Homework 9 (Chap 10-11)

1. Fig. 1 shows two masses connected by a cord passing over a pulley of radius $\boldsymbol{R_0}$ and moment of inertia $\boldsymbol{I}$. Mass $\boldsymbol{M_A}$ slides on a frictionless surface, and $\boldsymbol{M_B}$ hangs freely. Determine the acceleration of the masses.

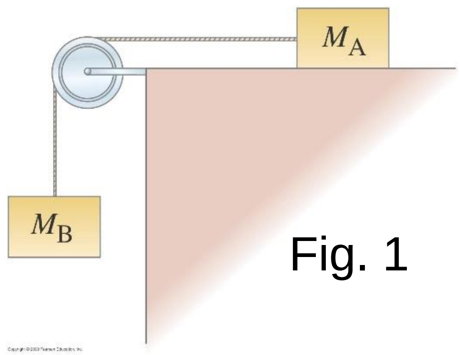


Fig. 1

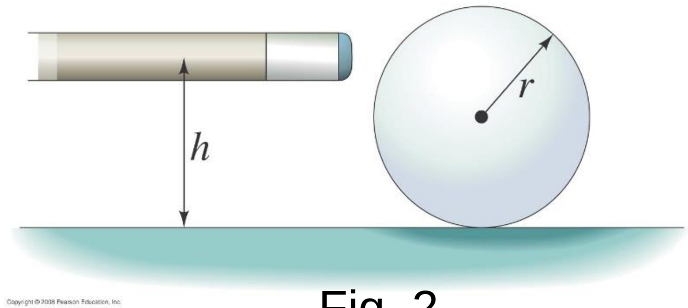


Fig. 2

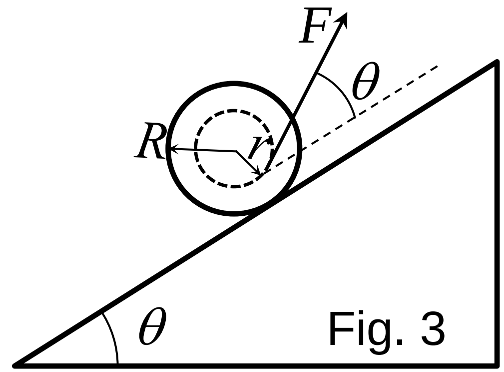
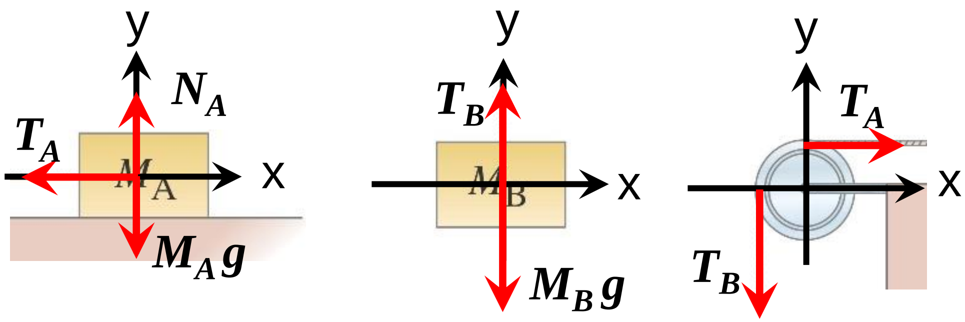


