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Math 170
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Math 170 Zombie Apocalypse Project

The idea of a zombie apocalypse, though fictional, offers a compelling opportunity to explore mathematical modeling of dynamic systems. In this report, we examine how a zombie outbreak could unfold by analyzing four distinct scenarios, each based on different assumptions about infection rates, human survival, and intervention strategies. To model these scenarios, we apply well-established mathematical frameworks, including the Predator-Prey (Lotka-Volterra) model and SIR (Susceptible-Infected-Recovered) models, adapting them to fit the unique features of a zombie apocalypse.

Each scenario is supported with derived mathematical equations, computational simulations of population dynamics, and graphical interpretations that illustrate how the outbreak evolves over time. By combining theoretical models with code-based simulations, this report highlights how small changes in conditions can significantly impact the outcome of a crisis. Through this project, we not only gain a deeper appreciation for mathematical modeling techniques but also explore how they can be extended to imaginative and unconventional scenarios.

The system of Differential Equations I am using is below:

$$\frac{dS}{dt} = -\beta SI + \gamma S\left(1 - \frac{S}{K}\right) - \mu S$$

$$\frac{dI}{dt} = \beta SI - \gamma I - cS\left(\frac{I}{K + I}\right)$$

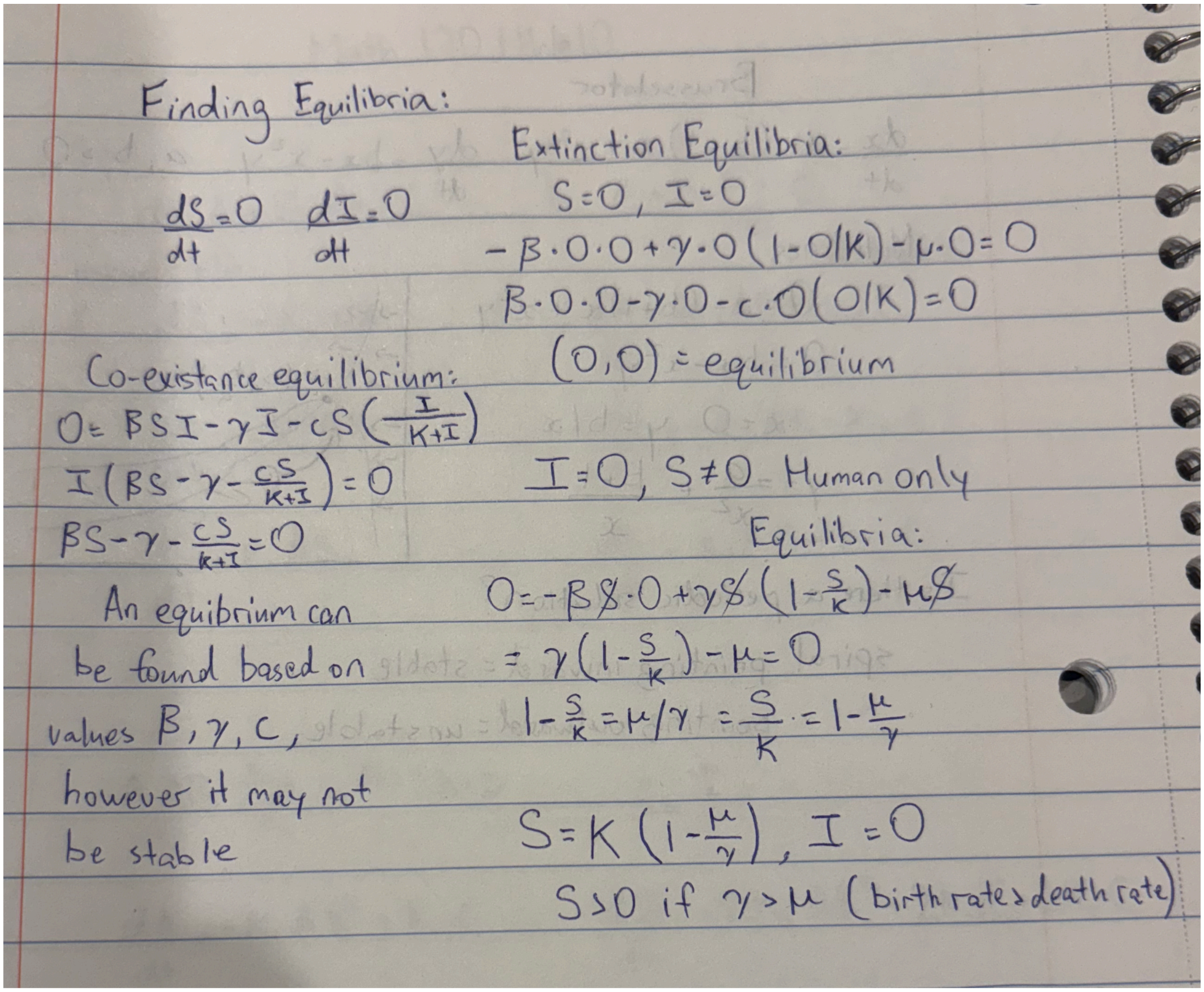
Variables:

- $S(t)$: Humans
- $I(t)$: Zombies

Parameters:

- β: Infection rate
- γ: Human birth rate
- μ: Human natural death rate
- K: Carrying capacity
- c: Human attack rate on zombies

The first thing I did was try to find equilibria based on the differential equation. I was able to find an extinction and human-only equilibria, along with a possible coexistence equilibria, which may not be stable/feasible. My calculations are below.



Finding Equilibria:

$\frac{dS}{dt}=0 \quad \frac{dI}{dt}=0$

Co-existance equilibrium:

$0 = \beta S I - \gamma I - cS\left(\frac{I}{K+I}\right)$

$I\left(\beta S - \gamma - \frac{cS}{K+I}\right)=0$

$\beta S - \gamma - \frac{cS}{K+I} = 0$

An equibrium can be found based on values β, γ, c, however it may not be stable

Extinction Equilibria:

$S=0, I=0$

$-\beta\cdot 0\cdot 0+\gamma\cdot 0(1-0/K)-\mu\cdot 0=0$

$\beta\cdot 0\cdot 0-\gamma\cdot 0-c\cdot 0(0/K)=0$

$(0,0)$ = equilibrium

$I=0, S\neq 0$ Human only Equilibria:

$0=-\beta \cancel{S}\cdot 0+\gamma\cancel{S}\left(1-\frac{S}{K}\right)-\mu\cancel{S}$

