Modeling a Zombie Apocalypse: Simulation and Analysis

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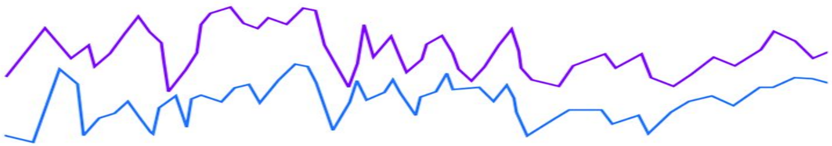
Why I chose this topic:

- Real World Applications
- Rich in Variables
- Interdisciplinary Thinking Approach
- Engaging and Relatable Scenario

How can we simulate a zombie apocalypse?



- Use of Dynamical Systems
- Locate Equilibrium Points
- Create code based simulations (MATLAB)
- Model out different scenarios
- Analyze Results / Make Tweaks

Understanding the System

$$\frac{dS}{dt} = -\beta SI + \gamma S\left(1 - \frac{S}{K}\right) - \mu S$$

$$\frac{dI}{dt} = \beta SI - \gamma I - cS\left(\frac{I}{K+I}\right)$$

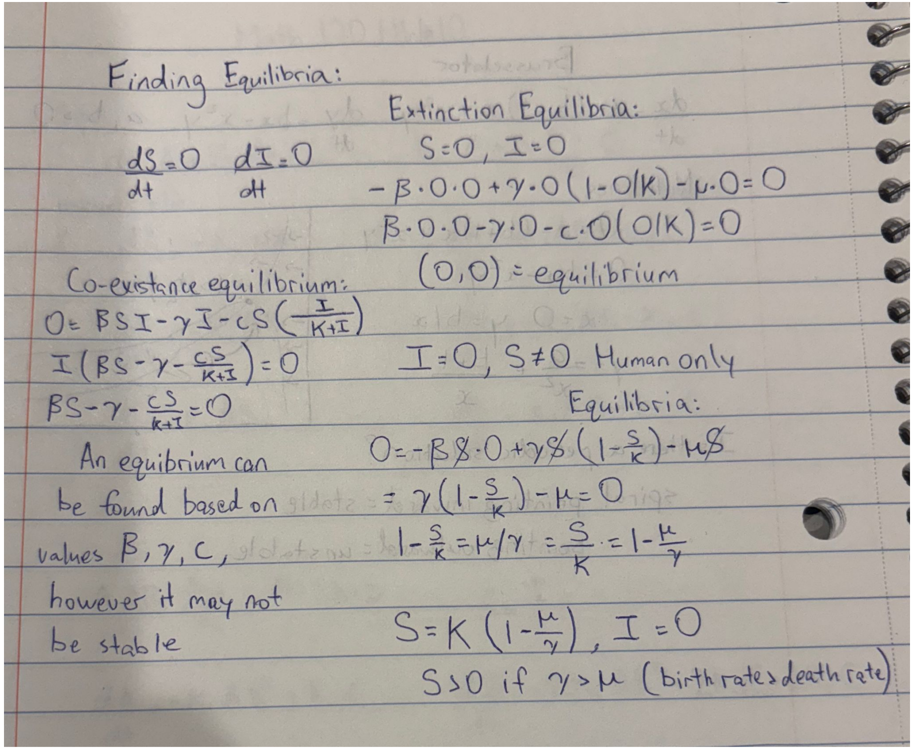
Lotka Volterra Model (predator prey model)

- **S(t): Human Population**
- **I(t): Zombie Population**

Parameters:

- **β: Infection rate**
- **γ: Human birth rate**
- **μ: Human natural death rate**
- **K: Carrying capacity**
- **c: Human attack rate on zombies**

Finding Equilibria



Finding Equilibria:

$\frac{dS}{dt}=0$ $\frac{dI}{dt}=0$

Extinction Equilibria:

$S=0,\ I=0$

$-\beta\cdot 0\cdot 0+\gamma\cdot 0(1-0/K)-\mu\cdot 0=0$

$\beta\cdot 0\cdot 0-\gamma\cdot 0-c\cdot 0(0/K)=0$

$(0,0)$ = equilibrium

Co-existance equilibrium:

$0=\beta SI-\gamma I-cS\left(\frac{I}{K+I}\right)$

$I\left(\beta S-\gamma-\frac{cS}{K+I}\right)=0$

$\beta S-\gamma-\frac{cS}{K+I}=0$

An equibrium can be found based on values β, γ, c, however it may not be stable

$I=0,\ S\neq 0$ Human only Equilibria:

$0=-\beta \cancel{S}\cdot 0+\gamma \cancel{S}\left(1-\frac{S}{K}\right)-\mu \cancel{S}$

$=\gamma\left(1-\frac{S}{K}\right)-\mu=0$

$1-\frac{S}{K}=\mu/\gamma$; $\frac{S}{K}=1-\frac{\mu}{\gamma}$

$S=K\left(1-\frac{\mu}{\gamma}\right),\ I=0$

$S>0$ if $\gamma>\mu$ (birth rate > death rate)

- **Extinction Equilibria** ✓
- **Human Only Equilibria** ✓
- **Coexistence Equilibria**

Zombie Apocalypse Scenarios:

Baseline Parameters

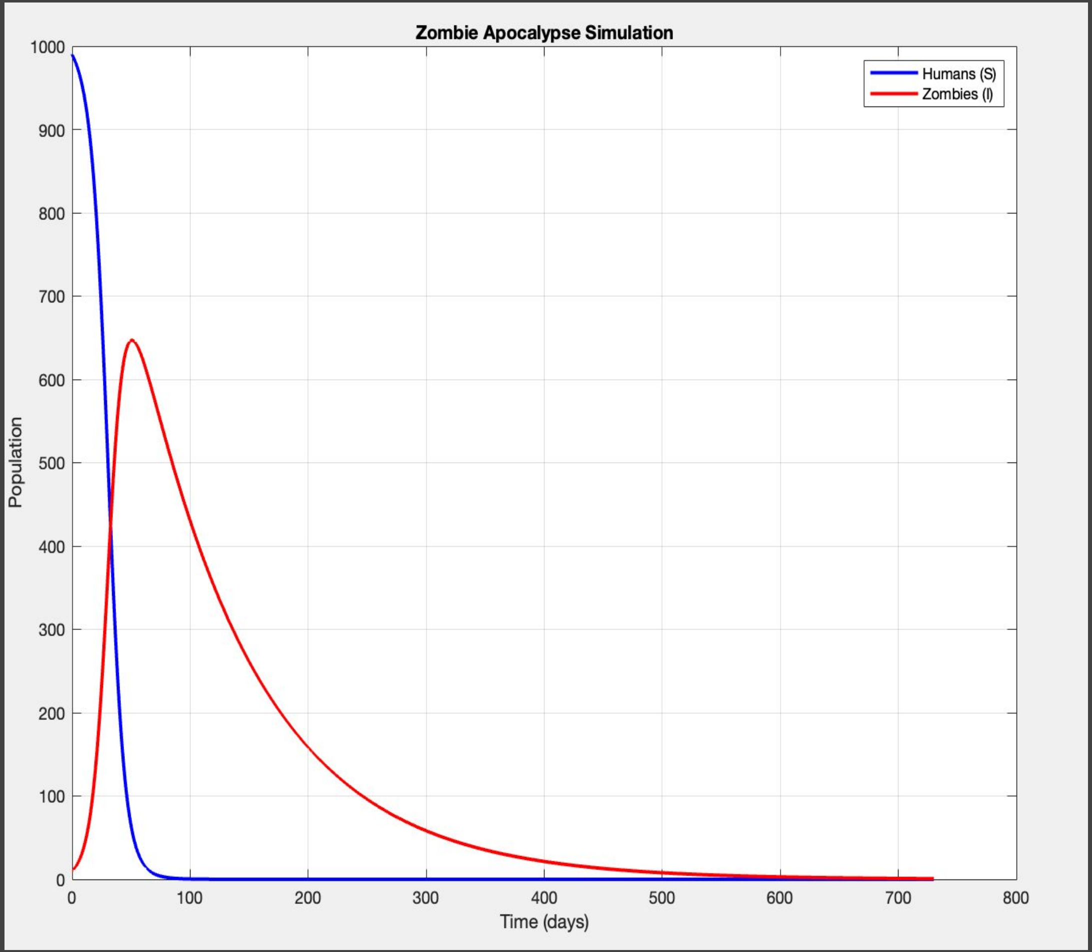
```
% Parameters
beta = 0.0002;    % Infection rate
gamma = 0.01;     % Human birth rate (logistic growth)
K = 1000;         % Carrying capacity for humans
mu = 0.001;       % Natural human death rate
c = 0.05;         % Initial human attack rate on zombies
```

