (8 pts) As shown in Fig. x(a), there is an uniform charge distribution with density r (>0) in the region $0 \le x \le 2R$, and infinite in the y- and z- directions , Determine the magnitude and the direction of the electric field on the x-axis, in the range $0 \le x \le 4R$. (b) (7 pts) As shown in Fig. x(b), a infinitely long cylindrical section with the z-axis as its central axis and with radius R in the charge distribution was shifted to the position right next to the boundary of the original charge distribution. Determine the magnitude and the direction of the electric field on the x-axis in the range $2R \le x \le 4R$.

(a) For $0 \le x \le 2R$



$$\left.\begin{aligned}\Phi_E &= \oiint \vec{E}(\vec{r})\cdot d\vec{A} = 2E(d)A \\ Q_{in} &= \rho\cdot 2dA\end{aligned}\right\} ①$$

According to Gauss's law,

$$2E(d)A = \frac{\rho\cdot 2dA}{\varepsilon_0}, \Rightarrow E(d) = \frac{\rho}{\varepsilon_0}d \quad ①$$

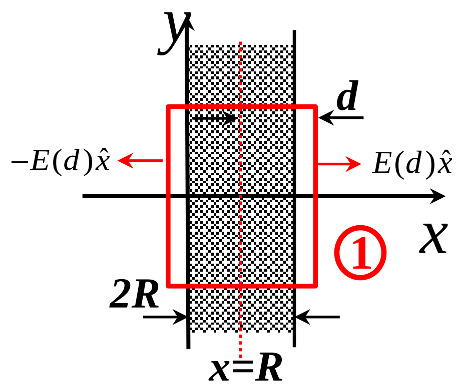
For $0 \le x \le R$,

$$d = R - x, \Rightarrow \vec{E}(x) = -\frac{\rho}{\varepsilon_0}(R-x)\hat{x} = \frac{\rho}{\varepsilon_0}(x-R)\hat{x} \quad ①$$

For $R \le x \le 2R$,

$$d = x - R, \Rightarrow \vec{E}(x) = \frac{\rho}{\varepsilon_0}(x-R)\hat{x} \quad ①$$

