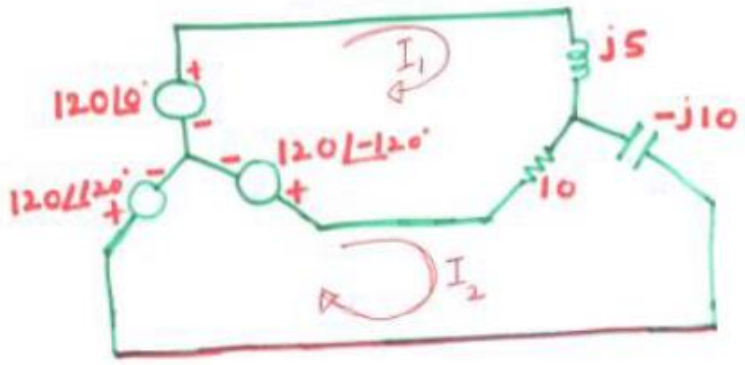
Unbalanced Circuit Analysis Example

Find currents using loop analysis.



Using Mesh analysis. Finding currents I_1 & I_2

$$-120\angle 120^\circ + 120\angle 0^\circ - (10+j5)I_1 + 10I_2 = 0$$

$$(10+j5)I_1 - 10I_2 = 120\sqrt{3}\angle 30^\circ \longrightarrow (1)$$

for mesh 2.

$$-120\angle 120^\circ + 120\angle -120^\circ - (10+j10)I_2 + 10I_1 = 0$$

$$-10I_1 + (10-j10)I_2 = 120\sqrt{3}\angle -90^\circ \longrightarrow (2)$$

Matrix Equation

$$\begin{bmatrix} 10+j5 & -10 \\ -10 & 10-j10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 120\sqrt{3}\angle 30^\circ \\ 120\sqrt{3}\angle -90^\circ \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 10+j5 & -10 \\ -10 & 10-j10 \end{bmatrix} = 50-j50 = 70.71\angle -45^\circ$$

$$\Delta_1 = \begin{bmatrix} 120\sqrt{3}\angle 30^\circ & -10 \\ 120\sqrt{3}\angle -90^\circ & 10-j10 \end{bmatrix} = 207.85(13.66-j13.66) = 4015\angle -45^\circ$$

$$\Delta_2 = \begin{bmatrix} 10+j5 & 120\sqrt{3}\angle 30^\circ \\ -10 & 120\sqrt{3}\angle -90^\circ \end{bmatrix}$$

$$= 207.85(13.66 - j13.66) = 3023.4\angle -20.1^\circ$$

$$= 4015\angle -45^\circ$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{4015.23\angle -45^\circ}{70.71\angle -45^\circ} = 56.78\ A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{3023.4\angle -20.1^\circ}{70.71\angle -45^\circ} = 42.75\angle 24.9^\circ\ A$$

(1)

$I_A = I_1 = 56.78\ \text{A}$

$I_C = 42.75\angle -155.1^\circ\ \text{A}$

$I_B = I_2 - I_1 = 25.46\angle 135^\circ\ \text{A}$

(2)

$I_A + I_B + I_C \approx 0$

(3)

$I_A * \sqrt{2} * \cos(120\pi) = 80.3\ \text{A}$

2.

A three-phase motor draws 20 kVA at 0.707 power factor lagging from a 220-V source. Determine the kilovoltampere rating of capacitors to make the combined power factor 0.90 lagging, and determine the line current before and after the capacitors are added.

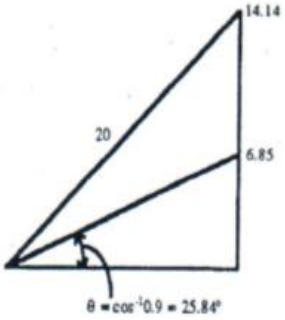
Solution:

From the figure,

$$\theta = \cos^{-1} 0.9 = 25.84^\circ$$

$$14.14 \tan 25.84^\circ = 6.85$$

$$14.14 - 6.85 = 7.29 \text{ kvar}$$



Without capacitors:

$$|I| = \frac{20,000}{\sqrt{3} \times 220} = 52.5 \text{ A}$$

With capacitors:

$$|I| = \frac{|14.14 + j6.85| \times 1000}{\sqrt{3} \times 220} = 41.2 \text{ A}$$