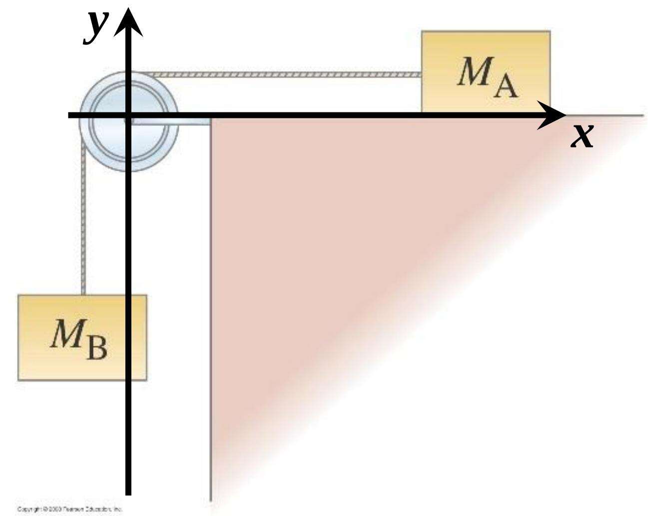
Fig. 3

2. In Fig. 2, if a billiard ball is hit in just the right way by a cue stick, the ball will roll without slipping immediately after losing contact with the stick. Consider a billiard ball (radius *r*, mass *M*) at rest on a horizontal pool table. A cue stick exerts a constant horizontal force *F* on the ball for a time ***t*** at a point that is a height ***h*** above the table's surface (see the figure below). Assume that the coefficients of kinetic friction and the static friction between the ball and table are μ_k and μ_S, respectively. Determine the range for ***h*** so that the ball will roll without slipping immediately after losing contact with the stick.

3. As shown in Fig. 3, on an inclined surface, a dumbbell with outer diameter ***R*** and inner diameter ***r*** ($\boldsymbol{r} = 3/5\boldsymbol{R}$) is pulled by a force ***F*** with a string winded around its inner post. Give that the inclined angle of the surface $\boldsymbol{\theta} = 37^\circ$, ***m*** the mass of the dumbbell, the moment of inertia of the dumbbell $\boldsymbol{I_c} = 4/5\, \boldsymbol{mR^2}$, the static friction coefficient $\mu_s = 3/5$. For a given magnitude of force ***F*** that makes the dumbbell to execute pure roll motion, draw the free-body diagram and determine the direction and magnitude of the acceleration of the dumbbell, and the friction force.

1. Fig. 1 shows two masses connected by a cord passing over a pulley of radius $\boldsymbol{R_0}$ and moment of inertia $\boldsymbol{I}$. Mass $\boldsymbol{M_A}$ slides on a frictionless surface, and $\boldsymbol{M_B}$ hangs freely. Determine the acceleration of the masses.





From the freebody diagram of M_A, we have

$$\vec{T}_A + \vec{N}_A + \vec{M}_A g = M_A \vec{a}_A$$

$$\Rightarrow \begin{array}{ll} x: \ -T_A = M_A a_A & (1) \\ y: N_A - M_A g = 0 & (2) \end{array}$$

From the freebody diagram of M_B, we have

$$\vec{T}_B + \vec{M}_B g = M_B \vec{a}_B \quad (3)$$

$$\Rightarrow y: T_B - M_B g = M_B a_B \quad (4)$$

From the freebody diagram of the pulley, we have

$$\vec{R}_0 \times \vec{T}_A + \vec{R}_0 \times \vec{T}_B = I\vec{\alpha}$$

$$\Rightarrow -R_0 T_A + R_0 T_B = I\alpha \quad (5)$$

The physical relation between a_A, a_B, and α is (for α, the positive direction is counter clockwise)

$$a_A = a_B = -R_0\alpha \equiv a \quad (6)$$

From (1),(4),(5), and (6) we get

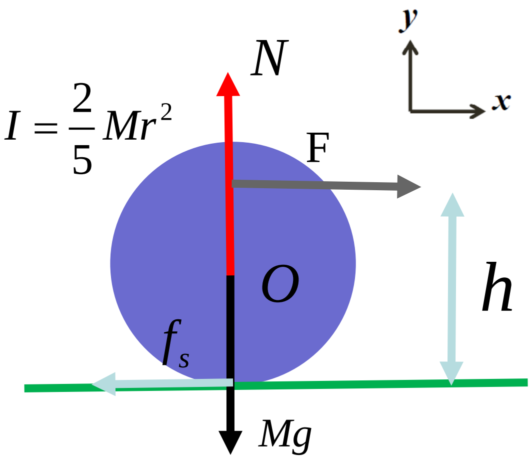
$$\Rightarrow R_0 M_A a + R_0(M_B g + M_B a) = -I\frac{a}{R_0}$$

$$\Rightarrow a = \frac{-M_B g}{M_A + M_B + I/R_0^2}$$

Problem 1 (solution 1, +torque for rollin to the same direction as *a*)

