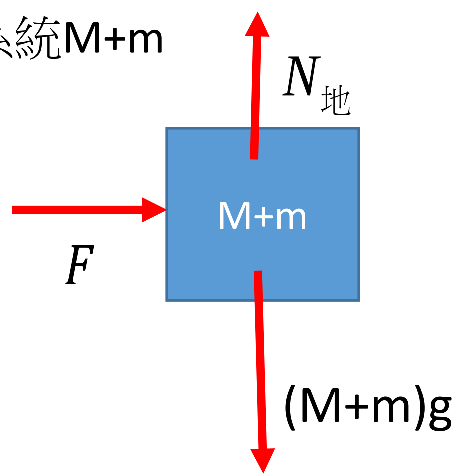
Problem 1

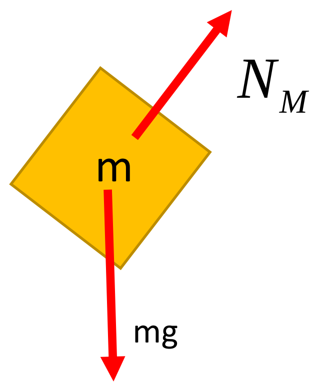
A small block of mass ***m*** rests on the sloping side of a wedge of mass ***M*** which itself rests on a horizontal frictionless table as shown in Fig. 1.

(i) Assuming all surfaces are frictionless, draw the free-body diagram and determine the magnitude of the force F that must be applied to ***M*** so that m remains in a fixed position relative to ***M***.

Sol:　因M,m無相對運動
➔分析系統

系統M+m





$$F = (M+m)a \quad \text{—— } 1$$

$$(M+m)g - N_{地} = 0$$

$$N_M = N_m$$

$$N_M \cos\theta = mg$$

$$N_M \sin\theta = ma$$

$$\Rightarrow \quad a = g\tan\theta \quad \text{—— } 2$$

由 1 2

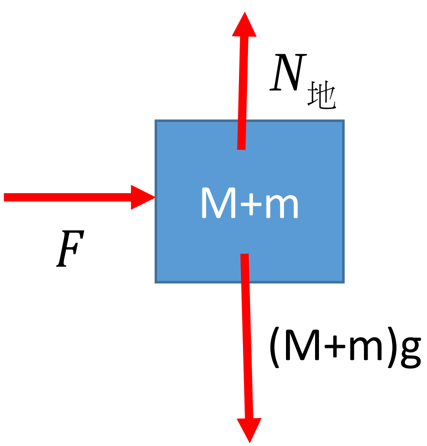
$$F = (M+m)a = (M+m)g\tan\theta$$

Problem 1

(ii) Now consider the coefficient of static friction is $\boldsymbol{\mu_s}$ between the small block and the wedge (the table is still frictionless), draw the free-body diagram for each block and determine the maximum horizontal force $\boldsymbol{F}$ applied to $\boldsymbol{M}$ such that the small block $\boldsymbol{m}$ to remain at a constant height above the table. Assume $\boldsymbol{M = 3m = 2}$ ***Kg***, $\boldsymbol{\mu_s = 1.0}$, $\boldsymbol{\mu_k = 0.6}$, And $\boldsymbol{\theta = 37^0}$.

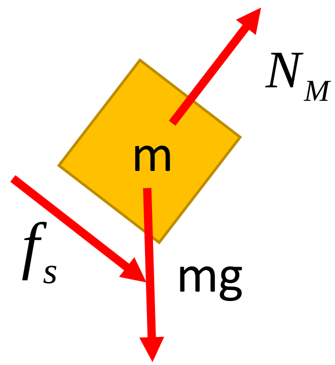
因為M,m無相對運動

➔ 系統M+m



➔ $F = (M+m)a$ ——— 1

$(M+m)g - N_{地} = 0$



$f_s\cos\theta + N_m\sin\theta = ma$ ——— 2

$f_s\sin\theta + mg - N_m\cos\theta = 0$ — 3

欲求最大F ➔ $f_s = N_m \bullet \mu_s$ — 4

由 3 4

➔ $N_m = \dfrac{mg}{(\cos\theta - \mu_s\sin\theta)}$ — 5

由 5 2

$$a = \frac{N_m(\mu_s\cos\theta + \sin\theta)}{m} = \frac{g(\mu_s\cos\theta + \sin\theta)}{(\cos\theta - \mu_s\sin\theta)}$$

