

HW3-1: There are three infinite lines with line charge density λ_0 . O is the origin of the coordinate and line B crosses the x -axis at $(-2,0,0)$.

- (a) What is the electric field at position $\vec{r} = (2, 3, 5)$ due to line B?
- (b) What is the electric field at position $\vec{r} = (2, 3, 5)$ due to line C?
- (c) What is the total electric field at position $\vec{r} = (2, 3, 5)$ due to line A, B and C?

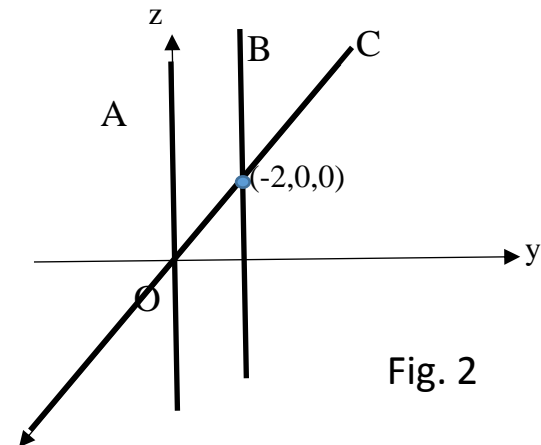


Fig. 2

HW3-2: Fig. 3 shows a cross sectional view of uniform charge distribution in an infinitely long cylindrical shell. The charge density is ρ ($\rho > 0$), the inner radius is $2R$, and the outer radius $3R$, the axis of the shell coincides with the z -axis. A second uniform cylindrical charge distribution is added to the system, with the axis of symmetry parallel to the z -axis but passing $(R, 0, 0)$. The radius of the cylinder is R , and the charge density is $-\rho$. Determine the magnitude and direction of the \mathbf{E} -field along the x -axis ($0 \leq x < 4R$).

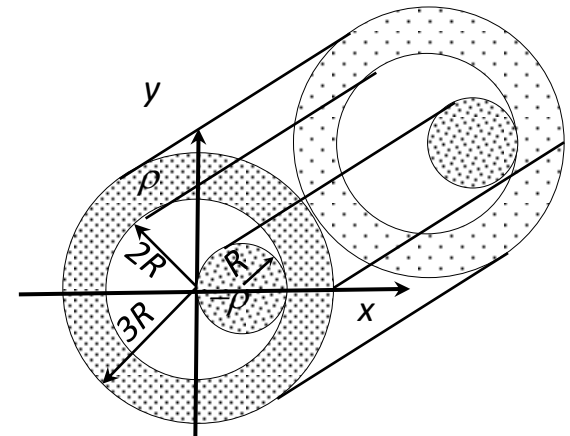
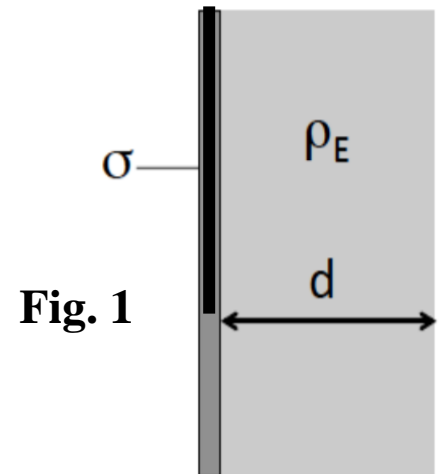


Fig. 3

HW 3-3: A very large thin plane has uniform surface charge density σ . Touching it on the right (see the Fig. 1) is a long wide slab of thickness d with uniform volume charge density ρ_E .

Determine the electric field (a) to the left of the plane, (b) to the right of the slab, and (c) everywhere inside the slab.