For $2R < x \le 4R$



$$\Phi_E = \oiint \vec{E}(\vec{r})\cdot d\vec{A} = 2E(d)A$$

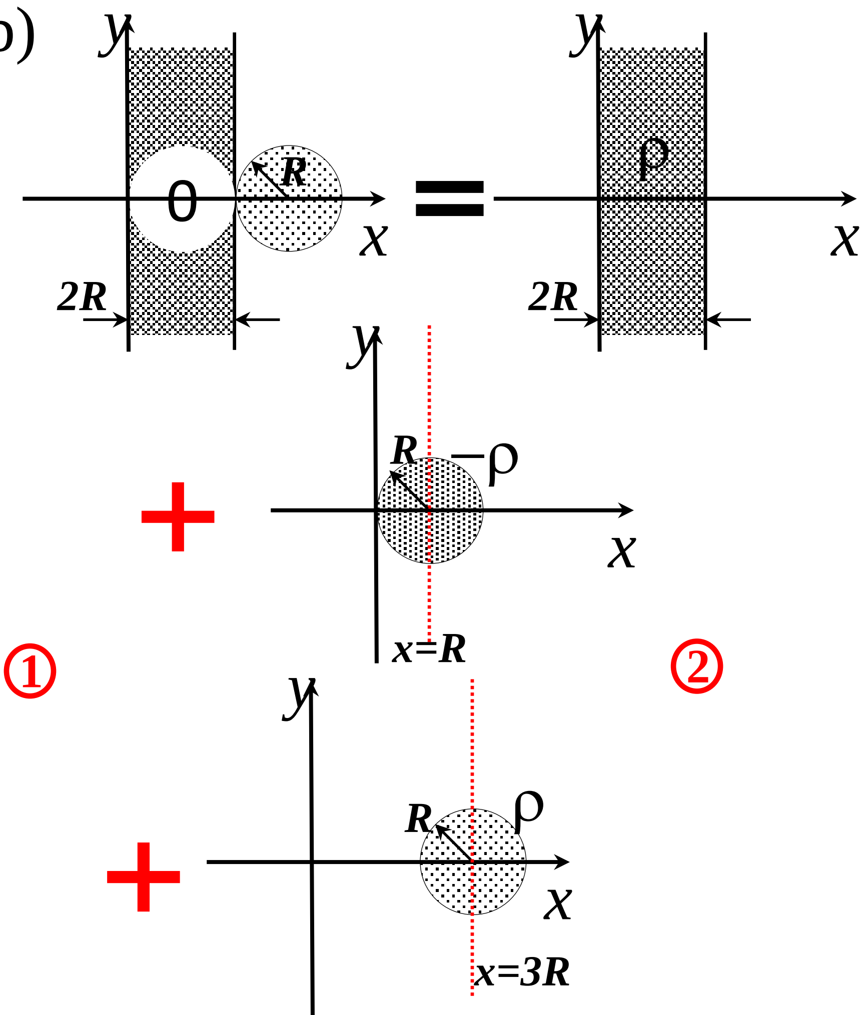
$$Q_{in} = \rho\cdot 2RA$$

$$2E(d)A = \frac{\rho\cdot 2RA}{\varepsilon_0}, \Rightarrow E(d) = \frac{\rho R}{\varepsilon_0} \quad ①$$

For $2R < x \le 4R$,

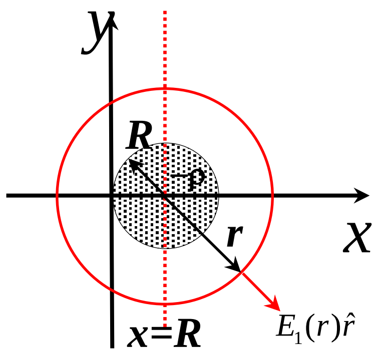
$$\vec{E}(x) = \frac{\rho R}{\varepsilon_0}\hat{x} \quad ①$$

(b)



For 2R<x ≤4R

The length of the cylindrical surface is ℓ.



$$\Phi_E = \oiint \vec{E}_1(\vec{r})\cdot d\vec{A} = 2\pi r\ell E_1(r)$$

$$Q_{in} = -\rho\cdot\pi R^2\ell$$

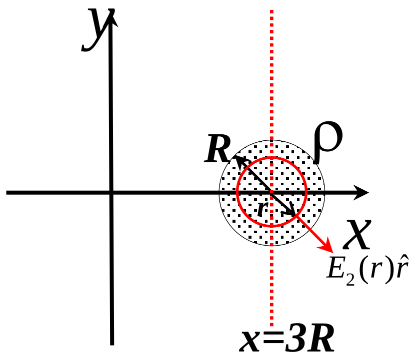
$$2\pi r\ell E_1(r) = -\frac{\rho\cdot\pi R^2\ell}{\varepsilon_0}$$

① $$E_1(r) = -\frac{\rho\cdot R^2}{2r\varepsilon_0}, \vec{r} = (x,y,z)-(R,0,z)$$

On x-axis, $\vec{r} = (x-R,0,0)$

$$\vec{E}_1(x) = -\frac{\rho\cdot R^2}{2(x-R)\varepsilon_0}\hat{x}$$ ①

The length of the cylindrical surface is ℓ.



$$\Phi_E = \oiint \vec{E}_2(\vec{r})\cdot d\vec{A} = 2\pi r\ell E_2(r)$$

$$Q_{in} = \rho\cdot\pi r^2\ell$$

$$2\pi r\ell E_2(r) = \frac{\rho\cdot\pi r^2\ell}{\varepsilon_0}$$

① $$E_2(r) = \frac{\rho\cdot r}{2\varepsilon_0}, \vec{r} = (x,y,z)-(3R,0,z)$$

On x-axis, $\vec{r} = (x-3R,0,0)$

$$\vec{E}_2(x) = \frac{\rho\cdot(x-3R)}{2\varepsilon_0}\hat{x}$$ ①

$$\vec{E}(x) = \frac{\rho R}{\varepsilon_0}\hat{x} + \vec{E}_1(x) + \vec{E}_2(x)$$

$$\vec{E}(x) = \frac{\rho R}{\varepsilon_0}\hat{x} - \frac{\rho\cdot R^2}{2(x-R)\varepsilon_0}\hat{x} + \frac{\rho\cdot(x-3R)}{2\varepsilon_0}\hat{x}$$ ①

