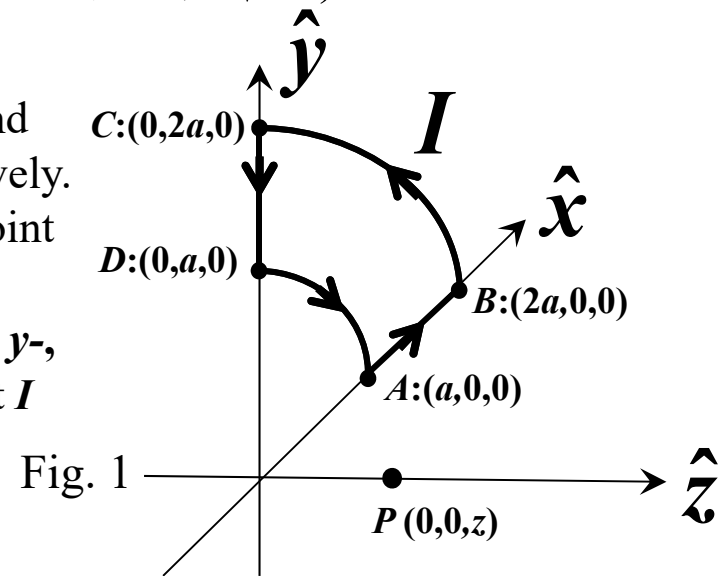


試卷請註明、姓名、班級、學號，請遵守考場秩序

I. 計算題(50 points) (所有題目必須有計算過程,否則不予計分)

- (10 pts) Fig. 1 shows a wire loop with current I from $A \rightarrow B \rightarrow C \rightarrow D \rightarrow A$ in x - y plane. The sections from $B \rightarrow C$ and $D \rightarrow A$ are two quarter circles with radius $2a$ and a , respectively. Evaluate the magnetic field (x -, y -, and z -components) at point P on the z -axis due to the current I from $A \rightarrow B$ and $C \rightarrow D$.
- (10 pts) same as problem 1, evaluate the magnetic field (x -, y -, and z -components) at point P on the z -axis due to the current I from $B \rightarrow C$ and $D \rightarrow A$.

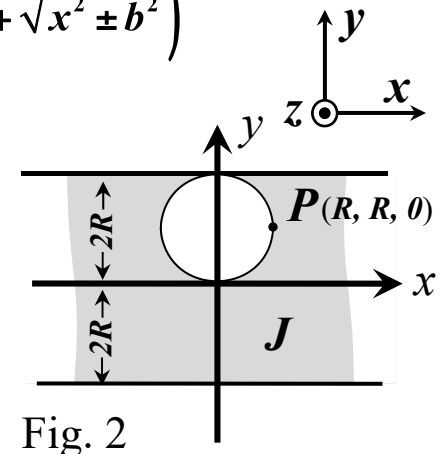


Integration Formula for reference

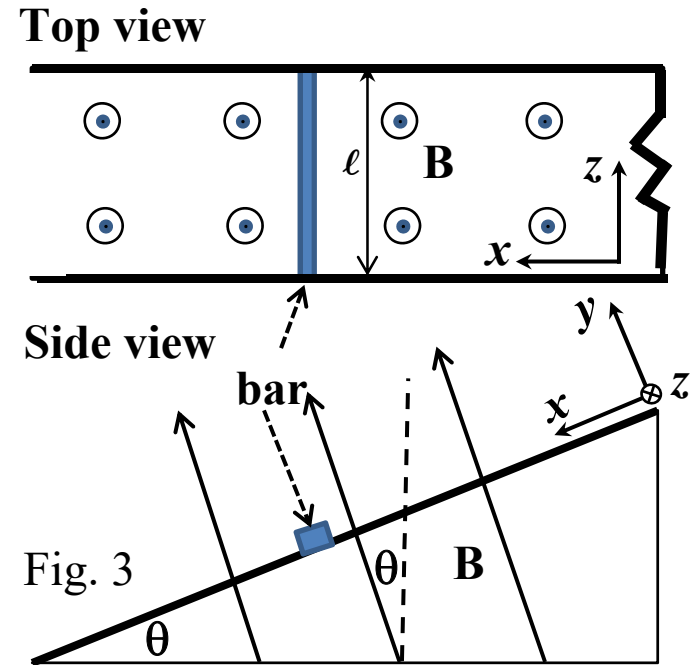
$$\int \frac{dx}{\sqrt{x^2 \pm b^2}} = \ln\left(x + \sqrt{x^2 \pm b^2}\right) \quad \int \frac{dx}{(x^2 \pm b^2)^{3/2}} = \frac{\pm x}{b^2 \sqrt{x^2 \pm b^2}} \quad \int \frac{xdx}{(x^2 \pm a^2)^{3/2}} = \frac{-1}{\sqrt{x^2 \pm a^2}}$$

$$\int \frac{x^2 dx}{\sqrt{x^2 \pm b^2}} = \frac{x\sqrt{x^2 \pm b^2}}{2} - \frac{b^2}{2} \ln\left(x + \sqrt{x^2 \pm b^2}\right) \quad \int \frac{x^2 dx}{(x^2 \pm b^2)^{3/2}} = \frac{-x}{\sqrt{x^2 \pm b^2}} + \ln\left(x + \sqrt{x^2 \pm b^2}\right)$$

- (15 pts) As shown in Fig. 2, an infinite conducting plate with thickness $4R$ carries a uniform current density J in $+z$ -direction, and in the plate there is an infinitely long hollow cylindrical region with radius R and its cylindrical axis passes the y -axis at $(0, R, 0)$. Determine (a) (10 pts) The magnitude and the direction of the B-field on the y -axis for $(0 \leq y < \infty)$. (b) (5 pts) The direction and magnetic of the B-field at point P.

