University of Chicago(Project Lab) with Mizuho Securities LLC

Estimation of CVA with rating triggered CSA

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Outline

1. Project Background
2. Data Pre-processing & Process Simulation
3. Dynamics with Rating Transition
4. Validation/Benchmarking
5. Conclusions
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Project Background

Project Background

- ## Project Overview
    - Collateralized OTC contracts, between two parties, are subject to credit rating migrations.
    - Under the CSA agreement, threshold structure specifies the amount of collateral required by the counterparty
    - Exotic CSAs designed to capture the counterparty credit migration risk require careful considerations in computation of CVA prices.
    - Threshold level, which is one of the primary factor driving exposure of a credit derivative, can be impacted by changes in counterparty credit ratings during the contract tenor.
    - But how do we estimate the impact of rating migrations on CVA?

Project Background

- ## FX Contract Parameters
  - Long FX Forward contract in USDJPY for 10 year term, with notional $100 million and forward rate of 99 USD/JPY
- ## Starting Assumptions
  - Problem focused on calculating the impact of rating changes only on CVA(FVA, KVA, etc. not under the scope of the project)
  - Instantaneous interest rates, assuming a piecewise continuous term structure, were presumed to be generate best estimates for discount factors.
  - Construction of forward volatility term structure using bootstrapping of implied volatilities was presumed appropriate input for running Monte Carlo simulations.
  - Credit Default Swap curve provided as an input starting the first simulation.
  - FX spot rate is assumed to follow a Black Scholes dynamics, log normally distributed

Project Background

- ## Data provided
    - Interest rate term structure(both USD and JPY)
    - Term structure of cross-currency basis
    - FX volatility term structure
    - FX spot rate
    - CDS proxy curves (both self and counterparty)
    - Corporate Rating Transition Matrix*
    - Threshold levels (both self and counterparty)

*Source: S&P Global, 2021 Annual Corporate Default and Rating Transition Study

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Project Background

- ## The Challenge
  - Simulating threshold levels based on rating transition matrix requires sampling of r.v. as paths for rating transitions over the contract tenor.
  - Rating transitions are published by agencies on annual basis whereas the frequency of simulating exposures has no such constraint.
  - Credit Default Swap(CDS), a typical credit quality indicator, is marked based on the current credit ratings. Separate stochastic model is required for appropriate mapping of probability of default.
  - Transition probabilities populated under physical measure, not risk-neutral measure.

Project Background

- CVA definition

$$\text{CVA}(t) \simeq -(1 - R_{\text{cp}}) \sum_{i=1}^{N} D_{s_0}(s_i) \text{EPE}(s_i) \text{SP}_{\text{us}}(s_i) \text{PD}_{\text{cp}}(s_{i-1}, s_i)$$

$$\text{EPE}(t) = \mathbb{E}_t\{\min(\text{FV}^+(t), T_{cp})\}$$

- CVA with rating dependency(ρ)

$$\text{CVA}(t) \simeq -(1 - R_{\text{cp}}) \sum_{i=1}^{N} D_{s_0}(s_i) \text{EPE}(s_i, \rho_{s_i}) \text{SP}_{\text{us}}(s_i, \rho_{s_i}) \text{PD}_{\text{cp}}(s_{i-1}, s_i, \rho_{s_i})$$

$$\text{EPE}(t, \rho) = \mathbb{E}_t\{\min(\text{FV}^+(t), T_{cp}(\rho))\}$$

Note: 'cp' refers to counterparty, 'us' refers to self
R–recovery rate, T-Threshold level, SP–Survival Probability, PD-Probability of Default, D(t)– discount factor of zero coupon domestic bond maturing in time t. EPE–Expected Positive Exposure

Data Pre-processing & Process Simulation

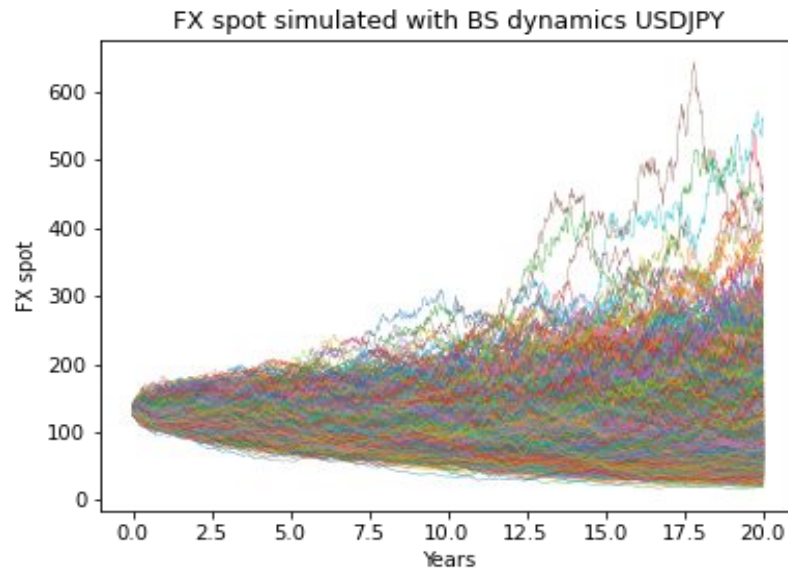# Data Preprocessing & Process Simulation

- ● Simulation of FX spot rate
  - ○ Using the following BS model to simulate FX spot rates

$$\frac{dX_t}{X_t} = (r_t^d - r_t^f)dt + \sigma_t dW_t$$

and by using Ito's calculus,

$$\Delta(\ln X_t) = \left(r_t^d - r_t^f - \frac{\sigma^2}{2} \right) \Delta t + \sigma\sqrt{\Delta t} \cdot B_t$$

FX spot simulated with BS dynamics USDJPY

Note: 'd' refers to domestic(USD), 'f' refers to foreign(JPY)
r– risk free rate, σ– Forward volatility, W/B– Brownian random variable

Data Preprocessing & Process Simulation

- ● Simulating Future value and Expected Exposures
  - ○ Calculating the Future Value of FX forward contract at time-t and the respective positive, negative, and mean exposure profiles defined by the equations below:
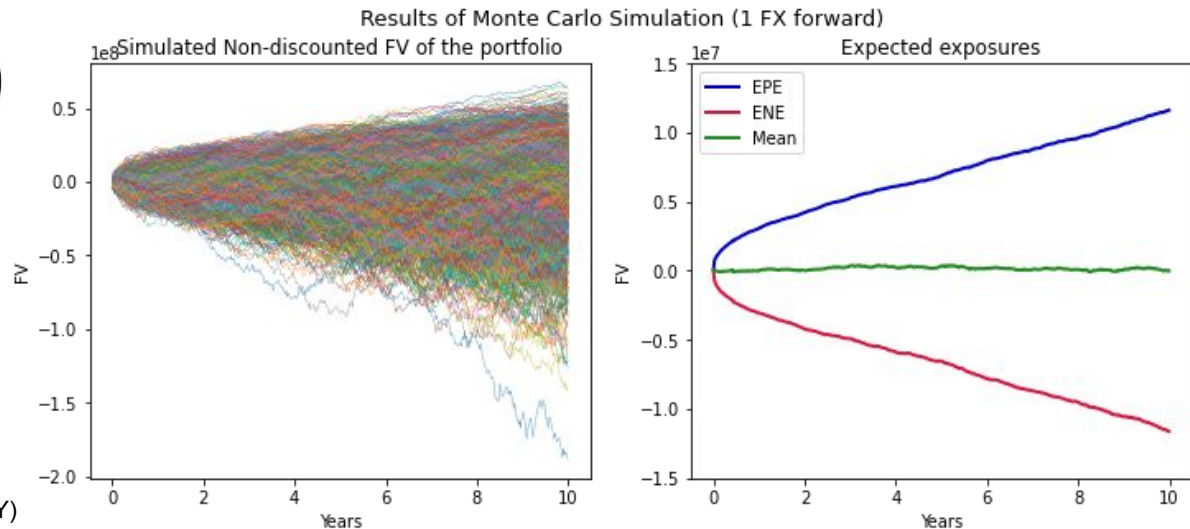
$$FV(t) = \pm N^d \left(\mathrm{DF}_t^d(T) - \frac{\bar{X}}{X_t}\mathrm{DF}_t^f(T) \right)$$

$$\mathrm{EPE}(t) = \mathbb{E}_t\left\{\min(\mathrm{FV}^+(t), T_{\mathrm{CP}})\right\}$$
$$\mathrm{ENE}(t) = \mathbb{E}_t\left\{\max(\mathrm{FV}^-(t), -T_{\mathrm{US}})\right\}$$
$$\mathrm{EE}(t) = \mathrm{EPE}(t) + \mathrm{ENE}(t)$$

Results of Monte Carlo Simulation (1 FX forward)

Note: 'd' refers to domestic(USD), 'f' refers to foreign(JPY)
DF–discounting factor, N–Notional, X–Spot rate

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Data Preprocessing & Process Simulation

- Simulating Future value and Expected Exposures considering CSA
  - Need to now adjust for thresholds(cp: $10 mil, us: $50 mil) and constant minimum transfer amount ($250k)
  - With these parameters, the following exposure profiles has been simulated