HW3-1sol (a) For line B, we choose the cylindrical Gauss's surface with radius r and height h , as shown in figure on the right.

$$\Rightarrow \oint \vec{E} \cdot d\vec{A} = E(r) \cdot 2\pi r \cdot h = \frac{q_{in}}{\epsilon_0} = \frac{\lambda \cdot h}{\epsilon_0}$$

$$\vec{E}_B = \frac{1}{2\pi\epsilon_0} \cdot \frac{\lambda}{r_B} \hat{r}_B \quad \text{in cylindrical coordinate } (r, \theta, z).$$

For point (2,3,5), the electric field is same as above with vector:

$$\vec{r}_B = (2,3,5) - (-2,0,5) = (4,3,0) \quad , \quad \hat{r}_B = \frac{(4,3,0)}{5} = \left(\frac{4}{5}, \frac{3}{5}, 0\right)$$

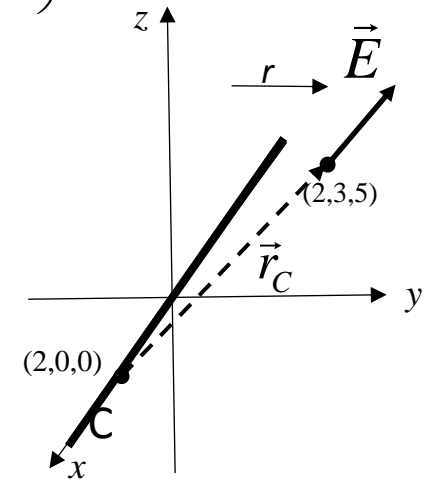
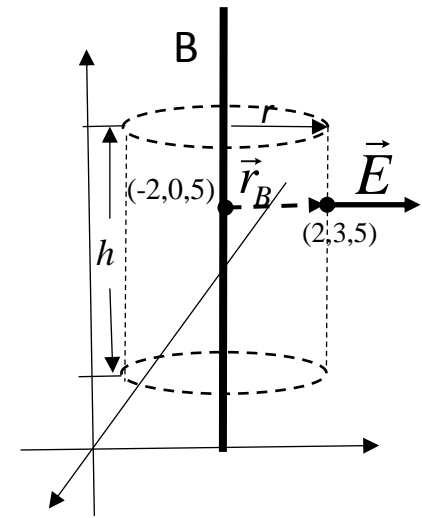
$$\vec{E}_B = \frac{\lambda}{2\pi\epsilon_0} \cdot \left(\frac{4}{25}, \frac{3}{25}, 0\right)$$

(b). Similar as above: $\vec{r}_C = (2,3,5) - (2,0,0) = (0,3,5)$

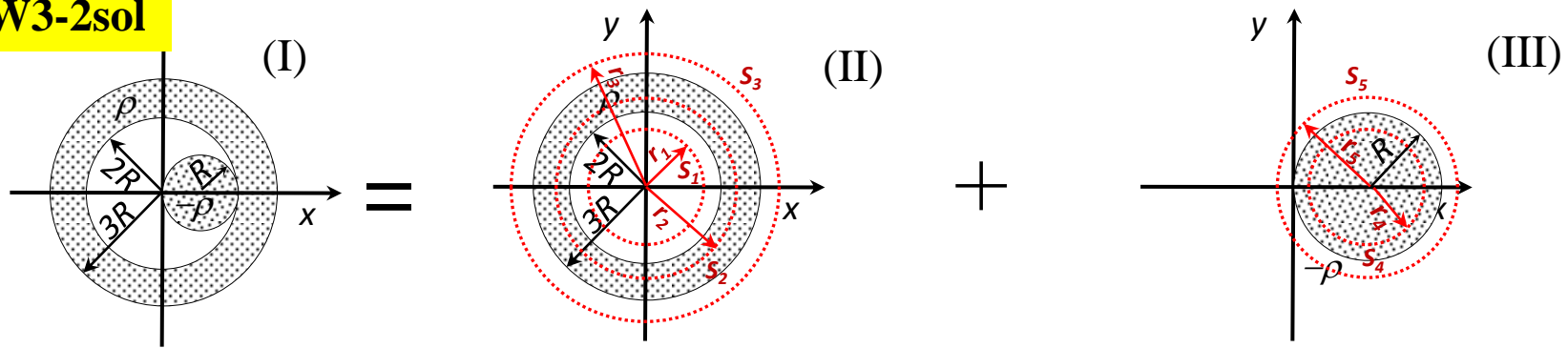
$$\vec{E}_C = \frac{\lambda}{2\pi\epsilon_0} \cdot \left(0, \frac{3}{34}, \frac{5}{34}\right)$$

