

Calculus (I) : Midterm Exam I (10/26/2020, 8:15 - 11:45 AM)

* *The Exam includes 9 problems with 114 points in total.*

* *Please show your work for partial credits.*

1. (10 pts.) Let $f(x) = \frac{x^3}{x(x-1)}$.

- (a) Find the domain and asymptotes of $f(x)$.
- (b) Find the equations of its tangent line and normal line at $x = -1$.
- (c) Is there a point on the graph with $f''(x) = 0$?

2. (10 pts.)

- (a) Use the definition of the derivative to find the derivative of $\ln x$.
- (b) Find the derivative of $y = (\sin x)^{\ln x}$. (Simply your answer in terms of x .)

3. (35 pts.) Evaluate the followings and simply your answers.

(a) $\lim_{x \rightarrow 0^+} \tan^{-1}(\ln x)$ (b) $\lim_{x \rightarrow -\infty} (\sqrt{x^2 + x} + x)$ (c) $\lim_{x \rightarrow -1} \frac{\sin(x+1)}{2x^2 + x - 1}$
(d) $\frac{d^{35}}{dx^{35}}(x \cos x)$ (e) $\frac{d^n}{dx^n} \left(\frac{1}{1-x} \right)$ (f) $\frac{d}{dx} \ln \left| \frac{x^2-1}{(2x+1)^5} \right|$ (g) $\frac{d}{dx} \sqrt[20]{\frac{1+\tanh x}{1-\tanh x}}$

4. (15 pts.) (a) State the Intermediate Value Theorem.

(b) Apply the Intermediate Value Theorem to show that the equation $e^x = 3 - 2x$ has a solution in the interval $(0,1)$.

(c) Let $f(x) = \begin{cases} x - 4 & \text{if } x \leq 5 \\ kx^2 - 24x + 46 & \text{if } x > 5 \end{cases}$. Find the value of k that will make f continuous on $(-\infty, \infty)$.

5. (10 pts.) Use the precise definition of limit to prove the followings.

(a) $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$ (b) $\lim_{x \rightarrow 2} \frac{1}{x} = \frac{1}{2}$.

6. (10 pts.) (a) If $g(x) + x \cos g(x) = x^2$, find $g'(0)$ and $g''(0)$.

(b) If $f(x) = x + e^x$, find $(f^{-1})'(1)$.

7. (8 pts.) The angle of elevation of the sun is decreasing at a rate of 0.25 rad/h . How fast is the shadow cast by a 400-ft-tall building increasing when the angle of elevation of the sun is $\frac{\pi}{6}$?
8. (8 pts.) The radius of a circular disk is given as 20 cm with a maximum error in measurement of 0.1 cm.
- (a) Use differential to estimate the maximum error in the calculated area of the disk.
 - (b) Find relative error and percentage error.
9. (8 pts.) In a murder investigation, the temperature of the corpse was 32°C at 1:00 PM and 30°C an hour later. Normal body temperature is 37°C and the ambient temperature was 20°C . When did the murder take place? ($\ln 2 \approx 0.6931$, $\ln 3 \approx 1.0986$, $\ln 5 \approx 1.6094$, $\ln 17 \approx 2.8332$)