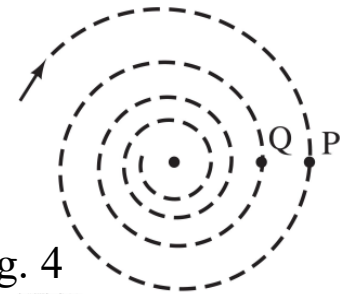


4. (15 pts) A conducting bar of mass m slides down two frictionless conducting rails which make an angle θ with respect to the horizontal and one end with a resistor R , and the distance between two rails is ℓ , as shown in Fig. 3. A uniform magnetic field \mathbf{B} is applied with an angle θ with respect to vertical. The bar is released from the top with zero velocity. Draw the free body diagram of the conducting bar as it slides down the rail. Find the velocity of the metal bar as a function of time. what is the terminal velocity v_T of the bar?



II. 選擇題(50 points)

1. (5 pts) A proton (質子) follows a spiral path through a gas in a magnetic field of 0.04 T , perpendicular to the plane of the spiral, as shown in Fig. 4. In two successive loops, at points P and Q , the radii are 9.0 mm and 7.0 mm , respectively. The change in the kinetic energy of this particle as it travels from P to Q is $a \text{ eV}$. What is the value a ? ($m_p = 1.6 \times 10^{-26} \text{ kg}$, $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$, Proton Charge = $1.6 \times 10^{-19} \text{ C}$)



- (A) $a < -10$ (B) $-10 \leq a < -5$ (C) $-5 \leq a < -1$ (D) $-1 \leq a < -0.5$ (E) $-0.5 \leq a < -0.25$
 (F) $-0.25 \leq a < 0.25$ (G) $0.25 \leq a < 0.5$ (H) $0.5 \leq a < 1$ (J) $1 \leq a < 5$ (K) $5 \leq a < 10$ (L) $10 \leq a$
2. (5 pts) A R-C circuit, shown in Fig. 5, consists of $R_1 = R = 5 \Omega$, $R_2 = 10 \Omega$, two batteries $\varepsilon_1 = 30 \text{ V}$ and $\varepsilon_2 = 10 \text{ V}$, and a capacitor $C = 3 \mu\text{F}$. The capacitor is initially uncharged. The switch S is closed at $t = 0$. Immediately after the switch is closed, what are the currents i_1 , i_2 ? During this charging process, what is the time constant τ of the circuit?

- (A) $i_1 = 1 \text{ A}$, $i_2 = 1 \text{ A}$, $\tau = 10 \mu\text{s}$; (B) $i_1 = 1 \text{ A}$, $i_2 = 2 \text{ A}$, $\tau = 10 \mu\text{s}$;
 (C) $i_1 = 2 \text{ A}$, $i_2 = 1 \text{ A}$, $\tau = 10 \mu\text{s}$; (D) $i_1 = 2 \text{ A}$, $i_2 = 2 \text{ A}$, $\tau = 10 \mu\text{s}$;
 (E) $i_1 = 1 \text{ A}$, $i_2 = 1 \text{ A}$, $\tau = 25 \mu\text{s}$; (F) $i_1 = 1 \text{ A}$, $i_2 = 2 \text{ A}$, $\tau = 25 \mu\text{s}$;
 (G) $i_1 = 2 \text{ A}$, $i_2 = 1 \text{ A}$, $\tau = 25 \mu\text{s}$; (H) $i_1 = 2 \text{ A}$, $i_2 = 2 \text{ A}$, $\tau = 25 \mu\text{s}$.

3. (5 pts) As shown in Fig. 6(a), an infinite plane parallel to the y - z plane crossing the x -axis at x_1 contains a constant current density $J_1 \cdot \hat{z}$, and another infinite plane at x_2 contains a constant current density $J_2 \cdot \hat{z}$. Fig. 6(b) shows the y -component of the B field as a function of x -coordinate in space. Let $\alpha = J_1/J_2$, which of the following is correct?

- (A) $\alpha < -3$ (B) $-3 \leq \alpha < -1$ (C) $-1 \leq \alpha < -1/2$ (D) $-1/2 \leq \alpha < 0$
 (E) $0 \leq \alpha < 1/2$ (F) $1/2 \leq \alpha < 1$ (G) $1 \leq \alpha < 3$ (H) $3 \leq \alpha$

4. (5 pts) A wire loop is situated inside a large solenoid, with the plane of the loop initially perpendicular to the axis of the solenoid. Current can be generated to flow through the loop of wire if

- (A) A constant current is flowing through the solenoid coil.
 (B) the current flowing through the solenoid is decreasing with time.
 (C) The wire loop is rotating within the solenoid, with a constant current flowing through the solenoid coil.
 (D) (A) and (B). (E) (A) and (C). (F) (B) and (C)
 (G) All above. (H) None above.