6 pts Problem 1

(a) $\vec{r} = (0,0,z)$

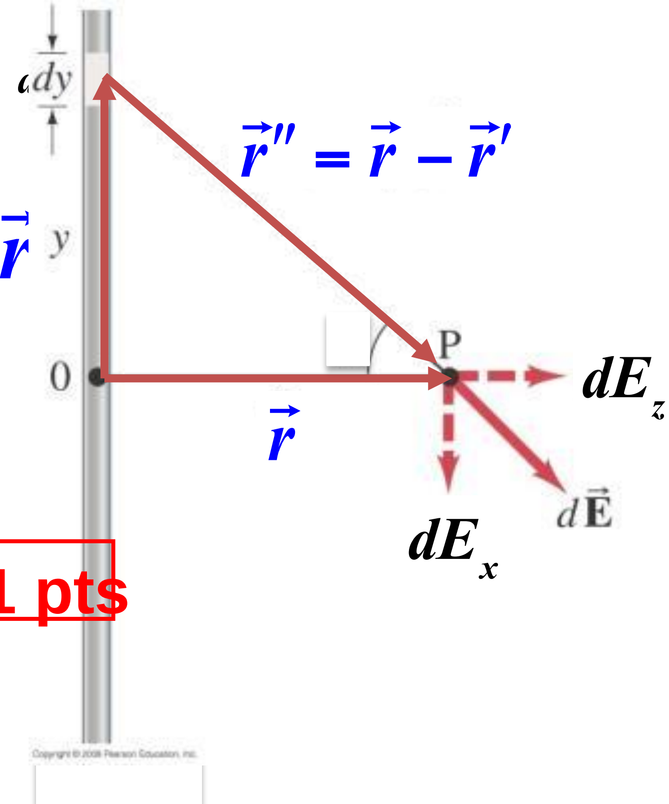
1 pts

$\vec{r}\,' = (x',0,0)$

$\vec{r}\,'' = \vec{r} - \vec{r}\,' = (-x',0,z)$

$$d\vec{E} = \frac{kdq}{\left|\vec{r}-\vec{r}\,'\right|^2}\hat{r}'' = \frac{k\left(\lambda_1 dx'\right)\left(-x'\hat{x}+z\hat{z}\right)}{\left(x'^2+z^2\right)^{3/2}}$$

2 pts



1 pts

$$E_x = -\frac{\lambda_1}{4\pi\varepsilon_0}\int_{x'=0}^{R}\frac{x'dx'}{\left(x'^2+z^2\right)^{3/2}} = -\frac{\lambda_1}{4\pi\varepsilon_0}\left(\frac{1}{z}-\frac{1}{\sqrt{R^2+z^2}}\right)$$

$$E_z = \frac{\lambda_1 z}{4\pi\varepsilon_0}\int_{x'=0}^{R}\frac{dx'}{\left(x'^2+z^2\right)^{3/2}} = \frac{\lambda_1}{4\pi\varepsilon_0}\left(\frac{R}{z}\frac{1}{\sqrt{R^2+z^2}}\right)$$

1 pts

$$\int\frac{xdx}{\left(x^2+z^2\right)^{3/2}} = \frac{-1}{\sqrt{x^2+z^2}} + \text{constant}$$

1 pts

$$\int\frac{dx}{\left(x^2+z^2\right)^{3/2}} = \frac{x}{z^2\sqrt{x^2+z^2}} + \text{ constant}$$

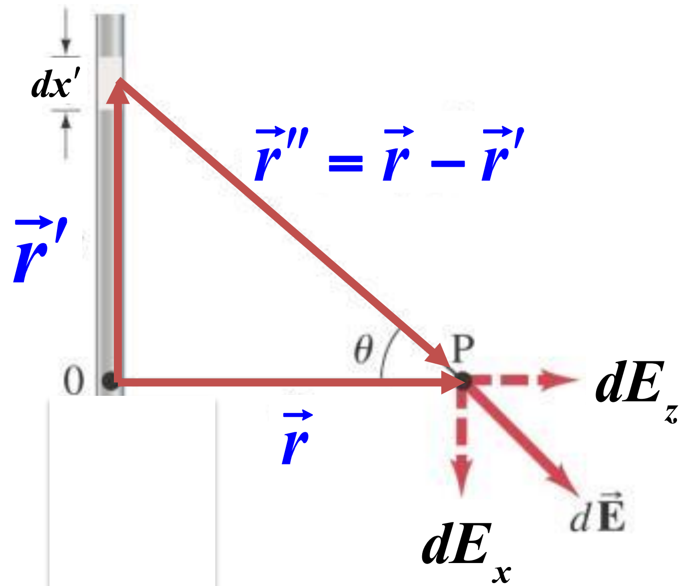
3 pts

(b) $\vec{r} = (0,0,z)$

$\vec{r}\,' = (0,y',0)$

$\vec{r}\,'' = \vec{r} - \vec{r}\,' = (0,-y',z)$

1 pts



$$d\vec{E} = \frac{kdq}{\left|\vec{r}-\vec{r}\,'\right|^2}\hat{r}'' = \frac{k\left(\lambda_1 dy'\right)\left(-y'\hat{y}+z\hat{z}\right)}{\left(y'^2+z^2\right)^{3/2}}$$

