Instructions

- ☐ You have 45 minutes for this quiz. You must stop after 45 minutes.
- ☐ The quiz must be taken in one continuous period of time.
- ☐ You may use 1 page of notes (front and back) as a resource for this quiz.
- ☐ Please scan and submit your notes with your completed exam.
- ☐ No other resources are allowed. No people, books, internet, software, etc.
- ☐ Please do not discuss the exam with anyone before *everyone* has submitted it.
- ☐ Please do not share copies of your exam solutions for *any reason*.

This exam contains two questions closely related to the following topics:

- ☐ Questions taken *exactly* from the 2.1 Discussion Problems.
- ☐ Solving an initial value problem for a wave-like equation, like Problem 2.3.
- ☐ Finding and sketching a domain of dependence.

Name:

Date:

Start time:

End time:

1. (a) The function $u(t, x) = t^2 + x^2$ is a solution to the wave equation $u_{tt} = u_{xx}$, as you can easily check. Can this solution be written in the form $F(x - t) + G(x + t)$, as we have claimed every solution to the wave equation can?

   (b) An observer sits at $x = 5$ along an infinite string obeying the wave equation $u_{tt} = 2u_{xx}$. If an initial displacement (a small pluck of the string) is concentrated at position $x = 0$ at time $t = 0$, when will the observer feel the effect of the displacement? For how long will they continue to feel the effect of the displacement? Answer these questions again, but this time with an initial velocity (a small strike of the string) concentrated at position $x = 0$ and time $t = 0$.

   (c)

2. Consider the initial value problem for the wave-like equation

$$\begin{aligned} u_{tt} + 3u_{xt} + 2u_{xx} &= 0, && -\infty < x < \infty, \quad t > 0, \\ u(0,x) &= e^{-x^2}, && -\infty < x < \infty, \\ u_t(0,x) &= 1, && -\infty < x < \infty. \end{aligned}$$

   (a) Solve this initial value problem. (Hint: think about factoring the operator like you did in your homework.)

   (b) Give a qualitative description of your solution. (Words like *bump*, *moving*, *speed*, etc. are appropriate for this qualitative description.)

   (c) Sketch the domain of dependence for a point (t, x) with $t > 0$.