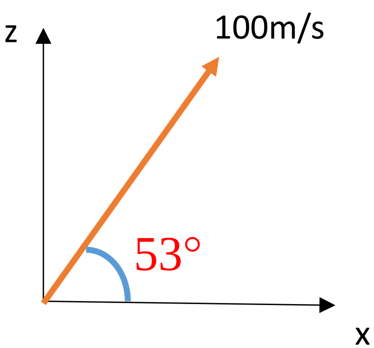
Problem 1

As shown in Fig. 1, a cannon located at the origin of the coordinate system launched a iron ball in the x-direction with a take off angle θ = 53^0, and the initial speed of the ball is 100 m/s. Assume the gravitational acceleration $\boldsymbol{g}$ = $10 m/s^2$. (sin 53^0= 4/5)

(A) A cannonball is fired at ***t = 0*** sec. In the meantime, a strong wind begins to blow in the ***+y-*** direction, and it accelerated the cannonball in the same direction with the acceleration $\boldsymbol{a_y = 0.06t}$ (m/s^2). If the wind stops at ***t =2.0*** sec. Find the coordinate of the position where the ball lands.



$$v_{x(t=0)} = 60m/s$$

$$v_{z(t=0)} = 80m/s$$

x座標:

$$t = 2\frac{v_{z(t=0)}}{g} = 16s$$

$$x = v_{x(t=0)}t = 960m$$

y座標:

當 $t \le 2$

$$a_y = 0.06t(\mathrm{m/s^2})$$

$$v_y(t) = \int_0^t 0.06t dt + v_y(0)$$

$$= 0.03t^2(m/s)$$

當 $t > 2$

$$a_y = 0(\mathrm{m/s^2})$$

$$v_y(t) = v_y(2) = 0.12(m/s)$$

$$y = \int_0^{16} v_y dt = \int_0^2 0.03t^2 dt + \int_2^{16} 0.12 dt + y(0)$$

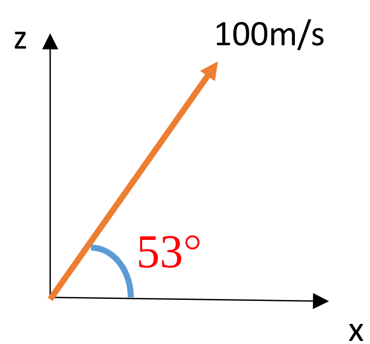
$$= \frac{0.03}{3}t^3\Big|_0^2 + 0.12t\Big|_2^{16} + 0$$

$$= 1.76m$$

$$\Rightarrow (x, y) = (960m, 1.76m)$$

Problem 1

(B) A second cannon ball is fired again at $\boldsymbol{t = 0}$ sec. Now the wind blows very strongly in the $\boldsymbol{-x}$ direction such that it accelerates the cannon ball with $\boldsymbol{a_x = -30.0\,t}$ (m/s^2). At $\boldsymbol{t = t_1}$, the winds stops and the cannon ball also stops moving in the x-direction and then reach ground at $\boldsymbol{t = t_2}$. Determine $\boldsymbol{t_1}$ and $\boldsymbol{t_2}$ and how far it lands from the origin?



$$v_{x(t=0)} = 60m/s$$

$$v_{z(t=0)} = 80m/s$$

$$t_2 = 2\frac{v_{z(t=0)}}{g} = 16s$$

當 $t \le t_1$

$$v_x(t) = \int_0^t a_x dt + v_x(0)$$

$$= \int_0^t -30t dt + 60$$

$$= -15t^2 + 60$$

$$\Rightarrow v_x(t_1) = -15t_1^2 + 60 = 0$$

$$t_1 = 2s$$

當 $t > t_1$

$$a_x = 0(\mathrm{m/s^2})$$

$$\Rightarrow v_x(t) = v_x(t_1) = 0(m/s)$$

$$x(16) = \int_0^{16} v_x dt + x(0)$$

$$= \int_0^2 (-15t^2 + 60)dt + \int_2^{16} 0dt + x(0)$$

$$= -5t^3 + 60t \Big|_0^2$$

$$= 80m$$

Problem 2

The water in the ***300-m-wide*** river has a speed of ***6.00 km/h*** due east relative to the Earth. A boat with a speed of ***12.0 km/h*** relative to the water is to travel from point A to point B, where B is 37^0 E of N relative to A, as shown in Fig. 2. (a) What should boat heading be? (b) How long will it take.