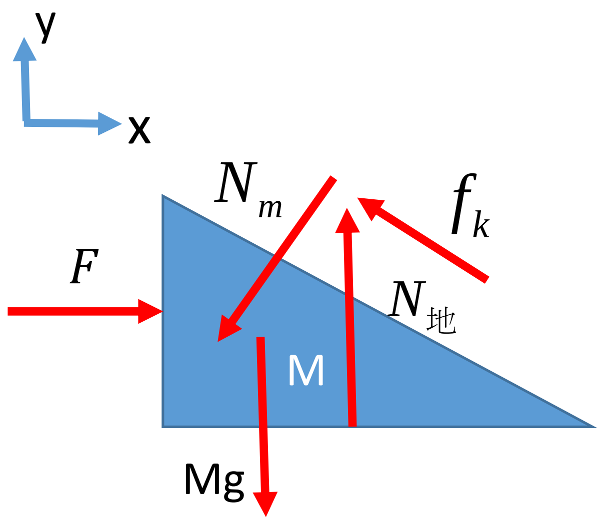
➔ 帶入 1

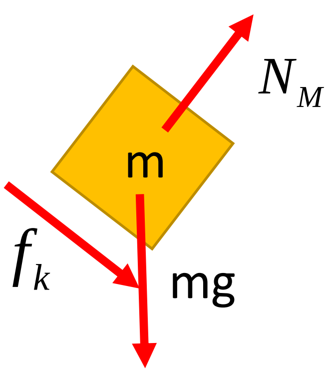
$$F = (M+m)a = \frac{g(M+m)(\mu_s\cos\theta + \sin\theta)}{(\cos\theta - \mu_s\sin\theta)} = 28mg$$

Problem 1

(iii) If $\boldsymbol{F = 30\,mg}$, determine the acceleration of each block.

$30mg > 28mg \Rightarrow$ M,m有相對運動





a' : acceleration of m relative to M $\Rightarrow a_y' / a_x' = -\tan\theta$

$$a_x = A_x + a_x'$$

$$a_y = a_y' = -a_x' \tan\theta$$

$$N_M \sin\theta + f_k \cos\theta = m(A_x + a_x')$$

$$N_M \cos\theta - f_k \sin\theta - mg = ma_y' = -ma_x' \tan\theta$$

$$F - N_m \sin\theta - f_k \cos\theta = MA_x$$

$$f_k = \mu_k N_M$$

$$\frac{27}{25} N_m = mA_x + ma_x'$$

$$\frac{11}{25} N_m - mg = -\frac{3}{4} ma_x'$$

$$30mg - \frac{27}{25} N_m = 3mA_x$$

$\Rightarrow$

$$A_x = \frac{607}{76}\,\mathrm{g}$$

$$a_x' = -\frac{444}{228}\,\mathrm{g}$$

$$a_y' = \frac{111}{76}\,\mathrm{g}$$

$$N_m = \frac{425}{76} mg$$

$\Rightarrow$

$$a_x = a_x' + A_x = \frac{1377}{228}\,\mathrm{g}$$

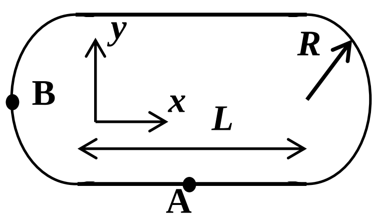
$$a_y = a_y' = \frac{111}{76}\,\mathrm{g}$$