Scenario 1: Early Zombie Overrun

- Very high infection rate β (e.g., zombie virus is super contagious).

- Low human birth rate γ, minimal human defense against zombies, low attack rate (low c value)

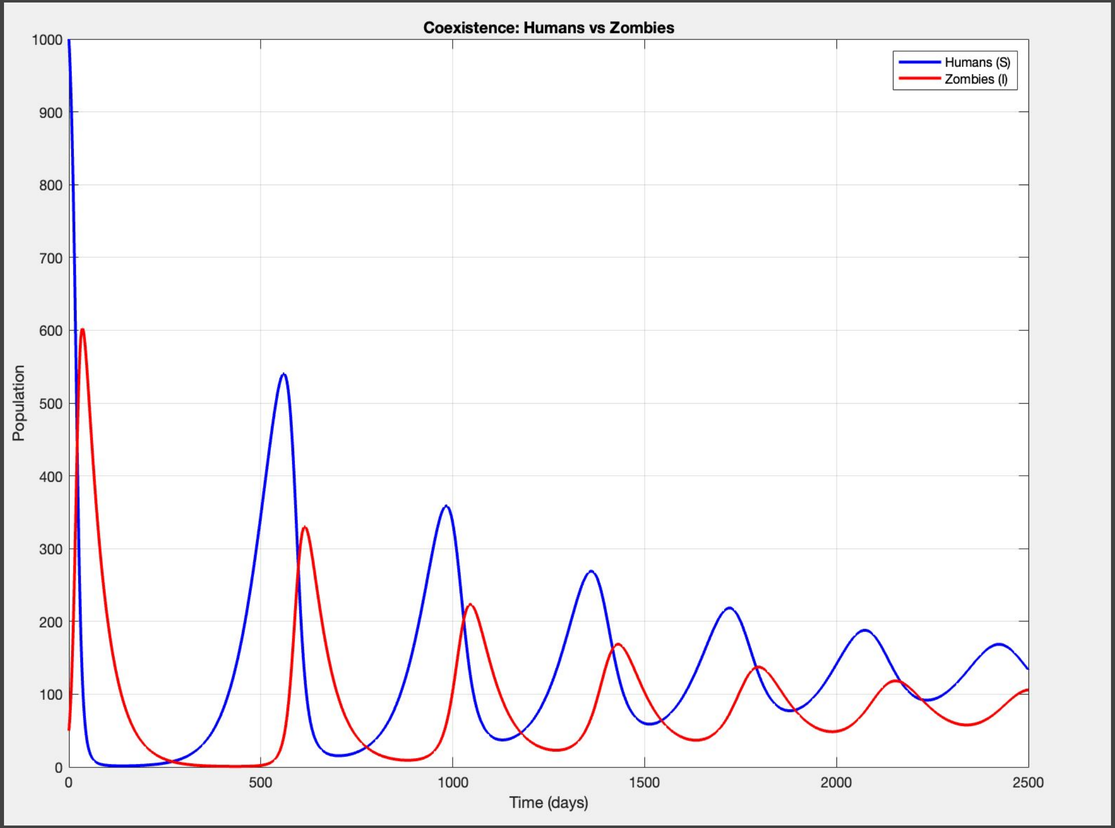
- Result: Zombies spread *extremely fast*, human population collapses almost immediately, zombie population follows because there are no more humans to infect and they slowly die out



Scenario 2: Coexistence

- Balanced parameters: neither zombies nor humans win completely. Humans still do not have a proper defense against the zombies.