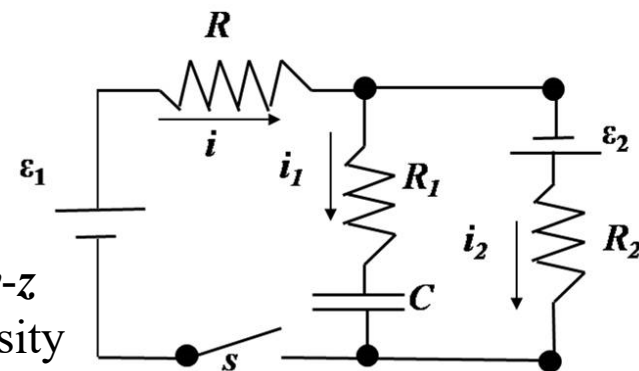


Fig. 5

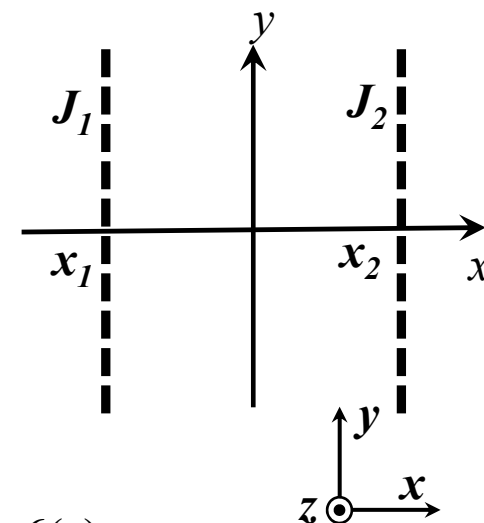


Fig. 6(a)

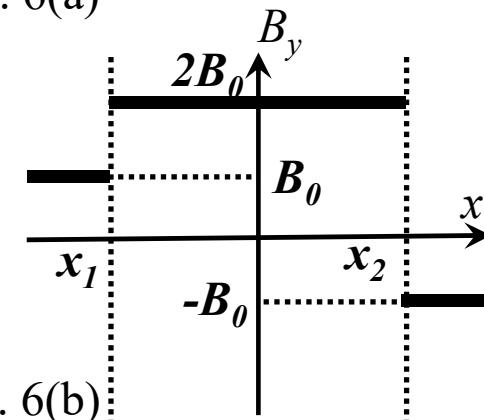


Fig. 6(b)

5. (5pts) Fig.7 shows the cross section of an infinitely long conducting rod along the z-axis (out of page), which carries a uniform current density J ($J > 0$) in the +z-direction. Consider two closed loops (dotted loops in the figure) labelled as **1** and **2** on the x-y plane : Loop **1** has the same elliptical shape as the conductor, and Loop **2** is a circle with the center at the origin. Let $\vec{B}_1(x,y,z)$ and $\vec{B}_2(x,y,z)$ be the B-field at each point along loop **1** and loop **2**, respectively, and

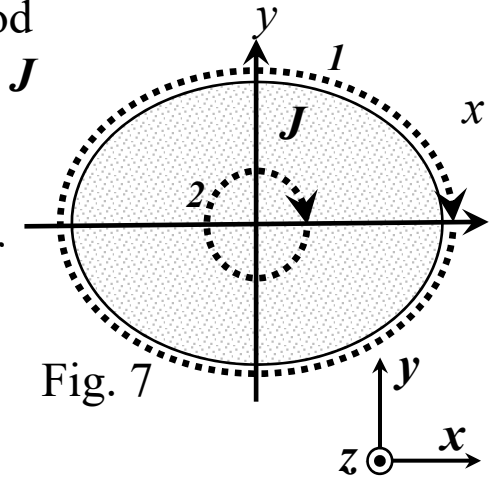


Fig. 7

$$K_1 = \oint_{\text{Loop1}} \vec{B}_1 \cdot d\vec{\ell}, \text{ and } K_2 = \oint_{\text{Loop2}} \vec{B}_2 \cdot d\vec{\ell}$$

Which of the following statement is correct?