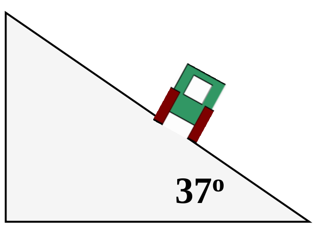
Problem 2

Car racing track : The length of straight track is $\boldsymbol{L = 1000\ m}$ and the radius of the two curved track is $\boldsymbol{R = 100\ m}$. The whole track is banked inward as shown in Fig. 2 with angle $\boldsymbol{37^o}$. A car racer is driving a car , mass $\boldsymbol{m = 2000\ kg}$, with constant speed $\boldsymbol{v_0 = 10\ m/s}$ along the track. The static frictional coefficient μ_s of the track is ***0.8***. Assume $\boldsymbol{g = 10\ m/s^2}$. Consider two positions A and B on the track.

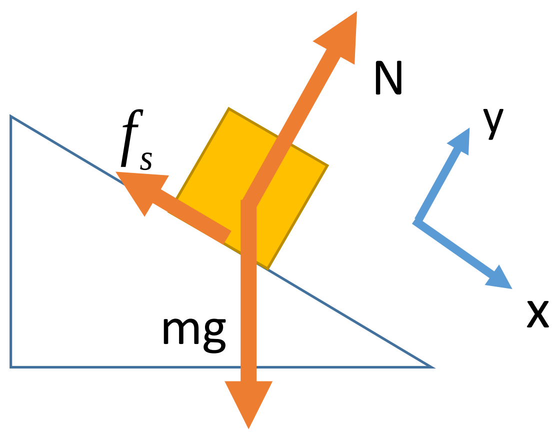
(A) What is the frictional force, magnitude and direction, at the position A?

A點：直線運動





Point A



$$x : mg\sin 37° - f_s = 0$$

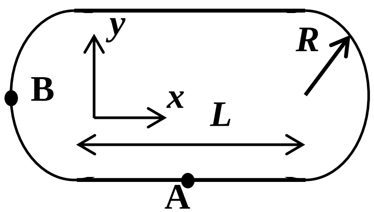
$$y : mg\cos 37° - N = 0$$

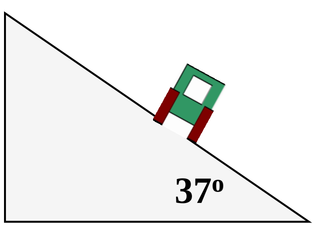
$$f_s = mg\sin 37° = 12000(\mathrm{N})$$ 延斜面朝上

Problem 2

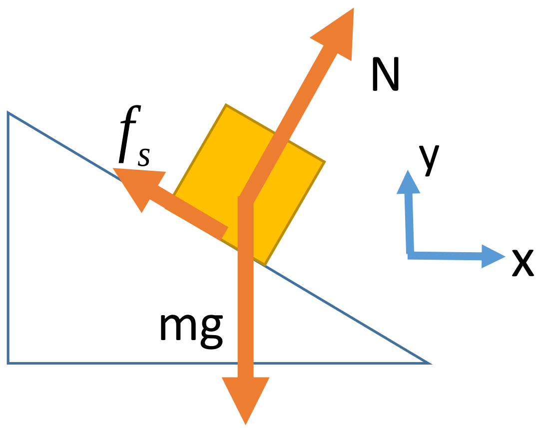
(B) What is the frictional force, magnitude and direction, at the position B?