(c). $\vec{E}_A = \frac{\lambda}{2\pi\epsilon_0} \cdot \left(\frac{2}{13}, \frac{3}{13}, 0\right)$

$$\vec{E}_{total} = \vec{E}_A + \vec{E}_B + \vec{E}_C = \frac{\lambda}{2\pi\epsilon_0} \cdot \left(\frac{2}{13} + \frac{4}{25}, \frac{2}{13} + \frac{3}{25} + \frac{3}{34}, \frac{5}{34}\right)$$



HW3-2sol



The calculation for part (III):

For inner cylinder, and $|x-R| < R$,
Choose a cylindrical surface S_3 with radius r_3
length ℓ_3 , and apply Gauss's law, i.e.

$$\Phi_E = \oiint_{S_3} \vec{E} \cdot d\vec{A} = E(r_3) 2\pi r_3 \ell = \frac{\rho \pi (r_3^2) \ell}{\epsilon_0}$$

$$\Rightarrow E(r_3) = \frac{\rho r_3}{2\epsilon_0} \Rightarrow E(x) = \frac{\rho |x-R|}{2\epsilon_0}$$

For $0 \leq x < R$, $|x - R| = -(x - R)$

$$\Rightarrow \vec{E}(x) = \frac{-\rho(x-R)}{2\epsilon_0} \hat{x} \quad \text{----- equ. (1)}$$

For $R \leq x < 2R$, $|x - R| = (x - R)$

$$\Rightarrow \vec{E}(x) = \frac{-\rho(x-R)}{2\epsilon_0} \hat{x} \quad \text{----- equ. (2)}$$

For $|x-R| \geq R$,

Choose a cylindrical surface S_4 with radius r_4
length ℓ_4 , and apply Gauss's law, i.e.

$$\Phi_E = \oiint_{S_4} \vec{E} \cdot d\vec{A} = E(r_4) 2\pi r_4 \ell = \frac{\rho \pi (R^2) \ell}{\epsilon_0}$$

$$\Rightarrow E(r_4) = \frac{\rho R^2}{2\epsilon_0 r_4} \Rightarrow E(x) = \frac{\rho R^2}{2\epsilon_0 |x-R|}$$

For $2R \leq x < 4R$, $|x - R| = (x - R)$

$$\Rightarrow E(x) = \frac{-\rho R^2}{2\epsilon_0 (x-R)} \hat{x} \quad \text{----- equ. (3)}$$

The calculation for part (II):

For $x < 2R$,

Choose a cylindrical surface S_1 with length ℓ_1 , and apply Gauss's law, i.e.

$$\Phi_E = \oiint_{S_1} \vec{E} \cdot d\vec{A} = E(r_1)2\pi r_1 \ell = 0$$

$$\Rightarrow E(r_1) = 0 \Rightarrow E(x) = 0, \text{ for } 0 \leq x < 2R \text{ ----- equ. (4)}$$

For $2R \leq x < 3R$,

Choose a cylindrical surface S_2 with length ℓ_2 .

$$\Phi_E = \oiint_{S_2} \vec{E} \cdot d\vec{A} = E(r_2)2\pi r_2 \ell = \frac{\rho\pi(r_2^2 - 4R^2)\ell}{\epsilon_0}$$

$$\Rightarrow E(r_2) = \frac{\rho(r_2^2 - 4R^2)}{2\pi r_2 \epsilon_0}$$

$$\Rightarrow \vec{E}(x) = \frac{\rho(x^2 - 4R^2)}{2x\epsilon_0} \hat{x} \text{ ----- equ. (5)}$$

For $3R < x$,

Choose a cylindrical surface S_3 with length ℓ_3 .

$$\Phi_E = \oiint_{S_3} \vec{E} \cdot d\vec{A} = E(r_2)2\pi r_2 \ell = \frac{\rho\pi(r_2^2 - 4R^2)\ell}{\epsilon_0}$$

The final result for part (I)

= sum of results of part (II) and Part (III):