- (A) $|\vec{B}_1| = \text{constant}$, $K_1 = 0$, $|\vec{B}_2| = \text{constant}$, $K_2 = 0$ (B) $|\vec{B}_1| = \text{constant}$, $K_1 \neq 0$, $|\vec{B}_2| \neq \text{constant}$, $K_2 = 0$
 (C) $|\vec{B}_1| = \text{constant}$, $K_1 \neq 0$, $|\vec{B}_2| \neq \text{constant}$, $K_2 \neq 0$ (D) $|\vec{B}_1| = \text{constant}$, $K_1 \neq 0$, $|\vec{B}_2| = \text{constant}$, $K_2 \neq 0$
 (E) $|\vec{B}_1| \neq \text{constant}$, $K_1 \neq 0$, $|\vec{B}_2| \neq \text{constant}$, $K_2 = 0$ (F) $|\vec{B}_1| \neq \text{constant}$, $K_1 \neq 0$, $|\vec{B}_2| = \text{constant}$, $K_2 \neq 0$
 (G) $|\vec{B}_1| \neq \text{constant}$, $K_1 \neq 0$, $|\vec{B}_2| \neq \text{constant}$, $K_2 \neq 0$

6. (5pts) Fig. 8 shows a wood cylinder of mass $m = 0.25 \text{ kg}$ and length $L = 0.1 \text{ m}$, with $N = 10$ turns of the wire wrapped around it longitudinally, so that the plane of the wire contains the long central axis of the cylinder. The cylinder is released on a plane inclined at an angle θ to the horizontal, with the plane of the coil parallel to the incline plane. If there is a vertical uniform magnetic field of magnitude 0.5 T , what is the least current i through the coil that keeps the cylinder from rolling down the plane? ($g = 10 \text{ m/s}^2$)

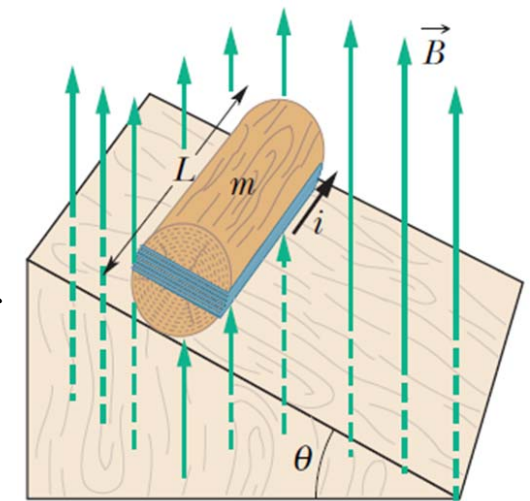


Fig. 8

- (A) 0.5; (B) 1.0; (C) 1.5; (D) 2.0; (E) 2.5; (F) 3.0 A.

1	2	3	4	5	6	7	8	9	10
E	H	D	F	G	E	B	C	G	B
11	12	13	14	15	16				
F	E	C	A	D	B				

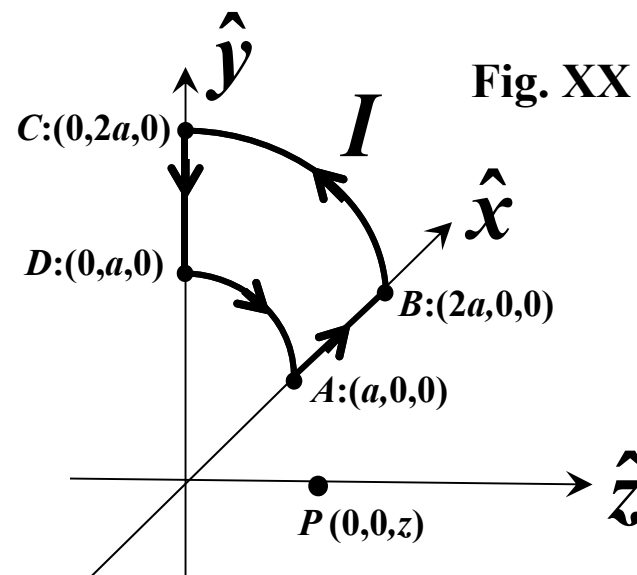
- (10 pts) Fig. XX shows a wire with current I from $A \rightarrow B \rightarrow C \rightarrow D \rightarrow A$ in x - y plane. The sections from $B \rightarrow C$ and $D \rightarrow A$ are two quarter circles with radius $2a$ and a . Evaluate the magnetic field (x -, y -, and z -components) at point P on the z -axis due to the current I from $A \rightarrow B$ and $C \rightarrow D$.
- (10 pts) Same as problem 1, evaluate the magnetic field (x -, y -, and z -components) at point P on the z -axis due to the current I from $B \rightarrow C$ and $D \rightarrow A$.

$$\int \frac{dx}{\sqrt{x^2 \pm b^2}} = \ln\left(x + \sqrt{x^2 \pm b^2}\right)$$

$$\int \frac{dx}{(x^2 \pm b^2)^{3/2}} = \frac{\pm x}{b^2 \sqrt{x^2 \pm b^2}} \quad \int \frac{x dx}{(x^2 \pm a^2)^{3/2}} = \frac{-1}{\sqrt{x^2 \pm a^2}}$$

$$\int \frac{x^2 dx}{\sqrt{x^2 \pm b^2}} = \frac{x \sqrt{x^2 \pm b^2}}{2} - \frac{b^2}{2} \ln\left(x + \sqrt{x^2 \pm b^2}\right)$$

$$\int \frac{x^2 dx}{(x^2 \pm b^2)^{3/2}} = \frac{-x}{\sqrt{x^2 \pm b^2}} + \ln\left(x + \sqrt{x^2 \pm b^2}\right)$$