B點：圓週運動





Point B



$$x: N\sin 37^\circ - f_s\cos 37^\circ = ma_c = m\frac{v_0^{\ 2}}{R}$$

$$y: mg - N\cos 37^\circ - f_s\sin 37^\circ = 0$$

$$\Rightarrow \frac{4}{5}N + \frac{3}{5}f_s = 10m$$

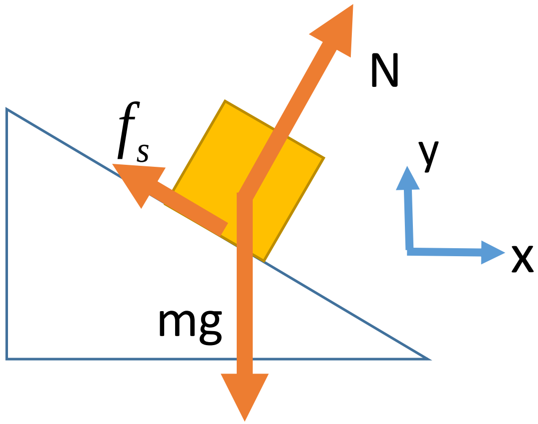
$$\frac{3}{5}N - \frac{4}{5}f_s = m$$

$$\Rightarrow N = \frac{43}{5}m = 17200(N)$$

$$f_s = \frac{26}{5}m = 10400(N)$$

Problem 2

(C)Now this car racer wants to increase the speed. What is the maximum speed $\boldsymbol{v_{max}}$ he can reach?



$$x: f_s \cos 37° + N \sin 37° = ma_c = m\frac{{v_{\max}}^2}{R}$$

$$y: mg + f_s \sin 37° - N \cos 37° = 0$$

$$v_{\max} \Rightarrow f_s = N \bullet \mu_S$$

$$\text{y}: \frac{12}{25}N - \frac{4}{5}N = -10m \Rightarrow N = 62500\ (\text{N})$$

$$\text{x}: \frac{16}{25}N + \frac{3}{5}N = m\frac{{v_{\max}}^2}{100} \Rightarrow v_{\max} = 5\sqrt{155}(\text{m/s}^2)$$

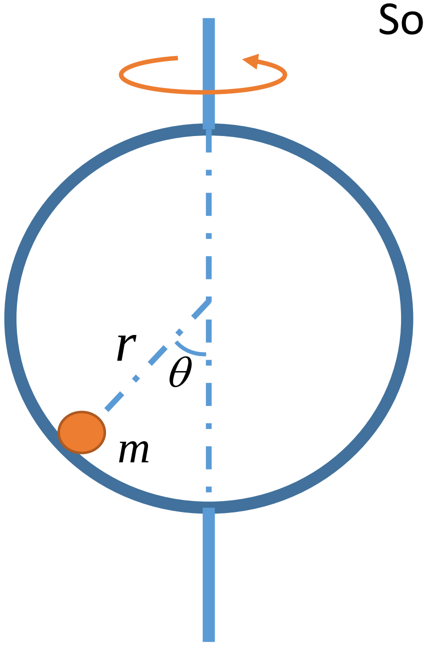
Problem 3

A small bead of mass m is constrained to slide without friction inside a circular vertical hoop of radius r which rotates about a vertical axis (Fig.3) at a frequency f .

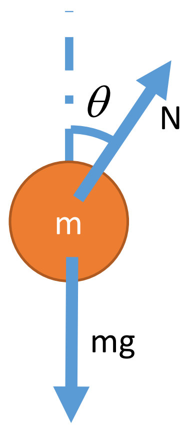
(a) Determine the angle θ where the bead will be in equilibrium – that is , where it will have no tendency to move up or down along the hoop.

(b) If $f = 2.00 rev/s$ and $r = 22.0cm$,what is θ ?

(c) Can the bead ride as high as the center of the circle $(\theta = 90°)$? Explane.



Sol:



(a)

$$x: N\sin\theta = ma_c$$

$$y: N\cos\theta = mg$$

$$\Rightarrow \tan\theta = \frac{a_c}{g}$$

$$\Rightarrow a_c = g\tan\theta = 4\pi^2 f^2 (r\sin\theta)$$

$$\Rightarrow \cos\theta = \frac{g}{4\pi^2 f^2 r}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{g}{4\pi^2 f^2 r}\right)$$

(b)

$$\theta = \cos^{-1}\left(\frac{10}{4 \bullet \pi^2 \bullet 4 \bullet 0.22}\right) \approx 73.24$$

(c)

不行

$$\cos\theta = \frac{g}{4\pi^2 f^2 r}$$

$$\cos 90° = 0 \Rightarrow f \to \infty$$

其角度隨旋轉頻率增大而增大，
當頻率趨近於無限時，
角度方可接近90度