If a billiard ball is hit in just the right way by a cue stick, the ball will roll without slipping immediately after losing contact with the stick. Consider a billiard ball (radius *r*, mass *M*) at rest on a horizontal pool table. A cue stick exerts a constant horizontal force *F* on the ball for a time ***t*** at a point that is a height ***h*** above the table's surface (see the figure below). Assume that the coefficients of kinetic friction and the static friction between the ball and table are μ_k and μ_S, respectively. Determine the range for ***h*** so that the ball will roll without slipping immediately after losing contact with the stick.

$$I = \frac{2}{5}Mr^2$$



$$\sum \vec{F} = M\vec{a}$$

$$\sum \vec{\tau} = \vec{r} \times \vec{F} = I\vec{\alpha}$$

$$\Rightarrow \quad \sum F_x = F - f_s = Ma$$

$$\sum F_y = N - Mg = 0$$

$$\sum \tau = F(h-r) + f_s \cdot r = I\alpha$$

\+ torque for rolling to the same direction as ***a***

Roll without slipping $\Rightarrow a = r\alpha$

$$\Rightarrow \frac{F - f_s}{M} = r\frac{F(h-r) + f_s \cdot r}{I}$$

$$\Rightarrow \frac{2}{5}Mr^2(F - f_s) = Mr\left[F(h-r) + f_s \cdot r\right]$$

$$\Rightarrow \frac{2}{5}r(F - f_s) = \left[F(h-r) + f_s \cdot r\right]$$

$$\Rightarrow -\frac{7}{5}rf_s = F(h - \frac{7}{5}r)$$

$$\Rightarrow f_s = F(1 - \frac{5}{7r}h) \qquad (\left|f_s\right| \le Mg\mu_s)$$

$$\Rightarrow \boxed{\frac{7}{5}(1 + \frac{Mg}{F}\mu_s) \ge \frac{h}{r} \ge \frac{7}{5}(1 - \frac{Mg}{F}\mu_s)}$$

Problem 1(solution 2, direction of torque and the angular acceleration follows the coordinate system of the free body diagram)

If a billiard ball is hit in just the right way by a cue stick, the ball will roll without slipping immediately after losing contact with the stick. Consider a billiard ball (radius *r*, mass *M*) at rest on a horizontal pool table. A cue stick exerts a constant horizontal force *F* on the ball for a time ***t*** at a point that is a height ***h*** above the table's surface (see the figure below). Assume that the coefficients of kinetic friction and the static friction between the ball and table are μ_k and μ_S , respectively. Determine the range for ***h*** so that the ball will roll without slipping immediately after losing contact with the stick.

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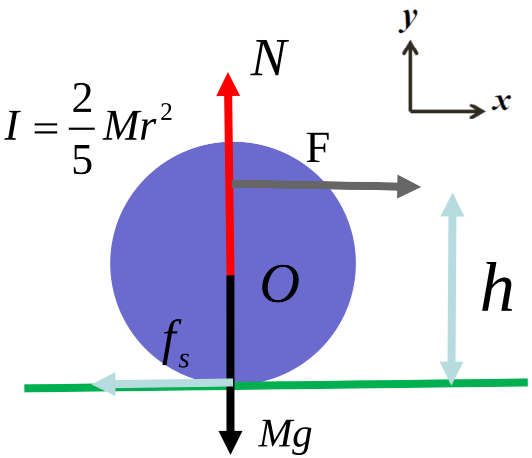