Results of Monte Carlo Simulation (1 FX forward) under a sample CSA

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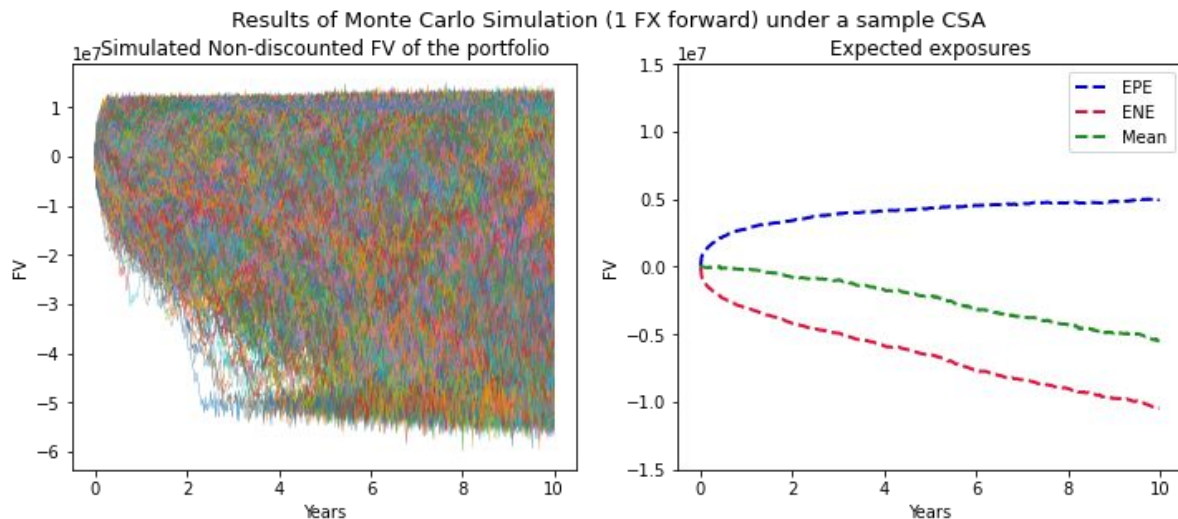
Data Preprocessing & Process Simulation

● Comparison between Exposure profiles with and without CSA

Exposure Comparison without or with CSA
Expected exposures
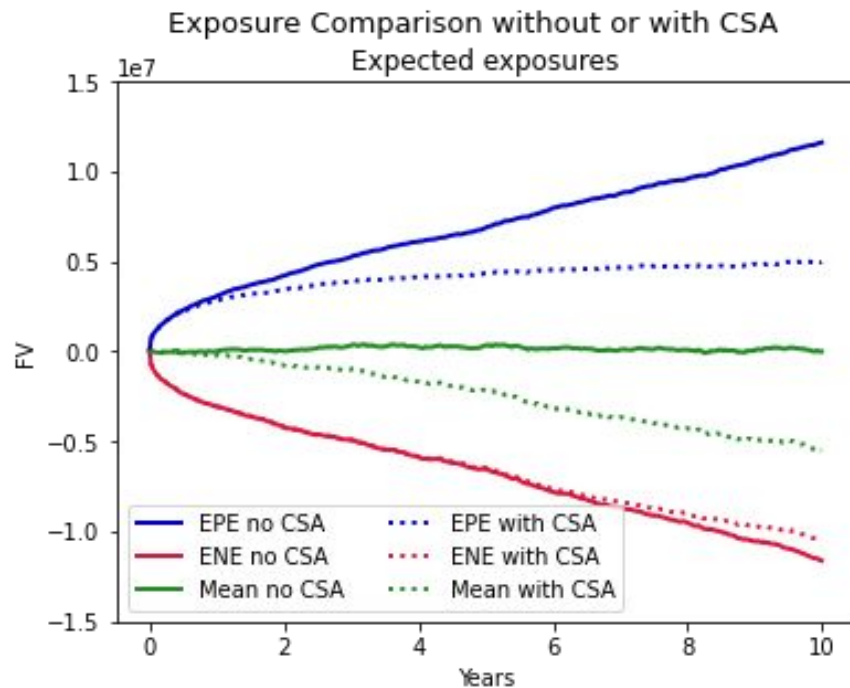
Data Preprocessing & Process Simulation

- Next step, calculating the survival probability and probability of default by using CDS curves
  - Construct the credit curves(term structure of probability) by bootstrapping a hazard rate curve from the observed CDS spreads
- CVA calculation with simulated exposures and credit curves
  - Using the equation described in slide-8, workout the value for CVA
  - Key parameters in the calculation are expected positive exposures and probability of default

Dynamics with Rating Transitions

Dynamics with Rating Transitions

- Rating changes can be thought of threshold changes
  - Threshold is main component in CSA that determines collateral
- Determine how the threshold may change over time
  - Use yearly transition matrix, assuming rating transition as a Markovian process

	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC
AAA	87.09%	5.86%	2.50%	0.68%	0.16%	0.24%	0.13%	0.00%	0.05%	0.00%	0.03%	0.05%	0.03%	0.00%	0.03%	0.00%	0.05%
AA+	2.21%	79.68%	10.59%	3.38%	0.68%	0.32%	0.18%	0.05%	0.09%	0.05%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AA	0.41%	1.31%	80.99%	8.42%	2.63%	1.12%	0.35%	0.38%	0.13%	0.08%	0.05%	0.03%	0.02%	0.02%	0.00%	0.02%	0.05%
AA-	0.04%	0.10%	3.68%	78.99%	9.74%	2.18%	0.57%	0.24%	0.14%	0.06%	0.03%	0.00%	0.00%	0.03%	0.08%	0.00%	0.00%
A+	0.00%	0.05%	0.41%	4.23%	79.16%	8.54%	2.04%	0.58%	0.32%	0.08%	0.05%	0.08%	0.01%	0.06%	0.03%	0.00%	0.00%
A	0.03%	0.04%	0.21%	0.39%	5.19%	79.36%	6.61%	2.32%	0.81%	0.25%	0.09%	0.10%	0.06%	0.08%	0.02%	0.00%	0.01%
A-	0.03%	0.01%	0.05%	0.14%	0.38%	6.14%	78.87%	7.27%	1.85%	0.53%	0.12%	0.12%	0.10%	0.10%	0.03%	0.01%	0.03%
BBB+	0.00%	0.01%	0.05%	0.06%	0.19%	0.69%	6.77%	76.65%	7.98%	1.50%	0.34%	0.26%	0.12%	0.14%	0.09%	0.02%	0.06%
BBB	0.01%	0.01%	0.04%	0.02%	0.09%	0.28%	0.95%	7.26%	76.68%	6.20%	1.28%	0.63%	0.26%	0.20%	0.10%	0.03%	0.05%
BBB-	0.01%	0.01%	0.02%	0.04%	0.06%	0.13%	0.23%	1.08%	9.03%	72.87%	5.57%	2.01%	0.81%	0.36%	0.21%	0.15%	0.19%
BB+	0.04%	0.00%	0.00%	0.03%	0.03%	0.08%	0.08%	0.37%	1.49%	10.78%	66.10%	7.70%	2.57%	0.97%	0.51%	0.22%	0.33%
BB	0.00%	0.00%	0.03%	0.01%	0.00%	0.05%	0.04%	0.15%	0.56%	1.87%	9.50%	65.34%	8.65%	2.36%	1.03%	0.35%	0.48%
BB-	0.00%	0.00%	0.00%	0.01%	0.01%	0.01%	0.04%	0.09%	0.22%	0.32%	1.61%	9.40%	63.89%	8.60%	3.01%	0.79%	0.71%
B+	0.00%	0.01%	0.00%	0.03%	0.00%	0.03%	0.06%	0.04%	0.05%	0.10%	0.29%	1.38%	8.23%	62.53%	9.47%	2.58%	1.78%
B	0.00%	0.00%	0.01%	0.01%	0.00%	0.03%	0.03%	0.01%	0.05%	0.03%	0.09%	0.20%	1.05%	7.26%	61.60%	10.01%	3.96%
B-	0.00%	0.00%	0.00%	0.00%	0.01%	0.03%	0.00%	0.05%	0.05%	0.08%	0.07%	0.16%	0.38%	2.02%	9.65%	55.28%	12.15%
CCC	0.00%	0.00%	0.00%	0.00%	0.02%	0.00%	0.07%	0.04%	0.07%	0.04%	0.02%	0.13%	0.34%	0.85%	2.53%	10.04%	43.91%