Problem 1

10 pts

$$\vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi} \int \frac{Id\vec{r}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \quad 1 \text{ pts}$$

(a). For the B-field results from the current from $A \rightarrow B$,

$$\vec{r} = (0, 0, z), \quad \vec{r}' = (x', 0, 0), \quad d\vec{r}' = \hat{x}dx' \quad 1 \text{ pts}$$

$$d\vec{r}' \times (\vec{r} - \vec{r}') = \hat{x}dx' \times (-x'\hat{x} + z\hat{z}) = \hat{y}(-zdx') \quad 1 \text{ pts}$$

$$|\vec{r} - \vec{r}'| = |(-x', 0, z)| = \sqrt{x'^2 + z^2} \quad 1 \text{ pts}$$

$$\vec{B}_{A \rightarrow B} = \int d\vec{B} = \frac{\mu_0}{4\pi} \int_a^{2a} \frac{Id\vec{r}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} = \frac{\mu_0 I}{4\pi} \hat{y} \int_a^{2a} \frac{-zdx'}{(x'^2 + z^2)^{3/2}} \quad 1 \text{ pts}$$

$$\int \frac{dx}{(x^2 \pm b^2)^{3/2}} = \frac{\pm x}{b^2 \sqrt{x^2 \pm b^2}}$$

$$= \frac{\mu_0 I}{4\pi} \hat{y} \left(\frac{-zx'}{z^2 \sqrt{x'^2 + z^2}} \right)_{x'=a}^{2a} = \frac{\mu_0 I}{4\pi z} \hat{y} \left[\frac{a}{\sqrt{z^2 + a^2}} - \frac{2a}{\sqrt{z^2 + 4a^2}} \right]$$

1 pts

(b). For the B-field results from the current from $C \rightarrow D$,

$$\vec{r} = (0, 0, z), \quad \vec{r}' = (0, y', 0), \quad d\vec{r}' = \hat{y} dy' \quad \boxed{1 \text{ pts}}$$

$$d\vec{r}' \times (\vec{r} - \vec{r}') = \hat{y} dy' \times (-y' \hat{y} + z \hat{z}) = \hat{x} (z dy') \quad \boxed{1 \text{ pts}}$$

$$|\vec{r} - \vec{r}'| = |(0, y', z)| = \sqrt{y'^2 + z^2} \quad \boxed{1 \text{ pts}}$$

$$\vec{B}_{C \rightarrow D} = \int d\vec{B} = \frac{\mu_0}{4\pi} \int_{2a}^a \frac{I d\vec{r}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} = \frac{\mu_0 I}{4\pi} \hat{x} \int_{2a}^a \frac{z dy'}{(y'^2 + z^2)^{3/2}} \quad \boxed{1 \text{ pts}} \quad \int \frac{dx}{(x^2 \pm b^2)^{3/2}} = \frac{\pm x}{b^2 \sqrt{x^2 \pm b^2}}$$

$$= \frac{\mu_0 I}{4\pi} \hat{x} \left(\frac{zy'}{z^2 \sqrt{y'^2 + z^2}} \right)_{y'=2a}^a = \frac{\mu_0 I}{4\pi z} \hat{x} \left[\frac{a}{\sqrt{z^2 + a^2}} - \frac{2a}{\sqrt{z^2 + 4a^2}} \right] \quad \boxed{1 \text{ pts}}$$

Problem 2

10 pts

(a). For the B-field results from the current from $B \rightarrow C$, (θ : angular from +x-axis

$$\vec{r} = (0, 0, z), \quad \vec{r}' = (x', y', 0), \quad d\vec{r}' = \hat{x}dx' + \hat{y}dy' \quad \boxed{1 \text{ pts}}$$

$$x' = 2a \cos \theta \quad dx' = -2a \sin \theta d\theta \quad \boxed{1 \text{ pts}}$$

$$y' = 2a \sin \theta \quad dy' = 2a \cos \theta d\theta \quad |\vec{r} - \vec{r}'| = |(-x', -y', z)| = \sqrt{4a^2 + z^2} \quad \boxed{1 \text{ pts}}$$

$$d\vec{r}' \times (\vec{r} - \vec{r}') = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ dx' & dy' & 0 \\ -x' & -y' & z \end{vmatrix} = \hat{x}(z dy') + \hat{y}(-z dx') + \hat{z}(x' dy' - y' dx') \quad \boxed{1 \text{ pts}}$$

$$= \hat{x}(2az \cos \theta d\theta) + \hat{y}(2az \sin \theta d\theta) + \hat{z}(4a^2 d\theta)$$

$$\vec{B}_{B \rightarrow C} = \frac{\mu_0 I}{4\pi} \frac{2a}{(4a^2 + z^2)^{3/2}} \left[\hat{x} \int_0^{\pi/2} z \cos \theta d\theta + \hat{y} \int_0^{\pi/2} z \sin \theta d\theta + \hat{z} \int_0^{\pi/2} 2a d\theta \right] \quad \boxed{1 \text{ pts}}$$

$$= \frac{\mu_0 I}{4\pi} \frac{2a}{(4a^2 + z^2)^{3/2}} [\hat{x}z + \hat{y}z + \hat{z}(\pi a)] \quad \boxed{1 \text{ pts}}$$