$=\gamma\left(1-\frac{S}{K}\right)-\mu=0$

$1-\frac{S}{K}=\mu/\gamma = \frac{S}{K}=1-\frac{\mu}{\gamma}$

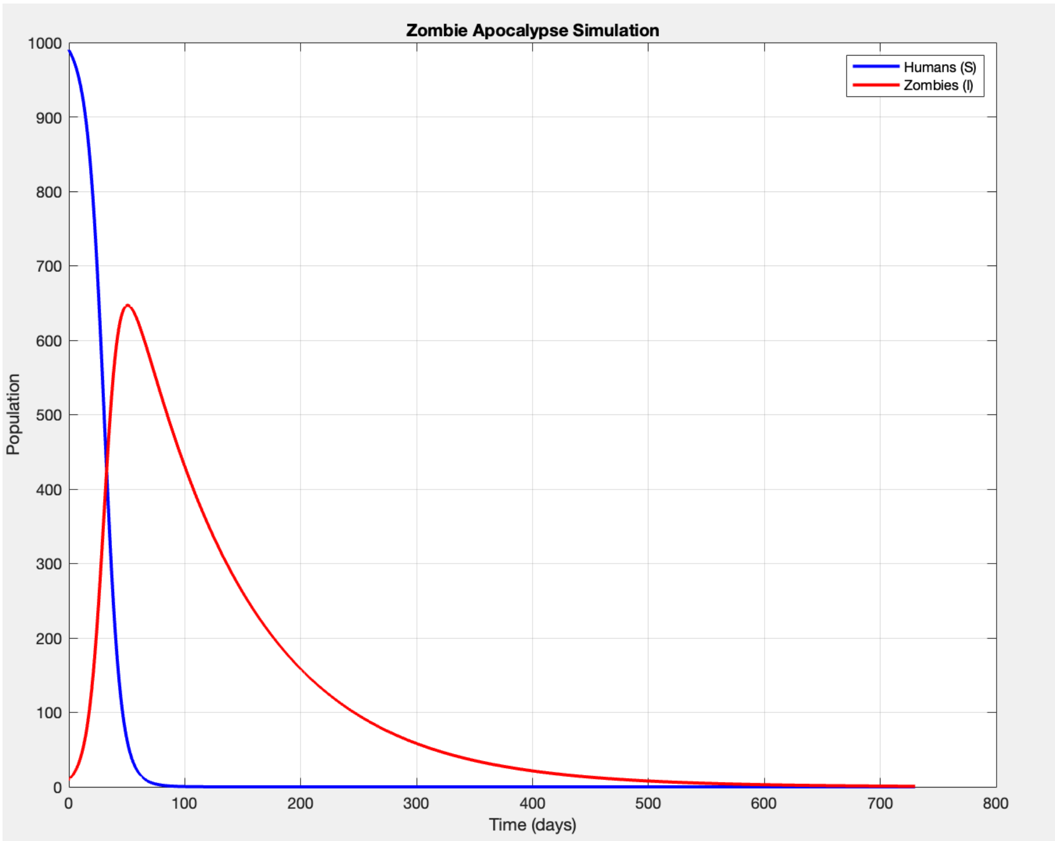
$S=K\left(1-\frac{\mu}{\gamma}\right), I=0$

$S>0$ if $\gamma>\mu$ (birth rate > death rate)

In my project, I modeled out 4 different scenarios that could play out for the zombie apocalypse.