Dynamics with Rating Transitions

- Simulate a rating change once a year, and then use the new threshold to calculate collateral values
    - The transition is only used once a year, and the result from the transition is used for the remainder of the year in the simulation
- Rating transitions were first validated with a provided dataset to ensure that rating validations made sense
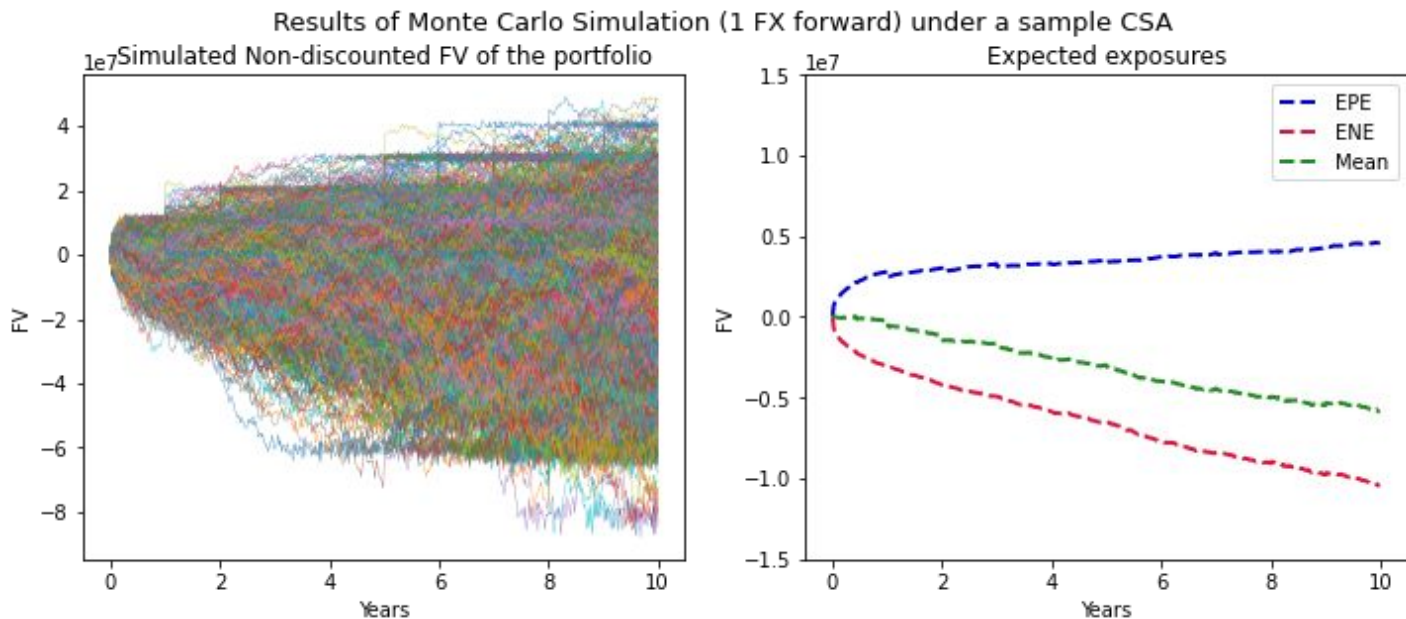
Dynamics with Rating Transitions

- Validation of transition rating changes
  - Simulated proportions of ratings are on the left and validation dataset is on the right
  - Paired t-test was used to statistically validate the two datasets:
    - p-value: 0.98 -> evidence that there is no statistical difference between the datasets

	Observed	10-year Validation
A+	0.1400	0.139630
B	0.0065	0.004901
A-	0.1870	0.200087
A	0.2520	0.275962
B+	0.0110	0.007096
BBB	0.0995	0.086081
BB	0.0125	0.011879
BBB+	0.1500	0.135117
BBB-	0.0375	0.037309
AA-	0.0500	0.042781
AA	0.0130	0.015758
BB+	0.0175	0.014930
BB-	0.0100	0.008420
CCC	0.0045	0.001955
AAA	0.0025	0.001964
B-	0.0045	0.002499
AA+	0.0020	0.002607

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Dynamics with Rating Transitions

- Subplots for expected exposure profile, considering dynamic rating regime starting with counterparty rating as BB- and self rating as A

	CP	US
AAA	200000000	200000000
AA+	200000000	200000000
AA	100000000	100000000
AA-	100000000	100000000
A+	80000000	80000000
A	60000000	60000000
A-	60000000	60000000
BBB+	50000000	50000000
BBB	50000000	50000000
BBB-	40000000	40000000
BB+	30000000	30000000
BB	20000000	20000000
BB-	10000000	10000000
B+	0	0
B	0	0
B-	0	0
CCC	0	0

Results of Monte Carlo Simulation (1 FX forward) under a sample CSA

Simulated Non-discounted FV of the portfolio

Expected exposures

- - EPE
- - ENE
- - Mean

Dynamics with Rating Transitions

- Comparison between static & dynamic ratings

Exposure Comparison with static and dynamic ratings

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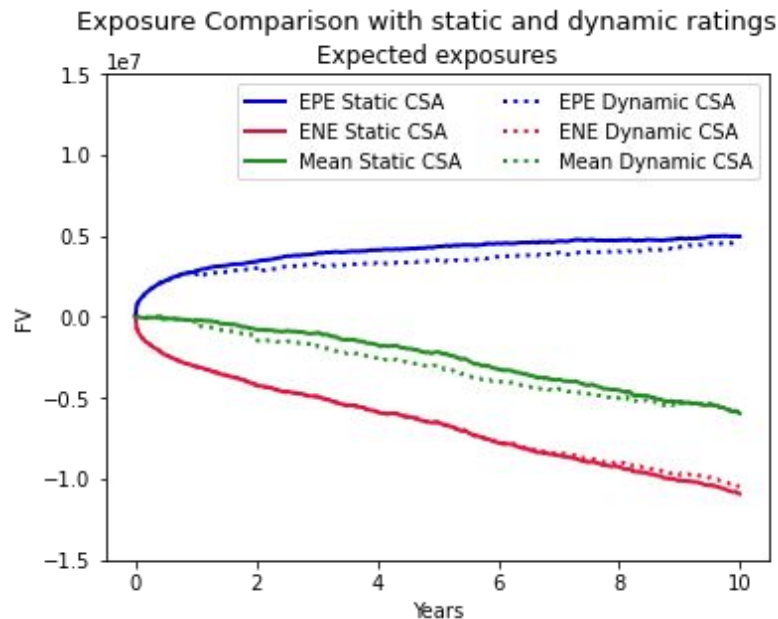
Dynamics with Rating Transitions

- ● CVA Calculation
  - ○ Calculation needs to be modified as the probabilities will change depending on the current rating -> no longer static
    - ■ For ease of calculation, PD ladder is used as is with values *jumping* from rating to rating for each time period
  - ○ Limitations: CDS was given for only the letter of ratings (missing +-), so AA+ = AA- = AA
  - ○ CVA becomes less negative, though difference is more marginal compared to CSA vs. no CSA

```
CVA with static rating CSA      : -1,627,685
CVA with dynamic rating CSA     : -1,387,958
```

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Validation/Benchmarking

Validation/Benchmarking

- ## Is CVA validation accurate?
  - If all the ratings become a singular rating, it should become the same as in the static case
  - Reflects the method having the same value as the static rating CSA when all ratings are changed to a single one

```
Static Method    : -1,627,685
Dynamic Validation       : -1,627,685
```

Validation/Benchmarking

- ## CSA validation in the extreme cases: infinite thresholds
  - If thresholds are changed to infinite, the thresholds should reflect that of no CSA… assuming the same initial ratings

	CP	US
AAA	inf	inf
AA+	inf	inf
AA	inf	inf
AA-	inf	inf
A+	inf	inf
A	inf	inf
A-	inf	inf
BBB+	inf	inf
BBB	inf	inf
BBB-	inf	inf
BB+	inf	inf
BB	inf	inf
BB-	inf	inf
B+	inf	inf
B	inf	inf
B-	inf	inf
CCC	inf	inf

Exposure Comparison with static and dynamic ratings

Expected exposures

- EPE Static CSA
- ENE Static CSA
- Mean Static CSA
- EPE Dynamic CSA
- ENE Dynamic CSA
- Mean Dynamic CSA

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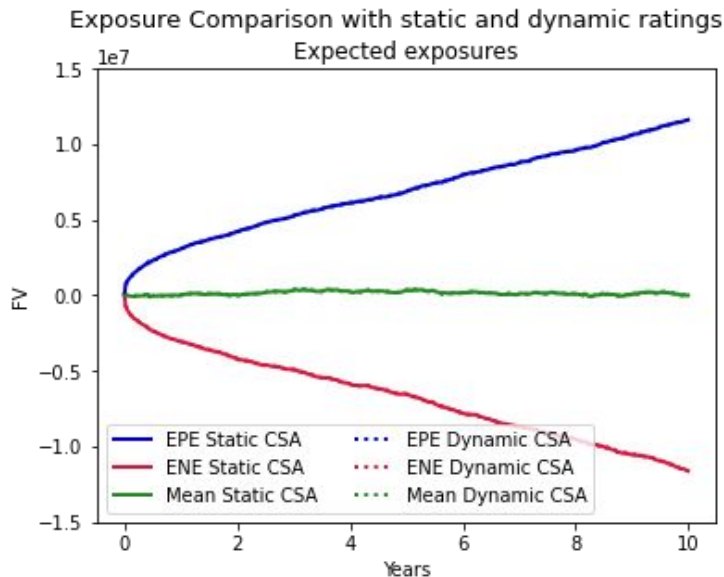
Validation/Benchmarking: Sensitivity to Contract Tenor

- **CSA validation** : If the tenor of the contract is changed from previously assumed 10y, the CVA results with static rating against dynamic ratings follow the same trend.