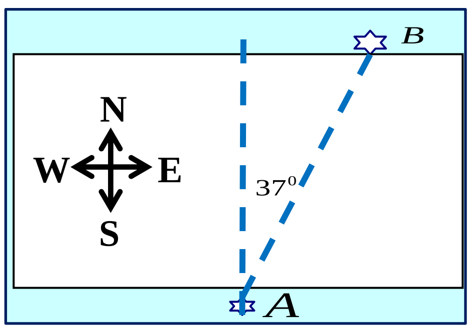
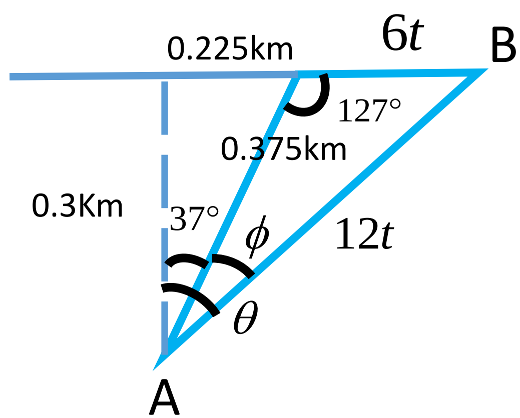


Fig. 2



Sol: 由正弦定理

$$\frac{6t}{\sin\phi} = \frac{12t}{\sin 127°} = \frac{0.375}{\sin(\pi-\theta)}$$

$\frac{\pi}{2}$

$$\sin\phi = \frac{\sin 127°}{2} = \frac{2}{5}$$

$$\cos\theta = \cos(\phi+37°) = \cos\phi\cos 37° - \sin\phi\sin 37°$$

$$= \frac{4\sqrt{21}-6}{25} = \frac{0.3}{12t}$$

$$t = \frac{5}{8(4\sqrt{21}-6)} \approx 0.0507\text{hr} \approx 3.04\text{min}$$

Problem 3

A person stands at the base of a hill that is a straight incline making an angle ϕ with the horizontal. (Fig3) For a given a initial speed v_0 , at what angle θ (to the horizontal) should objects be thrown so that the distance d they land up the hill is a large as possible?

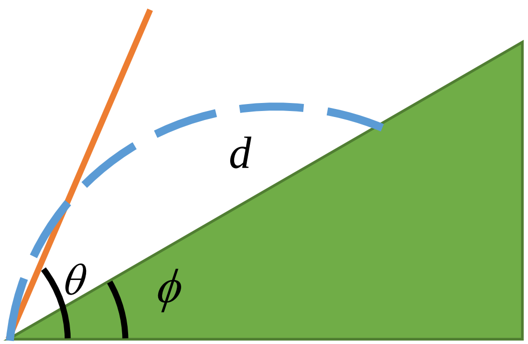


Fig3

$$x:(v_0\cos\theta)t = d\cos\phi$$

$$y:(v_0\sin\theta)t - \frac{1}{2}gt^2 = d\sin\phi$$

➔ $$t = \frac{d\cos\phi}{v_0\cos\theta}$$

$$(v_0\sin\theta)\left(\frac{d\cos\phi}{v_0\cos\theta}\right) - \frac{1}{2}g\left(\frac{d\cos\phi}{v_0\cos\theta}\right)^2 = d\sin\phi$$

$$d = \frac{2v_0^{\ 2}\cos^2\theta}{g\cos^2\phi}\left(\frac{\sin\theta\cos\phi - \sin\phi\cos\theta}{\cos\theta}\right)$$

$$d = \frac{2v_0^{\ 2}\cos\theta\sin(\theta-\phi)}{g\cos^2\phi}$$

➔

當d有最大時
d的一次微分為0

$$\frac{d}{d\theta}(d) = \frac{2v_0^{\ 2}(\cos\theta\cos(\theta-\phi) - \sin\theta\sin(\theta-\phi))}{g\cos^2\phi}$$

$$\cos(2\theta-\phi) = 0$$

➔ $$2\theta - \phi = 90^\circ$$

$$\theta = 45^\circ + \frac{\phi}{2}$$