(b). For the B-field results from the current from $D \rightarrow A$,

$$\vec{r} = (0, 0, z), \quad \vec{r}' = (x', y', 0), \quad d\vec{r}' = \hat{x}dx' + \hat{y}dy'$$

$$\begin{aligned} x' &= a \cos \theta & dx' &= -a \sin \theta d\theta & |\vec{r} - \vec{r}'| &= |(-x', -y', z)| = \sqrt{a^2 + z^2} \\ y' &= a \sin \theta & dy' &= a \cos \theta d\theta \end{aligned} \quad \boxed{1 \text{ pts}}$$

$$\begin{aligned} d\vec{r}' \times (\vec{r} - \vec{r}') &= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ dx' & dy' & 0 \\ -x' & -y' & z \end{vmatrix} = \hat{x}(z dy') + \hat{y}(-z dx') + \hat{z}(x' dy' - y' dx') \\ &= \hat{x}(az \cos \theta d\theta) + \hat{y}(az \sin \theta d\theta) + \hat{z}(a^2 d\theta) \end{aligned} \quad \boxed{1 \text{ pts}}$$

$$\vec{B}_{D \rightarrow A} = \frac{\mu_0 I}{4\pi} \frac{a}{(a^2 + z^2)^{3/2}} \left[\hat{x} \int_{\pi/2}^0 z \cos \theta d\theta + \hat{y} \int_{\pi/2}^0 z \sin \theta d\theta + \hat{z} \int_{\pi/2}^0 a d\theta \right] \quad \boxed{1 \text{ pts}}$$

$$= \frac{\mu_0 I}{4\pi} \frac{-a}{(a^2 + z^2)^{3/2}} \left[\hat{x}z + \hat{y}z + \hat{z} \left(\frac{\pi a}{2} \right) \right] \quad \boxed{1 \text{ pts}}$$

3. (15 pts) As shown in Fig. 2, an infinite conducting plate with thickness $4R$ carries a uniform current density \mathbf{J} in $+\mathbf{z}$ -direction, and in the plate there is a infinitely long hollow cylindrical region with radius R and its cylindrical axis passes the y -axis at $(0, R, 0)$. Determine (a) (10 pts) The magnitude and the direction of the B-field on the y -axis for $(0 \leq y < \infty)$. (b) (5 pts) The direction and magnetic of the B-field at point P.

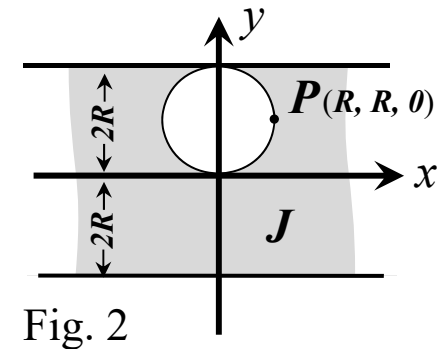
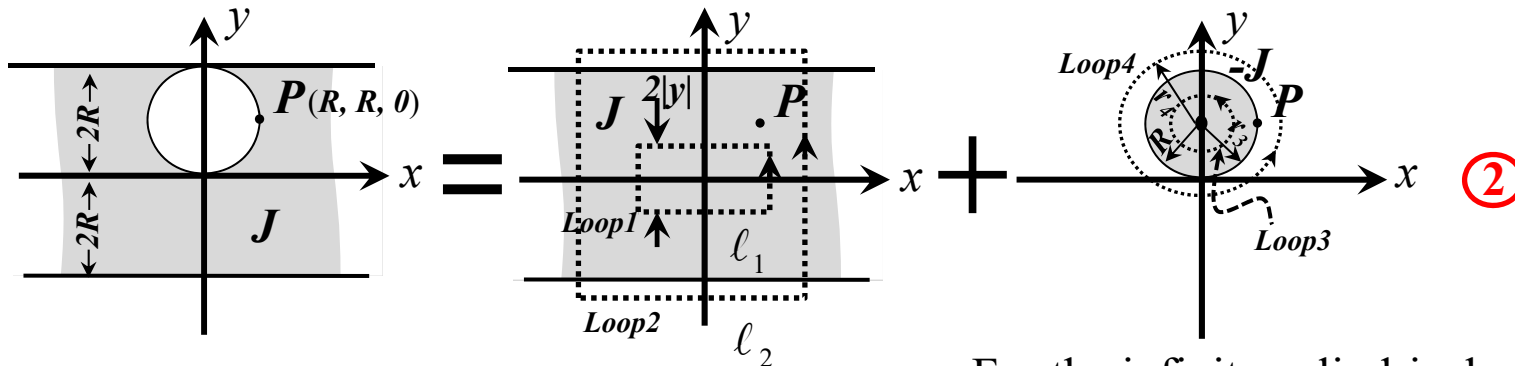


