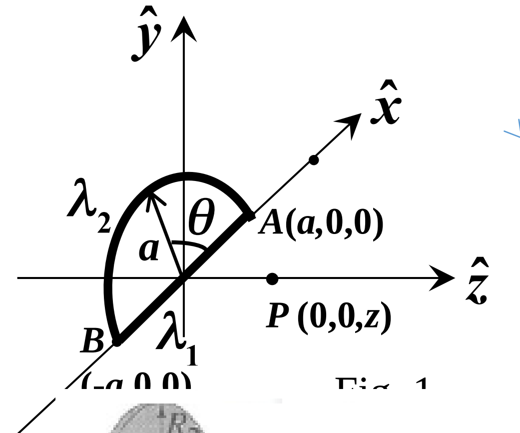
General Physics II: Hw 2

Problem 1

Fig. 1 shows two line charge distributions in the ***x-y*** plane. The charge density is $\boldsymbol{\lambda_1 = \lambda_0(1 - x/a)}$ for the rod on ***x***-axis (**-*a*** < ***x*** < ***a***) and $\boldsymbol{\lambda_2 = \lambda_0 \sin\theta}$ for the semicircle. Here ***a*** is the radius of the semi-circle, λ_0 is a positive constant and θ is the angle from +***x***-axis.

(a) (11 pts) Evaluate the electric field (***x*-, *y*-,** and ***z***-components) and the potential at point ***P*** on the ***z***-axis due to the AB line segment .

(b) (9 pts) Evaluate the electric field (***x*-, *y*-,** and ***z***-components) and the potential at point ***P*** on the ***z***-axis due to the semicircle in Fig. 1.



Problem 2

The x axis is the symmetry axis of a non-conducting flat ring of inner radius $R_1$and outer radius R_2, carrying a uniform surface charge density ($\sigma > 0$) (as show in Fig. 2). Find the electric field at point P.

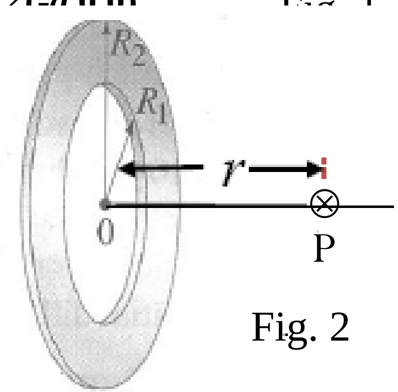


Fig. 2

Problem 3

(a) Determine the electric field at the point $\mathbf{P_1}$ and $\mathbf{P_2}$ as shown in Fig. 3 . The two charges are separated by a distance of **2a**.

(b) When the distance from the field point ($\mathbf{P_1}$ or $\mathbf{P_2}$) to the center of the dipole (O) are very large **y >> a** for $\mathbf{P_1}$ or **x>>a** for $\mathbf{P_2}$, show that the magnitude of the electric field is given by $\mathbf{2kp/r^3}$ and $\mathbf{kp/r^3}$ for $\mathbf{P_1}$ and $\mathbf{P_2}$, respectively. (**p = 2qa** is the dipole moment of the dipole)

Hint : use $(1+x)^n \approx 1+nx$ for **x << 1** .

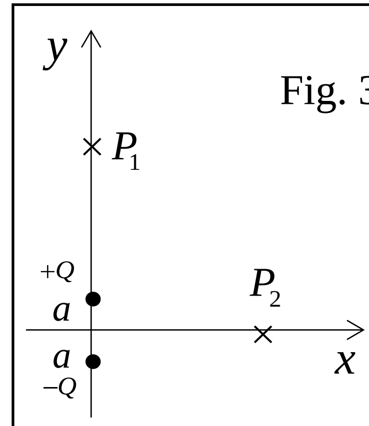


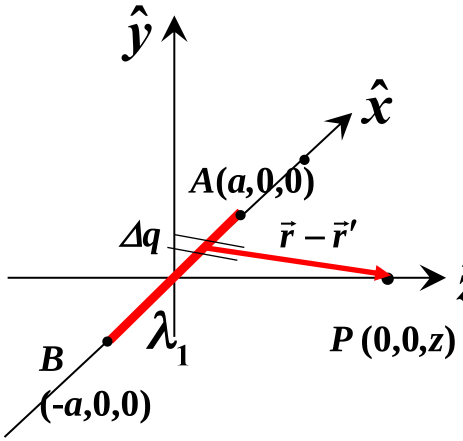
Fig. 3

Problem 1:

$$d\vec{E} = \frac{kdq}{\left|\vec{r} - \vec{r}'\right|^3}\left(\vec{r} - \vec{r}'\right)$$