Contract Tenor	Strike	CVA with Static rating	CVA with Dynamic rating
3y	120.10	-351,511	-336,094
5y	113.36	-792,829	-743,957
7y	107.10	-1,220,166	-1,081,025
9y	101.90	-1,485,007	-1,234,153
10y	99.00	-1,627,685	-1,361,154
12y	93.71	-1,814,668	-1,442,472
15y	87.00	-2,009,753	-1,511,301

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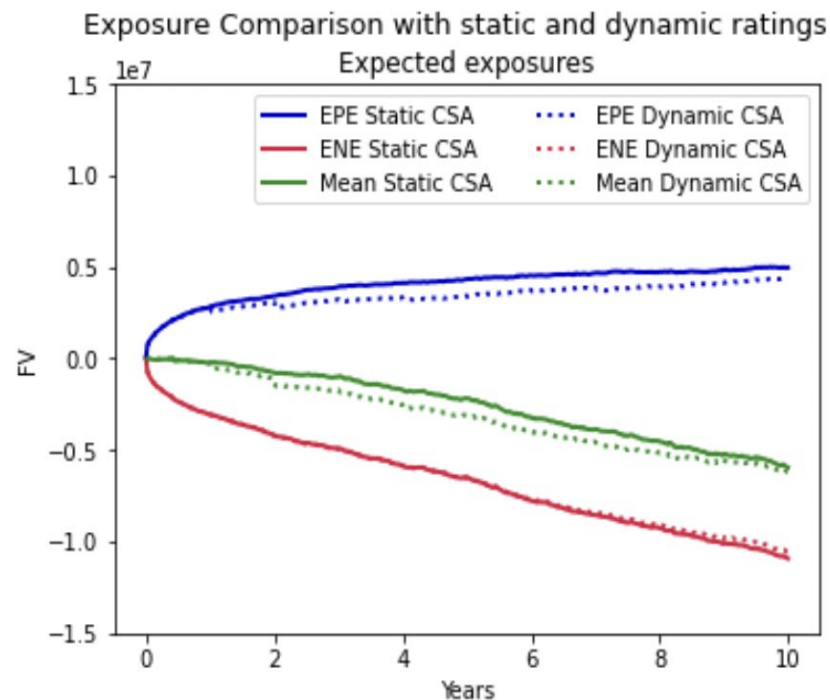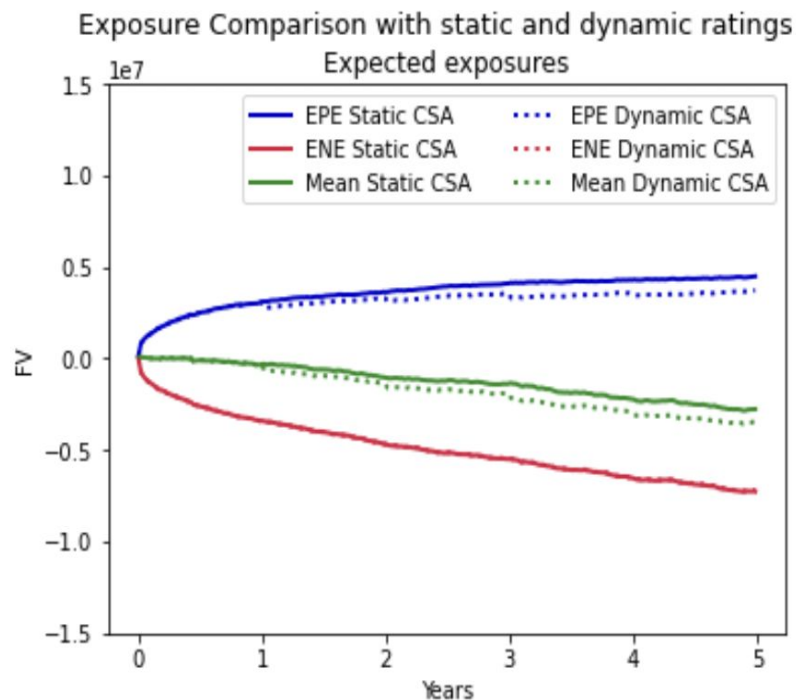
Validation/Benchmarking: Sensitivity to Contract Tenor

Validation/Benchmarking: Sensitivity to Contract Tenor

Exposure Comparison with static and dynamic ratings — Expected exposures

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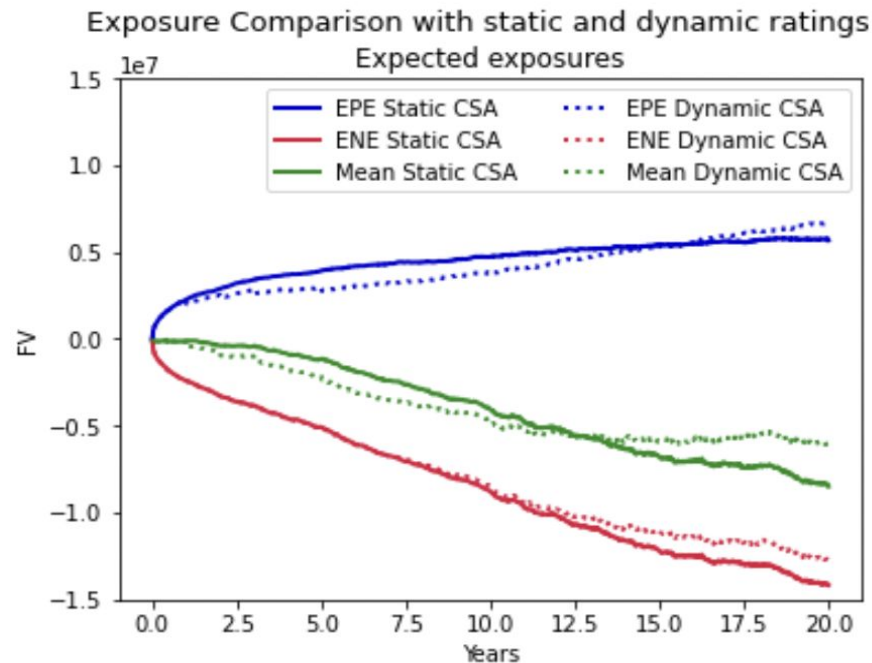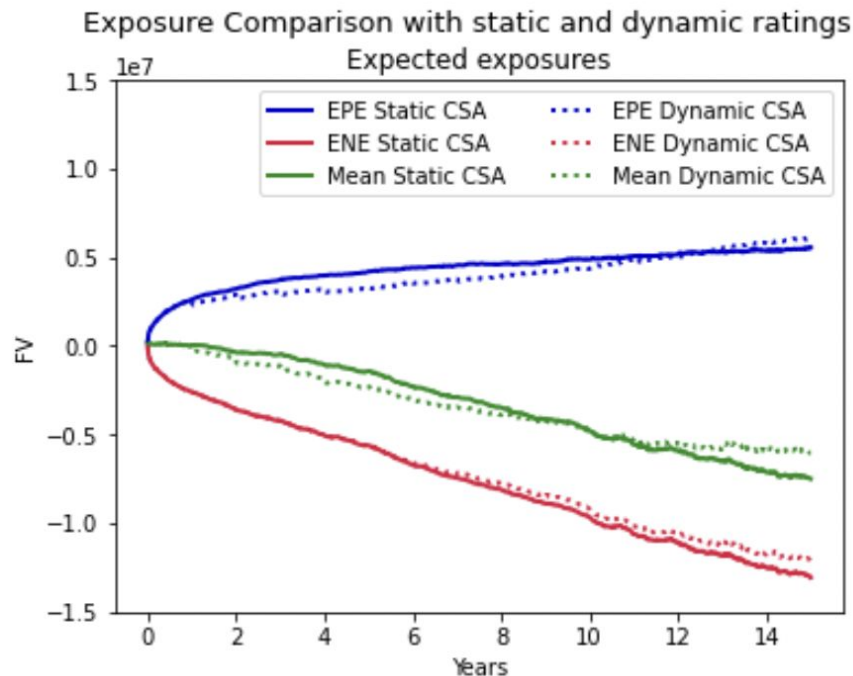
Validation/Benchmarking: Zero Threshold case

- If thresholds are changed to 0, then there should be no exposure (maximum exposure will be equal to 0)

	CP	US
AAA	0	0
AA+	0	0
AA	0	0
AA-	0	0
A+	0	0
A	0	0
A-	0	0
BBB+	0	0
BBB	0	0
BBB-	0	0
BB+	0	0
BB	0	0
BB-	0	0
B+	0	0
B	0	0
B-	0	0
CCC	0	0