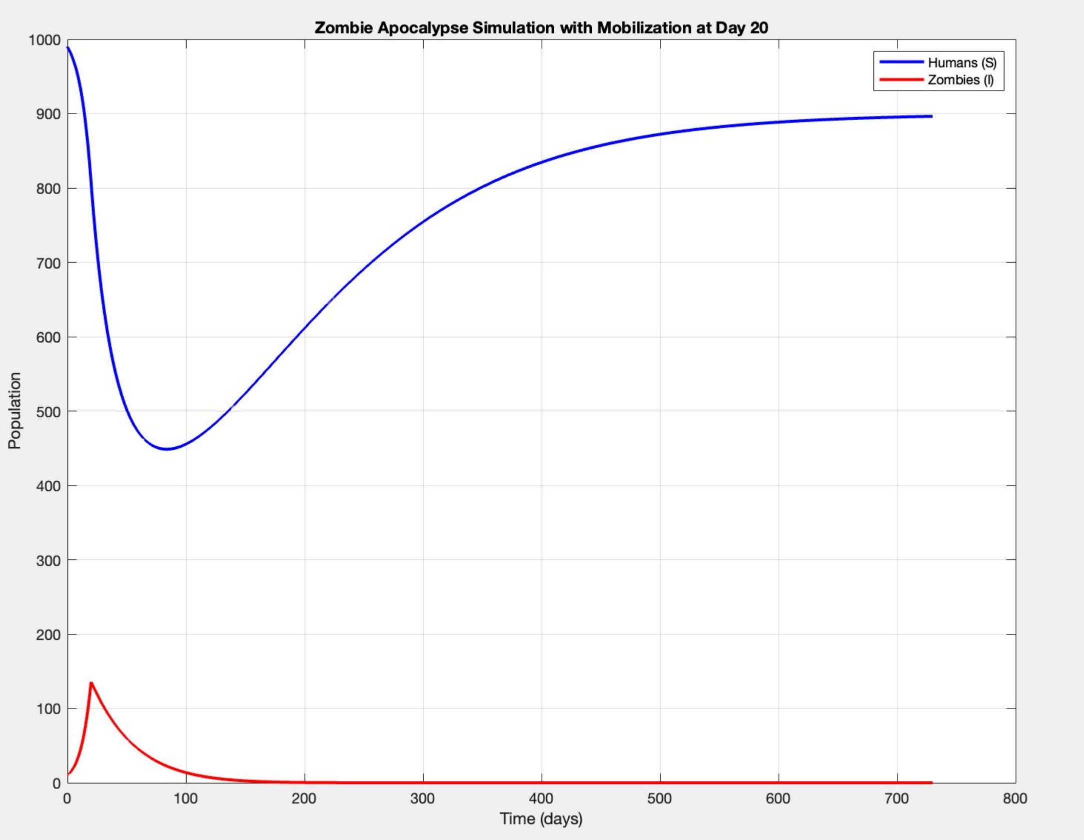
- Result: Small numbers of humans and zombies exist *forever;* no species can completely defeat the other. Even after 2500 days, the human population still has not stabilized, nor will it ever be high enough to rebuild their civilization.



Scenario 3: Human Mobilization

- High attack rate c (humans mobilize and figure out how to efficiently kill zombies at 5x the rate as before, 15 days into the apocalypse).
- Moderate reproduction γ to repopulate.