Similar calculation to part (a)

$$E_x = 0 \qquad E_y = -\frac{\lambda_1}{4\pi\varepsilon_0}\int_{y'=0}^{R}\frac{y'dy'}{\left(y'^2+z^2\right)^{3/2}} = -\frac{\lambda_1}{4\pi\varepsilon_0}\left(\frac{1}{z}-\frac{1}{\sqrt{R^2+z^2}}\right)$$

1 pts

$$E_z = \frac{\lambda_1 z}{4\pi\varepsilon_0}\int_{y'=0}^{R}\frac{dy'}{\left(y'^2+z^2\right)^{3/2}} = \frac{\lambda_1}{4\pi\varepsilon_0}\left(\frac{R}{z}\frac{1}{\sqrt{R^2+z^2}}\right)$$

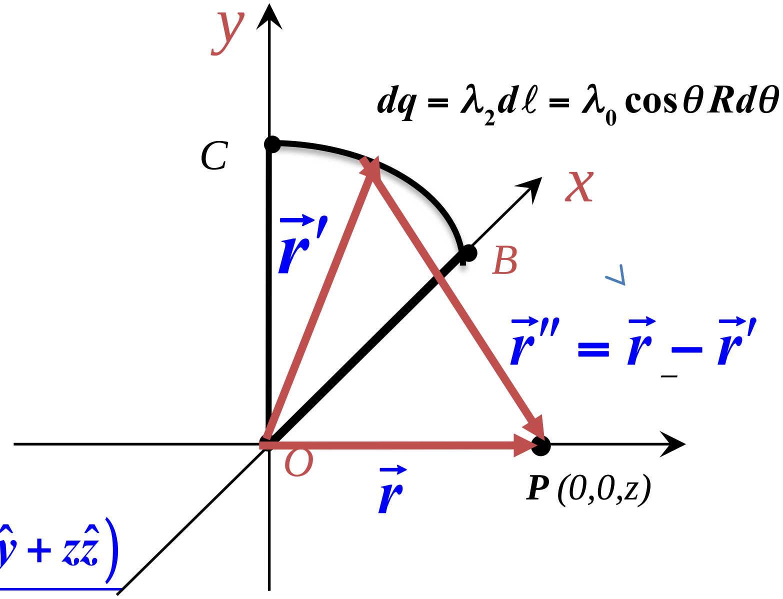
1 pts

6 pts

(c) $\vec{r} = (0,0,z)$ 1 pts

$$\vec{r}\,' = (x', y', 0) = (R\cos\theta, R\sin\theta, 0)$$

$$\vec{r}\,'' = \vec{r} - \vec{r}\,' = (-R\cos\theta, -R\sin\theta, z)$$



$$d\vec{E} = \frac{k\,dq}{\left|\vec{r} - \vec{r}\,'\right|^2}\hat{r}'' = \frac{k\left(\lambda_0 \cos\theta R d\theta\right)\left(-R\cos\theta\,\hat{x} - R\sin\theta\,\hat{y} + z\hat{z}\right)}{\left(R^2+z^2\right)^{3/2}}$$

2 pts

$$E_x = -\frac{\lambda_0}{4\pi\varepsilon_0}\frac{R^2}{\left(R^2+z^2\right)^{3/2}}\int_0^{\pi/2}\cos^2\theta\,d\theta = -\frac{\lambda_0}{4\pi\varepsilon_0}\frac{R^2}{\left(R^2+z^2\right)^{3/2}}\frac{\pi}{4}$$

1 pts

$$E_y = -\frac{\lambda_0}{4\pi\varepsilon_0}\frac{R^2}{\left(R^2+z^2\right)^{3/2}}\int_0^{\pi/2}\cos\theta\sin\theta\,d\theta = -\frac{\lambda_0}{4\pi\varepsilon_0}\frac{R^2}{\left(R^2+z^2\right)^{3/2}}\frac{1}{2}$$

1 pts

$$E_z = \frac{\lambda_0}{4\pi\varepsilon_0}\frac{zR}{\left(R^2+z^2\right)^{3/2}}\int_0^{\pi/2}\cos\theta\,d\theta = \frac{\lambda_0}{4\pi\varepsilon_0}\frac{zR}{\left(R^2+z^2\right)^{3/2}}$$

1 pts