Exposure Comparison with static and dynamic ratings

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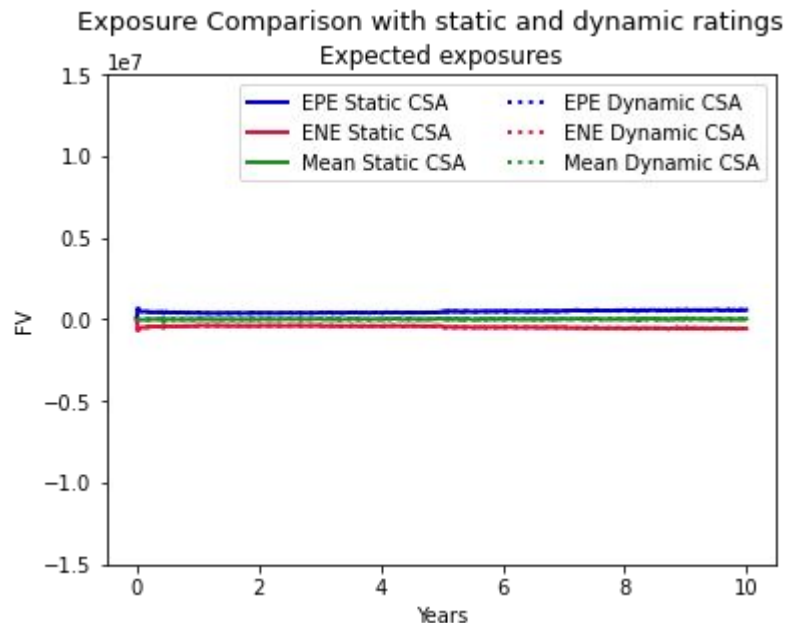
Result Exploration: Varying of CP Rating

- ## We will assume the following threshold table:
  - ### Thresholds are symmetric
  - ### How does varying the initial CP rating affect CVA estimates?
- ## We will first show 3 different outcomes
  - ### US: A, CP: A (same ratings)
  - ### US: A, CP: BBB (different ratings)
  - ### US: A, CP: BB (different ratings - max difference)

	CP	US
AAA	inf	inf
AA+	inf	inf
AA	inf	inf
AA–	inf	inf
A+	inf	inf
A	inf	inf
A–	inf	inf
BBB+	25000000.0	25000000.0
BBB	25000000.0	25000000.0
BBB–	25000000.0	25000000.0
BB+	0.0	0.0
BB	0.0	0.0
BB–	0.0	0.0
B+	0.0	0.0
B	0.0	0.0
B–	0.0	0.0
CCC	0.0	0.0

Varying of CP Rating: A, A

Static Rating Simulated FV

Dynamic Rating Simulated FV
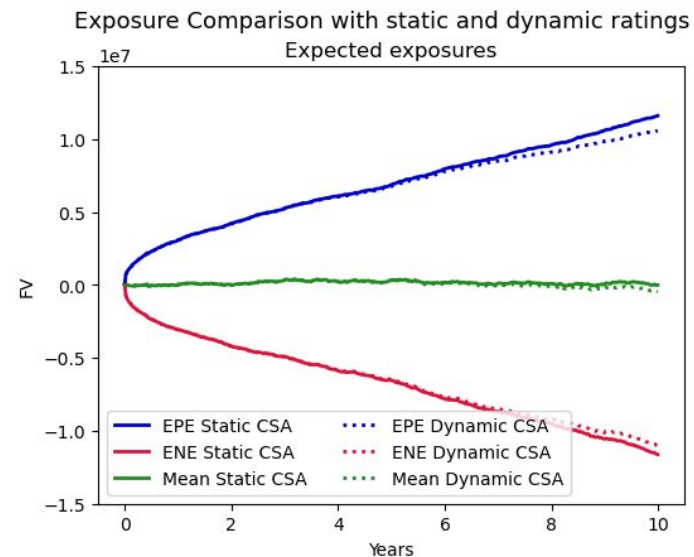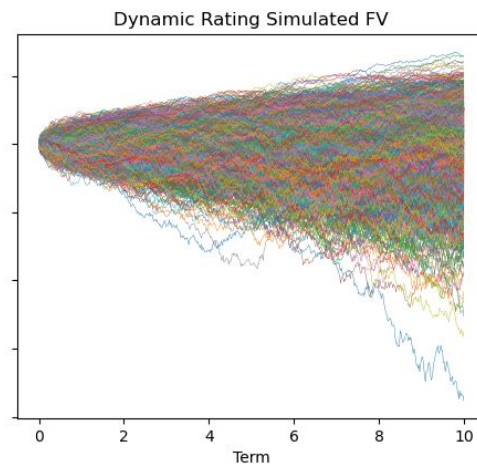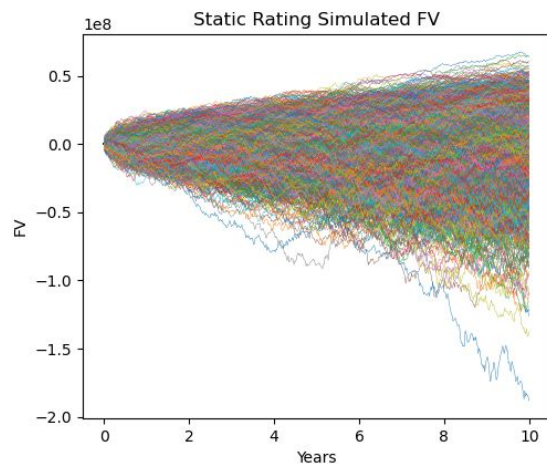
Exposure Comparison with static and dynamic ratings
Expected exposures
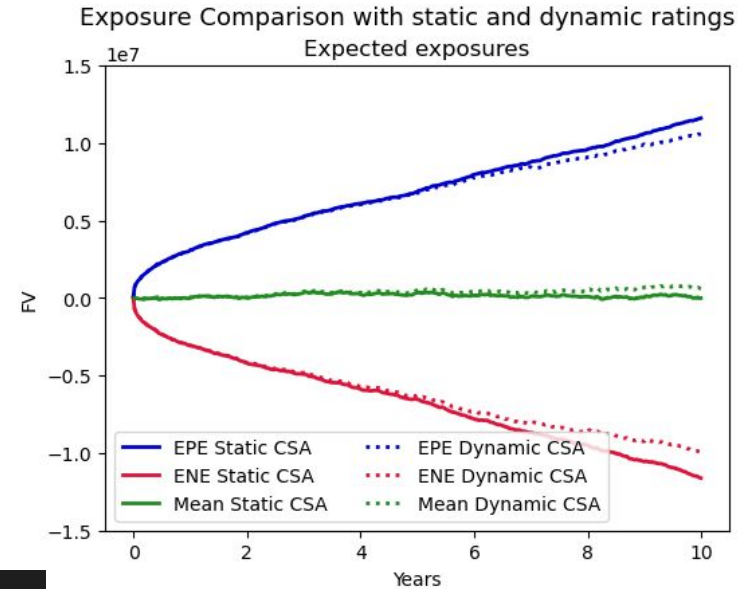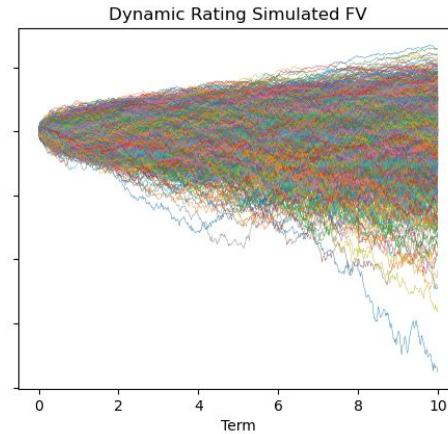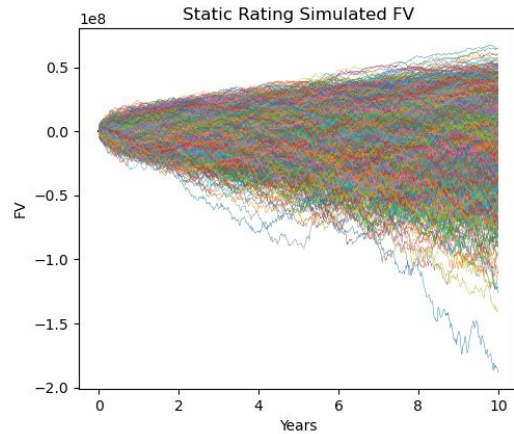
CVA with static rating CSA/CDS : -936,674
CVA with dynamic rating CSA/CDS : -987,949
CVA with dynamic rating CSA/static CDS : -901,344

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Varying of CP Rating: A, BBB