1. For the E-field results from the charge on AB line segment,

$$dq = \lambda_1 dx' = \lambda_0\left(1 - \frac{x'}{a}\right)dx' \quad ;$$



$$\vec{r} = (0,0,z), \quad \vec{r}' = (x',0,0)$$

$$\left|\vec{r} - \vec{r}'\right| = \left|(-x',0,z)\right| = \sqrt{x'^2 + z^2}$$

$$\vec{E}^{(1)} = \int d\vec{E} = \int_{-a}^{a} \frac{kdq}{\left|\vec{r} - \vec{r}'\right|^3}\left(\vec{r} - \vec{r}'\right) = k\int_{-a}^{a} \frac{\lambda_0\left(1 - \frac{x'}{a}\right)dx'}{\left(x'^2 + z^2\right)^{3/2}}(-x',0,z)$$

0, odd function

$$E_x^{(1)} = k\int_{-a}^{a}\frac{-\lambda_0 x' dx'}{\left(x'^2+z^2\right)^{3/2}} + \frac{k\lambda_0}{a}\int_{-a}^{a}\frac{x'^2 dx'}{\left(x'^2+z^2\right)^{3/2}} = \frac{2k\lambda_0}{a}\int_{0}^{a}\frac{x'^2 dx'}{\left(x'^2+z^2\right)^{3/2}}$$

$$= \frac{2k\lambda_0}{a}\left[\frac{-a}{\sqrt{a^2+z^2}} + \ln\left(\frac{a+\sqrt{a^2+z^2}}{z}\right)\right]$$

$$\int\frac{x^2 dx}{\left(x^2\pm a^2\right)^{3/2}} = \frac{-x}{\sqrt{x^2\pm a^2}} + \ln\left(x+\sqrt{x^2\pm a^2}\right)$$

$$\boxed{E_y^{(1)} = 0}$$

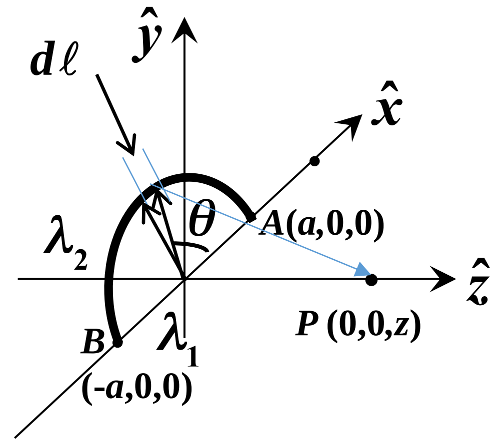
$$E_z^{(1)} = k\int_{-a}^{a}\frac{\lambda_0 z dx'}{\left(x'^2+z^2\right)^{3/2}} - \frac{k\lambda_0 z}{a}\int_{-a}^{a}\frac{x' dx'}{\left(x'^2+z^2\right)^{3/2}} = 2k\lambda_0 z\int_{0}^{a}\frac{dx'}{\left(x'^2+z^2\right)^{3/2}}$$

$$\boxed{= 2k\lambda_0\frac{a}{z\sqrt{a^2+z^2}}}$$

0, odd function

$$\int\frac{dx}{\left(x^2\pm a^2\right)^{3/2}} = \frac{\pm x}{a^2\sqrt{x^2\pm a^2}}$$

2. For the E-field results from the charge on AB semi-circle,



$$dq = \lambda_2 d\ell = \lambda_0 \sin\theta (a d\theta)$$

$$\vec{r} = (0,0,z), \quad \vec{r}' = (x',y',0) = (a\cos\theta, a\sin\theta, 0)$$

$$\left|\vec{r} - \vec{r}'\right| = \left|(-x', -y', z)\right| = \sqrt{a^2 + z^2}$$

$$\vec{E}^{(2)} = \int_0^\pi \frac{k\,dq}{\left|\vec{r} - \vec{r}'\right|^3}\left(\vec{r} - \vec{r}'\right) = k\int_0^\pi \frac{\lambda_0 a \sin\theta\, d\theta}{\left(a^2 + z^2\right)^{3/2}}(-a\cos\theta, -a\sin\theta, z)$$

$$E_x^{(2)} = \frac{-k\lambda_0 a^2}{\left(a^2 + z^2\right)^{3/2}}\int_0^\pi \cos\theta \sin\theta\, d\theta = 0$$

$$E_z^{(2)} = \frac{k\lambda_0 a z}{\left(a^2 + z^2\right)^{3/2}}\int_0^\pi \sin\theta\, d\theta = \frac{2k\lambda_0 a z}{\left(a^2 + z^2\right)^{3/2}}$$

$$E_y^{(2)} = \frac{-k\lambda_0 a^2}{\left(a^2 + z^2\right)^{3/2}}\int_0^\pi \sin^2\theta\, d\theta = \frac{-\pi k\lambda_0 a^2}{2\left(a^2 + z^2\right)^{3/2}}$$

Problem 2:

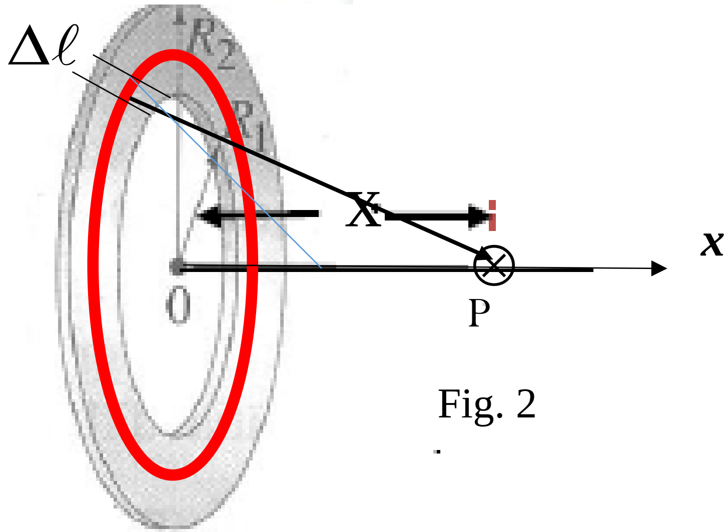


Fig. 2

For line segment $\Delta\ell$, only the x – component remains after integration over a circle. So the electric field due to the ring is

$$\left(\vec{E}\right)_x = \frac{k\,x 2\pi R\cdot\lambda}{(x^2+R^2)^{3/2}} = \frac{k\,xQ}{(x^2+R^2)^{3/2}}$$

Now we need to transform this equation for the ring with width ΔR, shown in Fig. 2,

$$Q \rightarrow \Delta Q = \sigma\Delta A = 2\pi R dR$$

$$\Rightarrow d\vec{E} = \frac{kx}{(x^2+R^2)^{3/2}}(\sigma 2\pi r dr)\hat{x}$$

$$E = kz\pi\sigma\int_{R_1}^{R_2}\frac{2RdR}{(x^2+R^2)^{3/2}}$$

變數變換:

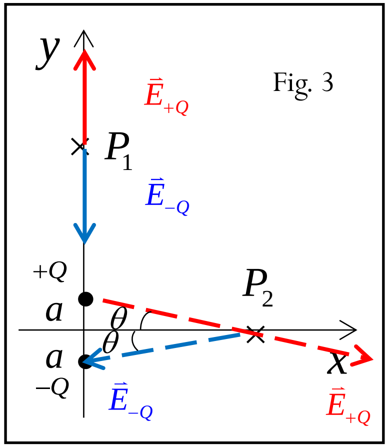
$$\begin{cases} u = x^2+R^2 \\ du = 2RdR \end{cases}$$

$$= kz\pi\sigma\int_{r=0}^{r=R}\frac{du}{u^{3/2}} = kz\pi\sigma\left[\frac{u^{-1/2}}{-1/2}\right]_{R=R_1}^{R=R_2}$$

$$\Rightarrow \vec{E} = 2\pi k\sigma\left[\frac{x}{\sqrt{x^2+R_2{}^2}} - \frac{x}{\sqrt{x^2+R_1{}^2}}\right]\hat{x}$$

Problem 3

$$(1+x)^n \approx 1+nx \quad \text{For } 0\le x \ll 1$$



Sol(a):

Suppose $P_1(0,y)$ $P_2(x,0)$

$$P_1 : \vec{E} = \vec{E}_{+Q} + \vec{E}_{-Q} = \frac{kQ}{(y-a)^2}\hat{y} - \frac{kQ}{(y+a)^2}\hat{y}$$

$$P_2 : \vec{E} = \vec{E}_{+Q} + \vec{E}_{-Q} = \frac{kQ}{x^2+a^2}\sin\theta(-\hat{y}) + \frac{kQ}{x^2+a^2}\sin\theta(-\hat{y}) = \frac{2akQ}{\left(x^2+a^2\right)^{\frac{3}{2}}}(-\hat{y})$$

Sol(b):

$$P_1 : \vec{E} = E\hat{y} = \left\{\frac{akQ}{(y-a)^2} - \frac{akQ}{(y+a)^2}\right\}\hat{y}$$

$$\approx \left\{\frac{akQ}{y^2}\left(1+2\frac{a}{y}\right) - \frac{akQ}{y^2}\left(1-2\frac{a}{y}\right)\right\}\hat{y}$$

$$= \frac{4akQ}{y^3}\hat{y} = \frac{2kp}{y^3}\hat{y}$$

$$P_2 : \vec{E} = \frac{2akQ}{\left(x^2+a^2\right)^{\frac{3}{2}}}(-\hat{y}) = \frac{2akQ}{x^3}\left(1+\frac{a^2}{x^2}\right)^{\frac{3}{2}}(-\hat{y})$$

$$\approx \frac{2akQ}{x^3}\left(1-\frac{3}{2}\frac{a^2}{x^2}\right)(-\hat{y}) \approx \frac{2akQ}{x^3}(-\hat{y})$$

$$= \frac{kp}{x^3}(-\hat{y})$$