- Result: Zombies die out after an initial outbreak. Humans bounce back and are able to mediate the apocalypse after fighting back



New System for Scenario 4: Cure Development

$$\frac{dS}{dt} = -\beta SI + \gamma S\left(1 - \frac{S+R}{K}\right) - \mu S - \rho S + \epsilon I$$

$$\frac{dI}{dt} = \beta SI - \gamma I - cS\left(\frac{I}{K+I}\right) - \epsilon I$$

$$\frac{dR}{dt} = \rho S$$

R(t): Number of vaccinated or recovered humans

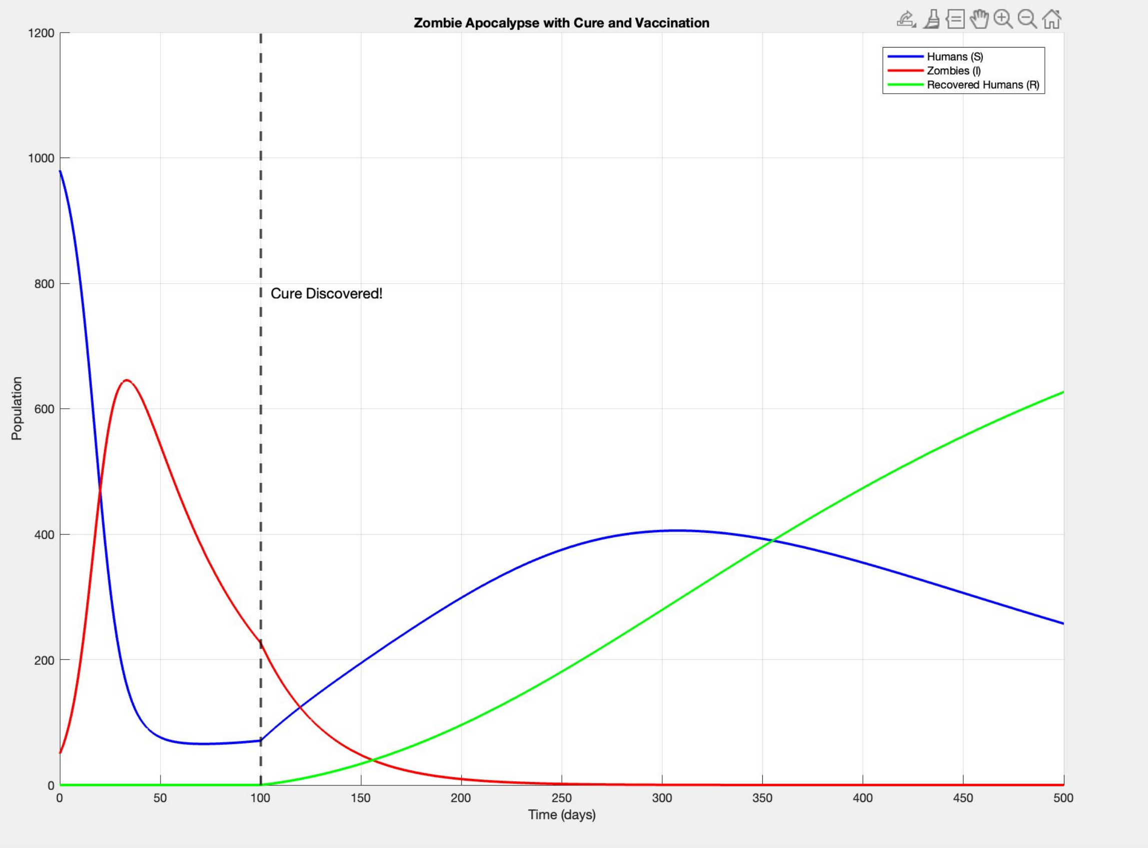
New Parameters:

- **ρ: Vaccination Rate (S →R)**
- **ε: Cure Rate (I →S)**

SIR Model (Susceptible Infected Recovered)

Scenario 4: Cure Development

- This model can convert zombies back into humans and also convert humans to officially recovered.
- Result: Gradual recovery of humanity after initial drop, while zombie population reaches 0 after initial spike.
- human population also starts to drop after vaccination



Conclusion:

Mathematical modeling (using adapted Lotka-Volterra and SIR models) was used to analyze a fictional zombie apocalypse.

Four distinct scenarios were explored, each showing how key parameters — infection rates, human response, and interventions — influence outbreak outcomes.

A combination of equations, simulations, and graphical analysis provided a comprehensive understanding of population dynamics.

Though fictional, the modeling techniques have practical applications in epidemiology, ecology, and systems modeling.

The study demonstrates the power of mathematical tools in predicting and understanding complex, dynamic systems.