Static Rating Simulated FV

Dynamic Rating Simulated FV

Exposure Comparison with static and dynamic ratings
Expected exposures
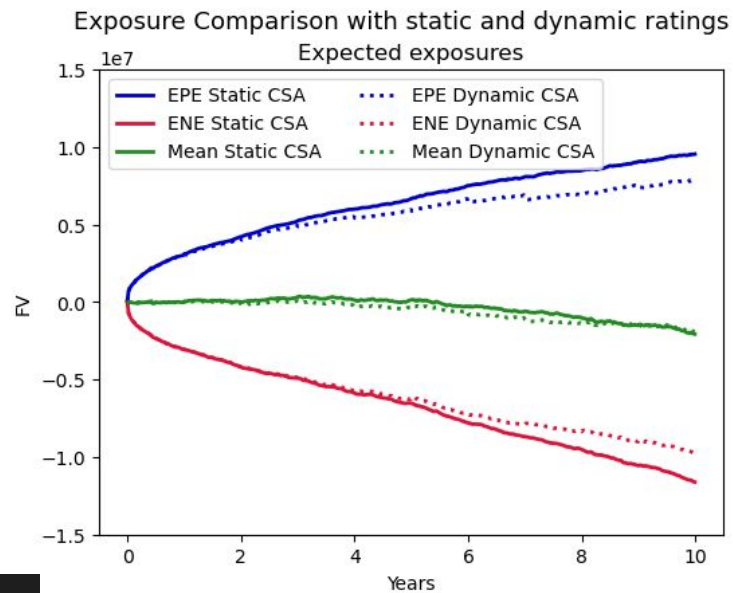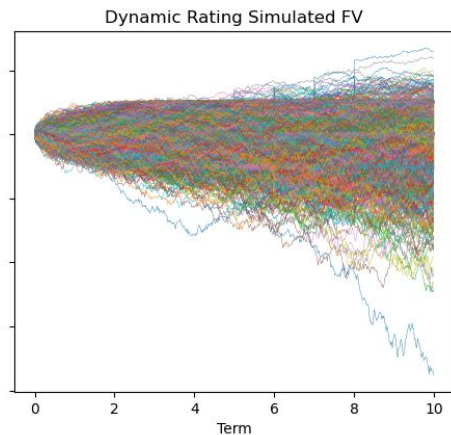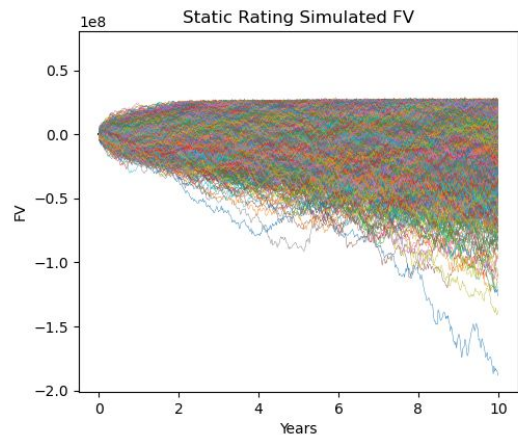
```
CVA with static rating CSA/CDS  : -1,280,168
CVA with dynamic rating CSA/CDS : -1,189,898
CVA with dynamic rating CSA/static CDS  : -1,112,141
```

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Varying of CP Rating: A, BB

Static Rating Simulated FV

Dynamic Rating Simulated FV

Exposure Comparison with static and dynamic ratings
Expected exposures
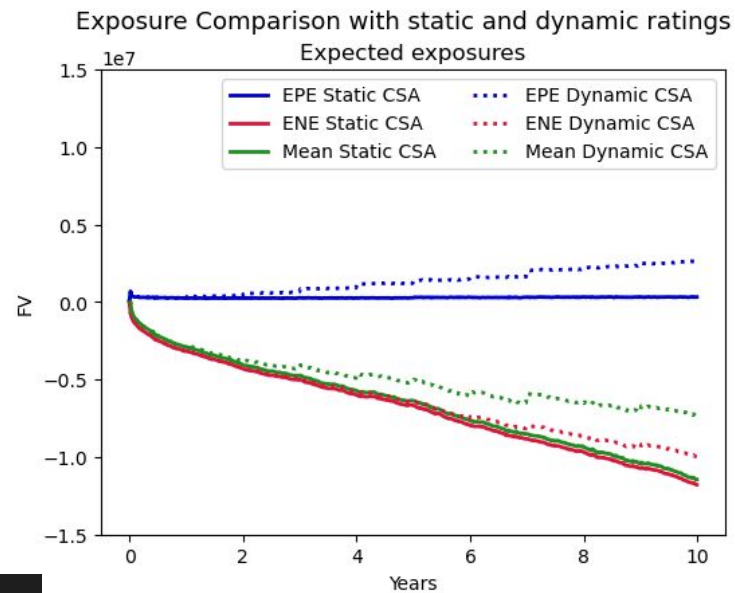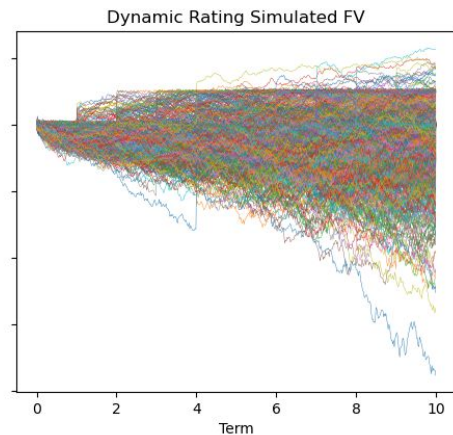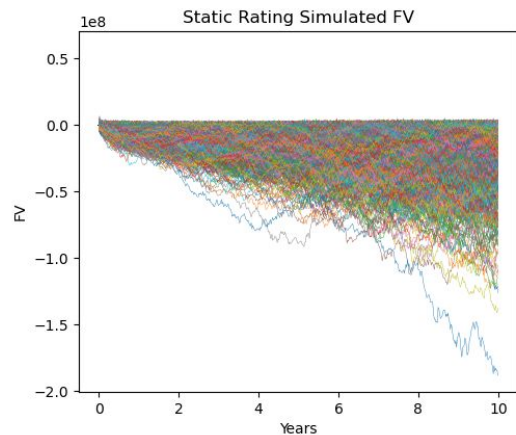
```
CVA with static rating CSA/CDS   : -113,855
CVA with dynamic rating CSA/CDS  : -479,609
CVA with dynamic rating CSA/static CDS  : -513,720
```

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Result Exploration: Threshold Gradient

- We will use a threshold table that is much smoother in its transitions
  - How does this affect results?
  - We will compare with the results from the previous table (less continuous transitions)
- We will show the following outcomes:
  - US: AA, CP: A
  - US: AA, CP: BBB
  - US: AA, CP: BB
  - US: AA, CP: B

	CP	US
AAA	inf	inf
AA+	inf	inf
AA	inf	inf
AA-	inf	inf
A+	100000000.0	100000000.0
A	75000000.0	75000000.0
A-	50000000.0	50000000.0
BBB+	25000000.0	25000000.0
BBB	25000000.0	25000000.0
BBB-	25000000.0	25000000.0
BB+	10000000.0	10000000.0
BB	10000000.0	10000000.0
BB-	10000000.0	10000000.0
B+	0.0	0.0
B	0.0	0.0
B-	0.0	0.0
CCC	0.0	0.0

Threshold Gradient: AA, A

CVA with static rating CSA/CDS   : -959,501

CVA with dynamic rating CSA/CDS : -1,026,737

CVA with dynamic rating CSA/static CDS   : -924,809
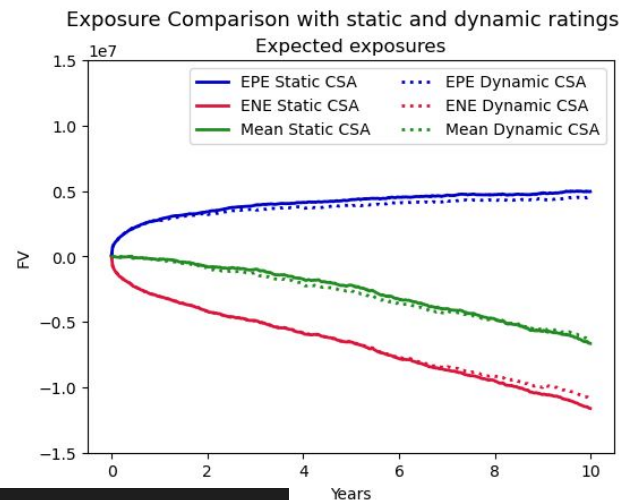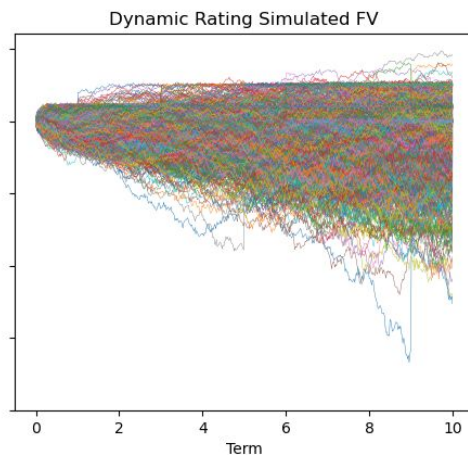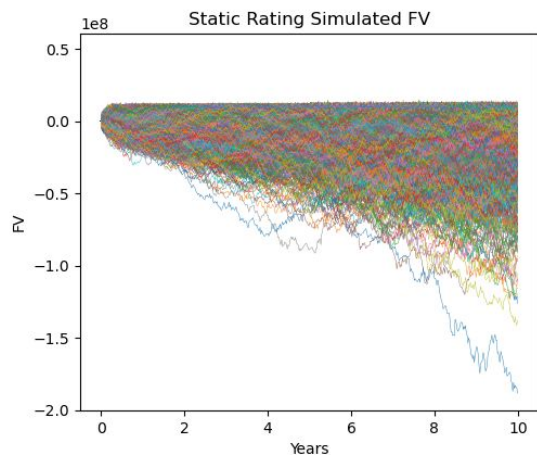
Threshold Gradient: AA, BBB