Fig. 2



(a) For the infinite plane block current and for $0 \leq y < 2R$, choose loop 1 (running counter clockwise) to apply Ampere's Law,

$$\oint_{\text{Loop1}} \vec{B} \cdot d\vec{\ell} = \mu_o I_{in}$$

$$\Rightarrow \oint_{\text{Loop1}} \vec{B} \cdot d\vec{\ell} = 2B\ell_1 = \mu_o J \cdot 2|y|\ell_1$$

$$\Rightarrow B = \mu_o J \cdot |y| = \mu_o Jy \Rightarrow \vec{B}(y) = \mu_o Jy(-\hat{x})$$

① ①

For $2R \leq y$, choose loop 2,

$$\oint_{\text{Loop2}} \vec{B} \cdot d\vec{\ell} = \mu_o I_{in} \Rightarrow 2B\ell_2 = \mu_o J(4R)\ell_2$$

$$\Rightarrow B(y) = \mu_o JR \Rightarrow \vec{B}(y) = 2\mu_o JR(-\hat{x})$$

① ①

For the infinite cylindrical current and for $(0, y)$ with $0 \leq |y-R| < R$, choose loop 3,

$$\oint_{\text{Loop3}} \vec{B} \cdot d\vec{\ell} = \mu_o I_{in} \Rightarrow B \cdot 2\pi \cdot r_3 = -\mu_o J\pi \cdot r_3^2$$

$$\Rightarrow B = -\frac{\mu_o J r_3}{2} = -\frac{\mu_o J |y-R|}{2}$$

①

for $0 \leq y < R$, $\vec{B}(y) = \frac{\mu_o J |R-y|}{2} (-\hat{x}) = \frac{\mu_o J (y-R)}{2} \hat{x}$ ①

for $R \leq y < 2R$, $\vec{B}(y) = \frac{\mu_o J |R-y|}{2} \hat{x} = \frac{\mu_o J (y-R)}{2} \hat{x}$

For $(0, y)$ with $2R \leq |y-R|$, choose loop 4,

$$\oint_{\text{Loop4}} \vec{B} \cdot d\vec{\ell} = \mu_o I_{in} \Rightarrow B \cdot 2\pi \cdot r_4 = -\mu_o J\pi \cdot R^2$$

$$\Rightarrow B = -\frac{\mu_o J \cdot R^2}{2r_4} = -\frac{\mu_o J \cdot R^2}{2|y-R|}$$

$$\text{For } 2R \leq y, \quad \vec{B}(y) = \frac{\mu_o J \cdot R^2}{2|y-R|} \hat{x} = \frac{\mu_o J \cdot R^2}{2(y-R)} \hat{x} \quad \textcircled{1}$$

For the total B-field,

for $0 \leq y < 2R$,

$$\vec{B}_{tot}(y) = \frac{\mu_o J(y-R)}{2} \hat{x} + \mu_o Jy(-\hat{x}) = -\frac{\mu_o J(y+R)}{2} \hat{x}$$

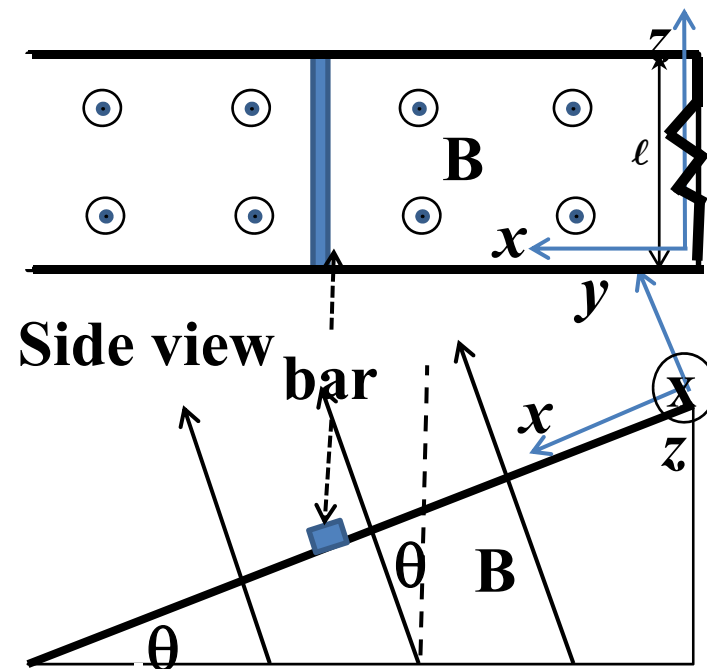
for $2R \leq y$,

$$\vec{B}_{tot}(y) = \frac{\mu_o J \cdot R^2}{2(y-R)} \hat{x} + 2\mu_o JR(-\hat{x}) = \mu_o J \frac{5R^2 - 4yR}{2(y-R)} \hat{x}$$