$$\sum \vec{F} = M\vec{a}$$

$$\sum \vec{\tau} = \vec{r} \times \vec{F} = I\vec{\alpha}$$

$$\sum F_x = F - f_s = Ma$$

$$\Rightarrow \sum F_y = N - Mg = 0$$

$$\sum \tau = -F(h-r) - f_s \cdot r = I\alpha$$

Roll without slipping ➔ $a = -r\alpha$

$$a = \frac{F - f_s}{M}, \quad \alpha = -\frac{F(h-r) + f_s \cdot r}{I}$$

$$\Rightarrow \frac{F - f_s}{M} = r\frac{F(h-r) + f_s \bullet r}{I}$$

$$\Rightarrow \frac{2}{5}Mr^2(F - f_s) = Mr\left[F(h-r) + f_s \bullet r\right]$$

$$\Rightarrow \frac{2}{5}r(F - f_s) = \left[F(h-r) + f_s \bullet r\right]$$

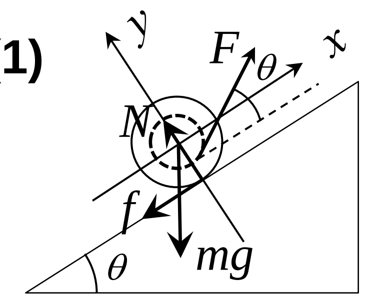
$$\Rightarrow -\frac{7}{5}rf_s = F(h - \frac{7}{5}r)$$

$$\Rightarrow f_s = F(1 - \frac{5}{7r}h) \qquad (\left|f_s\right| \le Mg\mu_s)$$

$$\Rightarrow \boxed{\frac{7}{5}(1 + \frac{Mg}{F}\mu_s) \ge \frac{h}{r} \ge \frac{7}{5}(1 - \frac{Mg}{F}\mu_s)}$$

2.As shown in Fig. x, on a inclined surface, a dumbbell with outer diameter ***R*** and inner diameter ***r*** (***r*** = 3/5***R***) is pulled by a force ***F*** with a string winded around its inner post. Give that the inclined angle of the surface $\boldsymbol{\theta} = 37^\circ$, ***m*** the mass of the dumbbell, the moment of inertia of the dumbbell $\boldsymbol{I_c} = 4/5\, \boldsymbol{mR^2}$, the static friction coefficient $\boldsymbol{\mu_s} = 3/5$. For a given magnitude of force ***F*** that makes the dumbbell to execute pure roll motion , draw the free-body diagram and determine the direction and magnitude of the acceleration of the dumbbell, and the friction force. ($\sin 37^\circ = 3/5$)

(1)



$\sum \vec{F} = m\vec{a}$

x: $F\cos\theta - mg\sin\theta - f = ma$ **(1)**

y: $F\sin\theta + N - mg\cos\theta = 0$ **(2)**

$\sum \vec{\tau} = I\vec{\alpha}$

$\vec{r}\times\vec{F} + \vec{R}\times\vec{f} = I\vec{\alpha}$

z: $rF - Rf = I\alpha$ **(3)**

For pure roll, $a = -R\alpha$ **(4)**

(1) → $\frac{4}{5}F - \frac{3}{5}mg - f = ma$ **(5)**

(3),(4) → $\frac{3}{5}RF - Rf = \frac{4}{5}mR^2\alpha$

$\Rightarrow \frac{3}{5}F - f = -\frac{4}{5}ma$ **(6)**

(5),(6) → $\frac{1}{5}F - \frac{3}{5}mg = \frac{9}{5}ma$

$\Rightarrow a = \frac{F}{9m} - \frac{g}{3}$, **in x-direction,(7)**

(6),(7) → $f = \frac{3}{5}F + \frac{4}{5}ma$

$\Rightarrow f = \frac{3}{5}F + \frac{4}{5}m\left(\frac{F}{9m} - \frac{g}{3}\right)$

$\Rightarrow f = \frac{31}{45}F - \frac{4}{15}mg$ **(8)**