Static Rating Simulated FV

Dynamic Rating Simulated FV

Exposure Comparison with static and dynamic ratings
Expected exposures
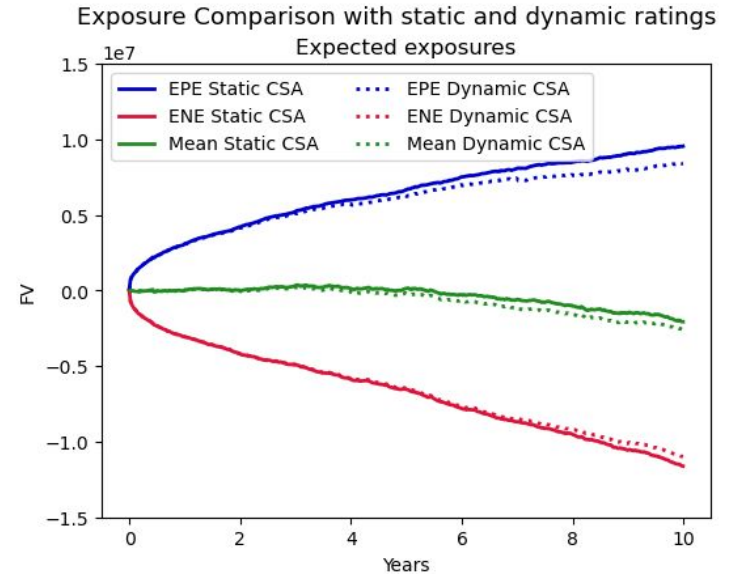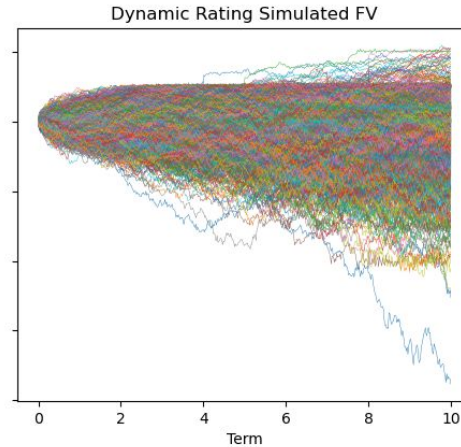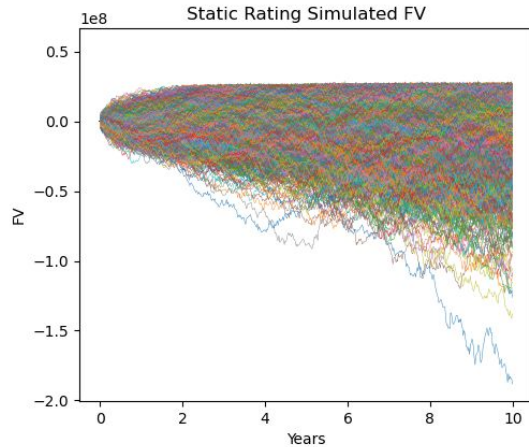
CVA with static rating CSA/CDS   : −1,309,595
CVA with dynamic rating CSA/CDS : −1,311,260
CVA with dynamic rating CSA/static CDS : −1,209,872

Threshold Gradient: AA, BB

Static Rating Simulated FV

Dynamic Rating Simulated FV

Exposure Comparison with static and dynamic ratings
Expected exposures

CVA with static rating CSA/CDS : -1,658,115
CVA with dynamic rating CSA/CDS : -1,437,998
CVA with dynamic rating CSA/static CDS : -1,519,190

Threshold Gradient: AA, B
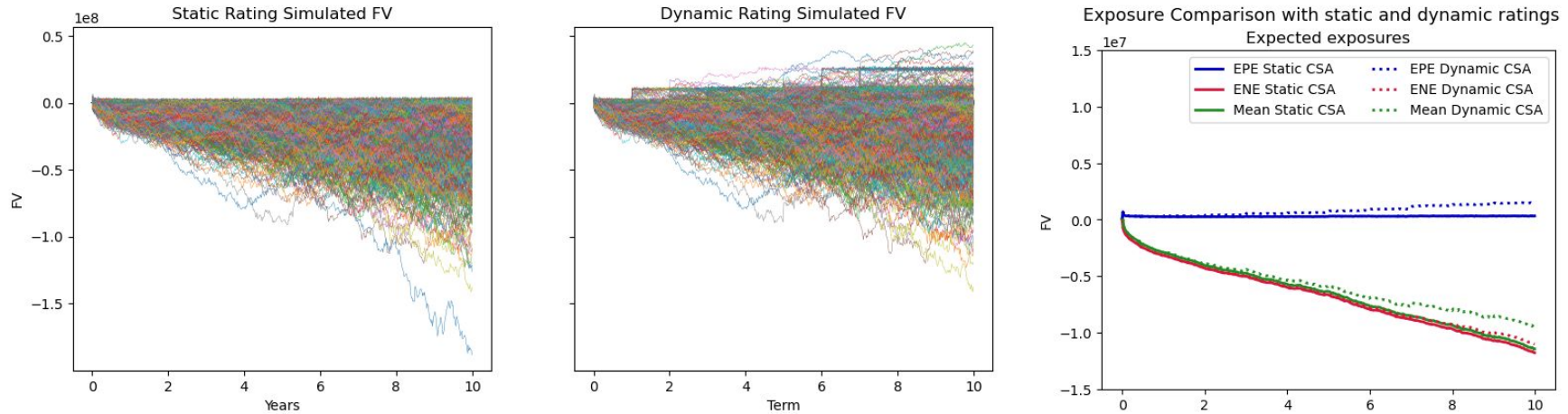

```
CVA with static rating CSA/CDS  : -145,974
CVA with dynamic rating CSA/CDS : -353,539
CVA with dynamic rating CSA/static CDS  : -367,995
```

Result Exploration: Unilateral Transitions I

- We will explore a transition table that only has one transition
  - How does this compare to the bilateral table that was seen earlier?
  - The thresholds will be finite
- We will show the following outcomes
  - US: A, CP: A
  - US: A, CP: BBB

	CP	US
AAA	50000000.0	50000000.0
AA+	50000000.0	50000000.0
AA	50000000.0	50000000.0
AA-	50000000.0	50000000.0
A+	50000000.0	50000000.0
A	50000000.0	50000000.0
A-	50000000.0	50000000.0
BBB+	50000000.0	50000000.0
BBB	50000000.0	50000000.0
BBB-	50000000.0	50000000.0
BB+	0.0	0.0
BB	0.0	0.0
BB-	0.0	0.0
B+	0.0	0.0
B	0.0	0.0
B-	0.0	0.0
CCC	0.0	0.0

Unilateral Transitions I: A, A
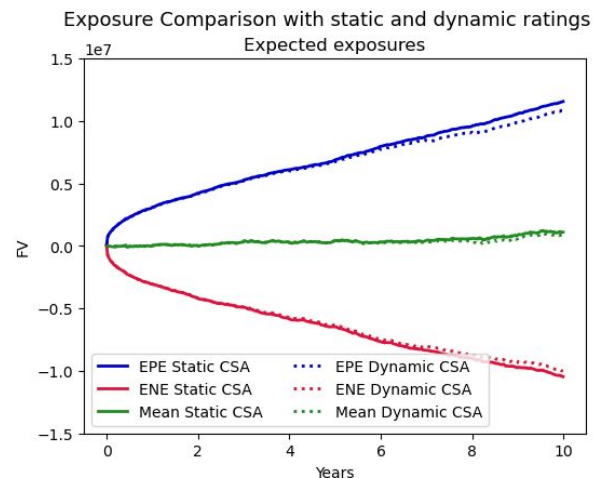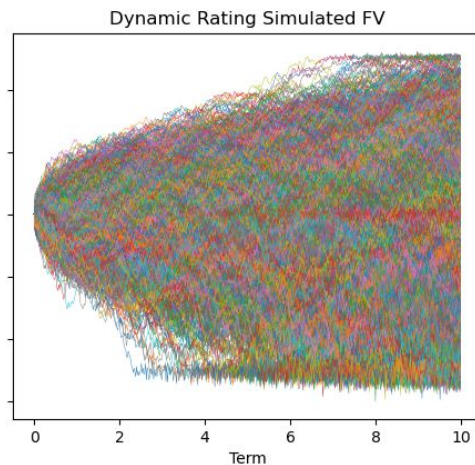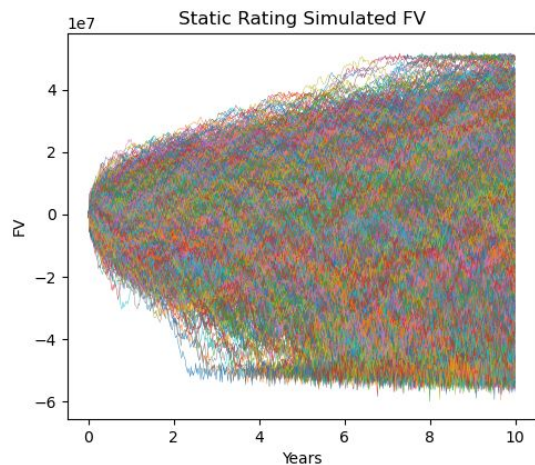
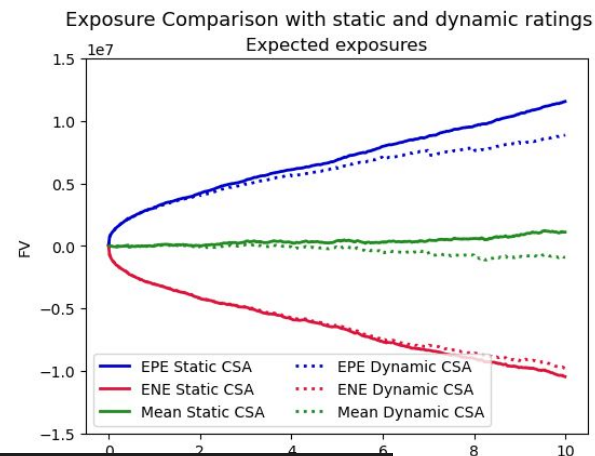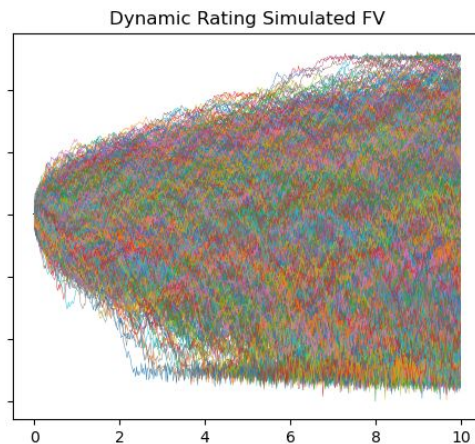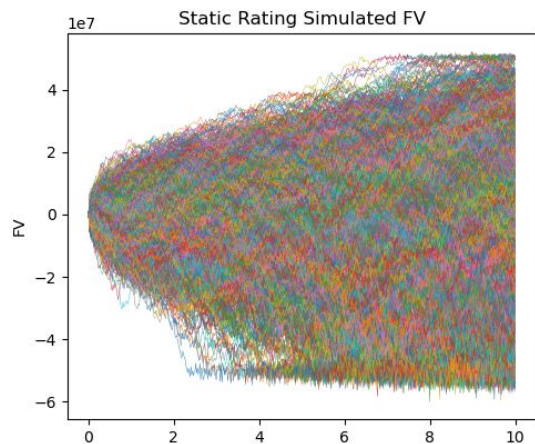
CVA with static rating CSA/CDS   : -935,967
CVA with dynamic rating CSA/CDS : -1,010,767
CVA with dynamic rating CSA/static CDS : -901,110