For the total E-field with $0 \leq x < R$:

$$= \text{equ. (1)} + \text{equ. (4)}$$

$$\vec{E}(x) = \frac{-\rho(x-R)}{2\epsilon_0} \hat{x}$$

For the total E-field with $R \leq x < 2R$,

$$= \text{equ. (2)} + \text{equ. (4)}$$

$$\vec{E}(x) = \frac{-\rho(x-R)}{2\epsilon_0} \hat{x}$$

For the total E-field with $2R \leq x < 3R$,

$$= \text{equ. (3)} + \text{equ. (5)}$$

$$E(x) = \frac{-\rho R^2}{2\epsilon_0(x-R)} \hat{x} + \frac{\rho(x^2 - 4R^2)}{2x\epsilon_0} \hat{x}$$

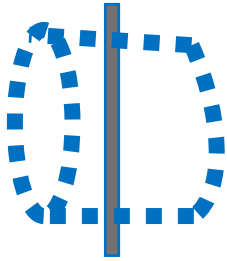
For the total E-field with $3R < x$,

$$= \text{equ. (3)} + \text{equ. (6)}$$

$$E(x) = \frac{-\rho R^2}{2\epsilon_0(x-R)} \hat{x} + \frac{\rho(x^2 - 4R^2)}{2x\epsilon_0} \hat{x}$$

Solution HW3-3 :

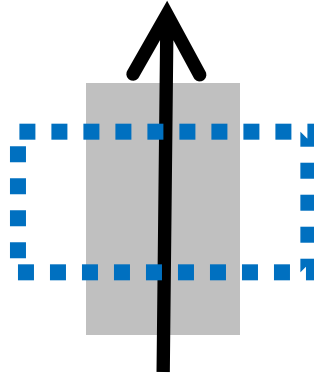
把兩個東西拆開來分析，令 *slab* 中心為 *x* 軸



$$\Phi_E = \oiint \vec{E} \cdot d\vec{A} = 2EA$$

$$= \frac{Q_{in}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

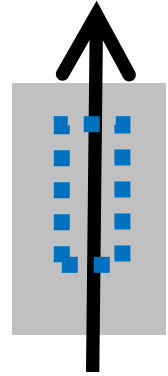
$$\vec{E} = \frac{\sigma}{2\epsilon_0} \vec{n}$$



$$\Phi_E = \oiint \vec{E} \cdot d\vec{A} = 2EA$$

$$= \frac{Q_{in}}{\epsilon_0} = \frac{\rho_E (Ad)}{\epsilon_0}$$

$$\vec{E} = \frac{\rho_E d}{2\epsilon_0} \vec{n}$$



$$\Phi_E = \oiint \vec{E} \cdot d\vec{A} = 2EA$$

$$= \frac{Q_{in}}{\epsilon_0} = \frac{\rho_E (A|x|)}{\epsilon_0}$$

$$\vec{E} = \frac{\rho_E x}{2\epsilon_0} \vec{x}$$

$$\vec{E} = -\frac{\sigma}{2\epsilon_0} x$$

$$\vec{E} = \frac{\sigma}{2\epsilon_0} x$$

$$\vec{E} = -\frac{\rho_E d}{2\epsilon_0} x$$

$$\vec{E} = \frac{\rho_E d}{2\epsilon_0} x$$

$$\vec{E} = \frac{\rho_E x}{2\epsilon_0} x$$

$$\begin{aligned}\vec{E} &= \frac{-\sigma}{2\epsilon_0} x + \frac{-\rho_E d}{2\epsilon_0} x \\ &= -\frac{\sigma + \rho_E d}{2\epsilon_0} x\end{aligned}$$

$$\begin{aligned}\vec{E} &= \frac{\sigma}{2\epsilon_0} x + \frac{\rho_E x}{2\epsilon_0} x \\ &= \frac{\sigma + \rho_E x}{2\epsilon_0} x\end{aligned}$$

$$\begin{aligned}\vec{E} &= \frac{\sigma}{2\epsilon_0} x + \frac{\rho_E d}{2\epsilon_0} x \\ &= \frac{\sigma + \rho_E d}{2\epsilon_0} x\end{aligned}$$