1. Early Zombie Overrun

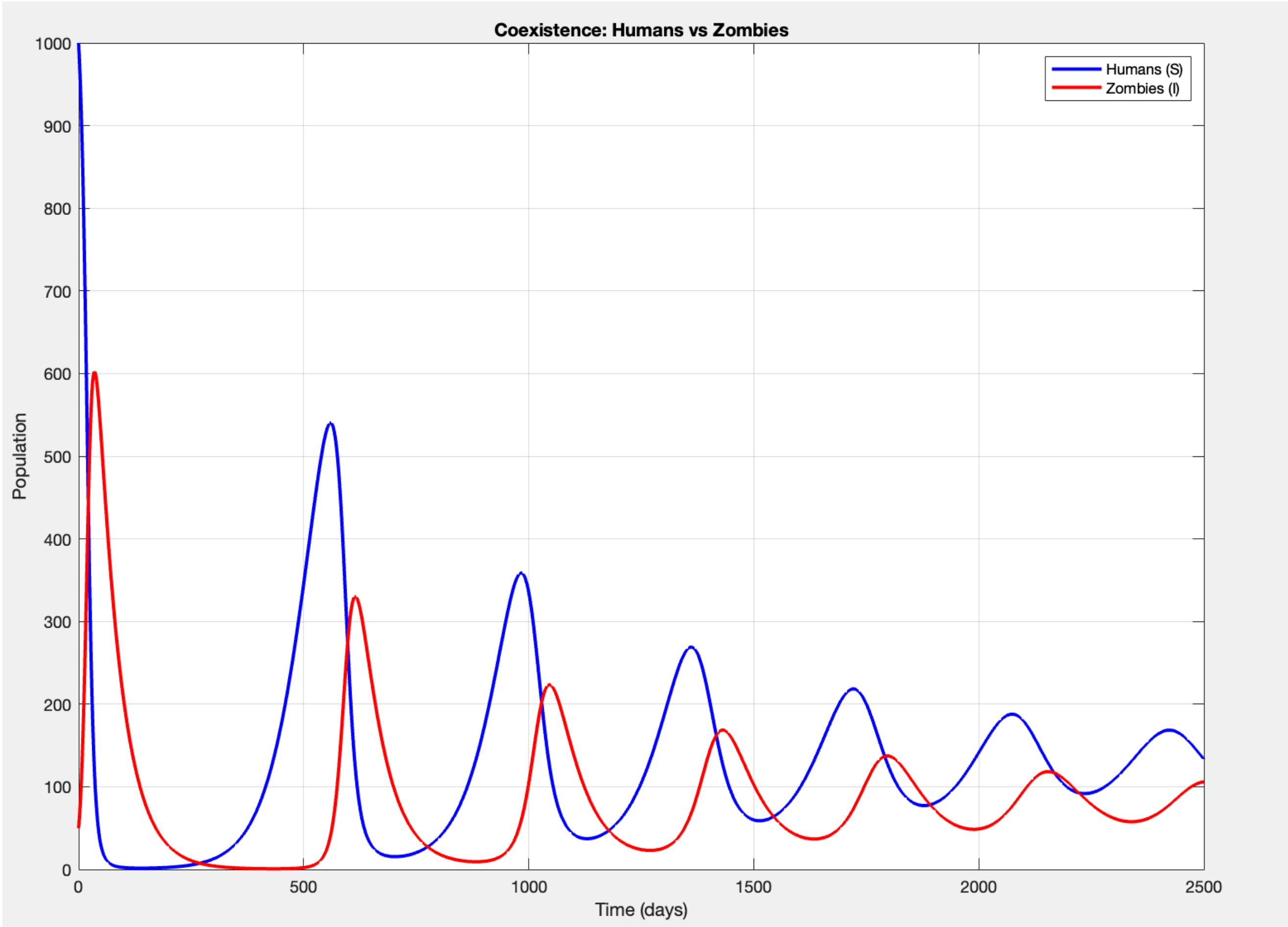
- Very high infection rate β (e.g., zombie virus is super contagious).
- Low human birthrate γ, minimal human defense against zombies
- Result: Zombies spread *extremely fast*, human population collapses almost immediately, zombie population follows because there are no more humans to infect and they slowly die out



2. Coexistence

- Balanced parameters: neither zombies nor humans win completely. Humans still do not have a proper defense against the zombies,