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Unilateral Transitions I: A, BBB


```
CVA with static rating CSA/CDS  : -1,383,427
CVA with dynamic rating CSA/CDS : -1,288,845
CVA with dynamic rating CSA/static CDS  : -1,191,212
```

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Result Exploration: Unilateral Transitions II

- We will change the transition table such that 50,000,000 is the lower bound
    - How does this compare?
- We will show the same outcomes
    - US: A, CP: A
    - US: A, CP: BBB

	CP	US
AAA	inf	inf
AA+	inf	inf
AA	inf	inf
AA-	inf	inf
A+	inf	inf
A	inf	inf
A-	inf	inf
BBB+	50000000.0	50000000.0
BBB	50000000.0	50000000.0
BBB-	50000000.0	50000000.0
BB+	50000000.0	50000000.0
BB	50000000.0	50000000.0
BB-	50000000.0	50000000.0
B+	50000000.0	50000000.0
B	50000000.0	50000000.0
B-	50000000.0	50000000.0
CCC	50000000.0	50000000.0

Unilateral Transitions II: A, A
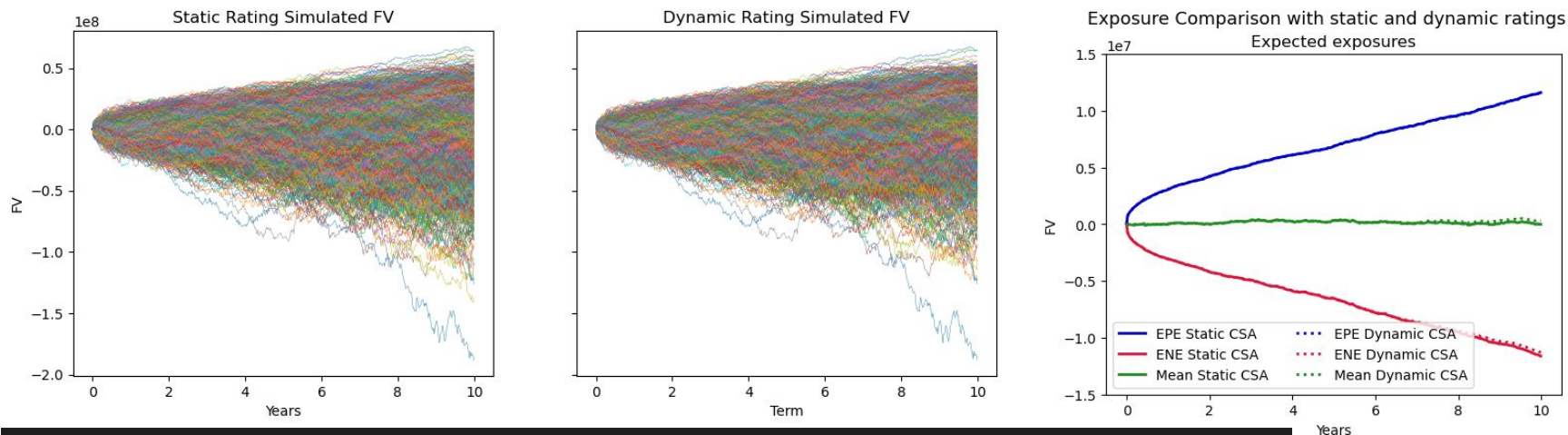
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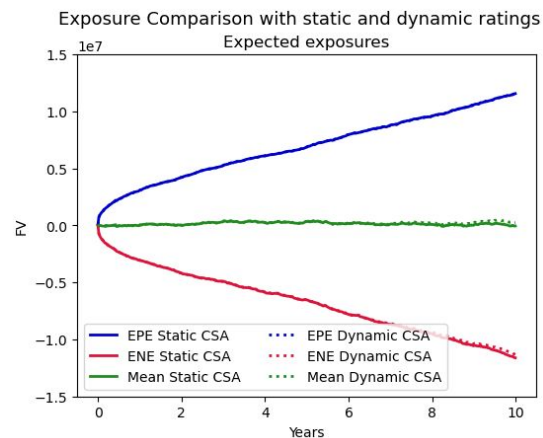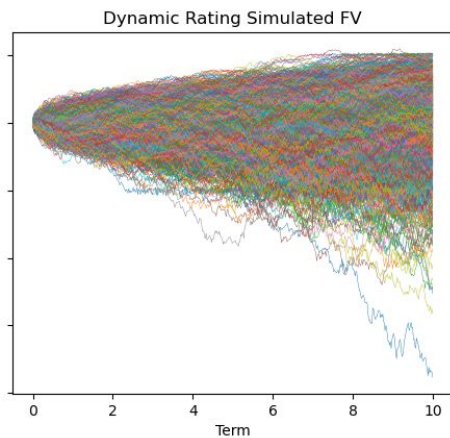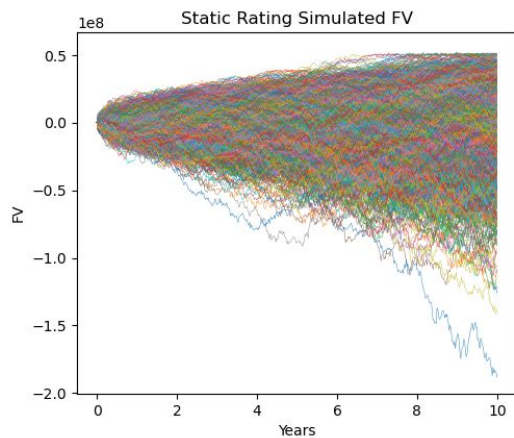
Unilateral Transitions II: A, BBB


```
CVA with static rating CSA/CDS   : -1,383,427
CVA with dynamic rating CSA/CDS  : -1,480,086
CVA with dynamic rating CSA/static CDS  : -1,383,428
```

Observations & Conclusions

Observations & Conclusions

- Comparison of 3 different approaches to calculate CVA
    - (A) CVA with static rating CSA/CDS
    - (B) CVA with dynamic rating CSA/CDS
    - (C) CVA with dynamic rating CSA and static CDS
- Results become more noticeable with a greater gap in thresholds
    - Most observable in cases with a distinct threshold table
    - Less observable in cases with a smaller number of transitions
- Typically observed |A| > |B| > |C| or |B| > |A| > |C|
    - Differences can arise when using +- ratings

Path Forward

Path Forward

- Rating transition matrices are published on annual basis
  - Convert the matrix into a weekly matrix so that transitions have the potential to occur on a more frequent basis
  - We tried to scale the transition matrix to weekly but failed (observed negative probability), because S&P data is purely estimated historically and does not guarantee any mathematical features (e.g. positive semi-definite)
- Market based measures
  - Backward looking nature of transition matrix lays ground for alternatives
  - Leverage ratio(Total liability/asset market value) , Cash flow(EBITDA/ asset market value) etc.

Thank you