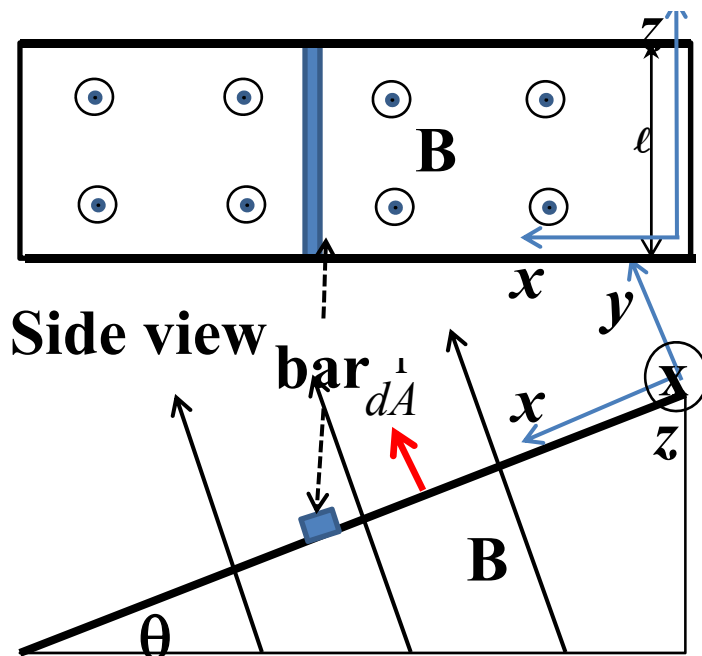
(b) For the B-field at P (R,R),

$$\begin{aligned} \vec{B}_P &= \frac{\mu_o J r_3}{2} (-\hat{y}) + \mu_o JR(-\hat{x}) \\ &= \frac{\mu_o JR}{2} (-\hat{y}) + \mu_o JR(-\hat{x}) \\ &= (-\mu_o JR, -\frac{\mu_o JR}{2}, 0) \end{aligned}$$

4. (15 pts) A conducting bar of mass m slides down two frictionless conducting rails which make an angle θ with the horizontal and one end with a resistor R , and the distance between two rails is ℓ , as shown in Fig. 4. A uniform magnetic field \mathbf{B} is applied with an angle θ with respect to vertical. The bar is released from the top with velocity $\mathbf{0}$. Draw the free body diagram of the conducting bar as it slides down the rail. Find the velocity of the metal bar as a function of time. What is the terminal velocity v_T of the bar?



4.



The movement of the bar $\rightarrow \Phi_B$ changes

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = B(\ell \cdot x(t)) \quad \boxed{1 \text{ pts}}$$

The change of Φ_B induces emf \mathcal{E} :

$$\left| \frac{d\Phi_B}{dt} \right| = \mathcal{E} = B\ell \frac{dx(t)}{dt} = B\ell v(t) \quad \boxed{1 \text{ pts}}$$

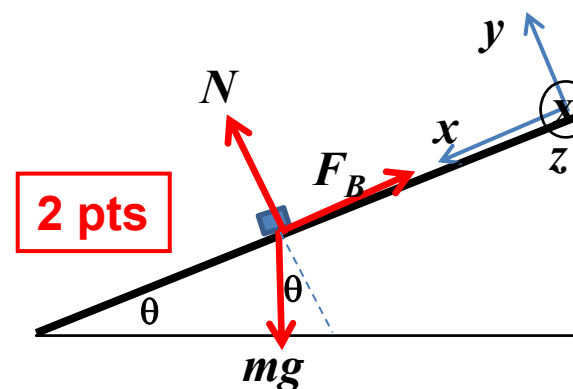
\mathcal{E} induces current I :

$$I = \frac{\mathcal{E}}{R} = \frac{B\ell v(t)}{R} \quad \boxed{1 \text{ pt}} \quad \text{Top view: 順時針}$$

the bar with I in $B \rightarrow$ magnetic force. $\boxed{1 \text{ pt}}$

$$\vec{F}_B = I\vec{\ell} \times \vec{B} = \frac{B^2 \ell^2 v(t)}{R} \quad \boxed{2 \text{ pt}}$$

Side view
Direction:



$$x: \quad mg \sin \theta - F_B = ma = m \frac{dv}{dt} \quad \boxed{1 \text{ pts}}$$

$$y: \quad N - mg \cos \theta = 0$$

When the bar has reach “terminal velocity” v_T , there is no acceleration.

$$\text{i.e.} \quad mg \sin \theta - \frac{B^2 \ell^2}{R} v_T = 0 \rightarrow v_T = \frac{R \cdot mg \sin \theta}{B^2 \ell^2} \quad \boxed{2 \text{ pts}}$$

Solve the equation of motion:

$$mg \sin \theta - \frac{B^2 \ell^2}{mR} v(t) = \frac{v(t) - v_0}{\tau}$$

$$\text{where} \quad \tau = \frac{mR}{B^2 \ell^2}, \quad v_0 = \frac{mgR \sin \theta}{B^2 \ell^2} = v_T \quad \boxed{2 \text{ pt}}$$

$$\int_0^v \frac{dv}{v - v_T} = - \int_0^t \frac{dt}{\tau} \rightarrow \ln \left| \frac{v - v_T}{-v_T} \right| = - \frac{t}{\tau}$$

$$\rightarrow v(t) = v_T (1 - e^{-t/\tau}) \quad \boxed{2 \text{ pts}}$$