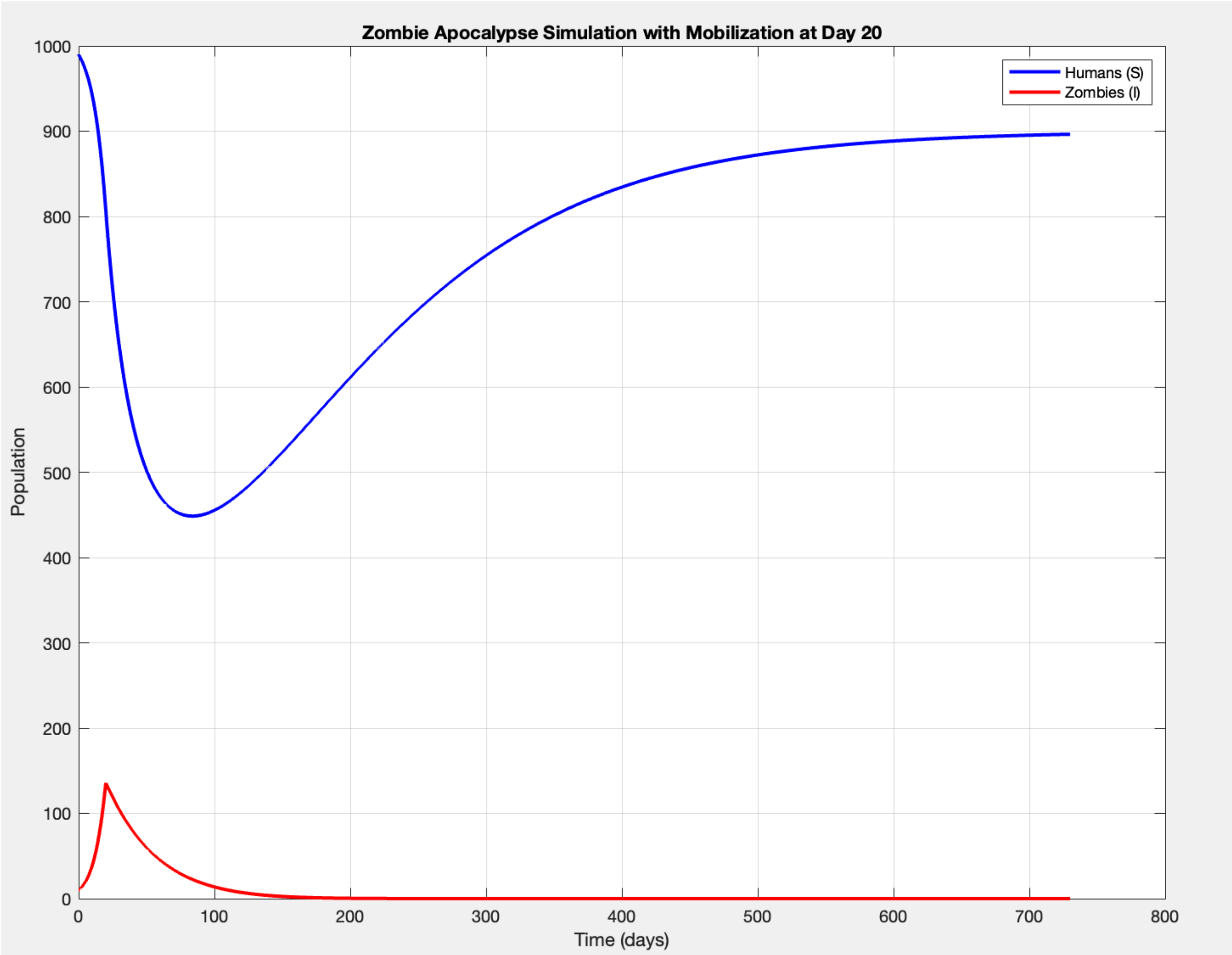
- Result: Small numbers of humans and zombies exist *forever*, no species can completely defeat the other. Even after 2500 days, the human population still has not stabilized, nor will it ever be high enough to rebuild their civilization.



3. Human Victory

- High attack rate c (humans mobilize and figure out how to efficiently kill zombies at 5x the rate as before, 15 days into the apocalypse
- Moderate reproduction γ to repopulate.

- Result: Zombies die out after an initial outbreak. Humans bounce back and are able to mediate the apocalypse after fighting back



4. Cure Development

- After a long enough time (after day 100), a cure is developed, saving the human population
- For this scenario, I used a different system of differential equations, which now has 3 variables instead of 2 (newly added recovered variable)
- I used an SIR model to model this scenario

  The equations are below

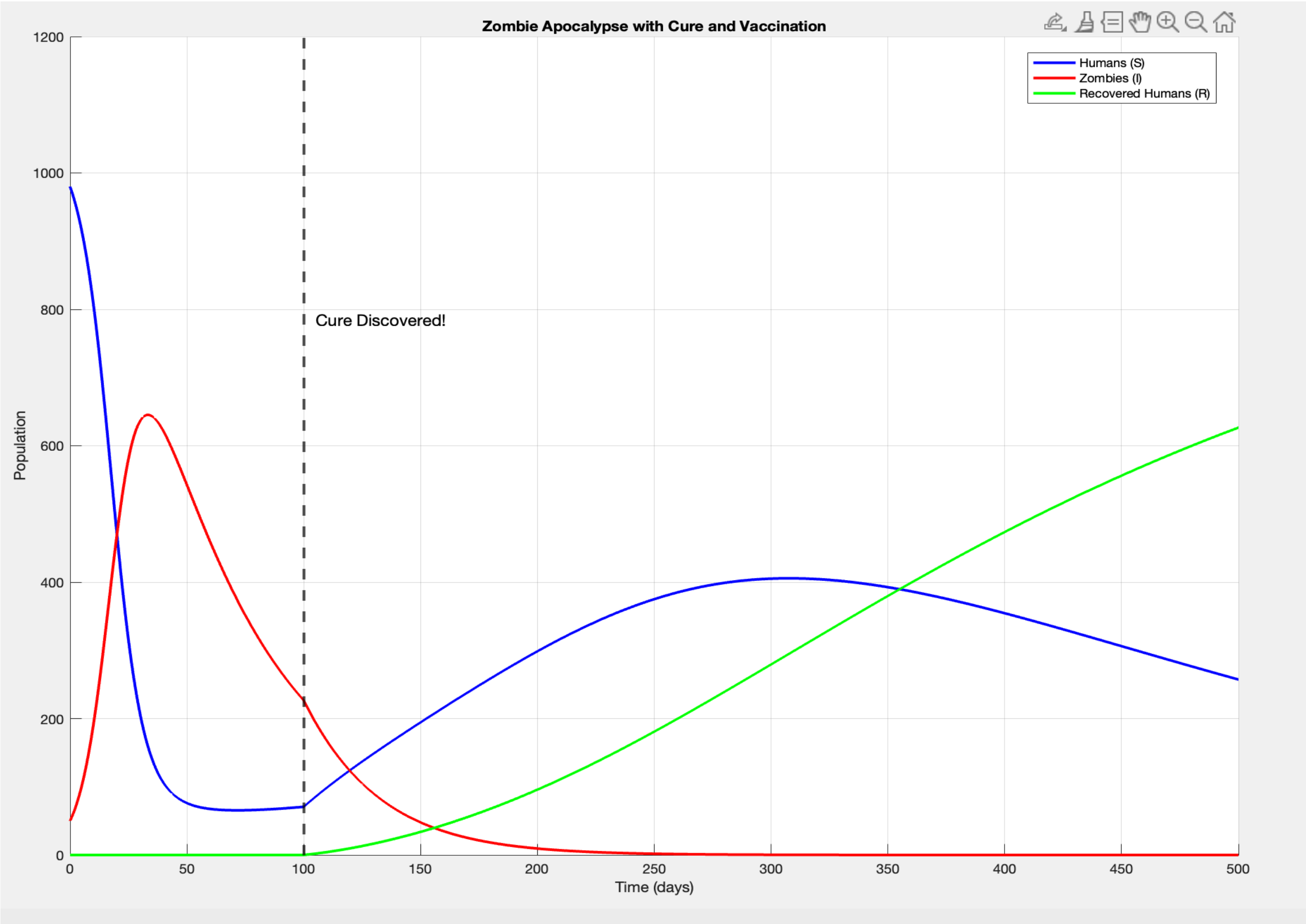
$$\frac{dS}{dt} = -\beta SI + \gamma S\left(1 - \frac{S+R}{K}\right) - \mu S - \rho S + \epsilon I$$

$$\frac{dI}{dt} = \beta SI - \gamma I - cS\left(\frac{I}{K+I}\right) - \epsilon I$$

$$\frac{dR}{dt} = \rho S$$

This model can convert zombies back into humans and also convert humans to officially recovered.

- Result: Gradual recovery of humanity after initial drop, while zombie population reaches 0 after initial spike. In the graph, the human population also starts to drop after vaccination due to it becoming a part of the recovered category.



Through the mathematical modeling of a zombie apocalypse, we have demonstrated how complex population dynamics can be understood using adapted versions of the Lotka-Volterra and SIR models. By exploring four distinct scenarios, we highlighted how varying key parameters — such as infection rates, human response, and external interventions — can dramatically alter the course of an outbreak. The combination of mathematical equations, computational simulations, and graphical analyses provided a comprehensive view of each possible outcome. While the subject matter is fictional, the methods and insights gained from this project have real-world applications in fields such as epidemiology, ecology, and systems modeling. Ultimately, this study reinforces the power of mathematical tools in predicting and understanding the behavior of dynamic, real-world systems — no matter how unusual the scenario may be.